



For  
2021

**CBSE**  
**PREVIOUS**  
**10 YEARS**  
**PAPERS**  
**with**  
**SOLUTIONS**

*Mathematics*  
**CLASS X**  
**2010 - 2020**



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# CBSE Mathematics 2020

General Instructions :

- (i) All questions are compulsory.
- (ii) This question paper consists of 30 questions divided into four sections—A, B, C and D.
- (iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculator is not permitted.

# Questions

## Section-A (1 Mark Each)

1. If one of the zeroes of the quadratic polynomial  $x^2 + 3x + k$  is 2, then the value of  $k$  is:

- (a) 10                      (b) -10                      (c) -7                      (d) -2

2. The total number of factors of a prime number is:

- (a) 1                      (b) 0                      (c) 2                      (d) 3

3. The quadratic polynomial, the sum of whose zeroes is  $-5$  and their product is 6, is

- (a)  $x^2 + 5x + 6$       (b)  $x^2 - 5x + 6$       (c)  $x^2 - 5x - 6$       (d)  $-x^2 + 5x + 6$

4. The value of  $k$  for which the system of equations  $x + y - 4 = 0$  and  $2x + ky = 3$  has no solution, is

- (a) -2                      (b)  $\neq 2$                       (c) 3                      (d) 2

5. The HCF and the LCM of 12, 21, 15 respectively are

- (a) 3140                      (b) 12,420                      (c) 3,420                      (d) 4,203

6. The value of  $x$  for which  $2x$ ,  $(x + 10)$  and  $(3x + 2)$  are the three consecutive terms of an AP, is

- (a) 6                      (b) -6                      (c) 18                      (d) -18

7. The first term of an AP is  $p$  and the common difference is  $q$ , then its 10th term is

- (a)  $q + 9p$                       (b)  $p - 9q$                       (c)  $p + 9q$                       (d)  $2p + 9q$

8. The distance between the points  $(a \cos \theta + b \sin \theta, 0)$  and  $(0, a \sin \theta - b \cos \theta)$ , is

- (a)  $a^2 + b^2$                       (b)  $a^2 - b^2$                       (c)  $\sqrt{a^2 + b^2}$                       (d)  $\sqrt{a^2 - b^2}$

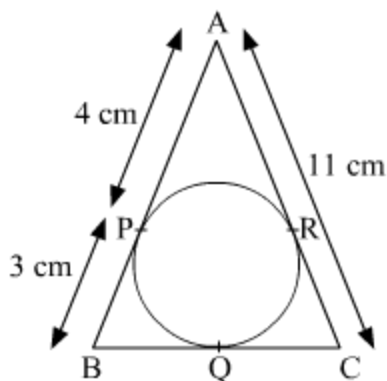
9. Find the ratio in which the segment joining the points  $(1, -3)$  and  $(4, 5)$  is divided by  $x$ -axis? Also, find the coordinates of this point on the  $x$ -axis.

- (a) 1                      (b) 2                      (c) -2                      (d) -1

10. The value of  $p$ , for which the points  $A(3, 1)$ ,  $B(5, p)$  and  $C(7, -5)$  are collinear, is

- (a) -2                      (b) 2                      (c) -1                      (d) 1

11. In Fig. 1,  $\triangle ABC$  is circumscribing a circle, the length of  $BC$  is \_\_\_\_\_cm.



12. Given  $\Delta ABC \sim \Delta PQR$ , if  $AB/PQ = \frac{1}{2}$ , then  $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} =$

13. ABC is an equilateral triangle of side  $2a$ , then length of one of its altitude is \_\_\_\_\_

14.  $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ =$  \_\_\_\_\_.

15. The value of  $(\sin^2 \theta + 1/1 + \tan^2 \theta) =$  \_\_\_\_\_.

**OR**

The value of  $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) =$  \_\_\_\_\_.

16. The ratio of the length of a vertical rod and the length of its shadow is 1:3. Find the angle of elevation of the sun at that moment?



17. Two cones have their heights in the ratio 1:3 and radii in the ratio 3:1. What is the ratio of their volumes?

18. A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant.

19. A die is thrown once. What is the probability of getting a number less than 3?

OR

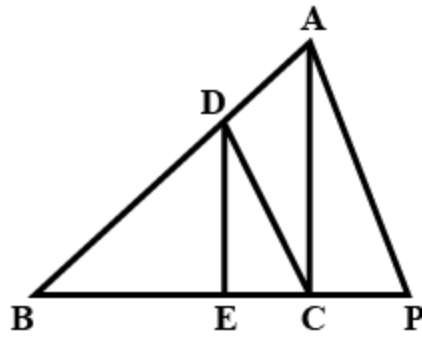
If the probability of winning a game is 0.07, what is the probability of losing it?

20. If the mean of first  $n$  natural number is 15, then find  $n$ .

## Section-B (2 Marks Each)

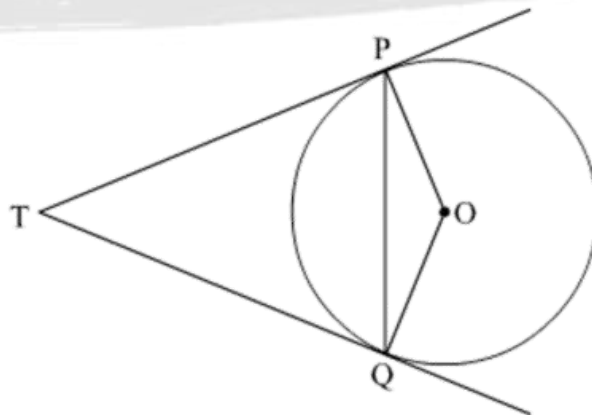
21. Show that  $(a - b)^2$ ,  $(a^2 + b^2)$  and  $(a + b)^2$  are in AP.

22. In the given Fig.  $DE \parallel AC$  and  $DC \parallel AP$ . Prove that  $\frac{BE}{EC} = \frac{BC}{CP}$

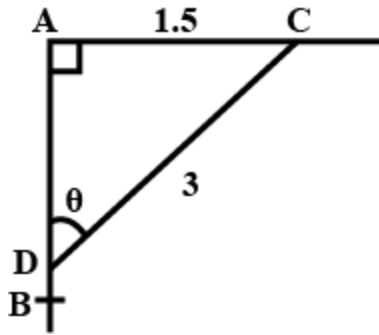


OR

In the given Fig. , two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2\angle OPQ$ .



23. The rod AC of a TV disc antenna is fixed at right angle to the wall AB and a rod CD is supporting the disc as shown in Fig. 4. If AC = 1.5m long and CD = 3m, find (i)  $\tan\theta$  (ii)  $\sec\theta + \operatorname{cosec}\theta$ .



24. A piece of wire 22 cm long is bent into the form of an arc of circle subtending an angle of  $60^\circ$  at its centre. Find the radius of the circle. ( Use  $\pi = 22/7$  )

25. If a number  $x$  is chosen at random from the numbers  $-3, -2, -1, 0, 1, 2, 3$ . What is the probability that  $x^2 \leq 4$ ?

26. Find the mean of the following distribution:

Class:	3-5	5-7	7-9	9-11	11-13
Frequency:	5	10	10	7	8

## Section- C (3 Marks Each)

27. Find the quadratic polynomial whose zeroes are reciprocal of the zeroes of the polynomial  $f(x) = ax^2 + bx + c$ ,  $a \neq 0$ ,  $c \neq 0$ .

OR

Divide the polynomial  $f(x) = 3x^2 - x^3 - 3x + 5$  by the polynomial  $g(x) = x - 1 - x^2$  and verify the division algorithm.

28. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by  $2y - x = 8$ ,  $5y - x = 14$  and  $y - 2x = 1$ .

**OR**

If 4 is the zero of the cubic polynomial  $x^3 - 3x^2 - 10x + 24$ , find its other two zeroes.

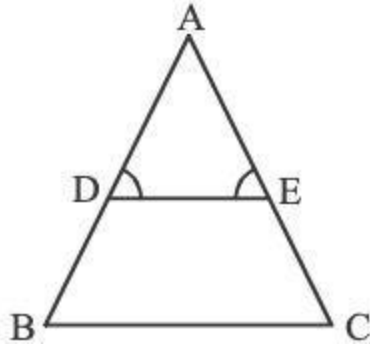
29. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced to 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

30. Find the area of triangle PQR formed by the points  $P(-5, 7)$ ,  $Q(-4, -5)$  and  $R(4, 5)$ .

**OR**

If the point  $C(-1, 2)$  divides internally the line segment joining  $A(2, 5)$  and  $B(x, y)$  in the ratio 3 : 4, find the coordinates of B.

31. In Fig.5,  $\angle D = \angle E$  and  $AD/DB = AE/EC$ , prove that BAC is an isosceles triangle.



32. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

33. If  $\sin\theta + \cos\theta = \sqrt{3}$ , then prove that  $\tan\theta + \cot\theta = 1$

34. A cone of base radius 4 cm is divided into two parts by drawing a plane through the midpoint of its height and parallel to its base. Compare the volume of the two parts.

### Section- D (4 Marks Each)

35. Show that the square of any positive integer cannot be of form  $(5q + 2)$  or  $(5q + 3)$  for any integer  $q$ .

**OR**

Prove that one of every three consecutive positive integers is divisible by 3.

36. The sum of four consecutive numbers in AP is 32 and the ratio of product of the first and last terms to the product of two middle terms is 7:15. Find the numbers.

37. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

38. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are  $30^\circ$  and  $45^\circ$  respectively. Find the height of the tower. (Take  $\sqrt{3} = 1.73$ )

39. A bucket in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the capacity of the bucket. Also find the total cost of milk that can completely fill the bucket at the rate of Rs. 40 per litre. ( Use  $\pi = 22/7$  )

40. +The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village:

Production yield/hect.	40-45	45-50	50-55	55-60	60-65	65-70
No. of farms	4	6	16	20	30	24

**Ans 1). (b)  $-10$**

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**Ans 2). (c)  $2$**

---

**Ans 3). (a)  $x^2 + 5x + 6$**

---

**Ans 4). (d)  $2$**

---

**Ans 5). (c)  $3,420$**

---

**Ans 6). (a)  $6$**

---

**Ans 7). (c)  $p + 9q$**

---

**Ans 8). (c)  $\sqrt{a^2 + b^2}$**

---

**Ans 9). (d)  $-1$**

---

**Ans 10). (a)  $-2$**

---

**Ans 11).  $10\text{cm}$  is the length of  $BC$ .**

---

**Ans 12).**

**We know that area of similar triangles are equal to the proportion of the squares of proportional Sides.**

**→ So,  $\text{ar}\triangle ABC/\text{ar}\triangle PQR=$**

**→  $AB^2 \div PQ^2 = (1 \div 3)^2 = 1 \div 9 = 1/9$**

---

**Ans 13).**

→ **Given :** ABC is an equilateral triangle of side 2a.

In equilateral triangle,  $\triangle ADB = \triangle ADC$

&  $\angle ADB = \angle ADC$  (Both 90 degree as  $AD \perp BC$ )

Thus,  $\triangle ADB \cong \triangle ADC$  (By R. H. S. Congruency)

→ As per CPCT i.e. they are corresponding parts of congruent triangle,

→ **BD = DC**

$$BD = DC = \frac{1}{2}BC$$

$$BD = DC = \frac{2a}{2}$$

$$BD = DC = a$$

→ **Using Pythagoras theorem,**

$$(\text{Hypotenuse})^2 = (\text{Height})^2 + (\text{Base})^2$$

$$(AB)^2 = (AD)^2 + (BD)^2$$

$$(2a)^2 = (AD)^2 + (a)^2$$

$$4a^2 = (AD)^2 + (a)^2$$

$$4a^2 - a^2 = AD^2$$

$$3a^2 = AD^2$$

**Hence,**



$$AD = a\sqrt{3}$$

$$BE = a\sqrt{3}$$

$$CF = a\sqrt{3}$$

Thus, the altitudes of the given equilateral triangle is equal to  $a\sqrt{3}$

**Ans → Hence, length of one of the altitude is  $a\sqrt{3}$**

**Ans 14).**

$$\rightarrow \frac{\cos(90-10)}{\sin 10} + \frac{\cos 59}{\operatorname{cosec} 31}$$

$$\rightarrow \frac{\cos(90-10)}{\sin 10} + \frac{\cos 59}{\operatorname{cosec} 31}$$

$$\rightarrow \frac{\sin 10}{\sin 10} + \frac{\cos 59}{\operatorname{cosec} 31}$$

$$\rightarrow 1 + \cos 59 / \sin(90-31)$$

$$\rightarrow 1 + \cos 59 / \cos 59$$

$$\rightarrow 1 + 1 = 2$$

**Hence proved.**

---

**Ans 15).**

$$\left( \sin^2 \theta + \frac{1}{1+\tan^2 \theta} \right)$$

$$= \left( \sin^2 \theta + \frac{1}{\sec^2 \theta} \right)$$

$$= (\sin^2 \theta + \cos^2 \theta)$$

$$= 1$$


---

**Ans 16).**

→ **Given:** The ratio of the length of a vertical rod and the length of its shadow is  $1:\sqrt{3}$ .

→ **Solution:**  $\tan \theta = 1/\sqrt{3}$

$$\theta = 30^\circ$$


---

**Ans 17).** → **Given:** Two cones have their heights in the ratio 1:3 and radii in the ratio 3:1.

→ **Solution:**  $\frac{r_1}{r_2} = \frac{3}{1} = \frac{1}{3}$

→ **Ratio of volumes:**  $\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = 3:1$

---

**Ans 18).** There are 26 letters. So, the probability that the chosen letter is a consonant is:

$$\rightarrow P(\text{consonant}) = \frac{21}{26}$$

---

**Ans 19).**

**We know in a die, the total no. of outcomes doesn't exceed 6.**

**So,**

$$\rightarrow P(\text{for number less than 3}) = \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$$

$$\rightarrow P(\text{for number less than 3}) = \frac{2}{6} = \frac{1}{3}$$

**OR**

**Ans 20).**

$$\rightarrow P(\text{losing}) = \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$$

$$\rightarrow P(\text{losing}) = 1 - 0.07 = 0.93$$

---

**Ans 20).**

**→ Given : Mean of first n natural number is 15**

$$\rightarrow \text{We know, Sum of first n natural numbers} = \frac{[n(n+1)]}{2}$$

Mean of first n natural numbers = Sum of observations / Total number of observations =  $\frac{[n(n+1)]}{2n}$

$$= (n + 1)/2 = 15$$

$$n + 1 = 30$$

$$n = 29$$

---

Ans 21).

→ Given:  $(a-b)^2$ ,  $(a^2 + b^2)$  and  $(a+b)^2$  are in A.P.

So, →  $(a^2 + b^2) - (a-b)^2 = 2ab$

$$\rightarrow (a+b)^2 - (a^2 + b^2) = 2ab$$

As we can see, Common difference is same.

Ans → Hence, the given terms are in AP

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Ans 22). → Given: In  $\Delta ABC$ ,  $DE \parallel AC$

→ To Prove:  $BE/EC = BC/CP$

→ Solution:

$$\rightarrow \text{In } \Delta ABC, DE \parallel AC, \frac{BD}{DA} = \frac{BE}{EC}$$

$$\rightarrow \text{In } \Delta ABP, DC \parallel AP, \frac{BD}{DA} = \frac{BC}{CP}$$

→ Now, from (i) & (ii),  $\frac{BE}{EC} = \frac{BC}{CP}$

**Ans → Hence, proved.**

**OR**

→ Let  $\angle OPQ = \theta$

→  $\angle TPQ = \angle TQP = 90^\circ - \theta$

→ In  $\triangle TPQ$ ,  $2(90^\circ - \theta) + \angle PTQ = 180^\circ$

→  $\angle PTQ = 2\theta = 2\angle OPQ$

**Ans → Hence, Proved.**

---

**Ans 23). → Given: AC = 1.5m long and CD = 3m**

In  $\triangle ACD$ ,  $CD^2 = AC^2 + AD^2$

$$\rightarrow \tan \theta = \frac{AC}{AD}$$

$$\rightarrow AD = \sqrt{CD^2 - AC^2} = \sqrt{3^2 - 1.5^2} = \sqrt{9 - 2.25} = \sqrt{6.75} = \frac{3\sqrt{3}}{2}$$

$$\rightarrow \tan \theta = \frac{1.5}{\frac{3\sqrt{3}}{2}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

(ii)  $\sec \theta + \operatorname{cosec} \theta$

$$= \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{3(\sqrt{3})/2} + \frac{3}{1.5}$$

$$= \frac{2}{\sqrt{3}} + 2 = \frac{2 + 2\sqrt{3}}{\sqrt{3}} = 2 \left( \frac{1 + \sqrt{3}}{\sqrt{3}} \right)$$

**Ans → Hence, the value of  $\tan\theta = 1/\sqrt{3}$  and the value of  $\sec\theta + \operatorname{cosec}\theta$  is**

$$2 \left( \frac{1 + \sqrt{3}}{\sqrt{3}} \right)$$

**Ans 24). → Given: Length of the wire = 22 cm, subtends an angle of  $60^\circ$  at its centre**

**Solution:  $2 \times \left( \frac{22}{7} \right) \times r \times \frac{60^\circ}{360^\circ} = 22$**

$$2 \times \left( \frac{22}{7} \right) \times r \times \frac{1}{6} = 22$$

**→  $r = 21\text{cm}$**

**Ans → The radius of the circle is 21cm.**

**Ans 25).**

**→ Total number of outcomes = 7**

→ Favourable outcomes are  $-2, -1, 0, 1, 2$ , i.e., 5

→  $P(x^2 \leq 4) = 5/7$

---

Ans 26).

Class	Frequency (f)	X	X*f
3 - 5	5	4	20
5 - 7	10	6	60
7 - 9	10	8	80
9 - 11	7	10	70
11 - 13	8	12	96
<u><math>\Sigma f = 40</math></u>		<u><math>\Sigma X*f = 326</math></u>	

→ Mean of the data,  $\bar{x} = \frac{\sum fix_i}{\sum fi} = \frac{326}{40} = 8.15$

**OR**

→ Given according to the table:

→ Modal class : 60 – 80

→ Formula for mode:  $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

→ On putting the values :  $60 + \frac{12 - 10}{24 - 10 - 6} \times 20$

$$= 60 + 5$$

$$= 65$$

Ans → The mode of the following data is 65.

---

Ans 27).

→ Let  $\alpha$  and  $\beta$  are the zeroes of the polynomial  $f(x) = ax^2 + bx + c$ .

→ According to the question,  $1/\alpha$  and  $1/\beta$  are the zeroes of the required quadratic polynomial

$$\begin{aligned} S' &= \frac{1}{\alpha} + \frac{1}{\beta} \\ &= \frac{\alpha + \beta}{\alpha\beta} \\ &= \frac{-b}{c} \quad \dots(\text{iii}) \end{aligned}$$

[From equation (i) and (ii)]

and product of zeroes of

$$\text{required polynomial} = \frac{1}{\alpha} \times \frac{1}{\beta}.$$

$$\begin{aligned} P' &= \frac{1}{\alpha\beta} \\ &= \frac{a}{c} \quad \dots(\text{iv}) \end{aligned}$$

[From equation (ii)]



→ Equation of the required quadratic polynomial

$$= k(x^2 - S'x + p'),$$

where k is any non-zero constant

$$= k\left(x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c}\right)$$

[From equation (iii) and (iv)]

$$= k\left(x^2 + \frac{b}{c}x + \frac{a}{c}\right)$$

Or  $(cx^2 + bx + a)$

---

Ans 28).

→ Let  $2y - x = 8$  ..... → eq<sup>n</sup> 1

→  $5y - x = 14$  ..... → eq<sup>n</sup> 2

→  $2x - y = -1$  ..... → eq<sup>n</sup> 3

On solving 1 and 2, we get  $x = -4$  and  $y = 2$

→ Coordinates of B =  $(-4, 2)$

**On solving 2 and 3:**

→ **We obtain  $x=1$  and  $y=3$**

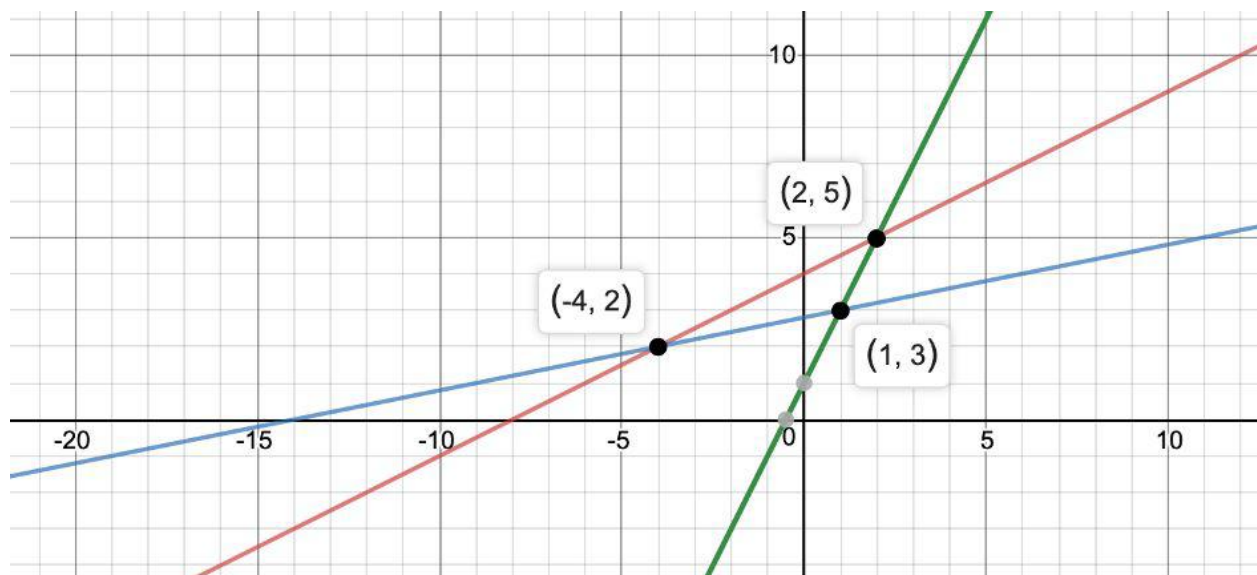
→ **Coordinates of C =  $(1,3)$**

**On solving 1 and 3:**

**$x=2$  and  $y=5$**

→ **Hence Coordinates of A= $(2, 5)$**

**Ans → So, Coordinates of the vertices of the triangle are A $(-4, 2)$ , B $(1, 3)$  and C $(2, 5)$**



**OR**

→ **Given:**

1). Given that the cubic polynomial is  $x^3 - 3x^2 - 10x + 24$

2). 4 is a zero of the given polynomial

→ **To find : It's other zeroes.**

→ **Solution:** Since, 4 is a zero of the given polynomial,  $(x - 4)$  is it's factor.

$$\begin{array}{r} x-4 \overline{) x^3 - 3x^2 - 10x + 24} \quad (x^2 + x - 6) \\ \underline{x^3 - 4x^2} \phantom{+ 24} \\ -x^2 - 10x + 24 \\ \underline{-x^2 - 4x} \phantom{+ 24} \\ -6x + 24 \\ \underline{-6x + 24} \\ 0 \end{array}$$

→ On further solving,  $x^2 + x + 6 = (x + 3)(x - 2)$

**Ans → The other zeroes of the polynomial are -3 and 2.**

---

**Ans 29). Let the speed of aircraft be x km/hr**

$$\rightarrow \frac{600}{x-200} - \frac{600}{x} = 30/60$$

$$\rightarrow x^2 - 200x - 240000 = 0$$

$$\rightarrow (x - 600) (x + 400) = 0$$

$$\rightarrow x = 600,$$

→ Since, -400 cannot be accepted 1/2 Speed of aircraft = 600 km/hr ∴

Duration of flight = 1 hr

Ans 30).

→ Solution:

$$P = (-5, 7)$$

$$Q = (-4, -5)$$

$$R = (4, 5)$$

Area of Triangle PQR

$$= (1/2) | -5(-5 - 5) - 4(5 - 7) + 4(7 + 5) | \text{ sq.units}$$

$$= (1/2) | 50 + 8 + 48 |$$

$$= (1/2) | 106 |$$

$$= 106/2$$

$$= 53$$

→ Area of Triangle PQR = 53 sq units

Ans 31).

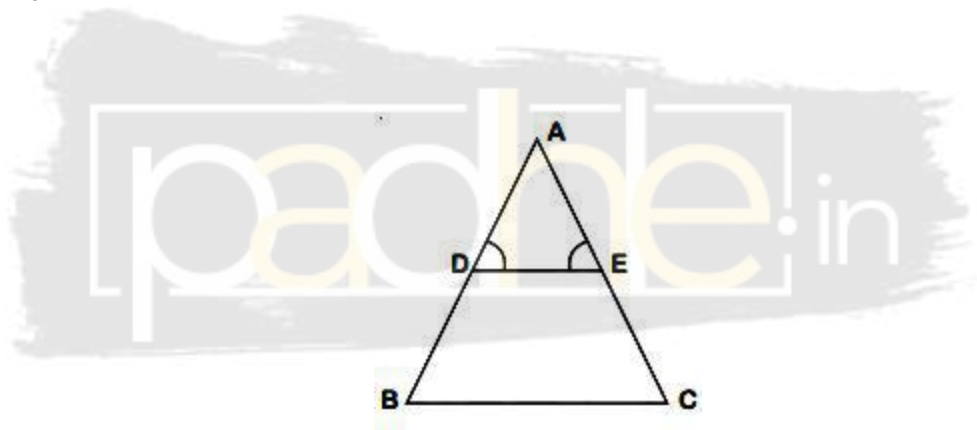
→ Coordinates of C are  $(\frac{3x+8}{7}, \frac{3y+20}{7}) = (1, -2)$

→  $x = -5, y = -2$

So, → Coordinates of B are  $(-5, -2)$

---

Ans 31).



→ Given:  $\angle D = \angle E$ , which means  $AE = ED$

$$\frac{AD}{DB} = \frac{AE}{EC} \rightarrow DB = EC$$

→  $AD + DB = AE + EC$

→  $AB = AC$

As,  $AB = AC$ ,  $\triangle BAC$  is an isosceles triangle.

Ans →  $\triangle BAC$  is an isosceles triangle.

---

**Ans 32). Given:- ABC is a triangle**

$$\mathbf{AC^2 = AB^2 + BC^2}$$

**To prove:-  $\angle B = 90^\circ$**

**→ Construction:- Construct a triangle PQR right angled at Q such that,  
 $PQ = AB$  and  $QR = BC$**

**→ Proof:-**

**In  $\triangle PQR$**

$$\mathbf{PR^2 = PQ^2 + QR^2}$$

**(By pythagoras theorem)**

$$\mathbf{\rightarrow PR^2 = AB^2 + BC^2 \dots\dots(1) \quad \text{as } AB = PQ \text{ and } QR = BC}$$

$$\mathbf{AC^2 = AB^2 + BC^2 \dots\dots(2) \text{ (Given)}}$$

**→ From equation (1)&(2), we have**

$$\mathbf{AC^2 = PR^2}$$

$$\mathbf{\rightarrow AC = PR \dots\dots(3)}$$

**Now, in  $\triangle ABC$  and  $\triangle PQR$**

$$\mathbf{AB = PQ}$$

$$\mathbf{BC = QR}$$

$$\mathbf{AC = PR \text{ (From (3))}}$$

**→  $\triangle ABC \cong \triangle PQR$  (By SSS congruency)**

**Therefore, by C.P.C.T.,**

$$\mathbf{\angle B = \angle Q}$$

$$\mathbf{\rightarrow \angle Q = 90^\circ}$$

$$\mathbf{\rightarrow \angle B = 90^\circ}$$

**Hence proved.**

---

**Ans 32).** → Given  $\sin \theta + \cos \theta = \sqrt{3}$

→ To prove:  $\tan \theta + \cot \theta = 1$ .

**Solution:**  $\sin \theta + \cos \theta = \sqrt{3}$

→  $(\sin \theta + \cos \theta)^2 = (\sqrt{3})^2$

→  $\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = 3$

→  $\sin \theta \cos \theta = 1$

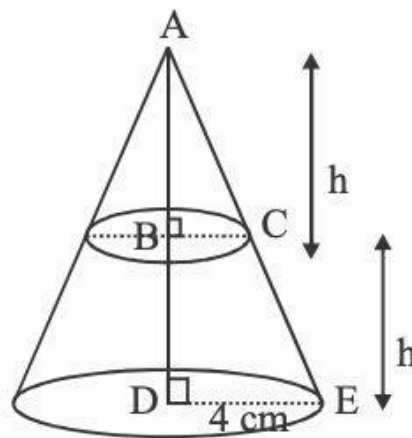
**L.H.S** =  $\tan \theta + \cot \theta =$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{1}{\cos \sin \theta} = 1 \rightarrow \text{RHS}$$

→ Hence, Proved.

---

**Ans 33).**



→ Let height of the cone is h

Now B is the midpoint of AC.

→ Let  $V_1$  and  $V_2$  be the volume of upper and lower part of the cone respectively

In  $\triangle ABE$  and  $\triangle ACD$

$$\rightarrow \angle B = 90^\circ = \angle C$$

$$\rightarrow \angle A = \angle A \quad (\text{Common angle})$$

$$\rightarrow \triangle ABE \sim \triangle ACD$$

$$\rightarrow \frac{AB}{BE} = \frac{AC}{CD}$$

$$\rightarrow \frac{h/2}{BE} = h/4$$

$$\rightarrow BE = 2 \text{ cm}$$

Ratio of volumes of two parts:

$$\frac{\frac{1}{3}\pi \times 2^2 \times h}{\frac{1}{3}\pi \times (2^2 + 4^2 + 2 \times 4) \times h}$$

$$= 4/28 = 1/7$$

Hence, the ratio of volumes of two parts is 1:7.

---

Ans 35).

Let  $a$  be any positive integer.

Take  $b = 5$  as the divisor.



$$\rightarrow a = 5m + r, r = 0, 1, 2, 3, 4$$

$$\rightarrow \text{Case-1 : } a = 5m \Rightarrow a^2 = 25m^2 = 5(5m^2) = 5q$$

$$\rightarrow \text{Case-2 : } a = 5m+1 \Rightarrow a^2 = 5(5m^2 + 2m) + 1 = 5q + 1$$

$$\rightarrow \text{Case-3 : } a = 5m+2 \Rightarrow a^2 = 5(5m^2 + 4m) + 4 = 5q + 4$$

$$\rightarrow \text{Case-4 : } a = 5m+3 \Rightarrow a^2 = 5(5m^2 + 6m + 1) + 4 = 5q + 4$$

$$\rightarrow \text{Case-5 : } a = 5m+4 \Rightarrow a^2 = 5(5m^2 + 8m + 3) + 1 = 5q + 1$$

Ans  $\rightarrow$  Hence proved,

square of any positive integer cannot be of the form  $(5q + 2)$  or  $(5q + 3)$  for any integer  $q$ .

OR

$\rightarrow$  Let  $n$  be any positive integer. Divide it by 3.

$$\rightarrow n = 3q + r$$

$$\rightarrow r = 0, 1, 2$$

$\rightarrow$  Case-1 :  $n = 3q$  (divisible by 3)

$$n + 1 = 3q + 1$$

$$n + 2 = 3q + 2$$

$\rightarrow$  Case-2 :  $n = 3q + 1$

$$n + 1 = 3q + 2$$

$$n + 2 = 3q + 3 \text{ (divisible by 3)}$$

$\rightarrow$  Case-3 :  $n = 3q + 2$

$$n + 1 = 3q + 3 \text{ (divisible by 3)}$$

$$n + 2 = 3q + 4$$

**Ans 36).**

→ **Given: Sum of four consecutive numbers in AP is 32**

**Let the four consecutive numbers in AP be  $(a-3d), (a-d), (a+d)$  and  $(a+3d)$**   
 **$a-3d+a-d+a+d+a+3d=32$**

$$4a=32$$

$$a=32/4$$

$$a=8 \quad \text{----> eq}^n (1)$$

$$\text{Now, } (a-3d)(a+3d)/(a-d)(a+d)=7/15$$

$$\rightarrow 15(a^2-9d^2)=7(a^2-d^2)$$

$$\rightarrow 15a^2-135d^2=7a^2-7d^2$$

$$\rightarrow 15a^2-7a^2=135d^2-7d^2$$

$$8a^2=128d^2$$

→ **Putting the value of  $a=8$  in above we get.**

$$8(8)^2=128d^2$$

$$128d^2=512$$

$$d^2=512/128$$

$$d^2=4$$

$$d=2$$

**So, the four consecutive numbers are**

$$8-(3 \times 2)$$

$$8-6=2$$

$$8-2=6$$

$$8+2=10$$

$$8+(3 \times 2)$$

$$8+6=14$$

Four consecutive numbers are 2,6,10and14

**OR**

Given :  $a = 1, d = 3$

We know that,

$$S(n) = n/2 [2a + (n - 1)d]$$

Putting all the values, we get

$$\rightarrow 287 = n/2[2 \times 1 + (n - 1) (3) ]$$

$$\rightarrow 287 = n/2[2 + (n - 1) 3]$$

$$\rightarrow 574 = 3n^2 - n$$

$$\rightarrow 3n^2 - n - 574 = 0$$

$$\rightarrow 3n^2 - 42n + 41n - 574 = 0$$

$$\rightarrow 3n(n - 14) + 41(n - 14) = 0$$

$$\rightarrow n = 14, - 41/3 \text{ ( Since, } n \text{ can't be negative)}$$

$$\rightarrow n = 14$$

We know that,

$$a + (n - 1)d = x$$

$$\rightarrow 1 + (14 - 1) (3) = x$$

$$\rightarrow 1 + 13 (3) = x$$

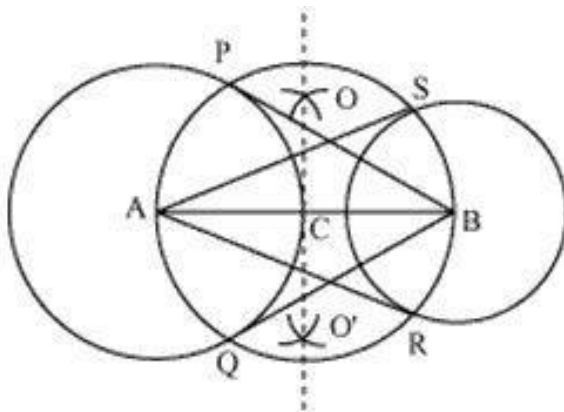
$$\rightarrow x = 40.$$

**Ans → The value of x is 40.**

---

**Ans 37). Following are the steps of construction:**

- 1. Take  $AB = 7$  cm.**
- 2. With A as centre and 3 cm as radius, draw a circle.**
- 3. Similarly, with B as centre and 2 cm as radius, draw a circle.**
- 4. Now, draw the perpendicular bisector of AB and mark the point of intersection O.**
- 5. With O as centre and OA as radius, draw a circle. Mark the 2 points where the circles with centre O and A meet as Q and R. Similarly, mark the points where the circles with centres O and B meet as S and T respectively.**
- 6. Join BR and BQ as well as AS and AT. Now, BR, BQ, AS and AT are the required tangents.**



**Ans 38).**

Let BC be the height of the tower and DC be the height of the flag - staff.

In rt.  $\triangle ABC$ ,

$$\rightarrow AB = BC \cot 30^\circ$$

$$\rightarrow AB = BC \sqrt{3} \quad \text{-----} \rightarrow \text{eq}^n \text{ (i)}$$

$\rightarrow$  In rt.  $\triangle ABD$ ,

$$AB = BD \cot 45^\circ$$

$$AB = (BC + CD) \cot 45^\circ$$

$$AB = (BC + 6) \quad \text{-----} \rightarrow \text{eq}^n \text{ (ii)}$$

$\rightarrow$  Equating (i) and (ii)

$$(BC + 6) = BC \sqrt{3}$$

$$BC(\sqrt{3}-1) = 6$$

$$BC = 6/0.73 = 9.58 \text{ m}$$

Therefore the height of the tower is 9.58 m.

---

Ans 39).

→ Given: Radius of the upper end of the frustum of cone =  $R = 20 \text{ cm}$

radius of the lower end of the frustum of cone =  $r = 10 \text{ cm}$

$H = 30 \text{ cm}$

→Solution: Volume =  $\frac{1}{3} \pi h [R^2 + r^2 + R \cdot r]$

$$= \frac{1}{3} \times \frac{22}{7} \times 30 \times [20^2 + 10^2 + 20 \cdot 10]$$

$$= 660/21 \times [400 + 100 + 200]$$

$$= (660 \times 700)/21$$

$$= 22000 \text{ cm}^3$$

Hence the capacity = 22L

$$\text{So, the Cost of milk} = ₹ 40 \times ₹ 22 = ₹ 880$$

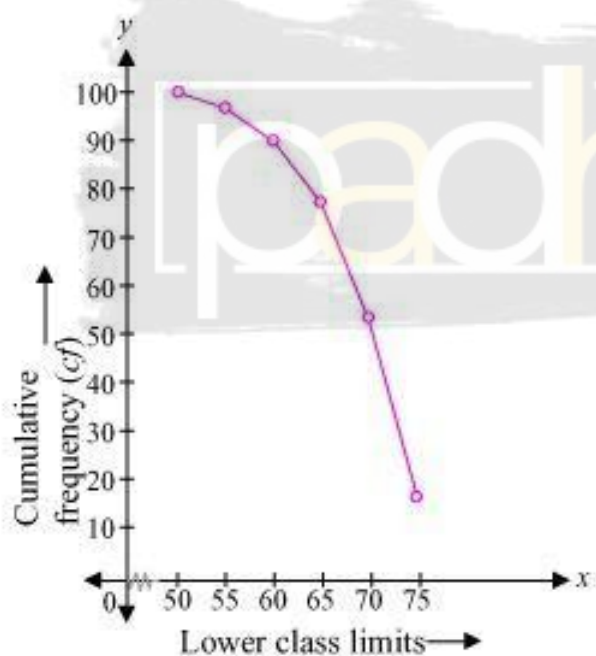
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Ans 40).

Production yield / hectare

No. of forms

More than or equal to 40	100
More than or equal to 45	96
More than or equal to 50	90
More than or equal to 55	74
More than or equal to 60	54
More than or equal to 65	24



# CBSE Mathematics 2019

General Instructions :

- (i) All questions are compulsory.
- (ii) This question paper consists of 30 questions divided into four sections—A, B, C and D.
- (iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculator is not permitted.



# Questions

## Section-A (1 Mark Each)

1. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, -3) and B is the point (1, 4).

2. For what values of k, the roots of the equation  $x^2 + 4x + k = 0$  are real? [1]

OR

Find the value of k for which the roots of the equation  $3x^2 - 10x + k = 0$  are reciprocal of each other.

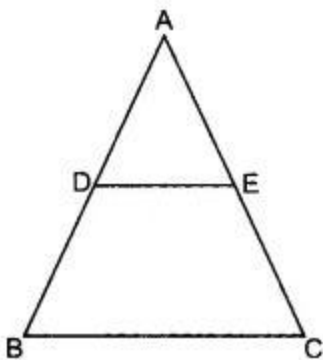
3. Find A if  $\tan 2A = \cot (A - 24^\circ)$

OR

Find the value of  $(\sin^2 33^\circ + \sin^2 57^\circ)$

4. How many two digits numbers are divisible by 3?

5. In Fig.,  $DE \parallel BC$ ,  $AD = 1$  cm and  $BD = 2$  cm. what is the ratio of the ar ( $\triangle ABC$ ) to the ar ( $\triangle ADE$ )



6. Find a rational number between  $\sqrt{2}$  and  $\sqrt{3}$ .

## Section-B (2 Marks Each)

7. Find the HCF of 1260 and 7344 using Euclid's algorithm.

OR

Show that every positive odd integer is of the form  $(4q + 1)$  or  $(4q + 3)$ , where  $q$  is some integer.

8. Which term of the A.P. 3, 15, 27, 39, ..... will be 120 more than its 21st term? [2]

OR

If  $S_n$ , the sum of first  $n$  terms of an A.P. is given by  $S_n = 3n^2 - 4n$ , find the  $n$ th term.

9. Find the ratio in which the segment joining the points  $(1, -3)$  and  $(4, 5)$  is divided by  $x$ -axis? Also, find the coordinates of this point on the  $x$ -axis.

10. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

11. A die is thrown once. Find the probability of getting a number which

(i) is a prime number

(ii) lies between 2 and 6.

12. Find  $c$  if the system of equations  $cx + 3y + (3 - c) = 0$ ,  $12x + cy - c = 0$  has infinitely many solutions?

## Section- C (3 Marks Each)

13. Prove that  $\sqrt{2}$  is an irrational number.

14. Find the value of  $k$  such that the polynomial  $x^2 - (k + 6)x + 2(2k - 1)$  has sum of its zeros equal to half to their product

15. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

OR

A fraction becomes  $\frac{1}{3}$  when 2 is subtracted from the numerator and it becomes  $\frac{1}{2}$  when 1 is subtracted from the denominator. Find the fraction.

16. Find the point on  $y$ -axis which is equidistant from the points  $(5, -2)$  and  $(-3, 2)$ .

OR

The line segment joining the points  $A(2, 1)$  and  $B(5, -8)$  is trisected at the points  $P$  and  $Q$  such that  $P$  is nearer to  $A$ . If  $P$  also lies on the line given by  $2x - y + k = 0$ , find the value of  $k$ .

17. Prove that  $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$ . [3]

OR

Prove that  $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$ .

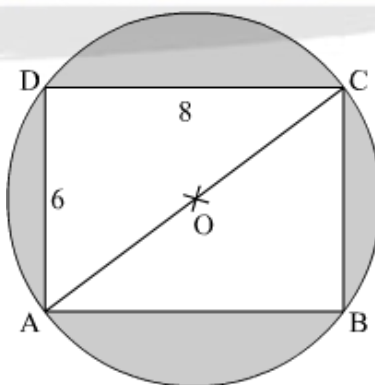
18. In Fig. PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.

19. In Fig.  $\angle ACB = 90^\circ$  and  $CD \perp AB$ , prove that  $CD^2 = BD \times AD$ .

OR

If P and Q are the points on side CA and CB respectively of  $\triangle ABC$ , right-angled at C, prove that  $(AQ^2 + BP^2) = (AB^2 + PQ^2)$ .

20. Find the area of the shaded region in Fig. if ABCD is a rectangle with sides 8 cm and 6 cm and D is the centre of the circle.



21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes, if 8 cm standing water is needed?

22. Find the mode of the following frequency distribution.

Class	Frequency
0 – 10	8
10 – 20	10
20 – 30	10
30 – 40	16
40 – 50	12
50 – 60	6
60 – 70	7

## Section- D (4 Marks Each)

23. Two water taps together can fill a tank in  $1\frac{7}{8}$  hours. The tap with longer diameter takes 2 hours less than the tap with a smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

24. If the sum of first four terms of an A.P. is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

25. Prove that  $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

26. A man in a boat rowing away from a lighthouse 100 m high takes 2 minutes to change the angle of elevation of the top of the lighthouse from  $60^\circ$  to  $30^\circ$ . Find the speed of the boat in metres per minute. [Use  $\sqrt{3} = 1.732$ ]

27. Construct a  $\Delta ABC$  in which  $CA = 6$  cm,  $AB = 5$  cm and  $\angle BAC = 45^\circ$ . Then construct a triangle whose sides are  $\frac{1}{2}$  of the corresponding sides of  $\Delta ABC$ .

28. A bucket open at the top is in the form of a frustum of a cone with a capacity of  $12308.8 \text{ cm}^3$ . The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it.

29. Prove that in a right-angle triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

30.

If the median of the following frequency distribution is 32.5. Find the values of  $f_1$  and  $f_2$ .

Class	Frequency
0 – 10	$f_1$
10 – 20	5
20 – 30	9
30 – 40	12
40 – 50	$f_2$
50 – 60	3
60 – 70	2
Total	40

OR

The marks obtained by 100 students of a class in an examination are given below.

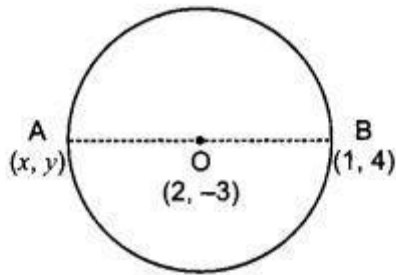
Marks	No. of Students
0 – 5	2
5 – 10	5
10 – 15	6
15 – 20	8
20 – 25	10
25 – 30	25
30 – 35	20
35 – 40	18
40 – 45	4
45 – 50	2

Draw 'a less than' type cumulative frequency curves (ogive). Hence find the median.

## Answers

Ans 1). Let the coordinates of point A be  $(x, y)$  and point O  $(2, -3)$  be point the centre, then

By midpoint formula,



$$\frac{x+1}{2} = 2 \quad \text{and} \quad \frac{y+4}{2} = -3$$

$$\text{or} \quad \begin{array}{l} x = 4 - 1 \\ x = 3 \end{array} \quad \text{and} \quad \begin{array}{l} y = -6 - 4 \\ y = -10 \end{array}$$

The coordinates of point A are (3, -10)

**Ans 2).** → The given equation is  $x^2 + 4x + k = 0$

On comparing the given equation with  $ax^2 + bx + c = 0$ , we get

$a = 1, b = 4$  and  $c = k$

→ For real roots,  $D \geq 0$

or  $b^2 - 4ac \geq 0$

or  $16 - 4k \geq 0$

or  $k \leq 4$

For  $k \leq 4$ , equation  $x^2 + 4x + k$  will have real roots.

**OR**

→ The given equation is  $3x^2 - 10x + k = 0$

On comparing it with  $ax^2 + bx + c = 0$ , we get

$a = 3, b = -10, c = k$

Let the roots of the equation are  $\alpha$  and  $1/\alpha$

Product of the roots =  $c/a$

$\alpha \cdot 1/\alpha = k/3$



or  $k = 3$

---

**Ans 3).**

→ Given,  $\tan 2A = \cot (A - 24^\circ)$

or  $\cot (90^\circ - 2A) = \cot (A - 24^\circ)$  [  $\rightarrow \tan \theta = \cot (90^\circ - \theta)$  ]

or  $90^\circ - 2A = A - 24^\circ$

or  $3A = 90^\circ + 24^\circ$

or  $3A = 114^\circ$

$A = 38^\circ$

**OR**

→  $\sin^2 33^\circ + \sin^2 57^\circ$

$= \sin^2 33^\circ + \cos^2 (90^\circ - 57^\circ)$

$= \sin^2 33^\circ + \cos^2 33^\circ$  [  $\rightarrow \sin^2 \theta + \cos^2 \theta = 1$  ]

$= 1$

---

**Ans 4).** The two-digit numbers divisible by 3 are 12, 15, 18, ..... 99

→ This is an A.P. in which  $a = 12$ ,  $d = 3$ ,  $a_n = 99$

→  $a_n = a + (n - 1) d$

$99 = 12 + (n - 1) \times 3$

$87 = (n - 1) \times 3$

or  $n - 1 = 29$

or  $n = 30$

So, there are 30 two-digit numbers divisible by 3.

