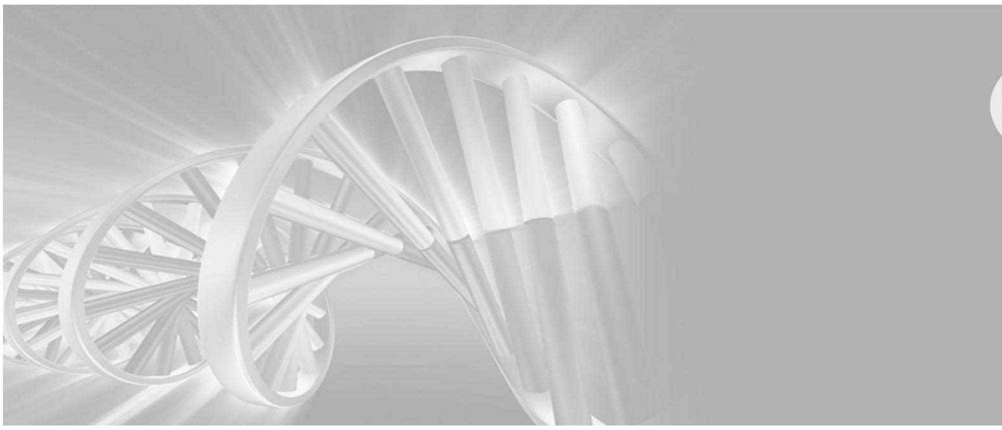
NCERT - MCQ

MATHS

CLASS 11 & 12

Teacher's GUIDE

CONTENTS



CHAPTER 1
SETS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The set of intelligent students in a class is :
(a) a null set (b) a singleton set
(c) a finite set (d) not a well defined collection

2. If the sets A and B are given by A = {1, 2, 3, 4}, B = {2, 4, 6, 8, 10} and the universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then
(a) $(A \cup B)' = \{5, 7, 9\}$
(b) $(A \cap B)' = \{1, 3, 5, 6, 7\}$
(c) $(A \cap B)' = \{1, 3, 5, 6, 7, 8\}$
(d) None of these

3. If A = {1, 2, 3, 4}, B = {2, 3, 5, 6} and C = {3, 4, 6, 7}, then
(a) $A - (B \cap C) = \{1, 3, 4\}$
(b) $A - (B \cap C) = \{1, 2, 4\}$
(c) $A - (B \cup C) = \{2, 3\}$
(d) $A - (B \cup C) = \{\phi\}$

4. Which of the following is correct?
(a) A È B ¹ A È A'
(b) (A Ç B)' = A' È B'
(c) (A' È B') ¹ A' È A
(d) (A Ç B)' = A' Ç B'

5. The number of the proper subset of {a, b, c} is:
(a) 3 (b) 8
(c) 6 (d) 7

6. Which one is different from the others ?
(i) empty set (ii) void set (iii) zero set (iv) null set :
(a) (i) (b) (ii)
(c) (iii) (d) (iv)

7. If the sets A and B are as follows :
$A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$, then
(a) $A - B = \{1, 2\}$
(b) $B - A = \{5\}$
(c) $[(A - B) - (B - A)] \cap A = \{1, 2\}$
(d) $[(A - B) - (B - A)] \cup A = \{3, 4\}$

8. If A = {x, y} then the power set of A is :
(a) $\{x^x, y^y\}$ (b) $\{\phi, x, y\}$
(c) $\{\phi, \{x\}, \{2y\}\}$ (d) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

9. The set {x : x is an even prime number} can be written as
(a) {2} (b) {2, 4}
(c) {2, 14} (d) {2, 4, 14}

10. Given the sets
$A = \{1, 3, 5\}, B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Which of the following may be considered as universal set for all the three sets A, B and C?
(a) {0, 1, 2, 3, 4, 5, 6}
(b) ϕ
(c) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(d) {1, 2, 3, 4, 5, 6, 7, 8}

11. If $A \cup B \neq \phi$, then $n(A \cup B) = ?$
(a) $n(A) + n(B) - n(A \cap B)$
(b) $n(A) - n(B) + n(A \cap B)$
(c) $n(A) - n(B) - n(A \cap B)$
(d) $n(A) + n(B) + n(A \cap B)$

12. Which of the following collections are sets ?
(a) The collection of all the days of a week
(b) A collection of 11 best hockey player of India.
(c) The collection of all rich person of Delhi
(d) A collection of most dangerous animals of India.

13. Which of the following properties are associative law ?
(a) $A \cup B = B \cup A$
(b) $A \cup C = C \cup A$
(c) $A \cup D = D \cup A$
(d) $(A \cup B) \cup C = A \cup (B \cup C)$

14. Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Value of $V - B$ and $B - V$ are respectively
(a) {*e*, o} and {*k*} (b) {*e*} and {*k*}
(c) {o} and {*k*} (d) {*e*, o} and {*k*, *i*}

15. Let $A = \{a, b\}, B = \{a, b, c\}$. What is $A \cup B$?
(a) {*a*, *b*} (b) {*a*, *c*}
(c) {*a*, *b*, *c*} (d) {*b*, *c*}

16. If A and B are finite sets, then which one of the following is the correct equation?
(a) $n(A-B) = n(A) - n(B)$
(b) $n(A-B) = n(B-A)$
(c) $n(A-B) = n(A) - n(A \cap B)$
(d) $n(A-B) = n(B) - n(A \cap B)$
[n (A) denotes the number of elements in A]

17. If ϕ denotes the empty set, then which one of the following is correct ?
(a) $\phi \in \phi$ (b) $\phi \in \{\phi\}$
(c) $\{\phi\} \in \{\phi\}$ (d) $0 \in \phi$

18. Which one of the following is an infinite set ?
(a) The set of human beings on the earth
(b) The set of water drops in a glass of water
(c) The set of trees in a forest
(d) The set of all primes

19. Let $A = \{x : x$ is a multiple of $3\}$ and $B = \{x : x$ is a multiple of $5\}$. Then A Ç B is given by:
(a) $\{15, 30, 45, ...\}$
(b) $\{3, 6, 9, ...\}$
(c) $\{15, 10, 15, 20...\}$
(d) $\{5, 10, 20, ...\}$

20. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ equals
(a) ϕ (b) $\{14, 3, 4\}$
(c) $\{3\}$ (d) $\{4\}$

21. $A = \{x : x \neq x\}$ represents
(a) $\{x\}$ (b) $\{1\}$
(c) $\{\ \}$ (d) $\{0\}$

22. Which of the following is a null set ?
(a) $\{0\}$
(b) $\{x : x > 0 \text{ or } x < 0\}$
(c) $\{x : x^2 = 4 \text{ or } x = 3\}$
(d) $\{x : x^2 + 1 = 0, x \in \mathbf{R}\}$

23. In a group of 52 persons, 16 drink tea but not coffee, while 33 drink tea. How many persons drink coffee but not tea ?
(a) 17 (b) 36
(c) 23 (d) 19

24. There are 600 student in a school. If 400 of them can speak Telugu, 300 can speak Hindi, then the number of students who can speak both Telugu and Hindi is:
(a) 100 (b) 200
(c) 300 (d) 400

25. In a group of 500 students, there are 475 students who can speak Hindi and 200 can speak Bengali. What is the number of students who can speak Hindi only ?
(a) 275 (b) 300
(c) 325 (d) 350

26. The set builder form of given set $A = \{3, 6, 9, 12\}$ and $B = \{1, 4, 9, ..., 100\}$ is
(a) $A = \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 5\}$,
$B = \{x : x = n^2, n \in N \text{ and } 1 \leq n \leq 10\}$
(b) $A = \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$,
$B = \{x : x = n^2, n \in N \text{ and } 1 \leq n \leq 10\}$
(c) $A = \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$,
$B = \{x : x = n^2, n \in N \text{ and } 1 < n < 10\}$
(d) None of these

27. Which of the following sets is a finite set?
(a) $A = \{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\}$
(b) $B = \{x : x \in Z \text{ and } x^2 \text{ is even}\}$
(c) $D = \{x : x \in Z \text{ and } x > -10\}$
(d) All of these

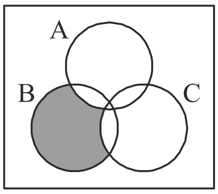
28. Which of the following is a singleton set?
(a) $\{x : |x| = 5, x \in N\}$
(b) $\{x : |x| = 6, x \in Z\}$
(c) $\{x : x^2 + 2x + 1 = 0, x \in N\}$
(d) $\{x : x^2 = 7, x \in N\}$

29. Which of the following is not a null set?
(a) Set of odd natural numbers divisible by 2
(b) Set of even prime numbers
(c) $\{x : x$ is a natural number, $x < 5$ and $x > 7\}$
(d) $\{y : y$ is a point common to any two parallel lines$\}$

30. If $A = \{x : x = n^2, n = 1, 2, 3\}$, then number of proper subsets is
(a) 3 (b) 8
(c) 7 (d) 4

31. Which of the following has only one subset?
(a) $\{\ \}$ (b) $\{4\}$
(c) $\{4, 5\}$ (d) $\{0\}$

32. The shaded region in the given figure is



(a) $B \cap (A \cup C)$ (b) $B \cup (A \cap C)$
(c) $B \cap (A - C)$ (d) $B - (A \cup C)$

33. If $A = \{x : x$ is a multiple of $3\}$ and $B = \{x : x$ is a multiple of $5\}$, then $A - B$ is equal to
(a) $\overline{A} \cap B$ (b) $A \cap \overline{B}$
(c) $\overline{A} \cap \overline{B}$ (d) $\overline{A \cap B}$

34. If A and B be any two sets, then $A \cap (A \cup B)'$ is equal to
(a) A (b) B
(c) ϕ (d) None of these

35. A survey shows that 63% of the people watch a news channel whereas 76% watch another channel. If x% of the people watch both channel, then
(a) $x = 35$ (b) $x = 63$
(c) $39 \leq x \leq 63$ (d) $x = 39$

36. The set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set-builder form is

(a) $\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 < n < 6\right\}$

(b) $\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 \le n < 6\right\}$

(c) $\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 \le n \le 6\right\}$

(d) None of the above

37. The set {x : x is a positive integer less than 6 and $3^x - 1$ is an even number} in roster form is

(a) {1, 2, 3, 4, 5} (b) {1, 2, 3, 4, 5, 6}
(c) {2, 4, 6} (d) {1, 3, 5}

38. If B = {x : x is a student presently studying in both classes X and XI}. Then, the number of elements in set B are

(a) finite (b) infinite
(c) zero (d) None of these

39. Consider:
X = Set of all students in your school.
Y = Set of all students in your class.
Then, which of the following is true?

(a) Every element of Y is also an element of X
(b) Every element of X is also an element of Y
(c) Every element of Y is not an element of X
(d) Every element of X is not an element of Y

40. If $A \subset B$ and $A \neq B$, then

(a) A is called a proper subset of B
(b) A is called a super set of B
(c) A is not a subset of B
(d) B is a subset of A

41. The set of real numbers {x : a < x < b} is called

(a) open interval (b) closed interval
(c) semi-open interval (d) semi-closed interval

42. Which of the following is true?

(a) $a \in \{\{a\}, b\}$ (b) $\{b, c\} \subset \{a, \{b, c\}\}$
(c) $\{a, b\} \subset \{a, \{b, c\}\}$ (d) None of these

43. The interval [a, b) is represented on the number line as

(a) 

(b) 

(c) 

(d) 

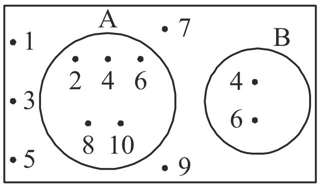
44. The interval represented by

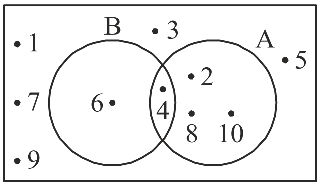
(a) (a, b) (b) [a, b]
(c) [a, b) (d) (a, b]

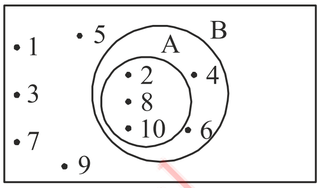
45. The number of elements in $P[P(P(\phi))]$ is

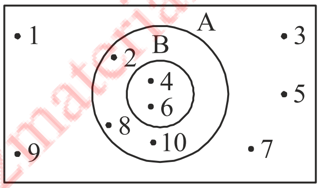
(a) 2 (b) 3
(c) 4 (d) 5

46. If U = {1, 2, 3, 4, ..., 10} is the universal set of A, B and A = {2, 4, 6, 8, 10}, B = {4, 6} are subsets of U, then given sets can be represented by Venn diagram as

(a) 

(b) 

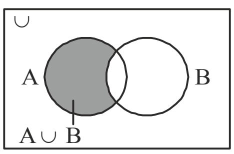
(c) 

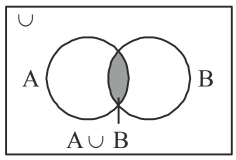
(d) 

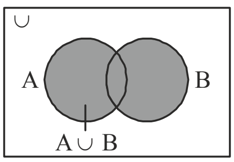
47. Most of the relationships between sets can be represented by means of diagrams which are known as

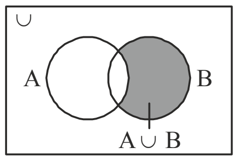
(a) rectangles (b) circles
(c) Venn diagrams (d) triangles

48. Which of the following represent the union of two sets A and B?

(a) 

(b) 

(c) 

(d) 

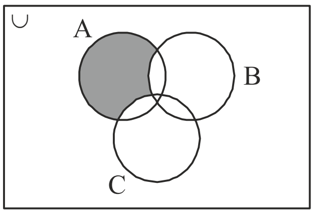
49. Let X = {Ram, Geeta, Akbar} be the set of students of Class XI, who are in school hockey team and Y = {Geeta, David, Ashok} be the set of students from Class XI, who are in the school football team. Then, $X \cap Y$ is

(a) {Ram, Geeta} (b) {Ram}
(c) {Geeta} (d) None of these

50. Which of the following represent A – B?

(a) $\{x : x \in A \text{ and } x \in B\}$
(b) $\{x : x \in A \text{ and } x \notin B\}$
(c) $\{x : x \in A \text{ or } x \in B\}$
(d) $\{x : x \in A \text{ or } x \notin B\}$

51. The shaded region in the given figure is

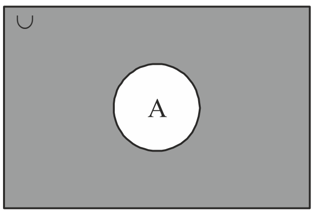


(a) $A \cap (B \cup C)$ (b) $A \cup (B \cap C)$
(c) $A \cap (B - C)$ (d) $A - (B \cup C)$

52. If A and B are non-empty subsets of a set, then $(A - B) \cup (B - A)$ equals to
(a) $(A \cap B) \cup (A \cup B)$ (b) $(A \cup B) - (A - B)$
(c) $(A \cup B) - (A \cap B)$ (d) $(A \cup B) - B$

53. Let A, B, C are three non-empty sets. If $A \subset B$ and $B \subset C$, then which of the following is true?
(a) $B - A = C - B$ (b) $A \cap B \cap C = B$
(c) $A \cup B = B \cap C$ (d) $A \cup B \cup C = A$

54. In the Venn diagram, the shaded portion represents



(a) complement of set A (b) universal set
(c) set A (d) None of these

55. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 5}, B = {2, 4, 6, 7} and C = {2, 3, 4, 8}, then which of the following is true?

(a) $(B \cup C)' = \{1, 5, 9, 10\}$

(b) $(C - A)' = \{1, 2, 3, 5, 6, 7, 9, 10\}$

(c) Both (a) and (b)
(d) None of the above

56. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to
(a) A (b) B
(c) ϕ (d) $A \cap B^c$

57. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to
(a) A (b) B
(c) A^c (d) B^c

58. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is
(a) {2, 3, 5} (b) {3, 5, 9}
(c) {1, 2, 5, 9} (d) None of these

59. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to
(a) A (b) B
(c) ϕ (d) None of these

60. If A and B are sets, then $A \cap (B - A)$ is
(a) ϕ (b) A
(c) B (d) None of these

61. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then $(A - B) \times (B - C)$ is
(a) {(1, 2), (1, 5), (2, 5)} (b) {(1, 4)}
(c) (1, 4) (d) None of these

62. If n(A) = 3, n(B) = 6 and $A \subseteq B$. Then, the number of elements in $A \cup B$ is equal to
(a) 3 (b) 9
(c) 6 (d) None of these

63. In a battle 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs. The minimum value of x is
(a) 10 (b) 12
(c) 15 (d) None of these

64. If A = {x : x is a multiple of 4} and B = {x : x is a multiple of 6}, then $A \cap B$ consists of all multiples of
(a) 16 (b) 12
(c) 8 (d) 4

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

65. Let P be a set of squares, Q be set of parallelograms, R be a set of quadrilaterals and S be a set of rectangles. Consider the following :
I. $P \subset Q$ II. $R \subset P$
III. $P \subset S$ IV. $S \subset R$
Which of the above are correct?
(a) I, II and III (b) I, III and IV
(c) I, II and IV (d) III and IV

66. Consider the following statements
I. $\phi \in \{\phi\}$ II. $\{\phi\} \subseteq \phi$
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

67. Consider the following sets.
I. A = {1, 2, 3}
II. $B = \{x \in R : x^2 - 2x + 1 = 0\}$
III. C = {1, 2, 2, 3}
IV. $D = \{x \in R : x^3 - 6x^2 + 11x - 6 = 0\}$
Which of the following are equal?
(a) A = B = C (b) A = C = D
(c) A = B = D (d) B = C = D

68. Consider the following relations:
I. $A - B = A - (A \cap B)$
II. $A = (A \cap B) \cup (A - B)$
III. $A - (B \cup C) = (A - B) \cup (A - C)$
Which of these is/are correct?
(a) Both I and III (b) Only II
(c) Both II and III (d) Both I and II

69. Consider the following statements

I. The vowels in the English alphabet.

II. The collection of books.

III. The rivers of India.

IV. The collection of most talented batsmen of India.

Which of the following is/are well-defined collections?

(a) I and II (b) Only I

(c) I and III (d) I and IV

70. The set of all letters of the word 'SCHOOL' is represented by

I. {S, C, H, O, O, L}

II. {S, C, H, O, L}

III. {C, H, L, O, S}

IV. {S, C, H, L}

The correct code is

(a) I and II (b) I, II and III

(c) II and III (d) I, II, III and IV

71. I. The collection of all months of a year beginning with the letter J.

II. The collection of ten most talented writers of India.

III. A team of eleven best cricket batsmen of the world.

IV. The collection of all boys in your class.

Which of the above are the sets?

(a) I and II (b) I and III

(c) I and IV (d) I, II and III

72. **Statement - I :** The set D = {x : x is a prime number which is a divisor of 60} in roster form is {1, 2, 3, 4, 5}.

Statement - II : The set E = the set of all letters in the word 'TRIGONOMETRY', in the roster form is {T, R, I, G, O, N, M, E, Y}.

(a) Statement I is true (b) Statement II is true

(c) Both are true (d) Both are false

73. The empty set is represented by

I. ϕ II. $\{\phi\}$

III. { } IV. {{ }}

(a) I and II (b) I and III

(c) II and III (d) I and IV

74. **Statement - I :** The set $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$ is the empty set.

Statement - II : The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is an empty set.

(a) Statement I is true (b) Statement II is true

(c) Both are true (d) Both are false

75. State which of the following is/are true?

I. The set of animals living on the Earth is finite.

II. The set of circles passing through the origin (0, 0) is infinite.

(a) Only I (b) Only II

(c) I and II (d) None of these

76. **Statement - I :** The set of positive integers greater than 100 is infinite.

Statement - II : The set of prime numbers less than 99 is finite.

(a) Statement I is true (b) Statement II is true

(c) Both are true (d) Both are false

77. Select the infinite set from the following:

I. The set of lines which are parallel to the X-axis.

II. The set of numbers which are multiples of 5.

III. The set of letters in the English alphabet.

(a) I and II (b) II and III

(c) I and III (d) None of these

78. Consider the following sets.

A = {0},

$B = \{x : x > 15 \text{ and } x < 5\}$,

$C = \{x : x - 5 = 0\}$,

$D = \{x : x^2 = 25\}$,

E = {x : x is an integral positive root of the equation $x^2 - 2x - 15 = 0$}

Choose the pair of equal sets

(a) A and B (b) C and D

(c) C and E (d) B and C

79. **Statement - I :** The set of concentric circles in a plane is infinite.

Statement - II : The set $\{x : x^2 - 3 = 0 \text{ and } x \text{ is rational}\}$ is finite.

(a) Statement I is true (b) Statement II is true

(c) Both are true (d) Both are false

80. Which of the following is/are true?

I. Every set A is a subset of itself.

II. Empty set is a subset of every set.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) None of these

81. Let A = {1, 3, 5} and B = {x : x is an odd natural number less than 6}. Then, which of the following are true?

I. $A \subset B$ II. $B \subset A$

III. A = B IV. $A \not\subset B$

(a) I and II are true (b) I and III are true

(c) I, II and III are true (d) I, II and IV are true

82. Given the sets A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}. Then, which of the following may be considered as universal set(s) for all the three sets A, B and C?

I. {0, 1, 2, 3, 4, 5, 6}

II. ϕ

III. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

IV. {1, 2, 3, 4, 5, 6, 7, 8}

(a) Only I (b) Only III

(c) I and III (d) III and IV

83. Which of the following is/are the universal set(s) for the set of isosceles triangles?
I. Set of right angled triangles.
II. Set of scalene triangles.
III. Set of all triangles in a plane.
(a) Only I (b) Only III
(c) II and III (d) None of these

84. **Statement - I :** In the union of two sets A and B, the common elements being taken only once.
Statement - II : The symbol '$\cap$' is used to denote the union.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

85. **Statement - I :** Let A = {a, b} and B = {a, b, c}. Then, $A \not\subset B$.
Statement - II : If $A \subset B$, then $A \cup B = B$.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

86. Which of the following are correct?
I. $A - B = A - (A \cap B)$.
II. $A = (A \cap B) \cup (A - B)$.
III. $A - (B \cup C) = (A - B) \cup (A - C)$.
(a) I and II (b) II and III
(c) I, II and III (d) None of these

87. Which of the following is/are true?
I. If A is a subset of the universal set U, then its complement A' is also a subset of U.
II. If U = {1, 2, 3, ..., 10} and A = {1, 3, 5, 7, 9}, then $(A')' = A$.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) None of these

88. **Statement-I :** Let U be the universal set and A be the subset of U. Then, complement of A is the set of element of A.
Statement-II : The complement of a set A can be represented by A'.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

89. **Statement-I :** The Venn diagram of $(A \cup B)'$ and $A' \cap B'$ are same.
Statement-II : The Venn diagram of $(A \cap B)'$ and $A' \cup B'$ are different.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

90. **Statement-I :** If A, B and C are finite sets, then $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.
Statement-II : If A, B and C are mutually pairwise disjoint, then $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

91. In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Then, which of the following is/are true?
I. 150 students were taking at least one juice.
II. 225 students were taking neither apple juice nor orange juice.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) None of these

92. Suppose A be a non-empty set, then the collection of all possible subsets of set A is a power set P(A).
Which of the following is correct?
I. $P(A) \cap P(B) = P(A \cap B)$
II. $P(A) \cup P(B) = P(A \cup B)$
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

93. Which of the following is correct?
I. Number of subsets of a set A having n elements is equal to 2^n.
II. The power set of a set A contains 128 elements then number of elements in set A is 7.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

94. Which of the following is correct?
I. Number of non-empty subsets of a set having n elements are $2^n - 1$.
II. The number of non-empty subsets of the set {a, b, c, d} are 15.
(a) Only I is false (b) Only II is false
(c) Both I and II are false (d) Both I and II are true

95. **Statement-I :** If A = {1, 2, 3, 4, 5}, B = {2, 4, 6}, C = {3, 4, 6}, then $(A \cup B) \cap C = \{3, 4, 6\}$
Statement-II : $(A \cup B)' = A' \cap B'$
(a) Only I is true
(b) Only II is true
(c) Both I and II are true.
(d) Both I and II are false.

96. Let A = {3, 6, 9, 12, 15, 18, 21}
B = {4, 8,12, 16, 20}
C = {2, 4, 6, 8, 10, 12, 14, 16}
and D = {5, 10, 15, 20}
Which of the following is incorrect?
I. A – B = {4, 8, 16, 20}
II. $(C - B) \cap (D - B) = \phi$
III. $B - C \neq B - D$
(a) Only I & II (b) Only II & III
(c) Only III & I (d) None of these

97. Which of the following is correct?

I. $n(S \cup T)$ is maximum when $n(S \cap T)$ is least.

II. If $n(U) = 1000$, $n(S) = 720$, $n(T) = 450$, then least value of $n(S \cap T) = 170$.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Both I and II are false

98. Which of the following is correct?

I. Three sets A, B, C are such that $A = B \cap C$ and $B = C \cap A$, then $A = B$.

II. If $A = \{a, b\}$, then $A \cap P(A) = A$

(a) Only I is true (b) Only II is true

(c) Both are true (d) Both are false

99. Consider the following relations :

I. $A = (A \cap B) \cup (A - B)$

II. $A - B = A - (A \cap B)$

III. $A - (B \cup C) = (A - B) \cup (A - C)$

Which of these is correct?

(a) I and III (b) I and II

(c) Only II (d) II and III

100. Consider the following statements.

I. If A_n is the set of first n prime numbers, then $\bigcup_{n=2}^{10} A_n$ is equal to $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

II. If A and B are two sets such that $n(A \cup B) = 50$, $n(A) = 28$, $n(B) = 32$, then $n(A \cap B) = 10$.

Which of these is correct?

(a) Only I is true (b) Only II is true

(c) Both are true (d) Both are false

101. Consider the following statements.

I. Let A and B be any two sets. The union of A and B is the set containing the elements of A and B both.

II. The intersection of two sets A and B is the set which consists of common elements of A and B.

Which of the statement is correct?

(a) Only statement-I is true.

(b) Only statement-II is true.

(c) Both statements are true.

(d) Neither I nor II are true.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

102. Match the following statements in column-I with their symbolic forms in column-II.

Column – I	Column – II
A. A is a subset of B	1. if and only if
B. If $A \subset B$ and $B \subset A$, then	2. $A \subset B$
C. A is not a subset of B	3. $A = B$
D. If $a \in A \Rightarrow a \in B$, then	4. $A \not\subset B$
E. The symbol "$\Leftrightarrow$" means	

Codes:

	A	B	C	D	E
(a)	4	3	1	2	3
(b)	2	3	4	2	1
(c)	1	2	3	4	3
(d)	4	3	2	1	4

103. Match the following sets in column -I with the intervals in column -II.

Column – I	Column – II
A. $\{x : x \in R,\ a < x < b\}$	1. $(a, b]$
B. $\{x \in R : a \le x \le b\}$	2. $[a, b)$
C. The set of real numbers x such that $a \le x < b$	3. (a, b)
D. $\{x : x \in R \text{ and } a < x \le b\}$	4. $[a, b]$

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	2	3	4	1
(c)	1	2	3	4
(d)	3	4	2	1

104. Match the following sets in column -I with the equal sets in column-II.

Column – I	Column – II
A. $A \cap B$	1. $(A \cap B) \cup (A \cap C)$
B. $(A \cap B) \cap C$	2. A
C. $\phi \cap A$	3. $A \cap (B \cap C)$
D. $U \cap A$	4. $B \cap A$
E. $A \cap A$	5. ϕ
F. $A \cap (B \cup C)$	

Codes:

	A	B	C	D	E	F
(a)	5	1	4	3	1	2
(b)	3	4	2	1	5	4
(c)	4	3	5	2	2	1
(d)	1	2	3	4	5	2

105. Match the following sets in column -I equal with the sets in column-II.

Column – I	Column – II
A. $A \cup A'$	1. $A' \cap B'$
B. $A \cap A'$	2. $A' \cup B'$
C. $(A \cup B)'$	3. U
D. $(A \cap B)'$	4. ϕ
E. ϕ'	5. A
F. U'	
G. $(A')'$	

Codes:

	A	B	C	D	E	F	G
(a)	1	2	3	4	5	3	2
(b)	3	4	1	2	3	4	5
(c)	4	3	2	1	4	5	3
(d)	5	4	3	2	1	4	1

106.

Column - I (Set)	**Column - II (Roster-form)**
(A) $\{x \in N : x^2 < 25\}$	1. $\{1, 2, 3, 4, 5\}$
(B) Set of integers between – 5 and 5	2. $\{2, 3, 5\}$
(C) {x : x is a natural number less than 6}	3. $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$
(D) {x : x is a prime number which is a divisor of 60}	4. $\{1, 2, 3, 4\}$

Codes :

	A	B	C	D
(a)	4	2	1	3
(b)	1	3	4	2
(c)	1	2	3	4
(d)	4	3	1	2

107.

Column - I	**Column - II**
(A) If $A \cup B = A \cap B$, then	1. $A = B$
(B) Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then	2. $A \cup B$
(C) If $P(A) = P(B)$, then	3. $A \subset B$
(D) $A \cup (B - A)$ is equal to	4. $(A \cap B \cap C)'$
(E) Let U be the universal set and $A \cup B \cup C = U$. Then, $\{(A - B) \cup (B - C) \cup (C - A)\}$ is equal to	5. $B = C$
(F) The set $(A \cap B')' \cup (B \cup C)$ is equal to	6. $A' \cup B$

Codes :

	A	B	C	D	E	F
(a)	1	2	3	4	5	6
(b)	3	2	1	5	6	4
(c)	2	1	5	4	6	2
(d)	3	5	1	2	4	6

108. If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, $B = \{3, 5\}$ and $C = \{1, 2, 4, 7\}$, then match the columns.

Column-I	**Column-II**
(A) $A \cup (B \cap C)$	1. $\{1, 2, 4, 7\}$
(B) $(A \cap B) \cup C$	2. $\{6\}$
(C) $A \cap (B \cup C)'$	3. $\{1, 3, 5, 7\}$
(D) $A' \cup (B \cap C')$	4. $\{1, 7\}$
(E) $A' \cap B'$	5. $\{2, 4, 6\}$

Codes :

	A	B	C	D	E
(a)	1	5	3	2	4
(b)	5	1	2	3	4
(c)	5	1	3	4	2
(d)	3	4	5	1	2

109. Match the complement of sets of the following sets in column-I with the sets in column-II.

Column - I	**Column - II**
(A) {x: x is a prime number}	1. {x : x is not divisible by 15}
(B) {x : x is a multiple of 3}	2. {x : x is an odd natural number}
(C) {x : x is a natural number divisible by 3 and 5}	3. {x : x is not a prime number}
(D) {x : x is an even natural number}	4. {x : x is not a multiple of 3}

Codes :

	A	B	C	D
(a)	3	4	2	1
(b)	1	2	3	4
(c)	3	4	1	2
(d)	4	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

110. If $X = \{1, 2, 3, \ldots, 10\}$ and 'a' represents any element of X, then the set containing all the elements satisfy $a + 2 = 6$, $a \in X$ is

(a) {4} (b) {3}
(c) {2} (d) {5}

111. If a set is denoted as $B = \phi$, then the number of element in B is

(a) 3 (b) 2
(c) 1 (d) 0

112. Let $X = \{1, 2, 3, 4, 5\}$. Then, the number of elements in X are

(a) 3 (b) 2
(c) 1 (d) 5

113. If $X = \{1, 2, 3\}$, then the number of proper subsets is

(a) 5 (b) 6
(c) 7 (d) 8

114. The number of non-empty subsets of the set {1, 2, 3, 4} is $3 \times a$. The value of 'a' is

(a) 3 (b) 4
(c) 5 (d) 6

115. If $A = \phi$, then the number of elements in P(A) is

(a) 3 (b) 2
(c) 1 (d) 0

116. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 = 144\}$ then the number of points, $A \cap B$ contains is

(a) 1 (b) 2
(c) 3 (d) 4

117. The cardinality of the set $P\{P[P(\phi)]\}$ is
(a) 0 (b) 1
(c) 2 (d) 4

118. If $n(A) = 8$ and $n(A \cap B) = 2$, then $n[(A \cap B)' \cap A]$ is equal to
(a) 8 (b) 6
(c) 4 (d) 2

119. In a school, there are 20 teachers who teach Mathematics or Physics of these, 12 teach Mathematics and 4 teach both Maths and Physics. Then the number of teachers teaching only Physics are
(a) 4 (b) 8
(c) 12 (d) 16

ASSERTION-REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

120. Assertion : The number of non-empty subsets of the set {a, b, c, d} are 15.
Reason : Number of non-empty subsets of a set having n elements are $2^n - 1$.

121. Suppose A, B and C are three arbitrary sets and U is a universal set.
Assertion : If $B = U - A$, then $n(B) = n(U) - n(A)$.
Reason : If $C = A - B$, then $n(C) = n(A) - n(B)$.

122. Assertion : Let $A = \{1, \{2, 3\}\}$, then $P(A) = \{\{1\}, \{2, 3\}, \phi, \{1, \{2, 3\}\}\}$.
Reason : Power set is set of all subsets of A.

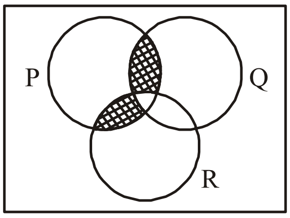
123. Assertion : The subsets of the set $\{1, \{2\}\}$ are $\{\ \}, \{1\}, \{\{2\}\}$ and $\{1, \{2\}\}$.
Reason : The total number of proper subsets of a set containing n elements is $2^n - 1$.

124. Assertion : For any two sets A and B, $A - B \subset B'$.
Reason : If A be any set, then $A \cap A' = \phi$.

CRITICAL THINKING TYPE QUESTIONS

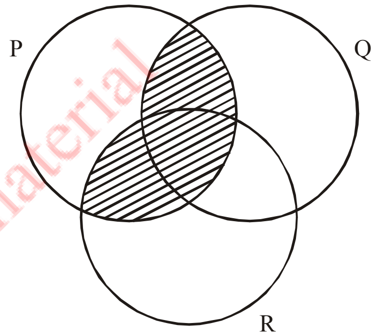
Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

125. What does the shaded portion of the Venn diagram given below represent?



(a) $(P \cap Q) \cap (P \cap R)$
(b) $((P \cap Q) - R) \cup ((P \cap R) - Q)$
(c) $((P \cup Q) - R) \cap ((P \cap R) - Q)$
(d) $((P \cap Q) \cup R) \cap ((P \cup Q) - R)$

126. What does the shaded region represent in the figure given below ?



(a) $(P \cup Q) - (P \cap Q)$
(b) $P \cap (Q \cap R)$
(c) $(P \cap Q) \cap (P \cap R)$
(d) $(P \cap Q) \cup (P \cap R)$

127. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are:
(a) 7, 6 (b) 6, 3
(c) 5, 1 (d) 8, 7

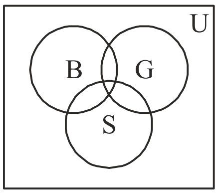
128. If A is the set of the divisors of the number 15, B is the set of prime numbers smaller than 10 and C is the set of even numbers smaller than 9, then $(A \cup C) \cap B$ is the set
(a) {1, 3, 5} (b) {1, 2, 3}
(c) {2, 3, 5} (d) {2, 5}

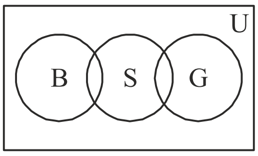
129. Let S = the set of all triangles, P = the set of all isosceles triangles, Q = the set of all equilateral triangles, R = the set of all right-angled triangles. What do the sets $P \cap Q$ and $R - P$ represents respectively ?
(a) The set of isosceles triangles; the set of non- isosceles right angled triangles
(b) The set of isosceles triangles; the set of right angled triangles
(c) The set of equilateral triangles; the set of right angled triangles
(d) The set of isosceles triangles; the set of equilateral triangles

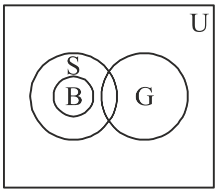
130. If A and B are non-empty sets, then $P(A) \cup P(B)$ is equal to
(a) $P(A \cup B)$ (b) $P(A \cap B)$
(c) $P(A) = P(B)$ (d) None of these

131. Let $A = \{(1, 2), (3, 4), 5\}$, then which of the following is incorrect?
(a) $\{3, 4\} \not\subset A$ as $(3, 4)$ is an element of A
(b) $\{5\}, \{(3, 4)\}$ are subsets of A but not elements of A
(c) $\{1, 2\}, \{5\}$ are subsets of A
(d) $\{(1, 2), (3, 4), 5\}$ are subset of A

132. Let U be the set of all boys and girls in school. G be the set of all girls in the school. B be the set of all boys in the school and S be the set of all students in the school who take swimming. Some but not all students in the school take swimming.

(a) 

(b) 

(c) 

(d) None of these

133. If $A = \{a, \{b\}\}$, then P(A) equals.
(a) $\{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$
(b) $\{\phi, \{a\}\}$
(c) $\{\{a\}, \{b\}, \phi\}$
(d) None of these

134. If A and B are two sets, then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to
(a) Only A (b) $A \cup B$
(c) $(A \cup B)'$ (d) None of these

135. A market research group conducted a survey of 2000 consumers and reported that 1720 consumers like product P_1 and 1450 consumers like product P_2. What is the least number that must have liked both the products?
(a) 1150 (b) 2000
(c) 1170 (d) 2500

136. In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all of three newspapers, then the number of families which buy A only, is
(a) 4400 (b) 3300
(c) 2000 (d) 500

137. A class has 175 students. The following data shows the number of students opting one or more subjects. Maths–100, Physics–70, Chemistry–40, Maths and Physics–30, Maths and Chemistry–28, Physics and Chemistry–23, Maths, Physics and Chemistry–18. How many have offered Maths alone?
(a) 35 (b) 48
(c) 60 (d) 22

138. If $aN = \{ax : x \in \mathbf{N}\}$, then the set $3N \cap 7N$ is
(a) 21 N (b) 10 N (c) 4 N (d) None

139. If $A = \{x \in \mathbf{R} : 0 < x < 3\}$ and $B = \{x \in \mathbf{R} : 1 \le x \le 5\}$ then $A \Delta B$ is
(a) $\{x \in \mathbf{R} : 0 < x < 1\}$ (b) $\{x \in \mathbf{R} : 3 \le x \le 5\}$
(c) $\{x \in \mathbf{R} : 0 < x < 1 \text{ or } 3 \le x \le 5\}$ (d) ϕ

140. Let A, B, C be finite sets. Suppose that n (A) = 10, n (B) = 15, n (C) = 20, n $(A \cap B)$ = 8 and n $(B \cap C)$ = 9. Then the possible value of n $(A \cup B \cup C)$ is
(a) 26
(b) 27
(c) 28
(d) Any of the three values 26, 27, 28 is possible

141. A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products ?
(a) 170 (b) 280
(c) 220 (d) None

142. Each student in a class of 40, studies at least one of the subjects English, Mathematics and Economics. 16 study English, 22 Economics and 26 Mathematics, 5 study English and Economics, 14 Mathematics and Economics and 2 study all the three subjects. The number of students who study English and Mathematics but not Economics is
(a) 7 (b) 5
(c) 10 (d) 4

143. A survey of 500 television viewers produced the following information, 285 watch football, 195 watch hockey, 115 watch basket-ball, 45 watch football and basket ball, 70 watch football and hockey, 50 watch hockey and basket ball, 50 do not watch any of the three games. The number of viewers, who watch exactly one of the three games are
(a) 325 (b) 310
(c) 405 (d) 372

144. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total 64 played both basketball and hockey, 80 played cricket and basketball and 40 played cricket and hockey, 24 played all the three games. The number of boys who did not play any game is:

(a) 128 (b) 216
(c) 240 (d) 160

145. Let A, B, C be three sets. If $A \in B$ and $B \subset C$, then

(a) $A \subset C$ (b) $A \not\subset C$
(c) $A \in C$ (d) $A \notin C$

146. Let $V = \{a, e, i, o, u\}$, $V - B = \{e, o\}$ and $B - V = \{k\}$. Then, the set B is

(a) $\{a, i, u\}$ (b) $\{a, e, k, u\}$
(c) $\{a, i, k, u\}$ (d) $\{a, e, i, k, u\}$

147. From 50 students taking examination in Mathematics, Physics and Chemistry, each of the students has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. Atmost 19 passed Mathematics and Physics, atmost 29 Mathematics and Chemistry and atmost 20 Physics and Chemistry. Then, the largest numbers that could have passed all three examinations, are

(a) 12 (b) 14
(c) 15 (d) 16

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(d)** **2.** **(a)** **3.** **(b)**

4. **(b)** **Note:** $(A \cup B)' = A' \cap B'$ (By De-morgan's law) and

$(A \cap B)' = A' \cup B'$

5. **(d)** The number of proper subsets of $\{1, 2, 3, ... n\}$ is $2^n - 1$.

Hence the number of proper subset of $\{a, b, c\}$ is $2^3 - 1 = 7$

6. **(c)** A set which does not contain any element is called an empty or void or null set.

But zero set contain 0.

7. **(a)** Given $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$

$\therefore A - B = \{1, 2\}$

8. **(d)** Let $A = \{x, y\}$

Power set = Set of all possible subsets of A

$\therefore \ P(A) = \{\phi, \{x\}, \{y\}, \{x, y\}\}$

9. **(a)** **10.** **(c)** **11.** **(a)**

12. **(a)** The days of a week are well defined.

Hence, the collection of all the days of a week, is a set.

13. **(d)**

14. **(a)** We have, $V = \{a, e, i, o, u\}$ and

$B = \{a, i, k, u\}$

$\therefore \ V - B = \{e, o\}$

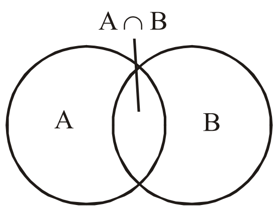
$\because$ the element e, o belong to V but not to B

$\therefore \ B - V = \{k\}$

$\because$ the element k belong to $B - V$ but not to $V - B$.

15. **(c)** $A \cup B = \{a, b\} \cup \{a, b, c\} = \{a, b, c\}$

16. **(c)** If A and B are finite sets, then



$A - B = A - (A \cap B)$

From the Venn diagram

$\Rightarrow \ n(A - B) = n(A) - n(A \cap B)$

17. **(b)** Since, ϕ is an empty set, $\phi \in \{\phi\}$

18. **(d)** In the given sets, the set of all primes is an infinite set.

19. **(a)** Given : $A = \{3, 6, 9, 15....\}$ and

$B = \{5, 10, 15, 20,.....\}$

$A \cap B = \{x : x \text{ is multiple of 3 and 5}\}$

$\Rightarrow \quad A \cap B = \{x : x \text{ is multiple of 15}\}$

$\Rightarrow \quad A \cap B = \{15, 30, 45,....\}$

20. **(a)** We have $x^2 = 16 \Rightarrow x = \pm 4$

Also, $2x = 6 \Rightarrow x = 3$

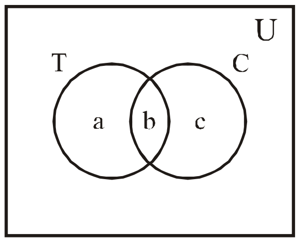
There is no value of x which satisfies both the above equations. Thus the set A contains no elements

$\therefore \ A = \phi$

21. **(c)** Clearly $A = \phi = \{\}$

22. **(d)** $x^2 + 1 = 0$ has no solution in R

23. **(d)** Let T denotes tea drinkers and C denotes coffee drinkers in universal set U.



From the diagram, we get

$a + b + c = 52$ ...(i)

$a = 16$ ...(ii)

$a + b = 33$ ...(iii)

Put $a = 16$ in equation (iii), we have

$16 + b = 33 \Rightarrow \ b = 17$

Now, substitute the values of a and b in equation (i), we get

$16 + 17 + c = 52$

$c = 52 - 33 = 19$

24. **(a)** Let $A \equiv$ Set of Tamil speaking students and

$B \equiv$ Hindi speaking students

$n(A) = 400, n(B) = 300$ and $n(A \cup B) = 600$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \ n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 400 + 300 - 600 = 100$

25. **(b)** Total number of students = 500

Let H be the set showing number of students who can speak Hindi = 475 and B be the set showing number of students who can speak Bengali = 200

So, n (H) = 475 and n (B) = 200 and given that

$n(B \cup H) = 500$

we have

$n(B \cup H) = n(B) + n(H) - n(B \cap H)$

$\Rightarrow 500 = 200 + 475 - n(B \cap H)$

so, $n(B \cap H) = 175$

Hence, persons who speak Hindi only = $n(H) - n(B \cap H)$

$= 475 - 175 = 300$

26. **(b)** Given, A = {3, 6, 9, 12}

$= \{x : x = 3n, n \in N \text{ and } 1 \le n \le 4\}$

and B = {1, 4, 9, ..., 100}

$= \{x : x = n^2, n \in N \text{ and } 1 \le n \le 10\}$

27. **(a)** (a) $A = \{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\} = \{2, 3\}$

So, A is a finite set

(b) $B = \{x : x \in Z \text{ and } x^2 \text{ is even}\}$

$= \{....., -6, -4, -2, 0, 2, 4, 6, ...\}$

Clearly, B is an infinite set.

(c) $D = \{x : x \in Z \text{ and } x > -10\}$

$= \{-9, -8, -7, ...\}$

Clearly, D is an infinite set.

28. **(a)**

(a) $|x| = 5 \Rightarrow x = 5$ $[\because x \in N]$

$\therefore$ Given set is singleton.

(b) $|x| = 6 \Rightarrow x = -6, 6$ $[\because x \in Z]$

$\therefore$ Given set is not singleton.

(c) $x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0$

$\Rightarrow$ x = –1, –1

Since, $-1 \notin N$, $\therefore$ given set = ϕ

(d) $x^2 = 7 \Rightarrow x = \pm\sqrt{7}$.

29. **(b)**

(a) There is no odd natural number divisible by 2, so there will be no element in this set, hence it is a null set.

(b) There is only one even prime number which is 2, i.e. there is an element, so it is not a null set.

(c) There is no natural number which is less than 5 and greater than 7, i.e. there is no element, so it is a null set.

(d) Since, parallel lines never intersect each other, so they have no common point, i.e. no element, so it is a

null set.

30. **(c)** Given that $A = \{x : x = n^2, n = 1, 2, 3\} = \{1, 4, 9\}$

$\therefore$ Number of elements in A is 3.

So, number of proper subsets $= 2^3 - 1 = 7$.

31. **(a)** Subset of { } i.e., ϕ is ϕ.

Subsets of {4} are ϕ, {4}.

Subsets of {4, 5} are ϕ, {4}, {5}, {4, 5}.

Subsets of {0} are ϕ, {0}.

32. **(d)** It is clear from the figure that set $A \cup C$ is not shaded and set B is shaded other than $A \cup C$, i.e., $B - (A \cup C)$.

33. **(b)**

34. **(c)** $A \cap (A \cup B)' = A \cap (A' \cap B') = (A \cap A') \cap B'$

$= \phi \cap B' = \phi$.

35. **(c)** Let A and B be the two sets of news channel such that n(A) = 63, n(B) = 76, $n(A \cup B) = 100$

Also, $n(A \cap B) = x$

Using, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow$ 100 = 63 + 76 – x

$\Rightarrow$ x = 139 – 100 = 39

Again, $n(A \cap B) \le n(A)$

$\Rightarrow$ $x \le 63$

$\therefore$ $39 \le x \le 63$.

36. **(c)** We see that each member in the given set has the numerator one less than the denominator. Also, the numerator begins from 1 and do not exceed 6. Hence, in the set-builder form, the given set is $\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 \le n \le 6\right\}$.

37. **(a)** Since, $3^x - 1$ is an even number for all $x \in Z^+$. So, the given set in roster form is {1, 2, 3, 4, 5}.

38. **(c)** A student cannot study simultaneously in both classes X and XI. Thus, the set B contains no element at all.

39. **(a)** We note that every element of Y is also an element of X, as if a student is in your class, then he is also in your school.

40. **(a)** If $A \subset B$ and $A \ne B$, then A is called a proper subset of B and B is called a super set of A.

41. **(a)** Let a, b $\in$ R and a < b. Then, the set of real numbers {x : a < x < b} is called an open interval. And a, b do not belong to this interval.

42. **(d)** a is not an element of {{a}, b}

$\therefore$ $a \notin \{\{a\}, b\}$

{b, c} is the element of {a, {b, c}}

$\therefore$ $\{b, c\} \in \{a, \{b, c\}\}$

$b \in \{a, b\}$ but $b \notin \{a, \{b, c\}\}$

$\therefore$ $\{a, b\} \not\subset \{a, \{b, c\}\}$.

43. **(b)**

represents [a, b).

represents (a, b].

represents (a, b).

represents [a, b].

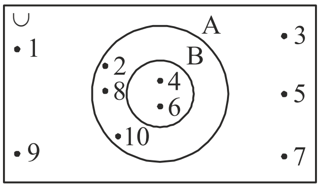
44. **(b)** The interval in the figure is [a, b].

45. **(c)** $n[P(\phi)] = 2^0 = 1$ $[\because n(\phi) = 0]$

$n[P(P(\phi))] = 2^1 = 2$

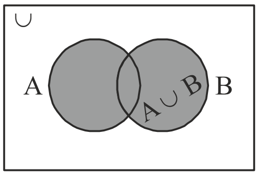
$n[P\{P(P(\phi))\}] = 2^2 = 4$.

46. (d) U = {1, 2, 3, 4, ..., 10}
A = {2, 4, 6, 8, 10}
B = {4, 6}
∵ All the elements of B are also in A.
∴ $B \subset A$
⇒ Set B lies inside A in the Venn diagram.



47. (c) Most of the relationships between sets can be represented by Venn diagrams.

48. (c) The union of two sets A and B can be represented by a Venn diagram as



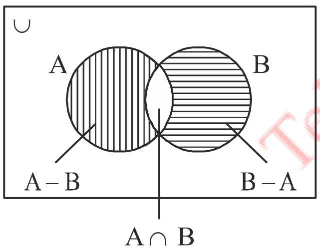
49. (c) Here, X = {Ram, Geeta, Akbar}
and Y = {Geeta, David, Ashok}
Then, $X \cap Y$ = {Geeta}

50. (b) Using the set-builder form, we can write the definition of difference as

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

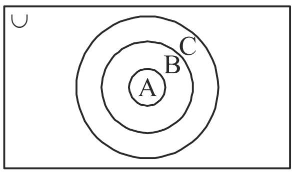
51. (d) The shaded region in the figure is $A - (B \cup C)$.

52. (c)



Clearly, $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

53. (c) If $A \subset B$ and $B \subset C$, then these sets is represented in Venn diagram as



Clearly, $A \cup B = B$
and $B \cap C = B$
Hence, $A \cup B = B \cap C$.

54. (a) In the given figure, the shaded portion represents complement of set A.

55. (c) $B \cup C = \{2, 3, 4, 6, 7, 8\}$

$(B \cup C)' = U - (B \cup C) = \{1, 5, 9, 10\}$

$C - A = \{4, 8\}$

$(C - A)' = \{1, 2, 3, 5, 6, 7, 9, 10\}$.

56. (d) $A \cap (A \cap B)^c = A \cap (A^c \cup B^c)$
$= (A \cap A^c) \cup (A \cap B^c) = \phi \cup (A \cap B^c) = A \cap B^c$.

57. (a) $A \cap B \subseteq A$. Hence, $A \cup (A \cap B) = A$.

58. (b) Given $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$.
Hence, A = {3, 5, 9}

59. (c) $A \cap (A \cup B)' = A \cap (A' \cap B')$,

$[\because (A \cup B)' = A' \cap B']$

$= (A \cap A') \cap B'$, [By associative law]

$= \phi \cap B'$, $[\because A \cap A' = \phi]$

$= \phi$.

60. (a) $A \cap (B - A) = \phi$ $[\because x \in B - A \Rightarrow x \notin A]$

61. (b) A – B = {1} and B – C = {4}
$(A - B) \times (B - C) = \{(1, 4)\}$.

62. (c) Since $A \subseteq B$,
∴ $A \cup B = B$
So, $n(A \cup B) = n(B) = 6$.

63. (a) Minimum value of x = 100 – (30 + 20 + 25 + 15)
= 100 – 90 = 10.

64. (b) A = {4, 8, 12, 16, 20, 24, ...}
B = {6, 12, 18, 24, 30, ...}
∴ $A \subset B$ = {12, 24, ...} = {x : x is a multiple of 12}.

STATEMENT TYPE QUESTIONS

65. (b) As given, P ≡ set of square, Q ≡ set of parallelogram, R ≡ set of quadrilaterals and S ≡ set of rectangles.
Since all squares are parallelogram

⇒ $P \subset Q$

Since, all squares are rectangles, ∴ $P \subset S$ and also all rectangles are quadrilateral, ∴ $S \subset R$

⇒ 1, 3 and 4 are correct

66. (d) Both statements are incorrect.

67. (b)

68. (d) Let us consider the sets
A = {1, 2, 4}, B = {2, 5, 6} and C = {1, 5, 7}
I. A – B = {1, 4} and $A - (A \cap B)$
= {1, 2, 4} – {2} = {1, 4}
∴ $A - B = A - (A \cap B)$
II. $(A \cap B) \cup (A - B)$
= {2} ∪ {1, 4} = {1, 2, 4} = A

III. $A - (B \cup C) = \{1, 2, 4\} - \{1, 2, 5, 6, 7\} = \{4\}$ and $(A - B) \cup (A - C) = \{1, 4\} \cup \{2, 4\} = \{1, 2, 4\}$

$\therefore$ $A - (B \cup C) \neq (A - B) \cup (A - C)$.

69. **(c)** In (i) and (iii), we can definitely decide whether a given particular object belongs to a given collection or not. For example, we can say that the river Nile does not belong to the collection of rivers of India. On the other hand, the river Ganga belongs to this collection.

Again, the collection of most talented batsmen of India and the collection of books is not well-defined, because the criterion for determining most talented batsman and collection of particular kind of books may vary from person-to-person.

70. **(c)** While writing the set in roster form, an element is not generally repeated, i.e. all elements are taken as distinct. The set of letters forming the word 'SCHOOL' is {S, C, H, O, L} or {H, O, L, C, S}. Here, the order of listing elements has no relevance. We can also express it as {S, C, H, O, L}.

71. **(c)** The collection of all months of a year beginning with the letter J and the collection of all boys in your class are well-defined. But the collection of ten most talented writers of India and a team of eleven best cricket batsmen of the world may vary from person-to-person, so these are not well defined. Hence, I and IV represent the sets.

72. **(b)** We can write $60 = 2 \times 2 \times 3 \times 5$

$\therefore$ Prime factors of 60 are 2, 3 and 5.

Hence, the set D in roster form is {2, 3, 5}.

There are 12 letters in the word 'TRIGONOMETRY' out of which three letters T, R and O are repeated.

Hence, set E in the roster form is {T, R, I, G, O, N, M, E, Y}.

73. **(b)** The empty set is denoted by the symbol ϕ or { }.

74. **(b)** The set of real numbers which satisfy $x^2 - 1 = 0$ is $\{-1, 1\}$.

So, Statement I is false.

Given, $x^2 = 16$ and $2x = 6$

$x = 4, -4$ and $x = 3$

$\therefore$ There is no real x which simultaneously satisfied $x^2 = 16$ and $2x = 6$.

So, Statement II is true.

75. **(c)** We do not know the number of animals living on the Earth, but it is some natural number. So, the set of animals living on the Earth is finite. There are infinite circles passing through the origin (0, 0). So, the set of circles passing through the origin (0, 0) is infinite.

76. **(c)** There are infinite positive integer greater than 100. So, the set of positive integers greater than 100 is infinite.

There are 25 prime numbers less than 99.

So, the set of prime numbers less than 99 is finite.

77. **(a)** There are infinite lines parallel to X-axis. So, the set of lines parallel to X-axis is infinite.

There are infinite numbers which are multiple of 5. So, the set of numbers, which are multiple of 5, is infinite.

There are 26 letters in the English alphabet. So, the set of letters in the English alphabet is finite.

78. **(c)** Since, $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that $A \neq B$, $A \neq C$, $A \neq D$, $A \neq E$.

Since, $B = \phi$, but none of the other sets are empty. Therefore $B \neq C$, $B \neq D$ and $B \neq E$. Also, $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.

Since, $E = \{5\}$, $C = E$. Further, $D = \{-5, 5\}$ and $E = \{5\}$, we find that $D \neq E$. Thus, the only one pair of equal sets is C and E.

79. **(c)** There are infinite concentric circles in a plane. So, the given set is infinite.

Now, $x^2 - 3 = 0$

or $x^2 = 3$

or $x = \pm\sqrt{3}$

Thus, there is no rational number satisfied $x^2 - 3 = 0$. So, given set is null set.

80. **(c)** From the definition of subset, it follows that every set is a subset of itself. Since, the empty set ϕ has no element, we agree to say that ϕ is a subset of every set.

81. **(c)** $A = \{1, 3, 5\}$

$B = \{x : x$ is an odd natural number less than $6\}$

$= \{1, 3, 5\}$

Since, every element of A is in B, so $A \subset B$.

Every element of B is in A, so $B \subset A$.

Then, $A = B$.

82. **(b)** The universal set must contain the elements 0, 1, 2, 3, 4, 5, 6 and 8.

83. **(b)** From all the three sets, set of all triangles in a plane is the universal set for set of isosceles triangle.

84. **(a)** Let A and B be two sets. Symbolically, the union of A and B write as $A \cup B$ and the common elements of A and B being taken only once.

85. **(b)** $A = \{a, b\}$, $B = \{a, b, c\}$

Since, all the elements of A are in B.

So, $A \subset B$.

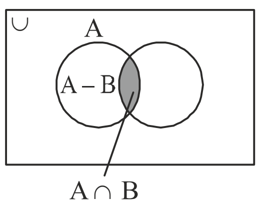
Hence, Statement I is false.

$\because$ $A \subset B$

$\Rightarrow$ $A \cup B = B$

Therefore, Statement II is true.

86. **(a)** I.

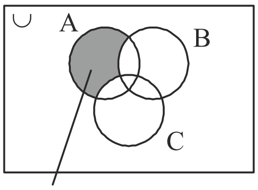


It is clear from the Venn diagram

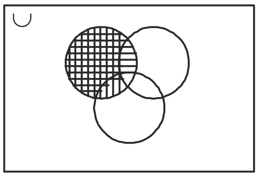
$A - B = A - (A \cap B)$

II. Also, it is clear from above diagram

$A = (A \cap B) \cup (A - B)$

III. 

$A - (B \cup C)$



≡ (horizontal) lines : A – C

||| (vertical) lines : A – B

It is clear from the diagrams

$A - (B \cup C) = (A - B) \cap (A - C)$

87. **(c)** If A is a subset of the universal set U, then its complement A' is also a subset of U.

We have, $A' = \{2, 4, 6, 8, 10\}$

Hence, $(A')' = \{x : x \in U \text{ and } x \notin A'\}$

$= \{1, 3, 5, 7, 9\} = A$

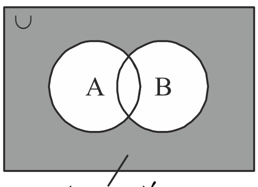
It is clear from the definition of the complement that for any subset of the universal set U, we have

$$(A')' = A$$

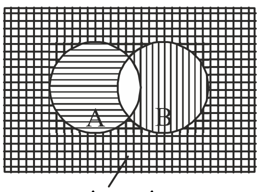
88. **(b)** Let U be the uni versal set and A is a subset of U. Then, the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U. Thus,

$A' = \{x : x \in U \text{ and } x \notin A\}$

Obviously, $A' = U - A$

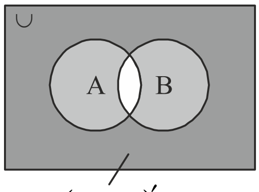
89. **(c)** 

$(A \cup B)'$

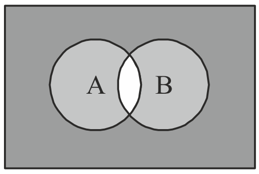


$A' \cap B'$

Clearly, $(A \cup B)'$ and $A' \cap B'$ are same.



$(A \cap B)'$



The shaded region is $A' \cup B'$

Clearly, $(A \cap B)'$ and $A' \cup B'$ are same.

90. **(a)** If A, B and C are finite sets, then

$n(A \cup B \cup C) = n(A) + n(B \cup C) - n[A \cap (B \cup C)]$

$[\because n(A \cup B) = n(A) + n(B) - n(A \cap B)]$

$= n(A) + n(B) + n(C) - n(B \cap C)$

$- n[A \cap (B \cup C)]$ ... (i)

Since, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, we get

$n[A \cap (B \cup C)] = n(A \cap B) + n(A \cap C)$

$- n[(A \cap B) \cap (A \cap C)]$

$= n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)$

Therefore, $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$- n(A \cap B) - n(B \cap C) - n(A \cap C)$

$+ n(A \cap B \cap C)$

Now, if A, B and C are mutually pairwise disjoint, then

$A \cap B = \phi = B \cap C = A \cap C = A \cap B \cap C$

$\therefore$ $n(A \cup B \cup C) = n(A) + n(B) + n(C)$.

91. **(b)** Let U denote the set of surveyed students and X denote the set of students taking apple juice and Y denote the set of students taking orange juice. Then,

$n(U) = 400, n(X) = 100, n(Y) = 150$

and $n(X \cap Y) = 75$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 100 + 150 - 75$

$= 175$

$\therefore$ 175 students were taking at least one juice.

$n(X' \cap Y') = n(X \cup Y)'$

$= n(U) - n(X \cup Y)$

$= 400 - 175$

$= 225$

Hence, 225 students were taking neither apple juice nor orange juice.

92. **(a)** Let $X \in P(A \cap B)$ …(i)

$\Leftrightarrow$ $x \subset A$ and $x \subset B$

$\Leftrightarrow$ $x \in P(A)$ and $x \in P(B)$

$\Leftrightarrow$ $x \in [P(A) \cap P(B)]$ …(ii)

Hence, from (i) and (ii)

$P(A) \cap P(B) = P(A \cap B)$

Now, $P(A) \cup P(B) \neq P(A \cup B)$, we can prove it by an example.

93. **(c)** Let $A = \{1, 2, 3, \ldots, n\}$

No. of subsets of $A = 2^n$

∴ $2^n = 128 \Rightarrow 2^n = 2^7 \Rightarrow n = 7$

∴ Number of elements in set A = 7

94. **(d)** Let X = {a, b, c, d}

$n(X) = 4$

No. of subsets of X = $2^4 = 16$

No. of non-empty subsets of A = 16 – 1 = 15

(∵ Only one set is empty set)

95. **(c)** I. $A \cup B = \{1, 2, 3, 4, 5, 6\}$

$(A \cup B) \cap C = \{3, 4, 6\}$

II. De-Morgan's law.

96. **(a)** Only I and II statements are incorrect.

I. $A - B = \{3, 6, 9, 15, 18, 21\}$

II. $C - B = \{2, 6, 10, 14, 20\}$

$D - B = \{5, 10, 15\}$

$(C - B) \cap (D - B) = \{10\}$

97. **(c)** Both the statements are true.

II. $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$= 720 + 450 - n(S \cap T)$

$= 1170 - n(S \cap T)$

$1170 - n(S \cap T) \le n(U)$

$1170 - n(S \cap T) \le 1000$

$\Rightarrow n(S \cap T) \ge 170$

98. **(a)** Only statement-I is true.

I. Consider $A = B \cap C$

$= (C \cap A) \cap C \Rightarrow A = C \cap A \Rightarrow A = B$

II. A = {a, b}

$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

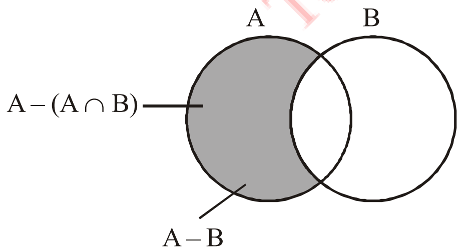
$A \cap P(A) = \phi$

99. **(b)** I and II are the correct statements.

$A - B = A - (A \cap B)$ is correct.

$A = (A \cap B) \cup (A - B)$ is correct.

Statement-III is false.



100. **(c)** I. $\bigcup_{n=2}^{10} A_n$ is the set of first 10 prime numbers

$= \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

II. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$50 = 28 + 32 - n(A \cap B)$

$n(A \cap B) = 60 - 50 = 10$

101. **(c)** By definition of union and intersection of two sets, both the statements are true.

MATCHING TYPE QUESTIONS

102. **(b)** If A is a subset of B, we write $A \subset B$ and if A is not a subset of B, then we write $A \not\subset B$.

In other words, $A \subset B$ if $a \in A$, then $a \in B$.

Now, if $A \subset B \Rightarrow$ Every element of A is in B

and $B \subset A \Rightarrow$ Every element of B is in A, then we can say A and B are the same set, so that we have $A \subset B$ and $B \subset A \Leftrightarrow A = B$,

where '⇔' is a symbol for two way implications and usually read as if and only if (briefly written as "iff").

103. **(d)** The open interval $a < x < b$ is represented by (a, b) or]a, b[. The interval $a \le x \le b$ contain end points also is called closed interval and is denoted by [a, b]. The interval $a \le x < b$ closed at the end a and open at the end b, i.e. [a, b). Similarly, the interval $a < x \le b$ is represented by (a, b].

104. **(c)** Some properties of operation of intersection are as follows:

A. $A \cap B = B \cap A$ [commutative law]

B. $(A \cap B) \cap C = A \cap (B \cap C)$ [associative law]

C. $\phi \cap A = \phi$ [law of ϕ]

D. $U \cap A = A$ [law of U]

E. $A \cap A = A$ [idempotent law]

F. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [distributive law]

105. **(b)** By properties of complement of a set,

A. $A \cup A' = U$

B. $A \cap A' = \phi$

By De-Morgan's laws,

C. $(A \cup B)' = A' \cap B'$

D. $(A \cap B)' = A' \cup B'$

By laws of empty set and universal set,

E. $\phi' = U$ and

F. $U' = \phi$

By law of double complementation,

G $(A')' = A$.

106. **(d)**

107. **(d)**

(A) Let $x \in A$, then $x \in A \cup B$

$\Rightarrow x \in A \cap B$ (∵ $A \cup B = A \cap B$)

$\Rightarrow x \in B$

$\therefore A \subset B$ …(i)

Similarly, if $y \in B$, then $y \in A \cap B$

$\Rightarrow y \in A$

$\therefore B \subset A$ …(ii)

From (i) & (ii), A = B

(C) Let $a \in A$, then there exists $x \in P(A)$ such that $a \in X$.

$\Rightarrow x \in P(B)$ $\quad (\because P(A) = P(B))$

$\Rightarrow a \in B$

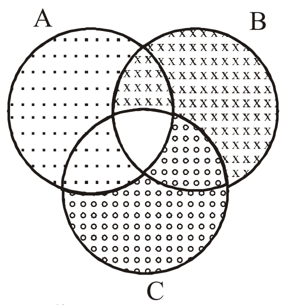
$\Rightarrow A \subset B$ ...(i)

Similarly, we can prove $B \subset A$ ...(ii)

from (i) and (ii), we have $A = B$

(D) $A \cup (B - A) = A \cup (B \cap A') = A \cup B$

(E)



From Venn - diagram

$(A - B) \cup (B - C) \cup (C - A) = (A \cap B \cap C)$.

108. **(b)** **109.** **(c)**

INTEGER TYPE QUESTIONS

110. **(a)** Since, $a + 2 = 6 \Rightarrow a = 4$

$\therefore$ the given set is $\{4\}$.

111. **(d)** An empty set does not contain any element.

112. **(d)** Number of elements in $X = 5$

113. **(c)** $n(X) = 3$

Number of proper subset $= 2^{n(x)} - 1$

$= 2^3 - 1 = 8 - 1 = 7$

114. **(c)** Total number of subset of given set $\{1, 2, 3, 4\} = 2^4 = 16$

Since, ϕ is the subset of every set.

$\therefore$ Number of non-empty subsets $= 16 - 1 = 15 = 3 \times 5$

115. **(c)** $n(A) = 0$

$n[P(A)] = 2^0 = 1$

116. **(d)** A is the set of points on circle.

B is the set of points on ellipse. These two intersects at four points.

$\therefore$ $A \cap B$ contains four points.

117. **(d)** $P(\phi)$ is the power set of the set ϕ.

$\therefore$ Cardinality $= P\{P[P(\phi)]\} = 4$

118. **(b)** $n[(A \cap B)' \cap A] = n[(A' \cup B') \cap A]$

$= n[(A' \cap A) \cup (B' \cap A)]$ (Distributive Law)

$= n[\phi \cup (B' \cap A)] = n(A \cap B') = n(A) - n(A \cap B)$

119. **(b)** Let M = set of Mathematics teachers

P = set of Physics teachers

n(only Maths teacher) $= n(M) - n(M \cap P) = 12 - 4 = 8$

Also, $n(M \cup P)$ = n (only Math teachers)

+ n(Only Physics teachers) + $n(M \cap P)$

$20 = 8 + 4 +$ n (only Physics teachers)

$\Rightarrow n = 8$.

ASSERTION-REASON TYPE QUESTIONS

120. **(a)** $A = \{a, b, c, d\}$

$\therefore$ $n(A) = 4$

$\therefore$ Number of subsets of $A = 2^4 = 16$, out of which only one set is empty set because empty set is subset of every set.

$\therefore$ Number of non-empty subsets of $A = 2^4 - 1 = 15$.

121. **(c)** If U is a universal set, then $B = U - A = A'$, for which $n(B) = n(A') = n(U) - n(A)$.

But for any three arbitrary sets A, B and C, we cannot always have $n(C) = n(A) - n(B)$, if $C = A - B$ as it is not specified here whether A is universal set or not. In case if A is not universal set, then we cannot conclude.

$n(C) = n(A) - n(B)$.

Hence, Assertion is true but Reason is false.

122. **(d)** As $A = \{1, \{2, 3\}\}$

$\therefore$ Subsets of $A = \phi, \{1\}, \{\{2, 3\}\}, \{1, \{2, 3\}\}$

Now, $\{\{2, 3\}\} \subset A$

$\therefore$ $\{\{2, 3\}\} \in P(A)$

$\therefore$ Assertion is false but Reason is obviously true.

123. **(b)** $\{1\}$ and $\{2\}$ are the element of $\{1, \{2\}\}$.

So, the subsets of the set $\{1, \{2\}\}$ are $\phi, \{1\}, \{\{2\}\}$ and $\{1, \{2\}\}$.

Hence, Assertion is true.

We know, total number of proper subsets of a set containing n elements is $2^n - 1$.

Hence, Reason is true. But Reason is not the correct explanation of Assertion.

124. **(b)** Let $x \in A - B$

$\Rightarrow$ $x \in A$ and $x \notin B$

$\Rightarrow$ $x \in A$ and $x \in B'$

$\Rightarrow$ $x \in B'$

$\therefore$ $A - B \subset B'$

It is true $A \cap A' = \phi$ [by complement laws]

Hence, both Assertion and Reason are correct but Reason is not a correct explanation of Assertion.

CRITICAL THINKING TYPE QUESTIONS

125. **(b)** In the given Venn diagram, shaded area between sets P an Q is $(P \cap Q) - R$ and shaded area between P and R is $(P \cap R) - Q$. So, both the shaded area is union of these two area and is represented by

$((P \cap Q) - R) \cup ((P \cap R) - Q)$.

126. **(d)** The shaded region represents $(P \cap Q) \cup (P \cap R)$.

127. **(b)** Given : Two finite sets have m and n elements

$\therefore$ $2^m - 2^n = 56$

$\Rightarrow \quad 2^m - 2^n = 64 - 8$

$\Rightarrow \quad 2^m - 2^n = 2^6 - 2^3$

$\Rightarrow \quad m = 6, n = 3$

128. (c) $A = \{1, 3, 5, 15\}, B = \{2, 3, 5, 7\}, C = \{2, 4, 6, 8\}$

$\therefore A \cup C = \{1, 2, 3, 4, 5, 6, 8, 15\}$

$(A \cup C) \cap B = \{2, 3, 5\}$

129. (a) As given :

S = the set of all triangles

P = the set of all isosceles triangles

Q = the set of all equilateral triangles

R = the set of all right angled triangles

$\therefore$ $P \cap Q$ represents the set of isosceles triangles and $R - P$ represents the set of non-isosceles right angled triangles.

130. (d) Let $A = \{1\}$, $B = \{2, 3\}$, then

$A \cup B = \{1, 2, 3\}$ and $A \cap B = \phi$

Now, $P(A) = \{\phi, \{1\}\}$, $P(B) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$

$\therefore$ $P(A) \cup P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{2, 3\}\}$

$P(A \cup B) = \{\phi, \{1\}, \{2\}, \{3\}, \{2, 3\}, \{1, 2\}, \{3, 1\}, \{1, 2, 3\}\}$

and $P(A \cap B) = \{\phi\}$.

131. (c) $\{5\}$ is a subset of A as $5 \in A$

But, $\{1, 2\}$ is not a subset of A as elements $1, 2 \notin A$.

132. (b)

133. (a) Let $B = \{b\}$. Then, $A = \{a, B\}$.

$\therefore$ $P(A) = \{\phi, \{a\}, \{B\}, \{a, B\}\}$

$= \{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$.

134. (b) $(A - B) \cup (B - A) \cup (A \cap B)$

= only A $\cup$ only B $\cup$ Both A and B

$= A \cup B$.

135. (c) Let U be the set of all consumers who were questioned, A be the set of consumers who liked product P_1 and B be the set of consumers who liked product P_2.

It is given that $n(U) = 2000, n(A) = 1720, n(B) = 1450$,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 1720 + 1450 - n(A \cap B)$

$= 3170 - n(A \cap B)$

Since, $A \cup B \subseteq U$

$\therefore$ $n(A \cup B) \le n(U)$

$\Rightarrow$ $3170 - n(A \cap B) \le 2000$

$\Rightarrow$ $3170 - 2000 \le n(A \cap B)$

$\Rightarrow$ $n(A \cap B) \ge 1170$

Thus, the least value of $n(A \cap B)$ is 1170.

Hence, the least number of consumers who liked both the products is 1170.

136. (b) $n(A) = 40\%$ of $10000 = 4000$, $n(B) = 2000$,

$n(C) = 1000$, $n(A \cap B) = 500$, $n(B \cap C) = 300$,

$n(C \cap A) = 400$, $n(A \cap B \cap C) = 200$

$\therefore \quad n\left(A \cap \bar{B} \cap \bar{C}\right) = n\left\{A \cap \left(B \cup C\right)'\right\}$

$= n(A) - n\{A \cap (B \cup C)\}$

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

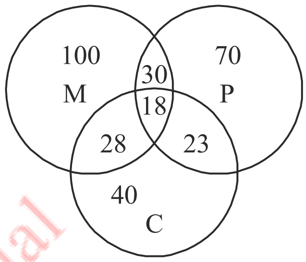
$= 4000 - 500 - 400 + 200 = 3300.$

137. (c) $n(M \cap P' \cap C')$

$= n(M) - [n(M \cap P) + n(M \cap C) - n(M \cap C \cap P)]$

$= 100 - 30 - 28 + 18 = 60$

[This can be solved directly by seeing the Venn Diagram]



138. (a) We have,

$3N = \{3x : x \in \mathbf{N}\} = \{3, 6, 9, 12, 15, 18, 21, 24.....\}$

$= \{x \in \mathbf{N} : x \text{ is a multiple of } 3\}$

and $7N = \{7x : x \in \mathbf{N}\} = \{7, 14, 21, 28.....\}$

$= \{x \in \mathbf{N} : x \text{ is a multiple of } 7\}$

$\therefore 3N \cap 7N = \{x \in \mathbf{N} : x \text{ is a multiple of } 3 \text{ and } 7\}$

$= \{x \in N : x \text{ is a multiple of } 21\} = \{21, 42, ...\}$

$= 21N$

139. (c) From the given we have in interval notation $A = (0, 3)$ and $B = [1, 5]$

Clearly $A - B = (0, 1) = \{x \in \mathbf{R} : 0 < x < 1\}$

and $B - A = [3, 5] = \{x \in \mathbf{R} : 3 \le x \le 5\}$

$\therefore A \Delta B = (A - B) \cup (B - A) = (0, 1) \cup [3, 5]$

$= \{x \in \mathbf{R} : 0 < x < 1 \text{ or } 3 \le x \le 5\}$

140. (d) We have

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 10 + 15 + 20 - 8 - 9 - n(C \cap A) + n(A \cap B \cap C)$

$= 28 - \{n(C \cap A) - n(A \cap B \cap C)\}$ ...(i)

Since $n(C \cap A) \ge n(A \cap B \cap C)$

We have $n(C \cap A) - n(A \cap B \cap C) \ge 0$ ...(ii)

From (i) and (ii)

$n(A \cup B \cup C) \le 28$ ...(iii)

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 10 + 15 - 8 = 17$

and $n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$= 15 + 20 - 9 = 26$

Since, $n(A \cup B \cup C) \ge n(A \cup C)$ and

$n(A \cup B \cup C) \geq n(B \cup C)$, we have

$n(A \cup B \cup C) \geq 17$ and $n(A \cup B \cup C) \geq 26$

Hence $n(A \cup B \cup C) \geq 26$ ...(iv)

From (iii) and (iv) we obtain

$26 \leq n(A \cup B \cup C) \leq 28$

Also $n(A \cup B \cup C)$ is a positive integer

$\therefore n(A \cup B \cup C) = 26$ or 27 or 28

141. (a) Let U be the set of consumers questioned X, the set of consumers who liked the product A and Y, the set of consumers who liked the product B. Then $n(U) = 1000$, $n(X) = 720$, $n(Y) = 450$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y) = 1170 - n(X \cap Y)$

$\therefore n(X \cap Y) = 1170 - n(X \cup Y)$

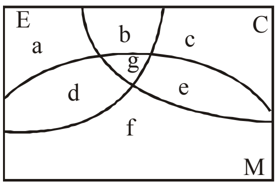
Clearly, $n(X \cap Y)$ is least when $n(X \cup Y)$ is maximum.

Now, $X \cup Y \subset U$

$\therefore n(X \cup Y) \leq n(U) = 1000$

$\therefore$ the maximum value of $n(X \cup Y)$ is 1000.

142. (b) C stands for set of students taking economics



$a + b + c + d + e + f + g = 40;\ a + b + d + g = 16$

$b + c + e + g = 22;\ d + e + f + g = 26$

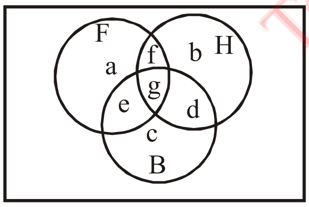
$b + g = 5;\ e + g = 14;\ g = 2$

Go by backward substitution

$e = 12, b = 3, d + f = 12, c + e = 17 \Rightarrow c = 5; a + d = 11$

$a + d + f = 18 \Rightarrow f = 7 \quad \therefore d = 12 - 7 = 5$

143. (a)



$a + e + f + g = 285, \quad b + d + f + g = 195$

$c + d + e + f = 115, \quad e + g = 45, f + g = 70, \quad d + g = 50$

$a + b + c + d + e + f + g = 500 - 50 = 450$

As in previous question, we obtain

$a + f = 240, b + d = 125, c + e = 65$

$a + e = 215, b + f = 145; b + c + d = 165$

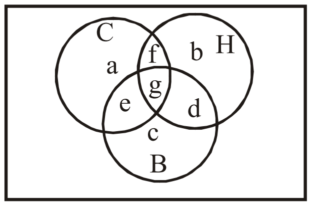
$a + c + e = 255; a + b + f = 335$

Solving we get

$b = 95, c = 40, a = 190, d = 30, e = 25, f = 50$ and $g = 20$

Desired quantity $= a + b + c = 325$

144. (d) $a + e + f + g = 224$



$b + d + f + g = 240$

$c + d + e + g = 336$

$d + g = 64, e + g = 80$

$f + g = 40, g = 24$

$\Rightarrow d = 40$

$e = 56, f = 16$

$a = 128, b = 160, c = 216$

$\therefore$ Boys who did not play any game

$= 800 - (a + b + c + d + e + f + g) = 160$

145. (b) Let $A = \{1\}$, $B = \{\{1\}, 2\}$ and $C = \{\{1\}, 2, 3\}$.

Here, $A \in B$ as $A = \{1\}$ and $B \subset C$ but $A \not\subset C$ as $1 \in A$ but $1 \notin C$.

146. (c) $V = \{a, e, i, o, u\}$

$V - B = \{e, o\}$

i.e., e and o are the elements belong to V but not to B

$B - V = \{k\}$

i.e., k is the element belongs to B but not to V.

$\therefore \quad B = \{a, i, u, k\}$

147. (b) Let M be the set of students passing in Mathematics, P be the set of students passing in Physics and C be the set of students passing in Chemistry.

Now, $n(M \cup P \cup C) = 50$, $n(M) = 37$, $n(P) = 24$, $n(C) = 43$

$n(M \cap P) \leq 19$, $n(M \cap C) \leq 29$, $n(P \cap C) \leq 20$ [given]

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P)$
$- n(M \cap C) - n(P \cap C) + n(M \cap P \cap C) \leq 50$

$\Rightarrow 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$

$\Rightarrow n(M \cap P \cap C) \leq 50 - 36$

$\Rightarrow n(M \cap P \cap C) \leq 14$

Thus, the largest possible number that could have passed all the three examinations, is 14.

CHAPTER

2

RELATIONS AND FUNCTIONS-I

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If $A \times B = \{(5,5), (5,6), (5,7), (8,6), (8,7), (8,5)\}$, then the value A is

(a) $\{5\}$ (b) $\{8\}$ (c) $\{5, 8\}$ (d) $\{5, 6, 7, 8\}$

2. Which of the following relation is a function ?

(a) $\{(a, b)\,(b, e)\,(c, e)\,(b, x)\}$

(b) $\{(a, d)\,(a, m)\,(b, e)\,(a, b)\}$

(c) $\{(a, d)\,(b, e)\,(c, d)\,(e, x)\}$

(d) $\{(a, d)\,(b, m)\,(b, y)\,(d, x)\}$

3. The relation R defined on the set of natural numbers as $\{(a, b) : \text{a differs from b by } 3\}$ is given

(a) $\{(1, 4), (2, 5), (3, 6), ...\}$ (b) $\{(4, 1), (5, 2), (6, 3), ...\}$

(c) $\{(1, 3), (2, 6), (3, 9), ...\}$ (d) None of these

4. If $f(x) = \dfrac{x}{x-1}$, then $\dfrac{f(a)}{f(a+1)}$ is equal to:

(a) $f(a^2)$ (b) $f\left(\dfrac{1}{a}\right)$ (c) $f(-a)$ (d) $f\left[\dfrac{-a}{a-1}\right]$

5. If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function described by the formula, $g(x) = \alpha x + \beta$ then what values should be assigned to α and β?

(a) $\alpha = 1, \beta = 1$ (b) $\alpha = 2, \beta = -1$

(c) $\alpha = 1, \beta = -2$ (d) $\alpha = -2, \beta = -1$

6. If $f : R \to R$ is defined by $f(x) = 3x + |x|$, then $f(2x) - f(-x) - 6x =$

(a) $f(x)$ (b) $2f(x)$

(c) $-f(x)$ (d) $f(-x)$

7. If $f(x) = x^3 - \dfrac{1}{x^3}$, then $f(x) + f\left(\dfrac{1}{x}\right)$ is equal to

(a) $2x^3$ (b) $2\dfrac{1}{x^3}$

(c) 0 (d) 1

8. The domain of $f(x) = \dfrac{1}{\sqrt{2x-1}} - \sqrt{1-x^2}$ is:

(a) $\left]\dfrac{1}{2}, 1\right[$ (b) $[-1, \infty[$

(c) $[1, \infty[$ (d) None of these

9. If $f(x+1) = x^2 - 3x + 2$, then $f(x)$ is equal to:

(a) $x^2 - 5x - 6$ (b) $x^2 + 5x - 6$

(c) $x^2 + 5x + 6$ (d) $x^2 - 5x + 6$

10. If $f(x) = \dfrac{1-x}{1+x}$, then $f\left(\dfrac{1-x}{1+x}\right)$ is equal to:

(a) x (b) $\dfrac{1-x}{1+x}$

(c) $\dfrac{1+x}{1-x}$ (d) $1/x$

11. The Cartesian product of two sets P and Q, i.e., $P \times Q = \phi$, if

(a) either P or Q is the null set

(b) neither P nor Q is the null set

(c) Both (a) and (b)

(d) None of the above

12. A relation is represented by

(a) Roster method (b) Set-builder method

(c) Both (a) and (b) (d) None of these

13. Let A = {x, y, z} and B = {a, b, c, d}. Then, which one of the following is not a relation from A to B?

(a) {(x, a), (x, c)} (b) {(y, c), (y, d)}

(c) {(z, a), (z, d)} (d) {(z, b), (y, b), (a, d)}

14. Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, a - b \text{ is an integer}\}$. Then

(a) domain of R is {2, 3, 4, 5, ...}

(b) range of R is Z

(c) Both (a) and (b)

(d) None of the above

15. There are three relations R_1, R_2 and R_3 such that

$R_1 = \{(2, 1), (3, 1), (4, 2)\}$,

$R_2 = \{(2, 2), (2, 4), (3, 3), (4, 4)\}$ and

$R_3 = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$

Then,

(a) R_1 and R_2 are functions

(b) R_2 and R_3 are functions

(c) R_1 and R_3 are functions

(d) Only R_1 is a function

16. Let N be the set of natural numbers and the relation R be defined such that $\{R = (x, y) : y = 2x, x, y \in N\}$. Then,

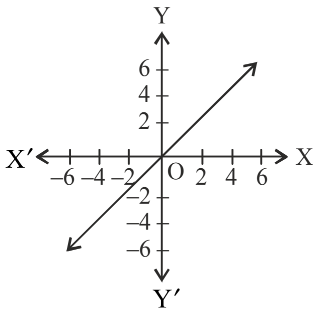
(a) R is a function

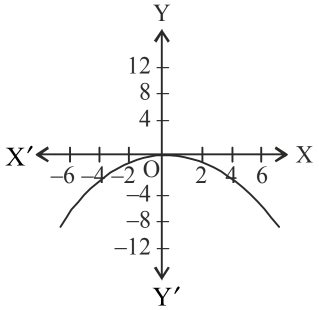
(b) R is not a function

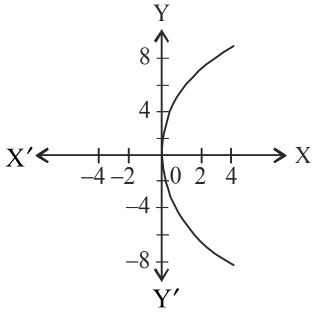
(c) domain, range and co-domain is N

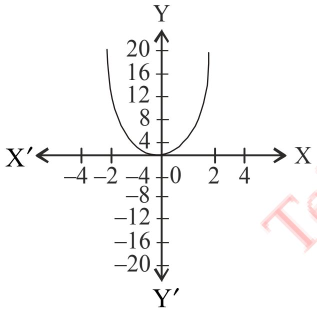
(d) None of the above

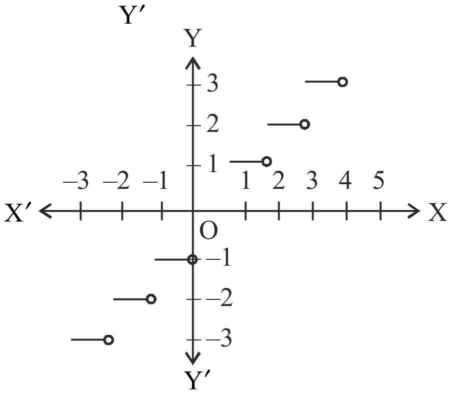
17. The graph of the function $f : R \to R$ and $f(x) = x^2, x \in R$, is

(a) 

(b) 

(c) 

(d) 

18. 

$f(x) = [x]$

Which of the following options identify the above graph?
(a) Modulus function
(b) Greatest integer function
(c) Signum function
(d) None of these

19. The domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ is equal, i.e. $f(x) = g(x)$, is
(a) $\{0, 2\}$ (b) $\left\{\frac{1}{2}, -2\right\}$
(c) $\left\{-\frac{1}{2}, 2\right\}$ (d) $\left\{\frac{1}{2}, 2\right\}$

20. If $g(x) = 1 + \sqrt{x}$ and $f[g(x)] = 3 + 2\sqrt{x} + x$, then $f(x) =$
(a) $1 + 2x^2$ (b) $2 + x^2$
(c) $1 + x$ (d) $2 + x$

21. Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Then,
(a) f is a relation from A to B
(b) f is a function from A to B
(c) Both (a) and (b)
(d) None of these

22. If A = {2, 3, 4, 5} and B = {3, 6, 7, 10}. R is a relation defined by R = {(a, b) : a is relatively prime to b, $a \in A$ and $b \in B$}, then domain of R is
(a) {2, 3, 5} (b) {3, 5}
(c) {2, 3, 4} (d) {2, 3, 4, 5}

23. The domain of relation
$R = \{(x, y) : x^2 + y^2 = 16, x, y \in Z\}$ is
(a) {0, 1, 2, 3, 4}
(b) {–4, –3, –2, –1}
(c) {–4, –3, –2, –1, 0, 1, 2, 3, 4}
(d) None of the above

24. If A = {1, 2}, B = {1, 3}, then $(A \times B) \cup (B \times A)$ is equal to
(a) {(1, 3), (2, 3), (3, 1), (3, 2), (1, 1), (2, 1), (1, 2)}
(b) {(1, 3), (3, 1), (3, 2), (2, 3)}
(c) {(1, 3), (2, 3), (3, 1), (3, 2), (1, 1)}
(d) None of these

25. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then $(A - C) \times (B - C)$ is equal to
(a) {(1, 4)} (b) {(1, 4), (4, 4)}
(c) {(4, 1), (4, 4)} (d) {(1, 2), (2, 5)}

26. Let set X = {a, b, c} and Y = ϕ. The number of ordered pairs in $X \times Y$ are
(a) 0 (b) 1
(c) 2 (d) 3

27. Let A = {x, y, z} and B = {a, b, c, d}. Which one of the following is not a relation from A to B?
(a) {(x, a), (x, c)} (b) {(y, c), (y, d)}
(c) {(z, a), (z, d)} (d) {(z, b), (y, b), (a, d)}

28. A relation R is defined in the set Z of integers as follows $(x, y) \in R$ iff $x^2 + y^2 = 9$. Which of the following is false?
(a) R = {(0, 3), (0, –3), (3, 0), (–3, 0)}
(b) Domain of R = {–3, 0, 3}
(c) Range of R = {–3, 0, 3}
(d) None of these

29. The domain and range of the relation R given by $R = \{(x, y) : y = x + \frac{6}{x};$ where $x, y \in N$ and $x < 6\}$ is
(a) {1, 2, 3}, {7, 5} (b) {1, 2}, {7, 5}
(c) {2, 3}, {5} (d) None of these

30. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, then $f\left(\frac{1}{2}\right) \times g(14)$ is
(a) $\frac{1336}{5}$ (b) $\frac{1363}{4}$
(c) 1251 (d) 1608

31. Let $f(x) = 1 + x$, $g(x) = x^2 + x + 1$, then $(f + g)(x)$ at $x = 0$ is
(a) 2 (b) 5
(c) 6 (d) 9

32. If $\phi(x) = a^x$, then $[\phi(p)]^3$ is equal to
(a) $\phi(3p)$ (b) $3\phi(p)$
(c) $6\phi(p)$ (d) $2\phi(p)$

33. Domain of $\sqrt{a^2 - x^2}$, $(a > 0)$ is
(a) $(-a, a)$ (b) $[-a, a]$
(c) $[0, a]$ (d) $(-a, 0]$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

34. Consider the following statements :
I. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$
II. $A \times \phi = \phi$
III. In general, $A \times B \neq B \times A$
Which of the above statements are true ?
(a) Only I (b) Only II
(c) Only III (d) All of the above

35. Consider the following statements:
Statement-I: The Cartesian product of two non-empty sets P and Q is denoted as P × Q and $P \times Q = \{(p, q) : p \in P, q \in Q\}$.
Statement-II: If A = {red, blue} and B = {b, c, s}, then A × B = {(red, b), (red, c), (red, s), (blue, b), (blue, c), (blue, s)}.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

36. Which of the following is/are true?
I. If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}.
II. If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y), such that $x \in A$ and $y \in B$.
III. If A = {1, 2} and B = {3, 4}, then $A \times (B \cap \phi) = \phi$.
(a) I and II are true (b) II and III are true
(c) I and III are true (d) All are true

37. Consider the following statements:
Statement-I: If R is a relation from A to B, then domain of R is the set A.
Statement-II: The set of all second elements in a relation R from a set A to a set B is called co-domain of R.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

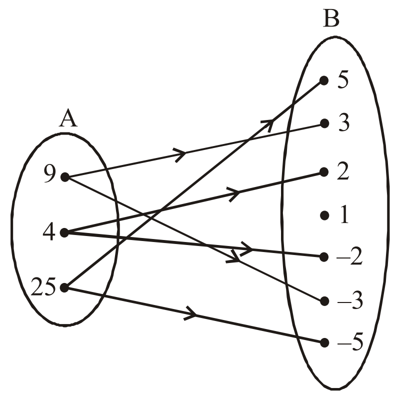
38. Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$. Then, which of the following is/are true?
I. $(a, a) \in R$ for all $a \in N$.
II. $(a, b) \in R$ implies $(b, a) \in R$.
III. $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$.
(a) I and II are true (b) II and III are true
(c) All are true (d) None of these

39. Consider the following statements:
Statement-I: The domain of the relation $R = \{(a, b) : a \in N, a < 5, b = 4\}$ is {1, 2, 3, 4}.
Statement-II: The range of the relation $S = \{(a, b) : b = |a - 1|, a \in Z \text{ and } |a| \leq 3\}$ is {1, 2, 3, 4}.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

40. Consider the following statements.
Let A = {1, 2, 3, 4} and B = {5, 7, 9}
I. A × B = B × A
II. n (A × B) = n (B × A)
Choose the correct option.
(a) Statement-I is true. (b) Statement-II is true.
(c) Both are true. (d) Both are false.

41. Consider the following statements.
I. Let A and B are non-empty sets such that $A \subseteq B$. Then, $A \times C \subseteq B \times C$.
II. For any two sets A and B, A × B = B × A
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

42. The figure given below shows a relation R between the sets A and B.



Then which of the following is correct?
I. The relation R in set-builder form is {(x,y) : x is the square of y, $x \in A$, $y \in B$}
II. The domain of the relation R is {4, 9, 25}
III. The range of the relation R is {–5, –3, –2, 2, 3, 5}
(a) Only I and II are true. (b) Only II and III are true.
(c) I, II and III are true (d) Neither I, II nor III are true.

43. Consider the following statements.
I. If (a, 1), (b, 2) and (c, 1) are in A × B and n(A) = 3, n (B) = 2, then A = {a, b, c} and B = {1, 2}
II. If A = {1, 2} and B = {3, 4}, then $A \times (B \cap \phi)$ is equal to A × B.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Neither I nor II is true

44. Consider the following statements.
I. Relation R = {(2, 0), (4, 8), (2, 1), (3, 6)} is not a function.
II. If first element of each ordered pair is different with other, then the given relation is a function.

Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Neither I nor II is true.

45. Consider the following statements.
I. If the set A has 3 elements and set B = {3, 4, 5}, then the number of elements in $A \times B = 9$.
II. The domain of the relation R defined by $R = \{(x, x+5) : x \in (0, 1, 2, 3, 4, 5)\}$ is {5, 6, 7, 8, 9, 10}.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Both I and II are false.

46. Consider the following statements.
I. If X = {p, q, r, s} and Y = {1,2, 3, 4, 5}, then {(p, 1), (q, 1), (r, 3), (s, 4)} is a function.
II. Let A = {1, 2, 3, 4, 6}. If R is the relation on A defined by {(a, b) : a, b ∈ A, b is exactly divisible by a}.
The relation R in Roster form is {(6, 3), (6, 2), (4, 2)}
Choose the correct option.
(a) Only I is false. (b) Only II is false.
(c) Both I and II are false. (d) Neither I nor II is false.

47. Consider the following statements.
I. Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a linear function from Z to Z. Then, f(x) is 2x – 1.
II. If $f(x) = x^3 - \dfrac{1}{x^3}$, then $f(x) + f\left(\dfrac{1}{x}\right)$ is equal to 0.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

48. Consider the following statements.
I. The relation $R = \{(x, x^3) : x$ is a prime number less than $10\}$ in Roster form is {(3, 27), (5, 125), (7, 343)}
II. The range of the relation $R = \{(x+2, x+4) : x \in N, x < 8\}$ is {1, 2, 3, 4, 5, 6, 7}.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

49. Consider the following statements
I. Let n(A) = m and n(B) = n. Then the total number of non-empty relations that can be defined from A to B is $2^{mn} - 1$
II. If A = {1, 2, 3}, B = {3, 8}, then $(A \cup B) \times (A \cap B)$ is equal to {(1, 3), (2, 3), (3, 3), (8, 3)}.
III. If $\left(\dfrac{x}{2} - 1, \dfrac{y}{9} + 1\right) = (2, 1)$, then the values of x and y respectively are 6 and 0.
Choose the correct option.
(a) Only I and II are false.
(b) Only II and III are true.
(c) Only I and III are true.
(d) All the three statements are true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

50. Let A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}. Then, match the following in column-I with the sets of ordered pairs in column-II.

Column -I	Column -II
A. $A \times (B \cap C)$	1. {(1, 4), (2, 4), (3, 4)}
B. $(A \times B) \cap (A \times C)$	2. {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}
C. $A \times (B \cup C)$	3. {(3, 4)}
D. $(A \times B) \cup (A \times C)$	
E. $(A \cap B) \times (B \cap C)$	

Codes:

	A	B	C	D	E
(a)	1	2	1	2	3
(b)	2	2	1	1	3
(c)	1	1	2	2	3
(d)	2	1	2	3	2

51. Let A = {1, 2, 3, 4, ..., 14}. A relation R from A to A is defined by R = {(x, y) : 3x – y = 0, where x, y ∈ A}.

Column-I	Column-II
A. In Roster form, the relation R is	1. {1, 2, 3, 4}
B. The domain of R is	2. {3, 6, 9, 12}
C. The range of R is	3. {1, 2, 3, 4, ..., 14}
D. The co-domain of R is	4. {(1, 3), (2, 6), (3, 9), (4, 12)}

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	4	1	2	3
(c)	4	2	1	3
(d)	4	3	1	2

52. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Then, match the functions given in column-I with the expressions in column-II.

Column- I	Column -II
A. $(f + g)(x)$	1. $x^2 - 2x - 1$
B. $(f - g)(x)$	2. $x^2 + 2x + 1$
C. $(fg)(x)$	3. $\dfrac{x^2}{2x+1}, x \neq -\dfrac{1}{2}$
D. $\left(\dfrac{f}{g}\right)(x)$	4. $2x^3 + x^2$

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	4	1	2	3
(c)	2	1	3	4
(d)	2	4	1	3

53. Let $f(x) = 2x + 5$ and $g(x) = x^2 + x$. Then, match the functions given in column-I with the expressions in column-II.

Column-I	Column-II
A. $(f+g)x$	1. $2x^3 + 7x^2 + 5x$
B. $(f-g)x$	2. $x^2 + 3x + 5$
C. $(fg)(x)$	3. $\frac{2x+5}{x^2+x}$, $x \neq 0, -1$
D. $\left(\frac{f}{g}\right)(x)$	4. $5 + x - x^2$

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	4	1	2	3
(c)	2	1	4	3
(d)	1	4	2	3

54.

Column - I	Column - II
(A) The range of the function $f(x) = x$, x is a real number, is	1. $[0, \infty)$
(B) The domain of the real function $f(x) = \frac{1}{\sqrt{4-x^2}}$ is	2. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(C) The range of the function $f(x) = \frac{x}{1+x^2}$ is	3. R (Real numbers)
(D) The range of the function defined by $f(x) = \sqrt{x-1}$ is	4. $(-2, 2)$

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	3	4	2	1
(c)	3	2	4	1
(d)	1	4	2	3

55.

Column - I	Column - II
(A) Every function is a	1. real function.
(B) If f is a function from A to B and $(a, b) \in f$, then b is called	2. linear function
(C) If the domain of a function is either R or a subset of R, then it is called a	3. relation
(D) The function f defined by $f(x) = mx + c$, where m and c are constants, $x \in R$ is called	4. the image of 'a' under f.

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	3	4	1	2
(c)	3	1	4	2
(d)	2	1	4	3

56. If f is the identity function and g is the modulus function, then match the column-I with column-II.

Column - I	Column -II
(A) $(f+g)(x) =$	1. $\begin{cases} 0, & x \geq 0 \\ 2x, & x < 0 \end{cases}$
(B) $(f-g)(x) =$	2. $\begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$
(C) $(fg)(x) =$	3. $\begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$
(D) $\left(\frac{f}{g}\right)(x) =$	4. $\begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

Codes:

(a)	1	4	3	2
(b)	4	1	2	3
(c)	4	1	3	2
(d)	2	1	3	4

57. If $A = \{3, 4\}$ and $B = \{5, 6, 7\}$. Then match the column-I with the column-II.

Column-I	Column-II
(A) Number of relations from A to A is	1. 2^6
(B) Number of relations from B to B is	2. 2^9
(C) Number of relations from A to B is	3. 2^4

Codes:

	A	B	C
(a)	1	2	3
(b)	1	3	2
(c)	2	3	1
(d)	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

58. If $(4x + 3, y) = (3x + 5, -2)$, then the sum of the values of x and y is

(a) 0 (b) 2 (c) –2 (d) 1

59. If $(x + 3, 4 - y) = (1, 7)$, then the value of $4 + y$ is

(a) 3 (b) 4 (c) 5 (d) 1

60. The number of elements in the set $\{(x, y) : 2x^2 + 3y^2 = 35, x, y \in Z\}$, where Z is the set of all integers,

(a) 8 (b) 2 (c) 4 (d) 6

61. If the set A has 3 elements and the set $B = \{3, 4\}$, then the number of elements in $A \times B$ is

(a) 6 (b) 9 (c) 8 (d) 2

62. If $n(X) = 5$ and $n(Y) = 7$, then the number of relations on $X \times Y$ is 2^{5m}. The value of 'm' is

(a) 5 (b) 7
(c) 6 (d) 8

63. If $f(x) = 4x - x^2$, $x \in R$, then $f(b + 1) - f(b - 1)$ is equal to $m(2 - b)$. The value of 'm' is

(a) 2 (b) 3
(c) 4 (d) 5

64. If $f(y) = 2y^2 + by + c$ and $f(0) = 3$ and $f(2) = 1$, then the value of $f(1)$ is
(a) 0 (b) 1
(c) 2 (d) 3

65. Let X = {1, 2, 3}. The total number of distinct relations that can be defined over X is 2^n. The value of 'n' is
(a) 9 (b) 6
(c) 8 (d) 2

66. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then the value of 'a' is
(a) 3 (b) 0
(c) 2 (d) 1

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

67. Let A = {1, 2, 3, 4, 6}. If R is the relation on A defined by {(a, b) : a, b ∈ A, b is exactly divisible by a}.
Assertion : The relation R in Roster form is {(6, 3), (6, 2), (4, 2)}.
Reason : The domain and range of R is {1, 2, 3, 4, 6}.

68. **Assertion :** If (x + 1, y – 2) = (3, 1), then x = 2 and y = 3.
Reason : Two ordered pairs are equal, if their corresponding elements are equal.

69. **Assertion :** Let f and g be two real functions given by f = {(0, 1), (2, 0), (3, –4), (4, 2), (5, 1)} and g = {(1, 0), (2, 2), (3, –1), (4, 4), (5, 3)}
Then, domain of $f \cdot g$ is given by {2, 3, 4, 5}.
Reason : Let f and g be two real functions. Then, $(f \cdot g)(x) = f\{g(x)\}$.

70. **Assertion :** If $f(x) = \frac{1}{x-2}$, $x \neq 2$ and $g(x) = (x-2)^2$, then
$(f + g)(x) = \frac{1+(x-2)^3}{x-2}$, $x \neq 2$.
Reason : If f and g are two functions, then their sum is defined by $(f + g)(x) = f(x) + g(x)$ $\forall\ x \in D_1 \cap D_2$, where D_1 and D_2 are domains of f and g, respectively.

71. **Assertion :** If A = {x, y, z} and B = {3, 4}, then number of relations from A to B is 2^5.
Reason : Number of relations from A to B is $2^{n(A) \times n(B)}$.

72. Let A = {a, b, c, d, e, f, g, h} and R = {(a, a), (b, b), (a, g), (b, a), (b, g), (g, a), (g, b), (g, g), (b, b)}
Consider the following statements:
Assertion : $R \subset A \times A$.
Reason : R is not a relation on A.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

73. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is
(a) m^n (b) $n^m - 1$
(c) $mn - 1$ (d) $2^{mn} - 1$

74. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is
(a) 2^9 (b) 9^2
(c) 3^2 (d) $2^9 - 1$

75. If $f(x) = \frac{2^x + 2^{-x}}{2}$, then $f(x + y) \cdot f(x - y) =$
(a) $\frac{1}{2}[f(2x) + f(2y)]$ (b) $\frac{1}{4}[f(2x) + f(2y)]$
(c) $\frac{1}{2}[f(2x) - f(2y)]$ (d) $\frac{1}{4}[f(2x) - f(2y)]$

76. $f(x) = \frac{x(x-p)}{q-p} + \frac{x(x-q)}{p-q}$, $p \neq q$. What is the value of $f(p) + f(q)$?
(a) $f(p - q)$ (b) $f(p + q)$
(c) $f(p(p + q))$ (d) $f(q(p - q))$

77. If $f(x) = x$ and $g(x) = |x|$, then $(f + g)(x)$ is equal to
(a) 0 for all $x \in R$ (b) 2x for all $x \in R$
(c) $\begin{cases} 2x, \text{for } x \geq 0 \\ 0, \ \text{for } x < 0 \end{cases}$ (d) $\begin{cases} 0, \ \text{for } x \geq 0 \\ 2x, \text{for } x < 0 \end{cases}$

78. Let A = {1, 2}, B = {3, 4}. Then, number of subsets of A × B is
(a) 4 (b) 8
(c) 18 (d) 16

79. If A, B and C are three sets, then
(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(b) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
(c) Both (a) and (b)
(d) None of the above

80. If A = {8, 9, 10} and B = {1, 2, 3, 4, 5}, then the number of elements in A × A × B are
(a) 15 (b) 30
(c) 45 (d) 75

81. If A, B and C are any three sets, then $A \times (B \cup C)$ is equal to
(a) $(A \times B) \cup (A \times C)$ (b) $(A \cup B) \times (A \cup C)$
(c) $(A \times B) \cap (A \times C)$ (d) None of these

82. If the set A has p elements, B has q elements, then the number of elements in A × B is
(a) p + q (b) p + q + 1
(c) pq (d) p^2

83. If A = {a, b, c}, B = {b, c, d} and C = {a, d, c}, then $(A - B) \times (B \cap C) =$
(a) {(a, c), (a, d)} (b) {(a, b), (c, d)}
(c) {(c, a), (a, d)} (d) {(a, c), (a, d), (b, d)}

84. If A = {a, b}, B = {c, d}, C = {d, e}, then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} is equal to
(a) $A \cap (B \cup C)$ (b) $A \cup (B \cap C)$
(c) $A \times (B \cup C)$ (d) $A \times (B \cap C)$

85. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in A × B is 7. Then $p^2 + q^2 =$
(a) 42 (b) 49
(c) 50 (d) 51

86. The cartesian product of A × A has 9 elements, two of which are (–1, 0) and (0, 1), the remaining elements of A × A is given by
(a) {(–1, 1), (0, 0), (–1, –1), (1, –1), (0, –1)}
(b) {(–1, –1), (0, 0), (–1, 1), (1, –1), (1, 0), (1, 1), (0, –1)}
(c) {(1, 0), (0, –1), (0, 0), (–1, –1), (1, –1), (1, 1)}
(d) None of these

87. Let A = {1, 2, 3}. The total number of distinct relations that can be defined over A, is
(a) 2^9 (b) 6
(c) 8 (d) 2^6

88. The relation R defined on the set A = {1, 2, 3, 4, 5} by R = {(x, y) : $|x^2 - y^2| < 16$} is given by
(a) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}
(b) {(2, 2), (3, 2), (4, 2), (2, 4)}
(c) {(3, 3), (4, 3), (5, 4), (3, 4)}
(d) None of these

89. The relation R defined on set A = {x : |x| < 3, x ∈ I} by R = {(x, y) : y = |x|} is
(a) {(–2, 2), (–1, 1), (0, 0), (1, 1), (2, 2)}
(b) {(–2, –2), (–2, 2), (–1, 1), (0, 0), (1, –2), (1, 2), (2, –1), (2, –2)}
(c) {(0, 0), (1, 1), (2, 2)}
(d) None of these

90. The domain of the function $f(x) = \frac{|x+3|}{x+3}$ is
(a) {–3} (b) R – {–3}
(c) R – {3} (d) R

91. Let n(A) = 8 and n(B) = p. Then, the total number of non-empty relations that can be defined from A to B is
(a) 8^p (b) $n^p - 1$
(c) 8p – 1 (d) $2^{8p} - 1$

92. The domain of the real valued function
$f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$ is
(a) (– 5, 1) (b) $-5 \le x$ and $x \ge 1$
(c) (– 4, 1] (d) ϕ

93. The domain of the function
$f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is
(a) (– 1, 0) ∪ (1, 2)
(b) (1, 2) ∪ (2, ∞)
(c) (– 1, 0) ∪ (1, 2) ∪ (2, ∞)
(d) (1, 2)

94. The domain of the function $f(x) = \frac{1}{\sqrt{9 - x^2}}$ is
(a) $-3 \le x \le 3$ (b) $-3 < x < 3$
(c) $-9 \le x \le 9$ (d) $-9 < x < 9$

95. The domain and range of the real function f defined by f(x) = |x – 1| is
(a) R, [0, ∞) (b) R, (–∞, 0)
(c) R, R (d) (–∞, 0), R

96. Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a linear function from Z into Z, then f(x) =
(a) 2x – 1 (b) 2x
(c) 2x + 1 (d) –2x + 1

97. The domain of the function f defined by $f(x) = \frac{1}{\sqrt{x - |x|}}$ is
(a) R (b) R^+
(c) R^- (d) {ϕ}

98. The domain of the function $f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$ is
(a) R (b) R – {1, 4}
(c) R – {1} (d) (1, 4)

99. The domain and range of the function f given by f(x) = 2 – |x – 5| is
(a) Domain = R^+, Range = (–∞, 1]
(b) Domain = R, Range = (–∞, 2]
(c) Domain = R, Range = (–∞, 2)
(d) Domain = R^+, Range = (–∞, 2]

100. If P = {a, b, c} and Q = {r}, then
(a) P × Q = Q × P (b) P × Q ≠ Q × P
(c) P × Q ⊂ Q × P (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** $\{5, 8\}$

2. **(c)** Since in (c) each element is associated with unique element. While in (a) element *b* is associated with two elements, in (b) element *a* is associated with three elements and in (d) element *b* is associated with two elements so relation given in option (c) is function.

3. **(b)** The set is $\{(a, b): a - b = 3, a, b \in N\}$

   Here $a = b + 3$

   For $b = 1, a = 4$

   For $b = 2, a = 5$

   For $b = 3, a = 6$.

   and so, on

   Hence the given set is

   $\{(4, 1), (5, 2), (6, 3)...\}$

4. **(a)** Given $f(x) = \frac{x}{x-1}$

   Then, $f(a) = \frac{a}{a-1}$

   and $f(a+1) = \frac{a+1}{a}$

   So, $\frac{f(a)}{f(a+1)} = \frac{a}{a-1} \times \frac{a}{a+1} = \frac{a^2}{a^2-1} = f(a^2)$

5. **(b)** (1, 1) satisfies $g(x) = \alpha x + \beta \therefore \alpha + \beta = 1$

   (2, 3) satisfies $g(x) = \alpha x + \beta \therefore 2\alpha + \beta = 3$

   Solving the two equation, we get $\alpha = 2, \ \beta = -1$

   It can be checked that other ordered pairs satisfy $g(x) = 2x - 1$

6. **(a)** $f(x) = 3x + |x|$

   $\therefore f(2x) - f(-x) - 6x$

   $= 6x + |2x| - 3(-x) - |-x| - cx$

   $= 3x + 2|x| - |x| \ (\because |x| = |-x|)$

   $= 3x + |x| = f(x)$

7. **(c)** Since $f(x) = x^3 - \frac{1}{x^3}$

   $$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3$$

   Hence,

   $$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$$

8. **(a)** Given, $f(x) = \frac{1}{\sqrt{2x-1}} - \sqrt{1-x^2} = p(x) - q(x)$

   where $p(x) = \frac{1}{\sqrt{2x-1}}$ and $q(x) = \sqrt{1-x^2}$

   Now, Domain of p(x) exist when $2x - 1 \neq 0$

   $\Rightarrow \quad x = \frac{1}{2}$ and $2x - 1 > 0$

   $\Rightarrow \quad x = \frac{1}{2}$ and $x > \frac{1}{2}$

   $\therefore \quad x \in \left(\frac{1}{2}, \infty\right)$

   and domain of q(x) exists when

   $1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow |x| \leq 1$

   $\therefore \quad -1 \leq x \leq 1$

   $\therefore$ Common domain is $\left]\frac{1}{2}, 1\right[$

9. **(d)** Given function is :

   $f(x+1) = x^2 - 3x + 2$

   This function is valid for all real values of x. So, putting x – 1 in place of x, we get

   $f(x) = f(x - 1 + 1)$

   $\Rightarrow \quad f(x) = (x-1)^2 - 3(x-1) + 2$

   $\Rightarrow \quad f(x) = x^2 - 2x + 1 - 3x + 3 + 2$

   $f(x) = x^2 - 5x + 6$

10. **(a)** Given function is :

    $f(x) = \frac{1-x}{1+x}$

    Putting $\frac{1-x}{1+x}$ in place of x,

    $$\Rightarrow f\left(\frac{1-x}{1+x}\right) = \frac{1 - \left(\frac{1-x}{1+x}\right)}{1 + \left(\frac{1-x}{1+x}\right)} = \frac{1+x-1+x}{1+x+1-x} = \frac{2x}{2}$$

    So, $f\left(\frac{1-x}{1+x}\right) = x$

11. **(a)** If either P or Q is the null set, then $P \times Q$ will be an empty set, i.e. $P \times Q = \phi$.

12. **(c)** A relation may be represented algebraically either by the Roster method or by the Set-builder method.

An arrow diagram is a visual representation of a relation.

13. **(d)** In option (d), $a \notin A$

$\therefore$ It it not a relation.

14. **(d)** The difference of two integers is also an integer.

$\therefore$ Domain of R = Z

Range of R = Z

15. **(c)** Since 2, 3, 4 are the elements of domain of R_1 having their unique images, this relation R_1 is a function.

Since, the same first element 2 corresponds to two different images 2 and 4, this relation R_2 is not a function.

Since, every element has one and only one image, this relation R_3 is a function.

16. **(a)** $R = \{(1, 2), (2, 4), (3, 6), (4, 8), ...\}$

Since, every natural number N has one and only one image, this relation R is a function.

The domain of R is the set of natural number, i.e. N. The co-domain is also N, and the range is the set of even natural numbers.

17. **(d)** (2,4) is an order pair of the function $f(x) = x^2, x \in R$

But point (2, 4) only lies on the graph given in option (d).

18. **(b)** **Greatest Integer Function:** The function $f : R \to R$ defined by $f(x) = [x], x \in R$ assumes the value of the greatest integer, less than or equal to x. Such a function is called the greatest integer function.

From the definition of [x], we can see that

$[x] = -1$ for $-1 \le x < 0$

$[x] = 0$ for $0 \le x < 1$

$[x] = 1$ for $1 \le x < 2$

$[x] = 2$ for $2 \le x < 3$ and so on.

The graph of the function is given in the question.

19. **(b)** For $f(x) = g(x)$

$\Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow 2x^2 + 4x - x - 2 = 0$

$\Rightarrow 2x(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 1) = 0$

$\Rightarrow x = -2, \frac{1}{2}$

$\therefore$ The domain for which the function $f(x) = g(x)$ is $\left\{-2, \frac{1}{2}\right\}$.

20. **(b)** We have, $g(x) = 1 + \sqrt{x}$ and

$f[g(x)] = 3 + 2\sqrt{x} + x$ ...(i)

Also, $f[g(x)] = f(1 + \sqrt{x})$ ...(ii)

By (i) and (ii), we get

$f(1 + \sqrt{x}) = 3 + 2\sqrt{x} + x$

Let $1 + \sqrt{x} = y$ or $x = (y - 1)^2$.

$\therefore \quad f(y) = 3 + 2(y - 1) + (y - 1)^2$

$= 3 + 2y - 2 + y^2 - 2y + 1 = 2 + y^2$

$\therefore \quad f(x) = 2 + x^2$

21. **(a)** Since, first elements of the ordered pairs in f belongs to A and second elements of the ordered pairs belongs to B. So, f is a relation from A to B.

Now, 2 has two different images 9 and 11.

So, f is not a function.

22. **(d)** In Roster form relation R is,

$R = \{(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)\}$

$\therefore$ Domain of R = {2, 3, 4, 5}.

23. **(d)** We have, $(x, y) \in R$, if $x^2 + y^2 = 16$

i.e. $y = \pm\sqrt{16 - x^2}$

For, $x = 0, y = \pm 4$

For, $x = \pm 4, y = 0$

We observe that no other values of $x, y \in Z$, which satisfy $x^2 + y^2 = 16$

$R = \{(0, 4), (0, -4), (4, 0), (-4, 0)\}$

$\therefore$ Domain of R = {0, 4, – 4}.

24. **(a)** $A \times B = \{1, 2\} \times \{1, 3\} = \{(1, 1), (1, 3), (2, 1), (2, 3)\}$

$B \times A = \{1, 3\} \times \{1, 2\} = \{(1, 1), (1, 2), (3, 1), (3, 2)\}$

$\therefore \quad (A \times B) \cup (B \times A)$

$= \{(1, 1), (1, 3), (2, 1), (2, 3), (1, 2), (3, 1), (3, 2)\}$

25. **(b)** $A - C = \{1, 4\}$ and $B - C = \{4\}$

$\therefore \quad (A - C) \times (B - C) = \{1, 4\} \times \{4\} = \{(1, 4), (4, 4)\}$.

26. **(a)** $X \times Y = \{a, b, c\} \times \{\ \} = \phi$

Hence, there are no ordered pairs formed in $X \times Y$.

27. **(d)** $R \subseteq A \times B$

For given A = {x, y, z} and B = {a, b, c, d}

$$A \times B = \begin{Bmatrix} (x, a), (x, b), (x, c), (x, d), (y, a), (y, b), \\ (y, c), (y, d), (z, a), (z, b), (z, c), (z, d) \end{Bmatrix}$$

Clearly, {(z, b), (y, b), (a, d)} is not the subset of $A \times B$.

$\therefore$ It is not a relation.

28. **(d)** $x^2 + y^2 = 9 \Rightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9 - x^2}$

$x = 0 \Rightarrow y = \pm\sqrt{9 - 0} = \pm 3 \in Z$

$x = \pm 1 \Rightarrow y = \pm\sqrt{9 - 1} = \pm\sqrt{8} \notin Z$

$x = \pm 2 \Rightarrow y = \pm\sqrt{9-4} = \pm\sqrt{5} \notin Z$

$x = \pm 3 \Rightarrow y = \pm\sqrt{9-9} = 0 \in Z$

$x = \pm 4 \Rightarrow y = \pm\sqrt{9-16} = \pm\sqrt{-7} \notin Z$ and so on.

$\therefore$ R = {(0, 3), (0, –3), (3, 0), (–3, 0)}

Domain of R = {x : (x, y) ∈ R} = {0, 3, –3}

Range of R = {y : (x, y) ∈ R} = {3, –3, 0}.

29. **(a)** When x = 1, y = 7 ∈ N, so (1, 7) ∈ R

When x = 2, y = 2 + 3 = 5 ∈ N, so (2, 5) ∈ R

Again for x = 3, y = 3 + 2 = 5 ∈ N, (3, 5) ∈ R

Similarly for x = 4, $y = 4 + \frac{6}{4} \notin N$ and for x = 5,

$y = 5 + \frac{6}{5} \notin N$.

Thus, R = {(1, 7), (2, 5), (3, 5)}

$\therefore$ Domain of R = {1, 2, 3}

and Range of R = {7, 5}.

30. **(b)** We have $f(x) = x^2 + 7$ and $g(x) = 3x + 5$

$\therefore f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 7 = \frac{1}{4} + 7 = \frac{29}{4}$

g (14) = 14 × 3 + 5 = 42 + 5 = 47

So, $f\left(\frac{1}{2}\right) \times g(14) = \frac{29}{4} \times 47 = \frac{1363}{4}$.

31. **(a)** We have $f(x) = 1 + x,\ g(x) = x^2 + x + 1$

$\therefore (f + g)(x) = f(x) + g(x)$

$= 1 + x + x^2 + x + 1 = x^2 + 2x + 2$

$\therefore (f + g)(0) = (0)^2 + 2(0) + 2 = 2.$

32. **(a)** $[\phi(p)]^3 = (a^p)^3 = a^{3p} = \phi(3p)$.

33. **(b)** Let $f(x) = \sqrt{a^2 - x^2}$

For f(x) to be defined $a^2 - x^2 \geq 0$

$\Rightarrow x^2 \leq a^2$

$\Rightarrow x \in [-a, a]$.

STATEMENT TYPE QUESTIONS

34. **(d)**

35. **(c)** P and Q are two non-empty sets. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, i.e. P × Q = {(p, q) : p ∈ P and q ∈ Q}.

Now, A = {red, blue}, B = {b, c, s}

A × B = Set of all ordered pairs

= {(red, b), (red, c), (red, s), (blue, b), (blue, c), (blue, s)}.

36. **(b)** I. P = {m, n} and Q = {n, m}

P × Q = {(m, n), (m, m), (n, n), (n, m)}

II. True

III. A = {1, 2}, B = {3, 4}

$B \cap \phi = \phi$

$\therefore A \times (B \cap \phi) = A \times \phi = \phi$

37. **(d)** The set of first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R ⇒ Domain = A.

The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the co-domain of the relation R.

Note that range ⊆ co-domain.

38. **(d)** $\therefore 2 \neq 2^2 \Rightarrow (2, 2) \notin R$

Hence, (a, a) ∉ R for all a ∈ N

Let (a, b) ∈ R for all a, b ∈ N

$\Rightarrow a = b^2$ [by definition]

$\Rightarrow b \neq a^2$

⇒ (b, a) ∉ R for all a, b ∈ N

Let (a, b) ∈ R and (b, c) ∈ R for all a, b, c ∈ N

$\Rightarrow a = b^2$ and $b = c^2$

$\Rightarrow a = c^4$

⇒ (a, c) ∉ R, for all a, b, c ∈ N.

39. **(a)** I. In Roster form,

R = {(1, 4), (2, 4), (3, 4), (4, 4)}

$\therefore$ Domain of R = {1, 2, 3, 4}

II. Here, |a| ≤ 3

⇒ –3 ≤ a ≤ 3

Hence, a = –3, –2, –1, 0, 1, 2, 3

In Roster form,

S = {(–3, 4), (–2, 3), (–1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}

$\therefore$ Range of S = {0, 1, 2, 3, 4}

40. **(b)** A × B = {(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9), (3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)}

$$B \times A = \begin{Bmatrix} (5,1), (5,2), (5,3), (5,4) \\ (7,1), (7,2), (7,3), (7,4) \\ (9,1)\ (9,2), (9,3), (9,4) \end{Bmatrix}$$

A × B ≠ B × A but n(A × B) = n(B × A) = 12

41. **(a)** Only I is true.

Let (x, y) ∈ A × C

⇒ x ∈ A and y ∈ C

⇒ x ∈ B and y ∈ C (∵ A ⊆ B)

⇒ (x, y) ∈ B × C

⇒ A × C ⊆ B × C

42. **(c)** R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

Domain = Set of first elements of ordered pairs in R.

Range = Set of second elements of ordered pairs in R.

43. **(a)** I. A = Set of first elements = {a, b, c}

B = Set of second elements = {1, 2}

II. $B \cap \phi = \phi \quad \therefore A \times \phi = \phi$

44. **(c)** Both the given statements are true.

I. R is not a function as 2 has two images 0 and 1.

45. **(a)** Only statement-I is true.

II. $R = \{(x, x+5) : x \in (0,1,2,3,4,5)\}$

$R = \{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

$\therefore$ Domain of $R = \{0,1,2,3,4,5\}$

46. **(b)** Only statement-II is false.

II. In Roster form,

$R = \{(1,1),(1,2),(1,3),(1,4),(1,6),(2,4),(2,6),(2,2),(4,4),(6,6),(3,3),(3,6)\}$

47. **(c)** Both the statements are true.

I. Since f(x) is a linear function

$\therefore$ $f(x) = mx + c$

$(1,1)$ and $(0,-1) \in R$.

$f(1) = m + c$, $f(0) = c$

$1 = m + c$, $-1 = c$

$\Rightarrow$ $m = 2$ and $c = -1$

Thus, $f(x) = 2x - 1$

II. $f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$

$\therefore$ $f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0.$

48. **(d)** Both the given statements are false.

I. Correct Roster form is $\{(2,8),(3,27),(5,125),(7,343)\}$

II. Given relation in Roster form is,

$R = \{(3,5),(4,6),(5,7),(6,8),(7,9),(8,10),(9,11)\}$

Range $= \{5,6,7,8,9,10,11\}$.

49. **(d)** II. $A \cup B = \{1,2,3,8\}$

$A \cap B = \{3\}$

$(A \cup B) \times (A \cap B) = \{(1,3),(2,3),(3,3),(8,3)\}$

III. $\frac{x}{2} - 1 = 2 \Rightarrow x = 6$ and $\frac{y}{9} + 1 = 1 \Rightarrow y = 0$

MATCHING TYPE QUESTIONS

50. **(c)** Given, $A = \{1,2,3\}$, $B = \{3,4\}$ and $C = \{4,5,6\}$

A. $B \cap C = \{4\}$

$\therefore$ $A \times (B \cap C) = \{(1,4),(2,4),(3,4)\}$

B. $A \times B = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$

$A \times C = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

$\therefore$ $(A \times B) \cap (A \times C) = \{(1,4),(2,4),(3,4)\}$

C. $B \cup C = \{3,4,5,6\}$

$\therefore$ $A \times (B \cup C) = \{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$

D. $(A \times B) \cup (A \times C) = \{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$

E. $A \cap B = \{3\}$, $B \cap C = \{4\}$

$\therefore$ $(A \cap B) \times (B \cap C) = \{3, 4\}$

51. **(b)** We have $3x - y = 0 \Rightarrow y = 3x$

For,

$x = 1, y = 3 \in A$

$x = 2, y = 6 \in A$

$x = 3, y = 9 \in A$

$x = 4, y = 12 \in A$

$x = 5, y = 15 \notin A$

A. In Roster form,

$R = \{(1,3),(2,6),(3,9),(4,12)\}$

B. Domain of R = Set of first element of ordered pairs in R

$= \{1,2,3,4\}$

C. Range of R = Set of second element of ordered pairs in R

$= \{3,6,9,12\}$

D. Co-domain of R is the set A.

52. **(a)** Since, domain of f = domain of g

We have,

$(f+g)(x) = x^2 + 2x + 1$

$(f-g)(x) = x^2 - 2x - 1$

$(fg)(x) = x^2(2x+1) = 2x^3 + x^2$

$\left(\frac{f}{g}\right)(x) = \frac{x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

53. **(a)** Domain of f = R

Domain of g = R

Domain of f $\cap$ Domain of g = R

A. $f + g : R \to R$ is given by

$(f+g)(x) = f(x) + g(x)$

$= 2x + 5 + x^2 + x$

$= x^2 + 3x + 5$

B. $f - g : R \to R$ is defined as

$(f-g)(x) = f(x) - g(x)$

$= 2x + 5 - x^2 - x$

$= 5 + x - x^2$

C. $(fg)(x) = f(x) \cdot g(x)$

$= (2x+5)(x^2+x)$

$= 2x^3 + 2x^2 + 5x^2 + 5x$

$= 2x^3 + 7x^2 + 5x$

D. $g(x) = 0$

$\therefore$ $x^2 + x = 0$

$\Rightarrow$ $x(x+1) = 0$

$\Rightarrow$ $x = 0, -1$

Domain of $\left(\frac{f}{g}\right)$ = Domain of f $\cap$ Domain of g $- \{0, -1\}$

$= R - \{0, -1\}$

Thus, $\frac{f}{g} : R - \{0,-1\} \to R$ is given by

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+5}{x^2+x}$.

54. **(b)** **(B)** f(x) assumes real values if $4 - x^2 > 0$

$\Rightarrow\ x^2-4<0 \Rightarrow (x+2)(x-2)<0$

$\Rightarrow\ x \in (-2, 2)$

$\Rightarrow$ Domain of f = (–2, 2)

(C) $f(x)=\frac{x}{1+x^2}=y\text{ (say)}$

$\Rightarrow\ y=\frac{x}{1+x^2} \quad \Rightarrow\ yx^2-x+y=0$

x assumes real values if

$(-1)^2-4(y^2)\geq 0 \Rightarrow 4y^2-1\leq 0$

$\Rightarrow\ (2y+1)(2y-1)\leq 0$

$\Rightarrow\ y\in\left[-\frac{1}{2},\frac{1}{2}\right]$

$\therefore$ Range of $f=\left[-\frac{1}{2},\frac{1}{2}\right]$

(D) Let f(x) = y $\Rightarrow y=\sqrt{x-1} \Rightarrow y^2=x-1$

$\Rightarrow\ x=y^2+1$

Since, $y\geq 0$ and $x\in[1,\infty) \Rightarrow$ Range of $f=[0,\infty)$

55. **(b)**

56. **(b)** (A) $f(x)+g(x)=x+|x|=\begin{cases}2x, & x\geq 0\\ 0, & x<0\end{cases}$

(B) $f(x)-g(x)=x-|x|=\begin{cases}0, & x\geq 0\\ 2x, & x<0\end{cases}$

(C) $f(x).g(x)=x.|x|=\begin{cases}x^2, & x\geq 0\\ -x^2, & x<0\end{cases}$

(D) $\frac{f(x)}{g(x)}=\frac{x}{|x|}=\begin{cases}1, & x>0\\ -1, & x<0\end{cases}$

57. **(d)** (A) Number of relations from A to A = $2^{n(A)\times n(A)}$

(B) Number of relations from B to B = $2^{n(B)\times n(B)}$

(C) Number of relations from A to B = $2^{n(A)\times n(B)}$

INTEGER TYPE QUESTIONS

58. **(a)** $4x+3=3x+5 \Rightarrow x=5-3=2$ and $y=-2$

$\therefore\ x+y=2-2=0$

59. **(d)** $4-y=7 \Rightarrow y=-3$

$\therefore\ 4+y=4-3=1$

60. **(a)** Elements in the given set are (2, 3), (–2, –3), (4, 1), (–4, –1), (2, –3), (–2, 3), (–4, 1) and (4, –1). So, number of elements in the set is 8.

61. **(a)** n(A) = 3, n(B) = 2

$n(A\times B)=n(A)\times n(B)=3\times 2=6$

62. **(b)** Total number of relations from X to Y is 2^{mn}

$\Rightarrow$ No. of relations = $2^{5\times 7}$

63. **(c)** $f(b+1)=4(b+1)-(b+1)^2$

$=4b+4-b^2-1-2b$

$=2b-b^2+3$

$f(b-1)=4(b-1)-(b-1)^2$

$=4b-4-b^2-1+2b$

$=6b-b^2-5$

$f(b+1)-f(b-1)=-4b+8$

$=4(2-b)\equiv m(2-b)$

64. **(a)** $f(y)=2y^2+by+c$

$f(0)=c$, $3=c$ | $f(2)=2(2)^2+b(2)+c$

$1=8+2b+c$

$2b+c=-7$

$2b+3=-7$

$2b=-10$

$b=-5$

Now, $f(1)=2(1)^2+b(1)+c$

$=2+b+c=2-5+3=0$

65. **(a)** $n(X\times X)=n(X).n(X)=3^2=9$

So, the total number of subsets of X × X is 2^9 and a subset of X × X is a relation over the set X.

66. **(c)** $f(x)=ax+b$

$f(-1)=-a+b$, $-5=-a+b$ | $f(3)=3a+b$, $3=3a+b$

On solving both the equations, we get a = 2

ASSERTION - REASON TYPE QUESTIONS

67. **(d)** In Roster form R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 4), (2, 6), (2, 2), (4, 4), (6, 6), (3, 3), (3, 6)}

Domain of R = Set of first element of ordered pairs in R = {1, 2, 3, 4, 6}

Range of R = {1, 2, 3, 4, 6}.

68. **(a)** Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

Given, (x + 1, y – 2) = (3, 1)

Then, by the definition

x + 1 = 3 and y – 2 = 1

$\Rightarrow$ x = 2 and y = 3.

69. **(c)** Domain of f = {0, 2, 3, 4, 5}

Domain of g = {1, 2, 3, 4, 5}

Domain of f · g = Domain of f $\cap$ Domain of g = {2, 3, 4, 5}

Hence, Assertion is true.

If f and g be two real functions.

Then, (f · g) (x) = f(x) · g(x)

Hence, Reason is false.

70. **(a)** Given functions are $f(x)=\frac{1}{x-2}$, $x\neq 2$ and $g(x)=(x-2)^2$

$\therefore\ (f+g)(x) = f(x) + g(x) = \dfrac{1}{x-2} + (x-2)^2,\ x \neq 2$

$= \dfrac{1+(x-2)^3}{(x-2)},\ x \neq 2.$

71. (d) We have

$A = \{x, y, z\},\ B = \{3, 4\} \Rightarrow n(A) = 3,\ n(B) = 2$

$\therefore\ n(A \times B) = n(A) \times n(B) = 6$

Therefore, the number of subsets of $A \times B$ is 2^6.

So, the number of relations from A to B is 2^6.

72. (c) We know that every subset of $A \times A$ is a relation on A.

So, Assertion is true but Reason is false.

CRITICALTHINKING TYPE QUESTIONS

73. (d)

74. (a) $A = \{2, 4, 6\}, B = \{2, 3, 5\}$

No. of relations from A to B $= 2^{3 \times 3} = 2^9$

75. (a) We have, $f(x) = \dfrac{2^x + 2^{-x}}{2}$

$\therefore f(x+y).\,f(x-y)$

$= \dfrac{1}{2}(2^{x+y} + 2^{-x-y}).\dfrac{1}{2}(2^{x-y} + 2^{-x+y})$

$= \dfrac{1}{4}\left[(2^{2x} + 2^{-2x}) + (2^{2y} + 2^{-2y})\right]$

$= \dfrac{1}{2}[f(2x) + f(2y)]$

76. (b) In the definition of function

$f(x) = \dfrac{x(x-p)}{q-p} + \dfrac{x(p-q)}{(p-q)} = p$

Putting p and q in place of x, we get

$f(p) = \dfrac{p(p-p)}{q-p} + \dfrac{p(p-q)}{(p-q)} = p$

$\Rightarrow\quad f(p) = p$

and $f(q) = \dfrac{q(q-p)}{q-p} + \dfrac{q(p-q)}{(p-q)} = q$

$\Rightarrow\quad f(q) = q$

Putting $x = (p+q)$

$f(p+q) = \dfrac{(p+q)(p+q-p)}{(q-p)} + \dfrac{(p+q)(p+q-q)}{(p-q)}$

$= \dfrac{(p+q)q}{(q-p)} + \dfrac{(p+q)(p)}{(p-q)} = \dfrac{pq + q^2 - p^2 - pq}{(q-p)}$

$= \dfrac{q^2 - p^2}{q-p} = \dfrac{(q-p)(q+p)}{(q-p)}$

$= p + q = f(q) + f(p)$

So, $f(p) + f(q) = f(p+q)$

77. (c) Given functions are : $f(x) = x$ and $g(x) = |x|$

$\therefore\ (f+g)(x) = f(x) + g(x) = x + |x|$

According to definition of modulus function,

$$(f+g)(x) = \begin{cases} x+x, & x \geq 0 \\ x-x, & x < 0 \end{cases} = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

78. (d) $n(A) = 2$ and $n(B) = 2$

$n(A \times B) = n(A) \times n(B) = 2 \times 2 = 4$

$\therefore$ Number of subset of $A \times B = 2^{n(A \times B)} = 2^4 = 16$

79. (c) Let $(a, b) \in A \times (B \cap C)$

$\Rightarrow\ a \in A$ and $b \in (B \cap C)$

$\Rightarrow\ a \in A$ and ($b \in B$ and $b \in C$)

$\Rightarrow\ (a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow\ (a, b) \in A \times B$ and $(a, b) \in (A \times C)$

$(a, b) \in (A \times B) \cap (A \times C)$

$\Rightarrow\ A \times (B \cap C) \subset (A \times B) \cap (A \times C)$ … (i)

Again, let $(x, y) \in (A \times B) \cap (A \times C)$

$(x, y) \in A \times B$ and $(x, y) \in A \times C$

$\Rightarrow\ (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow\ x \in A$ and ($y \in B$ and $y \in C$)

$\Rightarrow\ x \in A$ and $y \in (B \cap C)$

$\Rightarrow\ (x, y) \in A \times (B \cap C)$

$\Rightarrow\ (A \times B) \cap (A \times C) \subset A \times (B \cap C)$ … (ii)

From equations (i) and (ii), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$ … (iii)

Now, $A \times (B' \cup C')' = A \times \left[(B')' \cap (C')'\right]$

[by De-Morgan's law]

$= A \times (B \cap C)$ $\left[\because (A')' = A\right]$

$= (A \times B) \cap (A \times C)$ [by equation (iii)]

80. (c) $n(A \times B \times C \times .....) = n(A) \times n(B) \times n(C) \times .....$

$\therefore\ n(A \times A \times B) = n(A) \times n(A) \times n(B)$

$[\because n(A) = 3,\ n(B) = 5]$

$= 3 \times 3 \times 5 = 45$

81. (a) It is distributive law.

82. (c) $n(A \times B) = pq$.

83. (a) If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$

$A - B = \{a\},\ B \cap C = \{c, d\}$

Then, $(A - B) \times (B \cap C) = \{a\} \times \{c, d\}$

$= \{(a, c), (a, d)\}$

84. (c) $B \cup C = \{c, d\} \cup \{d, e\} = \{c, d, e\}$

$\therefore A \times (B \cup C) = \{a, b\} \times \{c, d, e\}$

$= \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$

85. (c) $n(A) = p,\ n(B) = q$

$n(A \times B) = pq = 7$

So, possible values of p, q are 7, 1

$\Rightarrow\ p^2 + q^2 = (7)^2 + (1)^2 = 50.$

86. (b) $(-1, 0) \in A \times A$ and $(0, 1) \in A \times A$

$\therefore$ $(-1, 0) \in A \times A \Rightarrow -1, 0 \in A$

and $(0, 1) \in A \times A \Rightarrow 0, 1 \in A$

$\therefore$ $A = \{-1, 0, 1\}$

$\therefore$ $A \times A = \{-1, 0, 1\} \times \{-1, 0, 1\}$

$= \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$

Since, $(-1, 0), (0, 1)$ already exist.

$\therefore$ Remaining 7 ordered pairs are

$\{(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)\}$

87. (a) $\because n(A \times A) = n(A) \times n(A) = 3^2 = 9$. So, the total number of subsets of $A \times A$ is 2^9.

88. (d) We have $R = \{(x, y) : |x^2 - y^2| < 16\}$

Let x = 1,

$|x^2 - y^2| < 16 \Rightarrow |1 - y^2| < 16$

$\Rightarrow$ $|y^2 - 1| < 16 \Rightarrow y = 1, 2, 3, 4$

Let x = 2,

$|x^2 - y^2| < 16 \Rightarrow |4 - y^2| < 16$

$\Rightarrow$ $|y^2 - 4| < 16 \Rightarrow y = 1, 2, 3, 4$

Let x = 3,

$|x^2 - y^2| < 16 \Rightarrow |9 - y^2| < 16$

$\Rightarrow$ $|y^2 - 9| < 16 \Rightarrow y = 1, 2, 3, 4$

Let x = 4,

$|x^2 - y^2| < 16 \Rightarrow |16 - y^2| < 16$

$\Rightarrow$ $|y^2 - 16| < 16 \Rightarrow y = 1, 2, 3, 4, 5$

Let x = 5,

$|x^2 - y^2| < 16 \Rightarrow |25 - y^2| < 16$

$\Rightarrow$ $|y^2 - 25| < 16 \Rightarrow y = 4, 5$

$\therefore$ $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 4), (5, 5)\}$.

89. (a) Given, $A = \{x : |x| < 3, x \in I\}$

$A = \{x : -3 < x < 3, x \in I\} = \{-2, -1, 0, 1, 2\}$

Also, $R = \{(x, y) : y = |x|\}$

$\therefore$ $R = \{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$.

90. (b) Here, f(x) is defined only when $x + 3 \neq 0$, i.e. when $x \neq -3$

$\therefore$ $D(f) = R - \{-3\}$.

91. (d) Given n(A) = 8 and n(B) = p

$\therefore$ Total number of relations from A to B $= 2^{8p}$

$\therefore$ Total number of non-empty relations from A to B $= 2^{8p} - 1$.

92. (c) $f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$

$\Rightarrow$ $5 - 4x - x^2 \geq 0, x + 4 > 0$

$\Rightarrow$ $(x + 5)(x - 1) \leq 0, x > -4$

$\Rightarrow$ $-5 \leq x \leq 1, x > -4$

$\Rightarrow$ $-4 < x \leq 1$.

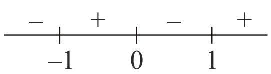
93. (c) Let $g(x) = \dfrac{3}{4 - x^2}$ $\therefore$ $x \neq \pm 2$

$\therefore$ $D(g(x)) = R - \{-2, 2\}$

$h(x) = \log_{10}(x^3 - x)$

$\therefore$ $x^3 - x > 0$

$\Rightarrow$ $x(x + 1)(x - 1) > 0$



$\therefore$ $x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Domain of f(x) is $(-1, 0) \cup (1, 2) \cup (2, \infty)$.

94. (b) $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

Clearly, $9 - x^2 > 0 \Rightarrow x^2 - 9 < 0$

$\Rightarrow$ $(x + 3)(x - 3) < 0$

Thus, domain of f(x) is $x \in (-3, 3)$.

95. (a) We have $f(x) = |x - 1|$

Here, f(x) is a modulus function and since modulus of a real number is uniquely defined $\forall$ real positive number.

$\therefore$ The domain of f(x) is R

We see that $f(x) = |x - 1|$

$$f(x) = \begin{cases} x - 1, & \text{if } x \geq 1 \\ -(x - 1), & \text{if } x < 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x - 1, & \text{if } x \geq 1 \\ 1 - x, & \text{if } x < 1 \end{cases}$$

From above, we observe that in both cases $f(x) \geq 0$.

Hence, range of f(x) is $[0, \infty)$.

96. (a) Since f is a linear function, f(x) = mx + c.

Also, since $(1, 1), (0, -1) \in R$,

f(1) = m + c = 1 and f(0) = c = –1

This gives m = 2

$\therefore$ f(x) = 2x – 1.

97. (d) Given that $f(x) = \dfrac{1}{\sqrt{x - |x|}}$,

where $x - |x| = \begin{cases} x - x = 0, & \text{if } x \geq 0 \\ x - (-x) = 2x, & \text{if } x < 0 \end{cases}$.

Thus, $\dfrac{1}{\sqrt{x - |x|}}$ is not defined for any $x \in R$.

Hence, f is not defined for any $x \in R$, i.e. domain of $f = \{\phi\}$.

98. (b) Since $x^2 - 5x + 4 = (x - 4)(x - 1)$, the function f is defined for all real numbers except x = 4 and x = 1.

Hence, the domain of f is $R - \{1, 4\}$.

99. (b) Given $f(x) = 2 - |x - 5|$

Domain of f(x) is defined for all real values of x.

Since, $|x - 5| \geq 0 \Rightarrow -|x - 5| \leq 0$

$\Rightarrow$ $2 - |x - 5| \leq 2 \Rightarrow f(x) \leq 2$

Hence, range of f(x) is $(-\infty, 2]$.

100. (b) Given P = {a, b, c} and Q = {r}

$P \times Q = \{(a, r), (b, r), (c, r)\}$

$Q \times P = \{(r, a), (r, b), (r, c)\}$

Since, by the definition of equality of ordered pairs, the pair (a, r) is not equal to the pair (r, a), we conclude that

$$P \times Q \neq Q \times P$$

However, the number of elements in each set will be the same.

CHAPTER

3

TRIGONOMETRIC FUNCTIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The value of $\tan^2\theta\ \sec^2\theta\ (\cot^2\theta - \cos^2\theta)$ is

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

2. Value of cot 5° cot 10° ..... cot 85° is

(a) 0 (b) –1 (c) 1 (d) 2

3. Value of sin 10° + sin 20° + sin 30° +......+sin360° is

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

4. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then value of A + B is

(a) π (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

5. If $\sin 2\theta + \sin 2\phi = 1/2$, $\cos 2\theta + \cos 2\phi = 3/2$, then value of $\cos^2(\theta - \phi)$ is

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $-\frac{5}{8}$ (d) $\frac{3}{5}$

6. If $0 < \theta < 360°$, then solutions of $\cos\theta = -1/2$ are

(a) 120º, 360º (b) 240º, 90º

(c) 60º, 270º (d) 120º, 240º

7. If $\tan\theta = -\frac{1}{\sqrt{3}}$, then general solution of the equation is

(a) $2n\pi + \frac{\pi}{6}, n \in I$ (b) $n\pi + \frac{\pi}{6}, n \in I$

(c) $2n\pi - \frac{\pi}{6}, n \in I$ (d) $n\pi - \frac{\pi}{6}, n \in I$

8. If $2\tan^2\theta = \sec^2\theta$, then general value of θ are

(a) $n\pi \pm \frac{\pi}{4}, n \in I$ (b) $n\pi \pm \frac{\pi}{6}, n \in I$

(c) $2n\pi + \frac{\pi}{4}, n \in I$ (d) $2n\pi \pm \frac{\pi}{6}, n \in I$

9. If $\sin 5x + \sin 3x + \sin x = 0$ and $0 \le x \le \pi/2$, then value of x is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

10. If $y = \frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha}$, then value of $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ is

(a) $\frac{y}{3}$ (b) y (c) 2y (d) $\frac{3}{2}y$

11. The number of solution of $\tan x + \sec x = 2\cos x$ in $(0, 2\pi)$ is

(a) 2 (b) 3 (c) 0 (d) 1

12. If $\sin A = \frac{3}{5}, 0 < A < \frac{\pi}{2}$ and $\cos B = \frac{-12}{13}, \pi < B < \frac{3\pi}{2}$, then value of sin (A – B) is

(a) $-\frac{13}{82}$ (b) $-\frac{15}{65}$ (c) $-\frac{13}{75}$ (d) $-\frac{16}{65}$

13. Value of tan15°. tan45° tan75° is

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) –1

14. Value of

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

15. The large hand of a clock is 42 cm long. How much distance does its extremity move in 20 minutes?

(a) 88 cm (b) 80 cm (c) 75 cm (d) 77 cm

16. The angle in radian through which a pendulum swings and its length is 75 cm and tip describes an arc of length 21 cm, is

(a) $\frac{7}{25}$ (b) $\frac{6}{25}$ (c) $\frac{8}{25}$ (d) $\frac{3}{25}$

17. The length of an arc of a circle of radius 3 cm, if the angle subtended at the centre is 30° is (π = 3.14)

(a) 1.50 cm (b) 1.35 cm (c) 1.57 cm (d) 1.20 cm

18. A circular wire of radius 7 cm is cut and bent again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is

(a) 50° (b) 210° (c) 100° (d) 60°

19. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. The angle in degrees which is subtended at the centre of hoop is

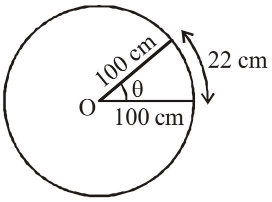
(a) 21.5° (b) 23.5° (c) 22.5° (d) 24.5°

20. The radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm is $\left(\text{Use } \pi = \frac{22}{7}\right)$

(a) 37.5 cm (b) 32.8 cm (c) 35.7 cm (d) 34.5 cm

21. The degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm as shown in figure, is $\left[\text{Use } \pi = \frac{22}{7}\right]$

(a) $12° 30'$
(b) $12° 36'$
(c) $11° 36'$
(d) $11° 12'$



22. If $\tan\theta = 3$ and θ lies in III[rd] quadrant, then the value of $\sin\theta$ is

(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$ (c) $\frac{-3}{\sqrt{10}}$ (d) $\frac{-5}{\sqrt{10}}$

23. If $\frac{\sin x}{\cos x} \times \frac{\sec x}{\operatorname{cosec} x} \times \frac{\tan x}{\cot x} = 9$, where $x \in \left(0, \frac{\pi}{2}\right)$, then the value of x is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

24. Find x from the equation:
$\operatorname{cosec}(90° + \theta) + x\cos\theta\cot(90° + \theta) = \sin(90° + \theta)$.
(a) $\cot\theta$ (b) $\tan\theta$ (c) $-\tan\theta$ (d) $-\cot\theta$

25. If A + B = 45°, then (cot A – 1) (cot B – 1) is equal to

(a) 1 (b) $\frac{1}{2}$ (c) –1 (d) 2

26. If $\sin A = \frac{3}{5}$ and A is in first quadrant, then the values of sin 2A, cos 2A and tan 2A are

(a) $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ (b) $\frac{1}{25}, \frac{7}{25}, \frac{1}{7}$
(c) $\frac{24}{25}, \frac{1}{25}, \frac{24}{7}$ (d) $\frac{1}{25}, \frac{24}{25}, \frac{1}{24}$

27. The value of $\tan(\alpha + \beta)$, given that $\cot\alpha = \frac{1}{2}$, $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\sec\beta = \frac{-5}{3}$, $\beta \in \left(\frac{\pi}{2}, \pi\right)$ is

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{5}{11}$ (d) $\frac{3}{11}$

28. The value of tan 75° – cot 75° is equal to

(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $2-\sqrt{3}$ (d) 1

29. The value of tan 3A – tan 2A – tan A is equal to
(a) tan 3A tan 2A tan A
(b) –tan 3A tan 2A tan A
(c) tan A tan 2A – tan 2A tan 3A – tan 3A tan A
(d) None of these

30. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then tan(2A + B) is equal to
(a) 1 (b) 2 (c) 3 (d) 4

31. If $\tan\theta = \frac{a}{b}$, then $b\cos 2\theta + a\sin 2\theta$ is equal to

(a) a (b) b (c) $\frac{a}{b}$ (d) None of these

32. Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, \pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3

33. If $\cos A = \frac{4}{5}$, $\cos B = \frac{12}{13}$, $\frac{3\pi}{2} < A, B < 2\pi$, the value of the $\cos(A+B)$ is

(a) $\frac{65}{33}$ (b) $\frac{33}{65}$ (c) $\frac{30}{65}$ (d) $\frac{65}{30}$

34. What is the value of radian measures corresponding to the 25° measures ?

(a) $\frac{5\pi}{36}$ (b) $\frac{2\pi}{36}$ (c) $\frac{3\pi}{36}$ (d) $\frac{4\pi}{36}$

35. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$
(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these

36. cos (A + B). cos (A – B) is given by:
(a) $\cos^2 A - \cos^2 B$ (b) $\cos(A^2 - B^2)$
(c) $\cos^2 A - \sin^2 B$ (d) $\sin^2 A - \cos^2 B$

37. If $\sin\theta = \frac{24}{25}$ and $0° < \theta < 90°$ then what is the value of $\sin\left(\frac{\theta}{2}\right)$?

(a) $\frac{12}{25}$ (b) $\frac{7}{25}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

38. What is the value of $\sin\left(\frac{5\pi}{12}\right)$?

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{\sqrt{6}+\sqrt{2}}{4}$
(c) $\frac{\sqrt{3}+\sqrt{2}}{4}$ (d) $\frac{\sqrt{6}+1}{2}$

39. If $x + \frac{1}{x} = 2\cos\theta$, then $x^3 + \frac{1}{x^3}$ is:

(a) $\frac{1}{2}\cos 3\theta$ (b) $2\cos 3\theta$
(c) $\cos 3\theta$ (d) $\frac{1}{3}\cos 3\theta$

40. If $1 + \cot\theta = \operatorname{cosec}\theta$, then the general value of θ is

(a) $n\pi + \frac{\pi}{2}$ (b) $2n\pi - \frac{\pi}{2}$
(c) $2n\pi + \frac{\pi}{2}$ (d) $2n\pi \pm \frac{\pi}{2}$

41. If $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$, then $x =$

(a) $n\pi \pm \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{3}$
(c) $n\pi \pm \frac{\pi}{4}$ (d) $n\pi \pm \frac{\pi}{2}$

42. The general value of θ satisfying the equation $\tan\theta + \tan\left(\frac{\pi}{2} - \theta\right) = 2$, is

(a) $n\pi \pm \frac{\pi}{4}$ (b) $n\pi + \frac{\pi}{4}$
(c) $2n\pi \pm \frac{\pi}{4}$ (d) $n\pi + (-1)^n \frac{\pi}{4}$

43. The general solution of $\sin^2\theta\sec\theta+\sqrt{3}\tan\theta=0$ is

(a) $\theta=n\pi+(-1)^{n+1}\frac{\pi}{3},\ \theta=n\pi;\ n\in I$

(b) $\theta=n\pi;\ n\in I$

(c) $\theta=\frac{n\pi}{2},\ n\in I$

(d) $\theta=n\pi+(-1)^{n+1}\frac{\pi}{2},\ \theta=n\pi;\ n\in I$

44. If $\tan\theta+\tan 2\theta+\sqrt{3}\tan\theta\tan 2\theta=\sqrt{3}$, then

(a) $\theta=(6n+1)\frac{\pi}{18},\ \forall\ n\in I$

(b) $\theta=(6n+1)\frac{\pi}{9},\ \forall\ n\in I$

(c) $\theta=(3n+1)\frac{\pi}{9},\ \forall\ n\in I$

(d) $\theta=(3n+1)\frac{\pi}{18}$

45. The most general value of θ satisfying the equations $\sin\theta=\sin\alpha$ and $\cos\theta=\cos\alpha$ is

(a) $2n\pi+\alpha$ (b) $2n\pi-\alpha$
(c) $n\pi+\alpha$ (d) $n\pi-\alpha$

46. If $\sec 4\theta-\sec 2\theta=2$, then the general value of θ is

(a) $(2n+1)\frac{\pi}{4}$ (b) $(2n+1)\frac{\pi}{10}$

(c) $n\pi+\frac{\pi}{2}$ or $\frac{n\pi}{5}+\frac{\pi}{10}$ (d) $(2n+1)\frac{\pi}{2}$

47. General solution of the equation $\tan\theta\tan 2\theta=1$ is given by

(a) $(2n+1)\frac{\pi}{4},\ n\in I$ (b) $n\pi+\frac{\pi}{6},\ n\in I$

(c) $n\pi-\frac{\pi}{6},\ n\in I$ (d) $n\pi\pm\frac{\pi}{6},\ n\in I$

48. If $\cot\theta+\cot\left(\frac{\pi}{4}+\theta\right)=2$, then the general value of θ is

(a) $2n\pi\pm\frac{\pi}{6}$ (b) $2n\pi\pm\frac{\pi}{3}$

(c) $n\pi\pm\frac{\pi}{3}$ (d) $n\pi\pm\frac{\pi}{6}$

49. If $2\cos^2 x+3\sin x-3=0,\ 0\le x\le 180°$, then $x=$

(a) 30°, 90°, 150° (b) 60°, 120°, 180°
(c) 0°, 30°, 150° (d) 45°, 90°, 135°

50. If $\cot\theta+\tan\theta=2\operatorname{cosec}\theta$, the general value of θ is

(a) $n\pi\pm\frac{\pi}{3}$ (b) $n\pi\pm\frac{\pi}{6}$

(c) $2n\pi\pm\frac{\pi}{3}$ (d) $2n\pi\pm\frac{\pi}{6}$

51. If $\sqrt{3}\tan 2\theta+\sqrt{3}\tan 3\theta+\tan 2\theta\tan 3\theta=1$, then the general value of θ is

(a) $n\pi\pm\frac{\pi}{5}$ (b) $\left(n+\frac{1}{6}\right)\frac{\pi}{5}$

(c) $\left(2n\pm\frac{1}{6}\right)\frac{\pi}{5}$ (d) $\left(n+\frac{1}{3}\right)\frac{\pi}{5}$

52. If $\cos 7\theta=\cos\theta-\sin 4\theta$, then the general value of θ is

(a) $\frac{n\pi}{4},\frac{n\pi}{3}+\frac{\pi}{18}$ (b) $\frac{n\pi}{3},\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$

(c) $\frac{n\pi}{4},\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$ (d) $\frac{n\pi}{6},\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$

53. Which among the following is/are correct?

(a) The angle is called negative, if the rotation is clockwise
(b) The angle is called positive, if the rotation is anti-clockwise
(c) The amount of rotation performed to get the terminal side from the initial side is called the measure of an angle
(d) All the above are correct

54. Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of

(a) 1 degree (b) 1 grade (c) 1 radian (d) 1 arc

55. Radian measure of $40°\,20'$ is equal to

(a) $\frac{120\pi}{504}$ radian (b) $\frac{121\pi}{540}$ radian

(c) $\frac{121\pi}{3}$ radian (d) None of these

56. π radian in degree measure is equal to

(a) 18° (b) 180° (c) 200° (d) 360°

57. The value of $\sin\frac{31\pi}{3}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

58. The value of $\cot\left(\frac{-15\pi}{4}\right)$ is

(a) $\frac{-1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

59. If $\cos\theta=\frac{-3}{5}$ and $\pi<\theta<\frac{3\pi}{2}$, then the value of $\left(\frac{\operatorname{cosec}\theta+\cot\theta}{\sec\theta-\tan\theta}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{13}{2}$ (d) None of these

60. $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$ is equal to

(a) $\sqrt{2}\sin x$ (b) $-2\sin x$

(c) $-\sqrt{2}\sin x$ (d) None of these

61. $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$ is equal to

(a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$ (d) None of these

62. The solution of $\sin x=-\frac{\sqrt{3}}{2}$ is

(a) $x=n\pi+(-1)^n\frac{4\pi}{3}$, where $n\in Z$

(b) $x=n\pi+(-1)^n\frac{2\pi}{3}$, where $n\in Z$

(c) $x=n\pi+(-1)^n\frac{3\pi}{3}$, where $n\in Z$

(d) None of the above

63. If $x = \sec\theta + \tan\theta$, then $x + \frac{1}{x} =$

(a) 1 (b) $2\sec\theta$ (c) 2 (d) $2\tan\theta$

64. The value of $\frac{\tan 70° - \tan 20°}{\tan 50°} =$

(a) 1 (b) 2 (c) 3 (d) 0

65. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} =$

(a) 0 (b) 1 (c) 2 (d) 4

66. The value of $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\frac{5\pi}{12}$ is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3+\sqrt{3}}{2}$ (d) $\frac{2}{3+\sqrt{3}}$

67. $1 + \cos 2x + \cos 4x + \cos 6x =$

(a) $2\cos x \cos 2x \cos 3x$
(b) $4\sin x \cos 2x \cos 3x$
(c) $4\cos x \cos 2x \cos 3x$
(d) None of these

68. $\text{cosec } A - 2\cot 2A \cos A =$

(a) $2\sin A$ (b) $\sec A$
(c) $2\cos A \cot A$ (d) None of these

69. If $\sin x + \cos x = \frac{1}{5}$, then $\tan 2x$ is

(a) $\frac{25}{17}$ (b) $\frac{7}{25}$ (c) $\frac{25}{7}$ (d) $\frac{24}{7}$

70. If $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta \tan 3\theta = 1$, then the general value of θ is

(a) $n\pi + \frac{\pi}{5}$ (b) $\left(n + \frac{1}{6}\right)\frac{\pi}{5}$

(c) $\left(2n \pm \frac{1}{6}\right)\frac{\pi}{5}$ (d) $\left(n + \frac{1}{3}\right)\frac{\pi}{5}$

71. If $\tan\theta - \sqrt{2}\sec\theta = \sqrt{3}$, then the general value of θ is

(a) $n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}$ (b) $n\pi + (-1)^n\frac{\pi}{3} - \frac{\pi}{4}$

(c) $n\pi + (-1)^n\frac{\pi}{3} + \frac{\pi}{4}$ (d) $n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{3}$

72. The general solution of $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ is

(a) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}$, $\theta = n\pi$, $n \in Z$
(b) $\theta = n\pi$, $n \in Z$
(c) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}$, $n \in Z$
(d) $\theta = \frac{n\pi}{2}$, $n \in Z$

73. The value of $\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$ is

(a) 2 (b) 3 (c) 1 (d) 0

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

74. I : $\cos\alpha + \cos\beta + \cos\gamma = 0$
II : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

(a) I is false and II is true (b) I and II both are true
(c) I and II both are false (d) I is true and II is false

75. Consider the statements given below:

I. $\sin x$ is positive in first and second quadrants.
II. $\text{cosec } x$ is negative in third and fourth quadrants.
III. $\tan x$ and $\cot x$ are negative in second and fourth quadrants.
IV. $\cos x$ and $\sec x$ are positive in first and fourth quadrants.

Choose the correct option.

(a) All are correct
(b) Only I and IV are correct
(c) Only III and IV are correct
(d) None is correct

76. Which among the following is/are true?

I. The values of $\text{cosec } x$ repeat after an interval of 2π.
II. The values of $\sec x$ repeat after an interval of 2π.
III. The values of $\cot x$ repeat after an interval of π.

(a) I is true (b) II is true
(c) III is true (d) All are true

77. Consider the following statements.

I. $\cot x$ decreases from 0 to $-\infty$ in first quadrant and increases from 0 to ∞ in third quadrant.
II. $\sec x$ increases from $-\infty$ to -1 in second quadrant and decreases from ∞ to 1 in fourth quadrant.
III. $\text{cosec } x$ increases from 1 to ∞ in second quadrant and decreases from -1 to $-\infty$ in fourth quadrant.

Choose the correct option.

(a) I is incorrect (b) II is incorrect
(c) III is incorrect (d) IV is incorrect

78. Consider the statements given below:

I. $2\cos x \cdot \cos y = \cos(x + y) - \cos(x - y)$.
II. $-2\sin x \cdot \sin y = \cos(x + y) - \cos(x - y)$.
III. $2\sin x \cdot \cos y = \sin(x + y) - \sin(x - y)$.
IV. $2\cos x \cdot \sin y = \sin(x + y) + \sin(x - y)$.

Choose the correct statements.

(a) I is correct
(b) II is correct
(c) Both I and II are correct
(d) III is correct

79. If $\sin 2x + \cos x = 0$, then which among the following is/are true?

I. $\cos x = 0$
II. $\sin x = -\frac{1}{2}$
III. $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$
IV. $x = n\pi + (-1)^n\frac{7\pi}{6}$, $n \in Z$

(a) I is true (b) I and II are true
(c) I, II and III are true (d) All are true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

80.

Column-I (Degree Measure)	Column-II (Radian Measure)
A. 25°	1. $\frac{26\pi}{9}$
B. $-47° 30'$	2. $\frac{4\pi}{3}$
C. 240°	3. $\frac{-19\pi}{72}$
D. 520°	4. $\frac{5\pi}{36}$

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	3	2	1
(c)	1	3	2	4
(d)	1	4	3	2

81. $\left[\text{Use } \pi = \frac{22}{7}\right]$.

Column-I (Radian Measure)	Column-II (Degree Measure)
A. $\frac{11}{16}$	1. 300°
B. -4	2. 210°
C. $\frac{5\pi}{3}$	3. 39° 22′ 30″
D. $\frac{7\pi}{6}$	4. $-229°\ 5'\ 27''$

Codes:

	A	B	C	D
(a)	3	4	2	1
(b)	1	4	2	3
(c)	3	4	1	2
(d)	2	4	1	3

82.

Column-I	Column-II
A. $\sin\frac{25\pi}{3}$	1. $-\sqrt{3}$
B. $\cos\frac{41\pi}{4}$	2. $\frac{\sqrt{3}}{2}$
C. $\tan\left(\frac{-16\pi}{3}\right)$	3. 1
D. $\cot\frac{29\pi}{4}$	4. $\frac{1}{\sqrt{2}}$

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	2	1	4	3
(c)	3	1	4	2
(d)	3	4	1	2

83.

Column-I	Column-II
A. $\cos(\pi - x)$	1. $-\cos x$
B. $\sin(\pi - x)$	2. $-\sin x$
C. $\sin(\pi + x)$	3. $\cos x$
D. $\cos(\pi + x)$	4. $\sin x$
E. $\cos(2\pi - x)$	
F. $\sin(2\pi - x)$	

Codes:

	A	B	C	D	E	F
(a)	1	4	2	1	3	2
(b)	1	2	4	1	3	1
(c)	2	4	1	2	3	2
(d)	1	2	2	4	3	1

84.

Column-I	Column-II
A. 1 radian is equal to	1. 0.01746 radian
B. 1° is equal to	2. 57°16′ (approx.)
C. 3° 45′ is equal to	3. $\frac{9\pi}{32}$ radian
D. 50° 37′ 30″ is equal to	4. $\frac{\pi}{48}$ radian

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	2	4	1	3
(c)	2	1	4	3
(d)	3	1	4	2

85.

Column-I (Degree measure)	Column-II (Radian measure)
(A) 25°	1. $\frac{-19\pi}{72}$
(B) –47° 30′	2. $\frac{4\pi}{3}$
(C) 240°	3. $\frac{26\pi}{9}$
(D) 520°	4. $\frac{5\pi}{36}$

Codes:

	A	B	C	D
(a)	4	2	1	3
(b)	3	1	2	4
(c)	4	1	2	3
(d)	3	2	1	4

86.

Column-I	Column-II
(A) $\sin x =$	1. $\frac{1}{\sqrt{3}}$
(B) $\tan x =$	2. -2
(C) $\cot x =$	3. $\frac{-\sqrt{3}}{2}$
(D) $\sec x =$	4. $\frac{-2}{\sqrt{3}}$
(E) $\text{cosec } x =$	5. $\sqrt{3}$

Codes:

	A	B	C	D	E
(a)	3	5	1	2	4
(b)	1	5	3	2	4
(c)	3	5	1	4	2
(d)	3	1	5	4	2

87.

Column-I (Trigonometric Equation)	Column-II (General Solution)
(A) $\cos 4x = \cos 2x$	1. $x = n\pi \pm \frac{\pi}{3}$
(B) $\cos 3x + \cos x - \cos 2x = 0$	2. $x = \frac{n\pi}{2} + \frac{3\pi}{8}$
(C) $\sin 2x + \cos x = 0$	3. $x = 2n\pi \pm \frac{\pi}{3}$
(D) $\sec^2 2x = 1 - \tan 2x$	4. $x = \frac{n\pi}{3}$ or $x = n\pi,\ n \in Z$
(E) $\sin x + \sin 3x + \sin 5x = 0$	5. $x = (2n+1)\frac{\pi}{2}$ or $x = n\pi + (-1)^n.\frac{7\pi}{6}$, $n \in Z$

Codes:

	A	B	C	D	E
(a)	4	3	5	1	2
(b)	4	3	5	2	1
(c)	3	4	5	1	2
(d)	1	3	5	2	4

88.

Column-I	Column-II
(A) $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} =$	1. $\tan 2x$
(B) $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} =$	2. $\tan \frac{x-y}{2}$
(C) $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} =$	3. $\frac{-\sin 2x}{\cos 10x}$
(D) $\frac{\sin x - \sin y}{\cos x + \cos y} =$	4. $2 \sin x$
(E) $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} =$	5. $\tan 4x$

Codes:

	A	B	C	D	E
(a)	3	5	2	1	4
(b)	3	5	1	4	2
(c)	3	1	2	5	4
(d)	3	5	1	2	4

89. Let $\sin x = \frac{3}{5}$, x lies in second quadrant.

Column-I (Trigonometric Function)	Column-II (Value)
(A) $\cos x =$	1. $-4/3$
(B) $\sec x =$	2. $-3/4$
(C) $\tan x =$	3. $-4/5$
(D) $\text{cosec } x =$	4. $-5/4$
(E) $\cot x =$	5. $5/3$

Codes:

	A	B	C	D	E
(a)	3	4	2	5	1
(b)	3	4	1	5	2
(c)	3	2	4	5	1
(d)	1	2	5	4	3

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

90. The value of cosec $(-1410)°$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) None of these

91. The expression $\cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is equal to

(a) -1 (b) 0 (c) 1 (d) None of these

92. If $\sin\theta + \cos\theta = 1$, then $\sin\theta\cos\theta =$

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

93. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}$, $\frac{-\pi}{2} < A < 0$ $\frac{-\pi}{2} < B < 0$, then value of $2 \sin A + 4 \sin B$ is $-$ a. The value of 'a' is

(a) 4 (b) 2 (c) 3 (d) 0

94. The value of $\sin 765°$ is $\frac{1}{\sqrt{n}}$. Value of n is

(a) 2 (b) 3 (c) 4 (d) 0

95. The value of cosec $(-1410)°$ is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) None of these

96. The value of $\tan\frac{19\pi}{3}$ is $\sqrt{n}$. Value of 'n' is

(a) 1 (b) 2 (c) 3 (d) 5

97. The value of $\sin\left(\frac{-11\pi}{3}\right)$ is $\frac{\sqrt{3}}{m}$. Value of 'm' is

(a) 1 (b) 2 (c) 3 (d) 5

98. The value of $\left(1+\cos\frac{\pi}{6}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)\left(1+\cos\frac{7\pi}{6}\right)$ is $\frac{m}{16}$. Value of m is

(a) 1 (b) 2 (c) 3 (d) 8

99. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\left(\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}\right)$ is equal to $\frac{m}{m+1}$. The value of m is

(a) 1 (b) 2 (c) 3 (d) 4

100. If $\sin x = \frac{-2\sqrt{6}}{5}$ and x lies in III quadrant, then the value of $\cot x$ is $\frac{1}{m\sqrt{6}}$. Value of m is

(a) 1 (b) 2 (c) 3 (d) 5

101. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\left(\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}\right)$ is equal to $\frac{1}{m}$. Value of m is

(a) 2 (b) 4 (c) 5 (d) 6

102. The value of

$3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$ is equal to

(a) 2 (b) 1 (c) 3 (d) 4

103. Value of $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cdot\cos^2\frac{\pi}{3}$ is $\frac{m}{m-1}$. The value of 'm' is

(a) 3 (b) 2 (c) 4 (d) None of these

104. $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$ is equal to

(a) 1 (b) 5 (c) 3 (d) 6

105. Value of

$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$ is

(a) 0 (b) 1 (c) 2 (d) 3

106. $\cot x\cot 2x - \cot 2x\cot 3x - \cot 3x\cot x$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

107. Assertion : The ratio of the radii of two circles at the centres of which two equal arcs subtend angles of 30° and 70° is 21 : 10.

Reason : Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.

108. Assertion : If $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$, then $\sin\theta + \cos\theta = \pm\sqrt{2}$.

Reason : $-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$.

109. Assertion : The solution of the equation

$\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right) + \tan\left(\theta+\frac{2\pi}{3}\right) = 3$

is $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$.

Reason : If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha$, $n \in I$.

110. Assertion : The degree measure corresponding to (–2) radian is –114° 19 min.

Reason : The degree measure of a given radian measure $= \frac{180}{\pi} \times$ Radian measure.

111. Assertion : $\frac{\cos(\pi+x)\cdot\cos(-x)}{\sin(\pi-x)\cdot\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$

Reason : $\cos(\pi+\theta) = -\cos\theta$ and $\cos(-\theta) = \cos\theta$. Also, $\sin(\pi-\theta) = \sin\theta$ and $\sin(-\theta) = -\sin\theta$.

112. Assertion : If $\tan 2x = -\cot\left(x+\frac{\pi}{3}\right)$, then $x = n\pi + \frac{5\pi}{6}$, $n \in Z$.

Reason : $\tan x = \tan y \Rightarrow x = n\pi + y$, where $n \in Z$.

113. Assertion : The measure of rotation of a given ray about its initial point is called an angle.

Reason : The point of rotation is called a vertex.

114. Assertion : In a unit circle, radius of circle is 1 unit.

Reason : 1 min (or 1′) is divided into 60s.

115. Assertion : Area of unit circle is π unit^2.

Reason : Radian measure of 40° 20′ is equal to $\frac{|2|\pi}{540}$ radian.

116. Assertion : The second hand rotates through an angle of 180° in a minute.

Reason : The unit of measurement is degree in sexagesimal system.

117. Assertion: cosec x is negative in third and fourth quadrants.

Reason : cot x decreases from 0 to $-\infty$ in first quadrant and increases from 0 to ∞ in third quadrant.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

118. The value of tan 20° + 2 tan 50° – tan 70° is equal to

(a) 1 (b) 0
(c) tan 50° (d) None of these

119. If α and β lies between 0 and $\frac{\pi}{2}$ and if $\cos(\alpha+\beta)=\frac{12}{13}$ and $\sin(\alpha-\beta)=\frac{3}{5}$, then value of $\sin 2\alpha$ is

(a) $\frac{55}{56}$ (b) $\frac{13}{58}$ (c) 0 (d) $\frac{56}{65}$

120. The most general value of θ satisfying the equation $\cos\theta=\frac{1}{\sqrt{2}}$ and $\tan\theta=-1$ is

(a) $2n\pi-7\frac{\pi}{4}$ (b) $n\pi-\frac{\pi}{4}$

(c) $n\pi+\frac{\pi}{2}$ (d) $2n\pi+\frac{7\pi}{4}$

121. Value of $\sqrt{3}$ cosec 20° – sec 20° is

(a) 3 (b) $\frac{3}{2}$ (c) 1 (d) 4

122. The solution of the equation $\cos^2\theta+\sin\theta+1=0$, lies in the interval

(a) $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

(c) $\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$ (d) $\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$

123. The number of values of x in the interval $[0,3\pi]$ satisfying the equation $2\sin^2 x+5\sin x-3=0$ is

(a) 4 (b) 6 (c) 1 (d) 2

124. Value of $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$ is

(a) $-\frac{1}{2}$ (b) 0 (c) 1 (d) $\frac{\sqrt{3}}{2}$

125. Value of sin 47° + sin 61° – sin 11° – sin 25° is

(a) cos 7° (b) sin 7° (c) sin 61° (d) –sin 25°

126. The value of expression $\sin\theta+\cos\theta$ lies between

(a) –2 and 2 both inclusive

(b) 0 and $\sqrt{2}$ both inclusive

(c) $-\sqrt{2}$ and $\sqrt{2}$ both inclusive

(d) 0 and 2 both inclusive

127. The solution of $\tan 2\theta\tan\theta=1$ is

(a) $2n\pi+\frac{\pi}{3}$ (b) $n\pi+\frac{\pi}{4}$

(c) $2n\pi-\frac{\pi}{6}$ (d) $(2n+1)\frac{\pi}{6}$

128. Number of solutions of equation, sin 5x cos 3x = sin 6x cos 2x, in the interval $[0,\pi]$ is

(a) 4 (b) 5 (c) 3 (d) 2

129. If tan (cot x) = cot (tan x), then

(a) $\sin 2x=\frac{2}{(2n+1)\pi}$ (b) $\sin x=\frac{4}{(2n+1)\pi}$

(c) $\sin 2x=\frac{4}{(2n+1)\pi}$ (d) None of these

130. Find the distance from the eye at which a coin of a diameter 1 cm be placed so as to hide the full moon, it is being given that the diameter of the moon subtends an angle of 31′ at the eye of the observer.

(a) 110 cm (b) 108 cm

(c) 110.9 cm (d) 112 cm

131. A wheel rotates making 20 revolutions per second. If the radius of the wheel is 35 cm, what linear distance does a point of its rim travel in three minutes? $\left(\text{Take }\pi=\frac{22}{7}\right)$

(a) 7.92 km (b) 7.70 km

(c) 7.80 km (d) 7.85 km

132. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use π = 3.14)

(a) 2.68 cm (b) 6.28 cm

(c) 6.82 cm (d) 7.42 cm

133. If the arcs of the same lengths in two circles subtend angles 65° and 110° at the centre, the ratio of their radii is

(a) 12 : 13 (b) 22 : 31 (c) 22 : 13 (d) 21 : 13

134. If tan A + cot A = 4, then $\tan^4 A+\cot^4 A$ is equal to

(a) 110 (b) 191 (c) 80 (d) 194

135. If $\frac{\sin A}{\sin B}=m$ and $\frac{\cos A}{\cos B}=n$, then the value of tan B; $n^2<1<m^2$, is

(a) n^2 (b) $\pm\sqrt{\frac{1-n^2}{m^2-1}}$

(c) $\frac{n^2}{(m^2-1)}$ (d) m^2

136. If $\tan(A-B)=1$, $\sec(A+B)=\frac{2}{\sqrt{3}}$, the smallest positive value of B is

(a) $\frac{25\pi}{24}$ (b) $\frac{19\pi}{24}$ (c) $\frac{13\pi}{24}$ (d) $\frac{7\pi}{24}$

137. The value of $4\sin\alpha\,\sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha+\frac{2\pi}{3}\right)=$

(a) $\sin 3\alpha$ (b) $\sin 2\alpha$ (c) $\sin\alpha$ (d) $\sin^2\alpha$

138. The solution of the equation $[\sin x+\cos x]^{1+\sin 2x}=2,\ -\pi\le x\le\pi$ is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

139. If $\tan\theta+\sec\theta=p$, then what is the value of $\sec\theta$?

(a) $\frac{p^2+1}{p^2}$ (b) $\frac{p^2+1}{\sqrt{p}}$

(c) $\frac{p^2+1}{2p}$ (d) $\frac{p+1}{2p}$

140. The number of solutions of the given equation $\tan\theta+\sec\theta=\sqrt{3}$, where $0\le\theta\le 2\pi$ is

(a) 0 (b) 1 (c) 2 (d) 3

141. If n is any integer, then the general solution of the equation $\cos x-\sin x=\frac{1}{\sqrt{2}}$ is

(a) $x=2n\pi-\frac{\pi}{12}$ or $x=2n\pi+\frac{7\pi}{12}$

(b) $x=n\pi\pm\frac{\pi}{12}$

(c) $x=2n\pi+\frac{\pi}{12}$ or $x=2n\pi-\frac{7\pi}{12}$

(d) $x=n\pi+\frac{\pi}{12}$ or $x=n\pi-\frac{7\pi}{12}$

142. If $4\sin^2\theta+2\left(\sqrt{3}+1\right)\cos\theta=4+\sqrt{3}$, then the general value of θ is

(a) $2n\pi\pm\frac{\pi}{3}$ (b) $2n\pi+\frac{\pi}{4}$

(c) $n\pi\pm\frac{\pi}{3}$ (d) $n\pi-\frac{\pi}{3}$

143. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x+5\sin x-3=0$ is

(a) 4 (b) 6

(c) 1 (d) 2

144. If $\sin\theta+\cos\theta=1$, then the general value of θ is

(a) $2n\pi$ (b) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$

(c) $2n\pi+\frac{\pi}{2}$ (d) $(2n-1)+\frac{\pi}{4}$

145. $\sin 6\theta+\sin 4\theta+\sin 2\theta=0$, then $\theta=$

(a) $\frac{n\pi}{4}$ or $n\pi\pm\frac{\pi}{3}$ (b) $\frac{n\pi}{4}$ or $n\pi\pm\frac{\pi}{6}$

(c) $\frac{n\pi}{4}$ or $2n\pi\pm\frac{\pi}{6}$ (d) None of these

146. If $\sqrt{2}\sec\theta+\tan\theta=1$, then the general value of θ is

(a) $n\pi+\frac{3\pi}{4}$ (b) $2n\pi+\frac{\pi}{4}$

(c) $2n\pi-\frac{\pi}{4}$ (d) $2n\pi\pm\frac{\pi}{4}$

147. If $12\cot^2\theta-31\operatorname{cosec}\theta+32=0$, then the value of $\sin\theta$ is

(a) $\frac{3}{5}$ or 1 (b) $\frac{2}{3}$ or $\frac{-2}{3}$

(c) $\frac{4}{5}$ or $\frac{3}{4}$ (d) $\pm\frac{1}{2}$

148. If $\sec^2\theta=\frac{4}{3}$, then the general value of θ is

(a) $2n\pi\pm\frac{\pi}{6}$ (b) $n\pi\pm\frac{\pi}{6}$

(c) $2n\pi\pm\frac{\pi}{3}$ (d) $n\pi\pm\frac{\pi}{3}$

149. General solution of $\tan 5\theta=\cot 2\theta$ is

(a) $\theta=\frac{n\pi}{7}+\frac{\pi}{14}$ (b) $\theta=\frac{n\pi}{7}+\frac{\pi}{5}$

(c) $\theta=\frac{n\pi}{7}+\frac{\pi}{2}$ (d) $\theta=\frac{n\pi}{7}+\frac{\pi}{3}$

150. If none of the angles x, y and $(x + y)$ is a multiple of π, then

(a) $\cot(x + y) = \dfrac{\cot x \cdot \cot y - 1}{\cot y + \cot x}$

(b) $\cot(x - y) = \dfrac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$

(c) (a) and (b) are true

(d) (a) and (b) are not true

151. Solution of the equation $3\tan(\theta - 15) = \tan(\theta + 15)$ is

(a) $\theta = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{4}$ (b) $\theta = n\pi + (-1)^n \dfrac{\pi}{3}$

(c) $\theta = n\pi - \dfrac{\pi}{3}$ (d) $\theta = n\pi - \dfrac{\pi}{4}$

152. If angle θ is divided into two parts such that the tangent of one part is K times the tangent to other and ϕ is their difference, then $\sin\theta$ is equal to

(a) $\dfrac{K+1}{K-1}\sin\dfrac{\theta}{2}$ (b) $\dfrac{K+1}{K-1}\sin\dfrac{\phi}{2}$

(c) $\dfrac{K+1}{K-1}\sin\phi$ (d) $\dfrac{K-1}{K+1}\sin\phi$

153. If $m\sin\theta = n\sin(\theta + 2\alpha)$, then $\tan(\theta + \alpha)\cdot\cot\alpha$ is equal to

(a) $\dfrac{m+n}{m-n}$ (b) $\dfrac{m-n}{m+n}$

(c) $\dfrac{m+n}{mn}$ (d) $\dfrac{m-n}{mn}$

154. If $5\tan\theta = 4$, then $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$

(a) 0 (b) 1 (c) $\dfrac{1}{6}$ (d) 6

155. $\dfrac{1 + \sin A - \cos A}{1 + \sin A + \cos A} =$

(a) $\sin\dfrac{A}{2}$ (b) $\cos\dfrac{A}{2}$ (c) $\tan\dfrac{A}{2}$ (d) $\cot\dfrac{A}{2}$

156. $\dfrac{1}{4}\left[\sqrt{3}\cos 23° - \sin 23°\right] =$

(a) $\cos 43°$ (b) $\cos 7°$ (c) $\cos 53°$ (d) None of these

157. If $\cos x + \cos y + \cos\alpha = 0$ and $\sin x + \sin y + \sin\alpha = 0$, then $\cot\left(\dfrac{x+y}{2}\right) =$

(a) $\sin\alpha$ (b) $\cos\alpha$ (c) $\cot\alpha$ (d) $\sin\left(\dfrac{x+y}{2}\right)$

158. $\sin 12° \sin 24° \sin 48° \sin 84° =$

(a) $\cos 20° \cos 40° \cos 60° \cos 80°$

(b) $\sin 20° \sin 40° \sin 60° \sin 80°$

(c) $\dfrac{3}{15}$

(d) None of these

159. If $\dfrac{2\sin\alpha}{\{1 + \cos\alpha + \sin\alpha\}} = y$, then $\dfrac{\{1 - \cos\alpha + \sin\alpha\}}{1 + \sin\alpha} =$

(a) $\dfrac{1}{y}$ (b) y (c) $1 - y$ (d) $1 + y$

160. If $\sin 2\theta + \sin 2\phi = \dfrac{1}{2}$ and $\cos 2\theta + \cos 2\phi = \dfrac{3}{2}$, then $\cos^2(\theta - \phi) =$

(a) $\dfrac{3}{8}$ (b) $\dfrac{5}{8}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{5}{4}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (b) $\tan^2\theta\sec^2\theta(\cot^2\theta-\cos^2\theta)$
$=\sec^2\theta(\tan^2\theta\cot^2\theta-\tan^2\theta\cos^2\theta)$
$=\sec^2\theta\left(1-\dfrac{\sin^2\theta}{\cos^2\theta}\cos^2\theta\right)=\sec^2\theta(1-\sin^2\theta)$
$=\sec^2\theta.\cos^2\theta=1$

2. (c) $\cot 5°\cot 10°\,.........\,\cot 85°$
$=\cot 5°\cot 10°....\cot(90°-10°)\cot(90°-5°)$
$=\cot 5°\cot 10°......\tan 10°\tan 5°$
$=(\tan 5°\cot 5°)(\tan 10°\cot 10°)........$
$=(1)(1)(1).......=1$

3. (b) $\because \sin 190°=\sin(180°+10°)=-\sin 10°$
$\sin 200°=-\sin 20°$
$\sin 210°=-\sin 30°$
....
$\sin 360°=\sin 180°=0$
$\therefore$ given expression $=0$

4. (d) $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}=\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}=\dfrac{5/6}{5/6}=1$
$\therefore A+B=45°=\dfrac{\pi}{4}$

5. (a) Using cosine formula
$\sin 2\theta+\sin 2\phi=2\sin(\theta+\phi)\cos(\theta-\phi)=1/2$ .....(i)
$\cos 2\theta+\cos 2\phi=2\cos(\theta+\phi)\cos(\theta-\phi)=3/2$ .....(ii)
Squaring (i) and (ii) and then adding
$4\cos^2(\theta-\phi)=\dfrac{1}{4}+\dfrac{9}{4}=\dfrac{5}{2}$
$\Rightarrow \cos^2(\theta-\phi)=\dfrac{5}{8}$

6. (d) $\cos\theta=-1/2=\cos 120°$ or $\cos 240°$ $[0<\theta<360°]$
$\therefore \theta=120°, 240°$

7. (d) $\tan\theta=-\dfrac{1}{\sqrt{3}}=\tan\left(-\dfrac{\pi}{6}\right)$
$\therefore \theta=n\pi-\dfrac{\pi}{6}$

8. (a) $2\tan^2\theta=\sec^2\theta=1+\tan^2\theta$
$\tan^2\theta=1=(1)^2=\tan^2\dfrac{\pi}{4}$
$\theta=n\pi\pm\dfrac{\pi}{4}, n\in I$

9. (c) $\sin 5x+\sin x=-\sin 3x$
$\Rightarrow 2\sin 3x\cos 2x+\sin 3x=0$
$\Rightarrow \sin 3x(2\cos 2x+1)=0$
$\Rightarrow \sin 3x=0, \cos 2x=-1/2$
$\Rightarrow x=n\pi, x=n\pi\pm(\pi/3)$
So, $x=\pi/3$

10. (b) $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}.\dfrac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$
$=\dfrac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$
$=\dfrac{(1+\sin^2\alpha+2\sin\alpha)-(1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$
$=\dfrac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}=\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$

11. (b) The given equation is $\tan x+\sec x=2\cos x$;
$\Rightarrow \sin x+1=2\cos^2 x \Rightarrow \sin x+1=2(1-\sin^2 x)$;
$\Rightarrow 2\sin^2 x+\sin x-1=0$;
$\Rightarrow (2\sin x-1)(\sin x+1)=0 \Rightarrow \sin x=\dfrac{1}{2},-1$
$\Rightarrow x=30°, 150°, 270°$.

12. (d) We have : $\sin A=\dfrac{3}{5}$, where $0<A<\dfrac{\pi}{2}$
$\therefore \quad \cos A=\pm\sqrt{1-\sin^2 A}$
$\Rightarrow \quad \cos A=+\sqrt{1-\sin^2 A}=\sqrt{1-\dfrac{9}{25}}=\dfrac{4}{5}$
[$\because$ cos is positive in first quadrant]
It is given that : $\cos B=\dfrac{-12}{13}$ and $\pi<B<\dfrac{3\pi}{2}$
$\therefore \quad \sin B=\pm\sqrt{1-\cos^2 B}$
$\Rightarrow \quad \sin B=-\sqrt{1-\cos^2 B}$
[$\because$ Sine is negative in the third quadrant]
$\Rightarrow \quad \sin B=-\sqrt{1-\left(\dfrac{-12}{13}\right)^2}=-\dfrac{5}{13}$
Now, $\sin(A-B)=\sin A\cos B-\cos A\sin B$
$=\dfrac{3}{5}\times\dfrac{-12}{13}-\dfrac{4}{5}\times\dfrac{-5}{13}=-\dfrac{16}{65}$

13. (b) $\tan 15°.\tan 45°\tan 75°$
$=\tan 15°.\tan(60°-15°).\tan(60°+15°)$
$=\tan(3\times 15°)=\tan 45°=1$

14. (a) $\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1+\cos\left(\pi-\dfrac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\dfrac{\pi}{8}\right)\right)$
$=\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1-\cos\dfrac{3\pi}{8}\right)\left(1-\cos\dfrac{\pi}{8}\right)$
$=\left(1-\cos^2\dfrac{\pi}{8}\right)\left(1-\cos^2\dfrac{3\pi}{8}\right)$

$$= \frac{1}{4}\left(2 - 1 - \cos\frac{\pi}{4}\right)\left(2 - 1 - \cos\frac{3\pi}{4}\right)$$

$$= \frac{1}{4}\left(1 - \cos\frac{\pi}{4}\right)\left(1 - \cos\frac{3\pi}{4}\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{\sqrt{2}}\right) = \frac{1}{4}\left(1 - \frac{1}{2}\right) = \frac{1}{8}$$

15. (a) The large hand of the clock makes a complete revolution in 60 minutes.

$\therefore$ Angle traced out by the large hand in 20 minutes (of time)

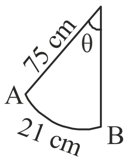
$$= \frac{360° \times 20}{60} = 120° = \frac{120\pi}{180} \text{ radian} = \frac{2\pi}{3} \text{ radian}$$

Hence, the distance moved by the extremity of the large hand = (42) × $\frac{2\pi}{3}$ = 88 cm. ($\because l = r\theta$)

16. (a) Given, length of pendulum = 75 cm

Radius (r) = length of pendulum = 75 cm

Length of arc (l) = 21 cm



Now, $\theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25}$ radian.

17. (c) Let l be the length of the arc. We know that,

Angle $\theta = \frac{l}{r}$, where θ is in radian.

Given, r = 3 cm

$$\theta = 30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6} \text{ rad}$$

On putting the values of r and θ, we get

$$\frac{\pi}{6} = \frac{l}{3} \Rightarrow l = \frac{\pi}{2} = \frac{3.14}{2} = 1.57 \text{ cm.}$$

18. (b) Circumference of a circular wire of radius 7 cm is = 2π × 7 = 14π

As we know, $\theta = \frac{l}{r}$

$$\Rightarrow \theta = \frac{14\pi}{12} = \frac{7\pi \times 180°}{6\pi} = 210°.$$

19. (c) Length of wire = 2π × 3 = 6π cm and r = 48 cm is the radius of the circle. Therefore, the angle θ (in radian) subtended at the centre of the circle is given by

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{6\pi}{48} = \frac{\pi}{8} = 22.5°.$$

20. (c) Here, l = 37.4 cm and θ = 60° = $\frac{60\pi}{180}$ radian = $\frac{\pi}{3}$

Hence, by $r = \frac{l}{\theta}$, we have

$$r = \frac{37.4 \times 3}{\pi} = \frac{37.4 \times 3 \times 7}{22} = 35.7 \text{ cm.}$$

21. (b) Given radius, r = 100 cm and arc length, l = 22 cm

We know that, $l = r\theta$

$$\theta = \frac{l}{r} = \frac{\text{Arc length}}{\text{Radius}}$$

$$= \frac{22}{100} = 0.22 \text{ rad} = 0.22 \times \frac{180}{\pi} \text{ degree}$$

$$= 0.22 \times \frac{180 \times 7}{22} = \frac{22}{100} \times \frac{180 \times 7}{22}$$

$$= \frac{126}{10} = 12\frac{6°}{10} = 12° + \frac{6}{10} \times 60' \quad [\because 1° = 60']$$

= 12° + 36′ = 12° 36′

Hence, the degree measure of the required angle is 12°36′.

22. (c) Given, $\tan\theta = \frac{3}{1}$ and θ lies in III quadrant.

We know that $\sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{3}{1}\right)^2 = 10$

$\Rightarrow \sec\theta = \pm\sqrt{10}$

Since, θ lies in III quadrant, so $\sec\theta = -\sqrt{10}$

$$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\sqrt{10}}$$

Also,

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\frac{1}{\sqrt{10}}\right)^2$$

$$= 1 - \frac{1}{10} = \frac{9}{10}$$

$$\Rightarrow \sin\theta = \pm\sqrt{\frac{9}{10}}$$

Since, θ lies in III quadrant so $\sin\theta = -\sqrt{\frac{9}{10}} = \frac{-3}{\sqrt{10}}$.

23. (b) $\frac{\sin x}{\cos x} \times \frac{\sec x}{\operatorname{cosec} x} \times \frac{\tan x}{\cot x} = 9$

$$\Rightarrow \tan x \times \tan x \times \frac{\tan x}{\cot x} = 9$$

$\Rightarrow \tan^4 x = 9$

$\Rightarrow \tan x = \pm\sqrt{3}$

$$\Rightarrow x = \frac{\pi}{3} \in \left(0, \frac{\pi}{2}\right).$$

24. (b) The given equation is

cosec(90° + θ) + x cos θ cot(90° + θ) = sin(90° + θ)

$\Rightarrow \sec\theta + x\cos\theta(-\tan\theta) = \cos\theta$

$$\Rightarrow \sec\theta - x\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \cos\theta$$

$\Rightarrow \sec\theta - x\sin\theta = \cos\theta$

$$\Rightarrow x\sin\theta = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$\Rightarrow x\sin\theta = \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

$\Rightarrow x = \tan\theta$.

25. (d) We have $A + B = \frac{\pi}{4}$

$$\Rightarrow \cot(A + B) = \cot\frac{\pi}{4} \Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = 1$$

$\Rightarrow \cot A \cot B - 1 = \cot A + \cot B$

$\Rightarrow$ cot A cot B – cot A – cot B – 1 = 0
$\Rightarrow$ cot A cot B – cot A – cot B + 1 = 2
$\Rightarrow$ cot A(cot B – 1) – 1(cot B – 1) = 2
$\Rightarrow$ (cot A – 1) (cot B – 1) = 2.

26. (a) We have, $\sin A = \frac{3}{5}$

$$\Rightarrow \cos A = \sqrt{1-\sin^2 A}$$

$$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

and $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

Now, $\sin 2A = 2\sin A \cdot \cos A = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

$\cos 2A = 1 - 2\sin^2 A = 1 - 2 \times \frac{9}{25} = 1 - \frac{18}{25} = \frac{7}{25}$

and $\tan 2A = \frac{24}{7}$.

27. (b) Given, $\cot\alpha = \frac{1}{2} \Rightarrow \tan\alpha = 2$ and $\sec\beta = \frac{-5}{3}$

Then, $\tan\beta = \sqrt{\sec^2\beta - 1}$

$$\Rightarrow \tan\beta = \pm\sqrt{\frac{25}{9}-1} = \pm\sqrt{\frac{16}{9}}$$

$$\Rightarrow \tan\beta = \pm\frac{4}{3}$$

But, $\tan\beta = \frac{-4}{3}$

[$\because$ tan β is negative in IInd quadrant]

$$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta} = \frac{2+\left(-\frac{4}{3}\right)}{1-(2)\left(\frac{-4}{3}\right)}$$

$$= \frac{\left(2-\frac{4}{3}\right)}{\left(1+\frac{8}{3}\right)} = \frac{2}{11}.$$

28. (a) tan 75° – cot 75° = tan(45° + 30°) – cot(45° + 30°)

$$= \frac{\tan 45° + \tan 30°}{1-\tan 45°\tan 30°} - \frac{\cot 45°\cot 30° - 1}{\cot 45° + \cot 30°}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} - \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} - \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(3+1+2\sqrt{3})}{3-1} - \frac{(3+1-2\sqrt{3})}{3-1} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}.$$

29. (a) tan 3A = tan(2A + A)

$$\Rightarrow \tan 3A = \frac{\tan 2A + \tan A}{1-\tan 2A\tan A}$$

$\Rightarrow$ tan 3A – tan 3A tan 2A tan A = tan 2A + tan A
$\Rightarrow$ tan 3A – tan 2A – tan A = tan 3A tan 2A tan A

30. (c) Given, $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ … (i)

Now, tan(2A + B)

$$= \frac{\tan 2A + \tan B}{1-\tan 2A\tan B} = \frac{\frac{2\tan A}{1-\tan^2 A} + \tan B}{1-\frac{2\tan A}{1-\tan^2 A}\times\tan B}$$

$$= \frac{\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right)+\frac{1}{3}}{1-\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right)\times\frac{1}{3}} = \frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\times\frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3.$$

31. (b) We have, $\tan\theta = \frac{a}{b}$

Now, b cos 2θ + a sin 2θ

$$= b\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + a\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= b\left(\frac{1-\frac{a^2}{b^2}}{1+\frac{a^2}{b^2}}\right) + a\left(\frac{2\times\frac{a}{b}}{1+\frac{a^2}{b^2}}\right)$$

$$= b\left(\frac{b^2-a^2}{b^2+a^2}\right) + \left(\frac{2\frac{a^2}{b}\times b^2}{b^2+a^2}\right)$$

$$= \frac{1}{b^2+a^2}[b^3 - a^2 b + 2a^2 b]$$

$$= \frac{1}{(b^2+a^2)} \times b(a^2+b^2) = b.$$

32. (c) Given, equation is $\tan x + \sec x = 2\cos x$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\Rightarrow 1 + \sin x = 2\cos^2 x$$

$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$
$\Rightarrow 2\sin^2 x + 2\sin x - \sin x - 1 = 0$
$\Rightarrow 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$
$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow$ either $\sin x = \frac{1}{2}$ or $\sin x = -1$

$\Rightarrow$ either $x = \frac{\pi}{6}, \frac{5\pi}{6} \in [0, \pi]$ or $x = \frac{3\pi}{2}$

But, $x = \frac{3\pi}{2}$ can not be possible.

$\therefore$ Number of solutions are 2.

33. **(b)** Since A and B both lie in the IV quadrant, it follows that $\sin A$ and $\sin B$ are negative. Therefore,

$\sin A = -\sqrt{1-\cos^2 A}$

$\Rightarrow \sin A = -\sqrt{1-\frac{16}{25}} = -\frac{3}{5}$

and, $\sin B = -\sqrt{1-\cos^2 B}$

$\Rightarrow \sin B = -\sqrt{1-\frac{144}{169}} = -\frac{5}{13}$

Now, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \frac{4}{5}\times\frac{12}{13} - \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right) = \frac{33}{65}$

34. **(a)** π radians $= 180°$

$1° = \frac{\pi}{180}$ radians

$\therefore \quad 25° = 25\times\frac{\pi}{180} = \frac{5\pi}{36}$

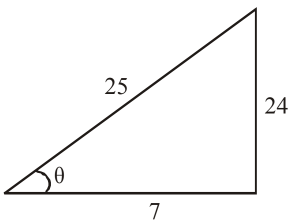
35. **(b)** Since $\tan\theta = -\frac{4}{3}$ is negative, θ lies either in second quadrant or in fourth quadrant. Thus $\sin\theta = \frac{4}{5}$ if θ lies in the second quadrant

or $\sin\theta = -\frac{4}{5}$, if θ lies in the fourth quadrant.

36. **(c)** $\cos(A+B).\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B.$
$= \cos^2 A(1-\sin^2 B) - \sin^2 A\sin^2 B$
$= \cos^2 A - \cos^2 A\sin^2 B - \sin^2 A\sin^2 B$
$= \cos^2 A - \sin^2 B(\cos^2 A + \sin^2 A)$
$= \cos^2 A - \sin^2 B$

37. **(c)** We have, $\sin\theta = \frac{24}{25}, 0° < \theta < 90°$



$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{24}{25}\right)^2$

Since θ lies in first quadrant $\Rightarrow \cos\theta = \frac{7}{25}$

$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$2\sin^2\frac{\theta}{2} = 1 - \cos\theta = 1 - \frac{7}{25}$

$2\sin^2\frac{\theta}{2} = \frac{18}{25}$

$\sin^2\frac{\theta}{2} = \frac{9}{25} \Rightarrow \sin\frac{\theta}{2} = \pm\frac{3}{5}$

$\Rightarrow \sin\frac{\theta}{2} = \frac{3}{5}$

[Negative sign discarded since θ is in first quadrant]

38. **(b)** $\sin\frac{5\pi}{12} = \sin 75°$

$= \sin(45° + 30°) = \sin 45°\cos 30° + \cos 45°\sin 35°$

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$

39. **(b)** Given : $x + \frac{1}{x} = 2\cos\theta$ ...(i)

Cubic both sides in eq^n (i) we get

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8\cos^3\theta$

$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3(2\cos\theta) = 8\cos^3\theta$

$\Rightarrow \quad x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$

$\Rightarrow \quad x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$

40. **(c)** $1 + \cot\theta = \operatorname{cosec}\theta$

$\Rightarrow \frac{1}{\sin\theta} = 1 + \frac{\cos\theta}{\sin\theta} \Rightarrow \sin\theta + \cos\theta = 1$

$\sin\theta\sin\frac{\pi}{4} + \cos\theta\cos\frac{\pi}{4} = \cos\frac{\pi}{4}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} \Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

Hence, $\theta = 2n\pi$ or $\theta = 2n\pi + \frac{\pi}{2}$

But, $\theta = 2n\pi$ is ruled out

41. **(b)** $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$

$\therefore \quad \sin 3\alpha = 4\sin\alpha(\sin^2 x\cos^2\alpha - \cos^2 x\sin^2\alpha)$

$\therefore \quad 3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha)$

$\therefore \quad \sin^2 x = \left(\frac{3}{4}\right) \Rightarrow \sin^2 x = \sin^2\frac{\pi}{3}$

$\therefore \quad x = n\pi \pm \frac{\pi}{3}.$

42. **(b)** $\tan\theta + \tan\left(\frac{\pi}{2} - \theta\right) = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4} \Rightarrow \theta = n\pi + \frac{\pi}{4}.$

43. (b) $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$

$\therefore \left(\sin^2\theta + \sqrt{3}\sin\theta\right)\sec\theta = 0$

$\therefore \sin\theta\left(\sin\theta + \sqrt{3}\right)\sec\theta = 0$

$\Rightarrow \sin\theta = 0$

$\therefore \theta = n\pi,\ n \in I \quad \left[\because \sin\theta \neq -\sqrt{3},\ \sec\theta \neq 0\right]$

44. (c) $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\therefore \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\therefore \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3} \Rightarrow \tan 3\theta = \tan\frac{\pi}{3}$

$\therefore 3\theta = n\pi + \frac{\pi}{3} \Rightarrow \theta = (3n+1)\frac{\pi}{9}.$

45. (a)

46. (c) $\sec 4\theta - \sec 2\theta = 2$

$\therefore \cos 2\theta - \cos 4\theta = 2\cos 4\theta\cos 2\theta$

$\therefore -\cos 4\theta = \cos 6\theta \Rightarrow 2\cos 5\theta\cos\theta = 0$

$\therefore \theta = n\pi + \frac{\pi}{2}$ or $\frac{n\pi}{5} + \frac{\pi}{10}.$

47. (d) $\tan\theta\tan 2\theta = 1$

$\therefore \tan\theta\,\frac{2\tan\theta}{1 - \tan^2\theta} = 1$

$\therefore 2\tan^2\theta = 1 - \tan^2\theta$

$\therefore 3\tan^2\theta = 1$

$\therefore \tan^2\theta = \frac{1}{3} = \tan^2\left(\frac{\pi}{6}\right)$

$\therefore \theta = n\pi \pm \frac{\pi}{6}.$

48. (d) $\cot\theta + \cot\left(\frac{\pi}{4} + \theta\right) = 2$

$\therefore \frac{\cos\theta}{\sin\theta} + \frac{\cos\left(\frac{\pi}{4}+\theta\right)}{\sin\left(\frac{\pi}{4}+\theta\right)} = 2$

$\therefore \sin\left(\frac{\pi}{4} + 2\theta\right) = 2\sin\theta\sin\left(\frac{\pi}{4}+\theta\right)$

$= \cos\left(\theta - \frac{\pi}{4} - \theta\right) - \cos\left(\theta + \frac{\pi}{4} + \theta\right)$

$\therefore \sin\left(\frac{\pi}{4} + 2\theta\right) = \cos\left(\frac{-\pi}{4}\right) - \cos\left(2\theta + \frac{\pi}{4}\right)$

$\Rightarrow \sin\left(\frac{\pi}{4} + 2\theta\right) + \cos\left(\frac{\pi}{4} + 2\theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\cos 2\theta + \frac{1}{\sqrt{2}}\sin 2\theta\right)$

$+ \left(\frac{1}{\sqrt{2}}\cos 2\theta - \frac{1}{\sqrt{2}}\sin 2\theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{2}{\sqrt{2}}\cos 2\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 2\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}.$

49. (a) $2\cos^2 x + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$, i.e. 30°, 150°, 90°.

50. (c) $\cot\theta + \tan\theta = 2\,\text{cosec}\,\theta$

$\Rightarrow \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}.$

51. (b) $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$

$\Rightarrow \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta\tan 3\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan 5\theta = \tan\frac{\pi}{6}$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \left(n + \frac{1}{6}\right)\frac{\pi}{5}.$

52. (c) $\cos 7\theta = \cos\theta - \sin 4\theta$

$\sin 4\theta = \cos\theta - \cos 7\theta$

$\Rightarrow \sin 4\theta = 2\sin(4\theta)\sin(3\theta)$

$\Rightarrow \sin 4\theta = 0 \Rightarrow 4\theta = n\pi$ or

$\sin 3\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right) \Rightarrow 3\theta = n\pi + (-1)^n\frac{\pi}{6}$

$\therefore \theta = \frac{n\pi}{4} = \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}.$

53. (d) In anti-clockwise rotation, the angle is said to be positive.
In clockwise rotation, the angle is said to be negative.
The measure of an angle is the amount of rotation performed to get the terminal side from the initial side.
So, all the statements are correct.

54. (c) Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian.

55. (b) We know that, 180° = π radian

Hence, $40°\,20' = 40\frac{1}{3}$ degree $[\because\ 1° = 60']$

$= \frac{\pi}{180} \times \frac{121}{3}$ radian $= \frac{121\pi}{540}$ radian

Therefore, $40°\,20' = \frac{121\pi}{540}$ radian.

56. (b) π radian = 180°.

57. (a) We know that, values of $\sin x$ repeats after an interval of 2π. Therefore,

$\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

58. **(b)** $\cot\left(-\dfrac{15\pi}{4}\right) = -\cot\left(\dfrac{15\pi}{4}\right)$ [$\because$ $\cot(-\theta) = -\cot\theta$]

$= -\cot\left(4\pi - \dfrac{\pi}{4}\right) = -\cot\left(2\pi \times 2 - \dfrac{\pi}{4}\right)$

$= -\left(-\cot\dfrac{\pi}{4}\right) = \cot\dfrac{\pi}{4} = 1$

[$\because$ $\cot(2n\pi - \theta) = -\cot\theta$]

59. **(d)** In III quadrant, only $\tan\theta$ and $\cot\theta$ are positive.

$\sin^2\theta = (1 - \cos^2\theta) = \left(1 - \dfrac{9}{25}\right) = \dfrac{16}{25}$

$\Rightarrow \quad \sin\theta = \pm\sqrt{\dfrac{16}{25}} = \pm\dfrac{4}{5}$

$\Rightarrow \quad \sin\theta = \dfrac{-4}{5}$ (as $\sin\theta$ is negative in 3rd quadrant)

$\therefore \quad \tan\theta = \left(\dfrac{-4}{5} \times \dfrac{5}{-3}\right) = \dfrac{4}{3}$

and $\cot\theta = \dfrac{3}{4} \Rightarrow \operatorname{cosec}\theta = \dfrac{-5}{4}$

and $\sec\theta = \dfrac{-5}{3}$

$$\therefore \quad \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\sec\theta - \tan\theta)} = \frac{\left(\dfrac{-5}{4} + \dfrac{3}{4}\right)}{\left(\dfrac{-5}{3} - \dfrac{4}{3}\right)} = \frac{\left(\dfrac{-2}{4}\right)}{\left(\dfrac{-9}{3}\right)}$$

$= \dfrac{-2}{4} \times \dfrac{3}{-9} = \dfrac{1}{6}.$

60. **(c)** $\cos\left(\dfrac{3\pi}{4} + x\right) - \cos\left(\dfrac{3\pi}{4} - x\right)$

$$= -2\sin\frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2}\sin\frac{\frac{3\pi}{4} + x - \frac{3\pi}{4} + x}{2}$$

$= -2\sin\dfrac{3\pi}{4}\sin x = -2\sin\left(\pi - \dfrac{\pi}{4}\right)\sin x$

$= -2\sin\dfrac{\pi}{4}\sin x = -2 \times \dfrac{1}{\sqrt{2}}\sin x = -\sqrt{2}\sin x$

61. **(d)** $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \dfrac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x}$

$$= \frac{2\cos\frac{3x + x}{2} \cdot \sin\frac{3x - x}{2}}{\cos 2x} = \frac{2\cos 2x \cdot \sin x}{\cos 2x}$$

$= 2\sin x$

62. **(a)** We have,

$\sin x = -\dfrac{\sqrt{3}}{2} = -\sin\dfrac{\pi}{3} = \sin\left(\pi + \dfrac{\pi}{3}\right) = \sin\dfrac{4\pi}{3}$

Hence, $\sin x = \sin\dfrac{4\pi}{3}$, which gives

$x = n\pi + (-1)^n\dfrac{4\pi}{3}$, where $n \in Z$.

Note: $\dfrac{4\pi}{3}$ is one such value of x for which $\sin x = -\dfrac{\sqrt{3}}{2}$. One may take any other value of x for which $\sin x = -\dfrac{\sqrt{3}}{2}$. The solutions obtained will be the same although these may apparently look different.

63. **(b)** Given that, $x = \sec\theta + \tan\theta$

$\Rightarrow \quad x + \dfrac{1}{x} = \sec\theta + \tan\theta + \dfrac{1}{\sec\theta + \tan\theta}$

$= \sec\theta + \tan\theta + \sec\theta - \tan\theta = 2\sec\theta$

Aliter: $x = \dfrac{1 + \sin\theta}{\cos\theta}$

$\therefore \quad x + \dfrac{1}{x} = \dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta}$

$\Rightarrow \quad \dfrac{2(1 + \sin\theta)}{\cos\theta\,(1 + \sin\theta)} = 2\sec\theta.$

64. **(b)**
$$\frac{\tan 70^\circ - \tan 20^\circ}{\tan 50^\circ} = \frac{\dfrac{\sin 70^\circ}{\cos 70^\circ} - \dfrac{\sin 20^\circ}{\cos 20^\circ}}{\dfrac{\sin 50^\circ}{\cos 50^\circ}}$$

$$= \frac{\dfrac{\sin 70^\circ\cos 20^\circ - \cos 70^\circ\sin 20^\circ}{\cos 70^\circ\cos 20^\circ}}{\dfrac{\sin 50^\circ}{\cos 50^\circ}}$$

$$= \frac{2}{2} \times \frac{\sin(70^\circ - 20^\circ)\cos 50^\circ}{\cos 70^\circ\cos 20^\circ\sin 50^\circ}$$

$$= \frac{2\sin 50^\circ\cos 50^\circ}{2\cos 70^\circ\cos 20^\circ\sin 50^\circ}$$

$$= \frac{2\cos 50^\circ}{\cos 90^\circ + \cos 50^\circ} = \frac{2\cos 50^\circ}{0 + \cos 50^\circ} = 2.$$

65. **(d)**
$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = \frac{\left[\cos 10^\circ - \sqrt{3}\sin 10^\circ\right]}{\sin 10^\circ\cos 10^\circ}$$

$$= \frac{2\left[\dfrac{1}{2}\cos 10^\circ - \dfrac{\sqrt{3}}{2}\sin 10^\circ\right]}{\sin 10^\circ\cos 10^\circ}$$

$$= \frac{2\left[\sin 30^\circ\cos 10^\circ - \cos 30^\circ\sin 10^\circ\right]}{\sin 10^\circ\cos 10^\circ}$$

$$= \frac{2[\sin(30° - 10°)]}{\sin 10° \cos 10°}$$

$$= \frac{2 \cdot 2\sin(30° - 10°)}{2\sin 10° \cos 10°} = \frac{4\sin 20°}{\sin 20°} = 4.$$

66. **(a)** $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\frac{5\pi}{12}$

$$= 1 - \sin^2\left(\frac{\pi}{12}\right) + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\left(\frac{5\pi}{12}\right)$$

$$= 1 + \frac{1}{2} + \left(\cos^2\frac{5\pi}{12} - \sin^2\frac{\pi}{12}\right)$$

$$= \frac{3}{2} + \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

$$= \frac{3}{2} + \cos\frac{\pi}{2}\cos\frac{\pi}{3} = \frac{3}{2} + 0 \cdot \frac{1}{2} = \frac{3}{2}.$$

67. **(c)** $1 + \cos 2x + \cos 4x + \cos 6x$
$= (1 + \cos 6x) + (\cos 2x + \cos 4x)$
$= 2\cos^2 3x + 2\cos 3x \cos x$
$= 2\cos 3x(\cos 3x + \cos x)$
$= 4\cos x \cos 2x \cos 3x.$

68. **(a)** $\text{cosec } A - 2\cot 2A \cos A = \frac{1}{\sin A} - \frac{2\cos A\cos 2A}{\sin 2A}$

$$= \frac{1}{\sin A} - \frac{2\cos A\cos 2A}{2\sin A\cos A} = \frac{1 - \cos 2A}{\sin A} = \frac{2\sin^2 A}{\sin A}$$

$= 2\sin A.$

69. **(d)** $\sin x + \cos x = \frac{1}{5}$

$$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{25}$$

$$\sin 2x = \frac{-24}{25} \Rightarrow \cos 2x = \frac{-7}{25} \Rightarrow \tan 2x = \frac{24}{7}.$$

70. **(b)** $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$

$$\Rightarrow \sqrt{3}(\tan 2\theta + \tan 3\theta) = 1 - \tan 2\theta\tan 3\theta$$

$$\Rightarrow \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta\tan 3\theta} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan 5\theta = \tan\frac{\pi}{6}$$

$$\Rightarrow 5\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \left(n + \frac{1}{6}\right)\frac{\pi}{5}.$$

71. **(d)** $\tan\theta - \sqrt{2}\sec\theta = \sqrt{3}$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} - \frac{\sqrt{2}}{\cos\theta} = \sqrt{3}$$

$$\Rightarrow \sin\theta - \sqrt{2} = \sqrt{3}\cos\theta$$

$$\Rightarrow \sin\theta - \sqrt{3}\cos\theta = \sqrt{2}$$

$$\Rightarrow \sin\left(\theta - \frac{\pi}{3}\right) = \sin\frac{\pi}{4}$$

$$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{3}.$$

72. **(b)** The given equation can be written as

$$\frac{\sin^2\theta}{\cos\theta} + \sqrt{3}\tan\theta = 0$$

$$\Rightarrow \tan\theta\sin\theta + \sqrt{3}\tan\theta = 0$$

$$\tan\theta(\sin\theta + \sqrt{3}) = 0 \text{ as } \sin\theta \neq -\sqrt{3}$$

Hence, $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in Z.$

73. **(a)** $\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$

Therefore, $\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$

$$= \frac{\cot 54°}{\tan(90° - 54°)} + \frac{\tan 20°}{\cot(90° - 20°)}$$

$$= \frac{\cot 54°}{\cot 54°} + \frac{\tan 20°}{\tan 20°} = 1 + 1 = 2.$$

STATEMENT TYPE QUESTIONS

74. **(b)** We have

$$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$$

$\Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] + 3 = 0$

$\Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)]$
$+ \sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + \sin^2\gamma + \cos^2\alpha = 0$

$\Rightarrow [\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma$
$+ 2\sin\gamma\sin\alpha] + [\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta$
$+ 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha] = 0$

$\Rightarrow [\sin\alpha + \sin\beta + \sin\gamma]^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$

$\Rightarrow \sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$

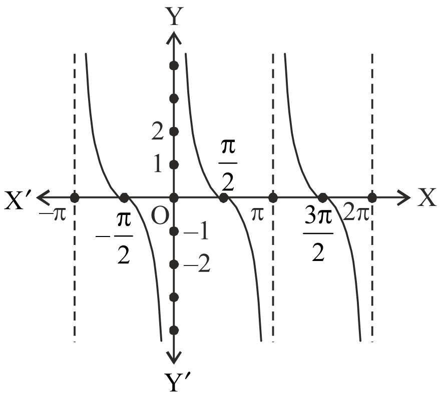
$\therefore$ I and II both are true.

75. **(a)** The signs of trigonometric functions in different quadrants are shown below

	I	II	III	IV
$\sin x$	+	+	–	–
$\cos x$	+	–	–	+
$\tan x$	+	–	+	–
$\text{cosec } x$	+	+	–	–
$\sec x$	+	–	–	+
$\cot x$	+	–	+	–

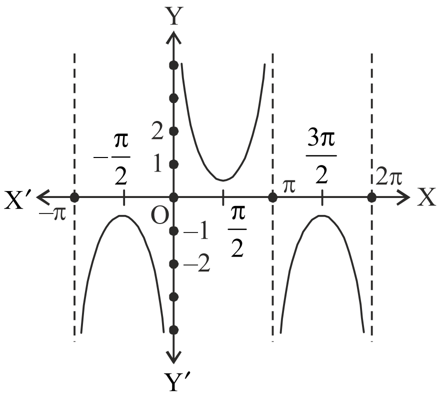
According to the above table, option (a) is correct.

76. **(d)** Using behaviour of trigonometric functions we can draw the graphs of $y = \cot x$, $y = \text{cosec } x$ and $y = \sec x$ as shown below.



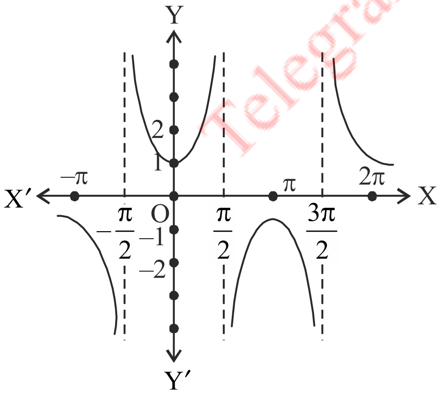
y = cot x

So, we see that the values of cot x repeat after an interval of π.



y = cosec x

Also, we can see that the values of cosec x repeat after an interval of 2π by using above graph of y = cosec x. Similarly, we can say that the values of sec x repeat after an interval of 2π by using the graph of y = sec x as shown below.



y = sec x

Hence, it is concluded that all the given statements are true.

77. **(a)** Only option (a) is incorrect.

78. **(b)** As a part of identities from above, we can also show that

I. $2\cos x\cos y = \cos(x+y) + \cos(x-y)$

II. $-2\sin x\sin y = \cos(x+y) - \cos(x-y)$

III. $2\sin x\cos y = \sin(x+y) + \sin(x-y)$

IV. $2\cos x\sin y = \sin(x+y) - \sin(x-y)$

Hence, option (b) is correct.

79. **(d)** $\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x\cos x + \cos x = 0$ $\quad [\because \sin 2x = 2\sin x\cos x]$

$\Rightarrow \cos x(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0$ or $\sin x = -\frac{1}{2}$

When $\cos x = 0$,

Then, $x = (2n+1)\frac{\pi}{2}$

When $\sin x = -\frac{1}{2}$,

Then, $\sin x = -\sin\frac{\pi}{6}$

$\sin x = \sin\left(\pi + \frac{\pi}{6}\right)$ $\quad [\because \sin(\pi+\theta) = -\sin\theta]$

$\sin x = \sin\frac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n\frac{7\pi}{6}$. $\quad [n \in Z]$

MATCHING TYPE QUESTIONS

80. **(b)** We know that,

Radian measure $= \frac{\pi}{180} \times$ Degree measure

A. Radian measure of $25° = \frac{\pi}{180} \times 25° = \frac{5\pi}{36}$.

B. We know that, $30' = \left(\frac{1}{2}\right)^o$ $\quad [\because 60' = 1°]$

$\therefore \; -47°\,30' = -\left(47\frac{1}{2}\right)^o = \left(\frac{-95}{2}\right)^o$

$\therefore$ Radian measure of $(-47°\,30') = \frac{\pi}{180} \times \left(\frac{-95}{2}\right)$

$= \frac{-19\pi}{72}$.

C. Radian measure of $240° = \frac{\pi}{180} \times 240 = \frac{4\pi}{3}$.

D. Radian measure of $520° = \frac{\pi}{180} \times 520 = \frac{26\pi}{9}$

81. **(c)** We know that

Degree measure $= \frac{180}{\pi} \times$ Radian measure

A. Degree measure of $\frac{11}{16} = \left(\frac{180}{\pi} \times \frac{11}{16}\right)^o$

$= \left(\frac{180}{22} \times \frac{11}{16} \times 7\right)^o$ $\quad \left[\because \pi = \frac{22}{7}\right]$

$= \left(\frac{90 \times 7}{16}\right)^o = \left(\frac{315}{8}\right)^o$

$= \left(39\frac{3}{8}\right)^o = 39°\left(\frac{3}{8}\times 60\right)'$ $[\because 1° = 60']$

$= 39°\left(22\frac{1}{2}\right)' = 39°\,22'\left(\frac{1}{2}\times 60\right)''$ $[\because 1' = 60'']$

$= 39°\,22'\,30''$.

B. Degree measure of $-4 = \left(\frac{180}{\pi}\times -4\right)^o$

$= \left(\frac{180}{22}\times -4\times 7\right)^o$ $\left[\because \pi = \frac{22}{7}\right]$

$= \left(\frac{90\times(-28)}{11}\right)^o = -\left(\frac{2520}{11}\right)^o = -\left(229\frac{1}{11}\right)^o$

$= -229°\left(\frac{1}{11}\times 60'\right)$ $[\because 1° = 60']$

$= -229°\left(5\frac{5}{11}\right)'$

$= -229°\,5'\left(\frac{5}{11}\times 60\right)''$ $[\because 1' = 60'']$

$\approx -229°\,5'\,27.3'' \approx -229°\,5'\,27''$ (approx.)

C. Degree measure of $\frac{5\pi}{3} = \left(\frac{180}{\pi}\times\frac{5\pi}{3}\right)^o = 300°$.

D. Degree measure of $\frac{7\pi}{6} = \left(\frac{180}{\pi}\times\frac{7\pi}{6}\right)^o = 210°$

82. (a) A. $\sin\frac{25\pi}{3} = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$[\because \sin(2n\pi + \theta) = \sin\theta]$

B. $\cos\frac{41\pi}{4} = \cos\left(10\pi + \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$[\because \cos(2n\pi + \theta) = \cos\theta]$

C. $\tan\left(\frac{-16\pi}{3}\right) = -\tan\frac{16\pi}{3}$ $[\because \tan(-\theta) = -\tan\theta]$

$= -\tan\left(5\pi + \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$

$[\because \tan(n\pi + \theta) = \tan\theta]$

D. $\cot\frac{29\pi}{4} = \cot\left(7\pi + \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1$

$[\because \cot(n\pi + \theta) = \cot\theta]$

83. (a) By taking suitable values of x and y in the identities, we get the following results:

$\cos(\pi - x) = -\cos x$; $\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$; $\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$; $\sin(2\pi - x) = -\sin x$

84. (c) A. 1 radian $= \frac{180°}{\pi} = 57°\,16'$ (approx.)

B. $1° = \left(\frac{\pi}{180}\right)^c = 0.01746$ radian (approx.)

C. $3°\,45' = \left(3\frac{45}{60}\right)^o = \left(3\frac{3}{4}\right)^o = \left(\frac{15}{4}\right)^o$

Also, $180° = \pi$ radian

$\Rightarrow 1° = \frac{\pi}{180}$ radian

$\Rightarrow \left(\frac{15}{4}\right)^o = \frac{\pi}{180}\times\frac{15}{4} = \frac{\pi}{48}$ radian

D. $50°\,37'\,30'' = 50° + \left(37\frac{30}{60}\right)'$

$= 50° + \left(\frac{75}{2}\right)' = 50° + \left(\frac{75}{2\times 60}\right)^o$

$= \left(\frac{405}{8}\right)^o = \left(\frac{\pi}{180}\times\frac{405}{8}\right)^c = \frac{9\pi}{32}$ radian

85. (c) π radians $= 180°$

$\Rightarrow 1° = \frac{\pi}{180}$ radians

(A) $25° = 25\times\frac{\pi}{180} = \frac{5\pi}{36}$

(B) $60' = 1°$ $\therefore 30' = \frac{30°}{60} = \frac{1}{2}°$

$\therefore 47°30' = \left(47 + \frac{1}{2}\right)^o = \left(\frac{95}{2}\right)^o$

$\therefore 180° = \pi$ radian

$-\frac{95°}{2} = \frac{-\pi}{180}\times\frac{95}{2}$ radians $= \frac{-19\pi}{72}$ radians

(C) $240° = 240\times\frac{\pi}{180} = \frac{4\pi}{3}$ radians.

(D) $180° = \pi$ radians

$520° = \frac{\pi}{180}\times 520$ radians $= \frac{26\pi}{9}$ radians

86. (a) Since x lies in the 3rd quadrant

$\cos x = -\frac{1}{2}$

$\therefore \sin x = -\sqrt{1-\cos^2 x}$ ($\because x$ lies in III rd quadrant)

$= -\sqrt{1-\frac{1}{4}} = -\sqrt{3}/2$

$\tan x = \sqrt{3}$, $\cot x = \frac{1}{\sqrt{3}}$

$\sec x = \left(\frac{1}{\cos x}\right) = -2$, $\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}}$

87. (b) A. $\cos 4x = \cos 2x$

$\Rightarrow 4x = 2n\pi \pm 2x$

Taking + ve sign, we get

$4x = 2n\pi + 2x$

$\Rightarrow 4x - 2x = 2n\pi$

$\Rightarrow \quad x = n\pi, n \in Z$

Taking – ve sign

$4x = 2n\pi - 2x$

$\Rightarrow \quad 4x + 2x = 2n\pi$

$\Rightarrow \quad 6x = 2n\pi$

$\Rightarrow \quad x = \frac{n\pi}{3}, n \in Z$

$\therefore$ General solution is $x = \frac{n\pi}{3}$ or $x = n\pi, n \in Z$

B. $\cos 3x + \cos x - \cos 2x = 0$

or $2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2} - \cos 2x = 0$

or $2\cos 2x\cos x - \cos 2x = 0$

or $\cos 2x\,(2\cos x - 1) = 0$

If $\cos 2x = 0,\ 2x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{4}$

If $2\cos x - 1 = 0$, $\cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3}$

$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{3}$

C. $\sin 2x + \cos x = 0$

$\Rightarrow \quad 2\cos x\cos x\cos x = 0$

$\Rightarrow \quad \cos x\,(2\sin x + 1) = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad 2\sin x + 1 = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = -\frac{1}{2}$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = \sin\left(\pi + \frac{\pi}{6}\right)$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = \sin\frac{7\pi}{6}$

$\Rightarrow \quad x = (2n+1)\frac{\pi}{2}$

or $\quad x = n\pi + (-1)^n\frac{7\pi}{6}, n \in Z$

Hence, general solution is

$x = (2n+1)\frac{\pi}{2}$ or $x = n\pi + (-1)^n\frac{7\pi}{6}$

where $n \in Z$

D. $\sec^2 2x = 1 - \tan 2x$

$\Rightarrow \quad 1 + \tan^2 2x = 1 - \tan 2x = 0$

$\Rightarrow \quad \tan^2 2x + \tan 2x = 0$

$\Rightarrow \quad \tan 2x(\tan 2x + 1) = 0$

If $\tan 2x = 0 \Rightarrow 2x = n\pi$ or $x = \frac{n\pi}{2}$

If $\tan 2x + 1 = 0 \Rightarrow \tan 2x = -1$

$= \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow \quad 2x = n\pi + \frac{3\pi}{4}$ or $x = \frac{n\pi}{2} + \frac{3\pi}{8}$

E. We have, $(\sin 5x + \sin x) + \sin 3x = 0$

$\Rightarrow \quad 2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} + \sin 3x = 0$

or $\quad 2\sin 3x\cos 2x + \sin 3x = 0$

or $\quad \sin 3x\,(2\cos 2x + 1) = 0$

If $\sin 3x = 0 \Rightarrow 3x = n\pi$ or $x = \frac{n\pi}{3}$

If $2\cos 2x + 1 = 0$, $\cos 2x = \frac{-1}{2}$

$= \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\therefore\ 2x = 2n\pi \pm \frac{2\pi}{3}$ or $x = n\pi \pm \frac{\pi}{3}$

88. (d) A. LHS $= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$= \frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$

$= \frac{-\sin 7x.\sin 2x}{\cos 10x\,.\,\sin 7x} = -\frac{\sin 2x}{\cos 10x} = \text{RHS}$

B. L.H.S. $= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$= \frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$

$= \frac{\sin 4x}{\cos 4x} = \tan 4x = \text{R.H.S.}$

C. L.H.S. $= \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \frac{\sin 3x + \sin x}{\cos 3x + \cos x}$

$= \frac{2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}}{2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}} = \frac{\sin 2x\cos x}{\cos 2x\cos x}$

$= \frac{\sin 2x}{\cos 2x} = \tan 2x$

D. L.H.S. $= \frac{\sin x - \sin y}{\cos x + \cos y} = \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$

$= \frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}} = \tan\frac{x-y}{2} = \text{R.H.S.}$

E. L.H.S. $= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{-(\sin 3x - \sin x)}{-(\cos^2 x - \sin^2 x)}$

$= \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x} = \frac{2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}}{\cos 2x}$

$= \frac{2\cos 2x \times \sin x}{\cos 2x}$ [$\because \cos 2x = \cos^2 x - \sin^2 x$]

$= 2\sin x$

89. (a) Since x lies in the second quadrant

$\sin x = 3/5$ given

$\cos x = -\sqrt{1-\sin^2 x}$ ($\because x$ lies in II quadrant)

$= -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}$

$\sec x = -\frac{5}{4}$, $\tan x = -\frac{3}{4}$

$\text{cosec } x = \frac{5}{3}$, $\cot x = -\frac{4}{3}$

INTEGER TYPE QUESTIONS

90. (c) As, we know that

$\text{cosec}(-\theta) = -\text{cosec } \theta$

$\therefore$ $\text{cosec } (-1410°) = -\text{cosec } (360 \times 4 - 30)°$

$= -(-\text{cosec } 30°)$

$= \text{cosec } 30°$ [$\because \text{cosec } (2n\pi - \theta) = -\text{cosec } \theta$]

$= 2.$

91. (b) Given expression

$= \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= \left(\cos\frac{10\pi}{13} + \cos\frac{3\pi}{13}\right) + \left(\cos\frac{8\pi}{13} + \cos\frac{5\pi}{13}\right)$

$= 2\cos\left(\frac{13\pi}{2\times 13}\right)\cdot\cos\left(\frac{7\pi}{2\times 13}\right)$

$+ 2\cos\left(\frac{13\pi}{2\times 13}\right)\cos\left(\frac{3\pi}{2\times 13}\right)$

$= 2\cos\frac{\pi}{2}\left(\cos\frac{7\pi}{26} + \cos\frac{3\pi}{26}\right)$ $\left[\because \cos\frac{\pi}{2} = 0\right]$

$= 0.$

92. (a) $\sin\theta + \cos\theta = 1$

Squaring on both sides, we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\therefore$ $\sin\theta\cos\theta = 0.$

93. (a) $\cos A = \frac{3}{5}$, $\cos B = \frac{4}{5}$

$\sin A = -\sqrt{1-\frac{9}{25}} = \frac{-4}{5}$

$\sin B = -\sqrt{1-\frac{16}{25}} = \frac{-3}{5}$

($\because \angle A$ and $\angle B$ in the 4^{th} quad.)