**Ans 5).**

→ Given,  $AD = 1$  cm,  $BD = 2$  cm

$AB = 1 + 2 = 3$  cm

→ Also,  $DE \parallel BC$  (Given)

$\angle ADE = \angle ABC$  ... (i) (corresponding angles)

→ In  $\triangle ABC$  and  $\triangle ADE$

$\angle A = \angle A$  (common)

$\angle ABC = \angle ADE$  [by equation (i)]

$\triangle ABC \sim \triangle ADE$  (by AA rule)

Now,

$$\frac{ar(ABC)}{ar(ADE)} = (AB/AD)^2$$

→  $\triangle ABC \sim \triangle ADE$

$$\frac{ar(ABC)}{ar(ADE)} = (3/1)^2 = 9:1$$

---

Ans 6).

We know,  $\sqrt{2}=1.414$  and

$\sqrt{3}=1.732$

so, rational number between  $\sqrt{2}$  and  $\sqrt{3}$

$=1.432$

$=1.563$

$=1.576$

$=1.657$

$=1.711$  etc.

---

Ans 7).

→ **Step-by-step explanation:**  
**In euclid's division algorithm,**

$$7344 = 1260 \times 5 + 1044$$

$$\text{Also, } 1260 = 1044 \times 1 + 216$$

$$1044 = 216 \times 4 + 180$$

$$216 = 180 \times 1 + 36$$

$$180 = 36 \times 5 + 0$$

**We will follow the following steps,**

→ **Step 1 : divide 7344 by 1260,**

**Quotient = 5 remainder = 1044,**

→ **Step 2: divide divisor 1260 by remainder,**

**Quotient = 1 remainder = 216,**

→ **Step 3: Repeat the above steps until we get remainder 0.**

**Thus, the last quotient which gives the remainder zero is 36.**

**OR**

→ **Any positive integer is of the form  $4q+1$  or  $4q+3$**

**As per Euclid's Division lemma.**

**If a and b are two positive integers, then,**

$$\rightarrow a=bq+r$$

**Where  $0 \leq r < b$ .**

→ **Let positive integers be a and b=4**

$$\text{Hence, } a=bq+r$$

Where,  $(0 \leq r < 4)$

R is an integer greater than or equal to 0 and less than 4

Hence, r can be either 0,1,2 and 3

→ Now, If  $r=1$

Then, our equation becomes

$$a = bq + r$$

$$a = 4q + 1$$

This will always be odd integer.

→ Now, If  $r=3$

Then, our equation becomes

$$a = bq + r$$

$$a = 4q + 3$$

This will always be an odd integer.

Hence proved.

---

Ans 8). The given A.P. is 3, 15, 27, 39,...

Here  $a = 3$ ,  $d = 12$

$$\rightarrow a_{21} = a + 20d = 3 + 20 \times 12 = 3 + 240 = 243$$

$$\text{Now, } a_n = a_{21} + 120 = 243 + 120 = 363$$

$$a_n = a + (n - 1)d$$

$$363 = 3 + (n - 1) \times 12$$

$$\text{or } 360 = (n - 1) \times 12$$

$$\text{or } n - 1 = 30$$

$$\rightarrow n = 31$$

Hence, the term which is 120 more than its 21st term will be its 31st term.

OR

→ Given,  $S_n = 3n^2 - 4n$

We know that

→  $a_n = S_n - S_{n-1}$

$$= 3n^2 - 4n - [3(n-1)^2 - 4(n-1)]$$

$$= 3n^2 - 4n - [3(n^2 - 2n + 1) - 4n + 4]$$

$$= 3n^2 - 4n - (3n^2 - 6n + 3 - 4n + 4)$$

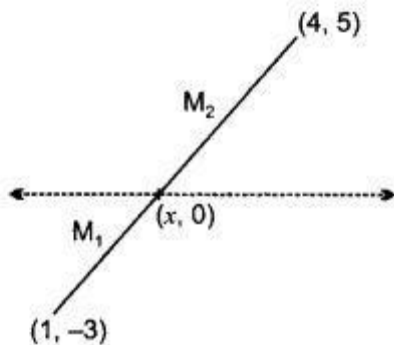
$$= 3n^2 - 4n - 3n^2 + 10n - 7$$

$$= 6n - 7$$

So,  $n$ th term will be  $6n - 7$

---

Ans 9). Let the given points be A (1, -3) and B (4, -5) and the line-segment joining by these points is divided by the x-axis, so the co-ordinate of the point of intersection will be P(x, 0)



Let the ratio be  $m_1 : m_2$

So, By section formula

$$0 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$0 = \frac{5m_1 - 3m_2}{m_1 + m_2}$$

or  $5m_1 - 3m_2 = 0$

or  $\frac{m_1}{m_2} = \frac{3}{5}$

∴ Required ratio is 3 : 5

Now to find the co-ordinates of this point on x-axis

∴  $x = \frac{3 \times 4 + 5 \times 1}{3 + 5}$

$$x = \frac{12 + 5}{8}$$

$$x = \frac{17}{8}$$

∴ The required point is  $\left(\frac{17}{8}, 0\right)$

Ans 10). **The possible outcomes of 3 times tossing a coin**

**1st= HHH**

**2nd= HHT or HTH or THH**

**3rd= TTH or THT or HTT**

**4th= TTT**

→ **Total number of events=8**

→ **No of favourable events= 6**

→ **P(E)=  $\frac{\text{no. of favourable outcomes}}{\text{total outcomes}}$**

$$P(E) = 6/8 = 3/4$$

Ans → Hence, the probability of losing the game is 3/4.

---

Ans 11). Possible numbers of events on throwing a dice = 6

Numbers on dice = 1, 2, 3, 4, 5 and 6

(i) Prime numbers = 2, 3 and 5

Favourable number of events = 3

Probability that it will be a prime number

$$= \frac{\text{no. of favourable outcomes}}{\text{total outcomes}} = 3/6 = 1/2$$

(ii) Numbers lying between 2 and 6 = 3, 4 and 5

Favourable number of events = 3

Probability that a number between 2 and 6 =  $3/6 = 1/2$

---

Ans 12). Given System of equations are

$$-cx + 3y + (3 - c) = 0 \quad \text{and} \quad 12x - cy - c = 0$$

To find: Value of c when system of equation has infinitely many solution

Condition for infinitely many solution is given by,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

From given system of equation,

$$a_1 = -c, \quad b_1 = 3, \quad c_1 = 3 - c \quad \text{and} \quad a_2 = 12, \quad b_2 = -c, \quad c_2 = -c$$

Putting these value in condition we get,

$$\frac{-c}{12} = \frac{3}{-c} = \frac{3-c}{-c}$$

→ To find value of c

consider first equality.i.e.,

$$-c/12 = 3/-c$$

$$(-c) \times (-c) = 3 \times 12$$

$$c^2 = 36$$

$$c = \pm \sqrt{36}$$

$$c = \pm 6$$

→ Here c = -6 is rejected as it does not satisfy the 2nd equality.

Therefore, c = 6

---

**Ans 13).** Let  $\sqrt{2}$  is a rational number.

So,  $\sqrt{2} = \frac{a}{b}$  where a and b are coprime integers and  $b \neq 0$

$$\text{or } \sqrt{2} b = a$$

→ Squaring on both sides, we get

$$2b^2 = a^2$$

→ Therefore, 2 divides  $a^2$

or 2 divides a (from theorem)

→ Let  $a = 2c$ , for some integer c

From equation (i)

$$2b^2 = (2c)^2$$

$$\text{or } 2b^2 = 4c^2$$

$$\text{or } b^2 = 2c^2$$



→ It means that 2 divides  $b^2$  and so 2 divides  $b$

Therefore  $a$  and  $b$  have at least 2 as a common factor.

→ But this contradicts the fact that  $a$  and  $b$  are co-prime.

This contradiction is due to our wrong assumption that  $\sqrt{2}$  is rational.

So, we conclude that  $\sqrt{2}$  is irrational.

Hence Proved.

---

Ans 14).  $x^2 - (k+6)x + 2(2k-1)$

As sum of zeros

$$= -b/a$$

$$= -(-(k+6))/1$$

$$= k+6$$

Product of zeros

$$= c/a$$

$$= 2(2k-1)/1$$

$$= 2(2k-1)$$

According to the question

$$\text{Sum of zeros} = 1/2 \text{ Product of zeros}$$

$$\rightarrow -b/a = 1/2 \times c/a$$

$$\rightarrow k + 6 = 1/2 \times 2(2k - 1)$$

$$\rightarrow k + 6 = 2k - 1$$

$$\rightarrow 6 + 1 = 2k - k$$

$$\rightarrow 7 = k$$

So Value of  $k = 7$

---

**Ans 15).**

$\rightarrow$  Let the present age of father be  $x$  years and sum of ages of his two children be  $y$  years

According to question

$$x = 3y \dots(i)$$

$\rightarrow$  After 5 years

Father's age =  $(x + 5)$  years

Sum of ages of two children =  $(y + 5 + 5)$  years =  $(y + 10)$  years

$\rightarrow$  In 2nd case

According to question

$$x + 5 = 2(y + 10)$$

$$\text{or } x + 5 = 2y + 20$$

$$\text{or } x - 2y = 15$$

$$\text{or } 3y - 2y = 15 \text{ (Using equations (i))}$$

$$y = 15$$

Now from equation (i)

$$x = 3y \text{ (Put } y = 15)$$

$$\text{or } x = 3 \times 15$$
$$x = 45$$

**Ans → Present age of father is 45 years.**

**OR**

**Let the fraction be  $x/y$**

**According to question  $\frac{x-2}{y} = 1/3$**

$$\text{or } 3(x - 2) = y$$

$$\text{or } 3x - y = 6 \dots (i)$$

**again, According to question**

$$\frac{x}{y-1} = 1/2$$

$$\text{or } 2x = y - 1$$

$$\text{or } 2x - y = -1 \dots (ii)$$

**On solving equation (i) and (ii), we get**

$$x = 7, y = 15$$

**Ans → The required fraction is  $7/15$**

---

**Ans 16).**

**We know that a point on the y-axis is of the form  $(0, y)$ .**

**So, let the point  $P(0, y)$  be equidistant from  $A(5, -2)$  and  $B(-3, 2)$**

**→ Then  $AP = BP$**

$$\text{→ or } AP^2 = BP^2$$

$$\text{→ or } (5 - 0)^2 + (-2 - y)^2 = (-3 - 0)^2 + (2 - y)^2$$

$$\text{or } 25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$$

$$8y = -16$$

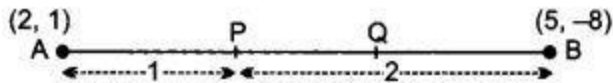
$$y = -2$$

**So, the required point is  $(0, -2)$**

OR

→ The line segment AB is trisected at the points P and Q and P is nearest to A

So, P divides AB in the ratio 1 : 2



Then co-ordinates of P, by section formula

$$\begin{aligned} &= P \left[ \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right] \\ &= P \left[ \frac{1(5) + 2(2)}{1 + 2}, \frac{1(-8) + 2(1)}{1 + 2} \right] \\ &= P \left[ \frac{5 + 4}{2}, \frac{-8 + 2}{3} \right] = P(3, -2) \end{aligned}$$

P lies on the line  $2x - y + k = 0$

It will satisfy the equation.

→ On putting  $x = 3$  and  $y = -2$  in the given equation, we get

$$2(3) - (-2) + k = 0$$

$$6 + 2 + k = 0$$

$$k = -8$$

Hence,  $k = -8$

---

Ans 17).

We know the trigonometric identities ,

$$1) \sin^2 A + \cos^2 A = 1$$

$$2) \sec^2 A = 1 + \tan^2 A$$

$$3) \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$4) \sin A \operatorname{cosec} A = 1$$

$$5) \cos A \sec A = 1$$

Now ,

$$\text{LHS} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A$$

$$= (\sin^2 A + \cos^2 A) + 2 + 2 + \operatorname{cosec}^2 A + \sec^2 A$$

$$= 1 + 2 + 2 + (1 + \cot^2 A) + (1 + \tan^2 A)$$

$$= 7 + \cot^2 A + \tan^2 A$$

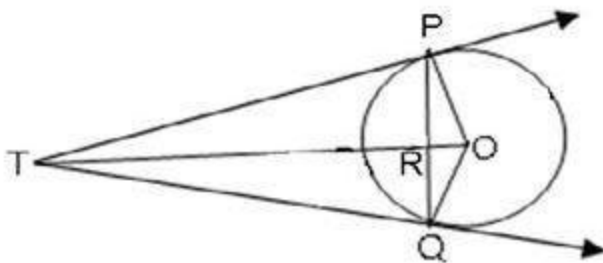
→ Hence Proved.

OR

$$\begin{aligned}
 \text{L.H.S.} &= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\
 &= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \\
 &= \left(\frac{\sin A + \cos A - 1}{\sin A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right) \\
 &\quad [\because (a+b)(a-b) = a^2 - b^2] \\
 &= \frac{(\sin A + \cos A)^2 - 1}{\sin A \cdot \cos A} \\
 &= \frac{\sin^2 A + \cos^2 A + 2 \cdot \sin A \cdot \cos A - 1}{\sin A \cdot \cos A} \\
 &= \frac{1 + 2 \sin A \cdot \cos A - 1}{\sin A \cdot \cos A} = \frac{2 \sin A \cdot \cos A}{\sin A \cdot \cos A} \\
 &= 2 \text{ (R.H.S.)} \qquad \qquad \text{Hence Proved.}
 \end{aligned}$$

---

Ans 18).



Let  $TR = y$

Since  $OT$  is perpendicular bisector of  $PQ$ .

Therefore,  $PR=QR=4\text{cm}$

In right triangle OTP and PTR, we have:

$$\rightarrow TP^2=TR^2+PR^2$$

$$\text{Also, } OT^2=TP^2+OP^2$$

$$OT^2=(TR^2+PR^2) + OP^2$$

$$(y+3)^2=y^2+16+25 \text{ (OR = 3, as } OR^2 = OP^2 - PR^2)$$

$$\rightarrow 6y=32$$

$$\rightarrow y = \frac{16}{3}$$

$$\rightarrow TP^2=TR^2+PR^2$$

$$\rightarrow TP^2 = \left(\frac{16}{3}\right)^2 + 4^2 = \frac{256}{9} + 16 = \frac{400}{9}$$

$$\rightarrow TP = \frac{20}{3} \text{ cm}$$

Ans 19).

$\rightarrow$  To prove  $CD^2 = BD \times AD$

In  $\Delta CAD$ ,  $CA^2 = CD^2 + AD^2 \dots (1)$

$\rightarrow$  Also in  $\Delta CDB$ ,  $CB^2 = CD^2 + BD^2 \dots (2)$

(1) + (2) we get:

$$\rightarrow CA^2 + CB^2 = 2CD^2 + AD^2 + BD^2$$

$$AB^2 = 2CD^2 + AD^2 + BD^2$$

$$AB^2 - AD^2 = BD^2 + 2CD^2$$

$$(AB + AD)(AB - AD) - BD^2 = 2CD^2$$

$$(AB + AD)BD - BD^2 = 2CD^2$$

$$BD(AB + AD - BD) = 2CD^2$$

$$BD(AD + AD) = 2CD^2$$

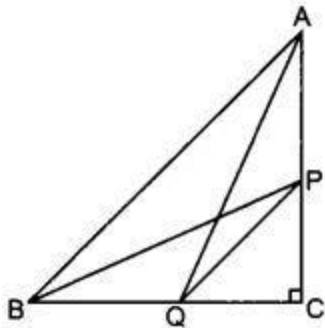
$$BD \times 2AD = 2CD^2$$

$$\rightarrow CD^2 = BD \times AD$$

Hence proved.

OR

$\rightarrow$  Given, ABC is a right-angled triangle in which  $\angle C = 90^\circ$



To prove :  $AQ^2 + BP^2 = AB^2 + PQ^2$

Construction: Join AQ, PB and PQ

Proof: In  $\triangle AQC$ ,  $\angle C = 90^\circ$

$AQ^2 = AC^2 + CQ^2$  ...(i) (Using Pythagoras theorem)

In  $\triangle PBC$ ,  $\angle C = 90^\circ$

$BP^2 = BC^2 + CP^2$  ...(ii) (Using Pythagoras theorem)

Adding equation (i) and (ii)

$$AQ^2 + BP^2 = AC^2 + CQ^2 + BC^2 + CP^2 = AC^2 + BC^2 + CQ^2 + CP^2$$



or

$$AQ^2 + BP^2 = AB^2 + PQ^2$$

Hence Proved.

---

Ans 20).

→ Given, ABCD is a rectangle with sides AB = 8 cm and BC = 6 cm

In  $\triangle ABC$

$$AC^2 = 8^2 + 6^2 \text{ (By Pythagoras Theorem)}$$

$$\rightarrow AC^2 = 64 + 36$$

$$\rightarrow AC^2 = 100$$

$$\rightarrow AC = 10 \text{ cm}$$

The diagonal of the rectangle will be the diameter of the circle

$$\text{radius of the circle} = \frac{10}{2} = 5 \text{ cm}$$

Area of shaded portion = Area of circle – Area of Rectangle

$$= \pi r^2 - l \times b$$

$$= 3.14 \times 5 \times 5 - 8 \times 6$$

$$= 78.50 - 48$$

$$= 30.50 \text{ cm}^2$$

Hence, Area of shaded portion =  $30.5 \text{ cm}^2$

---

Ans 21).

→ Let b be the width and h be the depth of the canal

$$b = 6 \text{ m and } h = 1.5 \text{ m}$$

Water is flowing with a speed = 10 km/h = 10,000 m/h

→ Length of water flowing in 1 hr = 10,000 m

→ Length (l) of water flowing in  $\frac{1}{2}$  hr = 5,000 m

$$\text{Volume of water flowing in 30 min.} = l \times b \times h = 5000 \times 6 \times 1.5 \text{ m}^3$$

→ Let the area irrigated in 30 min ( $\frac{1}{2}$  hr) be  $x \text{ m}^2$

Volume of water required for irrigation = Volume of water flowing in 30 min.

$$\rightarrow x \times \frac{8}{100} = 5000 \times 6 \times 1.5$$

$$\rightarrow x = 562500 \text{ m}^2 = 56.25 \text{ hectares. (}\therefore 1 \text{ hectare} = 104 \text{ m}^2\text{)}$$

Hence, the canal will irrigate 56.25 hectares in 30 min.

---

**Ans 22).**

→ Here, the maximum class frequency is 16

→ Modal class = 30-40

→ Lower limit (l) of modal class = 30

→ Class size (h) = 10

→ Frequency (f<sub>1</sub>) of the modal class = 16

→ Frequency (f<sub>0</sub>) of preceding class = 10

→ Frequency (f<sub>2</sub>) of succeeding class = 12

$$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$30 + \left( \frac{16 - 10}{32 - 10 - 12} \right) \times 10$$

$$= 30 + 6/10 \times 10$$

**Ans → Hence, Mode is 36.**

---

**Ans 23).** Let the tap A with longer diameter take x hours and the tap B with smaller diameter take (x + 2) hours to fill the tank.

$$\rightarrow \text{Portion of tank filled by the tap A in 1 hr.} = \frac{1}{x}$$

$$\rightarrow \text{and Portion of tank filled by the tap B in 1 hr.} = \frac{1}{x+2}$$

$$\text{Portion of the tank filled by both taps in 1 hr.} = \frac{1}{x} + \frac{1}{x+2} = \frac{x+2+x}{x(x+2)}$$

$$\rightarrow \text{Time taken by both taps to fill the tank} = 1 \frac{7}{8} \text{ hrs} = \frac{15}{8} \text{ hrs}$$

**Portion of the tank filled by both in 1 hr. = 8/15**

**According to question,**

$$\frac{2x+2}{x(x+2)} = \frac{8}{15}$$

$$\rightarrow \frac{2(x+1)}{x(x+2)} = \frac{8}{15}$$

$$\rightarrow 15x + 15 = 4x^2 + 8x$$

$$\rightarrow 4x^2 - 12x + 5x - 15 = 0$$

$$\rightarrow 4x(x - 3) + 5(x - 3) = 0$$

$$\rightarrow (4x + 5)(x - 3) = 0$$

$$\rightarrow 4x + 5 = 0 \text{ or } x - 3 = 0$$

$$\rightarrow x = -5/4$$

**Since, time can not be negative hence, neglected this value is;  $x = 3$**

**Hence, the time taken with longer diameter tap = 3 hours**

**and the time taken with a smaller diameter tap = 5 hours.**

**OR**

**Let the speed of the boat in still water be  $x$  km/h and the speed of the stream be  $y$  km/h.**

$$\rightarrow \text{Speed upstream} = (x - y) \text{ km/h}$$

$$\rightarrow \text{Speed downstream} = (x + y) \text{ km/h}$$

$$\text{Let } \frac{1}{x-y} = a \text{ and } \frac{1}{x+y} = b$$

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \Rightarrow 30a + 44b = 10 \Rightarrow 120a + 176b = 40$$

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \Rightarrow 40a + 55b = 13 \Rightarrow 120a + 165b = 39$$

**$\rightarrow$  On subtracting, we get,**

$$b = \frac{1}{11}$$

$$\therefore 30a + 4 = 10 \Rightarrow 30a = 6 \Rightarrow a = \frac{1}{5}$$

$$\therefore x - y = 5 \text{ and } x + y = 11$$

On solving, we get,

$$x = 8 \text{ and } y = 3$$

→ Speed of boat in still water = 8 km/hr

→ And, Speed of stream = 3 km/hr

Ans 24).

Given:

$$\rightarrow S_4 = 40$$

$$\rightarrow S_{14} = 280$$

$$S_n = n/2[2a + (n-1)d]$$

$$s = 4/2[2a + (4-1)d]$$

$$40 = 2[2a + 3d]$$

$$40/2 = 2a + 3d$$

$$2a + 3d = 20 \rightarrow (1)$$

similarly,

$$S_{14} = 14/2[2a + (14-1)d]$$

$$280 = 7[2a + 13d]$$

$$280/7 = 2a + 13d$$

$$2a + 13d = 40. \rightarrow (2)$$

By subtracting eq(2) from(1)

we get,

$$-10d = -20$$

$$10d = 20$$

$$d = 20/10$$

$$d = 2$$

→ Now substitute  $d=2$  in eq(1)

$$a=7$$

→ We have , $a=7$  &  $d=2$

$$\rightarrow S_n = n/2 [2a+(n-1)d]$$

$$\rightarrow S_n = n/2 [2(7)+(n-1)2]$$

$$\rightarrow S_n = n/2 [14+2n-2]$$

$$\rightarrow S_n = n/2 [12+2n]$$

$$\rightarrow S_n = n/2 [2(6+n)]$$

$$\rightarrow S_n = n (6+n)$$

$$\rightarrow S_n = 6n + n^2$$

---

**Ans 25).**

$$LHS = (\tan A - 1 + \sec A)/(\tan A + 1 - \sec A)$$

Now

$$\sec^2 A = 1 + \tan^2 A$$

$$\sec^2 A - \tan^2 A = 1$$

Using above relation at denominator of LHS

$$LHS = (\tan A - 1 + \sec A)/(\tan A - \sec A + \sec^2 A - \tan^2 A)$$

$$LHS = (\tan A - 1 + \sec A)/((\sec A - \tan A)(-1 + \sec A + \tan A))$$

$$LHS = 1/(\sec A - \tan A)$$

$$LHS = RHS$$

---

**Ans 26).**

→ Consider  $\Delta ABC$

Where AB is Height of Tower

BC is distance from boat to tower

$$\rightarrow \tan C = AB/BC = \sqrt{3}$$

Now, as height of the tower is 100m,  $AB=100$

$$100/BC = \sqrt{3}$$

$$BC = 57.73$$

→ Now let D be the point the boat travelled

$$\tan D = AB/BD = 1/\sqrt{3} = 100/BD = 173.2$$

→ Distance travelled by boat  $CD=BD-BC=$

$$173.2-57.73 = 115.47\text{m}$$

Speed of the boat =  $115.47/2 = 57.73$  m/min.

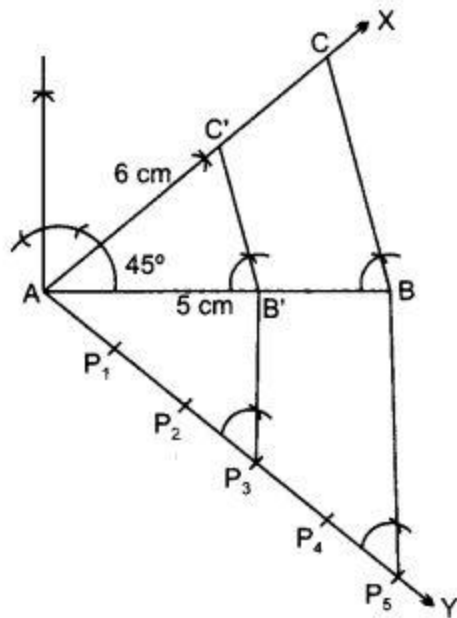
Ans → Speed of the boat is 57.73 m/min.

---

Ans 27).

Steps of Construction are as follows:

1. Draw  $AB = 5$  cm
2. At the point, A draw  $\angle BAX = 45^\circ$
3. From AX cut off  $AC = 6$  cm
4. Join BC,  $\Delta ABC$  is formed with given data.
5. Draw AY making an acute angle with AB as shown in the figure.



6. Draw 5 arcs  $P_1, P_2, P_3, P_4$ , and  $P_5$  with equal intervals.
7. Join  $BP_5$ .

8. Draw  $P3B' \parallel P5B$  meeting  $AB$  at  $B'$ .
9. From  $B'$ , draw  $B'C' \parallel BC$  meeting  $AC$  at  $C'$   
 $\Delta AB'C' \sim \Delta ABC$   
Hence  $\Delta AB'C'$  is the required triangle.

---

Ans 28). Given:  $\rightarrow$  Volume of the bucket -  $12308.8\text{cm}^3$

$\rightarrow r_1 = 20\text{cm}$  and  $r_2 = 12\text{cm}$



We have volume of bucket =  $\frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$$\Rightarrow 12308.8 = 3.14 \times \frac{h}{3}(20^2 + 12^2 + 20 \times 12)$$

$$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784} = 15 \text{ cm}$$

$\therefore$  height of the bucket,  $h = 15 \text{ cm}$

Surface area of the metal sheet used =  $\pi r_2^2 + \pi(r_1 + r_2)\ell$

$$\ell = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = 17 \text{ cm}$$

$\therefore$  Surface area of the metal sheet used =  $\pi r_2^2 + \pi(r_1 + r_2)\ell$

$$\begin{aligned} &= 3.14 \times 12^2 + \frac{22}{7} \times 32 \times 17 \\ &= 3.14(144 + 544) \end{aligned}$$

**Ans → Hence, height of the bucket = 15cm and Area of the metal sheet used is 2160.32 sq.cm.**

---

**Ans 29).**

**→ Given: A right angled  $\triangle ABC$ , right angled at B**

**→ To Prove-  $AC^2 = AB^2 + BC^2$**

**→ Construction: Draw perpendicular BD onto the side AC .**

**Proof:** We know that if a perpendicular is drawn from the vertex of a right angle of a right angled triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

We have,  $\triangle ADB \sim \triangle ABC$ . (by AA similarity)

Therefore,  $AD/AB = AB/AC$

(In similar Triangles corresponding sides are proportional)

$$AB^2 = AD \times AC \rightarrow (1)$$

→ Also,  $\triangle BDC \sim \triangle ABC$

Therefore,  $CD/BC = BC/AC$  (in similar Triangles corresponding sides are proportional)

$$\text{Or, } BC^2 = CD \times AC \rightarrow (2)$$

→ Adding the equations (1) and (2) we get,

$$AB^2 + BC^2 = AD \times AC + CD \times AC$$

$$AB^2 + BC^2 = AC(AD + CD)$$

( From the figure  $AD + CD = AC$ )

$$\rightarrow AB^2 + BC^2 = AC \cdot AC$$

$$\rightarrow AC^2 = AB^2 + BC^2$$

Ans  $\rightarrow$  Hence, Proved.

---

Ans 30).

Marks	No. of Students
0 – 5	2
5 – 10	5
10 – 15	6
15 – 20	8
20 – 25	10
25 – 30	25
30 – 35	20
35 – 40	18
40 – 45	4
45 – 50	2

→ Given: Median=32.5, then 30–40 is class interval.

→  $l = 30$  ,  $cf = 14 + f_1$

→  $f=12$  and  $h = 40-30 = 10$ .

→  $N = 40$  and  $N/2 = 20$

→  $31 + f_1 + f_2 = 40$ .

So,  $f_1 + f_2 = 9$

$$\rightarrow \text{Median} = \frac{\frac{N}{2} - cf}{f} \times h$$

$$\rightarrow 32.5 = 30 + \frac{20 - (14 + f_1)}{12} \times 10$$

$$\rightarrow 2.5 = \frac{6 - f_1}{6} \times 5$$

$$\rightarrow 15 = 30 - 5f_1$$

$$\rightarrow 5f_1 = 15$$

→ and on substituting values,  $f_1 = 3$  and  $f_2 = 6$

**OR**

Marks	Cumulative Frequency
less than 5	2
less than 10	7
less than 15	13
less than 20	21
less than 25	31

less than 30	56
less than 35	76
less than 40	94
less than 45	98
less than 50	100

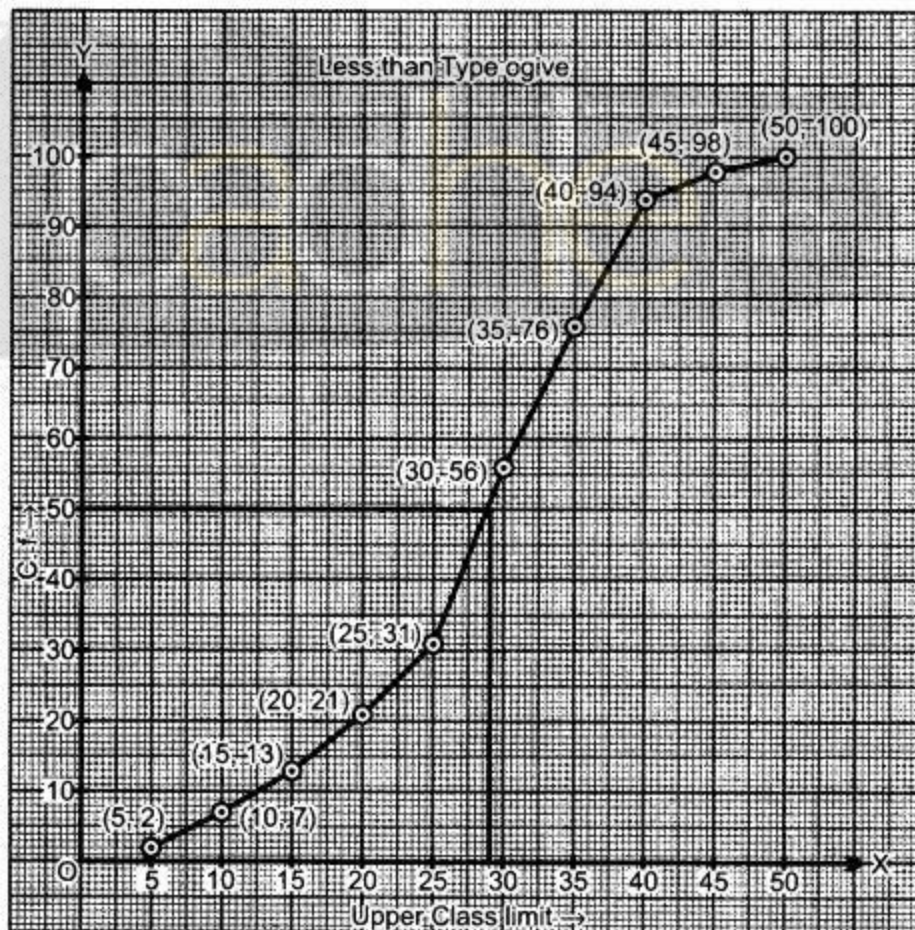
→ To draw a less than ogive,  
we mark the upper-class limits of the class intervals on the x-axis and their c.f. on the y-axis by taking a convenient scale.

→ Here,  $n = 100 \Rightarrow n/2 = 50$

→ To get median from graph From 50,

→ Draw a perpendicular, the point where this perpendicular meet on the x-axis will be the median.

→ Median = 29



# CBSE Mathematics 2018

General Instructions :

(i) All questions are compulsory.

(ii) This question paper consists of 30 questions divided into four sections—A, B, C and D.

(iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted.

# Questions

## Section-A (1 Mark Each)

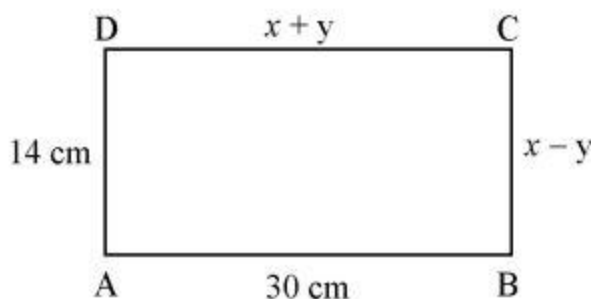
1. If  $x = 3$  is one root of the quadratic equation  $x^2 - 2kx - 6 = 0$ , then find the value of  $k$ .
2. What is the HCF of smallest prime number and the smallest composite number ?
3. Find the distance of a point  $P(x,y)$  from the origin.
4. In an AP, if the common difference  $(d) = -4$  and the seventh term  $(a_7)$  is 4, then find the first term.
5. What is the value of  $(\cos^2 67^\circ - \sin^2 23^\circ)$  ?
6. Given  $\triangle ABC \sim \triangle PQR$ , if  $\frac{AB}{PQ} = 1/3$ , then find  $\frac{ar\ ABC}{ar\ PQR}$  ?

## Section-B (2 Marks Each)

7. Given that  $\sqrt{2}$  is irrational, prove that  $(5 + 3\sqrt{2})$  is an irrational number



8. In fig. 1, ABCD is a rectangle. Find the values of  $x$  and  $y$ .



9. Find the sum of first 8 multiples of 3.

10. Find the ratio in which P (4,  $m$ ) divides the line segment joining the points A (2, 3) and B (6, -3). Hence find  $m$ .

11. Two different dice are tossed together. Find the probability : (i) of getting a doublet. (ii) of getting a sum 10, of the numbers on the two dice.

12. An integer is chosen at random between 1 and 100. Find the probability that it is : (i) divisible by 8  
(ii) not divisible by 8.

## Section- C (3 Marks Each)

13. Find HCF and LCM of 404 and 96 and verify that  $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$ .

14. Find all zeroes of the polynomial  $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$  if two of its zeroes are  $(2 + \sqrt{3})$  and  $(2 - \sqrt{3})$ .

15. If A (–2, 1), B (a, 0), C (4, b) and D (1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

OR

If A (–5, 7), B (–4, –5), C (–1, –6) and D (4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

16. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

17. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

OR

If the area of two similar triangles are equal, prove that they are congruent.

18. Prove that the lengths of tangents drawn from an external point to a circle are equal.

19. If  $4 \tan \theta = 3$ , evaluate  $\frac{4 \sin \theta - \cos \theta}{4 \sin \theta \cos \theta (-1)}$

OR

If  $\tan 2A = \cot (A - 18^\circ)$ , where  $2A$  is an acute angle, find the value of A.

20. Find the area of the shaded region in Fig. 2, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. [Use  $\pi = 3.14$ ]

21. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.

OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap ?

22. The table below shows the salaries of 280 persons :

Salary ( in Thousand ₹ )	No. of Persons
5-10	49
10-15	133
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4

40-45	2
45-50	1

Calculate the median salary of the data.

## Section- D (4 Marks Each)

23. A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete a total journey, what is the original average speed ?

24. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

25. In an equilateral ABC, D is a point on side BC such that  $BD = \frac{1}{3} BC$ . Prove that  $9(AD)^2 = 7(AB)^2$ .

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

26. Draw a triangle ABC with BC = 6 cm, AB = 5 cm and  $\angle ABC = 60^\circ$ . Then construct a triangle whose sides are  $\frac{3}{4}$  of the corresponding sides of the ABC.

27. Prove that :  $(\sin A - 2\sin^3 A) / (2\cos^3 A - \cos A) = \tan A$

28. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find :

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic ?

[Use  $\pi = 3.14$ ]

29. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are  $30^\circ$  and  $45^\circ$ . If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use  $\sqrt{3} = 1.732$ ]

30. The mean of the following distribution is 18. Find the frequency f of the class 19–21.

Class	11 –13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

OR

The following distribution gives the daily income of 50 workers of a factory :

Daily Income (in ₹)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

## Answers

Ans 1).

→ Given quadratic equation is:  $x^2 - 2kx - 6 = 0$   
:  $x = 3$  is a root of above equation

Then,

$$(3)^2 - 2k(3) - 6 = 0$$

$$9 - 6k - 6 = 0$$

$$3 - 6k = 0$$

$$3 = 6k$$

$$k = \frac{1}{2}$$

Ans 2). → Smallest prime number = 2

→ Smallest composite number = 4

→ Prime factorisation of 2 is  $1 \times 2$

→ Prime factorisation of 4 is  $1 \times 2^2$

Ans : HCF (2, 4) = 2

Ans 3). → Distance from P (x, y) to origin

→ Points of origin Q (0,0)

→ By applying distance formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(0 - x)^2 + (0 - y)^2}$$

$$\sqrt{x^2 + y^2} \text{ units.}$$

Ans → The distance of a point p(x,y) from the origin is  $\sqrt{x^2 + y^2}$  units.

---

Ans 4). Given: → common difference (d) = -4

→ the seventh term (a<sub>7</sub>) = 4

To find: The first term (a)

So,

$$d = -4$$

$$a_7 = 4$$

$$a + 6d = 4$$

$$a + 6(-4) = 4$$

$$a = 28$$

Ans → The first term (a) is 28

---

Ans 5).

Solution:

$$\begin{aligned} & \cos^2 67^\circ - \sin^2 23^\circ \\ &= \cos^2 67^\circ - \cos^2 (90^\circ - 23^\circ) \end{aligned}$$

→ We know,  $[\sin(90^\circ - \theta) = \cos \theta]$

$$= \cos^2 67^\circ - \cos^2 67^\circ = 0$$

**Ans** → the value of  $(\cos^2 67^\circ - \sin^2 23^\circ)$  is 0

---

**Ans 6).** Given,  $ABC \sim PQR$  &  $\frac{AB}{PQ} = \frac{1}{3}$

$$\begin{aligned} \text{So, } \frac{\text{ar}(ABC)}{\text{ar}(PQR)} &= \frac{\frac{AB^2}{PQ^2}}{1} \\ &= \left(\frac{1}{3}\right)^2 = 1/9 \end{aligned}$$

**Ans** →  $\frac{\text{ar}(ABC)}{\text{ar}(PQR)} = 1/9$

---

**Ans 7).** Given,  $\sqrt{2}$  is irrational number.

→ Let  $\sqrt{2} = m$  Suppose,  $5 + 3\sqrt{2}$  is a rational number.

**So,**  $5 + 3\sqrt{2} = a/b$

$$3\sqrt{2} = a/b - 5$$

$$3\sqrt{2} = \frac{a - 5b}{b}$$

$$\sqrt{2} = \frac{a - 5b}{3b}$$

$$\frac{a - 5b}{3b} = m$$

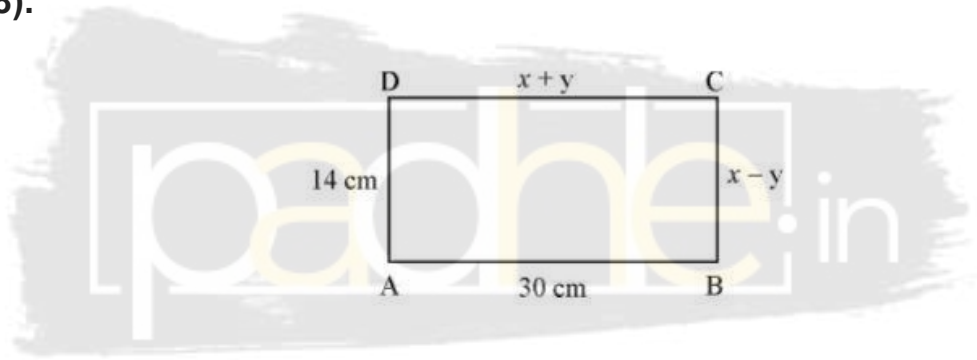


→ But  $\frac{a-5b}{3b}$  is rational number and so has to be m, but this contradicts the fact that  $m = \sqrt{2}$  which is irrational

Ans → Hence,  $5 + 3\sqrt{2}$  is also irrational.

---

Ans 8).



→ Since ABCD is a rectangle. its opposite sides are equal.

→  $AD = BC$  and  $AB = DC$

$$x - y = 14$$

$$x + y = 30$$

→ Solving both the equations by elimination method,

Add the equations to get:

$$2x = 44$$

$$x = 22$$

→ Put the value of x in any equation, we get

$$x + y = 30$$

$$22 + y = 30$$

$$y = 8$$

Hence,  $x = 22$  and  $y = 8$

---

**Ans 9).** We know, sum of  $n$  terms from 1st term is  $\frac{n}{2}[2a + (n - 1)d]$

→ In the given question,

**First term = 3**

**Common difference = 3**

**Then, applying formula from above.**

→ **Assuming the value of  $n$  as 8 and  $a$  as 3**

$$S_8 = \frac{8}{2}[2(3) + (8 - 1)3]$$

$$S_8 = 4[6 + 7(3)]$$

$$S_8 = 4[6 + 21]$$

$$S_8 = 4[27]$$

$$S_8 = 108$$

**Ans→ The sum of first 8 multiples of 3.**

---

**Ans 10).**

**Let P divides line segment AB in the ratio k : 1.**

**Let the ratio be k : 1**

**By section formula :**  $\frac{m_1x_2 - m_2x_1}{m_1 - m_2}$  ,  $\frac{m_1y_2 - m_2y_1}{m_1 - m_2}$

$$(x, y) = [ (kx_2 + x_1) / (k + 1) , (ky_2 + y_1) / (k + 1) ]$$

$$(4, m) = [ \{k(6) + 2\} / (k + 1) , \{k(-3) + 3\} / (k + 1) ]$$

$$4 = (6k + 2) / (k + 1) \text{ and } m = (-3k + 3) / (k + 1)$$

$$\rightarrow 4k + 4 = 6k + 2$$

$$\rightarrow 6k - 4k = 4 - 2$$

$$\rightarrow 2k = 2$$

$$\rightarrow k = 1$$

**Now, substituting k = 1 in m's value**

$$m = [-3(1) + 3] / (1 + 1)$$

$$\rightarrow -3+3 / 2$$

$$\rightarrow 0 / 2$$

$$\rightarrow 0$$

---

**Ans 11). Total numbers are 2, 3, 4, ....., 99**

**→ Let E be the event of getting a number divisible by 8.**

$$E = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\} = 12$$

$$P(E) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$$

$$\frac{12}{98} = 0.1224$$

**→ Let E' be the event of getting a number not divisible by 8.**

$$\text{Then, } P(E') = 1 - P(E)$$

$$= 1 - 0.1224 = 0.8756$$

---

**Ans 12).**

**→ HCF of 404 and 96**

$$404 = 2 \times 2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{Common Factors} = 2 \times 2$$

$$\text{HCF of 404 and 96} = 4$$

$$\text{LCM of 404 and 96}$$

$$404 = 2 \times 2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 101 = 9696$$

LCM of 404 and 96 is 9696

HCF  $\times$  LCM = Product of the two numbers

$$\rightarrow 4 \times 9696 = 404 \times 96$$

$$\rightarrow 38784 = 38784$$

LHS = RHS

---

Ans 14). Given: Two zeroes of the polynomials are:

$(2 + \sqrt{3})$  and  $(2 - \sqrt{3})$ .

$\rightarrow$  Quadratic polynomial with zeros is given by:

$$\rightarrow \{x - (2 + \sqrt{3})\} \cdot \{x - (2 - \sqrt{3})\}$$

$$\rightarrow (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$

$$\rightarrow (x-2)^2 - (\sqrt{3})^2$$

$$x^2 - 4x + 4 - 3$$

$$x^2 - 4x + 1 = g(x)$$

Now,  $g(x)$  will be a factor of  $p(x)$  so  $g(x)$  will be divisible by  $p(x)$

$$\begin{array}{r}
 2x^2 - x - 1 \\
 x^2 - 4x + 1 \overline{) 2x^4 - 9x^3 + 5x^2 + 3x - 1} \\
 \underline{2x^4 - 8x^3 + 2x^2} \phantom{+ 3x - 1} \\
 -x^3 + 3x^2 + 3x \phantom{- 1} \\
 \underline{-x^3 + 4x^2 - x} \phantom{- 1} \\
 +x^2 + 4x - 1 \\
 \underline{-x^2 + 4x - 1} \\
 0
 \end{array}$$

For other zeroes,  $2x^2 - x - 1 = 0$

$$2x^2 - 2x + x - 1 = 0$$

$$\text{or } 2x(x - 1) + 1(x - 1) = 0$$

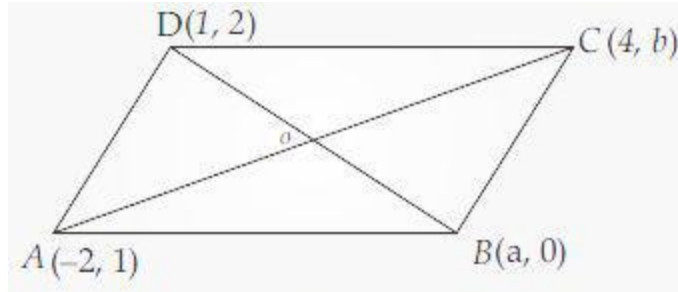
$$(x - 1)(2x + 1) = 0$$

$$x - 1 = 0, 2x + 1 = 0$$

$$x = 1, x = -\frac{1}{2}$$

→ Hence, Zeroes of  $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$  are  $1, -\frac{1}{2}, 2 + \sqrt{3}, 2 - \sqrt{3}$

Ans 15).