$\therefore 2\sin A + 4\sin B = 2\left(\frac{-4}{5}\right) + 4\left(\frac{-3}{5}\right) = -4 = -a$

94. (a) $\sin 765° = \sin(360\times 2 + 45)° = \sin 45° = \frac{1}{\sqrt{2}}$

95. (b) $\text{cosec } (-1410°) = -\text{cosec } (360 \times 4 - 30)°$

$= -(-\text{cosec } 30°) = \text{cosec } 30° = 2$

96. (c) $\tan\frac{19\pi}{3} = \tan\left(6\pi + \frac{\pi}{3}\right)$

$= \tan\left[2\pi\times 3 + \frac{\pi}{3}\right] = \tan\frac{\pi}{3} = \sqrt{3}$

97. (b) $\sin\left(\frac{-11\pi}{3}\right) = -\sin\left(\frac{11\pi}{3}\right) = -\sin\left(4\pi - \frac{\pi}{3}\right)$

$= -\sin\left(2\pi\times 2 - \frac{\pi}{3}\right) = -\left(-\sin\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

98. (c) $\left(1+\cos\frac{\pi}{6}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)$

$\left(1+\cos\frac{7\pi}{6}\right) = \left(1+\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{\sqrt{3}}{2}\right)$

$= \left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right) = \frac{1}{4}\times\frac{3}{4} = \frac{3}{16}$

99. (c) $\tan\theta = \frac{1}{\sqrt{7}} \Rightarrow \cot\theta = \sqrt{7}$

Given expression $= \frac{1+\cot^2\theta - 1 - \tan^2\theta}{1+\cot^2\theta + 1 + \tan^2\theta}$

$= \frac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta + \tan^2\theta} = \frac{(\sqrt{7})^2 - \left(\frac{1}{\sqrt{7}}\right)^2}{2+(\sqrt{7})^2 + \left(\frac{1}{\sqrt{7}}\right)^2}$

$= \frac{48}{64} = \frac{3}{4} = \frac{m}{m+1} \Rightarrow m = 3$

100. (b) $\cos^2 x = 1 - \sin^2 x = 1 - \frac{24}{25} = \frac{1}{25}$

$\Rightarrow \cos x = \frac{-1}{5}$

($\because \sin x$ and $\cos x$ are negative in III quad)

$\therefore \cot x = \frac{\cos x}{\sin x} = \frac{1}{2\sqrt{6}}$

101. (d) $\sin^2\theta = 1 - \cos^2\theta = \left(1-\frac{9}{25}\right) = \frac{16}{25}$

$\Rightarrow \sin\theta = \frac{-4}{5}$

$\therefore \tan\theta = \left(\frac{-4}{5}\times\frac{5}{-3}\right) = \frac{4}{3}$

$\cot\theta = \frac{3}{4}$

$\Rightarrow \text{cosec } \theta = \frac{-5}{4}$ and $\sec\theta = \frac{-5}{3}$

$\therefore \left(\frac{\text{cosec }\theta + \cot\theta}{\sec\theta - \tan\theta}\right) = \frac{-2}{4}\times\frac{3}{-9} = \frac{1}{6}$

102. (b) $3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$

$= 3\times\frac{1}{2}\times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right)\times 1$

$= 3 - 4\sin\frac{\pi}{6} = 3 - 4\times\frac{1}{2} = 1$

103. (a) $2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\times\left(\frac{1}{2}\right)^2 + \text{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$= \frac{2}{4} + \text{cosec}^2\frac{\pi}{6}\cos^2\frac{\pi}{3}$

$= \frac{1}{2}+(2)^2\left(\frac{1}{2}\right)^2 = \frac{3}{2} = \frac{m}{m-1}\quad \therefore\ m=3$

104. (d) $\cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= (\sqrt{3})^2 + \text{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + \text{cosec}\frac{\pi}{6} + 1 = 3+2+1 = 6$

105. (b) LHS $= \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

Now, $\cos\left(\frac{3\pi}{2}+x\right)=\sin x, \cos(2\pi+x)=\cos x$ and

$\cot\left(\frac{3\pi}{2}-x\right)=\tan x, \cot(2\pi+x)=\cot x$

L.H.S. $=\sin x.\cos x[\tan x+\cot x]$

$=\sin x.\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$

$=\sin x.\cos x\left[\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}\right]$

$=(\sin x.\cos x)\frac{1}{\cos x\sin x}=1\ [\because \sin^2 x+\cos^2 x=1]$

106. (b) L.H.S. $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x$

We have $3x=x+2x$

$\cot 3x=\cot(x+2x)=\frac{\cot x\cot 2x-1}{\cot x+\cot 2x}$

By cross multiplication

$\cot 3x(\cot x+\cot 2x)=\cot x\cot 2x-1$

$\cot x\cot 3x+\cot 2x\cot 3x=\cos x\cot 2x-1$

$\therefore\ \cos x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

ASSERTION - REASON TYPE QUESTIONS

107. (d) If the radii of the two circles are r_1 and r_2 and l is the length of arc in either case, then

$l = r_1$ (circular measure of 30°) $= r_1\left(\frac{30\pi}{180}\right)$

and also $l = r_2$ (circular measure of 70°) $= r_2\left(\frac{70\pi}{180}\right)$.

So, we must have $\frac{r_1\pi}{6}=\frac{7r_2\pi}{18}\Rightarrow\frac{r_1}{r_2}=\frac{7}{3}$.

108. (d) $\because\ \tan\left(\frac{\pi}{2}\sin\theta\right)=\cot\left(\frac{\pi}{2}\cos\theta\right)$

$=\tan\left(\frac{\pi}{2}-\frac{\pi}{2}\cos\theta\right)$

$\therefore\ \frac{\pi}{2}\sin\theta=n\pi+\frac{\pi}{2}-\frac{\pi}{2}\cos\theta$

$\Rightarrow\ \sin\theta+\cos\theta=2n+1,\ n\in I$

$\because\ -\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}$

$\therefore\ n=0,-1$

Then, $\sin\theta+\cos\theta=1,-1$.

109. (a) Given equation is

$\tan\theta+\tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta+\frac{2\pi}{3}\right)=3$

$\Rightarrow\ \tan\theta+\frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}+\frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}=3$

$\Rightarrow\ \tan\theta+\frac{\left(\tan\theta+\sqrt{3}\right)\left(1+\sqrt{3}\tan\theta\right)+\left(\tan\theta-\sqrt{3}\right)\times\left(1-\sqrt{3}\tan\theta\right)}{\left(1-\sqrt{3}\tan\theta\right)\left(1+\sqrt{3}\tan\theta\right)}=3$

$\Rightarrow\ \tan\theta+\frac{8\tan\theta}{1-3\tan^2\theta}=3$

$\Rightarrow\ \frac{\tan\theta-3\tan^3\theta+8\tan\theta}{1-3\tan^2\theta}=3$

$\Rightarrow\ \frac{3\left(3\tan\theta-\tan^3\theta\right)}{1-3\tan^2\theta}=3$

$\Rightarrow\ 3\tan 3\theta=3\Rightarrow\tan 3\theta=1$

$\Rightarrow\ \tan 3\theta=\tan\frac{\pi}{4}\Rightarrow 3\theta=n\pi+\frac{\pi}{4},\ n\in I$

$\Rightarrow\ \theta=\frac{n\pi}{3}+\frac{\pi}{12},\ n\in I.$

110. (d) Reason is true.

$\therefore$ Degree measure of (-2) radian $=\frac{180}{\pi}\times -2$

$=\frac{180}{22}\times -2\times 7\quad\left[\because\ \pi=\frac{22}{7}\right]$

$=\left(-\frac{1260}{11}\right)^{o}=-\left(114\frac{6}{11}\right)^{o}=-114^{\circ}\left(\frac{6}{11}\times 60\right)'$

$=-114^{\circ}32'\left(\frac{8}{11}\times 60\right)''=-114^{\circ}32'43.6''$

$=-114^{\circ}32'44''$ (approx.)

So, Assertion is false.

111. (a) I. $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}$

$\left[\begin{array}{r}\because\ \cos(\pi+\theta)=-\cos\theta\\ \cos(-\theta)=\cos\theta\\ \sin(\pi-\theta)=\sin\theta\\ \sin(-\theta)=-\sin\theta\end{array}\right]$

$= \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$

So, both the Assertion and Reason are true and Reason is the correct explanation of Assertion.

112. (a) We have,

$\tan 2x = -\cot\left(x + \dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{2} + x + \dfrac{\pi}{3}\right)$

$\Rightarrow \quad \tan 2x = \tan\left(x + \dfrac{5\pi}{6}\right)$

Therefore, $2x = n\pi + \left(x + \dfrac{5\pi}{6}\right)$, where $n \in Z$

$(\because \tan x = \tan y \Rightarrow x = n\pi + y,\ n \in Z)$

$\Rightarrow \quad x = n\pi + \dfrac{5\pi}{6}$, where $n \in Z$.

113. (b) Both are correct statements. Reason is not the correct explanation for the Assertion.

114. (b) Both Assertion and Reason is correct but Reason is not correct explanation.

115. (b) Both Assertion and Reason is correct. Reason is not the correct explanation for Assertion.

Reason : $40°20' = 40\dfrac{1}{3}$ degree

$= \dfrac{\pi}{180} \times \dfrac{|2|}{3}$ radian $= \dfrac{|2|\pi}{540}$ radian.

116. (d) Assertion is incorrect. The second hand rotates through 360° in a minute.

117. (c) Assertion is correct and Reason is incorrect.

CRITICAL THINKING TYPE QUESTIONS

118. (b) $\tan 20° + 2\tan 50° - \tan 70°$

$= \dfrac{\sin 20°}{\cos 20°} - \dfrac{\sin 70°}{\cos 70°} + 2\tan 50°$

$= \dfrac{\sin 20° \cos 70° - \cos 20° \sin 70°}{\cos 20° \cos 70°} + 2\tan 50°$

$= \dfrac{\sin(20° - 70°)}{\frac{1}{2}[\cos(70° + 20°) + \cos(70° - 20°)]} + 2\tan 50°$

$= \dfrac{2\sin(-50°)}{\cos 90° + \cos 50°} + 2\tan 50°$

$= \dfrac{-2\sin 50°}{0 + \cos 50°} + 2\tan 50°$

$= -2\tan 50° + 2\tan 50° = 0.$

119. (d) $\sin(2\alpha) = \sin(\alpha + \beta + \alpha - \beta)$

$= \sin(\alpha + \beta)\cos(\alpha - \beta) + \cos(\alpha + \beta)\sin(\alpha - \beta)$

$= \dfrac{5}{13}.\dfrac{4}{5} + \dfrac{12}{13}.\dfrac{3}{5} = \dfrac{56}{65}$

120. (d) $\cos\theta = \dfrac{1}{\sqrt{2}} = \cos\left(\dfrac{\pi}{4}\right)$

$\theta = 2n\pi \pm \dfrac{\pi}{4}$; $n \in I$

Put $n = 1$, $\theta = \dfrac{9\pi}{4}, \dfrac{7\pi}{4}$

$\tan\theta = -1 = \tan\left(\dfrac{-\pi}{4}\right) \Rightarrow \theta = n\pi - \pi/4,\ n \in I$

Put $n = 1$, $\theta = \dfrac{3\pi}{4}$

Put $n = 2$, $\theta = \dfrac{7\pi}{4}$

The common value which satisfies both these equation is $\left(\dfrac{7\pi}{4}\right)$. Hence the general value is $2n\pi + \dfrac{7\pi}{4}$

121. (d) The given expression

$= \dfrac{\sqrt{3}}{\sin 20°} - \dfrac{1}{\cos 20°} = \dfrac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$

$= \dfrac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\sin 20° \cos 20°}$

$= \dfrac{2(\sin 60° \cos 20° - \cos 60° \sin 20°)}{\sin 20° \cos 20°}$

$= \dfrac{2\sin(60° - 20°)}{\sin 20° \cos 20°} = \dfrac{2\sin 40°}{\sin 20° \cos 20°}$

$= \dfrac{4\sin 40°}{2\sin 20° \cos 20°} = \dfrac{4\sin 40°}{\sin 40°} = 4$

122. (d) We have $\sin^2\theta - \sin\theta - 2 = 0$

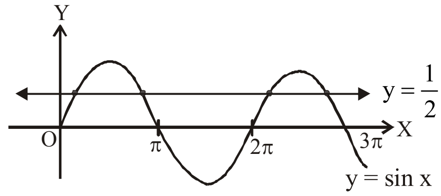
$\Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$

As $\sin\theta \neq 2$

$\therefore \ \sin\theta = -1 = \sin\dfrac{3\pi}{2}$

$\therefore \ \theta = \dfrac{3\pi}{2} = \dfrac{6\pi}{4} \in \left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right)$

123. (a)



$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow \ (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \ \sin x = \dfrac{1}{2}$ and $\sin x \neq -3$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

124. (b) $\text{LHS} = 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\left(\dfrac{9\pi}{13} + \dfrac{\pi}{13}\right) + \cos\left(\dfrac{9\pi}{13} - \dfrac{\pi}{13}\right) + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\dfrac{10\pi}{13} + \cos\dfrac{8\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\left(\pi - \dfrac{3\pi}{13}\right) + \cos\left(\pi - \dfrac{5\pi}{13}\right) + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= -\cos\dfrac{3\pi}{13} - \cos\dfrac{5\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= 0 = \text{RHS} \qquad [\because \cos(\pi - \theta) = -\cos\theta]$

125. (a) Given value

$= (\sin 47° + \sin 61°) - (\sin 11° + \sin 25°)$

$= 2 \sin 54° \cos 7° - 2 \sin 18° \cos 7°$

$= 2 \cos 7° (\sin 54° - \sin 18°)$

$= 2 \cos 7° \, 2 \cos 36° \sin 18°$

$= 2\cos 7° \dfrac{2 \sin 18° \cos 18°}{\cos 18°} \times \cos 36°$

$= \cos 7° \dfrac{2 \sin 36° \cos 36°}{\cos 18°}$

$= \cos 7° \dfrac{\sin 72°}{\cos 18°} = \cos 7°$ $\quad [\because \sin 72° = \cos 18°]$

126. (c) Since $\sin\theta + \cos\theta = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right]$

$= \sqrt{2}\left[\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right] = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

which lies between $-\sqrt{2}$ and $\sqrt{2}$

$[\because \sin\left(\theta + \frac{\pi}{4}\right)$ lies between -1 and $1]$

127. (d) $\tan 2\theta \tan\theta = 1 \Rightarrow \dfrac{2\tan\theta}{1-\tan^2\theta} \cdot \tan\theta = 1$

$\Rightarrow 2\tan^2\theta = 1 - \tan^2\theta \Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm\dfrac{1}{\sqrt{3}} = \tan\left(\pm\dfrac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi \pm \dfrac{\pi}{6}\ (n \in Z) = (6n \pm 1)\dfrac{\pi}{6}$

or $\tan 2\theta = \cot\theta = \tan\left(\dfrac{\pi}{2} - \theta\right)$

$\Rightarrow 2\theta = n\pi + \dfrac{\pi}{2} - \theta \Rightarrow 3\theta = n\pi + \dfrac{\pi}{2}$

$\Rightarrow \theta = \dfrac{n\pi}{3} + \dfrac{\pi}{6} = (2n+1)\dfrac{\pi}{6}$

128. (b) The given equation can be written as

$\dfrac{1}{2}(\sin 8x + \sin 2x) = \dfrac{1}{2}(\sin 8x + \sin 4x)$

or $\sin 2x - \sin 4x \Rightarrow -2\sin x \cos 3x = 0$

Hence $\sin x = 0$ or $\cos 3x = 0$.

That is, $x = n\pi\ (n \in I)$, or $3x = k\pi + \dfrac{\pi}{2}\ (k \in I)$.

Therefore, since $x \in [0, \pi]$, the given equation is satisfied if $x = 0, \pi, \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}$

129. (c) $\tan(\cot x) = \cot(\tan x) = \tan\left(\dfrac{\pi}{2} - \tan x\right)$

$\Rightarrow \cot x = n\pi + \dfrac{\pi}{2} - \tan x$

$[\because \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha]$

$\Rightarrow \cot x + \tan x = n\pi + \dfrac{\pi}{2}$

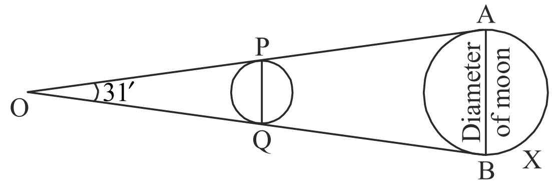
$\Rightarrow \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \dfrac{1}{\sin x \cos x} = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \dfrac{1}{\sin 2x} = \dfrac{(2n+1)\pi}{4}$

$\therefore \sin 2x = \dfrac{4}{(2n+1)\pi}$

130. (c) The coin will just hide the full moon if the lines joining the observer's eye O to the ends A and B of moon's diameter touch the coin at the ends P and Q of its diameter.



Here, $\angle POQ = \angle AOB = 31'$

$= \left(\dfrac{31}{60}\right)^{o} = \dfrac{31}{60} \times \dfrac{\pi}{180}$ radian.

Since, this angle is very small, the diameter PQ of the coin can be regarded as an arc of a circle whose centre is O and radius equal to the distance of the coin from O.

$\therefore \quad \dfrac{31\pi}{60 \times 180} = \dfrac{1}{r} \quad \left(\because \quad \theta = \dfrac{\ell}{r}\right)$

$\Rightarrow \quad r = \dfrac{60 \times 180}{31\pi}$

$\Rightarrow \quad r = \dfrac{60 \times 180 \times 7}{31 \times 22} = 110.9$ cm.

131. (a) Radius of the wheel = 35 cm

$\therefore$ Circumference of the wheel $= 2\pi \times 35$ cm

$= 2 \times \dfrac{22}{7} \times 35$ cm $= 220$ cm.

Hence, the linear distance travelled by a point of the rim in one revolution = 220 cm.

Number of revolutions made by the wheel in 3 minutes

$= 20 \times 3 \times 60 = 3600$

$\therefore$ The linear distance travelled by a point of the rim in 3 minutes $= 220 \times 3600 = 792000$ cm

$= \dfrac{792000}{100000}$ km $= 7.92$ km.

132. (b) In 60 minutes, the minute hand of a watch completes one revolution. Therefore, in 40 minutes, the minute hand turns through $\dfrac{2}{3}$ of a revolution. Therefore,

$\theta = \dfrac{2}{3} \times 360°$ or $\dfrac{4\pi}{3}$ radian. Hence, the required distance travelled is given by

$l = r\theta = 1.5 \times \dfrac{4\pi}{3} = 2\pi = 2 \times 3.14 = 6.28$ cm.

133. (c) Let r_1 and r_2 be the radii of the two circles. Given that

$\theta_1 = 65° = \frac{\pi}{180} \times 65 = \frac{13\pi}{36}$ radian

and $\theta_2 = 110° = \frac{\pi}{180} \times 110 = \frac{22\pi}{36}$ radian

Let l be the length of each of the arc.

Then, $l = r_1\,\theta_1 = r_2\,\theta_2$, which gives

$\frac{13\pi}{36} \times r_1 = \frac{22\pi}{36} \times r_2$, i.e. $\frac{r_1}{r_2} = \frac{22}{13}$

Hence, $r_1 : r_2 = 22 : 13$.

134. (d) $\tan A + \cot A = 4$ … (i)

Squaring (i) both sides, we get

$\tan^2 A + \cot^2 A + 2 = 16$

$\Rightarrow \tan^2 A + \cot^2 A = 14$ … (ii)

Squaring (ii) both sides, we get

$(\tan^2 A + \cot^2 A)^2 = 196$

$\Rightarrow \tan^4 A + \cot^4 A = 196 - 2$

$\Rightarrow \tan^4 A + \cot^4 A = 194.$

135. (b) Given, $\frac{\sin A}{\sin B} = m$

$\Rightarrow \sin A = m \sin B$ ... (i)

and $\frac{\cos A}{\cos B} = n$

$\Rightarrow \cos A = n \cos B$ ... (ii)

Squaring (i) and (ii) and then adding, we get

$1 = m^2 \sin^2 B + n^2 \cos^2 B$

$\Rightarrow \frac{1}{\cos^2 B} = m^2 \frac{\sin^2 B}{\cos^2 B} + n^2$ [Dividing by $\cos^2 B$]

$\Rightarrow \sec^2 B = m^2 \tan^2 B + n^2$

$\Rightarrow 1 + \tan^2 B = m^2 \tan^2 B + n^2$

$\Rightarrow 1 - n^2 = (m^2 - 1)\tan^2 B$

$\Rightarrow \tan^2 B = \frac{1-n^2}{m^2-1}$

$\Rightarrow \tan B = \pm\sqrt{\frac{1-n^2}{m^2-1}}.$

136. (d) $\tan(A - B) = 1 \Rightarrow A - B = 45°$ or $225°$

$\sec(A + B) = \frac{2}{\sqrt{3}} \Rightarrow A + B = 30°$ or $330°$

$A + B = 330° = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ … (i)

and $A - B = 225° = \frac{5\pi}{4}$ … (ii)

Solving (i) and (ii), we get

$2B = \frac{11\pi}{6} - \frac{5\pi}{4} \Rightarrow 2B = \frac{7\pi}{12} \Rightarrow B = \frac{7\pi}{24}.$

137. (a) $4 \sin\alpha \sin\left(\alpha + \frac{\pi}{3}\right)\sin\left(\alpha + \frac{2\pi}{3}\right)$

$= 2\sin\alpha\left\{2\sin\left(\alpha + \frac{2\pi}{3}\right)\sin\left(\alpha + \frac{\pi}{3}\right)\right\}$

$= 2\sin\alpha\,[2\sin(\alpha + 120°)\sin(\alpha + 60°)]$

$= 2\sin\alpha\,[\cos(\alpha + 120° - \alpha - 60°) - \cos(\alpha + 120° + \alpha + 60°)]$

$= 2\sin\alpha\,[\cos 60° - \cos(180° + 2\alpha)]$

$= 2\sin\alpha \cdot \frac{1}{2} - 2\sin\alpha(-\cos 2\alpha)$

$= \sin\alpha + 2\cos 2\alpha \sin\alpha$

$= \sin\alpha + \sin(2\alpha + \alpha) - \sin(2\alpha - \alpha)$

$= \sin\alpha + \sin 3\alpha - \sin\alpha = \sin 3\alpha.$

138. (c) $[\sin x + \cos x]^{1 + 2\sin x \cos x} = 2$

$\Rightarrow (\sin x + \cos x)^{(\sin x + \cos x)^2} = 2$

$\Rightarrow (\sin x + \cos x)^{(\sin x + \cos x)^2} = \left(\sqrt{2}\right)^{\left(\sqrt{2}\right)^2}$ … (i)

Comparing (i) both sides, we get

$\sin x + \cos x = \sqrt{2}$

$\Rightarrow \sin\left(x + \frac{\pi}{4}\right) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4}$

So, $x = \frac{\pi}{4}$, when $n = 0$.

139. (c) Given that

$\tan\theta + \sec\theta = p$ … (i)

and we know that

$\Rightarrow \sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)p$

(multiplying both the sides by $(\sec\theta - \tan\theta)$)

$\Rightarrow (\sec\theta - \tan\theta)\,p = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ … (ii)

On solving equations (i) and (ii), we get

$2\sec\theta = \frac{p^2+1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$

140. (c) We have,

$\sec\theta + \tan\theta = \sqrt{3}$ … (i)

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ … (ii)

[$\because \sec^2\theta - \tan^2\theta = 1$]

By solving (i) and (ii), we get

$\tan\theta = \frac{1}{2}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \tan\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi + \frac{\pi}{6}$

$\therefore$ Solutions for $0 \le \theta \le 2\pi$ are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$.

Hence, there are two solutions.

141. (c) Given equation is $\cos x - \sin x = \frac{1}{\sqrt{2}}$

Dividing equation by $\sqrt{2}$,

$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{2}$

$\Rightarrow \cos\left(\frac{\pi}{4} + x\right) = \cos\frac{\pi}{3}$

$\Rightarrow \frac{\pi}{4} + x = 2n\pi \pm \frac{\pi}{3}$

$x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi + \frac{\pi}{12}$

or $x = 2n\pi - \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi - \frac{7\pi}{12}$.

142. (a) $4\sin^2\theta + 2\left(\sqrt{3}+1\right)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4 - 4\cos^2\theta + 2\left(\sqrt{3}+1\right)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4\cos^2\theta - 2\left(\sqrt{3}+1\right)\cos\theta + \sqrt{3} = 0$

$\Rightarrow \cos\theta = \frac{2\left(\sqrt{3}+1\right) \pm \sqrt{4\left(\sqrt{3}+1\right)^2 - 16\sqrt{3}}}{8}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $\frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$ or $2n\pi \pm \frac{\pi}{3}$.

143. (a) $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow \sin x = \frac{-5 \pm \sqrt{25+24}}{4} = \frac{-5 \pm 7}{4} = -3, \frac{1}{2}$

But $\sin x \neq -3$

$\therefore \sin x = \frac{1}{2}$

$\therefore$ Number of solution in $[0, 3\pi]$ will be equal to 4.

144. (b) $\sin\theta + \cos\theta = 1$

Dividing by $\sqrt{1^2+1^2} = \sqrt{2}$,

$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$.

145. (a) $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow 2\sin 4\theta\cos 2\theta + \sin 4\theta = 0$

$\Rightarrow \sin 4\theta\,(2\cos 2\theta + 1) = 0$

$\Rightarrow 2\cos 2\theta = -1 \Rightarrow \cos 2\theta = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

and $\sin 4\theta = 0 \Rightarrow 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$

$\theta = \frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}$.

146. (c) $\sqrt{2}\sec\theta + \tan\theta = 1$

$\Rightarrow \frac{\sqrt{2}}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \sin\theta - \cos\theta = -\sqrt{2}$

Dividing by $\sqrt{2}$ on both sides, we get

$\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = -1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = 1$

$\Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \cos(0)$

$\Rightarrow \theta + \frac{\pi}{4} = 2n\pi \pm 0 \Rightarrow \theta = 2n\pi - \frac{\pi}{4}$.

147. (c) $12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12(\operatorname{cosec}^2\theta - 1) - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 31\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 16\operatorname{cosec}\theta - 15\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow (4\operatorname{cosec}\theta - 5)(3\operatorname{cosec}\theta - 4) = 0$

$\Rightarrow \operatorname{cosec}\theta = \frac{5}{4}, \frac{4}{3}$

$\therefore \sin\theta = \frac{4}{5}, \frac{3}{4}$.

148. (b) We have $\sec^2\theta = \frac{4}{3}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos^2\theta = \cos^2\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ .... $\left[\begin{array}{l}\text{If } \cos^2\theta = \cos^2\alpha \\ \Rightarrow \theta = n\pi \pm \alpha\end{array}\right]$

149. (a) We have $\tan 5\theta = \cot 2\theta$

$\Rightarrow \tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right) \ldots \left[\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right]$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta \Rightarrow 7\theta = n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}$.

150. (c) Since, none of the x, y and $(x+y)$ is multiple of π, we find that $\sin x$, $\sin y$ and $\sin(x+y)$ are non-zero. Now,

$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)} = \frac{\cos x\cos y - \sin x\sin y}{\sin x\cos y + \cos x\sin y}$

On dividing numerator and denominator by $\sin x \sin y$, we have

$\cot(x+y) = \frac{\cot x\cot y - 1}{\cot y + \cot x}$

On replacing y by (–y) in above identity, we get

$\cot(x-y) = \frac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$.

151. (a) Given, $3\tan(\theta - 15) = \tan(\theta + 15)$

$\frac{\tan A}{\tan B} = \frac{3}{1}$,

where $A = \theta + 15°$, $B = \theta - 15°$

On applying componendo and dividendo, we get

$\Rightarrow \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{3+1}{3-1} \Rightarrow \dfrac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}} = 2$

$\Rightarrow \dfrac{\sin(A+B)}{\sin(A-B)} = 2$

$\Rightarrow \sin 2\theta = 2 \sin 30°$

$\Rightarrow \sin 2\theta = 2 \cdot \dfrac{1}{2} = 1 = \sin \dfrac{\pi}{2}$

$\Rightarrow 2\theta = n\pi + (-1)^n \dfrac{\pi}{2}$

$\Rightarrow \theta = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{4}.$

152. (c) Let $\theta = \alpha + \beta$. Then, $\tan \alpha = K \tan \beta$

or $\dfrac{\tan \alpha}{\tan \beta} = \dfrac{K}{1}$

Applying componendo and dividendo, we have

$\dfrac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta} = \dfrac{K+1}{K-1}$

or $\dfrac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} = \dfrac{K+1}{K-1}$

i.e., $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \dfrac{K+1}{K-1}$

Given that, $\alpha - \beta = \phi$ and $\alpha + \beta = \theta$. Therefore,

$\dfrac{\sin \theta}{\sin \phi} = \dfrac{K+1}{K-1}$ or $\sin \theta = \dfrac{K+1}{K-1} \sin \phi.$

153. (a) We have, $m \sin \theta = n \sin(\theta + 2\alpha)$

$\Rightarrow \dfrac{\sin(\theta+2\alpha)}{\sin \theta} = \dfrac{m}{n}$

Using componendo and dividendo, we get

$\dfrac{\sin(\theta+2\alpha) + \sin \theta}{\sin(\theta+2\alpha) - \sin \theta} = \dfrac{m+n}{m-n}$

$\Rightarrow \dfrac{2 \sin\left(\frac{\theta+2\alpha+\theta}{2}\right) \cdot \cos\left(\frac{\theta+2\alpha-\theta}{2}\right)}{2 \cos\left(\frac{\theta+2\alpha+\theta}{2}\right) \cdot \sin\left(\frac{\theta+2\alpha-\theta}{2}\right)} = \dfrac{m+n}{m-n}$

$\Rightarrow \dfrac{2 \sin(\theta+\alpha) \cdot \cos \alpha}{2 \cos(\theta+\alpha) \cdot \sin \alpha} = \dfrac{m+n}{m-n}$

$\Rightarrow \tan(\theta+\alpha) \cdot \cot \alpha = \dfrac{m+n}{m-n}$

154. (c) $5 \tan \theta = 4 \Rightarrow \tan \theta = \dfrac{4}{5}$

$\therefore \sin \theta = \dfrac{4}{\sqrt{41}}$ and $\cos \theta = \dfrac{5}{\sqrt{41}}$

$\dfrac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta} = \dfrac{5 \times \frac{4}{\sqrt{41}} - 3 \times \frac{5}{\sqrt{41}}}{5 \times \frac{4}{\sqrt{41}} + 2 \times \frac{5}{\sqrt{41}}}$

$= \dfrac{20-15}{20+10} = \dfrac{5}{30} = \dfrac{1}{6}.$

155. (c) $\dfrac{1 + \sin A - \cos A}{1 + \sin A + \cos A}$

$= \dfrac{2\sin^2 \frac{A}{2} + 2 \sin \frac{A}{2} \cos \frac{A}{2}}{2\cos^2 \frac{A}{2} + 2 \sin \frac{A}{2} \cos \frac{A}{2}}$ $\left(\begin{array}{l} \text{as} \\ \sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} \\ \cos A = 2\cos^2 \frac{A}{2} - 1 \\ \cos A = 1 - 2\sin^2 \frac{A}{2} \end{array}\right)$

$= \dfrac{2 \sin \frac{A}{2}\left(\sin \frac{A}{2} + \cos \frac{A}{2}\right)}{2 \cos \frac{A}{2}\left(\cos \frac{A}{2} + \sin \frac{A}{2}\right)} = \tan \dfrac{A}{2}.$

Trick: Put $A = 60°$

Then, $\dfrac{1 + \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)}{1 + \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)} = \dfrac{1+\sqrt{3}}{3+\sqrt{3}} = \dfrac{1}{\sqrt{3}},$

which is given by option (c), i.e. $\tan \dfrac{60°}{2} = \dfrac{1}{\sqrt{3}}.$

156. (d) $\dfrac{1}{4}\left\{\sqrt{3} \cos 23° - \sin 23°\right\}$

$= \dfrac{1}{2}\{\cos 30° \cos 23° - \sin 30° \sin 23°\}$

$= \dfrac{1}{2}\cos\{30° + 23°\} = \dfrac{1}{2} \cos 53°.$

157. (c) Given equation $\cos x + \cos y + \cos \alpha = 0$ and $\sin x + \sin y + \sin \alpha = 0$.

The given equation may be written as

$\cos x + \cos y = -\cos \alpha$ and $\sin x + \sin y = -\sin \alpha$.

Therefore,

$2 \cos\left(\dfrac{x+y}{2}\right) \cos\left(\dfrac{x-y}{2}\right) = -\cos \alpha$ ... (i)

$2 \sin\left(\dfrac{x+y}{2}\right) \cos\left(\dfrac{x-y}{2}\right) = -\sin \alpha$ ... (ii)

Divide (i) by (ii), we get

$\dfrac{2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)}{2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)} = \dfrac{\cos \alpha}{\sin \alpha}$

$\Rightarrow \cot\left(\dfrac{x+y}{2}\right) = \cot \alpha.$

158. (a) $\sin 12° \sin 24° \sin 48° \sin 84°$

$= \dfrac{1}{4}(2 \sin 12° \sin 48°)(2 \sin 24° \sin 84°)$

$= \dfrac{1}{2}(\cos 36° - \cos 60°)(\cos 60° - \cos 108°)$

$= \dfrac{1}{4}\left(\cos 36° - \dfrac{1}{2}\right)\left(\dfrac{1}{2} + \sin 18°\right)$

$= \dfrac{1}{4}\left\{\dfrac{1}{4}\left(\sqrt{5}+1\right) - \dfrac{1}{2}\right\}\left\{\dfrac{1}{2} + \dfrac{1}{4}\left(\sqrt{5}-1\right)\right\} = \dfrac{1}{16}$

and $\cos 20° \cos 40° \cos 60° \cos 80°$

$= \frac{1}{2}[\cos(60° - 20°) \cos 20° \cos(60° + 20°)]$

$= \frac{1}{2}\left[\frac{1}{4}\cos 3(20°)\right] = \frac{1}{8}\cos 60° = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}.$

159. (b) We have, $\frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$

Then, $\frac{4 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{2 \cos^2 \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} = y$

$\Rightarrow \frac{2 \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}} \times \frac{\left(\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}\right)}{\left(\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}\right)} = y$

$\Rightarrow \frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = y.$

Trick: Put value of $\theta = 30°$ and check.

160. (b) Given, $\sin 2\theta + \sin 2\phi = \frac{1}{2}$ ... (i)

and $\cos 2\theta + \cos 2\phi = \frac{3}{2}$ ... (ii)

Square and adding,

$\therefore \quad (\sin^2 2\theta + \cos^2 2\theta) + (\sin^2 2\phi + \cos^2 2\phi)$

$+ 2[\sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi] = \frac{1}{4} + \frac{9}{4}$

$\Rightarrow \cos 2\theta \cos 2\phi + \sin 2\theta \sin 2\phi = \frac{1}{4}$

$\Rightarrow \cos(2\theta - 2\phi) = \frac{1}{4} \Rightarrow \cos 2(\theta - \phi) = \frac{1}{4}$

$\Rightarrow 2\cos^2 (\theta - \phi) - 1 = \frac{1}{4} \Rightarrow \cos^2 (\theta - \phi) = \frac{5}{8}$

CHAPTER

4

PRINCIPLE OF MATHEMATICAL INDUCTION

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Let P(n) be statement $2^n < n!$. Where n is a natural number, then P(n) is true for:
(a) all n (b) all $n > 2$
(c) all $n > 3$ (d) None of these

2. If $P(n) = 2 + 4 + 6 + ...+ 2n, n \in \mathbf{N}$, then $P(k) = k(k+1) + 2$ $\Rightarrow P(k+1) = (k+1)(k+2) + 2$ for all $k \in \mathbf{N}$. So we can conclude that $P(n) = n(n+1) + 2$ for
(a) all $n \in \mathbf{N}$ (b) $n > 1$
(c) $n > 2$ (d) nothing can be said

3. Let $T(k)$ be the statement $1 + 3 + 5 + ... + (2k - 1) = k^2 + 10$ Which of the following is correct?
(a) $T(1)$ is true
(b) $T(k)$ is true $\Rightarrow$ $T(k + 1)$ is true
(c) $T(n)$ is true for all $n \in \mathbf{N}$
(d) All above are correct

4. Let $S(K) = 1 + 3 + 5 ... + (2K - 1) = 3 + K^2$, then which of the following is true?
(a) Principle of mathematical induction can be used to prove the formula
(b) $S(K) \Rightarrow S(K+1)$
(c) $S(K) \not\Rightarrow S(K+1)$
(d) $S(1)$ is correct

5. Let P(n) : "$2^n < (1 \times 2 \times 3 \times ... \times n)$". Then the smallest positive integer for which P(n) is true is
(a) 1 (b) 2
(c) 3 (d) 4

6. A student was asked to prove a statement $P(n)$ by induction. He proved that $P(k+1)$ is true whenever $P(k)$ is true for all $k > 5 \in$ N and also that $P(5)$ is true. On the basis of this he could conclude that P(n) is true
(a) for all $n \in$ N
(b) for all $n > 5$
(c) for all $n \geq 5$
(d) for all $n < 5$

7. If $P(n) : 2 + 4 + 6 + ... + (2n), n \in N$, then $P(k) = k(k+1) + 2$ implies $P(k+1) = (k+1)(k+2) + 2$ is true for all $k \in N$. So statement $P(n) = n(n+1) + 2$ is true for:
(a) $n \geq 1$
(b) $n \geq 2$
(c) $n \geq 3$
(d) None of these

8. If $P(n)$: "$46^n + 16^n + k$ is divisible by 64 for $n \in$ N" is true, then the least negative integral value of k is.
(a) -1 (b) 1
(c) 2 (d) -2

9. Use principle of mathematical induction to find the value of k, where $(10^{2n-1} + 1)$ is divisible by k.
(a) 11 (b) 12
(c) 13 (d) 9

10. For all $n \geq 1$, $1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 =$
(a) $\frac{n(n+1)}{6}$
(b) $n(n + 1)(2n - 1)$
(c) $\frac{n(n-1)(2n+1)}{2}$
(d) $\frac{n(n+1)(2n+1)}{6}$

11. P(n) : $2.7^n + 3.5^n - 5$ is divisible by
(a) 24, $\forall$ n $\in$ N
(b) 21, $\forall$ n $\in$ N
(c) 35, $\forall$ n $\in$ N
(d) 50, $\forall$ n $\in$ N

12. For all $n \geq 1$, $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} =$
(a) n (b) $\frac{n}{n+1}$
(c) $\frac{(n+1)}{n}$ (d) $\frac{4n+3}{2n}$

13. For every positive integer n, $7^n - 3^n$ is divisible by
(a) 7 (b) 3
(c) 4 (d) 5

14. By mathematical induction,

$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+.....+\frac{1}{n(n+1)(n+2)}$ is equal to

(a) $\frac{n(n+1)}{4(n+2)(n+3)}$

(b) $\frac{n(n+3)}{4(n+1)(n+2)}$

(c) $\frac{n(n+2)}{4(n+1)(n+3)}$

(d) None of these

15. By using principle of mathematical induction for every natural number, $(ab)^n$ =

(a) $a^n b^n$ (b) $a^n b$
(c) ab^n (d) 1

16. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by

(a) 113 (b) 123
(c) 133 (d) None of these

17. For all $n \in N$, $41^n - 14^n$ is a multiple of

(a) 26 (b) 27
(c) 25 (d) None of these

18. The remainder when 5^{4n} is divided by 13, is

(a) 1 (b) 8
(c) 9 (d) 10

19. If m, n are any two odd positive integers with n < m, then the largest positive integer which divides all the numbers of the type $m^2 - n^2$ is

(a) 4 (b) 6
(c) 8 (d) 9

20. For natural number n, $2^n (n-1)! < n^n$, if

(a) n < 2 (b) n > 2 (c) n ≥ 2 (d) n > 3

21. For all $n \in N$, $3.5^{2n+1} + 2^{3n+1}$ is divisible by

(a) 19 (b) 17
(c) 23 (d) 25

22. Principle of mathematical induction is used

(a) to prove any statement
(b) to prove results which are true for all real numbers
(c) to prove that statements which are formulated in terms of n, where n is positive integer
(d) in deductive reasoning

23. For all $n \in N$, $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n$ is equal to

(a) $\frac{(2n+1)3^{n+1}+3}{4}$

(b) $\frac{(2n-1)3^{n+1}+3}{4}$

(c) $\frac{(2n+1)3^{n}+3}{4}$

(d) $\frac{(2n-1)3^{n+1}+1}{4}$

24. For all $n \in N$,

$1+\frac{1}{1+2}+\frac{1}{1+2+3}+.....+\frac{1}{1+2+3+.....+n}$

is equal to

(a) $\frac{3n}{n+1}$ (b) $\frac{n}{n+1}$

(c) $\frac{2n}{n-1}$ (d) $\frac{2n}{n+1}$

25. $10^n + 3(4^{n+2}) + 5$ is divisible by ($n \in N$)

(a) 7 (b) 5
(c) 9 (d) 17

26. The statement P(n)
"1 × 1! + 2 × 2! + 3 × 3! + ... + n × n! = (n + 1)! – 1" is

(a) True for all n > 1 (b) Not true for any n
(c) True for all $n \in N$ (d) None of these

27. If n is a natural number, then $\left(\frac{n+1}{2}\right)^n \geq n!$ is true when

(a) n > 1 (b) n ≥ 1
(c) n > 2 (d) n ≥ 2

28. For natural number n, $(n!)^2 > n^n$, if

(a) n > 3 (b) n > 4
(c) n ≥ 4 (d) n ≥ 3

STATEMENT TYPE QUESTIONS

Directions : Read the following statement and choose the correct option from the given below four options.

29. **Statement-I :** $1 + 2 + 3 + ... + n < \frac{1}{8}(2n+1)^2$, $n \in N$.

Statement-II : n(n + 1) (n + 5) is a multiple of 3, $n \in N$.

(a) Only Statement I is true
(b) Only Statement II is true
(c) Both Statements are true
(d) Both Statements are false

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

30. **Assertion :** For every natural number n ≥ 2,

$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+.....+\frac{1}{\sqrt{n}} > \sqrt{n}$

Reason : For every natural number n ≥ 2,

$\sqrt{n(n+1)} < n + 1$.

31. **Assertion :** $11^{m+2} + 12^{2m+1}$ is divisible by 133 for all $m \in N$.

Reason : $x^n - y^n$ is divisible by $x + y$, $\forall\ n \in N, x \neq y$.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

32. The greatest positive integer, which divides $n(n+1)(n+2)(n+3)$ for all $n \in \mathbf{N}$, is

(a) 2 (b) 6
(c) 24 (d) 120

33. Let P(n) : $n^2 + n + 1$ is an even integer. If $P(k)$ is assumed true then $P(k+1)$ is true. Therefore $P(n)$ is true.

(a) for $n > 1$ (b) for all $n \in N$
(c) for $n > 2$ (d) None of these

34. By the principle of induction $\forall\ n \in N$, 3^{2n} when divided by 8, leaves remainder

(a) 2 (b) 3
(c) 7 (d) 1

35. If n is a positive integer, then $5^{2n+2} - 24n - 25$ is divisible by

(a) 574 (b) 575
(c) 674 (d) 576

36. The greatest positive integer, which divides $(n + 1)(n + 2)(n + 3) \ldots\ldots (n + r)$ for all $n \in W$, is

(a) r (b) r!
(c) n + r (d) (r + 1)!

37. If $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$, then P(n) is true for

(a) $n \geq 1$ (b) $n > 0$
(c) $n < 0$ (d) $n \geq 2$

38. For all $n \in N$,

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\ldots\left(1+\frac{(2n+1)}{n^2}\right)$$

is equal to

(a) $\frac{(n+1)^2}{2}$ (b) $\frac{(n+1)^3}{3}$
(c) $(n+1)^2$ (d) None of these

39. For all $n \in N$, the sum of $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is

(a) a negative integer (b) a whole number
(c) a real number (d) a natural number

40. For given series:

$1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + \ldots\ldots,$

if S_n is the sum of n terms, then

(a) $S_n = \frac{n(n+1)^2}{2}$, if n is even

(b) $S_n = \frac{n^2(n+1)}{2}$, if n is odd

(c) Both (a) and (b) are true
(d) Both (a) and (b) are false

41. When 2^{301} is divided by 5, the least positive remainder is

(a) 4 (b) 8
(c) 2 (d) 6

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** Let $P(n) : 2^n < n!$

Then $P(1) : 2^1 < 1!$, which is true

Now $P(2) : 2^2 < 2!$, which is not true

Also $P(3) : 2^3 < 3!$, which is not true

$P(4) : 2^4 < 4!$, which is true

Let $P(k)$ is true if $k \geq 4$

That is $2^k < k!$, $k \geq 4$

$\Rightarrow 2.2^k < 2(k!) \Rightarrow 2^{k+1} < k(k!)$ $[\because k \geq 4 > 2]$

$\Rightarrow 2^{k+1} < (k+1)! \Rightarrow P(k+1)$ is true.

Hence, we conclude that $P(n)$ is not true for $n = 2, 3$ but holds true for $n \geq 4$.

2. **(d)** We note that $P(1) = 2$ and hence, $P(n) = n(n+1) + 2$ is not true for $n = 1$.

So the principle of mathematical induction is not applicable and nothing can be said about the validity of the statement $P(n) = n(n+1) + 2$.

3. **(b)** When $k = 1$, LHS = 1 but RHS = 1 + 10 = 11

$\therefore$ $T(1)$ is not true

Let $T(k)$ is true.

i.e., $1 + 3 + 5 + ... + (2k - 1) = k^2 + 10$

Now, $1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= k^2 + 10 + 2k + 1 = (k+1)^2 + 10$

$\therefore T(k+1)$ is true.

i.e., $T(k)$ is true $\Rightarrow T(k+1)$ is true.

But $T(n)$ is not true for all $n \in \mathbf{N}$, as $T(1)$ is not true.

4. **(b)** $S(K) = 1+3+5+...+(2K-1) = 3 + K^2$

$S(1) = 1 = 3 + 1$, which is not true

$\because$ $S(1)$ is not true.

$\therefore$ P.M.I cannot be applied

Let $S(K)$ is true, i.e. $1 + 3 + 5 + ... + (2K - 1) = 3 + K^2$

$\Rightarrow 1 + 3 + 5 + ... + (2K - 1) + 2K + 1$

$= 3 + K^2 + 2K + 1 = 3 + (K+1)^2$

$\therefore S(K) \Rightarrow S(K+1)$

5. **(d)** Since $P(1) : 2 < 1$ is false

$P(2) : 2^2 < 1 \times 2$ is false

$P(3) : 2^3 < 1 \times 2 \times 3$ is false

$P(4) : 2^4 < 1 \times 2 \times 3 \times 4$ is true

6. **(c)** Since $P(5)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

7. **(d)** $P(1) = 2$ and $k(k + 1) + 2 = 4$, So $P(1)$ is not true. Mathematical Induction is not applicable.

8. **(a)** For $n = 1$, $P(1)$: $65 + k$ is divisible by 64.

Thus k, should be –1

Since 65 – 1 = 64 is divisible by 64.

9. **(a)** Let P(n) be the statement given by

P(n) : $10^{2n-1} + 1$ is divisible by 11

For n = 1, P(1) : $10^{(2 \times 1) - 1} + 1 = 11$,

which is divisible by 11.

So, P(1) is true.

Let P(k) be true, i.e. $10^{2k-1} + 1$ is divisible by 11

$\Rightarrow$ $10^{2k-1} + 1 = 11\lambda$, for some $\lambda \in$ N ... (i)

We shall now show that P(k + 1) is true. For this, we have to show that $10^{2(k+1)-1} + 1$ is divisible by 11.

Now, $10^{2(k+1)-1} + 1 = 10^{2k-1}\ 10^2 + 1$

$= (11\lambda - 1)100 + 1$ [Using (i)]

$= 1100\lambda - 99 = 11(100\lambda - 9) = 11\mu$,

where $\mu = 100\lambda - 9 \in$ N

$\Rightarrow$ $10^{2(k+1)-1} + 1$ is divisible by 11

$\Rightarrow$ P(k + 1) is true.

Thus, P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(k) is true for all n $\in$ N, i.e. $10^{2n-1} + 1$ is divisible by 11 for all n $\in$ N.

10. **(d)** Let the given statement be P(n), i.e.

$$P(n) : 1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For n = 1,

$$P(1) : 1 = \frac{1(1+1)((2 \times 1) + 1)}{6} = \frac{1 \times 2 \times 3}{6} = 1,$$

which is true.

Assume that P(k) is true for some positive integer k,

$$\text{i.e. } 1^2 + 2^2 + 3^2 + 4^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6} \quad ... (i)$$

We shall now prove that P(k + 1) is also true, i.e. $1^2 + 2^2 + 3^2 + 4^2 + ... + k^2 + (k+1)^2$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Now, L.H.S. $= (1^2 + 2^2 + 3^2 + 4^2 + ... + k^2) + (k+1)^2$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \quad \text{[Using (i)]}$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6} = \text{R.H.S.}$$

Thus, P(k + 1) is true, whenever P(k) is true. Hence, from the principle of mathematical induction, the statement P(n) is true for all natural numbers n.

11. (a) $P(n) : 2.7^n + 3.5^n - 5$ is divisible by 24.

For n = 1,

P(1) : 2.7 + 3.5 – 5 = 24, which is divisible by 24.

Assume that P(k) is true,

i.e. $2.7^k + 3.5^k - 5 = 24q$, where $q \in N$ ... (i)

Now, we wish to prove that P(k + 1) is true whenever P(k) is true, i.e. $2.7^{k+1} + 3.5^{k+1} - 5$ is divisible by 24.

We have,

$2.7^{k+1} + 3.5^{k+1} - 5 = 2.7^k \cdot 7^1 + 3.5^k \cdot 5^1 - 5$

$= 7[2.7^k + 3.5^k - 5 - 3.5^k + 5] + 3.5^k \cdot 5 - 5$

$= 7[24q - 3.5^k + 5] + 15.5^k - 5$

$= (7 \times 24q) - 21.5^k + 35 + 15.5^k - 5$

$= (7 \times 24q) - 6.5^k + 30 = (7\times 24q) - 6(5^k - 5)$

$= (7 \times 24q) - 6(4p)$ [$\because$ $(5^k - 5)$ is a multiple of 4]

$= (7 \times 24q) - 24p = 24(7q - p)$

$= 24 \times r$; $r = 7q - p$, is some natural number ... (ii)

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

12. (b) Let $P(n) : \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

For n = 1,

$P(1) : \frac{1}{1.2} = \frac{1}{2} = \frac{1}{1+1}$, which is true.

Assume that P(k) is true for some natural number k,

i.e. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{k(k+1)} = \frac{k}{k+1}$ ... (i)

We shall now prove that P(k + 1) is true, i.e.

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\text{L.H.S.} = \left[\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{k(k+1)}\right] + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \quad \text{[Using (i)]}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k^2+2k+1)}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} = \text{R.H.S.}$$

Thus, P(k + 1) is true whenever P(k) is true. Hence, by the principle of mathematical induction, P(n) is true for all natural numbers.

13. (c) Let $P(n) : 7^n - 3^n$ is divisible by 4.

For n = 1,

$P(1) : 7^1 - 3^1 = 4$, which is divisible by 4. Thus, P(n) is true for n = 1.

Let P(k) be true for some natural number k,

i.e. $P(k) : 7^k - 3^k$ is divisible by 4.

We can write $7^k - 3^k = 4d$, where $d \in N$ ... (i)

Now, we wish to prove that P(k + 1) is true whenever P(k) is true, i.e. $7^{k+1} - 3^{k+1}$ is divisible by 4.

Now, $7^{(k+1)} - 3^{(k+1)} = 7^{(k+1)} - 7.3^k + 7.3^k - 3^{(k+1)}$

$= 7(7^k - 3^k) + (7 - 3)3^k = 7(4d) + 4.3^k$ [using (i)]

$= 4(7d + 3^k)$, which is divisible by 4.

Thus, P(k + 1) is true whenever P(k) is true. Therefore, by the principle of mathematical induction the statement is true for every positive integer n.

14. (b) Let $P(n) : \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ... + \frac{1}{n(n+1)(n+2)}$

$$= \frac{n(n+3)}{4(n+1)(n+2)}$$

For n = 1,

$$\text{L.H.S.} = \frac{1}{1\cdot2\cdot3} = \frac{1}{6}$$

and $\text{R.H.S.} = \frac{1(1+3)}{4(1+1)(1+2)} = \frac{1}{6}$

$\therefore$ P(1) is true.

Let P(k) is true, then

$$P(k) : \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ... + \frac{1}{k(k+1)(k+2)}$$

$$= \frac{k(k+3)}{4(k+1)(k+2)} \quad ... (i)$$

For n = k + 1,

$$P(k+1) : \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ...$$

$$+ \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

$$\text{L.H.S.} = \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ...$$

$$+ \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$ [from (i)]

$= \frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)} = \text{R.H.S.}$

Hence, P(k + 1) is true.

Hence, by principle of mathematical induction for all $n \in N$, P(n) is true.

15. (a) Let P(n) be the given statement,

i.e. $P(n) : (ab)^n = a^n b^n$

We note that P(n) is true for n = 1, since $(ab)^1 = a^1 b^1$

Let P(k) be true,

i.e. $(ab)^k = a^k b^k$ ... (i)

We shall now prove that P(k + 1) is true whenever P(k) is true.

Now, we have $(ab)^{k+1} = (ab)^k (ab)$

$= (a^k b^k)(ab)$ [by using (i)]

$= (a^k \cdot a^1)(b^k \cdot b^1) = a^{k+1} \cdot b^{k+1}$

Therefore, P(k + 1) is also true whenever P(k) is true.

Hence, by principle of mathematical induction, P(n) is true for all $n \in N$.

16. (c) On putting n = 1 in $11^{n+2} + 12^{2n+1}$, we get

$11^{1+2} + 12^{(2\times 1)+1} = 11^3 + 12^3 = 3059$,

which is divisible by 133 only.

17. (b) Let P(n) be the statement given by

$P(n) : 41^n - 14^n$ is a multiple of 27

For n = 1,

i.e. $P(1) = 41^1 - 14^1 = 27 = 1 \times 27$,

which is a multiple of 27.

$\therefore$ P(1) is true.

Let P(k) be true, i.e. $41^k - 14^k = 27\lambda$ ... (i)

For n = k + 1,

$41^{k+1} - 14^{k+1} = 41^k\, 41 - 14^k\, 14$

$= (27\lambda + 14^k)41 - 14^k\, 14$ [using (i)]

$= (27\lambda \times 41) + (14^k \times 41) - (14^k \times 14)$

$= (27\lambda \times 41) + 14^k(41 - 14)$

$= (27\lambda \times 41) + (14^k \times 27)$

$= 27(41\lambda + 14^k)$,

which is a multiple of 27.

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

18. (a) For n = 1,

$5^4 = 625 = (624 + 1) = (48 \times 13) + 1$,

i.e 5^4 leaves 1 as remainder when divided by 13.

19. (c) Let m = 2k + 1, n = 2k – 1 ($k \in N$)

$\therefore\ m^2 - n^2 = 4k^2 + 1 + 4k - 4k^2 + 4k - 1 = 8k$

Hence, all the numbers of the form $m^2 - n^2$ are always divisible by 8.

20. (b) The condition $2^n (n-1)! < n^n$ is satisfied for n > 2.

21. (b) $3.5^{2n+1} + 2^{3n+1}$

Put n = 1, we get

$(3 \times 5^3) + 2^4 = 391$, which is divisible by 17.

Put n = 2, we get

$(3 \times 5^5) + 2^7 = 9503$, which is divisible by 17 only.

22. (c) In algebra or in other discipline of Mathematics, there are certain results or statements that are formulated in terms of n, where n is a positive integer. To prove such statement, the well-suited principle, i.e. used-based on the specific technique is known as the principle of mathematical induction.

23. (b) Let the statement P(n) be defined as

$P(n) = 1.3 + 2.3^2 + 3.3^3 + ... + n.3^n$

$= \frac{(2n-1)3^{n+1} + 3}{4}$

Step I : For n = 1,

$P(1) : 1.3 = \frac{(2.1-1)3^{1+1} + 3}{4} = \frac{3^2 + 3}{4}$

$= \frac{9+3}{4} = \frac{12}{4} = 3 = 1.3$, which is true.

Step II : Let it is true for n = k,

i.e. $1.3 + 2.3^2 + 3.3^3 + ... + k.3^k$

$= \frac{(2k-1)3^{k+1} + 3}{4}$ ... (i)

Step III : For n = k + 1,

$(1.3 + 2.3^2 + 3.3^3 + ... + k.3^k) + (k+1)3^{k+1}$

$= \frac{(2k-1)3^{k+1} + 3}{4} + (k+1)3^{k+1}$

[Using equation (i)]

$= \frac{(2k-1)3^{k+1} + 3 + 4(k+1)3^{k+1}}{4}$

$= \frac{3^{k+1}(2k - 1 + 4k + 4) + 3}{4}$

[taking 3^{k+1} common in first and last term of numerator part]

$= \frac{3^{k+1}(6k+3) + 3}{4} = \frac{3^{k+1} \cdot 3(2k+1) + 3}{4}$

[taking 3 common in first term of numerator part]

$= \frac{3^{(k+1)+1}[2k + 2 - 1] + 3}{4}$

$= \frac{[2(k+1) - 1]3^{(k+1)+1} + 3}{4}$

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

24. (d) Let the statement P(n) be defined as

$$P(n) : 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + ...$$

$$+ \frac{1}{1+2+3+.....+n} = \frac{2n}{n+1}$$

i.e. $P(n) : 1+\frac{1}{1+2}+\frac{1}{1+2+3}+.....+\frac{2}{n(n+1)}=\frac{2n}{n+1}$

$$\left[\because \text{ sum of natural numbers} = \frac{n(n+1)}{2}\right]$$

Step I : For n = 1,

$P(1) : 1 = \frac{2\times 1}{1+1} = \frac{2}{2} = 1$, which is true.

Step II : Let it is true for n = k,

i.e. $1 + \frac{1}{1+2}+\frac{1}{1+2+3}+.....+\frac{2}{k(k+1)}=\frac{2k}{k+1}$ ... (i)

Step III : For n = k + 1,

$$\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+.....+\frac{2}{k(k+1)}\right)+\frac{2}{(k+1)(k+2)}$$

$= \frac{2k}{k+1} + \frac{2}{(k+1)(k+2)}$ [using equation (i)]

$= \frac{2k(k+2)+2}{(k+1)(k+2)} = \frac{2\left[k^2+2k+1\right]}{(k+1)(k+2)}$

[taking 2 common in numerator part]

$= \frac{2(k+1)^2}{(k+1)(k+2)}$ $[\because (a+b)^2 = a^2 + 2ab + b^2]$

$= \frac{2(k+1)}{k+2} = \frac{2(k+1)}{(k+1)+1}$

Therefore, P(k + 1) is true, when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

25. (c) $10^n + 3(4^{n+2}) + 5$

Taking n = 2;

$10^2 + 3 \times 4^4 + 5 = 100 + 768 + 5 = 873$

Therefore, this is divisible by 9.

26. (c) Check for n = 1, 2, 3, ..., it is true for all $n \in N$.

27. (b) Check through option, the condition $\left(\frac{n+1}{2}\right)^n \geq n!$ is true for $n \geq 1$.

28. (d) Check through option, condition $(n!)^2 > n^n$ is true when $n \geq 3$.

STATEMENT TYPE QUESTIONS

29. (c) I. Let the statement P(n) be defined as

$P(n) : 1 + 2 + 3 + ... + n < \frac{1}{8}(2n+1)^2$

Step I : For n = 1,

$P(1) : 1 < \frac{1}{8}(2.1+1)^2 \Rightarrow 1 < \frac{1}{8} \times 3^2$

$\Rightarrow 1 < \frac{9}{8}$, which is true.

Step II : Let it is true for n = k.

$1 + 2 + 3 + ... + k < \frac{1}{8}(2k+1)^2$ ... (i)

Step III : For n = k + 1,

$(1+2+3+.....+k)+(k+1) < \frac{1}{8}(2k+1)^2+(k+1)$

[using equation (i)]

$= \frac{(2k+1)^2}{8} + \frac{k+1}{1} = \frac{(2k+1)^2+8k+8}{8}$

$= \frac{4k^2+1+4k+8k+8}{8}$

$= \frac{4k^2+12k+9}{8} = \frac{(2k+3)^2}{8}$

$= \frac{(2k+2+1)^2}{8} = \frac{[2(k+1)+1]^2}{8}$

$\Rightarrow 1+2+3+.....+k+(k+1) < \frac{[2(k+1)+1]^2}{8}$

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

II. Let the statement P(n) be defined as

P(n) : n(n + 1) (n + 5) is a multiple of 3.

Step I : For n = 1,

$P(1) : 1(1 + 1)(1 + 5) = 1 \times 2 \times 6 = 12 = 3 \times 4$, which is a multiple of 3, that is true.

Step II : Let it is true for n = k,

i.e. $k(k + 1)(k + 5) = 3\lambda$

$\Rightarrow k(k^2 + 5k + k + 5) = 3\lambda$

$\Rightarrow k^3 + 6k^2 + 5k = 3\lambda$ ... (i)

Step III : For n = k + 1, (k + 1) (k + 1 + 1) (k + 1 + 5)

$= (k+1)(k+2)(k+6) = (k^2+2k+k+2)(k+6)$

$= (k^2 + 3k + 2)(k + 6)$

$= k^3 + 6k^2 + 3k^2 + 18k + 2k + 12$

$= k^3 + 9k^2 + 20k + 12$
$= (3\lambda - 6k^2 - 5k) + 9k^2 + 20k + 12$ [using equation (i)]
$= 3\lambda + 3k^2 + 15k + 12$
$= 3(\lambda + k^2 + 5k + 4)$, which is a multiple of 3.
Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.
Hence, both the statements are true.

ASSERTION - REASON TYPE QUESTIONS

30. **(a)** **Assertion :** Let P(n) : $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$

For n = 2,

P(2) : $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2}$, which is true.

Assume P(k) is true,

i.e. $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} > \sqrt{k}$ ... (i)

For n = k + 1, we have to show that

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ ... (ii)

L.H.S. $= \left(\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}}\right) + \frac{1}{\sqrt{k+1}}$ ... (iii)

Reason : For n = k,

$\sqrt{k(k+1)} < k + 1$

$\Rightarrow \sqrt{k}\sqrt{k+1} < \sqrt{k+1}\sqrt{k+1}$

$\Rightarrow \sqrt{k} < \sqrt{k+1}$

$\because \sqrt{k+1} > \sqrt{k}$ for $k \geq 2$

$\Rightarrow 1 > \frac{\sqrt{k}}{\sqrt{k+1}}$

$\Rightarrow \sqrt{k} > \frac{k}{\sqrt{k+1}}$, (Multiplying by $\sqrt{k}$)

$\Rightarrow \sqrt{k} > \frac{(k+1)-1}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \sqrt{k+1} - \frac{1}{\sqrt{k+1}}$

$\Rightarrow \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ ... (iv)

From (iii) and (iv),

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k}$

$+ \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ [Using (i)]

Hence, (ii) is true for n = k + 1
Hence, P(n) is true for $n \geq 2$
So, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

31. **(c)** If $11^{m+2} + 12^{2m+1}$ is divisible by 133, then
$11^{m+2} + 12^{2m+1} = 133\lambda, \lambda \in N$ ... (i)
Hence, $11^{(m+1)+2} + 12^{2(m+1)+1}$
$= (11^{m+2} \times 11) + (12^{2m+1} \times 12^2)$
$= (133\lambda - 12^{2m+1}) \times 11 + (144 \times 12^{2m+1})$ [using (i)]
$= (11 \times 133\lambda) - (11 \times 12^{2m+1}) + (144 \times 12^{2m+1})$
$= (11 \times 133\lambda) + (133 \times 12^{2m+1})$

CRITICALTHINKING TYPE QUESTIONS

32. **(c)** The product of *r* consecutive integers is divisible by *r* ! . Thus $n(n+1)(n+2)(n+3)$ is divisible by 4 ! = 24.

33. **(d)** *P*(1) is not true (Principle of induction is not applicable). Also $n(n+1)+1$ is always an odd integer.

34. **(d)** Let P(n) be the statement given by
P(n) : 3^{2n} when divided by 8, the remainder is 1.
or P(n) : $3^{2n} = 8\lambda + 1$ for some $\lambda \in N$
For n = 1,
P(1) : $3^2 = (8 \times 1) + 1 = 8\lambda + 1$, where $\lambda = 1$
$\therefore$ P(1) is true.
Let P(k) be true.
Then, $3^{2k} = 8\lambda + 1$ for some $\lambda \in N$ ... (i)
We shall now show that P(k + 1) is true, for which we have to show that $3^{2(k+1)}$ when divided by 8, the remainder is 1.
Now, $3^{2(k+1)} = 3^{2k} \cdot 3^2 = (8\lambda + 1) \times 9$ [Using (i)]
$= 72\lambda + 9 = 72\lambda + 8 + 1 = 8(9\lambda + 1) + 1$
$= 8\mu + 1$, where $\mu = 9\lambda + 1 \in N$
$\Rightarrow$ P(k + 1) is true.
Thus, P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction P(n) is true for all $n \in N$.

35. **(d)** Let P(n) be the statement given by
P(n) : $5^{2n+2} - 24n - 25$ is divisible by 576.
For n = 1,
P(1) : $5^{2+2} - 24 - 25 = 625 - 49 = 576$,
which is divisible by 576.
$\therefore$ P(1) is true.
Let P(k) be true,
i.e. P(k) : $5^{2k+2} - 24k - 25$ is divisible by 576.
$\Rightarrow 5^{2k+2} - 24k - 25 = 576\lambda$ ... (i)
We have to show that P(k + 1) is true,
i.e. $5^{2k+4} - 24k - 49$ is divisible by 576

Now, $5^{2k+4} - 24k - 49$

$= 5^{2k+2+2} - 24k - 49 = 5^{2k+2} \cdot 5^2 - 24k - 49$

$= (576\lambda + 24k + 25) \cdot 25 - 24k - 49$ [from (i)]

$= 576.25\lambda + 600k + 625 - 24k - 49$

$= 576.25\lambda + 576k + 576$

$= 576\{25\lambda + k + 1\}$, which is divisible by 576.

$\therefore$ P(k + 1) is true whenever P(k) is true.

So, P(n) is true for all $n \in N$.

36. **(b)** The product of r consecutive natural numbers is divisible by r! and not by (r + 1)!

37. **(d)** Let P(n) : $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$

For n = 1,

P(n) is not true.

For n = 2,

$$P(2) : \frac{4^2}{2+1} < \frac{4!}{(2)^2} \Rightarrow \frac{16}{3} < \frac{24}{4} \text{ which is true.}$$

Let for n = m > 2, P(n) is true, i.e.

$$\frac{4^m}{m+1} < \frac{(2m)!}{(m!)^2}$$

$$\text{Now, } \frac{4^{m+1}}{m+2} = \frac{4^m}{m+2} \cdot \frac{4(m+1)}{m+2} < \frac{(2m)!}{(m!)^2} \cdot \frac{4(m+1)}{(m+2)}$$

$$= \frac{(2m)!(2m+1)(2m+2)4(m+1)(m+1)^2}{(2m+1)(2m+2)(m!)^2(m+1)^2(m+2)}$$

$$= \frac{[2(m+1)]!}{[(m+1)!]^2} \cdot \frac{2(m+1)^2}{(2m+1)(m+2)} < \frac{[2(m+1)]!}{[(m+1)!]^2}$$

$$\left[\because \frac{2(m+1)^2}{(2m+1)(m+2)} < 1 \ \forall\ m > 2\right]$$

Hence, for $n \geq 2$, P(n) is true.

38. **(c)** Let the statement P(n) be defined as

$$P(n) : \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2$$

Step I : For n = 1,

$$\text{i.e. } P(1) : \left(1+\frac{3}{1}\right) = (1+1)^2 = 2^2 = 4 = \left(1+\frac{3}{1}\right),$$

which is true.

Step II : Let it is true for n = k,

$$\text{i.e. } \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2k+1}{k^2}\right) = (k+1)^2 \ \ldots \text{(i)}$$

Step III : For n = k + 1,

$$\left\{\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2k+1}{k^2}\right)\right\}\left(1+\frac{2k+1+2}{(k+1)^2}\right)$$

$$= (k+1)^2\left(1+\frac{2k+3}{(k+1)^2}\right) \quad \text{[using equation (i)]}$$

$$= (k+1)^2\left[\frac{(k+1)^2 + 2k + 3}{(k+1)^2}\right]$$

$= k^2 + 2k + 1 + 2k + 3$

$= (k+2)^2 = [(k+1)+1]^2$ $[\because (a+b)^2 = a^2 + 2ab + b^2]$

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

39. **(d)** Let the statement P(n) be defined as

$$P(n) : \frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15} \text{ is a natural number for all } n \in N.$$

Step I : For n = 1,

$$P(1) : \frac{1}{5} + \frac{1}{3} + \frac{7}{15} = 1 \in N$$

Hence, it is true for n = 1.

Step II : Let it is true for n = k,

$$\text{i.e. } \frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15} = \lambda \in N \quad \ldots \text{(i)}$$

Step III : For n = k + 1,

$$\frac{(k+1)^5}{5} + \frac{(k+1)^3}{3} + \frac{7(k+1)}{15}$$

$$= \frac{1}{5}(k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1)$$

$$+ \frac{1}{3}(k^3 + 3k^2 + 3k + 1) + \frac{7}{15}k + \frac{7}{15}$$

$$= \left(\frac{k^5}{5} + \frac{k^3}{3} + \frac{7}{15}k\right) + (k^4 + 2k^3 + 3k^2 + 2k)$$

$$+ \frac{1}{5} + \frac{1}{3} + \frac{7}{15}$$

$= \lambda + k^4 + 2k^3 + 3k^2 + 2k + 1$

[using equation (i)]

which is a natural number, since λ k $\in$ N.

Therefore, P(k + 1) is true, when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

40. (c) Let P(n) : $S_n = \begin{cases} \dfrac{n(n+1)^2}{2}, & \text{when n is even} \\ \dfrac{n^2(n+1)}{2}, & \text{when n is odd} \end{cases}$

Also, note that any term T_n of the series is given by

$$T_n = \begin{cases} n^2, & \text{if n is odd} \\ 2n^2, & \text{if n is even} \end{cases}$$

We observe that P(1) is true, since

$$P(1) : S_1 = 1^2 = 1 = \frac{1 \cdot 2}{2} = \frac{1^2 \cdot (1+1)}{2}$$

Assume that P(k) is true for some natural number k, i.e

Case I : When k is odd, then k + 1 is even. We have,

$$P(k+1) : S_{k+1} = 1^2 + 2 \times 2^2 + ... + k^2 + 2 \times (k+1)^2$$

$$= \frac{k^2(k+1)}{2} + 2 \times (k+1)^2$$

$$\left[\text{as k is odd, } 1^2 + 2 \times 2^2 + ... + k^2 = k^2 \frac{(k+1)}{2}\right]$$

$$= \frac{(k+1)}{2}[k^2 + 4(k+1)]$$

$$= \frac{k+1}{2}[k^2 + 4k + 4]$$

$$= \frac{k+1}{2}(k+2)^2$$

$$= (k+1)\frac{[(k+1)+1]^2}{2}$$

So, P(k + 1) is true, whenever P(k) is true, in the case when k is odd.

Case II : When k is even, then k + 1 is odd.

Now, $P(k+1) : S_{k+1} = 1^2 + 2 \times 2^2 + ... + 2 \cdot k^2 + (k+1)^2$

$$= \frac{k(k+1)^2}{2} + (k+1)^2$$

$$\left[\text{as k is even, } 1^2 + 2 \times 2^2 + ... + 2k^2 = k\frac{(k+1)^2}{2}\right]$$

$$= \frac{(k+1)^2(k+2)}{2} = \frac{(k+1)^2((k+1)+1)}{2}$$

Therefore, P(k + 1) is true, whenever P(k) is true for the case when k is even.

Thus, P(k + 1) is true whenever P(k) is true for any natural number k. Hence, P(n) true for all natural numbers n.

41. (c) $2^4 \equiv 1 \pmod 5 \Rightarrow (2^4)^{75} \equiv (1)^{75} \pmod 5$

i.e. $2^{300} \equiv 1 \pmod 5 \Rightarrow 2^{300} \times 2 \equiv (1.2) \pmod 5$

$\Rightarrow 2^{301} \equiv 2 \pmod 5$

$\therefore$ Least positive remainder is 2.

CHAPTER

5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Value of $\left(\frac{2i}{1+i}\right)^2$ is

(a) i (b) $2i$
(c) $1-i$ (d) $1-2i$

2. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$ then

(a) $a=2, b=-1$ (b) $a=1, b=0$
(c) $a=0, b=1$ (d) $a=-1, b=2$

3. $1 + i^2 + i^4 + i^6 + ... + i^{2n}$ is

(a) positive (b) negative
(c) 0 (d) cannot be determined

4. If $(x+iy)(2-3i)=4+i$, then

(a) $x=-14/13, y=5/13$ (b) $x=5/13, y=14/13$
(c) $x=14/13, y=5/13$ (d) $x=5/13, y=-14/13$

5. If $4x+i(3x-y)=3+i(-6)$, where x and y are real numbers, then the values of x and y are

(a) $x=\frac{3}{5}$ and $y=\frac{33}{4}$ (b) $x=\frac{3}{4}$ and $y=\frac{22}{3}$
(c) $x=\frac{3}{4}$ and $y=\frac{33}{4}$ (d) $x=\frac{3}{4}$ and $y=\frac{33}{5}$

6. If $z=x-i\ y$ and $z^{\frac{1}{3}}=p+iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to

(a) –2 (b) –1 (c) 2 (d) 1

7. The polar form of the complex number $(i^{25})^3$ is

(a) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (b) $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$
(c) $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$ (d) $\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$

8. If $z_1=\sqrt{3}+i\sqrt{3}$ and $z_2=\sqrt{3}+i$, then in which quadrant $\left(\frac{z_1}{z_2}\right)$ lies?

(a) I (b) II (c) III (d) IV

9. The solutions of the quadratic equation $ax^2+bx+c=0$, where $a, b, c \in R$, $a \neq 0$, $b^2-4ac<0$, are given by $x=?$

(a) $\frac{b\pm\sqrt{4ac-b^2 i}}{2a}$ (b) $\frac{-b\pm\sqrt{4ac+b^2 i}}{2a}$
(c) $\frac{-b\pm\sqrt{4ac-b^2 i}}{2a}$ (d) $\frac{-b\pm\sqrt{4ab-c^2 i}}{2a}$

10. If $x^2+x+1=0$, then what is the value of x?

(a) $\frac{1+\sqrt{3}i}{2}$ (b) $\frac{-1\pm\sqrt{3}i}{2}$
(c) $\frac{-1\pm\sqrt{3}i}{3}$ (d) $\frac{-1\pm\sqrt{2}i}{2}$

11. The solution of $\sqrt{3x^2-2}=2x-1$ are :

(a) $(2,4)$ (b) $(1,4)$ (c) $(3,4)$ (d) $(1,3)$

12. If α, β are roots of the equation $x^2-5x+6=0$, then the equation whose roots are $\alpha+3$ and $\beta+3$ is

(a) $2x^2-11x+30=0$ (b) $-x^2+11x=0$
(c) $x^2-11x+30=0$ (d) $2x^2-5x+30=0$

13. Value of k such that equations $2x^2+kx-5=0$ and $x^2-3x-4=0$ have one common root, is

(a) $-1, -2$ (b) $-3, -\frac{27}{4}$
(c) $3, \frac{4}{27}$ (d) $-2, -3$

14. If $a<b<c<d$, then the nature of roots of $(x-a)(x-c)+2(x-b)(x-d)=0$ is

(a) real and equal (b) complex
(c) real and unequal (d) None of these

15. For the equation $\frac{1}{x+a}-\frac{1}{x+b}=\frac{1}{x+c}$, if the product of roots is zero, then sum of roots is

(a) $-\frac{2bc}{b+c}$ (b) $\frac{2ca}{c+a}$
(c) $\frac{bc}{c+a}$ (d) $\frac{-bc}{b+c}$

16. Product of real roots of the equation $t^2x^2+|x|+9=0$

(a) is always positive (b) is always negative
(c) does not exist (d) None of these

17. If p and q are the roots of the equation $x^2+px+q=0$, then
(a) $p=1, q=-2$ (b) $p=0, q=1$
(c) $p=-2, q=0$ (d) $p=-2, q=1$

18. The roots of the given equation $(p-q)x^2+(q-r)x+(r-p)=0$ are :
(a) $\frac{p-q}{r-p}, 1$ (b) $\frac{q-r}{p-q}, 1$
(c) $\frac{r-p}{p-q}, 1$ (d) None of these

19. If α, β are the roots of $ax^2+bx+c=0$, then $\alpha\beta^2+\alpha^2\beta+\alpha\beta$ equals
(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $\frac{-bc}{a^2}$ (d) abc

20. The roots of equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ is
(a) one (b) two
(c) infinite (d) None of these

21. If $z_1=3+i$ and $z_2=i-1$, then
(a) $|z_1+z_2|>|z_1|+|z_2|$ (b) $|z_1+z_2|<|z_1|-|z_2|$
(c) $|z_1+z_2|\le|z_1|+|z_2|$ (d) $|z_1+z_2|<|z_1|+|z_2|$

22. Let z be any complex number such that $|z|=4$ and $\arg(z)=\frac{5\pi}{6}$, then value of z is
(a) $-2\sqrt{3}-2i$ (b) $2\sqrt{3}-i$
(c) $\sqrt{2}+3i$ (d) $-2\sqrt{3}+2i$

23. If $z=\frac{3-i}{2+i}+\frac{3+i}{2-i}$, then value of arg (zi) is
(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

24. If the complex numbers z_1, z_2, z_3 represents the vertices of an equilateral triangle such that $|z_1|=|z_2|=|z_3|$, then value of $z_1+z_2+z_3$ is
(a) 0 (b) 1 (c) 2 (d) $\frac{3}{2}$

25. $(1+i)^8+(1-i)^8$ equal to
(a) 2^8 (b) 2^5 (c) $2^4\cos\frac{\pi}{4}$ (d) $2^8\cos\frac{\pi}{8}$

26. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is equal to :
(a) $\frac{7-26i}{25}$ (b) $\frac{-7-26i}{25}$
(c) $\frac{-7+26i}{25}$ (d) $\frac{7+26i}{25}$

27. If $z=1+i$, then the multiplicative inverse of z^2 is (where, $i=\sqrt{-1}$)
(a) 2i (b) 1 – i
(c) $-\frac{i}{2}$ (d) $\frac{i}{2}$

28. $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$ is equal to :
(a) $\frac{1}{2}+\frac{9}{2}i$ (b) $\frac{1}{2}-\frac{9}{2}i$
(c) $\frac{1}{4}-\frac{9}{4}i$ (d) $\frac{1}{4}+\frac{9}{4}i$

29. The complex number $\left(\frac{1+2i}{1-i}\right)$ lies in:
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

30. Amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}+1}$ is :
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

31. The value of $(1+i)^4\left(1+\frac{1}{i}\right)^4$ is
(a) 12 (b) 2
(c) 8 (d) 16

32. Evaluate: $(1+i)^6+(1-i)^3$.
(a) –2 – 10i (b) 2 – 10i
(c) –2 + 10i (d) 2 + 10i

33. If $(x+iy)^{\frac{1}{3}}=a+ib$, where x, y, a, b $\in$ R, then $\frac{x}{a}-\frac{y}{b}=$
(a) a^2-b^2 (b) $-2(a^2+b^2)$
(c) $2(a^2-b^2)$ (d) a^2+b^2

34. The value of $\frac{i^{4n+1}-i^{4n-1}}{2}$. is
(a) i (b) 2i (c) –i (d) –2i

35. $\sqrt{-3}\sqrt{-6}$ is equal to
(a) $3\sqrt{2}$ (b) $-3\sqrt{2}$ (c) $3\sqrt{2}i$ (d) $-3\sqrt{2}i$

36. If $z(2-i)=(3+i)$, then z^{20} is equal to
(a) 2^{10} (b) -2^{10}
(c) 2^{20} (d) -2^{20}

37. The real part of $\frac{(1+i)^2}{(3-i)}$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{5}$
(c) $-\frac{1}{3}$ (d) None of these

38. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is
(a) $\frac{8}{25}-\frac{31}{25}i$ (b) $-\frac{8}{25}-\frac{31}{25}i$
(c) $-\frac{8}{25}+\frac{31}{25}i$ (d) None of these

39. What is the conjugate of $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$?

(a) $-3i$ (b) $3i$ (c) $\frac{3}{2}i$ (d) $-\frac{3}{2}i$

40. If $z = \dfrac{7-i}{3-4i}$, then $|z|^{14} =$

(a) 2^7 (b) $2^7 i$ (c) -2^7 (d) $-2^7 i$

41. Represent $z = 1+i\sqrt{3}$ in the polar form.

(a) $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$ (b) $\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$

(c) $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ (d) $4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

42. The modulus of $\dfrac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ is

(a) 2 (b) 4 (c) $3\sqrt{2}$ (d) $2\sqrt{2}$

43. The argument of the complex number $\left(\frac{i}{2}-\frac{2}{i}\right)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$

44. The square root of $(7-24i)$ is

(a) $\pm(3-5i)$ (b) $\pm(3+4i)$
(c) $\pm(3-4i)$ (d) $\pm(4-3i)$

45. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$.

(a) $\pm\frac{\sqrt{19}}{5}i$ (b) $\pm\frac{\sqrt{19}i}{2}$

(c) $\frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$ (d) $\frac{-1\pm\sqrt{19}i}{\sqrt{5}}$

46. If α and β are the roots of the equation $x^2 + 2x + 4 = 0$, then $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 32 (d) $\frac{1}{4}$

47. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$ equals

(a) $\frac{ac}{b}$ (b) 1 (c) $\frac{ab}{c}$ (d) $\frac{b}{ac}$

48. If $1-i$, is a root of the equation $x^2 + ax + b = 0$, where $a, b \in R$, then the values of a and b are,

(a) 2, 2 (b) −2, 2 (c) −2, −2 (d) 1, 2

49. Which of the following is correct for any two complex numbers z_1 and z_2?

(a) $|z_1 z_2| = |z_1||z_2|$
(b) $\arg(z_1 z_2) = \arg(z_1)\cdot\arg(z_2)$
(c) $|z_1 + z_2| = |z_1| + |z_2|$
(d) $|z_1 + z_2| \ge |z_1| - |z_2|$

50. A number z = a + ib, where a and b are real numbers, is called

(a) complex number (b) real number
(c) natural number (d) integer

51. If $ax^2 + bx + c = 0$ is a quadratic equation, then equation has no real roots, if

(a) $D > 0$ (b) $D = 0$
(c) $D < 0$ (d) None of these

52. If z = a + ib, then real and imaginary part of z are

(a) Re(z) = a, Im(z) = b (b) Re(z) = b, Im(z) = a
(c) Re(z) = a, Im(z) = ib (d) None of these

53. Which of the following options defined 'imaginary number'?

(a) Square root of any number
(b) Square root of positive number
(c) Square root of negative number
(d) Cube root of number

54. If $x = \sqrt{-16}$, then

(a) x = 4i (b) x = 4
(c) x = – 4 (d) All of these

55. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\frac{z_1}{z_2}$ is equal to

(a) $\frac{1}{5}(9+12i)$ (b) $9+12i$

(c) $3+2i$ (d) $\frac{1}{5}(12+9i)$

56. The value of $(1+i)^5 \times (1-i)^5$ is

(a) −8 (b) 8i (c) 8 (d) 32

57. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then value of $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$ is

(a) 2 (b) 2i (c) $\sqrt{2}$ (d) $\sqrt{2}i$

58. If $\frac{(1+i)^3}{(1-i)^3}-\frac{(1-i)^3}{(1+i)^3} = x + iy$

(a) x = 0, y = −2 (b) x = −2, y = 0
(c) x = 1, y = 1 (d) x = −1, y = 1

59. Additive inverse of 1 – i is

(a) 0 + 0i (b) −1 − i
(c) −1 + i (d) None of these

60. If z is a complex number such that $z^2 = (\bar{z})^2$, then

(a) z is purely real
(b) z is purely imaginary
(c) either z is purely real or purely imaginary
(d) None of these

61. If $|z| = 1$, $(z \neq -1)$ and $z = x + iy$, then $\left(\frac{z-1}{z+1}\right)$ is

(a) purely real (b) purely imaginary
(c) zero (d) undefined

62. If $\bar{z}$ be the conjugate of the complex number z, then which of the following relations is false?

(a) $|z| = |\bar{z}|$ (b) $z \cdot \bar{z} = |\bar{z}|^2$
(c) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ (d) $\arg z = \arg \bar{z}$

63. If $\sqrt{a+ib} = x + iy$, then possible value of $\sqrt{a-ib}$ is

(a) $x^2 + y^2$ (b) $\sqrt{x^2 + y^2}$
(c) $x + iy$ (d) $x - iy$

64. A value of k for which the quadratic equation $x^2 - 2x(1 + 3k) + 7(2k + 3) = 0$ has equal roots is

(a) 1 (b) 2 (c) 3 (d) 4

65. The roots of the equation $3^{2x} - 10.3^x + 9 = 0$ are

(a) 1, 2 (b) 0, 2 (c) 0, 1 (d) 1, 3

66. If $x^2 + y^2 = 25$, $xy = 12$, then x =

(a) {3, 4} (b) {3, –3}
(c) {3, 4, –3, –4} (d) {–3, –3}

67. If the roots of the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2(\sqrt{pr})x + q = 0$ be real, then

(a) $p = q$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $r^2 = pq$

68. If $a > 0$, $b > 0$, $c > 0$, then both the roots of the equation $ax^2 + bx + c = 0$.

(a) Are real and negative (b) Have negative real parts
(c) Are rational numbers (d) None of these

69. If a and b are the odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0$, $a \neq 0$, will be

(a) rational (b) irrational
(c) non-real (d) equal

70. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) =

(a) (–4, 7) (b) (4, –7) (c) (4, 7) (d) (–4, –7)

71. If the sum of the roots of the equation $x^2 + px + q = 0$ is equal to the sum of their squares, then

(a) $p^2 - q^2 = 0$ (b) $p^2 + q^2 = 2q$
(c) $p^2 + p = 2q$ (d) None of these

72. If a root of the equations $x^2 + px + q = 0$ and $x^2 + \alpha x + \beta = 0$ is common, then its value will be (where $p \neq \alpha$ and $q \neq \beta$)

(a) $\frac{q-\beta}{\alpha - p}$ (b) $\frac{p\beta - \alpha q}{q - \beta}$

(c) $\frac{q-\beta}{\alpha - p}$ or $\frac{p\beta - \alpha q}{q - \beta}$ (d) None of these

73. If $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$ have a common root, then $a^2 - b^2$ is equal to

(a) 10 (b) 20 (c) 30 (d) 40

74. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

(a) $a < 2$ (b) $2 \leq a \leq 3$
(c) $3 < a \leq 4$ (d) $a > 4$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

75. **Statement - I :** Roots of quadratic equation $x^2 + 3x + 5 = 0$ is $x = \frac{-3 \pm i\sqrt{11}}{2}$.

Statement - II : If $x^2 - x + 2 = 0$ is a quadratic equation, then its roots are $\frac{1 \pm i\sqrt{7}}{2}$.

(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Both are incorrect

76. **Statement - I :** Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 + i\bar{z}_2 = 0$ and $\arg(z_1 \cdot z_2) = \pi$, then $\arg(z_1)$ is $\frac{3\pi}{4}$.

Statement - II : $\arg(z_1 \cdot z_2) = \arg z_1 + \arg z_2$.

(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Neither I nor II is correct

77. Which of the following are correct?

I. Modulus of $\frac{1+i}{1-i}$ is 1.

II. Argument of $\frac{1+i}{1-i}$ is $\frac{\pi}{2}$.

III. Modulus of $\frac{1}{1+i}$ is $\sqrt{2}$.

IV. Argument of $\frac{1}{1+i}$ is $\frac{\pi}{4}$.

(a) I and II are correct (b) III and IV are correct
(c) I, II and III are correct (d) All are correct

78. **Statement - I :** If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.

Statement II : If $z = x + iy$, then $|z| = \sqrt{x^2 + y^2}$.

(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Neither I nor II is correct

79. Consider the following statements

I. Additive inverse of $(1 - i)$ is equal to $-1 + i$.

II. If z_1 and z_2 are two complex numbers, then $z_1 - z_2$ represents a complex number which is sum of z_1 and additive inverse of z_2.

III. Simplest form of $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ is $1 + 2\sqrt{2}$ i.

Choose the correct option.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) I, II and III are correct.
(d) I, II and III are incorrect.

80. Consider the following statements.
I. Representation of $z = x + iy$ in terms of r and θ is called polar form of the complex number.
II. $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Choose the correct option.
(a) Only I is incorrect.
(b) Only II is correct.
(c) Both I and II are incorrect.
(d) Both I and II are correct.

81. Consider the following statements.
I. Let z_1 and z_2 be two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg(z_1) - \arg(z_2) = 0$
II. Roots of quadratic equation $x^2 + 3x + 5 = 0$ is $x = \frac{-3 \pm i\sqrt{11}}{2}$

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Neither I nor II is correct.

82. Consider the following statements.
I. The value of $x^3 + 7x^2 - x + 16$, when $x = 1 + 2i$ is $-17 + 24i$.
II. If $iz^3 + z^2 - z + i = 0$ then $|z| = 1$

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Both are incorrect.

83. Consider the following statements.
I. If z, z_1, z_2 be three complex numbers then $z\bar{z} = |z|^2$
II. The modulus of a complex number $z = a + ib$ is defined as $|z| = \sqrt{a^2+b^2}$.
III. Multiplicative inverse of $z = 3 - 2i$ is $\frac{3}{13} + \frac{2}{13}i$

Choose the correct option.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) Only I and III are correct.
(d) All I, II and III are correct.

84. Consider the following statements.
I. Modulus of $\frac{1+i}{1-i}$ is 1.
II. Argument of $\frac{1+i}{1-i}$ is $\frac{\pi}{2}$.

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Both are incorrect.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

85.

Column - I (Complex Nos.)	Column - II (Multiplicative inverse)
(A) $4 - 3i$	(1) $\frac{\sqrt{5}}{14} - i\frac{3}{14}$
(B) $\sqrt{5} + 3i$	(2) $\frac{4}{25} + i\frac{3}{25}$
(C) $-i$	(3) $0 + i$

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	1	3	2
(d)	2	3	1

86. Simplify the complex numbers given in column-I and match with column-II.

Column - I	Column - II
(A) $(1-i)^4$	(1) $-\left(\frac{22}{3} + i\frac{107}{27}\right)$
(B) $\left(\frac{1}{3} + 3i\right)^3$	(2) $-4 + 0i$
(C) $\left(-2 - \frac{1}{3}i\right)^3$	(3) $-\frac{242}{27} - 26i$

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	3	1	2
(d)	2	3	1

87.

Column - I	Column -II
(A) i^{-1}	(1) -1
(B) i^{-2}	(2) $-i$
(C) i^{-3}	(3) i
(D) i^4	(4) 1

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	2	3	4	1
(d)	1	4	3	2

88.

Column - I (Complex Number)	Column - II (a + ib form)
(A) $(1-i)-(-1+6i)$	(1) $-\frac{21}{5}-\frac{21}{10}i$
(B) $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$	(2) -4
(C) $\left(\frac{1}{3}+3i\right)^3$	(3) $-\frac{22}{3}-\frac{107}{27}i$
(D) $(1-i)^4$	(4) $2-7i$
(E) $\left(-2-\frac{1}{3}i\right)^3$	(5) $\frac{-242}{27}-26i$

Codes:

	A	B	C	D	E
(a)	5	4	3	2	1
(b)	4	1	5	2	3
(c)	4	2	5	1	3
(d)	3	1	2	5	4

89.

Column – I (Complex Number)	Column - II (Multiplicative Inverse)
(A) $4-3i$	(1) $\frac{\sqrt{5}}{14}-\frac{3}{14}i$
(B) $\sqrt{5}+3i$	(2) $\frac{1}{49}-\frac{4\sqrt{3}\,i}{49}$
(C) $-i$	(3) $0+i.1$
(D) $\left(2+\sqrt{3}\,i\right)^2$	(4) $\frac{4}{25}+i\frac{3}{25}$

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	3	1	2
(c)	2	1	3	4
(d)	4	1	3	2

90.

Column - I (Quadratic Equation)	Column - II (Roots)
(A) $2x^2+x+1=0$	(1) $\frac{1\pm\sqrt{7}\,i}{2}$
(B) $x^2+3x+9=0$	(2) $\frac{-1\pm\sqrt{7}\,i}{4}$
(C) $-x^2+x-2=0$	(3) $\frac{-3\pm\sqrt{11}\,i}{2}$
(D) $x^2+3x+5=0$	(4) $\frac{-3\pm\left(3\sqrt{3}\right)i}{2}$

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	4	1	3
(d)	2	1	4	3

91.

Column - I (Complex Number)	Column - II (Polar form)
(A) $(1-i)$	(1) $\sqrt{2}\left[\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)\right]$
(B) $(-1+i)$	(2) $2\left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]$
(C) $(-1-i)$	(3) $\sqrt{2}\left[\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right]$
(D) $\left(\sqrt{3}+i\right)$	(4) $\sqrt{2}\left[\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right]$

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	3	1	4	2
(c)	2	4	1	3
(d)	2	1	4	3

92.

Column - I	Column - II
(A) If $z = x + iy$, then modulus z is	(1) $a + i\,0$
(B) The modulus of complex number $x + iy$ is	(2) $0 + bi$
(C) Complex numbers which lie on x-axis are in the form of	(3) $\sqrt{x^2+y^2}$
(D) Complex numbers which lie on y-axis are in the form of	(4) distance of the point from the origin

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	1	2
(c)	4	1	3	2
(d)	2	3	1	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

93. $i^{57}+\frac{1}{i^{25}}$, when simplified has the value

(a) 0 (b) $2i$

(c) $-2i$ (d) 2

94. If $z = 2 - 3i$, then the value of $z^2 - 4z + 13$ is

(a) 1 (b) –1 (c) 0 (d) None of these

95. If $\frac{c+i}{c-i} = a + ib$, where a, b, c are real, then $a^2 + b^2$ is equal to:

(a) 7 (b) 1 (c) c^2 (d) $-c^2$

96. If $x + iy = \frac{a+ib}{a-ib}$, then $x^2 + y^2 =$
(a) 1 (b) 2 (c) 0 (d) 4

97. If $z = x + iy$, $z^{\frac{1}{3}} = a - ib$ and $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$, then value of k equals
(a) 2 (b) 4 (c) 6 (d) 1

98. $2x^2 - (p + 1)x + (p - 1) = 0$. If $\alpha - \beta = \alpha\beta$, then what is the value of p?
(a) 1 (b) 2 (c) 3 (d) –2

99. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, then $z_1 + z_2$ equals to $a + ai$. Value of 'a' is equal to
(a) 3 (b) 4 (c) 5 (d) 2

100. If $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$, then $z_1 - z_2$ equals to $-1 + bi$. The value of 'b' is
(a) 1 (b) 2 (c) 3 (d) 5

101. If $z = 5i\left(\frac{-3}{5}i\right)$, then z is equal to $3 + bi$. The value of 'b' is
(a) 1 (b) 2 (c) 0 (d) 3

102. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\frac{z_1}{z_2}$ is equal to $\frac{1}{a}(9 + 12i)$. The value of 'a' is
(a) 1 (b) 2 (c) 4 (d) 5

103. Value of $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$ is
(a) 0 (b) 1 (c) 2 (d) 3

104. If $z = i^9 + i^{19}$, then z is equal to $a + ai$. The value of 'a' is
(a) 0 (b) 1 (c) 2 (d) 3

105. If $z = i^{-39}$, then simplest form of z is equal to $a + i$. The value of 'a' is
(a) 0 (b) 1 (c) 2 (d) 3

106. If $(1 - i)^n = 2^n$, then the value of n is
(a) 1 (b) 2
(c) 0 (d) None of these

107. The value of $(1 + i)^5 (1 - i)^5$ is 2^n. 'n' is equal to
(a) 2 (b) 3 (c) 4 (d) 5

108. The value of $(1 + i)^8 + (1 - i)^8$ is 2^n. Value of n is
(a) 2 (b) 3 (c) 4 (d) 5

109. Roots of $x^2 + 2 = 0$ are $\pm\sqrt{n}\, i$. The value of n is
(a) 1 (b) 2 (c) 3 (d) 4

110. If $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$ and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$, then $|z_1 z_2|$ is equal to $\sqrt{m}$. Value of m is
(a) 6 (b) 3 (c) 2 (d) 5

111. The modulus of $\sqrt{2i} - \sqrt{-2i}$ is:
(a) 2 (b) $\sqrt{2}$ (c) 0 (d) $2\sqrt{2}$

112. If $z = x + iy$, $z^{1/3} = a - ib$, then $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$ where k is equal to
(a) 1 (b) 2 (c) 3 (d) 4

113. If the equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 - 1) + px + 4x^2 + 2 = 0$ have both roots common, then the value of $(2r - p)$ is :
(a) 0 (b) 1/2
(c) 1 (d) None of these

114. $\arg \bar{z} + \arg z$; $z \neq 0$ is equal to :
(a) $\frac{\pi}{4}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

115. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to
(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

116. If $z = 2 - 3i$, then value of $z^2 - 4z + 13$ is
(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

117. Assertion : Let f(x) be a quadratic expression such that $f(0) + f(1) = 0$. If –2 is one of the root of $f(x) = 0$, then other root is $\frac{3}{5}$.
Reason : If α and β are the zeroes of $f(x) = ax^2 + bx + c$, then sum of zeroes $= -\frac{b}{a}$, product of zeroes $= \frac{c}{a}$.

118. Assertion : If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then $\frac{z_1}{z_2}$ is purely imaginary.
Reason : If z is purely imaginary, then $z + \bar{z} = 0$.

119. Assertion : The greatest integral value of λ for which $(2\lambda - 1)x^2 - 4x + (2\lambda - 1) = 0$ has real roots, is 2.
Reason : For real roots of $ax^2 + bx + c = 0$, $D \geq 0$.

120. Assertion : Consider z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$.
Reason : $\arg(z) = 0 \Rightarrow z$ is purely real.

121. Assertion : If P and Q are the points in the plane XOY representing the complex numbers z_1 and z_2 respectively, then distance $|PQ| = |z_2 - z_1|$.
Reason : Locus of the point P(z) satisfying $|z - (2 + 3i)| = 4$ is a straight line.

122. Assertion : The equation $ix^2 - 3ix + 2i = 0$ has non-real roots.
Reason : If a, b, c are real and $b^2 - 4ac \geq 0$, then the roots of the equation $ax^2 + bx + c = 0$ are real and if $b^2 - 4ac < 0$, then roots of $ax^2 + bx + c = 0$ are non-real.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

123. If $|z-4|<|z-2|$, its solution is given by
(a) $\text{Re}(z)>0$ (b) $\text{Re}(z)<0$
(c) $\text{Re}(z)>3$ (d) $\text{Re}(z)>2$

124. The equation whose roots are twice the roots of the equation, $x^2-3x+3=0$ is:
(a) $4x^2+6x+3=0$ (b) $2x^2-3x+3=0$
(c) $x^2-3x+6=0$ (d) $x^2-6x+12=0$

125. The roots of the equation $4^x-3 \,.\, 2^{x+3}+128=0$ are
(a) 4 and 5 (b) 3 and 4
(c) 2 and 3 (d) 1 and 2

126. If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots , then the value of 'q' is
(a) 4 (b) 12
(c) 3 (d) $\frac{49}{4}$

127. For the equation $3x^2+px+3=0$, $p>0$, if one of the root is square of the other, then p is equal to
(a) 1/3 (b) 1
(c) 3 (d) 2/3

128. Value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}-1$ is
(a) –2 (b) 0 (c) –1 (d) 1

129. Modulus of $z=\frac{(1+i\sqrt{3})(\cos\theta+i\sin\theta)}{2(1-i)(\cos\theta-i\sin\theta)}$ is
(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

130. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are
(a) $\sqrt{2}$ and $\frac{\pi}{6}$ (b) 1 and 0
(c) 1 and $\frac{\pi}{3}$ (d) 1 and $\frac{\pi}{4}$

131. If $|z^2-1|=|z|^2+1$, then z lies on
(a) imaginary axis (b) real axis
(c) origin (d) None of these

132. If $z=2+i$, then $(z-1)(\bar{z}-5)+(\bar{z}-1)(z-5)$ is equal to
(a) 2 (b) 7
(c) –1 (d) –4

133. If $z = r(\cos\theta + i\sin\theta)$, then the value of $\frac{z}{\bar{z}}+\frac{\bar{z}}{z}$ is
(a) $\cos 2\theta$ (b) $2\cos 2\theta$
(c) $2\cos\theta$ (d) $2\sin\theta$

134. The square root of i is
(a) $\pm\frac{1}{\sqrt{2}}(-1+i)$ (b) $\pm\frac{1}{\sqrt{2}}(1+i)$
(c) $\pm\frac{1}{\sqrt{2}}(1-i)$ (d) None of these

135. The number of real roots of $\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$ is
(a) 0 (b) 2
(c) 4 (d) 6

136. If the roots of the equation $\frac{a}{x-a}+\frac{b}{x-b}=1$ are equal in magnitude and opposite in sign, then
(a) $a = b$ (b) $a + b = 1$
(c) $a - b = 1$ (d) $a + b = 0$

137. Find the value of a such that the sum of the squares of the roots of the equation $x^2-(a-2)x-(a+1)=0$ is least.
(a) 4 (b) 2
(c) 1 (d) 3

138. If α, β are the roots of the equation $(x-a)(x-b)=5$, then the roots of the equation $(x-\alpha)(x-\beta)+5=0$ are
(a) a, 5 (b) b, 5
(c) a, α (d) a, b

139. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right|=1$ lies on
(a) circle $x^2+y^2=1$ (b) the x-axis
(c) the y-axis (d) the line $x+y=1$

140. The value of $(z+3)(\bar{z}+3)$ is equivalent to
(a) $|z+3|^2$ (b) $|z-3|$
(c) z^2+3 (d) None of these

141. $|z_1+z_2|=|z_1|+|z_2|$ is possible, if
(a) $z_2=\bar{z}_1$ (b) $z_2=\frac{1}{z_1}$
(c) $\arg(z_1)=\arg(z_2)$ (d) $|z_1|=|z_2|$

142. $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other for
(a) $x=n\pi$ (b) $x=\left(n+\frac{1}{2}\right)\frac{\pi}{2}$
(c) $x=0$ (d) No value of x

143. The modulus of the complex number z such that $|z+3-i|=1$ and $\arg(z)=\pi$ is equal to
(a) 3 (b) 2
(c) 9 (d) 4

144. If $Z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$, then polar form of Z is
(a) $\sqrt{2}\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$ (b) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$
(c) $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ (d) $\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

145. $(x - iy)(3 + 5i)$ is the conjugate of $(-6 - 24i)$, then x and y are
(a) $x = 3, y = -3$ (b) $x = -3, y = 3$
(c) $x = -3, y = -3$ (d) $x = 3, y = 3$

146. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then
(a) $|z| = 0$ (b) $|z| = 1$
(c) $|z| > 1$ (d) $|z| < 1$

147. The amplitude of $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$ is
(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{\pi}{15}$

148. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then $(x^2 + y^2)^2 =$
(a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{a+b}{c+d}$
(c) $\frac{c^2+d^2}{a^2+b^2}$ (d) $\left(\frac{a^2+b^2}{c^2+d^2}\right)^2$

149. If the equation $(m - n)x^2 + (n - l)x + l - m = 0$ has equal roots, then l, m and n satisfy
(a) $2l = m + n$ (b) $2m = n + l$
(c) $m = n + l$ (d) $l = m + n$

150. If the product of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ be 2, then the sum of roots is
(a) 1 (b) –1
(c) 2 (d) –2

151. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$
(a) $\frac{2}{a}$ (b) $\frac{2}{b}$ (c) $\frac{2}{c}$ (d) $-\frac{2}{a}$

152. If one root of $ax^2 + bx + c = 0$ be square of the other, then the value of $b^3 + ac^2 + a^2c$ is
(a) 3abc (b) –3abc
(c) 0 (d) None of these

153. If α, β are the roots of $(x - a)(x - b) = c$, $c \neq 0$, then the roots of $(x - \alpha)(x - \beta) + c = 0$ shall be
(a) a, c (b) b, c
(c) a, b (d) a + c, b + c

154. If the roots of the equation $ax^2 + bx + c = 0$ are α, β, then the value of $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ will be
(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $-\frac{bc}{a^2}$ (d) None of these

155. If α, β be the roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 \cdot (2\beta - 35)^3$ is equal to
(a) 1 (b) 64
(c) 8 (d) None of these

156. If the sum of the roots of the equation $x^2 + px + q = 0$ is three times their difference, then which one of the following is true?
(a) $9p^2 = 2q$ (b) $2q^2 = 9p$
(c) $2p^2 = 9q$ (d) $9q^2 = 2p$

157. If the ratio of the roots of $x^2 + bx + c = 0$ and $x^2 + qx + r = 0$ be the same, then
(a) $r^2c = b^2q$ (b) $r^2b = c^2q$
(c) $rb^2 = cq^2$ (d) $rc^2 = bq^2$

158. If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of equation $x^2 + px + q = 0$ are $\alpha^2 + \beta^2, \frac{\alpha\beta}{2}$, then
(a) p = 1, q = –56 (b) p = –1, q = –56
(c) p = 1, q = 56 (d) p = –1, q = 56

159. If A.M. of the roots of a quadratic equation is $\frac{8}{5}$ and A.M. of their reciprocals is $\frac{8}{7}$, then the equation is
(a) $5x^2 - 16x + 7 = 0$ (b) $7x^2 - 16x + 5 = 0$
(c) $7x^2 - 16x + 8 = 0$ (d) $3x^2 - 12x + 7 = 0$

160. If the roots of $4x^2 + 5k = (5k + 1)x$ differ by unity, then the negative value of k is
(a) –3 (b) –5
(c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

161. Sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is
(a) 2 (b) 4 (c) 5 (d) 6

162. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is
(a) 6 (b) 0 (c) 4 (d) 10

163. Value of $\frac{(\cos\theta + i\sin\theta)^4}{(\cos\theta - i\sin\theta)^3}$ is
(a) $\cos 5\theta + i\sin 5\theta$ (b) $\cos 7\theta + i\sin 7\theta$
(c) $\cos 4\theta + i\sin 4\theta$ (d) $\cos\theta + i\sin\theta$

164. The value of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots\ldots\ldots\ldots\infty}}$ is
(a) $1 - \sqrt{2}$ (b) $1 + \sqrt{2}$
(c) $1 \pm \sqrt{2}$ (d) None of these

165. If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots \text{ to } \infty}}}$, then
(a) x is an irrational number
(b) $2 < x < 3$
(c) $x = 3$
(d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** $\left(\frac{2i}{1+i}\right)^2 = \frac{4i}{1+i^2+2i} = \frac{-4}{1-1+2i} = \frac{-4}{2i}$

$= \frac{-2}{i} = 2i \left(\because \frac{1}{i} = -i\right)$

2. **(b)** $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1-i^2} = \frac{1+i^2-2i}{2} = -i$

$\therefore (-i)^{100} = (i)^{100} = (i^4)^{25} = 1$

$\Rightarrow 1 = a+ib$

$\Rightarrow a = 1, b = 0$

3. **(d)** Given expression $= 1 + i^2 + i^4 + ... + i^{2n}$

$= 1 - 1 + 1 - 1 + ... + (-1)^n$, which cannot be determined unless n is known.

4. **(b)** $x + iy = \frac{4+i}{2-3i} = \frac{(4+i)(2+3i)}{13} = \frac{5+14i}{13}$

$\therefore x = 5/13$, $y = 14/13$

5. **(c)** We have, $4x + i(3x - y) = 3 + i(-6)$

Now, equating the real and the imaginary parts of above equation, we get

$4x = 3$ and $3x - y = -6$

$\Rightarrow x = \frac{3}{4}$ and $3 \times \frac{3}{4} - y = -6$

or $\frac{9}{4} + 6 = y \Rightarrow \frac{9+24}{4} = y$

$\therefore y = \frac{33}{4}$

hence, $x = \frac{3}{4}$ and $y = \frac{33}{4}$

6. **(a)** $z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p+iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$

$y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$

7. **(b)** $z = (i^{25})^3 = (i)^{75} = i^{4 \times 18 + 3} = (i^4)^{18}(i)^3$

$= i^3 = -i = 0 - i$

Polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1\left\{\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right\}$

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

8. **(a)** $\frac{z_1}{z_2} = \frac{\sqrt{3}+i\sqrt{3}}{\sqrt{3}+i} = \left(\frac{3+\sqrt{3}}{4}\right) + \left(\frac{3-\sqrt{3}}{4}\right)i$

which is represented by a point in first quadrant.

9. **(c)** Quadratic equation

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b \pm \sqrt{4ac - b^2}i}{2a}$

10. **(b)** $b^2 - 4ac = 1^2 - 4 \times 1 \times 1$

$[\because a = 1, b = 1, c = 1]$

$b^2 - 4ac = 1 - 4 = -3$

$\therefore$ the solutions are given by

$x = \frac{-1 \pm \sqrt{-3}}{2 \times 1} = \frac{-1 \pm \sqrt{3}i}{2}$

11. **(d)** Given $\sqrt{3x^2 - 2} = 2x - 1$

squaring both the sides

$\Rightarrow 3x^2 - 2 = 4x^2 + 1 - 4x \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x-3)(x-1) = 0 \Rightarrow x = 1, 3.$

12. **(c)** Let $\alpha + 3 = x$

$\therefore \alpha = x - 3$ (replace x by $x - 3$)

So the required equation

$(x-3)^2 - 5(x-3) + 6 = 0$

$\Rightarrow x^2 - 6x + 9 - 5x + 15 + 6 = 0$

$\Rightarrow x^2 - 11x + 30 = 0$

13. **(b)** Let α be the common root

$\therefore \quad 2\alpha^2 + k\alpha - 5 = 0$

$\alpha^2 - 3\alpha - 4 = 0$

Solving both equations

$\frac{\alpha^2}{-4k - 15} = \frac{\alpha}{-5+8} = \frac{1}{-6-k}$

$\Rightarrow \alpha^2 = \frac{4k+15}{k+6}$ and $\alpha = \frac{-3}{k+6}$

$\Rightarrow \left(\frac{-3}{k+6}\right)^2 = \frac{4k+15}{k+6}$

$\Rightarrow (4k+15)(k+6) = 9$

$\Rightarrow 4k^2 + 39k + 81 = 0$

$\Rightarrow k = -3$ or $k = -27/4$

14. (c) Here, $3x^2 - (a+c+2b+2d)x + (ac+2bd) = 0$

$\therefore D = (a+c+2b+2d)^2 - 12(ac+2bd)$

$= [(a+2d)-(c+2b)]^2 + 4(a+2d)(c+2b) - 12(ac+2bd)$

$= [(a+2d)-(c+2b)]^2 + 8(c-b)(d-a) > 0.$

Hence roots are real and unequal.

15. (a) $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$

$\frac{b-a}{x^2+(b+a)x+ab} = \frac{1}{x+c}$

or $x^2 + (a+b)x + ab = (b-a)x + (b-a)c$

or $x^2 + 2ax + ab + ca - bc = 0$

Since product of the roots = 0

$ab + ca - bc = 0$

$a = \frac{bc}{b+c}.$

Thus, sum of roots $= -2a = \frac{-2bc}{b+c}$

16. (a) Product of real roots $= \frac{9}{t^2} > 0,\ \forall\ t \in R$

$\therefore$ Product of real roots is always positive.

17. (a) $p+q = -p$ and $pq = q \Rightarrow q(p-1) = 0$

$\Rightarrow q = 0$ or $p = 1$.

If $q = 0$, then $p = 0$. i.e. $p = q$

$\therefore p = 1$ and $q = -2$.

18. (c) Given equation is

$(p-q)x^2 + (q-r)x + (r-p) = 0$

By using formula for finding the roots

viz : $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$, we get

$$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p)(p-q)}}{2(p-q)}$$

$$\Rightarrow\ x = \frac{(r-q) \pm (q+r-2p)}{2(p-q)} = \frac{r-p}{p-q}, 1$$

19. (a) Given. $ax^2 + bx + c = 0$ and α, β are roots of given equation

$\therefore \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ ...... (i)

Now, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta+\alpha) + \alpha\beta$

$= \frac{c}{a} \cdot \left(-\frac{b}{a}\right) + \frac{c}{a}$ [Using equation (i)]

$= -\frac{cb}{a^2} + \frac{c}{a}$

$= \frac{-cb+ac}{a^2} = \frac{c(a-b)}{a^2}$

20. (b) Consider the given equation

$x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

By taking L.C.M, we get

$\frac{x(x-1)-2}{x-1} = \frac{x-1-2}{x-1}$

$\Rightarrow\ x(x-1) - 2 = x - 3$

$\Rightarrow\ x^2 - x - 2 = x - 3$

$\Rightarrow\ x^2 - 2x + 1 = 0$

$\Rightarrow\ (x-1)^2 = 0$

$\Rightarrow\ x = 1, 1$

Thus, the given equation has two roots.

21. (d) $z_1 + z_2 = 2 + 2i$

$\Rightarrow |z_1 + z_2| = \sqrt{4+4} = \sqrt{8}$

Now $|z_1| = \sqrt{10}$, $|z_2| = \sqrt{2}$.

It is clear that, $|z_1 + z_2| < |z_1| + |z_2|$

22. (d) Let $z = r(\cos\theta + i\sin\theta)$. Then $r = 4$, $\theta = \frac{5\pi}{6}$

$\therefore z = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= 4\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -2\sqrt{3} + 2i$

23. (d) $z = \frac{3-i}{2+i} + \frac{3+i}{2-i} = \frac{(3-i)(2-i)+(3+i)(2+i)}{(2+i)(2-i)}$

$\Rightarrow\ z = 2 \Rightarrow (iz) = 2i$, which is the positive imaginary quantity

$\therefore\ \arg(iz) = \frac{\pi}{2}$

24. (a) Let the complex number z_1, z_2, z_3 denote the vertices A, B, C of an equilateral triangle ABC. Then, if O be the origin we have $OA = z_1$, $OB = z_2$, $OC = z_3$,

Therefore $|z_1| = |z_2| = |z_3| \Rightarrow OA = OB = OC$

i.e. O is the circumcentre of ΔABC

Hence $z_1 + z_2 + z_3 = 0$.

25. (b) $(1+i)^8 + (1-i)^8$

$= \{(1+i)^2\}^4 + \{(1-i)^2\}^4$

$= \{1+2i+i^2\}^4 + \{1-2i+i^2\}^4$

$= (1+2i-1)^4 + (1-2i-1)^4$

$= 2^4 . i^4 + (-2)^4 . i^4$

$= 2^4 + 2^4$ [Since $i^4 = 1$]

$= 2 \times 2^4$

$= 2^5$

26. (c) Let $z=\frac{2+5i}{4-3i}$ Rationalize,

$=\frac{2+5i}{4-3i}\times\frac{4+3i}{4+3i}$

$=\frac{8+26i-15}{(4)^2-(3i)^2}=\frac{8+26i-15}{16+(9)}$ $(\because i^2=-1)$

$=\frac{-7+26i}{16+9}=\frac{-7+26i}{25}$

27. (c) Let $z=1+i$

then $z^2=(1+i)^2$

$=1^2+i^2+2.1.i$

$=1+i^2+2i$

$=1-1+2i$ $(\because i^2=-1)$

$=2i$

Now, $2i\times-\frac{i}{2}\Rightarrow -i^2=1$

Hence, $-\frac{i}{2}$ is multiplicative inverse of z^2.

28. (d) Let $z=\left(\frac{1}{1-2i}+\frac{2}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

$=\left[\frac{1+i+3-6i}{(1-2i)(1+i)}\right]\left[\frac{3+4i}{2-4i}\right]$

$=\left[\frac{4-5i}{3-i}\right]\left[\frac{3+4i}{2-4i}\right]=\left[\frac{32+i}{2-14i}\right]$

$=\frac{32+i}{2-14i}\times\frac{2+14i}{2+14i}=\frac{64+448i+2i-14}{4+196}$

$=\frac{50+450i}{200}=\frac{1}{4}+\frac{9}{4}i$

29. (b) Let $z=\frac{1+2i}{1-i}$ be the given complex number.

$\Rightarrow z=\frac{1+2i}{1-i}\times\frac{1+i}{1+i}=\frac{1+i+2i+2i^2}{1-i^2}$

$=\frac{-1+3i}{2}=\frac{-1}{2}+\frac{3}{2}i$

$\Rightarrow (x, y)=\left(\frac{-1}{2},\frac{3}{2}\right)$ which lies in II[nd] quadrant.

30. (c) Let $r(\cos\theta+i\sin\theta)=\frac{1+i\sqrt{3}}{\sqrt{3}+1}=\frac{1}{\sqrt{3}+1}+i\frac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow r\cos\theta=\frac{1}{\sqrt{3}+1}$; $r\sin\theta=\frac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow \tan\theta=\sqrt{3}\Rightarrow\theta=\frac{\pi}{3}$.

31. (d) $(1+i)^4\times\left(1+\frac{1}{i}\right)^4=(1+i)^4\times(1-i)^4$

$=(1-i^2)^4=(1+1)^4=2^4=16$.

32. (a) $(1+i)^6=\{(1+i)^2\}^3=(1+i^2+2i)^3=(1-1+2i)^3$

$=8i^3=-8i$ and

$(1-i)^3=1-i^3-3i+3i^2$

$=1+i-3i-3=-2-2i$

$\therefore$ $(1+i)^6+(1-i)^3=-8i-2-2i=-2-10i$

33. (b) $(x+iy)^{\frac{1}{3}}=a+ib$

$\Rightarrow x+iy=(a+ib)^3$

$\Rightarrow x+iy=a^3-ib^3+i3a^2b-3ab^2$

$=a^3-3ab^2+i(3a^2b-b^3)$

$\Rightarrow x=a^3-3ab^2$ and $y=3a^2b-b^3$

So, $\frac{x}{a}-\frac{y}{b}=a^2-3b^2-3a^2+b^2$

$=-2a^2-2b^2=-2(a^2+b^2)$

34. (a) $\frac{i^{4n+1}-i^{4n-1}}{2}=\frac{i^{4n}i-i^{4n}i^{-1}}{2}$

$=\frac{i-\frac{1}{i}}{2}=\frac{i^2-1}{2i}=\frac{-2}{2i}=i$

35. (b) $\sqrt{-3}=i\sqrt{3}$, $\sqrt{-6}=i\sqrt{6}$

So, $\sqrt{(-3)}\sqrt{(-6)}=i^2\,3\sqrt{2}=-3\sqrt{2}$

36. (b) We have, $z(2-i)=(3+i)$

$\Rightarrow z=\left(\frac{3+i}{2-i}\right)\times\left(\frac{2+i}{2+i}\right)=\frac{5+5i}{5}$

$\Rightarrow z=1+i$

$\Rightarrow z^2=2i\Rightarrow z^{20}=-2^{10}$

37. (d) $(1+i)^2=1+i^2+2i=2i$

$\therefore$ $\frac{(1+i)^2}{3-i}=\frac{2i(3+i)}{3^2-i^2}=\frac{6i-2}{10}=\frac{-1+3i}{5}$

$\therefore$ Real part $=\frac{-1}{5}$.

38. (b) Let $z=\frac{3+4i}{4-5i}\times\frac{4+5i}{4+5i}=-\frac{8}{41}+\frac{31}{41}i$

Then, $\bar{z}=-\frac{8}{41}-\frac{31}{41}i$

and $|z|=\sqrt{\left(-\frac{8}{41}\right)^2+\left(\frac{31}{41}\right)^2}=\frac{5}{\sqrt{41}}$

$\therefore$ Multiplicative inverse of z

$=\frac{\bar{z}}{|z|^2}=\frac{-\frac{8}{41}-\frac{31}{41}i}{\frac{25}{41}}=-\frac{8}{25}-\frac{31}{25}i$

39. (c) Let $z = \dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}} \times \dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}+\sqrt{5-12i}}$

$= \dfrac{5+12i+5-12i+2\sqrt{25+144}}{5+12i-5+12i}$

$= \dfrac{3}{2i} = \dfrac{3i}{-2} = 0 - \dfrac{3}{2}i$

Therefore, the conjugate of $z = 0 + \dfrac{3}{2}i$

40. (a) $z = \dfrac{7-i}{3-4i} = \dfrac{7-i}{3-4i} \times \dfrac{3+4i}{3+4i} = \dfrac{21+4+i(28-3)}{25}$

$= 1 + i$

$\therefore\ |z| = |1+i| = \sqrt{2}$

$\therefore\ |z|^{14} = \left(\sqrt{2}\right)^{14} = \left[\left(\sqrt{2}\right)^2\right]^7 = 2^7$

41. (c) Let $1 = r\cos\theta$, $\sqrt{3} = r\sin\theta$

By squaring and adding, we get

$r^2(\cos^2\theta + \sin^2\theta) = 4$

i.e, $r = \sqrt{4} = 2$

Therefore, $\cos\theta = \dfrac{1}{2}$, $\sin\theta = \dfrac{\sqrt{3}}{2}$, which gives $\theta = \dfrac{\pi}{3}$

Therefore, required polar form is

$z = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right).$

42. (d) $\dfrac{\left(1+i\sqrt{3}\right)(2+2i)}{\sqrt{3}-i} = \dfrac{2+2\sqrt{3}i+2i-2\sqrt{3}}{\sqrt{3}-i}$

$= \dfrac{\left(2-2\sqrt{3}\right)+\left(2\sqrt{3}+2\right)i}{\sqrt{3}-i} \times \dfrac{\sqrt{3}+i}{\sqrt{3}+i}$

$= \dfrac{2\sqrt{3}-6+2i-2\sqrt{3}i+6i+2\sqrt{3}i-2\sqrt{3}-2}{3+1}$

$= \dfrac{8i-8}{4} = -2 + 2i$

$\therefore$ Modulus $= \sqrt{(-2)^2+(2)^2} = 2\sqrt{2}.$

43. (d) Since $\left(\dfrac{i}{2} - \dfrac{2}{i}\right) = \dfrac{i}{2} - \dfrac{2i}{i^2} = \dfrac{i}{2} + 2i = \dfrac{5}{2}i$

So, argument is $\tan^{-1}\left(\dfrac{b}{a}\right) = \tan^{-1}\left(\dfrac{\frac{5}{2}}{0}\right) = \dfrac{\pi}{2}.$

44. (d) Let $z = 7 - 24i$

$= 7 - 2\cdot 4\cdot 3i = 16 - 9 - 2\cdot 4\cdot 3i$

$= (4)^2 + (-3i)^2 - 2\cdot 4\cdot 3i$

$= (4-3i)^2$

$\therefore\ \sqrt{7-24i} = \pm(4-3i)$

45. (c) Here, $b^2 - 4ac = 1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$

Therefore, the solutions are

$\dfrac{-1\pm\sqrt{-19}}{2\sqrt{5}} = \dfrac{-1\pm\sqrt{19}i}{2\sqrt{5}}$

46. (d) Given equation is $x^2 + 2x + 4 = 0$

Since α, β are roots of this equation

$\therefore\ \alpha + \beta = -2$ and $\alpha\beta = 4$

Now, $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} = \dfrac{\alpha^3+\beta^3}{(\alpha\beta)^3}$

$= \dfrac{(\alpha+\beta)\left(\alpha^2+\beta^2-\alpha\beta\right)}{(\alpha\beta)^3} = \dfrac{(-2)\left((\alpha+\beta)^2-3\alpha\beta\right)}{4\times4\times4}$

$= \dfrac{-2(4-12)}{4\times4\times4} = \dfrac{(-2)\times(-8)}{4\times4\times4} = \dfrac{1}{4}$

47. (d) Since α, β are roots of the equation $ax^2 + bx + c = 0$

$\therefore\ \alpha + \beta = \dfrac{-b}{a}$, $\alpha\beta = \dfrac{c}{a}$ ... (i)

Now, $\dfrac{1}{a\alpha+b} + \dfrac{1}{a\beta+b} = \dfrac{a(\alpha+\beta)+2b}{(a\alpha+b)(a\beta+b)}$

$= \dfrac{a(\alpha+\beta)+2b}{a^2\alpha\beta + ab(\alpha+\beta)+b^2}$

$= \dfrac{a\left(-\dfrac{b}{a}\right)+2b}{a^2\cdot\dfrac{c}{a} + ab\left(-\dfrac{b}{a}\right)+b^2} = \dfrac{b}{ac}.$ [using (i)]

48. (b) Since complex roots always occur in conjugate pair.

$\therefore$ Other conjugate root is $1 + i$.

Sum of roots $= \dfrac{-a}{1} = (1-i) + (1+i) \Rightarrow a = -2$

Product of roots $= \dfrac{b}{1} = (1-i)(1+i) \Rightarrow b = 2$

49. (a) (a) $|z_1 z_2| = |z_1||z_2|$

(b) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

(c) $|z_1 + z_2| \neq |z_1| + |z_2|$

(d) $|z_1 + z_2| \leq |z_1| - |z_2|$

50. (a) A number $z = a + ib$ where $a, b \in R$ is called complex number.

51. (c) For a quadratic equation $ax^2 + bx + c = 0$

$x = \dfrac{-b\pm\sqrt{D}}{2a}$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

For real roots $D \geq 0$. If roots are not real, then $D < 0$.

52. (a) Here, $z = a + ib$, then real part of z, i.e., $Re(z) = a$ and imaginary part of z, i.e., $Im(z) = b$.

53. (c) Square root of negative number is imaginary in general

$(a)^{\frac{1}{2n}}$, where $a < 0$ and $n \in N$ gives imaginary number.

54. (a) Here, $x = \sqrt{-16}$

$$x = \sqrt{-1 \times 16}$$

$$= \sqrt{-1} \times \sqrt{4 \times 4} = 4i$$

55. (a) Let $z_1 = 6 + 3i$ and $z_2 = 2 - i$

Then, $\frac{z_1}{z_2} = (6+3i)\frac{1}{2-i} = \frac{(6+3i)(2+i)}{(2-i)(2+i)}$

$$= (6+3i)\left(\frac{2}{2^2+(-1)^2} + i\frac{1}{2^2+(-1)^2}\right)$$

$$= (6+3i)\left(\frac{2}{5} + i\frac{1}{5}\right)$$

$$= (6+3i)\frac{(2+i)}{5}$$

$$= \frac{1}{5}[12 - 3 + i(6+6)]$$

$$= \frac{1}{5}(9 + 12i)$$

56. (d) $(1 + i)^5 (1 - i)^5 = (1 - i^2)^5$

$= 2^5 = 32$

57. (c) $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{2 - i + 1 + i + 1}{2 - i - (1+i) + 1}\right|$

[$\because z_1 = 2 - i$ and $z_2 = 1 + i$]

$$= \left|\frac{4}{2 - i - 1 - i + 1}\right| = \left|\frac{4}{2 - 2i}\right| = \left|\frac{2}{1-i}\right| = \frac{2}{|1-i|}$$

$\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$$= \frac{2}{\sqrt{(1)^2 + (-1)^2}} \quad \left[\because |z| = \sqrt{a^2 + b^2}\right]$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}.$$

58. (a) $\frac{(1+i)^3}{(1-i)^3} - \frac{(1-i)^3}{(1+i)^3} = x + iy$

$$\Rightarrow \frac{(1 + i^2 + 2i)^3 - (1 + i^2 - 2i)^3}{(1 - i^2)^3} = x + iy$$

$$\Rightarrow \frac{8i^3 + 8i^3}{2^3} = x + iy$$

$\Rightarrow 2i^3 = x + iy \Rightarrow -2i = x + iy$

$\Rightarrow x = 0, y = -2$

59. (c) If $z = x + iy$ is the additive inverse of $1 - i$, then

$(x + iy) + (1 - i) = 0$

$\Rightarrow x + 1 = 0, y - 1 = 0$

$\Rightarrow x = -1, y = 1$

$\therefore$ The additive inverse of $1 - i$ is $z = -1 + i$

Trick: Since $(1 - i) + (-1 + i) = 0$.

60. (c) Let $z = x + iy$, then its conjugate $\bar{z} = x - iy$

Given that $z^2 = (\bar{z})^2$

$\Rightarrow x^2 - y^2 + 2ixy = x^2 - y^2 - 2ixy$

$\Rightarrow 4ixy = 0$

If $x \neq 0$, then $y = 0$ and if $y \neq 0$, then $x = 0$.

61. (b) $z = x + iy$

$\Rightarrow |z|^2 = x^2 + y^2 = 1$ ... (i)

Now, $\left(\frac{z-1}{z+1}\right) = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$

$$= \frac{(x^2 + y^2 - 1) + 2iy}{(x+1)^2 + y^2} = \frac{2iy}{(x+1)^2 + y^2}$$

[By equation (i)]

Hence, $\left(\frac{z-1}{z+1}\right)$ is purely imaginary.

62. (d) Let $z = x + iy$, $\bar{z} = x - iy$

Since $\arg(z) = \theta = \tan^{-1}\frac{y}{x}$

$\arg(\bar{z}) = \theta = \tan^{-1}\left(\frac{-y}{x}\right)$

Thus, $\arg(z) \neq \arg(\bar{z})$.

63. (d) $\sqrt{a + ib} = x + yi$

$\Rightarrow \left(\sqrt{a+ib}\right)^2 = (x + yi)^2$

$\Rightarrow a = x^2 - y^2$, $b = 2xy$ and hence

$\sqrt{a - ib} = \sqrt{x^2 - y^2 - 2xyi} = \sqrt{(x - yi)^2} = x - iy$

Note: In the question, it should have been given that $a, b, x, y \in R$.

64. (b) Given equation is $x^2 - 2x(1 + 3k) + 7(2k + 3) = 0$

Since, it has equal roots.

$\therefore$ Discriminant $D = 0$

$\Rightarrow b^2 - 4ac = 0 \Rightarrow 4(1 + 3k)^2 - 4 \times 7(2k + 3) = 0$

$\Rightarrow 1 + 9k^2 + 6k - 14k - 21 = 0$

$\Rightarrow 9k^2 - 8k - 20 = 0$

$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow 9k(k - 2) + 10(k - 2) = 0$

$\Rightarrow k = \frac{-10}{9}, 2$

Only $k = 2$ satisfy given equation.

65. (b) Given equation is $3^{2x} - 10.3^x + 9 = 0$ can be written as $(3^x)^2 - 10(3^x) + 9 = 0$

Let $a = 3^x$, then it reduces to the equation

$a^2 - 10a + 9 = 0 \Rightarrow (a - 9)(a - 1) = 0$

$\Rightarrow a = 9, 1$

Now, $a = 3^x$

$\Rightarrow 9 = 3^x \Rightarrow 3^2 = 3^x \Rightarrow x = 2$

and $1 = 3^x \Rightarrow 3^0 = 3^x \Rightarrow x = 0$

Hence, roots are 0, 2.

66. (c) $x^2 + y^2 = 25$ and $xy = 12$

$\Rightarrow x^2 + \left(\frac{12}{x}\right)^2 = 25$

$\Rightarrow x^4 + 144 - 25x^2 = 0$

$\Rightarrow (x^2 - 16)(x^2 - 9) = 0$

$\Rightarrow x^2 = 16$ and $x^2 = 9$

$\Rightarrow x = \pm 4$ and $x = \pm 3$.

67. (b) Equations $px^2 + 2qx + r = 0$ and

$qx^2 - 2\left(\sqrt{pr}\right)x + q = 0$ have real roots, then

from first

$4q^2 - 4pr \geq 0 \Rightarrow q^2 - pr \geq 0$

$\Rightarrow q^2 \geq pr$ ... (i)

and from second

$4(pr) - 4q^2 \geq 0$ (for real root)

$\Rightarrow pr \geq q^2$ ... (ii)

From (i) and (ii), we get result

$q^2 = pr$.

68. (b) The roots of the equations are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) Let $b^2 - 4ac > 0$, $b > 0$

Now, if $a > 0$, $c > 0$, $b^2 - 4ac < b^2$

$\Rightarrow$ the roots are negative.

(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a}, \quad \left(i = \sqrt{-1}\right)$$

which are imaginary and have negative real part. $[\because b > 0]$

$\therefore$ In each case, the roots have negative real part.

69. (a) Given equation $2ax^2 + (2a + b)x + b = 0$, $(a \neq 0)$

Now, its discriminant $D = B^2 - 4AC$

$= (2a + b)^2 - 4.2ab = (2a - b)^2$

Hence, D is a perfect square. So, given equation has rational roots.

70. (a) Since $2 + i\sqrt{3}$ is a root, therefore, $2 - i\sqrt{3}$ will be other root. Now sum of the roots $= 4 = -p$ and product of roots $= 7 = q$. Hence $(p, q) = (-4, 7)$.

71. (c) Let the roots be α and β

$\Rightarrow \alpha + \beta = -p$, $\alpha\beta = q$

Given, $\alpha + \beta = \alpha^2 + \beta^2$

But $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow -p = (-p)^2 - 2q$

$\Rightarrow p^2 - 2q = -p \Rightarrow p^2 + p = 2q$

72. (c) Let the common root be y.

Then, $y^2 + py + q = 0$ and $y^2 + \alpha y + \beta = 0$

On solving by cross multiplication, we have

$$\frac{y^2}{p\beta - q\alpha} = \frac{y}{q - \beta} = \frac{1}{\alpha - p}$$

$$\therefore \; y = \frac{q - \beta}{\alpha - p} \text{ and } \frac{y^2}{y} = y = \frac{p\beta - q\alpha}{q - \beta}.$$

73. (d) Let α be a common root, then

$\alpha^2 + a\alpha + 10 = 0$ ... (i)

and $\alpha^2 + b\alpha - 10 = 0$ ... (ii)

From (i) – (ii),

$$(a - b)\alpha + 20 = 0 \Rightarrow \alpha = -\frac{20}{a - b}$$

Substituting the value of α in (i), we get

$$\left(-\frac{20}{a - b}\right)^2 + a\left(-\frac{20}{a - b}\right) + 10 = 0$$

$\Rightarrow 400 - 20a(a - b) + 10(a - b)^2 = 0$

$\Rightarrow 40 - 2a^2 + 2ab + a^2 + b^2 - 2ab = 0$

$\Rightarrow a^2 - b^2 = 40$.

74. (a) Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$

If roots are real, then $D \geq 0$

$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$

$\Rightarrow -a + 3 \geq 0$

$\Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$

As roots are less than 3, hence $f(3) > 0$.

$9 - 6a + a^2 + a - 3 > 0$

$\Rightarrow a^2 - 5a + 6 > 0$

$\Rightarrow (a - 2)(a - 3) > 0 \Rightarrow$ either $a < 2$ or $a > 3$

Hence, $a < 2$ satisfy all.

STATEMENT TYPE QUESTIONS

75. (c) I. Given $x^2 + 3x + 5 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 3$, $c = 5$

Now, $D = b^2 - 4ac$

$= (3)^2 - 4 \times 1 \times 5 = 9 - 20 = -11 < 0$

$$\Rightarrow x = \frac{-3 \pm \sqrt{-11}}{2 \times 1}$$

$$\therefore x = \frac{-3 \pm i\sqrt{11}}{2} \quad \left[\because \sqrt{-1} = i\right]$$

II. Given $x^2 - x + 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get a = 1, b = –1, c = 2

Now, $D = b^2 - 4ac$

$= (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$

$\Rightarrow x = \dfrac{-(-1) \pm \sqrt{-7}}{2 \times 1}$

$= \dfrac{1 \pm i\sqrt{7}}{2}$ $\quad [\because \sqrt{-1} = i]$

76. **(c)** Given that, $\overline{z_1} + i\,\overline{z_2} = 0$

$\Rightarrow z_1 = iz_2$, i.e. $z_2 = -iz_1$

Thus, $\arg(z_1 z_2) = \arg z_1 + \arg(-iz_1) = \pi$

$\because \arg(z_1 z_2) = \arg z_1 + \arg z_2$

$\Rightarrow \arg(-iz_1^{\,2}) = \pi$

$\Rightarrow \arg(-i) + 2\arg(z_1) = \pi$

$\Rightarrow \dfrac{-\pi}{2} + 2\arg(z_1) = \pi$

$\Rightarrow \arg(z_1) = \dfrac{3\pi}{4}$

77. **(a)** I. We have,

$\dfrac{1+i}{1-i} = \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{1-1+2i}{1+1} = i = 0 + i$

Now, let us put $0 = r\cos\theta$, $1 = r\sin\theta$

Squaring and adding,

$r^2 = 1$, i.e, $r = 1$

So, $\cos\theta = 0$, $\sin\theta = 1$

Therefore, $\theta = \dfrac{\pi}{2}$

Hence, the modulus of $\dfrac{1+i}{1-i}$ is 1 and the argument is $\dfrac{\pi}{2}$.

II. We have :

$\dfrac{1}{1+i} = \dfrac{1-i}{(1+i)(1-i)} = \dfrac{1-i}{1+1} = \dfrac{1}{2} - \dfrac{i}{2}$

Let $\dfrac{1}{2} = r\cos\theta$, $-\dfrac{1}{2} = r\sin\theta$

Proceeding as

$r = \dfrac{1}{\sqrt{2}}$; $\cos\theta = \dfrac{1}{\sqrt{2}}$, $\sin\theta = \dfrac{-1}{\sqrt{2}}$

Therefore, $\theta = \dfrac{-\pi}{4}$

[$\because \cos\theta > 0$ and $\sin\theta < 0$ is in IV quadrant]

Hence, the modulus of $\dfrac{1}{1+i}$ is $\dfrac{1}{\sqrt{2}}$ and the argument is $-\dfrac{\pi}{4}$.

78. **(c)** (a + ib) (c + id) (e + if) (g + ih) = A + iB

Taking modulus on both sides, we get

|(a + ib) (c + id) (e + if) (g + ih)| = |A + iB|

$\Rightarrow$ |a + ib| |c + id| |e + if| |g + ih| = |A + iB|

$[\because |z_1 z_2 \ldots z_n| = |z_1||z_2||z_3|\ldots|z_n|]$

$\Rightarrow \sqrt{a^2+b^2}\sqrt{c^2+d^2}\sqrt{e^2+f^2}\sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

$\left[\because \text{If } z = a + ib, \text{ then } |z| = \sqrt{a^2+b^2}\right]$

Squaring on both sides, we get

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

79. **(c)** I. Additive inverse of $(1-i) = -(1-i) = -1 + i$

II. Since, difference of two complex numbers is also a complex number and $z_1 - z_2$ can be written as $(z_1) + (-z_2)$ which is sum of z_1 and additive inverse of z_2.

III. $\dfrac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \dfrac{1+\sqrt{2}i}{1+\sqrt{2}i} = \dfrac{5+5\sqrt{2}i+\sqrt{2}i-2}{1+2}$

$= \dfrac{3+6\sqrt{2}i}{3} = 1 + 2\sqrt{2}i$

80. **(d)** By definition, both the statements are correct.

81. **(c)** II. $x^2 + 3x + 5 = 0$

$x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(5)}}{2}$

$= \dfrac{-3 \pm \sqrt{9-20}}{2}$

$= \dfrac{-3 \pm \sqrt{11}i}{2}$

82. **(c)** I. $x = 1 + 2i$

$\Rightarrow (x-1) = 2i$

$\Rightarrow (x-1)^2 = (2i)^2 \Rightarrow x^2 - 2x + 5 = 0$

Consider

$x^3 + 7x^2 - x + 16 = x(x^2 - 2x + 5) + 9(x^2 - 2x + 5) + (12x - 29)$

$= x(0) + 9(0) + 12x - 29$

$= -17 + 24i$

II. $iz^3 + z^2 - z + i = 0$

$z^3 - iz^2 + iz + 1 = 0$ (Dividing both side by i)

$\Rightarrow (z-i)(z^2+i) = 0$

$\Rightarrow z = i$ or $z^2 = -i$

Now, $z = i \Rightarrow |z| = |i| = 1$

$z^2 = -i \Rightarrow |z^2| = |-i| = 1$

$\Rightarrow |z|^2 = 1$

$\Rightarrow |z| = 1$

83. **(d)** I. $z\bar{z} = (a + ib)(a - ib)$

$= a^2 - (ib)^2 = a^2 + b^2$

$= |z|^2$

III. $z^{-1} = \frac{3}{(3)^2 + (-2)^2} + \frac{i(2)}{3^2 + (-2)^2}$

$= \frac{3}{13} + \frac{2}{13}i$

84. (c) $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1-i^2}$

$= \frac{1+i^2+2i}{2} = \frac{2i}{2} = i \equiv 0 + i$

$\left|\frac{1+i}{1-i}\right| = |i| = 1$

Now, $r\cos\theta = 0, r\sin\theta = 1$

$r^2 = 1 \Rightarrow r = 1$

$\therefore \cos\theta = 0$ and $\sin\theta = 1$

$\Rightarrow \theta = \frac{\pi}{2}$

MATCHING TYPE QUESTIONS

85. (b) A. Let $z = 4 - 3i$

Then, its multiplicative inverse is

$\frac{1}{z} = \frac{1}{4-3i} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i} = \frac{4+3i}{16-9i^2}$

[use $(a - b)(a + b) = a^2 - b^2$]

$= \frac{4+3i}{16+9}$ [$\because i^2 = -1$]

$= \frac{4+3i}{25} = \frac{4}{25} + \frac{3i}{25}$

B. Let $z = \sqrt{5} + 3i$

Then, its multiplicative inverse is

$\frac{1}{z} = \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$= \frac{\sqrt{5}-3i}{5-9i^2}$ [use $(a + b)(a - b) = a^2 - b^2$]

$= \frac{\sqrt{5}-3i}{5+9} = \frac{\sqrt{5}-3i}{14}$ [$\because i^2 = -1$]

$= \frac{\sqrt{5}}{14} - \frac{3i}{14}$

C. Let $z = -i$

Then, its multiplicative inverse is

$\frac{1}{z} = -\frac{1}{i} = -\frac{1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$ [$\because i^2 = -1$]

$= 0 + i.$

86. (d) A. $(1 - i)^4 = [(1 - i)^2]^2$

$= (1 + i^2 - 2i)^2$ [use $(a - b)^2 = a^2 + b^2 - 2ab$]

$= (1 - 1 - 2i)^2$ [$\because i^2 = -1$]

$= (-2i)^2 = (-2)^2 i^2$

$= 4(-1) = -4 + 0i$

B. $\left(\frac{1}{3} + 3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3 \times \frac{1}{3} \times 3i\left(\frac{1}{3} + 3i\right)$

$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3} + 3i\right)$

$= \frac{1}{27} - 27i + 3i \times \frac{1}{3} + 3i \times 3i$ [$\because i^3 = -i$]

$= \frac{1}{27} - 27i + i + 9i^2$

$= \frac{1}{27} - 27i + i - 9$ [$\because i^2 = -1$]

$= \left(\frac{1}{27} - \frac{9}{1}\right) - i(27 - 1)$

$= \left(\frac{1-243}{27}\right) - 26i = -\frac{242}{27} - 26i$

C. $(-1)^3\left(2 + \frac{1}{3}i\right)^3$

$= -\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3 \times 2 \times \frac{1}{3}i\left(2 + \frac{1}{3}i\right)\right]$

$= -\left[8 + \frac{1}{27}i^3 + 2i\left(2 + \frac{1}{3}i\right)\right]$

$= -\left[8 - \frac{1}{27}i + 4i + \frac{2}{3}i^2\right]$ [$\because i^3 = -i$]

$= -\left[8 - \frac{1}{27}i + 4i - \frac{2}{3}\right]$ [$\because i^2 = -1$]

$= -\left[\left(\frac{8}{1} - \frac{2}{3}\right) + i\left(\frac{4}{1} - \frac{1}{27}\right)\right]$

$= -\left[\left(\frac{24-2}{3}\right) + i\left(\frac{108-1}{27}\right)\right]$

$= -\left[\frac{22}{3} + i\frac{107}{27}\right]$

87. (b) We know that,

$i = \sqrt{-1},\ i^2 = -1$

$\Rightarrow i^{-1} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i$

$\Rightarrow i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$

$\Rightarrow$ $i^{-3} = \frac{1}{i^3} = \left\{\frac{i}{i^3 \times i}\right\}$

[multiplying numerator and denominator by i]

$\Rightarrow$ $\frac{i}{i^4} = i$

$\Rightarrow$ $i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$

88. **(b)** (A) $(1-i)-(-1+i.6) = (1-i)+(1-6i)$
$= 1+1-i-6i$
$= 2-7i = (a+ib)$,
where $a = 2, b = -7$

(B) $\left(\frac{1}{5}+i.\frac{2}{5}\right)-\left(4+i.\frac{5}{2}\right) = \left(\frac{1}{5}+\frac{2}{5}i\right)+\left(-4-\frac{5}{2}i\right)$

$= \frac{1}{5}-4+\frac{2}{5}i-\frac{5}{2}i = -\frac{19}{5}-\left(-\frac{2}{5}+\frac{5}{2}\right)i$

$= -\frac{21}{5}-\frac{21}{10}i$

(C) $\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3+3\left(\frac{1}{3}\right)^2(3i)+3.\left(\frac{1}{3}\right)(3i)^2+(3i)^3$

$= \frac{1}{27}+i+9(-1)+27i^3$

$= \frac{1}{27}+i+9(-1)+27i.(i^2)$

$= \frac{1}{27}+i+9(-1)+27i.(-1)$

$= \frac{1}{27}+i-9-27i = -\frac{242}{27}-26i$

(D) $(1-i)^4 = [(1-i)^2]^2$
$= (1+i^2-2i)^2 = (1-1-2i)^2$
$= (-2i)^2 = 4i^2 = 4(-1) = -4$

(E) $\left(-2-\frac{1}{3}i\right)^3$

$= (-2)^3-3(-2)^2.\left(\frac{1}{3}i\right)+3(-2)\left(-\frac{1}{3}i\right)^2-\left(\frac{1}{3}i\right)^3$

$= -8-4i-6\times\frac{1}{9}(i^2)-\frac{1}{27}i^3$

$= -8-4i-\frac{2}{3}(-1)-\frac{1}{27}i.i^2$

$= -8-4i+\frac{2}{3}-\frac{1}{27}i.(-1)$

$= -8-4i+\frac{2}{3}+\frac{1}{27}i$

$= -\frac{22}{3}-\frac{107}{27}i$

89. **(d)** (A) We have multiplicative inverse of $4-3i$

$= \frac{1}{4-3i}\times\frac{4+3i}{4+3i}$

$= \frac{4+3i}{4^2-9i^2} = \frac{4+3i}{16+9} = \frac{4+3i}{25} = \frac{4}{25}+i\frac{3}{25}$

(B) We have multiplicative inverse of $\sqrt{5}+3i$

$= \frac{1}{\sqrt{5}+3i}\times\frac{\sqrt{5}-3i}{\sqrt{5}-3i}$ (multiply by conjugate)

$= \frac{\sqrt{5}-3i}{5-9i^2} = \frac{\sqrt{5}-3i}{5+9} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14}-\frac{3}{14}i$
$[\because (a+ib)(a-ib) = a^2+b^2]$

(C) We have multiplicative inverse of $-i = \frac{1}{-i}$.

Multiply by conjugate

$= \frac{1}{-i}\times\frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i = 0+i.1$

(D) $z = (2+\sqrt{3}i)^2 = 4+3i^2+4\sqrt{3}i$

$= 1+4\sqrt{3}i$

$\therefore$ $\frac{1}{z} = \frac{1}{4+\sqrt{3}i} = \frac{1-4\sqrt{3}i}{1+48}$ (On rationalizing)

90. **(c)** (A) $2x^2+x+1 = 0$. Comparing with $ax^2+bx+c = 0$
$a = 2, b = 1, c = 1$
$b^2-4ac = 1^2-4.2.1 = 1-8 = -7$

$\therefore$ $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1\pm\sqrt{-7}}{2.2}$

$= \frac{-1\pm\sqrt{7}i}{4}$

(B) $x^2+3x+9 = 0$ $\therefore$ $a = 1, b = 3, c = 9$
$b^2-4ac = 3^2-4.1.9 = 9-36 = -27$

$\therefore$ $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-3\pm\sqrt{-27}}{2\times1}$

$= \frac{-3\pm(3\sqrt{3})i}{2}$

(C) $-x^2+x-2 = 0$ or $x^2-x+2 = 0$
$a = 1, b = -1, c = 2$

Hence, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-(-1)\pm\sqrt{(-1)^2-4\times1\times2}}{2\times1}$

$= \frac{1\pm\sqrt{1-8}}{2}$

$= \frac{1\pm\sqrt{-7}}{2} = \frac{1\pm\sqrt{7}i}{2}$

(D) $x^2+3x+5=0$

$a=1, b=3, c=5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{(3)^2-4\times1\times5}}{2\times1}$$

$$x=\frac{-3\pm\sqrt{9-20}}{2}$$

$$x=\frac{-3\pm\sqrt{-11}}{2}$$

$$x=\frac{-3\pm\sqrt{11}i}{2}$$

91. (a)

(A) We have $1-i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=1, r\sin\theta=-1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=1^2+(-1)^2$

$\Rightarrow r^2.1=1+1 \Rightarrow r^2=2$

$\therefore$ $r=\sqrt{2}$, By dividing $\frac{r\sin\theta}{r\cos\theta}=\frac{-1}{1}=-1$

$\Rightarrow$ $\tan\theta=-1$ i.e., θ lies in fourth quadrant.

$\Rightarrow$ $\theta=-45$

$\Rightarrow$ $\theta=-\frac{\pi}{4}$

$\therefore$ Polar form of $1-i$

$$=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$$

(B) We have $-1+i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=-1$ and $r\sin\theta=1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=(-1)^2+1^2 \Rightarrow r^2.1=1+1$

$\therefore$ $r^2=2$ $\therefore$ $r=\sqrt{2}$

By dividing, $\frac{r\sin\theta}{r\cos\theta}=\frac{1}{-1}=-1 \Rightarrow \tan\theta=-1$

$\therefore$ θ lies in second quadrant ;

$\theta=180°-45°=135°$ i.e $\theta=\frac{3\pi}{4}$

$\therefore$ Polar form of $-1+i$

$$=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

(C) we have $-1-i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=-1$ and $r\sin\theta=-1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=(-1)^2+(-1)^2$

$\Rightarrow$ $r^2.1=1+1$

$\Rightarrow$ $r^2=2$

$\therefore$ $r=\sqrt{2}$

By dividing $\frac{r\sin\theta}{r\cos\theta}=\frac{-1}{-1}=1 \Rightarrow \tan\theta=1$

$\therefore$ θ lies in III[rd] quadrant.

$\theta=-180°+45°=-135°$ or $\theta=-\frac{3\pi}{4}$

$\therefore$ Polar form of $-1-i$

$$=\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$$

(D) $r=\sqrt{3}+i=r(\cos\theta+i\sin\theta)$

$\therefore$ $r\cos\theta=\sqrt{3}$, $r\sin\theta=1$

Squaring and adding $r^2=3+1=4, r=2$

Also $\tan\theta=\frac{1}{\sqrt{3}}$, $\sin\theta$ and $\cos\theta$ both are positive.

$\therefore$ θ lies in the I quadrant

$\therefore \theta=30°=\frac{\pi}{6}$

$\therefore$ Polar form of z is $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

92. (b) $z=x+iy \Rightarrow |z|=\sqrt{x^2+y^2}$

INTEGER TYPE QUESTIONS

93. (a) $i^{57}+\frac{1}{i^{25}}=(i^4)^{14}.i+\frac{1}{(i^4)^6.i}$

$=i+\frac{1}{i}$ $(\because i^4=1)$

$=i-i$ $\left(\because \frac{1}{i}=-i\right)$

$=0$

94. (c) $z=2-3i \Rightarrow z-2=-3i$

Squaring, we get

$z^2-4z+4=-9 \Rightarrow z^2-4z+13=0$

95. (b) Given: $\frac{c+i}{c-i}=a+ib$

Then, $a+ib=\frac{c+i}{c-i}\cdot\frac{c+i}{c+i}=\frac{c^2-1+2ic}{c^2+1}$

$\Rightarrow$ $a=\frac{c^2-1}{c^2+1}$ and $b=\frac{2c}{c^2+1}$

$\Rightarrow$ $(a^2+b^2)=\frac{(c^2-1)^2+4c^2}{(c^2+1)^2}$

$$=\frac{c^4+1-2c^2+4c^2}{(c^2+1)^2}=\frac{(c^2+1)^2}{(c^2+1)^2}=1.$$

96. (a) We have,

$$x + iy = \frac{a + ib}{a - ib} \quad \text{... (i)}$$

Its conjugate,

$$x - iy = \frac{a - ib}{a + ib} \quad \text{... (ii)}$$

Multiply (i) and (ii),

$$(x + iy)(x - iy) = \frac{a + ib}{a - ib} \times \frac{a - ib}{a + ib}$$

$x^2 + y^2 = 1.$

97. (b) $(x + iy)^{\frac{1}{3}} = a - ib$

$x + iy = (a - ib)^3 = (a^3 - 3ab^2) + i(b^3 - 3a^2 b)$

$\Rightarrow x = a^3 - 3ab^2,\ y = b^3 - 3a^2 b$

$\Rightarrow \frac{x}{a} = a^2 - 3b^2,\ \frac{y}{b} = b^2 - 3a^2$

$\therefore \frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2$

$\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2) = k(a^2 - b^2)$

$\therefore k = 4.$

98. (b) $2x^2 - (p + 1)x + (p - 1) = 0$

Given $\alpha - \beta = \alpha\beta \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = \alpha^2 \beta^2$

$\Rightarrow \frac{(p-1)^2}{4} = \frac{(p+1)^2}{4} - \frac{4(p-1)}{2}$

$\Rightarrow 2(p - 1) = p \Rightarrow p = 2.$

99. (c) $z_1 = 2 + 3i,\ z_2 = 3 + 2i$

$z_1 + z_2 = (2 + 3i) + (3 + 2i) = 5 + 5i$

Hence, a = 5

100. (d) $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$

$z_1 - z_2 = (2 + 3i) - (3 - 2i) = -1 + 5i \equiv -1 + bi$

Hence, b = 5.

101. (c) $z = 5i\left(\frac{-3}{5}i\right) = -3i^2 = -3(-1) = 3 \equiv 3 + 0i$

Hence, b = 0

102. (d) $\frac{z_1}{z_2} = \frac{6+3i}{2-i} = \frac{6+3i}{2-i} \times \frac{2+i}{2+i}$

$= \frac{12+6i+6i-3}{4-i^2} = \frac{9+12i}{5}$

$= \frac{1}{5}(9+12i) \equiv \frac{1}{a}(9+12i)$

Hence, a = 5.

103. (a) Consider

$i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$

$= (i^4)^k + (i^4)^k . i + (i^4)^k . i^2 + (i^4)^k . i^3$

$= 1 + i + i^2 + i^3 = 1 + i - 1 - i = 0$

104. (a) $z = i^9 + i^{19} = (i^4)^2 . i + (i^4)^4 . i^3$

$= i + i^3 \quad (\because i^4 = 1)$

$= i - i = 0 \quad (\because i^3 = 1)$

$\equiv 0 + 0\,i$

Hence, a = 0

105. (a) $z = i^{-39} = \frac{1}{i^{39}} = \frac{1}{(i^4)^9} \cdot \frac{1}{i^3}$

$= 1 \cdot \frac{1}{i^3} \quad (\because i^4 = 1)$

$= \frac{1}{i^2 \cdot i} = \frac{-1}{i} \quad (\because i^2 = -1)$

$= i \quad \left(\because \frac{1}{i} = -i\right)$

$\equiv 0 + i$

Hence, value of a = 0.

106. (c) $(1 - i)^n = 2^n$

Take modulus, both the side $|(1 - i)^n| = |2^n|$

$|1 - i|^n = |2|^n$

$\Rightarrow \left[\sqrt{1^2 + (-1)^2}\right]^n = 2^n$

$\Rightarrow \left(\sqrt{2}\right)^n = 2^n \Rightarrow 2^{\frac{n}{2}} = 2^n$

$\Rightarrow \frac{n}{2} = n \quad \Rightarrow n = 0$

107. (d) $(1 + i)^5 (1 - i)^5 = [(1 + i)(1 - i)]^5$

$= (1 - i^2)^5 = [1 - (-1)]^5$

$= 2^5$

108. (d) $(1 + i)^8 + (1 - i)^8 = [\{(1 + i)^2\}^4 + \{(1 - i)^2\}^4]$

$= [(1 + i^2 + 2i)^4 + (1 + i^2 - 2i)^4]$

$= [(2i)^4 + (-2i)^4] = 16i^4 + 16i^4$

$= 32i^4 = 32 = 2^5$

109. (b) $x^2 + 2 = 0 \Rightarrow x^2 = -2$

$\Rightarrow x = \pm\sqrt{-2} = \pm\sqrt{2}\,i$

110. (a) $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = 1 + i$

$|z_1| = \sqrt{2}$

and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$

$= \sqrt{3}\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$

$|z_2| = \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3}$

$|z_1 z_2| = |z_1||z_2| = \sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

111. (a) As we know, if $z = a + ib$, then

$|z| = \sqrt{a^2 + b^2}$

Let $z = \sqrt{2i} - \sqrt{-2i}$

$= \sqrt{2i} - i\sqrt{2i} \quad (\because \sqrt{-1} = i)$

$= \sqrt{2i}(1 - i)$

Now, $|z| = |\sqrt{2}\sqrt{i}(1 - i)|$

$= \sqrt{2}|\sqrt{i}||1 - i| = \sqrt{2} \times 1 \times \sqrt{1^2 + (-1)^2}$

$= \sqrt{2} \times \sqrt{2} = 2$

112. (d) $z^{1/3} = a - ib \Rightarrow z = (a - ib)^3$

$\therefore x + iy = a^3 + ib^3 - 3ia^2b - 3ab^2$. Then

$x = a^3 - 3ab^2 \Rightarrow \frac{x}{a} = a^2 - 3b^2$

$y = b^3 - 3a^2b \Rightarrow \frac{y}{b} = b^2 - 3a^2$

So, $\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2)$

113. (a) Given equations are

$k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and

$6k(2x^2 - 1) + px + 4x^2 + 2 = 0$

$\Rightarrow (6k + 2)x^2 + rx + 3k - 1 = 0$ ...(i)

$\Rightarrow (12k + 4)x^2 + px - 6k + 2 = 0$ ...(ii)

Let α and β be the roots of both equations (i) and (ii).

$\therefore \alpha + \beta = \frac{-r}{6k + 2}$ (from (i))

and $\alpha + \beta = \frac{-p}{12k + 4}$ (from (ii))

$\therefore \frac{-r}{2(1 + 3k)} = \frac{-p}{4(1 + 3k)} \Rightarrow \frac{-r}{2} = \frac{-p}{4}$

$\Rightarrow -2r = -p \Rightarrow 2r - p = 0.$

114. (c) Let $z = r(\cos\theta + i\sin\theta)$

Then $r = |z|$ and $\theta = \arg(z)$

Now $z = r(\cos\theta + i\sin\theta)$

$\Rightarrow \bar{z} = r(\cos\theta - i\sin\theta)$

$= r[\cos(-\theta) + i\sin(-\theta)]$

$\therefore \arg(\bar{z}) = -\theta$

$\Rightarrow \arg(\bar{z}) = -\arg(z)$

$\Rightarrow \arg(\bar{z}) + \arg(z) = 0.$

115. (c) $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

116. (a) We have, $z = 2 - 3i$

$\Rightarrow z - 2 = -3i \Rightarrow (z - 2)^2 = (-3i)^2$

$\Rightarrow z^2 - 4z + 4 = 9i^2 \Rightarrow z^2 - 4z + 13 = 0$

ASSERTION - REASON TYPE QUESTIONS

117. (a) Since $x = -2$ is a root of $f(x)$.

$\therefore f(x) = (x + 2)(ax + b)$

But $f(0) + f(1) = 0$

$\therefore 2b + 3a + 3b = 0$

$\Rightarrow -\frac{b}{a} = \frac{3}{5}$.

118. (b) We have,

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2) = |z_1|^2 + |z_2|^2$

where $\theta_1 = \arg(z_1)$, $\theta_2 = \arg(z_2)$

$\Rightarrow \cos(\theta_1 - \theta_2) = 0$

$\Rightarrow \theta_1 - \theta_2 = \frac{\pi}{2}$

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$

$\Rightarrow \text{Re}\left(\frac{z_1}{z_2}\right) = 0$

$\therefore \frac{z_1}{z_2}$ is purely imaginary.

If z is purely imaginary, then $z + \bar{z} = 0$.

119. (d) For real roots, $D \geq 0$

$\Rightarrow (-4)^2 - 4(2\lambda - 1)(2\lambda - 1) \geq 0 \Rightarrow (2\lambda - 1)^2 \leq 4$

$\Rightarrow -2 \leq 2\lambda - 1 \leq 2$

$\Rightarrow -\frac{1}{2} \leq \lambda \leq \frac{3}{2}$

$\therefore$ Integral values of λ are 0 and 1

Hence, greatest integral value of $\lambda = 1$.

120. (a) We have, $\arg(z) = 0$

$\Rightarrow$ z is purely real

$\therefore$ Reason is true.

Also, $|z_1| = |z_2| + |z_1 - z_2|$

$\Rightarrow |z_1 - z_2|^2 = (|z_1| - |z_2|)^2$

$\Rightarrow |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \quad \Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow \arg(z_1) - \arg(z_2) = 0$

$\Rightarrow \arg\left(\dfrac{z_1}{z_2}\right) = 0$

$\Rightarrow \dfrac{z_1}{z_2}$ is purely real.

$\Rightarrow \text{Im}\left(\dfrac{z_1}{z_2}\right) = 0$

121. (c) Assertion is a standard result.

$|z - (2 + 3i)| = 4$

$\Rightarrow$ Distance of P(z) from the point (2, 3) is equal to 4.

$\Rightarrow$ Locus of P is a circle with centre at (2, 3) and radius 4.

122. (d) We have,

$ix^2 - 3ix + 2i = 0$

or $i(x^2 - 3x + 2) = 0$

$\Rightarrow x^2 - 3x + 2 = 0 \qquad [\because i \neq 0]$

$\Rightarrow x = 1, 2$, which are real.

CRITICALTHINKING TYPE QUESTIONS

123. (c) Given $|z-4| < |z-2|$ Let $z = x + iy$

$\Rightarrow |(x-4)+iy)| < |(x-2)+iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$

$\Rightarrow x > 3 \Rightarrow \text{Re}(z) > 3$

124. (d) The given equation is $x^2 - 3x + 3 = 0$

Let a, b be the roots of the given equation then,

$a + b = 3, ab = 3$

We know, $(a-b)^2 = (a+b)^2 - 4ab = 9 - 12$

$\Rightarrow a - b = \sqrt{3}i$

So, $a = \dfrac{3+\sqrt{3}i}{2}$ and $b = \dfrac{3-\sqrt{3}i}{2}$

If A and B are the roots of the new equation which are double of the founded roots then

$A = 3 + \sqrt{3}i$ and $B = 3 - \sqrt{3}i$

So, $A + B = 6$ and $AB = 9 + 3 = 12$

Thus the new equation is

$x^2 - 6x + 12 = 0$

125. (b) We have, $4^x - 3 \,.\, 2^{x+3} + 128 = 0$

$\Rightarrow 2^{2x} - 3 \,.\, 2^x \,.\, 2^3 + 128 = 0$

$\Rightarrow 2^{2x} - 24 \,.\, 2^x + 128 = 0$

$\Rightarrow y^2 - 24y + 128 = 0$ where $2^x = y$

$\Rightarrow (y-16)(y-8) = 0 \Rightarrow y = 16, 8$

$\Rightarrow 2^x = 16$ or $2^x = 8 \Rightarrow x = 4$ or 3

126. (d) 4 is a root of $x^2 + px + 12 = 0$

$\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

Now, the equation $x^2 + px + q = 0$ has equal roots.

$\therefore p^2 - 4q = 0 \Rightarrow q = \dfrac{p^2}{4} = \dfrac{49}{4}$

127. (c) Let α, α^2 be the roots of $3x^2 + px + 3$.

$\therefore \quad \alpha + \alpha^2 = -p/3$ and $\alpha^3 = 1$

$\Rightarrow \quad (\alpha - 1)(\alpha^2 + \alpha + 1) = 0$

$\Rightarrow \quad \alpha = 1$ or $\alpha^2 + \alpha = -1$

If $\alpha = 1$, $p = -6$ which is not possible as $p > 0$

If $\alpha^2 + \alpha = -1 \Rightarrow -p/3 = -1 \Rightarrow p = 3$.

128. (a) Given expression

$$= \frac{i^{10}\left(i^{582} + i^{580} + i^{578} + i^{576} + i^{574}\right)}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$$

$= i^{10} - 1 = (i^2)^5 - 1 = (-1)^5 - 1$

$= -1 - 1 = -2$

129. (c) $|z| = \dfrac{|1+i\sqrt{3}|\,|\cos\theta + i\sin\theta|}{2|1-i|\,|\cos\theta - i\sin\theta|} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

130. (b) Suppose, $z = \dfrac{1+2i}{1-(1-i)^2}$

$= \dfrac{1+2i}{1-(1^2+i^2-2i)} \qquad [\text{using } (a-b)^2]$

$= \dfrac{1+2i}{1+2i} \qquad (\because i^2 = -1)$

$= 1 = 1 + 0 \,.\, i$

$|z| = \sqrt{(\text{Real part})^2 + (\text{Img. Part})^2}$

and amp $(z) = \tan^{-1}\left[\dfrac{\text{Img. part}}{\text{Real part}}\right]$

$\therefore \quad |z| = 1$ and amp $(z) = \tan^{-1}\left(\dfrac{0}{1}\right) = 0$

131. (a) Let $z = x + iy$,

$\because \quad |z^2 - 1| = |z|^2 + 1$

$\Rightarrow \quad |x^2 - y^2 - 1 + i2xy| = |x + iy|^2 + 1$

$\Rightarrow \quad (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$

$\Rightarrow \quad 4x^2 = 0 \Rightarrow x = 0$

Hence, z lies on y-axis or imaginary axis.

132. (d) $(z-1)(\bar{z}-5) + (\bar{z}-1)(z-5)$

$= 2\text{Re}\left[(z-1)(\bar{z}-5)\right]$

$\left[\because z_1\bar{z}_2 + z_2\bar{z}_1 = 2\text{Re}(z_1 z_2)\right]$

$= 2\text{ Re}[(1+i)(-3-i)] = 2(-2) = -4$

[Given $z = 2 + i$]

133. (b) Given, $z = r(\cos\theta + i\sin\theta)$;

$\bar{z} = r(\cos\theta - i\sin\theta)$

$\therefore \frac{z}{\bar{z}} = \frac{r(\cos\theta + i\sin\theta)}{r(\cos\theta - i\sin\theta)}$

$= (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)^{-1}$

$= (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$

$= (\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

$\therefore \frac{\bar{z}}{z} = (\cos\theta - i\sin\theta)(\cos\theta + i\sin\theta)^{-1}$

$= (\cos\theta - i\sin\theta)(\cos\theta - i\sin\theta)$

$= (\cos\theta - i\sin\theta)^2 = (\cos 2\theta - i\sin 2\theta)$

$\therefore \frac{z}{\bar{z}} + \frac{\bar{z}}{z} = \cos 2\theta + i\sin 2\theta + \cos 2\theta - i\sin 2\theta$

$= 2\cos 2\theta$

134. (b) Let $z = i = \frac{1}{2} + \frac{1}{2}i^2 + 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}i$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}i\right)^2 + 2\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}i$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^2$

$\therefore \sqrt{i} = \frac{\pm 1}{\sqrt{2}}(1+i).$

135. (a) We have, $\left(x + \frac{1}{x}\right)^3 + \left(x + \frac{1}{x}\right) = 0$

$\Rightarrow \left(x + \frac{1}{x}\right)\left[\left(x + \frac{1}{x}\right)^2 + 1\right] = 0$

$\Rightarrow$ either $x + \frac{1}{x} = 0$

$\Rightarrow x^2 = -1 \Rightarrow x = \pm i$

or $\left(x + \frac{1}{x}\right)^2 + 1 = 0$

$\Rightarrow x^2 + \frac{1}{x^2} + 3 = 0$

$\Rightarrow x^4 + 3x^2 + 1 = 0$

$\Rightarrow x^2 = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2} < 0$

$\therefore$ There is no real root.

136. (d) Given equation is $\frac{a}{x-a} + \frac{b}{x-b} = 1$

$\Rightarrow a(x - b) + b(x - a) = (x - a)(x - b)$

$\Rightarrow x^2 - x(a + b) + ab = ax - ab + bx - ab$

$\Rightarrow x^2 - 2x(a + b) + 3ab = 0$

So, sum of roots $= \alpha + (-\alpha) = 2(a + b)$

or $a + b = 0$.

137. (c) Let α, β be the roots of the equation.

$\therefore \alpha + \beta = a - 2$ and $\alpha\beta = -(a + 1)$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a - 2)^2 + 2(a + 1)$

$= (a - 1)^2 + 5$

$\therefore \alpha^2 + \beta^2$ will be minimum if $(a - 1)^2 = 0$, i.e. $a = 1$.

138. (d) Since α, β are roots of the equation

$(x - a)(x - b) = 5$ or $x^2 - (a + b)x + (ab - 5) = 0$

$$\left.\begin{array}{l}\therefore\ \alpha + \beta = a + b \text{ or } a + b = \alpha + \beta \\ \text{and } \alpha\beta = ab - 5 \text{ or } ab = \alpha\beta + 5\end{array}\right\} \quad \text{... (i)}$$

Taking another equation

$(x - \alpha)(x - \beta) + 5 = 0$

or $x^2 - (\alpha + \beta)x + (\alpha\beta + 5) = 0$

or $x^2 - (a + b)x + ab = 0$ [using (i)]

$\therefore$ Its roots are a, b.

139. (b) Given, $\left|\frac{i+z}{i-z}\right| = 1$

Let $z = x + iy$

$\therefore \left|\frac{i + x + iy}{i - (x + iy)}\right| = 1$

$\Rightarrow \left|\frac{x + i(1+y)}{-x + i(1-y)}\right| = 1$

$\Rightarrow \sqrt{x^2 + (1+y)^2} = \sqrt{(-x)^2 + (1-y)^2}$

$\Rightarrow x^2 + 1 + y^2 + 2y = x^2 + 1 + y^2 - 2y$

$\Rightarrow 4y = 0 \Rightarrow y = 0$

Hence, z lies on x-axis.

140. (a) $(z+3)(\bar{z}+3) = z\bar{z} + 3z + 3\bar{z} + (3)^2$

$= |z|^2 + 3\left(\frac{z+\bar{z}}{2}\right) \times 2 + 3^2$ $\left[\because |z|^2 = z\bar{z}\right]$

$= |z|^2 + 2 \times 3 \times (\text{Re}(z)) + 3^2$

$= |z|^2 + 2\text{Re}(3z) + (3)^2 = |z + 3|^2$

141. (c) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$,

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$\because |z_1 + z_2| = |z_1| + |z_2|$ [given]

$\Rightarrow |(r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)|$

$= r_1 + r_2$

$\Rightarrow \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)} = r_1 + r_2$

$\Rightarrow r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2) = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1$

$\Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$

$\Rightarrow \arg(z_1) = \arg(z_2)$.

142. (d) Let $z = \sin x + i\cos 2x$

According to the given condition,

$\bar{z} = \cos x - i\sin 2x$

$\therefore$ sin x – i cos 2x = cos x – i sin 2x

$\Rightarrow$ (sin x – cos x) + i(sin 2x – cos 2x) = 0

On equating real and imaginary parts, we get

sin x – cos x = 0, sin 2x – cos 2x = 0

$\Rightarrow$ tan x = 1 and tan 2x = 1

$\Rightarrow x = \frac{\pi}{4}$ and $2x = \frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}$ and $x = \frac{\pi}{8}$

which is not possible.

143. (a) Let z = x + iy

$\therefore |z + 3 - i| = |(x + 3) + i(y - 1)| = 1$

$\Rightarrow \sqrt{(x+3)^2 + (y-1)^2} = 1$ ... (i)

$\because$ arg z = π

$\Rightarrow \tan^{-1}\frac{y}{x} = \pi$

$\Rightarrow \frac{y}{x} = \tan\pi = 0$

$\Rightarrow$ y = 0 ... (ii)

From equations (i) and (ii), we get

x = –3, y = 0

$\therefore$ z = –3

$\Rightarrow |z| = |-3| = 3$

144. (b) Given that :

$$Z = \frac{1-i}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}$$

$$= \frac{2(i-1)}{1+i\sqrt{(3)}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$= \frac{2\left(i+\sqrt{3}-1+i\sqrt{3}\right)}{1+3}$$

$$= \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$$

Now, put $\frac{\sqrt{3}-1}{2} = r\cos\theta$, $\frac{\sqrt{3}+1}{2} = r\sin\theta$

Squaring and adding, we obtain

$$r^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$= \frac{2\left(\sqrt{3}\right)^2+1}{4} = \frac{2\times 4}{4} = 2$$

Hence, $r = \sqrt{2}$ which gives :

$$\cos\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}, \ \sin\theta = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Therefore, $\theta = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$

Hence, the polar form is $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$.

145. (a) (x – iy) (3 + 5i)

$= 3x + 5xi - 3yi - 5yi^2$

$= 3x + (5x - 3y)i + 5y$ $[\because i^2 = -1]$

$= (3x + 5y) + (5x - 3y)i$ ... (i)

Given, (x – iy) (3 + 5i) = –6 + 24i

$\left[\begin{array}{l}\text{using equation (i), and } z = (a + ib) \\ \Rightarrow \bar{z} = (a - ib)\end{array}\right]$

On comparing the real and imaginary parts of both sides, we get

3x + 5y = –6 and 5x – 3y = 24

Solving the above equations by substitution or elimination method, we get

x = 3, y = –3

146. (b) Let z = x + iy, then $\frac{z-1}{z+1} = \frac{x-1+iy}{x+1+iy}$

$$= \frac{(x-1+iy)}{(x+1+iy)} \times \frac{(x+1-iy)}{(x+1-iy)}$$

$$= \frac{\left[\left(x^2-1\right) - i(x-1)y + i(x+1)y + y^2\right]}{\left[(x+1)^2 + y^2\right]}$$

For purely imaginary, real (z) = 0

$\Rightarrow x^2 + y^2 = 1, |z| = 1.$

147. (c) $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right) = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i\,2\sin^2\frac{\pi}{10}$

$$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$$

For amplitude, $\tan\theta = \frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \tan\frac{\pi}{10} \Rightarrow \theta = \frac{\pi}{10}$.

148. (a) $x + iy = \sqrt{\frac{a+ib}{c+id}} \Rightarrow x - iy = \sqrt{\frac{a-ib}{c-id}}$

Also, $x^2 + y^2 = (x + iy)(x - iy) = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$

$\Rightarrow (x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

149. (b) As sum of coefficients is zero, hence one root is 1 and other root is $\frac{l-m}{m-n}$.

Since roots are equal,

$\therefore \frac{l-m}{m-n} = 1 \Rightarrow 2m = n + l.$

150. (b) It is given that

$$\alpha\beta = 2 \Rightarrow \frac{3a+4}{a+1} = 2$$

$\Rightarrow$ $3a + 4 = 2a + 2 \Rightarrow a = -2$

Also, $\alpha + \beta = -\dfrac{2a+3}{a+1}$

Putting this value of a, we get sum of roots

$$= -\frac{2a+3}{a+1} = -\frac{-4+3}{-2+1} = -1.$$

151. (d) $\alpha + \beta = -\dfrac{b}{a}$, $\alpha\beta = \dfrac{c}{a}$ and $\alpha^2 + \beta^2 = \dfrac{(b^2 - 2ac)}{a^2}$

Now, $\dfrac{\alpha}{a\beta + b} + \dfrac{\beta}{a\alpha + b} = \dfrac{\alpha(a\alpha + b) + \beta(a\beta + b)}{(a\beta + b)(a\alpha + b)}$

$$= \frac{a(\alpha^2+\beta^2) + b(\alpha+\beta)}{\alpha\beta a^2 + ab(\alpha+\beta) + b^2} = \frac{a\dfrac{(b^2-2ac)}{a^2} + b\left(-\dfrac{b}{a}\right)}{\left(\dfrac{c}{a}\right)a^2 + ab\left(-\dfrac{b}{a}\right) + b^2}$$

$$= \frac{b^2 - 2ac - b^2}{a^2c - ab^2 + ab^2} = \frac{-2ac}{a^2c} = -\frac{2}{a}.$$

152. (a) Let α, α^2 be the two roots. Then,

$$\alpha + \alpha^2 = -\frac{b}{a} \quad \text{...(i)}$$

and $$\alpha \cdot \alpha^2 = \frac{c}{a} \quad \text{...(ii)}$$

On cubing both sides of (i),

$$\alpha^3 + \alpha^6 + 3\alpha\alpha^2(\alpha + \alpha^2) = -\frac{b^3}{a^3}$$

$$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} + 3\frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3} \quad \text{[by (i) and (ii)]}$$

$\Rightarrow$ $b^3 + ac^2 + a^2c = 3abc.$

153. (c) Given $(x - a)(x - b) = c$

$\therefore$ $\alpha + \beta = a + b$ and $\alpha\beta = ab - c$

Now, given equation $(x - \alpha)(x - \beta) + c = 0$

$\Rightarrow$ $x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

If its roots be p and q, then

$p + q = (\alpha + \beta) = a + b$

$pq = \alpha\beta + c = ab - c + c = ab$

So, it can be given by $x^2 - (a + b)x + ab = 0$

So, its roots will be a and b.

154. (a) $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

Now, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta + \alpha) + \alpha\beta$

$$= \alpha\beta(1 + \alpha + \beta) = \frac{c}{a}\left\{1 + \left(-\frac{b}{a}\right)\right\} = \frac{c(a-b)}{a^2}.$$

155. (b) Since α, β are the roots of the equation $2x^2 - 35x + 2 = 0$

Also, $\alpha\beta = 1$

$\therefore$ $2\alpha^2 - 35\alpha = -2$ or $2\alpha - 35 = \dfrac{-2}{\alpha}$

$2\beta^2 - 35\beta = -2$ or $2\beta - 35 = \dfrac{-2}{\beta}$

Now, $(2\alpha - 35)^3(2\beta - 35)^3 = \left(\dfrac{-2}{\alpha}\right)^3\left(\dfrac{-2}{\beta}\right)^3$

$$= \frac{8 \cdot 8}{\alpha^3\beta^3} = \frac{64}{1} = 64.$$

156. (c) Let α, β are roots of $x^2 + px + q = 0$

So, $\alpha + \beta = -p$ and $\alpha\beta = q$

Given that $(\alpha + \beta) = 3(\alpha - \beta) = -p$

$\Rightarrow$ $\alpha - \beta = \dfrac{-p}{3}$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow$ $\dfrac{p^2}{9} = p^2 - 4q$ or $2p^2 = 9q.$

157. (c) Let α, β be the roots of $x^2 + bx + c = 0$ and α', β' be the roots of $x^2 + qx + r = 0$.

Then, $\alpha + \beta = -b$, $\alpha\beta = c$, $\alpha' + \beta' = -q$, $\alpha'\beta' = r$

It is given that $\dfrac{\alpha}{\beta} = \dfrac{\alpha'}{\beta'} \Rightarrow \dfrac{\alpha+\beta}{\alpha-\beta} = \dfrac{\alpha'+\beta'}{\alpha'-\beta'}$

$$\Rightarrow \frac{(\alpha+\beta)^2}{(\alpha-\beta)^2} = \frac{(\alpha'+\beta')^2}{(\alpha'-\beta')^2} \Rightarrow \frac{b^2}{b^2-4c} = \frac{q^2}{q^2-4r}$$

$\Rightarrow$ $b^2r = q^2c.$

158. (b) Since roots of the equation $x^2 - 5x + 16 = 0$ are α, β.

$\Rightarrow$ $\alpha + \beta = 5$ and $\alpha\beta = 16$ and $\alpha^2 + \beta^2 + \dfrac{\alpha\beta}{2} = -p$

$\Rightarrow$ $(\alpha + \beta)^2 - 2\alpha\beta + \dfrac{\alpha\beta}{2} = -p$

$\Rightarrow$ $25 - 32 + 8 = -p$

$\Rightarrow$ $p = -1$ and $(\alpha^2 + \beta^2)\left(\dfrac{\alpha\beta}{2}\right) = q$

$\Rightarrow$ $[(\alpha + \beta)^2 - 2\alpha\beta]\left[\dfrac{\alpha\beta}{2}\right] = q$

$\Rightarrow$ $q = [25 - 32]\dfrac{16}{2} = -56$

So, $p = -1$, $q = -56$.

159. (a) Let the roots are α and β.

$\Rightarrow \dfrac{\alpha+\beta}{2}=\dfrac{8}{5}$

$\Rightarrow \alpha+\beta=\dfrac{16}{5}$ ... (i)

and $\dfrac{\frac{1}{\alpha}+\frac{1}{\beta}}{2}=\dfrac{8}{7} \Rightarrow \dfrac{\alpha+\beta}{2\alpha\beta}=\dfrac{8}{7} \Rightarrow \dfrac{\left(\frac{16}{5}\right)}{2\left(\frac{8}{7}\right)}=\alpha\beta$

$\Rightarrow \alpha\beta=\dfrac{7}{5}$ ... (ii)

$\therefore$ Equation is $x^2-\left(\dfrac{16}{5}\right)x+\dfrac{7}{5}=0$

$\Rightarrow 5x^2-16x+7=0$

160. (c) $4x^2+5k=(5k+1)x$

$\Rightarrow 4x^2-(5k+1)x+5k=0;\ (\alpha-\beta)=1$

$\therefore \alpha+\beta=\dfrac{(5k+1)}{4}$ and $\alpha\beta=\dfrac{5k}{4}$

Now, $\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$\Rightarrow \alpha-\beta=\sqrt{\dfrac{(5k+1)^2}{16}-\dfrac{4\cdot 5k}{4}}=1$

$\therefore 25k^2-70k-15=0$

$\Rightarrow (5k+1)(k-3)=0 \Rightarrow k=-\dfrac{1}{5},\ 3.$

161. (b) **Case I :** $x-2>0$, Putting $x-2=y,\ y>0$

$x>2$

$\therefore y^2+y-2=0 \Rightarrow y=-2,1$

$\Rightarrow x=0,3$

But $0<2$, Hence $x=3$ is the real root.

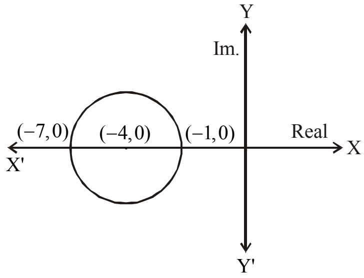
Case II : $x-2<0 \Rightarrow x<2,\ y<0$

$y^2-y-2=0 \Rightarrow y=2,-1 \Rightarrow x=4,\ x=1$

Since $4 \not< 2$, only $x=1$ is the real root.

Hence the sum of the real roots $=3+1=4$

162. (a) z lies on or inside the circle with centre (– 4, 0) and radius 3 units.



From the Argand diagram maximum value of $|z+1|$ is 6.

163. (b) $\dfrac{(\cos\theta+i\sin\theta)^4}{(\cos\theta-i\sin\theta)^3}=(\cos\theta+i\sin\theta)^4(\cos\theta-i\sin\theta)^{-3}$

$=(\cos 4\theta+i\sin 4\theta)\{\cos(-\theta)+i\sin(-\theta)\}^{-3}$

$=(\cos 4\theta+i\sin 4\theta)\{\cos(-3)(-\theta)+i\sin(-3)(-\theta)\}$

$=(\cos 4\theta+i\sin 4\theta)\{\cos 3\theta+i\sin 3\theta\}$

$=\cos 4\theta\cos 3\theta-\sin 4\theta\sin 3\theta$

$+\,i(\sin 4\theta\cos 3\theta+\sin 3\theta\cos 4\theta)$

$=\cos(4\theta+3\theta)+i\sin(4\theta+3\theta)=\cos 7\theta+i\sin 7\theta$

164. (b) Let $x=2+\dfrac{1}{2+\dfrac{1}{2+.....\infty}}$

$\Rightarrow x=2+\dfrac{1}{x}$ [On simplification]

$\Rightarrow x=1\pm\sqrt{2}$

But the value of the given expression cannot be negative or less than 2, therefore $1+\sqrt{2}$ is required answer.

165. (c) $x=\sqrt{6+x} \Rightarrow x^2=6+x$

$\Rightarrow x^2-x-6=0 \Rightarrow (x-3)(x+2)=0 \Rightarrow x=3,-2$

$x=-2$ will be rejected as $x>0$. Hence, $x=3$ is the solution.

CHAPTER 6

LINEAR INEQUALITY

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If x is real number and $|x| < 3$, then

(a) $x \geq 3$ (b) $-3 < x < 3$
(c) $x \leq -3$ (d) $-3 \leq x \leq 3$

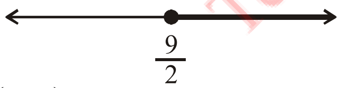
2. Given that x, y and b are real numbers and $x < y$, $b < 0$, then

(a) $\frac{x}{b} < \frac{y}{b}$ (b) $\frac{x}{b} \leq \frac{y}{b}$
(c) $\frac{x}{b} > \frac{y}{b}$ (d) $\frac{x}{b} \geq \frac{y}{b}$

3. Solution of a linear inequality in variable x is represented on number line is

(a) $x \in (-\infty, 5)$ (b) $x \in (-\infty, 5]$
(c) $x \in [5, \infty)$ (d) $x \in (5, \infty)$

4. Solution of linear inequality in variable x is represented on number line is



(a) $x \in \left(\frac{9}{2}, \infty\right)$ (b) $x \in [\frac{9}{2}, \infty)$
(c) $x \in (-\infty, \frac{9}{2})$ (d) $x \in (-\infty, \frac{9}{2}]$

5. If $|x+3| \geq 10$, then

(a) $x \in (-13, 7]$ (b) $x \in (-13, 7)$
(c) $x \in (-\infty, 13] \cup [-7, \infty)$ (d) $x \in (-\infty, -13] \cup [7, \infty)$

6. Let $\frac{C}{5} = \frac{F-32}{9}$. If C lies between 10 and 20, then :

(a) $50 < F < 78$ (b) $50 < F < 68$
(c) $49 < F < 68$ (d) $49 < F < 78$

7. The solution set of the inequality $4x + 3 < 6x + 7$ is

(a) $[-2, \infty)$ (b) $(-\infty, -2)$
(c) $(-2, \infty)$ (d) None of these

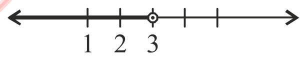
8. Which of the following is the solution set of $3x - 7 > 5x - 1 \ \forall \ x \in R$?

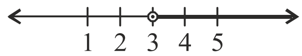
(a) $(-\infty, -3)$ (b) $(-\infty, -3]$
(c) $(-3, \infty)$ (d) $(-3, 3)$

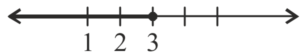
9. The solution set of the inequality $37 - (3x + 5) \geq 9x - 8(x - 3)$ is

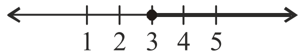
(a) $(-\infty, 2)$ (b) $(-\infty, -2)$
(c) $(-\infty, 2]$ (d) $(-\infty, -2]$

10. The graph of the solution on number line of the inequality $3x - 2 < 2x + 1$ is

(a) 

(b) 

(c) 

(d) 

11. The solution set of the inequalities $6 \leq -3(2x - 4) < 12$ is

(a) $(-\infty, 1]$ (b) $(0, 1]$
(c) $(0, 1] \cup [1, \infty)$ (d) $[1, \infty)$

12. Which of the following is the solution set of linear inequalities $2(x - 1) < x + 5$ and $3(x + 2) > 2 - x$?

(a) $(-\infty, -1)$ (b) $(-1, 1)$ (c) $(-1, 7)$ (d) $(1, 7)$

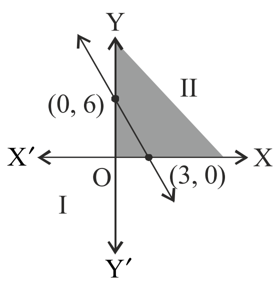
13. x and b are real numbers. If $b > 0$ and $|x| > b$, then

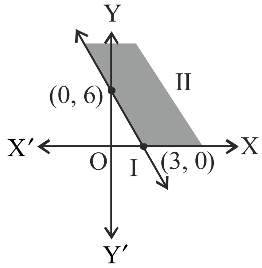
(a) $x \in (-b, \infty)$ (b) $x \in (-\infty, b)$
(c) $x \in (-b, b)$ (d) $x \in (-\infty, -b) \cup (b, \infty)$

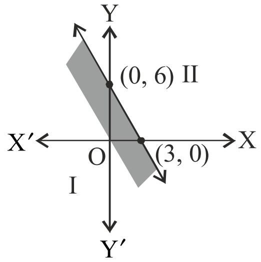
14. If $a < b$ and $c < 0$, then

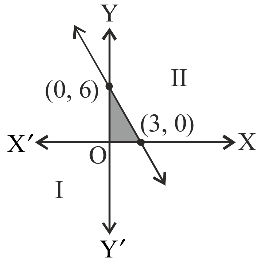
(a) $\frac{a}{c} = \frac{b}{c}$ (b) $\frac{a}{c} > \frac{b}{c}$
(c) $\frac{a}{c} < \frac{b}{c}$ (d) None of these

15. The graph of the inequality $40x + 20y \leq 120$, $x \geq 0$, $y \geq 0$ is

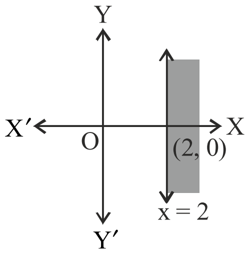
(a) 

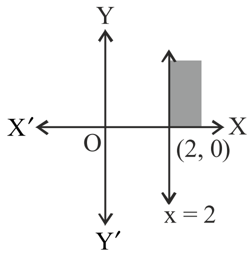
(b) 

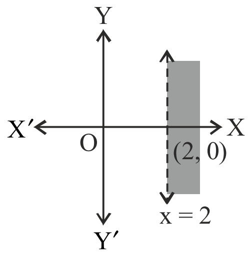
(c) 

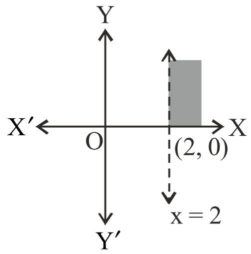
(d) 

16. The graphical solution of $3x - 6 \geq 0$ is

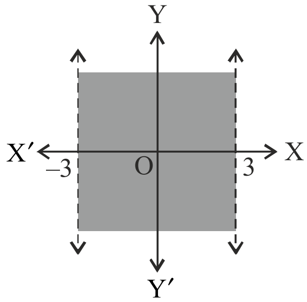
(a) 

(b) 

(c) 

(d) 

17. The inequality representing the following graph is



(a) $|x| < 3$ (b) $|x| \leq 3$ (c) $|x| > 3$ (d) $|x| \geq 3$

18. The solutions of the system of inequalities $3x - 7 < 5 + x$ and $11 - 5x \leq 1$ on the number line is

(a) 

(b) 

(c) 

(d) None of the above

19. The solution set of the inequalities $3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$, is

(a) $(-5, \infty)$ (b) $[5, \infty)$ (c) $(5, \infty)$ (d) $[-5, \infty)$

20. If $\frac{5-2x}{3} \leq \frac{x}{6} - 5$, then $x \in$

(a) $[2, \infty)$ (b) $[-8, 8]$ (c) $[4, \infty)$ (d) $[8, \infty)$

21. If $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$, then $x \in$

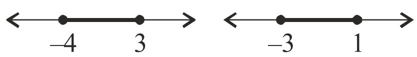
(a) $[1, \infty)$ (b) $(1, \infty)$ (c) $(-5, 5)$ (d) $[-5, 5]$

22. If $-5 \leq \frac{5-3x}{2} \leq 8$, then $x \in$

(a) $\left[-\frac{11}{3}, 5\right]$ (b) $[-5, 5]$

(c) $\left[-\frac{11}{3}, \infty\right)$ (d) $(-\infty, \infty)$

23. Solutions of the inequalities comprising a system in variable x are represented on number lines as given below, then



(a) $x \in (-\infty, -4] \cup [3, \infty)$
(b) $x \in [-3, 1]$
(c) $x \in (-\infty, -4) \cup [3, \infty)$
(d) $x \in [-4, 3]$

24. The inequality $\frac{2}{x} < 3$ is true, when x belongs to

(a) $\left[\frac{2}{3}, \infty\right)$ (b) $\left(-\infty, \frac{2}{3}\right]$

(c) $(-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$ (d) None of these

25. Solution of $|3x + 2| < 1$ is

(a) $\left[-1, -\frac{1}{3}\right]$ (b) $\left\{-\frac{1}{3}, -1\right\}$

(c) $\left(-1, -\frac{1}{3}\right)$ (d) None of these

26. Solution of $|x - 1| \geq |x - 3|$ is
(a) $x \leq 2$ (b) $x \geq 2$ (c) $[1, 3]$ (d) None of these

27. If $-3x + 17 < -13$, then
(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$
(c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

28. If $|x + 2| \leq 9$, then
(a) $x \in (-7, 11)$ (b) $x \in [-11, 7]$
(c) $x \in (-\infty, -7) \cup (11, \infty)$ (d) $x \in (-\infty, -7) \cup [11, \infty)$

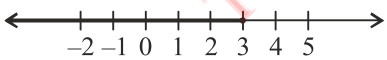
STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

29. Consider the following statements about Linear Inequalities :
I. Two real numbers or two algebraic expressions related by the symbols $<, >, \leq$ or $\geq$ form an inequality.
II. When equal numbers added to (or subtracted from) both sides of an inequality then the inequality does not changed.
III. When both sides of an inequality multiplied (or divided) by the same positive number then the inequality does not changed.
Which of the above statements are true ?
(a) Only I (b) Only II
(c) Only III (d) All of the above

30. Consider the following statements:
Statement-I : Consider the inequality $30x < 200$ such that x is not a negative integer or fraction. Then, the value of x, which make the inequality a true statement are 1, 2, 3, 4, 5, 6.
Statement-II : The solution of an inequality in one variable is the value of that variable which makes it a true statement.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

31. Consider the following statements:
Statement-I : The solution set of $7x + 3 < 5x + 9$ is $(-\infty, 3)$.
Statement-II : The graph of the solution of above inequality is represented by



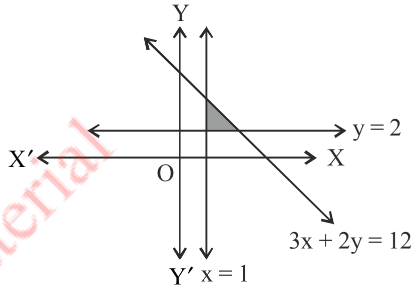
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

32. Consider the following statements:
Statement-I : The solution set of $5x - 3 < 7$, when x is an integer, is $\{....., -3, -2, -1\}$.
Statement-II : The solution of $5x - 3 < 7$, when x is a real number, is $(-\infty, 2)$.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

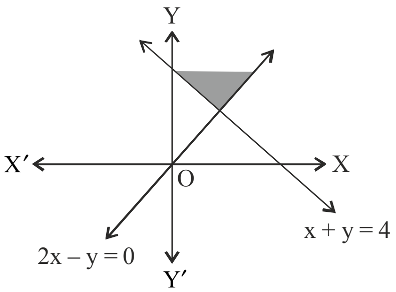
33. Consider the following statements:
Statement-I : The solution set of the inequality
$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3} \text{ is } (-\infty, 2).$$
Statement-II : The solution set of the inequality
$$\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6) \text{ is } (-\infty, 120].$$
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

34. Consider the following statements:
Statement-I : The region containing all the solutions of an inequality is called the solution region.
Statement-II : The half plane represented by an inequality is checked by taking any point on the line.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

35. Which of the following is/are true?
I. The graphical solution of the system of inequalities $3x + 2y \leq 12$, $x \geq 1$, $y \geq 2$ is

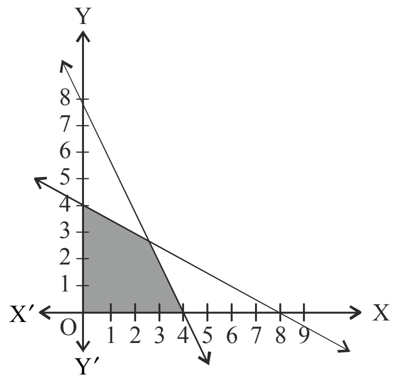


II. The region represented by the solution set of the inequalities $2x + y \geq 6$, $3x + 4y \leq 12$ is bounded.
III. The solution set of the inequalities $x + y \geq 4$, $2x - y > 0$ is



(a) Only I is true (b) I and II are true
(c) I and III are true (d) Only III is true

36. Which of the following linear inequalities satisfy the shaded region of the given figure.
I. $x + 2y \leq 8$ II. $x \geq 0, y \geq 0$
III. $x \leq 0, y \leq 0$ IV. $2x + y \leq 8$
V. $4x + 5y \leq 40$



(a) I, III and V (b) I, IV and V
(c) I, III and IV (d) I, II, and IV

37. Consider the following statements.

I. Inequalities involving the symbol ≥ or ≤ are called slack inequalities.

II. Inequalities which do not involve variables are called numerical inequalities.

Choose the correct option.

(a) Only I is true (b) Only II is true.
(c) Both are true. (d) Both are false.

38. Consider the following statements.

I. Solution set of the inequality $-15 < \frac{3(x-2)}{5} \le 0$ is $(-23, 2]$

II. Solution set of the inequality $7 \le \frac{3x+11}{2} \le 11$ is $\left[1, \frac{11}{3}\right]$

III. Solution set of the inequality $-5 \le \frac{2-3x}{4} \le 9$ is $[-1, 1] \cup [3, 5]$

Choose the correct option

(a) Only I and II are true. (b) Only II and III are true.
(c) Only I and III are true. (d) All are true.

39. Consider the following statements.

I. Equal numbers may be added to (or subtracted from) both sides of an inequality.

II. When both sides are multiplied (or divided) by a negative number, then the inequality is reversed.

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Both I and II are false.

40. Consider the following statements.

I. Solution set of 24x < 100 is {1, 2, 3, 4}, when x is a natural number.

II. Solution set of 24x < 100 is {…, –3, –2, –1, 0, 1, 2, 3, 4}, when x is an integer.

Choose the correct option.

(a) Only I is false. (b) Only II is false.
(c) Both are false. (d) Both are true.

41. I. When x is an integer, the solution set of 3x + 8 > 2 is {–1, 0, 1, 2, 3, …}.

II. When x is a real number, the solution set of 3x + 8 > 2 is {–1, 0, 1}.

Choose the correct option.

(a) Only I is incorrect.
(b) Only II is incorrect.
(c) Both I and II are incorrect.
(d) Both I and II are correct.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

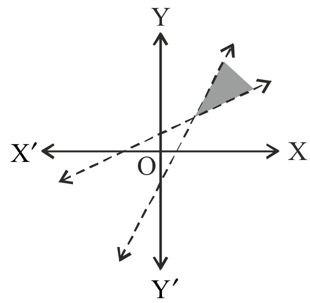
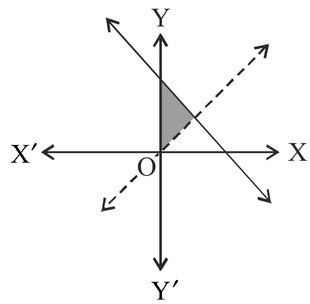
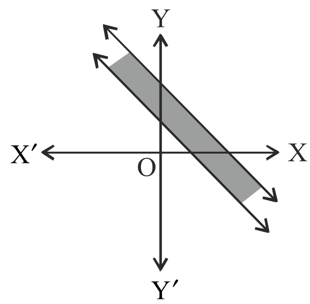
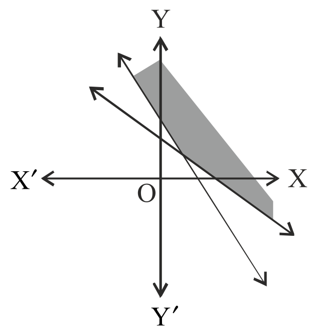
42.

Column - I (Linear inequations)	Column - II (Solution set)
(A) $2x-4 \le 0$	(1) $[3, \infty)$
(B) $-3x+12 < 0$	(2) $(3, \infty)$
(C) $4x-12 \ge 0$	(3) $(-\infty, 2]$
(D) $7x+9 > 30$	(4) $(4, \infty)$

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	3	1	4	2
(c)	2	4	1	3
(d)	2	1	4	3

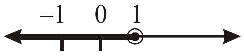
43. Match the linear inequalities given in column-I with solution set representing by graphs in column-II

Column-I	Column-II
A. $2x - y > 1$, $x - 2y < -1$	1. 
B. $x + y \le 6$, $x + y \ge 4$	2. 
C. $2x + y \ge 8$, $x + 2y \ge 10$	3. 
D. $x + y \le 9$, $y > x$, $x \ge 0$	4. 

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	4	3
(c)	1	3	4	2
(d)	3	4	2	1

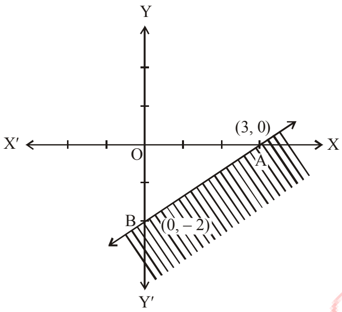
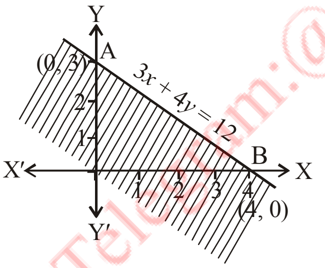
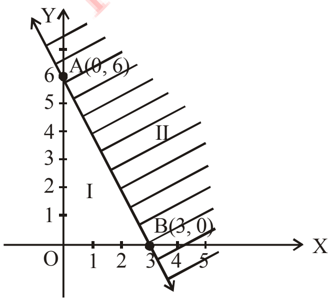
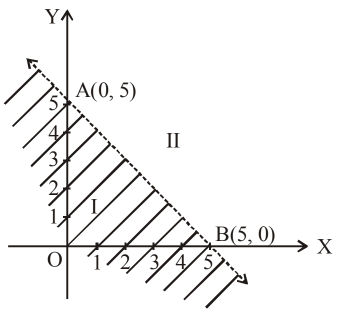
44.

Column - I (Linear inequations)	**Column - II (Solution on number line)**
(A) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$	(1) 
(B) $3x - 2 < 2x + 1$	(2)
(C) $3(1-x) < 2(x+4)$	(3) 
(D) $3x - 7 < 5 + x$ and $11 - 5x \leq 1$	(4) 

Codes

	A	B	C	D
(a)	4	2	3	1
(b)	1	3	2	4
(c)	4	3	2	1
(d)	1	2	3	4

45.

Column - I (Inequality)	**Column - II (Graph)**
(A) $x + y < 5$	(1) 
(B) $2x + y \geq 6$	(2) 
(C) $3x + 4y \leq 12$	(3) 
(D) $2x - 3y > 6$	(4) 

Codes

	A	B	C	D
(a)	4	2	3	1
(b)	4	3	2	1
(c)	1	2	3	4
(d)	1	3	2	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

46. The solution set of the inequality $4x + 3 < 6x + 7$ is $(-a, \infty)$. The value of 'a' is
(a) 1 (b) 4
(c) 2 (d) None of these

47. The set of real x satisfying the inequality $\frac{5-2x}{3} \leq \frac{x}{6} - 5$ is $[a, \infty)$. The value of 'a' is
(a) 2 (b) 4 (c) 6 (d) 8

48. The solution set of the inequality $3(2-x) \geq 2(1-x)$ is $(-\infty, a]$. The value of 'a' is
(a) 2 (b) 3 (c) 4 (d) 5

49. The solution set of $\frac{2x-1}{3} \geq \left(\frac{3x-2}{4}\right) - \left(\frac{2-x}{5}\right)$ is $(-\infty, a]$. The value of 'a' is
(a) 2 (b) 3 (c) 4 (d) 5

50. If $5x + 1 > -24$ and $5x - 1 < 24$, then $x \in (-a, a)$. The value of 'a' is
(a) 2 (b) 3 (c) 4 (d) 5

51. If x satisfies the inequations $2x - 7 < 11$ and $3x + 4 < -5$, then x lies in the interval $(-\infty, -m)$. The value of 'm' is
(a) 2 (b) 3 (c) 4 (d) 5

52. If $|x| < 3$ and x is a real number, then $-m < x < m$. The value of m is
(a) 3 (b) 4 (c) 2 (d) 1

53. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.
(a) 2 (b) 9 (c) 8 (d) 7

54. The solution of the inequality $-8 \leq 5x - 3 < 7$ is $[-a, b)$. Sum of 'a' and 'b' is
(a) 1 (b) 2 (c) 3 (d) 4

55. The number of pairs of consecutive odd natural numbers both of which are larger than 10, such that their sum is less than 40, is
(a) 4 (b) 6 (c) 3 (d) 8

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

56. **Assertion :** The inequality $ax + by < 0$ is strict inequality.
Reason : The inequality $ax + b \geq 0$ is slack inequality.

57. **Assertion :** If $a < b$, $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.
Reason : If both sides are divided by the same negative quantity, then the inequality is reversed.

58. **Assertion :** $|3x - 5| > 9 \Rightarrow x \in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$.
Reason : The region containing all the solutions of an inequality is called the solution region.

59. **Assertion :** A line divides the cartesian plane in two part(s).
Reason : If a point $P(\alpha, \beta)$ on the line $ax + by = c$, then $a\alpha + b\beta = c$.

60. **Assertion :** Each part in which a line divides the cartesian plane, is known as half plane.
Reason : A point in the cartesian plane will either lie on a line or will lie in either of half plane I or II.

61. **Assertion :** Two real numbers or two algebraic expressions related by the symbol $<, >, \leq$ or $\geq$ forms an inequality.
Reason : The inequality $ax + by < 0$ is strict inequality.

62. **Assertion :** The inequality $3x + 2y \geq 5$ is the linear inequality.
Reason : The solution of $5x - 3 < 7$, when x is a real number, is $(-\infty, 2)$.

63. **Assertion :** If $3x + 8 > 2$, then $x \in \{-1, 0, 1, 2, \ldots\}$, when x is an integer.
Reason : The solution set of the inequality $4x + 3 < 5x + 7$ $\forall\ x \in R$ is $[4, \infty)$.

64. **Assertion :** Graph of linear inequality in one variable is a visual representation.
Reason : If a point satisfying the line $ax + by = c$, then it will lie in upper half plane.

65. **Assertion :** The region containing all the solutions of an inequality is called the solution region.
Reason : The values of x, which make an inequality a true statement, are called solutions of the inequality.

66. **Assertion :** A non-vertical line will divide the plane into left and right half planes.
Reason : The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

67. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then what can you say about breadth?
(a) breadth = 20 (b) breadth ≤ 20
(c) breadth ≥ 20 (d) breadth $\neq 20$

68. The set of real values of x satisfying $|x - 1| \leq 3$ and $|x - 1| \geq 1$ is
(a) $[2, 4]$ (b) $(-\infty, 2] \cup [4, +\infty)$
(c) $[-2, 0] \cup [2, 4]$ (d) None of these

69. The marks obtained by a student of class XI in first and second terminal examinations are 62 and 48, respectively. The minimum marks he should get in the annual examination to have an average of at least 60 marks, are
(a) 70 (b) 50 (c) 74 (d) 48

70. Ravi obtained 70 and 75 marks in first two unit tests. Then, the minimum marks he should get in the third test to have an average of at least 60 marks, are
(a) 45 (b) 35 (c) 25 (d) None of these

71. The pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23, are
(a) (4, 6), (6, 8), (8, 10), (10, 12)
(b) (6, 8), (8, 10), (10, 12)
(c) (6, 8), (8, 10), (10, 12), (12, 14)
(d) (8, 10), (10, 12)

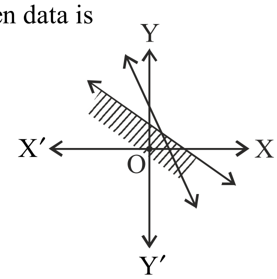
72. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. The possible length of the shortest board, if the third piece is to be at least 5 cm longer than the second, is
(a) less than 8 cm
(b) greater than or equal to 8 cm but less than or equal to 22 cm
(c) less than 22 cm
(d) greater than 22 cm

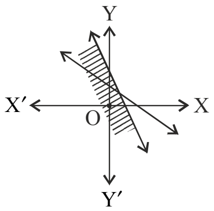
73. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then
(a) breadth > 20 cm (b) length < 20 cm
(c) breadth $\geq$ 20 cm (d) length $\leq$ 20 cm

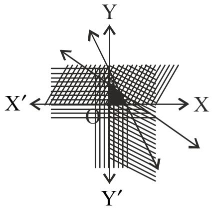
74. The set of values of x satisfying $2 \leq |x - 3| < 4$ is
(a) $(-1, 1] \cup [5, 7)$ (b) $-4 \leq x \leq 2$
(c) $-1 < x < 7$ or $x \geq 5$ (d) $x < 7$ or $x \geq 5$

75. IQ of a person is given by the formula
$$IQ = \frac{MA}{CA} \times 100$$
where, MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 years children, then the range of their mental age is
(a) $9.8 \leq MA \leq 16.8$ (b) $10 \leq MA \leq 16$
(c) $9.6 \leq MA \leq 16.8$ (d) $9.6 \leq MA \leq 16.6$

76. A furniture dealer deals in only two items — tables and chairs. He has ` 15,000 to invest and a space to store atmost 60 pieces. A table costs him ` 750 and chair ` 150. Suppose he makes x tables and y chairs
The graphical solution of the inequations representing the given data is

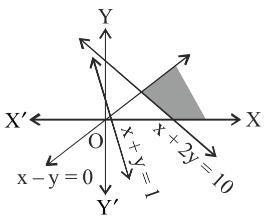
(a) 

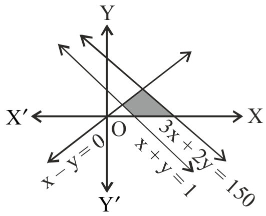
(b) 

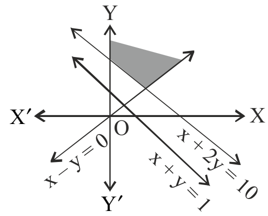
(c) 

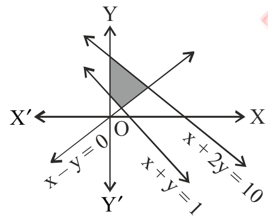
(d) None of these

77. The graphical solution of the inequalities $x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$ is

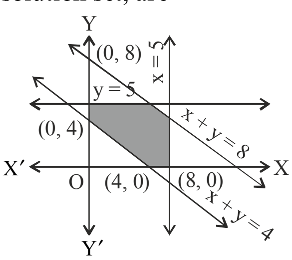
(a) 

(b) 

(c) 

(d) 

78. Linear inequalities for which the shaded region for the given figure is the solution set, are



(a) $x + y \le 8$, $x + y \le 4$, $x \le 5$, $y \le 5$, $x \ge 0$, $y \ge 0$
(b) $x + y \le 8$, $x + y \ge 4$, $x \le 5$, $y \le 5$, $x \ge 0$, $y \ge 0$
(c) $x + y \ge 8$, $x + y \ge 4$, $x \ge 5$, $y \ge 5$, $x \ge 0$, $y \ge 0$
(d) None of the above

79. A solution of 8% boric is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 L of the 8% solution, of the 2% solution will have to be added is
(a) more than 320 and less than 1000
(b) more than 160 and less than 320
(c) more than 320 and less than 1280
(d) more than 320 and less than 640

80. A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week.
The number of cassettes must be sold by the company to realise some profit, is
(a) more than 2000 (b) less than 2000
(c) more than 1000 (d) less than 1000

81. A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?
(a) more than 120 litres but less than 300 litres
(b) more than 140 litres but less than 600 litres
(c) more than 100 litres but less than 280 litres
(d) more than 160 litres but less than 500 litres

82. If $\dfrac{|x+3|+x}{x+2} > 1$, then $x \in$
(a) $(-5, -2)$ (b) $(-1, \infty)$
(c) $(-5, -2) \cup (-1, \infty)$ (d) None of these

83. If $|2x - 3| < |x + 5|$, then x belongs to
(a) $(-3, 5)$ (b) $(5, 9)$ (c) $\left(-\dfrac{2}{3}, 8\right)$ (d) $\left(-8, \dfrac{2}{3}\right)$

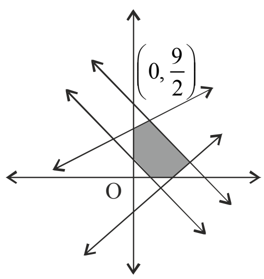
84. Solution of $(x - 1)^2 (x + 4) < 0$ is
(a) $(-\infty, 1)$ (b) $(-\infty, -4)$ (c) $(-1, 4)$ (d) $(1, 4)$

85. Solution of $\left|1 + \dfrac{3}{x}\right| > 2$ is
(a) $(0, 3]$ (b) $[-1, 0)$
(c) $(-1, 0) \cup (0, 3)$ (d) None of these

86. Solution of $|2x - 3| < |x + 2|$ is
(a) $\left(-\infty, \dfrac{1}{3}\right)$ (b) $\left(\dfrac{1}{3}, 5\right)$
(c) $(5, \infty)$ (d) $\left(-\infty, \dfrac{1}{3}\right) \cup (5, \infty)$

87. Solution of $\left|x + \dfrac{1}{x}\right| > 2$ is
(a) $R - \{0\}$
(b) $R - \{-1, 0, 1\}$
(c) $R - \{1\}$
(d) $R - \{-1, 1\}$

88. Which of the following linear inequalities satisfy the shaded region of the given figure?



(a) $2x + 3y \ge 3$
(b) $3x + 4y \le 18$
(c) $x - 6y \le 3$
(d) All of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** $|x| < 3 \Rightarrow -3 < x < 3$

2. **(c)** $x < y \Rightarrow \frac{x}{b} > \frac{y}{b}$

3. **(d)** **4.** **(b)**

5. **(d)** $|x+3| \geq 10,$

$\Rightarrow x+3 \leq -10$ or $x+3 \geq 10$

$\Rightarrow x \leq -13$ or $x \geq 7$

$\Rightarrow x \in (-\infty, -13] \cup [7, \infty)$

6. **(b)** Given : $\frac{C}{5} = \frac{F-32}{9}$ and $10 < C < 20$.

$\Rightarrow C = \frac{5F-(32)5}{9}$

Since, $10 < C < 20$

$\Rightarrow 10 < \frac{5F-160}{9} < 20$

$\Rightarrow 90 < 5F - 160 < 180$

$\Rightarrow 90 + 160 < 5F < 180 + 160$

$\Rightarrow 250 < 5F < 340$

$\Rightarrow \frac{250}{5} < F < \frac{340}{5}$

$\Rightarrow 50 < F < 68$

7. **(c)** We have, $4x + 3 < 6x + 7$

or $4x - 6x < 6x + 4 - 6x$

or $-2x < 4$ or $x > -2$

i.e. all the real numbers which are greater than –2, are the solutions of the given inequality. Hence, the solution set is $(-2, \infty)$.

8. **(a)** We have, $3x - 7 > 5x - 1$

Transferring the term 5x to L.H.S. and the term –7 to R.H.S.

Dividing both sides by 2,

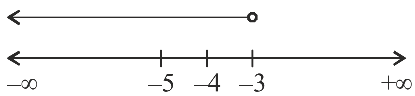
$3x - 5x > -1 + 7$

$\Rightarrow -2x > 6$

$\Rightarrow \frac{2x}{2} < -\frac{6}{2}$

$\Rightarrow x < -3$

With the help of number line, we can easily look for the numbers less than –3.



$\therefore$ Solution set is $(-\infty, -3)$, i.e. all the numbers lying between $-\infty$ and –3 but $-\infty$ and –3 are not included as $x < -3$.

9. **(c)** We have, $37 - (3x + 5) \geq 9x - 8(x - 3)$

$(37 - 3x - 5) \geq 9x - 8x + 24$

$\Rightarrow 32 - 3x \geq x + 24$

Transferring the term 24 to L.H.S. and the term (–3x) to R.H.S.

$32 - 24 \geq x + 3x$

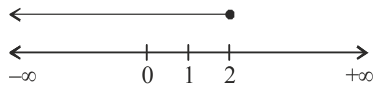
$\Rightarrow 8 \geq 4x$

$\Rightarrow 4x \leq 8$

Dividing both sides by 4,

$\frac{4x}{4} \leq \frac{8}{4}$

$\Rightarrow x \leq 2$

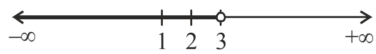


$\therefore$ Solution set is $(-\infty, 2]$.

10. **(a)** We have, $3x - 2 < 2x + 1$

Transferring the term 2x to L.H.S. and the term (–2) to R.H.S.

$3x - 2x < 1 + 2 \Rightarrow x < 3$



All the numbers on the left side of 3 will be less than it.

$\therefore$ Solution set is $(-\infty, 3)$.

11. **(b)** The given inequality $6 \leq -3(2x - 4) < 12$

$6 \leq -6x + 12 < 12$

Adding (–12) to each term,

$6 - 12 \leq -6x + 12 - 12 < 12 - 12$

$\Rightarrow -6 \leq -6x < 0$

Dividing by (–6) to each term,

$\frac{-6}{-6} \geq \frac{-6x}{-6} > \frac{0}{-6}$

$\Rightarrow 1 \geq x > 0 \quad \Rightarrow \quad 0 < x \leq 1$

$\therefore$ Solution set is (0, 1].

12. **(c)** We have the given inequalities as

$2(x - 1) < x + 5$ and $3(x + 2) > 2 - x$

Now, $2x - 2 < x + 5$

Transferring the term x to L.H.S and the term –2 to R.H.S.

$2x - x < 5 + 2$

$\Rightarrow x < 7$ ... (i)

and $3(x + 2) > 2 - x$

$\Rightarrow 3x + 6 > 2 - x$

Transferring the term (–x) to L.H.S. and the term 6 to R.H.S.,

$\Rightarrow 3x + x > 2 - 6$

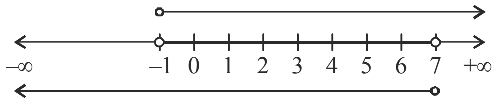
$\Rightarrow 4x > -4$

Dividing both sides by 4,

$x > \frac{-4}{4}$

$\Rightarrow x > -1$ ... (ii)

$\Rightarrow$ Draw the graph of inequalities (i) and (ii) on the number line.

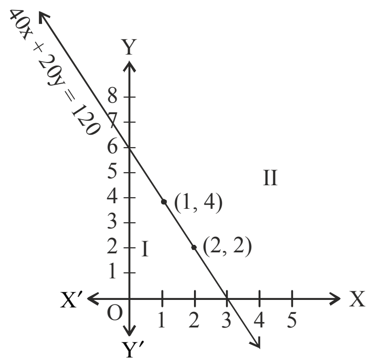


Hence, solution set of the inequalities are real numbers, x lying between –1 and 7 excluding 1 and 7.
i.e. $-1 < x < 7$
$\therefore$ Solution set is (–1, 7) or] –1, 7[.

13. (d) We have, $|x| > b, b > 0$
$\Rightarrow x < -b$ and $x > b \Rightarrow x \in (-\infty, -b) \cup (b, \infty)$

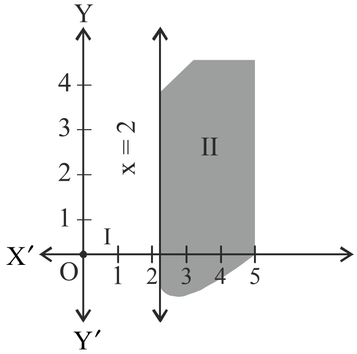
14. (b) We have,
$a < b$ and $c < 0$
Dividing both sides of $a < b$ by c. Since, c is a negative number, sign at inequality will get reversed.
Hence, $\frac{a}{c} > \frac{b}{c}$.

15. (d) We have,
$40x + 20y \leq 120, x \geq 0, y \geq 0$ ... (i)
In order to draw the graph of the inequality (i), we take one point say (0, 0), in half plane I and check whether values of x and y satisfy the inequality or not.



We observe that x = 0, y = 0 satisfy the inequality. Thus, we say that the half plane I is the graph of the inequality. Since, the points on the line also satisfy the inequality (i) above, the line is also a part of the graph. Thus, the graph of the given inequality is half plane I including the line itself. Clearly, half plane II is not the part of the graph. Hence, solutions of inequality (i) will consists of all the points of its graph (half plane I including the line).
Also, since it is given $x > 0, y > 0$, x and y can only take positive values in half plane I.

16. (a) Graph of $3x - 6 = 0$ is given in the figure,

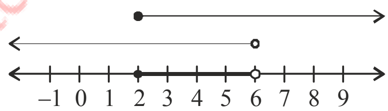


We select a point say (0, 0) and substituting it in given inequality, we see that
$3(0) - 6 \geq 0$ or $-6 \geq 0$, which is false.

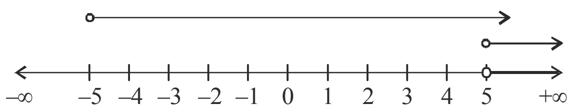
Thus, the solution region is the shaded region on the right hand side of the line x = 2.
Also, all the points on the line $3x - 6 = 0$ will be included in the solution. Hence, a dark line is drawn in the solution region.

17. (a) The shaded region in the figure lies between x = –3 and x = 3 not including the line x = –3 and x = 3 (lines are dotted).
Therefore, $-3 < x < 3$
$\Rightarrow |x| < 3$ [$\because |x| < a \Leftrightarrow -a < x < a$]

18. (b) Given inequalities are
$3x - 7 < 5 + x$ ... (i)
and $11 - 5x \leq 1$ ... (ii)
From inequality (i), we have
$3x - 7 < 5 + x$
or $x < 6$ ... (iii)
Also, from inequality (ii), we have
$11 - 5x \leq 1$
or $-5x \leq -10$, i.e. $x \geq 2$ ... (iv)
If we draw the graph of inequalities (iii) and (iv) on the number line, we see that the values of x, which are common to both, are shown by bold line in figure.



19. (c) We have $3x - 7 > 2(x - 6)$
$\Rightarrow 3x - 7 > 2x - 12$
Transferring the term 2x to L.H.S. and the term (–7) to R.H.S.,
$3x - 2x > -12 + 7$
$\Rightarrow x > -5$ ... (i)
and $6 - x > 11 - 2x$
Transferring the term (–2x) to L.H.S. and the term 6 to R.H.S.,
$-x + 2x > 11 - 6$
$\Rightarrow x > 5$ ... (ii)
Draw the graph of inequations (i) and (ii) on the number line,



Hence, solution set of the equations are real numbers, x lying on greater than 5 excluding 5.
i.e., $x > 5$
$\therefore$ Solution set is (5, ∞) or]5, ∞[.

20. (d) We have $\frac{5 - 2x}{3} \leq \frac{x}{6} - 5$
or $2(5 - 2x) \leq x - 30$ or $10 - 4x \leq x - 30$
or $-5x \leq -40$ or $x \geq 8$
Thus, all real numbers which are greater than or equal to 8 are the solutions of the given inequality, i.e., $x \in [8, \infty)$.

21. (a) We have $\frac{3x - 4}{2} \geq \frac{x + 1}{4} - 1$
or $\frac{3x - 4}{2} \geq \frac{x - 3}{4}$
or $2(3x - 4) \geq (x - 3)$

or $6x - 8 \geq x - 3$

or $5x \geq 5$ or $x \geq 1$

Thus, all real numbers which are greater than or equal to 1 is the solution set of the given inequality.

$\therefore \quad x \in [1, \infty)$.

22. **(a)** We have $-5 \leq \frac{5-3x}{2} \leq 8$

or $-10 \leq 5 - 3x \leq 16$ or $-15 \leq -3x \leq 11$

or $5 \geq x \geq -\frac{11}{3}$,

which can be written as $\frac{-11}{3} \leq x \leq 5$

$\therefore \quad x \in \left[\frac{-11}{3}, 5\right]$.

23. **(a)** Common solution of the inequalities is from $-\infty$ to -4 and 3 to ∞.

24. **(c)** **Case I :**

When $x > 0$, $\frac{2}{x} < 3 \Rightarrow 2 < 3x \Rightarrow \frac{2}{3} < x$ or $x > \frac{2}{3}$

Case II :

When $x < 0$, $\frac{2}{x} < 3 \Rightarrow 2 > 3x \Rightarrow \frac{2}{3} > x$ or $x < \frac{2}{3}$,

which is satisfied when $x < 0$.

$\therefore \quad x \in (-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$.

25. **(c)** $|3x + 2| < 1 \Leftrightarrow -1 < 3x + 2 < 1$

$\Leftrightarrow \quad -3 < 3x < -1 \Leftrightarrow -1 < x < -\frac{1}{3}$.

26. **(b)** $|x - 1|$ is the distance of x from 1.

$|x - 3|$ is the distance of x from 3.

The point $x = 2$ is equidistant from 1 and 3. Hence, the solution consists of all $x \geq 2$.

27. **(a)** $-3x < -13 - 17$

$-3x < -30 \Rightarrow x > 10$

$\Rightarrow \quad x \in (10, \infty)$.

28. **(b)** Given, $|x + 2| \leq 9$

$\Rightarrow \quad -9 \leq x + 2 \leq 9$

$\Rightarrow \quad -11 \leq x \leq 7$

STATEMENT TYPE QUESTIONS

29. **(d)**

30. **(b)** For $x = 0$,

L.H.S. = 30(0) = 0 < 200 (R.H.S.), which is true.

For $x = 1$,

L.H.S. = 30(1) = 30 < 200 (R.H.S.), which is true.

For $x = 2$,

L.H.S. = 30(2) = 60 < 200, which is true.

For $x = 3$,

L.H.S. = 30(3) = 90 < 200, which is true.

For $x = 4$,

L.H.S. = 30(4) = 120 < 200, which is true.

For $x = 5$,

L.H.S. = 30(5) = 150 < 200, which is true.

For $x = 6$,

L.H.S. = 30(6) = 180 < 200, which is true.

In the above situation, we find that the values of x, which makes the above inequality a true statement are 0, 1, 2, 3, 4, 5, 6. These values of x, which make above inequality a true statement are called solutions of inequality and the set {0, 1, 2, 3, 4, 5, 6} is called its solution set.

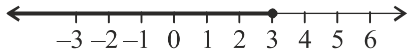
Thus, any solution of an inequality in one variable is a value of the variable which makes it a true statement.

31. **(a)** We have, $7x + 3 < 5x + 9$

or $2x < 6$ or $x < 3$

$\Rightarrow \quad x \in (-\infty, 3)$

The graphical representation of the solutions are given in figure.



32. **(b)** We have, $5x - 3 < 7$

Adding 3 on both sides,

$5x - 3 + 3 < 7 + 3$

$\Rightarrow \quad 5x < 10$

Dividing both sides by 5,

$\frac{5x}{5} < \frac{10}{5} \quad \Rightarrow \quad x < 2$

I. When x is an integer, the solution of the given inequality is {....., –1, 0, 1}.

II. When x is a real number, the solution of given inequality is $(-\infty, 2)$, i.e. all the numbers lying between $-\infty$ and 2 but ∞ and 2 are not included as $x < 2$.