→ Given sides of parallelogram  $A(-2,1)$ ,  $B(a,0)$ ,  $C(4,b)$ ,  $D(1,2)$

We know that diagonals of Parallelogram bisect each other.

Mid-point let say  $O$  of diagonal  $AC$  is given by

$$\rightarrow x = \left( \frac{x_1 + x_2}{2} \right) \text{ and } y = \left( \frac{y_1 + y_2}{2} \right)$$

$$O\left( \frac{-2+4}{2}, \frac{1+b}{2} \right)$$

→ Mid-point let say  $P$  of diagonal  $BD$  is given by

$$P\left( \frac{a+1}{2}, \frac{0+2}{2} \right)$$

Points  $O$  and  $P$  are same

→ Equating the corresponding coordinates of both midpoints, we get

$$\frac{-2+4}{2} = \frac{a+1}{2}$$

→  $a = 1$  and

$$\frac{1+b}{2} = \frac{0+2}{2}$$

→ **b = 1** and

**Now the Given coordinates of the parallelogram are written as**

**A(-2,1), B(1,0), C(4,1), D(1,2)**

→ **By distance formula,  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$**

**We can find the length of each side**

$$AB = \sqrt{(-2 - 1)^2 + (1 - 0)^2}$$

$$AB = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$$

→ **AB = CD** (pair of opposite sides of the parallelogram are parallel and equal)

$$BC = \sqrt{(4 - 1)^2 + (1 - 0)^2} = \sqrt{10}$$

$$BC = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$$

**BC=AD** (pair of opposite sides of the parallelogram are parallel and equal )

$$\rightarrow AB=BC=CD=AD = \sqrt{10}$$

→ **ABCD is a Rhombus**



OR

Given ABCD is quadrilateral.

→ By joining points A and C, the quadrilateral is divided into two triangles.

→ Now, Area of quad. ABCD = Area of ABC + Area of ACD

→ Area of  $\triangle ABC$

$$= \frac{1}{2} [x_1 y_2 - y_3 \ x_2 y_3 - y_1 \ x_3 y_1 - y_2]$$

$$= \frac{1}{2} [-5 -5 \ 6 -4 \ -6 -7 \ -1 \ 7 \ 5]$$

$$= \frac{1}{2} [-5(1) - 4(-13) - 1(12)]$$

$$= \frac{1}{2} (-5 + 52 - 12)$$

$$= \frac{1}{2} (35) \text{ sq. units}$$

→ Area of  $\triangle ADC$

$$= \frac{1}{2} [x_1 y_2 - y_3 \ x_2 y_3 - y_1 \ x_3 y_1 - y_2]$$

$$= \frac{1}{2} [-5 \ 5 \ 6 \ 4 -6 -7 \ (-1) \ 7 - 5]$$

$$= \frac{1}{2} [-5 \ 5 \ 6 \ 4 -6 -7 \ (-1) \ 7 - 5]$$

$$= \frac{1}{2} [ -5(11) - 4(-13) - 1(2) ]$$

$$= \frac{1}{2} (-55 + 52 - 2)$$

$$= -\frac{5}{2} \text{ sq. units.}$$

So, Area of quadrilateral ABCD =

$$\frac{1}{2} (35) - \frac{1}{2} (109)$$

$$= -\frac{37}{2} \text{ sq. units.}$$

Ans 16). Let the usual speed of the plane be  $x$  km/hr.

→ Increased speed = 100 km/hr

→ Time taken to cover 1500 km =  $\frac{1500}{x}$  hr.

Time taken to cover 1500 km with increased speed =  $\frac{1500}{x} + 100$  hr

According to the Question,

$$\rightarrow \frac{1500}{x} - \frac{1500}{x + 100} = \frac{30}{60}$$

$$\rightarrow x^2 + 100x - 300000 = 0$$

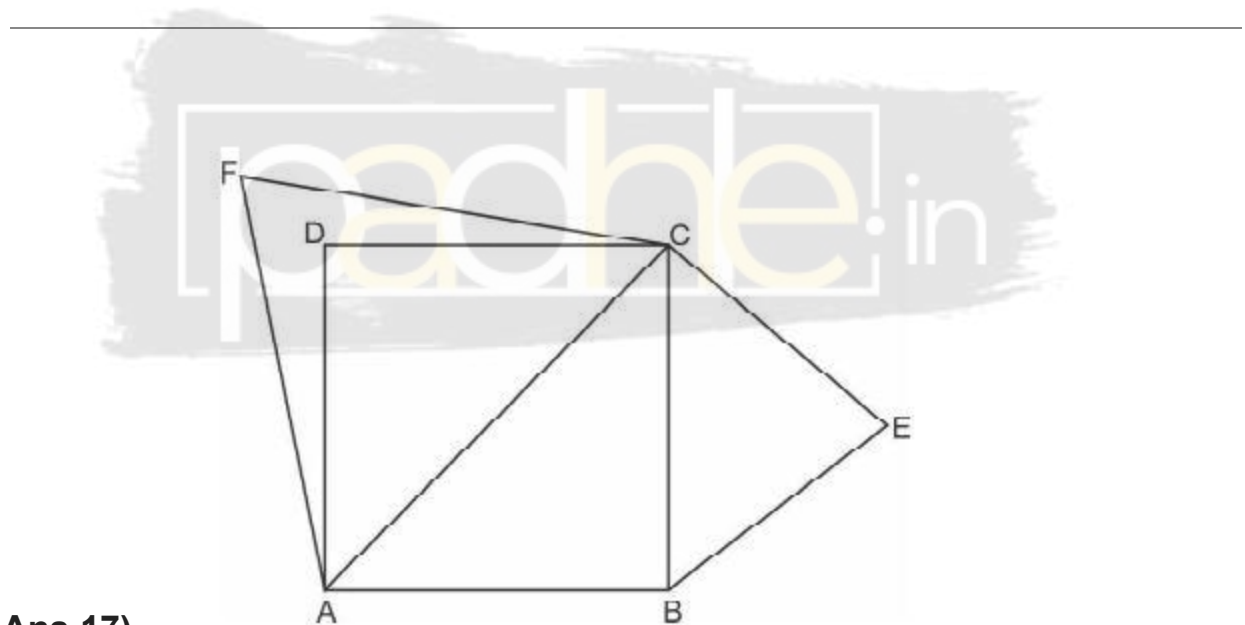
$$\rightarrow x^2 + 600x - 500x - 300000 = 0$$

$$\rightarrow (x + 600)(x - 500) = 0$$

→  $x = -600$  or  $500$  (Neglecting negative sign as speed cannot be negative)

→  $x = 500$

Ans → Hence, the usual speed of the plane is 500 km/hr.



Ans 17).

→ In  $\triangle ABC$ ,  $AC^2 = AB^2 + BC^2$

$$a^2 + a^2 = 2a^2$$

$$AC = \sqrt{2} a^2 = \sqrt{2} a$$

→ Area of equilateral BEC (formed on side BC of square ABCD)

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times a^2$$

→ Area of equilateral ACF (formed on diagonal AC of square ABCD)

$$= \frac{\sqrt{3}}{4} (\sqrt{2}a)^2$$

$$= \frac{\sqrt{3}}{4} 2a^2$$

$$= 2 \frac{\sqrt{3}}{4} a^2$$

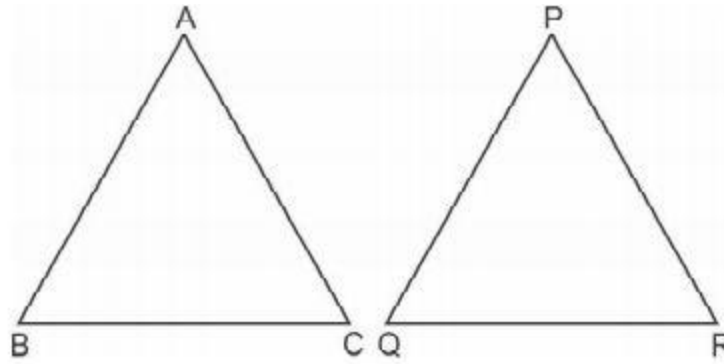
→ From eq. (i) and (ii), ar  $\triangle$  ACF = 2 × ar  $\triangle$  BCE

Or,

$$\rightarrow \text{ar}(\triangle \text{BCE}) = \frac{1}{2} \text{ar}(\triangle \text{ACF})$$

→ Hence Proved, area of triangle described on one side of square is half the area of triangle described on its diagonal.

OR



**Given,  $\triangle ABC \sim \triangle PQR$   
 $Ar(\triangle ABC) = ar(\triangle PQR)$**

**To Prove:  $\triangle ABC \cong \triangle PQR$**

$$\rightarrow \frac{ar(ABC)}{ar(PQR)} = AB^2/PQ^2 = BC^2/QR^2 = AC^2/PR^2$$

**: As Ratio of area of similar triangles is equal to the square of corresponding sides**

$$\rightarrow \text{But, as given, } \frac{ar(ABC)}{ar(PQR)} = 1$$

$$AB^2/PQ^2 = BC^2/QR^2 = AC^2/PR^2 = 1$$

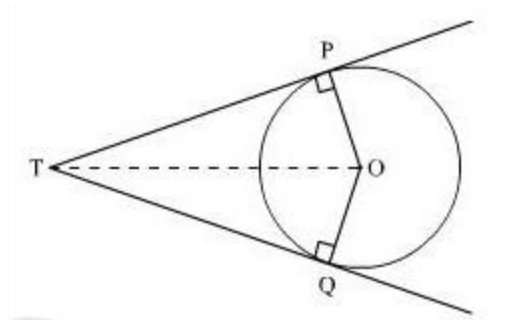
$$\rightarrow AB^2 = PQ^2 / AB = PQ$$

$$\rightarrow BC^2 = QR^2 / BC = QR$$

$$\rightarrow AC^2 = PR^2 / AC = PR$$

**: - By SSS congruence axiom  $\triangle ABC \cong \triangle PQR$ , hence Proved.**

Ans 18).



→ Let two tangent PT and QT are drawn to circle of centre O as shown in figure.

Both the given tangents PT and QT touch to the circle at P and Q respectively.

We have to prove :

→ length of PT = length of QT

→ Construction :- draw a line segment ,from centre O to external point T { touching point of two tangents } .

→ Now  $\triangle POT$  and  $\triangle QOT$

We know tangent makes the right angle with the radius of the circle.

→ Here, PO and QO are radii . So,  $\angle OPT = \angle OQT = 90^\circ$

$\rightarrow \triangle POT \text{ and } \triangle QOT$   
 $\angle OPT = OQT = 90^\circ$   
 Common hypotenuse OT  
 And  $OP = OQ$  [ OP and OQ are radii]  
 So, R - H - S rule of similarity  
 $\triangle POT \sim \triangle QOT$   
 Hence,  $OP/OQ = PT/QT = OT/OT$   
 $PT/QT = 1$   
 $PT = QT$  [ hence proved]

---

Ans 19). Given,  $4 \tan \theta = 3$

$$\rightarrow \tan \theta = \frac{3}{4} = \frac{P}{B}$$

$\rightarrow$  Now,  $P = 3K$ ,  $B = 4K$ ,

$\rightarrow$  We know,  $\tan \theta = (\text{perpendicular} / \text{base})$

so, after comparing , perpendicular = 3 and base = 4

$$\rightarrow \text{Hypotenuse} = \sqrt{(3^2 + 4^2)} = 5$$

Now,  $\sin \theta = \text{perpendicular} / \text{hypotenuse} = 3/5$

$\cos = \text{base} / \text{hypotenuse} = 4/5$

$\rightarrow$  Now,  $(4\sin - \cos + 1)/(4\sin + \cos - 1)$

$$= (4 \times 3/5 - 4/5 + 1)/(4 \times 3/5 + 4/5 - 1)$$

$$= (12 - 4 + 5)/(12 + 4 - 5)$$

$$= 13/11$$

---

**Ans 20). Given  $\tan 2A = \cot(A-18^\circ)$**

$$\rightarrow \cot(90-2A) = \cot(A-18^\circ)$$

$$[\rightarrow \tan \theta = \cot(90-\theta)]$$

**Comparing angles we get,**

$$90 - 2A = A - 18$$

$$\rightarrow 90 + 18 = A + 2A$$

$$\rightarrow 3A = 108$$

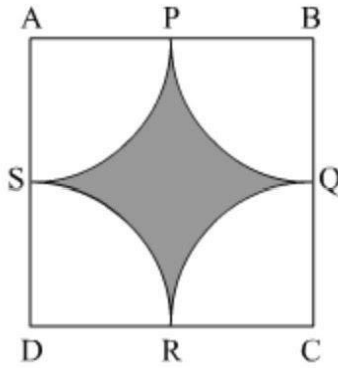
$$\rightarrow A = 108/3$$

$$\rightarrow A = 36^\circ$$

---

**Ans 20).**





→ Given, ABCD is a square of side = 12 cm.

P, Q, R and S are the mid points of sides AB, BC, CD and AD respectively.

→ Area of shaded region = Area of square – 4 × Area of quadrant =

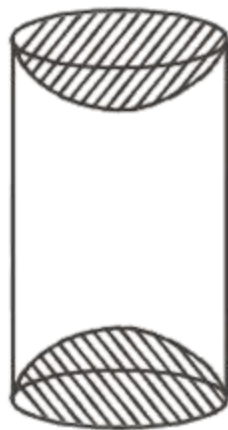
$$\begin{aligned} &\rightarrow a^2 - 4 \times \frac{1}{4} \pi r^2 \\ &= (12)^2 - 3.14 \times (6)^2 \end{aligned}$$

$$= 144 - 3.14 \times 36$$

$$= 144 - 113.04 = 30.96 \text{ cm}^2$$

---

Ans 21).



→ Let the original height of cylinder = 10cm

→ Base radius of cylinder = 3.5cm

so base radius of hemisphere = 3.5cm(same as that of cylinder)

→ The total surface area would be the sum of the curved surface area of cylinder and the surface areas of 2 hemispheres.

surface area of cylinder =  $2\pi rh$

surface area of one hemisphere =  $2\pi r^2$

$$TSA = 2\pi rh + 2(2\pi r^2)$$

$$TSA = 2\pi rh + 4\pi r^2$$

$$TSA = 2\pi r(h + 2r)$$

$$TSA = 2 * \frac{22}{7} * 3.5(10 + 2 * 3.5) = 22 * (10 + 7)$$

$$TSA = \boxed{374 \text{ cm}^2}$$

**OR**

→ Volume of rice = volume of cone =  $\frac{1}{3}\pi r^2 h$

Here , r = 12 [ ∵ diameter = 24 m ] and h = 3.5m

→ Now, volume of rice =  $\frac{1}{3} \times \frac{22}{7} \times (12)^2 \times 3.5 \text{ m}^3$

$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times \frac{7}{2} \text{ m}^3$$

$$= 22 \times 24 = 528 \text{ m}^3$$

Again, area of canvas cloth = curve surface area of cone =  $\pi rl$

→ Here  $l = \sqrt{r^2 + h^2}$

$$= \sqrt{\{12^2 + 3.5^2\}} = \sqrt{\{144 + 12.25\}} = \sqrt{\{156.25\}} = 12.5 \text{ m}$$

$$\therefore \text{Area of canvas cloth} = 22/7 \times 12 \times 12.5 \text{ m}^2$$

$$= 471.43 \text{ m}^2$$

---

$$\frac{N}{2} = \frac{280}{2} = 140$$

→ The cumulative frequency just greater than 140 is 182.

→ Median class is 10 – 15.

→  $l = 10$ ,  $h = 5$ ,  $N = 280$ , c.f. = 49 and  $f = 133$

$$\rightarrow \text{MEDIAN} = l + \frac{\frac{N}{2} - c.f.}{f} \times h$$

$$= 10 + \frac{140 - 49}{133} \times 5$$

$$= 10 + \frac{91 \times 5}{133}$$

$$= 10 + 455/133$$

$$= 13.42$$

**Ans → 13.42**

**Ans 23).**

→ **Given, speed of motor boat in still water = 18 km/hr.**

→ **Let speed of stream = x km/hr**

→ **Speed of boat downstream=(18 + x) km/hr.**

→ **And speed of boat upstream=(18 – x) km/hr.**

→ **Time of the upstream journey =  $\frac{24}{18 - x}$**

→ **Time of the upstream journey =  $\frac{24}{18 + x}$**

**Now, According to the question,**

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1$$

$$\frac{24(18 + x) - 24(18 - x)}{(18 - x)(18 + x)} = 1$$

$$\frac{24 * 18 + 24x - 24 * 18 - 24x}{324 - x^2} = 1$$

$$\rightarrow 48x / 324 - x^2 = 1$$

$$48x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

$$\rightarrow x^2 + 54x - 6x - 324 = 0$$

$$x(x + 54) - 6(x + 54) = 0$$

$$(x + 54)(x - 6) = 0$$

$$\rightarrow \text{Either } x + 54 = 0, x = -54$$

Now, as the speed cannot be negative,  $x - 6 = 0, x = 6$

Ans  $\rightarrow$  So, the speed of the stream is 6 km/hr

OR

$\rightarrow$  Let original speed of the train be  $x$  km/h.

Then, time taken to travel 63 km =  $63/x$  hours

$\rightarrow$  New speed =  $(x + 6)$  km/hr

Time taken to travel 72 km =  $72/(x + 6)$  hours

$\rightarrow$  According to the question,

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

$\rightarrow$  On solving,  $135x + 378 = 3x^2 + 18x$

$$x^2 - 39x - 126 = 0$$

$$(x - 42)(x + 3) = 0$$

$$x = -3 \text{ or } x = 42$$

As the speed cannot be negative,  $x = 42$

→ Thus, the average speed of the train is 42 km/hr.

Ans → The average speed of the train is 42 km/hr.

---

Ans 24).

Let the four consecutive numbers in AP be  $(a-3d), (a-d), (a+d)$  and  $(a+3d)$

So, according to the question.

$$\rightarrow a-3d+a-d+a+d+a+3d=32$$

$$4a=32$$

$$a=32/4$$

$$\rightarrow a=8 \dots (1)$$

$$\rightarrow \text{Now, } (a-3d)(a+3d)/(a-d)(a+d)=7/15$$

$$15(a^2-9d^2)=7(a^2-d^2)$$

$$15a^2-135d^2=7a^2-7d^2$$

$$15a^2-7a^2=135d^2-7d^2$$

$$8a^2=128d^2$$

→ Putting the value of  $a=8$  in above we get.

$$8(8)^2=128d^2$$

$$128d^2=512$$

$$d^2=512/128$$

$$d^2=4$$

$$d=2$$

→ So, the four consecutive numbers are

$$8-(3 \times 2)$$

$$8-6=2$$

$$8-2=6$$

$$8+2=10$$

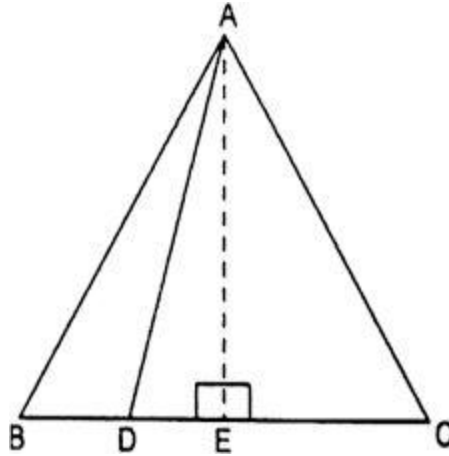
$$8+(3 \times 2)$$

$$8+6=14$$

Four consecutive numbers are 2, 6, 10 and 14.

---

**Ans 25).**



→  $ABC$  is an equilateral triangle , where  $D$  point on side  $BC$  in such a way that  $BD = BC/3$  . Let  $E$  is the point on side  $BC$  in such a way that  $AE \perp BC$  .

→ Now,  $\triangle ABE$  and  $\triangle AEC$

$$\angle AEB = \angle ACE = 90^\circ$$

$AE$  is common side of both triangles ,

$AB = AC$  [ all sides of equilateral triangle are equal ]

→ From R - H - S congruence rule ,

$$\triangle ABE \cong \triangle ACE$$

$$\therefore BE = EC = BC/2$$

Now, from Pythagoras theorem ,

$$\triangle ADE \text{ is right angle triangle } \therefore AD^2 = AE^2 + DE^2 \text{ -----(1)}$$

$$\triangle ABE \text{ is also a right angle triangle } \therefore AB^2 = BE^2 + AE^2 \text{ -----(2)}$$

From equation (1) and (2)

$$AB^2 - AD^2 = BE^2 - DE^2$$

$$= (BC/2)^2 - (BE - BD)^2$$

$$= BC^2/4 - \{(BC/2) - (BC/3)\}^2$$

$$= BC^2/4 - (BC/6)^2$$

$$= BC^2/4 - BC^2/36 = 8BC^2/36 = 2BC^2/9$$

$$\rightarrow AB = BC = CA$$



**So,  $AB^2 = AD^2 + 2AB^2/9$**

**→  $9AB^2 - 2AB^2 = 9AD^2$**

**→ Hence,  $9AD^2 = 7AB^2$**

**OR**

**→ Given: A right angled  $\triangle ABC$ , right angled at B**

**To Prove-  $AC^2 = AB^2 + BC^2$**

**Construction: draw perpendicular BD onto the side AC .**

**Proof:**

**We know that if a perpendicular is drawn from the vertex of a right angle of a right angled triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.**

**We have,**

**$\triangle ADB \sim \triangle ABC$ . (by AA similarity)**

Therefore,  $AD/AB = AB/AC$

(In similar Triangles corresponding sides are proportional)

$$AB^2 = AD \times AC \dots\dots (1)$$

Also,  $\triangle BDC \sim \triangle ABC$

Therefore,  $CD/BC = BC/AC$

(in similar Triangles corresponding sides are proportional)

$$\text{Or, } BC^2 = CD \times AC \dots\dots (2)$$

Adding the equations (1) and (2) we get,

$$AB^2 + BC^2 = AD \times AC + CD \times AC$$

$$AB^2 + BC^2 = AC(AD + CD)$$

( From the figure  $AD + CD = AC$ )

$$AB^2 + BC^2 = AC \cdot AC$$

Therefore,  $AC^2 = AB^2 + BC^2$

---

Ans 26).

Steps of construction:-

- (i) Draw a line segment BC with measurement of 6 cm.
- (ii) Now construct angle  $60^\circ$  from point B and draw  $AB = 5$  cm.
- (iii) Join the point C with point A. Thus  $\triangle ABC$  is the required triangle.
- (iv) Draw a line BX which makes an acute angle with BC and is opposite of vertex A.
- (v) Cut four equal parts of line BX namely BB<sub>1</sub>, BB<sub>2</sub>, BB<sub>3</sub>, BB<sub>4</sub>.
- (vi) Now join B<sub>4</sub> to C. Draw a line B<sub>3</sub>C' parallel to B<sub>4</sub>C.
- (vii) And then draw a line B'C' parallel to BC.

Hence  $\triangle AB'C'$  is the required triangle.



**Ans 28). Given:**

→ Diameter of upper end of bucket = 30 cm

→ Radius of the upper end of the frustum of cone,  $r_1 = 15$  cm

→ Diameter of lower end of bucket = 10 cm

→ Radius of the lower end of the frustum of cone,  $r_2 = 5$  cm

→ Height of the frustum of Cone,  $h = 24$  cm

→ Slant height of bucket,  $L = \sqrt{(h^2 + (r_1 - r_2)^2)}$

$$L = \sqrt{24^2 + (15 - 5)^2}$$

$$L = \sqrt{576 + 10^2}$$

$$L = 26 \text{ cm}$$

→ Area of metal sheet = Curved Surface Area of bucket + area of lower end

$$= \pi(r_1 + r_2)L + \pi r_2^2$$

$$= 3.14(15 + 5) \times 26 + \pi(5)^2$$

$$= 3.14 \times 20 \times 26 + 25 \times 3.14$$

$$= 1711.3 \text{ cm}^2$$

Hence, the Area of metal sheet used to make the bucket is  $1711.3 \text{ cm}^2$ .

ii). We should avoid bucket made by ordinary plastic because it is non-biodegradable. It is harmful for our environment.

Ans 29).

→ Height of light house = 100 m

The angles of depression of 2 ships are 30 and 45 degree.

→ To Find: If one ship is exactly behind the other on the same side of the light house, then find the distance between the 2 ships

Solution :

Refer the attached figure

In  $\triangle ABC$

AB = Height of tower = Perpendicular = 100 m

BC = Base

$\angle ACB = 45^\circ$

We will use trigonometric ratio to find the length of base :

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{100}{BC}$$

$$BC = 100$$

In  $\triangle ABD$

AB = Height of tower = Perpendicular = 100 m

BD = Base

$$\angle ADB = 30^\circ$$

We will use trigonometric ratio to find the length of base :

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{BD}$$

$$BD = \frac{100}{\frac{1}{\sqrt{3}}}$$

$$BD = 100\sqrt{3}$$

Use  $\sqrt{3} = 1.732$

$$\rightarrow BD = 100 * 1.732$$

$$\rightarrow BD = 173.2 \text{ m}$$

Now we are required to find the distance between the 2 ships i.e.  $CD = BD - BC$ .

So,  $CD = BD - BC$ .

$$CD = 173.2 \text{ m} - 100 \text{ m}$$

$$CD = 73.2 \text{ m}$$

Thus the distance between the 2 ships is 73.2 m.

Ans 30).

Class Interval	Mid Value $x_i$	Frequency $f_i$	$f_i x_i$
11 - 13	12	3	36
13 - 15	14	6	84
15 - 17	16	9	144
17 - 19	18	13	234
19 - 21	20	$f$	$20f$
21 - 23	22	5	110
23 - 25	24	4	6

$$\sum f = 40 + f \quad \sum fx = 704 + 20f$$

Given : Mean = 18

We know, Mean ( $\bar{x}$ ) =  $\frac{\sum fx}{\sum f}$

$$\Rightarrow 18 = \frac{704 + 20f}{40 + f}$$

$$\Rightarrow 18(40 + f) = 704 + 20f$$

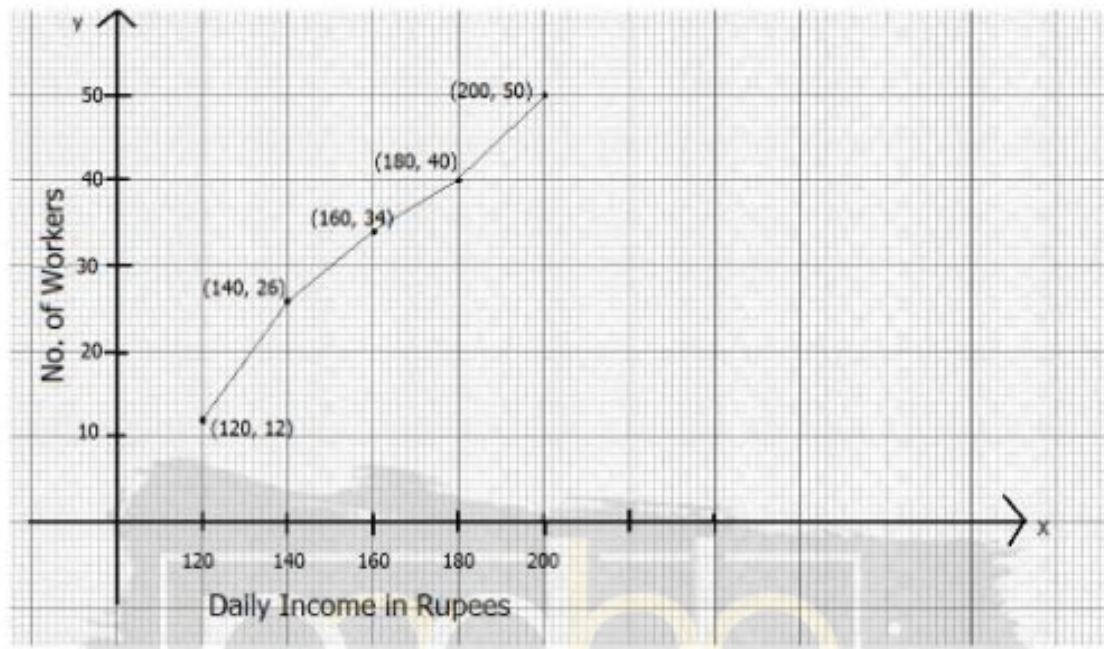
$$\Rightarrow 720 - 704 = 20f - 18f$$

$$\Rightarrow 2f = 16$$

$$\Rightarrow f = 8$$

**OR**





# **CBSE Mathematics 2017**

General Instructions :

- (i) All questions are compulsory.
- (ii) This question paper consists of 30 questions divided into four sections—A, B, C and D.
- (iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of a calculator is not permitted.

# Questions

## Section-A (1 Mark Each)

1. What is the common difference of an A.P. in which  $a_{21} - a_7 = 84$  ?
2. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is  $60^\circ$ , then find the length of OP
3. If a tower 30 m high, casts a shadow  $10\sqrt{3}$  m long on the ground, then what is the angle of elevation of the sun ?
4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap ?

## Section- B (2 Marks Each)

5. Find the value of p, for which one root of the quadratic equation  $px^2 - 14x + 8 = 0$  is 6 times the other.
6. Which term of the progression  $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots$  is the first negative term ?
7. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord

8. A circle touches all the four sides of a quadrilateral ABCD. Prove that  $AB + CD = BC + DA$

9. A line intersects the y-axis and x-axis at the points P and Q respectively. If  $(2, -5)$  is the midpoint of PQ

10. If the distances of  $P(x, y)$ , from  $A(5, 1)$  and  $B(-1, 5)$  are equal, then prove that  $3x = 2y$ .

### Section- C (3 Marks Each)

11. If  $ad \neq bc$ , then prove that the equation  $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$  has no real roots.

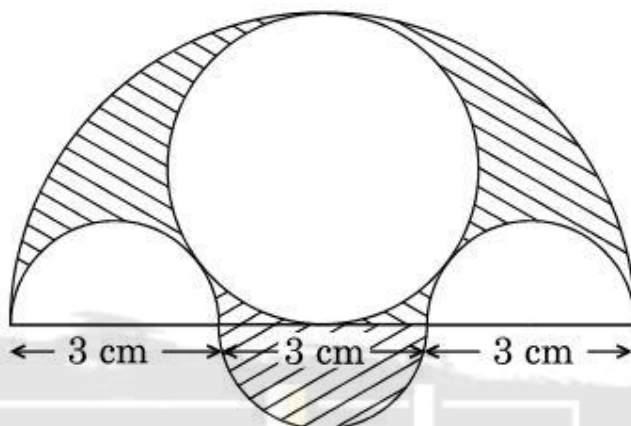
12. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P

13. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

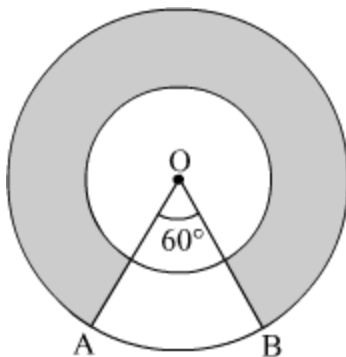
14. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

15. In what ratio does the point  $(\frac{24}{11}, y)$  divide the line segment joining the points  $P(2, -2)$  and  $Q(3, 7)$  ? Also find the value of y.

16. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



17. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If  $\angle AOB = 60^\circ$ , find the area of the shaded region. [Use  $\pi = 22/7$ ]



18. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?

19. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

20. The dimensions of a solid iron cuboid are 4.4 m × 2.6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

### Section- D (4 Marks Each)

21. Solve for x :

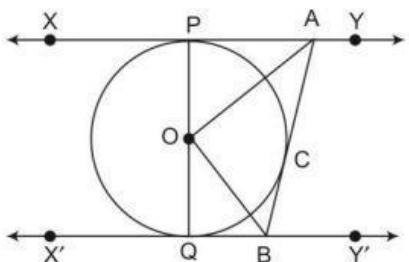
$$\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$$

22. Two taps running together can fill a tank in  $3\frac{1}{13}$  hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ?

23. If the ratio of the sum of the first n terms of two A.P.s is  $(7n + 1) : (4n + 27)$ , then find the ratio of their 9th terms

24. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

25. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, intersecting XY at X and X'Y' at Y. Prove that  $\angle AOB = 90^\circ$ .



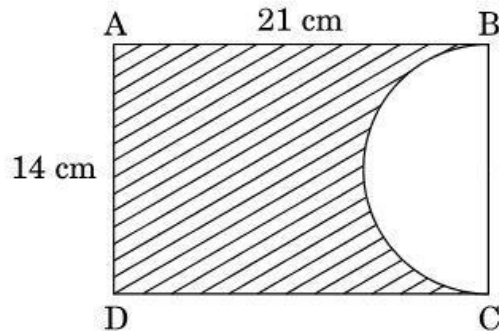
26. Construct a triangle ABC with side  $BC = 7$  cm,  $\angle B = 45^\circ$ ,  $\angle A = 105^\circ$ . Then construct another triangle whose sides are  $\frac{3}{4}$  times the corresponding sides of the  $\triangle ABC$

27. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are  $45^\circ$  and  $60^\circ$  respectively. Find the width of the river. [Use  $\sqrt{3} = 1.732$ ]

28. If the points  $A(k + 1, 2k)$ ,  $B(3k, 2k + 3)$  and  $C(5k - 1, 5k)$  are collinear, then find the value of  $k$ .

29. Two different dice are thrown together. Find the probability that the numbers obtained have (i) even sum, and (ii) even product

30. In the given figure, ABCD is a rectangle of dimensions 21 cm  $\times$  14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



31. In a rain-water harvesting system, the rain-water from a roof of  $22 \text{ m} \times 20 \text{ m}$  drains into a cylindrical tank having a diameter of base  $2 \text{ m}$  and height  $3.5 \text{ m}$ . If the tank is full, find the rainfall in  $\text{cm}$ . Write your views on water conservation.

## Answers

Ans 1). Given,  $a_{21} - a_7 = 84$

$$\rightarrow (a + 20d) - (a + 6d) = 84$$

$$\rightarrow a + 20d - a - 6d = 84$$

$$\rightarrow 20d - 6d = 84$$

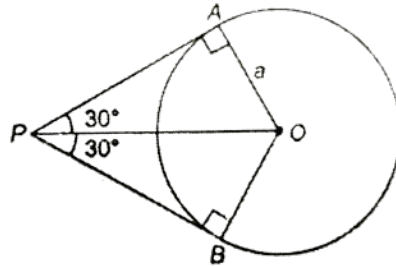
$$\rightarrow 14d = 84$$

$$\rightarrow d = 6$$

So, the common difference is 6



**Ans 2).**



**Given,  $\rightarrow \angle APB = 60^\circ$  and  $\angle APO = 30^\circ$**

**$\rightarrow$  In right angle  $\triangle OAP$ ,  $\frac{OP}{OA} = \operatorname{cosec} 30^\circ$**

$$\frac{OP}{a} = 2$$

**$\rightarrow OP = 2a$**

**Hence, the length of OP is 2a.**

---

**Ans 3). Given : Height of the tower - 30m**

**Length of shadow on the ground -  $10\sqrt{3}$**

**Solution:**

**$\rightarrow$  In  $\triangle ABC$ ,  $\tan \theta = \frac{AB}{BC}$**

$$\tan \theta = \frac{30}{10\sqrt{3}}$$

**$\tan \theta = \tan 60^\circ$**

$$\theta = 60^\circ$$

Hence, the angle of elevation is  $60^\circ$

Ans 4).

Given: Probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18

$$\rightarrow \text{Total apples} = 900$$

$$\rightarrow P(E) = 0.18$$

$$\rightarrow \frac{\text{No. of rotten apples}}{\text{Total no. of apples}} = 0.18$$

$$\rightarrow \frac{\text{No. of rotten apples}}{900} = 0.18$$

$$\text{No. of rotten apples} = 900 \times 0.18 = 162$$

Ans: Hence, the no. of rotten apples is 162.

---

Ans 5). Given: Quadratic equation -  $px^2 - 14x + 8 = 0$

$$\rightarrow \text{So, let one root} = \beta$$

$$\rightarrow \text{other root will be} = 6\beta$$

$$\rightarrow \text{Sum of roots} = \frac{-b}{a}$$

$$\beta + 6\beta = \frac{-(-14)}{p}$$

$$7\beta = \frac{14}{p}$$

$$\beta = \frac{2}{p}$$

→ We know, Product of roots =  $\frac{c}{a}$

$$\beta * 6\beta = \frac{8}{p}$$

$$6\beta^2 = \frac{8}{p}$$

→ On Putting value of  $\beta$  from eq. (i)

$$6 \left( \frac{2}{p} \right)^2 = \frac{8}{p}$$

$$6 * 4/p^2 = \frac{8}{p}$$

$$\rightarrow 24p = 8p^2$$

$$8p^2 - 24p = 0$$

$$\rightarrow 8p(p - 3) = 0$$

$$\rightarrow \text{Either } 8p = 0 \rightarrow p = 0$$

Or

$$p - 3 = 0 \rightarrow p = 3$$

→ For  $p = 0$ , the given condition will not be satisfied

→ SO, answer will be  $p = 3$

**Ans 6).**

**Given, A.P. is  $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots$**

**So, this is equal to  $\rightarrow 20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, \dots$**

**Here,  $a = 20, d = \frac{77}{4} - 20 = \frac{77-80}{4} = \frac{-3}{4}$**

**$\rightarrow$  Let  $a_n$  is first negative term**

$$\rightarrow a_n + (n-1)d < 0$$

$$\rightarrow 20 + (n-1)\left(\frac{-3}{4}\right) < 0$$

$$\rightarrow 20 - \left(\frac{3}{4}\right)n + \frac{3}{4} < 0$$

$$\rightarrow 20 + \frac{3}{4} < \frac{3}{4}n$$

$$\rightarrow \frac{83}{4} < \frac{3}{4}n$$

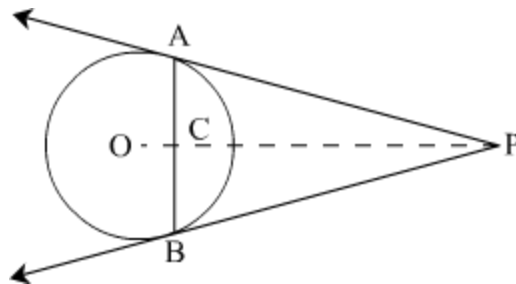
$$\rightarrow n > \frac{83}{4} \times \frac{4}{3}$$

$$\rightarrow n > \frac{83}{3} \text{ or } 27.66$$

**( Since term cannot be in fraction, round it off)**

**$\rightarrow$  Ans: 28<sup>th</sup> term will be the first negative term of the given A.P.**

Ans 7). Given, a circle of radius OA and centred at O with chord AB and tangents PQ & RS are drawn from point A and B respectively.



→ Let AB be a chord of a circle with center O and let AP and BP be the tangents at A and B.

→ Let the tangent meet at P. Join OP. Suppose OP meets AB at C.

→ To prove :  $\angle PAC = \angle PBC$

→ Proof : In  $\triangle PAC$  and  $\triangle PBC$

$PA = PB$  [Tangents from an external point to a circle are equal]

$\angle APC = \angle BPC$  [PA and PB are equally inclined to OP]

→  $PC = PC$  [Common]

$\triangle PAC \cong \triangle PBC$  [SAS Congruence]

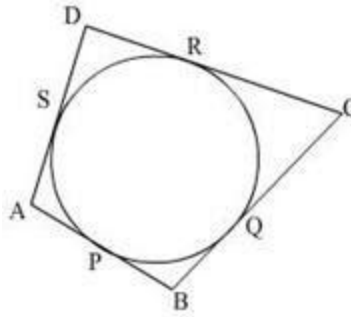
$\angle PAC = \angle PBC$  [C.P.C.T]

Hence Proved.

---

Ans 8). Given: A circle touches all the four sides of a quadrilateral ABCD

To Prove:  $AB + CD = BC + DA$



As it's known that the length of tangents drawn from an external point to the circle are equal, which means,  $AP=AS$

$$BP=BQ$$

$$CR=CQ$$

$$DR=DS$$

$$\rightarrow (AP+PB)+(CR+RD)=(AS+SD)+(BQ+QC)$$

$$\rightarrow AB+BC=AD+BC$$

**Ans 9).** Let the point

$P(x,0)$  and  $Q(0,y)$  (because on  $y$ -axis,  $y=0$  and on  $x$ -axis,  $x=0$ )

and midpoint  $(2,-5)$

Now, to find out midpoint

$$\rightarrow \frac{0+x}{2} = 2$$

$$\rightarrow \frac{0+y}{2} = -5$$

**So,  $x=4, y=-10$**

**→ Ans - The coordinates of P: (4,0)**

**Q: (4,0) and (0,-10)**

---

**Ans 10).**

**→ Given: Distances of P(x, y), from A(5, 1) and B(- 1, 5) are equal**

**→ To prove:  $3x = 2y$**

**As it is given,  $PA=PB$**

$$\sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$$

**→ On squaring both the sides:**

$$(x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$$

$$\rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$$

$$- 10x - 2y = 2x - 10y$$

$$\rightarrow - 10x - 2x = - 10y + 2y$$

$$\rightarrow 12x = 8y$$

$$\rightarrow 3x = 2y$$

**Hence, Proved!**

---

**Ans 11) Given,  $ad \neq bc$**

$$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$$

$$D = b^2 - 4ac$$

$$\begin{aligned}
&= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) \\
&= 4[a^2c^2 + b^2d^2 + 2abcd] - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) \\
&= 4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2] \\
&= 4[-a^2d^2 - b^2c^2 + 2abcd] \\
&= -4[a^2d^2 + b^2c^2 - 2abcd] \\
&= -4[ad - bc]^2
\end{aligned}$$

→ Hence, D is negative!

**Ans 12). Given:  $a = 5$ ,  $a_n = 45$ ,  $S_n = 400$**

**So, as we know,  $S_n = n/2 [a + l]$  → where  $l$  is the last term**

$$\rightarrow 400 = n/2 [5 + 45]$$

$$\rightarrow 400 = n/2 [50]$$

$$\rightarrow 25n = 400$$

$$\rightarrow n = 400/25 = 16$$

**Now, as we know,  $a_n = a + (n - 1) d$**

$$\rightarrow 45 = 5 + (16 - 1) d$$

$$\rightarrow 45 - 5 = 15d$$

$$\rightarrow 15d = 40$$

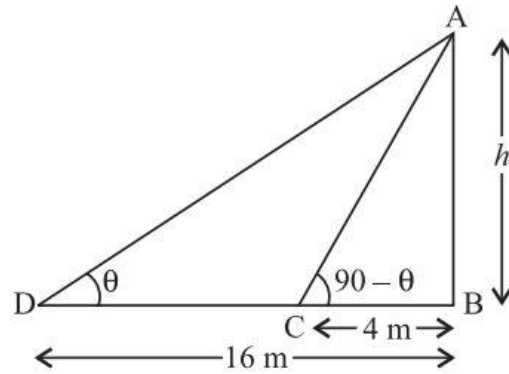
$$\rightarrow d = 8/3$$

**Hence,  $n = 16$  and  $d = 8/3$**

**Ans 13). Given: Distances of 4 m and 16 m**

**Let height AB of tower =  $h$**





→ Now, as we can see, in  $\triangle ABC$

$$\frac{AB}{AC} = \tan(90 - \theta)$$

$$\frac{h}{4} = \cot \theta$$

→ In  $\triangle ABD$ ,

$$\frac{AB}{AD} = \tan \theta$$

$$\frac{h}{16} = \tan \theta$$

→ Multiply eq. (i) and (ii)

$$\frac{h}{4} \times \frac{h}{16} = \cot \theta \times \tan \theta$$

$$h^2/64 = 1$$

$$[\rightarrow \cot \theta \times \tan \theta = 1/\tan \theta \times \tan \theta = 1]$$

$$\rightarrow h^2/64 \text{ so } h = 8\text{m}$$

→ Height of tower = 8 m

---

**Ans 14). Given, no. of white balls = 15**

**Let no. of black balls = x**

**Total balls = (15 + x)**

**According to the question, P (Black ball) = 3 × P (White ball)**

$$\frac{x}{(15 + x)} = 3 \times \frac{15}{(15 + x)}$$

**x = 45**

**→ Hence, no of black balls in the bag = 45**

---

**Ans 15). Let point R divides PQ in the ratio k : 1**

**→ We know that,  $R = \left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$**

$$(24/11, y) = \left( \frac{k(3) + 1(2)}{k + 1}, \frac{k(7) + 1(-2)}{k + 1} \right)$$

$$= \frac{3k + 2}{k + 1}, \frac{7k + -2}{k + 1}$$

$$= 3k + 2 = 24$$

$$11 (3k + 2) = 24 (k + 1)$$

$$\rightarrow 33k + 22 = 24k + 24$$

$$\rightarrow 33k - 24k = 24 - 22$$

$$\rightarrow 9k = 2$$

$$\rightarrow k = 2/9$$

**So,  $k:1 = 2:9$**

$$y = \frac{7k-2}{k+1} = \frac{7\left(\frac{2}{9}\right)-2}{\frac{2}{9}+1}$$

→ **On solving and putting the value of k**

$$y = \frac{-4}{11}$$

→ **Ans: Line PQ divides in the ratio 2 : 9 and value of  $y = \frac{-4}{11}$**

---

**Ans 16). Given: Radius of large semi-circle = 4.5 cm**

→ **Area of large semi-circle =  $\frac{1}{2} \pi R^2 = \frac{1}{2} \times \frac{22}{7} \times 4.5 \times 4.5$**

→ **Diameter of inner circle = 4.5 cm ,  $r = 4.5/2$  cm**

→ **Area of inner circle =  $\pi r^2 = \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2}$**

→ **Diameter of small semi-circle = 3 cm,  $r = \frac{3}{2}$**

→ **Area of small semi-circle =  $\frac{1}{2} \pi r^2$**

→  **$\frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$**

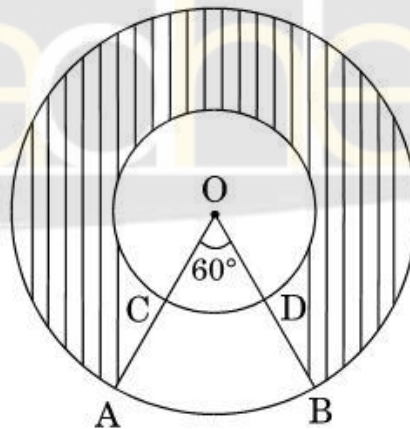
→ **Area of shaded region = Area of large semi circle + Area of 1 small semicircle – Area of inner circle – Area of 2 small semi circle**

$$\rightarrow \left( \frac{1}{2} \times \frac{22}{7} \times 4.5 \times 4.5 \right) + \left( \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} - \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \right) - \left( 2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \right)$$

$$\rightarrow \text{On solving,} = \frac{990 - 643.5}{28} = \frac{346.5}{28}$$

→ Ans - 12.37 cm<sup>2</sup> approximately

Ans 17).