33. **(b)** I. We have, $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

$\Rightarrow \frac{3x-6}{5} \leq \frac{10-5x}{3}$

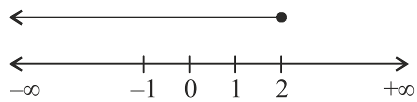
$\Rightarrow \quad 9x - 18 \leq 50 - 25x$

Transferring the terms $(-25x)$ to L.H.S. and the term (-18) to R.H.S.

$9x + 25x \leq 50 + 18$

$\Rightarrow \quad 34x \leq 68$

$\Rightarrow \quad x \leq \frac{68}{34} \Rightarrow x \leq 2$



$\therefore$ Solution set is $(-\infty, 2]$

II. We have, $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$

$\Rightarrow \frac{1}{2}\left(\frac{3x}{5} + \frac{4}{1}\right) \geq \frac{1}{3}(x - 6)$

Taking L.C.M. in L.H.S.,

$\frac{1}{2}\left(\frac{3x + 20}{5}\right) \geq \frac{1}{3}(x - 6)$

$\Rightarrow \frac{3x + 20}{10} \geq \frac{x - 6}{3}$

$\Rightarrow \quad 3(3x + 20) \geq 10(x - 6)$

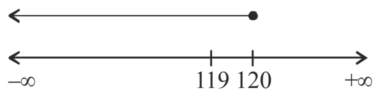
$\Rightarrow \quad 9x + 60 \geq 10x - 60$

Transferring the term 10x to L.H.S. and the term 60 to R.H.S.

$9x - 10x \geq -60 - 60 \Rightarrow -x \geq -120$

Multiplying both sides by –1,

$x \leq 120$



∴ Solution set is $(-\infty, 120]$.

34. **(a)** I. The region containing all the solutions of an inequality is called the solution region.

II. In order to identify the half plane represented by an inequality, it is just sufficient to take any point (a, b) (not on line) and check whether it satisfies the inequality or not. If it satisfies, then the inequality represents the half plane and shade the region, which contains the point, otherwise the inequality represents that half plane which does not contains the point within it. For convenience, the point (0, 0) is preferred.

35. **(a)** I. The given system of inequalities

$3x + 2y \leq 12$ ... (i)

$x \geq 1$ ... (ii)

$y \geq 2$ ... (iii)

Step I: Consider the given inequations as strict equations

i.e. $3x + 2y = 12$, $x = 1$, $y = 2$

Step II: Draw the table for $3x + 2y = 12$

x	0	4
y	6	0

(i.e., Find the points on x-axis and y-axis)

Step III: Plot the points and draw the graph

For $3x + 2y = 12$, and

Graph of $x = 1$ will be a line parallel to y-axis cutting x-axis at 1.

and Graph of $y = 2$ will be a line parallel to x-axis cutting y-axis at 2.

Step IV: Take a point (0, 0) and put it in the given inequations (i), (ii) and (iii).

i.e., $0 + 0 \leq 12$, $0 \leq 12$ [true]

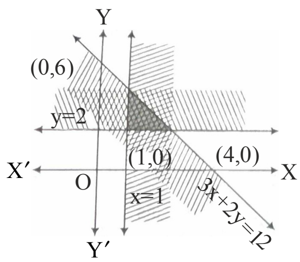
So, the shaded region will be towards the origin

$0 \geq 1$ [false]

So, the shaded region will be away from the origin

$0 \geq 2$ [false]

So, the shaded region will be away from the origin.



Thus, common shaded region shown the solution of the inequalities.

II. The given system of inequalities

$2x + y \geq 6$ ... (i)

$3x + 4y \leq 12$ ... (ii)

Step I: Consider the given inequations as strict equations

i.e., $2x + y = 6$

$3x + 4y = 12$

Step II: Find the points on the x-axis and y-axis for

$2x + y = 6$

x	0	3
y	6	0

and $3x + 4y = 12$

x	0	4
y	3	0

Step III: Plot the points and draw the graph using the above tables.

Step IV: Take a point (0, 0) and putting in the given inequations (i) and (ii),

i.e. $0 + 0 \geq 6$

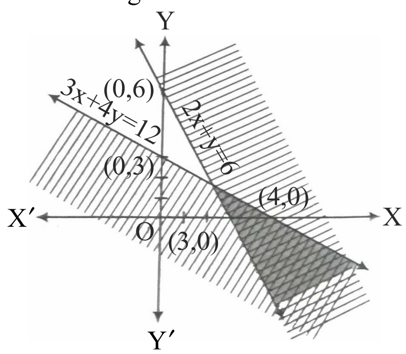
$0 \geq 6$ [false]

So, the shaded region will be away from the origin.

and $0 + 0 \leq 12$

$0 \leq 12$ (True)

So, the shaded region will be towards the origin.



Thus, common shaded region shows the solution of the inequality.

Since, common shaded region is not enclosed. So, it is not bounded.

III. The given system of inequalities

$x + y \geq 4$ ... (i)

$2x - y > 0$ ... (ii)

Step I: Consider the given inequations as strict equations

i.e., $x + y = 4$, $2x - y = 0$

Step II: Find the points on the x-axis and y-axis for

$x + y = 4$

x	0	4
y	4	0

and $2x - y = 0$

x	0	1
y	0	2

Step III: Plot the points to draw the graph using the above tables.

Step IV: Take a point (0, 0) and put it in the inequation (i)

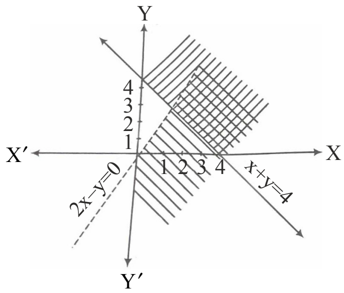
$0 + 0 \geq 4$ [false]

So, the shaded region will be away from the origin.

Take a point (1, 0) and put it in the inequation (ii)

$2 - 0 > 0$ [true]

So, the shaded region will be towards the point (1, 0)



Thus, the common shaded region shows the solution of the inequalities.

36. (d) **37.** (c)

38. (a) I. $-75 < 3x - 6 \Rightarrow -23 < x$

$3x - 6 \leq 0 \Rightarrow x \leq 2$

II. $14 \leq 3x + 11 \Rightarrow 3 \leq 3x \Rightarrow 1 \leq x$

$3x + 11 \leq 22 \Rightarrow 3x \leq 11 \Rightarrow x \leq \frac{11}{3}$

III. $-20 \leq 2 - 3x \Rightarrow x \leq \frac{22}{3}$

$2 - 3x \leq 36 \Rightarrow -34 \leq 3x \Rightarrow x \geq \frac{-34}{3}$

39. (c) Both the statements are correct.

40. (d) We are given :

$24x < 100$

or $\frac{24x}{24} < \frac{100}{24}$

or $x < \frac{100}{24}$

(I) When x is natural number, the following values of x make the statement true

$x = 1, 2, 3, 4.$

The solution set = {1, 2, 3, 4}

(II) When x is an integer, in this case the solutions of the given inequality are ......, –3, –2, –1, 0, 1, 2, 3, 4.

∴ The solution set of the inequality is {...., –3, –2, –1, 0, 1, 2, 3, 4}.

41. (b) Inequality is $3x + 8 > 2$

Transposing 8 to RHS $3x > 2 - 8 = -6$

Dividing by 3, $x > -2$

(I) When x is an integer the solution is {–1, 0, 1, 2, 3, ...}

(II) When x is real, the solution is $(-2, \infty)$.

MATCHING TYPE QUESTIONS

42. (a) (A) $2x - 4 \leq 0 \Rightarrow x \leq 2$

(B) $-3x + 12 < 0 \Rightarrow x > 4$

(C) $4x - 12 \geq 0 \Rightarrow x \geq 3$

(D) $7x + 9 > 30 \Rightarrow 7x > 21 \Rightarrow x > 3$

43. (c) A. The given system of inequalities

$2x - y > 1$ ... (i)

$x - 2y < -1$ ... (ii)

Step I : Consider the inequations as strict equations

i.e. $2x - y = 1$ and $x - 2y = -1$

Step II: Find the points on the x-axis and y-axis for $2x - y = 1$.

x	0	$\frac{1}{2}$
y	–1	0

and $x - 2y = -1$

x	0	–1
y	$\frac{1}{2}$	0

Step III: Plot the graph using the above tables.

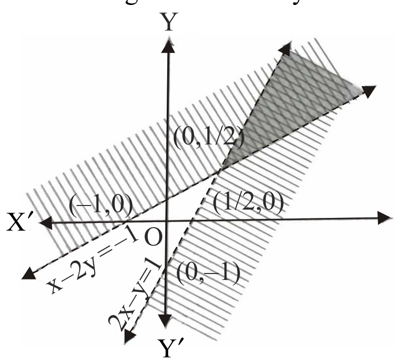
Step IV: Take a point (0, 0) and put it in the inequations (i) and (ii).

$0 - 0 > 1$, i.e., $0 > 1$ [false]

So, the shaded region will be away from the origin

and $0 - 0 < -1$, i.e., $0 < -1$ [false]

So, the shaded region will be away from the origin.



Thus, common shaded region shows the solution of the inequalities.

B. The given system of inequalities

$x + y \leq 6$ ... (i)

$x + y \geq 4$ ... (ii)

Step I: Consider the inequations as strict equations

i.e. $x + y = 6$ and $x + y = 4$

Step II: Find the points on the x-axis and y-axis for

$x + y = 6.$

x	0	6
y	6	0

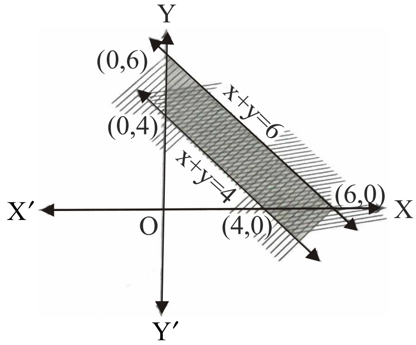
and $x + y = 4$

x	0	4
y	4	0

Step III: Plot the graph using the above tables.

Step IV: Take a point (0, 0) and put it in the inequations (i) and (ii),

i.e. $0 + 0 \leq 6$ i.e., $0 \leq 6$ [true]



So, the shaded region will be towards the origin.

and $0 + 0 \geq 4 \Rightarrow 0 \geq 4$ [false]

So, the shaded region will be away from the origin.

Thus, common shaded region shows the solution of the inequalities.

C. The given system of inequalities

$2x + y \geq 8$ ... (i)

$x + 2y \geq 10$ ... (ii)

Step I: Consider the inequations as strict equations

i.e. $2x + y = 8$ and $x + 2y = 10$

Step II: Find the points on the x-axis and y-axis for

$2x + y = 8$

x	0	4
y	8	0

and $x + 2y = 10$

x	0	10
y	5	0

Step III: Plot the points using the above tables and draw the graph.

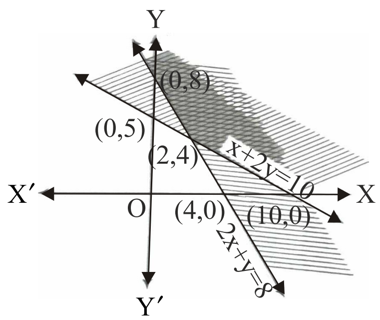
Step IV: Take a point (0, 0) and put it in the given inequations (i) and (ii),

i.e., $0 + 0 \geq 8$ i.e. $0 \geq 8$ [false]

So, the shaded region will be away from the origin.

i.e., $0 + 0 \geq 10$, i.e. $0 \geq 10$ [false]

So, the shaded region will be away from the origin.



Thus, common shaded region shows the solution of the inequalities.

D. The given system of inequalities

$x + y \leq 9$ ... (i)

$y > x$ ... (ii)

$x \geq 0$ ... (iii)

Step I: Consider the inequations as strict equations

i.e. $x + y = 9$, $y = x$, $x = 0$

Step II: Find the points on the x-axis and y-axis for

$x + y = 9$

x	0	9
y	9	0

and $y = x$

x	1	2	3
y	1	2	3

Step III: Plot the points using the above tables and draw the graph

For $x + y = 9$ and

For $y = x$

Graph of $x = 0$ will be the y-axis.

Step IV: Take a point (0, 0), put it in the inequations (i), (ii) and (iii), we get

$0 + 0 \leq 9$ [true]

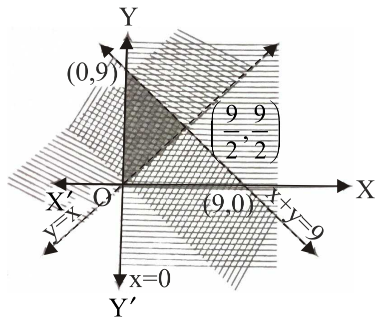
So, the shaded region will be towards the origin.

Take a point (0, 1), put in $y > x$, $1 > 0$ [true]

So, the shaded region will be towards the origin.

Take a point (1, 0), put it in $x \geq 0$, $1 \geq 0$ [true]

So, the shaded region will be towards the origin.



Thus, common shaded region shows the solution of the inequalities.

44. (c) (A) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

$\Rightarrow \frac{3x-4}{2} \geq \frac{x+1-4}{4}$

$\Rightarrow 3x - 4 \geq \frac{x-3}{2}$

$\Rightarrow 6x - 8 \geq x - 3$

$\Rightarrow 5x \geq 5 \Rightarrow x \geq 1$

(B) $3x - 2 < 2x + 1 \Rightarrow x < 3$

(C) $3(1-x) < 2(x+4) \Rightarrow 3 - 3x < 2x + 8$

$\Rightarrow -5 < 5x \Rightarrow x > -1$

(D) $3x - 7 < 5 + x \Rightarrow 2x < 12 \Rightarrow x < 6$

$11 - 5x \leq 1 \Rightarrow 10 \leq 5x \Rightarrow 2 \leq x$

45. (b) (A) We draw the graph of the equation

$x + y = 5$ ...(i)

Putting $y = 0$, $x = 5$, therefore the point on the x-axis is (5, 0). The point on the y-axis is (0, 5). AB is the graph of (i) (See Fig)

Putting $x = 0$, $y = 0$ in the given inequality, we have $0 + 0 < 5$ or $5 > 0$ which is true. Hence, origin lies in the half plane region I.

Clearly, any point on the line does not satisfy the given inequality.

Hence, the shaded region I excluding the points on the line is the solution region of the inequality.

(B) We draw the graph of the equation

$2x + y = 6$ ...(i)

Putting $x = 0$, $y = 6$, therefore the point on y-axis is (0, 6) and the point on x-axis is (3, 0). AB is the graph of (i).

Putting $x = 0$, $y = 0$ in the given inequality, we have $2(0) + 0 \geq 6$ or $0 \geq 6$, which is false.

Hence, origin does not lie in the half plane region I.

Clearly, any point on the line satisfy the given inequality.

Hence, the shaded region II including the points on the line is the solution region of the inequality.

(C) We draw the graph of the equation $3x + 4y = 12$.

The line passes through the points (4, 0), (0, 3). This line is represented by AB.

Now consider the inequality $3x + 4y \leq 12$

Putting $x = 0, y = 0$

$0 + 0 = 0 \leq 12$, which is true

$\therefore$ Origin lies in the region of $3x + 4y \leq 12$

The shaded region represents this inequality.

(D) We draw the graph of $2x - 3y = 6$

The line passes through (3, 0), (0, –2)

AB represents the equation $2x - 3y = 6$

Now consider the inequality $2x - 3y > 6$

Putting $x = 0, y = 0$

$0 = 0 > 6$ is not true.

$\therefore$ Origin does not lie in the region of $2x - 3y > 6$

The graph of $2x - 3y > 6$ is shown as shaded area.

INTEGER TYPE QUESTIONS

46. (c) $4x + 3 < 6x + 7$

$\Rightarrow -2x < 4$

$\Rightarrow -x < 2 \Rightarrow x > -2$

$\Rightarrow x \in (-2, \infty)$

47. (d) $\dfrac{5-2x}{3} \leq \dfrac{x}{6} - 5$

$\Rightarrow \dfrac{5-2x}{3} \leq \dfrac{x-30}{6}$

$\Rightarrow 5 - 2x \leq \dfrac{x-30}{2}$

$\Rightarrow 10 - 4x \leq x - 30 \Rightarrow 40 \leq 5x$

$\Rightarrow 8 \leq x \Rightarrow x \in [8, \infty)$

48. (c) $3(2 - x) \geq 2(1 - x)$

$\Rightarrow 6 - 3x \geq 2 - 2x$

$\Rightarrow -x \geq -4 \Rightarrow x \leq 4$

$\Rightarrow x \in (-\infty, 4]$

49. (a) $\dfrac{2x-1}{3} \geq \dfrac{15x - 10 - 8 + 4x}{20}$

$\Rightarrow \dfrac{2x-1}{3} \geq \dfrac{19x-18}{20}$

$\Rightarrow 40x - 20 \geq 57x - 54$

$\Rightarrow -17x \geq -34 \Rightarrow x \leq 2$

$\Rightarrow x \in (-\infty, 2]$

50. (d) Given inequality is $5x + 1 > -24$

$\Rightarrow 5x > -25 \Rightarrow x > -5$

Also, $5x - 1 < 24$

$\Rightarrow 5x < 25 \Rightarrow x < 5$

Hence, $-5 < x < 5 \Rightarrow x \in (-5, 5)$

51. (b) $2x - 7 < 11 \Rightarrow 2x < 18 \Rightarrow x < 9$

$3x + 4 < -5 \Rightarrow 3x < -9 \Rightarrow x < -3$

Hence, common solution is $x < -3$.

So, $x \in (-\infty, -3)$

52. (a) By definition of $|x|$, we have

$|x| < 3 \Rightarrow -3 < x < 3$

$\Rightarrow m = 3$.

53. (b) Let shortest side measure x cm. Therefore the longest side will be $3x$ cm and third side will be $(3x - 2)$ cm

According to the problem,

$x + 3x + 3x - 2 \geq 61$

$\Rightarrow 7x - 2 \geq 61$ or $7x \geq 63$

$\Rightarrow x \geq 9$cm

Hence, the minimum length of the shortest side is 9 cm and the other sides measure 27 cm and 25 cm.

54. (c) $-8 \leq 5x - 3 \Rightarrow -5 \leq 5x \Rightarrow -1 \leq x$

$5x - 3 < 7 \Rightarrow 5x < 10 \Rightarrow x < 2$

Hence, common sol is $-1 \leq x < 2$

$\Rightarrow x \in [-1, 2)$

$\Rightarrow a = 1, b = 2$ and $a + b = 3$

55. (a) Let x and $x + 2$ be two odd natural numbers.

we have, $x > 10$ …(i)

and $x + (x + 2) < 40$ …(ii)

On solving (i) and (ii), we get

$10 < x < 19$

So, required pairs are (11, 13), (13, 15), (15, 17) and (17, 19)

ASSERTION - REASON TYPE QUESTIONS

56. (b) Let us consider some inequalities :

$ax + b < 0$ … (i)

$ax + b > 0$ … (ii)

$ax + b \leq 0$ … (iii)

$ax + b \geq 0$ … (iv)

$ax + by > c$ … (v)

$ax + by \leq c$ … (vi)

$ax^2 + bx + c > 0$ … (vii)

$ax^2 + bx + c \leq 0$ … (viii)

Inequalities (i), (ii), (v) and (vii) are strict inequalities, while inequalities (iii), (iv), (vi) and (viii) are slack inequalities.

$\therefore$ Both Assertion and Reason are correct but Reason cannot explain Assertion.

57. **(d)** Assertion is false, Reason is true because if $a < b$, $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

58. **(b)** We have, $|3x - 5| > 9$

$\Rightarrow 3x - 5 < -9$ or $3x - 5 > 9$

$\Rightarrow 3x < -4$ or $3x > 14$

$\Rightarrow x < \frac{-4}{3}$ or $x > \frac{14}{3}$

$\therefore x \in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$.

59. **(b)** Both Assertion and Reason are correct but Reason is not correct explanation for the Assertion.

60. **(b)** Both are correct.

61. **(b)** Both are correct; Reason is not the correct explanation of Assertion.

62. **(b)** Both Assertion and Reason are correct but Reason is not the correct explanation.

Reason: $5x - 3 < 7$

$\Rightarrow 5x < 10 \Rightarrow x < 2$

$\Rightarrow x \in (-\infty, 2)$

63. **(c)** Assertion is correct.

$3x + 8 > 2 \Rightarrow 3x > -6$

$\Rightarrow x > -2$

$\Rightarrow x \in \{-1, 0, 1, 2, \ldots\}$

Reason is incorrect.

$4x + 3 < 5x + 7$

$-x < 4 \Rightarrow x > -4$

$\Rightarrow x \in (-4, \infty)$

64. **(c)** Assertion is correct. Reason is incorrect. If a point satisfying the line $ax + by = c$, then it will lie on the line.

65. **(b)** Both are correct but Reason is not the correct explanation.

66. **(d)** Assertion is incorrect. Reason is correct.

CRITICAL THINKING TYPE QUESTIONS

67. **(c)** If x cm is the breadth, then

$2(3x + x) \geq 160 \Rightarrow x \geq 20$

68. **(c)** $|x - 1| \leq 3 \Rightarrow -3 \leq x - 1 \leq 3 \Rightarrow -2 \leq x \leq 4$

and $|x - 1| \geq 1 \Rightarrow x - 1 \leq -1$ or $x - 1 \geq 1$

$\Rightarrow x \leq 0$ or $x \geq 2$

Taking the common values of x, we get

$x \in [-2, 0] \cup [2, 4]$

69. **(a)** Let x be the marks obtained by student in the annual examination. Then,

$\frac{62 + 48 + x}{3} \geq 60$

or $110 + x \geq 180$

or $x \geq 70$

Thus, the student must obtain a minimum of 70 marks to get an average of at least 60 marks.

70. **(b)** Let Ravi got x marks in third unit test.

$\therefore$ Average marks obtained by Ravi

$= \frac{\text{Sum of marks in all tests}}{\text{Number of tests}} = \frac{70 + 75 + x}{3} = \frac{145 + x}{3}$

Now, it is given that he wants to obtain an average of at least 60 marks.

At least 60 marks means that the marks should be greater than or equal to 60.

i.e. $\frac{145 + x}{3} \geq 60$

$\Rightarrow 145 + x \geq 60 \times 3$

$\Rightarrow 145 + x \geq 180$

Now, transferring the term 145 to R.H.S.,

$x \geq 180 - 145 \Rightarrow x \geq 35$

i.e. Ravi should get greater than or equal to 35 marks in third unit test to get an average of at least 60 marks.

$\therefore$ Minimum marks Ravi should get = 35.

71. **(b)** Let numbers are $2x$ and $2x + 2$

Then, according to the question,

$2x > 5 \Rightarrow x > \frac{5}{2}$

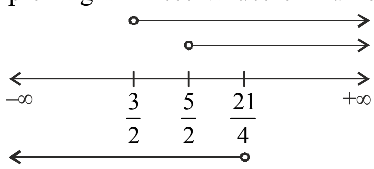
and $2x + 2 > 5 \Rightarrow 2x > 5 - 2$

$\Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2}$

and $2x + 2x + 2 < 23 \Rightarrow 4x < 23 - 2$

$\Rightarrow 4x < 21 \Rightarrow x < \frac{21}{4}$

Now, plotting all these values on number line



From above graph, it is clear that $x \in \left(\frac{5}{2}, \frac{21}{4}\right)$ in which integer values are $x = 3, 4, 5$.

When $x = 3$, pair is $(2 \times 3, 2 \times 3 + 2) = (6, 8)$

When $x = 4$, pair is $(2 \times 4, 2 \times 4 + 2) = (8, 10)$

When $x = 5$, pair is $(2 \times 5, 2 \times 5 + 2) = (10, 12)$

$\therefore$ Required pairs are (6, 8), (8, 10), (10, 12).

72. **(b)** Let the shortest side be x cm.

Then, by given condition, second length = x + 3 cm

Third length = 2x cm

Also given, total length = 91

Hence, sum of all the three lengths should be less than or equal to 91

$x + x + 3 + 2x \leq 91$

$\Rightarrow 4x + 3 \leq 91$

Subtracting (–3) to each term,

$-3 + 4x + 3 \leq 91 - 3$

$\Rightarrow 4x \leq 88$

$\Rightarrow \frac{4x}{4} \leq \frac{88}{4} \Rightarrow x \leq \frac{88}{4}$

$\Rightarrow x \leq 22$ cm ... (i)

Again, given that

Third length ≥ second length + 5

$\Rightarrow$ $2x \geq (x + 3) + 5$

$\Rightarrow$ $2x \geq x + (3 + 5)$

Transferring the term x to L.H.S.,

$2x - x \geq 8$

$\Rightarrow$ $x \geq 8$ ... (ii)

From equations (i) and (ii), length of shortest board should be greater than or equal to 8 but less than or equal to 22, i.e., $8 \leq x \leq 22$.

73. (c) Let breadth of rectangle be x cm.

$\therefore$ Length of rectangle = 3x

Perimeter of rectangle = 2(Length + Breadth)

$= 2(x + 3x) = 8x$

Given, Perimeter $\geq$ 160 cm

$8x \geq 160$

Dividing both sides by 8,

$x \geq 20$ cm

74. (a) We have, $2 \leq |x - 3| < 4$

Case I : If $x < 3$, then

$2 \leq |x - 3| < 4$

$\Rightarrow$ $2 \leq -(x - 3) < 4$

$\Rightarrow$ $2 \leq -x + 3 < 4$

Subtracting 3 from both sides,

$-1 \leq -x < 1$

Multiplying (–1) on both sides,

$-1 < x \leq 1$

$\Rightarrow$ $x \in (-1, 1]$

Case II : If $x > 3$, then

$2 \leq |x - 3| < 4$

$\Rightarrow$ $2 \leq x - 3 < 4$

Adding 3 on both sides,

$\Rightarrow$ $5 \leq x < 7$

Hence, the solution set of given inequality is $x \in (-1, 1] \cup [5, 7)$.

75. (c) We have

$$IQ = \frac{MA}{CA} \times 100$$

$$\Rightarrow IQ = \frac{MA}{12} \times 100 \quad [\because CA = 12 \text{ years}]$$

$$= \frac{25}{3} MA$$

Given, $80 \leq IQ \leq 140$

$$\Rightarrow 80 \leq \frac{25}{3} MA \leq 140$$

$$\Rightarrow 240 \leq 25MA \leq 420$$

$$\Rightarrow \frac{240}{25} \leq MA \leq \frac{420}{25}$$

$$\Rightarrow 9.6 \leq MA \leq 16.8$$

76. (c) The inequalities are :

$750x + 150y \leq 15000$

i.e. $5x + y \leq 100$ … (i)

$x + y \leq 60$ … (ii)

$x \geq 0$ … (iii)

$y \geq 0$ … (iv)

The lines corresponding to (i) and (ii) are

$5x + y = 100$ … (v)

$x + y = 60$ … (vi)

Table for $5x + y = 100$

x	0	20
y	100	0

Table for $x + y = 60$

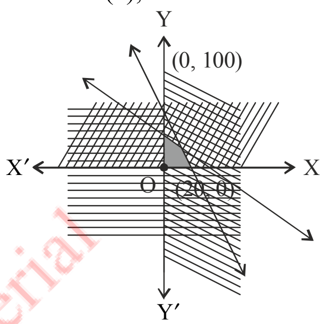
x	0	60
y	60	0

First, draw the lines (v) and (vi)

$\because$ $5(0) + 0 \leq 100$

i.e., $0 \leq 100$, which is true.

Therefore, inequality (i) represent the half plane made by the line (v), which contains the origin.



Again, $0 + 0 \leq 60$

i.e. $0 \leq 60$, which is true.

Therefore, inequality (ii) represent the half plane made by the line (vi) which contains origin. Inequality $x \geq 0$ represent the half plane on the right of y-axis. Inequality $y \geq 0$ represent the half plane above x-axis.

77. (d) The given system of inequalities

$x + 2y \leq 10$ ... (i)

$x + y \geq 1$ ... (ii)

$x - y \leq 0$ ... (iii)

$x \geq 0, y \geq 0$ ... (iv)

Step I: Consider the given inequations as strict equations,

i.e. $x + 2y = 10$, $x + y = 1$, $x - y = 0$

and $x = 0$, $y = 0$

Step II: Find the points on the x-axis and y-axis for

$x + 2y = 10$

x	0	10
y	5	0

and $x + y = 1$

x	0	1
y	1	0

For $x - y = 0$

x	1	2
y	1	2

Step III: Plot the graph of $x + 2y = 10$, $x + y = 1$, $x - y = 0$ using the above tables.

Step IV: Take a point (0, 0) and put it in the inequations (i) and (ii),

$0 + 0 \leq 10$ [true]

So, the shaded region will be towards origin,

and $0 + 0 \geq 1$ [false]

So, the shaded region will be away from the origin.

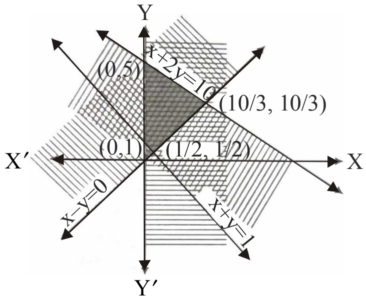
Again, take a point (2, 2) and put it in the inequation (iv), we get

$2 \geq 0, 2 \geq 0$ [true]

So, the shaded region will be towards point (2, 2). And take a point (0, 1) and put it in the inequation (iii), we get

$0 - 1 \leq 0$ [true]

So, the shaded region will be towards point (0, 1).



Thus, the common shaded region shows the solution of the inequalities.

78. **(b)** (i) Consider the line $x + y = 8$. We observe that the shaded region and origin lie on the same side of this line and (0, 0) satisfies $x + y \leq 8$. Therefore, $x + y \leq 8$ is the linear inequality corresponding to the line $x + y = 8$.

(ii) Consider $x + y = 4$. We observe that shaded region and origin are on the opposite side of this line and (0, 0) satisfies $x + y \leq 4$. Therefore, we must have $x + y \geq 4$ as linear inequalities corresponding to the line $x + y = 4$.

(iii) Shaded portion lie below the line $y = 5$. So, $y \leq 5$ is the linear inequality corresponding to $y = 5$.

(iv) Shaded portion lie on the left side of the line $x = 5$. So, $x \leq 5$ is the linear inequality corresponding to $x = 5$.

(v) Also, the shaded region lies in the first quadrant only. Therefore, $x \geq 0$, $y \geq 0$.

In view of (i), (ii), (iii), (iv) and (v) above the linear inequalities corresponding to the given solutions are:

$x + y \leq 8$, $x + y \geq 4$, $y \leq 5$, $x \leq 5$, $x \geq 0$ and $y \geq 0$.

79. **(c)** Let the 2% boric acid solution be x L.

$\therefore$ Mixture = (640 + x)L

Now, according to the question, two conditions arise :

I. 2% of x + 8% of 640 > 4% of (640 + x)

II. 2% of x + 8% of 640 < 6% of (640 + x)

From condition I,

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 > \frac{4}{100} \times (640 + x)$$

Multiplying both sides by 100,

$$100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640\right] > \frac{4}{100} \times (640 + x) \times 100$$

$\Rightarrow 2x + 8 \times 640 > 4 \times 640 + 4x$

Transferring the term 4x to L.H.S. and the term (8 × 640) to R.H.S.

$2x - 4x > 4 \times 640 - 8 \times 640$

$\Rightarrow -2x > 640(4 - 8)$

$\Rightarrow -2x > -4 \times 640$

Dividing both sides by –2,

$$\frac{-2x}{-2} < \frac{-4 \times 640}{-2}$$

$\Rightarrow x < 2 \times 640$

$\Rightarrow x < 1280$ ... (i)

From condition II,

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 < \frac{6}{100} \times (640 + x)$$

$$\Rightarrow 100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640\right] < [6 \times 640 + 6x] \times \frac{100}{100}$$

$\Rightarrow 2x + 8 \times 640 < 6 \times 640 + 6x$

Transferring the term 6x to L.H.S. and the term (8 × 640) to R.H.S.,

$2x - 6x < 6 \times 640 - 8 \times 640$

$\Rightarrow -4x < 640(6 - 8)$ $\Rightarrow -4x < -2 \times 640$

Dividing both sides by –4,

$$\frac{-4x}{-4} > \frac{-2 \times 640}{-4}$$

$\Rightarrow x > 320$ ... (ii)

Hence, from equations (i) and (ii),

$320 < x < 1280$ i.e., $x \in (320, 1280)$

The number of litres to be added should be greater than 320 L and less than 1280 L.

80. **(a)** Given, $C(x) = 26000 + 30x$

and $R(x) = 43x$

$\therefore$ Profit = R(x) – C(x)

$= 43x - (26000 + 30x) = 13x - 26000$

For some profit, $13x - 26000 > 0$

$\Rightarrow 13x > 26000$

$\Rightarrow x > 2000$

81. **(a)** Let x litres of 30% acid solution is required to be added. Then,

Total mixture = (x + 600) litres

$\therefore$ 30% of x + 12% of 600 > 15% of (x + 600)

and 30% of x + 12% of 600 < 18% of (x + 600)

or $$\frac{30x}{100} + \frac{12}{100}(600) > \frac{15}{100}(x + 600)$$

and $$\frac{30x}{100} + \frac{12}{100}(600) < \frac{18}{100}(x + 600)$$

or $30x + 7200 > 15x + 9000$

and $30x + 7200 < 18x + 10800$

or $15x > 1800$ and $12x < 3600$

or $x > 120$ and $x < 300$

i.e. $120 < x < 300$

Thus, the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.

82. **(c)** We have $\dfrac{|x + 3| + x}{x + 2} > 1$

$$\Rightarrow \frac{|x + 3| + x}{x + 2} - 1 > 0 \Rightarrow \frac{|x + 3| - 2}{x + 2} > 0$$

Now, two cases arise :

Case I : When $x + 3 \geq 0$, i.e. $x \geq -3$. Then,

$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{x+3-2}{x+2} > 0$

$\Rightarrow \frac{x+1}{x+2} > 0$

$\Rightarrow \{(x + 1) > 0 \text{ and } x + 2 > 0\}$

or $\{x + 1 < 0 \text{ and } x + 2 < 0\}$

$\Rightarrow \{x > -1 \text{ and } x > -2\}$ or $\{x < -1 \text{ and } x < -2\}$

$\Rightarrow x > -1$ or $x < -2$

$\Rightarrow x \in (-1, \infty)$ or $x \in (-\infty, -2)$

$\Rightarrow x \in (-3, -2) \cup (-1, \infty)$ [Since $x \geq -3$] ... (i)

Case II : When $x + 3 < 0$, i.e. $x < -3$

$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{-x-3-2}{x+2} > 0$

$\Rightarrow \frac{-(x+5)}{x+2} > 0 \Rightarrow \frac{x+5}{x+2} < 0$

$\Rightarrow (x+5<0 \text{ and } x+2>0)$ or $(x+5>0 \text{ and } x+2<0)$

$\Rightarrow (x < -5 \text{ and } x > -2)$ or $(x > -5 \text{ and } x < -2)$

it is not possible.

$\Rightarrow x \in (-5, -2)$ ... (ii)

Combining (i) and (ii), the required solution is $x \in (-5, -2) \cup (-1, \infty)$.

83. (c) We have, $|2x - 3| < |x + 5|$

$\Rightarrow |2x - 3| - |x + 5| < 0$

$$\Rightarrow \begin{cases} 3-2x+x+5<0, & x \leq -5 \\ 3-2x-x-5<0, & -5 < x \leq \frac{3}{2} \\ 2x-3-x-5<0, & x > \frac{3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x > 8, & x \leq -5 \\ x > -\frac{2}{3}, & -5 < x \leq \frac{3}{2} \\ x < 8, & x > \frac{3}{2} \end{cases}$$

$\Rightarrow x \in \left(-\frac{2}{3}, \frac{3}{2}\right] \cup \left(\frac{3}{2}, 8\right) \Rightarrow x \in \left(-\frac{2}{3}, 8\right)$

84. (b) $(x - 1)^2$ is always positive except when $x = 1$ (and then it is 0)

$\therefore$ Solution is when $x + 4 < 0$ and $x \neq 1$

i.e. $x < -4, x \neq 1$

$\therefore x \in (-\infty, -4)$.

85. (c) $\left|1 + \frac{3}{x}\right| > 2$

Case I : $1 + \frac{3}{x} > 2 \Rightarrow \frac{3}{x} > 1$ (Clearly $x > 0$)

$\Rightarrow 3 > x$ or $x < 3$

Case II : $1 + \frac{3}{x} < -2 \Rightarrow \frac{3}{x} < -3$ (Clearly $x < 0$)

$\Rightarrow 3 > -3x \Rightarrow -1 < x$ or $x > -1$

Hence, either $0 < x < 3$ or $-1 < x < 0$

86. (b) $|2x - 3| < |x + 2|$

$\Rightarrow -|x + 2| < 2x - 3 < |x + 2|$ ... (i)

Case I : $x + 2 \geq 0$. Then by (i),

$-(x + 2) < 2x - 3 < x + 2$

$\Rightarrow -x - 2 < 2x - 3 < x + 2$

$\Rightarrow 1 < 3x$ and $x < 5 \Rightarrow \frac{1}{3} < x < 5$

Case II : $x + 2 < 0$. Then by (i),

$(x + 2) < 2x - 3 < -(x + 2)$

$\Rightarrow -(x + 2) > 2x - 3 > (x + 2)$

$\Rightarrow 1 > 3x$ and $x > 5 \Rightarrow \frac{1}{3} \leq x$ and $x > 5$, Not possible.

87. (b) $\left|x + \frac{1}{x}\right| > 2$ [Clearly $x \neq 0$]

$\Rightarrow \left|\frac{x^2+1}{x}\right| > 2 \Rightarrow \frac{x^2+1}{|x|} > 2$ [$\because x^2 + 1 > 0$]

$\Rightarrow x^2 + 1 > 2|x|$

$\Rightarrow |x|^2 - 2|x| + 1 > 0 \Rightarrow (|x| - 1)^2 > 0$

$\Rightarrow |x| \neq 1 \Rightarrow x \neq -1, 1$

$\therefore x \in R - \{-1, 0, 1\}$.

88. (d) From the graph,

$-7x + 4y \leq 14$, $x - 6y \leq 3$

$3x + 4y \leq 18$, $2x + 3y \geq 3$

CHAPTER

7

PERMUTATIONS AND COMBINATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then the value of r is
(a) 41 (b) 14 (c) 10 (d) 51

2. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the value of n is:
(a) 20 (b) 19 (c) 18 (d) 17

3. If ${}^{30}C_{r+2} = {}^{30}C_{r-2}$, then r equals:
(a) 8 (b) 15 (c) 30 (d) 32

4. The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is
(a) 432 (b) 108 (c) 36 (d) 18

5. In an examination, there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is :
(a) 11 (b) 12 (c) 27 (d) 63

6. How many arrangements can be made out of the letters of the word " MOTHER" taken four at a time so that each arrangement contains the letter ´M´?
(a) 240 (b) 120 (c) 60 (d) 360

7. In how many ways can a bowler take four wickets in a single 6-ball over ?
(a) 6 (b) 15 (c) 20 (d) 30

8. There are four chairs with two chairs in each row. In how many ways can four persons be seated on the chairs, so that no chair remains unoccupied ?
(a) 6 (b) 12 (c) 24 (d) 48

9. If a secretary and a joint secretary are to be selected from a committee of 11 members, then in how many ways can they be selected ?
(a) 110 (b) 55 (c) 22 (d) 11

10. A bag contains 3 black, 4 white and 2 red balls, all the balls being different. Number of selections of atmost 6 balls containing balls of all the colours is
(a) 1008 (b) 1080 (c) 1204 (d) 1130

11. Number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour.
(a) $6 \times (9!)^2$ (b) 12!
(c) $4 \times (8!)^2$ (d) $5 \times (9!)^2$

12. Number of words each containing 3 consonants and 2 vowels that can be formed out of 5 consonants and 4 vowels is
(a) 3600 (b) 7200 (c) 6728 (d) 2703

13. Every body in a room shakes hands with every body else. If total number of hand-shaken is 66, then the number of persons in the room is
(a) 11 (b) 10 (c) 12 (d) 19

14. Number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is
(a) 652 (b) 650 (c) 651 (d) 640

15. Total number of eight digit numbers in which all digits are different is
(a) $\frac{8.7!}{3}$ (b) $\frac{5.8!}{3}$ (c) $\frac{8.9!}{2}$ (d) $\frac{9.9!}{2}$

16. Number of words from the letters of the words BHARAT in which B and H will never come together is
(a) 210 (b) 240 (c) 422 (d) 400

17. Four couples (husband and wife) decide to form a committee of four members, then the number of different committees that can be formed in which no couple finds a place is
(a) 15 (b) 16 (c) 20 (d) 21

18. Number of different ways in which 8 persons can stand in a row so that between two particular person A and B there are always two person is
(a) 11 (b) 13 (c) 15 (d) 16

19. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are
(a) 216 (b) 375 (c) 400 (d) 720

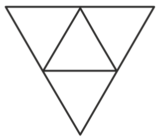
20. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number
(a) 601 (b) 600 (c) 603 (d) 602

21. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
(a) $8.{}^{6}C_4.{}^{7}C_4$ (b) $6.7.{}^{8}C_4$
(c) $6.8.{}^{7}C_4.$ (d) $7.{}^{6}C_4.{}^{8}C_4$

22. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ?
(a) 16 (b) 36 (c) 60 (d) 180

23. The number of ways in which four letters of the word MATHEMATICS can be arranged is given by
(a) 136 (b) 192 (c) 1680 (d) 2454

24. In how many ways can this diagram be coloured subject to the following two conditions?
(i) Each of the smaller triangle is to be painted with one of three colours: red, blue or green.
(ii) No two adjacent regions have the same colour.



(a) 20 (b) 24 (c) 28 (d) 30

25. There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?
(a) 72 (b) 144 (c) 14 (d) 19

26. In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together?
(a) 41472 (b) 41470 (c) 41400 (d) 41274

27. The number of ways of distributing 50 identical things among 8 persons in such a way that three of them get 8 things each, two of them get 7 things each, and remaining 3 get 4 things each, is equal to

(a) $\dfrac{(50!)(8!)}{(8!)^3(3!)^2(7!)^2(4!)^3(2!)}$

(b) $\dfrac{(50!)(8!)}{(8!)^3(7!)^3(4!)^3}$

(c) $\dfrac{(50!)}{(8!)^3(7!)^2(4!)^3}$

(d) $\dfrac{(8!)}{(3!)^2(2!)}$

28. If eleven members of a committee sit at a round table so that the President and Secretary always sit together, then the number of arrangements is
(a) $10!\times 2$ (b) $10!$ (c) $9!\times 2$ (d) $11!\times 2!$

29. ABC is a triangle. 4, 5, 6 points are marked on the sides AB, BC, CA, respectively, the number of triangles on different side is
(a) $(4+5+6)!$ (b) $(4-1)(5-1)(6-1)$
(c) $5!\,4!\,6!$ (d) $4\times 5\times 6$

30. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to
(a) 60 (b) 120 (c) 7200 (d) 720

31. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is
(a) 6 (b) 18 (c) 12 (d) 9

32. The number of ways in which 3 prizes can be distributed to 4 children, so that no child gets all the three prizes, are
(a) 64 (b) 62 (c) 60 (d) None of these

33. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then, what is the rank of the word RACHIT?
(a) 479 (b) 480 (c) 481 (d) 482

34. The number of chords that can be drawn through 21 points on a circle, is
(a) 200 (b) 190 (c) 210 (d) None of these

35. The number of ways a student can choose a programme out of 5 courses, if 9 courses are available and 2 specific courses are compulsory for every student is
(a) 35 (b) 40 (c) 24 (d) 120

36. The number of ways in which we can choose a committee from four men and six women so that the committee include at least two men and exactly twice as many women as men is
(a) 94 (b) 126 (c) 128 (d) None of these

37. A father with 8 children takes them 3 at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is
(a) 56 (b) 100 (c) 112 (d) None of these

38. 4 buses runs between Bhopal and Gwalior. If a man goes from Gwalior to Bhopal by a bus and comes back to Gwalior by another bus, then the total possible ways are
(a) 12 (b) 16 (c) 4 (d) 8

39. Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to the number of heads is
(a) 20 (b) 9 (c) 120 (d) 40

40. The figures 4, 5, 6, 7, 8 are written in every possible order. The number of numbers greater than 56000 is
(a) 72 (b) 96 (c) 90 (d) 98

41. There are 5 roads leading to a town from a village. The number of different ways in which a villager can go to the town and return back, is
(a) 25 (b) 20 (c) 10 (d) 5

42. The number of numbers that can be formed with the help of the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places, is
(a) 24 (b) 18 (c) 12 (d) 30

43. In a circus, there are ten cages for accommodating ten animals. Out of these, four cages are so small that five out of 10 animals cannot enter into them. In how many ways will it be possible to accommodate ten animals in these ten cages?
(a) 66400 (b) 86400 (c) 96400 (d) None of these

44. On the occasion of Deepawali festival, each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is
(a) ${}^{20}C_2$ (b) $2\cdot{}^{20}C_2$
(c) $2\cdot{}^{20}P_2$ (d) None of these

45. To fill 12 vacancies, there are 25 candidates of which five are from scheduled caste. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, then the number of ways in which the selection can be made
(a) $^5C_3 \times {}^{22}C_9$ (b) $^{22}C_9 - {}^5C_3$
(c) $^{22}C_3 + {}^5C_3$ (d) None of these

46. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is
(a) 9(10!) (b) 2(10!) (c) 45(8!) (d) 10!

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

47. The number of 3 letters words, with or without meaning which can be formed out of the letters of the word 'NUMBER'.
Statement I : When repetition of letters is not allowed is 120.
Statement II : When repetition of letters is allowed is 216.
Choose the correct option.
(a) Only Statement I is correct
(b) Only Statement II is correct
(c) Both I and II are correct
(d) Both I and II are false

48. The number of 4 letter words that can be formed from letters of the word 'PART', when:
Statement I : Repetition is not allowed is 24.
Statement II : Repetition is allowed is 256.
Which of the above statement(s) is/are true?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

49. Consider the following statements:
Statement I : The number of diagonals of n-sided polygon is $^nC_2 - n$.
Statement II : A polygon has 44 diagonals. The number of its sides are 10.
Choose the correct option from the choices given below.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

50. A committee of 7 has to be formed from 9 boys and 4 girls.
I. In 504 ways, this can be done, when the committee consists of exactly 3 girls.
II. In 588 ways, this can be done, when the committee consists of at least 3 girls.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

51. Consider the following statements.
I. $^nC_r + 2\,^nC_{r-1} + {}^nC_{r-2} = {}^{n+2}C_r$
II. $^nC_p = {}^nC_q \Rightarrow p = q \text{ or } p + q = n$
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

52. Consider the following statements.
I. Value of $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$ is zero.
II. The total number of 9 digit numbers which have all different digits is 9!
Choose the correct option.
(a) Only I is true (b) Only II is true.
(c) Both are true. (d) Both are false.

53. Consider the following statements.
I. Three letters can be posted in five letter boxes in 3^5 ways.
II. In the permutations of n things, r taken together, the number of permutations in which m particular things occur together is $^{n-m}P_{r-m} \times {}^rP_m$.
Choose the correct option.
(a) Only I is false. (b) Only II is false.
(c) Both are false. (d) Both are true.

54. Consider the following statements.
I. If some or all n objects are taken at a time, the number of combinations is $2^n - 1$.
II. An arrangement in a definite order which can be made by taking some or all of a number of things is called a permutation.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

55. Consider the following statements.
I. If there are n different objects, then $^nC_r = {}^nC_{n-r}$, $0 \le r \le n$.
II. If there are n different objects, then $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, $0 \le r \le n$
Choose the correct option.
(a) Both are false. (b) Both are true.
(c) Only I is true. (d) Only II is true.

56. Consider the following statements.
I. If $^nP_r = {}^nP_{r+1}$ and $^nC_r = {}^nC_{r-1}$, then the values of n and r are 3 and 2 respectively.
II. From a class of 32 students, 4 are to be chosen for a competition. This can be done in $^{32}C_2$ ways.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are false. (d) Both are true.

57. Consider the following statements.
I. If n is an even natural number, then the greatest among $^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ is $^nC_{n/2}$.
II. If n is an odd natural number, then the greatest among $^nC_0, {}^nC_1, {}^nC_2 \ldots, {}^nC_n$ is $^nC_{\frac{n-1}{2}}$ or $^nC_{\frac{n+1}{2}}$.
Choose the correct option.
(a) Only I is false. (b) Only II is true.
(c) Both are true. (d) Both are false.

58. Consider the following statements.

If n is a natural number and r is non-negative integer such that $0 \le r \le n$, then

I. $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

II. $^nC_r = \frac{n}{r}\cdot{}^{n-1}C_{r-1}$

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

59. Consider the following statements.

I. The continued product of first n natural numbers is called the permutation.

II. L.C.M of 4!, 5! and 6! is 720.

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

60.

Column - I	Column - II
A. $\frac{7!}{5!}$ equals	1. 28
B. $\frac{12!}{(10!)(2!)}$ equals	2. 42
C. $\frac{8!}{6!\times 2!}$ equals	3. 66

Codes:

	A	B	C
(a)	1	2	3
(b)	1	3	2
(c)	3	2	1
(d)	2	3	1

61. Using the digits 1, 2, 3, 4, 5, 6, 7, a number of 4 different digits is formed. Find :

Column - I	Column - II
A. How many numbers are formed?	1. 840
B. How many numbers are exactly divisible by 2?	2. 200
C. How many numbers are exactly divisible by 25?	3. 360
D. How many of these are exactly divisible by 4?	4. 40

Match the questions in column-I with column-II and choose the correct option from the codes given below.

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	3	1	4	2
(c)	1	3	4	2
(d)	4	2	3	1

62.

Column - I	Column - II
(A) Value of n, if $(n+2)! = 2550 \times n!$, is	(1) 5
(B) Value of n, if $(n+1)! = 12\,(n-1)!$, is	(2) 121
(C) Value of x, if $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$, is	(3) 2730
(D) Value of P(15, 3) is	(4) 49
(E) Value of n, if 2. P(5, 3) = P(n, 4), is	(5) 3

Codes

	A	B	C	D	E
(a)	4	5	2	3	1
(b)	1	3	5	2	4
(c)	4	2	5	3	1
(d)	1	5	3	2	4

63.

Column-I	Column-II
(A) If P(n, 4) = 20 . P(n, 2) then the value of n is	(1) 28
(B) $^5P_r = 2\,{}^6P_{r-1}$	(2) 4
(C) $^5P_r = {}^6P_{r-1}$	(3) 7
(D) Value of $\frac{8!}{6!\times 2!}$ is	(4) 3

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	3	4	1	2
(c)	4	2	3	1
(d)	3	4	2	1

64.

Column - I	Column - II
(A) If $^nC_8 = {}^nC_2$. Find $^nC_2$.	(1) 5
(B) Determine *n* if $^{2n}C_3 : {}^nC_2 = 12:1$	(2) 91
(C) Determine *n* if $^{2n}C_3 : {}^nC_3 = 11:1$	(3) 6
(D) If $^nC_8 = {}^nC_6$, then the value of $^nC_2$ is	(4) 45

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	1	3	4
(d)	2	3	1	4

65.

Column - I	Column - II
(A) If $^nP_r = 720$ and $^nC_r = 120$, then the value of 'r' is	(1) 3
(B) If $^{2n}C_3 : {}^nC_3 = 11:1$, then the value of 'n' is	(2) 4950
(C) If $^{n+2}C_8 : {}^{n-2}P_4 = 57:16$, then the value of 'n' is	(3) 19
(D) Value of $^{100}C_{98}$ is	(4) 6

Codes

	A	B	C	D
(a)	1	4	3	2
(b)	1	3	4	2
(c)	2	4	3	1
(d)	2	3	4	1

66. How many words (with or without dictionary meaning) can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

Column - I	Column - II
A. 4 letters are used at a time	1. 720
B. All letters are used at a time	2. 240
C. All letters are used but the first is a vowel	3. 360

Match the statements in column-I with column-II and choose the correct options from the codes given below.

Codes:

	A	B	C
(a)	1	2	3
(b)	3	1	2
(c)	2	1	3
(d)	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

67. If $^nC_9 = {}^nC_8$, what is the value of $^nC_{17}$?

(a) 1 (b) 0 (c) 3 (d) 17

68. If $^{10}C_x = {}^{10}C_{x+4}$, then the value of x is

(a) 5 (b) 4 (c) 3 (d) 2

69. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals`

(a) 5 (b) 7 (c) 6 (d) 4

70. Total number of ways of selecting five letters from letters of the word INDEPENDENT is

(a) 70 (b) 72 (c) 75 (d) 80

71. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, then the value of $x = 2^m$. The value of m is

(a) 2 (b) 4 (c) 6 (d) 5

72. Value of $\frac{n!}{(n-r)!}$ when n = 6, r = 2 is 5 m. The value of m is

(a) 2 (b) 4 (c) 6 (d) 5

73. Find n if $^{n-1}P_3 : {}^nP_4 = 1 : 9$

(a) 2 (b) 6 (c) 8 (d) 9

74. Determine n if $^{2n}C_3 : {}^nC_2 = 12 : 1$

(a) 5 (b) 3 (c) 4 (d) 1

75. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides.
If $T_{n+1} - T_n = 21$, then n equals`

(a) 5 (b) 7 (c) 6 (d) 4

76. The number of values of r satisfying the equation $^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is

(a) 1 (b) 2 (c) 3 (d) 4

77. What is the value of $^nP_0$?

(a) 0 (b) 1 (c) ∞ (d) $\frac{1}{2}$

78. What is the value of ^nC_n?

(a) 0 (b) ∞ (c) r (d) 1

79. What is the value of $^nC_0$?

(a) 0 (b) ∞
(c) 1 (d) None of these

80. If $^nC_9 = {}^nC_8$, what is the value of $^nC_{17}$?

(a) 1 (b) 0 (c) 3 (d) 17

81. If $^{10}C_x = {}^{10}C_{x+4}$, then the value of x is

(a) 5 (b) 4 (c) 3 (d) 2

82. If the ratio $^{2n}C_3 : {}^nC_3$ is equal to 11 : 1, n equals

(a) 2 (b) 6 (c) 8 (d) 9

83. The number of combinations of 4 different objects A, B, C, D taken 2 at a time is

(a) 4 (b) 6 (c) 8 (d) 7

84. If $^{12}P_r = {}^{11}P_6 + 6.\,{}^{11}P_5$, then r is equal to:

(a) 6 (b) 5
(c) 7 (d) None of these

85. $({}^8C_1 - {}^8C_2 + {}^8C_3 - {}^8C_4 + {}^8C_5 - {}^8C_6 + {}^8C_7 - {}^8C_8)$ equals:

(a) 0 (b) 1 (c) 70 (d) 256

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

86. Assertion : If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24th position.

Reason : The number of ways of arranging four distinct objects taken all at a time is C(4, 4).

87. Assertion : A number of four different digits is formed with the help of the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways. Then, number of ways which are exactly divisible by 4 is 200.

Reason : A number divisible by 4, if unit place digit is divisible by 4.

88. Assertion : Product of five consecutive natural numbers is divisible by 4!.

Reason : Product of n consecutive natural numbers is divisible by (n + 1)!

89. **Assertion :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$.

Reason : The number of ways of choosing any 3 places, from 9 different places is $^9C_3$.

90. **Assertion :** A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 with repetition. The total number formed are 216.

Reason : If sum of digits of any number is divisible by 3 then the number must be divisible by 3.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

91. $^nC_r + 2\,^nC_{r-1} + ^nC_{r-2}$ is equal to:
(a) $^{n+2}C_r$ (b) $^nC_{r+1}$
(c) $^{n-1}C_{r+1}$ (d) None of these

92. If ^nC_r denotes the number of combination of n things taken r at a time, then the expression $^nC_{r+1} + ^nC_{r-1} + 2\times ^nC_r$ equals
(a) $^{n+1}C_{r+1}$ (b) $^{n+2}C_r$
(c) $^{n+2}C_{r+1}$ (d) $^{n+1}C_r$

93. Given 12 points in a plane, no three of which are collinear. Then number of line segments can be determined, are:
(a) 76 (b) 66 (c) 60 (d) 80

94. There are 10 true-false questions in a examination. Then these questions can be answered in:
(a) 100 ways (b) 20 ways
(c) 512 ways (d) 1024 ways

95. The total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty paise coins and 7 twenty five paise coins is:
(a) $^{37}C_6$ (b) 56 (c) 28 (d) 29

96. In a chess tournament where the participants were to play one game with one another, two players fell ill having played 6 games each, without playing among themselves. If the total number of games is 117, then the number of participants at the beginning was :
(a) 15 (b) 16 (c) 17 (d) 18

97. In how many ways can 10 lion and 6 tigers be arranged in a row so that no two tigers are together?
(a) $10! \times {}^{11}P_6$ (b) $10! \times {}^{10}P_6$
(c) $6! \times {}^{10}P_7$ (d) $6! \times {}^{10}P_6$

98. In how many ways can the letters of the word CORPORATION be arranged so that vowels always occupy even places ?
(a) 120 (b) 2700 (c) 720 (d) 7200

99. What is $\dfrac{(n+2)!+(n+1)!(n-1)!}{(n+1)!(n-1)!}$ equal to ?
(a) 1 (b) Always an odd integer
(c) A perfect square (d) None of these

100. The number of numbers of 9 different non-zero digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than the digit in the middle is
(a) $2(4!)$ (b) $(4!)^2$
(c) $8!$ (d) None of these

101. Number of 6 digit numbers that can be made with the digits 1, 2, 3 and 4 and having exactly two pairs of digits is
(a) 978 (b) 1801 (c) 1080 (d) 789

102. Number of 5 digit numbers that can be made using the digits 1 and 2 and in which at least one digit is different.
(a) 30 (b) 25 (c) 28 (d) 31

103. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls, Number or ways in which we can place the balls in the boxes (order is not considered in the box) so that no box remains empty is
(a) 150 (b) 160 (c) 12 (d) 19

104. In an examination, there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is
(a) 11 (b) 12 (c) 27 (d) 63

105. Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?
(a) 12 (b) 13 (c) 14 (d) 15

106. Four writers must write a book containing 17 chapters. The first and third writer must write 5 chapters each, the second writer must write 4 chapters and fourth writer must write three chapters. The number of ways that can be found to divide the book between four writers, is
(a) $\dfrac{17!}{(5!)^2\,4!\,3!\,2!}$ (b) $\dfrac{17!}{5!\,4!\,3!\,2!}$
(c) $\dfrac{17!}{(5!)^2\,4!\,3!}$ (d) $\dfrac{17!}{(5!)^2\times 4\times 3}$

107. A student has to answer 10 questions, choosing at least 4 from each of parts A and B. If there are 6 questions in Part A and 7 in Part B, in how many ways can the student choose 10 questions?
(a) 266 (b) 260 (c) 256 (d) 270

108. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?
(a) $^{52}C_{18}\,^{35}C_2$
(b) $^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$
(c) $^{52}C_{18} + {}^{35}C_2 + {}^{52}C_{19}$
(d) $^{52}C_{18} \times {}^{35}C_2 + {}^{35}C_1 \times {}^{52}C_{19}$

109. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?
(a) 40 (b) 45 (c) 42 (d) 41

110. There were two women participants in a chess tournament. The number of games the men played between themselves exceeded by 52 the number of games they played with women. If each player played one game with each other, the number of men in the tournament, was
(a) 10 (b) 11 (c) 12 (d) 13

111. For a game in which two partners play against two other partners, six persons are available. If every possible pair must play with every other possible pair, then the total number of games played is
(a) 90 (b) 45 (c) 30 (d) 60

112. A house master in a vegetarian boarding school takes 3 children from his house to the nearby dhaba for non-vegetarian food at a time as often as he can, but he does not take the same three children more than once. He finds that he goes to the dhaba (road side hotel) 84 times more than a particular child goes with him. Then the number of children taking non-vegetarian food in his hostel, is
(a) 15 (b) 5 (c) 20 (d) 10

113. The number of circles that can be drawn out of 10 points of which 7 are collinear, is
(a) 120 (b) 113
(c) 85 (d) 86

114. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty?
(a) 50 (b) 100
(c) 150 (d) 200

115. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth player have just one card, is
(a) $\frac{52!}{(17!)^3}$ (b) 52!
(c) $\frac{52!}{17!}$ (d) None of these

116. The number of 3 digit numbers having at least one of their digit as 5 are
(a) 250 (b) 251
(c) 252 (d) 253

117. The number of 4-digit numbers that can be formed with the digits 1, 2, 3, 4 and 5 in which at least 2 digits are identical, is
(a) 505 (b) $4^5 - 5!$
(c) 600 (d) None of these

118. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is
(a) 324 (b) 341
(c) 359 (d) None of these

119. How many numbers lying between 999 and 10000 can be formed with the help of the digits 0, 2, 3, 6, 7, 8, when the digits are not repeated?
(a) 100 (b) 200
(c) 300 (d) 400

120. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side of the table. The number of ways in which the seating arrangement can be done equals
(a) ${}^{11}C_4\,(9!)^2$ (b) ${}^{11}C_6\,(9!)^2$
(c) ${}^6P_0 \times {}^5P_0$ (d) None of these

121. At an election, a voter may vote for any number of candidates not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is
(a) 6210 (b) 385
(c) 1110 (d) 5040

122. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is
(a) 140 (b) 196
(c) 280 (d) 346

123. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is
(a) $\frac{10!}{6}$ (b) 3! 7!
(c) ${}^{10}P_3 \cdot 7!$ (d) None of these

124. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then the number of ways in which the car can be filled is
(a) 10 (b) 20
(c) 30 (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) $\frac{56!}{(50-r)!} = 30800\left(\frac{54!}{(51-r)!}\right)$

$\Rightarrow 56 \times 55 = \frac{30800}{51-r}$

$\Rightarrow 51 - r = \frac{30800}{56 \times 55} \Rightarrow 51 - r = 10 \Rightarrow 41 = r$

2. (b) Given ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$

$\frac{{}^{n+2}C_8}{{}^{n-2}P_4} = \frac{57}{16}$ $\left[\because {}^nC_r = \frac{n!}{r!(n-r)!} \text{ and } {}^nP_r = \frac{n!}{(n-r)!}\right]$

$\Rightarrow \frac{(n+2)!}{8!(n+2-8)!} \times \frac{(n-2-4)!}{(n-2)!} = \frac{57}{16}$

$\Rightarrow \frac{(n+2)(n+1)n.(n-1)}{8.7.6.5.4.3.2.1} = \frac{57}{16}$

$\Rightarrow (n+2)(n+1)n(n-1) = 143640$

$\Rightarrow (n^2+n-2)(n^2+n) = 143640$

$\Rightarrow (n^2+n)^2 - 2(n^2+n) + 1 = 143640 + 1$

$\Rightarrow (n^2+n-1)^2 = (379)^2$

$\Rightarrow n^2+n-1 = 379$ $[\because n^2+n-1 > 0]$

$\Rightarrow n^2+n-1-379 = 0$

$\Rightarrow n^2+n-380 = 0$

$\Rightarrow (n+20)(n-19) = 0$

$\Rightarrow n = -20, n = 19$

$\because$ n is not negative.

$\therefore n = 19$

3. (b) Let ${}^{30}C_{r+2} = {}^{30}C_{r-2}$ ...(i)

We know, If ${}^nC_{r_1} = {}^nC_{r_2}$, then $r_1 + r_2 = n$

In above given equation (i), we have

$n = 30, r_1 = r+2, r-2 = r_2$

$\therefore r_1 + r_2 = r+2+r-2 = 2r$

and $n = 30$

$\therefore 2r = 30 \Rightarrow r = 15$

4. (b) The total number of numbers that can be formed with the digits 3, 4, 5, 6 taken all at a time = ${}^4P_4 = 4! = 24$. Consider the digits at the unit places in all these number. Each of the digits 3, 4, 5, 6 occurs in 3! = 6 times in unit's place. So, total of the digits at the unit places

$= (3+4+5+6)6 = 108.$

[Similary, the sum of the digits in the other places will also be 108]

5. (d) Student has 4 choices to answer the question.

$\therefore$ Total no. of ways to answer the question

$= 4 \times 4 \times 4 = 64$ $(\because$ total choices = 4)

But out of these there is only one way such that all answers are correct.

$\therefore$ Required number of ways of (student can fail to get all answers correct = 1 – 64 = 63.

6. (a) There are six letters in MOTHER, all different, i.e. arrangement can be made out of the letters of the word MOTHER taken four at a time with M present in every arrangement.

So, rest 3 letters can be arrangement from 5 letters

So, total number of ways = $4 \times {}^5P_3$

$= 4 \times \frac{5!}{(5-3)!} = \frac{4 \times 5 \times 4 \times 3 \times 2}{2} = 240$

7. (b) There are 6 balls in one over and 4 wickets are to be taken. So, 4 balls are to succeed. This can be done in ${}^6C_4$ ways.

$\Rightarrow$ Required number of ways = ${}^6C_4$

$= \frac{6!}{4!2!} = \frac{6 \times 5}{2} = 15$

8. (c) First chair can be occupied in 4 ways and second chair can be occupied in 3 ways, third chair can be occupied in 2 ways and last chair can be occupied in one way only. So total number of ways = $4 \times 3 \times 2 \times 1 = 24$.

9. (b) Selection of 2 members out of 11 has ${}^{11}C_2$ number of ways

So, ${}^{11}C_2 = 55$

10. (a) The required number of selections

$= {}^3C_1 \times {}^4C_1 \times {}^2C_1 ({}^6C_3 + {}^6C_2 + {}^6C_0) = 42 \times 4! = 1008$

11. (d) Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ Required number of ways = $\frac{1}{2} \times 9! \times 10! = 5(9!)^2$

12. (b) 3 consonants and 2 vowels from 5 consonants and 4 vowels can be selected in ${}^5C_3 \times {}^4C_2 = 60$ ways. But total number of words with 3 + 2 = 5 letters = 5! ways = 120

$\therefore$ The required number of words = $60 \times 120 = 7200$

13. (c) If number of persons = n.

Then total number of hand-shaken = ${}^nC_2 = 66$

$\Rightarrow n(n-1) = 132$

$\Rightarrow (n+11)(n-12) = 0$

$\therefore n = 12$ $(\because n \neq -11)$

14. (a) Other than 2 numbers, remaining five places are filled by 1 and 3 and for each place there is two conditions.
No. of ways for five places = $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
For 2 numbers, selecting 2 places out of 7 = ${}^7C_2$
$\therefore$ Required no. of ways = ${}^7C_2 . 2^5 = 652$

15. (d) There are ten digits 0, 1, 2, ..., 9. Permutations of these digits taken eight at a time = ${}^{10}P_8$ which includes permutations having 0 at the first. When 0 is fixed at the first place, then number of such permutations = ${}^9P_7$.
So, required number

$$= {}^{10}P_8 - {}^9P_7 = \frac{10!}{2} - \frac{9!}{2} = \frac{9.\,9!}{2}$$

16. (b) There are 6 letters in the word BHARAT, 2 of them are identical. Hence total number of words = 6!/2! = 360
Number of words in which B and H come together

$$= \frac{5!2!}{2!} = 120$$

$\therefore$ The required number of words = 360 – 120 = 240

17. (b) The number of committees of 4 gentlemen = ${}^4C_4 = 1$
The number of committees of 3 gentlemen, 1 wife
$= {}^4C_3 \times {}^1C_1$
($\because$ after selecting 3 gentlemen only 1 wife is left who can be included)
The number of committees of 2 gentlemen, 2 wives
$= {}^4C_2 \times {}^2C_2$
The number of committees of 1 gentleman, 3 wives
$= {}^4C_1 \times {}^3C_3$
The number of committees of 4 wives = 1
$\therefore$ The required number of committees = 1 + 4 + 6 + 4 + 1 = 16

18. (d) The number of 4 persons including A and B = ${}^6C_2$
Considering these four as a group, number of arrangements with the other four = 5!
But in each group the number of arrangements = 2! × 2!
$\therefore$ Required number of ways = ${}^6C_2 \times 5! \times 2! \times 2! + 1 = 16$

19. (d) Required number of numbers
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

20. (a) Alphabetical order is
A, C, H, I, N, S
No. of words starting with A = 5!
No. of words starting with C = 5!
No. of words starting with H = 5!
No. of words starting with I = 5!
No. of words starting with N = 5!
SACHIN – 1
$\therefore$ Sachin appears at serial no. 601

21. (d) First let us arrange M, I, I, I, I, P, P
Which can be done in $\frac{7!}{4!2!}$ ways
Now 4 S can be kept at any of the ticked places in ${}^8C_4$ ways so that no two S are adjacent.
Total required ways

$$= \frac{7!}{4!2!}\,{}^8C_4 = \frac{7!}{4!2!}\,{}^8C_4 = 7 \times {}^6C_4 \times {}^8C_4$$

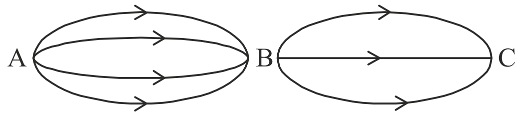
22. (c) X - X - X - X - X. The four digits 3, 3, 5,5 can be arranged at (-) places in $\frac{4!}{2!2!} = 6$ ways.
The five digits 2, 2, 8, 8, 8 can be arranged at (X) places in $\frac{5!}{2!3!}$ ways = 10 ways
Total no. of arrangements = $6 \times 10 = 60$ ways

23. (d) Two pairs of identical letters can be arranged in ${}^3C_2 \frac{4!}{2!2!}$ ways. Two identical letters and two different letters can be arranged in ${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$ ways. All different letters can be arranged in ${}^8P_4$ ways
$\therefore$ Total no. of arrangements

$$= {}^3C_2 \frac{4!}{2!2!} + {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} + \frac{8!}{4!} = 2454.$$

24. (b) These conditions are satisfied exactly when we do as follows: First paint the central triangle in any one of the three colours. Next, paint the remaining 3 triangles, with any one of the remaining two colours. By the fundamental principle of counting, this can be done in 3 × 2 × 2 × 2 = 24 ways.

25. (a) In the following figure :



There are 4 bus routes from A to B and 3 routes from B to C. Therefore, there are 4 × 3 = 12 ways to go from A to C. It is round trip so the man will travel back from C to A via B. It is restricted that man cannot use same bus routes from C to B and B to A more than once. Thus, there are 2 × 3 = 6 routes for return journey. Therefore, the required number of ways = 12 × 6 = 72.

26. (a) First, we take books of a particular subject as one unit. Thus, there are 4 units which can be arranged in 4! = 24 ways. Now, in each of the arrangements, mathematics books can be arranged in 3! ways, history books in 4! ways, chemistry books in 3! ways and biology books in 2! ways. Thus, the total number of ways = 4! × 3! × 4! × 3! × 2! = 41472.

27. (d) Number of ways of dividing 8 persons in three groups, first having 3 persons, second having 2 persons and third having 3 persons = $\frac{8!}{3!2!3!}$. Since all the 50 things are identical.

So, required number = $\frac{8!}{(3!)^2 \cdot (2!)}$

28. (c) Since, out of eleven members, two members sit together, then the number of arrangements = 9! × 2 ($\because$ two members can sit in two ways).

29. (d) Required number of such triangles
$= {}^4C_1 \times {}^5C_1 \times {}^6C_1 = 4 \times 5 \times 6$

30. (c) Given 4 vowels and 5 consonants
$\therefore$ Total number of words = ${}^4C_2 \times {}^5C_3 \times 5!$
$= 6 \times 10 \times 120 = 7200$.

31. (b) Total number of parallelograms formed
$= {}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

32. (c) Each of the three prizes can be given to any of the four children.
$\therefore$ Total number of ways of distributing prizes
$= 4 \times 4 \times 4 = 64$
Number of ways in which one child gets all prizes = 4
$\therefore$ Number of ways in which no child gets all the three prizes = 64 – 4 = 60

33. (c) In the word 'RACHIT', the number of words beginning with A, C, H, I is 5! and the next word we get RACHIT.
$\therefore$ Required number of words
$= 4 \times 5! + 1 = 4 \times 120 + 1 = 481$

34. (c) Number of chords that can be drawn through 21 points on circle = Number of ways of selecting 2 points from 21 points on circle

$$= {}^{21}C_2 = \frac{21 \times 20}{2 \times 1} = 210$$

35. (a) Total number of available courses = 9
Out of these, 5 courses have to be chosen. But it is given that 2 courses are compulsory for every student, i.e. you have to choose only 3 courses, out of 7.

It can be done in ${}^7C_3$ ways $= \frac{7 \times 6 \times 5}{6} = 35$ ways.

36. (a) There are two possibilities :

	Men	Women
(i)	2	4
(ii)	3	6

(i) Number of ways of choosing a committee of 2 men and 4 women $= {}^4C_2 \times {}^6C_4$

$$= \frac{4 \times 3}{2 \times 1} \times \frac{6 \times 5}{2 \times 1} = 90$$

(ii) Number of ways of choosing a committee of 3 men and 6 women $= {}^4C_3 \times {}^6C_6$
$= 4 \times 1 = 4$

$\therefore$ Required number of ways = 94

37. (a) Number of times he will go to garden
= Number of ways of selecting 3 children from 8 children

$$= {}^8C_3 = \frac{8 \times 7 \times 6}{3 \times 2} = 56$$

38. (a) Since the man can go in 4 ways and can back in 3 ways.
Therefore, total number of ways are $4 \times 3 = 12$ ways.

39. (a) Required number of ways $= \frac{6!}{3!\,3!} = \frac{720}{6 \times 6} = 20$

[Number of heads = 3, number of tails = 3 and coins are identical]

40. (c) Required number of ways = 5! – 4! – 3!
= 120 – 24 – 6 = 90
[Number will be less than 56000 only if either 4 occurs on the first place or 5, 4 occurs on the first two places].

41. (a) The man can go in 5 ways and he can return in 5 ways. Hence, total number of ways are $5 \times 5 = 25$.

42. (b) The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places in $\frac{4!}{2!\,2!} = 6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places in $\frac{3!}{2!}$ = 3 ways
Hence, the required number of ways $= 6 \times 3 = 18$.

43. (b) At first, we have to accommodate those 5 animals in cages which cannot enter in 4 small cages, therefore number of ways are ${}^6P_5$. Now, after accommodating 5 animals we left with 5 cages and 5 animals, therefore, number of ways are 5!. Hence, required number of ways $= {}^6P_5 \times 5! = 86400$.

44. (b) $2 \cdot {}^{20}C_2$ {Since two students can exchange cards each other in two ways}.

45. (a) The selection can be made in ${}^5C_3 \times {}^{22}C_9$ ways.
{Since 3 vacancies filled from 5 candidates in ${}^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9}.

46. (a) 12 persons can be seated around a round table in 11! ways. The total number of ways in which 2 particular persons sit side by side is $10! \times 2!$.
Hence, the required number of arrangements $= 11! - 10! \times 2! = 9 \times 10!$.

STATEMENT TYPE QUESTIONS

47. (c) I. Number of 3 letter words (repetition not allowed)
$= 6 \times 5 \times 4 = 120$
(as first place can be filled in 6 different ways, second place can be filled in 5 different ways and third place can be filled in 4 different ways)
II. Number of 3 letter words (repetition is allowed)
$= 6 \times 6 \times 6 = 216$
(as each of the place can be filled in 6 different ways)

48. (c) I. Number of 4 letter words that can be formed from alphabets of the word 'PART'
$= {}^4P_4 = 4! = 24$
II. Number of 4 letter words that can be formed when repetition is allowed $= 4^4 = 256$

49. (a) I. In n-sided polygon, the number of vertices = n
$\therefore$ Number of lines that can be formed using n points $= {}^nC_2$.
Out of these, ${}^nC_2$ lines, n lines from the polygon.
$\therefore$ Number of diagonals $= {}^nC_2 - n$
II. Let the number of sides of a polygon = n
Number of diagonal = Number of line segment joining any two vertices of polygon – Number of sides
$= {}^nC_2 - n$

$$= \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$$

Now, $\frac{n(n-3)}{2} = 44$

$\Rightarrow n^2 - 3n - 88 = 0$
$\Rightarrow (n - 11)(n + 8) = 0$
$\Rightarrow n = 11$
or $n = -8$ rejected.

50. (c) (I) A committee consisting of 3 girls and 4 boys can be formed in ${}^4C_3 \times {}^9C_4$ ways

$= {}^4C_1 \times {}^9C_4 = \frac{4}{1} \times \frac{9\times8\times7\times6}{1.2.3.4}$ ways

$= 504$ ways

(II) A committee having at least 3 girls will consists of (a) 3 girls 4 boys, (b) 4 girls 3 boys

This can be done in ${}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$ ways

$= \frac{4}{1} \times \frac{9\times8\times7\times6}{1\times2\times3\times4} + 1 \times \frac{9\times8\times7}{1\times2\times3}$ ways

$= 504 + 84$ ways $= 588$ ways

51. (c) (I) ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$

$= \left[{}^nC_r + {}^nC_{r-1}\right] + \left[{}^nC_{r-1} + {}^nC_{r-2}\right]$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r.$

(II) If ${}^nC_p = {}^nC_q \Rightarrow {}^nC_p = {}^nC_{n-q}$

$\Rightarrow p = q$ or $p = n - q \left[\because {}^nC_r = {}^nC_{n-r}\right]$

52. (a) I. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$
$= {}^{15}C_7 + {}^{15}C_6 - {}^{15}C_6 - {}^{15}C_7 = 0$

II. Total number $= 10! - 9! = 9 \times 9!$

53. (c) Both are false

I. Correct is 5^3.
($\because$ each one of the three letters can be posted in anyone of the five letter boxes.)

II. Statement will be true if m particular things always occur.

54. (c) Both are true statements.

55. (b) Both are true statements.

I. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!\,[n-(n-r)]!}$

$= {}^nC_{n-r}$

II. ${}^nC_r + {}^nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$

$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$

$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r\,(n-r+1)}\right] = \frac{(n+1)!}{r!(n+1-r)!}$

56. (a) I. ${}^nP_r = {}^nP_{r+1}$
$\Rightarrow n - r = 1$ ...(i)
and ${}^nC_r = {}^nC_{r-1}$
$\Rightarrow n - r + 1 = r \Rightarrow n - 2r = -1$ ...(ii)

On solving (i) and (ii), we get
$n = 3$ and $r = 2$

II. Required no. of ways $= {}^{32}C_4 = \frac{32!}{4!\,28!}$

57. (c) Both are true.

58. (c) Both statements are true.

59. (b) I. The continued product of first n natural numbers is called the 'n factorial'.

II. $5! = 5 \times 4!$
$6! = 6 \times 5 \times 4!$
$\therefore$ L.C.M. of $4!, 5!, 6!$ = L.C.M. $[4!, 5\times4!, 6\times5\times4!]$
$= 4! \times 5 \times 6 = 6! = 720$

MATCHING TYPE QUESTIONS

60. (d) A. $\frac{7!}{5!} = \frac{7\times6\times5!}{5!} = 42$

B. $\frac{12!}{10!\;2!} = \frac{12\times11\times10!}{10!\times2\times1} = 66$

C. $\frac{8!}{6!\;2!} = \frac{8\times7\times6!}{6!\times2\times1} = 28$

61. (c) A. The number of 4 different digits $= {}^7P_4$

$= \frac{7!}{(7-4)!}$

$= 7 \times 6 \times 5 \times 4 = 840$

B. The numbers exactly divisible by 2
= Number of ways of filling first 3 places
× Number of ways of filling unit's place
$= {}^6P_3 \times 3$

$= \frac{6!}{(6-3)!} \times 3 = \frac{6!}{(3!)} \times 3$

$= 6 \times 5 \times 4 \times 3 = 360$

C. Number of 4-digit numbers divisible by 25
= Numbers ending with 25 or 75

$= \underset{5\times4}{\square\square} \quad \underset{25 \text{ or } 75}{\square\square}$

$= 5 \times 4 \times 2 = 40$

($\because$ when numbers end with 25 or 75, the other two places can be filled in 5 and 4 ways)

D. Number of 4-digit numbers divisible by 4
= Numbers ending with 12, 16, 24, 32, 36, 64, 72, 76, 52, 56
Now, number ending with 12

$= \square\;\square\;\square\;\square = 20$
$\;\;4 \times 5 \times 1 \times 1$

Similarly, numbers ending with other number (16, 24, ...) = 20 each
$\therefore$ Required numbers $= 10 \times 20 = 200$

62. (a) (A) $(n+2)(n+1)\,n! = 2550 \times n!$
$\Rightarrow n^2 + 3n - 2548 = 0$
$\Rightarrow (n+52)(n-49) = 0$
$\Rightarrow n = 49$

(B) $(n+1)\,n\,(n-1)! = 12(n-1)!$
$\Rightarrow n^2 + n - 12 = 0 \Rightarrow (n+4)(n-3) = 0$
$\Rightarrow n = 3$

(C) $\frac{1}{9!}\left[1+\frac{1}{10}\right]=\frac{x}{11\times10}\times\frac{1}{9!}$

$\Rightarrow \frac{11}{10}=\frac{x}{11\times10} \Rightarrow x=11\times11=121$

(D) $P(15,3)=\frac{15!}{12!}=\frac{15\times14\times13\times12!}{12!}=2730$

(E) $P(n,4)=2.\,P(5,3)$

$\Rightarrow \frac{n!}{(n-4)!}=2\cdot\left[\frac{5!}{(5-3)!}\right]$

$\Rightarrow n(n-1)(n-2)(n-3)=\frac{2(5!)}{2!}$

$\Rightarrow n(n-1)(n-2)(n-3)=5\times4\times3\times2\times1$

$\Rightarrow n(n-1)(n-2)(n-3)=5\times(5-1)\times(5-2)\times(5-3)$

$\Rightarrow n=5$

63. (d) (A) $\frac{n!}{(n-4)!}=20\cdot\frac{n!}{(n-2)!}$

$\Rightarrow (n-2)!=20(n-4)!$

$\Rightarrow (n-2)(n-3)=5\times4$

$\Rightarrow n-3=4 \Rightarrow n=7$

(B) We have,

${}^5P_r=2\,{}^6P_{r-1}$

or $\frac{5!}{(5-r)!}=2\left[\frac{6!}{(6-r+1)!}\right]$

or $\frac{5!}{(5-r)!}=2\left[\frac{6\times5!}{(7-r)!}\right]$

or $\frac{1}{(5-r)!}=\frac{12}{(7-r)(6-r)(5-r)!}$

or $(7-r)(6-r)=12$

or $42-7r-6r+r^2=12$

or $r^2-13r+30=0$

or $r^2-10r-3r+30=0$

or $r(r-10)-3(r-10)=0$

or $(r-10)(r-3)=0$

or $r=10$ or $r=3$

Hence, $r=3$

[$r=10 \Rightarrow {}^5P_{10}$ which is meaningless]

(C) We have,

${}^5P_r={}^6P_{r-1}$

or $\frac{5!}{(5-r)!}=2\left[\frac{6!}{[6-(r-1)]!}\right]$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(6-r+1)!}$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(7-r)!}$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(7-r)(6-r)(5-r)!}$

or $(7-r)(6-r)=6$

or $r^2-13r+36=0$

or $r=4, 9$

or $r=4$

[$r=9 \Rightarrow {}^5P_r$ which is meaningless]

(D) $\frac{8!}{6!\times2!}=\frac{8\times7\times6\times5\times4\times3\times2\times1}{(6\times5\times4\times3\times2\times1)\times(2\times1)}$

$=\frac{8\times7}{2\times1}=28$

64. (b) (A) We have

${}^nC_r={}^nC_{n-r}$

${}^nC_2={}^nC_{n-2}$

${}^nC_8={}^nC_{n-2} \Rightarrow n-2=8$ or $n=10$

${}^nC_2={}^{10}C_2=\frac{10\times9}{1\times2}=45$

(B) ${}^{2n}C_3:{}^nC_2=12:1$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{1.2.3}\div\frac{n(n-1)}{1.2}=\frac{12}{1}$

$\left[{}^nC_r=\frac{n(n-1)......(n-r+1)}{1.2.3........n}\right]$

or $\frac{2n(2n-1)2(n-1)}{6}\times\frac{2}{n(n-1)}=\frac{12}{1}$

or $\frac{4n(2n-1)(n-1)}{3}\times\frac{1}{n(n-1)}=12$

$2n-1=9, 2n=10$ or $n=5$

(C) ${}^{2n}C_3:{}^nC_3=11:1$

or $\frac{2n(2n-1)(2n-2)}{1.2.3}\div\frac{n(n-1)(n-2)}{1.2.3}=\frac{11}{1}$

or $\frac{4n(n-1)(2n-1)}{6}\times\frac{6}{n(n-1)(n-2)}=\frac{11}{1}$

or $\frac{4(2n-1)}{n-2}=11$

$4(2n-1)=11(n-2)$ or $8n-4=11n-22$

or $3n=18 \quad \therefore n=6$

(D) ${}^nC_8={}^nC_6 \Rightarrow n=8+6=14$

$\therefore {}^nC_2={}^{14}C_2=\frac{14}{2}\times\frac{13}{1}\times{}^{12}C_0$

$=\frac{14}{2}\times\frac{13}{1}\times1=91$

65. (a) (A) $120=\frac{720}{r!} \Rightarrow r!=6 \Rightarrow r!=3! \Rightarrow r=3$

(B) $\frac{(2n)!}{(2n-3)!}\times\frac{(n-3)!}{n!}=\frac{11}{1}$

$\frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)}=\frac{11}{1}$

$\Rightarrow \frac{4(2n-1)}{n-2}=\frac{11}{1} \Rightarrow 3n=18 \Rightarrow n=6$

(C) $\frac{(n+2)!}{8!(n-6)!}\times\frac{(n-6)!}{(n-2)!}=\frac{57}{16}$

$\Rightarrow (n+2)(n+1)(n)(n-1)=\frac{57}{16}\times8!$

$\Rightarrow (n-1)n(n+1)(n+2)=18\times19\times20\times21$

$\Rightarrow n-1=18 \Rightarrow n=19$

(D) $^{100}C_{98} = {}^{100}C_{100-98} = {}^{100}C_2$

$= \frac{100}{2} \times \frac{99}{1} \times {}^{98}C_0 \left(\because {}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1} \right)$

$= 4950$

66. (b) A. Number of words using 4 letters out of 6 letters

$= {}^6P_4 = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$

B. Number of words using all letters
$= {}^6P_6 = 6! = 720$

C. Number of words starting with vowel
= Number of ways of choosing first letter (out of O and A) × Number of ways of arranging 5 alphabets

$= 2 \times 5! = 2 \times 120 = 240$

INTEGER TYPE QUESTIONS

67. (a) $\frac{n!}{9!(n-9)!} = \frac{n!}{8!(n-8)!}$

$\Rightarrow \frac{1}{9 \times 8!(n-9)!} = \frac{1}{8!(n-8)(n-9)!}$

$\Rightarrow \frac{1}{9} = \frac{1}{(n-8)} \Rightarrow 9 = n - 8$

$\Rightarrow 9 + 8 = n \Rightarrow n = 17$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \qquad [\because {}^nC_n = 1]$

68. (c) We have $^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow x + x + 4 = 10 \Rightarrow 2x = 6 \Rightarrow x = 3$

69. (b) $^{n+1}C_3 - {}^nC_3 = 21 \quad \because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^nC_2 = 21 \quad \Rightarrow \quad n = 7$

70. (b) There are 11 letters in the given word which are as follows (NNN) (EEE) (DD)IPT

Five letters can be selected in the following manners :

(i) All letters different : $^6C_5 = 6$

(ii) Two similar and three different : $^3C_1 \cdot {}^5C_3 = 30$

(iii) Three similar and two different : $^2C_1 \cdot {}^5C_2 = 20$

(iv) Three similar and two similar : $^2C_1 \cdot {}^2C_1 = 4$

(v) Two similar, two similar and one different :
$^3C_2 \cdot {}^4C_1 = 12$

$\therefore$ Total selections $= 6 + 30 + 20 + 4 + 12 = 72$

71. (d) $\frac{1}{6!} + \frac{1}{7!} = \frac{1}{6!} + \frac{1}{7.6!} = \frac{x}{8.7.6.!}$

$\Rightarrow \frac{1}{6!}\left(1 + \frac{1}{7}\right) = \frac{x}{8.7.6!}$

$\Rightarrow \frac{8}{7} = \frac{x}{8.7} \Rightarrow x = 64$

72. (c) n = 6, r = 2

$\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 6 \times 5$

73. (d) $\frac{^{n-1}P_3}{^nP_4} = \frac{1}{9} \Rightarrow \frac{^{n-1}P_3}{n.{}^{n-1}P_3} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$ or $n = 9$

74. (a) $^{2n}C_3 : {}^nC_2 = 12 : 1$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{1.2.3} \div \frac{n(n-1)}{1.2} = \frac{12}{1}$

$\left[{}^nC_r = \frac{n(n-1)......(n-r+1)}{1.2.3........n} \right]$

or $\frac{2n(2n-1)2(n-1)}{6} \times \frac{2}{n(n-1)} = \frac{12}{1}$

or $\frac{4n(2n-1)(n-1)}{3} \times \frac{1}{n(n-1)} = 12$

$2n - 1 = 9, 2n = 10$ or $n = 5$

75. (b) $^{n+1}C_3 - {}^nC_3 = 21$

$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^nC_2 = 21 \Rightarrow n = 7$

76. (b) $^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$\Rightarrow {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2-1} + {}^{39}C_{r^2}$

$\Rightarrow {}^{40}C_{3r} = {}^{40}C_{r^2}$

$\Rightarrow r^2 = 3r$ or $r^2 = 40 - 3r \Rightarrow r = 0, 3$ or $-8, 5$

3 and 5 are the values as the given equation is not defined by $r = 0$ and $r = -8$. Hence, the number of values of r is 2.

77. (b) $^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

78. (d) $^nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!} = 1$

79. (c) $^nC_0 = \frac{n!}{0!(n-0)} = \frac{n!}{0!n!} = \frac{n!}{n!} = 1$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \quad [\because {}^nC_n = 1]$

80. (a) $\frac{n!}{9!(n-9)!} = \frac{n!}{8!(n-8)!}$

$\Rightarrow \frac{1!}{9 \times 8!(n-9)!} = \frac{1!}{8!(n-8)(n-9)!}$

$\Rightarrow \frac{1}{9} = \frac{1}{(n-8)} \Rightarrow 9 = n - 8$

$\Rightarrow 9 + 8 = n \Rightarrow n = 17$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \qquad [\because {}^nC_n = 1]$

81. (c) We have $^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow x + x + 4 = 10 \Rightarrow 2x = 6 \Rightarrow x = 3$

82. (b) We have,

$^{2n}C_3 : {}^nC_3 = 11 : 1$

$\Rightarrow \frac{^{2n}C_3}{^nC_3} = \frac{11}{1} \Rightarrow \frac{\frac{(2n)!}{(2n-3)!3!}}{\frac{n!}{(n-3)!(3!)}} = \frac{11}{1}$

$\Rightarrow \frac{(2n)!}{(2n-3)!} \times \frac{(n-3)!}{n!} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!}$

$\times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$

$\Rightarrow \dfrac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \dfrac{11}{1}$

$\Rightarrow \dfrac{4(2n-1)}{n-2} = \dfrac{11}{1}$

$\Rightarrow 8n-4 = 11n-22 \Rightarrow 3n = 18 \Rightarrow n = 6$

83. (b) The combination will be AB, AC, AD, BC, BD and CD.

84. (a) Given: ${}^{12}P_r = {}^{11}P_6 + 6.{}^{11}P_5$

We know that

${}^{n-1}P_r + r\,.\,{}^{n-1}P_{r-1} = r!\,{}^{n}C_r$

$\therefore\ {}^{11}P_6 + 6.\ {}^{11}P_5 = 6!\ {}^{12}C_6$

$\Rightarrow\ {}^{12}P_6 = 6!\ {}^{12}C_6$

$\therefore\ \dfrac{12!}{6!} = 6!\dfrac{12!}{6!6!}$ which are equal

$\therefore\ r = 6$

85. (b) Let

$A = {}^8C_1 - {}^8C_2 + {}^8C_3 - {}^8C_4 + {}^8C_5 - {}^8C_6 + {}^8C_7 - {}^8C_8$

$= \dfrac{8!}{1!7!} - \dfrac{8!}{2!6!} + \dfrac{8!}{3!5!} - \dfrac{8!}{4!4!} + \dfrac{8!}{5!3!} - \dfrac{8!}{6!2!} + \dfrac{8!}{7!1!} - \dfrac{8!}{0!8!}$

Note: ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

Thus,

$A = 8 - \dfrac{8\times7}{2} + \dfrac{8\times7\times6}{3\times2} - \dfrac{8\times7\times6\times5}{4\times3\times2\times1}$

$+ \dfrac{8\times7\times6}{3\times2} - \dfrac{8\times7}{2} + 8 - 1$

And $A = 8 - 28 + 56 - 70 + 56 - 28 + 8 - 1 = 1$

ASSERTION - REASON TYPE QUESTIONS

86. (c) Number of ways of arranging four distinct objects in a line is ${}^4P_4 = 4! = 24$.

Hence, Statement II is false.

Again, when W, I, F, E are arranged in all possible ways, then number of words formed is 4! = 24 and WIFE occurs last of all as its letters are against alphabetical order.

87. (c) For the number exactly divisible by 4, then last two digits must be divisible by 4, the last two digits are viz.

12, 16, 24, 32, 36, 52, 56, 64, 72, 76

Total 10 ways. Now, the remaining two first places on the left of 4-digit numbers are to be filled from the remaining 5-digits and this can be done in ${}^5P_2$ = 20 ways.

$\therefore$ Required number of ways = 20 × 10 = 200.

88. (c) Product of n consecutive natural numbers

= (m + 1) (m + 2) (m + 3) ..... (m + n), m ∈ whole number

$= \dfrac{(m+n)!}{m!} = n! \times \dfrac{(m+n)!}{m!\ n!}$

$= n! \times {}^{m+n}C_m$

$\Rightarrow$ Product is divisible by n!, then it is always divisible by (n – 1)! but not by (n + 1)!

89. (a) Let the number of ways of distributing n identical objects among r persons such that each person gets at least one object is same as the number of ways of selecting (r – 1) places out of (n – 1) different places, i.e. ${}^{n-1}C_{r-1}$.

90. (d) Number form by using 1, 2, 3, 4, 5 = 5! = 120

Number formed by using 0, 1, 2, 4, 5

4	4	3	2	1

= 4.4.3.2.1 = 96

Total number formed, divisible by 3 (taking numbers without repetition) = 216

Statement 1 is false and statement 2 is true.

CRITICALTHINKING TYPE QUESTIONS

91. (a) Let $A = {}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$

$= \dfrac{n!}{r!(n-r)!} + \dfrac{2n!}{(r-1)!(n-r+1)!} + \dfrac{n!}{(r-2)!(n-r+2)!}$

$= \dfrac{n![(n-r+2.n-r+1) + 2(n-r+2)r + r(r-1)]}{r!(n-r+2)!}$

$= \dfrac{\left[\begin{array}{l} n![(n^2 - nr + n - nr + r^2 - r + 2n - 2r + 2 \\ +2nr - 2r^2 + 4r + r^2 - r] \end{array}\right]}{r!(n-r+2)!}$

$= \dfrac{(n^2+3n+2)n!}{r!(n-r+2)!} = \dfrac{(n+1)(n+2)n!}{r!(n-r+2)!}$

$= \dfrac{(n+2)!}{r!(n+2-r)!} = {}^{n+2}C_r.$

92. (c) ${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$

$= {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$

$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

93. (b) To find number of line segment we will have to draw the line segments joining two points. If n is the number of such lines segments, then

$n = {}^{12}C_2 = \dfrac{12!}{2!(12-2)!} = \dfrac{12\times11\times10!}{2\times10!} = 66.$

94. (d) There are 10 questions with options of false/ true. It means each question has two options. Thus the number of ways that these questions can be answered $= 2^{10} = 1024$ ways.

95. (c) Since we know that the total number of selections of r things from n things where each thing can be repeated as many times as one can, is ${}^{n+r-1}C_r$

Here r = 6 ($\because$ we have to select 6 coins)

and n = 3 ($\because$ it is repeated 3 times)

$\therefore$ Required number $= {}^{3+6-1}C_6 = 28$

96. (a) Let the no. of participants at the beginning was n.

Now, we have 6 games and each participant will play 2 games.

$\therefore$ Total no. of games played by 2 persons

$= 6 \times 2 = 12$

Since, two players fell ill having played 6 games each, without playing among them selves and total no. of games = 117

$\therefore \frac{n(n-1)}{2} = 117 - 12$

$\Rightarrow n(n-1) = 2(105) = 210$

$\Rightarrow n^2 - n - 210 = 0$

$\Rightarrow n^2 - 15n + 14n - 210 = 0$

$\Rightarrow n(n-15) + 14(n-15) = 0$

$\Rightarrow n = -14, 15$

But no. of participants can not be –ve

$\therefore n = 15.$

97. (a) There are 10 lions and there is no restrictions on arranging lions. They can be arranged in 10! ways. But there is a restriction in arrangements of tigers that no two tigers come together. So two tiger are to be arranged on the either side of a lion. This gives 11 places for tigers and there are 6 tigers. So, tigers can be arranged in ${}^{11}P_6$ ways.
So, total arrangemns are $10! \times {}^{11}P_6$

98. (d) In the word CORPORATION, there are 11 positions, there are 3 vowels O, A and I and they can occupy even places only (2^{nd} 4^{th}, 6^{th}, 8^{th} and 10^{th} positions), total 5 positions : This can be done in ${}^5C_3$ ways.
There are remaining 6 positions for odd numbered places (i.e. 1, 3, 5, 7, 9, 11) and these are be occupied by 5 consonants, namely, C, R, P, T, N.
This can be done in ${}^6C_5$ ways.
Total number of ways $= {}^5C_3 \times {}^6C_5 = 7200$

99. (c) Given expression is :

$$\frac{(n+2)!+(n+1)!(n-1)!}{(n+1)!(n-1)!} = x \text{ (let)}$$

$$\Rightarrow x = \frac{(n+2)(n+1)n(n-1)!+(n+1)(n-1)!}{(n+1)(n-1)!}$$

$= (n+2)n + 1 = n^2 + 2n + 1 = (n+1)^2$

Which is a perfect square.

100. (b) $x_1\, x_2\, x_3\, x_4$ (x_5) $x_6\, x_7\, x_8\, x_9$. Under the given situation x_5 can be 5 only. The selection for x_1, x_2, x_3, x_4 must be from 1, 2, 3, 4, so they can be arranged 4 ! ways. Again the selection of x_6, x_7, x_8, x_9 must be from 6, 7, 8, 9 so they can be arranged in 4! ways.
Desired number of ways $= (4!)\,(4!) = (4!)^2$

101. (c) The number will have 2 pairs and 2 different digits.
The number of selections $= {}^4C_2 \times {}^2C_2$, and for each selection, number of arrangements $= \frac{6!}{2!2!}$.

Thus, the required number $= {}^4C_2 \times {}^2C_2 \times \frac{6!}{2!2!} = 1080$

102. (a) Total number of numbers without restriction $= 2^5$
Two numbers have all the digits equal. So,
The required number $= 2^5 - 2 = 30$

103. (a) One possible arrangement = | 2 | 2 | 1 |
Three such arrangements are possible. Therefore, the number of ways $= ({}^5C_2)({}^3C_2)({}^1C_1)(3) = 90$
The other possible arrangements = | 1 | 1 | 3 |
Three such arrangements are possible.
Thus, the number of ways $= ({}^5C_1)({}^4C_1)({}^3C_3)(3) = 60$
Hence, the total number of ways $= 90 + 60 = 150$.

104. (d) There are three multiple choice questions, each has four possible answers. Therefore, the total number of possible answers will be $4 \times 4 \times 4 = 64$. Out of these, possible answers only one will be correct and hence the number of ways in which a student can fail to get all correct answers is $64 - 1 = 63$.

105. (a) There will be as many signals as there are ways of filling in 2 vacant places ☐ in succession by the 4 flags of different colours. The upper vacant place can be filled in 4 different ways by anyone of the 4 flags; following which, the lower vacant place can be filled in 3 different ways by anyone of the remaining 3 different flags. Hence, by the multiplication principle, the required number of signals $= 4 \times 3 = 12$.

106. (c) Evidently, (c) is correct option because we have to divide 17 into four groups each distinguishable into groups of 5, 5, 4 and 3.

107. (a) The possibilities are:
4 from Part A and 6 from Part B
or 5 from Part A and 5 from Part B
or 6 from Part A and 4 from Part B
Therefore, the required number of ways is
$= {}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$
$= 105 + 126 + 35 = 266$.

108. (b) The following are the number of possible choices:
${}^{52}C_{18} \times {}^{35}C_2$ (18 families having atmost 2 children and 2 selected from other type of families)
${}^{52}C_{19} \times {}^{35}C_1$ (19 families having atmost 2 children and 1 selected from other type of families)
${}^{52}C_{20}$ (All selected 20 families having atmost 2 children). Hence, the total number of possible choices is : $= {}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$

109. (d) Let us make the following cases :
Case I : Boy borrows Mathematics Part II, then he borrows Mathematics Part I also. So, the number of possible choices is ${}^6C_1 = 6$.
Case II : Boy does not borrow Mathematics Part II, then the number of possible choices is ${}^7C_3 = 35$.
Hence, the total number of possible choices is $= 35 + 6 = 41$.

110. (d) Let there were n men playing in the tournament with 2 women. According to the given condition,
${}^nC_2 - {}^nC_1 \times {}^2C_1 = 52$

$\Rightarrow \frac{n(n-1)}{2} - 2n = 52$

$\Rightarrow n^2 - n - 4n = 104$

$\Rightarrow n^2 - 5n - 104 = 0$

$\Rightarrow n = 13.$

111. (b) For one game four persons are required.
This can be done in ${}^6C_4 = 15$ ways.
Once a set of 4 persons are selected, number of games possible will be $\frac{{}^4C_2}{2} = 3$ games.

$\therefore$ Total number of possible games $= 3 \times 15 = 45$.

112. (d) The number of times the house master goes to dhaba is $^nC_3$. Let n be the number of children taking non-vegetarian food.

Now, $^nC_3 - {}^{n-1}C_2 = 84$

$$\Rightarrow \frac{n(n-1)(n-2)}{6} - \frac{(n-1)(n-2)}{2} = 84$$

$$\Rightarrow (n-1)(n-2)\left[\frac{n}{6} - \frac{1}{2}\right] = 84$$

$\Rightarrow (n-1)(n-2)(n-3) = 6 \times 6 \times 14$

$\Rightarrow (n-1)(n-2)(n-3) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$
$= 7 \times 8 \times 9$

$\Rightarrow (n-1) = 9 \Rightarrow n = 10.$

113. (c) Required number

$= {}^3C_3 + {}^3C_2 \times {}^7C_1 + {}^7C_2 \times {}^3C_1$

$= 1 + 3 \times 7 + 21 \times 3 = 1 + 21 + 63 = 85.$

114. (c) Let the boxes be marked as A, B and C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities :

(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways

$= A(1)\ B(1)\ C(3)$

$= {}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 = 5 \cdot 4 \cdot 1 = 20$

(ii) Any two box containing 2 balls each and third containing 1 ball, the number of ways

$= A(2)\ B(2)\ C(1) = {}^5C_2 \cdot {}^3C_2 \cdot {}^1C_1$

$= 10 \times 3 \times 1 = 30$

Since, the box containing 1 ball could be any of the three boxes A, B, C. Hence, the required number of ways = 30 × 3 = 90.

Hence, total number of ways = 60 + 90 = 150.

115. (a) For the first player, distribute the cards in $^{52}C_{17}$ ways. Now, out of 35 cards left, 17 cards can be put for second player in $^{35}C_{17}$ ways. Similarly, for third player put them in $^{18}C_{17}$ ways. One card for the last player can be put in $^1C_1$ way. Therefore, the required number of ways for the proper distribution

$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^1C_1$

$$= \frac{52!}{35!17!} \times \frac{35!}{18!17!} \times \frac{18!}{17!1!} \times 1! = \frac{52!}{(17!)^3}.$$

116. (c) Total number of 3-digit numbers having at least one of their digits as 5 = Total number of 3-digit numbers – (Total number of 3-digit numbers in which 5 does not appear at all)

$= 9 \times 10 \times 10 - 8 \times 9 \times 9$

$= 900 - 648 = 252$

117. (a) Total number of 4-digit numbers = 5 × 5 × 5 × 5 = 625 (as each place can be filled by anyone of the numbers 1, 2, 3, 4 and 5)

Numbers in which no two digits are identical

= 5 × 4 × 3 × 2 = 120 (i.e. repetition not allowed)

(as 1st place can be filled in 5 different ways, 2nd place can be filled in 4 different ways and so on)

Number of 4-digits numbers in which at least 2 digits are identical = 625 – 120 = 505

118. (a) The number of words starting from A are 5! = 120

The number of words starting from I are 5! = 120

The number of words starting from KA are 4! = 24

The number of words starting from KI are 4! = 24

The number of words starting from KN are 4! = 24

The number of words starting from KRA are 3! = 6

The number of words starting from KRIA are 2! = 2

The number of words starting from KRIN are 2! = 2

The number of words starting from KRISA are 1! = 1

The number of words starting from KRISNA are 1! = 1

Hence, rank of word 'KRISNA

= 2(120) + 3(24) + 6 + 2(2) + 2(1) = 324

119. (c) The numbers between 999 and 10000 are all 4-digit numbers. The number of 4-digit numbers formed by digits 0, 2, 3, 6, 7, 8 is $^6P_4 = 360$.

But here those numbers are also involved which begin from 0. So, we take those numbers as three-digit numbers.

Taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2, 3, 6, 7, 8 are $^5P_3 = 60$

So, the required numbers = 360 – 60 = 300.

120. (b) After sending 4 to one side and 3 to other side. We have to select 5 for one side and 6 for other side from remaining.

This can be done in $^{11}C_5 \times {}^6C_6$ ways $= {}^{11}C_5$

Now, there are 9 on each side of the long table and each can be arranged in 9! ways.

$\therefore$ Required number of ways $= {}^{11}C_5 \times 9! \times 9!$

$= {}^{11}C_6 \times (9!)^2$ $\qquad [\because {}^nC_r = {}^nC_{n-r}]$

121. (b) Total number of ways

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210 = 385$

122. (b) The number of choices available to him

$= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$

$$= \frac{5!}{4!\,1!} \times \frac{8!}{6!\,2!} + \frac{5!}{5!\,0!} \times \frac{8!}{5!\,3!}$$

$$= 5 \times \frac{8 \times 7}{2} + 1 \times \frac{8 \times 7 \times 6}{3 \times 2}$$

$= 5 \times 4 \times 7 + 8 \times 7$

$= 140 + 56 = 196$

123. (a) For A, B, C to speak in order of alphabets, 3 places out of 10 may be chosen first in $^{10}C_3$ ways.

The remaining 7 persons can speak in 7! ways.

Hence, the number of ways in which all the 10 persons can speak is $^{10}C_3 \cdot 7! = \frac{10!}{3!} = \frac{10!}{6}$.

124. (d) Since 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in $^2C_1$ ways. Now from the remaining 5 persons we have to select 2 which can be done in $^5C_2$ ways. But the front seat and the rear seat person can interchange among themselves. Therefore, the required number of ways in which the car can be filled is $^5C_2 \times {}^2C_1 \times 2!$

$= 20 \times 2 = 40.$

CHAPTER 8

BINOMIAL THEOREM

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. How many terms are present in the expansion of $\left(x^2+\dfrac{2}{x^2}\right)^{11}$?
(a) 11 (b) 12 (c) 10 (d) 11!

2. The total number of terms in the expansion of $(x+a)^{51}-(x-a)^{51}$ after simplification is
(a) 102 (b) 25 (c) 26 (d) None of these

3. The term independent of x in the expansion of $\left(2x+\dfrac{1}{3x^2}\right)^9$ is
(a) 2^{nd} (b) 3^{rd} (c) 4^{th} (d) 5^{th}

4. In the expansion of $\left(\sqrt[3]{\dfrac{x}{3}}-\sqrt{\dfrac{3}{x}}\right)^{10}$, $x>0$, the constant term is
(a) –70 (b) 70 (c) 210 (d) –210

5. The coefficient of x^{-12} in the expansion of $\left(x+\dfrac{y}{x^3}\right)^{20}$ is
(a) ${}^{20}C_8$ (b) ${}^{20}C_8\, y^8$ (c) ${}^{20}C_{12}$ (d) ${}^{20}C_{12}\, y^{12}$

6. In the binomial expansion of $(a-b)^n$, $n \geq 5$ the sum of the 5th and 6th terms is zero. Then a/b equals :
(a) $\dfrac{n-5}{6}$ (b) $\dfrac{n-4}{5}$ (c) $\dfrac{5}{n-4}$ (d) $\dfrac{6}{n-5}$

7. If the coefficients of x^7 and x^8 in $\left(2+\dfrac{x}{3}\right)^n$ are equal, then n is
(a) 56 (b) 55 (c) 45 (d) 15

8. The coefficient of the term independent of x in the expansion of $\left(\sqrt{\dfrac{x}{3}}+\dfrac{3}{2x^2}\right)^{10}$ is
(a) 5/4 (b) 7/4 (c) 9/4 (d) None of these

9. The coefficient of x^p and x^q (p and q are positive integers) in the expansion of $(1+x)^{p+q}$ are
(a) equal
(b) equal with opposite signs
(c) reciprocal of each other
(d) None of these

10. If t_r is the rth term in the expansion of $(1+x)^{101}$, then the ratio $\dfrac{t_{20}}{t_{19}}$ equal to
(a) $\dfrac{20x}{19}$ (b) $83x$ (c) $19x$ (d) $\dfrac{83x}{19}$

11. r and n are positive integers $r>1$, $n>2$ and coefficient of $(r+2)^{th}$ term and $3r^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals
(a) $3r$ (b) $3r+1$ (c) $2r$ (d) $2r+1$

12. In the expansion of $\left(x+\dfrac{2}{x^2}\right)^{15}$, the term independent of x is :
(a) ${}^{15}C_6.26$ (b) ${}^{15}C_5.2^5$
(c) ${}^{15}C_4.2^4$ (d) None of these

13. The formula
$(a+b)^m = a^m + ma^{m-1}b+\dfrac{m(m-1)}{1.2}a^{m-2}b^2+...$ holds when
(a) $b<a$ (b) $a<b$
(c) $|a|<|b|$ (d) $|b|<|a|$

14. $\dfrac{1}{\sqrt{5+4x}}$ can be expanded by binomial theorem, if
(a) $x<1$ (b) $|x|<1$
(c) $|x|<\dfrac{5}{4}$ (d) $|x|<\dfrac{4}{5}$

15. The expansion of $\dfrac{1}{(4-3x)^{1/2}}$ by binomial theorem will be valid, if
(a) $x<1$ (b) $|x|<1$
(c) $-\dfrac{2}{\sqrt{3}}<x<\dfrac{2}{\sqrt{3}}$ (d) None of these

16. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1+x)^n$ are in A.P., then value of n is
(a) 3 (b) 7 (c) 11 (d) 14

17. If in the binomial expansion of $(1+x)^n$ where n is a natural number, the coefficients of the 5th, 6th and 7th terms are in A.P., then n is equal to:
(a) 7 or 13 (b) 7 or 14 (c) 7 or 15 (d) 7 or 17

18. The coefficient of the middle term in the expansion of $(2+3x)^4$ is :
(a) 6 (b) 5! (c) 8! (d) 216

19. If the r^{th} term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4, then r is equal to
(a) 2 (b) 3 (c) 4 (d) 5

20. What is the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$?
(a) $C(12, 7)\, x^3 y^{-3}$ (b) $C(12, 6)\, x^{-3} y^3$
(c) $C(12, 7)\, x^{-3} y^3$ (d) $C(12, 6)\, x^3 y^{-3}$

21. If x^4 occurs in the rth term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then what is the value of r ?
(a) 4 (b) 8 (c) 9 (d) 10

22. What is the coefficient of x^3y^4 in $(2x + 3y^2)^5$?
(a) 240 (b) 360 (c) 720 (d) 1080

23. If the coefficient of x^7 in $\left[ax^2+\frac{1}{bx}\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
(a) $a-b=1$ (b) $a+b=1$ (c) $\frac{a}{b}=1$ (d) $ab=1$

24. If A and B are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ then :
(a) A = B (b) 2A = B (c) A = 2B (d) AB = 2

25. What is the coefficient of x^3 in $\frac{(3-2x)}{(1+3x)^3}$?
(a) –272 (b) –540 (c) –870 (d) –918

26. If 'n' is positive integer and three consecutive coefficient in the expansion of $(1 + x)^n$ are in the ratio 6 : 33 : 110, then n is equal to :
(a) 9 (b) 6 (c) 12 (d) 16

27. $\sqrt{5}\,[(\sqrt{5}+1)^{50}-(\sqrt{5}-1)^{50}]$ is
(a) an irrational number (b) 0
(c) a natural number (d) None of these

28. The number of term in the expansion of $[(x+4y)^3\,(x-4y)^3]^2$ is
(a) 6 (b) 7 (c) 8 (d) 32

29. The term independent of x in the expansion of $\left(\sqrt[6]{x}-\frac{1}{\sqrt[3]{x}}\right)^9$ is
(a) $-{}^9C_3$ (b) $-{}^9C_4$ (c) $-{}^9C_5$ (d) $-{}^8C_3$

30. If the coefficients of r^{th} and $(r + 1)^{th}$ terms in the expansion of $(3 + 7x)^{29}$ are equal, then the value of r is
(a) 31 (b) 11 (c) 18 (d) 21

31. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is
(a) 1594 (b) 792 (c) 924 (d) 2924

32. The coefficient of x^{-7} in the expansion of $\left[ax-\frac{1}{bx^2}\right]^{11}$ will be :
(a) $\frac{462}{b^5}a^6$ (b) $\frac{462a^5}{b^6}$ (c) $\frac{-462a^5}{b^6}$ (d) $\frac{-462a^6}{b^5}$

33. The coefficient of x^3 in the expansion of $\left(x-\frac{1}{x}\right)^7$ is :
(a) 14 (b) 21 (c) 28 (d) 35

34. Find the largest coefficient in the expansion of $(4 + 3x)^{25}$.
(a) $(3)^{25}\times{}^{25}C_{10}\left(\frac{4}{3}\right)^{11}$ (b) $20\times{}^{25}C_{11}\left(\frac{4}{3}\right)^{14}$
(c) $(2)^8\times{}^{25}C_{11}\left(\frac{5}{2}\right)^{11}$ (d) $(4)^{25}\times{}^{25}C_{11}\times\left(\frac{3}{4}\right)^{11}$

35. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then value of $\frac{(C_0+C_1)(C_1+C_2).....(C_{n-1}+C_n)}{C_0C_1C_2.....C_{n-1}}$ is
(a) $\frac{(n+3)^3}{(2n)!}$ (b) $\frac{(n+1)^n}{n!}$ (c) $\frac{(2n)!}{(n+1)!}$ (d) $\frac{(n-1)^n}{n!}$

36. Notation form of $(a + b)^n$ is
(a) $\sum_{k=0}^{n}{}^nC_k a^{n+k}b^k$ (b) $\sum_{k=0}^{n}{}^nC_k a^{n-k}b^k$
(c) $\sum_{k=0}^{n}{}^nC_k b^{n+k}a^k$ (d) None of these

37. In every term, the sum of indices of a and b in the expansion of $(a + b)^n$ is
(a) n (b) n + 1 (c) n + 2 (d) n – 1

38. The approximation of $(0.99)^5$ using the first three terms of its expansion is
(a) 0.851 (b) 0.751 (c) 0.951 (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

39. The largest term in the expansion of $(3 + 2x)^{50}$, where $x=\frac{1}{5}$, is
I. 5^{th} II. 3^{rd}
III. 7^{th} IV. 6^{th}
Choose the correct option
(a) Only I (b) Only II
(c) Both I and IV (d) Both III and IV

40. Consider the following statements.
I. Coefficient of x^r in the binomial expansion of $(1 + x)^n$ is ${}^nC_r$.
II. Coefficient of $(r + 1)^{th}$ term in the binomial expansion of $(1 + x)^n$ is ${}^nC_r$.
Choose the correct option.
(a) Only I is correct (b) Only II is correct
(c) Both are correct. (d) Both are incorrect.

41. Consider the following statements.

I. General term of the expansion of $(x+y)^n$ is ${}^nC_r x^{n-r} y^r$.

II. The coefficients ${}^nC_r$ occuring in the binomial theorem are known as binomial coefficients.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

42. Consider the following statements.

I. General term in the expansion of $(x^2-y)^6$ is $(-1)^r x^{12-2r} \cdot y^r$

II. 4^{th} term in the expansion of $(x-2y)^{12}$ is $-1760x^9y^3$.

Choose the correct option.

(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

43. Consider the following statements.

Binomial expansion of $(x+a)^n$ contains $(n+1)$ terms.

I. If n is even, then $\left(\frac{n}{2}+1\right)$ th term is the middle term.

II. If n is odd, then $\left(\frac{n+1}{2}\right)$ th is the middle term.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

44. Consider the following statements.

I. The number of terms in the expansion of $(x+a)^n$ is $n+1$.

II. The binomial expansion is briefly written as

$$\sum_{r=0}^{n} {}^nC_r x^{n-r} . a^r$$

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

45.

	Column I (Expression)	Column II (Expansion)
A.	$(1-2x)^5$	1. $\frac{x^5}{243}+\frac{5}{81}.x^3+\frac{10}{27}.x+\frac{10}{9}.\frac{1}{x}+\frac{5}{3}.\frac{1}{x^3}+\frac{1}{x^5}$
B.	$\left(\frac{2}{x}-\frac{x}{2}\right)^5$	2. $1-10x+40x^2-80x^3+80x^4-32x^5$
C.	$(2x-3)^6$	3. $32x^{-5}-40x^{-3}+20x^{-1}-5x+\frac{5}{8}x^3-\frac{1}{32}x^5$
D.	$\left(\frac{x}{3}+\frac{1}{x}\right)^5$	4. $64x^6-576x^5+2160x^4-4320x^3+4860x^2-2916x+729$

Codes

	A	B	C	D
(a)	2	4	3	1
(b)	2	3	4	1
(c)	1	3	4	2
(d)	1	4	3	2

46. Using Binomial Theorem, evaluate expression given in column-I and match with column-II.

Column I	Column II
A. $(96)^3$	1. 104060401
B. $(102)^5$	2. 9509900499
C. $(101)^4$	3. 11040808032
D. $(99)^5$	4. 884736

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	1	3	4
(d)	2	3	1	4

47.

Column-I	Column-II
A. Coefficient of x^5 in $(x+3)^8$ is	1. 18564
B. Coefficient of a^5b^7 in $(a-2b)^{12}$ is	2. $61236\, x^5y^5$
C. 13^{th} term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$, is	3. 1512
D. Middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$, is	4. -101376

Codes

	A	B	C	D
(a)	3	1	4	2
(b)	2	1	4	3
(c)	2	4	1	3
(d)	3	4	1	2

48.

Column-I	Column-II
A. Term independent of x in the expansion of $\left(x^2+\frac{1}{x}\right)^9$ is	1. 6^{th} term
B. Term independent of x in the expansion of $\left(x^2+\frac{1}{2x}\right)^{12}$ is	2. 10^{th} term
C. Term independent of x in the expansion of $\left(2x-\frac{1}{x}\right)^{10}$ is	3. 9^{th} term
D. Term independent of x in the expansion of $\left(x^3+\frac{3}{x^2}\right)^{15}$ is	4. 7^{th} term

Codes

	A	B	C	D
(a)	2	1	3	4
(b)	4	3	1	2
(c)	4	1	2	3
(d)	3	2	1	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

49. If the second, third and fourth terms in the expansion of $(a + b)^n$ are 135, 30 and 10/3 respectively, then the value of n is

(a) 6 (b) 5 (c) 4 (d) None of these

50. Coefficient of x^{13} in the expansion of $(1 - x)^5 (1 + x + x^2 + x^3)^4$ is

(a) 4 (b) 6 (c) 32 (d) 5

51. If x^4 occurs in the t^{th} term in the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, then the value of t is equal to :

(a) 7 (b) 8 (c) 9 (d) 10

52. In the expansion of $(1 + x)^{18}$, if the coefficients of $(2r + 4)^{th}$ and $(r - 2)^{th}$ terms are equal, then the value of r is :

(a) 12 (b) 10 (c) 8 (d) 6

53. A positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6, is

(a) 3 (b) 4 (c) 0 (d) None of these

54. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1 + x)^n$ are in A.P., then value of n is

(a) 3 (b) 7 (c) 11 (d) 14

55. If the coefficient of x in $(x^2 + k/x)^5$ is 270, then the value of k is

(a) 2 (b) 3 (c) 4 (d) 5

56. If the r^{th} term in the expansion of

$\left(\frac{x}{3} - \frac{2}{x^2}\right)^{10}$ contains x^4, then the value of r is

(a) 2 (b) 3 (c) 4 (d) 5

57. The number of zero terms in the expansion of

$(1+3\sqrt{2}x)^9 + (1-3\sqrt{2}x)^9$ is

(a) 2 (b) 3 (c) 4 (d) 5

58. Number of terms in the expansion of

$\left(1+5\sqrt{2}x\right)^9 + \left(1-5\sqrt{2}x\right)^9$ is

(a) 2 (b) 3 (c) 4 (d) 5

59. Value of 'a', if 17^{th} and 18^{th} terms in the expansion of $(2 + a)^{50}$ are equal, is

(a) 1 (b) 2 (c) 3 (d) 4

60. One value of α for which the coefficients of the middle terms in the expansion of $(1 + \alpha x)^4$ and $(1 - \alpha x)^6$ are equal, is $\frac{-3}{10}$. Other value of 'α' is

(a) 0 (b) 1 (c) 2 (d) 3

61. Number of terms involving x^6 in the expansion of

$\left(2x^2 - \frac{3}{x}\right)^{11}$, $r \neq 0$, is

(a) 1 (b) 2 (c) 6 (d) 0

62. If the fourth term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, then the value of $a \times n$ is

(a) 2 (b) 6 (c) 3 (d) 4

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

63. **Assertion :** The term independent of x in the expansion of

$\left(x + \frac{1}{x} + 2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$.

Reason : The coefficient of x^6 in the expansion of $(1 + x)^n$ is ${}^nC_6$.

64. **Assertion :** If $(1 + ax)^n = 1 + 8x + 24x^2 + ...$, then the values of a and n are 2 and 4 respectively.

Reason : $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ...$ for all $n \in Z^+$.

65. **Assertion :** If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ is equal to $\frac{n}{2}a_n$.

Reason : ${}^nC_r = {}^nC_{n-r}$.

66. If $(1 + x)^n = \sum_{r=0}^{n} C_r x^r$, then

Assertion : $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)...\left(1+\frac{C_n}{C_{n-1}}\right) = \frac{(n+1)^n}{n!}$

Reason : ${}^nC_r = \frac{n(n-1)...(n-r+1)}{r(r-1)...1}$

67. **Assertion :** The r^{th} term from the end in the expansion of $(x + a)^n$ is ${}^nC_{n-r+1} x^{r-1} a^{n-r+1}$.

Reason : The r^{th} term from the end in the expansion of $(x + a)^n$ is $(n - r + 2)^{th}$ term.

68. **Assertion :** In the expansion of $(x + 2y)^8$, the middle term is 4^{th} term.

Reason : If n is even in the expansion of $(a + b)^n$, then $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term.

69. **Assertion :** ${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = 2^{n-1}$

Reason : ${}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}$

70. **Assertion:** Number of terms in the expansion of $[(3x + y)^8 - (3x - y)^8]$ is 4.

Reason: If n is even, then $\{(x + a)^n - (x - a)^n\}$ has $\frac{n}{2}$ terms.

71. **Assertion:** Number of terms in the expansion of

$\left(\sqrt{x}+\sqrt{y}\right)^{10}+\left(\sqrt{x}-\sqrt{y}\right)^{10}$ is 6.

Reason: If n is even, then the expansion of

$\{(x+a)^n+(x-a)^n\}$ has $\left(\frac{n}{2}+1\right)$ terms.

72. **Assertion:** General term of the expansion $(x + 2y)^9$ is $^9C_r.2^r.x^{9-r}.y^r$.

Reason: General term of the expansion $(x + a)^n$ is given by $T_{r+1} = {}^nC_r\, x^{n-r}.a^r$

73. **Assertion.** The coefficients of the expansions are arranged in an array. This array is called Pascal's triangle.

Reason: There are 11^{th} terms in the expansion of $(4x + 7y)^{10} + (4x - 7y)^{10}$.

74. **Assertion.** In the binomial expansion $(a + b)^n$, r^{th} term is $^nC_r.a^{n-r}.b^r$.

Reason. If n is odd, then there are two middle terms.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

75. After simplification, what is the number of terms in the expansion of $[(3x + y)^5]^4 - [(3x-y)^4]^5$?

(a) 4 (b) 5 (c) 10 (d) 11

76. The term independent of x in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$, is 'a' times the corresponding binomial coefficient. Then 'a' is

(a) 3 (b) 1/3 (c) –1/3 (d) None of these

77. The term independent of x in the expansion of $\left(\frac{1-x}{1+x}\right)^2$ is

(a) 4 (b) 3 (c) 1 (d) None of these

78. The middle term in the expansion of

$\left(1+\frac{1}{x^2}\right)\left(1+x^2\right)^n$ is

(a) $^{2n}C_n\, x^{2n}$ (b) $^{2n}C_n\, x^{-2n}$

(c) $^{2n}C_n$ (d) $^{2n}C_{n-1}$

79. What are the values of k if the term independent of x in the expansion of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405?

(a) ±3 (b) ±6 (c) ±5 (d) ±4

80. If $7^9 + 9^7$ is divided by 64 then the remainder is

(a) 0 (b) 1 (c) 2 (d) 63

81. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is

(a) 6th term (b) 7th term

(c) 5th term (d) 8th term

82. The middle term in the expansion of $\left(1+\frac{1}{x^2}\right)^n\left(1+x^2\right)^n$ is

(a) $^{2n}C_n\, x^{2n}$ (b) $^{2n}C_n\, x^{-2n}$

(c) $^{2n}C_n$ (d) $^{2n}C_{n-1}$

83. The value of $^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$ is

(a) $^{55}C_4$ (b) $^{55}C_3$ (c) $^{56}C_3$ (d) $^{56}C_4$

84. In the expansion of $(1 + x)^{50}$, the sum of the coefficients of odd powers of x is :

(a) 0 (b) 2^{49} (c) 2^{50} (d) 2^{51}

85. Expand by using binomial and find the degree of polynomial

$\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$ is

(a) 7 (b) 6 (c) 5 (d) 4

86. Value of $\sum_{r=1}^{10} r.\frac{^nC_r}{^nC_{r-1}}$ is

(a) $10n - 45$ (b) $10n + 45$

(c) $10n - 35$ (d) $10n^2 - 35$

87. If $(1 + x)^{2n} = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then

(a) $a_0 + a_2 + a_4 + ... = \frac{1}{2}(a_0 + a_1 + a_2 + a_3 + ...)$

(b) $a_{n+1} < a_n$

(c) $a_{n-3} = a_{n+3}$

(d) All of these

88. If $(1 + ax)^n = 1 + 8x + 24x^2 + \ldots$ then the values of a and n are

(a) n = 4, a = 2 (b) n = 5, a = 1

(c) n = 8, a = 3 (d) n = 8, a = 2

89. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$

(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

90. The sum of the series

$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... -..... + {}^{20}C_{10}$ is

(a) 0 (b) $^{20}C_{10}$ (c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}{}^{20}C_{10}$

91. The coefficient of x^{32} in the expansion of :

$\left(x^4-\frac{1}{x^3}\right)^{15}$ is:

(a) $^{-15}C_3$ (b) $^{15}C_4$ (c) $^{-15}C_5$ (d) $^{15}C_2$

92. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$

(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** 12 terms. [$\because$ No. of terms in $(x+a)^n = n+1$)]

2. **(c)** Since the total number of terms are 52 of which 26 terms get cancelled.

3. **(c)** Suppose $(r+1)^{th}$ term is independent of x. We have

$$T_{r+1} = {}^9C_r\,(2x)^{9-r}\left(\frac{1}{3x^2}\right)^r = {}^9C_r\,2^{9-r}\,\frac{1}{3^r}\,.\,x^{9-3r}$$

This term is independent of x if $9-3r=0$
i.e., $r=3$.
Thus, 4th term is independent of x.

4. **(c)** The constant term

$$= {}^{10}C_6\left(\sqrt[3]{\frac{x}{3}}\right)^6\left(-\sqrt{\frac{3}{x}}\right)^4 = {}^{10}C_4\,\frac{1}{3^2}\,.\,3^2 = 210$$

5. **(b)** Suppose x^{-12} occurs is $(r+1)^{th}$ term.
We have

$$T_{r+1} = {}^{20}C_r\,x^{20-r}\left(\frac{y}{x^3}\right)^r = {}^{20}C_r\,x^{20-4r}\,y^r$$

This term contains x^{-12} if $20-4r=-12$ or $r=8$.
$\therefore$ The coefficient of x^{-12} is ${}^{20}C_8\,y^8$.

6. **(b)** Given,
$T_5 + T_6 = 0$
$\Rightarrow {}^nC_4\,a^{n-4}b^4 - {}^nC_5\,a^{n-5}b^5 = 0$
$\Rightarrow {}^nC_4\,a^{n-4}b^4 = {}^nC_5\,a^{n-5}b^5$

$$\Rightarrow \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$$

7. **(b)** Since $T_{r+1} = {}^nC_r\,a^{n-r}x^r$ in expansion of $(a+x)^n$,
Therefore,

$$T_8 = {}^nC_7(2)^{n-7}\left(\frac{x}{3}\right)^7 = {}^nC_7\,\frac{2^{n-7}}{3^7}x^7$$

and $$T_9 = {}^nC_8(2)^{n-8}\left(\frac{x}{3}\right)^8 = {}^nC_8\,\frac{2^{n-8}}{3^8}x^8$$

Therefore, $${}^nC_7\,\frac{2^{n-7}}{3^7} = {}^nC_8\,\frac{2^{n-8}}{3^8}$$

(since it is given that coefficient of x^7 = coefficient of x^8)

$$\Rightarrow \frac{n!}{7!\,(n-7)!}\times\frac{8!\,(n-8)!}{n!} = \frac{2^{n-8}}{3^8}\cdot\frac{3^7}{2^{n-7}}$$

$$\Rightarrow \frac{8}{n-7} = \frac{1}{6} \Rightarrow n = 55$$

8. **(a)** The $(r+1)$th term in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ is given by

$$T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r = {}^{10}C_r\,\frac{x^{5-(r/2)}}{3^{5-(r/2)}}\cdot\frac{3^r}{2^r\,x^{2r}}$$

$$= {}^{10}C_r\,\frac{3^{(3r/2)-5}}{2^r}\,x^{5-(5r/2)}.$$

For T_{r+1} to be independent of x, we must have
$5-(5r/2)=0$ or $r=2$.
Thus, the 3rd term is independent of x and is equal to

$${}^{10}C_2\,\frac{3^{3-5}}{2^2} = \frac{10\times 9}{2}\times\frac{3^{-2}}{4} = \frac{5}{4}$$

9. **(a)** Coefficient of x^p and x^q in the expansion of $(1+x)^{p+q}$ are ${}^{p+q}C_p$ and ${}^{p+q}C_q$.

and $${}^{p+q}C_p = {}^{p+q}C_q = \frac{(p+q)!}{p!\,q!}$$

10. **(d)** t_r is the rth term in the expansion of $(1+x)^{101}$.
$t_r = {}^{101}C_{r-1}.\,(x)^{(r-1)}$

$$\therefore \frac{t_{20}}{t_{19}} = \frac{{}^{101}C_{19}}{{}^{101}C_{18}}\cdot\frac{x^{19}}{x^{18}} = \frac{{}^{101}C_{19}\,x}{{}^{101}C_{18}} = \frac{\frac{101!}{19!82!}}{\frac{101!}{18!83!}}x = \frac{83x}{19}$$

11. **(c)** $t_{r+2} = {}^{2n}C_{r+1}\,x^{r+1}$; $t_{3r} = {}^{2n}C_{3r-1}\,x^{3r-1}$
Given ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;
$\Rightarrow {}^{2n}C_{2n-(r+1)} = {}^{2n}C_{3r-1}$
$\Rightarrow 2n-r-1 = 3r-1 \Rightarrow 2n = 4r \Rightarrow n = 2r$

12. **(b)** On comparing with the expansion of $(x+a)^n$, we get

$$x = x,\ a = \frac{2}{x^2},\ n = 15$$

Now, r^{th} term of $\left(x+\frac{2}{x^2}\right)^{15}$ is given as

$T_{r+1} = {}^nC_r\,x^{n-r}\,a^r$

$$= {}^{15}C_r\,(x)^{15-r}\left(\frac{2}{x^2}\right)^r$$

$$= {}^{15}C_r\,x^{15-r}\,2^r\,.\,x^{-2r} = {}^{15}C_r\,x^{15-3r}\,2^r$$

Now, in the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term is independent of x if $15-3r=0$
i.e., $r=5$
$\therefore$ Term independent of $x = {}^{15}C_5\,.\,2^5$

13. **(d)** The expression can be written as $a^m\left\{\left(1+\frac{b}{a}\right)^m\right\}$

14. **(c)** The given expression can be written as $5^{-1/2}\left(1+\frac{4}{5}x\right)^{-1/2}$

and it is valid only when $\left|\frac{4}{5}x\right| < 1 \Rightarrow |x| < \frac{5}{4}$

15. **(d)** The given expression can be written as $4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$

and it is valid only when $\left|\frac{3}{4}x\right|<1 \Rightarrow -\frac{4}{3}<x<\frac{4}{3}$

16. **(b)** $2\,{}^{n}C_2 = {}^{n}C_1 + {}^{n}C_3$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow n = 2$ or 7

17. **(b)** In the binomial expansion of $(1+x)^n$,

$T_r = {}^{n}C_{r-1}.(x)^{r-1}$

For $r = 5,\ T_5 = {}^{n}C_4 x^4$

$r = 6,\ T_6 = {}^{n}C_5 x^5$

and $r = 7,\ T_7 = {}^{n}C_6 x^6$

Since, the coefficients of these terms are in A.P.

$\Rightarrow T_5 + T_7 = 2T_6$

$\Rightarrow {}^{n}C_4 + {}^{n}C_6 = 2 \times {}^{n}C_5$

$$\Rightarrow \frac{n!}{(n-4)!4!}+\frac{n!}{(n-6)!6!}=\frac{2\times n!}{(n-5)!5!}$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{4!}+\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}=\frac{2n(n-1)(n-2)(n-3)(n-4)}{5!}$$

$$\Rightarrow \frac{1}{4!}+\frac{(n-4)(n-5)}{6!}=\frac{2(n-4)}{5!}$$

$$\Rightarrow \frac{1}{1}+\frac{(n-4)(n-5)}{5\times 6}=\frac{2(n-4)}{5}$$

$$\Rightarrow \frac{30+n^2-9n+20}{5\times 6}=\frac{2n-8}{5}$$

$\Rightarrow n^2 - 9n + 50 = 6(2n - 8)$

$\Rightarrow n^2 - 9n + 50 - 12n + 48 = 0$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow (n-7)(n-14) = 0$

$\Rightarrow n = 7$ or $n = 14$.

18. **(d)** When exponent is n then total number of terms are n + 1. So, total number of terms in $(2+3x)^4 = 5$

Middle term is 3rd.

$\Rightarrow T_3 = {}^{4}C_2(2)^2.(3x)^2$

$$= \frac{4\times 3\times 2\times 1}{2\times 1\times 2}\times 4\times 9x^2 = 216x^2$$

$\therefore$ Coefficient of middle term is 216

19. **(b)** $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-r+1}\left(\frac{-2}{x^2}\right)^{r-1}$

$$= [{}^{10}C_{r-1}]\,x^{13-r-2r}.(-2)^{r-1}\left(\frac{1}{3}\right)^{10-r}$$

r^{th} term contains x^4 when $13 - 3r = 4 \Rightarrow r = 3$

20. **(d)** In the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$, n = 12 (even)

then middle term is $\frac{12}{2}+1 = 7^{th}$ term.

$(r+1)^{th}$ term,

$$T_{r+1} = {}^{12}C_r\left[\frac{x\sqrt{y}}{3}\right]^{12-r}.\left(-\frac{3}{y\sqrt{x}}\right)^r$$

$$\therefore\ T_7 = T_{6+1} = {}^{12}C_6\left(\frac{x\sqrt{y}}{3}\right)^6\left(-\frac{3}{y\sqrt{x}}\right)^6$$

$$= {}^{12}C_6\frac{x^6y^3}{y^6x^3} = {}^{12}C_6x^3y^{-3} = C(12,6)x^3y^{-3}$$

21. **(c)** In the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, let T_r is the r^{th} term

$$T_r = {}^{15}C_{r-1}(x^4)^{15-r+1}\left(\frac{1}{x^3}\right)^{r-1}$$

$$= {}^{15}C_{r-1}x^{64-4r-3r+3} = {}^{15}C_{r-1}x^{67-7r}$$

x^4 occurs in this term

$\Rightarrow 4 = 67 - 7r \Rightarrow 7r = 63 \Rightarrow r = 9.$

22. **(c)** $T_r = {}^{n}C_{r-1}(2x)^{r-1}(3y^2)^{n-r+1}$

$T_4 = T_{3+1} = {}^{5}C_3(2x)^3(3y^2)^2$

$$= \frac{5!}{3!2!}2^3.x^3.9y^4 = \frac{5.4}{2.1}\times 8\times 9\times x^3y^4 = 720\,x^3y^4$$

$\therefore$ Coefficient of $x^3y^4 = 720$

23. **(d)** T_{r+1} in the expansion

$$\left[ax^2+\frac{1}{bx}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$$

$$= {}^{11}C_r\,(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$$

For the coefficient of x^7, we have

$22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^7 = {}^{11}C_5(a)^6\ (b)^{-5}$ ...(i)

Again T_{r+1} in the expansion

$$\left[ax-\frac{1}{bx^2}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$$

$$= {}^{11}C_r(a)^{11-r}(-1)^r\times(b)^{-r}(x)^{-2r}(x)^{11-r}$$

For the coefficient of x^{-7}, we have

$11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

$\therefore$ Coefficient of $x^{-7} = {}^{11}C_6\,a^5\times 1\times(b)^{-6}$

$\therefore$ Coefficient of x^7 = Coefficient of x^{-7}

$\Rightarrow {}^{11}C_5\,(a)^6\,(b)^{-5} = {}^{11}C_6\,a^5\times(b)^{-6}$

$\Rightarrow ab = 1.$

24. **(c)** We have

$$(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1\,x + {}^{2n}C_2\,x^2$$

$$+ \ldots {}^{2n}C_nx^n + \ldots + {}^{2n}C_{2n}x^{2n} \quad \ldots(i)$$

$(1+x)^{2n-1} = {}^{2n-1}C_0 + {}^{2n-1}C_1\, x + {}^{2n-1}C_2 x^2 +$
$.... + {}^{2n-1}C_n x^n + ... + {}^{2n-1}C_{2n-1}\, x^{2n-1}$ ...(ii)

According to the given data and equations (i) and (ii), we can claim that

$A = {}^{2n}C_n$ and $B = {}^{2n-1}C_n$

$$\Rightarrow \frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{2n!}{n!n!}}{\frac{(2n-1)!}{n!(n-1)!}}$$

$$\Rightarrow \frac{A}{B} = \frac{2n(2n-1)!}{n(n-1)!} \times \frac{(n-1)!}{(2n-1)!} = 2$$

$\Rightarrow A = 2B$

25. (d) $\frac{(3-2x)}{(1+3x)^3} = (3-2x)(1+3x)^{-3}$

$$= (3-2x)\left[1 - 9x + \frac{(-3)(-4)}{2!}.9x^2 + \frac{(-3)(-4)(-5)}{3!}.27x^3 + ...\right]$$

[Expanding $(1+3x)^{-3}$]

$= (3-2x)(1 - 9x + 54x^2 - 270x^3 + ...)$

$\therefore$ Coefficient of $x^3 = -270 \times 3 - 2 \times 54$
$= -810 - 108 = -918$

26. (c) Let the consecutive coefficient of $(1+x)^n$ are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

From the given condition,

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6 : 33 : 110$

Now ${}^nC_{r-1} : {}^nC_r = 6 : 33$

$$\Rightarrow \frac{n!}{(r-1)!\,(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{6}{33}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{2}{11} \Rightarrow 11r = 2n - 2r + 2$$

$\Rightarrow 2n - 13r + 2 = 0$ ....(i)

and ${}^nC_r : {}^nC_{r+1} = 33 : 110$

$$\Rightarrow \frac{n!}{r!(n-r)!} \times \frac{(r+1)!\,(n-r-1)!}{n!} = \frac{33}{110} = \frac{3}{10}$$

$$\Rightarrow \frac{(r+1)}{n-r} = \frac{3}{10} \Rightarrow 3n - 13r - 10 = 0$$ ...(ii)

Solving (i) & (ii), we get n = 12

27. (c) $\sqrt{5}\left[\left(\sqrt{5}+1\right)^{50} - \left(\sqrt{5}-1\right)^{50}\right]$

$$= 2\sqrt{5}\left[{}^{50}C_1\left(\sqrt{5}\right)^{49} + {}^{50}C_3\left(\sqrt{5}\right)^{47} + ...\right]$$

$$= 2\left[{}^{50}C_1(\sqrt{5})^{50} + {}^{50}C_3(\sqrt{5})^{48} + ...\right]$$

= a natural number

28. (b) $[(x+4y)^3 (x-4y)^3]^2 = [\{x^2 - (4y)^2\}]^6$
$= (x^2 - 16y^2)^6$

$\therefore$ No. of terms in the expansion = 7

29. (a) $T_{r+1} = {}^9C_r\left(\sqrt[6]{x}\right)^{9-r}\left(-\frac{1}{\sqrt[3]{x}}\right)^r$

$$= {}^9C_r(-1)^r . x^{\frac{9-r}{6}-\frac{r}{3}} = {}^9C_r . x^{\left(\frac{9-3r}{6}\right)}$$

Now $\frac{9-3r}{6} = 0 \Rightarrow r = 3$;

Thus, term independent of $x = -{}^9C_3$

30. (d) $T_{r+1} = {}^{29}C_r.3^{29-r}.(7x)^r = ({}^{29}C_r . 3^{29-r} . 7^r)\, x^r$

$\therefore$ a_r = coefficient of $(r+1)^{th}$ term $= {}^{29}C_r . 3^{29-r} . 7^r$

Now, $a_r = a_{r-1}$

$\Rightarrow {}^{29}C_r . 3^{29-r} . 7^r = {}^{29}C_{r-1} . 3^{30-r} . 7^{r-1}$

$$\Rightarrow \frac{{}^{29}C_r}{{}^{29}C_{r-1}} = \frac{3}{7} \Rightarrow \frac{30-r}{r} = \frac{3}{7} \Rightarrow r = 21$$

31. (c) We have $2^n = 4096 = 2^{12} \Rightarrow n = 12$;

the greatest coeff = coeff of middle term.

So, middle term = t_7

Coeff of $t_7 = {}^{12}C_6 = \frac{12!}{6!6!} = 924$.

32. (b) Suppose x^{-7} occurs in $(r+1)^{th}$ term.

we have $T_{r+1} = {}^nC_r\, x^{n-r} a^r$ in $(x+a)^n$.

In the given question, n = 1, x = ax, $a = \frac{-1}{bx^2}$

$$\therefore \quad T_{r+1} = {}^{11}C_r(ax)^{11-r}\left(\frac{-1}{bx^2}\right)^r$$

$= {}^{11}C_r\, a^{11-r}\, b^{-r}\, x^{11-3r}(-1)^r$

This term contains x^{-7} if $11 - 3r = -7$

$\Rightarrow r = 6$

Therefore, coefficient of x^{-7} is

$${}^{11}C_6\,(a)^5\left(\frac{-1}{b}\right)^6 = \frac{462}{b^6}a^5$$

33. (b) Given, $\left(x - \frac{1}{x}\right)^7$ and the $(r+1)^{th}$ term in the expansion of $(x+a)^n$ is $T_{(r+1)} = {}^nC_r(x)^{n-r} a^r$

$\therefore$ $(r+1)^{th}$ term in expansion of

$$\left(x - \frac{1}{x}\right)^7 = {}^7C_r\,(x)^{7-r}\left(-\frac{1}{x}\right)^r$$

$= {}^7C_r(x)^{7-2r}(-1)^r$

Since x^3 occurs in T_{r+1}

$\therefore$ $7 - 2r = 3 \Rightarrow r = 2$

thus the coefficient of $x^3 = {}^7C_2\,(-1)^2 = \frac{7\times 6}{2\times 1} = 21$.

34. (d) $(4+3x)^{25} = 4^{25}\left(1 + \frac{3}{4}x\right)^{25}$

Let $(r+1)^{th}$ term will have largest coefficient

$$\Rightarrow \frac{\text{Coefficient of } T_{r+1}}{\text{Coefficient of } T_r} \geq 1$$

$$\Rightarrow \frac{{}^{25}C_r\left(\frac{3}{4}\right)^r}{{}^{25}C_{r-1}\left(\frac{3}{4}\right)^{r-1}} \geq 1$$

$$\Rightarrow \left(\frac{25-r+1}{r}\right)\frac{3}{4} \geq 1 \Rightarrow r \leq \frac{78}{7}$$

Largest possible value of r is 11

$\therefore$ Coefficient of $T_{12} = 4^{25} \times {}^{25}C_{11} \times \left(\frac{3}{4}\right)^{11}$

35. (b) The given expression,

$$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right).....\left(1+\frac{C_n}{C_{n-1}}\right)$$

$$=\left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\left(1+\frac{n-2}{3}\right)...\left(1+\frac{1}{n}\right)$$

$(\because C_0 = C_n = 1)$

$$= \frac{(n+1)^n}{n!}$$

36. (b) The notation $\sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$ stands for

${}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + ... + {}^nC_r a^{n-r} b^r + ... + {}^nC_n a^{n-n} b^n$

where, $b^0 = 1 = a^{n-n}$.

Hence, the notation form of $(a + b)^n$ is

$$(a+b)^n = \sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$$

37. (a) In the expansion of $(a + b)^n$, the sum of the indices of a and b is $n + 0 = n$ in the first term, $(n - 1) + 1 = n$ in the second term and so on.

Thus, it can be seen that the sum of the indices of a and b is n in every term of the expansion.

38. (c) Now, $(0.99)^5 = (1 - 0.01)^5$

$= {}^5C_0(1)^5 - {}^5C_1(1)^4(0.01) + {}^5C_2(1)^3(0.01)^2$ (ignore the other terms)

$$= 1 - 5 \times 1 \times 0.01 + \frac{5\times4}{2} \times 1 \times 0.01 \times 0.01$$

$= 1 - 0.05 + 10 \times 0.0001 = 1 - 0.05 + 0.001$

$= 1.001 - 0.05 = 0.951$

STATEMENT TYPE QUESTIONS

39. (d) $\because (3+2x)^{50} = 3^{50}\left(1+\frac{2x}{3}\right)^{50}$

Here, $T_{r+1} = 3^{50}\ {}^{50}C_r\left(\frac{2x}{3}\right)^{r-1}$

and $T_r = 3^{50}\ {}^{50}C_{r-1}\left(\frac{2x}{3}\right)^{r-1}$

But $x = \frac{1}{5}$ [given]

$$\therefore \frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{{}^{50}C_r}{{}^{50}C_{r-1}}\frac{2}{3}\cdot\frac{1}{5} \geq 1$$

$\Rightarrow 102 - 2r \geq 15r \Rightarrow r \leq 6$

$\Rightarrow r = 6$

Therefore, there are two greatest terms T_r and T_{r+1} i.e., T_6 and T_7.

40. (c) Both are correct.

41. (c)

42. (a) I. General term $= T_{r+1} = {}^6C_r\,(x^2)^{6-r}\,(-y)^r$

$$= (-1)^r \frac{6!}{r!(6-r)!}.x^{12-2r}.y^r$$

II. 4^{th} term $= T_{3+1}$ in the expansion of $(x + (-2y))^{12}$

$= {}^{12}C_3 x^{12-3}[-2y]^3$

$$= \frac{12\,.\,11\,.\,10}{1\,.\,2\,.\,3}x^9(-1)^3.2^3.y^3$$

$= -220 \times 8.x^9.y^3 = -1760\,x^9\,y^3.$

43. (a) Statement II is false.

If n is odd, then $\left(\frac{n+1}{2}\right)$th and $\left(\frac{n+3}{2}\right)$th terms are the two middle terms.

44. (c)

MATCHING TYPE QUESTIONS

45. (b) (A) $(1 - 2x)^5$

$= {}^5C_0.1^5 + {}^5C_1.1^4.(-2x) + {}^5C_2.1^3(-2x)^2 + {}^5C_3.1^2(-2x)^3 + {}^5C_4.1^1(-2x)^4 + {}^5C_5 1^0(-2x)^5$

$$= 1.1 + 5.1.(-2x) + \frac{5.4}{1.2}.1.4x^2 + \frac{5.4}{1.2}.1(-8x^3) + \frac{5}{1}.1.16x^4 + (-32x^5)$$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5.$

(B) $\left[\frac{2}{x}+\left(-\frac{x}{2}\right)\right]^5$

$$= C(5,0)\left(\frac{2}{x}\right)^5 + C(5,1)\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + C(5,2)\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + C(5,3)\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + C(5,4)\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + C(5,5)\left(-\frac{x}{2}\right)^5$$

$$= 1\left(\frac{2}{x}\right)^5 + 5\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + 10\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + 10\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + 5\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + \left(-\frac{x}{2}\right)^5$$

$$= 32x^{-5} - 40x^{-3} + 20x^{-1} - 5x + \frac{5}{8}x^3 - \frac{1}{32}x^5$$

(C) $(2x-3)^6$

$$= {}^6C_0(2x)^6 + {}^6C_1(2x)^5(-3) + {}^6C_2(2x)^4(-3)^2 + {}^6C_3(2x)^3(-3)^3 + {}^6C_4(2x)^2(-3)^4 + {}^6C_5(2x)(-3)^5 + {}^6C_6(2x)^0(-3)^6$$

$$= 64x^6 + \frac{6}{1}(32x^5)(-3) + \frac{6\,.\,5}{1\,.\,2}(16x^4)9 + \frac{6\,.\,5\,.\,4}{1\,.\,2\,.\,3}(8x^3)(-27) + \frac{6\,.\,5}{1\,.\,2}(4x^2)81 + \frac{6}{1}(2x)(-243) + 729$$

$$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$$

(D) $$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5\left(\frac{1}{x}\right)^0 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{x}{3}\right)^0\left(\frac{1}{x}\right)^5$$

$$= \frac{x^5}{243} + \frac{5}{1}.\frac{x^4}{81}.\frac{1}{x} + \frac{5.4}{1.2}.\frac{x^3}{27}.\frac{1}{x^2} + \frac{5.4}{1.2}.\frac{x^2}{9}.\frac{1}{x^3} + \frac{5}{1}.\frac{x}{3}.\frac{1}{x^4} + \frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5}{81}.x^3 + \frac{10}{27}.x + \frac{10}{9}.\frac{1}{x} + \frac{5}{3}.\frac{1}{x^3} + \frac{1}{x^5}$$

46. (a) (A) $(96)^3 = (100-4)^3$

$$= {}^3C_0\,(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)\,(4)^2 - {}^3C_3(4)^3$$

(B) $(102)^5 = (100+2)^5$

(C) $(101)^4 = (100+1)^4$

(D) $(99)^5 = (100-1)^5$

47. (d) (A) General term in $(x+3)^8 = {}^8C_r x^{8-r}.\,3^r$

We have to find the coefficient of x^5

$8-r=5,\ r=8-5=3$

$\therefore$ Coefficient of x^5 (putting $r=3$)

$$= {}^8C_3\,.\,3^3 = \frac{8.7.6}{1.2.3}\,.27 = 56.27 = 1512$$

(B) $(a-2b)^{12} = [\,a+(-2b)]^{12}$

General term $T_{r+1} = C(12, r)\,a^{12-r}(-2b)^r$.

Putting $12-r=5$ or $12-5=r \Rightarrow r=7$

$T_{7+1} = C\,(12, 7)\,a^{12-7}\,(-2b)^7$

$= C(12, 7)\,a^5\,(-2b)^7 = C(12, 7)\,(-2)^7\,a^5b^7$

Hence required coefficient is $C(12, 7)\,(-2)^7$

$$= -\frac{12!}{7!\,5!}.2^7 = \frac{-12\times11\times10\times9\times8\times7!}{7!\times5\times4\times3\times2\times1}\times2^7$$

$= 8\times-11\times9\times2^7$

$= -99\times8\times128 = -101376$

(C) 13th term, $T_{13} = T_{12+1}$

$$= {}^{18}C_{12}\,(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= {}^{18}C_6 9^6.x^6(-1)^{12}.\frac{1}{3^{12}}\times\frac{1}{x^6}$$

$$= 18564\times(3^2)^6.\frac{1}{3^{12}}\times\frac{x^6}{x^6}$$

$$= 18564\times\frac{3^{12}}{3^{12}} = 18564$$

(D) Number of terms in the expansion is $10+1 = 11$ (odd)

Middle term of the expansion is $\left(\frac{n}{2}+1\right)^{th}$ term

$= (5+1)^{th}$ term $= 6^{th}$ term

$$T_6 = T_{5+1} = C(10, 5)\left(\frac{x}{3}\right)^{10-5}(9y)^5$$

$$= C(10, 5)\frac{x^5}{3^5}9^5y^5 = C(10, 5)3^5x^5y^5$$

$$= \frac{10!}{5!(10-5)!}3^5x^5y^5 = \frac{10!}{5!\,5!}3^5x^5y^5$$

$$= \frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!}3^5x^5y^5 = 61236x^5y^5$$

48. (b) (A) General term of

$\left(x^2+\frac{1}{x}\right)^9$ is $T_{r+1} = {}^9C_r\,(x^2)^{9-r}\left(\frac{1}{x}\right)^r$

$= (x^{18-2r}.\,x^{-r}).{}^9C_r = x^{18-3r}.{}^9C_r$

Term independent of x $\Rightarrow 18-3r=0 \Rightarrow r=6$ i.e. 7^{th} term.

(B) General term $= {}^{12}C_r(x^2)^{12-r}(2x)^{-r}$

$= {}^{12}C_r\,x^{24-2r-r}\bullet\,2^{-r}$

Term independent of x $\Rightarrow 24-3r=0 \Rightarrow r=8$ i.e. 9^{th} term.

(C) General term $= {}^{10}C_r\,(2x)^{10-r}\left(-\frac{1}{x}\right)^r$

$= {}^{10}C_r\,2^{10-r}.x^{10-r}.(-1)^r.x^{-r}$

Term independent of x $\Rightarrow 10-2r=0 \Rightarrow r=5$ i.e. 6^{th} term.

(D) General term $= {}^{15}C_r\,(x^3)^{15-r}\left(\frac{3}{x^2}\right)^r$

$= {}^{15}C_r.x^{45-3r}.3^r.x^{-2r}$

$= {}^{15}C_r.x^{45-5r}.3^r$

Term independent of x $\Rightarrow 45-5r=0 \Rightarrow r=9$ i.e., 10^{th} term

INTEGER TYPE QUESTIONS

49. (b) $T_2 = {}^nC_1\, ab^{n-1} = 135$ ...(i)

$T_3 = {}^nC_2\, a^2b^{n-2} = 30$ ...(ii)

$T_4 = {}^nC_3\, a^3b^{n-3} = \frac{10}{3}$ ...(iii)

Dividing (i) by (ii)

$$\frac{{}^nC_1ab^{n-1}}{{}^nC_2a^2b^{n-2}} = \frac{135}{30}$$

$$\frac{n}{\frac{n}{2}(n-1)}\frac{b}{a} = \frac{9}{2} \quad ...(iv)$$

$$\frac{b}{a} = \frac{9}{4}(n-1) \quad ...(v)$$

Dividing (ii) by (iii)

$$\frac{\frac{n(n-1)}{2}}{\frac{n(n-1)(n-2)}{3.2}}\cdot\frac{b}{a} = 9 \quad ...(vi)$$

Eliminating a and b from (v) and (vi), we get

n = 5

50. (a) Expression $= (1-x)^5.(1+x)^4(1+x^2)^4$

$= (1-x)(1-x^2)^4(1+x^2)^4$

$= (1-x)(1-x^4)^4$

$\therefore$ Coefficient of $x^{13} = -{}^4C_3(-1)^3 = 4$

51. (c) The binomial expansion of $(x+a)^n$ gives $(t+1)^{th}$ term $= T_{t+1} = {}^nC_t x^{n-t} a^t$

We have expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$.

On comparing with $(x+a)^n$, we get

$x = x^4$, $a = \frac{1}{x^3}$, $n = 15$

$\therefore$ t^{th} term

$= T_t = {}^{15}C_{t-1}(x^4)^{15-(t-1)}.\left(\frac{1}{x^3}\right)^{t-1}$

$= {}^{15}C_{t-1}(x)^{60-4t+4}.(x)^{-3t+3}$

$= {}^{15}C_{t-1}(x)^{67-7t}$

Since, x^4 occurs in the t^{th} term

$\therefore$ $67 - 7t = 4 \Rightarrow 7t = 63 \Rightarrow t = 9$

52. (d) Since the coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^n = {}^nC_r$

$\therefore$ In the expansion of $(1+x)^{18}$

coefficient of $(2r+4)^{th}$ term $= {}^{18}C_{2r+3,}$

Similarly, coefficient of $(r-2)^{th}$ term in the expansion of $(1+x)^{18} = {}^{18}C_{r-3}$

If ${}^nC_r = {}^nC_s$ then $r + s = n$

So, ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$ gives

$2r + 3 + r - 3 = 18$

$\Rightarrow 3r = 18 \Rightarrow r = 6.$

53. (a) Given expansion is $(1+x)^m$. Now,

General term $= T_{r+1} = {}^mC_r\, x^r$

Put r = 2, we have

$T_3 = {}^mC_2.x^2$

According to the question $C(m, 2) = 6$

or $\frac{m(m-1)}{2!} = 6$

$\Rightarrow m^2 - m = 12$

or $m^2 - m - 12 = 0$

$\Rightarrow m^2 - 4m + 3m - 12 = 0$

or $(m-4)(m+3) = 0$

$\therefore$ $m = 4$, since $m \neq -3$

54. (b) $2\,{}^nC_2 = {}^nC_1 + {}^nC_3 \Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow n = 2$ or 7

55. (b) **Hint :** $T_{r+1} = {}^5C_r(x^2)^{5-r}(k/x)^r = {}^5C_r\, k^r x^{10-3r}$

For coefficient of x, $10 - 3r = 1 \Rightarrow r = 3$

coefficient of $x = {}^5C_3\, k^3 = 270$

$\Rightarrow k^3 = \frac{270}{10} = 27 \therefore k = 3$

56. (b) **Hint :** $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-(r-1)}\left(-\frac{2}{x^2}\right)^{r-1}$

$= {}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r}.(-2)^{r-1}x^{13-3r}$

for coefficient of x^4, $13 - 3r = 4 \Rightarrow r = 3$

57. (d) **Hint :** Given expression

$= 2[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4 + {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8]$

$\therefore$ the number of non-zero terms is 5

58. (d) If n is odd, then the expansion of $(x+a)^n + (x-a)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. So, the expansion of $\left(1+5\sqrt{2}x\right)^9 + \left(1-5\sqrt{2}x\right)^9$ has $\left(\frac{9+1}{2}\right) = 5$ terms.

59. (a) $T_{17} = {}^{50}C_{16} \times 2^{34} \times a^{16}$

$T_{18} = {}^{50}C_{17} \times 2^{33} \times a^{17}$

Given $T_{17} = T_{18}$

$\Rightarrow \frac{{}^{50}C_{16}}{{}^{50}C_{17}} \times 2 = \frac{a^{17}}{a^{16}}$

$\Rightarrow a = \frac{50!}{34!16!} \times \frac{33!17! \times 2}{50!} = \frac{17}{34} \times 2 = 1$

60. (a) In the expansion of $(1+\alpha x)^4$

Middle term $= {}^4C_2(\alpha x)^2 = 6\alpha^2x^2$

In the expansion of $(1-\alpha x)^6$,

Middle term $= {}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$

It is given that

Coefficient of the middle term in $(1+\alpha x)^4$ = Coefficient of the middle term in $(1-\alpha x)^6$

$\Rightarrow 6\alpha^2 = -20\alpha^3$

$\Rightarrow \alpha = 0, \alpha = -\frac{3}{10}$

61. (d) Suppose x^6 occurs in $(r+1)^{th}$ term in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$

Now, $T_{r+1} = {}^{11}C_r\,(2x^2)^{11-r}\left(-\frac{3}{x}\right)^r$

$= {}^{11}C_r(-1)^r\,2^{11-r}\,3^r x^{22-3r}$

For this term to contain x^6, we must have

$22 - 3r = 6 \Rightarrow r = \frac{16}{3}$, which is a fraction.

But, r is a natural number. Hence, there is no term containing x^6.

62. (c) $T_{3+1} = \frac{5}{2}$

$\Rightarrow {}^nC_3\,(ax)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$

$\Rightarrow {}^nC_3\,a^{n-3}.x^{n-6} = \frac{5}{2}$ …(i)

$\Rightarrow n - 6 = 0 \Rightarrow n = 6$

($\because$ RHS of above equality is independent of x)

Put n = 6 in (i), we get

${}^6C_3\,a^3 = \frac{5}{2} \Rightarrow a^3 = \frac{1}{8}$

$\Rightarrow a = \frac{1}{2}$ and $n = 6$

Hence, $a \times \frac{1}{n} = \frac{1}{2} \times 6 = 3$

ASSERTION - REASON TYPE QUESTIONS

63. (d) $\left(x + \frac{1}{x} + 2\right)^m = \left(\frac{x^2 + 2x + 1}{x}\right)^m = \frac{(1+x)^{2m}}{x^m}$

Term independent of x is coefficient of x^m in the expansion of $(1+x)^{2m} = {}^{2m}C_m = \frac{(2m)!}{(m!)^2}$

Coefficient of x^6 in the expansion of $(1+x)^n$ is ${}^nC_6$

64. (a) Given that, $(1 + ax)^n = 1 + 8x + 24x^2 + \ldots$

$\Rightarrow 1 + \frac{n}{1}ax + \frac{n(n-1)}{1.2}a^2x^2 + \ldots = 1 + 8x + 24x^2 + \ldots$

On comparing the coefficient of x, x^2, we get

$na = 8, \frac{n(n-1)}{2}a^2 = 24$

$\Rightarrow na\,(na - a) = 48 \Rightarrow 8(8 - a) = 48$

$\Rightarrow 8 - a = 6 \Rightarrow a = 2 \therefore n \times 2 = 8 \Rightarrow n = 4$

65. (a) Let $b = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{{}^nC_r}$

$= n\sum_{r=0}^{n} \frac{1}{{}^nC_r} - \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$

$= na_n - \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}}$ $\left(\because {}^nC_r = {}^nC_{n-r}\right)$

$= n\,a_n - b$

$\therefore 2b = na_n \Rightarrow b = \frac{n}{2}a_n$

66. (b) We have, $\left(1 + \frac{C_1}{C_0}\right)\left(1 + \frac{C_2}{C_1}\right)\ldots\left(1 + \frac{C_n}{C_{n-1}}\right)$

$= \left(1 + \frac{n}{1}\right)\left[1 + \frac{\frac{n(n-1)}{2!}}{n}\right]\ldots\left(1 + \frac{1}{n}\right)$

$= \frac{(1+n)}{1}.\frac{(1+n)}{2}.\frac{(1+n)}{3}\ldots\frac{(1+n)}{n} = \frac{(1+n)^n}{n!}$

67. (a) There are (n + 1) terms in the expansion of $(x + a)^n$. Observing the terms, we can say that the first term from the end is the last term, i.e., $(n + 1)^{th}$ term of the expansion and n + 1 = (n + 1) – (1 – 1). The second term from the end is the n^{th} term of the expansion and n = (n + 1) – (2 – 1).

The third term from the end is the $(n - 1)^{th}$ term of the expansion and n – 1 = (n + 1) – (3 – 1), and so on. Thus, r^{th} term from the end will be term number (n + 1) – (r – 1) = (n – r + 2) of the expansion and the $(n - r + 2)^{th}$ term is ${}^nC_{n-r+1}x^{r-1}a^{n-r+1}$.

68. (d) In the expansion of $(x + 2y)^8$, the middle term is $\left(\frac{8}{2} + 1\right)^{th}$ i.e., 5th term.

69. (a) In the binomial expression, we have

$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots + {}^nC_nb^n$ ...(i)

The coefficients ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ are known as binomial or combinatorial coefficients.

Putting a = b = 1 in (i), we get

${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

Thus, the sum of all binomial coefficients is equal to 2^n.

Again, putting a = 1 and b = – 1 in Eq. (i), we get

${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots$

Thus, the sum of all the odd binomial coefficients is equal to the sum of all the even binomial coefficients and each is equal to $\frac{2^n}{2} = 2^{n-1}$.

$\Rightarrow {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$

70. (a) Both Assertion and Reason are correct. Also, Reason is the correct explanation for the Assertion.

71. (a) Both are correct and Reason is the correct explanation.

72. (a) **Assertion:** $(x + 2y)^9$

n = 9, a = 2y

$\therefore\ T_{r+1} = {}^9C_r x^{9-r}.(2y)^r$

$= {}^9C_r.2^r.x^{9-r}.y^r$

73. **(c)** Assertion is correct. Reason is false.

Total number of terms $= \left(\frac{n}{2}+1\right) = 5+1 = 6$

74. **(d)** Assertion is false and Reason is true.

CRITICALTHINKING TYPE QUESTIONS

75. **(c)** Given expression is :
$[(3x+y)^5]^4 - [(3x-y)^4]^5 = [(3x+y)]^{20} - [(3x-y)]^{20}$
First and second expansion will have 21 terms each but odd terms in second expansion be Ist, 3rd, 5th,.....21st will be equal and opposite to those of first expansion.
Thus, the number of terms in the expansion of above expression is 10.

76. **(d)** $T_{r+1} = {}^{18}C_r (9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

$= (-1)^r \, {}^{18}C_r \, 9^{18-\frac{3r}{2}} \, x^{18-\frac{3r}{2}}$
is independent of x provided $r = 12$ and then $a = 1$.

77. **(c)** $(1-x)^2(1+x)^{-2} = (1-2x+x^2)(1-2x+3x^2+..........)$
The term independent of x is 1.

78. **(c)**

79. **(a)** Given expansion is $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$

$(r+1)_{th}$ term, $T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r}\left(\frac{k}{x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^{10}C_r x^{5-r/2}.(k)^r.x^{-2r}$

$\therefore T_{r+1} = {}^{10}C_r x^{(10-5r)/2}(k)^r$

Since, T_{r+1} is independent of x

$\therefore \frac{10-5r}{2} = 0 \Rightarrow r = 2$

$\therefore 405 = {}^{10}C_2 (k)^2$
$405 = 45 \times k^2$
$\Rightarrow k^2 = 9 \Rightarrow k = \pm 3$

80. **(a)** We have
$7^9 + 9^7 = (8-1)^9 + (8+1)^7 = (1+8)^7 - (1-8)^9$

$= [1 + {}^7C_1 8 + {}^7C_2 8^2 + ... + {}^7C_7 8^7]$

$- [1 - {}^9C_1 8 + {}^9C_2 8^2 - ..... - {}^9C_9 8^9]$

$= {}^7C_1 8 + {}^9C_1 8 + [{}^7C_2 + {}^7C_3.8 + ... - {}^9C_2 + {}^9C_3.8 - ...]8^2$

$= 8(7+9) + 64k = 8..16 + 64k = 64q$,
where $q = k+2$
Thus, $7^9 + 9^7$ is divisible by 64.

81. **(d)** $T_{r+1} = \frac{n(n-1)(n-2).........(n-r+1)}{r!}(x)^r$

For first negative term,

$n - r + 1 < 0 \Rightarrow r > n+1$

$\Rightarrow r > \frac{32}{5} \therefore r = 7. \left(\because n = \frac{27}{5}\right)$

Therefore, first negative term is T_8.

82. **(c)** $\left(1+\frac{1}{x^2}\right)^n (1+x^2)^n = \frac{(1+x^2)^{2n}}{x^{2n}}$,
numerator has $(2n+1)$ terms.

$\therefore$ The middle terms is $\frac{1}{x^{2n}}\left[{}^{(2n)}C_n (x^2)^n\right] = {}^{(2n)}C_n$.

83. **(d)** ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

$= {}^{50}C_4 + \left[{}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3\right]$

We know $\left[{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

Proceeding in the same way, we get

${}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$.

84. **(b)** Binomial expansion of
$(1+x)^{50} = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots + C_{50}x^{50}$
and in given expression
Putting x = 1, we get
$2^{50} = C_0 + C_1 + C_2 + C_3 \ldots + C_{50}$ ... (i)
and putting x = – 1
$0 = C_0 - C_1 + C_2 - C_3 \ldots.. + C_{50}$ ... (ii)
Subtracting (ii) from (i), we get
$2^{50} = 2(C_1 + C_3 + C_5 + \ldots C_{49})$

$\Rightarrow C_1 + C_3 + C_5 + \ldots C_{49} = \frac{2^{50}}{2} = 2^{49}$

Sum of the coefficient of odd powers of x = 2^{49}

85. **(a)** $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= 2[x^5 + {}^5C_2 x^3(x^3-1) + {}^5C_4 x(x^3-1)^2]$
$= 2[x^5 + 10x^3(x^3-1) + 5x(x^6 - 2x^3 + 1]$
$= 10x^7 + 20x^6 + 2x^5 - 20x^4 - 20x^3 + 10x$
$\therefore$ polynomial has degree 7.

86. **(a)** $\frac{r.{}^nC_r}{{}^nC_{r-1}} = \frac{n.{}^{n-1}C_{r-1}}{{}^nC_{r-1}}$

$= n.\frac{(n-1)!}{(r-1)!(n-r)!} \times \frac{(r-1)!(n-r+1)!}{n!}$

$= n - r + 1$

Sum = n + (n – 1) + ... + (n – 9) = 10 n – 45

87. **(d)** $a_0 + a_1 + a_2 + ... = 2^{2n}$ and $a_0 + a_2 + a_4 + ... = 2^{2n-1}$
$a_n = {}^{2n}C_n$ = the greatest coefficient, being the middle coefficient
$a_{n-3} = {}^{2n}C_{n-3} = {}^{2n}C_{2n-(n-3)} = {}^{2n}C_{n+3} = a_{n+3}$

88. (a) $na = 8 \Rightarrow n^2a^2 = 64, \frac{n(n-1)}{2}a^2 = 24$

since $\frac{2n}{n-1} = \frac{8}{3} \Rightarrow 6n = 8n - 8$

$\Rightarrow n = 4, a = 2$

89. (b) Coeff. of x^n in $(1+x)(1-x)^n$ = coeff. of x^n in

$(1+x)(1-{}^nC_1x+{}^nC_2x^2-....+(-1)^n\,{}^nC_nx^n)$

$= (-1)^n\,{}^nC_n + (-1)^{n-1}\,{}^nC_{n-1} = (-1)^n + (-1)^{n-1}.n$

$= (-1)^n(1-n)$

90. (d) We know that, $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + ...\ {}^{20}C_{10}x^{10} + ...\ {}^{20}C_{20}x^{20}$

Put $x = -1$, $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} - {}^{20}C_{11} .... + {}^{20}C_{20}$

$\Rightarrow 0 = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9] + {}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10}]$

$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$

91. (b) We know by Binomial expansion, that $(x+a)^n$

$= {}^nC_0x^na^0 + {}^nC_1x^{n-1}.a + {}^nC_2x^{n-2}a^2 + {}^nC_3x^{n-3}a^3. + {}^nC_4x^{n-4}.a^4 + ... + {}^nC_nx^0a^n$

Given expansion is $\left(x^4 - \frac{1}{x^3}\right)^{15}$

On comparing we get $n = 15, x = x^4, a = \left(-\frac{1}{x^3}\right)$

$\therefore$ We have

$\left(x^4 - \frac{1}{x^3}\right)^{15} = {}^{15}C_0(x^4)^{15}\left(-\frac{1}{x^3}\right)^0 + {}^{15}C_1(x^4)^{14}\left(-\frac{1}{x^3}\right) + {}^{15}C_2(x^4)^{13}\left(-\frac{1}{x^3}\right)^2 + {}^{15}C_3(x^4)^{12}\left(-\frac{1}{x^3}\right)^3 + {}^{15}C_4(x^4)^{11}\left(-\frac{1}{x^3}\right)^4 + ...$

$T_{r+1} = {}^{15}C_r(x^4)^{15-r}.\left(-\frac{1}{x^3}\right)^r = -{}^{15}C_r\,x^{60-7r}$

$\Rightarrow x^{60-7r} = x^{32} \Rightarrow 60 - 7r = 32$

$\Rightarrow 7r = 28 \Rightarrow r = 4$

So, 5th term, contains x^{32}

$= {}^{15}C_4(x^4)^{11}\left(-\frac{1}{x^3}\right)^4 = {}^{15}C_4x^{44}x^{-12} = {}^{15}C_4x^{32}.$

Thus, coefficient of $x^{32} = {}^{15}C_4$.

92. (c) $\because$ x^3 and higher powers of x may be neglected

$\therefore \frac{(1+x)^{\frac{3}{2}} - \left(1+\frac{x}{2}\right)^3}{(1-x)^{\frac{1}{2}}}$

$= (1-x)^{\frac{-1}{2}}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}.\frac{1}{2}}{2!}x^2\right) - \left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$

$= \left[1+\frac{x}{2}+\frac{\frac{1}{2}.\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right] = \frac{-3}{8}x^2$

(as x^3 and higher powers of x can be neglected)

CHAPTER 9

SEQUENCES AND SERIES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Let a_1, a_2, a_3, ...be the sequence, then the sum expressed as $a_1 + a_2 + a_3 + ... + a_n$ is called ....
(a) Sequence (b) Series
(c) Finite (d) Infinite

2. The third term of a geometric progression is 4. The product of the first five terms is :
(a) 4^3 (b) 4^5 (c) 4^4 (d) 4^7

3. In an AP. the pth term is q and the $(p + q)$th term is 0. Then the qth term is
(a) $-p$ (b) p (c) $p+q$ (d) $p-q$

4. If a, b, c, d, e, f are in A.P., then $e - c$ is equal to:
(a) $2(c-a)$ (b) $2(d-c)$ (c) $2(f-d)$ (d) $(d-c)$

5. The fourth, seventh and tenth terms of a G.P. are p, q, r respectively, then :
(a) $p^2 = q^2 + r^2$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $pqr + pq + 1 = 0$

6. If 1, a and P are in A. P. and 1, g and P are in G. P., then
(a) $1+2a+g^2=0$ (b) $1+2a-g^2=0$
(c) $1-2a-g^2=0$ (d) $1-2a+g^2=0$

7. For a, b, c to be in G.P. What should be the value of $\frac{a-b}{b-c}$?
(a) ab (b) bc
(c) $\frac{a}{b}$ or $\frac{b}{c}$ (d) None of these

8. What is the sum of terms equidistant from the beginning and end in an A.P. ?
(a) First term – Last term (b) First term × Last term
(c) First term + Last term (d) First term ÷ Last term

9. The first and eight terms of a G.P. are x^{-4} and x^{52} respectively. If the second term is x^t, then t is equal to:
(a) –13 (b) 4 (c) $\frac{5}{2}$ (d) 3

10. If the p^{th}, q^{th} and r^{th} terms of a G.P. are again in G.P., then which one of the following is correct?
(a) p, q, r are in A.P.
(b) p, q, r are in G.P.
(c) p, q, r are in H.P.
(d) p, q, r are neither in A.P. nor in G.P. nor in H.P.

11. If $5(3^{a-1}+1)$, $(6^{2a-3}+2)$ and $7(5^{a-2}+5)$ are in AP, then what is the value of a?
(a) 7 (b) 6
(c) 5 (d) None of these

12. If p^{th} term of an AP is q, and its q^{th} term is p, then what is the common difference ?
(a) – 1 (b) 0 (c) 2 (d) 1

13. If a, b, c are in geometric progression and a, 2b, 3c are in arithmetic progression, then what is the common ratio r such that $0 < r < 1$?
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

14. If 1, x, y, z, 16 are in geometric progression, then what is the value of x + y + z ?
(a) 8 (b) 12 (c) 14 (d) 16

15. The product of first nine terms of a GP is, in general, equal to which one of the following?
(a) The 9th power of the 4th term
(b) The 4th power of the 9th term
(c) The 5th power of the 9th term
(d) The 9th power of the 5th term

16. In a G.P. if $(m+n)^{th}$ term is p and $(m-n)^{th}$ term is q, then m^{th} term is:
(a) $\frac{p}{q}$ (b) $\frac{q}{p}$ (c) pq (d) $\sqrt{pq}$

17. The following consecutive terms
$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ of a series are in:
(a) H.P. (b) G.P.
(c) A.P. (d) A.P., G.P.

18. The series $(\sqrt{2}+1), 1, (\sqrt{2}-1)$ ... is in :
(a) A.P. (b) G.P.
(c) H.P. (d) None of these

19. Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of the G.P. is:
(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $\sqrt{3}-2$ (d) $3+\sqrt{2}$

20. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61......., then n is equal to
(a) 10 (b) 12 (c) 11 (d) 13

21. There are four arithmetic means between 2 and –18. The means are
(a) –4, –7, –10, –13 (b) 1, –4, –7, –10
(c) –2, –5, –9, –13 (d) –2, –6, –10, –14

22. The arithmetic mean of three observations is x. If the values of two observations are y, z; then what is the value of the third observation ?
(a) x (b) $2x - y - z$
(c) $3x - y - z$ (d) $y + z - x$

23. What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...?$
(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

24. $\frac{1}{q+r}, \frac{1}{r+p}, \frac{1}{p+q}$ are in A.P. then,
(a) p, q, r are in A.P (b) p^2, q^2, r^2 are in A.P
(c) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in A.P (d) $p + q + r$ are in A.P

25. If G be the geometric mean of x and y, then $\frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} =$
(a) G^2 (b) $\frac{1}{G^2}$ (c) $\frac{2}{G^2}$ (d) $3G^2$

26. In a Geometric Progression with first term a and common ratio r, what is the Arithmetic Mean of the first five terms?
(a) a + 2r (b) a r^2
(c) a $(r^5 - 1)/[5(r - 1)]$ (d) a $(r^4 - 1)/[5(r - 1)]$

27. If p, q, r are in A.P., a is G.M. between p & q and b is G.M. between q and r, then a^2, q^2, b^2 are in
(a) G.P. (b) A.P.
(c) H.P (d) None of these

28. Sum of n terms of series 1.3+3.5+5.7+.......... is
(a) $\frac{1}{3}n(n+1)(2n+1) - n$ (b) $\frac{3}{2}n(n+1)(2n+1) - n$
(c) $\frac{4}{5}n(n+1)(2n+1) - n$ (d) $\frac{2}{3}n(n+1)(2n+1) - n$

29. Let a_1, a_2, a_3........... be terms of an A.P. If $\frac{a_1 + a_2 + ...a_p}{a_1 + a_2 + ...+ a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals
(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

30. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression equals
(a) $\sqrt{5}$ (b) $\frac{1}{2}\left(\sqrt{5} - 1\right)$
(c) $\frac{1}{2}\left(1 - \sqrt{5}\right)$ (d) $\frac{1}{2}\sqrt{5}$.

31. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
(a) –4 (b) –12 (c) 12 (d) 4

32. The harmonic mean of $\frac{a}{1 - ab}$ and $\frac{a}{1 + ab}$ is :
(a) a (b) $\frac{a}{1 - a^2b^2}$
(c) $\frac{1}{1 - a^2b^2}$ (d) $\frac{a}{1 + a^2b^2}$

33. If arithmetic mean of a and b is $\frac{(a^{n+1} + b^{n+1})}{a^n + b^n}$, then the value of n is equal to
(a) –1 (b) 0 (c) 1 (d) 2

34. The H. M between roots of the equation $x^2 - 10x + 11 = 0$ is equal to :
(a) $\frac{1}{5}$ (b) $\frac{5}{21}$ (c) $\frac{21}{20}$ (d) $\frac{11}{5}$

35. If m arithmetic means are inserted between 1 and 31 so that the ratio of the 7^{th} and $(m - 1)^{th}$ means 5 : 9, then the value of m is
(a) 10 (b) 11 (c) 12 (d) 14

36. Let S_n denote the sum of first n terms of an A.P. If $S_{2n} = 3\ S_n$, then the ratio $S_{3n}/\ S_n$ is equal to :
(a) 4 (b) 6 (c) 8 (d) 10

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

37. Consider the following statements
I. If $a_1, a_2, ..., a_n$ ... is a sequence, then the expression $a_1 + a_2 + ... + a_n + ...$ is called a series.
II. Those sequences whose terms follow certain patterns are called progressions.
Choose the correct option.
(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

38. Consider the following statements.
I. A sequence is called an arithmetic progression if the difference of a term and the previous term is always same.
II. Arithmetic Mean (A.M.) A of any two numbers a and b is given by $\frac{1}{2}(a + b)$ such that a, A, b are in A.P.
The arithmetic mean for any n positive numbers a_1, $a_2, a_3, ..., a_n$ is given by
$$\text{A.M.} = \frac{a_1 + a_2 + ... + a_n}{n}$$
Choose the correct option.
(a) Only I is true (b) Both are true
(c) Only II is true (d) Both are false

39. **Statement I:** Three numbers a, b, c are in A.P., then b is called the arithmetic mean of a and c.
Statement II: Three numbers a, b, c are in A.P. iff $2b = a + c$.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

40. **Statement I:** If 'a' is the first term and 'd' is the common difference of an A.P., then its n^{th} term is given by
$a_n = a - (n - 1)d$
Statement II: The sum S_n of n terms of an A.P. with first term 'a' and common difference 'd' is given by $S_n = \frac{n}{2}\{2a + (n - 1)d\}$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

41. Consider the following statements.
I. The n^{th} term of a G.P. with first term 'a' and common ratio 'r' is given by $a_n = a.r^{n-1}$.
II. Geometric mean of a and b is given by $(ab)^{1/3}$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

42. I. Three numbers a, b, c are in G.P. iff $b^2 = ac$
II. The reciprocals of the terms of a given G.P. form a G.P.
III. If $a_1, a_2, ..., a_n, ...$ is a G.P., then the expression $a_1 + a_2 + ... + a_n + ...$ is called a geometric series.
Choose the correct option.
(a) Only I and II are true
(b) Only II and III are true
(c) All are true
(d) Only I and III are true

43. I. If each term of a G.P. be raised to the same power, the resulting sequence also forms a G.P.
II. 25^{th} term of the sequence 4, 9, 14, 19, ... is 124.
Choose the correct option.
(a) Both are true (b) Both are false
(c) Only I is true (d) Only II is true

44. I. 18^{th} term of the sequence 72, 70, 68, 66, ... is 40.
II. 4^{th} term of the sequence $8 - 6i, 7 - 4i, 6 - 2i, ...$ is purely real.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

45. I. 37 terms are there in the sequence 3, 6, 9, 12, ..., 111.
II. General term of the sequence 9, 12, 15, 18, ... is $3n + 8$.
Choose the correct option.
(a) Only I is true (b) Only II is true.
(c) Both are true (d) Both are false

46. I. 11^{th} terms of the G.P. 5, 10, 20, 40, ... is 5120
II. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtained quadratic equation is $x^2 - 16x + 25 = 0$
Choose the correct option.
(a) Only I is true (b) Only II is true.
(c) Both are true (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

47.

Column - I	Column - II
A. Sum of 20 terms of the A.P. 1, 4, 7, 10, ... is	1. 70336
B. Sum of the series 5+13+21+...+181 is	2. 156375
C. The sum of all three digit natural numbers, which are divisible by 7, is	3. 2139
D. The sum of all natural numbers between 250 and 1000 which are exactly divisible by 3, is	4. 590

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	3	1	4
(d)	2	1	3	4

48.

Column - I	Column - II
A. Sum of 7 terms of the G.P. 3, 6, 12, ... is	1. $\frac{10}{9}[10^n - 1] + n^2$
B. Sum of 10 terms of the G.P. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$ is	2. $\frac{1023}{512}$
C. Sum of the series $2 + 6 + 18 + ... + 4374$ is	3. 381
D. Sum to n terms of the series $11 + 103 + 1005 + ...$ is	4. 6560

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	1	4	2	3
(c)	3	4	2	1
(d)	3	2	4	1

49.

Column - I	Column - II
A. Sum to infinity of the G.P. $\frac{-5}{4}, \frac{5}{16}, \frac{-5}{64}, ...$ is	1. $\frac{2}{3}$
B. Value of $6^{1/2}.6^{1/4}.6^{1/8}$ .... ∞ is	2. $\frac{1}{3}$
C. If the first term of a G.P. is 2 and the sum to infinity is 6 then the common ratio is	3. -1
D. If each term of an infinite G.P. is twice the sum of the terms following it, then the common ratio of the G.P. is	4. 6

Codes

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	4	1	2
(d)	3	1	4	2

50. If the sequence is defined by $a_n = n(n + 2)$, then match the columns.

Column - I	Column - II
A. $a_1 =$	1. 35
B. $a_2 =$	2. 24
C. $a_3 =$	3. 8
D. $a_4 =$	4. 3
E. $a_5 =$	5. 15

Codes

	A	B	C	D	E
(a)	4	3	5	2	1
(b)	4	2	5	3	1
(c)	1	3	2	5	4
(d)	3	4	5	1	2

51. If the n^{th} term of the sequence is defined as $a_n = \frac{2n-3}{6}$, then match the columns.

Column - I	Column - II
A. $a_1 =$	1. 1/6
B. $a_2 =$	2. 1/2
C. $a_3 =$	3. 5/6
D. $a_4 =$	4. –1/6
E. $a_5 =$	5. 7/6

Codes

	A	B	C	D	E
(a)	4	1	3	2	5
(b)	5	3	2	1	4
(c)	4	3	3	1	5
(d)	4	1	2	3	5

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

52. If the j^{th} term and k^{th} term of an A.P. are k and j respectively, the (k + j) th term is
(a) 0 (b) 1
(c) k + j + 1 (d) k + j –1

53. Third term of the sequence whose n^{th} term is $a_n = 2^n$, is
(a) 2 (b) 4 (c) 8 (d) 3

54. The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}$, n > 2. Then value of $\frac{a_{n+1}}{a_n}$ for n = 2, is
(a) 1 (b) 2 (c) 3 (d) 4

55. If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116. then the last term is
(a) 0 (b) 2 (c) 4 (d) 6

56. If the sum of first p terms of an A.P. is equal to the sum of the first q terms then the sum of the first $(p + q)$ terms, is
(a) 0 (b) 1 (c) 2 (d) 3

57. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then the value of n is
(a) 1 (b) 2 (c) 3 (d) 4

58. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°. The number of the sides of the polygon is
(a) 6 (b) 9 (c) 8 (d) 5

59. Which term of the following sequence $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$.......is $\frac{1}{19683}$?
(a) 3 (b) 9
(c) 6 (d) None of these

60. How many terms of G.P. 3, 3^2, 3^3, ... are needed to give the sum 120?
(a) 3 (b) 4 (c) 5 (d) 6

61. If f is a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in N$. such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, find the value of n.
(a) 2 (b) 4 (c) 6 (d) 8

62. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then the common ratio is
(a) 5 (b) 1 (c) 4 (d) 3

63. How many terms of the geometric series 1 + 4 + 16 + 64 + ... will make the sum 5461?
(a) 3 (b) 4 (c) 5 (d) 7

64. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n,\ m \neq n,\ T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals
(a) $\frac{1}{m} + \frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

65. **Assertion:** For x = ± 1, the numbers $\frac{-2}{7}$, x, $\frac{-7}{2}$ are in G.P.
Reason: Three numbers a, b, c are in G.P. if $b^2 = ac$.

66. **Assertion:** Sum to n terms of the geometric progression $x^3, x^5, x^7, ... (x \neq \pm 1)$ is $\dfrac{x^3(1-x^{2n})}{(1-x^2)}$.

Reason: If 'a' is the first term and r is common ratio of a G.P. then sum to n terms is given as

$S_n = \dfrac{a(r^n-1)}{r-1}$ or $= \dfrac{a(1-r^n)}{1-r}$ if $r \neq 1$.

67. **Assertion:** Value of a_{17}, whose n^{th} term is $a_n = 4n - 3$, is 65.
Reason: Value of a_9, whose n^{th} term is $a_n = (-1)^{n-1}.n^3$.

68. **Assertion:** If each term of a G.. P. is multiplied or divided by some fixed non-zero number, the resulting sequence is also a G P.
Reason: If $-1 < r < 1$, i.e. $|r| < 1$, then the sum of the infinite G. P., $a + ar + ar^2 + ...= \dfrac{a}{1-r}$

i.e., $S_\infty = \dfrac{a}{1-r}$

69. **Assertion:** If the third term of a G.P. is 4, then the product of its first five terms is 4^5.
Reason: Product of first five terms of a G.P. is given as $a\,(ar)\,(ar^2)\,(ar^3)\,(ar^4)$

70. **Assertion:** If a, b, c are in A.P., then b + c, c + a, a + b are in A.P.
Reason: If a, b, c are in A.P., then $10^a, 10^b, 10^c$ are in G.P.

71. **Assertion:** If $\dfrac{2}{3}, k, \dfrac{5}{8}$ are in A.P, then the value of k is $\dfrac{31}{48}$.
Reason: Three numbers a, b, c are in A.P. iff $2b = a + c$

72. **Assertion:** If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, then the value of m is 27.

Reason: 20^{th} term of the G.P. $\dfrac{5}{2}, \dfrac{5}{4}, \dfrac{5}{8}, ...$ is $\dfrac{5}{2^{20}}$

73. **Assertion:** The 20^{th} term of the series $2 \times 4 + 4 \times 6 + 6 \times 8 + ... +$ n terms is 1680.
Reason: If the sum of three numbers in A.P. is 24 and their product is 440. Then the numbers are 5, 8, 11 or 11, 8, 5.

74. **Assertion:** Sum of n terms of the A.P., whose k^{th} term is $5k + 1$, is $\dfrac{n(5n+7)}{2}$.
Reason: Sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 980.

75. **Assertion:** : The sum of n terms of two arithmetic progressions are in the ratio (7n + 1) : (4n + 17), then the ratio of their n^{th} terms is 7:4.
Reason: If $S_n = ax^2 + bx + c$, then $T_n = S_n - S_{n-1}$

76. Let sum of n terms of a series $S_n = 6n^2 + 3n + 1$.
Assertion: The series S_n is in A.P.
Reason: Sum of n terms of an A.P. is always of the form $an^2 + bn$.

77. **Assertion:** The arithmetic mean (A.M.) between two numbers is 34 and their geometric mean is 16.
The numbers are 4 and 64.

Reason: For two numbers a and b, A.M. = $A = \dfrac{a+b}{2}$

G.M = G = $\sqrt{ab}$.

78. **Assertion:** The ratio of sum of m terms to the sum of n terms of an A.P is $m^2 : n^2$. If T_k is the k^{th} term, then $T_5/T_2 = 3$.
Reason: For n^{th} term, $t_n = a + (n - 1)d$, where 'a' is first term and 'd' is common difference.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

79. Consider an infinite geometric series with first term *a* and common ratio *r*. If its sum is 4 and the second term is $\dfrac{3}{4}$, then :

(a) $a = \dfrac{4}{7}, r = \dfrac{3}{7}$ (b) $a = 2, r = \dfrac{3}{8}$

(c) $a = \dfrac{3}{2}, r = \dfrac{1}{2}$ (d) $a = 3, r = \dfrac{1}{4}$

80. If roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then its common difference is
(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

81. 4^{th} term from the end of the G.P. 3, 6, 12, 24., ..., 3072 is
(a) 348 (b) 843
(c) 438 (d) 384

82. If $a^x = b^y = c^z$, where a, b, c are in G.P. and $a, b, c, x, y, z \neq 0$; then $\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}$ are in:
(a) A.P. (b) G.P. (c) H.P (d) None of these

83. The value of $3 - 1 + \dfrac{1}{3} - \dfrac{1}{9} + ...$ is equal to:

(a) $\dfrac{20}{9}$ (b) $\dfrac{9}{20}$ (c) $\dfrac{9}{4}$ (d) $\dfrac{4}{9}$

84. $5^{1+x} + 5^{1-x}, \dfrac{a}{2}, 5^{2x} + 5^{-2x}$ are in A.P., then the value of a is:
(a) $a < 12$ (b) $a \leq 12$
(c) $a \geq 12$ (d) None of these

85. The product of *n* positive numbers is unity, then their sum is :
(a) a positive integer (b) divisible by *n*
(c) equal to $n + \dfrac{1}{n}$ (d) never less than *n*

86. An infinite G.P has first term *x* and sum 5, then
(a) $x < -10$ (b) $-10 < x < 0$
(c) $0 < x < 10$ (d) $x > 10$

87. Sum of the first *n* terms of the series

$\dfrac{1}{2} + \dfrac{3}{4} + \dfrac{7}{8} + \dfrac{15}{16} + ...$ is equal to :

(a) $2^n - n - 1$ (b) $1 - 2^{-n}$
(c) $n + 2^{-n} - 1$ (d) $2^n + 1$

88. In a G.P. of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the G.P. is

(a) $\frac{-4}{5}$ (b) $\frac{1}{5}$
(c) 4 (d) None of these

89. The sum of 11 terms of an A.P. whose middle term is 30,

(a) 320 (b) 330 (c) 340 (d) 350

90. The first term of an infinite G.P. is 1 and each term is twice the sum of the succeeding terms. then the sum of the series is

(a) 2 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

91. There are four numbers of which the first three are in G.P. and the last three are in A.P., whose common difference is 6. If the first and the last numbers are equal then two other numbers are

(a) –2, 4 (b) –4, 2
(c) 2, 6 (d) None of these

92. If in a series $S_n = an^2 + bn + c$, where S_n denotes the sum of n terms, then

(a) The series is always arithmetic
(b) The series is arithmetic from the second term onwards
(c) The series may or may not be arithmetic
(d) The series cannot be arithmetic

93. If the sum of the first ten terms of an arithmetic progression is four times the sum of the first five terms, then the ratio of the first term to the common difference is :

(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

94. If the nth term of an arithmetic progression is 3n + 7, then what is the sum of its first 50 terms?

(a) 3925 (b) 4100
(c) 4175 (d) 8200

95. Let x be one A.M and g_1 and g_2 be two G.Ms between y and z. What is $g_1^3 + g_2^3$ equal to ?

(a) xyz (b) xy^2z
(c) xyz^2 (d) 2xyz

96. What is the sum of the first 50 terms of the series $(1 \times 3) + (3 \times 5) + (5 \times 7) + ...$?

(a) 1,71,650 (b) 26,600
(c) 26,650 (d) 26,900

97. The A.M. of the series 1, 2, 4, 8, 16, ..., 2^n is :

(a) $\frac{2^n - 1}{n}$ (b) $\frac{2^{n+1} - 1}{n+1}$
(c) $\frac{2^n + 1}{n}$ (d) $\frac{2^n - 1}{n+1}$

98. The 10 th common term between the series $3 + 7 + 11 + ...$ and $1 + 6 + 11 + ...$ is

(a) 191 (b) 193 (c) 211 (d) None of these

99. A man saves ` 135/- in the first year, ` 150/- in the second year and in this way he increases his savings by ` 15/- every year. In what time will his total savings be ` 5550/-?

(a) 20 years (b) 25 years
(c) 30 years (d) 35 years

100. Let a, b, c, be in A.P. with a common difference d. Then $e^{1/c}, e^{b/ac}, e^{1/a}$ are in :

(a) G.P. with common ratio e^d
(b) G.P with common ratio $e^{1/d}$
(c) G.P. with common ratio $e^{d/(b^2-d^2)}$
(d) A.P.

101. If $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P. then $9^{ax+1}, 9^{bx+1}, 9^{cx+1}, x \neq 0$ are in :

(a) G.P. (b) G.P. only if $x < 0$
(c) G.P. only if $x > 0$ (d) None of these

102. The value of x + y + z is 15 if a, x, y, z, b are in A.P. while the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is $\frac{5}{3}$ if a, x, y, z, b are in H.P. Then the value of a and b are

(a) 2 and 8 (b) 1 and 9
(c) 3 and 7 (d) None of these

103. The A. M. between two positive numbers a and b is twice the G. M. between them. The ratio of the numbers is

(a) $(\sqrt{2}+3) : (\sqrt{2}-3)$
(b) $(2+\sqrt{3}) : (2-\sqrt{3})$
(c) $(\sqrt{3}+1) : (\sqrt{3}-1)$
(d) None of these

104. If S_n denotes the sum of n terms of a G.P. whose first term is a and the common ratio r, then value of $S_1 + S_3 + S_5 + ...+ S_{2n-1}$ is

(a) $\frac{a}{1+r}\left[n + r.\frac{1-r^{2n}}{1-r^2}\right]$ (b) $\frac{2a}{1+r}\left[n + r.\frac{1-r^{2n}}{1+r^2}\right]$
(c) $\frac{a}{1+r}\left[n - r.\frac{1-r^{2n}}{1-r^2}\right]$ (d) $\frac{a}{1-r}\left[n - r.\frac{1-r^{2n}}{1-r^2}\right]$

105. If S_1, S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms respectively of an A.P., then value of

$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2)$ is

(a) $\frac{1}{2}$ (b) 0 (c) $-\frac{1}{2}$ (d) $\frac{3}{2}$

106. Find the sum up to 16 terms of the series

$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + ...$

(a) 448 (b) 445 (c) 446 (d) None of these

107. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$
(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

108. If sum of the infinite G.P. is $\frac{4}{3}$ and its first term is $\frac{3}{4}$ then its common ratio is :

(a) $\frac{7}{16}$ (b) $\frac{9}{16}$ (c) $\frac{1}{9}$ (d) $\frac{7}{9}$

109. If sixth term of a H. P. is $\frac{1}{61}$ and its tenth term is $\frac{1}{105}$, then the first term of that H.P. is

(a) $\frac{1}{28}$ (b) $\frac{1}{39}$ (c) $\frac{1}{6}$ (d) $\frac{1}{17}$

110. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are the p^{th}, q^{th}, r^{th} terms respectively of an A.P. then the value of $ab(p-q) + bc(q-r) + ca(r-p)$ is

(a) –1 (b) 2 (c) 0 (d) –2

111. If the sum of an infinitely decreasing GP is 3, and the sum of the squares of its terms is 9/2, then sum of the cubes of the terms is

(a) $\frac{107}{12}$ (b) $\frac{105}{17}$ (c) $\frac{108}{13}$ (d) $\frac{97}{12}$

112. If x,y,z are in G.P. and $a^x = b^y = c^z$, then

(a) $\log_b a = \log_a c$ (b) $\log_c b = \log_a c$

(c) $\log_b a = \log_c b$ (d) None of these

113. The sum to infinite term of the series

$1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$ is

(a) 3 (b) 4 (c) 6 (d) 2

114. The fifth term of the H.P., $2, 2\frac{1}{2}, 3\frac{1}{3}$, ... will be

(a) $5\frac{1}{5}$ (b) $3\frac{1}{5}$

(c) $\frac{1}{10}$ (d) 10

115. If the 7^{th} term of a H.P. is $\frac{1}{10}$ and the 12^{th} term is $\frac{1}{25}$, then the 20^{th} term is

(a) $\frac{1}{37}$ (b) $\frac{1}{41}$

(c) $\frac{1}{45}$ (d) $\frac{1}{49}$

116. The harmonic mean of $\frac{a}{1-ab}$ and $\frac{a}{1+ab}$ is

(a) $\frac{a}{\sqrt{1-a^2b^2}}$ (b) $\frac{a}{1-a^2b^2}$

(c) a (d) $\frac{1}{1-a^2b^2}$

117. If the arithmetic, geometric and harmonic means between two distinct positive real numbers be A, G and H respectively, then the relation between them is

(a) $A > G > H$ (b) $A > G < H$

(c) $H > G > A$ (d) $G > A > H$

118. If the arithmetic, geometric and harmonic means between two positive real numbers be A, G and H, then

(a) $A^2 = GH$ (b) $H^2 = AG$

(c) $G = AH$ (d) $G^2 = AH$

119. If b^2, a^2, c^2 are in A.P., then $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ will be in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

120. If the arithmetic mean of two numbers be A and geometric mean be G, then the numbers will be

(a) $A \pm (A^2 - G^2)$

(b) $\sqrt{A} \pm \sqrt{A^2 - G^2}$

(c) $A \pm \sqrt{(A+G)(A-G)}$

(d) $\frac{A \pm \sqrt{(A+G)(A-G)}}{2}$

121. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$, then a, b, c are in

(a) A.P. (b) G.P.

(c) H.P. (d) In G.P. and H.P. both

122. If a, b, c are in A.P. and a, b, d in G.P., then a, a – b, d – c will be in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

123. If the ratio of H.M. and G.M. of two quantities is 12 : 13, then the ratio of the numbers is

(a) 1 : 2 (b) 2 : 3

(c) 3 : 4 (d) None of these

124. If the ratio of H.M. and G.M. between two numbers a and b is 4 : 5, then the ratio of the two numbers will be

(a) 1 : 2 (b) 2 : 1

(c) 4 : 1 (d) 1 : 4

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)**

2. **(b)** Here,
$t_3 = 4 \Rightarrow ar^2 = 4$
$\therefore$ Product of first five terms
$= a.\ ar.\ ar^2.\ ar^3.\ ar^4$
$= a^5 r^{10} = (ar^2)^5 = (4)^5$

3. **(b)** Let a, d be the first term and common difference respectively.
Therefore, $T_p = a + (p-1)d = q$ and ...(i)
$T_{p+q} = a + (p+q-1)d = 0$ ...(ii)
Subtracting (i), from (ii) we get $qd = -q$
Substituting in (i), we get
$a = q - (p-1)(-1) = q + p - 1$
Now $T_q = a + (q-1)d = q + p - 1 + (q-1)(-1)$
$= p + q - 1 - q + 1 = p$

4. **(b)** Let x be the common difference of the A.P.
$a, b, c, d, e, f.$
$\therefore e = a + (5-1)x \quad [\because a_n = a + (n-1)d]$
$\Rightarrow e = a + 4x$ ...(i)
and $c = a + 2x$ ...(ii)
$\therefore$ Using equations (i) and (ii), we get
$e - c = a + 4x - a - 2x$
$\Rightarrow e - c = 2x = 2(d - c).$

5. **(b)** Let a be the first term and r be common ratio.
Fourth term of G.P. : $p = T_4 = ar^3$ ...(i)
Seventh term of G.P. : $q = T_7 = ar^6$ ...(ii)
Tenth term of G.P. : $r = T_{10} = ar^9$ ...(iii)
Equ. (i) × Equ. (iii) :
$pr = ar^3 \times ar^9 \Rightarrow pr = a^2r^{12} \Rightarrow pr = (ar^6)^2 \Rightarrow pr = q^2$

6. **(d)** $2a = 1 + P$ and $g^2 = P$
$\Rightarrow g^2 = 2a - 1 \Rightarrow 1 - 2a + g^2 = 0$

7. **(c)** $\frac{a}{b}$ or $\frac{b}{c}$

8. **(c)** First term + last term

9. **(b)** Let a be the first term and r be the common ratio so, general term of G.P is $T_n = ar^{n-1}$
As given,
$T_1 = x^{-4} = a$ and, $T_8 = ar^7 = x^{52}$ $\therefore ar^7 = x^{52}$
$\Rightarrow x^{-4} r^7 = x^{52} \Rightarrow r^7 = x^{56}$
$\Rightarrow r^7 = (x^8)^7 \Rightarrow r = x^8$
$\therefore T_2 = ar^1 = x^{-4}.x^8$
$T_2 = x^4$
But $T_2 = t\,x \Rightarrow x^t = x^4 \Rightarrow t = 4$

10. **(a)** Let R be the common ratio of this GP and a be the first term. pth term is aR^{p-1}, qth term is aR^{q-1} and rth term is aR^{r-1}.
Since p, q and r are in G.P. then
$(aR^{q-1})^2 = aR^{p-1}.\ aR^{r-1}$
$\Rightarrow a^2R^{2q-2} = a^2R^{p+r-2}$
$\Rightarrow R^{2q-2} = R^{p+r-2}$
$\Rightarrow 2q - 2 = p + r - 2$
$\Rightarrow 2q = p + r \Rightarrow$ p, q, r are in A.P.

11. **(d)** None of the options a, b or c satisfy the condition.

12. **(a)** Let first term and common difference of an AP are a and d respectively.
Its p^{th} term $= a + (p-1)d = q$ ...(i)
and q^{th} term $= a + (q-1)d = p$ ...(ii)
Solving eqs. (i) and (ii), we find
$a = p + q - 1,\ d = -1$

13. **(a)** Given that a, b, c, are in GP.
Let r be common ratio of GP.
So, a = a, b = ar and $c = ar^2$
Also, given that a, 2b, 3c are in AP.
$\Rightarrow 2b = \frac{a + 3c}{2}$
$\Rightarrow 4b = a + 3c$ ... (i)
From eq. (i)
$4ar = a + 3ar^2$
$\Rightarrow 3r^2 - 4r + 1 = 0$
$\Rightarrow 3r^2 - 3r - r + 1 = 0$
$\Rightarrow 3r(r-1) - 1(r-1) = 0$
$\Rightarrow (r-1)(3r-1) = 0$
$\Rightarrow r = 1$ or $r = \frac{1}{3}$

14. **(c)** As given 1, x, y, z, 16 are in geometric progression.
Let common ratio be r,
$x = 1.\ r = r$
$y = 1.\ r^2 = r^2$
$z = 1.\ r^3 = r^3$
and $16 = 1.r^4 \Rightarrow 16 = r^4$
$\Rightarrow r = 2$
$\therefore x = 1.r = 2,\ y = 1.r^2 = 4,\ z = 1.r^3 = 8$
$\therefore x + y + z = 2 + 4 + 8 = 14$

15. **(d)** Let a be the first term and r, the common ratio
First nine terms of a GP are a, ar, ar^2, ... ar^8.
$\therefore P = a.ar.ar^2 .... ar^8$
$= a^9.r^{1+2+...+8}$
$= a^9.r^{\frac{8.9}{2}} = a^9r^{36}$
$= (ar^4)^9 = (T_5)^9$
= 9th power of the 5th term

16. **(d)** For a G. P, $a_{m+n} = p$ and $a_{m-n} = q$,
We know that $a_n = AR^{n-1}$ (in G.P.)
where A = first term and R = ratio

$\therefore a_{m+n} = p$

$\Rightarrow AR^{m+n-1} = p$ ...(i)

and $a_{m-n} = q$

$\Rightarrow AR^{m-n-1} = q$ ...(ii)

On multiply equations (i) and (ii), we have

$(AR^{m+n-1}).(AR^{m-n-1}) = pq$

$\Rightarrow A^2.R^{2(m-1)} = pq$

$\Rightarrow (AR^{m-1})^2 = pq$

$\Rightarrow AR^{m-1} = \sqrt{pq}$

$\Rightarrow a_m = \sqrt{pq}$

17. **(c)** The following consecutive terms

$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ are in A.P because

$2\left(\frac{1}{1-x}\right) = \frac{1}{1+\sqrt{x}} + \frac{1}{1-\sqrt{x}} = \frac{2}{1-x}$

(i.e. 2b = a + c)

18. **(b)** Consider series $(\sqrt{2}+1), 1, (\sqrt{2}-1)$,........

$a = \sqrt{2}+1, r = \sqrt{2}-1$

Common ratios of this series are equal. Therefore series is in G.P.

19. **(b)** In G.P., let the three numbers be $\frac{a}{r}, a, ar$

If the middle number is double, then new numbers are in A.P.

i.e., $\frac{a}{r}, 2a, ar$, are in A.P.

$\therefore \quad 2a - \frac{a}{r} = ar - 2a$

$\Rightarrow a\left[2 - \frac{1}{r}\right] = a[r-2]$

$\Rightarrow \quad 2 - \frac{1}{r} = r - 2$

$\Rightarrow \quad r + \frac{1}{r} = 4$

$\Rightarrow r^2 - 4r + 1 = 0$

$\Rightarrow \quad r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

$\because \quad r < 1$ not possible

$\therefore \quad r = 2 + \sqrt{3}$

20. **(c)** Given, $\frac{2n}{2}\{2.2 + (2n-1)3\} = \frac{n}{2}\{2.57 + (n-1)2\}$

or $2(6n+1) = 112 + 2n$ or $10n = 110$

$\therefore n = 11$

21. **(d)** Let the means be X_1, X_2, X_3, X_4 and the common difference be b; then $2, X_1, X_2, X_3, X_4, -18$ are in A.P.;

$\Rightarrow -18 = 2 + 5b \Rightarrow 5b = -20 \Rightarrow b = -4$

Hence, $X_1 = 2 + b = 2 + (-4) = -2;$

$X_2 = 2 + 2b = 2 - 8 = -6$

$X_3 = 2 + 3b = 2 - 12 = -10;$

$X_4 = 2 + 4b = 2 - 16 = -14$

The required means are –2, –6, –10, –14.

22. **(c)** We take third observation as w

So, $x = \frac{y+z+w}{3}$

$\Rightarrow \quad 3x = y + z + w$

$\Rightarrow \quad w = 3x - y - z$

23. **(d)** $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$ can be written as

$1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \ldots$

[$\because$ This is a GP with first term = 1

and common ratio $= -\frac{1}{2}$]

So, sum of the series

$= \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$

24. **(b)** $1/(q+r), 1/(r+p), 1/(p+q)$ are in A.P.

$\Rightarrow \quad \frac{1}{r+p} - \frac{1}{q+r} = \frac{1}{p+q} - \frac{1}{r+p}$

$\Rightarrow \quad q^2 - p^2 = r^2 - q^2$

$\Rightarrow \quad p^2, q^2, r^2$ are in A.P.

25. **(b)** As given $G = \sqrt{xy}$

$\therefore \frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} = \frac{1}{xy - x^2} + \frac{1}{xy - y^2}$

$= \frac{1}{x-y}\left\{-\frac{1}{x} + \frac{1}{y}\right\} = \frac{1}{xy} = \frac{1}{G^2}$

26. **(c)** First five terms of given geometric progression are a, ar, ar^2 , ar^3, ar^4

A.M. of these five terms

$= \frac{a + ar + ar^2 + ar^3 + ar^4}{5} = \frac{a(r^5 - 1)}{5(r-1)}$

27. **(b)** Since p, q, r are in A.P.

$\therefore \quad q = \frac{p+r}{2}$ ...(i)

Since a is the G.M. between p, q

$\therefore \quad a^2 = pq$ ...(ii)

Since b is the G.M. between q, r

$\therefore \quad b^2 = qr$ ...(iii)

From (ii) and (iii)

$p = \frac{a^2}{q}, \quad r = \frac{b^2}{q}$

$\therefore$ (i) gives $2q = \frac{a^2}{q} + \frac{b^2}{q}$

$\Rightarrow \quad 2q^2 = a^2 + b^2 \Rightarrow a^2, q^2, b^2$ are in A.P.

28. (d) $T_n = [n^{th} \text{ term of } 1.3.5......] \times [n^{th} \text{ term of } 3.5.7....]$

or $T_n = [1 + (n-1) \times 2] \times [3 + (n-1) \times 2]$

or $T_n = (2n-1)(2n+1) = (4n^2 - 1)$

$$S_n = \sum T_n = \sum (4n^2 - 1)$$

$$= 4.\sum n^2 - \sum 1$$

$$= \frac{4 \times n(n+1)(2n+1)}{6} - n = \frac{2}{3} n\,(n+1)(2n+1) - n$$

29. (d) $$\frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2} \Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$$\Rightarrow \frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, $p = 11, q = 41 \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

30. (b) Let the series a, ar, ar^2, ... are in geometric progression.

given, $a = ar + ar^2$

$\Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1 - 4 \times -1}}{2} \Rightarrow r = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow r = \frac{\sqrt{5} - 1}{2}$$ [$\because$ terms of G.P. are positive

$\therefore$ r should be positive]

31. (b) As per question,

$a + ar = 12$ ...(i)

$ar^2 + ar^3 = 48$ ...(ii)

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$$

($\because$ terms are +ve and –ve alternately)

$\Rightarrow a = -12$

32. (a) Let C be the required harmonic mean such that

$\frac{a}{1-ab}, C, \frac{a}{1+ab}$ are in H.P.

$\Rightarrow \frac{1-ab}{a}, \frac{1}{C}, \frac{1+ab}{a}$ are in A.P.

$$\Rightarrow \frac{2}{C} = \frac{1-ab}{a} + \frac{1+ab}{a} \Rightarrow \frac{2}{C} = \frac{2}{a} \Rightarrow C = a.$$

33. (b) Arithmetic mean between a and b is given by $\frac{a+b}{2}$

$$\therefore \frac{a+b}{2} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$$

$\Rightarrow 2a^{n+1} + 2b^{n+1} = a^{n+1} + a^n b + b^n a + b^{n+1}$

$\Rightarrow (a^{n+1} - a^n b) + (b^{n+1} - ab^n) = 0$

$\Rightarrow a^n (a - b) + b^n (b - a) = 0$

$\Rightarrow (a^n - b^n)(a - b) = 0$

$\Rightarrow a^n - b^n = 0 \quad (\because a - b \neq 0)$

$$\Rightarrow \left(\frac{a}{b}\right)^n = 1 \Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^0$$

$\Rightarrow n = 0$

34. (d) Let α and β be the root of equation

$x^2 - 10x + 11 = 0$

$\therefore \alpha + \beta = 10, \alpha\beta = 11$

$$\therefore HM = \frac{2\alpha\beta}{\alpha + \beta} = \frac{2.11}{10} = \frac{+22}{10} = \frac{11}{5}$$

35. (d) Let the means be $x_1, x_2, ...x_m$ so that $1, x_1, x_2, ...x_m, 31$ is an A.P. of (m + 2) terms.

Now, $31 = T_{m+2} = a + (m+1)d = 1 + (m+1)d$

$\therefore d = \frac{30}{m+1}$ Given : $\frac{x_7}{x_{m-1}} = \frac{5}{9}$

$$\therefore \frac{T_8}{T_m} = \frac{a + 7d}{a + (m-1)d} = \frac{5}{9}$$

$\Rightarrow 9a + 63d = 5a + (5m - 5)d$

$$\Rightarrow 4.1 = (5m - 68)\frac{30}{m+1}$$

$\Rightarrow 2m + 2 = 75m - 1020 \Rightarrow 73m = 1022$

$$\therefore m = \frac{1022}{73} = 14$$

36. (b) Since, S_n denote the sum of an A.P. series.

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$ where 'a' is the first term and 'd' is the common difference of an A.P.

Given, $S_{2n} = 3S_n$

Now, $S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$

$\therefore$ From given equation, we have

$$\frac{2n}{2}[2a + (2n-1)d] = \frac{3n}{2}[2a + (n-1)d]$$

$\Rightarrow 2[2a + (2n-1)d] = 3[2a + (n-1)d]$

$\Rightarrow 4a + 2(2n-1)d = 6a + 3(n-1)d$

$\Rightarrow (4n-2)d = 2a + (3n-3)d$

$\Rightarrow 2a = (n+1)d$

Now, consider

$$\frac{S_{3n}}{S_n} = \frac{\frac{1}{2}(3n)[2a + (3n-1)d]}{\frac{1}{2}(n)[2a + (n-1)d]}$$

$$= \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{3[2a + 3nd - d]}{[2a + nd - d]}$$

Put value of $2a = (n+1)d$, we get

$$\frac{S_{3n}}{S_n} = \frac{3[(n+1)d + 3nd - d]}{(n+1)d + nd - d}$$

$$= \frac{3[nd + d + 3nd - d]}{nd + d + nd - d} = \frac{3(4nd)}{2nd} = 6$$

STATEMENT TYPE QUESTIONS

37. (d) By definition, both the given statements are true.

38. (b)

39. (c) Both are statements are true.

40. (b) I. n^{th} term is $a_n = a + (n - 1)d$

41. (a) II. Geometric mean of 'a' and 'b' = $\sqrt{ab}$

42. (c) All the given statements are true.

43. (a) Both the given statements are true

II. $a = 4, d = 5$

$a_n = 124 \Rightarrow a + (n - 1)d = 124$

$\Rightarrow 4 + (n - 1)5 = 124$

$\Rightarrow n = 25$

44. (b)

I. $a = 72, d = -2$

$a + (n - 1)d = 40$

$\Rightarrow 72 + (n - 1)(-2) = 40$

$\Rightarrow 2n = 34 \Rightarrow n = 17$

Hence, 17^{th} term is 40.

II. $a = 8 - 6i, d = -1 + 2i$

$a_n = (8 - 6i) + (n - 1)(-1 + 2i)$

$= (9 - n) + i(2n - 8)$

a_n is purely real if $2n - 8 = 0 \Rightarrow n = 4$

Hence, 4^{th} term is purely real.

45. (a) I. $a = 3, d = 3$

$a + (n - 1)d = 111 \Rightarrow 3 + (n - 1)(3) = 111$

$\Rightarrow n = 37$

II. $a = 9, d = 3$

$a_n = a + (n - 1)d = 9 + (n - 1)3 = 3n + 6$

46. (c) I. $a.r^{n-1} = 5120 \Rightarrow 5(2^{n-1}) = 5120$

$\Rightarrow 2^{n-1} = 1024 \Rightarrow 2^{n-1} = 2^{10}$

$\Rightarrow n = 11$

II. Let α, β be the roots of the quadratic equation.

A.M. of $\alpha, \beta = \dfrac{\alpha+\beta}{2} = 8$;

G. M. of $\alpha, \beta = \sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 5^2$

$\alpha + \beta = 16, \ \alpha\beta = 25$

Equation whose roots are α, β, is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 16x + 25 = 0$

MATCHING TYPE QUESTIONS

47. (a) (A) $S_{20} = \dfrac{20}{2}[2 \times 1 + (20 - 1)3]$

$= 10 \times 59 = 590$

(B) $a + (n - 1)d = 181$

$\Rightarrow 5 + (n - 1)8 = 181 \Rightarrow n = 23$

$\therefore$ Required sum $= \dfrac{n}{2}[a + l] = \dfrac{23}{2}[5 + 181]$

$= 2139$

(C) 105, 112, 119, ..., 994

$a_n = 994 \Rightarrow a + (n - 1)d = 994$

$\Rightarrow 105 + (n - 1)7 = 994$

$\Rightarrow n = 128$

$\therefore$ Required sum $= \dfrac{128}{2}[2 \times 105 + (128 - 1)7]$

$= 70336$

(D) 252, 255, 258, ..., 999

$a_n = 999 \Rightarrow 252 + (n - 1)3 = 999$

$\Rightarrow n = 250$

$S_n = \dfrac{250}{2}[252 + 999] = 156375$

48. (d) (A) $S_7 = a\left(\dfrac{r^7 - 1}{r - 1}\right) = 3\left(\dfrac{2^7 - 1}{2 - 1}\right)$

$= 3(128 - 1) = 381$

(B) $S_{10} = 1\left[\dfrac{\left(\frac{1}{2}\right)^{10} - 1}{\left(\frac{1}{2}\right) - 1}\right] = 2\left(1 - \dfrac{1}{210}\right)$

$= \dfrac{1024 - 1}{512} = \dfrac{1023}{512}$

(C) $a = 2, r = 3, l = 4374$

Required sum $= \dfrac{lr - a}{r - 1} = \dfrac{(4374 \times 3) - 2}{3 - 1}$

$= 6520$

(D) $S_n = 11 + 103 + 1005 + \ldots$ to n terms

$= (10 + 1) + (10^2 + 3) + (10^3 + 5) + \ldots + \{10^n + (2n - 1)\}$

$= (10 + 10^2 + 10^3 + \ldots + 10^n) + \{1 + 3 + 5 + \ldots + (2n - 1)\}$

$= \dfrac{10(10^n - 1)}{(10 - 1)} + \dfrac{n}{2}(1 + 2n - 1)$

$= \dfrac{10}{9}(10^n - 1) + n^2$

49. (c) (A) $S = \dfrac{a}{1 - r} = \dfrac{\frac{-5}{4}}{1 - \left(-\frac{1}{4}\right)} = -1$

(B) $6^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\infty\right)} = 6^{\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)} = 6^1 = 6$

(C) $S_\infty = 6 \Rightarrow \dfrac{2}{1 - r} = 6 \Rightarrow r = \dfrac{2}{3}$

(D) $a_n = 2[a_{n+1} + a_{n+2} + \ldots\infty] \ \forall \ n \in N$

$\Rightarrow a.r^{n-1} = 2[a.r^n + a.r^{n+1} + \ldots\infty]$

$= \dfrac{2.a.r^n}{1 - r}$

$\Rightarrow 1 = \dfrac{2r}{1 - r} \Rightarrow r = \dfrac{1}{3}$

50. **(a)** $a_n = n(n+2)$

For $n=1$, $a_1 = 1(1+2) = 3$

For $n=2$, $a_2 = 2(2+2) = 8$

For $n=3$, $a_3 = 3(3+2) = 15$

For $n=4$, $a_4 = 4(4+2) = 24$

For $n=5$, $a_5 = 5(5+2) = 35$

Thus first five terms are 3, 8, 15, 24, 35.

51. **(d)** Here $a_n = \frac{2n-3}{6}$

Putting $n = 1, 2, 3, 4, 5$, we get

$a_1 = \frac{2\times1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$;

$a_2 = \frac{2\times2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$;

$a_3 = \frac{2\times3-3}{6} = \frac{6-3}{6} = \frac{3}{6} = \frac{1}{2}$;

$a_4 = \frac{2\times4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$;

and $a_5 = \frac{2\times5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$

$\therefore$ The first five terms are $-\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$ and $\frac{7}{6}$

INTEGER TYPE QUESTIONS

52. **(a)** Let a, be the first term and d, the common difference.

General term (n_{th} term) of the AP is

$T_n = a + (n-1)d$

As given, $T_j = a + (j-1)d = k$ ....(i)

$T_k = a + (k-1)d = j$ ....(ii)

Subtracting (ii) from (i), we get

$(j-k)d = k - j \Rightarrow d = -1$

On putting $d = -1$ in (i), we get

$a + (j-1)(-1) = k$

$\Rightarrow a = k + j - 1$

Now, $T_{k+j} = a + (k+j-1)d = k + j - 1 + [(k+j)-1](-1)$

$= (k+j-1) - (k+j-1) = 0$

53. **(c)** $a_n = 2^n \Rightarrow a_3 = 2^3 = 8$

54. **(b)** For n = 1, $\frac{a_{n+1}}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$ $(\because a_1 = a_2 = 1)$

and $a_n = a_{n-1} + a_{n-2}, n > 2$ ...(A)

n = 3 in equation (A) $a_3 = a_2 + a_1 = 1 + 1 = 2$

for n = 2, $\frac{a_{n+1}}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2$;

55. **(c)** $a = 25, d = 22 - 25 = -3$. Let n be the no. of terms

Sum = 116; Sum $= \frac{n}{2}[2a + (n-1)d]$

$116 = \frac{n}{2}[50 + (n-1)(-3)]$

or $232 = n[50 - 3n + 3] = n[53 - 3n]$

$= -3n^2 + 53n$

$\Rightarrow 3n^2 - 53 + 232 = 0$

$\Rightarrow (n-8)(3n-29) = 0$

$\Rightarrow n = 8$ or $n = \frac{29}{3}$, $n \neq \frac{29}{3}$ $\therefore n = 8$

$\therefore$ Now, $T_8 = a + (8-1)d = 25 + 7\times(-3)$

$= 25 - 21$

$\therefore$ Last term = 4

56. **(a)** Let a be the first term and d be the common difference of A.P.

Sum of first p terms $= \frac{p}{2}[2a + (p-1)d]$ ... (i)

Sum of first q terms $= \frac{q}{2}[2a + (q-1)d]$ ... (ii)

Equating (i) & (ii)

$\frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

Transposing the term of R.H.S to L.H.S

or $2a(p-q) + p(p-1) - q(q-1)d = 0$

$\Rightarrow 2a(p-q) + [(p^2 - q^2 - (p-q)d] = 0$

or $2a(p-q) + (p-q)[(p+q) - d] = 0$

$\Rightarrow (p-q)[2a + (p+q-1)d] = 0$

$\Rightarrow 2a + (p+q-1)d = 0$ ...(iii)

$(\because p \neq q)$

Sum of first $(p+q)$ term $= \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2} \times 0 = 0$

$\therefore 2a + (p+q-1)d = 0$ [from (iii)]

57. **(a)** A. M. between a and b $= \frac{a+b}{2}$

$\therefore \frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$

$2a^n + 2b^n = a^n + ab^{n-1} + a^{n-1}b + b^n$

$\Rightarrow a^n a^{n-1}b - ab^{n-1} + b^n = 0$

$\Rightarrow a^{n-1}(a-b) - b^{n-1}(a-b) = 0$

$\Rightarrow (a-b)(a^n - b^{n-1}) + b^n = 0$ $[\because a \neq 0]$

$\Rightarrow a^{n-1} - b^{n-1} = 0 \Rightarrow a^{n-1} = b^{n-1}$

$\left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0 \Rightarrow n - 1 = 0 \Rightarrow n = 1$

58. (b) The angles of a polygon of n sides form an A.P. whose first term is 120° and common difference is 5°.
The sum of interior angles

$$= \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[2\times 120+(n-1)5]$$

$$= \frac{n}{2}[240+5n-5] = \frac{n}{2}(235+5n)$$

Also the sum of interior angles = 180 × n – 360

$$\therefore \frac{n}{2}(235+5n) = 180n-360$$

Multiplying by $\frac{2}{5}$, $n(47+n) = 2(36n-72)$

$n(47+n) = 72n - 144$
$\Rightarrow n^2 + (47-72)n + 144 = 0$
$\Rightarrow n^2 - 25n + 144 = 0$
$\Rightarrow (n-16)(n-9) = 0$
$\Rightarrow n \neq 16 \ \therefore n = 9$

59. (b) $a = \frac{1}{3}$, $r = \frac{1/9}{1/3} = \frac{1}{9}\times\frac{3}{1} = \frac{1}{3}$

Let $T_n = \frac{1}{19683} \Rightarrow ar^{n-1} = \frac{1}{19683}$

$$\Rightarrow \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^9 \Rightarrow n = 9$$

60. (b) Let n be the number of terms of the G.P. 3, 3^2, 3^3, ... makes the sum = 120
we have $a = 3$, $r = 3$

$$S = \frac{a(r^n-1)}{r-1},\ r>1;\ \text{Sum} = \frac{3(3^n-1)}{3-1} = 120$$

or $\frac{3}{2}(3^n-1) = 120$

Multiplying both sides by $\frac{3}{2}$

$\therefore 3^n - 1 = 80$
$\therefore 3^n = 80 + 1 = 81 = 3^4 \Rightarrow n = 4$
$\therefore$ Required number of terms of given G. P. is 4

61. (b) $f(1) = 3, f(x+y) = f(x)f(y)$
$f(2) = f(1+1) = f(1)f(1) = 3.3 = 9$
$f(3) = f(1+2) = f(1)f(2) = 3.9 = 27$
$f(4) = f(1+3) = f(1)f(3) = 3.27 = 81$
Thus we have

$$\sum_{1}^{n} f(x) = f(1)+f(2)+f(3)+........+f(n) = 120$$

$\Rightarrow 3 + 9 + 27 + ...$ to n term = 120

or $\frac{3(3^n-1)}{3-1} = 120 \qquad [a=3, r=3]$

$$\therefore \frac{3(3^n-1)}{2} = 120 \Rightarrow 3^n - 1 = 120\times\frac{2}{3} = 80$$

$3^n = 80 + 1 = 81 = 3^4 \Rightarrow n = 4$

62. (c) Let the G.P. be a, ar, ar^2, ...
$S = a + ar + ar^2 + ...+$ to $2n$ term

$$= \frac{a(r^{2n}-1)}{r-1}$$

The series formed by taking term occupying odd places is $S_1 = a + ar^2 + ar^4 + ...$ to n terms

$$S_1 = \frac{a\left[(r^2)^n-1\right]}{r^2-1} \Rightarrow S_1 = \frac{a(r^{2n}-1)}{r^2-1}$$

Now, $S = 5S_1$

or $\frac{a(r^{2n}-1)}{r-1} = 5\frac{a(r^{2n}-1)}{r^2-1}$

$$\Rightarrow 1 = \frac{5}{r+1}$$

$\Rightarrow r + 1 = 5 \ \therefore r = 4$

63. (d) $a\left(\frac{r^n-1}{r-1}\right) = 5461 \Rightarrow \frac{4^n-1}{4-1} = 5461$

$\Rightarrow 4^n = 4^7$
$\Rightarrow n = 7$

64. (d) $T_m = a + (m-1)d = \frac{1}{n}$ ...(i)

$T_n = a + (n-1)d = \frac{1}{m}$ ...(ii)

(i) - (ii) $\Rightarrow (m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (i) $a = \frac{1}{mn} \Rightarrow a - d = 0$

ASSERTION - REASON TYPE QUESTIONS

65. (a) The numbers $\frac{-2}{7}, x, \frac{-7}{2}$ will be in G.P.

If $\frac{x}{-\frac{2}{7}} = \frac{-\frac{7}{2}}{x} \Rightarrow x^2 = -\frac{7}{2}\times -\frac{2}{7} = 1 \Rightarrow x = \pm 1$

66. (a) Here $a = x^3$, $r = \frac{x^5}{x^3} = x^2, x \neq \pm 1$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{x^3(1-x^{2n})}{(1-x^2)}$$

67. (b) **Assertion:** $a_n = 4n - 3$

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Reason: $a_n = (-1)^{n-1}.n^3$

$a_9 = (-1)^{9-1}.(9)^3 = (-1)^8(729) = 729$

68. (b) Both are true but Reason is not the correct explanation for the Assertion.

69. (a) **Assertion:** $a_3 = 4 \Rightarrow ar^2 = 4$

$\therefore$ Product of first five terms $= a\ (ar)\ (ar^2)\ (ar^3)\ (ar^4)$
$= a^5.r^{10} = (ar^2)^5 = 4^5$

70. (b) **Assertion:** b + c, c + a, a + b will be in A.P.

if $(c + a) - (b + c) = (a + b) - (c + a)$

i.e. if $2b = a + c$

i.e. if a, b, c are in A.P.