→ Solution : Angle for shaded region =  $360^\circ - 60^\circ = 300^\circ$

Area of shaded region

$$= \frac{\pi\theta}{360} (R^2 - r^2)$$

$$= \frac{22}{7} \times \frac{300}{360} [42^2 - 21^2]$$

$$= \frac{22}{7} \times \frac{5}{6} \times 63 \times 21$$

Hence, the area of the shaded region =  $3465 \text{ cm}^2$

Ans 18).

Width of canal =  $5.4 \text{ m}$

Depth of canal =  $1.8 \text{ m}$

In  $60 \text{ min.}$ ,  $25 \text{ km}$  of water flows through it.

Which means. in  $40 \text{ mins}$ , there will be

$$\rightarrow \frac{25}{60} \times 40 = \frac{50}{3} \text{ km, will be available for irrigation.}$$

→ Hence, the volume of water in the canal = volume of land irrigated.

→ Volume of canal =  $l \times b \times h$

$$= 5.4 \times 1.8 \times \frac{50000}{3} \text{ m}^3$$

→ Area of land irrigated if  $10 \text{ cm}$  standing water is required

$$= 5.4 \times 1.8 \times \frac{50000}{3} \times \frac{1}{0.1} \text{ m}^3$$

$$= 16,20,000 \text{ m}^2$$

---

Ans 19). Solution :

→ Slant height of frustum ( $l$ ) =  $4 \text{ cm}$

→ Perimeter of upper top = 18 cm

$$\rightarrow 2\pi R = 18 \text{ cm} ; R = 9/\pi \text{ cm}$$

→ Perimeter of lower bottom = 6 cm

$$\rightarrow 2\pi r = 6 ; r = 3/\pi \text{ cm}$$

→ Curved S.A. of frustum =  $\pi l [R + r]$

$$= \pi \times 4 \times \left[ \frac{9+3}{\pi} \right]$$

$$= \pi \times 4 \times \frac{12}{\pi} = 48 \text{ cm}^2$$

Hence, the curved surface area of the frustum is 48 cm<sup>2</sup>

---

Ans 20). Given:

→ Inner radius of pipe 'r' = 30 cm

→ Thickness of pipe = 5 cm

→ Outer radius = 30 + 5 ; R = 35 cm

Now, according to the question, a solid iron cuboid is melted and recast into a hollow cylindrical pipe.

So, Vol. of hollow pipe = Vol. of cuboid

$$\rightarrow \pi h(R^2 - r^2) = l \times b \times h$$

$$\rightarrow \frac{22}{7} \times h[35^2 - 30^2] = 4.4 \times 2.6 \times 1 \times 100 \times 100 \times 100$$

$$\rightarrow \frac{22}{7} \times h \times 65 \times 5 = 44 \times 26 \times 1 \times 100 \times 100$$

→ On solving h,

$$h = \frac{44 \times 26 \times 100 \times 100 \times 7}{22 \times 65 \times 5} = 11200 \text{ cm} / 112\text{m}$$

---

Ans 21).

$$\rightarrow \text{Given: } \frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}$$

$$\frac{1}{x+1} - \frac{5}{x+4} = -\frac{3}{5x+1}$$

$$\frac{(x+4) - 5(x+1)}{(x+1)(x+4)} = -\frac{3}{5x+1}$$

$$\frac{(x+4-5x-5)}{(x^2+5x+4)} = -\frac{3}{5x+1}$$

$$\frac{(-4x-1)}{(x^2+5x+4)} = -\frac{3}{5x+1}$$

$$(4x+1)(5x+1) = 3(x^2+5x+4)$$

$$\rightarrow 20x^2 + 4x + 5x + 1 = 3x^2 + 15x + 12$$

$$\rightarrow 17x^2 - 6x - 11 = 0$$

$$\rightarrow 17x^2 - 17x + 11x - 11 = 0$$

$$\rightarrow 17x(x-1) + 11(x-1) = 0$$

$$\rightarrow (x-1)(17x+11) = 0$$

**Ans**  $\rightarrow$  Either  $x = 1$  or  $x = -11/17$

---

**Ans 22).** Let tank fill by one tap =  $x$  hrs

Let tank fill by other tap =  $(x+3)$  hrs

$\rightarrow$  As given, together they fill by  $\frac{40}{13}$  hrs

$$\rightarrow \text{Which means } \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$$

$$\frac{x+3+x}{(x)(x+3)} = \frac{13}{40}$$

$$(2x+3) / (x^2+3x) = \frac{13}{40}$$

$$13x^2 + 39x = 80x + 120$$

$$\rightarrow 13x^2 - 41x - 120 = 0$$

$$\rightarrow 13x^2 - 65x + 24x - 120 = 0$$

$$\rightarrow 13x(x - 5) + 24(x - 5) = 0$$

$$\rightarrow (x - 5)(13x + 24) = 0$$

Either  $x - 5 = 0$  or  $13x + 24 = 0$

$$x = 5, x = -24/13$$

$\rightarrow x$  cannot be  $-24/13$  since we are talking about time.

$\rightarrow$  So,  $x = 5$

$\rightarrow$  One tap fill the tank in 5 hrs so other tap fills the tank in  $5 + 3 = 8$  hrs

Ans 23).

We know, Ratio of sum of first  $n$  terms of two A.P.s are:

$$\rightarrow \frac{\frac{n}{2} [2a + (n-1)d]}{\frac{n}{2} [2A + (n-1)D]} = \frac{7n + 1}{4n + 27}$$

$\rightarrow$  Now, put  $n = 17$

$$\frac{[2a + 16d]}{[2A + (16)D]} = \frac{120}{95} = 24/19$$

On simplifying,

$$\rightarrow \frac{a + 8d}{A + 8D} = 24/19$$

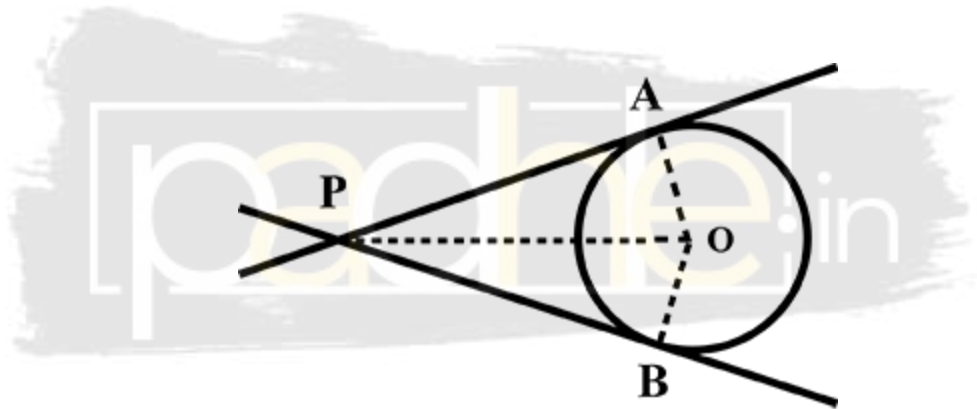


**Ans → Ratio of 9th terms of two A.P.s is 24 : 19**

---

**Ans 24).**

→ **Statement:** The tangents drawn from an external point to a circle are equal.



→ **To prove :**  $PA = PB$

→ **Const<sup>n</sup> :** Join radius OA and OB also join O to P.

**Proof :→** In  $\triangle OAP$  and  $\triangle OBP$   $OA = OB$  (Radii)

→  $\angle A = \angle B$  (Each  $90^\circ$ )

→  $OP = OP$  (Common)

→  $\triangle AOP \cong \triangle BOP$  (RHS cong.)

→  $PA = PB$  (cpct)

**Ans → Hence, the statement is proved.**

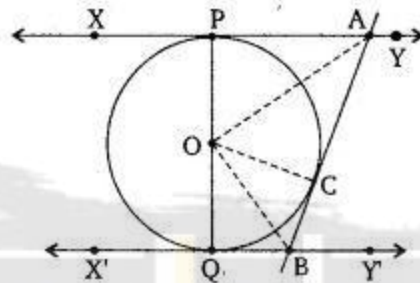
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**Ans 25).**

→ **Given:**

XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and X'Y' at B

→ Let us join point O to C



In  $\triangle OPA$  and  $\triangle OCA$

→  $OP = OC$  (Radii of the same circle)

→  $AP = AC$  (Tangent from point A)

→  $AO = AO$  (Common side)

→  $\triangle OPA \cong \triangle OCA$  (SSS congruence criterion)

So, →  $\angle POA = \angle COA$ .....(1)

Similarly,

→  $\angle QOB \cong \angle OCB$

→  $\angle QOB = \angle COB$ .....(2)

Since, POQ is the diameter of the circle, it is a straight line.

Therefore, →  $\angle POA + \angle COA + \angle COB + \angle QOB = 180$

◦

So, from equation (1) and equation (2)

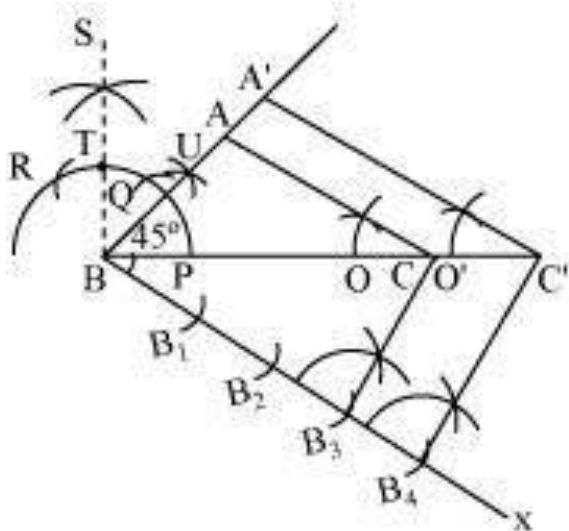
$$\rightarrow 2\angle COA + 2\angle COB = 180^\circ$$

$$\rightarrow \angle COA + \angle COB = 90^\circ$$

$$\rightarrow \angle AOB = 90^\circ$$

Hence, Proved.

Ans 26).  $\rightarrow$  Given:  $BC = 7$  cm,  $\angle B = 45^\circ$ ,  $\angle A = 105^\circ$ .



$$\angle A + \angle B + \angle C = 180^\circ$$

[Angle sum property]

$$105^\circ + 45^\circ + \angle C = 180^\circ$$

$$150^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 150^\circ$$

$$\angle C = 30^\circ$$

### Steps of Construction:

→ Draw a line segment  $BC = 7$  cm.

→ Make an angle of  $45^\circ$  at point B and  $30^\circ$  at point C, which intersect each other at point A. Thus, ABC is a given Triangle.

→ Now from B draw a ray BY by making an acute  $\angle CBY$  with base BC on the side opposite to the vertex A.

→ Now, locate 4 points  $B_1, B_2, B_3, B_4$  on BY such that  $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$ .

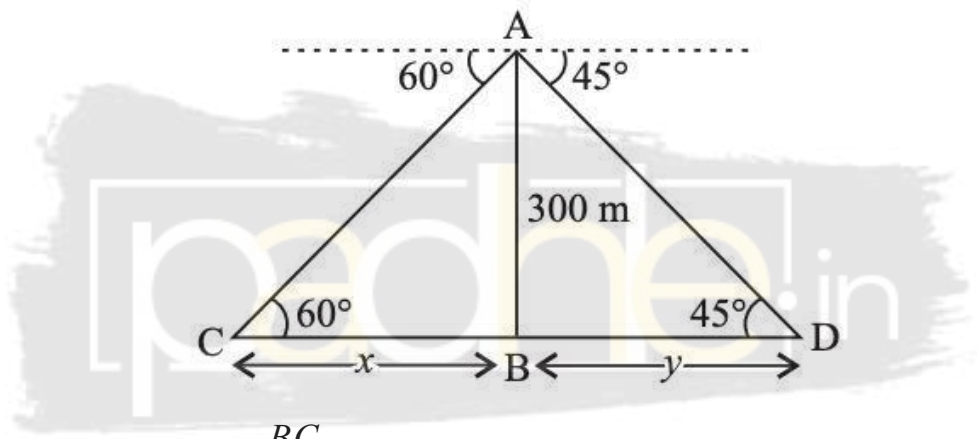
→ Join  $B_3C$ , and draw a line through  $B_4M \parallel B_3C$  intersecting the extended line BC at M.

→ From M draw  $OM \parallel AC$  intersecting the extended line BA at O.

Ans → Thus,  $\triangle OBM$  is the required triangle whose sides are  $\left(\frac{3}{4}\right)$  times of the corresponding sides of  $\triangle ABC$ .

**Ans 27).**

**Let aeroplane is at A, 300 m high from a river. C and D are opposite banks of river.**



**So, In right  $\triangle ABC$ ,  $\frac{BC}{AB} = \cot 60^\circ$**

$$\rightarrow \frac{x}{300} = 1/\sqrt{3}$$

$$\rightarrow x = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 100\sqrt{3} \text{ m}$$

$$= 100 \times 1.732 = 173.2 \text{ m}$$

→ In right  $\triangle ABD$ ,

$$\frac{BD}{AB} = \cot 45^\circ$$

$$\rightarrow \frac{y}{300} = 1$$

$$\rightarrow y = 300$$

$$\rightarrow \text{Width of river} = x + y = 173.2 + 300 = 473.2 \text{ m}$$

**Ans** → Width of river is 473.2 m.

---

**Ans 28).** → Given:  $A(k + 1, 2k)$ ,  $B(3k, 2k + 3)$  and  $C(5k - 1, 5k)$  are collinear

→ To find: the value of  $k$ .

if  $A(x_1, y_1)$ ,  $B(x_2, y_2)$ ,  $C(x_3, y_3)$  are three collinear points then

$$\rightarrow \text{Area of triangle } ABC = 0$$

$$\rightarrow \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

→ Given  $A(k+1, 2k)$ ,  $B(3k, 2k+3)$ ,  $C(5k-1, 5k)$ , therefore

$$\rightarrow \frac{1}{2} (k+1)[2k+3-5k] + 3k[5k-2k] + (5k-1)[2k-(2k+3)] = 0$$

$$\rightarrow (k+1)[3-3k] + 3k \cdot 3k + (5k-1)(-3) = 0$$

$$\rightarrow 3k - 3k^2 + 3 - 3k + 9k^2 - 15k + 3 = 0$$

$$\rightarrow 6k^2 - 15k + 6 = 0$$

Divide each term with 2

$$\rightarrow 2k^2 - 5k + 2 = 0$$

$$\rightarrow 2k^2 - 4k - k + 2 = 0$$

$$\rightarrow 2k(k-2) - (k-2) = 0$$

$$\rightarrow (k-2)(2k-1) = 0$$

$$\rightarrow k-2 = 0 \text{ or } 2k-1 = 0$$

$$\rightarrow k=2 \text{ or } k = \frac{1}{2}$$

**Ans 29). When two different dice are thrown together**

$$\rightarrow \text{Total outcomes} = 6 \times 6 = 36$$

(i) For even sum—Favourable outcomes are (1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)

$$\rightarrow \text{No. of favourable outcomes} = 18$$

$$\rightarrow P(\text{even sum}) = \frac{\text{favourable outcomes}}{\text{Total outcomes}} = \frac{18}{36} = \frac{1}{2}$$

(ii) For even product—Favourable outcomes are (1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

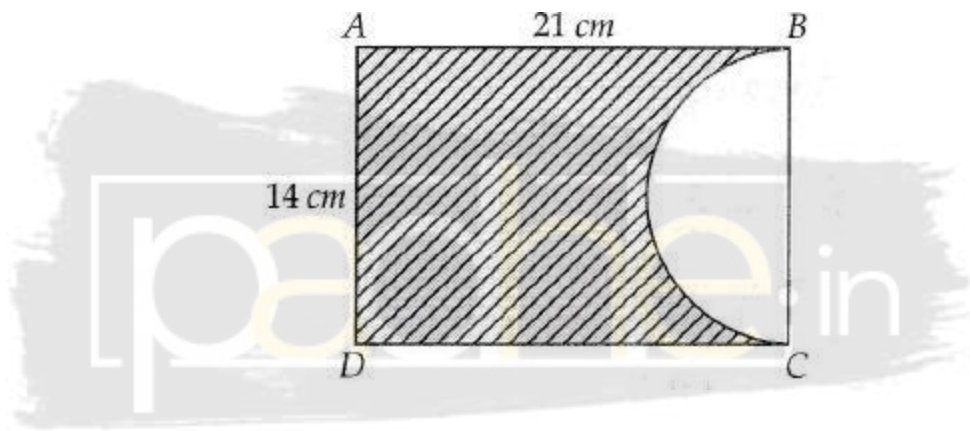
$$\rightarrow \text{No. of favourable outcomes} = 27$$

$$\rightarrow P(\text{even sum}) = \frac{\text{favourable outcomes}}{\text{Total outcomes}} = \frac{27}{36} = \frac{3}{4}$$


---

**Ans 30).**

**Area of shaded region = Area of rectangle - Area of semi-circle**



$$= (21 \times 14) - \frac{1}{2} \times \pi \times 7 \times 7$$

$$= 294 - 77$$

$$= 217 \text{ cm}^2$$

$$\text{Perimeter of shaded region} = 21 + 14 + 21 + \pi \times 7$$

$$= 56 + 22 = 78 \text{ cm}$$

**Ans  $\rightarrow$  Area of shaded region = 78 cm**

**Ans 31). Given: Base  $\rightarrow$  2 m and Height  $\rightarrow$  3.5 m.**



→ Rectangular roof :  $l=22\text{m}, b=20\text{m}$

→ Cylindrical tank :  $r=1\text{m}, h=3.5\text{m}$

→ The volume of tank =  $\pi r^2 h$

$$= \frac{22}{7} \times 1 \times 1 \times 3.5$$

$$= 11\text{m}^3$$

→ Area of roof =  $l \times b = 22 \times 20 = 440\text{m}^2$

→ Rainfall =  $\frac{\text{Volume of tank}}{\text{Area of the roof}}$

→ Rainfall =  $11\text{m}^3/440\text{m}^2$

$$= 0.025\text{m or } 25 \text{ cm}$$

# CBSE Mathematics 2016

General Instructions :

(i) All questions are compulsory.

(ii) This question paper consists of 30 questions divided into four sections—A, B, C and D.

(iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.

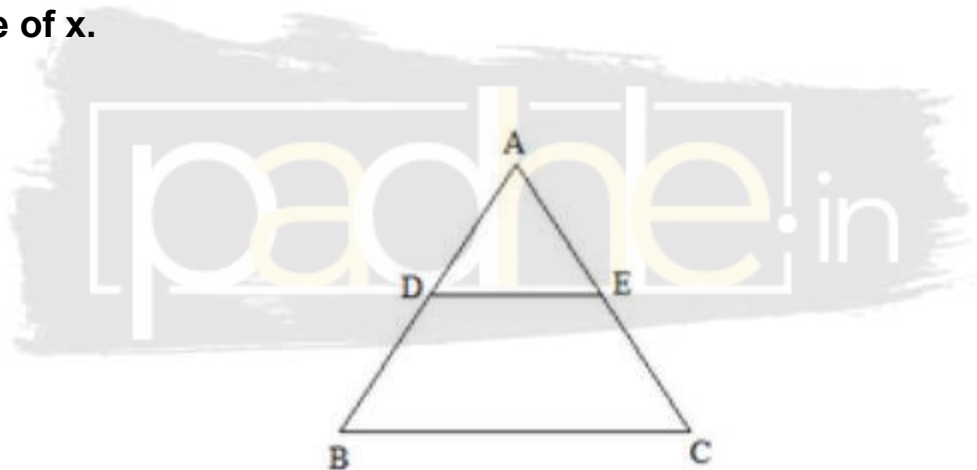
(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted.

# Questions

## Section-A (1 Mark Each)

1. In  $\triangle ABC$ , D and E are points AC and BC respectively such that  $DE \parallel AB$ . If  $AD = 2x$ ,  $BE = 2x - 1$ ,  $CD = x + 1$  and  $CE = x - 1$ , then find the value of x.



2. In  $\triangle ABC$ , A, B and C are interior angles of  $\triangle ABC$ , then prove that :

$$\frac{\sin(B+C)}{2} = \frac{\cos A}{2}$$

3. If  $x = 3 \sin q$  and  $y = 4 \cos q$ , find the value of  $\sqrt{16x^2 + 9y^2}$
4. If empirical relationship between mean, median and mode is expressed as  $\text{mean} = k(3 \text{ median} - \text{mode})$ , then find the value of k.

## Section-B (2 Marks Each)

5. Express 23150 as product of its prime factors. Is it unique ?
6. State whether the real number 52.0521 is rational or not. If it is rational express it in the form  $p/q$ , where  $p, q$  are co-prime, integers and  $q \neq 0$ . What can you say about prime factorisation of  $q$  ?
7. Given the linear equation  $x - 2y - 6 = 0$ , write another linear equation in these two variables, such that the geometrical representation of the pair so formed is : (i) coincident lines (ii) intersection lines
8. In an isosceles  $\triangle ABC$  right angled at B, prove that  $AC^2 = 2AB^2$ .
9. Prove the following identity :

$$\left[ \frac{1 - \tan A}{1 - \cot A} \right]^2 = \tan^2 A \quad \angle A \text{ is acute}$$

10. Given below is a cumulative frequency distribution table. Corresponding to it, make an ordinary frequency distribution table

x	cf
More than or equal to 0	45
More than or equal to 10	38
More than or equal to 20	29
More than or equal to 30	17
More than or equal to 40	11
More than or equal to 50	6

## Section-C (3 Marks Each)

11. Find LCM and HCF of 3930 and 1800 by prime factorisation method.

12. Using division algorithm, find the quotient and remainder on dividing  $f(x)$  by  $g(x)$  where  $f(x) = 6x^3 + 13x^2 + x - 2$  and  $g(x) = 2x + 1$ .

13. If three zeroes of a polynomial  $x^4 - x^3 - 3x^2 + 3x$  are 0,  $\sqrt{3}$  and  $-\sqrt{3}$ , then find the fourth zero.

14. Solve the following pair of equations by reducing them to a pair of linear equations :

$$\frac{1}{x} - \frac{4}{y} = 2$$

$$\frac{1}{x} + \frac{3}{y} = 9$$

15.  $\triangle ABC$  is a right angled triangle in which  $\angle B = 90^\circ$ . D and E are any point on AB and BC respectively. Prove that  $AE^2 + CD^2 = AC^2 + DE^2$ .

16. In the given figure, RQ and TP are perpendicular to PQ, also  $TS \perp PR$  prove that  $ST.RQ = PS.PQ$ .

17. If  $\sec A = \frac{2}{\sqrt{3}}$ , find the value

**Solution :** Given,  $n = \frac{\cos B}{\sin A}; m = \frac{\cos B}{\cos A}$

So,  $n^2 = \frac{\cos^2 B}{\sin^2 A}; m^2 = \frac{\cos^2 B}{\cos^2 A}$

L.H.S.  $= (m^2 + n^2) \cos^2 A = \left( \frac{\cos^2 B}{\cos^2 A} + \frac{\cos^2 B}{\sin^2 A} \right) \cos^2 A$

$= \frac{(\sin^2 A \cos^2 B + \cos^2 A \cos^2 B)}{\cos^2 A \sin^2 A} \times \cos^2 A$

of:

$\frac{\tan A}{\cos A} + \frac{1 + \sin A}{\tan A}$

18. Prove that :  $\sec^2 q - \cot^2(90^\circ - q) = \cos^2(90^\circ - q) + \cos^2 q$ .

19. For the month of February, a class teacher of Class IX has the following absentee record for 45 students. Find the mean number of days, a student was absent.

Number of days of absent	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24
Number of students	18	3	6	2	0	1

20. Find the missing frequency (x) of the following distribution, if mode is 34.5 :

C.I.	Frequency
------	-----------

0 – 10	4
10 – 20	$8 = f_0$
$l = 20 - 30$	$10 = f_1$
30 – 40	$x = f_2$
40 – 50	8

## Section-C (4 Marks Each)

21. Prove that  $\sqrt{5}$  is an irrational number. Hence show that  $3 + 2\sqrt{5}$  is also an irrational number.

22. Obtain all other zeroes of the polynomial  $x^4 + 6x^3 + x^2 - 24x - 20$ , if two of its zeroes are  $+2$  and  $-5$ .

23. Draw graph of following pair of linear equations :  $y = 2(x - 1)$  and  $4x + y = 4$ . Also write the coordinate of the points where these lines meet the x-axis and y-axis.

24. A boat goes 30 km upstream and 44 km downstream in 10 hours. The same boat goes 40 km upstream and 55 km downstream in 13 hours. On this information some student guessed the speed of the boat in still water as 8.5 km/h and speed of the stream as 3.8 km/h. Do you agree with their guess ? Explain what do we learn from the incident ?

25. In an equilateral  $\triangle ABC$ , E is any point on BC such that  $BE = \frac{1}{4}BC$ . Prove that  $16AE^2 = 13AB^2$ .

26. In the figure, if  $\angle ABD = \angle XYD = \angle CDB = 90^\circ$ .  $AB = a$ ,  $XY = c$  and  $CD = b$ , then prove that  $c(a + b) = ab$ .

27. In the  $\triangle ABC$  (see figure),  $\angle A = \text{right angle}$ ,  $AB = \sqrt{x}$  and  $BC = \sqrt{x+5}$  Evaluate  $\sin C \cdot \cos C \cdot \tan C + \cos^2 C \cdot \sin A$

28. If  $\frac{\cos B}{\sin A} = n$  and  $\frac{\cos B}{\cos A} = m$ , then show that  $(m^2 + n^2) \cos^2 A = n^2$ .

29. Prove that :  $\frac{\sec A - 1}{\sec A + 1} = \left( \frac{\sin A}{1 + \cos A} \right)^2$

30. Following table shows marks (out of 100) of students in a class test :

Marks	No. of students
More than or equal to 0	80
More than or equal to 10	77
More than or equal to 20	72
More than or equal to 30	65
More than or equal to 40	55
More than or equal to 50	43
More than or equal to 60	28
More than or equal to 70	16
More than or equal to 80	10
More than or equal to 90	8



More than or equal to 100	0
---------------------------	---

Draw a 'more than type' ogive. From the curve, find the median. Also, check the value of the median by actual calculation.



**Ans 1):**

**Given:**

1.  $DE \parallel BC$
2.  $AD = x$
3.  $DB = x - 2$
4.  $AE = x + 2$  and  $EC = x - 1$ .

**To find: Value of x**

→ In  $\triangle ABC$ , we have

$DE \parallel BC$  so, by Thale's theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

→  $AD \times EC = AE \times DB$

$$x(x-1) = (x-2)(x+2)$$

$$x^2 - x = x^2 - 4$$

$$x = 4$$

**Answer:** The value of x is 4.

---

**Ans 2):**

**Given:** A,B and C are interior angles of ABC

→ Now,  $\angle A + \angle B + \angle C = 180$  for interior angles of triangle ABC

So,  $\angle B + \angle C = 180 - \angle A$ .

→ Now, Multiply both sides by  $\frac{1}{2}$

$$\frac{1}{2} (\angle B + \angle C) = \frac{1}{2} (180 - \angle A) = (90 - A/2) \text{ [opened the bracket]}$$

→ Now, take sine of  $\frac{(\angle B + \angle C)}{2}$

$$\frac{\sin (\angle B + \angle C)}{2} = \frac{\sin (90 - A)}{2} \text{ [ Since, } \frac{(\angle B + \angle C)}{2} = \frac{(90 - A)}{2} \text{ ]}$$

→ Now, we know  $\sin (90 - \theta) = \cos \theta$

**So,**

$$\frac{\sin (90 - A)}{2} = \frac{\cos A}{2}$$

**Hence, Proved.**

---

**Ans 3)**

**Given:**

$$x = 3\sin a$$

$$y = 4\cos a$$

**To find,**  $\sqrt{16(x^2) + 9(y)^2}$

**Now, substitute '3sin a' at the place of x and '4cosa' at the place of y**

$$\sqrt{(16(3\sin a)^2) + 9(4\cos a)^2}$$

$$= \sqrt{(16)(9)\sin^2 a + (9)(16)\cos^2 a}$$

$$= \sqrt{(16)(9)(\sin^2 a + \cos^2 a)}$$

**Now, we know that  $\sin^2 x + \cos^2 x = 1$**

**On applying this in the solution**

$$= \sqrt{(16)(9)}$$

$$= 4 \times 3$$

$$= 12 \quad (\text{since } \sin^2 a + \cos^2 a \text{ becomes } 1)$$

**Ans 4).**

**Given: mean =  $k(3 \text{ median} - \text{mode})$**

**We know, the Empirical formula is**

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

**So,**

$$\rightarrow 2 \text{ Mean} = 3 \text{ Median} - \text{Mode}$$

$$\rightarrow \text{Mean} = \frac{3 \text{ Median} - \text{Mode}}{2}$$

**We are given, mean =  $k(3 \text{ median} - \text{mode})$**

**So, on substituting the value**

$$k(3 \text{ median} - \text{mode}) = \frac{3 \text{ Median} - \text{Mode}}{2}$$

$$\rightarrow k = \frac{1}{2}$$

**$\rightarrow$  Hence, the value of  $k$  is  $\frac{1}{2}$**

---

**Ans 5).**

**Fundamental theorem of arithmetic states that the expansion of prime factorization of any number is unique.**

**So, the expansion of prime factorization of 23150:**

$$23150 = 2 \times 5 \times 5 \times 463$$

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**Ands 6). Yes, 52.0521 is rational as it can be expressed in the p/q form**

$$52.0521 = \frac{520521}{10000}$$

---

**Ans 7).**

→ **The lines**

**$a_1x + b_1y + c_1$  and  $a_2x + b_2y + c_2 = 0$  are coincident if**

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

**So, another line can be  $2x-4y-12=0$**

→ **Condition for lines to be intersecting :**

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

**So, another line can be  $x-4y-6=0$**

---

**Ans 8).**

→ **Given: In triangle ABC angle B= 90°**

**To Prove:  $AC^2 = 2AB^2$**

→ **We know, By Pythagoras theorem**

$$AB^2 + BC^2 = AC^2$$

**And, by isosceles triangle property,**

$$AB = BC$$

$$\text{So, } AB^2 + AB^2 = AC^2 \text{ ( Since, } AB = BC \text{)}$$

$$\text{Therefore, } 2AB^2 = AC^2$$

$$\text{Hence, proved } 2AB^2 = AC^2$$

$$\text{Ans 9). To prove, } \left[ \frac{1 - \tan A}{1 - \cot A} \right]^2 = \tan^2 A$$

$$\left[ \frac{1 - \tan A}{1 - \cot A} \right]^2 \text{ (We know, } \cot A = 1/\tan A \text{)}$$

$$\text{So, } \left[ \frac{1 - \tan A}{1 - 1/\tan A} \right]^2$$

$$\left[ \frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}} \right]^2$$

$$\left[ \frac{\tan A (1 - \tan A)}{\tan A - 1} \right]^2$$

→ From denominator, take (-) out

$$\left[ \frac{\tan A (1 - \tan A)}{-(1 - \tan A)} \right]^2$$

→ Reduce to the simplest form

$$[-\tan A]^2$$

$$= \tan^2 A$$

Hence, Proved!

---

Ans 10). We know to find out the ordinary frequency (fi)

$$45-38=7$$

$$38-29=9$$

$$29-17=12$$

$$17-11=6$$

$$11-6=5$$

$$6=6$$

### TABLE

x	cf	fi
More than or equal to 0	45	7
More than or equal to 10	38	9
More than or equal to 20	29	12
More than or equal to 30	17	6
More than or equal to 40	11	5
More than or equal to	6	6

50		
----	--	--

Ans 11).  $HCF(3930, 1800) = 30$

$LCM(3930, 1800) = 235800$

Step-by-step explanation:

We have ,

2 | 3930

\_\_\_\_\_

3 | 1965

\_\_\_\_\_

5 | 655

\_\_\_\_\_

\* 131

2 | 1800

\_\_\_\_\_

2 | 900

\_\_\_\_\_

2 | 450

\_\_\_\_\_

5 | 225

\_\_\_\_\_

5 | 45



---

$$3 \mid 9$$

---

3

Now,

$$3930 = 2 \times 3 \times 5 \times 131$$

and

$$1800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

$$= 2^3 \times 3^2 \times 5^2$$

$$\text{HCF}(3930, 1800) = 2 \times 3 \times 5 = 30$$

Product of the smallest power of each common prime factors of the numbers

$$\text{LCM}(3930, 1800) = 2^3 \times 3^2 \times 5^2 \times 131$$

$$= 235800$$

---

$$\text{Ans 12). Dividend} = 6x^3 + 13x^2 + x - 2$$

$$\text{Divisor} = 2x + 1$$

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

$$6x^3 + 13x^2 + x - 2 = (2x + 1 \times 3x^2) + (10x^2 + x - 2)$$

$$6x^3 + 13x^2 + x - 2 = (2x + 1 \times 3x^2 + 5x) + (-4x - 2)$$

$$6x^3 + 13x^2 + x - 2 = (2x + 1 \times 3x^2 + 5x - 2) + 0$$

Hence, the remainder comes out to be 0 and quotient is  $3x^2 + 5x - 2$

**Ans 13). Given:**

**Polynomial:**  $x^4 - x^3 - 3x^2 + 3x$

**Roots:** 0,  $\sqrt{3}$  and  $-\sqrt{3}$

**To find: 4th root**

→ If 0,  $\sqrt{3}$  and  $-\sqrt{3}$  are roots of the polynomial,  $x$ ,  $(x-\sqrt{3})$  and  $(x+\sqrt{3})$  would be the factors.

→ Let  $a$  is the fourth root of the equation, which means,  $(x-a)$  would be the factor

So,

$$x(x-\sqrt{3})(x+\sqrt{3})(x-a) = x^4 - x^3 - 3x^2 + 3x$$

$$x(x^2-3)(x-a) = x^4 - x^3 - 3x^2 + 3x \quad [\rightarrow (a+b)(a-b) = a^2-b^2]$$

$$x^4 - ax^3 - 3x^2 + 3ax = x^4 - x^3 - 3x^2 + 3x$$

→ On comparing on both the sides, we get  $a=1$

Hence, the fourth root is  $a = 1$ .

---

**Ans 14).**

---

**Ans 15).** As  $\triangle ABE$  is right angled triangle, then,

$$\rightarrow (AE)^2 = (AB)^2 + (BE)^2 \text{ -----} \rightarrow \text{Eq}^n (1)$$

Similarly,  $\triangle DBC$  is right angled triangle, so,

$$\rightarrow (CD)^2 = (BD)^2 + (BC)^2 \text{ -----} \rightarrow \text{Eq}^n (2)$$

On adding equation (1) and (2),

$$\rightarrow (AE)^2 + (CD)^2 = (AB)^2 + (BE)^2 + (BD)^2 + (BC)^2$$

$$\rightarrow (AE)^2 + (CD)^2 = (AB^2 + BC^2) + (BE^2 + BD^2) \text{ -----} \rightarrow \text{Eq}^n (3)$$

$\rightarrow$  In  $\triangle ABC$ ,

$$AC^2 = AB^2 + BC^2$$

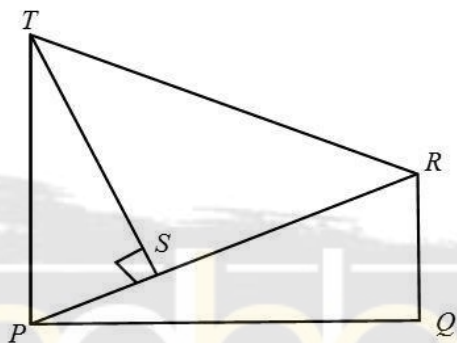
$\rightarrow$  In  $\triangle DBE$

$$DE^2 = BE^2 + BD^2 \text{ -----} \rightarrow \text{Eq}^n (4)$$

On putting the values of Eq<sup>n</sup> (4) in Eq<sup>n</sup> (3)

$$(AE)^2 + (CD)^2 = (AC^2) + (DE^2) \text{ ----- Hence Proved}$$

Ans 16).



**Given:** RQ and TP are perpendicular to PQ, also  $TS \perp PR$

**To Prove:**  $ST.RQ = PS.PQ$ .

**Sol<sup>n</sup>:** By angle sum property

$$\rightarrow \angle R + \angle P + \angle Q = 180^\circ$$

$$\rightarrow \angle 1 + \angle 2 + \angle 3 = 180^\circ$$

**Given that,**  $\angle 3 = 90^\circ$ . So,

$$\rightarrow \angle 1 + \angle 2 + 90^\circ = 180^\circ$$

$$\rightarrow \angle 1 + \angle 2 = 90^\circ \text{ .....(eq 1)}$$

TS is a perpendicular drawn on line PR such that  $\angle TSP = \angle 5 = 90^\circ$

Also, TP is perpendicular on line PQ such that  $\angle TPQ = 90^\circ$

$$\rightarrow \angle 4 + \angle 2 = 90^\circ \text{ .....(eq 2)}$$

On comparing (eq 1) and (eq 2) we get,

$$\rightarrow \angle 1 + \angle 2 = \angle 4 + \angle 2$$

$$\rightarrow \angle 1 = \angle 4$$

In  $\triangle RQP$  and  $\triangle TSP$

$$\rightarrow \angle 3 = \angle 5 \text{ (each } 90^\circ)$$

$$\rightarrow \angle 1 = \angle 4 \text{ (from above)}$$

By AA

$$\triangle RQP \sim \triangle TSP$$

Therefore,

$$\rightarrow ST/PQ = PS/RQ$$

Cross -multiply them

$$\rightarrow ST.RQ = PS.PQ$$

Hence, proved

Ans 17).

Given:  $\sec A = \frac{2}{\sqrt{3}}$

To find:  $\frac{\tan A}{\cos A} + \frac{1 + \sin A}{\tan A}$

$$\rightarrow \text{By trigonometry identity : } \sec A = \frac{2}{\sqrt{3}} = \frac{H}{B}$$

$$\rightarrow H = 2 \text{ and } B = \sqrt{3}$$

$$\rightarrow \text{We know, } H^2 = P^2 + B^2$$

$$H^2 = P^2 + B^2$$

$$2^2 = P^2 + (\sqrt{3})^2$$

On solving,  $P=1$

We already have,  $H=2$ ,  $B= \sqrt{3}$

→ We also know,  $\sin A = \frac{P}{H}$

→ So,  $\sin A = \frac{1}{2}$

→  $\cos A = \frac{B}{H}$

→  $\cos A = \frac{\sqrt{3}}{2}$

→  $\tan A = \frac{P}{B}$

→  $\tan A = \frac{1}{\sqrt{3}}$

→ On substituting these values in  $\frac{\tan A}{\cos A} + \frac{1 + \sin A}{\tan A}$

→ We get,  $\frac{4 + 9\sqrt{3}}{6}$

→ So,  $\frac{\tan A}{\cos A} + \frac{1 + \sin A}{\tan A} = \frac{4 + 9\sqrt{3}}{6}$

**[In exam, write down the solution also]**

---

Ans 18). To prove:  $\sec^2 q - \cot^2(90^\circ - q) = \cos^2(90^\circ - q) + \cos^2 q$ .

$$\rightarrow \text{LHS} = \sec^2 q - \cot^2(90^\circ - q)$$

$$= \sec^2 q - \tan^2 q$$

$$= 1/\cos^2 q - \sin^2 q/\cos^2 q$$

$$= (1 - \sin^2 q)/\cos^2 q$$

$$= \cos^2 q/\cos^2 q = 1$$

$$\rightarrow \text{RHS} = \cos^2(90^\circ - q) + \sin^2 q$$

$$\sin^2 q + \cos^2 q = 1$$

**LHS = RHS, Hence Proved**

---

**Ans 19).**

---

**Ans 20).**

**Given: Mode ---> 34.5 , l = 20**

**To find: x**

**So, according to the formula of mode**

$$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right)h$$

$$34.5 = 20 + \left( \frac{10 - 8}{20 - 8 - x} \right)10$$

$$34.5 = 20 + \left( \frac{2}{12 - x} \right)10$$

$$34.5 - 20 = \left( \frac{2}{12 - x} \right)10$$

$$14.5 = \left( \frac{20}{12 - x} \right)$$

$$14.5 (12 - x) = 20$$

$$\frac{20}{14.5} = 12 - x$$

$$\rightarrow \text{On simplifying} \rightarrow \frac{40}{29} = 12 - x$$

$$x = 12 - \frac{40}{29}$$

$$x = \frac{348 - 40}{29}$$

$$x = \frac{308}{29}$$

$$x = 10.62$$



---

**Ans 21).**

→  $\sqrt{5}$  has no common factor

**Let's assume that  $\sqrt{5}$  is rational and  $p/q$  has no common factor**

$$\sqrt{5} = p/q$$

→ **Squaring on both sides**

$$(\sqrt{5})^2 = (p/q)^2$$

$$5 = p^2/q^2$$

$$5q^2 = p^2$$

**so 5 is a factor of  $p^2$**

**also 5 is a factor of  $p$**

$$\text{let } p = 5m$$

$$5q^2 = (5m)^2$$

$$5q^2 = 25m^2$$

$$q^2 = 5m^2$$

**So 5 is a factor of  $q^2$**

**also 5 is a factor of  $q$**

**Hence we said that  $\sqrt{5}$  is rational but according to our contradiction  $\sqrt{5}$  is irrational.**

$$3 + \sqrt{5} = k$$

$$\sqrt{5} = k - 3$$

→ **LHS is irrational since  $\sqrt{5}$  is irrational as proved**

**Hence, RHS is irrational too.**

---

**Ans 22). → Polynomial  $p(x) = x^4 + 6x^3 + x^2 - 24x - 20$**

**→ The given two zeroes are: 2 and (-5)**

**→ According to Factor Theorem**

**$(x-2)$  and  $(x+5)$  are factors of  $p(x)$**

**→ Also,**

$$(x-2)(x+5) = x^2 + 5x - 2x - 10$$

$$x^2 + 3x - 10$$

**$x^2 + 3x - 10$  is also a factor of  $p(x)$**

**→ Now,**

**Dividing  $p(x) = x^4 + 6x^3 + x^2 - 24x - 20$  by  $x^2 + 3x - 10$**

**We get  $Q(x) = x^2 + 3x + 2$**

$$r(x) = 0$$

$$p(x) = (x^2 + 3x - 10)(x^2 + 3x + 2)$$

$$= (x-2)(x+5)(x^2 + x + 2x + 2)$$

$$= (x-2)(x+5)(x+1)(x+2)$$

**So, the zeroes of  $p(x)$  are:**

$$x - 2 = 0, x = 2$$

$$x + 5 = 0, x = -5$$

$$x + 1 = 0, x = -1$$

$$x + 2 = 0, x = -2$$

---

**Ans 23). \*GRAPH\***



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**Ans 24).**

Let the speed of boat in still water =  $x$  km/hr and The speed of stream =  $y$  km/hr

Speed of boat at downstream

$$\rightarrow (x+y) \text{ km/hr}$$

Speed of boat at upstream

$$\rightarrow (x-y) \text{ km/hr}$$

$$\rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$$

→ Time taken to cover 30 km upstream =  $\frac{30}{x-y}$

→ Time taken to cover 44 km downstream =  $\frac{44}{x+y}$

→ So, according to the question:

$$\frac{30}{x-y} + \frac{44}{x+y} = 10$$

→ Time taken to cover 40 km upstream =  $\frac{40}{x-y}$

→ Time taken to cover 55 km downstream =  $\frac{55}{x+y}$

→ So, according to the question:

$$\frac{40}{x-y} + \frac{55}{x+y} = 13$$

→ Let  $\frac{1}{x-y} = u$  and  $\frac{1}{x+y} = v$

⇒  $30u + 44v = 10$  -----> eq<sup>n</sup>1

⇒  $40u + 55v = 13$  -----> eq<sup>n</sup>2

⇒  $(150u + 220v = 50) - (160u + 220v = 52)$  [ Multiplied by 5 and subtracted both the eq<sup>n</sup>]

So,  $u = \frac{1}{5}$  and  $v = \frac{1}{4}$

Now, as we know  $\frac{1}{x-y} = u$  and  $\frac{1}{x+y} = v$

So,  $\frac{1}{x-y} = u = \frac{1}{5}$ ,  $x-y = 5$

And,  $\frac{1}{x+y} = v$ ,  $x+y=11$

On subtracting these 2 equations  $\rightarrow$

$x= 8$  and  $y = 3$

Hence, the speed of the boat in still water= $8\text{km/hr}$

The speed of stream= $3\text{km/hr}$

(Show the solutions also in exam)

---

Ans 25).

Join A to midpoint of BC at D.

So,  $ED = BE = \left(\frac{1}{4}\right)BC \rightarrow \text{eq}^n1$

In triangle AED,  $AE^2 = AD^2 + ED^2 \rightarrow \text{eq}^n2$

In triangle ABD,  $AD^2 = AB^2 - BD^2 \rightarrow \text{eq}^n3$

Putting value of  $AD^2$  from (3) into (2),

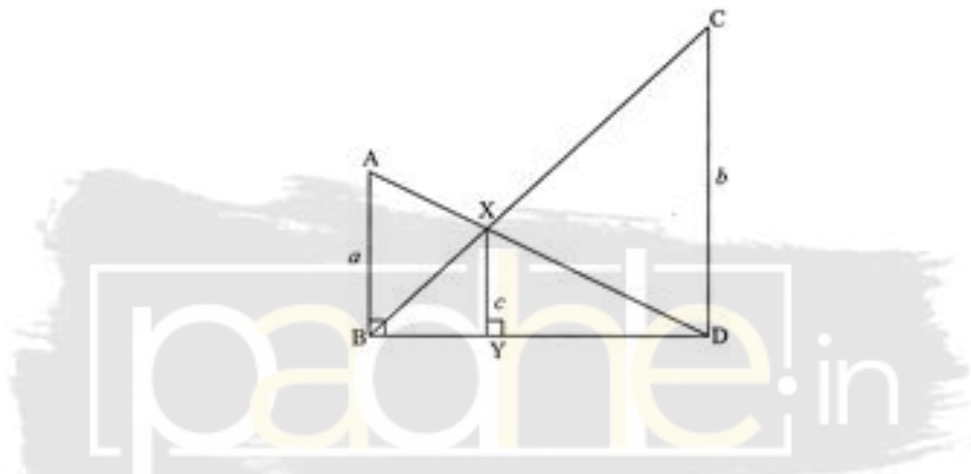
$$AE^2 = AB^2 - BD^2 + ED^2 = AB^2 - \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{4}\right)^2$$

as  $BD = (1/2)BC$  and  $ED = (1/4)BC$  from (1).....,

So we get .... $16AE^2 = 13AB^2$

---

Ans 26).