Reason: $10^a, 10^b, 10^c$ are in G.P. if $\frac{10^b}{10^a} = \frac{10^c}{10^b}$

i.e. if $10^{b-a} = 10^{c-b}$

i.e. if $b - a = c - b \Rightarrow 2b = a + c$ which is true.

71. (a) **Assertion:** $2k = \frac{2}{3} + \frac{5}{8} = \frac{16+15}{24}$

$2k = \frac{31}{24}$

$k = \frac{31}{24 \times 2} = \frac{31}{48}$

72. (b) **Assertion:** Let the sum of n term is denoted by S_n

$\therefore\ S_n = 3n^2 + 5n$

Put $n = 1, 2.\quad T_1 = S_1 = 3.1^2 + 5.1 = 3 + 5 = 8;$

$S_2 = T_1 + T_2 = 3.\,2^2 + 5\,.\,2 = 12 + 10 = 22$

$\therefore\ T_2 = S_2 - S_1 = 22 - 8 = 14$

$\therefore$ Common difference $d = T_2 - T_1 = 14 - 8 = 6$

$a = 8,\ d = 6$

m^{th} term $= a + (m-1)d = 164 \Rightarrow 8 + (m-1)\,.\,6 = 164$

$6m + 2 = 164 \Rightarrow 6m = 164 - 2 = 162$

$\therefore\ m = \frac{162}{6} = 27$

Reason: $T_n = ar^{n-1}$

$T_{20} = \frac{5}{2}\left(\frac{1}{2}\right)^{20-1} = \frac{5}{2}.\frac{1}{2^{19}} = \frac{5}{2^{20}}$

73. (b) **Assertion:** First factor of the terms are

2, 4, 6, ...

$\therefore$ First factor of n^{th} term $= 2n$ ...(i)

Second factor of the term are 4, 6, 8 ....

$\therefore$ Second factor of n^{th} term

$= 4 + (n-1)2 = 2(n+1)$ ...(ii)

$\therefore\ n^{th}$ term of the given series

$= 2n \times 2(n+1) = 4n(n+1)$

$\therefore$ putting $n = 20$

20^{th} term of the given series $= 4 \times 20 \times 21$
$= 80 \times 21 = 1680$

Reason: Let three number in A.P be $a - d, a, a + d$

Their sum $= a - d + a + a + d = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Their product $= (a-d)\,(a)\,(a+d) = 440$

$a(a^2 - d^2) = 440 \Rightarrow 8\,(64 - d^2) = 440$

$\Rightarrow\ 64 - d^2 = 55 \Rightarrow d^2 = 64 - 55 = 9$

$\Rightarrow\ d = \pm 3$

Hence, the numbers are $8 - 3, 8, 8 + 3$ or $8 + 3, 8, 8 - 3$

i.e., 5, 8, 11, or 11, 8, 5

74. (c) **Assertion:** $T_k = 5k + 1$ Putting $k = 1, 2$

$T_1 = 5 \times 1 + 1 = 5 + 1 = 6$;

$T_2 = 5 \times 2 + 1 = 10 + 1 = 11$

$\therefore\ d = T_2 - T_1 = 11 - 6 = 5$

$a = 6,\ d = 5$

Sum of n term $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 6 + (n-1)5]$

$= \frac{n}{2}[12 + 5n - 5] = \frac{n(5n+7)}{2}$

Reason: We have to find the sum

$105 + 110 + 115 + ... + 995$

Let $995 = n^{th}$ term

$\therefore\ a + [n-1]d = 995$ or $105 + [n-1]5 = 995$

Dividing by 5,

$21 + (n-1) = 199$ or $n = 199 - 20 = 179$

$\therefore\ 105 + 110 + 115 + ... + 995$

$= \frac{n}{2}[2a + (n-1)d]$

$= \frac{179}{2}[2 \times 105 + (179-1)5]$

$= \frac{179}{2}[2 \times 105 + 5 \times 178] = 98450$

75. (a) $\because \frac{S_n}{S'_n} = \frac{(7n+1)}{(4n+17)} = \frac{n(7n+1)}{n(4n+17)}$

$\therefore S_n = (7n^2 + n)\lambda,\ S'_n = (4n^2 + 17n)\lambda$

Then, $\frac{T_n}{T'_n} = \frac{S_n - S_{n-1}}{S'_n - S'_{n-1}} = \frac{7(2n-1)+1}{4(2n-1)+17} = \frac{14n-6}{8n+13}$

$\Rightarrow T_n : T'_n = (14n - 6) : (8n + 13)$

76. (d) We have, $S_n = 6n^2 + 3n + 1$

$\therefore S_1 = 6 + 3 + 1 = 10$

$S_2 = 24 + 6 + 1 = 31$

$S_3 = 54 + 9 + 1 = 64$ and so on.

So, $T_1 = 10$

$T_2 = S_2 - S_1 = 31 - 10 = 21$

$T_3 = S_3 - S_2 = 64 - 31 = 33$

So, the sequence is 10, 21, 33,...

Now, $21 - 10 = 11$ and $33 - 21 = 12 \neq 11$

$\therefore$ The given series is not in A.P.

So, Assertion is false and Reason is true.

77. **(a)** Let the numbers be a and b.

Then, $A.M. = \frac{a+b}{2} = 34 \Rightarrow a + b = 68$ ...(i)

Also, $G.M. = \sqrt{ab} = 16 \Rightarrow ab = 256$ ...(ii)

Now, $a-b = \pm\sqrt{(a+b)^2 - 4ab}$

$= \pm\sqrt{(68)^2 - 4\times 256} = \pm\sqrt{4624 - 1024} = \pm\sqrt{3600}$

$\Rightarrow a - b = \pm 60$

$\therefore$ a – b = 60 or a – b = – 60 ...(iii)

when a – b = 60, then solving (i) and (iii), we get

a = 64 and b = 4.

Then, numbers are 64 and 4.

When a – b = –60, then solving (i) and (iii), we get

a = 4, b = 64

$\therefore$ Numbers are 4 and 64.

78. **(b)** $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ (given)

Also

$$\frac{T_m}{T_n} = \frac{S_m - S_{m-1}}{S_n - S_{n-1}} = \frac{m^2 - (m-1)^2}{n^2 - (n-1)^2} = \frac{2m-1}{2n-1}$$

Substituting m = 5 and n = 2, we get

$$\frac{T_5}{T_2} = \frac{2(5)-1}{2(2)-1} = \frac{9}{3} = 3$$

CRITICALTHINKING TYPE QUESTIONS

79. **(d)** Since, sum = 4

and second term $= \frac{3}{4}$

$\Rightarrow \frac{a}{1-r} = 4$, and $ar = \frac{3}{4}$

$\Rightarrow \frac{a}{1 - \frac{3}{4a}} = 4$

$\Rightarrow (a-1)(a-3) = 0$

$\Rightarrow a = 1$ or $a = 3$

80. **(c)** Let roots be α, β, γ and $a = a - d, b = a, c = a + d$.

Then $\alpha + \beta + \gamma = 3a = -(-12) \Rightarrow a = 4$

$\alpha\beta\gamma = a(a^2 - d^2) = -(-28) \Rightarrow d = \pm 3$

81. **(d)** Clearly, the given progression is a G.P. with common ratio $r = 2$.

$\therefore$ 4^{th} term from the end $= \ell\left(\frac{1}{r}\right)^{4-1}$

$= (3072)\left(\frac{1}{2}\right)^{4-1} = 384$

82. **(a)** As given : $a^x = b^y = c^z$

Let, $a^x = b^y = c^z = k$ (say)

$\Rightarrow a = k^{1/x}, b = k^{1/y}, c = k^{1/z}$

As given : a, b, c are in G.P.

$\Rightarrow b^2 = ac$

i.e., $k^{2/y} = k^{1/x}\, k^{1/z} = k^{\left(\frac{1}{x}+\frac{1}{z}\right)}$

$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

83. **(c)** The given sereis $3 - 1 + \frac{1}{3} - \frac{1}{9}.......\infty$ is in G.P.

Its common ratio $r = -\frac{1}{3}$ and first term a = 3

$$S_\infty = \frac{a}{1-r} = \frac{3}{1+\frac{1}{3}} = \frac{3\times 3}{4} = \frac{9}{4}$$

84. **(d)** Given : $5^{1+x} + 5^{1-x}, \frac{a}{2}, 5^{2x} + 5^{-2x}$ are in A.P.

We know that if a, b, c are in A.P. then 2b = a + c

$\therefore\ 2.\frac{a}{2} = 5^{1+x} + 5^{1-x} + 5^{2x} + 5^{-2x}$

$\Rightarrow a = 5.5^x + 5(5^x)^{-1} + (5^x)^2 + (5^x)^{-2}$

Let $5^x = t$

$\therefore\ a = 5t + \frac{5}{t} + t^2 + \frac{1}{t^2}$

$\Rightarrow a = t^2 + \frac{1}{t^2} + 5\left(t + \frac{1}{t}\right)$

$\Rightarrow a = \left(t + \frac{1}{t}\right)^2 - 2 + 5\left(t + \frac{1}{t}\right)$

Put $t + \frac{1}{t} = A$

$\therefore\ a = A^2 + 5A - 2$ [add & subtract $\left(\frac{b}{2a}\right)^2$]

$\Rightarrow a = \left[A^2 + 5A - \left(\frac{5}{2}\right)^2\right] + \left(\frac{5}{2}\right)^2 - 2$

$\Rightarrow a = \left(A - \frac{5}{2}\right)^2 + \frac{17}{4}$

$\Rightarrow a \geq \frac{17}{4}$.

85. **(d)** Since, product of n positive number is unity.

$\Rightarrow x_1 x_2 x_3 ....x_n = 1$ ...(i)

Using A.M. $\geq$ GM

$\Rightarrow \frac{x_1 + x_2 + ... + x_n}{n} \geq (x_1 x_2 .....x_n)^{\frac{1}{n}}$

$\Rightarrow x_1 + x_n + ... + x_n \geq n\ (1)^{\frac{1}{n}}$ [From eq^n (i)]

$\Rightarrow$ Sum of n positive number is never less than n.

86. (c) We know that, the sum of infinite terms of GP is

$$S_\infty = \begin{cases} \frac{a}{1-r}, & |r|<1 \\ \infty, & |r|\ge 1 \end{cases}$$

$$\therefore S_\infty = \frac{x}{1-r} = 5 \quad (\because |r|<1)$$

or, $1-r = \frac{x}{5}$

$\Rightarrow r = \frac{5-x}{5}$ exists only when $|r|<1$

i.e., $-1 < \frac{5-x}{5} < 1$

or, $-10 < -x < 0$

or, $0 < x < 10$

87. (c) Sum of the n terms of the series

$\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.......$ upto n terms, can be written as

$\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)....$ upto n terms

$$= n-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....+n \text{ terms}\right)$$

$$= n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}$$

$$= n+2^{-n}-1$$

88. (c) Let us consider a G.P. $a, ar, ar^2, ...$with $2n$ terms.

We have $\frac{a(r^{2n}-1)}{r-1} = \frac{5a\left[(r^2)^n-1\right]}{(r^2-1)}$

(Since common ratio of odd terms will be r^2 and number of terms will be n)

$$\Rightarrow \frac{a(r^{2n}-1)}{r-1} = 5\frac{a(r^{2n}-1)}{(r^2-1)}$$

$\Rightarrow a(r+1) = 5a$, i.e., $r = 4$

89. (b) Middle term = 6^{th} term = 30

$\Rightarrow a + 5d = 30$

$$S_{11} = \frac{11}{2}[2a+10d] = \frac{11}{2}\times 2[a+5d] = 11\times 30 = 330$$

90. (c) Let the G.P. be $1, r, r^2, ... \infty$

Given $x_n = 2(x_{n+1}+x_{n+2}+.....\text{ to }\infty)$

$\therefore x_n = 2\frac{x_{n+1}}{1-r}$ [common ratio is r]

$\therefore \frac{x_{n+1}}{x_n} = \frac{1-r}{2} \Rightarrow r = \frac{1-r}{2} \quad \therefore r = \frac{1}{3}$

The sum of required series is

$$1+\frac{1}{3}+\left(\frac{1}{3}\right)^2+.....\infty = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

91. (b) Let the last three numbers in A.P. be $a, a+6, a+12$, then the first term is also $a+12$.

But $a+12, a, a+6$ are in G.P.

$\therefore a^2 = (a+12)(a+6) \Rightarrow a^2 = a^2+18a+72$

$\therefore a = -4$.

$\therefore$ The numbers are 8, –4, 2, 8.

92. (b) $S_n = an^2+bn+c$

$\therefore S_{n-1} = a(n-1)^2+b(n-1)+c$ for $n\ge 2$

$\therefore t_n = S_n - S_{n-1}$

$= a\{n^2-(n-1)^2\}+b\{n-(n-1)\}$

$= a(2n-1)+b$

$\therefore t_n = 2an+b-a, \; n\ge 2$

$\therefore t_{n-1} = 2a(n-1)+b-a$ for $n\ge 3$

$\therefore t_n - t_{n-1} = 2a(n-n+1) = 2a$ for $n\ge 3$

$\therefore t_3-t_2 = t_4-t_3 = ...2a$

Now $t_2-t_1 = (S_2-S_1)-S_1 = S_2-2S_1$

$= (a.2^2+b.2+c)-\{a.1^2+b.1+c\}$

$= 2a-c \ne 2a$

$\therefore$ Series is arithmetic from the second term onwards.

93. (a) Sum of n terms of A.P with first term = a and common difference, = d is given by

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore \quad S_{10} = 5[2a+9d]$

$S_5 = \frac{5}{2}[2a+4d]$

According to the given condition,

$S_{10} = S_5 \Rightarrow 5[2a+9d] = 4\times\frac{5}{2}[2a+4d]$

$\Rightarrow 2a+9d = 2[2a+4d]$

$\Rightarrow 2a+9d = 4a+8d \Rightarrow d = 2a$

$\Rightarrow \frac{a}{d} = \frac{1}{2} \Rightarrow a : d = 1 : 2$

94. (c) As given, n^{th} term is

$T_n = 3n+7$

Sum of n term, $S_n = \sum T_n$

$= \sum(3n+7) = 3\sum n + 7\sum 1$

$= \frac{3n(n+1)}{2}+7n = n\left[\frac{3n+3+14}{2}\right]$

$= n\left[\frac{3n+17}{2}\right]$

Sum of 50 terms $= S_{50} = 50\left[\frac{3\times 50+17}{2}\right]$

$= 50\left[\frac{167}{2}\right] = 25\times 167 = 4175$

95. (d) Since x is A.M

$\Rightarrow \quad x = \frac{y+z}{2},$

$\Rightarrow \quad 2x = y + z \qquad ...(i)$

and y, g_1, g_2, z ....are in G.P.

$\Rightarrow \quad \frac{g_1}{y} = \frac{g_2}{g_1} = \frac{z}{g_2}$

$\Rightarrow \quad g_1^2 = g_2 y$

$\Rightarrow \quad g_1^3 = g_1 g_2 y \qquad ...(ii)$

Also, $g_2^2 = g_1 z$

$g_2^3 = g_1 g_2 z \qquad ...(iii)$

$\Rightarrow \quad g_1^2 g_2^2 = g_1 g_2 yz$

$\Rightarrow \quad yz = g_1 g_2 \qquad ...(iv)$

Adding equations (ii) and (iii)

$g_1^3 + g_2^3 = y g_1 g_2 + z g_1 g_2 = g_1 g_2 (y+z)$

$= yz \,.\, 2x = 2xyz$

96. (a) The given series is

$(1\times 3) + (3\times 5) + (5\times 7) + ...$

Its general term is given by

$T_n = (2n-1)(2n+1) = 4n^2 - 1$

Sum of series $= 4\Sigma n^2 - \Sigma 1$

$S_n = \frac{4n(n+1)(2n+1)}{6} - n$

$S_n = n\left[\frac{2(2n^2+3n+1)}{3} - 1\right]$

$S_n = n\left[\frac{4n^2+6n+2-3}{3}\right]$

$S_n = \left[\frac{n(4n^2+6n-1)}{3}\right]$

For sum of first 50 terms of the series,

n = 50,

$S_{50} = \frac{50[4(50)^2+6(50)-1]}{3}$

$= \frac{50\times(10000+300-1)}{3}$

$= \frac{50\times 10299}{3} = 171650$

97. (b) We know that A.M. $= \frac{S_n}{n+1}$

Given sequence 1, 2, 4, 8, 16,....., 2^n.

$\Rightarrow \quad S_n = 1 + 2 + 2^2 + 2^3 + 2^4 + ... + 2^n$

$= \frac{2^{n+1}-1}{2-1} = 2^{n+1} - 1 \quad \left[\because S_n = \frac{a(r^n-1)}{(r-1)}\right]$

$\therefore \quad$ A.M. $= \frac{2^{n+1}-1}{n+1}$.

98. (a) The first common term is 11.

Now the next common term is obtained by adding L.C.M. of the common difference 4 and 5, i.e., 20.

Therefore, 10^{th} common term = T_{10} of the AP whose $a = 11$ and $d = 20$

$T_{10} = a + 9d = 11 + 9(20) = 191$

99. (a) Given statement makes an AP series where, a = ` 135, d = ` 15 and S_n = ` 5550

Let total savings be ` 5550 in n years

So, $S_n = \frac{n}{2}[2a + (n-1)d]$

$5550 = \frac{n}{2}[2\times 135 + (n-1)15]$

$\Rightarrow \quad 11100 = n[270 + 15n - 15]$

$\Rightarrow \quad 15n^2 + 255n - 11100 = 0$

$\Rightarrow \quad n^2 + 17n - 740 = 0$

$\Rightarrow \quad n^2 + 37n - 20n - 740 = 0$

$\Rightarrow \quad (n+37)(n-20) = 0$

$n = 20 \quad (\because n \neq -37)$

100. (c) a, b, c are in A.P. $\Rightarrow$ 2b = a + c

Now,

$e^{1/c} \times e^{1/a} = e^{(a+c)/ac} = e^{2b/ac} = (e^{b/ac})^2$

$\therefore e^{1/c}, \; e^{b/ac}, e^{1/a}$ in G.P. with common ratio

$= \frac{e^{b/ac}}{e^{1/c}} = e^{(b-a)/ac} = e^{d/(b-d)(b+d)}$

$= e^{d/(b^2-d^2)}$

[$\because$ a, b, c are in A.P. with common difference d

$\therefore$ b – a = c – b = d]

101. (a) $\frac{2}{\sqrt{c}+\sqrt{a}} = \frac{1}{\sqrt{b}+\sqrt{c}} + \frac{1}{\sqrt{a}+\sqrt{b}}$

$= \frac{2\sqrt{b}+\sqrt{a}+\sqrt{c}}{(\sqrt{b}+\sqrt{c})(\sqrt{a}+\sqrt{b})}$

$\Rightarrow 2\sqrt{ab} + 2b + 2\sqrt{ac} + 2\sqrt{bc}$

$= 2\sqrt{bc} + 2\sqrt{ac} + c + 2\sqrt{ab} + a$

$\Rightarrow 2b = a + c$

$\therefore$ a, b, c, are in A.P.

$\Rightarrow$ ax, bx, cx, are in A.P.

$\Rightarrow$ ax + 1, bx + 1, cx + 1, are in A.P.

$\Rightarrow 9^{ax+1}, 9^{bx+1}, 9^{cx+1}$ are in G.P.

102. (b) As x, y, z, are A.M. of a and b

$\therefore x + y + z = 3\left(\frac{a+b}{2}\right)$

$\therefore 15 = \frac{3}{2}(a+b) \Rightarrow a + b = 10 \qquad ...(i)$

Again $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are A.M. of $\frac{1}{a}$ and $\frac{1}{b}$

$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\therefore \frac{5}{3}=\frac{3}{2}.\frac{a+b}{ab}$

$\Rightarrow \frac{10}{9}=\frac{10}{ab} \Rightarrow ab=9$ ...(ii)

Solving (i) and (ii), we get
$a=9,\ 1,\ b=1, 9$

103. (b) Given $2\sqrt{ab}=\frac{a+b}{2}$

$\Rightarrow \sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}=4$

$\Rightarrow t^2-4t+5=0$, where $\sqrt{\frac{a}{b}}=t$

$\therefore \quad t=2\pm\sqrt{3} \Rightarrow \sqrt{\frac{a}{b}}=2\pm\sqrt{3}$

$\therefore \quad \frac{a}{b}=\frac{(2\pm\sqrt{3})^2}{4-3}=\frac{(2\pm\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$

$\therefore$ a : b = $2+\sqrt{3} : 2-\sqrt{3}$

or $2-\sqrt{3} : 2+\sqrt{3}$

104. (d) We have $S_n=\frac{a(1-r^n)}{1-r}$

$\therefore \quad S_{2n-1}=\frac{a}{1-r}[1-r^{2n-1}]$

Putting 1, 2, 3,........, n for n in it and summing up we have

$S_1+S_3+S_5+.........+S_{2n-1}$

$=\frac{a}{1-r}[(1+1+....\text{n term})-(r+r^3+r^5+.....\text{n term})]$

$=\frac{a}{1-r}\left[n-\frac{r\{1-(r^2)^n\}}{1-r^2}\right]=\frac{a}{1-r}\left[n-r.\frac{1-r^{2n}}{1-r^2}\right]$

105. (b) We have ,

$S_1=\frac{n_1}{2}[2a+(n_1-1)d] \Rightarrow \frac{2S_1}{n_1}=2a+(n_1-1)d$

$S_2=\frac{n_2}{2}[2a+(n_2-1)d] \Rightarrow \frac{2S_2}{n_2}=2a+(n_2-1)d$

$S_3=\frac{n_3}{2}[2a+(n_3-1)d] \Rightarrow \frac{2S_3}{n_3}=2a+(n_3-1)d$

$\therefore \frac{2S_1}{n_1}(n_2-n_3)+\frac{2S_2}{n_2}(n_3-n_1)+\frac{2S_3}{n_3}(n_1-n_2)$

$=[2a+(n_1-1)d](n_2-n_3)+[2a+(n_2-1)d](n_3-n_1)$
$+[2a+(n_3-1)d](n_1-n_2)=0$

106. (c) We have $t_n=\frac{1^3+2^3+3^3+.......+n^3}{1+3+5+.....\text{upto n terms}}$

$=\frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\frac{n}{2}\{2+2(n-1)\}}=\frac{\frac{n^2(n+1)^2}{4}}{n^2}$

$=\frac{(n+1)^2}{4}=\frac{n^2}{4}+\frac{n}{2}+\frac{1}{4}$

$\therefore \quad S_n=\Sigma t_n=\frac{1}{4}\Sigma n^2+\frac{1}{2}\Sigma n+\frac{1}{4}\Sigma 1$

$=\frac{1}{4}\frac{n(n+1)(2n+1)}{6}+\frac{1}{2}\frac{n(n+1)}{2}+\frac{1}{4}n$

$S_{16}=\frac{16.17.33}{24}+\frac{16.17}{4}+\frac{16}{4}=446$

107. (b) If n is odd, the required sum is

$1^2+2.2^2+3^2+2.4^2+......+2.(n-1)^2+n^2$

$=\frac{(n-1)(n-1+1)^2}{2}+n^2$

[$\because$ (n–1) is even
$\therefore$ using given formula for the sum of (n–1) terms.]

$=\left(\frac{n-1}{2}+1\right)n^2=\frac{n^2(n+1)}{2}$

108. (a) $S_\infty=\frac{a}{1-r}$ where 'a' be the first term and r be the common ratio of G.P.

$\therefore \quad \frac{4}{3}=\frac{3/4}{1-r}$

$\Rightarrow \quad 1-r=\frac{3/4}{4/3} \Rightarrow \quad 1-\frac{9}{16}=r \Rightarrow \quad r=\frac{7}{16}$

109. (c) Let six term of H.P. = $\frac{1}{61}$

$\Rightarrow$ six term of AP = 61
Similarly tenth term of A.P. = 105
Let first term of AP is a and common diff. = d
$\therefore \quad a+5d=61$
and $a+9d=105$
solving these equation, we get
$a=6, d=11$

Hence, first term of H.P. = $\frac{1}{6}$

110. (c) Let x be the first term and y be the c.d. of corresponding A.P., then

$\frac{1}{a}=x+(p-1)y$ ...... (i)

$\frac{1}{b}=x+(q-1)y$ ...... (ii)

$\frac{1}{c}=x+(r-1)y$ ...... (iii)

Multiplying (i), (ii) and (iii) respectively by abc (q – r), abc (r – p), abc (p – q) and then adding, we get, bc (q – r) + ca (r – p)+ ab (p – q) = 0

111. (c) Let the GP be a, ar, ar^2 ,........, where $0 < r < 1$.
Then, $a + ar + ar^2 + ... = 3$
and $a^2 + a^2r^2 + a^2r^4 + ... = 9/2$.

$$\Rightarrow \frac{a}{1-r} = 3 \text{ and } \frac{a^2}{1-r^2} = \frac{9}{2}$$

$$\Rightarrow \frac{9(1-r)^2}{1-r^2} = \frac{9}{2} \Rightarrow \frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$$

Putting $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get a = 2

Now, the required sum of the cubes is

$$a^3 + a^3r^3 + a^3r^6 + ... = \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$$

112. (c) x,y,z are in G.P. $\Rightarrow y^2 = xz$ ......(i)
We have, $ax = b^y = c^z = \lambda$ (say)
$\Rightarrow x \log a = y \log b = z \log c = \log \lambda$

$$\Rightarrow x = \frac{\log\lambda}{\log a}, y = \frac{\log\lambda}{\log b}, z = \frac{\log\lambda}{\log c}$$

Putting x,y,z in (i), we get

$$\left(\frac{\log\lambda}{\log b}\right)^2 = \frac{\log\lambda}{\log a}\cdot\frac{\log\lambda}{\log c}$$

$(\log b)^2 = \log a . \log c$
or $\log_a b = \log_b c \Rightarrow \log_b a = \log_c b$

113. (a) We have

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...\infty \quad ....(i)$$

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...\infty \quad ....(ii)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3}\times\frac{3}{2} \Rightarrow S = 3$$

114. (d) Series $2, 2\frac{1}{2}, 3\frac{1}{3}$, ... are in H.P.

$\Rightarrow \frac{1}{2}, \frac{2}{5}, \frac{3}{10}$, ... will be in A.P.

Now, first term $a = \frac{1}{2}$

and common difference $d = -\frac{1}{10}$

So, 5^{th} term of the A.P. $= \frac{1}{2} + (5-1)\left(-\frac{1}{10}\right) = \frac{1}{10}$.

Hence, 5^{th} term in H.P. is 10.

115. (d) Considering corresponding A.P.
$a + 6d = 10$ and $a + 11d = 25$
$\Rightarrow d = 3, a = -8$
$\Rightarrow T_{20} = a + 19d = -8 + 57 = 49$

Hence, 20^{th} term of the corresponding H.P. $= \frac{1}{49}$.

116. (c)
$$\text{H.M.} = \frac{2\left(\frac{a}{1-ab}\right)\left(\frac{a}{1+ab}\right)}{\frac{a}{1-ab}+\frac{a}{1+ab}}$$

$$= \frac{2\left(\frac{a^2}{1-a^2b^2}\right)}{\frac{a}{1-ab}+\frac{a}{1+ab}} = \frac{2a^2}{2a} = a.$$

117. (a) It is a fundamental concept.

118. (d) Let $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.

Then, $G^2 = ab$ ...(i)

and $AH = \left(\frac{a+b}{2}\right)\cdot\frac{2ab}{a+b} = ab$ ...(ii)

From (i) and (ii), we have $G^2 = AH$

119. (a) Given that b^2, a^2, c^2 are in A.P.
$\therefore a^2 - b^2 = c^2 - a^2$
$\Rightarrow (a-b)(a+b) = (c-a)(c+a)$

$$\Rightarrow \frac{1}{b+c} - \frac{1}{a+b} = \frac{1}{c+a} - \frac{1}{b+c}$$

$$\Rightarrow \frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a} \text{ are in A.P.}$$

120. (c) A.M. $= \frac{a+b}{2} = A$ and G.M. $= \sqrt{ab} = G$

On solving a and b are given by the values $A \pm \sqrt{(A+G)(A-G)}$.

Trick: Let the numbers be 1, 9. Then, A = 5 and G = 3. Now, put these values in options.

Here, (c) $\Rightarrow 5 \pm \sqrt{8\times 2}$, i.e. 9 and 1.

121. (c) Since the reciprocals of a and c occur on R.H.S., let us first assume that a, b, c are in H.P.

So, that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b}$ = d, say

$\Rightarrow \frac{a-b}{ab}=d=\frac{b-c}{bc} \Rightarrow a-b=abd$ and $b-c=bcd$

Now, L.H.S. $= -\frac{1}{a-b}+\frac{1}{b-c}=-\frac{1}{abd}+\frac{1}{bcd}$

$=\frac{1}{bd}\left(\frac{1}{c}-\frac{1}{a}\right)=\frac{1}{bd}(2d) \Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}$ = R.H.S.

$\therefore$ a, b, c are in H.P. is verified.

Aliter: $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$

$\frac{1}{b-a}-\frac{1}{c}=\frac{1}{a}-\frac{1}{b-c}$

$\Rightarrow \frac{c-b+a}{c(b-a)}=\frac{b-c-a}{a(b-c)} \Rightarrow -\frac{1}{c(b-a)}=\frac{1}{a(b-c)}$

$\Rightarrow ac-bc=ab-ac \Rightarrow b=\frac{2ac}{a+c}$

$\therefore$ a, b, c are in H.P.

122. (b) Given that a, b, c are in A.P.

$\Rightarrow b=\frac{a+c}{2}$ ... (i)

and $b^2=ad$ ... (ii)

Hence, a, a – b, d – c are in G.P. because

$(a-b)^2=a^2-2ab+b^2=a(a-2b)+ad$

$\Rightarrow a(-c)+ad=ad-ac.$

123. (d) Given that $\frac{\text{H.M.}}{\text{G.M.}}=\frac{12}{13}$

$\Rightarrow \frac{\frac{2ab}{a+b}}{\sqrt{ab}}=\frac{12}{13}$ or $\frac{a+b}{2\sqrt{ab}}=\frac{13}{12}$

$\Rightarrow \frac{(a+b)+2\sqrt{ab}}{(a+b)-2\sqrt{ab}}=\frac{13+12}{13-12}=\frac{25}{1}$

$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{5^2}{1} \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{5}{1}$

$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)}=\frac{5+1}{5-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}}=\frac{6}{4} \Rightarrow \left(\frac{a}{b}\right)^{\frac{1}{2}}=\frac{6}{4} \Rightarrow a:b=9:4$

124. (c) We have H.M. $=\frac{2ab}{a+b}$ and G.M. $=\sqrt{ab}$

So, $\frac{\text{H.M.}}{\text{G.M.}}=\frac{4}{5} \Rightarrow \frac{\frac{2ab}{(a+b)}}{\sqrt{ab}}=\frac{4}{5}$

$\Rightarrow \frac{2\sqrt{ab}}{(a+b)}=\frac{4}{5} \Rightarrow \frac{a+b}{2\sqrt{ab}}=\frac{5}{4}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{5+4}{5-4} \Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{9}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{3}{1}$

$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)}=\frac{3+1}{3-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}}=\frac{4}{2} \Rightarrow \left(\frac{a}{b}\right)=2^2=4$

$\Rightarrow a:b=4:1$

CHAPTER 10

STRAIGHT LINES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Slope of non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by :

(a) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (b) $m = \dfrac{x_2 - x_1}{y_2 - y_1}$

(c) $m = \dfrac{x_2 + x_1}{y_2 + y_1}$ (d) $m = \dfrac{y_2 + y_1}{x_2 + x_1}$

2. If a line makes an angle α in anti-clockwise direction with the positive direction of x-axis, then the slope of the line is given by :

(a) $m = \sin\alpha$ (b) $m = \cos\alpha$

(c) $m = \tan\alpha$ (d) $m = \sec\alpha$

3. The point (x, y) lies on the line with slope m and through the fixed point (x_0, y_0) if and only if its coordinates satisfy the equation $y - y_0$ is equal to ....

(a) $m(x - x_0)$ (b) $m(y - x_0)$

(c) $m(y - x)$ (d) $m(x - y_0)$

4. If a line with slope m makes x-intercept d. Then equation of the line is :

(a) $y = m(d - x)$ (b) $y = m(x - d)$

(c) $y = m(x + d)$ (d) $y = mx + d$

5. The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by :

(a) $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ (b) $d = \dfrac{|Ax_1 - By_1 + C|}{\sqrt{A^2 + B^2}}$

(c) $d = \dfrac{\sqrt{A^2 + B^2}}{|Ax_1 + By_1 + C|}$ (d) $d = \dfrac{\sqrt{A^2 + B^2}}{|Ax_1 - By_1 + C|}$

6. Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by:

(a) $d = \dfrac{\sqrt{A^2 + B^2}}{|C_1 - C_2|}$ (b) $d = \dfrac{\sqrt{A^2 - B^2}}{|C_1 - C_2|}$

(c) $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$ (d) $d = \dfrac{|C_1 + C_2|}{\sqrt{A^2 + B^2}}$

7. The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is

(a) 45° (b) 135° (c) –45° (d) –135°

8. Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) –1 (b) 0 (c) 2 (d) $\sqrt{3}$

9. Which of the following lines is farthest from the origin?

(a) $x - y + 1 = 0$ (b) $2x - y + 3 = 0$

(c) $x + 2y - 2 = 0$ (d) $x + y - 2 = 0$

10. Equation of the straight line making equal intercepts on the axes and passing through the point (2, 4) is :

(a) $4x - y - 4 = 0$ (b) $2x + y - 8 = 0$

(c) $x + y - 6 = 0$ (d) $x + 2y - 10 = 0$

11. A line passes through $P(1, 2)$ such that its intercept between the axes is bisected at P. The equation of the line is

(a) $x + 2y = 5$ (b) $x - y + 1 = 0$

(c) $x + y - 3 = 0$ (d) $2x + y - 4 = 0$

12. The tangent of angle between the lines whose intercepts on the axes are $a, -b$ and $b, -a$ respectively, is

(a) $\dfrac{a^2 - b^2}{ab}$ (b) $\dfrac{b^2 - a^2}{2}$

(c) $\dfrac{b^2 - a^2}{2ab}$ (d) None of these

13. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2) then the equation of the line will be

(a) $2x + 3y = 12$ (b) $3x + 2y = 12$

(c) $4x - 3y = 6$ (d) $5x - 2y = 10$

14. The intercept cut off by a line from y-axis twice than that from x-axis, and the line passes through the point (1, 2). The equation of the line is

(a) $2x + y = 4$ (b) $2x + y + 4 = 0$

(c) $2x - y = 4$ (d) $2x - y + 4 = 0$

15. Line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and $(x, 24)$. Find the value of x.

(a) 2 (b) 3 (c) 4 (d) 5

16. Let the perpendiculars from any point on the line $7x + 56y = 0$ upon $3x + 4y = 0$ and $5x - 12y = 0$ be p and p', then

(a) $2p = p'$ (b) $p = 2p'$

(c) $p = p'$ (d) None of these

17. The lines $x + 2y - 5 = 0$, $2x - 3y + 4 = 0$, $6x + 4y - 13 = 0$

(a) are concurrent.

(b) form a right angled triangle.

(c) form an isosceles triangle.

(d) form an equilateral triangle.

18. A triangle ABC is right angled at A has points A and B as (2, 3) and (0, –1) respectively. If $BC = 5$, then point C may be
(a) (–4, 2) (b) (4, –2) (c) (0, 4) (d) (0, –4)

19. The relation between a, b, a' and b' such that the two lines $ax + by = c$ and $a'x + b'y = c'$ are perpendicular is
(a) $aa' - bb' = 0$ (b) $aa' + bb' = 0$
(c) $ab + a'b' = 0$ (d) $ab - a'b' = 0$

20. The equation of a straight line which cuts off an intercept of 5 units on negative direction of y-axis and makes an angle of 120° with the positive direction of x-axis is
(a) $\sqrt{3}x + y + 5 = 0$ (b) $\sqrt{3}x + y - 5 = 0$
(c) $\sqrt{3}x - y - 5 = 0$ (d) $\sqrt{3}x - y + 5 = 0$

21. The equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$ is
(a) $2x + 3y + 6 = 0$ (b) $2x - 3y - 6 = 0$
(c) $2x - 3y + 6 = 0$ (d) $2x + 3y - 6 = 0$

22. Which one of the following is the nearest point on the line $3x - 4y = 25$ from the origin?
(a) (–1, –7) (b) (3, –4)
(c) (–5, –8) (d) (3, 4)

23. If the mid-point of the section of a straight line intercepted between the axes is (1, 1), then what is the equation of this line?
(a) $2x + y = 3$ (b) $2x - y = 1$
(c) $x - y = 0$ (d) $x + y = 2$

24. What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?
(a) 60° (b) 45° (c) 30° (d) 15°

25. If the points (x, y), (1, 2) and (–3, 4) are collinear, then
(a) $x + 2y - 5 = 0$ (b) $x + y - 1 = 0$
(c) $2x + y - 4 = 0$ (d) $2x - y + 10 = 0$

26. If p be the length of the perpendicular from the origin on the straight line $x + 2by = 2p$, then what is the value of b?
(a) $\frac{1}{p}$ (b) p (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

27. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is
(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

28. The coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$ are
(a) (–6, 5) (b) (5, 6) (c) (–5, 6) (d) (6, 5)

29. The length of the perpendicular from the origin to a line is 7 and line makes an angle of 150° with the positive direction of y-axis then the equation of the line is
(a) $4x + 5y = 7$ (b) $-x + 3y = 2$
(c) $\sqrt{3}x - y = 10\sqrt{2}$ (d) $\sqrt{3}x + y = 14$

30. A straight line makes an angle of 135° with x-axis and cuts y-axis at a distance of – 5 from the origin. The equation of the line is
(a) $2x + y + 5 = 0$ (b) $x + 2y + 3 = 0$
(c) $x + y + 5 = 0$ (d) $x + y + 3 = 0$

31. The equation of a line through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$ is
(a) $2x + y - 5 = 0$ (b) $x - 3y + 6 = 0$
(c) $x + 2y - 7 = 0$ (d) $x + 3y + 8 = 0$

32. The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other
(a) $a_1b_1 - b_1a_2 = 0$ (b) $a_1^2b_2 + b_1^2a_2 = 0$
(c) $a_1b_1 + a_2b_2 = 0$ (d) $a_1a_2 + b_1b_2 = 0$

33. If the coordinates of the points A and B be (3, 3) and (7, 6), then the length of the portion of the line AB intercepted between the axes is
(a) $\frac{5}{4}$ (b) $\frac{\sqrt{10}}{4}$ (c) $\frac{\sqrt{13}}{3}$ (d) None of these

34. The line $(3x - y + 5) + \lambda(2x - 3y - 4) = 0$ will be parallel to y-axis, if λ =
(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

35. The equation of a straight line passing through (–3, 2) and cutting an intercept equal in magnitude but opposite in sign from the axes is given by
(a) $x - y + 5 = 0$ (b) $x + y - 5 = 0$
(c) $x - y - 5 = 0$ (d) $x + y + 5 = 0$

36. The points A(1, 3) and C(5, 1) are the opposite vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is
(a) $2x + y - 8 = 0$ (b) $2x - y - 4 = 0$
(c) $2x - y + 4 = 0$ (d) $2x + y + 7 = 0$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

37. Consider the following statements about straight lines :
I. Slope of horizontal line is zero and slope of vertical line is undefined.
II. Two lines are parallel if and only if their slopes are equal.
III. Two lines are perpendicular if and only if product of their slope is – 1.
Which of the above statements are true ?
(a) Only I (b) Only II
(c) Only III (d) All the above

38. The distances of the point (1, 2, 3) from the coordinate axes are A, B and C respectively. Now consider the following equations:
I. $A^2 = B^2 + C^2$ II. $B^2 = 2C^2$
III. $2A^2C^2 = 13B^2$
Which of these hold(s) true?
(a) Only I (b) I and III (c) I and II (d) II and III

39. Consider the following statements.
I. Equation of the line passing through (0, 0) with slope m is $y = mx$
II. Equation of the x-axis is $x = 0$.

Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

40. Consider the following statements.
I. The distance between two points (x_1, y_1) and (x_2, y_2) is given by
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
II. The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2)
$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

41. Consider the following statements.
The three given points A, B, C are collinear i.e., lie on the same straight line, if
I. area of ΔABC is zero.
II. slope of AB = Slope of BC.
III. any one of the three points lie on the straight line joining the other two points.
Choose the correct option
(a) Only I is true (b) Only II is true
(c) Only III is true (d) All are true

42. Consider the following statements.
I. Slope of horizontal line is zero and slope of vertical line is undefined.
II. Two lines whose slopes are m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$
Choose the correct option.
(a) Both are true (b) Both are false
(c) Only I is true (d) Only II is true

43. Consider the following statements.
I. The length of perpendicular from a given point (x_1, y_1) to a line $ax + by + c = 0$ is
$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$
II. Three or more straight lines are said to be concurrent lines, if they meet at a point.
Choose the correct option
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

44. Consider the following statements.
I. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle then centroid is
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$
II. If the point $P(x, y)$ divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ (internally), then
$$x = \frac{mx_2 + nx_1}{m + n}, \quad y = \frac{my_2 + ny_1}{m + n}$$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

45. Consider the following statements.
I. The equation of a straight line passing through the point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$
II. Equation of the y-axis is x = 0.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false.

46. Consider the following statements.
I. The equation of a straight line making intercepts a and b on x and y-axis respectively is given by
$$\frac{x}{a} + \frac{y}{b} = 1$$
II. If $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ be two parallel lines, then distance between two parallel lines, $d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

47. Consider the following statements.
I. If (a, b), (c, d) and (a – c, b – d) are collinear, then bc – ad = 0
II. If the points A (1, 2), B (2, 4) and C (3, a) are collinear, then the length BC = 5 unit.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

48. Consider the following statements.
I. Centroid of a triangle is a point where angle bisectors meet.
II. If value of area after calculations is negative then we take its negative value.
Choose the correct option
(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

49. Consider the following statements.
I. Two lines are parallel if and only if their slopes are equal.
II. Two lines are perpendicular if and only if product of their slopes is 1.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false.

50. Equation of a line is $3x - 4y + 10 = 0$
I. Slope of the given line is $\frac{3}{4}$.
II. x-intercept of the given line is $-\frac{10}{3}$.
III. y-intercept of the given line is $\frac{5}{2}$.
Choose the correct option.
(a) Only I and II are true
(b) Only II and III are true
(c) Only I and III are true
(d) All I, II and III are true

51. Consider the equation $\sqrt{3}x + y - 8 = 0$

I. Normal form of the given equation is $\cos 30°x + \sin 30°\ y = 4$

II. Values of p and w are 4 and 30° respectively.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

52. Slope of the lines passing through the points

I. (3, – 2) and (– 1, 4) is $-\frac{3}{2}$

II. (3, – 2) and (7, – 2) is 0.

III. (3, – 2) and (3, 4) is 1.

Choose the correct option.

(a) Only I and III are true
(b) Only I and II are true
(c) Only II and III are true
(d) None of these

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

53. The value of x for which the points $(x, –1)$, (2, 1) and (4, 5) are collinear, is

(a) 1 (b) 2 (c) 3 (d) 4

54. The distance of the point (–1, 1) from the line $12(x + 6) = 5(y – 2)$ is

(a) 2 (b) 3 (c) 4 (d) 5

55. The perpendicular from the origin to the line $y = mx + c$ meets it at the point (–1, 2). Find the value of m + c.

(a) 2 (b) 3 (c) 4 (d) 5

56. The values of k for which the line $(k – 3)x – (4 – k^2)y + k^2 – 7k + 6 = 0$ is parallel to the x-axis, is

(a) 3 (b) 2 (c) 1 (d) 4

57. The line joining (–1, 1) and (5, 7) is divided by the line x + y = 4 in the ratio 1 : k. The value of 'k' is

(a) 2 (b) 4 (c) 3 (d) 1

58. If three points (h, 0), (a, b) and (0, k) lies on a line, then the value of $\frac{a}{h} + \frac{b}{k}$ is

(a) 0 (b) 1 (c) 2 (d) 3

59. Value of x so that 2 is the slope of the line through (2, 5) and (x, 3) is

(a) 0 (b) 1 (c) 2 (d) 3

60. What is the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (– 1, 4) and (0, 6)?

(a) 6 (b) 7 (c) 5 (d) 9

61. Reduce the equation $\sqrt{3}x + y - 8 = 0$ into normal form. The value of p is

(a) 2 (b) 3 (c) 4 (d) 5

62. The distance between the parallel lines 3x – 4y + 7 = 0 and 3x – 4y + 5 = 0 is $\frac{a}{b}$. Value of a + b is

(a) 2 (b) 5 (c) 7 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

63. **Assertion:** If θ is the inclination of a line l, then the slope or gradient of the line l is tan θ.

Reason: The slope of a line whose inclination is 90°, is not defined.

64. **Assertion:** The inclination of the line l may be acute or obtuse.

Reason: Slope of x-axis is zero and slope of y-axis is not defined.

65. **Assertion:** Slope of the line passing through the points (3, –2) and (3, 4) is 0.

Reason: If two lines having the same slope pass through a common point, then these lines will coincide.

66. **Assertion:** If A (– 2, –1), B (4, 0), C (3, 3) and D (–3, 2) are the vertices of a parallelogram, then mid-point of AC = Mid-point of BD

Reason: The points A, B and C are collinear ⇔ Area of ΔABC = 0.

67. **Assertion:** Pair of lines x + 2y – 3 = 0 and – 3x – 6y + 9 = 0 are coincident.

Reason: Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

68. **Assertion:** If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, are parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

Reason: If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular, then $a_1a_2 – b_1b_2 = 0$.

69. **Assertion:** The equation of the line making intercepts a and b on x and y-axis respectively is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Reason: The slope of the line ax + by + c = 0 is $\frac{b}{a}$.

70. **Assertion:** The equation of a line parallel to the line ax + by + c = 0 is ax – by – λ = 0, where λ is a constant.

Reason: The equation of a line perpendicular to the line ax + by + c = 0 is bx – ay + λ = 0, where λ is a constant.

71. **Assertion:** The distance between the parallel lines 3x – 4y + 9 = 0 and 6x – 8y – 15 = 0 is $\frac{33}{10}$.

Reason: Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

72. Assertion: Equation of the horizontal line having distance 'a' from the x-axis is either y = a or y = –a.

Reason: Equation of the vertical line having distance b from the y-axis is either x = b or x = –b.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

73. In what ratio does the line $y - x + 2 = 0$ cut the line joining (3, –1) and (8, 9)?

(a) 2 : 3 (b) 3 : 2 (c) 3 : –2 (d) 1 : 2

74. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is:

(a) $\frac{3}{2}$ (b) $\frac{3}{10}$ (c) 6 (d) $\frac{9}{4}$

75. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y – intercept is:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

76. If the area of the triangle with vertices (x, 0), (1, 1) and (0, 2) is 4 square unit, then the value of x is :

(a) – 2 (b) – 4 (c) – 6 (d) 8

77. The distance of the line $2x + y = 3$ from the point (–1, 3) in the direction whose slope is 1, is

(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{2\sqrt{5}}{3}$

78. The straight lines $x + 2y - 9 = 0$, $3x + 5y - 5 = 0$ and $ax + by = 1$ are concurrent if the straight line $35x - 22y + 1 = 0$ passes through :

(a) (a, b) (b) (b, a)

(c) (a, – b) (d) (– a, b)

79. The reflection of the point (4, – 13) in the line $5x + y + 6 = 0$ is

(a) (–1, –14) (b) (3, 4)

(c) (0, 0) (d) (1, 2)

80. If *a, b, c* are in A.P., then the straight lines $ax + by + c = 0$ will always pass through

(a) (1, – 2) (b) (1, 2)

(c) (– 1, 2) (d) (– 1, – 2)

81. What is the image of the point (2, 3) in the line $y = – x$?

(a) (–3, –2) (b) (– 3, 2)

(c) (–2, –3) (d) (3, 2)

82. If p be the length of the perpendicular from the origin on the straight line $ax + by = p$ and $b = \frac{\sqrt{3}}{2}$, then what is the angle between the perpendicular and the positive direction of x-axis?

(a) 30° (b) 45° (c) 60° (d) 90°

83. If (– 4, 5) is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is

(a) $x + 3y = 21$ (b) $2x - 3y = 7$

(c) $x + 7y = 31$ (d) $2x + 3y = 21$

84. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The co-ordinates of the point A is

(a) $\left(\frac{13}{5}, 0\right)$ (b) $\left(\frac{5}{13}, 0\right)$

(c) (–7, 0) (d) None of these

85. The vertices of a triangle ABC are (1, 1), (4, – 2) and (5, 5) respectively. Then equation of perpendicular dropped from C to the internal bisector of angle A is

(a) $y - 5 = 0$ (b) $x - 5 = 0$

(c) $2x + 3y - 7 = 0$ (d) None of these

86. The line L has intercepts a and b on the coordinate axes. When keeping the origin fixed, the coordinate axes are rotated through a fixed angle, then same line has intercepts p and q on the rotated axes, then

(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

(c) $a^2 + \beta^2 = b^2 + q^2$ (d) $b^2 + q^2 = \frac{1}{b^2} + \frac{1}{q^2}$

87. The equation of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, – 10), then the equation of the third side is (are)

(a) $3x + y + 7 = 0$, $x - 3y - 31 = 0$

(b) $2x + y + 5 = 0$, $x - 2y + 3 = 0$

(c) $3x + y + 7 = 0$, $x + y = 0$

(d) $3x - y = 7$, $x + 3y = 15$

88. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for

(a) exactly one value of p

(b) exactly two values of p

(c) more than two values of p

(d) no value of p

89. The bisector of the acute angle formed between the lines $4x - 3y + 7 = 0$ and $3x - 4y + 14 = 0$ has the equation

(a) $x + y + 3 = 0$ (b) $x - y - 3 = 0$

(c) $x - y + 3 = 0$ (d) $3x + y - 7 = 0$

90. The equations of the lines which cuts off an intercept –1 from y-axis and equally inclined to the axes are

(a) $x - y + 1 = 0$, $x + y + 1 = 0$

(b) $x - y - 1 = 0$, $x + y - 1 = 0$

(c) $x - y - 1 = 0$, $x + y + 1 = 0$

(d) None of these

91. If the coordinates of the points A, B, C be (–1, 5), (0, 0) and (2, 2) respectively and D be the middle point of BC, then the equation of the perpendicular drawn from B to the line AD is

(a) $x + 2y = 0$ (b) $2x + y = 0$

(c) $x - 2y = 0$ (d) $2x - y = 0$

92. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(a) Above the x-axis at a distance of $\frac{3}{2}$ from it

(b) Above the x-axis at a distance of $\frac{2}{3}$ from it

(c) Below the x-axis at a distance of $\frac{3}{2}$ from it

(d) Below the x-axis at a distance of $\frac{2}{3}$ from it

93. Equation of angle bisector between the lines $3x + 4y - 7 = 0$ and $12x + 5y + 17 = 0$ are

(a) $\frac{3x+4y-7}{\sqrt{25}} = \pm\frac{12x+5y+17}{\sqrt{169}}$

(b) $\frac{3x+4y+7}{\sqrt{25}} = \frac{12x+5y+17}{\sqrt{169}}$

(c) $\frac{3x+4y+7}{\sqrt{25}} = \pm\frac{12x+5y+17}{\sqrt{169}}$

(d) None of these

94. The equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$, is

(a) $\left(4-\sqrt{5}\right)x - \left(3-2\sqrt{5}\right)y + \left(2-4\sqrt{5}\right) = 0$

(b) $\left(4+\sqrt{5}\right)x - \left(3+2\sqrt{5}\right)y + \left(2+4\sqrt{5}\right) = 0$

(c) $\left(4+\sqrt{5}\right)x + \left(3+2\sqrt{5}\right)y + \left(2+4\sqrt{5}\right) = 0$

(d) None of these

95. Choose the correct statement which describe the position of the point $(-6, 2)$ relative to straight lines $2x + 3y - 4 = 0$ and $6x + 9y + 8 = 0$.

(a) Below both the lines (b) Above both the lines

(c) In between the lines (d) None of these

96. If A and B are two points on the line $3x + 4y + 15 = 0$ such that $OA = OB = 9$ units, then the area of the triangle OAB is

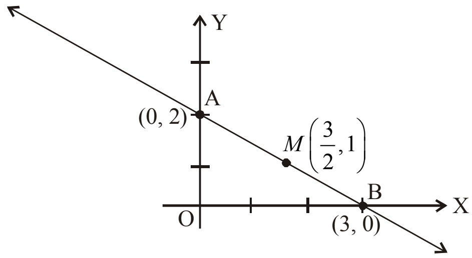
(a) 18 sq. units (b) $18\sqrt{2}$ sq. units

(c) $\frac{18}{\sqrt{2}}$ sq. units (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$, $x_1 \neq x_2$
2. (c) $m = \tan\alpha$, $\alpha \neq 90°$
3. (a) $y - y_0 = m(x - x_0)$
4. (b) $y = m(x - d)$
5. (a) $d = \left|\dfrac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$
6. (c) $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$
7. (a) The equation of the line $x - y + 3 = 0$ can be rewritten as $y = x + 3$
   $\Rightarrow m = \tan\theta = 1$ and hence $\theta = 45°$.
8. (a) Equation of line in intercept form is $\dfrac{x}{a} + \dfrac{y}{a} = 1$
   ($\because$ Intercept has equal length)
   $\Rightarrow x + y = a$
   $\Rightarrow y = -x + a$
   $\Rightarrow$ slope $= -1$
9. (d) Let d_1, d_2, d_3, d_4 are distances of equations $x - y + 1 = 0$, $2x - y + 3 = 0$, $x + 2y - 2 = 0$ and $x + y - 2 = 0$ respectively from the origin.
   $$d_1 = \left|\frac{-0 + 1}{\sqrt{1^2 + (-1)^2}}\right| = \frac{1}{\sqrt{2}}$$
   $$d_2 = \left|\frac{2(0) - 0 + 3}{\sqrt{2^2 + (-1)^2}}\right| = \frac{3}{\sqrt{5}}$$
   $$d_3 = \left|\frac{1(0) + 2(0) - 2}{\sqrt{1^2 + 2^2}}\right| = \frac{2}{\sqrt{5}}$$
   $$d_4 = \left|\frac{0 + 0 - 2}{\sqrt{1^2 + 1^2}}\right| = \frac{2}{\sqrt{2}} = \sqrt{2}$$
   Hence, line corresponding to d_4 (1.414) is farthest from origin.
10. (c) Let intercept on x-axis and y-axis be a and b respectively so that the equation of line is
    $\dfrac{x}{a} + \dfrac{y}{b} = 1$
    But $a = b$ [given]
    so; $x + y = a$
    Also it passes through (2, 4) (given)
    Thus $2 + 4 = a$
    $\Rightarrow a = 6$
    Now, the reqd. equation of the straight line
    $x + y = 6$
    or, $x + y - 6 = 0$.
11. (d) We know that the equation of a line making intercepts a and b with x-axis and y-axis, respectively, is given by
    $\dfrac{x}{a} + \dfrac{y}{b} = 1.$
    Here we have $1 = \dfrac{a + 0}{2}$ and $2 = \dfrac{0 + b}{2}$,
    which give $a = 2$ and $b = 4$. Therefore, the required equation of the line is given by
    $\dfrac{x}{2} + \dfrac{y}{4} = 1$ or $2x + y - 4 = 0$
12. (c) Equations of lines are
    $\dfrac{x}{a} + \dfrac{y}{-b} = 1$ and $\dfrac{x}{b} + \dfrac{y}{-a} = 1$
    $\Rightarrow bx - ay = ab$ and $ax - by = ab$
    $\Rightarrow m_1 = \dfrac{b}{a}$ and $m_2 = \dfrac{a}{b}$ (slopes)
    $$\therefore \tan\theta = \frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{b}{a} \times \frac{a}{b}} = \frac{b^2 - a^2}{2ab}$$
13. (a) Equation of line AB is $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y - y_1}{x - x_1}$



    $\Rightarrow -\dfrac{2}{3} = \dfrac{y - 2}{x - 3}$
    $\Rightarrow -2(x - 3) = 3(y - 2)$
    $\Rightarrow 2x + 3y = 12$
14. (a) Let the line make intercept 'a' on x-axis. Then, it makes intercept '$2a$' on y-axis. Therefore, the equation of the line is given by
    $\dfrac{x}{a} + \dfrac{y}{2a} = 1$
    It passes through (1, 2), so, we have
    $\dfrac{1}{a} + \dfrac{2}{2a} = 1$ or $a = 2$
    Therefore, the required equation of the line is given by
    $\dfrac{x}{2} + \dfrac{y}{4} = 1$ or $2x + y = 4$
15. (c) Slope of the line through the points (–2, 6) and (4, 8) is,
    $m_1 = \dfrac{8 - 6}{4 - (-2)} = \dfrac{2}{6} = \dfrac{1}{3}$
    Slope of the line through the points (8, 12) and (x, 24) is :
    $m_2 = \dfrac{24 - 12}{x - 8} = \dfrac{12}{x - 8}$

Since, two lines are perpendicular,
$m_1 m_2 = -1$, which gives

$$\frac{1}{3} \times \frac{12}{x-8} = -1$$

$\Rightarrow -x + 8 = 4 \Rightarrow 8 - 4 = x \Rightarrow x = 4$

16. (c) Any point on the line $7x + 56y = 0$ is

$\left(x_1, -\frac{7x_1}{56}\right)$, *i.e.*, $\left(x_1, -\frac{x_1}{8}\right)$

$\therefore$ The perpendicular distance p and p' are

$$p = \frac{3x_1 - \frac{4x_1}{8}}{5} = \frac{x_1}{2}$$

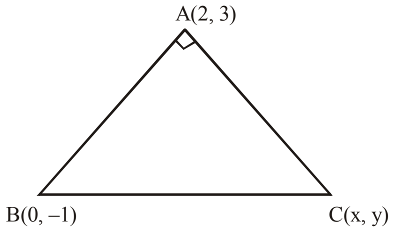
and $p' = \dfrac{5x_1 + \frac{12x_1}{8}}{13} = \dfrac{x_1}{2} \Rightarrow p = p'$

17. (b) Lines II and III are at right angles

$\left[\because \left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1\right]$

Lines I and II intersect at the point (1, 2) and (1, 2) does not belong to III. Hence, the lines are not concurrent, i.e., they form a right angled triangle.

18. (c) Slope of $AB = 2 \Rightarrow$ slope of $AC = -\frac{1}{2}$



$\Rightarrow \frac{y-3}{x-2} = -\frac{1}{2} \Rightarrow x + 2y - 8 = 0$ ....(i)

Also $x^2 + (y+1)^2 = 25$

$\Rightarrow (8-2y)^2 + (y+1)^2 = 25$ [from (i)]

$\Rightarrow y = 2$ *or* 4 and correspondingly $x = 4$ and $x = 0$.
Hence, C is (0, 4) or (4, 2).

19. (b) Slope of the line $ax + by = c$ is $\frac{-a}{b}$, and the slope of the line $a'x + b'y = c'$ is $\frac{-a'}{b'}$. The lines are perpendicular if $\left(\frac{-a}{b}\right)\left(\frac{-a'}{b'}\right) = -1$ or $aa' + bb' = 0$

20. (a) Here, m = tan 120° = tan (90 + 30°) = – cot 30° = $-\sqrt{3}$
and c = –5
So, the equation of the line is

$y = -\sqrt{3}\,x - 5$ [Using : $y = mx + c$]

$\Rightarrow \sqrt{3}\,x + y + 5 = 0$

21. (c) The equation of a line perpendicular to
$3x + 2y + 5 = 0$ is
$2x - 3y + \lambda = 0$ ...(i)
This passes through the point (3, 4).
$\therefore 3 \times 2 - 3 \times 4 + \lambda = 0 \Rightarrow \lambda = 6$
Putting $\lambda = 6$ in (i), we get $2x - 3y + 6 = 0$, which is the required equation.

22. (b) Only two point A (–1, – 7) and B (3, 4) satisfy the given equation of the line 3x – 4y = 25
Distance of A (– 1, – 7) from the origin O.

$= \sqrt{(0+1)^2 + (0+7)^2} = \sqrt{50} = 5\sqrt{2}$

Distance of B (3, – 4) from the origin O.

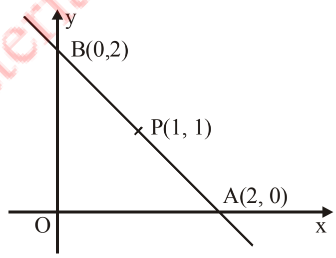
$= \sqrt{(0-3)^2 + (0+4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

The nearest point is (3, – 4)

23. (d) Let intercept on x-axis be a and that on y axis be b, the coordinate of these end points are (a, 0) and (b, 0).

Since, P(1, 1) is the mid point therefore $1 = \frac{a+0}{2}$ and

$1 = \frac{0+b}{2} \Rightarrow a = 2, b = 2.$



So, equation of straight line is $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{2} + \frac{y}{2} = 1 \Rightarrow x + y = 2$

24. (a) The given lines are

$y = (2 - \sqrt{3})\,x + 5$

and $y = (2 + \sqrt{3})\,x - 7$

Therefore, slope of first line = $m_1 = 2 - \sqrt{3}$ and slope of second line = $m_2 = 2 + \sqrt{3}$

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$$

$$= \left|\frac{2\sqrt{3}}{2}\right| = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3} = 60°$$

25. (a) If (x, y), (1, 2) and (–3, 4) are collinear then slope of line joining (x, 4) and (1, 2) is same as line joining points (1, 2) and (–3, 4) or line joining (x, 4) to (– 3, 4).

So, $\frac{2-y}{1-x} = \frac{4-2}{-3-1} = \frac{4-y}{-3-x}$

$\Rightarrow \frac{2-y}{1-x} = -\frac{1}{2} \Rightarrow \frac{y-2}{1-x} = \frac{1}{2}$

$\Rightarrow -4 + 2y = 1 - x \Rightarrow x + 2y - 5 = 0$

26. (d) Length of perpendicular from the origin on the straight line x + 2by – 2p = 0 is

$$\left|\frac{0 + 2b \times 0 - 2p}{\sqrt{1^2 + (2b)^2}}\right| = p$$

or $p = \left|\frac{-2p}{\sqrt{1^2 + 4b^2}}\right|$ or $p^2 = \frac{4p^2}{1+4b^2}$

$\Rightarrow \quad \frac{4}{1+4b^2} = 1$

$\Rightarrow \quad 1 + 4b^2 = 4$ or $4b^2 = 3 \Rightarrow b^2 = \frac{3}{4}$

$\Rightarrow \quad b = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \quad b = \frac{\sqrt{3}}{2}$ matches with the given option.

27. (a) Let the required line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

then $a + b = -1$ ....(ii)

(i) passes through (4, 3), $\Rightarrow \frac{4}{a} + \frac{3}{b} = 1$

$\Rightarrow 4b + 3a = ab$ ....(iii)

Eliminating b from (ii) and (iii), we get

$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1

$\therefore$ Equations of straight lines are

$\frac{x}{2} + \frac{y}{-3} = 1$ or $\frac{x}{-2} + \frac{y}{1} = 1$

28. (b) Let (h, k) be the coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$. Then, the slope of the perpendicular line is $\frac{k-3}{h-2}$. Again the slope of the given line $x + y - 11 = 0$ is -1

Using the condition of perpendicularity of lines, we have

$\left(\frac{k-3}{h-2}\right)(-1) = -1$

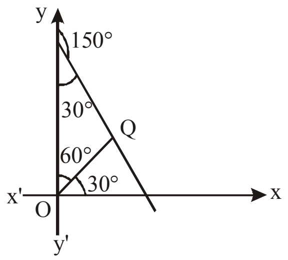
or $k - h = 1$ ...(i)

Since (h, k) lies on the given line, we have,

$h + k - 11 = 0$ or $h + k = 11$ ...(ii)

Solving (i) and (ii), we get $h = 5$ and $k = 6$. Thus (5, 6) are the required coordinates of the foot of the perpendicular.

29. (d) Here $p = 7$ and $\alpha = 30°$



$\therefore$ Equation of the required line is

$x \cos 30° + y \sin 30° = 7$

or $x\frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 7$

or $\sqrt{3}x + y = 14$

30. (c) The equation of a line making an angle θ with positive x-axis and cutting intercept c on y-axis is given by $y = \tan\theta \; x + c$

Here, $\theta = 135° \Rightarrow \tan\theta = -1$ and $c = -5$

$\therefore \; y = -x - 5 \Rightarrow x + y + 5 = 0$

31. (a) Let the required line by method $P + \lambda Q = 0$ be $(x - 3y + 1) + \lambda(2x + 5y - 9) = 0$

$\therefore$ Perpendicular from $(0, 0) = \sqrt{5}$ gives

$\frac{1 - 9\lambda}{\sqrt{(1+2\lambda)^2 + (5-3\lambda)^2}} = \sqrt{5}$

Squaring and simplifying, $(8\lambda - 7)^2 = 0 \Rightarrow \lambda = 7/8$

Hence the line required is

$(x - 3y + 1) + 7/8\,(2x + 5y - 9) = 0$

or $22x + 11y - 55 = 0 \Rightarrow 2x + y - 5 = 0$

32. (d) The two lines having the slopes m_1 and m_2 are perpendicular iff $m_1.m_2 = -1$

Now $a_1x + b_1y + c_1 = 0$

$\Rightarrow \; y = \frac{-a_1}{b_1}x - \frac{c_1}{b_1} \Rightarrow$ slope $(m_1) = \frac{-a_1}{b_1}$

Similarly, $a_2x + b_2y + c_2 = 0$

Gives the slope, $m_2 = \frac{-a_2}{b_2}$

Now, we know the lines $\perp$ when $m_1 \,.\, m_2 = -1$

$\Rightarrow \; \frac{-a_1}{b_1} \cdot \frac{-a_2}{b_2} = -1$

$\Rightarrow \; a_1a_2 = -b_1b_2 \Rightarrow a_1a_2 + b_1b_2 = 0.$

33. (a) Equation of line AB is $y - 3 = \frac{6-3}{7-3}(x - 3)$

$\Rightarrow \; 3x - 4y + 3 = 0 \Rightarrow \frac{x}{-1} + \frac{y}{\frac{3}{4}} = 1$

Hence, required length is $\sqrt{(-1)^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{4}.$

34. (b) The given line can be written in this form $(3 + 2\lambda)x + (-1 - 3\lambda)y + (5 - 4\lambda) = 0$

It will be parallel to y-axis, if

$-1 - 3\lambda = 0 \Rightarrow \lambda = -\frac{1}{3}.$

35. (a) Let the equation be $\frac{x}{a} + \frac{y}{-a} = 1$

$\Rightarrow \; x - y = a$

But it passes through $(-3, 2)$, hence $a = -3 - 2 = -5$.

Hence, the equation is $x - y + 5 = 0$.

36. (b) Mid point $\equiv (3, 2)$. Equation is $2x - y - 4 = 0$.

STATEMENT TYPE QUESTIONS

37. (d)

38. (d) Given: A = distance of point from x-axis

$A^2 = 2^2 + 3^2 = 4 + 9 = 13$

$B^2 = 3^2 + 1^2 = 9 + 1 = 10$

$C^2 = 1^2 + 2^2 = 1 + 4 = 5$

From above, we get

$B^2 = 10 = 2 \times 5 = 2C^2$

$\Rightarrow \quad B^2 = 2C^2 \qquad [\because C^2 = 5]$

and $\; 2A^2C^2 = 2.13.5 = 13.10 = 13B^2 \qquad [\because B^2 = 10]$

$\Rightarrow \quad 2A^2C^2 = 13B^2$

39. (a) I. Equation of line is

$y - 0 = m(x - 0)$

$\Rightarrow y = mx$

II. Equation of the x-axis is $y = 0$.

40. (c) Both are true.

41. (d) All are true statements.

42. (a) Both the given statements are true.

43. (c) 1

44. (c) Both the given statements are true.

45. (c) Both the given statements are true.

46. (c) Both the given statements are true.

47. (a) I. Let A, B and C having coordinates (a, b), (c, d) and {(a – c), (b – d)} respectively be the points

If these points are collinear then

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

On solving this expression we get

1. $\{a(d-b) - b(c-a)\} = 0$

$\Rightarrow \quad ad - ab - bc + ab = 0$

$\Rightarrow \quad bc - ad = 0$

II. Since the points are collinear.

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 3 & a & 1 \end{vmatrix} = 0$$

Expanding the above expression

$\Rightarrow \quad 1\begin{vmatrix} 4 & 1 \\ a & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 3 & a \end{vmatrix} = 0$

$\Rightarrow \quad (4 - a) - 2(2 - 3) + 1(2a - 12) = 0$

$\Rightarrow \quad 4 - a + 2 + 2a - 12 = 0$

$\Rightarrow \quad a - 6 = 0$

$\Rightarrow \quad a = 6$

Thus, coordinates of C are (3, 6).

Thus, BC $= \sqrt{(3-2)^2 + (6-4)^2}$

$= \sqrt{1+4} = \sqrt{5}$ unit

48. (c) I. Centroid of a triangle is a point where medians meet.

II. If value of area after calculations is negative then we take its absolute value.

49. (a) II. Product of slopes = –1

50. (d) $y = \frac{3}{4}x + \frac{5}{2} \;\Rightarrow\;$ Slope $= \frac{3}{4}$

Also, $3x - 4y = -10$

$\Rightarrow \quad \frac{x}{-\frac{10}{3}} + \frac{y}{\frac{5}{2}} = 1$

$\Rightarrow$ x-intercept $= \frac{-10}{3}$ and y-intercept $= \frac{5}{2}$

51. (c) $\sqrt{3}x + y - 8 = 0$

$\Rightarrow \quad \frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$ (on dividing by 2)

$\Rightarrow \quad \cos 30°\, x + \sin 30° y = 4$

52. (b) I. Slope $= \frac{4-(-2)}{-1-3} = \frac{-3}{2}$

II. Slope $= \frac{-2-(-2)}{7-3} = \frac{0}{4} = 0$

III. Slope $= \frac{4-(-2)}{3-3} = \frac{6}{0}$ which is not defined.

INTEGER TYPE QUESTIONS

53. (a) We have the points $A(x, -1)$, $B(2, 1)$, $C(4, 5)$.

A, B, C are collinear if the slope of AB = Slope of BC.

Slope of $AB = \frac{1+1}{2-x} = \frac{2}{2-x}$;

Slope of $BC = \frac{5-1}{4-2} = \frac{4}{2} = 2$

$\therefore \frac{2}{2-x} = 2$ or $2 - x = 1$ or $x = 1$

54. (d) The given line is $12(x + 6) = 5(y - 2)$

$\Rightarrow \quad 12x + 72 = 5y - 10$

or $\quad 12x - 5y + 72 + 10 = 0$

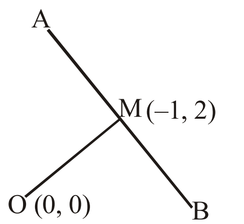
$\Rightarrow \quad 12x - 5y + 82 = 0$

The perpendicular distance from (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{(ax_1 + by_1 + c)}{\sqrt{a^2 + b^2}}$.

The point (x_1, y_1) is (–1, 1), therefore, perpendicular distance from (–1, 1) to the line $12x - 5y + 82 = 0$ is

$= \frac{|-12 - 5 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{65}{\sqrt{144+25}} = \frac{65}{\sqrt{169}} = \frac{65}{13} = 5$

55. (b) Let the perpendicular OM is drawn from the origin to AB.



M is the foot of the perpendicular

Slope of $OM = \frac{2-0}{-1-0} = \frac{2}{-1}$;

Slope of $AB = m$

$OM \perp AB \;\; \therefore m \times (-2) = -1 \;\; \therefore m = \frac{1}{2}$

$M(-1, 2)$ lies on AB whose equation is

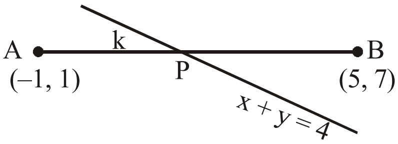
$y = mx + c$ or $y = \frac{1}{2}x + c$

$2 = \frac{1}{2} \times (-1) + c \Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore \quad m = \frac{1}{2}$ or $c = \frac{5}{2} \Rightarrow m + c = \frac{6}{2} = 3$

56. (a) Any line parallel to x-axis of the form $y = p$
i.e. coefficient of $x = 0$
$\therefore$ In equation $(k-3)x-(4-k^2)\,y+k^2-7k+6=0$
Coefficient of $x = k-3 = 0 \;\therefore\; k = 3$

57. (a) The line joining the points A (–1, 1) and B(5, 7) is divided by $P(x, y)$ in the ratio $k : 1$



$\therefore$ Point P is $\left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$

This point lies on the line $x + y = 4$

$\therefore \frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4$

$\Rightarrow 5k-1+7k+1 = 4k+4 \Rightarrow 8k = 4 \Rightarrow k = \frac{1}{2}$

$\therefore$ P divides AB in the ratio 1 : 2

58. (b) The given points are $A(h, 0)$, $B(a, b)$, $C(0, k)$, they lie on the same plane.
$\therefore$ Slope of AB = Slope of BC

$\therefore$ Slope of $AB = \frac{b-0}{a-h} = \frac{b}{a-h}$;

Slope of $BC = \frac{k-b}{0-a} = \frac{k-b}{-a}$

$\therefore \frac{b}{a-h} = \frac{k-b}{-a}$ or by cross multiplication

$-ab = (a-h)(k-b)$
or $-ab = ak - ab - hk + hb$
or $0 = ak - hk + hb$
or $ak + hb = hk$

Dividing by hk $\Rightarrow \frac{ak}{hk} + \frac{hb}{hk} = 1$ or $\frac{a}{h} + \frac{b}{k} = 1$

Hence proved.

59. (b) Slope of line through (2, 5) and (x, 3) is $\frac{3-5}{x-2}$

We have, $\frac{3-5}{x-2} = 2 \Rightarrow x = 1$

60. (d) Let A(3, y), B(2, 7), C(–1, 4) and D(0, 6) be the given points.

m_1 = slope of AB $= \frac{7-y}{2-3} = (y-7)$

m_2 = slope of CD $= \frac{6-4}{0-(-1)} = 2$

Since AB and CD are parallel
$\therefore \; m_1 = m_2 \Rightarrow y = 9.$

61. (c) Given equation is

$\sqrt{3}x + y - 8 = 0$

Divide this by $\sqrt{(\sqrt{3})^2 + 1^2} = 2$,

we get, $\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$

Which is in the normal form. Hence, p = 4.

62. (c) Given parallel lines are
$3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$

Required distance $= \frac{|7-5|}{\sqrt{(3)^2+(-4)^2}} = \frac{2}{5}$

$\Rightarrow a = 2, b = 5$

ASSERTION - REASON TYPE QUESTIONS

63. (b) Assertion is correct and Reason is also correct

64. (b) Both the Assertion and Reason are true.

65. (c) Assertion is false and Reason is true.

Assertion: Slope $= \frac{4-(-2)}{3-3} = \frac{6}{0}$ which is not defined.

66. (b) Mid-point of AC $= \left(\frac{1}{2}, \frac{2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Mid-point of BD $= \left(\frac{1}{2}, 1\right)$

67. (a) **Assertion:**
$a_1 = 1, b_1 = 2, c_1 = -3$
$a_2 = -3, b_2 = -6, c_2 = 9$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{-1}{3}$

So, the given lines are coincident.

68. (c) Assertion is correct but Reason is incorrect.
Correct Reason is given lines are perpendicular, if $a_1a_2 + b_1b_2 = 0$.

69. (c) Assertion is correct. Reason is incorrect.

Reason: The slope of the given line is $\frac{-a}{b}$.

70. (d) Assertion is incorrect Reason is correct.
Assertion: The equation of a line parallel to the line $ax + by + c = 0$ is $ax + by + \lambda = 0$ where λ is a constant.

71. (a) **Assertion:** A = 3, B = – 4

$C_1 = 9, C_2 = -\frac{15}{2}$

$d = \frac{\left|-\frac{15}{2}-9\right|}{\sqrt{9+16}} = \frac{\left|-\frac{33}{2}\right|}{5} = \frac{33}{10}$

72. (b) Both are correct.

CRITICALTHINKING TYPE QUESTIONS

73. (a) Let the point of intersection divide the line segment joining points, (3, – 1) and (8, 9) in k : 1 ratio then

The point is $\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$

Since this point lies on the line $y - x + 2 = 0$

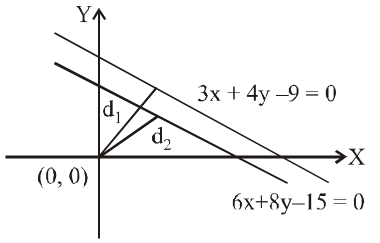
We have, $\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$

$\Rightarrow \frac{9k-1-8k-3}{k+1} + 2 = 0 \Rightarrow \frac{k-4}{k+1} + 2 = 0$

$\Rightarrow k-4+2k+2=0 \Rightarrow 3k-2=0$

$k=\frac{2}{3}:1$ i.e. $2:3$

74. (b) Let d_1 and d_2 be the distances of two lines $3x+4y-9=0$ and $6x+8y-15=0$ respectively from origin.



$\therefore \quad d_1=\frac{|3(0)+4(0)-9|}{\sqrt{3^2+4^2}} \Rightarrow d_1=\frac{9}{5}$

and $d_2=\frac{|6(0)+8(0)-15|}{\sqrt{36+64}}=\frac{15}{10}=\frac{3}{2}$

$\therefore$ distance between these lines is, $d=d_1-d_2$

$\Rightarrow \quad d=\frac{9}{5}-\frac{3}{2}=\frac{18-15}{10}=\frac{3}{10}$

75. (d) Given line is $3x+y=3$

Let the equation of line which is perpendicular to above line is

$x-3y+\lambda=0$.

This line is passing through point (2, 2)

$\therefore \quad 2-3\times 2+\lambda=0$

$\Rightarrow \quad 2-6+\lambda=0 \Rightarrow \lambda=4$

$\therefore$ Equation of line is $x-3y+4=0$

$\Rightarrow \quad 3y=x+4 \Rightarrow y=\frac{1}{3}x+\frac{4}{3}$

Compare the above equation with $y=mx+c$,

We get $c=\frac{4}{3}$

Thus, y–intercept is $\frac{4}{3}$.

76. (c) **Note:** If the vertices of a triangle are $A(a_1, b_1)$, $B(a_2, b_2)$ and $C(a_3, b_3)$, then the area of the triangle ABC

$$=\frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

Here in the given question:

we have A(x, 0), B(1, 1), C(0, 2).

and $\frac{1}{2}\begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix}=4$

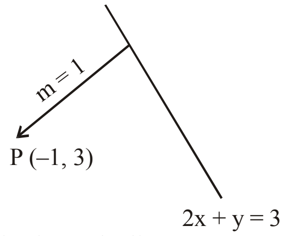
$\Rightarrow \frac{1}{2}[x(1-2)+1(2)]=4$

$\Rightarrow -x+2=8 \Rightarrow x=-6.$

77. (c) The equation of the line through (–1, 3) and having the slope 1 is

$$\frac{x+1}{\cos\theta}=\frac{y-3}{\sin\theta}=r.$$

Any point on this line at a distance r from $P(-1, 3)$ is $(-1+r\cos\theta,\ 3+r\sin\theta)$



This point is on the line $2x+y=3$ if

$2(-1+r\cos\theta)+3+r\sin\theta=3$ ...(i)

But $\tan\theta=1$; $\Rightarrow \theta=45°$

(i) becomes,

$-2+2r.\frac{1}{\sqrt{2}}+3+r.\frac{1}{\sqrt{2}}=3$

$\Rightarrow \frac{3r}{\sqrt{2}}=2; \quad r=\frac{2\sqrt{2}}{3}$

Hence the required distance $=\frac{2\sqrt{2}}{3}$.

78. (a) Given equation of straingh lines are

$x+2y-9=0,\ 3x+5y-5=0$

and $ax+by-1=0$

They are concurrent, if

$-5+5b-2(-3+5a)-9(3b-5a)=0$

$\Rightarrow -5+5b+6-10a-27b+45a=0$

$\Rightarrow 35a-22b+1=0$

Thus, given straight lines are concurrent if the straight line $35x-22y+1=0$ passes through (a, b).

79. (a) Let (h, k) be the point of reflection of the given point (4, –13) about the line $5x+y+6=0$. The mid-point of the line segment joining points (h, k) and (4, –13) is given by

$$\left(\frac{h+4}{2},\frac{k-13}{2}\right)$$

This point lies on the given line, so we have

$$5\left(\frac{h+4}{2}\right)+\frac{k-13}{2}+6=0$$

or $\quad 5h+k+19=0$ …(i)

Again the slope of the line joining points (h, k) and (4, –13) is given by $\frac{k+13}{h-4}$. This line is perpendicular to the given line and hence

$$(-5)\left(\frac{k+13}{h-4}\right)=-1$$

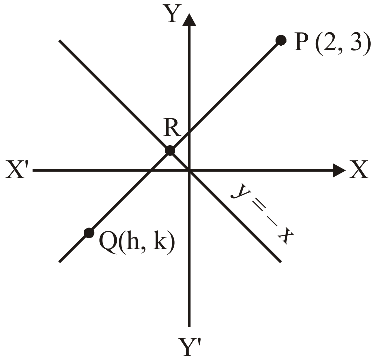
This gives $5k+65=h-4$

or $\quad h-5k-69=0$ …(ii)

On solving (i) and (ii), we get $h=-1$ and $k=-14$. Thus the point (–1, – 14) is the reflection of the given point.

80. (a) (1, – 2)

81. (a) Let there be a point P(2,3) on cartesian plane. Image of this point in the line $y=-x$ will lie on a line which is perpendicular to this line and distance of this point from y = –x will be equal to distance of the image from this line.



Let Q be the image of p and let the co-ordinate of Q be (b, k)

Slope of line y = –x is –1

Line joining P, Q will be perpendicular to y = –x so, its slope = 1.

Let the equation of the line be y = x + c since this passes through point (2, 3)

$3 = 2 + c \Rightarrow c = 1$

and the equation y = x + 1

The point of intersection R lies in the middle of P & Q.

Point of intersection of line $y = -x$ and $y = x + 1$ is

$2y = 1, \Rightarrow y = \frac{1}{2}$ and $x = -\frac{1}{2}$

Hence, $\frac{h+2}{2} = -\frac{1}{2}$ and $\frac{k+3}{2} = \frac{1}{2}$

$\Rightarrow \quad h = -3$ and $k = -2$

So, the image of the point (2, 3) in the line y = – x is (–3, –2).

82. (c) Equation of line is ax + by – p = 0, then length of perpendicular, from the origin is

$$p = \left|\frac{a \times 0 + b \times 0 - p}{\sqrt{a^2 + b^2}}\right| \text{ or } p = \left|\frac{-p}{\sqrt{a^2+b^2}}\right|$$

$\Rightarrow a^2 + b^2 = 1$

$b = \frac{\sqrt{3}}{2}$ or $b^2 = \frac{3}{4} \Rightarrow a^2 + \frac{3}{4} = 1$

$a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$

[$a = -\frac{1}{2}$ not taken since angle is with + ve direction to x-axis.]

Equation is $\frac{1}{2}x + \frac{\sqrt{3}}{2}y = p$ or $x\cos 60° + y \sin 60° = p$

Angle = 60°

83. (c) One vertex of square is (– 4, 5) and equation of one diagonal is 7x – y + 8 = 0

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be y = mx + c where m is the slope of the line and c is the y-intercept.

Since this line passes through (– 4, 5)

$\therefore \quad 5 = -4m + c \quad \ldots$(i)

Since this line is at right angle to the line 7x – y + 8 = 0 or y = 7x + 8, having slope = 7,

$\therefore \quad 7 \times m = -1$ or $m = \frac{-1}{7}$

Putting this value of m in equation (i) we get

$$5 = -4 \times \left(\frac{-1}{7}\right) + c$$

or $5 = \frac{4}{7} + c$ or $c = 5 - \frac{4}{7} = \frac{31}{7}$

Hence equation of the other diagonal is

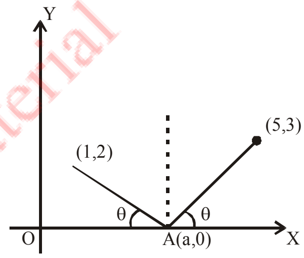
$$y = -\frac{1}{7}x + \frac{31}{7}$$

or $\quad 7y = -x + 31$

or $\quad x + 7y - 31 = 0$

or $\quad x + 7y = 31.$

84. (a) Let the coordinates of A be (a, 0). Then the slope of the reflected ray is



$\frac{3-0}{5-a} = \tan\theta$ (say) $\quad \ldots$(i)

Then the slope of the incident ray

$= \frac{2-0}{1-a} = \tan(\pi - \theta)$. $\quad \ldots$(ii)

from (i) and (ii) $\quad \tan\theta + \tan(\pi - \theta) = 0$

$$\Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0 \Rightarrow 3 - 3a + 10 - 2a = 0$$

$$\Rightarrow a = \frac{13}{5}$$

Thus, the co-ordinates of A are $\left(\frac{13}{5}, 0\right)$.

85. (b) $AB = 3\sqrt{2}$, $AC = 4\sqrt{2}$, $BC = 5\sqrt{2}$

$\therefore \frac{AB}{AC} = \frac{3}{4}$. That is the internal bisector of angle A cuts the side BC in ratio 3 : 4 at D. The coordinates of D are

$$\left(\frac{4\times 4 + 3\times 5}{4+3}, \frac{4\times -2 + 3\times 5}{4+3}\right) \equiv \left(\frac{31}{7}, 1\right)$$

Slope of AD = 0

$\therefore$ Equation of perpendicular from C(5, 5) to AD is x = 5

86. (b) Since the line L has intercepts a and b on the coordinate axes, therefore its equation is

$\frac{x}{a} + \frac{y}{b} = 1 \quad \ldots$(i)

When the axes are rotated, its equation with respect to the new axes and same origin will become

$\frac{x}{p}+\frac{y}{q}=1$ ...(ii)

In both the cases, the length of the perpendicular from the origin to the line will be same.

$\therefore \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$ or $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

87. (a) Third side passes through (1, – 10), so let its equation be $y+10=m(x-1)$

If it makes equal angle, say θ with given two sides, then

$\tan\theta=\frac{m-7}{1+7m}=\frac{m-(-1)}{1+m(-1)} \Rightarrow m=-3$ or $1/3$

Hence possible equations of third side are

$y+10=-3(x-1)$ and $y+10=\frac{1}{3}(x-1)$

or $3x+y+7=0$ and $x-3y-31=0$

88. (a) If the lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore m_1=m_2 \Rightarrow -\frac{p(p^2+1)}{-1}=-\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2+1)(p+1)=0 \Rightarrow p=-1$

$\therefore p$ can have exactly one value.

89. (c) On comparing given equations with $ax+by+c=0$

We get $a_1=4$, $a_2=3$, $b_1=-3$, $b_2=-4$

Now $a_1a_2+b_1b_2=(4\times3+3\times4)=24>0$ (Positive)

Since, $a_1a_2+b_1b_2$ is +ve

∴ Origin lies in obtuse angle

For acute angle, we find the bisector

Now, equation of bisectors of given lines are

$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

The equation of the bisector is

$\left[\frac{4x-3y+7}{5}\right]=-\left[\frac{3x-4y+14}{5}\right] \Rightarrow x-y+3=0$

90. (c) Here, c = –1 and m = tan θ = tan 45° = 1

(Since the line is equally inclined to the axes, so θ = 45°)

Also, m = tan 135° = –1 ⇒ m = ± 1

∴ θ = 45° and 135°

Hence, equation of straight line is $y=\pm(1\cdot x)-1$

⇒ $x-y-1=0$ and $x+y+1=0$

91. (c) Here, D(1, 1), therefore, equation of line AD is given by $2x+y-3=0$. Thus, the line perpendicular to AD is $x-2y+k=0$ and it passes through B, so k = 0. Hence, required equation is $x-2y=0$.

92. (c) The lines passing through the intersection of the lines $ax+2by+3b=0$ and $bx-2ay-3a=0$ is

$ax+2by+3b+\lambda(bx-2ay-3a)=0$

$\Rightarrow (a+b\lambda)x+(2b-2a\lambda)y+3b-3\lambda a=0$ ... (i)

Line (i) is parallel to x-axis,

$\therefore a+b\lambda=0 \Rightarrow \lambda=\frac{-a}{b}=0$

Put the value of λ in (i),

$ax+2by+3b-\frac{a}{b}(bx-2ay-3a)=0$

$y\left(2b+\frac{2a^2}{b}\right)+3b+\frac{3a^2}{b}=0,$

$y\left(\frac{2b^2+2a^2}{b}\right)=-\left(\frac{3b^2+3a^2}{b}\right)$

$y=\frac{-3(a^2+b^2)}{2(b^2+a^2)}=\frac{-3}{2},\ y=-\frac{3}{2}$

So, it is $\frac{3}{2}$ unit below x-axis.

93. (a) By direct formulae,

$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

$\frac{3x+4y-7}{\sqrt{3^2+4^2}}=\pm\frac{12x+5y+17}{\sqrt{(12)^2+(5)^2}}$

$\frac{3x+4y-7}{5}=\pm\frac{12x+5y+17}{13}$.

94. (a) The equations of the bisectors of the angles between the lines are $\frac{x-2y+4}{\sqrt{1+4}}=\pm\frac{4x-3y+2}{\sqrt{16+9}}$

Taking positive sign, then

$(4-\sqrt{5})x-(3-2\sqrt{5})y-(4\sqrt{5}-2)=0$ ... (i)

and negative sign gives

$(4+\sqrt{5})x-(2\sqrt{5}+3)y+(4\sqrt{5}+2)=0$ ... (ii)

Let θ be the angle between the line (i) and one of the given line, then

$\tan\theta=\left|\frac{\frac{1}{2}-\frac{4-\sqrt{5}}{3-2\sqrt{5}}}{1+\frac{1}{2}\cdot\frac{4-\sqrt{5}}{3-2\sqrt{5}}}\right|=\sqrt{5}+2>1$

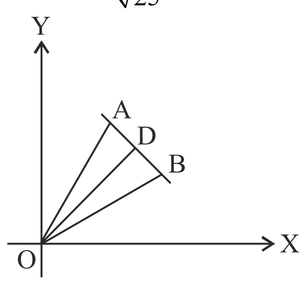
Hence, the line (i) bisects the obtuse angle between the given lines.

95. (a) $L \equiv 2x+3y-4=0$, $L_{(-6,2)}=-12+6-4<0$

$L'=6x+9y+8=0$, $L'_{(-6,2)}=-36+18+8<0$

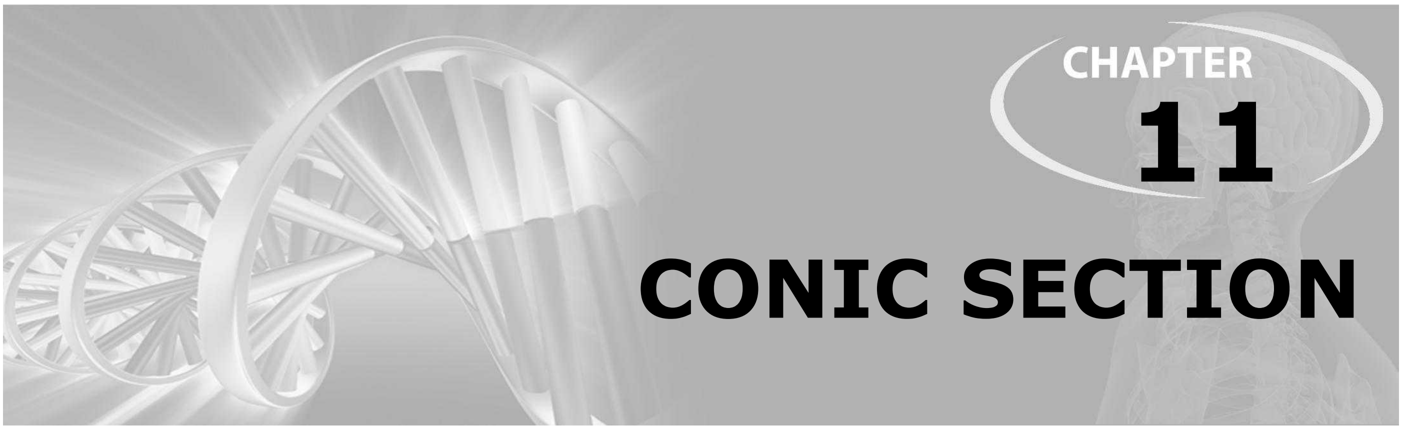
Hence, the point is below both the lines.

96. (b) OA = OB = 9, OD = $\frac{15}{\sqrt{25}}=3$



Therefore, $AB=2AD=2\sqrt{81-9}=2\sqrt{72}=12\sqrt{2}$

Hence, $\Delta=\frac{1}{2}(3\times12\sqrt{2})=18\sqrt{2}$ sq. units



CHAPTER 11

CONIC SECTION

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The equation of the circle which passes through the point (4, 5) and has its centre at (2, 2) is
(a) $(x-2)+(y-2)=13$ (b) $(x-2)^2+(y-2)^2=13$
(c) $(x)^2+(y)^2=13$ (d) $(x-4)^2+(y-5)^2=13$

2. Point (1, 2) relative to the circle $x^2+y^2+4x-2y-4=0$ is a/an
(a) exterior point
(b) interior point, but not centre
(c) boundary point
(d) centre

3. A conic section with eccentricity e is a parabola if:
(a) $e=0$ (b) $e<1$ (c) $e>1$ (d) $e=1$

4. For the ellipse $3x^2+4y^2=12$ length of the latus rectum is:
(a) 3 (b) 4 (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

5. The focal distance of a point on the parabola $y^2=12x$ is 4. What is the abscissa of the point ?
(a) 1 (b) -1
(c) $2\sqrt{3}$ (d) -2

6. What is the difference of the focal distances of any point on the hyperbola?
(a) Eccentricity
(b) Distance between foci
(c) Length of transverse axis
(d) Length of semi-transverse axis

7. The equation of an ellipse with foci on the x-axis is
(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (b) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$
(c) $\frac{x}{a}+\frac{y}{b}=1$ (d) $\frac{a}{x}+\frac{b}{y}=1$

8. Length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
(a) $\frac{b^2}{a^2}$ (b) $\frac{2b}{a}$
(c) $\frac{2b^2}{a}$ (d) $\frac{2a^2}{b}$

9. The equation of a hyperbola with foci on the x-axis is
(a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (b) $\frac{x}{a}-\frac{y}{b}=1$
(c) $\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$ (d) $\frac{a}{x}-\frac{b}{y}=1$

10. Length of the latus rectum of the hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is
(a) $\frac{b^2}{a}$ (b) $\frac{2b^2}{a}$ (c) $\frac{a^2}{b}$ (d) $\frac{2a^2}{b}$

11. What is the length of the smallest focal chord of the parabola $y^2=4ax$?
(a) a (b) 2a (c) 4a (d) 8a

12. The equation of the hyperbola with vertices (3, 0), (–3, 0) and semi-latus rectum 4 is given by :
(a) $4x^2-3y^2+36=0$ (b) $4x^2-3y^2+12=0$
(c) $4x^2-3y^2-36=0$ (d) $4x^2+3y^2-25=0$

13. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is
(a) $x^2-y^2=32$ (b) $\frac{x^2}{4}-\frac{y^2}{9}=1$
(c) $2x-3y^2=7$ (d) None of these

14. If the equation of a circle is $(4a-3)x^2+ay^2+6x-2y+2=0$, then its centre is
(a) (3, –1) (b) (3, 1) (c) (–3, 1) (d) None of these

15. The equation of the parabola with vertex at origin, which passes through the point (–3, 7) and axis along the x-axis is
(a) $y^2=49x$ (b) $3y^2=-49x$
(c) $3y^2=49x$ (d) $x^2=-49y$

16. The length of the semi-latus rectum of an ellipse is one third of its major axis, its eccentricity would be

(a) $\frac{2}{3}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

17. The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is

(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$

(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

18. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

19. Eccentricity of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if it passes through point (9, 5) and (12, 4) is

(a) $\sqrt{3/4}$ (b) $\sqrt{4/5}$ (c) $\sqrt{5/6}$ (d) $\sqrt{6/7}$

20. The equation of the ellipse with focus at (± 5, 0) and $x = \frac{36}{5}$ as one directrix is

(a) $\frac{x^2}{36}+\frac{y^2}{25}=1$ (b) $\frac{x^2}{36}+\frac{y^2}{11}=1$

(c) $\frac{x^2}{25}+\frac{y^2}{11}=1$ (d) None of these

21. The foci of the ellipse $25(x+1)^2+9(y+2)^2=225$ are at :

(a) (–1, 2) and (–1, –6) (b) (–2, 1) and (–2, 6)

(c) (–1, –2) and (–2, –1) (d) (–1, –2) and (–1, –6)

22. The eccentricity of the hyperbola $x^2-3y^2=2x+8$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

23. The equation of the hyperbola with vertices at (0, ±6) and $e=\frac{5}{3}$ is

(a) $\frac{x^2}{36}-\frac{y^2}{64}=1$ (b) $\frac{y^2}{36}-\frac{x^2}{64}=1$

(c) $\frac{x^2}{64}-\frac{y^2}{36}=1$ (d) $\frac{y^2}{64}-\frac{x^2}{36}=1$

24. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is:

(a) $4x^2+3y^2=1$ (b) $3x^2+4y^2=12$

(c) $4x^2+3y^2=12$ (d) $3x^2+4y^2=1$

25. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

26. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is

(a) $5x^2+3y^2-48=0$ (b) $3x^2+5y^2-15=0$

(c) $5x^2+3y^2-32=0$ (d) $3x^2+5y^2-32=0$

27. The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by :

(a) $x^2-3y^2=3$ (b) $3x^2-y^2=3$

(c) $-x^2+3y^2=3$ (d) $-3x^2+y^2=3$

28. For what value of k, does the equation

$9x^2+y^2=k(x^2-y^2-2x)$

represent equation of a circle ?

(a) 1 (b) 2 (c) –1 (d) 4

29. The eccentricity of the ellipse whose major axis is three times the minor axis is:

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{2}{\sqrt{3}}$

30. The focal distance of a point on the parabola $y^2 = 8x$ is 4. Its ordinates are:

(a) ±1 (b) ±2 (c) ±3 (d) ±4

31. If the eccentricity and length of latus rectum of a hyperbola are $\frac{\sqrt{13}}{3}$ and $\frac{10}{3}$ units respectively, then what is the length of the transverse axis?

(a) $\frac{7}{2}$ unit (b) 12 unit (c) $\frac{15}{2}$ unit (d) $\frac{15}{4}$ unit

32. The equation of a circle with centre at (1, 0) and circumference 10π units is

(a) $x^2+y^2-2x+24=0$ (b) $x^2+y^2-x-25=0$

(c) $x^2+y^2-2x-24=0$ (d) $x^2+y^2+2x+24=0$

33. If the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$, then

(a) transverse axis is along x-axis of length 6

(b) transverse axis is along y-axis of length 8

(c) conjugate axis is along y-axis of length 6

(d) None of the above

34. The length of transverse axis of the hyperbola $3x^2 - 4y^2 = 32$, is

(a) $\frac{8\sqrt{2}}{\sqrt{3}}$ (b) $\frac{16\sqrt{2}}{\sqrt{3}}$ (c) $\frac{3}{32}$ (d) $\frac{64}{3}$

35. The length of the transverse axis along x-axis with centre at origin of a hyperbola is 7 and it passes through the point (5, – 2). Then, the equation of the hyperbola is

(a) $\frac{4}{49}x^2-\frac{196}{51}y^2=1$ (b) $\frac{49}{4}x^2-\frac{51}{196}y^2=1$

(c) $\frac{4}{49}x^2-\frac{51}{196}y^2=1$ (d) None of these

36. The equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13, is
(a) $25x^2 - 144y^2 = 900$ (b) $144x^2 - 25y^2 = 900$
(c) $144x^2 + 25y^2 = 900$ (b) $25x^2 + 144y^2 = 900$

37. Which one of the following points lies outside the ellipse $(x^2/a^2) + (y^2/b^2) = 1$?
(a) (a, 0) (b) (0, b)
(c) (– a, 0) (d) (a, b)

38. The equation of an ellipse with one vertex at the point (3, 1), the nearer focus at the point (1, 1) and $e = \frac{2}{3}$ is :
(a) $\frac{(x+3)^2}{36} + \frac{(y-1)^2}{20} = 1$ (b) $\frac{(x-3)^2}{20} + \frac{(y+1)^2}{36} = 1$
(c) $\frac{(x-3)^2}{36} + \frac{(y+1)^2}{20} = 1$ (d) $\frac{(x-3)^2}{36} + \frac{(y-1)^2}{20} = 1$

39. The vertex of the parabola $(x-4)^2 + 2y = 9$ is :
(a) (2, 8) (b) (7, 2) (c) $\left(4, \frac{9}{2}\right)$ (d) $\left(-4, -\frac{9}{2}\right)$

40. The equations of the lines joining the vertex of the parabola $y^2 = 6x$ to the points on it which have abscissa 24 are
(a) $y \pm 2x = 0$ (b) $2y \pm x = 0$
(c) $x \pm 2y = 0$ (d) $2x \pm y = 0$

41. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and e_2 is the eccentricity of the hyperbola passing through the foci of the ellipse and $e_1 e_2 = 1$, then equation of the hyperbola is :
(a) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (b) $\frac{x^2}{16} - \frac{y^2}{9} = -1$
(c) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (d) $\frac{x^2}{9} - \frac{y^2}{36} = 1$

42. A circle has radius 3 and its centre lies on the line $y = x - 1$. The equation of the circle, if it passes through (7, 3), is
(a) $x^2 + y^2 + 8x - 6y + 16 = 0$
(b) $x^2 + y^2 - 8x + 6y + 16 = 0$
(c) $x^2 + y^2 - 8x - 6y - 16 = 0$
(d) $x^2 + y^2 - 8x - 6y + 16 = 0$

43. The equation $y^2 + 3 = 2(2x + y)$ represents a parabola with the vertex at
(a) $\left(\frac{1}{2}, 1\right)$ and axis parallel to y-axis
(b) $\left(1, \frac{1}{2}\right)$ and axis parallel to x-axis
(c) $\left(\frac{1}{2}, 1\right)$ and focus at $\left(\frac{3}{2}, 1\right)$
(d) $\left(1, \frac{1}{2}\right)$ and focus at $\left(\frac{3}{2}, 1\right)$

44. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

45. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
(a) $d^2 + (3b - 2c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$
(c) $d^2 + (2b - 3c)^2 = 0$ (d) $d^2 + (2b + 3c)^2 = 0$

46. The eccentricity of the curve $2x^2 + y^2 - 8x - 2y + 1 = 0$ is:
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

47. The focus of the curve $y^2 + 4x - 6y + 13 = 0$ is
(a) (2, 3) (b) (–2, 3)
(c) (2, –3) (d) (–2, –3)

48. The eccentricities of the ellipse $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$, $\alpha > \beta$; and $\frac{x^2}{9} + \frac{y^2}{16} = 1$ are equal. Which one of the following is correct ?
(a) $4\alpha = 3\beta$ (b) $\alpha\beta = 12$
(c) $4\beta = 3\alpha$ (d) $9\alpha = 16\beta$

49. The vertex of the parabola $x^2 + 8x + 12y + 4 = 0$ is:
(a) (–4, 1) (b) (4, –1) (c) (–4, –1) (d) (4, 1)

50. The equation of the conic with focus at (1, – 1) directrix along $x - y + 1 = 0$ and with eccentricity $\sqrt{2}$ is
(a) $x^2 - y^2 = 1$ (b) $xy = 1$
(c) $2xy - 4x + 4y + 1 = 0$ (d) $2xy + 4x - 4y - 1 = 0$

51. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is
(a) (3, –4) (b) (–3, 4) (c) (–3, –4) (d) (3, 4)

52. A circle of radius 5 touches another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5) then its equation is :
(a) $x^2 + y^2 + 18x + 16y + 120 = 0$
(b) $x^2 + y^2 - 18x - 16y + 120 = 0$
(c) $x^2 + y^2 - 18x + 16y + 120 = 0$
(d) None of these

53. The circle $x^2 + y^2 - 8x + 4y + 4 = 0$ touches :
(a) x-axis only (b) y-axis only
(c) both (a) and (b) (d) None of these

54. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct point, then
(a) $r > 2$ (b) $2 < r < 8$ (c) $r < 2$ (d) $r = 2$

55. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is
(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3 (d) 2

56. The conic represented by
$x = 2 (\cos t + \sin t), y = 5 (\cos t - \sin t)$ is
(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

57. Equation of the ellipse whose axes are along the coordinate axes, vertices are $(\pm 5, 0)$ and foci at $(\pm 4, 0)$ is
(a) $\frac{x^2}{16}+\frac{y^2}{9}=1$ (b) $\frac{x^2}{25}+\frac{y^2}{9}=1$
(c) $\frac{x^2}{4}+\frac{y^2}{25}=1$ (d) $\frac{x^2}{25}+\frac{y^2}{16}=1$

58. Equation of the hyperbola whose directrix is $2x + y = 1$, focus $(1, 2)$ and eccentricity $\sqrt{3}$ is
(a) $7x^2 - 2y^2 + 12xy - 2x + 9y - 22 = 0$
(b) $5x^2 - 2y^2 + 10xy + 2x + 5y - 20 = 0$
(c) $4x^2 + 8y^2 + 8xy + 2x - 2y + 10 = 0$
(d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

59. I. Equation of conjugate hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
II. Length of latus rectum of the conjugate hyperbola is $\frac{2a^2}{b}$.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

60. I. The straight line passing through the focus and perpendicular to the directrix is called the axis of the conic section.
II. The points of intersection of the conic section and the axis are called vertices of the conic section.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

61. An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.
I. The two fixed points are called the foci of the ellipse.
II. The mid point of the line segment joining the foci is called the centre of the ellipse.
III. The end points of the major axis are called the vertices of the ellipse.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) Only I and III are correct.
(d) All are correct.

62. If the equation of the circle is $x^2 + y^2 - 8x + 10y - 12 = 0$, then
I. Centre of the circle is $(4, -5)$.
II. Radius of the circle is $\sqrt{53}$.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

63. If equation of the ellipse is $\frac{x^2}{100}+\frac{y^2}{400}=1$, then
I. Vertices of the ellipse are $(0, \pm 20)$
II. Foci of the ellipse are $(0, \pm 10\sqrt{3})$
III. Length of major axis is 40.
IV. Eccentricity of the ellipse is $\frac{\sqrt{3}}{2}$.
(a) I and II are true. (b) III and IV are true.
(c) II, III, IV are true. (d) All are true.

64. If the equation of the hyperbola is $\frac{y^2}{9}-\frac{x^2}{27}=1$, then
I. the coordinates of the foci are $(0, \pm 6)$
II. the length of the latus rectum is 18 units.
III. the eccentricity is $\frac{4}{5}$.
(a) Only I is true. (b) Only II is true.
(c) Only I and II is true. (d) Only II and III is true.

65. Consider the following statements.
I. A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
II. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.
(a) Only I is true (b) Only II is true.
(c) Both are true. (d) Both are false.

66. If the equation of the hyperbola is
$9y^2 - 4x^2 = 36$, then
I. the coordinates of foci are $(0, \pm\sqrt{13})$
II. the eccentricity is $\frac{2}{\sqrt{13}}$.
III. the length of the latus rectum is 8.
(a) Only I is true. (b) Only II is true.
(c) Only III is true. (d) None of them is true.

67. Consider the following statements.
I. The equation of a circle with centre (h, k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.
II. The equation of the parabola with focus at $(a, 0)$, $a > 0$ and directrix $x = -a$ is $y^2 = -4ax$
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Botha re false.

68. Consider the following statements.

I. Length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$

II. A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

69. Match the foci, centre, transverse axis, conjugate axis and vertices of hyperbola given in column-I with their corresponding meaning given in column-II

Column-I	Column-II
A. Foci	1. Mid-point of the line segment joining the foci.
B. Centre	2. Points at which the hyperbola intersects the transverse axis.
C. Transverse axis	3. Line through the foci.
D. Conjugate axis	4. Two fixed points.
E. Vertices	5. Line through the centre and perpendicular to the transverse axis

Codes

	A	B	C	D	E
(a)	4	3	1	5	2
(b)	1	4	3	5	2
(c)	4	1	5	3	2
(d)	4	1	3	5	2

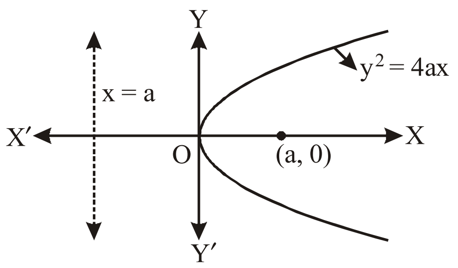
70.

Column - I	Column - II
(A) If e = 1, the conic is called	(1) Hyperbola
(B) If e < 1, the conic is called	(2) Parabola
(C) If e > 1, the conic is called	(3) Circle
(D) If e = 0, the conic is called	(4) Ellipse

Codes

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	1	4	2
(d)	3	4	1	2

71. Match the columns for the parabola given in the graph.



Column - I	Column - II
(A) Eccentricity	(1) x + a = 0
(B) Focus	(2) 4a
(C) Equation of directrix	(3) x – a = 0
(D) Length of latus rectum	(4) (a, 0)
(E) Equation of latus rectum	(5) 1
(F) Equation of axis	(6) y = 0

Codes

	A	B	C	D	E	F
(a)	5	4	1	2	3	6
(b)	5	4	2	1	6	3
(c)	6	1	4	2	3	5
(d)	6	1	2	3	4	5

72.

Column - I (Centre and radius of circle)	Column - II (Equation of circle)
(A) Centre (–3, 2), radius = 4	(1) $x^2+y^2+4x-6y-3=0$
(B) Centre (–4, –5), radius = 7	(2) $x^2+y^2-4y=0$
(C) Centre (0, 2), radius = 2	(3) $x^2+y^2+8x+10y-8=0$
(D) Centre (–2, 3), radius = 4	(4) $(x+3)^2+(y-2)^2=16$

Codes

	A	B	C	D
(a)	4	2	3	1
(b)	1	2	3	4
(c)	1	3	2	4
(d)	4	3	2	1

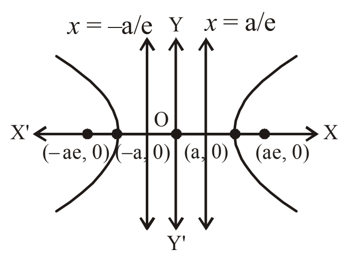
73. If the equation of ellipse is $9x^2+4y^2=36$, then

Column - I	Column - II
(A) The foci are	(1) $(0, \pm 3)$
(B) The vertices are	(2) $\frac{\sqrt{5}}{3}$
(C) The length of major axis is	(3) 6
(D) The eccentricity is	(4) $(0, \pm\sqrt{5})$

Codes

	A	B	C	D
(a)	4	1	3	2
(b)	2	1	3	4
(c)	4	3	1	2
(d)	2	3	1	4

74. Read the graph of the hyperbola. Match the column - I with column - II.



Column - I	Column - II
(A) Equation of the directrix is	(1) 2a
(B) Vertices are	(2) 2ae
(C) Foci are	(3) 2b
(D) Distance between foci is	(4) $(\pm a, 0)$
(E) Length of transverse axis is	(5) $x = \pm\frac{a}{e}$
(F) Length of conjugate axis is	(6) $(\pm ae, 0)$

Codes

	A	B	C	D	E	F
(a)	5	6	4	2	3	1
(b)	4	5	2	6	3	1
(c)	5	4	6	2	1	3
(d)	5	4	2	6	3	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

75. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is

(a) 9 (b) 1 (c) 5 (d) 7

76. Tangents are drawn from the point (–2, –1) to the parabola $y^2 = 4x$. If α is the angle between these tangent then the value of $\tan \alpha$ is

(a) 3 (b) 4 (c) –5 (d) 5

77. The focal distance of a point on the parabola $y^2 – 12x$ is 4. The abscissa of this point is

(a) 0 (b) 1 (c) 2 (d) 4

78. Radius of the circle $(x + 5)^2 + (y – 3)^2 = 36$ is

(a) 2 (b) 3 (c) 6 (d) 5

79. The equation of the circle with centre (0, 2) and radius 2 is $x^2 + y^2 – my = 0$. The value of m is

(a) 1 (b) 2 (c) 4 (d) 3

80. The equation of parabola whose vertex (0, 0) and focus (3, 0) is $y^2 = 4ax$. The value of 'a' is

(a) 2 (b) 3 (c) 4 (d) 1

81. The equation of the hyperbola whose vertices are (± 2, 0) and foci are (± 3, 0) is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Sum of a^2 and b^2 is

(a) 5 (b) 4 (c) 9 (d) 1

82. For the parabola $y^2 = 8x$, the length of the latus-rectum is

(a) 4 (b) 2 (c) 8 (d) None of these

83. For the parabola $y^2 = –12x$, equation of directrix is x = a. The value of 'a' is

(a) 3 (b) 4 (c) 2 (d) 6

84. The foci of an ellipse are (±2, 0) and its eccentricity is $\frac{1}{2}$ then the equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{12}=1$. The value of 'a' is

(a) 3 (b) 4 (c) 6 (d) 2

85. The equation of the ellipse whose axes are along the co-ordinate axes, vertices are (±5, 0) and foci at (±4, 0), is $\frac{x^2}{25}+\frac{y^2}{b^2}=1$. The value of b^2 is

(a) 3 (b) 5 (c) 9 (d) 4

86. If y = 2x is a chord of the circle $x^2 + y^2 – 10x = 0$, then the equation of a circle with this chord as diameter, is $x^2 + y^2 – ax – by = 0$. Sum of a and b is

(a) 4 (b) 2 (c) 6 (d) 0

ASSERTION- REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

87. Assertion : Length of focal chord of a parabola $y^2 = 8x$ making an angle of 60° with x-axis is 32.

Reason : Length of focal chord of a parabola $y^2 = 4ax$ making an angle α with x-axis is $4a\,\text{cosec}^2\,\alpha$.

88. Assertion : If $P\left(\frac{3\sqrt{3}}{2},1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5}, 0)$

Reason : Circle drawn with focal radius as diameter touches the auxiliary circle.

89. Assertion : Ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and $12x^2 – 4y^2 = 27$ intersect each other at right angle.

Reason : Whenever focal conics intersect, they intersect each other orthogonally.

90. Assertion : Centre of the circle $x^2 + y^2 – 6x + 4y – 12 = 0$ is (3,–2).

Reason : The coordinates of the centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are $(-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of y)

91. Assertion : Radius of the circle $2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0$ is 1.

Reason : Radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$\sqrt{\left(\frac{1}{2}\text{coeff. of }x\right)^2+\left(\frac{1}{2}\text{coeff. of }y\right)^2-\text{constant term}}$$

92. **Assertion :** Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola.

Reason : The equation of a hyperbola with foci on the y-axis is : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

93. **Assertion :** A hyperbola in which a = b is called a rectangular hyperbola.

Reason : The eccentricity of a hyperbola is the ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of the hyperbola.