Given:  $\angle ABD = \angle XYD = \angle CDB = 90^\circ$ .  $AB = a$ ,  $XY = c$  and  $CD = b$ ,

As  $\angle ABD = \angle XYD = \angle CDB = 90^\circ$

$\Rightarrow AB \parallel XY \parallel CD$

In  $\triangle ABD$

As  $AB \parallel XY$

So,  $\frac{DY}{BD} = \frac{c}{a}$

$\Rightarrow DY = \frac{c}{a} (BD)$  .-----> eq<sup>n</sup>(1)

In  $\triangle BDC$

As  $CD \parallel XY$

So,

$$\frac{BY}{BD} = \frac{c}{b}$$

$$BY = \frac{c}{b} (BD) \dots\dots(2)$$

Adding (1) and (2), we get

$$DY + BY = \frac{c}{a} (BD) + \frac{c}{b} (BD)$$

$$BD = c \left( \frac{1}{a} + \frac{1}{b} \right) BD$$

$$\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{1}{c} = \frac{a+b}{ab}$$

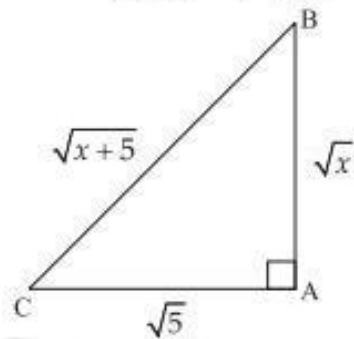
$$c(a+b) = ab$$

Hence proved.

Ans 27). In ABC, using the pythagoras theorem,

$$(\sqrt{x+5})^2 = (\sqrt{x})^2 + AC^2$$

$$x + 5 = x + AC^2$$



$$5 = AC^2$$

or

$$AC = \sqrt{5}$$

$$\sin C = \frac{\sqrt{x}}{\sqrt{x+5}}; \cos C = \frac{\sqrt{5}}{\sqrt{x+5}};$$

$$\tan C = \frac{\sqrt{x}}{\sqrt{5}}$$

and

$$\sin A = \sin 90^\circ$$

$$= 1$$

Then,  $\sin C \cos C \tan C + \cos^2 C \sin A$

$$= \frac{\sqrt{x}}{\sqrt{x+5}} \frac{\sqrt{5}}{\sqrt{x+5}} \frac{\sqrt{x}}{\sqrt{5}} + \left( \frac{\sqrt{5}}{\sqrt{x+5}} \right)^2 \cdot 1$$

$$= \frac{x}{x+5} + \frac{5}{x+5}$$

$$= \frac{x+5}{x+5}$$

**Ans = 1**



---

**Ans 28).**

**Solution :** Given,  $n = \frac{\cos B}{\sin A}; m = \frac{\cos B}{\cos A}$

So,  $n^2 = \frac{\cos^2 B}{\sin^2 A}; m^2 = \frac{\cos^2 B}{\cos^2 A}$

$$\text{L.H.S.} = (m^2 + n^2) \cos^2 A = \left( \frac{\cos^2 B}{\cos^2 A} + \frac{\cos^2 B}{\sin^2 A} \right) \cos^2 A$$

$$= \frac{(\sin^2 A \cos^2 B + \cos^2 A \cos^2 B)}{\cos^2 A \sin^2 A} \times \cos^2 A$$

$$\cos^2 B (\sin^2 A + \cos^2 A) / \sin^2 A$$

$$\cos^2 B / \sin^2 A = n^2 = \text{RHS}$$

---

**Ans 29).  $\sec A - 1 \mid \sec A + 1$**

**Multiplying and dividing by  $\sec A - 1$**

$$\Rightarrow (\sec A - 1)^2 / (\sec^2 A - 1)$$

$$\Rightarrow 1 \times (\sec^2 A + 1 - 2\sec A) \rightarrow [\sec^2 A - 1 = \tan^2 A]$$

Divide the equation by  $\tan^2 A$

$$\Rightarrow \cot^2 A (\tan^2 A + 2 - 2\sec A) \rightarrow \left[ \frac{1}{\tan A} = \cot A \right]$$

$$\Rightarrow 1 + 2\cot^2 A - 2 \times \frac{1}{\cos A} \times \frac{\cos A}{\sin A} \times \frac{\cos A}{\sin A} \rightarrow [\text{on multiplying}]$$

$$\Rightarrow \left[ \frac{\cos A}{\sin A} = \cot A \times \frac{1}{\cos A} = \sec A \right]$$

$$\Rightarrow (1 + \cot^2 A) + \cot^2 A - 2\cot A \operatorname{cosec} A$$

$$\Rightarrow \operatorname{cosec}^2 A + \cot^2 A - 2\operatorname{cosec}^2 A \cot^2 A = \operatorname{cosec}^2 A$$

$$\Rightarrow (\cot A - \operatorname{cosec} A)^2$$

Hence Proved!

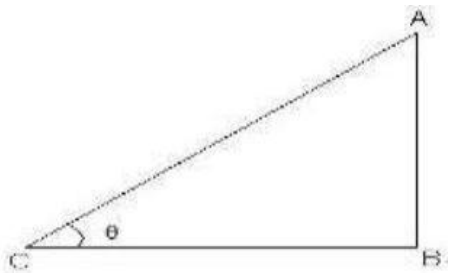
# CBSE Maths 2015

1. All questions are compulsory.
2. The question paper consists of 31 questions divided into four sections – A, B, C and D.
3. Section A contains 4 questions of 1 mark each.
4. Section B contains 6 questions of 2 marks each.
5. Section C contains 10 questions of 3 marks each
6. Section D contains 11 questions of 4 marks each.

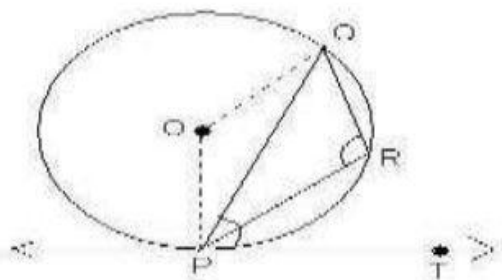
Questions -

## SECTION A

1. If the quadratic equation  $px^2 - 2\sqrt{5}px + 15 = 0$  has two equal roots then find the value of  $p$ .
2. In the following figure, a tower  $AB$  is 20 m high and  $BC$ , its shadow on the ground, is  $20\sqrt{3}$  m long. Find the Sun's altitude.

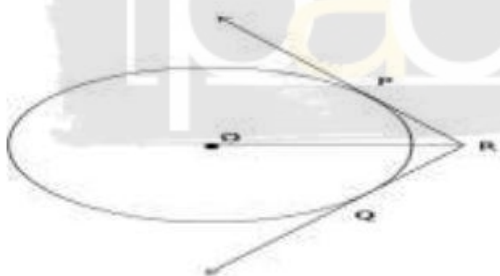


3. Two different dice are tossed together. Find the probability that the product of the two numbers on the top of the dice is 6.
4. In the following figure,  $PQ$  is a chord of a circle with center  $O$  and  $PT$  is a tangent. If  $\angle QPT = 60^\circ$ , find  $\angle PRQ$ .

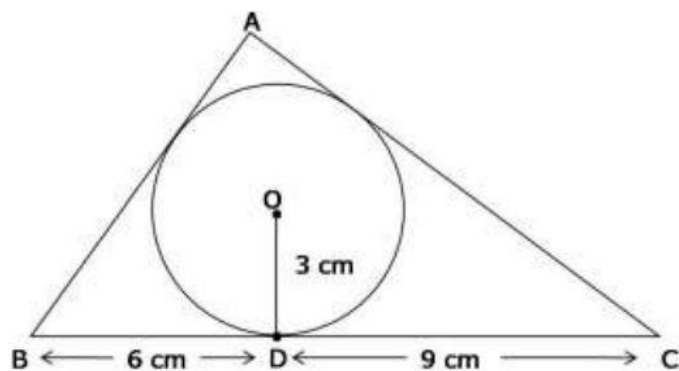


## SECTION B

5. In the following figure, two tangents RQ and RP are drawn from an external point R to the circle with center O. If  $\angle PRQ = 120^\circ$ , then prove that  $OR = PR + RQ$ .



6. In the following figure, a  $\triangle ABC$  is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of  $\triangle ABC$  is  $54 \text{ cm}^2$ , then find the lengths of sides AB and AC.



7. Solve the following quadratic equation for x:  $4x^2 + 4bx - (a^2 - b^2) = 0$ .

8. In an AP, if  $S_5 + S_7 = 167$  and  $S_{10} = 235$ , then find the A. P., where  $S_n$  denotes the sum of its first  $n$  terms.

9. The points A (4, 7), B (P, 3) and C (7, 3) are the vertices of a right triangle, right-angled at B, Find the values of P.

10. Find the relation between  $x$  and  $y$  if the points A( $x$ ,  $y$ ), B (-5, 7) and C(-4, 5) are collinear.

## SECTION C

11. The 14<sup>th</sup> term of an A. P. is twice its 8<sup>th</sup> term. If its 6<sup>th</sup> term is -8, then find the sum of its first 20 terms.

12. Solve for  $x$ :  $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ .

13. The angle of elevation of an aeroplane from point A on the ground is  $60^\circ$ . After flight of 15 seconds, the angle of elevation changes to  $30^\circ$ . If the aeroplane is flying at a constant height of  $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

14. If the coordinates of points A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that  $AP = \frac{3}{7} AB$ , where P lies on the line segment AB.

15. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is  $\frac{1}{4}$ . The probability of selecting a blue ball at random from the same jar is  $\frac{1}{3}$ . If the jar contains 10 orange balls, find the total number of balls in the jar.

16. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is  $60^\circ$ . Also find the area of the corresponding major segment. [ Use  $\pi = \frac{22}{7}$  ].

17. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but height 2.8 m, and the canvas to be used costs Rs. 100 per sq. m, find the amount, the associations will have to pay. What values are shown by these associations? [ Use  $\pi = 22/7$  ].

18. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer.

19. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs. 5 per sq. cm. [ Use  $\pi = 22/7$  ].

20. 504 Cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere, Find the diameter of the sphere and hence find its surface area. [ Use  $\pi = 22/7$  ].

## SECTION D

21. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field.

22. Find the 60 th term of the AP 8, 10, 12, ... .., if it has a total of 60 terms and hence find the sum of its last 10 terms.

23. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

24. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

25. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

26. Construct a  $\Delta ABC$  in which  $AB = 6$  cm,  $\angle A = 30^\circ$  and  $\angle B = 60^\circ$ , Construct another  $\Delta AB'C'$  similar to  $\Delta ABC$  with base  $AB' = 8$  cm.

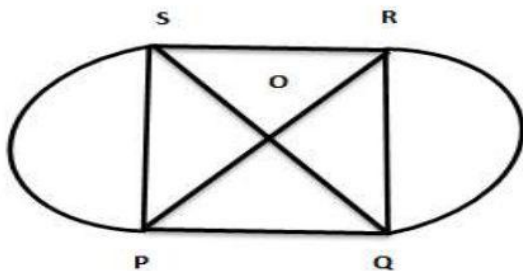
27. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is  $30^\circ$ . The angle of depression of the reflection of the cloud in the lake, at A is  $60^\circ$ . Find the distance of the cloud from A.

28. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

- i. a card of spade or an ace.
- j. a black king.
- k. neither a jack nor a king
- l. either a king or a queen.

29. Find the values of  $k$  so that the area of the triangle with vertices  $(1, -1)$ ,  $(-4, 2k)$  and  $(-k, -5)$  is 24 sq. units.

30. In the following figure, PQRS is square lawn with side  $PQ = 42$  metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersections of its diagonals. Find the total area of the two flower beds (shaded parts).



31. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of the same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. [ Use  $\pi = 22/7$  ]





# CBSE Maths 2015

## Solutions :

1. We have one formula if a quadratic equation of the form  $ax^2 + bx + c = 0$  have equal roots then  $b^2 - 4ac = 0$ .

In the given equation  $a = p$ ,  $b = -2\sqrt{5}p$  and  $c = 15$ ,

$$(-2\sqrt{5}p)^2 - 4 \times p \times 15 = 0$$

$$20p^2 - 60p = 0$$

$$p = 0 \text{ or } p = 30$$

$p = 0$  is not possible so  $p = 30$

2. From trigonometric expressions clearly

$$\tan\theta = AB/AC$$

$$\tan\theta = 20/20\sqrt{3}$$

$$\tan\theta = 1/\sqrt{3}$$

$$\theta = 30^\circ$$

3. When two dice are tossed together there will be 36 possibilities. Possibilities for product of the numbers is (1,6), (2,3), (3,2) and (6,1). Required probability is  $= 4/36$ .

4.  $\angle OPT = 90^\circ$

From the figure  $\angle OPQ = \angle OPT - \angle QPT = 90^\circ - 60^\circ = 30^\circ$

From the figure  $\angle POQ = 2\angle QPT = 2 \times 60^\circ = 120^\circ$

Reflex angle of  $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$\angle PRQ = 1/2 \times \text{reflex angle } \angle POQ = 1/2 \times 240^\circ = 120^\circ$

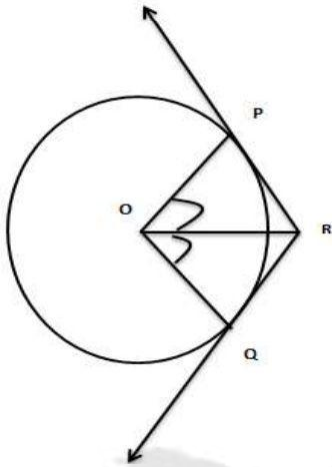
5.  $\angle OPT = 90^\circ$

From the figure  $\angle OPQ = \angle OPT - \angle QPT = 90^\circ - 60^\circ = 30^\circ$

From the figure  $\angle POQ = 2\angle QPT = 2 \times 60^\circ = 120^\circ$

Reflex angle of  $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$\angle PRQ = \frac{1}{2} \times \text{reflex angle } \angle POQ = \frac{1}{2} \times 240^\circ = 120^\circ$ .



Given that  $\angle PRQ = 120^\circ$

From the property of the circle one can get

$$\angle PRO = \angle QRO = 120/2 = 60^\circ$$

We know that lengths of tangents from an external points are equal.

Thus, from diagram  $PR = RQ$ .

After  $OP$  and  $OQ$ .

Both are the radii from the center  $O$ ,

$OP$  is perpendicular to  $PR$  and  $OQ$  is perpendicular to  $RQ$ .

Thus,  $\triangle OPR$  and  $\triangle OQR$  are right angled congruent triangles.

$$\text{Hence, } \angle POR = 90^\circ - \angle PRO = 90^\circ - 60^\circ = 30^\circ .$$

$$\angle QOR = 90^\circ - \angle QRO = 90^\circ - 60^\circ = 30^\circ$$

$$\sin \angle QRO = \sin 30^\circ = \frac{1}{2}$$

From the diagram

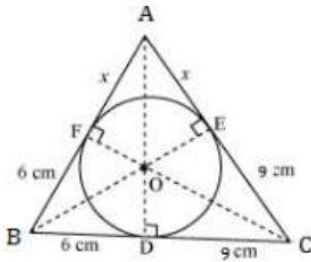
$$\sin 30^\circ = PR/OR \text{ implies } PR/OR = 1/2$$

$$OR = 2PR$$

$$OR = PR + PR$$

$$OR = PR + QR.$$

6.



Let the given circle touch the sides  $AB$  and  $AC$  of the triangle at points  $R$  and  $Q$  respectively and let the length of line segment  $AR$  be  $y$ .

Now, it can be observed that:

$$BR = BP = 6 \text{ cm (tangents from point B)}$$

$$CQ = CP = 9 \text{ cm (tangents from point C)}$$

$$AR = AQ = y \text{ (tangents from point A)}$$

$$AB = AR + RB = y + 6$$

$$BC = BP + PC = 6 + 9 = 15$$

$$CA = CQ + QA = 9 + x$$

$$2s = AB + BC + CA = y + 6 + 15 + 9 + y = 30 + 2y$$

$$s = 15 + y$$

$$s - a = 15 + y - 15 = y$$

$$s - b = 15 + y - (y + 9) = 6$$

$$s - c = 15 + y - (6 + y) = 9$$

$$\text{Area of the triangle } ABC = \sqrt{[s(s - a)(s - b)(s - c)]}$$

$$54 = \sqrt{[(15 + y)y(6)(9)]}$$

$$18 = \sqrt{[(15y + y^2)(6)]}$$

$$x^2 + 15x - 54 = 0$$

$$(x + 18)(x - 3) = 0$$

Distance can't be negative then  $x = 3$

$$AC = 3 + 9 = 12$$

$$AB = AR + RB = 6 + x = 6 + 3 = 9$$

$$7. \quad 4x^2 + 4bx - (a^2 - b^2) = 0$$

$$\therefore \Rightarrow x^2 + bx - \left(\frac{a^2 - b^2}{4}\right) = 0$$

$$\therefore \Rightarrow x^2 + 2\left(\frac{b}{2}\right)x = \frac{a^2 - b^2}{4}$$

$$\therefore \Rightarrow x^2 + 2\left(\frac{b}{2}\right)x + \left(\frac{b}{2}\right)^2 = \frac{a^2 - b^2}{4} + \left(\frac{b}{2}\right)^2$$

$$\therefore \Rightarrow \left(x + \frac{b}{2}\right)^2 = \frac{a^2}{4}$$

$$\therefore \Rightarrow x + \frac{b}{2} = \pm \frac{a}{2}$$

$$\therefore \Rightarrow x = -\frac{b}{2} \pm \frac{a}{2}$$

$$\therefore \Rightarrow x = \frac{-b-a}{2}, \frac{-b+a}{2}$$

$$\text{Hence, the roots are } \Rightarrow -\left(\frac{a+b}{2}\right) \text{ and } \left(\frac{a-b}{2}\right)$$

8.  $S_{10} = 10/2 (2a+9d) = 235$

$$5(2a+9d) = 235$$

$$10a+45d = 235$$

Dividing whole by 5

$$2a+9d = 47$$

Multiplying by 6

$$12a+54d = 282 \sim \{1\}$$

$$S_5 + S_7 = 5/2 [2a+(5-1)d] + 7/2 [2a+(7-1)d]$$

$$5/2 (2a+4d) + 7/2 (2a+6d)$$

$$5(a+2d) + 7(a+3d) = 167$$

$$5a+10d+7a+21d = 167$$

$$12a + 31d = 167 \quad \text{--- (1)}$$

Subtracting (2) from (1), we get,

$$23d = 115$$

$$d = 115/23$$

$$d = 5$$

$$2a + 9d = 47$$

$$2a + 9 \times 5 = 47$$

$$2a = 47 - 45$$

$$2a = 2$$

$$a = 1$$

A.P., is 1, 6, 11, 16...

9.

Given  $A(4, 7)$ ,  $B(p, 3)$  and  $C(7, 3)$

$$\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(p - 4)^2 + (3 - 7)^2} = \sqrt{(p - 4)^2 + 16}$$

$$BC = \sqrt{(7 - p)^2 + (3 - 3)^2} = \sqrt{(7 - p)^2}$$

$$AC = \sqrt{(7 - 4)^2 + (3 - 7)^2} = \sqrt{(3)^2 + (4)^2} = 5$$

Triangle  $ABC$  is right angle at  $B$ , hence

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\therefore \left( \sqrt{(p - 4)^2 + 16} \right)^2 + \left( \sqrt{(7 - p)^2} \right)^2 = 25$$

$$\Rightarrow p^2 - 8p + 16 + 16 + 49 - 14p + p^2 = 25$$

$$\Rightarrow 2p^2 - 22p - 81 = 25$$

$$\Rightarrow 2p^2 - 22p + 56 = 0$$

$$\Rightarrow p^2 - 11p + 28 = 0$$

$$\Rightarrow p^2 - 7p - 4p + 28 = 0$$

$$\Rightarrow p(p - 7) - 4(p - 7) = 0$$

$$\text{Or } (p - 7)(p - 4) = 0$$

If  $p - 7 = 0$  then  $p = 7$ , which is not possible as  $B$  and  $C$  will be same.

10.

$$\begin{aligned}\text{Area of triangle} &= \frac{1}{2} [x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_3)] \\ &= \frac{1}{2} [x(7 - 5) + 5(5 - y) - 4(4 - 7)]\end{aligned}$$

Given  $A, B$  and  $C$  are collinear, then area of triangle must be zero.

$$\therefore \frac{1}{2} [x(7 - 5) + 5(5 - y) - 4(4 - 7)] = 0$$

$$\Rightarrow \frac{1}{2} [2x - 25 + 5y - 4y + 25] = 0$$

$$\Rightarrow \frac{1}{2} (2x + y + 3) = 0$$

Then, relation between  $x$  and  $y$  is  $2x + y + 3 = 0$

11. let's take the first term as 'a' and let the common difference be d, then given

$$a_{14} = 2a_8 \text{ -----eq (1)}$$

also,

$$a_{14} = a + 13d \quad \text{and} \quad a_8 = a + 7d$$

then eq(1) becomes

$$a + 13d = 2(a + 7d)$$

$$a + 13d = 2a + 14d$$

$$a - 2a = 14d - 13d$$

$$-a = 1d \text{ or } a = -d \text{ -----eq(2)}$$

$$\text{also given } a_6 = -8$$

i.e,  $a+5d = -8$  ----- eq (3)

from eq(2)  $a = -d$

then eq (3) becomes

$$-d + 5d = -8 \quad \text{or} \quad 4d = -8 \quad \text{or} \quad d = -8/4 \quad \text{or} \quad d = -2$$

now,

$$a = -(-2) \quad \text{i.e} \quad a = 2 \quad d = -2$$

we are asked to find the sum of first 20 terms i.e  $n=20$

$$\text{we have, } S_n = \frac{n}{2}[2a + (n-1)d]$$

so,

$$\begin{aligned} S_{20} &= \frac{20}{2}[2 + (20-1)(-2)] \\ &= 10[2 + 19(-2)] \\ &= 10[2 - 38] \\ &= 10[-36] \\ &= -360 \end{aligned}$$

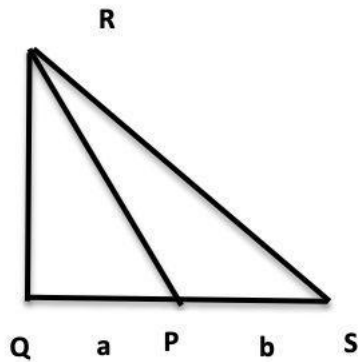
Sum of 1st 20 terms is -360

12.

$$\begin{aligned} \sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} &= 0 \\ \Rightarrow \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} &= 0 \\ \sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) &= 0 \\ \Rightarrow (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) &= 0 \\ \Rightarrow \sqrt{3}x + \sqrt{2} = 0 \quad \text{or} \quad x - \sqrt{6} &= 0 \\ \Rightarrow x = \frac{-\sqrt{2}}{\sqrt{3}} \quad \text{or} \quad x = \sqrt{6} \end{aligned}$$



13.



Let  $QR$  be the height then  $QR = 1500\sqrt{3}$ . In 15 seconds, the aeroplane moves from  $P$  to  $S$ .  $P$  and  $S$  are the points where angles of elevations are  $60^\circ$  and  $30^\circ$ . Let  $QP = a$  and  $PS = b$

$$QR = a + b$$

In triangle  $RQP$ ,

$$\tan 60^\circ = QR/QP$$

$$\sqrt{3} = 1500\sqrt{3}/a$$

$$a = 1500m$$

In triangle  $RQS$ ,

$$\tan 30^\circ = QR/QS$$

$$1/\sqrt{3} = 1500\sqrt{3}/(a + b)$$

$$b = 3000m$$

14.

$$\text{Given: } \frac{AP}{AB} = \frac{3}{7}$$

$$\Rightarrow \frac{AP}{AP + AB} = \frac{3}{7}$$

$$\Rightarrow 7AP = 3AP + 3PB$$

$$\Rightarrow 4AP = 3PB$$

$$\Rightarrow \frac{AP}{PB} = \frac{3}{4}$$

Therefore, P divides the line segment AB internally in the ratio 3: 4

Let  $(x, y)$  be the coordinates of points P.

Using the section formula, we get

$$x = \frac{3 \times 2 + 4 \times (-2)}{3 + 4} = -\frac{2}{7}$$

$$y = \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} = -\frac{20}{7}$$

Hence, the coordinates of point P are  $\left(-\frac{2}{7}, -\frac{20}{7}\right)$

15.

Given that the jar contains red, blue and orange balls.

For suppose " $a$ " be the number of red balls

and " $b$ " be the number of blue balls

Number of orange balls is given as 10

Total number of balls will be  $= a + b + 10$

Given that

Probability of selecting red ball is  $= \frac{1}{4}$  implies  $\frac{a}{a+b+10} = \frac{1}{4}$

$4a = a + b + 10$  implies  $3a - b = 10$ .....(1)

Probability of selecting blue ball is  $= \frac{1}{3}$  implies  $\frac{b}{a+b+10} = \frac{1}{3}$

$3b = a + b + 10$  implies  $2b - a = 10$ .....(2)

Solving (1) and (2) we get,

Multiply eq. (2) by 3 then add it with equation (1)

$5b = 40$  implies  $b = 8$

If  $b = 8$  then  $a = 6$ .

Total number of balls in the jar will be equal to  $6 + 8 + 10 = 24$ .

16.

Given radius of the circle is  $14\text{cm}$  and center angle is  $60^\circ$ . We have the formula for finding the area.

$$\begin{aligned} &= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin \theta \\ &= \frac{60}{360} \times \pi \times 14 \times 14 - \frac{1}{2} \times 14 \times 14 \times \sin 60^\circ \\ &= \frac{1}{60} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$= \frac{22 \times 14}{3} - \frac{147\sqrt{3}}{3}$$

So area will be equal to  $\frac{308-147\sqrt{3}}{3}$ .

17.

Given parameters are

Diameter of the tent =  $4.2\text{m}$  then radius will be =  $2.1\text{m}$ ,

Height of the cylindrical part is  $(h) = 4\text{m}$

Height of the conical part is  $(h_1) = 2.8\text{m}$

We have the formula for slant height

$$l = \sqrt{h_1^2 + r^2}$$

$$l = \sqrt{2.8^2 + 2.1^2} = \sqrt{12.25} = 3.5\text{m}$$

$$\begin{aligned} \text{Surface area will be} &= 2\pi rh = 2 \times \frac{22}{7} \times 2.1 \times 4 = 22 \times 0.3 \times 8 \\ &= 52.8\text{m}^2 \end{aligned}$$

18.

volume of hemisphere =  $\frac{2}{3}\pi r^3$  = volume of liquid present in bowl

$$= \frac{2}{3} \times \frac{22}{7} \times (18)^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 18 \times 18 \times 18 \text{ cm}^3$$

now, according to question 10% liquid is wasted by transfer .

so, rest volume of liquid = 90% of volume of liquid present in bowl

again ,

90% of volume of liquid present in bowl = 72 x volume of each bottle

$$\frac{9}{10} \times \frac{2}{3} \times \frac{22}{7} \times 18 \times 18 \times 18 = 72 \times \frac{22}{7} \times 3 \times 3 \times H$$

$$H = 5.4 \text{ cm}$$

19.

The greatest diameter a hemisphere can have 10 cm.

Total surface area of the solid = Surface area of the cube + CSA of the hemisphere - Area of the base of the hemisphere.

$$= 6 \times (10)^2 + 2 \times \frac{22}{7} (5)^2 - \frac{22}{7} (5)^2 = 600 + \frac{22}{7} \times 25 = 678.57 \text{ cm}^2$$

Cost of printing the block at the rate of Rs 5 per sq. cm

$$\text{The cost of } 678.57 \text{ sq cm} = 678.57 \times 5 = 3392.85$$

Hence, the amount is Rs 3393.

20.

$$\text{Given } r = \frac{3.5}{2} \text{ cm and } h = 3 \text{ cm}$$

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of cone} = \frac{1}{3} \pi \left( \frac{3.5}{2} \right)^2 \times 3 = \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3 = \frac{19.25}{2} \text{ cm}^3$$

$$\text{Volume of 504 cones} = 504 \times \frac{19.25}{2} = 4851 \text{ cm}^3$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

Cones are melted and recast in sphere

$$\therefore \frac{4}{3} \times \frac{22}{7} r^3 = 4851$$

$$\Rightarrow r^3 = \frac{4851 \times 21}{88} = \frac{441 \times 21}{8}$$

$$\Rightarrow r = \frac{21}{2} = 10.5 \text{ cm}$$

$$\text{Diameter of sphere} = 2 \times 10.5 = 21 \text{ cm}$$

21.

. Let us assume that  $a$  be the longer length and  $b$  be the shorter length Given that diagonal of a rectangular field is 16 metres more than the shorter side Implies diagonal =  $16 + b$ .

Longer side is 14 metres more than the shorter side implies  $a = 14 + b$ . By using Pythagoras theorem

$$(16 + b)^2 = (14 + b)^2 + b^2$$

$$256 + b^2 + 32b = 196 + b^2 + 28b + b^2$$

$$b^2 - 4b - 60 = 0$$

$$(b + 6)(b - 10) = 0$$

$$b = -6 \text{ or } b = 10$$

Breadth will be equal to 10m and other length is = 24m.

22.

$$AP = 8, 10, 12, \dots$$

$$a = 8, d = 10 - 8 = 2$$

as we know,  $a_n = a + (n-1) \times d$

$$a_{60} = 8 + (60-1) \times 2$$

$$= 8 + 118 = 126$$

so, now we find the sum of n terms

$$s_{60} = 60/2 [2 \times 8 + (60-1) \times 2]$$

$$= 30 [16 + 118]$$

$$= 4020$$

as we taken last ten terms =  $60 - 10 = 50$

so,

$$s_{50} = 50/2[2 \times 8 + (50-1)2]$$

$$= 25[16 + 98]$$

$$= 2850$$

$$\text{Sum of last ten terms} = s_{60} - s_{50} = 4020 - 2850 = 1170$$

23.

Let  $y$  be the required first speed of the train

We have  $\frac{\text{Distance}}{\text{Speed}} = \text{Time}$

Thus we have,  $\frac{54}{y} + \frac{63}{y+6} = 3$

$$\frac{54(y+6) + 63y}{y(y+6)} = 3$$

$$54y + 324 + 63y = 3y^2 + 18y$$

$$3y^2 - 99y - 324 = 0$$

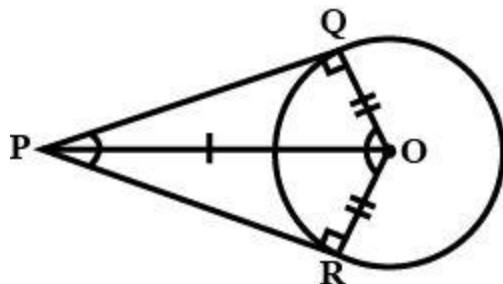
$$y^2 - 33y - 108 = 0$$

$$(y+3)(y-36) = 0$$

$$y = -3 \text{ or } y = 36$$

Speed can't be negative and hence initial speed of the train is 36 km/h.

24.



In  $\triangle PQO$  and  $\triangle PRO$

$PO =$  common

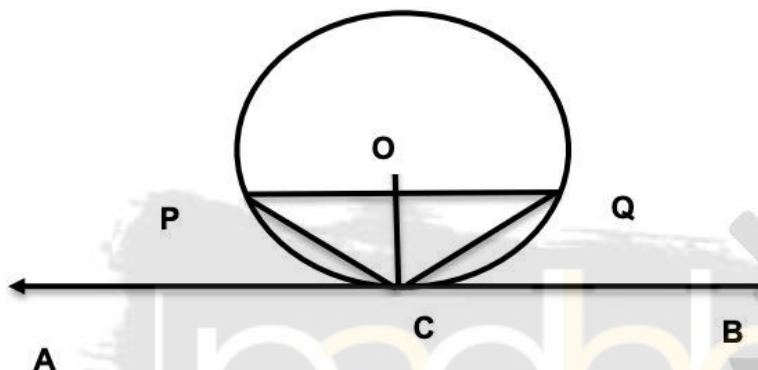
$QO = RO =$  Radius of circle

$\angle PQO = \angle PRO = 90^\circ$

(Radius  $\perp$  tangent)

$\therefore \triangle PQO \cong \triangle PRO$  (RHS)  
 and hence  $PQ = PR$   
 Hence proved (by CPCT)

25.



In the figure,  $C$  is the midpoint of the minor arc  $PQ$ ,  $O$  is the center of the circle and  $AB$  is tangent to the circle through point  $C$ .

We have to show the tangent drawn at the midpoint of the *arc PQ* of a circle is parallel to the chord joining the end points of the *arc PQ*.

We will show  $PQ$  is parallel to  $AB$ .

It is given that  $C$  is the midpoint point of the *arc PQ*.

So,  $\text{arc } PC = \text{arc } CQ$

Implies  $PC = CQ$

This shows that the triangle  $PQC$  is an isosceles triangle.

Thus, the perpendicular bisector of the side  $PQ$  of triangle  $PQC$  passes through *vertex C*.

The perpendicular bisector of a chord passes through the center of the circle.

So the perpendicular bisector of  $PQ$  passes through the center  $O$  of the circle.

Thus perpendicular bisector of  $PQ$  passes through the points  $O$  and  $C$ .

Implies  $PQ$  is perpendicular to  $OC$

$AB$  is the tangent to the circle through the point  $C$  on the circle.

Implies  $AB$  is perpendicular to  $OC$

The chord  $PQ$  and the tangent  $PQ$  of the circle are perpendicular to the same line  $OC$ .

$PQ$  is perpendicular to  $AB$ .

26.

Construct the  $\triangle ABC$  as per given measurements.

In the half plane of  $\overline{AB}$  which does not contain  $C$ , draw  $\overline{AX}$  such that  $\angle BAX$  is an acute angle.

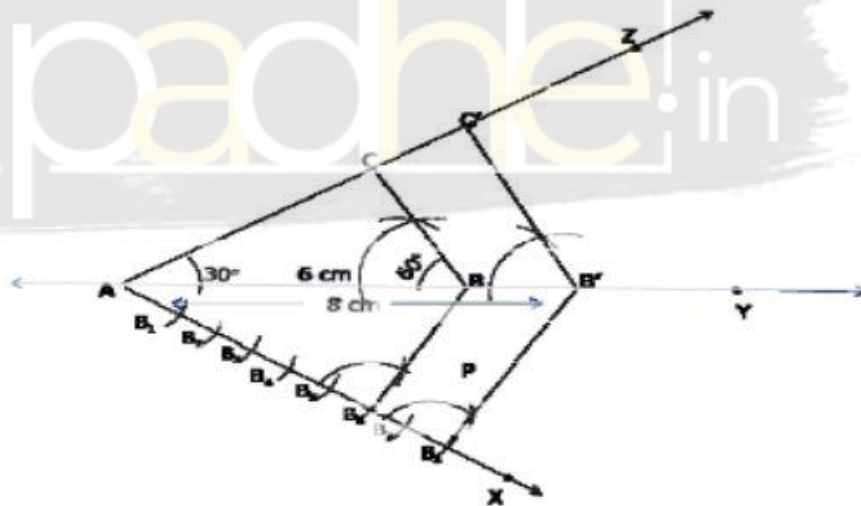
3) With some appropriate radius and centre  $A$ , Draw an arc to intersect  $\overline{AX}$  at  $B_1$ . Similarly, with center  $B_1$  and the same radius, draw an arc to intersect  $\overline{BX}$  at  $B_2$  such that  $B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7 = B_7B_8$ .

4) Draw  $\overline{B_8B}$ .

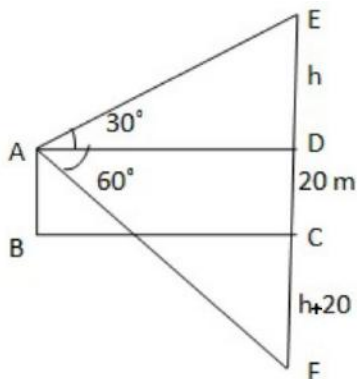
5) Through  $B_8$  draw a ray parallel to  $\overline{B_8B}$  to intersect  $\overline{AY}$  at  $B'$ .

6) Through  $B'$  draw a ray parallel to  $\overline{BC}$  to intersect  $\overline{AZ}$  at  $C'$ .

Thus,  $\triangle AB'C'$  is the required triangle.



27.



if we assume the distance of the cloud from point  $A$ .



e.g.,  $DE = h$  and  $CF = h - 20$

now, from  $\triangle ADE$ ,

$$\tan 30^\circ = DE/AD$$

$$1/\sqrt{3} = h/AD$$

$$AD = \sqrt{3}h \dots\dots\dots(1)$$

from triangle  $\triangle ADF$ ,

$$\tan 60^\circ = DF/AD$$

$$\sqrt{3} = (FC + CD)/\sqrt{3}h$$

$$\sqrt{3} \times \sqrt{3}h = (h + 20 + 20)$$

$$3h = h + 40$$

$$h = 20\text{m.}$$

hence, Distance of cloud from A = 20m

28.

Let us assume that S be the sample space of drawing a card from a well-shuffled deck.

$$n(S) = {}^{52}_1C$$

i) In a deck we have 13 spade 4 ace card a spade can be chosen in 13 ways. An ace can be chosen in 4 ways. But we have already chosen one ace card which is included in spade.

So required probability =  $13+4-1/52 = 16/52 = 4/13$ .

ii) There are only two black king in a deck. A black king card is drawn in 2 ways. So required probability =  $2/52 = 1/26$ .

iii) There are 4 jack and 4 king cards in deck. So there are 44 cards which are neither jacks nor kings So required probability =  $44/52 = 2/13$ .

iv) There are 4 kings and 4 queen cards in a deck. So there are  $4 + 4 = 8$  card which are either king or queen. So required probability =  $8/52 = 1/26$ .

29.

Area of the triangle formed by  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$24 = \frac{1}{2} [1(2k + 5) + (-4)(-5 + 1) + (-k)(-1 - 2k)]$$

$$48 = 2k + 5 + 16 + k + 2k^2$$

$$2k^2 + 3k - 27 = 0$$

$$(2k + 9)(k - 3) = 0$$

$$k = -4.5 \text{ or } k = 3$$

30.

Given that  $PQRS$  is a square. So each side is equal and angle between the adjacent sides is a right angle. Also the diagonals perpendicularly bisect each other.

In triangle  $PQR$

$$PR^2 = PQ^2 + QR^2$$

$$PR^2 = 42^2 + 42^2$$

$$PR = 42\sqrt{2}$$

$$OR = \frac{1}{2}PR = \frac{42}{\sqrt{2}} = OQ.$$

$$\text{Area of sector } ORQ = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \times \frac{42}{\sqrt{2}} \times \frac{42}{\sqrt{2}}$$

$$\text{Area of triangle } ROQ = \frac{1}{2} \times RO \times OQ = \frac{1}{2} \times \frac{42}{\sqrt{2}} \times \frac{42}{\sqrt{2}} = \left(\frac{42}{2}\right)^2$$

Area of the shape  $ORQ = \text{Area of sector } ORQ - \text{Area of triangle } ROQ$

$$= \left(\frac{42}{2}\right)^2 \left(\frac{\pi}{2} - 1\right)$$

$$= 441(0.57) = 251.37 \text{ cm}^2$$

Area of the flower bed  $ORQ = \text{Area of the flower bed } OPS = 251.37 \text{ cm}^2$

31.

First, we need to find out the volume of the previous solid metal cylinder.

$$h = 10 \text{ cm}$$

$$r = 4.2 \text{ cm}$$

Volume of cylinder before scooping out =  $\pi r^2 h$

$$= 176.4\pi \text{ cm}^3$$

Volume of scooped part =

$$\frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times (4.2)^3$$

$$= 98.784\pi \approx 98.8\pi \text{ cm}^3$$

Therefore volume of the scooped metal cylinder =  $176.4\pi - 98.8\pi = 77.6\pi \text{ cm}^3$

Now For wire,

Diameter = 1.4 cm

Radius =  $1.4/2 = 0.7$

The volume of the scooped metal cylinder = The volume of the wire

$$77.6\pi = \pi r^2 h$$

Cut  $\pi$  from both the sides

$$77.6 = (0.7)^2 h$$

$$h = 77.6/0.49$$

$$h = 158.36 \text{ cm} \approx 158.4 \text{ cm}$$

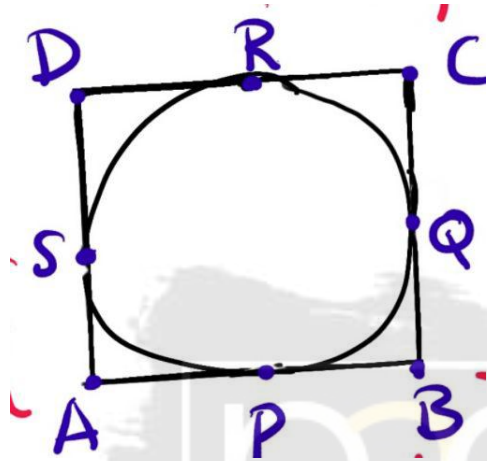
# CBSE Maths 2014

- 1) All questions are compulsory.
- 2) The question paper consists of thirty questions divided into 4 sections A, B, C and D. Section A comprises of ten questions of 01 mark each, Section B comprises of five questions of 02 marks each, Section C comprises ten questions of 03 marks each and Section D comprises of five questions of 06 marks each.
- 3) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- 4) There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.
- 5) In question on construction, drawing should be near and exactly as per the given measurements.
- 6) Use of calculators is not permitted.

Questions -

1. If the height of a vertical pole is 3 times the length of its shadow on the ground, then the angle of elevation of the Sun at that time is \_\_\_\_\_.
2. A bag contains cards numbered from 1 to 25. A card is drawn at random from the bag. The probability that the number on this card is divisible by both 2 and 3 is \_\_\_\_\_.
3. Two different coins are tossed simultaneously. The probability of getting at least one head is \_\_\_\_\_.
4. Two concentric circles are of radii 5 cm and 3 cm. Length of the chord of the larger circle (in cm), which touches the smaller circle is \_\_\_\_\_.

5. In figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If  $AB = x$  cm,  $BC = 7$  cm,  $CR = 3$  cm and  $AS = 5$  cm, find  $x$ .



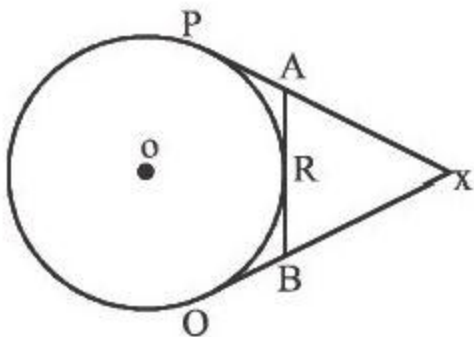
6. The perimeter of a triangle with vertices  $(0, 4)$ ,  $(0, 0)$  and  $(3, 0)$  is \_\_\_\_\_.

7. A rectangular sheet of paper  $40 \text{ cm} \times 22 \text{ cm}$ , is rolled to form a hollow cylinder of height  $40 \text{ cm}$ . The radius of the cylinder (in cm) is \_\_\_\_\_.

8. The next term of the A.P.  $7, 28, 63, \dots$  is

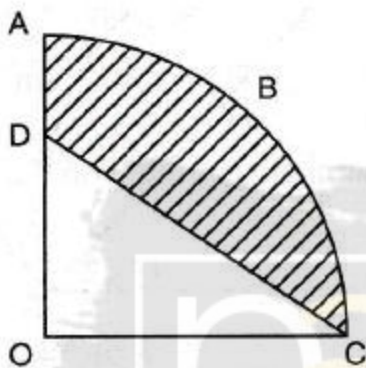
## SECTION B

9. In figure,  $XP$  and  $XQ$  are two tangents to the circle with centre  $O$ , drawn from an external point  $X$ .  $ARB$  is another tangent, touching the circle at  $R$ . Prove that  $XA + AR = XB + BR$ .



10. Prove that the tangents drawn at the ends of any diameter of a circle are parallel.

12. In Figure , OABC is a quadrant of a circle of radius 7 cm. If OD = 4 cm, find the area of the shaded region. [Use  $\pi = 22/7$  ]



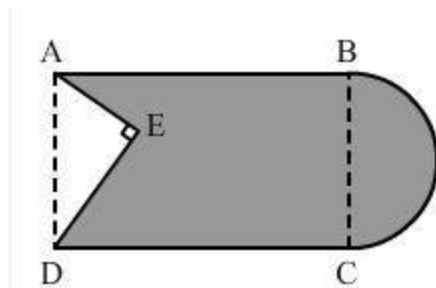
13. Solve for x:  $3x^2 - 22x - 23 = 0$ .

14. The sum of the first n terms of an AP is  $5n - n^2$ . Find the nth term of this A.P.

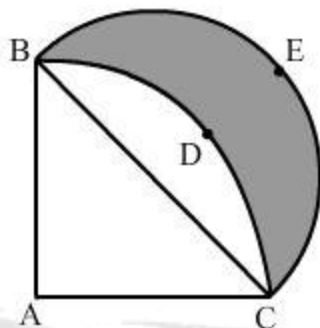
### SECTION C

15. Points P, Q, R and S divide the line segment joining the points A (1, 2) and B (6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

16. In the figure, from a rectangular region ABCD with AB = 20 cm, a right triangle AED with AE = 9 cm and DE = 12 cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region.  
[Use  $\pi = 3.14$ ]



17. In the figure, ABCD is a quadrant of a circle of radius 28 cm and a semi circle BEC is drawn with BC as diameter. Find the area of the shaded region. [Use  $\pi = 22/7$ ]



18. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs 25 per metre. [Use  $\pi = 22/7$ ]

19. A girl empties a cylindrical bucket, full of sand, of base radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct up to one place of decimal.

20. The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P.

21. Two ships are approaching a light-house from opposite directions. The angles of depression of the two ships from the top of the light-house are  $30^\circ$  and  $45^\circ$ . If the distance between the two ships is 100 m, find the height of the light-house. [Use  $\sqrt{3} = 1.732$ ]

22. If 2 is a root of the quadratic equation  $3x^2 + px - 8 = 0$  and the quadratic equation  $4x^2 - 2px + k = 0$  has equal roots, find the value of k.

23. Construct a triangle PQR, in which PQ = 6 cm, QR = 7 cm and PR = 8 cm. Then construct another triangle whose sides are  $\frac{4}{5}$  times the corresponding sides of  $\Delta PQR$ .

24. Find the value(s) of  $p$  for which the points  $(p + 1, 2p - 2)$ ,  $(p - 1, p)$  and  $(p - 3, 2p - 6)$  are collinear.

### SECTION D

25. The mid-point  $P$  of the line segment joining the points  $A (-10, 4)$  and  $B (-2, 0)$  lies on the line segment joining the points  $C (-9, -4)$  and  $D (-4, y)$ . Find the ratio in which  $P$  divides  $CD$ . Also find the value of  $y$ .

26. A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal.

27. The angle of elevation of the top of a chimney from the foot of a tower is  $60^\circ$  and the angle of depression of the foot of the chimney from the top of the tower is  $30^\circ$ . If the height of the tower is 40 m, find the height of the chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m. State if the height of the above mentioned chimney meets the pollution norms. What value is discussed in this question?

28. A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid. [Use  $\pi = 22/7$ ].

29. If  $S_n$  Denotes the sum of the first  $n$  terms of an A.P., prove that  $S_{30} = 3 (S_{20} - S_{10})$ .

30. A metallic bucket, open at the top, of height 24 cm is in the form of the frustum of a cone, the radii of whose lower and upper circular ends are 7 cm and 14 cm respectively. Find



(i) the volume of water which can completely fill the bucket.

(ii) the area of the metal sheet used to make the bucket.

[Use  $\pi = 22/7$ ]

31. The sum of the squares of two consecutive even numbers is 340. Find the numbers.

32. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

33. A dice is rolled twice. Find the probability that

(i) 5 will not come up either time.

(ii) 5 will come up exactly one time.

34.

**solve for x :**  $3 \left( \frac{3x - 1}{2x + 3} \right) - 2 \left( \frac{2x + 3}{3x - 1} \right) = 5; x \neq \frac{1}{3}, -\frac{3}{2}$

# CBSE Maths 2014

Solutions :-

1. Assume OA as the pole and OB is its shadow.

Let the length of the shadow be  $x$ .

Let  $\theta$  be the angle of elevation of the top of the pole from the ground.

Given that the height of the pole ( $h$ ) =  $3 \times$  length of its shadow

$$\Rightarrow h = 3x$$

In  $\triangle OAB$

$$\tan \theta = \frac{AO}{OB} = \frac{h}{x} = \frac{3x}{x} = \sqrt{3}$$

$$\Rightarrow \tan \theta = \tan 60^\circ$$

$$\Rightarrow \theta = 60^\circ$$

$\therefore$  The angle of elevation of the Sun is  $60^\circ$ .

2. Total number of cards = 25

Total number of possible outcomes = 25

Assume  $E$  as the event that the number on the card drawn is divisible by both 2 and 3.

$$\therefore E = \{6, 12, 18, 24\}$$

$\therefore$  Number of possible outcomes favourable to  $E = 4$

$$P(E) = \frac{\text{Number of possible outcomes favourable to } E}{\text{Total number of outcomes}}$$

$$P(E) = \frac{4}{25}$$

3. When two coins are tossed simultaneously, the possible outcomes are  $\{(H, H), (H, T), (T, H), (T, T)\}$ .

$\therefore$  Total number of possible outcomes = 4

Assume the event of getting at least one head as 'E'.

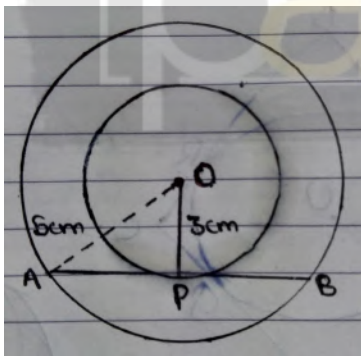
Number of possible outcomes favourable to E is  $\{(H, H), (H, T), (T, H)\}$ .

$\therefore$  Number of favourable outcomes = 3

$P(E) = \frac{\text{Number of favourable outcomes to E}}{\text{Total number of possible outcomes}}$

$P(E) = \frac{3}{4}$ .

4.



In the figure, O is the common centre, of the given concentric circles.

AB is a chord of the bigger circle such that it is a tangent to the smaller circle P.

Since, OP is the radius of the smaller circle.

$\therefore OP \perp AB \Rightarrow \angle APO = 90^\circ$

Also, radius perpendicular to a chord bisects the chord

$\therefore$  OP bisects AB

$\Rightarrow AP =$

Now, in right  $\triangle APO$ ,

$$OA^2 = AP^2 + OP^2$$

$$\Rightarrow 5^2 = AP^2 + 3^2 \Rightarrow AP^2 = 5^2 - 3^2$$

$$\Rightarrow AP^2 = 4^2 \Rightarrow AP = 4\text{cm}$$

$$\Rightarrow AB = 4 \Rightarrow AB = 2 \times 4 = 8\text{cm}$$

Hence, the required length of the chord AB is 8cm.

5. Tangents drawn from an external point to a circle are always equal.

The quadrilateral, ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. Therefore A,B,C and D are external points from which tangents AS=AP=5, BP=BQ, CQ=CR=3 and DR=DS are drawn

$$BC=7. \Rightarrow BQ + CQ = BC=7 \Rightarrow BQ = 7-3=4 \therefore BP=4$$

$$AB=x, \Rightarrow AP + BP = x \Rightarrow 5+4 = x \Rightarrow x=9$$

6. A(0,4)

B(0,0)

C(3,0)

Therefore perimeter of triangle ABC is = AB+BC+AC

$$AB = \sqrt{(0-0)^2 + (0-4)^2}$$

$$= \sqrt{(0)+(16)}$$

$$= +4, -4$$

$$BC = \sqrt{(3-0)^2 + (0-0)^2}$$

$$= \sqrt{9+0}$$

$$= +3, -3$$

$$AC = \sqrt{(3-0)^2 + (0-4)^2}$$

$$= \sqrt{9+16}$$

$$= +5, -5$$

Measurement of sides cannot be rejected therefore -4,-3,-5 is rejected

Therefore AB=4 cm BC=3cm AC=5cm

Perimeter= 12 cm

7. Area of the rectangular sheet =  $l \times b = (40 \times 22) \text{ cm}^2 = 880 \text{ cm}^2$

Assume the radius of the cylinder as 'r'.

Given that height (h) of the cylinder = 40 cm

Area of the rectangular sheet is same as the curved surface area of the cylinder.

$\therefore 2\pi rh = \text{Area of the rectangular sheet}$

$\Rightarrow 2 \times 22/7 \times r \times 40 = 880$

$\Rightarrow r = 7/2 = 3.5$

$\therefore$  Radius of the cylinder is 3.5 cm.

8. 7, 28, 63...

$= 7, (7 \times 2 \times 2)\sqrt{\phantom{x}}, (7 \times 3 \times 3) \dots$

$= (7 \times 1 \times 1)\sqrt{\phantom{x}}, (7 \times 2 \times 2), (7 \times 3 \times 3) \dots$

$= 7, 27, 37 \dots$

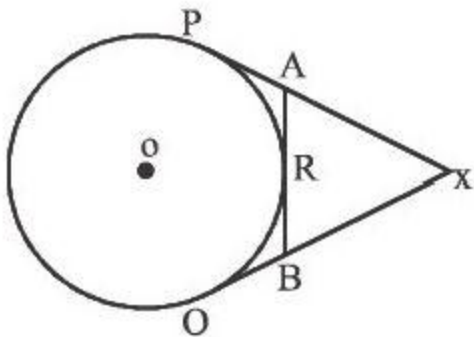
Given that the first term of the given AP is 7 and the common difference is  $(27-7) = 20$

So,  $a = 7, d = 20$

The next term of the sequence, i.e. the 4th term =  $a + 3d$

$= 7 + 3 \times 20 = 7 + 60 = 67$

9.



GIVEN:

XP and XQ are tangents of the circle with centre O and R is a point on the circle.

TO PROVE :  $XA + AR = XB + BR$ .

PROOF:

$XP = XQ$  [X is an external point].....(1)

$AP = AR$  [A is an external point].....(2)

$BQ = BR$  [B is an external point].....(3)

From eq 1

$XP = XQ$

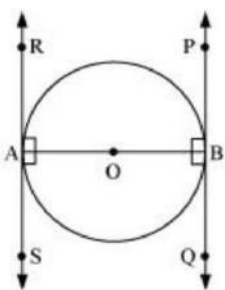
$(XA + AP) = (XB + BQ)$

$XA + AR = XB + BR$

[From eq 2 & 3 ,  $AP = AR$  ,  $BQ = BR$  ]

Hence proved.

10.



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus,  $OA \perp RS$  and  $OB \perp PQ$

$$\angle OAR = 90^\circ$$

$$\angle OAS = 90^\circ$$

$$\angle OBP = 90^\circ$$

$$\angle OBQ = 90^\circ$$

It can be observed that

$$\angle OAR = \angle OBQ \text{ (Alternate interior angles)}$$

$$\angle OAS = \angle OBP \text{ (Alternate interior angles)}$$

Since alternate interior angles are equal, lines PQ and RS will be parallel.

11. The numbers that two dice could show are 1, 2, 3, 4, 5, 6.

The possible outcomes when two dice are rolled are:

$\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$

$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$

$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$\therefore$  Number of possible outcomes =  $6 \times 6 = 36$ .

Assume 'E' as the event that the sum of numbers appearing on the two dice is 10.

The possible outcomes favourable to event, E are  $\{(4, 6), (5, 5), (6, 4)\}$ .

$\therefore$  Number of favourable outcomes to event E = 3.

$P(E) = \frac{\text{Number of possible outcomes favourable to E}}{\text{Total number of possible outcomes}} = \frac{3}{36} = \frac{1}{12}$ .

12. Given that, radius of the circle = 7 cm

$$\text{Area of the quadrant OABC} = \frac{1}{4} \times \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{77}{2} \text{ cm}^2 = 33.5 \text{ cm}^2$$

$$\text{Area of } \triangle ODC = \frac{1}{2} \times OD \times OC$$

$$= \left( \frac{1}{2} \times 4 \times 7 \right) \text{ cm}^2 \text{ (Since OC is the radius of the circle)}$$

$$= 14 \text{ cm}^2$$

$$\text{Area of the shaded region} = \text{Area of the quadrant OABC} - \text{Area of } \triangle ODC$$

$$\text{Area of the shaded region} = 33.5 - 14 = 24.5 \text{ cm}^2$$

13. Given the quadratic equation as,

$$3x^2 - 22x - 23 = 0$$

$$\Rightarrow 3x^2 - 32x + 2x - 23 = 0$$

$$\Rightarrow 3x(x-6) + 2(x-6) = 0$$

$$\Rightarrow (3x+2)(x-6) = 0$$

$$\Rightarrow x = -\frac{2}{3} \text{ or } x = 6$$

14. The sum of n terms of an A.P. is given by  $5n - n^2$

$$\text{But we know that } S_n = \frac{n}{2}(a + tn)$$



where  $a$  = first term  $T_n$  =  $n$ th term

So, we have  $5n - n^2 = n^2 (a + tn)$

$$\Rightarrow n(5-n) = n^2(a + tn)$$

$$\Rightarrow 10 - 2n = a + tn \dots (1)$$

Now, we have sum of the first term  $S_1 = a$

$$\Rightarrow a = (5(1) - 1^2) = 4$$

Substitute the value of  $a$  in equation (1), we get

$$10 - 2n = 4 + tn$$

$$\Rightarrow tn = 6 - 2n$$

Therefore,  $n$ th term of the A.P. is  $6 - 2n$ .

15. It is given that P, Q, R and S divide the line segment joining the points A(1, 2) and B(6, 7) in 5 equal parts.

Section formula,

$$(x, y) = \left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

P divides the line AB in 1:4.

$$P(x, y) = \left( \frac{1(6) + 4(1)}{1+4}, \frac{1(7) + 4(2)}{1+4} \right) = (2, 3)$$

Q divides the line AB in 2:3.

$$Q(x, y) = \left( \frac{2(6) + 3(1)}{2+3}, \frac{2(7) + 3(2)}{2+3} \right) = (3, 4)$$

R divides the line AB in 3:2.

$$R(x, y) = \left( \frac{3(6) + 2(1)}{3+2}, \frac{3(7) + 2(2)}{3+2} \right) = (4, 5)$$

Therefore the coordinates of the points P, Q and R are P(2,3), Q(3,4) and R(4,5).

16. AED is an right angle triangle,

$$AD^2 = AE^2 + ED^2.$$

$$\Rightarrow AD^2 = (9^2 + 12^2)$$

$$= (81 + 144)$$

$$= 225 \text{ cm}^2$$

$$\Rightarrow AD = 15 \text{ cm}$$

Area of the rectangular region ABCD

$$= AB \times AD$$

$$= (20 \times 15)$$

$$= 300 \text{ cm}^2$$

Area of  $\triangle AED$

$$= \frac{1}{2} \times AE \times DE = \left(\frac{1}{2} \times 9 \times 12\right) \text{ cm}^2 = 54 \text{ cm}^2$$

In a rectangle

$$AD = BC = 15 \text{ cm}$$

Since, BC is the diameter of the circle, radius of the circle = 15 cm

$$\text{Area of the semi-circle} = \frac{1}{2} \times \pi \times r^2$$

$$= \left(\frac{1}{2} \times 3.14 \times \frac{15}{2} \times \frac{15}{2}\right) = 88.3125 \text{ cm}^2$$

Area of the shaded region = Area of the rectangle + Area of the semi-circle - Area of the triangle

$$= (300 + 88.3125 - 54) \text{ cm}^2$$

$$= 334.3125 \text{ cm}^2.$$

17. Given:

Radius (r) of the circle = AB = AC = 28 cm

Area of quadrant ABPC =  $\frac{1}{4} \times \pi \times r^2$

$$= \left( \frac{1}{4} \times \frac{22}{7} \times 28 \times 28 \right) \text{ cm}^2$$

$$= 22 \times 28 = 616 \text{ cm}^2$$

Area of  $\triangle ABC = \frac{1}{2} \times AC \times AB = \left( \frac{1}{2} \times 28 \times 28 \right)$

$$= 392 \text{ cm}^2$$

Area of segment BPC = Area of quadrant ABPC – Area of  $\triangle ABC$

$$= (616 - 392) \text{ cm}^2$$

$$= 224 \text{ cm}^2 \dots\dots\dots(1)$$

In a right-angled  $\triangle BAC$

$BC^2 = BA^2 + AC^2$  (By Pythagoras theorem)

$$BC^2 = (28^2 + 28^2) \text{ cm}^2$$

$$BC^2 = 784 + 784 \text{ cm}^2$$

$$BC = \sqrt{16 \times 98} = \sqrt{16 \times 49 \times 2}$$

$$BC = 4 \times 7\sqrt{2} = 28\sqrt{2}$$

$$BC = 28\sqrt{2} \text{ cm}$$

$$BC(\text{Diameter}) = 28\sqrt{2}$$

$$\text{Radius of semicircle} = \frac{28\sqrt{2}}{2} = 14\sqrt{2} \text{ cm}$$

$$\text{Area of semicircle BEC} = \frac{1}{2} \times \pi \times r^2$$

$$= \left( \frac{1}{2} \times 22 \times 7 \times 14 \sqrt{2} \times 14 \sqrt{2} \right) \text{ cm}^2$$

$$= 22 \times \sqrt{2} \times 14 \sqrt{2} = 22 \times 14 \times 2 = 44 \times 14$$

$$= 616 \text{ cm}^2$$

Area of the shaded portion = Area of semicircle BEC – Area of segment BPC

$$= 616 - 224 \text{ cm}^2 = 392 \text{ cm}^2$$

Hence, the area of the shaded region = 392 cm<sup>2</sup>

18. For Conical tent,

$$b=5\text{m}$$

$$\text{radius } r = \text{diameter}/2 = 14/2 = 7\text{m}$$

$$\text{height } h = 24 \text{ m}$$

$$\text{slant height } l = \sqrt{(h^2 + (r^2))} = \sqrt{(576) + (49)} = \sqrt{625} = 25$$

$$\text{C.S.A of tent} = \pi r l$$

$$= (22/7) \times 7 \times 25$$

$$= 550 \text{ m}^2$$

Area of cloth required = C.S.A of tent

$$l \times b = 550$$

$$l \times 5 = 550$$

$$l = 110\text{m}$$

$$\text{Cost of 1 m cloth} = ₹25$$

$$\text{Cost of 110 m cloth} = ₹(110 \times 25) = ₹2750$$

19.

$$\begin{aligned}\text{Volume of sand} &= \pi r^2 h \\ &= \frac{22}{7} \times 18 \times 18 \times 32 \\ &= 32585.1429\end{aligned}$$

Volume of cylindrical bucket will be equal to the Volume of heap so formed.

Therefore, Volume of conical heap =  $\frac{1}{3} \pi r^2 h$

Thus,  $\frac{1}{3} \times \frac{22}{7} \times r^2 \times h = 32585.1429$

$$r^2 = \frac{32585.1429 \times 7 \times 1}{22 \times 1/8}$$

$$r = \sqrt{1296}$$

$$r = 36 \text{ cm.}$$

$$l = \sqrt{r^2 + h^2} \text{ (Whole Root)}$$

$$= \sqrt{1296 + 576}$$

$$= \sqrt{1872}$$

$$l = 43.26$$

20. Given :

$S_7 = 63$  and sum of its next 7 terms is 161

By using the formula ,Sum of nth terms ,  $S_n = \frac{n}{2} [2a + (n - 1) d]$

$$S_7 = \frac{7}{2} [2a + (7 - 1)d]$$

$$63 = \frac{7}{2} [2a + 6d]$$

$$63 \times \frac{2}{7} = [2a + 6d]$$

$$9 \times 2 = 2a + 6d$$

$$2a + 6d = 18 \text{ .....(1)}$$

And

Sum of its next 7 terms = 161(Given)

Sum of first 14 terms = Sum of first 7 terms + Sum of next 7 terms.

$$S_{14} = 63 + 161 = 224$$

$$S_{14} = 224$$

By using the formula ,Sum of nth terms ,  $S_n = n/2 [2a + (n - 1) d]$

$$S_{14} = (14/2) [ 2a + (14 - 1)d ]$$

$$224 = 7 [ 2a + 13d ]$$

$$224/7 = 2a + 13d$$

$$2a + 13d = 32 \dots\dots\dots (2)$$

On subtracting eq (1) from (2)

$$2a + 13d = 32$$

$$2a + 6d = 18$$

$$(-) \quad (-) \quad (-)$$

-----

$$7d = 14$$

-----

$$d = 14/7$$

$$d = 2$$

On putting the value of  $d = 2$  in eq (1),

$$2a + 6d = 18$$

$$2a + 6(2) = 18$$

$$2a + 12 = 18$$

$$2a = 18 - 12$$

$$2a = 6$$

$$a = 6/2$$

$$a = 3$$

For 28th term :

By using the formula ,  $a_n = a + (n - 1)d$

$$a_{28} = a + (28 - 1) d$$

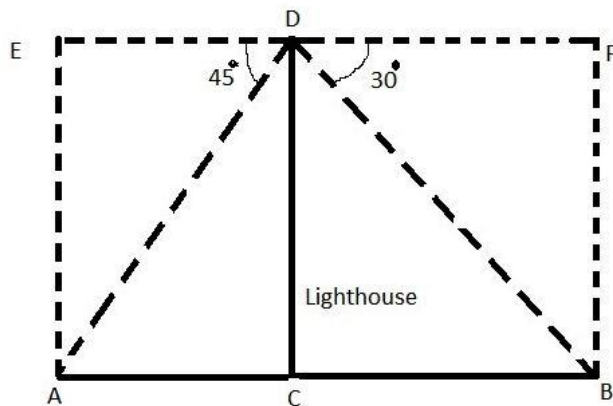
$$a_{28} = 3 + (27) 2$$

$$a_{28} = 3 + 54$$

$$a_{28} = 57$$

Hence, 28th term of this A.P is 57.

21.



$$AB = 100$$

$$\text{Let } AC = x$$

$$\text{Let } BC = 100 - x$$

Let  $CD = h$

In  $\triangle EAD$ ,

$$EA = DC$$

$$ED = AC$$

$$\tan 45^\circ = EA/ED$$

$$1 = h/x$$

$$x = h$$

In  $\triangle DFB$

$$FB = CD$$

$$DF = CB$$

$$\tan 30^\circ = FB/DF$$

$$1/\sqrt{3} = h/100-x$$

$$1/\sqrt{3} = h/100-h$$

$$100-h = \sqrt{3}h$$

$$100-h = 1.732h$$

$$2.732h = 100$$

$$h = 100/2.732$$

$$h = 36.603\text{m}$$

$$h = 36.6\text{m}$$

22. Given :  $3x^2 + px - 8 = 0$ .....(1)

$$4x^2 - 2px + k = 0$$
..... (2)

On putting the value of given root i.e  $x = 2$  in eq 1 .

$$3x^2 + px - 8 = 0$$

$$3(2)^2 + p(2) - 8 = 0$$

$$3 \times 4 + 2p - 8 = 0$$

$$12 + 2p - 8 = 0$$



$$4 + 2p = 0$$

$$2p = -4$$

$$p = -4/2 = -2$$

$$p = -2$$

Hence the value of p is - 2.

On putting the value of p = - 2 in eq 2,

$$4x^2 - 2px + k = 0$$

$$4x^2 - 2(-2)x + k = 0$$

$$4x^2 + 4x + k = 0$$

On comparing the given equation with  $ax^2 + bx + c = 0$

Here,  $a = 4$ ,  $b = 4$ , and  $c = k$

$$D(\text{discriminant}) = b^2 - 4ac$$

Given : Quadratic equation has equal roots i.e  $D = 0$

$$b^2 - 4ac = 0$$

$$4^2 - 4(4)(k) = 0$$

$$16 - 16k = 0$$

$$16 = 16k$$

$$k = 16/16 = 1$$

$$k = 1$$

23. Steps of construction:

Step 1: Draw a ray PX.

Step 2: PQ = 6 cm as the radius, draw an arc from point P intersecting PX at Q.

Step 3: Q as the centre and radius equal to 7 cm, draw an arc.

Step 4: P as the centre and radius equal to 8 cm, draw another arc, cutting the previously drawn arc at R.

Step 5: Join PR and QR.

Therefore, PQR is the required triangle.

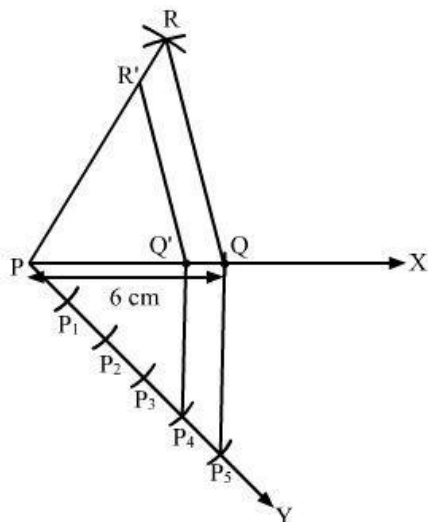
Step 6: Draw any ray PY making an acute angle with PQ.

Step 7: Now locate 5 points P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub>, P<sub>4</sub> and P<sub>5</sub> on PY so that PP<sub>1</sub>=P<sub>1</sub>P<sub>2</sub>=P<sub>2</sub>P<sub>3</sub>=P<sub>3</sub>P<sub>4</sub>=P<sub>4</sub>P<sub>5</sub>.

Step 8: Join P<sub>5</sub>Q and draw a line through P<sub>4</sub>, which is parallel to P<sub>5</sub>Q, to intersect PQ at Q'.

Step 9: Draw a line through Q', which is parallel to line QR, to intersect PR at R'.

Then we get, PQ'R' is the required triangle.



24. Given ,

Points =  $(p+1, 2p-2)$ ,  $(p-1, p)$  and  $(p-3, 2p-6)$

For the given points  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  to be collinear then

$$[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

Here,

$$x_1 = p + 1 \quad y_1 = 2p - 2$$

$$x_2 = p - 1 \quad y_2 = p$$

$$x_3 = p - 3 \quad y_3 = 2p - 6$$

Substituting the values in the formula ,

$$(p + 1)(p - (2p - 6)) + (p - 1)(2p - 6 - (2p - 2)) + (p - 3)(2p - 2 - (p)) = 0$$

$$(p + 1)(p - 2p + 6) + (p - 1)(2p - 6 - 2p + 2) + (p - 3)(2p - 2 - p) = 0$$

$$(p + 1)(-p + 6) + (p - 1)(-4) + (p - 3)(p - 2) = 0$$

$$-p^2 - p + 6p + 6 - 4p + 4 + p^2 - 3p - 2p + 6 = 0$$

$$-4p + 16 = 0$$

$$4p = 16$$

Dividing both the sides by 4

$$4p / 4 = 16 / 4$$

$$p = 4$$

Hence,

For the points to be collinear ,  $p = 4$

25.

Given ,

Points =  $(p+1, 2p-2)$ ,  $(p-1, p)$  and  $(p-3, 2p-6)$

For the given points  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  to be collinear then

$$[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

Here,

$$x_1 = p+1 \quad y_1 = 2p-2$$

$$x_2 = p-1 \quad y_2 = p$$

$$x_3 = p-3 \quad y_3 = 2p-6$$

Substituting the values in the formula ,

$$(p+1)(p - (2p-6)) + (p-1)(2p-6 - (2p-2)) + (p-3)(2p-2 - (p)) = 0$$

$$(p+1)(p - 2p+6) + (p-1)(2p-6 - 2p+2) + (p-3)(2p-2 - p) = 0$$

$$(p+1)(-p+6) + (p-1)(-4) + (p-3)(p-2) = 0$$

$$-p^2 - p + 6p + 6 - 4p + 4 + p^2 - 3p - 2p + 6 = 0$$

$$-4p + 16 = 0$$

$$4p = 16$$

Dividing both the sides by 4

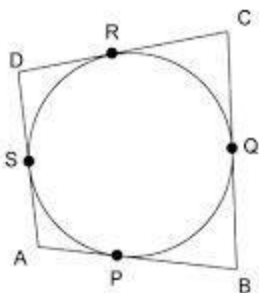
$$4p / 4 = 16 / 4$$

$$p = 4$$

Hence,

For the points to be collinear ,  **$p = 4$**

26.



Given:- Let ABCD be the quadrilateral circumscribing the circle with centre O. The quadrilateral touches the circle at point P,Q,R and S.

To prove:-  $AB+CD=AD+BC$

Proof:-

As we know that, length of tangents drawn from the external point are equal.  
Therefore,

$$AP=AS.....(1)$$

$$BP=BQ.....(2)$$

$$CR=CQ.....(3)$$

$$DR=DS.....(4)$$

Adding equation (1),(2),(3) and (4), we get

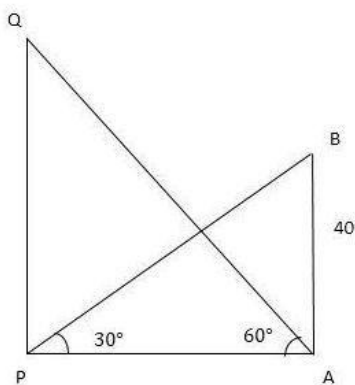
$$AP+BP+CR+DR=AS+BQ+CQ+DS$$

$$(AP+BP)+(CR+DR)=(AS+DS)+(BQ+CQ)$$

$$\Rightarrow AB+CD=AD+BC$$

Hence Proved.

27.



Let PQ be the chimney

Let AB be the tower where AB = 40 m

Also, given that  $\angle APB = 30^\circ$  and  $\angle PAQ = 60^\circ$

Let us consider the triangle ABP

Thus, we have,

$$\tan 30^\circ = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{AP}$$

$$AP = 40\sqrt{3}m$$

Now, we shall consider the triangle APQ

Hence, we have,

$$\tan 60^\circ = \frac{PQ}{AP}$$

$$\sqrt{3} = \frac{PQ}{40\sqrt{3}}$$

$$120 = PQ$$

Thus, the **height of the chimney is 120 m**

Since, the minimum height of a smoke emitting chimney should be 100 m and the height of the chimney is 120 m.

Thus, the height of the chimney meets the pollution norms.

28. For the cubical block : –

Edge of the cube,  $a = 7$  cm

---

For the hemisphere : –

Diameter of the hemisphere = Length of the side of the cube

$$\text{Radius of the hemisphere} = \frac{7}{2}$$

Surface area of the remaining solid = Surface area of the cube - Surface area of the circle on one of the faces of the cube + Surface area of the hemisphere.

$$\rightarrow 6a^2 - \pi r^2 + 2\pi r^2$$

$$\rightarrow 6a^2 + \pi r^2$$

$$\rightarrow 6 \times 7^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$\rightarrow 294 + 38.5 \text{ cm}^2$$

$$\rightarrow 332.5 \text{ cm}^2$$

29.

In an A.P , the sum of n terms is  $S_n = n[2a + (n-1)d]$ .

where,

a = First term

d = Common difference

n = Number of terms in an A.P.

We have to prove that,  $S_{30} = 3(S_{20} - S_{10})$ .

Take R.H.S.,  $3(S_{20} - S_{10})$

$$= 3 [20\{2a + (20-1)d\} - 10\{2a + (10-1)d\}]$$

$$= 3[10(2a + 19d) - 5(2a + 9d)]$$

$$= 3[20a + 190d - 10a - 45d]$$

$$= 3[10a + 145d]$$

$$= 15[2a + 29d]$$

Now take L.H.S., we get

$$S_{30} = 30 \cdot 2[2a + (30-1)d]$$

$$S_{30} = 15[2a + 29d]$$

$\therefore$  L.H.S. = R.H.S.

Hence, proved.

30. (i). volume of frustum of cone

$$= \frac{1}{3} \cdot \pi \cdot h \cdot (R^2 + Rr + r^2)$$

$$= \frac{1}{3} \cdot 22/7 \cdot 24 \cdot (14^2 + 14 \cdot 7 + 7^2)$$

$$= 22 \cdot 8/7 \cdot (196 + 98 + 49)$$

$$= 22 \cdot 8/7 \cdot 343$$

$$= 22 \cdot 8 \cdot 49$$

$$= 8624 \text{ cm}^3$$

(ii). slant height of frustum l

$$= \sqrt{(R - r)^2 + h^2}$$

$$= \sqrt{(14 - 7)^2 + 24^2}$$

$$= \sqrt{7^2 + 24^2}$$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625}$$

$$= 25 \text{ cm}$$

area of metallic sheet = CSA of frustum + area of base

$$= \pi \cdot l \cdot (R + r) + \pi \cdot r^2$$

$$= 22/7 \cdot 25 \cdot (14 + 7) + 22/7 \cdot 7^2$$

$$= 550/7 \cdot 21 + 22 \cdot 7$$

$$= 550 \cdot 3 + 154$$

$$= 1650 + 154$$

$$= 1804 \text{ cm}^2$$



31. Let the consecutive even numbers be  $x$  &  $x + 2$ .

A/q

Square of these number is 340.

$$(x)^2 + (x + 2)^2 = 340$$

$$x^2 + x^2 + 2^2 + 2 \cdot x \cdot 2 = 340$$

$$2x^2 + 4x = 340 - 4$$

$$2x(x + 2) = 336$$

$$x(x + 2) = 336/2$$

$$x^2 + 2x = 168$$

$$x^2 + 2x - 168 = 0$$

Now, By using the middle term splitting method.

$$x^2 + 2x - 168$$

$$x^2 + 14x - 12x - 168$$

$$x(x + 14) - 12(x + 14)$$

$$(x + 14)(x - 12)$$

$$(x + 14) = 0 \text{ or, } (x - 12) = 0$$

$$x = -14 \text{ or } x = 12$$

Reject the negative value of  $x$  and take  $x = 12$

Hence, Required number are

$$\Rightarrow x = 12 \text{ and } (x + 2) = 12 + 2 = 14$$

Verification -

$$12^2 + 14^2 = 340$$

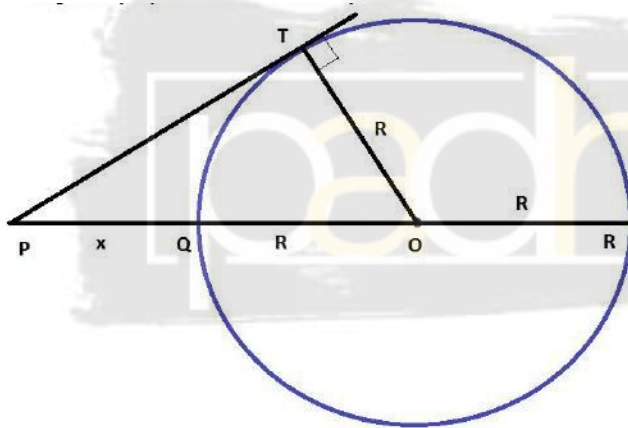
$$144 + 196 = 340$$

$$340 = 340$$

32.

Tangent is a line which touches a circle exactly at one point. All other lines intersect the circle at two points. Take a secant PQOR. Tangent be PT.

Let the radius be R. Let PQ = x.



powerpoint of P wrt circle:

$$\begin{aligned} &= PQ * PR \\ &= x (x + 2R) \\ &= x^2 + 2 R x \\ &= (x + R)^2 - R^2 \\ &= PO^2 - R^2 \end{aligned}$$

So powerpoint of P wrt circle depends only on distance PO and Radius.

Powerpoint of P calculated along PT, (that is Q and R merge at T):

$$= PT * PT$$

Hence, we have :  $PT^2 = PO^2 - R^2$

In the triangle PTO, this means Pythagoras theorem is valid. So the triangle is right angled.

Hence  $PT \perp OT$ .

33. In a die has 6 faces numbered 1,2,3,4,5 and 6.

Number of possible outcomes on rolling the dice twice are

$\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$   
 $(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$

$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

Total number of outcomes = 36.

(i) Favourable outcomes for the event that 5 will not show up either time are

$\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4),$   
 $(3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 6)\}$

Total number of favourable outcomes = 25

$\therefore$  Probability of not getting 5 either time =  $\frac{\text{Number of favorable outcomes}}{\text{Total Number of outcomes}} = \frac{25}{36}$

(ii) Favourable outcomes for the event that 5 will show up exactly one time are

$\{(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 5)\}$

Total number of favourable outcomes = 10

$\therefore$  Probability of getting 5 such that it will come up exactly one time

=  $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{10}{36}$

34.

$$3\left(\frac{3x-1}{2x+3}\right) - 2\left(\frac{2x+3}{3x-1}\right) = 5$$

$$\text{Let } \left(\frac{3x-1}{2x+3}\right) = m$$

$$\text{i.e. } 3m - 2/m = 5$$

$$3m^2 - 2 = 5m$$

$$3m^2 - 5m - 2 = 0$$

$$3m^2 - 6m + m - 2 = 0$$

$$3m(m-2) + (m-2) = 0$$

$$(3m+1)(m-2) = 0$$

$$m = 2 \text{ or } m = -\frac{1}{3}$$

$$\left(\frac{3x-1}{2x+3}\right) = 2 \quad \text{OR} \quad \left(\frac{3x-1}{2x+3}\right) = -\frac{1}{3}$$

$$3x-1/2x+3 = 2$$

$$x = -7$$

$$9x-3 = -2x-3$$

$$x = 0$$

Answer :  $x = -7, 0$

# CBSE Maths 2013

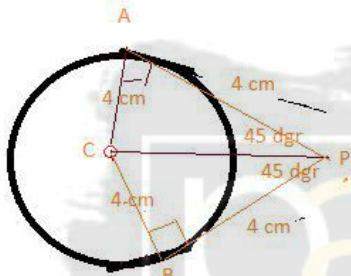
- 1) All questions are compulsory.
- 2) The question paper consists of thirty questions divided into 4 sections A, B, C and D. Section A comprises of ten questions of 01 mark each, Section B comprises of five questions of 02 marks each, Section C comprises ten questions of 03 marks each and Section D comprises of five questions of 06 marks each.
- 3) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- 4) There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.
- 5) In question on construction, drawing should be near and exactly as per the given measurements.
- 6) Use of calculators is not permitted.

Questions -

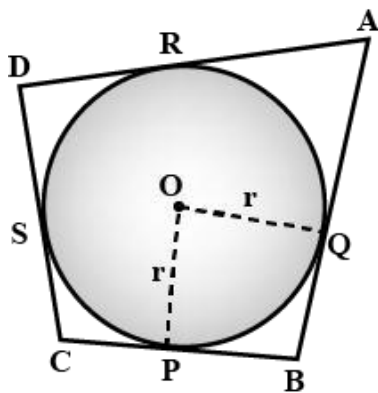
## SECTION A

1. The common difference of the A.P.  $1/p, 1-p/p, 1-2p/p, \dots$  is:

2. In Fig. 1, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If  $PA \perp PB$ , then the length of each tangent is:



3. In Figure,, a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively, If  $AB = 29$  cm,  $AD = 23$  cm,  $\angle B = 90^\circ$  and  $DS = 5$  cm, then the radius of the circle (in cm.) is:

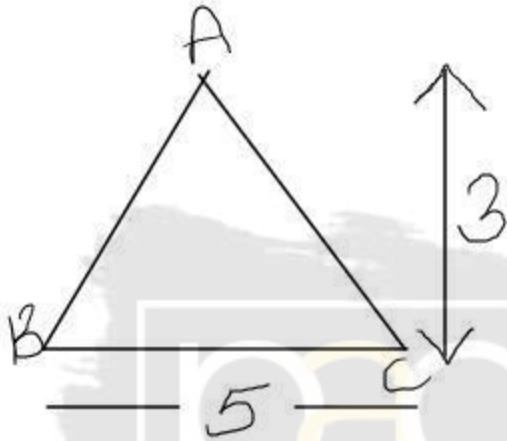


4. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is  $30^\circ$ . The distance of the car from the base of the tower (in m.) is:

5. The probability of getting an even number, when a die is thrown once, is :

6. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a prime-number less than 23, is :

7. In Fig. 3, Find the area of triangle ABC (in sq. units) is:



8. If the difference between the circumference and the radius of a circle is 37 cm, then using  $\pi = 22/7$ , the circumference (in cm) of the circle is:

(A) 154

(B) 44

(C) 14

(D) 7

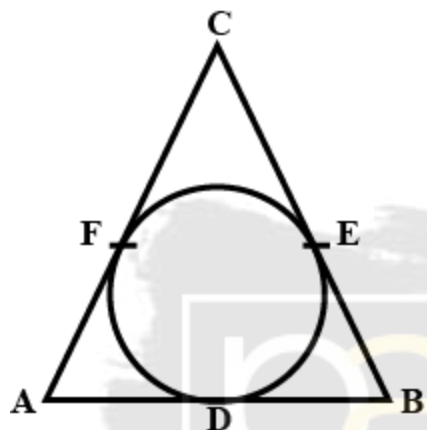
## SECTION B

9. Solve the following quadratic equation for x:

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

10. How many three-digit natural numbers are divisible by 7?

11. In Fig. 4, a circle inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If  $AB = 12$  cm,  $BC = 8$  cm and  $AC = 10$  cm, then find the lengths of AD, BE and CF.



12. Prove that the parallelogram circumscribing a circle is a rhombus.

13. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.

14. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions  $14$  cm  $\times$   $7$  cm. Find the area of the remaining card board. [ Use  $\pi = 22/7$  ]

## SECTION C

15. For what value of  $k$ , are the roots of the quadratic equation  $kx(x - 2) + 6 = 0$  equal?

16. Find the number of terms of the A.P.  $18, 15\frac{1}{2}, 13, \dots, -49\frac{1}{2}$  and find the sum of all its terms.



17. Construct a triangle with sides 5 cm, 4 cm and 6 cm. Then construct another triangle whose sides are  $\frac{2}{3}$  times the corresponding sides of first triangle.

18. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is  $30^\circ$ . If the height of the second pole is 24 m, find the height of the first pole. [ $\sqrt{3} = 1.732$ ]

19. Prove that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.

20. Find the ratio in which the y-axis divides the line segment joining the points (-4, -6) and (10, 12). Also find the coordinates of the point of division.

21. In Fig.5, AB and CD are two diameters of a circle with centre O, which are perpendicular to each other. OB is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.[ use  $\pi = \frac{22}{7}$ ]

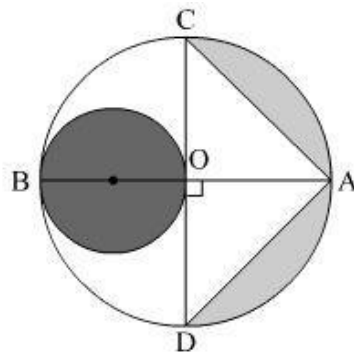


Fig. 5

22. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of the same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. [ $\pi = \frac{22}{7}$ ]

23. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. [ $\pi = \frac{22}{7}$ ]

24. In a circle of radius 21 cm, an arc subtends an angle of  $60^\circ$  at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc. [ Use  $\pi = 22/7$  ]

25. Solve the following for x:

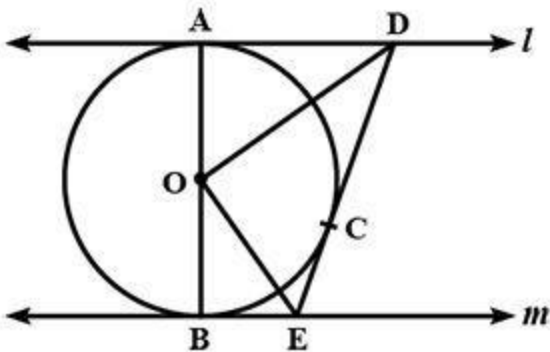
$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

26. Sum of the areas of two squares is  $400 \text{ cm}^2$ . If the difference of their perimeters is 16 cm, find the sides of the two squares.

27. If the sum of first 7 terms of an A.P. is 49 and that of first 17 terms is 289, find the sum of its first n terms.

28. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

29. In fig, l and m are two parallel tangents to a circle with centre O, touching the circle at A and B respectively. Another tangent at C intersects the line l at D and m at E. Prove that  $\angle DOE = 90^\circ$



30. The angle of elevation of the top of a building from the foot of the tower is  $30^\circ$  and the angle of elevation of the top of the tower from the foot of the building is  $60^\circ$ . If the tower is 60 m high, find the height of the building.

31. A group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest. Which of the above values you prefer more?

32. The three vertices of a parallelogram ABCD are A(3, -4), B(-1, -3) and C(-6, 2). Find the coordinates of vertex D and find the area of ABCD.

33. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in the level of water in the tank in half an hour.

34. A bucket open at the top, and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are 30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of Rs 10 per  $100\text{cm}^2$ . [Use  $\pi = 3.14$ ]

## CBSE Maths 2013

1.

The common difference is - 1.

2.

Given that,  $AP \perp PB$ ,  $CA \perp AP$ ,  $CB \perp BP$ .

$$\Rightarrow \angle ACB = 90^\circ.$$

And  $AC = CB$  ( radius of the circle)

$\therefore$  APBC is a square .

Each side of the square is equal to 4 cm.

Therefore, length of each tangent is 4 cm.

3.

The lengths of the tangents drawn from an external point to a circle are equal.

$$DS = DR = 5 \text{ cm}$$

$$\therefore AR = AD - DR = 23 - 5 = 18 \text{ cm}$$

$$AQ = AR = 18 \text{ cm}$$

$$\therefore QB = AB - AQ = 29 - 18 = 11 \text{ cm}$$

$$QB = BP = 11 \text{ cm.}$$

$$\angle PBQ = 90^\circ \text{ [Given]}$$

$\angle OPB = 90^\circ$  [Angle between the tangent and the radius at the point of contact is a right angle.]

$\angle OQB = 90^\circ$  [Angle between the tangent and the radius at the point of contact is a right angle.]

$\angle POQ = 90^\circ$  [Angle sum property of a quadrilateral.]

So, OQBP is a square.

$\therefore QB = BP = r = 11 \text{ cm}$

$\therefore$  the radius of the circle is 11 cm.

4.

in Tri. ABC

$$\tan 30^\circ = AB/BC$$

$$1/\sqrt{3} = 75/BC$$

$$BC = 75\sqrt{3}$$

5.

When a die is thrown once, the sample space S is given by { 1, 2, 3, 4, 5, 6}.

Number of possible outcomes = 6

Let E be the event of getting an even number.  $E = \{2, 4, 6\}$

Number of favourable outcomes = 3

$\therefore$  Probability of getting an even number =  $P(E) =$

Number of favourable outcomes/Number of possible outcomes =  $3/6 = \frac{1}{2}$

6.

Given that the box contains 90 discs.

As one disc is drawn at random from the box, numbered from 1 to 90 i.e 1,2,3,.....90.

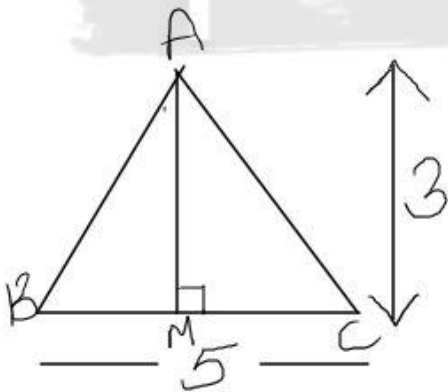
Number of possible outcomes = 90

Let E be the event of getting a prime number less than 23 are 2, 3, 5, 7, 11, 13, 17, and 19.

Number of favourable outcomes = 8

$\therefore P(E) = \text{Number of favourable outcomes} / \text{Number of possible outcomes} = 8/90$   
 $= 4/45.$

7.



Given :  $BC = 5$  unit and  $AM = 3$  unit.

In  $\triangle ABC$ ,  $BC$  is the base and  $AM$  is the height.

$\therefore \text{Area of triangle } ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times BC \times AM$$

$$= \frac{1}{2} \times 5 \times 3$$

$$= 7.5 \text{ sq. units}$$

8.

Given that the difference between circumference and radius of the circle is 37 cm.

Assume the radius of the circle as 'r'.

$$\Rightarrow 2\pi r - r = 37$$

$$\Rightarrow r(2\pi - 1) = 37$$

$$\Rightarrow r(2 \times 22/7 - 1) = 37$$

$$\Rightarrow r \times 37/7 = 37$$

$$\Rightarrow r = 7 \text{ cm}$$

$$\therefore \text{Circumference of the circle} = 2\pi r = 2 \times 22/7 \times 7 = 44 \text{ cm}$$

Ans -> B

9.

Given quadratic equation is

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$\Rightarrow (4x - \sqrt{3})(\sqrt{3}x + 2) = 0$$

$$\therefore x = \frac{\sqrt{3}}{4} \text{ or } x = -\frac{2}{\sqrt{3}}$$

10.

Let three digit numbers from 100 to 999.

All the three-digit natural numbers that are divisible by 7 will be of the form  $7n$ .

$$\therefore 100 \leq 7n \leq 999$$

$$100/7 \leq n \leq 999/7$$

$$14^2/7 \leq n \leq 142^5/7$$

As  $n$  is an integer, number of 3-digit natural numbers that are divisible by 7 are  $142 - 14$  i.e. 128.

11.

We know that  $AD = AF$

$$BD = BE$$

$$CE = CF$$

$$\text{Let } AD = AF = x$$

$$BD = BE = y$$

$$CE = CF = z$$

$$\text{Then } x + y = 12$$

$$y + z = 8$$

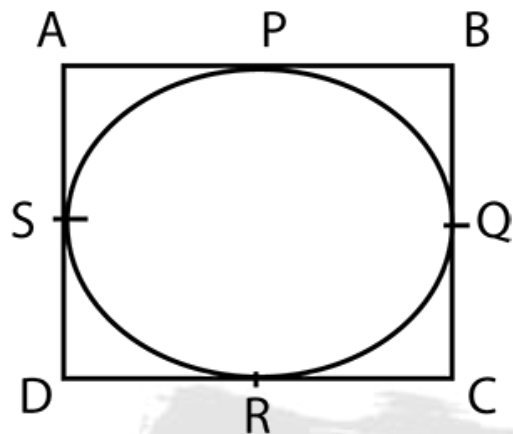
$$x + z = 10$$

On Solving above equation we get  $x = 7, y = 5, z = 3$

So  $AD = 7, BE = 5, CF = 3$



12.



Since ABCD is a parallelogram circumscribed in a circle

$$AB=CD\ldots\ldots(1)$$

$$BC=AD\ldots\ldots(2)$$

$$DR=DS \text{ (Tangents on the circle from same point D)}$$

$$CR=CQ \text{ (Tangent on the circle from same point C)}$$

$$BP=BQ \text{ (Tangent on the circle from same point B)}$$

$$AP=AS \text{ (Tangents on the circle from same point A)}$$

Adding all these equations we get

$$DR+CR+BP+AP=DS+CQ+BQ+AS$$

$$(DR+CR)+(BP+AP)=(CQ+BQ)+(DS+AS)$$

$$CD+AB=AD+BC$$

Putting the value of equation 1 and 2 in the above equation we get

$$2AB=2BC$$

$$AB=BC\ldots\ldots(3)$$

From equation (1), (2) and (3) we get

$$AB=BC=CD=DA$$

∴ ABCD is a Rhombus

13.

Let E be the event of that the drawn card is neither a king nor a queen.

Total number of possible outcomes = 52.

Total number of cards that are king or queen in the pack of playing cards = 4 + 4 = 8.

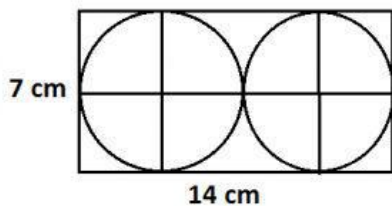
∴ Number of cards that are neither a king nor a queen = 52 - 8 = 44.

Total number of favourable outcomes = 44.

∴ Required probability =  $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{44}{52} = \frac{11}{13}$

∴ The probability that the drawn card is neither a king nor a queen is 11/13.

14.



Dimension of the rectangular card board = 14 cm × 7 cm

Two circular pieces of equal radii and maximum area touching each other are cut from the rectangular card board, therefore, the diameter of each circular piece is 14/2 = 7 cm.

Radius of each circular piece =  $d/2 = 7/2$  cm

∴ Sum of area of two circular pieces =  $2 \times \pi \times \left(\frac{7}{2}\right)^2 = 2 \times \frac{22}{7} \times \frac{49}{4} = 77 \text{ cm}^2$ .

Area of the remaining card board = Area of the card board - Area of two circular pieces.

$$= (14 \times 7) - (77) = 98 - 77 = 21 \text{ cm}^2.$$

∴ The area of the remaining card board is  $21 \text{ cm}^2$ .

15.

Value of  $k = (?)$  equation  $kx(x - 2) + 6 = 0$  roots is equal.

→ equation  $kx(x - 2) + 6 = 0$

$$kx^2 - 2kx + 6 = 0$$

→ two roots of quadratic equation

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha, \beta = \frac{+2k \pm \sqrt{(-2k)^2 - 4 \times 6 \times k}}{2 \times k}$$

$$\alpha, \beta = 2k \frac{\pm \sqrt{4k^2 - 24k}}{2k}$$

→ root is equal,  $\alpha = \beta$

$$\therefore \frac{2k + \sqrt{4k^2 - 24k}}{2k} = \frac{2k - \sqrt{4k^2 - 24k}}{2k}$$

$$\therefore 2\sqrt{4k^2 - 24k} = 0$$

$$\therefore 4k^2 - 24k$$

$$\therefore 4k = 24$$

$$\therefore k = \frac{24}{4}$$

$$\therefore k = 6$$

16.

Given : A.P.  $18, 15\frac{1}{2}, 13, \dots, -49\frac{1}{2}$

First term  $a = 18$ , common difference,  $d = 15\frac{1}{2} - 18 = -2\frac{1}{2}$  and the last term of the A.P.  $= -49\frac{1}{2}$

Assume that A.P. has  $n$  terms.

$$\therefore a_n = a + (n - 1) d$$

$$\Rightarrow -49\frac{1}{2} = 18 - (n - 1) \times 2\frac{1}{2}$$

$$\Rightarrow -\frac{99}{2} = 18 - \frac{5}{2}(n - 1)$$

$$\Rightarrow -99 = 36 - 5(n - 1)$$

$$\Rightarrow 5(n - 1) = 135$$

$$\Rightarrow 5n = 140$$

$$\therefore n = \frac{140}{5} = 28$$

$\therefore$  The given A.P. has 28 terms.

Sum of all the terms ( $S_n$ ):

$$S_n = \frac{n}{2} [2a + (n - 1) d]$$

$$S_{28} = \frac{28}{2} [2 \times 18 + (28 - 1) \times -(\frac{5}{2})]$$

$$S_{28} = 14 [36 - \frac{135}{2}]$$

$$S_{28} = -441$$

$\therefore$  The sum of all the terms of the A.P. is  $= -441$ .

17.

Step 1: Draw a line segment  $AB = 4$  cm. Consider the point A as centre, draw an arc of 5 cm radius.

Similarly, consider the point B as its centre, draw an arc of 6 cm radius.

Both the arcs will intersect each other at C and the required triangle ABC is formed.

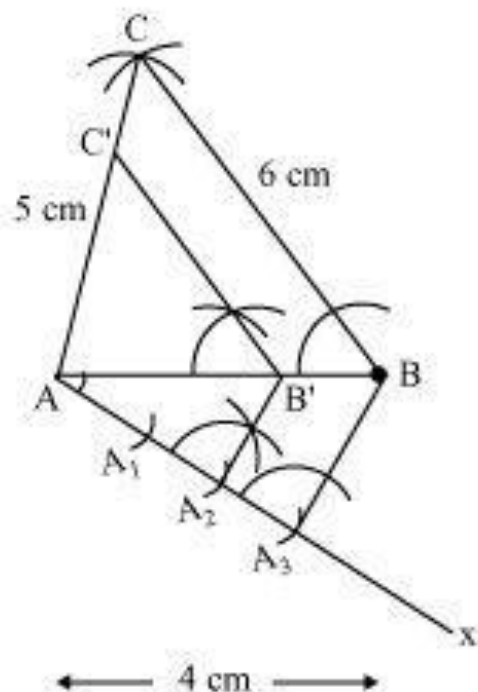
Step 2: Draw a ray AX making an acute angle with the line segment AB on the opposite side of the vertex C.

Step 3: Locate 3 points  $A_1, A_2, A_3$  on line AX such that  $AA_1 = A_1A_2 = A_2A_3$ .

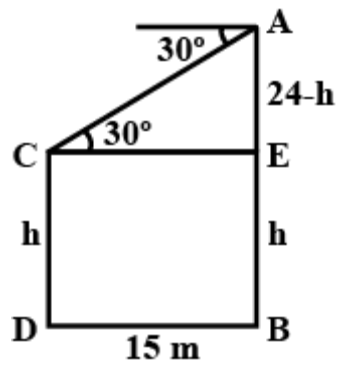
Step 4: Join B and  $A_3$  i.e.  $BA_3$  and draw a line through  $A_2$  parallel to  $BA_3$  to intersect AB at point  $B'$ .

Step 5: Draw a line segment through  $B'$  parallel to BC to intersect AC at  $C'$ .

$\therefore \triangle AB'C'$  is the required triangle.



18.



Distance between two poles is  $BD = 15$  m

Hence,  $CE = 15$  m

Height of second pole,  $AB = 24$  m

Let, height of first pole be  $CD = h$  m

Hence,  $BE = h$  m

Therefore,  $AE = 24 - h$  m

In right angled triangle  $AEC$ ,

$$\tan 30^\circ = \frac{AE}{CE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24 - h}{15}$$

$$\Rightarrow 15 = 24\sqrt{3} - \sqrt{3}h$$

$$\Rightarrow \sqrt{3}h = 24 - \frac{15}{\sqrt{3}}$$

$$\Rightarrow h = 24 - 5\sqrt{3} = 24 - 5 \times 1.732 = 24 - 8.66 = 15.34 \text{ m}$$

Therefore,  $h = 15.34 \text{ m}$

19.

Given : Points (7, 10), (-2, 5) and (3, -4) are vertices of a triangle.

Solution :

Let A(7, 10), B (-2, 5) and C (3, -4)

By using distance formula :  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Vertices : A(7, 10), B (-2, 5)

Length of side AB =  $\sqrt{(-2 - 7)^2 + (5 - 10)^2}$

$$AB = \sqrt{(-9)^2 + (-5)^2}$$

$$AB = \sqrt{81 + 25}$$

$$AB = \sqrt{106} \text{ units}$$

Vertices : B (-2, 5) and C (3, -4)

Length of side BC =  $\sqrt{(3 + 2)^2 + (-4 - 5)^2}$

$$BC = \sqrt{(5)^2 + (-9)^2}$$

$$BC = \sqrt{25 + 81}$$

$$BC = \sqrt{106} \text{ units}$$

Vertices : A(7, 10), C (3, - 4 )

Length of side AC =  $\sqrt{(3 - 7)^2 + (- 4 - 10)^2}$

$$AC = \sqrt{(-4)^2 + (-14)^2}$$

$$AC = \sqrt{16 + 196}$$

$$AC = \sqrt{212} \text{ units}$$

Since the 2 sides AB = BC =  $\sqrt{106}$ .

Therefore  $\Delta$  is an isosceles.

Now, in  $\Delta ABC$ , by using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$(\sqrt{212})^2 = (\sqrt{106})^2 + (\sqrt{106})^2$$

$$212 = 106 + 106$$

$$212 = 212$$

$$\text{Since } AC^2 = AB^2 + BC^2$$

Hence, the given vertices of a triangle is an isosceles right triangle.

20.

Let  $y$ -axis divides the line joining points A(-4, -6) and B(10, 12) in ratio  $y : 1$

Then, as per section formula the coordinates of point which divides the line is  $\frac{10y - 4}{y + 1}, \frac{12y - 6}{y + 1}$

We know that coordinate at  $y$ -axis of point of  $x$  is zero

$$\text{Then, } \frac{10y - 4}{y + 1} = 0$$

$$\Rightarrow 10y - 4 = 0$$

$$\Rightarrow 10y = 4$$



$$\Rightarrow y = \frac{10}{4} = \frac{5}{2}$$

Then, ratio is  $\frac{2}{5} : 1 \Rightarrow 2 : 5$

Substitute the value of  $y$  in  $y$ -coordinates, we get

$$\frac{12\frac{2}{5} - 6}{\frac{2}{5} + 1} = \frac{24 - 30}{2 - 5} = \frac{-6}{-3} = 2$$

Then, coordinates of point which divides the line joining  $A$  and  $B$  is  $(0, 2)$  and ratio  $\frac{2}{5}$ .

21.

Given that  $AB$  and  $CD$  are the diameters of a circle with centre  $O$ .

$\therefore OA = OB = OC = OD = 7$  cm (Radius of the circle)

Area of the shaded region

= Area of the circle with diameter  $OB$  + (Area of the semi-circle  $ACDA$  - Area of  $\triangle ACD$ )

=  $\pi r^2 + (1/2\pi r^2 - 1/2 \times CD \times OA)$

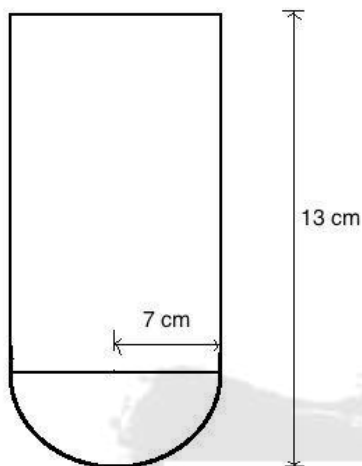
=  $22/7 \times (7/2)^2 + (1/2 \times 22/7 \times 49 - 1/2 \times 14 \times 7)$

=  $(77/2 + 77 - 49)$  cm<sup>2</sup>

=  $66.5$  cm<sup>2</sup>

$\therefore$  The area of the shaded region is  $66.5$  cm<sup>2</sup>.

22.



Radius = 7 cm

Height of cylindrical portion =  $13 - 7 = 6$  cm

Curved surface area of cylindrical portion can be calculated as follows:

$$\begin{aligned} & 2\pi rh \\ &= 2 \times \frac{22}{7} \times 7 \times 6 \\ &= 264 \text{ cm}^2 \end{aligned}$$

Curved surface area of hemispherical portion can be calculated as follows:

$$\begin{aligned} & 2\pi r^2 \\ &= 2 \times \frac{22}{7} \times (7)^2 \\ &= 308 \text{ cm}^2 \end{aligned}$$

Then, total surface area of the vessel =  $308 + 264 = 572$  sq cm

23.

Given height of cylinder = 10 cm

Radius of cylinder = 3.5 cm

The volume of toy = Volume of cylinder - 2 × Volume of a hemisphere

$$\begin{aligned} &= \pi r^2 h - (2 \times \frac{2}{3} \pi r^3) \\ &= [\pi (3.5)^2 \times 10] - (2 \times \frac{2}{3} \times \pi (3.5)^3) = 205.251 \text{ cm}^3 \end{aligned}$$

24.

Given that the radius of the circle = 21 cm.

Measure of the angle subtended by the arc at the centre =  $60^\circ$ .

(i) Length of the arc

$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

where  $r = 21$  cm and  $\theta = 60^\circ$

$$= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$$

$$= 22 \text{ cm.}$$

(ii) Area of the sector formed by the arc

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

$r = 21$  cm and  $\theta = 60^\circ$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (21)^2$$

$$= 231 \text{ cm}^2.$$

25.

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

$$\frac{2x}{2x(2a+b+2x)} - \frac{1(2a+b+2x)}{2x(2a+b+2x)} = \frac{b}{2a(b)} + \frac{2a}{b(2a)}$$

$$\frac{2x-2a-b-2x}{4ax+2bx+4x^2} = \frac{2a+b}{2ab}$$

$$\frac{-2a-b}{4ax+2bx+4x^2} = \frac{2a+b}{2ab}$$

$$\frac{(-1)(2a+b)}{4ax+2bx+4x^2} = \frac{2a+b}{2ab}$$

$$\frac{-(2ab)}{1} = \frac{2a+b}{2a+b} \times \frac{4ax+2bx+4x^2}{1}$$

$$-2ab = 4ax + 4x^2 + 2xb$$

$$0 = 4ax + 4x^2 + 2xb + 2ab$$

$$0 = 4x(a + x) + 2b(x + a)$$

$$0 = (x + a)(4x + 2b)$$

[By Zero Product Rule]

$$x + a = 0 \quad \text{OR} \quad 4x + 2b = 0$$

$$x = -a \quad \text{OR} \quad 4x = -2b$$

$$x = -a \quad \text{OR} \quad x = -b/2$$

26.

Assume that the sides of the two squares be  $x$  cm and  $y$  cm where  $x > y$ .

The areas of the squares are  $x^2$  and  $y^2$  and their perimeters are  $4x$  and  $4y$ .

Given that,  $x^2 + y^2 = 400$  ... (1)

$$\text{and } 4x - 4y = 16$$

$$\Rightarrow 4(x - y) = 16 \Rightarrow x - y = 4 \dots (2)$$

$$\Rightarrow x = y + 4$$

Substituting the value of  $x$  in eqn (1), we get  $(y+4)^2 + y^2 = 400$

$$\Rightarrow y^2 + 16 + 8y + y^2 = 400$$

$$\Rightarrow 2y^2 + 8y + 16 = 400$$

$$\Rightarrow y^2 + 4y - 192 = 0$$

$$\Rightarrow y^2 + 16y - 12y - 192 = 0$$

$$\Rightarrow y(y + 16) - 12(y + 16) = 0$$

$$\Rightarrow (y + 16)(y - 12) = 0$$

$$\Rightarrow y = -16 \text{ or } y = 12$$

Since the value of  $y$  cannot be negative so the value of  $y = 12$ .

$$\text{Value of } x = y + 4 = 12 + 4 = 16.$$

$\therefore$  The sides of the two squares are 16 cm and 12 cm.

27.

Sum of first 7 terms = 49

$$\Rightarrow 7/2(2a+6d)=49$$

$$\Rightarrow a+3d=7$$

$\Rightarrow$  Sum of first 17 terms = 289

$$\Rightarrow 17/2(2a+16d)=289$$

$$\Rightarrow a+8d=17$$

$$\Rightarrow (a+8d)-(a+3d)=17-7$$

$$\Rightarrow 5d=10$$

$$\Rightarrow d=2$$

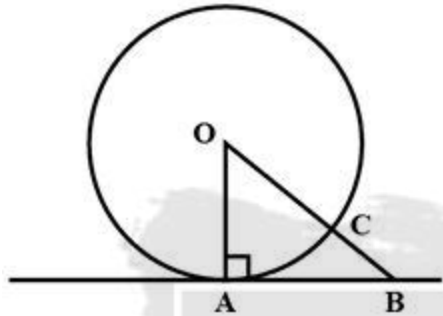
$$\Rightarrow a=1$$

Sum of first  $n$  terms

$$=2n(2+2n-2)$$

$$=n^2$$

28.



Referring to the figure:

$OA = OC$  (Radii of circle)

Now  $OB = OC + BC$

$\therefore OB > OC$  (OC being radius and B any point on tangent)

$\Rightarrow OA < OB$

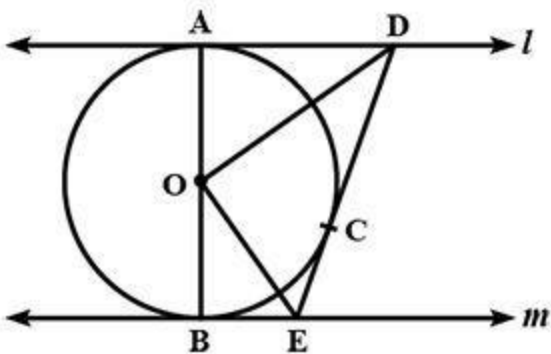
B is an arbitrary point on the tangent.

Thus, OA is shorter than any other line segment joining O to any point on tangent.

Shortest distance of a point from a given line is the perpendicular distance from that line.

Hence, the tangent at any point of circle is perpendicular to the radius.

29.



Join OC

In triangle ODA and triangle ODC

OA=OC (Radii of the same circle )

AD=DC (Length of tangent drawn from an external point to a circle are equal)

DO=OD (common side)

$$\triangle DOA \cong \triangle ODC$$

$$\therefore \angle DOA = \angle COD$$

$$\triangle DOA \cong \triangle ODC$$

$$\therefore \angle DOA = \angle COD$$

Similarly  $\triangle OEB \cong \triangle OEC$

$$\therefore \angle EOB = \angle COE$$

AOB is a diameter of the circle.

Hence, it is a straight line.

$$\therefore \angle DOA + \angle COD + \angle COE + \angle EOB = 180$$

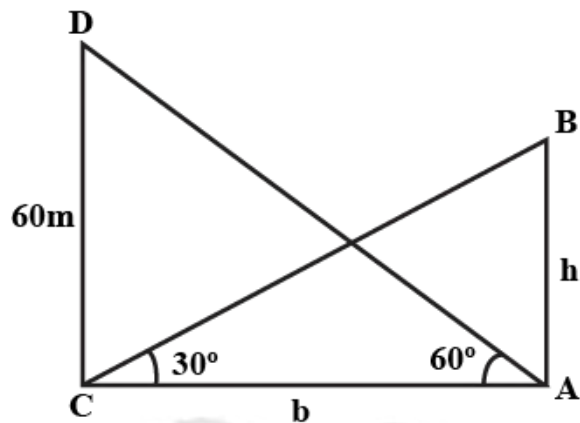
$$\Rightarrow 2\angle COD + 2\angle COE = 180$$

$$\Rightarrow \angle COD + \angle COE = 90$$

$$\Rightarrow \angle DOE = 90^\circ$$

Hence proved.

30.



Angle of elevation of the top of a building from the foot of a tower  $= 30^\circ$

Angle of elevation of the top of a the tower from the foot of the building  $= 60^\circ$

Height of tower  $= 60\text{m}$

Let the height of the building be ' $h$ 'm

Let the distance the tower and building be ' $b$ 'm

In  $\triangle ABC$

$$\tan 30^\circ = \frac{h}{b}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{b}$$

$$h = \frac{b}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \times \frac{60}{\sqrt{3}}\text{m}$$

$$= \frac{60}{3}\text{m}$$

$$= 20\text{m}$$

In  $\triangle ADC$

$$\tan 60^\circ = \frac{60\text{m}}{b}$$

$$\sqrt{3} = \frac{60\text{m}}{b}$$

$$b = \frac{60}{\sqrt{3}}\text{m}$$



∴ height of the building = 20m

31.

Given that the group consists of 12 persons.

∴ Total number of possible outcomes = 12

Assume  $E_1$  as the event of selecting persons who are extremely patient.

∴ Number of favourable outcomes to  $E_1$  is 3.

Assume  $E_2$  as the event of selecting persons who are extremely kind or honest.

Number of people who are extremely honest is 6.

Number of persons who are extremely kind =  $12 - (6 + 3) = 3$

∴ Number of favourable outcomes to  $E_2 = 6 + 3 = 9$ .

(i) Probability of selecting a person who is extremely patient is  $P(E_1)$ .

$P(E_1) = \text{Number of outcomes favourable to } E_1 / \text{Total number of possible outcomes} = 3/12 = 1/4$ .

(ii) Probability of selecting a person who is extremely kind or honest is

$P(E_2) = \text{Number of outcomes favourable to } E_2 / \text{Total number of possible outcomes} = 9/12 = 3/4$ .

32.

Given  $A(3, -4), B(-1, -3), C(-6, 2)$

$$O \text{ is M.P of } AC = \left( \frac{3-6}{2}, \frac{-4+2}{2} \right) = \left( -\frac{3}{2}, -1 \right)$$

$$\therefore O = \left(-\frac{3}{2}, -1\right)$$

Similarly,  $O$  is MP of  $BD$

$$\Rightarrow \frac{-3}{2} = \frac{-1+a}{2}, -1 = \frac{-3+6}{2}$$

$$\therefore a = -2, b = +1$$

$$\therefore D = (-2, 1)$$

$$\therefore \text{Area of parallelogram } ABCD = \text{Ar of } \triangle ABC + \text{Ar } \triangle ADC$$

$$\text{Area of } \triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[3(-3 - 2) - 1(2 + 4) - 6(4 + 3)]$$

$$= \frac{1}{2}[-15 - 6 + 6] = \frac{|15|}{2} = \frac{15}{2}$$

$$\text{Area of } \triangle ADC = \frac{1}{2}[3(1 - 2) - 2(2 + 4) - 6(-4 - 1)]$$

$$= \frac{1}{2}[-3 - 12 + 30] = \frac{15}{2}$$

$$\therefore \text{Area of parallelogram } ABCD = \frac{15}{2} + \frac{15}{2} = 15$$

33.

Given:

Water is flowing through a cylindrical pipe of internal diameter 2cm into a cylindrical tank of base radius 40cm, at rate of 0.4m/second.

To find:

The rise in level of water in the tank in half an hour.

We have,

Internal diameter of cylindrical pipe= 2cm

Radius of cylindrical tank,  $[R] = 40\text{cm}$

The length of water flowing in 1 second= 0.4m

We know that 1m= 100cm

So,

$$0.4\text{m} = (0.4 \times 100)\text{cm}$$

40cm length of water flowing in 1 second.

Internal radius of cylindrical pipe,  $[r] = \frac{2}{2}\text{cm}$

Internal radius of cylindrical pipe,  $[r] = 1\text{cm}$ .

According to the question:

Length of water flow in half an hour,  $[h] = 30\text{minutes}$

Length of water flow in second=  $(30 \times 40 \times 60)\text{cm}$

Length of water flow in second= 72000cm

Now,

We know that formula of the volume of cylinder:  $\pi r^2 h$  cubic units

Volume of water in cylindrical tank = Volume of water flow from pipe

$$\pi R^2 H = \pi r^2 h$$

$$R^2 H = r^2 h$$

$$(40\text{cm})^2 \times H = (1\text{cm})^2 \times 72000\text{cm}$$

$$1600\text{cm}^2 \times H = 72000\text{cm}^2$$

$$H = \frac{72000}{1600} \text{cm}$$

$$H = 45\text{cm}$$

34.

GIVEN :

Diameter of upper circular end of a bucket = 30 cm

Radius of the upper end of the bucket ,  $R = 30/2 = 15$  cm

Diameter of lower circular end of a bucket = 10 cm

Radius of the lower end of the bucket ,  $R = 10/2 = 5$  cm

Height of the bucket,  $h = 24$  cm

Slant height of a bucket ,  $l = \sqrt{(R - r)^2 + h^2}$

$$l = \sqrt{(15 - 5)^2 + 24^2}$$

$$l = \sqrt{10^2 + 576}$$

$$l = \sqrt{100 + 576}$$

$$l = \sqrt{676}$$

$$l = 26 \text{ cm}$$

Slant height of a bucket,  $l = 26$  cm

Surface area of the bucket = Curved surface area of a bucket + Area of the smaller circular base

$$= \pi(R+r)l + \pi r^2$$

$$= \pi(15+5) \times 26 + \pi \times 5^2$$

$$= \pi(20 \times 26 + 25)$$

$$= \pi(520 + 25)$$

$$= \pi \times 545$$

$$= 3.14 \times 545$$

$$= 1711.3 \text{ cm}^2$$

Surface area of the bucket =  $1711.3 \text{ cm}^2$

Cost of metal sheet used @ of ₹10 per  $100 \text{ cm}^2 = 1711.3 \text{ cm}^2 \times 10/100$

$$= 1711.3 \times 0.1 = ₹ 171.13$$

Hence, the cost of metal sheet used is ₹ 171.13.

-----X-----X-----

# CBSE Maths 2012

- 1) All questions are compulsory.
- 2) The question paper consists of thirty questions divided into 4 sections A, B, C and D. Section A comprises of ten questions of 01 mark each, Section B comprises of five questions of 02 marks each, Section C comprises ten questions of 03 marks each and Section D comprises of five questions of 06 marks each.
- 3) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- 4) There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.
- 5) In question on construction, drawing should be near and exactly as per the given measurements.
- 6) Use of calculators is not permitted.

## Questions -

1. The length of shadow of a tower on the plane ground is 3 times the height of the tower. The angle of elevation of sun is: (A) 45 (B) 30 (C) 60 (D) 90
2. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, then the diameter of the larger circle (in cm) is: (A) 34 (B) 26 (C) 17 (D) 14
3. If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is: (A) 1 : 2 (B) 2 : 1 (C) 1 : 4 (D) 4 : 1
4. Two dice are thrown together. The probability of getting the same number on both dice is: (A)  $\frac{1}{2}$  (B)  $\frac{1}{3}$  (C)  $\frac{1}{6}$  (D)  $\frac{1}{12}$
5. The coordinates of the point P dividing the line segment joining the points A(1,3) and B(4,6) in the ratio 2 : 1 are: (A) (2,4) (B) (3,5) (C) (4,2) (D) 5,3

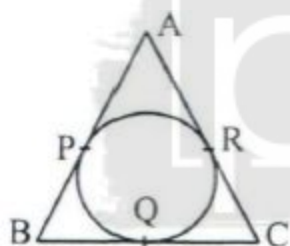
6. If the coordinates of the one end of a diameter of a circle are (2,3) and the coordinates of its centre are (-2,5), then the coordinates of the other end of the diameter are: (A) (-6,7) (B) ( 6,-7) (C) (6,7) (D) (-6,-7)

7. The sum of first 20 odd natural number is : (A) 100 (B) 210 (C) 400 (D) 420

8. If 1 is a root of the equations  $ay^2 + ay + 3 = 0$  and  $y^2 + y + b = 0$ , then ab equals:

(A) 3 (B) -7/2 (C) 6 (D) -3

9. In Fig., the sides AB, BC and CA of a triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then the length of BC (in cm) is: (A) 11 (B) 10 (C) 14 (D) 15



10. In Fig., a circle touches the side DF of EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of EDF (in cm) is: (A) 18 (B) 13.5 (C) 12 (D) 9



## SECTION B

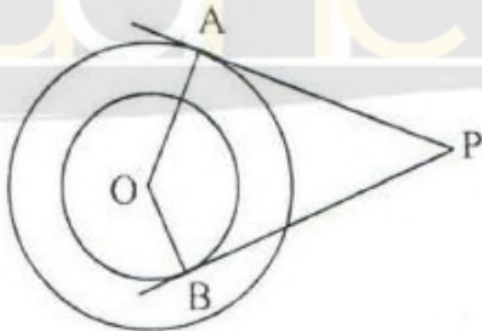
11. If a point  $A(0,2)$  is equidistant from the points  $B(3,p)$  and  $C(p,5)$  then find the value of  $p$ .

12. A number is selected at random from the first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

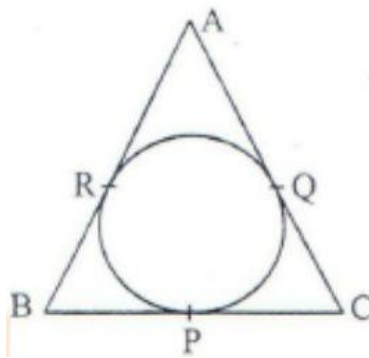
13. The volume of a hemisphere is  $2425\frac{1}{2} \text{ cm}^3$ . Find its curved surface area.

[Use  $\pi = 22/7$ ]

14. Tangents  $PA$  and  $PB$  are drawn from an external point  $P$  to two concentric circles with centre  $O$  and radii 8 cm and 5 cm respectively, as shown in Fig., If  $AP = 15$  cm, then find the length of  $BP$ .



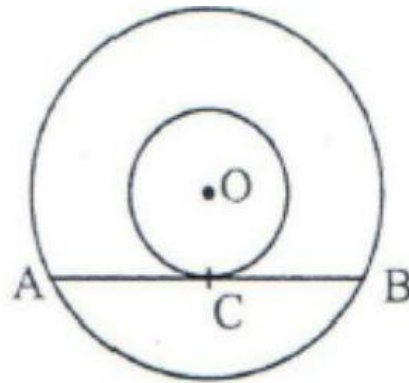
15. In fig., an isosceles triangle  $ABC$ , with  $AB = AC$ , circumscribes a circle. Prove that the point of contact  $P$  bisects the base  $BC$ .



OR

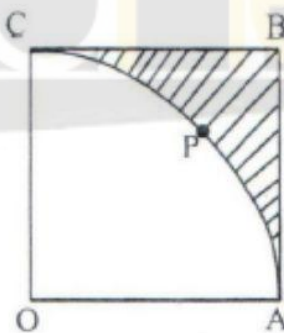
In fig., the chord  $AB$  of the larger of the two concentric circles, with centre  $O$ , touches the smaller circle at  $C$ . Prove that  $AC = CB$





16. In fig., OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region.

[Use  $\pi = 22/7$ ]



17. Find the sum of all three digit natural numbers, which are multiples of 7.

18. Find the values (s) of k so that the quadratic equation  $3x^2 - 2kx + 12 = 0$  has equal roots.

### SECTION C

19. A point P divides the line segment joining the points A(3,-5) and B(-4,8) such that  $AP/PB = K/1$ . If P lies on the line  $x + y = 0$ , then find the value of K.

20. If the vertices of a triangle are (1,-3), (4,p) and (-9,7) and its area is 15 sq. units, find the value (s) of p.

21. Prove that the parallelogram circumscribing a circle is a rhombus.

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

22. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid.

[Use  $\pi = 22/7$ ]

OR

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.

23. In fig., PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If  $\angle POQ = 30^\circ$ , then the area of the shaded region is?

24. Solve for x:  $4x^2 - 4ax + (a^2 - b^2) = 0$

25. A kite is flying at a height of 45 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is  $60^\circ$ . Find the length of the string assuming that there is slack in the string.

26. Draw a triangle ABC with side BC = 6 cm,  $\angle C = 30^\circ$  and  $\angle A = 105^\circ$ . Then construct another triangle whose sides are  $2/3$  times the corresponding sides of triangle ABC.

27. The 16th term of an AP is 1 more than twice its 8th term. If the 12th term of the AP is 47, then find its  $n^{\text{th}}$  term.

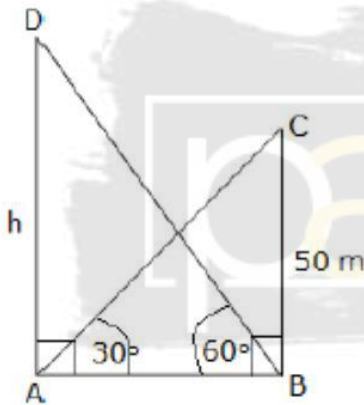
28. A card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) the queen of diamonds.

#### SECTION D

29. A bucket is in the form of a frustum of a cone and its can hold 28.49 litres of water. If the radii of its circular ends are 28 cm and 21 cm, find the height of the bucket.

[Use  $\pi = 22/7$ ]

30. The angle of elevation of the top of a hill at the foot of a tower is  $60^\circ$  and the angle of depression from the top of the tower of the foot of the hill is  $30^\circ$ . If the tower is 50 m high, find the height of the hill.



31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

32. A shopkeeper buys some books for ₹80. If he had bought 4 more books for the same amount, each book would have cost ₹1 less. Find the number of books he bought.

33. Sum of the first 20 terms of an AP is -240, and its first term is 7. Find its 24th term.

34. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid.

[Use  $\pi = 22/7$ ]

# CBSE Maths 2012

Solutions -

1. B
2. B
3. C
4. C
5. B
6. A
7. C
8. A
9. B
10. A

11. It is given that the point A (0, 2) is equidistant from the points B(3, p) and C(p, 5).

So,  $AB = AC$ ,  $AB^2 = AC^2$

Using distance formula, we have:

$$\Rightarrow (0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$$

$$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$$

$$\Rightarrow 4 - 4p = 0$$

$$\Rightarrow 4p = 4$$

$$\Rightarrow p = 1$$

Hence, the value of  $p = 1$ .

12. The total number of outcomes is 50. Favourable outcomes = {12, 24, 36, 48}

Required probability =  $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{50} = \frac{2}{25}$

13. Given volume of a hemisphere:  $2425\frac{1}{2} \text{ cm}^3 = 4851/2 \text{ cm}^3$

Now, let  $r$  be the radius of the hemisphere

Volume of a hemisphere  $= \frac{2}{3} \times \pi r^3$

$$= \frac{2}{3} \times \pi r^3 = 4851/2$$

$$= \frac{2}{3} \times \frac{22}{7} \times r^3 = 4851/2$$

$$R^3 = 4851/2 \times 3/2 \times 7/22 = (21/2)^3$$

Therefore  $r = 21/2 \text{ cm}$

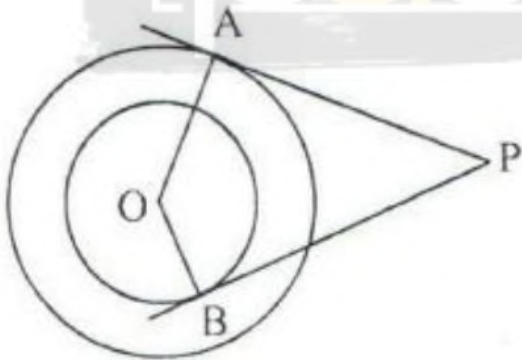
So, Curved surface area of the hemisphere  $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 693 \text{ sq.cm}$$

14. Given: Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii OA = 8 cm, OB = 5 cm respectively. Also, AP = 15 cm

To find: Length of BP

Construction: We join the points O and P.



Solution:  $OA \perp AP$  ;  $OB \perp BP$

[Using the property that radius is perpendicular to the tangent at the point of contact of a circle]

In right angled triangle OAP,

$$OP^2 = OA^2 + AP^2 \quad [\text{Using Pythagoras Theorem}]$$

$$= (8)^2 + (15)^2 = 64 + 225 = 289$$

$$\therefore OP = 17 \text{ cm}$$

In right angled triangle OBP,

$$OP^2 = OB^2 + BP^2$$

$$BP^2 = OP^2 - OB^2$$

$$= (17)^2 - (5)^2 = 289 - 25 = 264$$

$$\therefore BP = \sqrt{264} = 2\sqrt{66} \text{ cm}.$$

15. Given: ABC is an isosceles triangle, where  $AB = AC$ , circumscribing a circle.

To prove: The point of contact P bisects the base BC.

i.e.  $BP = PC$

Proof: It can be observed that

BP and BR ; CP and CQ; AR and AQ are pairs of tangents drawn to the circle from the external points B , C and A respectively.

So, applying the result that the tangents drawn from an external point to a circle, we get

$$BP = BR \text{ --- (i)}$$

$$CP = CQ \text{ --- (ii)}$$

$$AR = AQ \text{ --- (iii)}$$

Given that  $AB = AC$

$$\Rightarrow AR + BR = AQ + CQ$$

$$\Rightarrow BR = CQ \text{ [from (iii)]}$$

$$\Rightarrow BP = CP \text{ [from (i) and (ii)]}$$

$\therefore$  P bisects BC.

Hence proved.

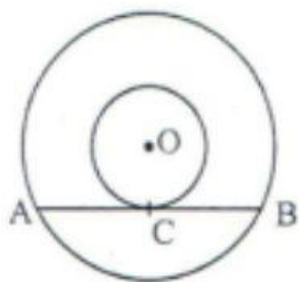
OR

Given: The chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at

C.

To prove:  $AC = CB$

Construction: Let us join OC.



Proof: In the smaller circle, AB is a tangent to the circle at the point of contact C.

$$\therefore OC \perp AB \text{ ---- (i)}$$

(Using the property that the radius of a circle is perpendicular to the tangent at the point of contact)

For the larger circle, AB is a chord and from (i) we have  $OC \perp AB$

$\therefore$  OC bisects AB

(Using the property that the perpendicular drawn from the centre to a chord of a circle bisects the chord)

$$\therefore AC = CB$$

16. Given, OABC is a square of side 7 cm

i.e.  $OA = AB = BC = OC = 7\text{cm}$

$$\therefore \text{Area of square OABC} = (\text{side})^2 = 7^2 = 49 \text{ sq.cm}$$

Given, OAPC is a quadrant of a circle with centre O.

$$\therefore \text{Radius of the sector} = OA = OC = 7 \text{ cm.}$$

Sector angle =  $90^\circ$

$$\therefore \text{Area of quadrant OAPC} = \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 = \frac{77}{2} \text{ sq. cm} = 38.5 \text{ sq. cm}$$

$$\therefore \text{Area of shaded portion} = \text{Area of Square} - \text{OABC Area of quadrant OAPC}$$

$$= (49 - 38.5) \text{ sq. cm} = 10.5 \text{ sq. cm}$$

17. First three- digit number that is divisible by 7 = 105

$$\text{Next number} = 105 + 7 = 112$$

Therefore the series is 105, 112, 119,...

The maximum possible three digit number is 999.

When we divide by 7, the remainder will be 5.

Clearly,  $999 - 5 = 994$  is the maximum possible three – digit number divisible by 7.

The series is as follows:

$$105, 112, 119, \dots, 994$$

$$\text{Here } a = 105, d = 7$$

Let 994 be the  $n$ th term of this A.P

$$a_n = a + (n-1)d$$

$$994 = 105 + (n-1)7$$

$$(n-1)7 = 889$$

$$(n-1) = 127$$

$$n = 128$$

So, there are 128 terms in the A.P.

$$\text{Sum} = n/2 \{\text{first term} + \text{last term}\}$$

$$= 128/2 \{a_1 + a_{128}\}$$

$$= 64\{105+994\} = (64)(1099) = 70336$$

18. Given quadratic equation is  $3x^2 - 2kx + 12 = 0$

$$\text{Here } a = 3, b = -2k \text{ and } c = 12$$

The quadratic equation will have equal roots if  $\Delta = 0$

$$b^2 - 4ac = 0$$

Putting the values of a,b and c we get

$$(2k)^2 - 4(3)(12) = 0$$

$$\Rightarrow 4k^2 - 144 = 0$$

$$\Rightarrow 4k^2 = 144$$

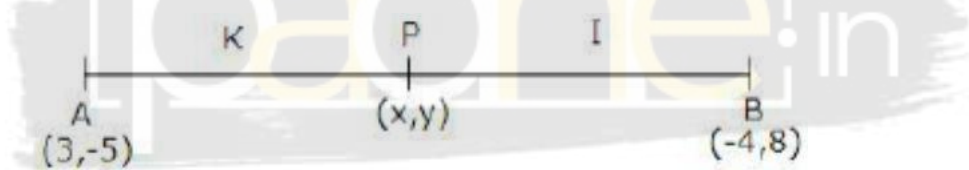
$$\Rightarrow k^2 = \frac{144}{4} = 36$$

Considering square root on both sides,

$$k = \sqrt{36} = \pm 6$$

Therefore, the required values of k are 6 and -6.

19.



Let the co-ordinates of point P be (x, y)

Then using the section formula co-ordinates of P are.

$$x = \frac{-4K+3}{K+1} \quad y = \frac{8K-5}{K+1}$$

Since P lies on  $x+y=0$

$$\frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$$

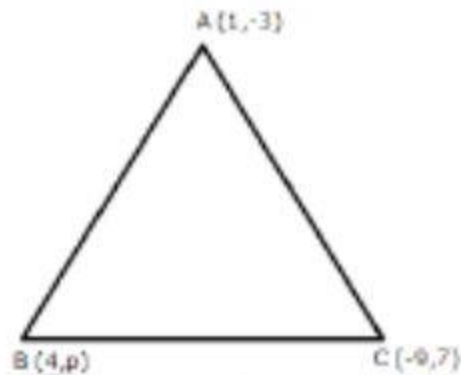
$$4K - 2 = 0$$

$$K = \frac{2}{4}$$

$$K = \frac{1}{2}$$



20.



The area of a triangle, whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} |x_1 y_2 - x_2 y_1 + x_2 y_3 - x_3 y_2 + x_3 y_1 - x_1 y_3|$$

Substituting the given coordinates

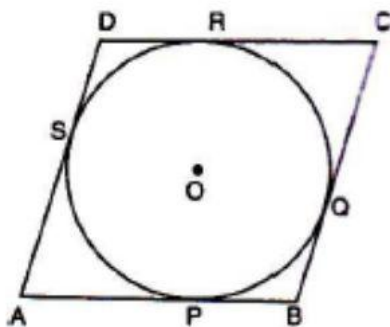
$$\text{Area of triangle} = \frac{1}{2} |1(p-7) + 4(7+3) + (-9)(-3-p)|$$

On solving -

$$p = -3 \text{ or } -9$$

Ans hence the value of  $p = -3$  or  $-9$

21. Let ABCD be a parallelogram such that its sides touching a circle with centre O. We know that the tangents to a circle from an exterior point are equal in length.



$$AP = AS \text{ [From A] ... (i)}$$

$$BP = BQ \text{ [From B] ... (ii)}$$

$$CR = CQ \text{ [From C] ... (iii)}$$

$$\text{and, } DR = DS \text{ [From D] ... (iv)}$$

Adding (i), (ii), (iii) and (iv), we get

$$AP+BP+CR+DR=AS+BQ+CQ+DS$$

$$(AP+BP)+(CR+DR)=(AS+DS)+(BQ+CQ)$$

$$AB+CD = AD+BC$$

$$2 AB = 2 BC$$

[ ABCD is a parallelogram therefore  $AB=CD$  and  $BC = AD$ ]

$$AB=BC$$

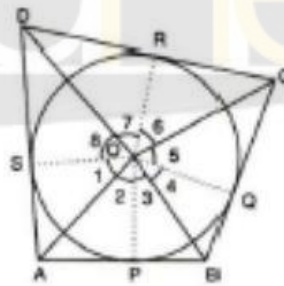
Thus,  $AB=BC=CD=AD$

Hence, ABCD is a rhombus.

OR

A circle with centre O touches the sides AB, BC, CD, and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

TO PROVE :  $\angle AOB + \angle COD = 180$  and,  $\angle AOD + \angle BOC = 180$



CONSTRUCTION

Join OP, OQ, OR and OS.

PROOF

Since the two tangents drawn from an external point to a circle subtend equal angles at the centre.

Therefore  $\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Now  $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

[Sum of all the angles subtended at a point is  $360^\circ$ ]

$$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360 \text{ and } 2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^\circ$$

$$[\angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD]$$

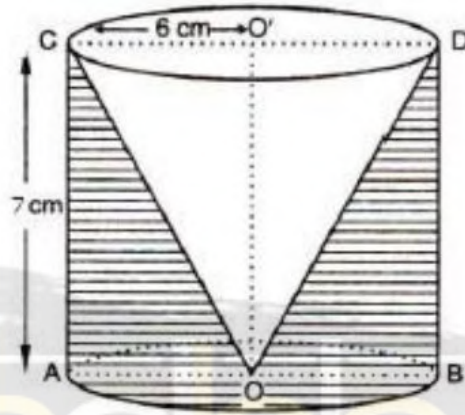
$$[\angle 1 + \angle 8 = \angle AOD \text{ and } \angle 4 + \angle 5 = \angle BOC]$$

$$\angle AOB + \angle COD = 180^\circ$$

$$\angle AOD + \angle BOC = 180^\circ$$

Hence Proved

22.



Given: radius of cyl=radius of cone= $r=6\text{cm}$

Height of the cylinder=height of the cone= $h=7\text{cm}$

Slant height of the cone= $l$

$$\sqrt{7.7 + 6.6}$$

$$= \sqrt{85}$$

Total surface area of the remaining solid

= curved surface area of the cylinder + area of the base of the cylinder + curved surface area of the cone

$$(2\pi rh + \pi r^2 + \pi rl)$$

$$= 2 \times \frac{22}{7} \times 6 \times 7 + \frac{22}{7} \times 6.6 + \frac{22}{7} \times 6 \times \sqrt{85}$$

$$= 377.1 + 132/7 \sqrt{85} \text{ cm}^2$$

OR

Volume of the conical heap=volume of the sand emptied from the bucket.

Volume of the conical heap=

$$\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 \times 24 \text{ cm}^2 \text{ (height of the cone is 24)} \text{-----(1)}$$

Volume of the sand in the bucket =  $\pi r^2 h$

$$= \pi (18)^2 32 \text{ cm}^2 \text{ -----(2)}$$

Equating 1 and 2

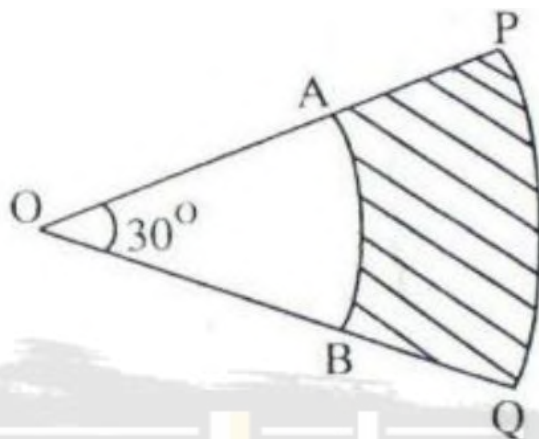
$$\frac{1}{3} \pi r^2 \times 24 = \pi (18)^2 32$$

$$r^2 = \frac{(18)^2 32 \times 3}{24}$$

$$24$$

$$r = 36 \text{ cm}$$

23. [Use  $\pi = 22/7$ ]



Area of the shaded region =  
Area of sector POQ - Area of sector AOB

$$\frac{\theta}{360} \pi R^2 - \frac{\theta}{360} \pi r^2$$

$$\begin{aligned} \text{Area of Shaded region} &= 30/360 \times 22/7 \times (7^2 - 3.5^2) \\ &= 77/8 \text{ cm}^2 \end{aligned}$$

24.

$$4x^2 - 4ax + (a^2 - b^2) = 0$$

$$\Rightarrow (4x^2 - 4ax + a^2) - b^2 = 0$$

$$\Rightarrow [(2x^2) - 2.2x.a + a^2] - b^2 = 0$$

$$\Rightarrow [(2x - a)^2] - b^2 = 0$$

$$\Rightarrow [(2x - a)^2 - b^2][(2x - a) + b] = 0$$

$$[(2x - a) - b] = 0 \text{ or } [(2x - a) + b] = 0$$

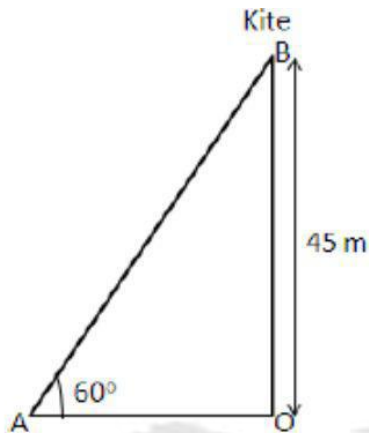
$$x = (a+b)/2 ; \quad x = (a-b)/2$$

25. Given: Position of kite is B.

Height of kite above ground = 45 m

Angle of inclination = 60

Required length of string = AB



In right angled triangle AOB,

$$\sin A = \frac{OB}{AB}$$

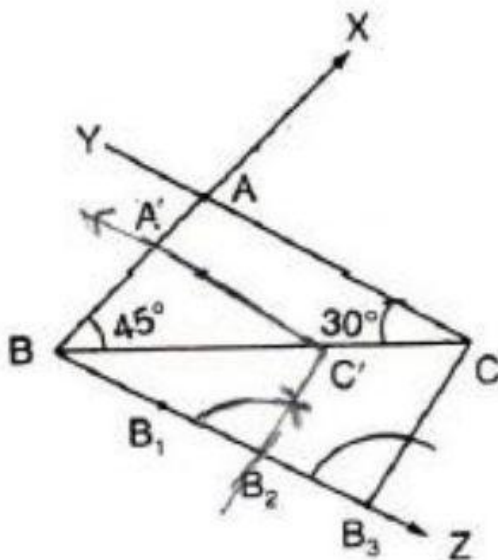
$$\Rightarrow \sin 60^\circ = \frac{45}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{45}{AB}$$

$$\Rightarrow AB = \frac{45 \times 2}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 30\sqrt{3}m$$

Hence, the length of the string is  $30\sqrt{3}$  m

26.



27. Let  $a$  and  $d$  respectively be the first term and the common difference of the AP.

We know that the  $n$ th term of an AP is given by  $a^n = a + (n - 1)d$

According to the given information,

$$A_{16} = 1 + 2 a_8$$

$$a + (16 - 1)d = 1 + 2[a + (8 - 1)d]$$

$$a + 15d = 1 + 2a + 14d$$

$$-a + d = 1 \dots (1)$$

Also, it is given that,  $a_{12} = 47$

$$a + (12 - 1)d = 47$$

$$a + 11d = 47 \dots (2)$$

Adding (1) and (2), we have:

$$12d = 48$$

$$d = 4$$

From (1),

$$-a + 4 = 1$$

$$a = 3$$

$$\text{Hence, } a_n = a + (n - 1)d = 3 + (n - 1)(4) = 3 + 4n - 4 = 4n - 1$$

Hence, the  $n^{\text{th}}$  term of the AP is  $4n - 1$ .

28. Total number of outcomes=52

(i) Probability of getting a red king

Here the number of favourable outcomes=2

$$\text{Probability} = \frac{\text{no.of favourable outcomes}}{\text{total number of outcomes}}$$

$$= 2/52$$

$$= 1/26$$

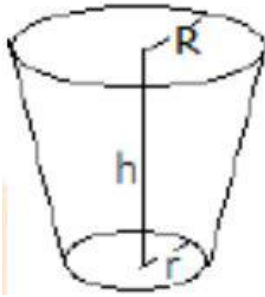
(iii)Probability of queen of diamonds

number of queens of diamond=1,hence

$$\text{Probability} = \frac{\text{no.of favourable outcomes}}{\text{total number of outcomes}}$$

$$1/52$$

29.



Here,  $R = 28$  cm and  $r = 21$  cm, we need to find  $h$ .

Volume of frustum =  $28.49$  L =  $28.49 \times 1000 \text{ cm}^3 = 28490 \text{ cm}^3$

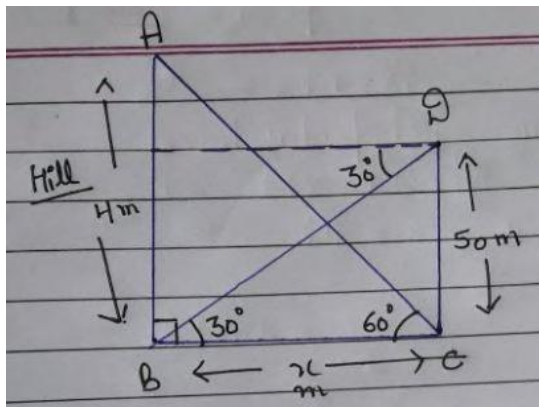
Now, Volume of frustum =  $\frac{\pi h}{3} (R^2 + Rr + r^2)$

On putting values -

$$h = \frac{28490 \times 3}{\pi (28^2 + 28 \times 21 + 21^2)} = 15$$

Height of bucket is  $15$  cm.

30. Given :-



AB is the hill having  $h$  metre height

DC the tower having  $50$  meter height.

Angle ACB is  $60^\circ$  and Angle DBC is  $30^\circ$ .

The height of hill  $H = 50 + y$  m

Taking the triangle DBC .

Here Perpendicular is  $50$  m ( DC ) and Base is  $x$  m (CB)

Angle DCB is  $90^\circ$  and angle DBC is  $30^\circ$ .

$$\Rightarrow \tan(\theta) = \frac{\text{perpendicular}}{\text{base}}$$

$$\Rightarrow \tan(30) = \frac{50}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$$

$$\Rightarrow 50\sqrt{3} = x$$

Now base of triangle DBC is equivalent to the base of triangle ABC.

Now taking triangle ABC

Here Perpendicular is AB( H meter ) and base is CB( $50\sqrt{3}$  m)

Angle ABC is  $90^\circ$  and Angle ACB is  $60^\circ$

$$\Rightarrow \tan(\theta) = \frac{\text{perpendicular}}{\text{base}}$$

$$\Rightarrow \tan(60) = \frac{H}{50\sqrt{3}}$$

$$\Rightarrow \sqrt{3} = \frac{H}{50\sqrt{3}}$$

$$\Rightarrow \sqrt{3} \times 50\sqrt{3} = H$$

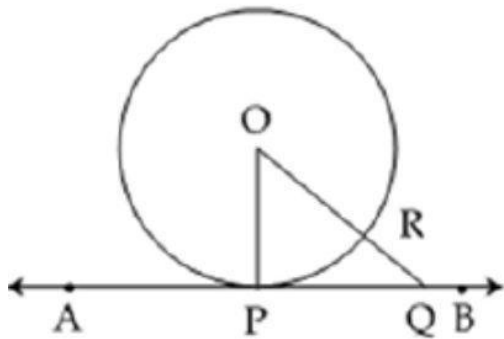
$$\Rightarrow 3 \times 50 = H$$

$$\bullet \Rightarrow \underline{\underline{150 = H}}$$

31. Given: AB is a tangent to a circle with centre O.

To prove: OP is perpendicular to AB.





Construction: Take a point Q on AB and join OQ.

Proof: Since Q is a point on the tangent AB, other than the point of contact P, so Q will be outside the circle.

Let OQ intersect the circle at R.

Now  $OQ = OR + RQ$

$OQ > OR$   $OQ > OP$  [as  $OR = OP$ ]

$OP < OQ$

Thus OP is shorter than any other segment among all and the shortest length is the perpendicular from O on AB.

OP  $\perp$  AB. Hence proved.

32. Let the number of books bought be ' $x$ '.

The cost of each book = Rs.  $\frac{80}{x}$

If he would have bought 4 more books,

The cost of each book = Rs.  $\frac{80}{(x+4)}$

$$\frac{80}{x} - \frac{80}{(x+4)} = 1$$

$$\frac{(80x+320-80x)}{(x^2+4x)} = 1$$

$$x^2 + 4x = 320$$

$$x^2 + 4x - 320 = 0$$

$$x^2 + 20x - 16x - 320 = 0$$

$$x(x + 20) - 16(x + 20) = 0$$

$$(x + 20)(x - 16) = 0$$

$$x + 20 = 0 \text{ OR } x - 16 = 0$$

$$x = 0 - 20 \text{ OR } x = 0 + 16$$

$$x = -20 \text{ OR } x = 16$$

As, the number of books cannot be negative, so, -20 is discarded.

The number of books he bought = 16

33.

Given:  $S_{20} = -240$  and  $a = 7$

$$S_n = n/2 [2a + (n-1)d]$$

$$S_{20} = 20/2 [2 \times 7 + 19d] = -240$$

On solving,

$$10(14 + 19d) = -240$$

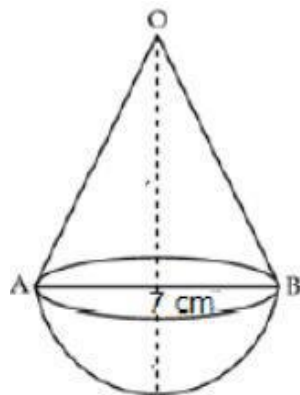
$$19d = -38$$

$$d = -2$$

$$a_{24} = a + 23d = 7 + 23(-2)$$

$$a_{24} = -39$$

34.



Radius of hemi-sphere = 7 cm

Radius of cone = 7 cm

Height of cone = diameter = 14 cm

Volume of solid = Volume of cone + Volume of Hemi-sphere

$$\begin{aligned} &= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \\ &= \frac{1}{3} \pi r^2 (h+2r) \\ &= \frac{1}{3} \times \frac{22}{7} \times 49 (14+14) \\ &= \frac{1}{3} \times \frac{22}{7} \times 49 \times 28 \\ &= 4312/2 \text{ cm}^3 \end{aligned}$$



# CBSE Maths 2011

- 1) All questions are compulsory.

The question paper consists of 34 questions divided into 4 sections A, B, C and D.

- 2) Section A comprises of ten questions of 01 mark each, Section B comprises of eight questions of 02 marks each, Section C comprises ten questions of 03 marks each and Section D comprises of six questions of 06 marks each.

- 3) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

There is no overall choice. However, internal choice has been provided in one

- 4) question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.

- 5) In question on construction, drawing should be near and exactly as per the given measurements.

- 6) Use of calculators is not permitted.

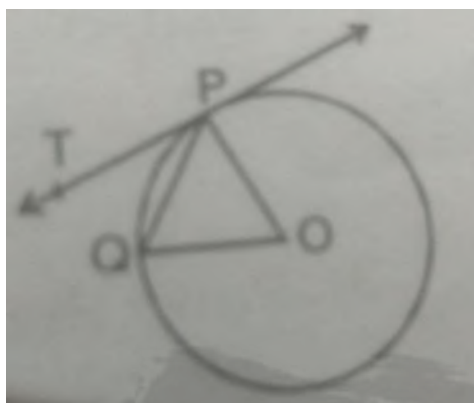
Questions -

## SECTION A

1. The roots of the equation  $x^2 - 3x - m(m+3) = 0$ , where  $m$  is a constant, are:

2. If the common differences of an A.P. is 3, then  $a_{20} - a_{15}$  is:

3. In figure, O is the centre of a circle, PQ is a chord and PT is the tangent at P. If  $\angle POQ = 70^\circ$ , then  $\angle TPQ$  is equal to \_\_\_\_\_.



4. In Figure,, AB and AC are tangents to the circle with centre O such that  $\angle BAC = 40^\circ$ . Then  $\angle BOC$  is equal to \_\_\_\_\_.

5. The perimeter (in cm) of a square circumscribing a circle of radius a cm, is \_\_\_\_\_.

6. The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is \_\_\_\_\_.

7. A tower stands vertically on the ground. From a point on the ground which is 25 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be  $45^\circ$ . Then the height (in meters) of the tower is \_\_\_\_\_ m.

8. If P(a2, 4) is the mid-point of the line-segment joining the points A (-6, 5) and B(-2, 3), then the value of a is \_\_\_\_\_.

9. If A and B are the points  $(-6, 7)$  and  $(-1, -5)$  respectively, then the distance.  $2AB$  is equal to \_\_\_\_\_.

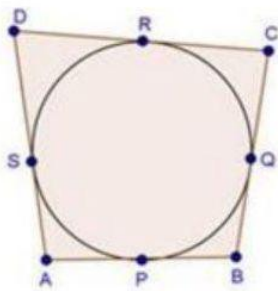
10. A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the card will not be an ace is \_\_\_\_\_.

## SECTION B

11. Find the value of  $m$  so that the quadratic equation  $mx(x - 7) + 49 = 0$  has two equal roots.

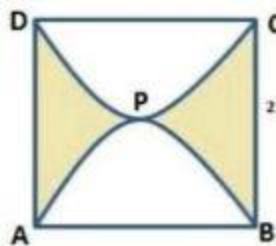
12. Find how many two-digit numbers are divisible by 6.

13. In Figure 3, a circle touches all the four sides of a quadrilateral ABCD whose sides are  $AB = 6$  cm,  $BC = 9$  cm and  $CD = 8$  cm. Find the length of the side AD.



14. Draw a line segment AB of length 7 cm. Using ruler and compasses, find a point P on AB such that  $AP/AB = 3/5$ .

15. Find the perimeter of the shaded region in Figure 4, if ABCD is a square of side 14 cm and APB and CPD are semicircles. [Use  $\pi = 22/7$ ]



16. Two cubes each of volume  $27 \text{ cm}^3$  are joined end to end to form a solid. Find the surface area of the resulting cuboid.

(EITHER Q.15 OR Q.16)

17. Find the value of  $y$  for which the distance between the points  $A(3, -1)$  and  $B(11, y)$  is 10 units.

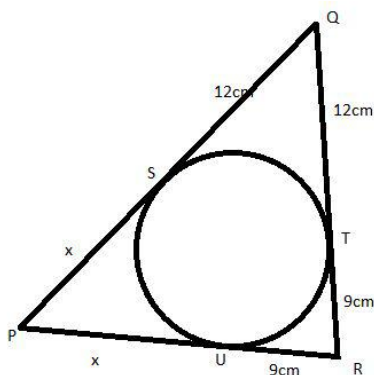
18. A ticket is drawn at random from a bag containing tickets numbered from 1 to 40. Find the probability that the selected ticket has a number which is a multiple of 5.

## SECTION C

19. Find the roots of the following quadratic equation:  $x^2 - 35x + 10 = 0$

20. Find the A.P. whose fourth term is 9 and the sum of its sixth term and thirteenth term is 40.

21. In Figure 5, a triangle PQR is drawn to circumscribe a circle of radius 6 cm such that the segments QT and TR into which QR is divided by the point of contact T, are of lengths 12 cm and 9 cm respectively. If the area of  $\Delta PQR = 189 \text{ cm}^2$ , then find the lengths of sides PQ and PR.



22. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of  $60^\circ$ .

23. A chord of a circle of radius 14 cm subtends an angle of  $120^\circ$  at the centre. Find the area of the corresponding minor segment of the circle. Use  $\pi = \frac{22}{7}$  and  $\sqrt{3} = 1.73$ .

24. An open metal bucket is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at Rs. 30 per litre. [Use  $\pi = \frac{22}{7}$ ]

25. Point  $P(x, 4)$  lies on the line segment joining the points  $A(-5, 8)$  and  $B(4, -10)$ . Find the ratio in which point P divides the line segment AB. Also find the value of x.

26. Find the area of the quadrilateral ABCD, whose vertices are  $A(-3, -1)$ ,  $B(-2, -4)$ ,  $C(4, -1)$  and  $D(3, 4)$ .

27. From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be  $45^\circ$  and  $60^\circ$ . If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [Use  $\sqrt{3} = 1.73$ ]



28. Two dice are rolled once. Find the probability of getting such numbers on the two dice, whose product is 12.

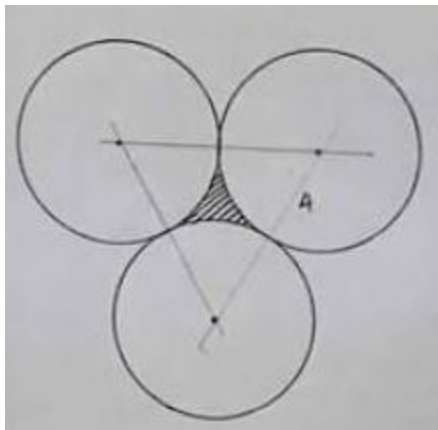
## SECTION D

29. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

30. The first and the last terms of an A.P. are 8 and 350 respectively. If its common difference is 9, how many terms are there and what is their sum?

31. A train travels 180 km at a uniform speed. If the speed had been 9 km/hour more, it would have taken 1 hour less for the same journey. Find the speed of the train.

32. In the given figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these three circles (shaded region). [Use  $\pi = 22/7$ ]



33. Water is flowing at the rate of 15 km/hour through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

34. The angle of elevation of the top of a vertical tower from a point on the ground is  $60^\circ$ . From another point 10 m vertically above the first, its angle of elevation is  $30^\circ$ . Find the height of the tower.



# CBSE Maths 2011

Solutions:

1.

Given quadratic equation:  $x^2 - 3x - m(m+3) = 0$ .

It can be written as

$$x^2 - \{(m+3) - m\}x - m(m+3) = 0$$

$$x^2 - (m+3)x + mx - m(m+3) = 0$$

$$x\{x - (m+3)\} + m\{x - (m+3)\} = 0$$

$$(x+m)\{x - (m+3)\} = 0$$

Therefore,  $x = -m$  and  $x = (m+3)$  are the zeroes of the equation.

2.

Let  $a$  be the first term of the A.P. and  $d$  be the common difference.

$$n\text{th term of an A.P.} = a_n = a + (n-1)d$$

$$a_{20} - a_{15} = [a + (20-1)d] - [a + (15-1)d]$$

$$= 19d - 14d$$

$$= 5d$$

$$= 5 \times 3$$

$$= 15.$$

3.

Consider the triangle in the circle. i.e.,  $\triangle OPQ$ :

$OP = OQ$  (as both are radii of the circle)

$\Rightarrow \angle OQP = \angle OPQ$  (Equal sides of a triangle have equal angles opposite to them)

$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$  (based on the angle sum property of triangles)

$\Rightarrow 70^\circ + 2\angle OPQ = 180^\circ$

$\Rightarrow 2\angle OPQ = 180^\circ - 70^\circ = 110^\circ$

$\Rightarrow \angle OPQ = 55^\circ$

The tangent is perpendicular to the radius through the point of contact.

$\therefore \angle OPT = 90^\circ$

$\Rightarrow \angle OPQ + \angle TPQ = 90^\circ$

$\Rightarrow 55^\circ + \angle TPQ = 90^\circ$

$\Rightarrow \angle TPQ = 90^\circ - 55^\circ = 35^\circ$ .

4.

The tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \angle ABO = \angle ACO = 90^\circ$

In quadrilateral ABOC, according to the angle sum property:

$\angle ABO + \angle BOC + \angle ACO + \angle BAC = 360^\circ$

$\Rightarrow 90^\circ + \angle BOC + 90^\circ + 40^\circ = 360^\circ$

$\Rightarrow \angle BOC + 220^\circ = 360^\circ$

$\Rightarrow \angle BOC = 360^\circ - 220^\circ = 140^\circ$ .

5.

Let the radius of the circle be 'a' cms

The diameter of the circle becomes two times the radius

$$2 \times a \text{ cm} = 2a \text{ cm.}$$

Therefore, the side of the circumscribing square = Diameter of the circle =  $2a \text{ cm}$ .

$$\therefore \text{Perimeter of the circumscribing square} = 4 \times 2a \text{ cm} = 8a \text{ cm.}$$

6.

In a cube, the largest right circular cone has a diameter that is equal to the side of a cube.

Similarly, its height is equal to the side of the cube.

So, if the radius of the cone is  $r \text{ cm}$ ,

$$\therefore 2r = 4.2 \text{ cm.}$$

$$\Rightarrow r = 4.22 \text{ cm} = 2.1 \text{ cm.}$$

7.

In the figure above AB is the tower and C is the point on the ground 25 m away from the foot of the tower in such a way that  $\angle ACB = 45^\circ$ .

Therefore, in  $\triangle ABC$ ,

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\Rightarrow 1 = \frac{AB}{25}$$

$$\Rightarrow AB = 25 \text{ m}$$

Therefore, the height of the tower AB is 25 m.

8.

P(a, 4) is the mid-point of the line segment joining points A (-6, 5) and B (-2, 3).

∴ Coordinates of the point P can be found by using the midpoint formula.

$$P = \left( \frac{-6 + (-2)}{2}, \frac{5 + 3}{2} \right) = (-4, 4)$$

Given that P(a, 4) is the mid-point, therefore

$$(-4, 4) = (a, 4)$$

$$\therefore a = -4$$

$$\Rightarrow a = -4$$

So, the value of a is -4.

9.

Given that the coordinates of the points A and B are (-6, 7) and (-1, -5) respectively.

$$AB = \sqrt{(-6 - (-1))^2 + (7 - (-5))^2}$$

$$= \sqrt{(-6 + 1)^2 + (7 + 5)^2}$$

$$= \sqrt{(-5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

$$\therefore 2AB = 2 \times 13 = 26.$$

Hence, the distance 2AB is 26 units.

10.

Number of ace cards in a deck of 52 playing cards is 4.

∴ Number of non-ace cards in the pack =  $52 - 4 = 48$

Assume E as the event of drawing a non-ace card.

Total number of possible outcomes = 52

Number of favourable outcomes = 48

$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{48}{52} = \frac{12}{13}$ .

11.

we have to find the value of m so that the quadratic equation  $mx(x-7)+49=0$  has two equal roots.

$$mx(x-7)+49=0$$

$$mx^2 - 7mx + 49 = 0$$

Compare above equation with the general quadratic equation  $ax^2 + bx + c = 0$ , we get

$a=m$ ,  $b=-7m$ ,  $c=49$

As we know if the quadratic equation has equal roots then the discriminant will be equals to 0 i.e

$$b^2 - 4ac = 0$$

$$\Rightarrow (-7m)^2 - 4(m)(49) = 0$$

$$\Rightarrow 49m(m-4) = 0$$

implies  **$m=4, 0$**

12.

The two digit numbers that are divisible by 6 are 12, 18, 24, ..., 96

Here, First term =  $a = 12$

Common difference  $d = 18 - 12 = 6$ ,

Last term =  $a_n = 96$

Since,  $n$ th term is given by:

$$a_n = a + (n - 1)d$$

$$96 = 12 + (n - 1)6$$

$$96 - 12 = (n - 1)6$$

$$84 = 6n - 6$$

$$84 + 6 = 6n$$

$$90 = 6n$$

$$n = 15$$

There are 15 two-digit numbers divisible by 6.

13.

Here, ABCD is a quadrilateral.

Let us assume that the quadrilateral ABCD touches the circle at points P, Q, R and S.

Thus, AB, BC, CD and AD are tangents to the circle

Property of tangent states that "*Tangent drawn from an external point to a circle are equal in length*".

Thus, we get

$$AP = AS$$

$$\text{Let } AP = AS = x$$

$$\text{Also, } BP = BQ, CQ = CR \text{ and } DR = DS$$



Since  $AP = x$

Thus,  $BP = AB - AP = 6 - x$

Now,  $BQ = BP = 6 - x$

Also,  $CQ = CB - BQ = 9 - (6 - x) = 3 + x$

Now,  $CR = CQ = 3 + x$

Also,  $DR = CD - CR = 8 - (3 + x) = 5 - x$

Now,  $DS = DR = 5 - x$

Finally,

$AD = AS + DS$

$AD = x + 5 - x$

$AD = 5 \text{ cm}$

Thus, length of side AD is 5cm

14.

Step of Construction :

1. Draw a line segment  $AB = 7\text{cm}$

2. Draw a line from  $A$  making an acute angle with line segment  $AB$ .

3. Taking  $A$  as centre draw an arc cutting  $A_1$  on the line.

And with the same radius consider  $A_1$  as a centre and draw another arc cutting line at  $A_2$ .

Repeat the same procedure and divide the line  $AX$  from  $A$  into 8 equal parts :

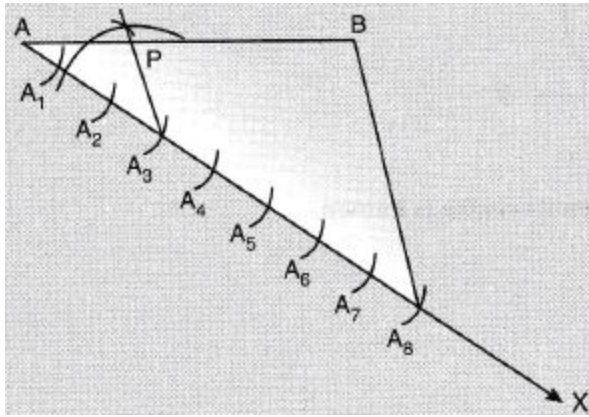
$AA_1, A_1A_2, A_2A_3, A_3A_4, A_4A_5, A_5A_6, A_6A_7$  and  $A_7A_8$

4. Join  $A_8$  and  $B$  by drawing a line.

5. Draw a parallel line to  $A_8B$  from  $A_3$  which divides line segment  $AB$  at point  $P$

6.  $P$  is the required point such that  $AP/AB = 3/5$ .

Construction given below :



15.

ABCD is a square of side 14 cm.  $\therefore$

$\therefore AB = BC = CD = AD = 14$  cm

There are two semicircles in the square of radius =  $14/2 = 7$  cm

Perimeter of the shaded region =  $BC + AD + \text{Perimeter of semi-circle APB} + \text{Perimeter of semi-circle CPD}$

Perimeter of semicircle APB =  $1/2 \times 2\pi \times \text{radius} = 1/2 \times 2\pi \times 7 = 22/7 \times 7 = 22$  cm.

Perimeter of semicircle CPD =  $1/2 \times 2\pi \times \text{radius} = 1/2 \times 2\pi \times 7 = 22/7 \times 7 = 22$  cm.

$\therefore$  Hence, the perimeter of the shaded region =  $22 + 22 + 14 + 14 = 72$  cm.

16.

Volume of one cube,  $v = 27 \text{ cm}^3$

Therefore, side of a cube  $s = v^{1/3} = 27^{1/3} = 3 \text{ cm}$

Two cubes of the same volume joined together to form a cuboid.

Therefore, the length of cuboid,  $L = 2s = 6 \text{ cm}$

Width of cuboid (B) = Height of cuboid (H) =  $s = 3 \text{ cm}$

Surface Area of cuboid,

$$S.A = 2(LB + BH + LH) = 2(6 \times 3 + 3 \times 3 + 6 \times 3) = 2(18 + 9 + 18) = 90 \text{ cm}^2$$

17.

Coordinates of the given points are A (3, -1) and B (11, y).

Distance between the points A and B = AB = 10 units

$$\therefore (11-3)^2 + [y-(-1)]^2 = 10$$

$$\Rightarrow (64) + (y+1)^2 = 10$$

$$\Rightarrow 64 + (y+1)^2 = 100$$

$$\Rightarrow (y+1)^2 = 100 - 64$$

$$\Rightarrow (y+1)^2 = 36$$

$$\Rightarrow (y+1) = \pm 6$$

$$\Rightarrow (y+1) = 6 \text{ or } (y+1) = -6$$

$$\Rightarrow y = 6 - 1 = 5 \text{ or } -6 - 1 = -7$$

Hence, the values of  $y = 5$  or  $-7$ .

18.

Assume the event of drawing a multiple of 5 as E.

Since there are 40 tickets, the total number of outcomes = 40

The outcomes of event E are 5, 10, 15, 20, 25, 30, 35 and 40.

So, the number of favourable outcomes of event E = 8

Probability that the selected ticket has a number that is a multiple of 5:

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{8}{40}$$

$$= \frac{1}{5}$$

Hence, the probability of drawing a ticket which is a multiple of 5 =  $\frac{1}{5}$ .

19.

Compare the given quadratic equation with the standard form of quadratic equation i.e.,  $ax^2 + bx + c = 0$ .

$$a = 1, b = -35, c = 10$$

$$D = b^2 - 4ac$$

$$= (-35)^2 - 4 \times 1 \times 10$$

$$= 1225 - 40$$

$$= 1185$$

$$\therefore x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{35 \pm \sqrt{1185}}{2 \times 1}$$

$$= \frac{35 + \sqrt{1185}}{2} \quad \text{OR} \quad \frac{35 - \sqrt{1185}}{2}$$

20.

Assume the first term of the A.P. as 'a' and the common difference as 'd'.

4th term of the A.P. =  $t_4 = 9$ .

The sum of the sixth and thirteenth term is 40  $\Rightarrow t_6 + t_{13} = 40$ .

If  $t_4 = 9$

$$\Rightarrow a + (4-1)d = 9 \quad [t_n = a + (n-1)d]$$

$$\Rightarrow a + 3d = 9 \text{ ----- (1)}$$

$$t_6 + t_{13} = 40$$

$$\Rightarrow \{a + (6-1)d\} + \{a + (13-1)d\} = 40$$

$$\Rightarrow \{a + 5d\} + \{a + 12d\} = 40$$

$$\Rightarrow 2a + 17d = 40 \text{ ----- (2)}$$

Solving the linear equations (1) and (2)

From (1):

$$a = 9 - 3d$$

Substituting the value of a in (2):

$$2(9 - 3d) + 17d = 40$$

$$\Rightarrow 18 + 11d = 40$$

$$\Rightarrow 11d = 22$$

$$\Rightarrow d = 2$$

$$\therefore a = 9 - 3 \times 2 = 3$$

Hence, the given A.P. = a, a + d, a + 2d ..., where a = 3 and d = 2.

So, the A.P. is 3, 5, 7, 9 ...

21.

Since the triangle is circumscribing the circle, the circle is the incircle of the triangle. All the sides of the triangle are tangents to the circle.

Given  $RT = 9\text{cm}$  and  $QT = 12\text{cm}$

Since  $RT$  and  $RU$  are tangents to a circle from a point  $R$ , they are equal

Thus  $RT = RU = 9\text{cm}$

similarly,  $QT = QS = 12\text{cm}$

Let  $PS = PU = x\text{ cm}$

radius of circle,  $r = 6\text{cm}$

sides of triangle are:

$PQ = (12+x)\text{ cm}$

$QR = 12+9 = 21\text{cm}$

$PR = (9+x)\text{ cm}$

$$s = \frac{PQ+QR+PR}{2} = \frac{12+x+21+9+x}{2} = \frac{42+2x}{2} = 21 + x$$

Given that area =  $189\text{cm}^2$

$$\text{area} = sr$$

$$\Rightarrow 189 = s \times 6$$

$$\Rightarrow s = \frac{189}{6} = 31.5\text{cm}$$

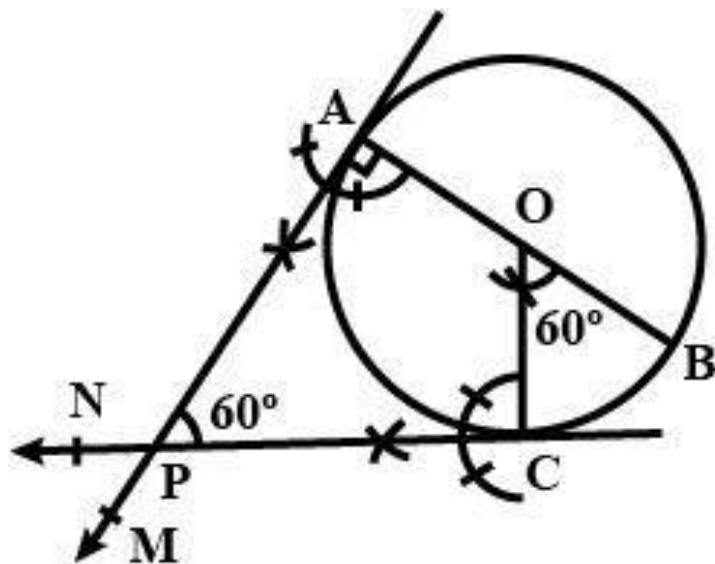
$$\Rightarrow x = 10.5$$

$$PQ = (12+x) \text{ cm} = 12+10.5 = 22.5\text{cm}$$

$$QR = 12 + 9 = 21 \text{ cm}$$

$$PR = (9+x) \text{ cm} = 9+10.5 = 19.5\text{cm}$$

22.



Steps of construction :

1) Draw a circle with  $O$  as centre and radius  $= 3\text{cm}$ .

2) Draw any diameter  $AOB$  of this circle.

3) Construct  $\angle BOC = 60^\circ$  such that radius  $OC$  meets the circle at  $C$ .

4) Draw  $AM \perp AB$  and  $CN \perp OC$ .

Let  $AM$  and  $CN$  intersect each other at  $P$ . Then,  $PA$  and  $CN$  intersect each other at  $P$ .

Then,  $PA$  and  $PC$  are the required tangents to the given circle, inclined at an angle of  $60^\circ$ .

23.

The radius of the circle is 14 cm. The subtends angle is 120 degree at the centre.

The area of minor sector is

$$A_1 = \pi r^2 \times \frac{\theta}{360}$$

$$A_1 = \frac{22}{7} \times (14)^2 \times \frac{120}{360}$$

$$A_1 = 205.33$$

Area of triangle is

$$A_2 = \frac{1}{2} \times r^2 \sin \theta$$

$$A_2 = \frac{1}{2} \times (14)^2 \times \sin(120)$$

$$A_2 = 84.87$$

The formula to find the area of minor segment is

$$A = A_1 - A_2$$

$$A = 205.33 - 84.87 = 120.46$$



Therefore the area of minor segment of the circle is 120.46 cm<sup>2</sup>.

24.

Height of frustum of cone,  $h = 21$  cm

Radius of lower end,  $r = 10$  cm

Radius of upper end,  $R = 20$  cm

Volume of frustum of cone =  $\frac{1}{3}\pi h(R^2 + r^2 + Rr)$

Volume of milk required to fill the bucket

$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times [(20)^2 + (10)^2 + 20 \times 10] \text{ cm}^3$$

$$= 22 \times (400 + 100 + 200)$$

$$= 22 \times 700$$

$$= 15400 \text{ cm}^3$$

$$= 15400/1000 \text{ litres}$$

$$= 15.4 \text{ litres}$$

Cost of milk = Rs 30 per litre

Hence, the cost of milk for filling the bucket is Rs  $(30 \times 15.4)$  i.e. Rs 462.

25.

**Given points are**

A(-5,8) and B(4,-10)

since A,P and B are collinear, we have

Slope of AP = Slope of BP

$$\frac{8-4}{-5-x} = \frac{4+10}{x-4}$$

$$\frac{4}{-5-x} = \frac{14}{x-4}$$

$$\frac{2}{-5-x} = \frac{7}{x-4}$$

$$2x - 8 = -35 - 7x$$

$$9x = -27$$

$$\implies \boxed{x = -3}$$

$\therefore$  Point P is (-3,4)

Let the point P divides the line segment joining A and B in the ratio m:n

Then, the coordinates of P is given by

$$\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$\left( \frac{4m-5n}{m+n}, \frac{-10m+8n}{m+n} \right)$$

$$\text{But, } \left( \frac{4m-5n}{m+n}, \frac{-10m+8n}{m+n} \right) = (-3, 4)$$

$$\implies \frac{4m-5n}{m+n} = -3 \quad \frac{-10m+8n}{m+n} = 4$$

$$\implies 7m = 2n \text{ and } -14m = -4n$$

$$\implies \frac{m}{n} = 27$$

$$\implies \boxed{m : n = 2 : 7}$$

26.

The coordinates of  $ABCD$  are  $A(-3, -1), B(-2, -4), C(4, -1), D(3, 4)$ .

Area of quadrilateral  $ABCD$  = Area of triangle  $ABC$  + Area of triangle  $ACD$

$$\text{Area of triangle} = \frac{1}{2} | [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] |$$

$$\text{Area of triangle } ABC = \frac{1}{2} | [-3(-4 + 1) - 2(-1 + 1) + 4(-1 + 4)] |$$

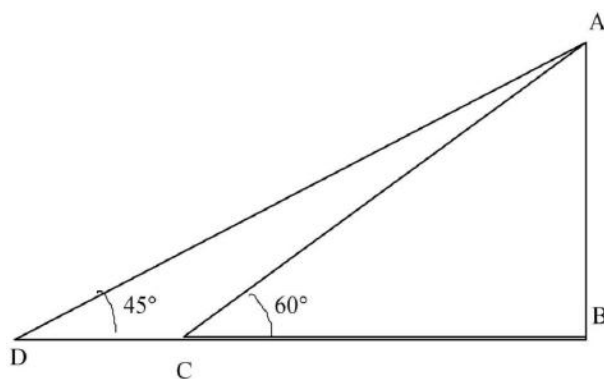
$$= \frac{1}{2} | [9 + 12] | = \frac{21}{2} \text{ sq. units}$$

$$\text{Area of triangle } ACD = \frac{1}{2} | [-3(-1 - 4) + 4(4 + 1) + 3(-1 + 1)] |$$

$$= \frac{1}{2} | [15 + 20] | = \frac{35}{2} \text{ sq. units}$$

$$\text{Therefore, Area of quadrilateral } ABCD = \frac{21}{2} + \frac{35}{2} = 28 \text{ sq. units}$$

27.



AB is the tower

The angle of depression of two car in the same straight line with the base of the tower at an instant are found to be 45 degree and 60 degree i.e.  $\angle ACB = 60^\circ$  and  $\angle ADB = 45^\circ$

The cars are 100 m apart i.e.  $CD = 100$  m

Let CB be  $x$

So,  $DB = DC + CB = 100 + x$

In  $\triangle ABC$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{x}$$

$$\sqrt{3}x = AB \quad \text{---1}$$

In  $\triangle ABD$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{AB}{BD}$$

$$1 = \frac{AB}{100+x}$$

$$100 + x = AB \quad \text{----2}$$

With 1 and 2

$$100 + x = \sqrt{3}x$$

$$100 = (\sqrt{3} - 1)x$$

$$\frac{100}{(\sqrt{3}-1)} = x$$

$$136.6025 = x$$

$$\text{So, } \sqrt{3}(136.6025) = 236.602 = AB$$

Hence the height of tower is 236.602 m

28.

List out the sample space of the given experiment

$S = \{(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)\}$

Therefore,  $n(S) = 36$

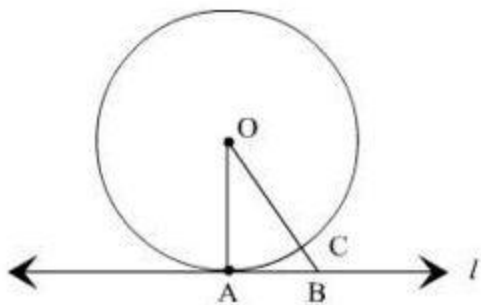
Assume the event of getting the numbers whose product is 12 as E.

$E = \{(2, 6), (3, 4), (4, 3), (6, 2)\}$

Therefore,  $n(E) = 4$

Hence, the required probability =  $\frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$ .

29.



Given : A circle C (O, r) and a tangent l at point A.

To prove :  $OA \perp l$

Construction : Take a point B, other than A, on the tangent l. Join OB. Suppose OB meets the circle in C.

Proof: We know that, among all line segment joining the point O to a point on l, the perpendicular is shortest to l.

$OA = OC$  (Radius of the same circle)

Now,  $OB = OC + BC$ .

$$\therefore OB > OC$$

$$\Rightarrow OB > OA$$

$$\Rightarrow OA < OB$$

B is an arbitrary point on the tangent l. Thus, OA is shorter than any other line segment joining O to any point on l.

Here,  $OA \perp l$

30.

nth term of AP =  $a + (n-1)d$

where a is the first term, d is the common difference.

$$350 = 8 + (n-1)9$$

$$342 = (n-1)9$$

$$n-1 = 38$$

$$n = 39$$

Number of terms = 39.

$$\text{Sum} = (39/2)(8+350)$$

$$= (39/2)(358)$$

$$= 39 \times 179$$

$$= 6981$$

31.

Let the speed of the train be  $x \text{ kmph}$

The time taken by the train to travel  $180 \text{ km}$  is  $\frac{180}{x} \text{ h}$

The increased speed is  $x + 9$

The time taken is  $\frac{180}{x + 9}$

According to the question,

The time taken is  $\frac{180}{x} - 1$

$$\Rightarrow \frac{180}{x} - 1 = \frac{180}{x + 9}$$

$$\Rightarrow \frac{180 - x}{x} = \frac{180}{x + 9}$$

$$\Rightarrow 180x - x^2 + 1680 - 9x = 180x$$

$$\Rightarrow x^2 + 9x - 1680 = 0$$

$$\Rightarrow x^2 + 45x - 36x - 1680 = 0$$

$$\Rightarrow x(x + 45) - 36(x + 45) = 0$$

$$x = 36, -45$$

Speed cannot be negative (In this case,)

So The speed of the train is  $36 \text{ kmph}$

32.

- Construction:- Draw a line connecting the centres of each pair of adjacent circles creating an equilateral triangle of side length 7 cm.

Now,

Area of triangle =

$$\frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$\frac{\sqrt{3}}{4} \times 7^2$$

$$\frac{49\sqrt{3}}{4}$$

There are four areas inside of this equilateral triangle.

[Three 60° circle sector.]

Area of sector =

$$\frac{?}{360} \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{77}{12} \text{ cm}^2$$

$$\text{Area of three sectors} = \frac{77}{4} \text{ cm}^2$$

Thus, area of Shaded portion = area of equilateral triangle - area of three sectors

$$\frac{49\sqrt{3}}{4} - \frac{77}{4} = 1.97 \text{ cm}^2$$



33.

$$\text{radius} = 7\text{cm} = 7/100$$

$$h = 21\text{cm} = 21/100\text{m}$$

volume of pipe = volume of pond

$$\pi r^2 h = l \times b \times h$$

$$22/7 \times 7/100 \times 7/100 \times h = 50 \times 44 \times 21/100$$

$$22/100 \times 7/100 \times h = 50 \times 44 \times 21/100$$

$$h = \frac{50 \times 44 \times 21 \times 100 \times 100}{22 \times 7}$$

$$22 \times 7 \times 100$$

$$h = 30000\text{m}$$

$$h = 30\text{km}$$

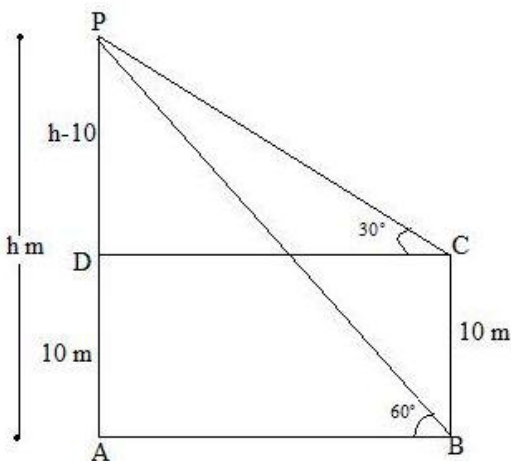
height of pipe = distance travelled by water

$$\text{time} = \text{distance} / \text{speed}$$

$$\text{time} = 30/15$$

$$\text{time} = 2 \text{ hours}$$

34.



Let the height of the tower be  $h$  cm.

Now, In  $\triangle PAB$ ,

$$\tan 60^\circ = \frac{AP}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AB}$$

$$\Rightarrow AB = \frac{h}{\sqrt{3}} \quad \dots\dots (1)$$

And, In  $\triangle PCD$ ,

$$\tan 30^\circ = \frac{PD}{CD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-10}{CD}$$

$$\Rightarrow CD = \sqrt{3}(h-10) \quad \dots\dots (2)$$

Since,  $AB = CD$ , SO, equation (2) becomes,

$$AB = \sqrt{3}(h-10) \quad \dots\dots (3)$$

Equating equation (1) and (3), we get,

$$\frac{h}{\sqrt{3}} = \sqrt{3}(h-10)$$

$$\Rightarrow h = 3(h-10)$$

$$\Rightarrow 2h = 30$$

$$\Rightarrow h = 15 \text{ cm}$$

Hence, the height of the tower will be 15 cm.