94. **Assertion :** Eccentricity of conjugate hyperbola is equal to $\sqrt{\frac{b^2+a^2}{b^2}}$

Reason : Equation of directrix of conjugate hyperbola is $y=\pm\frac{b}{e}$

95. **Assertion:** The area of the ellipse $2x^2+3y^2=6$ is more than the area of the circle $x^2+y^2-2x+4y+4=0$.

Reason: The length of semi-major axis of an ellipse is more than the radius of the circle.

96. Parabola is symmetric with respect to the axis of the parabola.

Assertion: If the equation has a term y^2, then the axis of symmetry is along the x-axis.

Reason: If the eqution has a term x^2, then the axis of symmetry is along the x-axis.

97. Let the centre of an ellipse is at (0, 0)

Assertion: If major axis is on the y-axis and ellipse passes through the points (3, 2) and (1, 6), then the equation of ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$.

Reason: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is an equation of ellipse if major axis is along y-axis.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

98. The equation of the directrix of the parabola $y^2+4y+4x+2=0$ is :

(a) $x=-1$ (b) $x=1$ (c) $x=\frac{-3}{2}$ (d) $x=\frac{3}{2}$

99. The value of p such that the vertex of $y=x^2+2px+13$ is 4 units above the y-axis is

(a) 2 (b) ± 4 (c) 5 (d) ± 3

100. A parabola has the origin as its focus and the line $x=2$ as the directrix. Then the vertex of the parabola is at

(a) (0, 2) (b) (1, 0) (c) (0, 1) (d) (2, 0)

101. What is the radius of the circle passing through the points (0, 0), (a, 0) and (0, b) ?

(a) $\sqrt{a^2-b^2}$ (b) $\sqrt{a^2+b^2}$

(c) $\frac{1}{2}\sqrt{a^2+b^2}$ (d) $2\sqrt{a^2+b^2}$

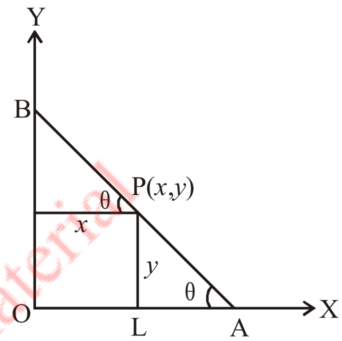
102. If (2, 0) is the vertex and the y-axis is the directrix of a parabola, then its focus is

(a) (0, 0) (b) (–2, 0) (c) (4, 0) (d) (–4, 0)

103. The latus rectum of parabola $y^2=5x+4y+1$ is:

(a) 10 (b) 5 (c) $\frac{5}{4}$ (d) $\frac{5}{2}$

104. A bar of given length moves with its extremities on two fixed straight lines at right angles. Any point of the bar describes



(a) parabola (b) ellipse

(c) hyperbola (d) circle

105. The equation of the circle, which touches the line $y=5$ and passes through (–1, 2) and (1, 2) is

(a) $9x^2+9y^2-60y+75=0$

(b) $9x^2+9y^2-60x-75=0$

(c) $9x^2+9y^2+60y-75=0$

(d) $9x^2+9y^2+60x+75=0$

106. Which points on the curve $x^2=2y$ are closest to the point (0, 5) ?

(a) $(\pm 2\sqrt{2},4)$ (b) $(\pm 2,2)$

(c) $(\pm 3,9/2)$ (d) $(\pm\sqrt{2},1)$

107. The latus rectum of the parabola $y^2=4ax$ whose focal chord is PSQ such that SP = 3 and SQ = 2 is given by :

(a) $\frac{24}{5}$ (b) $\frac{12}{5}$ (c) $\frac{6}{5}$ (d) $\frac{1}{5}$

108. The eccentric angles of the extremities of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are given by

(a) $\tan^{-1}\left(\pm\frac{ae}{b}\right)$ (b) $\tan^{-1}\left(\pm\frac{be}{a}\right)$

(c) $\tan^{-1}\left(\pm\frac{b}{ae}\right)$ (d) $\tan^{-1}\left(\pm\frac{a}{be}\right)$

109. The two conics $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ and $y^2=-\frac{b}{a}x$ intersect if and only if

(a) $0<a\le\frac{1}{2}$ (b) $0<b\le\frac{1}{2}$

(c) $b^2>a^2$ (d) $b^2<a^2$

110. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$, then the equation of the pair of tangent are

(a) $x^2 + y^2 - 5xy = 0$ (b) $x^2 + y^2 + 2x + y = 0$

(c) $x^2 + y^2 - xy + 7 = 0$ (d) $2x^2 + 2y^2 + 5xy = 0$

111. Equation of the circle passing through the origin and through the points of intersection of the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ and the line $x + y - 1 = 0$ is

(a) $x^2 + y^2 - 20x + 15y = 0$

(b) $x^2 + y^2 + 33x + 33y = 0$

(c) $x^2 + y^2 - 22x - 16y = 0$

(d) $2x^2 + 2y^2 - 4x - 5y = 0$

112. Equation of the circle concentric with the circle $x^2 + y^2 - 3x + 4y - c = 0$ and passing through the point $(-1, -2)$, is

(a) $x^2 + y^2 - 3x - 4y = 0$

(b) $x^2 + y^2 - 3x + 4y = 0$

(c) $x^2 + y^2 + 3x + 4y = 0$

(d) $x^2 + y^2 - 7x + 7y = 0$

113. If the line $x + y = 1$ is a tangent to a circle with centre (2, 3), then its equation is

(a) $x^2 + y^2 + 2x + 2y + 5 = 0$

(b) $x^2 + y^2 - 4x - 6y + 5 = 0$

(c) $x^2 + y^2 - x - y + 3 = 0$

(d) $x^2 + y^2 + 5x + 2y = 0$

114. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then radius of the circle is

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{5}{2}$

115. A.M. of the slopes of two tangents which can be drawn from the point (3, 1) to the circle $x^2 + y^2 = 4$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$

(c) $\frac{3}{5}$ (d) $\frac{1}{7}$

116. Equation of the circle which passes through the intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ whose centre lies on $13x + 30y = 0$ is

(a) $x^2 + y^2 + 5x + y = 0$

(b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

(c) $2x^2 + 2y^2 + 3x - 4y = 0$

(d) $4x^2 + 4y^2 - 8x + 7y + 10 = 0$

117. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq.units. Then the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y = 62$

(b) $x^2 + y^2 + 2x - 2y = 62$

(c) $x^2 + y^2 + 2x - 2y = 47$

(d) $x^2 + y^2 - 2x + 2y = 47$.

118. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 23 = 0$

(b) $x^2 + y^2 - 2x - 2y - 23 = 0$

(c) $x^2 + y^2 + 2x + 2y - 23 = 0$

(d) $x^2 + y^2 - 2x + 2y - 23 = 0$

119. Intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is

(a) $x^2 + y^2 + x - y = 0$ (b) $x^2 + y^2 - x + y = 0$

(c) $x^2 + y^2 + x + y = 0$ (d) $x^2 + y^2 - x - y = 0$

120. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation:

(a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$

(c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$

121. Two circles $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cut each other orthogonally, then :

(a) $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ (b) $2g_1g_2 - 2f_1f_2 = c_1 + c_2$

(c) $2g_1g_2 + 2f_1f_2 = c_1 - c_2$ (d) $2g_1g_2 - 2f_1f_2 = c_1 - c_2$

122. If the straight line $ax + by = 2$; $a, b \neq 0$ touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$, then the values of a and b are

(a) $\frac{3}{2}, 2$ (b) $-\frac{4}{3}, 1$

(c) $\frac{1}{4}, 2$ (d) $\frac{2}{3}, -1$

123. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis . The locus of the other end of the diameter through A is

(a) $(y - q)^2 = 4px$ (b) $(x - q)^2 = 4py$

(c) $(y - p)^2 = 4qx$ (d) $(x - p)^2 = 4qy$

124. The value of λ does the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$ is/are

(a) $\pm 2\sqrt{2}$ (b) $2 \pm \sqrt{3}$ (c) ± 5 (d) $5 \pm \sqrt{2}$

125. The equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represents a hyperbola
(a) The length of the transverse axes is 4
(b) Length of latus rectum is 9
(c) Equation of directrix is $x = \frac{21}{5}$ and $x = -\frac{11}{5}$
(d) None of these

126. The sum of the minimum distance and the maximum distance from the point (4, – 3) to the circle $x^2 + y^2 + 4x - 10y - 7 = 0$ is
(a) 20 (b) 12 (c) 10 (d) 16

127. If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\theta = 4$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 16$, then the value of θ equals
(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

128. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are
(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$

129. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
(a) an ellipse (b) a circle
(c) a parabola (d) a hyperbola

130. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is
(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

131. If $x + y = k$ is normal to $y^2 = 12x$, then the value of k is
(a) 3 (b) 9
(c) –9 (d) –3

132. A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point P(x, y) is taken on the rod in such a way that AP = 6 cm. Then, the locus of P is a/an.
(a) circle (b) ellipse
(c) parabola (d) hyperbola

133. If a parabolic reflector is 20 cm in diameter and 5 cm deep, then the focus is
(a) (2, 0) (b) (3, 0)
(c) (4, 0) (d) (5, 0)

134. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Then, the length of supporting wire attached to the roadway 18 m from the middle is
(a) 10.02 m (b) 9.11 m
(c) 10.76 m (d) 12.06 m

135. The length of the line segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where the line segment makes an angle θ to the x-axis is $\frac{4am}{n}$. Here, m and n respectively are
(a) $\sin\theta$, $\cos\theta$ (b) $\cos\theta$, $\sin\theta$
(c) $\cos\theta$, $\sin^2\theta$ (d) $\sin^2\theta$, $\cos\theta$

136. An arch is in the form of semi-ellipse. It is 8 m wide and 2 m high at the centre. Then, the height of the arch at a point 1.5 m from one end is
(a) 1.56 m (b) 2.4375 m
(c) 2.056 m (d) 1.086 m

137. A man running a race course notes that sum of its distance from two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Then, the equation of the posts traced by the man is
(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (b) $x^2 + y^2 = 25$
(c) $x^2 + y^2 = 9$ (d) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

138. The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is
(a) $x^2 + y^2 - 2x - 2y + 1 = 0$
(b) $x^2 + y^2 - 2x - 2y - 1 = 0$
(c) $x^2 + y^2 - 2x - 2y = 0$
(d) $x^2 + y^2 - 2x + 2y - 1 = 0$

139. Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for
(a) only one value of k (b) $0 < k < 1$
(c) $k < 0$ (d) all integral values of k

140. Find the equation of a circle which passes through the origin and makes intercepts 2 units and 4 units on x-axis and y-axis respectively.
(a) $x^2 + y^2 - 2x - 4y = 0$ (b) $x^2 + y^2 - 4y = 0$
(c) $x^2 + y^2 + 2x = 0$ (d) $x^2 + y^2 - 4x - 2y = 0$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** As the circle is passing through the point (4, 5) and its centre is (2, 2) so its radius is $\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$.
Therefore, the required equation is
$(x-2)^2+(y-2)^2=13$

2. **(a)** We put the co-ordinates of the given point in the given equation of circle
$x^2+y^2+4x-2y-4=0$
At (1, 2)
$(1)^2+(2)^2+4(1)-2(2)-4=1+4+4-4-4=1>0$
$\Rightarrow$ Point (1, 2) lies out side the circle i.e, an exterior point.

3. **(d)** A conic section is a parabola if e = 1.

4. **(a)** We know that length of latus rectum of ellipse $=\dfrac{2b^2}{a}$
Given, $3x^2+4y^2=12 \Rightarrow \dfrac{x^2}{4}+\dfrac{y^2}{3}=1$
$\Rightarrow a=2, b=\sqrt{3}$
$\therefore$ Length of latus rectum $=\dfrac{2\times 3}{2}=3$

5. **(a)** Focal distance of a point (x_1, y_1) on the parabola $y^2=4ax$ is equal to its distance from directrix $x+a=0$ is x_1+1.
For $y^2=12x$; $a=3$,
so $x_1+3=4$
$\Rightarrow x_1=1$

6. **(c)** In case of hyperbola difference between two focal points from any point $P(x_1, y_1)$ of the hyperbola having eccentricity = e is equal to the length of transverse axis.
i.e., $S'P-SP=(ex_1+a)-(ex_1-a)$,
[where S′ and S are two focal points = 2a]

7. **(a)** $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

8. **(c)** $\dfrac{2b^2}{a}$

9. **(a)** $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

10. **(b)** $\dfrac{2b^2}{a}$

11. **(c)** The length of smallest focal chord of the parabola is its latus rectum and for parabola $y^2=4ax$, it is 4a.

12. **(c)** We have $a=3$ and $\dfrac{b^2}{a}=4 \Rightarrow b^2=12$
Hence, the equation of the hyperbola is $\dfrac{x^2}{9}-\dfrac{y^2}{12}=1$
$\Rightarrow 4x^2-3y^2=36 \Rightarrow 4x^2-3y^2-36=0$

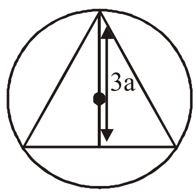
13. **(a)** $(-ae-ae)^2=(16)^2$
$\Rightarrow 4a^2e^2=256 \Rightarrow a^2=32\ (\because e=\sqrt{2})$
Now, $e=\sqrt{\dfrac{a^2+b^2}{a^2}} \Rightarrow b^2=32$
$\therefore$ Required hyperbola is $x^2-y^2=32$

14. **(c)** Since the given equation represents a circle, therefore, $4a-3=a$ *i.e.,* $a=1$
($\because$ coefficients of x^2 and y^2 must be equal)
So, the circle becomes
$x^2+y^2+6x-2y+2=0$
$\therefore$ The coordinates of centre are (– 3, 1)

15. **(b)** Let a parabola with vertex at origin and axis along the x-axis be $y^2=4ax$. It passes through(–3, 7),
hence $(7)^2=4a(-3) \Rightarrow a=-\dfrac{49}{12}$.
$\therefore$ The required equation of the parabola is
$y^2=4\left(-\dfrac{49}{12}\right)x$ or $3y^2=-49x$.

16. **(c)** Let eq. of ellipse be $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$,
length of semi-latus rectum
$=\dfrac{b^2}{a}=\dfrac{a^2(1-e^2)}{a}=a(1-e^2)$
Given $a(1-e^2)=\dfrac{1}{3}(2a)$
$\Rightarrow 1-e^2=\dfrac{2}{3} \Rightarrow e^2=1-\dfrac{2}{3}=\dfrac{1}{3} \Rightarrow e=\dfrac{1}{\sqrt{3}}$

17. **(c)** Let equation of circle having centre (0, 0) be
$x^2+y^2=r^2$ ...(i)
Since, in an equilateral triangle, the centroid coincides with the centre of the circle.
$\therefore$ Radius of circle, $r=\dfrac{2}{3}(3a)=2a$



On putting r = 2a in (i), we get
$x^2+y^2=(2a)^2 \Rightarrow x^2+y^2=4a^2$

18. **(a)** $2ae=6 \Rightarrow ae=3$; $2b=8 \Rightarrow b=4$
$b^2=a^2(1-e^2)$; $16=a^2-a^2e^2$
$\Rightarrow a^2=16+9=25 \Rightarrow a=5$
$\therefore e=\dfrac{3}{a}=\dfrac{3}{5}$

19. **(d)** $e=\sqrt{1-\dfrac{1}{7}}=\sqrt{\dfrac{6}{7}}$

20. **(b)** We have $ae = 5$ [Since focus is (±ae, 0)]

and $\frac{a}{e} = \frac{36}{5}$ $\left[\text{since directrix is } x = \pm\frac{a}{e}\right]$

On solving we get $a = 6$ and $e = \frac{5}{6}$

$\Rightarrow b^2 = a^2(1-e^2) = 36\left(1-\frac{25}{36}\right) = 11$

Thus, the required equation of the ellipse is

$\frac{x^2}{36}+\frac{y^2}{11} = 1$.

21. **(a)** The given eq. is $25(x+1)^2 + 9(y+2)^2 = 225$

$\Rightarrow \frac{(x+1)^2}{9}+\frac{(y+2)^2}{25} = 1$

centre of the ellipse is (–1, –2) and $a = 3$, $b = 5$, so that $a < b$.

$\Rightarrow 3 = 5\sqrt{1-e^2} \Rightarrow e^2 = 1-\frac{9}{25} = \frac{16}{25} \Rightarrow e = \frac{4}{5}$

Hence, foci are $\left(-1, -2-5\times\frac{4}{5}\right)$ and $\left(-1, -2+5\times\frac{4}{5}\right)$,

i.e., foci are (–1, –6) and (–1, 2).

22. **(c)** The given equation reduces to

$\frac{(x-1)^2}{9}-\frac{y^2}{3} = 1$. Thus $a^2 = 9$, $b^2 = 3$

Using $b^2 = a^2(e^2-1)$,

we get $3 = 9(e^2-1) \Rightarrow e = \frac{2}{\sqrt{3}}$.

23. **(b)** Since the vertices are on the y-axis (with origin at the mid point), the equation is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2} = 1$.

As vertices are $(0, \pm 6)$, $a = 6$,

$b^2 = a^2(e^2-1) = 36\left(\frac{25}{9}-1\right) = 64$, so the required equation of the hyperbola is

$\frac{y^2}{36}-\frac{x^2}{64} = 1$

24. **(b)** $e = \frac{1}{2}$. Directrix, $x = \frac{a}{e} = 4$

$\therefore a = 4\times\frac{1}{2} = 2$

$\therefore b = 2\sqrt{1-\frac{1}{4}} = \sqrt{3}$

Equation of ellipse is

$\frac{x^2}{4}+\frac{y^2}{3} = 1 \Rightarrow 3x^2+4y^2 = 12$

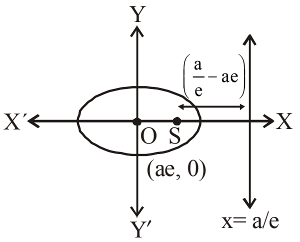
25. **(a)** Perpendicular distance of directrix from focus

$= \frac{a}{e} - ae = 4$

$\Rightarrow a\left(2-\frac{1}{2}\right) = 4$

$\Rightarrow a = \frac{8}{3}$

$\therefore$ Semi-major axis = 8/3



26. **(d)** Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

It passes through (– 3, 1)

so $\frac{9}{a^2}+\frac{1}{b^2} = 1$ ...(i)

Also, $b^2 = a^2(1-2/5)$

$\Rightarrow 5b^2 = 3a^2$ ...(ii)

Solving (i) and (ii) we get $a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$

So, the equation of the ellipse is $3x^2+5y^2 = 32$

27. **(b)** $ae = 2$ and $e = 2$

$\therefore a = 1$

$b^2 = a^2(e^2-1)$

$\Rightarrow b^2 = 1(4-1) \Rightarrow b^2 = 3$

Equation of hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{1}-\frac{y^2}{3} = 1$

$\Rightarrow 3x^2-y^2 = 3$

28. **(d)** The given equation $9x^2 + y^2 = k(x^2 - y^2 - 2x)$ can be written as $9x^2+y^2-kx^2+ky^2+2kx = 0$

$\Rightarrow (9-k)x^2+(1+k)y^2+2kx = 0$

This equation represents a circle, if coefficients of x^2 and y^2 are equal. so,

$9-k = 1+k$

$\Rightarrow 2k = 8 \Rightarrow k = 4$

29. **(c)** Let a be the major axis and b, the minor axis of the ellipse, then 3 minor axis = major axis.

$\Rightarrow 3b = a$

Eccentricity is given by

$b^2 = a^2(1-e^2)$

$\Rightarrow b^2 = 9b^2(1-e^2)$

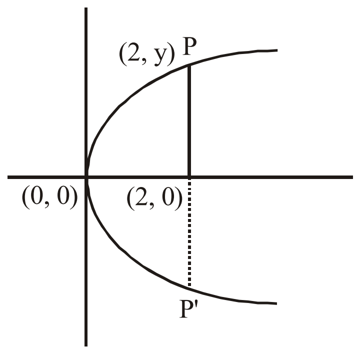
$\frac{1}{9} = (1-e^2) \Rightarrow e^2 = \frac{8}{9} \Rightarrow e = \frac{2\sqrt{2}}{3}$

30. **(d)** Given parabola is $y^2 = 8x$ and focal distance = 4

Comparing this with standard parabola, $y^2 = 4ax$

$a = 2$, co-ordinate of focus is (0, 2).

Focal distance of any point (x, y) = x + 2



$\Rightarrow x+2 = 4 \Rightarrow x = 2$

$y^2 = 8x = 8\times 2 = 16 \Rightarrow y = \pm 4$

31. (c) Length of latus rectum of a hyperbola is $\frac{2b^2}{a}$ where a is the half of the distance between two vertex of the hyperbola.

Latus rectum = $\frac{2b^2}{a} = \frac{10}{3}$

or, $b^2 = \frac{5a}{3}$ ...(i)

In case of hyperbola,

$b^2 = a^2(e^2 - 1)$ ...(ii)

Putting value of b^2 from equation (i) and $e = \frac{\sqrt{13}}{3}$ in equation (ii),

$\frac{5a}{3} = a^2\left(\frac{13}{9} - 1\right)$ or, $\frac{5a}{3} = \frac{4a^2}{9}$

$\Rightarrow$ $4a^2 - 15a = 0$ or, $a(4 - 15a) = 0$

$a \neq 0$, hence, $a = \frac{15}{4}$

Length of transverse axis = $2a = 2 \times \frac{15}{4} = \frac{15}{2}$

32. (c) Centre (1, 0), circumference = 10π (given)

$\therefore$ $2\pi r = 10\pi \Rightarrow r = 5$

So, equation of circle is $(x - 1)^2 + (y - 0)^2 = 25$

$\Rightarrow$ $x^2 + y^2 - 2x - 24 = 0$

33. (a) The foci are always on the transverse axis. It is the positive term whose denominator gives the transverse axis.

Thus, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ has transverse axis along x-axis of length 6.

34. (a) The given equation may be written as

$\frac{x^2}{32/3} - \frac{y^2}{8} = 1$ or $\frac{x^2}{\left(4\sqrt{2}/\sqrt{3}\right)^2} - \frac{y^2}{\left(2\sqrt{2}\right)^2} = 1$

Comparing the given equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get $a^2 = \left(\frac{4\sqrt{2}}{\sqrt{3}}\right)^2$ or $a = \frac{4\sqrt{2}}{\sqrt{3}}$. Therefore, length of transverse axis of a hyperbola = $2a = 2 \times \frac{4\sqrt{2}}{\sqrt{3}} = \frac{8\sqrt{2}}{\sqrt{3}}$

35. (c) Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ represent the hyperbola. Then, according to the given condition, the length of transverse axis, i.e., $2a = 7 \Rightarrow a = \frac{7}{2}$.

Also, the point (5, – 2) lies on the hyperbola. So, we have $\frac{4}{49}(25) - \frac{4}{b^2} = 1$, which gives $b^2 = \frac{196}{51}$.

Hence, the equation of the hyperbola is

$\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1$

36. (a) Conjugate axis is 5 and distance between foci = 13

$\Rightarrow 2b = 5$ and $2ae = 13$.

Now, also we know for hyperbola

$b^2 = a^2(e^2 - 1) \Rightarrow \frac{25}{4} = \frac{(13)^2}{4e^2}\left(e^2 - 1\right)$

$\Rightarrow$ $\frac{25}{4} = \frac{169}{4} - \frac{169}{4e^2}$ or $e^2 = \frac{169}{144}$ $\Rightarrow$ $e = \frac{13}{12}$

or $a = 6$, $b = \frac{5}{2}$ or hyperbola is $\frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$

$\Rightarrow$ $25x^2 - 144y^2 = 900$

37. (d) The equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

The point for which $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 > 0$ is outside ellipse.

Since, at (a, 0) : $1 + 0 - 1 = 0$

It lies on the ellipse.

At (0, b) : $0 + 1 - 1 = 0$

It lies on the ellipse.

At (– a, 0) : $1 + 0 - 1 = 0$

It lies on the ellipse.

At (a, b) : $1 + 1 - 1 > 0$

So, the point (a, b) lies outside the ellipse.

38. (d) Given, $e = \frac{2}{3}$

So, $a = \frac{3}{2}$ $(\because\ ae = 1)$

We know, $b^2 = a^2(1 - e^2)$

$\Rightarrow$ $b^2 = \frac{9}{4}\left[1 - \frac{4}{9}\right] = \frac{5}{4}$

So equation of the ellipse with vertex (3, 1) is

$\frac{(x-3)^2}{36} + \frac{(y-1)^2}{20} = 1$.

39. (c) The given parabola can be written as:

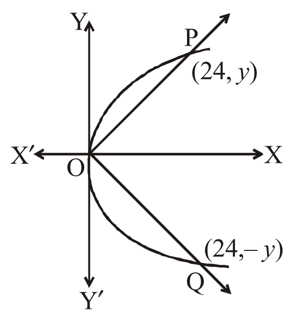
$(x - 4)^2 = -2(y - 9/2)$

which is of the form $x^2 = 4ay$

Thus, the vertex is (4, 9/2).

40. (b) Let P and Q be points on the parabola $y^2 = 6x$ and OP, OQ be the lines joining the vertex O to the points P and Q whose abscissa are 24 $\Rightarrow$ $y = \pm 12$.

Therefore, the coordinates of the points P and Q are (24, 12) and (24, – 12), respectively. Hence, the lines are $y = \pm\frac{12}{24}x \Rightarrow 2y = \pm x$.



41. **(b)** The eccentricity of $\frac{x^2}{16}+\frac{y^2}{25}=1$ is

$$e_1=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

$\therefore\ e_2=\frac{5}{3}\ (\because e_1e_2=1)$

$\Rightarrow$ foci of ellipse $=(0, \pm 3)$

$\Rightarrow$ Equation of hyperbola is

$$\frac{x^2}{16}-\frac{y^2}{9}=-1$$

42. **(d)** Let the centre of the circle be (h, k).

Since the centre lies on the line $y=x-1$

$\therefore\ k=h-1$ ...(i)

Since the circle passes through the point (7, 3), therefore, the distance of the centre from this point is the radius of the circle.

$\therefore\ 3=\sqrt{(h-7)^2+(k-3)^2}$

$\Rightarrow\ 3=\sqrt{(h-7)^2+(h-1-3)^2}$ [using (i)]

$\Rightarrow\ h=7$ or $h=4$

For $h=7$, we get $k=6$

and for $h=4$, we get $k=3$

Hence, the circles which satisfy the given conditions are

$(x-7)^2+(y-6)^2=9$

or $x^2+y^2-14x+12y+76=0$

and $(x-4)^2+(y-3)^2=9$

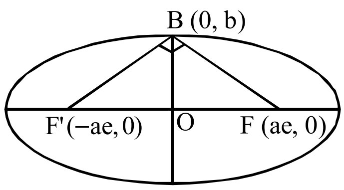
or $x^2+y^2-8x-6y+16=0$

43. **(c)** The given equation can be rewritten as $(y-1)^2=4\left(x-\frac{1}{2}\right)$ which is a parabola with its vertex $\left(\frac{1}{2},1\right)$ axis along the line $y=1$, hence axis parallel to x-axis. Its focus is $\left(\frac{1}{2}+1,1\right)$, i.e., $\left(\frac{3}{2},1\right)$.

44. **(a)** $\because \angle FBF'=90° \Rightarrow FB^2+F'B^2=FF'^2$

$\therefore\left(\sqrt{a^2e^2+b^2}\right)^2+\left(\sqrt{a^2e^2+b^2}\right)^2=(2ae)^2$

$\Rightarrow 2(a^2e^2+b^2)=4a^2e^2 \Rightarrow e^2=\frac{b^2}{a^2}$



Also $e^2=1-b^2/a^2=1-e^2$

$\Rightarrow 2e^2=1 \Rightarrow e=\frac{1}{\sqrt{2}}$.

45. **(d)** Solving equations of parabolas

$y^2=4ax$ and $x^2=4ay$

we get (0, 0) and $(4a, 4a)$

Substituting in the given equation of line

$2bx+3cy+4d=0$,

we get $d=0$ and $2b+3c=0$

$\Rightarrow d^2+(2b+3c)^2=0$

46. **(b)** The given curve is :

$2x^2-8x+y^2-2y+1=0$

$\Rightarrow\ 2(x^2-4x+4-4)+(y^2-2y+1)=0$

$\Rightarrow\ 2(x-2)^2-8+(y-1)^2=0$

$\Rightarrow\ 2(x-2)^2+(y-1)^2=8$

$\Rightarrow\ \frac{(x-2)^2}{4}+\frac{(y-1)^2}{8}=1$

This is equation of ellipse with centre (2, 1)

$\Rightarrow\ a^2=4, b^2=8$

Eccentricity $e=\sqrt{\frac{b^2-a^2}{b^2}}=\sqrt{\frac{8-4}{8}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$

47. **(b)** The given equation of curve is

$y^2+4x-6y+13=0$

which can be written as :

$y^2-6y+9+4x+4=0$

$\Rightarrow\ (y^2-6y+9)=-4(x+1)$

$\Rightarrow\ (y-3)^2=-4(x+1)$

Put $Y=y-3$ and $X=x+1$

On comparing $Y^2=4aX$

Length of focus from vertex, $a=-1$

At focus $X=a$ and $Y=0$

$\Rightarrow\ x+1=-1\ \Rightarrow\ x=-2$

$y-3=0\ \Rightarrow\ y=3$

$\therefore$ Focus is $(-2, 3)$.

48. **(a)** Let eccentricity of both the parabolas be e.

Then in the given ellipse

$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$

We have $a^2=\alpha^2, b^2=\beta^2$

$b^2=a^2(1-e^2)$

$\beta^2=\alpha^2(1-e^2)$ $(\because\ \alpha>\beta)$

$\Rightarrow\ \frac{\beta^2}{\alpha^2}=1-e^2$

$\Rightarrow\ e^2=1-\frac{\beta^2}{\alpha^2}$ ...(i)

From equation $\frac{x^2}{9}+\frac{y^2}{16}=1$

$a^2=9, b^2=16$

Then $b^2=a^2(1-e^2)$, $b>a$

$\frac{16}{9}=1-e^2$

$e^2=1-\frac{16}{9}$ ...(ii)

From equations (i) and (ii) we get

$1-\frac{16}{9}=1-\frac{\beta^2}{\alpha^2}$

$\Rightarrow \dfrac{16}{9} = \dfrac{\beta^2}{\alpha^2} \Rightarrow \dfrac{\beta}{\alpha} = \pm\dfrac{4}{3} \Rightarrow 4\alpha = 3\beta$

or $4\alpha = -3\beta$

$4\alpha = 3\beta$ is in the option.

49. (a) Given the equation of parabola

$x^2 + 8x + 12y + 4 = 0$

Make it perfect square

$\Rightarrow x^2 + 8x + 16 + 12y + 4 - 16 = 0$

$\Rightarrow (x+4)^2 + 12y - 12 = 0$

$\Rightarrow (x+4)^2 = -12(y-1)$

$\Rightarrow X^2 = -12Y$

where $X = x + 4$ and $Y = y - 1$

vertex $X = 0$ and $Y = 0$

$\Rightarrow x + 4 = 0$ and $y - 1 = 0$

$\Rightarrow x = -4, y = 1$ i.e., $(-4, 1)$

50. (c) From the definition of conic; If P(x, y) is the point on a conic then ratio of its distance from focus to its distance from directrix is a fixed ratio e, called eccentricity.

Here focus is (1, – 1) and directrix is $x - y + 1 = 0$.

Distance of this point from focus

$= \sqrt{(x-1)^2 + (y+1)^2}$

Distance of this point from directrix. $= \left|\dfrac{x - y + 1}{\sqrt{1^2 + (-1)^2}}\right|$

So, from the definition of conic

$\sqrt{(x-1)^2 + (y+1)^2} = e.\left|\dfrac{x-y+1}{\sqrt{2}}\right|$ ...(i)

Squaring both sides of equation (i), we get

$(x-1)^2 + (y+1)^2 = e^2.\dfrac{(x-y+1)^2}{2}$

$= (\sqrt{2})^2.\dfrac{(x-y+1)^2}{2}$ $\left[e = \sqrt{2}, \text{given}\right]$

$= (x - y + 1)^2$

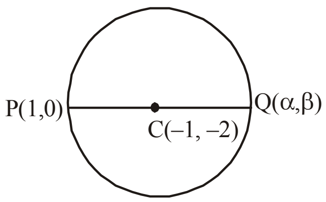
$\Rightarrow (x-1)^2 + (y+1)^2 = (x - y + 1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 + 2y + 1$

$= x^2 - 2xy + y^2 + 2x - 2y + 1$

$\Rightarrow 2xy - 4x + 4y + 1 = 0.$

51. (c) The given circle is $x^2 + y^2 + 2x + 4y - 3 = 0$



Centre (–1, –2)

Let $Q(\alpha, \beta)$ be the point diametrically opposite to the point $P(1, 0)$,

then $\dfrac{1+\alpha}{2} = -1$ and $\dfrac{0+\beta}{2} = -2 \Rightarrow \alpha = -3, \beta = -4$

So, Q is (–3, –4)

52. (b) We consider the options. Since, the required equation of circle has radius 5 and touches another circle at (5, 5)

$\therefore$ point (5, 5) satisfies the equation of required circle.

Point (5, 5) lies only on the circle

$x^2 + y^2 - 18x - 16y + 120 = 0$

and also radius of this circle is 5.

53. (b) We have circle $x^2 + y^2 - 8x + 4y + 4 = 0$

$x^2 - 8x + 16 + y^2 + 4y + 4 = -4 + 20$

$(x-4)^2 + (y+2)^2 = 4^2$

Its centre is (4, – 2) and radius is 4.

Clearly this touches y-axis.

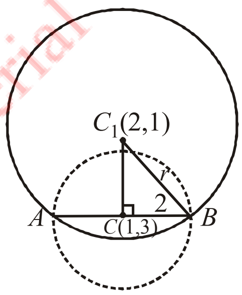
54. (b) $|r_1 - r_2| < C_1C_2$ for intersection

$\Rightarrow r - 3 < 5 \Rightarrow r < 8$ ...(i)

and $r_1 + r_2 > C_1C_2,\ r + 3 > 5 \Rightarrow r > 2$ ...(ii)

From (i) and (ii), $2 < r < 8$.

55. (c) The given circle is $x^2 + y^2 - 2x - 6y + 6 = 0$ with centre $C(1, 3)$ and radius $= \sqrt{1 + 9 - 6} = 2$. Let AB be one of its diameter which is the chord of other circle with centre at C_1 (2, 1).



Then in ΔC_1CB,

$C_1B^2 = CC_1^2 + CB^2$

$r^2 = [(2-1)^2 + (1-3)^2] + (2)^2$

$\Rightarrow r^2 = 1 + 4 + 4 \Rightarrow r^2 = 9 \Rightarrow r = 3.$

56. (c) From given equations

$\dfrac{x}{2} = \cos t + \sin t$ ...(i)

$\dfrac{y}{5} = \cos t - \sin t$ ...(ii)

Eliminating t from (i) and (ii), we have

$\dfrac{x^2}{4} + \dfrac{y^2}{25} = 2 \Rightarrow \dfrac{x^2}{8} + \dfrac{y^2}{50} = 1$ which is an ellipse.

57. (b) Let the equation of the required ellipse be

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ...(i)

The coordinates of its vertices and foci are (± a, 0) and (± ae, 0) respectively.

$\therefore$ a = 5 and ae = 4 ⇒ e = 4/5

Now, $b^2 = a^2(1 - e^2) \Rightarrow b^2 = 25\left(1 - \dfrac{16}{25}\right) = 9$

Substituting the values of a^2 and b^2 in (i), we get

$\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$, which is the equation of the required ellipse.

58. (a) Let P (x, y) be any point on the hyperbola and PM is perpendicular from P on the directrix,
Then by definition, SP = e PM
$\Rightarrow (SP)^2 = e^2 (PM)^2$

$$\Rightarrow (x-1)^2 + (y-2)^2 = 3\left\{\frac{2x+y-1}{\sqrt{4+1}}\right\}^2$$

$\Rightarrow 5(x^2 + y^2 - 2x - 4y + 5)$
$= 3(4x^2 + y^2 + 1 + 4xy - 2y - 4x)$
$\Rightarrow 7x^2 - 2y^2 + 12xy - 2x + 9y - 22 = 0$
Which is the required hyperbola.

STATEMENT TYPE QUESTIONS

59. **(c)** Both are true statements.
60. **(c)** Both are true statements.
61. **(d)** By definition of ellipse, all statements are correct.
62. **(c)** The given equation is

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

or $(x^2 - 8x) + (y^2 + 10) = 12$
or $(x^2 - 8x + 16) + (y^2 + 10y + 25) = 12 + 16 + 25$
or $(x-4)^2 + (y+5)^2 = 53$
Therefore, the given circle has centre at (4, –5) and radius $\sqrt{53}$.

63. **(d)** $\dfrac{x^2}{100} + \dfrac{y^2}{400} = 1$ is the equation of ellipse.
Major axis is along y-axis
$a^2 = 400, \therefore a = 20, b^2 = 100 \ \therefore b = 10$
$c^2 = a^2 - b^2 = 400 - 100 = 300 \ \therefore c = 10\sqrt{3}$
Vertices are $(0, \pm a)$ i.e., $(0, \pm 20)$
$\therefore$ Foci are $(0, \pm c)$ i.e., $(0, \pm 10\sqrt{3})$
Length of major axis $= 2a = 2 \times 20 = 40$
Length of minor axis $= 2b = 2 \times 10 = 20$

Eccentricity, $e = \dfrac{c}{a} = \dfrac{10\sqrt{3}}{20} = \dfrac{\sqrt{3}}{2}$

Length of Latus rectum $= \dfrac{2b^2}{a} = \dfrac{2 \times 100}{20} = 10$

64. **(c)** Comparing the equation $\dfrac{y^2}{9} - \dfrac{x^2}{27} = 1$ with the standard equation.
we have, $a = 3, \ b = 3\sqrt{3}$
and $c = \sqrt{a^2 + b^2} = \sqrt{9 + 27} = \sqrt{36} = 6$
Therefore, the coordinates of the foci are (0, ± 6) and that of vertices are (0, ± 3). Also, the eccentricity $e = \dfrac{c}{a} = \dfrac{5}{4}$ and the length of latus rectum $= \dfrac{2b^2}{a}$

$$= \frac{2(3\sqrt{3})^2}{3} = \frac{54}{3} = 18 \text{ units}$$

65. **(c)** Both the statements are definitions.

66. **(a)** $9y^2 - 4x^2 = 36$ is the equation of hyperbola
i.e., $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$
$\therefore a^2 = 4, b^2 = 9,$
$\therefore c^2 = a^2 + b^2 = 4 + 9 = 13, a = 2, b = 3, c = \sqrt{3}$
Axis is y-axis
Foci $(0, \pm\sqrt{13})$, vertices $= (0, \pm 2)$

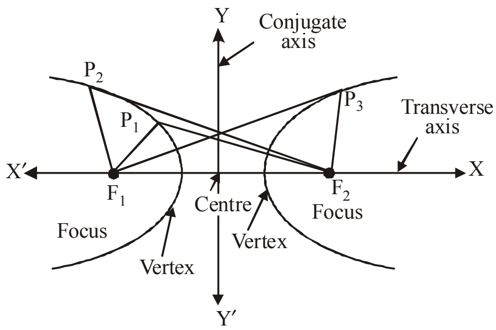
Eccentricity $= e = \dfrac{c}{a} = \dfrac{\sqrt{13}}{2}$,

Latus rectum $= \dfrac{2b^2}{a} = \dfrac{2 \times 9}{2} = 9.$

67. **(a)** Only I is true.
68. **(c)** Both the statements are true.

MATCHING TYPE QUESTIONS

69. **(d)** The two fixed points are called the foci of the hyperbola. The mid-point of the line segment joining the foci is called the centre of the hyperbola. The line through the foci is called the transverse axis and the line through the centre and perpendicular to the transverse axis is called the conjugate axis. The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.



$$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2$$

70. **(b)** e = 1 $\Rightarrow$ Parabola
e < 1 $\Rightarrow$ Ellipse
e > 1 $\Rightarrow$ Hyperbola
e = 0 $\Rightarrow$ Circle

71. **(a)**

72. **(d)** (A) h = –3, k = 2, r = 4
Required circle is $(x+3)^2 + (y-2)^2 = 16$
(B) $(x^2 + 8x) + (y^2 + 10y) = 8$
$\Rightarrow (x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$
$\Rightarrow (x+4)^2 + (y+5)^2 = 49$
(C) $(x-0)^2 + (y-2)^2 = 4$
$\Rightarrow (x^2 + y^2 + 4 - 4y = 4$
$\Rightarrow x^2 + y^2 - 4y = 0$

73. **(a)** $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1 \Rightarrow \dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$
$\Rightarrow$ a = 3, b = 2

Now, $c = \sqrt{a^2 - b^2} = \sqrt{5}$, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Hence, foci $(0, \pm\sqrt{5})$, Vertices $(0, \pm 3)$,

Length of major axis = 6 units

Eccentricity = $\frac{\sqrt{5}}{3}$

74. (c) By definition of hyperbola, we have
$A \to 5; B \to 4; C \to 6; D \to 2; E \to 1; F \to 3$

INTEGER TYPE QUESTIONS

75. (d) $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$a = \sqrt{\frac{144}{25}}, b = \sqrt{\frac{81}{25}}, \quad e = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$

$\therefore$ Foci = $(\pm 3, 0)$

$\therefore$ foci of ellipse = foci of hyperbola

$\therefore$ for ellipse $ae = 3$ but $a = 4$,

$\therefore \quad e = \frac{3}{4}$

Then $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 16\left(1 - \frac{9}{16}\right) = 7$

76. (a) Any tangent to $y^2 = 4x$ is $y = mx + 1/m$
If it is drawn from (–2, –1), then
$-1 = -2m + 1/m$
$\Rightarrow 2m^2 - m - 1 = 0$
If $m = m_1, m_2$ then $m_1 + m_2 = 1/2$,
$m_1 m_2 = -1/2$

$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$

$= \frac{\sqrt{1/4 + 2}}{1 - 1/2} = 3$

77. (b) $a = 3$
So, focal distance is $x + 3$.
$\therefore x + 3 = 4 \Rightarrow x = 1$
Hence, the abscissa = 1

78. (c) Comparing the equation of the circle
$(x + 5)^2 + (y - 3)^2 = 36$
with $(x - h)^2 + (y - k)^2 = 2$
$\therefore -h = 5$ or $h = -5, k = 3, r^2 = 36 \Rightarrow r = 6$
$\therefore$ Centre of the circle is (–5, 3) and radius = 6

79. (c) Here $h = 0$, $k = 2$ and $r = 2$. Therefore, the required equation of the circle is
$(x - 0)^2 + (y - 2)^2 = (2)^2$
or $x^2 + y^2 - 4y + 4 = 4$
or $x^2 + y^2 - 4y = 0$

80. (b) Vertex (0, 0), Focus is (3, 0)
$a = 3$
$\therefore 4a = 12$
$\therefore$ Equation of parabola is $y^2 = 12x$

81. (c) Since the foci are on x-axis, the equation of the hyperbola is of the form

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given : vertices are $(\pm 2, 0)$, $a = 2$
Also, since foci are $(\pm 3, 0)$, $c = 3$ and
$b^2 = c^2 - a^2 = 9 - 4 = 5$
Therefore, the equation of the hyperbola is

$\frac{x^2}{4} - \frac{y^2}{5} = 1$

82. (c) $y^2 = 8x \Rightarrow y^2 = 4.2.x \Rightarrow a = 2$
Length of the latus-rectum = 4a = 8

83. (a) $y^2 = -12x \Rightarrow 4a = 12 \Rightarrow a = 3$
So, equation of the directrix is x = 3.

84. (b) Coordinates of foci are (± ae, 0)
$\therefore ae = 2 \Rightarrow a.\frac{1}{2} = 2 \Rightarrow a = 4$

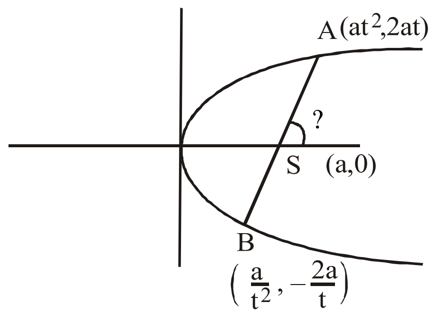
85. (c) Coordinates of vertices and foci are (± a, 0) and (± ae, 0) respectively.
$\therefore a = 5$ and $ae = 4 \Rightarrow e = \frac{4}{5}$

Now, $b^2 = a^2(1 - e^2) \Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9$

86. (c) On solving $y = 2x$ and $x^2 + y^2 - 10x = 0$ simultaneously, we get x = 0, 2
Putting x = 0 and x = 2 respectively in y = 2x, we get y = 0 and y = 4. Thus, the points of intersection of the given line and the circle are A (0, 0) and B (2, 4).
Required equation is $x^2 + y^2 - 2x - 4y = 0$
$a = 2, b = 4 \Rightarrow a + b = 6$

ASSERTION- REASON TYPE QUESTIONS

87. (d)



Let AB be a focal chord.

Slope of AB = $\frac{2t}{t^2 - 1} = \tan\alpha$

$\Rightarrow \tan\frac{\alpha}{2} = \frac{1}{t} \Rightarrow t = \cot\frac{\alpha}{2}$

Length of AB = $a\left(t + \frac{1}{t}\right)^2 = 4a\,\mathrm{cosec}^2\,\alpha$

$\Rightarrow$ Reason is correct but Assertion is false.

88. **(a)** The ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\therefore$ Auxiliary circle is $x^2+y^2=9$ and $(-\sqrt{5},0)$ and $(\sqrt{5},0)$ are focii.

$\therefore$ Assertion is true. Reason is true.

89. **(a)** $e=\frac{5}{3}, a=5$

$\therefore$ Focii are $(\pm 3, 0)$

For hyperbola $\frac{x^2}{\frac{27}{12}}-\frac{y^2}{\frac{27}{4}}=1$

$e=\sqrt{\frac{12+4}{4}}=2,\ a=\frac{3}{2}$

$\therefore$ focii are $(\pm 3, 0)$

$\therefore$ The two conics are confocal.

90. **(a)** Given circle is

$x^2+y^2-6x+4y-12=0$

$\text{Centre}=\left(-\frac{1}{2}\times(-6), -\frac{1}{2}\times 4\right)=(3,-2)$

91. **(a)** Given circle can be written as

$x^2+y^2+\frac{3}{2}x+2y+\frac{9}{16}=0$

$\text{Radius}=\sqrt{\left(\frac{3}{4}\right)^2+(1)^2-\frac{9}{16}}=1$

92. **(c)** Assertion is correct but Reason is incorrect.

93. **(d)** Assertion is incorrect. Reason is correct.

Assertion : A hyperbola in which a = b is called an equilateral hyperbola.

94. **(b)** Both Assertion and Reason are correct.

95. **(b)** Given ellipse is $\frac{x^2}{3}+\frac{y^2}{2}=1$, whose area is

$\pi\sqrt{3}\sqrt{2}=\pi\sqrt{6}$. Circle is $x^2+y^2-2x+4y+4=0$

or $(x-1)^2+(y+2)^2=1$.

Its area is π. Hence, Assertion is true.

Also, Reason is true (as length of semi-major axis $=\sqrt{3}>1$ (radius of circle) but it is not the correct explanation of Assertion.

96. **(c)** Parabola is symmetric with respect to the axis of the parabola. If the equation has a y^2 term, then the axis of symmetry is along the x-axis and if the equation has an x^2 term, then the axis of symmetry is along the y-axis.

97. **(a)** **Assertion**: Since, major axis is along y-axis. Hence, equation of ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \quad \text{...(i)}$$

Given that (i) passes through the points (3, 2) and (1, 6) i.e, they will satisfy it

$$\therefore \frac{3^2}{b^2}+\frac{2^2}{a^2}=1 \Rightarrow \frac{9}{b^2}+\frac{4}{a^2}=1 \quad \text{...(ii)}$$

and $\frac{1^2}{b^2}+\frac{6^2}{a^2}=1 \Rightarrow \frac{1}{b^2}+\frac{36}{a^2}=1$ ...(iii)

Multiplying (ii) by 9 and then subtracting (iii) from it,

we get $\frac{80}{b^2}=8 \Rightarrow b^2=\frac{80}{8}\ \therefore b^2=10$

from eq (ii), we get $\frac{9}{10}+\frac{4}{a^2}=1 \Rightarrow \frac{4}{a^2}=1-\frac{9}{10} \Rightarrow a^2=40$

putting the value of $a^2=40$ and $b^2=10$ in (i), we get

$\frac{x^2}{10}+\frac{y^2}{40}=1$

CRITICALTHINKING TYPE QUESTIONS

98. **(d)** Given $y^2+4y+4x+2=0$

$\Rightarrow (y+2)^2+4x-2=0$

$\Rightarrow (y+2)^2=-4\left(x-\frac{1}{2}\right)$

Replace, $y+2=y,\ x-\frac{1}{2}=x$

we have, $y^2=-4x$

This is a parabola with directrix at $x=1$

$\Rightarrow x-\frac{1}{2}=1 \Rightarrow x=\frac{3}{2}$

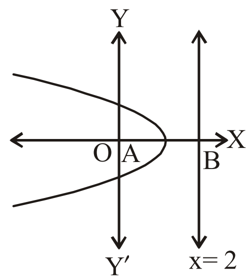
99. **(d)** Given, $y=x^2+2px+13$

$\Rightarrow y-(13-p^2)=(x+p)^2$

$\therefore$ vertex is at $(-p, 13-p^2)$

$\Rightarrow 13-p^2=4 \Rightarrow p^2=9 \Rightarrow p=\pm 3$

100. **(b)** Vertex of a parabola is the mid-point of focus and the point



where directrix meets the axis of the parabola.

Here focus is O (0, 0) and directxix meets the axis at B (2, 0)

$\therefore$ Vertex of the parabola is (1, 0)

101. **(c)** Let (h, k) be the centre of the circle.

Since, circle is passing through (0, 0), (a, 0) and (0, b), distance between centre and these points would be same and equal to radius.

Hence, $h^2+k^2=(h-a)^2+k^2=h^2+(k-b)^2$

$\Rightarrow h^2+k^2=h^2+k^2+h^2-2ah=h^2+k^2+b^2-2bk$

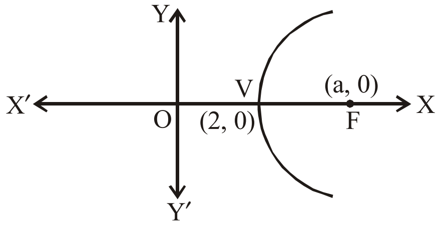
$\Rightarrow h^2+k^2=h^2+k^2+a^2-2ah$

$\Rightarrow h=\frac{a}{2}$

Similarly, $k=\frac{b}{2}$

$\therefore$ Radius of circle $=\sqrt{h^2+k^2}=\frac{1}{2}\sqrt{a^2+b^2}$

102. (c) Vertex is (2, 0). Since, y-axis is the directrix of a parabola.
$\therefore$ Equation of directrix is $x = 0$. So, axis of parabola is x-axis. Let the focus be (a, 0)



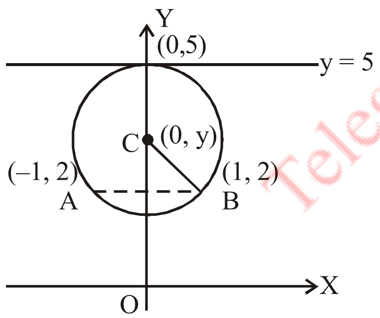
Distance of the vertex of a parabola from directrix = its distance from focus
So, $OV = VF \Rightarrow 2 = a - 2$
$\Rightarrow$ Focus is (4, 0)

103. (b) Given the equation of parabola:
$y^2 = 5x + 4y + 1$
$\Rightarrow y^2 - 4y = 5x + 1$
$\Rightarrow (y-2)^2 = 5x + 5 = 5(x+1)$
(By adding 4 on each side)
Put $y - 2 = Y$ and $x + 1 = X$
Then we get $Y^2 = 5X$
which is in the form of $y^2 = 4ax$
where 4a is the latus rectum
Thus length of latus rectum = 5.

104. (b) Let P (x, y) be any point on the bar such that PA = a and PB = b, clearly from the figure.
$x = OL = b\cos\theta$ and $y = PL = a\sin\theta$

This gives $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. Which is an ellipse.

105. (a)



The centre of the circle is on the perpendicular bisector of the line joining (–1, 2) and (1, 2), which is the y-axis. The ordinate of the centre is given by

$$(5-y)^2 = 1 + (y-2)^2 \Rightarrow y = \frac{10}{3}$$

Hence, eq. of the circle is

$$x^2 + \left(y - \frac{10}{3}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\Rightarrow 9x^2 + 9y^2 - 60y + 75 = 0$$

106. (a) Since all of the points

$$(\pm 2\sqrt{2}, 4), (\pm 2, 2), \left(\pm 3, \frac{9}{2}\right)$$

and $(\pm\sqrt{2}, 1)$ lie on the curve $x^2 = 2y$

And the distance between $(\pm 2\sqrt{2}, 4)$ and $(0, 5)$ is shortest distance. Thus $(\pm 2\sqrt{2}, 4)$ on the curve are closest to the point (0, 5).

107. (a) We know the relationship between semi latus rectum and focal chord which is given as

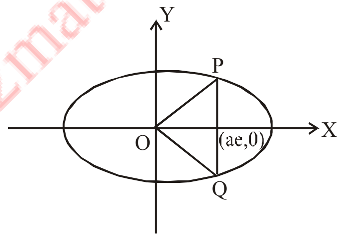
$$\frac{2}{2a} = \frac{1}{SP} + \frac{1}{SQ} \Rightarrow \frac{2(SP)(SQ)}{SP + SQ} = 2a$$

Given : SP = 3, SQ = 2

$$\therefore 2a = \frac{2(3)(2)}{3+2} \Rightarrow 2a = \frac{12}{5}$$

Now, latus rectum $= 2[2a] = \frac{24}{5}$.

108. (c) Let eq. of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
If the latus rectum is PQ then for the points P and Q



$$x = ae, \ \frac{y^2}{b^2} = 1 - e^2$$

$$\Rightarrow y^2 = b^2(1-e^2) \Rightarrow y = \pm b\sqrt{1-e^2}$$

Hence, P is $(ae, b\sqrt{1-e^2})$ and Q is $(ae, -b\sqrt{1-e^2})$.
If eccentric angle of the extremities be θ,

then, $a\cos\theta = ae$ and $b\sin\theta = \pm b\sqrt{1-e^2}$

$$\Rightarrow \tan\theta = \pm\frac{\sqrt{1-e^2}}{e} = \pm\left(\frac{b}{ae}\right) \Rightarrow \theta = \tan^{-1}\left(\pm\frac{b}{ae}\right)$$

109. (b) The x co-ordinates of the points of intersection are given by $\frac{x^2}{a^2} + \frac{1}{ab}x + 1 = 0$
and the roots are real if and only if

$$\frac{1}{a^2b^2} - \frac{4}{a^2} \ge 0 \Rightarrow \frac{1}{b^2} - 4 \ge 0$$

$$\Rightarrow 0 < b^2 \le \frac{1}{4} \Rightarrow 0 < b \le \frac{1}{2}$$

110. (d) Equation of pair of tangents is given by $SS_1 = T^2$,
or $S = x^2 + y^2 + 20(x+y) + 20, S_1 = 20,$
$T = 10(x+y) + 20 = 0$
$\therefore SS_1 = T^2$
$\Rightarrow 20(x^2 + y^2 + 20(x+y) + 20) = 10^2(x+y+2)^2$
$\Rightarrow 4x^2 + 4y^2 + 10xy = 0 \Rightarrow 2x^2 + 2y^2 + 5xy = 0$

111. (c) Let the required equation be
$(x^2+y^2-2x+4y-20)+\lambda(x+y-1)=0$
Since, it passes through (0, 0), so we have
$-20-\lambda=0 \Rightarrow \lambda=-20$
Hence the required equation is
$(x^2+y^2-2x+4y-20)-20(x+y-1)=0$
$\Rightarrow x^2+y^2-22x-16y=0$

112. (b) The equation of two concentric circles differ only in constant terms. So let the equation of the required circle be
$x^2+y^2-3x+4y+\lambda=0$
It passes through (–1, – 2), so we have
$1+4+3-8+\lambda=0 \Rightarrow \lambda=0,$
Hence required equation is $x^2+y^2-3x+4y=0$

113. (b) Radius of the circle = perpendicular distance of (2, 3) from $x+y=1$ is $\frac{4}{\sqrt{2}}=2\sqrt{2}$
$\therefore$ The required equation will be
$(x-2)^2+(y-3)^2=8 \Rightarrow x^2+y^2-4x-6y+5=0$

114. (a) The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x-4y+4=0$ and $3x-4y-\frac{7}{2}=0$ and so it is equal to

$$\frac{4}{\sqrt{9+16}}+\frac{7/2}{\sqrt{9+16}}=\frac{3}{2}$$

Hence radius is $\frac{3}{4}$.

115. (c) Any tangent to the given circle, with slope m is
$y=mx+2\sqrt{1+m^2}$
since it passes through the point (3, 1) ; so
$1=3m+2\sqrt{1+m^2}$
$\Rightarrow 4m^2+4=(3m-1)^2 \Rightarrow 5m^2-6m-3=0$
If $m=m_1, m_2$, then
AM of slopes $=\frac{1}{2}(m_1+m_2)=\frac{1}{2}(6/5)=3/5$

116. (b) The equation of required circle is $s_1+\lambda s_2=0$
$=x^2(1+\lambda)+y^2(1+\lambda)+x(2+13\lambda)-y\left(\frac{7}{2}+3\lambda\right)-\frac{25}{2}=0$

Centre $=\left(\frac{-(2+13\lambda)}{2},\frac{7/2+3\lambda}{2}\right)$

$\because$ Centre lies on $13x+30y=0$

$$\Rightarrow -13\left(\frac{2+13\lambda}{2}\right)+30\left(\frac{7/2+3\lambda}{2}\right)=0 \Rightarrow \lambda=1$$

117. (d) $\pi r^2=154 \Rightarrow r=7$
For centre:
On solving equation
$2x-3y=5 \,\&\, 3x-4y=7,$ we get $x=1, y=-1$
$\therefore$ centre = (1, –1)
Equation of circle, $(x-1)^2+(y+1)^2=7^2$
$x^2+y^2-2x+2y=47$

118. (d) Two diameters are along
$2x+3y+1=0$ and $3x-y-4=0$
solving we get centre (1, –1)
circumference $=2\pi r=10\pi$
$\therefore r=5.$
Required circle is, $(x-1)^2+(y+1)^2=5^2$
$\Rightarrow x^2+y^2-2x+2y-23=0$

119. (d) Solving $y=x$ and the circle
$x^2+y^2-2x=0$, we get
$x=0, y=0$ and $x=1, y=1$
$\therefore$ Extremities of diameter of the required circle are (0, 0) and (1, 1). Hence, the equation of circle is
$(x-0)(x-1)+(y-0)(y-1)=0$
$\Rightarrow x^2+y^2-x-y=0$

120. (d) The given circle is $x^2+y^2-6x+14=0$, centre (3, 3), radius = 2
Let (h, k) be the centre of touching circle. Then radius of touching circle = h [as it touches y-axis also]
$\therefore$ Distance between centres of two circles
= sum of the radii of two circles
$\Rightarrow \sqrt{(h-3)^2+(k-3)^2}=2+h$
$\Rightarrow (h-3)^2+(k-3)^2=(2+h)^2$
$\Rightarrow h^2-6h+9+k^2-6k+9=4+4h+h^2$
$\Rightarrow k^2-10h-6k+14=0$
$\therefore$ locus of (h, k) is
$y^2-10x-6y+14=0$

121. (a) If two circles intersect at right angle i.e. the tangent at their point of intersection are at right angles, then the circles are called orthogonal circles.
The circles
$x^2+y^2+2gx+2fy+c=0$ and
$x^2+y^2+2g_1x+2f_1y+c_1=0$
are orthogonal, if
$2gg_1+2ff_1=c+c_1$
Thus, in the given question, the condition will be
$2g_1g_2+2f_1f_2=c_1+c_2$.

122. (b) Given $x^2+y^2-2x=3$
$\therefore$ Centre = (1, 0) and radius = 2
And $x^2+y^2-4y=6$
$\therefore$ Centre = (0, 2) and radius = $\sqrt{10}$.
Since line $ax+by=2$ touches the first circle.

$$\therefore \frac{a(1)+b(0)-2}{\sqrt{a^2+b^2}}=2 \text{ or } (a-2)=[2\sqrt{a^2-b^2}] \text{ ...(i)}$$

Also the given line is normal to the second circle. Hence it will pass through the centre of the second circle.
$\therefore a(0)+b(2)=2$ or $2b=2 \Rightarrow b=1$
Putting the value in equation (i) we get
$a-2=2\sqrt{a^2+1}$ or $(a-2)^2=4(a^2+1)$

or $a^2 + 4 - 4a = 4a^2 + 4$ or, $3a^2 + 4a = 0$

or $a(3a + 4) = 0$ or $a = 0, -4/3$

$\therefore$ values of a and b are –4/3, 1 respectively.

123. (d) Let the variable circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{....(i)}$$

$$\therefore p^2 + q^2 + 2gp + 2fq + c = 0 \quad \text{....(ii)}$$

Circle (i) touches x-axis,

$\therefore g^2 - c = 0 \Rightarrow c = g^2$. From (ii)

$$p^2 + q^2 + 2gp + 2fq + g^2 = 0 \quad \text{....(iii)}$$

Let the other end of diameter through (p, q) be (h, k), then

$$\frac{h+p}{2} = -g \text{ and } \frac{k+q}{2} = -f$$

Put in (iii)

$$p^2 + q^2 + 2p\left(-\frac{h+p}{2}\right) + 2q\left(-\frac{k+q}{2}\right) + \left(\frac{h+p}{2}\right)^2 = 0$$

$$\Rightarrow \quad h^2 + p^2 - 2hp - 4kq = 0$$

$\therefore$ locus of (h, k) is $x^2 + p^2 - 2xp - 4yq = 0$

$$\Rightarrow \quad (x - p)^2 = 4qy$$

124. (c) $\because$ Equation of ellipse is $9x^2 + 16y^2 = 144$ or $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then we get $a^2 = 16$ and $b^2 = 9$ and comparing the line $y = x + \lambda$ with $y = mx + c$

$\therefore$ m = 1 and c = λ

If the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$, then $c^2 = a^2m^2 + b^2$

$\Rightarrow \lambda^2 = 16 \times 1^2 + 9 \Rightarrow \lambda^2 = 25$

$\therefore \lambda = \pm 5$

125. (c) We have, $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$\Rightarrow 9(x^2 - 2x) - 16(y^2 - 2y) = 151$

$\Rightarrow 9(x^2 - 2x + 1) - 16(y^2 - 2y + 1) = 144$

$\Rightarrow 9(x - 1)^2 - 16(y - 1)^2 = 144$

$$\Rightarrow \frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$$

Shifting the origin at (1, 1) without rotating the axes

$\frac{X^2}{16} - \frac{Y^2}{9} = 1$, where $x = X + 1$ and $y = Y + 1$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

where $a^2 = 16$ and $b^2 = 9$

so the length of the transverse axes = 2a = 8

The length of the latus rectum = $\frac{2b^2}{a} = \frac{a}{2}$

The equation of the directrix, $x = \pm \frac{a}{e}$

$x - 1 = \pm \frac{16}{5} \Rightarrow x = \pm \frac{16}{5} + 1 \Rightarrow x = \frac{21}{5}$; $x = -\frac{11}{5}$

126. (a) Centre of the given circle $\equiv$ C(– 2, 5)

Radius of the circle CN = CT = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{2^2 + 5^2 + 7} = \sqrt{36} = 6$

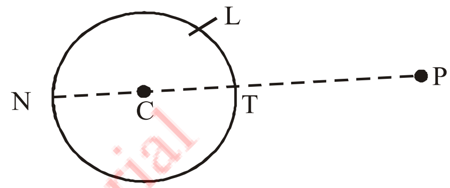
Distance between (4, – 3) and (– 2, 5) is

$PC = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

We join the external point, (4, – 3) to the centre of the circle (– 2, 5). Then PT is the minimum distance, from external point P to the circle and PN is the maximum distance. Minimum distance = PT = PC – CT = 10 – 6 = 4.

Maximum distance = PN = PC + CN = (10 + 6 = 16)

So, sum of minimum and maximum distance = 16 + 4 = 20.



127. (b) Given: $x^2 - y^2 \sec^2\theta = 4$ and $x^2 - \sec^2\theta + y^2 = 16$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{4\cos^2\theta} = 1 \text{ and } \frac{x^2}{16\cos^2\theta} + \frac{y^2}{16} = 1$$

According to problem

$$\frac{4 + 4\cos^2\theta}{4} = 3\left(\frac{16 - 16\cos^2\theta}{16}\right)$$

$\Rightarrow 1 + \cos^2\theta = 3(1 - \cos^2\theta) \Rightarrow 4\cos^2\theta = 2$

$$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

128. (b) Any tangent to the parabola $y^2 = 8ax$ is

$$y = mx + \frac{2a}{m} \quad \text{...(i)}$$

If (i) is a tangent to the circle, $x^2 + y^2 = 2a^2$ then,

$$\sqrt{2}a = \pm\frac{2a}{m\sqrt{m^2 + 1}}$$

$\Rightarrow m^2(1 + m^2) = 2 \Rightarrow (m^2 + 2)(m^2 - 1) = 0; \Rightarrow m = \pm 1.$

So from (i), $y = \pm(x + 2a)$.

129. (d) Tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Given that $y = \alpha x + \beta$ is the tangent of hyperbola

$\Rightarrow m = \alpha$ and $a^2m^2 - b^2 = \beta^2$

$\therefore a^2\alpha^2 - b^2 = \beta^2$

Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

130. (b) $\frac{dy}{dx} = 2x - 5 \therefore m_1 = (2x - 5)_{(2,0)} = -1$,

$m_2 = (2x - 5)_{(3,0)} = 1 \Rightarrow m_1m_2 = -1$

i.e. the tangents are perpendicular to each other.

131. (b) $y = mx + c$ is normal to the parabola

$y^2 = 4ax$ if $c = -2am - am^3$

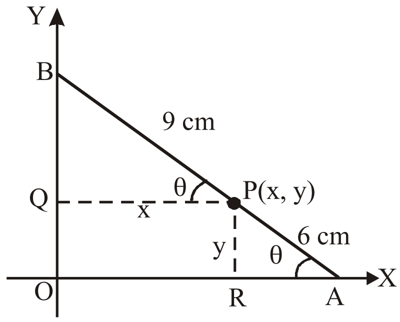
Here $m = -1$, $c = k$ and $a = 3$

$\therefore \quad c = k = -2(3)(-1) - 3(-1)^3 = 9$

132. (b) Let AB be the rod making an angle θ with OX as shown in figure and P(x, y) the point on it such that AP = 6 cm.

Since, AB = 15 cm, we have

PB = 9 cm



From P, draw PQ and PR perpendiculars on y-axis and x-axis, respectively.

From Δ PBQ, $\cos\theta = \dfrac{x}{9}$

From ΔPRA, $\sin\theta = \dfrac{y}{6}$

Since, $\cos^2\theta + \sin^2\theta = 1$

$\left(\dfrac{x}{9}\right)^2 + \left(\dfrac{y}{6}\right)^2 = 1$ or $\dfrac{x^2}{81} + \dfrac{y^2}{36} = 1$

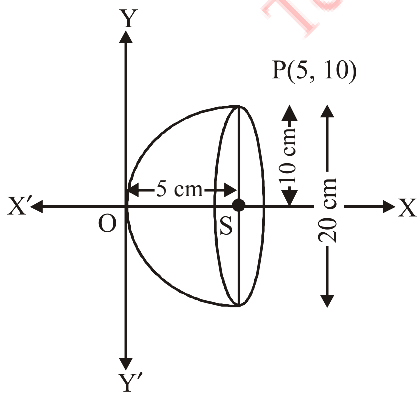
Thus, the locus of P is an ellipse.

133. (d) Taking vertex of the parabolic reflector at origin, x-axis along the axis of parabola. The equation of the parabola is $y^2 = 4ax$. Given depth is 5 cm, diameter is 20 cm.

$\because$ Point P(5, 10) lies on parabola.

$\therefore \ (10)^2 = 4a(5) \Rightarrow a = 5$

Clearly, focus is at the mid-point of given diameter.

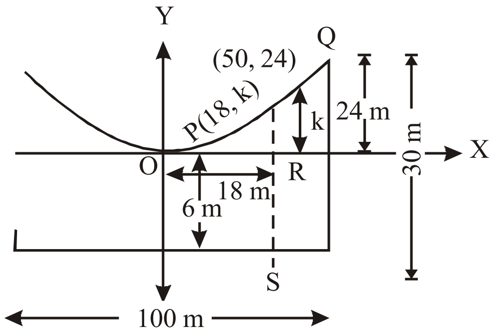


i.e., S = (5, 0)

134. (b) Since, wires are vertical. Let equation of the parabola is in the form

$x^2 = 4ay$ …(i)

Focus is at the middle of the cable and shortest and longest vertical supports are 6 m and 30 m and roadway is 100 m long.



$x^2 = 4ay$

Clearly, coordinate of Q(50, 24) will satisfy eq. (i)

$(50)^2 = 4a \times 24$

$\Rightarrow 2500 = 96a \Rightarrow a = \dfrac{2500}{96}$

Hence, from eq. (i), $x^2 = 4 \times \dfrac{2500}{96} y \Rightarrow x^2 = \dfrac{2500}{24} y$

Let PR = k m

$\therefore$ Point P(18, k) will satisfy the equation of parabola i.e.,

From eq. (i), $(18)^2 = \dfrac{2500}{24} k$

$\Rightarrow 324 = \dfrac{2500}{24} k$

$\Rightarrow k = \dfrac{324 \times 24}{2500} = \dfrac{324 \times 6}{625} = \dfrac{1944}{625}$

$\Rightarrow k = 3{:}11$

$\therefore$ Required length = 6 + k = 6 + 3.11 = 9.11 m (approx.)

135. (c) Let any point P(h, k) will satisfy

$y^2 = 4ax$ i.e, $k^2 = 4ah$ ...(i)

Let a line OP makes an angle θ from the x-axis.

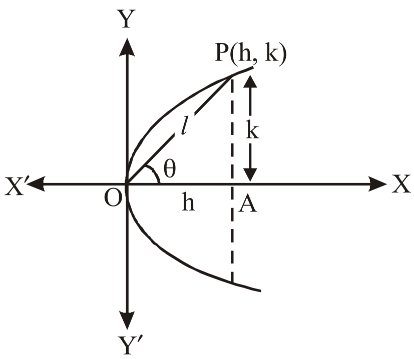
$\therefore$ In ΔOAP, $\sin\theta = \dfrac{PA}{OP}$

$\sin\theta = \dfrac{k}{l}$

$\Rightarrow k = l \sin\theta$

and $\cos\theta = \dfrac{OA}{OP}$

$\Rightarrow \cos\theta = \dfrac{h}{l} \Rightarrow h = \ell \cos\theta$



Hence, from eq.(i), we get

$l^2 \sin^2\theta = 4a \times l \cos\theta$ (put $k = l\sin\theta$, $h = l\cos\theta$)

$\Rightarrow \quad l = \dfrac{4a\cos\theta}{\sin^2\theta}$

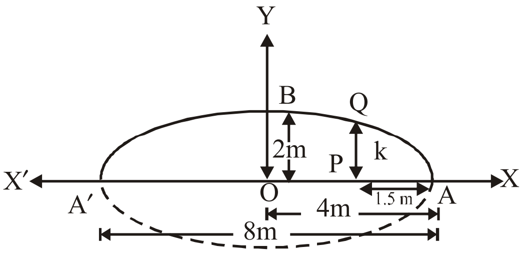
136. (a) Clearly, equation of ellipse takes the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \ldots(i)$$

Here, it is given $2a = 8$ and $b = 2 \Rightarrow a = 4, b = 2$

Put the values of a and b in eq. (i), we get

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$



Given, AP = 1.5 m

$\Rightarrow$ OP = OA – AP = 4 – 1.5

$\Rightarrow$ OP = 2.5 M

Let PQ = k

$\therefore$ Coordinate Q = (2.5, k) will satisfy the equation of ellipse.

i.e., $\frac{(2.5)^2}{16}+\frac{k^2}{4}=1 \Rightarrow \frac{6.25}{16}+\frac{k^2}{4}=1$

$\Rightarrow \frac{k^2}{4}=\frac{1}{1}-\frac{6.25}{16}=\frac{16-6.25}{16}$

$\Rightarrow \frac{k^2}{4}=\frac{9.75}{16} \Rightarrow k^2=\frac{9.75}{4}$

$\Rightarrow k^2 = 2.4375$

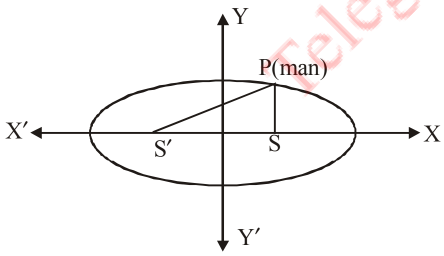
k = 1.56 m (approx.)

137. (a) Clearly, path traced by the man will be ellipse.

Given, SP + S′P = 10

i.e., 2a = 10

$\Rightarrow$ a = 5



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Since, the coordinates of S and S′ are (c, 0) and (– c, 0), respectively. Therefore, distance between S and S′ is

$2c = 8 \Rightarrow c = 4$

$\because \quad c^2 = a^2 - b^2$

$\Rightarrow \quad 16 = 25 - b^2 \Rightarrow b^2 = 25 - 16$

$\Rightarrow \quad b^2 = 9 \Rightarrow b = 3$

Hence, equation of path (ellipse) is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{25}+\frac{y^2}{9}=1 \ (\because a = 5, b = 3)$$

138. (a) Since the equation can be written as $(x-1)^2+(y-1)^2=1$ or $x^2+y^2-2x-2y+1=0$, which represents a circle touching both the axes with its centre (1, 1) and radius one unit.

139. (a) The equation of the circle through (1, 0), (0, 1) and (0, 0) is $x^2+y^2-x-y=0$

It passes through (2k, 3k)

So, $4k^2+9k^2-2k-3k=0$ or $13k^2-5k=0$

$\Rightarrow \quad k(13k-5)=0 \Rightarrow k=0$ or $k=\frac{5}{13}$

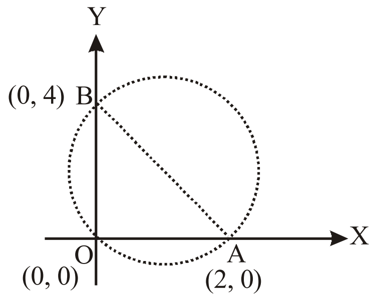
But $k \neq 0$ [$\because$ all the four points are distinct)

$\therefore \quad k=\frac{5}{13}$.

140. (a) As the circle passes through origin and makes intercept 2 units and 4 units on x-axis and y-axis respectively, it passes through the points A(2, 0) and B(0, 4).

Since axes are perpendicular to each other, therefore, $\angle AOB = 90°$ and hence AB becomes a diameter of the circle.

So, the equation of the required circle is



$(x-2)(x-0)+(y-0)(y-4)=0$

or $x^2+y^2-2x-4y=0$.

CHAPTER 12

INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. For every point $P(x, y, z)$ on the xy-plane,
(a) $x = 0$ (b) $y = 0$
(c) $z = 0$ (d) None of these

2. For every point $P(x, y, z)$ on the x-axis (except the origin),
(a) $x = 0, y = 0, z \neq 0$ (b) $x = 0, z = 0, y \neq 0$
(c) $y = 0, z = 0, x \neq 0$ (d) None of these

3. The distance of the point $P(a, b, c)$ from the x-axis is
(a) $\sqrt{b^2 + c^2}$ (b) $\sqrt{a^2 + c^2}$
(c) $\sqrt{a^2 + b^2}$ (d) None of these

4. Point (–3, 1, 2) lies in
(a) Octant I (b) Octant II
(c) Octant III (d) Octant IV

5. The three vertices of a parallelogram taken in order are (–1, 0), (3, 1) and (2, 2) respectively. The coordinate of the fourth vertex is
(a) (2,1) (b) (–2,1)
(c) (1,2) (d) (1,–2)

6. The point equidistant from the four points (0,0, 0), (3/2, 0, 0), (0,5/2, 0) and (0, 0, 7/2) is:
(a) $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{5}\right)$ (b) $\left(3, 2, \frac{3}{5}\right)$
(c) $\left(\frac{3}{4}, \frac{5}{4}, \frac{7}{4}\right)$ (d) $\left(\frac{1}{2}, 0, -1\right)$

7. The perpendicular distance of the point $P(6, 7, 8)$ from xy-plane is
(a) 8 (b) 7
(c) 6 (d) None of these

8. The ratio in which the join of points (1, –2, 3) and (4, 2, –1) is divided by XOY plane is
(a) 1 : 3 (b) 3 : 1
(c) –1 : 3 (d) None of these

9. The ratio in which the line joining the points (2,4, 5) and (3, 5, – 4) is internally divided by the xy-plane is
(a) 5 : 4 (b) 3 : 4
(c) 1 : 2 (d) 7 : 5

10. L is the foot of the perpendicular drawn from a point $P(6, 7, 8)$ on the xy-plane. The coordinates of point L is
(a) (6, 0, 0) (b) (6, 7, 0)
(c) (6, 0, 8) (d) None of these

11. If the sum of the squares of the distance of the point (x, y, z) from the points (a, 0, 0) and (–a , 0, 0) is $2c^2$, then which one of the following is correct?
(a) $x^2 + a^2 = 2c^2 - y^2 - z^2$ (b) $x^2 + a^2 = c^2 - y^2 - z^2$
(c) $x^2 - a^2 = c^2 - y^2 - z^2$ (d) $x^2 + a^2 = c^2 + y^2 + z^2$

12. The equation of set points P such that $PA^2 + PB^2 = 2K^2$, where A and B are the points (3, 4, 5) and (–1, 3, –7), respectively is
(a) $K^2 - 109$ (b) $2K^2 - 109$
(c) $3K^2 - 109$ (d) $4K^2 - 10$

13. The ratio in which the join of (2, 1, 5) and (3, 4, 3) is divided by the plane $(x + y - z) = \frac{1}{2}$ is:
(a) 3 : 5 (b) 5 : 7
(c) 1 : 3 (d) 4 : 5

14. The octant in which the points (– 3, 1, 2) and (– 3, 1, – 2) lies respectively is
(a) second, fourth (b) sixth, second
(c) fifth, sixth (d) second, sixth

15. Let L, M, N be the feet of the perpendiculars drawn from a point P(7, 9, 4) on the x, y and z-axes respectively. The coordinates of L, M and N respectively are
(a) (7, 0, 0), (0, 9, 0), (0, 0, 4) (b) (7, 0, 0), (0, 0, 9), (0, 4, 0)
(c) (0, 7, 0), (0, 0, 9), (4, 0, 0) (d) (0, 0, 7), (0, 9, 0), (4, 0, 0)

16. If a parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then the length of the diagonal is
(a) 7 units (b) 5 units
(c) 8 units (d) 3 units

17. The points A(4, – 2, 1), B(7, – 4, 7), C (2, – 5, 10) and D (– 1, – 3, 4) are the vertices of a
(a) tetrahedron (b) parallelogram
(c) rhombus (d) square

18. x-axis is the intersection of two planes are
(a) xy and xz (b) yz and zx
(c) xy and yz (d) None of these

19. The point (– 2, – 3, – 4) lies in the
(a) first octant (b) seventh octant
(c) second octant (d) eighth octant

20. A plane is parallel to yz-plane, so it is perpendicular to:
(a) x-axis (b) y-axis
(c) z-axis (d) None of these

21. The locus of a point for which x = 0 is
(a) xy-plane (b) yz-plane
(c) zx-plane (d) None of these

22. If L, M and N are the feet of perpendiculars drawn from the point P(3, 4, 5) on the XY, YZ and ZX-planes respectively, then
(a) distance of the point L from the point P is 5 units.
(b) distance of the point M from the point P is 3 units.
(c) distance of the point N from the point P is 4 units.
(d) All of the above.

23. If the point A (3, 2, 2) and B(5, 5, 4) are equidistant from P, which is on x-axis, then the coordinates of P are
(a) $\left(\frac{39}{4}, 2, 0\right)$ (b) $\left(\frac{49}{4}, 2, 0\right)$
(c) $\left(\frac{39}{4}, 0, 0\right)$ (d) $\left(\frac{49}{4}, 0, 0\right)$

24. The points (0, 7, 10), (– 1, 6, 6) and (– 4, 9, 6) form
(a) a right angled isosceles triangle
(b) a scalene triangle
(c) a right angled triangle
(d) an equilateral triangle

25. The point in YZ-plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1) is
(a) (0, 3, 1) (b) (0, 1, 3)
(c) (1, 3, 0) (d) (3, 1, 0)

26. Perpendicular distance of the point P(3, 5, 6) from y-axis is
(a) $\sqrt{41}$ (b) 6
(c) 7 (d) None of these

27. The coordinates of the point R, which divides the line segment joining P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) in the ratio k : 1, are
(a) $\left(\frac{kx_2 - x_1}{1-k}, \frac{ky_2 - y_1}{1-k}, \frac{kz_2 - z_1}{1-k}\right)$
(b) $\left(\frac{kx_2 + x_1}{1+k}, \frac{ky_2 + y_1}{1+k}, \frac{kz_2 + z_1}{1+k}\right)$
(c) $\left(\frac{kx_2 + x_1}{1-k}, \frac{ky_2 + y_1}{1-k}, \frac{kz_2 + z_1}{1-k}\right)$
(d) None of these

28. The ratio, in which YZ-plane divides the line segment joining the points (4, 8, 10) and (6, 10, – 8), is
(a) 2 : 3 (externally) (b) 2 : 3 (internally)
(c) 1 : 2 (externally) (d) 1 : 2 (internally)

29. The ratio in which YZ-plane divides the line segment formed by joining the points (– 2, 4, 7) and (3, – 5, 8), is
(a) 2 : 3 (externally) (b) 2 : 3 (internally)
(c) 1 : 3 (externally) (d) 1 : 3 (internally)

30. If the origin is the centroid of a ΔABC having vertices A(a, 1, 3), B(– 2, b, – 5) and C(4, 7, c), then
(a) a = – 2 (b) b = 8
(c) c = – 2 (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

31. P(a, b, c); Q (a + 2, b + 2, c – 2) and R (a + 6, b + 6, c – 6) are collinear.
Consider the following statements :
I. R divides PQ internally in the ratio 3 : 2
II. R divides PQ externally in the ratio 3 : 2
III. Q divides PR internally in the ratio 1 : 2
Which of the statements given above is/are correct ?
(a) Only I (b) Only II
(c) I and III (d) II and III

32. Consider the following statements
I. The x-axis and y-axis together determine a plane known as xy-plane.
II. Coordinates of points in xy-plane are of the form (x_1, y_1, 0).
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

33. Consider the following statement
I. Any point on *X*-axis is of the form (*x*, 0, 0)
II. Any point on *Y*-axis is of the form (0, *y*, 0)
III. Any point on *Z*-axis is of the form (0, 0, *z*)
Choose the correct option.
(a) Only I and II are true. (b) Only II and III are true.
(c) Only I and III are true. (d) All are true.

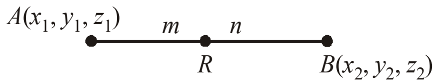
34. I. The distance of the point (*x*, *y*, *z*) from the origin is given by $\sqrt{x^2 + y^2 + z^2}$.
II. If a point *R* divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio *m* : *n* externally, then
$$R = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

35. I. The (0, 7, – 10), (1, 6, – 6) and (4, 9, –6) are the vertices of an isosceles triangle.
II. Centroid of the triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

36. I. The coordinates of the mid-point of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

II. If a point R divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$ internally, then

$$R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$



Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

37. Distance between the points (2, 3, 5) and (4, 3, 1) is $a\sqrt{5}$. The value of 'a' is
(a) 2 (b) 3 (c) 9 (d) 5

38. The perpendicular distance of the point $P(6, 7, 8)$ from xy-plane is
(a) 8 (b) 7
(c) 6 (d) None of these

39. The ratio in which the YZ-plane divide the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8) is 2 : m. The value of m is
(a) 2 (b) 3 (c) 4 (d) 1

40. Given that A(3, 2, – 4), B(5, 4 – 6) and C(9, 8, – 10) are collinear. Ratio in which B divides AC is 1 : m. The value of m is
(a) 2 (b) 3 (c) 4 (d) 5

41. If the origin is the centroid of the triangle with vertices A (2a, 2, 6), B (– 4, 3b, –10) and C (8, 14, 2c), then the sum of value of a and c is
(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

42. **Assertion:** The coordinates of the point which divides the join of A (2, –1, 4) and B (4, 3, 2) in the ratio 2 : 3 externally is C (–2, –9, 8)
Reason : If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points, and let R be a point on PQ produced dividing it externally in the ratio $m_1 : m_2$. Then the coordinates of R are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

43. **Assertion :** If three vertices of a parallelogram ABCD are A(3, –1, 2), B (1, 2, –4) and C (–1, 1, 2), then the fourth vertex is (1, –2, 8).
Reason : Diagonals of a parallelogram bisect each other and mid-point of AC and BD coincide.

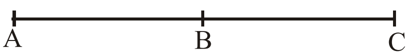
44. **Assertion :** The distance of a point P(x, y, z) from the origin O(0, 0, 0) is given by $OP = \sqrt{x^2 + y^2 + z^2}$.
Reason : A point is on the x-axis. Its y-coordinate and z-coordinate are 0 and 0 respectively.

45. **Assertion :** Coordinates (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.
Reason : Opposite sides of a parallelogram are equal and diagonals are not equal.

46. **Assertion :** If P (x, y, z) is any point in the space, then x, y and z are perpendicular distances from YZ, ZX and XY-planes, respectively.
Reason : If three planes are drawn parallel to YZ, ZX and XY-planes such that they intersect X, Y and Z-axes at (x, 0, 0), (0, y, 0) and (0, 0, z), then the planes meet in space at a point P(x, y, z).

47. **Assertion :** The distance between the points P(1, – 3, 4) and Q(– 4, 1, 2) is $\sqrt{5}$ units.
Reason : $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
where, P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2).

48. **Assertion :** Points (– 2, 3, 5), (1, 2, 3) and (7, 0, – 1) are collinear.
Reason : Three points A, B and C are said to be collinear, if AB + BC = AC (as shown below).



49. **Assertion :** Points (– 4, 6, 10), (2, 4, 6) and (14, 0, – 2) are collinear.
Reason : Point (14, 0, – 2) divides the line segment joining by other two given points in the ratio 3 : 2 internally.

50. **Assertion :** The XY-plane divides the line joining the points (– 1, 3, 4) and (2, – 5, 6) externally in the ratio 2 : 3.
Reason : For a point in XY-plane, its z-coordinate should be zero.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

51. What is the locus of a point which is equidistant from the points (1, 2, 3) and (3, 2, – 1) ?
(a) $x + z = 0$ (b) $x - 3z = 0$
(c) $x - z = 0$ (d) $x - 2z = 0$

52. What is the shortest distance of the point (1, 2, 3) from x-axis ?

(a) 1 (b) $\sqrt{6}$

(c) $\sqrt{13}$ (d) $\sqrt{14}$

53. The equation of locus of a point whose distance from the *y*-axis is equal to its distance from the point (2, 1, –1) is

(a) $x^2 + y^2 + z^2 = 6$ (b) $x^2 - 4x^2 + 2z^2 + 6 = 0$

(c) $y^2 - 2y^2 - 4x^2 + 2z + 6 = 0$ (d) $x^2 + y^2 - z^2 = 0$

54. ABC is a triangle and AD is the median. If the coordinates of A are (4, 7, – 8)and the coordinates of centroid of the triangle ABC are (1, 1, 1), what are the coordinates of D?

(a) $\left(-\frac{1}{2}, 2, 11\right)$ (b) $\left(-\frac{1}{2}, -2, \frac{11}{2}\right)$

(c) (–1, 2, 11) (d) (–5, –11, 19)

55. In three dimensional space the path of a point whose distance from the x-axis is 3 times its distance from the yz-plane is:

(a) $y^2 + z^2 = 9x^2$ (b) $x^2 + y^2 = 3z^2$

(c) $x^2 + z^2 = 3y^2$ (d) $y^2 - z^2 = 9x^2$

56. Let (3, 4, – 1) and (– 1, 2, 3) be the end points of a diameter of a sphere. Then, the radius of the sphere is equal to

(a) 2 units (b) 3 units

(c) 6 units (d) 7 units

57. Find the coordinates of the point which is three fifth of the way from (3, 4, 5) to (– 2, – 1, 0).

(a) (1, 0, 2) (b) (2, 0, 1)

(c) (0, 2, 1) (d) (0, 1, 2)

58. The coordinates of the points which trisect the line segment joining the points P(4, 2, – 6) and Q(10, – 16, 6) are

(a) (6, – 4, – 2) and (8, 10, – 2)

(b) (6, – 4, – 2) and (8, – 10, 2)

(c) (–6, 4, 2) and (– 8, 10, 2)

(d) None of these

59. If A(3, 2, 0), B(5, 3, 2) and C(– 9, 6, – 3) are three points forming a triangle and AD, the bisector of ∠BAC, meets BC in D, then the coordinates of the point D are

(a) $\left(\frac{17}{8}, \frac{57}{8}, \frac{17}{8}\right)$ (b) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

(c) $\left(\frac{8}{17}, \frac{8}{19}, \frac{17}{8}\right)$ (d) None of these

60. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1), then the vertices are

(a) (7, 2, 5), (3, 12, 17), (– 3, 4, – 7)

(b) (7, 2, 5), (3, 12, 17), (3, 4, 7)

(c) (7, 2, 5), (–3, 11, 15), (3, 4, 8)

(d) None of the above

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** On xy-plane, z-co-ordinate is zero.
2. **(c)** On x-axis, y and z-co-ordinates are zero.
3. **(a)** Let $(a, 0, 0)$ be a point on x-axis.

   Required distance $=\sqrt{(a-a^2)+(b-0)^2+(c-0)^2}$

   $=\sqrt{b^2+c^2}$
4. **(b)** $(-3, 1, 2)$ lies in second octant.
5. **(b)** Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

   $\therefore$ Coordinates of the mid point of AC
   = Coordinates of the mid-point of BD

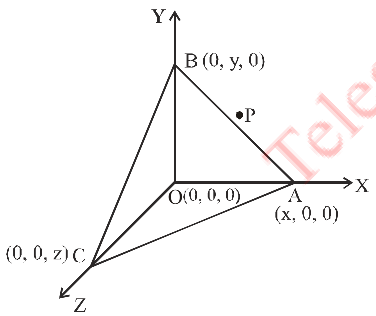
   $\Rightarrow \left(\frac{-1+2}{2},\frac{0+2}{2}\right)=\left(\frac{3+x}{2},\frac{1+y}{2}\right)$

   $\Rightarrow \left(\frac{1}{2},1\right)=\left(\frac{3+x}{2},\frac{y+1}{2}\right)$

   $\Rightarrow \frac{3+x}{2}=\frac{1}{2}$ and $\frac{y+1}{2}=1$

   $\Rightarrow$ $x=-2$ and $y=1$.

   Hence the fourth vertex of the parallelogram is $(-2, 1)$
6. **(c)**



   We know the co-ordinate of P which is equidistant from four points A(x, 0, 0), B (0, y, 0), C (0, 0, z), O (0, 0, 0) is $\frac{1}{2}(x, y, z)$

   $\therefore$ Given: points are $(0, 0, 0)$, $\left(\frac{3}{2},0,0\right),\left(0,\frac{5}{2},0\right)$ and $\left(0,0,\frac{7}{2}\right)$

   $\therefore$ Co-ordinate of point $P=\frac{1}{2}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2}\right)=\left(\frac{3}{4},\frac{5}{4},\frac{7}{4}\right)$
7. **(a)** Let L be the foot of perpendicular drawn from the point $P(6, 7, 8)$ to the xy-plane and the distance of this foot L from P is z-coordinate of P, i.e., 8 units.
8. **(b)** Let $A(1, -2, 3)$ and $B(4, 2, -1)$. Let the plane XOY meet the line AB in the point C such that C divides AB in the ratio $k : 1$, then $C\equiv\left(\frac{4k+1}{k+1},\frac{2k-2}{k+1},\frac{-k+3}{k+1}\right)$.

   Since C lies on the plane XOY i.e. the plane $z = 0$,

   therefore, $\frac{-k+3}{k+1}=0\Rightarrow k=3$.
9. **(a)** Let the line joining the points $(2, 4, 5)$ and $(3, 5, -4)$ is internally divided by the xy - plane in the ratio $k : 1$.

   $\therefore$ For xy plane, $z = 0$

   $\Rightarrow 0=\frac{-k\times 4+5}{k+1}\Rightarrow 4k=5\Rightarrow k=\frac{5}{4}$.

   so, ratio is 5 : 4
10. **(b)** Since L is the foot of perpendicular from P on the xy-plane, z-coordinate is zero in the xy-plane. Hence, coordinates of L are $(6, 7, 0)$.
11. **(b)** Let the point be $P(x, y, z)$ and two points, $(a, 0, 0)$ and $(-a, 0, 0)$ be A and B

    As given in the problem,

    $PA^2 + PB^2 = 2c^2$

    so, $(x+a)^2+(y-0)^2+(z-0)^2+(x-a)^2+(y-0)^2+(z-0)^2=2c^2$

    or, $(x+a)^2+y^2+z^2+(x-a)^2+y^2+z^2=2c^2$

    $\Rightarrow x^2+2a+a^2+y^2+z^2+x^2-2a+a^2+y^2+z^2=2c^2$

    $\Rightarrow 2(x^2+y^2+z^2+a^2)=2c^2$

    $\Rightarrow x^2+y^2+z^2+a^2=c^2$

    $\Rightarrow x^2+a^2=c^2-y^2-z^2$
12. **(b)** Let the coordinates of point P be (x, y, z).

    Here, $PA^2=(x-3)^2+(y-4)^2+(z-5)^2$

    $PB^2=(x+1)^2+(y-3)^2+(z+7)^2$

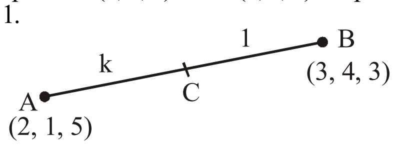
    By the given condition

    $PA^2+PB^2=2K^2$

    We have

    $(x-3)^2+(y-4)^2+(z-5)^2+(x+1)^2+(y-3)^2+(z+7)^2=2K^2$

    i.e. $2x^2+2y^2+2z^2-4x-14y+4z=2K^2-109$
13. **(b)** As given plane $x+y-z=\frac{1}{2}$ divides the line joining the points A(2, 1, 5) and B(3, 4, 3) at a point C in the ratio $k:1$.



    Then coordinates of C

    $\left(\frac{3k+2}{k+1},\frac{4k+1}{k+1},\frac{3k+5}{k+1}\right)$

    Point C lies on the plane,

    $\Rightarrow$ Coordinates of C must satisfy the equation of plane.

So, $\left(\frac{3k+2}{k+1}\right)+\left(\frac{4k+1}{k+1}\right)-\left(\frac{3k+5}{k+1}\right)=\frac{1}{2}$

$\Rightarrow 3k+2+4k+1-3k-5=\frac{1}{2}(k+1)$

$\Rightarrow 4k-2=\frac{1}{2}(k+1)$

$\Rightarrow 8k-4=k+1 \Rightarrow 7k=5$

$\Rightarrow k=\frac{5}{7}$

Ratio is 5 : 7.

14. **(d)** The point (– 3, 1, 2) lies in second octant and the point (– 3, 1, – 2) lies in sixth octant.

15. **(a)** Since L is the foot of perpendicular from P on the x-axis, its y and z-coordinates are zero. So, the coordinates of L is (7, 0, 0). Similarly, the coordinates of M and N are (0, 9, 0) and (0, 0, 4), respectively.

16. **(a)** Length of edges of the parallelopiped are 5 – 2, 9 – 3, 7 – 5 i.e., 3, 6, 2.

$\therefore$ Length of diagonal is $\sqrt{3^2+6^2+2^2}=7$ units.

17. **(b)** Here, the mid-point of AC is

$\left(\frac{4+2}{2},\frac{-2-5}{2},\frac{1+10}{2}\right)=\left(3,-\frac{7}{2},\frac{11}{2}\right)$

and that of BD is

$\left(\frac{7-1}{2},\frac{-4-3}{2},\frac{7+4}{2}\right)=\left(3,-\frac{7}{2},\frac{11}{2}\right)$

So, the diagonals AC and BD bisect each other.

$\Rightarrow$ ABCD is a parallelogram.

As $|AB|=\sqrt{3^2+2^2+6^2}=7$ and

$|AD|=\sqrt{5^2+1^2+3^2}=\sqrt{35}\neq|AB|$,

Therefore, ABCD is not a rhombus and naturally, it cannot be a square.

18. **(a)**

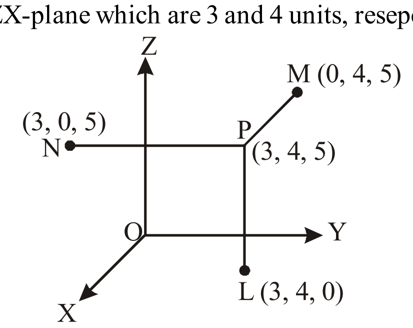
19. **(b)** The point (– 2, – 3, – 4) lies on negative of x, y and z-axis.

$\therefore$ It lies in seventh octant.

20. **(a)** A plane is parallel to yz-plane which is always perpendicular to x-axis.

21. **(b)** For yz-plane x = 0, locus of point for which x = 0 is yz-plane.

22. **(d)** L is the foot of perpendicular drawn from the point P(3, 4, 5) to the XY-plane. Therefore, the coordinates of the point L is (3, 4, 0). The distance between the point (3, 4, 5) and (3, 4, 0) is 5. Similarly, we can find the lengths of the foot of perpendiculars on YZ and ZX-plane which are 3 and 4 units, resepctively.



23. **(d)** The point on the x-axis is of the form P(x, 0, 0). Since, the points A and B are equidistant from P. Therefore, $PA^2 = PB^2$,

i.e., $(x-3)^2+(0-2)^2+(0-2)^2$
$=(x-5)^2+(0-5)^2+(0-4)^2$

$\Rightarrow \quad 4x=25+25+16-17$ i.e., $x=\frac{49}{4}$

Thus, the point P on the x-axis is $\left(\frac{49}{4},0,0\right)$ which is equidistant from A and B

24. **(a)** Let P(0, 7, 10), Q(– 1, 6, 6) and R(– 4, 9, 6) be the vertices of a triangle

Here, $PQ=\sqrt{1+1+16}=3\sqrt{2}$

$QR=\sqrt{9+9+0}=3\sqrt{2}$

$PR=\sqrt{16+4+16}=6$

Now, $PQ^2+QR^2=\left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2$
$=18+18=36=(PR)^2$

Therefore, ΔPQR is a right angled triangle at Q. Also, OQ = QR, Hence, ΔPQR is a right angled isosceles triangle.

25. **(b)** Since x-coordinate of every point in YZ-plane is zero. Let P(0, y, z) be a point on the YZ-plane such that PA = PB = PC.

Now, PA = PB

$\Rightarrow \quad (0-2)^2+(y-0)^2+(z-3)^2$
$=(0-0)^2+(y-3)^2+(z-2)^2,$

i.e., $z-3y=0$

and PB = PC

$\Rightarrow \quad y^2+9-6y+z^2+4-4z=y^2+z^2+1-2z,$

i.e., $3y+z=6$

On simplifying the two equations, we get y = 1 and z = 3. Here, the coordinate of the point P are (0, 1, 3).

26. **(d)** Let M is the foot of perpendicular from P on the y-axis, therefore its x and z-coordinates are zero. The coordinates of M is (0, 5, 0). Therefore, the perpendicular distance of the point P from y-axis $=\sqrt{3^2+6^2}=\sqrt{45}$.

27. **(b)** The coordinates of the point R which divides PQ in the ratio k : 1 where coordinates of P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) are obtained by taking $k=\frac{m}{n}$ in the coordinates of the point R which divides PQ internally in the ratio m : n, which are as given below.

$\left(\frac{kx_2+x_1}{1+k},\frac{ky_2+y_1}{1+k},\frac{kz_2+z_1}{1+k}\right)$

28. **(a)** Let YZ-plane divides the line segment joining A (4, 8, 10) and B(6, 10, –8) at P(x, y, z) in the ratio k : 1. Then, the coordinates of P are

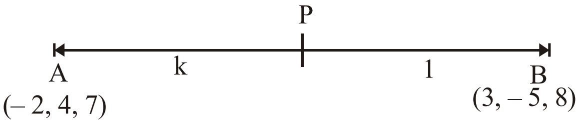
$\left(\frac{4+6k}{k+1},\frac{8+10k}{k+1},\frac{10-8k}{k+1}\right)$

Since, P lies on the YZ-plane, its x-coordinates is zero.

i.e., $\frac{4+6k}{k+1}=0$ or $k=-\frac{2}{3}$

Therefore, YZ-plane divides AB externally in the ratio 2 : 3

29. **(b)** The given points are A(–2, 4, 7) and B(3, – 5, 8).
Let the point P(0, y, z) in YZ-plane divides AB in the ratio k : 1. Then,



x-coordinate of point $P = \frac{mx_2 + nx_1}{m+n}$

$\frac{k \times 3 + 1 \times (-2)}{k+1} = 0$ ($\because$ x-coordinate of P is zero)

$\Rightarrow \quad 3k - 2 = 0$

$\Rightarrow \quad k = \frac{2}{3}$

$\Rightarrow \quad k : 1 = 2 : 3$

$\therefore$ YZ-plane divides the segment internally in the ratio 2 : 3

30. **(a)** For centroid of ΔABC,

$$x = \frac{a-2+4}{3} = \frac{a+2}{3}$$

$$y = \frac{1+b+7}{3} = \frac{b+8}{3}$$

and $\quad z = \frac{3-5+c}{3} = \frac{c-2}{3}$

But given centroid is (0, 0, 0).

$\therefore \quad \frac{a+2}{3} = 0 \Rightarrow a = -2$

$\frac{b+8}{3} = 0 \Rightarrow b = -8$

$\frac{c-2}{3} = 0 \Rightarrow c = 2$

STATEMENT TYPE QUESTIONS

31. **(d)** Given that P (a, b, c), Q (a + 2, b + 2, c – 2) and R (a + 6, b + 6, c – 6) are collinear, one point must divide, the other two points externally or internally.
Let R divide P and Q in ratio k : 1 so,
taking on x-coordinates

$\frac{k(a+2)+a}{k+1} = a+6$

$\Rightarrow \quad k(a+2)+a = (k+1)(a+6)$

$\Rightarrow \quad ka + 2k + a = ka + 6k + a + 6 \Rightarrow -4k = 6$

or $\quad k = -\frac{3}{2}$

Negative sign shows that this is external division in ratio 3 : 2. So, R divides P and Q externally in 3 : 2 ratio.
Putting this value for y - and z - coordinates satisfied :
for y - coordinate

$\frac{3(b+2)-2b}{3-2} = 3b + 6 - 2b = b + 6$

and for z-coordinate :

$\frac{3(c-2)-2c}{3-2} = \frac{3c-6-2c}{1} = c - b$

Statement II is correct.
Also, let Q divide P and R in ratio p : 1 taking an x-coordinate:

$\frac{p(a+6)+a}{p+1} = a+2$

$\frac{p.a+6p+a}{p+1} = a+2$

$\Rightarrow \quad pa + 6p + a = pa + a + 2p + 2$

$\Rightarrow \quad 4p = 2 \Rightarrow p = \frac{1}{2}$.

Positive sign shows that the division is internal and in the ratio 1 : 2
Verifying for y - and z- coordinates, satisfies this results.
For y coordinate,

$\frac{(b+6)\times 1 + 2b}{3} = \frac{3b+6}{3} = b+2$

and for z-coordinate,

$\frac{c-6+2c}{3} = \frac{3c-6}{3} = c-2$

values are satisfied.
So, statement III is correct.

32. **(c)** **33.** **(d)**

34. **(c)** Both the statements are true.

35. **(c)** Both the statements are true.

36. **(c)** Both the given statements are true.

INTEGER TYPE QUESTIONS

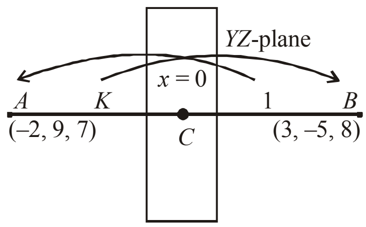
37. **(a)** The given points are (2, 3, 5) and (4, 3, 1).
$\therefore$ Required distance

$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} = \sqrt{4+0+16}$

$= \sqrt{20} = 2\sqrt{5}$

38. **(a)** Let *L* be the foot of perpendicular drawn from the point *P*(6, 7, 8) to the *xy*-plane and the distance of this foot *L* from *P* is *z*-coordinate of *P*, i.e., 8 units.

39. **(b)** Let the points be *A*(–2, 4, 7) and *B*(3, –5, 8) on *YZ*-plane, *x*-coordinate = 0.



Let the ratio be *K* : 1.
The coordinates of *C* are

$\left(\frac{3K-2}{K+1}, \frac{-5K+4}{K+1}, \frac{8K+7}{K+1}\right)$

Clearly $\frac{3K-2}{K+1} = 0 \Rightarrow 3K = 2 \Rightarrow K = \frac{2}{3}$

Hence required ratio is 2 : 3.

40. **(a)** Suppose B divides AC in the ratio λ : 1.

$\therefore \quad B = \left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right) = (5, 4, -6)$

$$\Rightarrow \frac{9\lambda+3}{\lambda+1}=5, \frac{8\lambda+2}{\lambda+1}=4, \frac{-10\lambda-4}{\lambda+1}=-6$$

$$\Rightarrow \lambda=\frac{1}{2}$$

So, required ratio is 1 : 2.

41. (a) Centriod of Δ ABC are $\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

Given centroid = (0, 0, 0)

$\therefore \quad 2a+4=0, 16+3b=0, 2c-4=0$

$\Rightarrow \quad a=-2, b=\frac{-16}{3}, c=2$

Hence, a + c = 0

ASSERTION - REASON TYPE QUESTIONS

42. (a) Assertion :

$$x=\frac{2\times4-3\times2}{2-3}, y=\frac{2\times3-3(-1)}{2-3},$$

$$z=\frac{2\times2-3\times4}{2-3}$$

$\Rightarrow x=-2, y=-9, z=8$

43. (a) Since diagonals of a parallelogram bisect each other therefore, mid-point of AC and BD coincide.

$$\therefore \quad (1, 0, 2)=\left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2}=1, \frac{y+2}{2}=0, \frac{z-4}{2}=2$$

$\Rightarrow \quad x=1, y=-2, z=8$

44. (b) Both Assertion and Reason is correct.

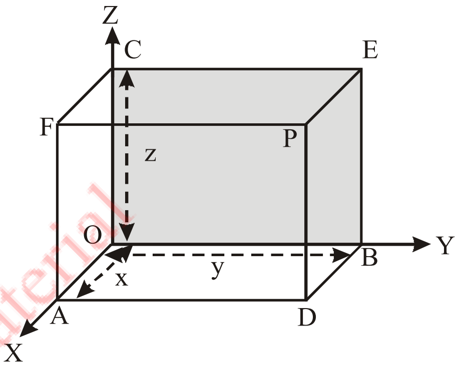
45. (a) **Assertion:** The given points are $A(-1, 2, 1)$, $B(1, -2, 5)$, $C(4, -7, 8)$ and $D(2, -3, 4)$, then by distance formula

$$AB=\sqrt{(1+1)^2+(-2-2)^2+(5-1)^2}$$
$$=\sqrt{4+16+16}=\sqrt{36}=6$$
$$BC=\sqrt{(4-1)^2+(-7+2)^2+(8-5)^2}$$
$$=\sqrt{9+25+9}=\sqrt{43}$$
$$CD=\sqrt{(2-4)^2+(-3+7)^2+(4-8)^2}$$
$$=\sqrt{4+16+16}=\sqrt{36}=6$$
$$DA=\sqrt{(-1-2)^2+(2+3)^2+(1-4)^2}$$
$$=\sqrt{9+25+9}=\sqrt{43}$$
$$AC=\sqrt{(4+1)^2+(-7-2)^2+(8-1)^2}$$
$$=\sqrt{25+81+49}=\sqrt{155}$$
$$BD=\sqrt{(2-1)^2+(-3+2)^2+(4-5)^2}$$
$$=\sqrt{1+1+1}=\sqrt{3}$$

Now, since $AB = CD$ and $BC = DA$ i.e., opposite sides are equal and $AC \neq BD$ i.e. the diagonals are not equal. So, points are the vertices of parallelogram.

46. (b) **Assertion :** Through, the point P in the space, we draw three planes, parallel to the coordinates planes, meeting the X-axis, Y-axis and Z-axis in the points A, B and C, respectively. We observe that OA = x, OB = y and OC = z. Thus, if P(x, y, z) is any point in the space, then x, y and z are perpendicular distances from YZ, ZX and XY-planes, respectively.

Reason : Given x, y and z, we locate the three points A, B and C on the three coordinate axes. Through the points A, B and C, we draw planes parallel to the YZ-plane, ZX-plane and XY-plane, respectively. The point of intersection of these three planes, namely ADPF, BDPE and CEPF is obviously the point P, corresponding to the ordered triplet (x, y, z).



47. (d) The distance PQ between the points P(1, – 3, 4) and Q(– 4, 1, 2) is

$$PQ=\sqrt{(-4-1)^2+(1+3)^2+(2-4)^2}$$
$$=\sqrt{25+16+4}=\sqrt{45}=3\sqrt{5} \text{ units}$$

48. (a) The given points are

A(–2, 3, 5), B(1, 2, 3), C(7, 0, – 1)

Distance between A and B

$$AB=\sqrt{(-2-1)^2+(3-2)^2+(5-3)^2}$$
$$=\sqrt{(-3)^2+(1)^2+(2)^2}=\sqrt{9+1+4}=\sqrt{14}$$

Distance between B and C

$$BC=\sqrt{(1-7)^2+(2-0)^2+(3+1)^2}$$
$$=\sqrt{(-6)^2+(2)^2+(4)^2}$$
$$=\sqrt{36+4+16}=\sqrt{56}=2\sqrt{14}$$

Distance between A and C

$$AC=\sqrt{(-2-7)^2+(3-0)^2+(5+1)^2}$$
$$=\sqrt{(-9)^2+(3)^2+(6)^2}$$
$$=\sqrt{81+9+36}=\sqrt{126}=3\sqrt{14}$$

Clearly, AB + BC = AC

Hence, the given points are collinear.

49. (c) Let A(– 4, 6, 10), B(2, 4, 6) and C(14, 0, – 2) be the given points. Let the point P divides AB in the ratio k : 1. Then, coordinates of the point P are

$$\frac{2k-4}{k+1}, \frac{4k+6}{k+1}, \frac{6k+10}{k+1}$$

Let us examine whether for some value of k, the point P coincides with point C.

On putting $\frac{2k-4}{k+1}=14$, we get $k=-\frac{3}{2}$

When $k=-\frac{3}{2}$, then $\frac{4k+6}{k+1}=\frac{4\left(-\frac{3}{2}\right)+6}{-\frac{3}{2}+1}=0$

and $\frac{6k+10}{k+1}=\frac{6\left(-\frac{3}{2}\right)+10}{-\frac{3}{2}+1}=-2$

Therefore, C (14, 0, – 2) is a point which divides AB externally in the ratio 3 : 2 and is same as P. Hence A, B, C are collinear.

50. (a) Suppose xy-plane divides the line joining the given points in the ratio $\lambda : 1$. The coordinates of the points of division are $\left[\frac{2\lambda-1}{\lambda+1},\frac{-5\lambda+3}{\lambda+1},\frac{6\lambda+4}{\lambda+1}\right]$. Since, the points lies on the XY-plane.

$\therefore \quad \frac{6\lambda+4}{\lambda+1}=0 \Rightarrow \lambda=\frac{-2}{3}$

CRITICALTHINKING TYPE QUESTIONS

51. (d) Let (h, k, ℓ) be the point which is equidistant from the points (1, 2, 3) and (3, 2, –1)

$\Rightarrow \sqrt{(h-1)^2+(k-2)^2+(\ell-3)^2}$

$=\sqrt{(h-3)^2+(k-2)^2+(\ell+1)^2}$

$\Rightarrow (h-1)^2+(\ell-3)^2=(h-3)^2+(\ell+1)^2$

$\Rightarrow h^2+1-2h+\ell^2-6\ell+9=h^2-6h+9+\ell^2+2\ell+1$

$\Rightarrow -2h-6\ell=-6h+2\ell$

$\Rightarrow 6h-2h-6\ell-2\ell=0 \Rightarrow 4h-8\ell=0$

$\Rightarrow h-2\ell=0$

Putting h = x and ℓ = z

We get locus of points (h, k, ℓ)

as, x – 2z = 0

52. (c) Any point on x-axis has y = z = 0

Distance of the point (1, 2, 3) from x-axis is the distance between point (1, 2, 3) and point (1, 0, 0)

$=\sqrt{(1-1)^2+(2-0)^2+(3-0)^2}=\sqrt{2^2+3^2}$

$=\sqrt{4+9}=\sqrt{13}$

53. (c) The variable point is $P(x, y, z)$.

Its distance from the y-axis $=\sqrt{x^2+z^2}$

Its distance from (2, 1, –1)

$=\sqrt{(x-2)^2+(y-1)^2+(z+1)^2}$

Given

$\sqrt{x^2+z^2}=\sqrt{(x-2)^2+(y-1)^2+(z+1)^2}$

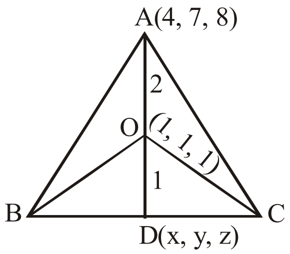
$\Rightarrow y^2-2y-4x+2z+6=0$

54. (b) Let coordinates of D be (x, y, z)

Co-ordinates of centroid is (1, 1, 1), and of A, is (4, 7, 8)

Centroid divides median in 2 : 1 ratio

So, $\frac{AO}{OD}=2:1$



For x :

$1=\frac{2\times x+1\times 4}{1+2}$

$\Rightarrow \quad x=-1/2$

For y :

$1=\frac{2y+1\times 7}{1+2} \quad \Rightarrow y=-2$

For z :

$1=\frac{2\times z+1\times -8}{3} \quad \Rightarrow z=+11/2$

$\therefore$ Coordinates of D are (–1/2, –2, 11/2)

55. (a) Let $P(x_1, y_1, z_1)$ be the point.

Then distance of P from x-axis $=\sqrt{y_1^2+z_1^2}$

Given plane is x = 0 (yz-plane)

Distance of $P(x_1, y_1, z_1)$ from yz-plane is $\frac{x_1}{\sqrt{1}}$

From the given condition,distance of P from x-axis = 3 × distance of P from yz-plane

$\sqrt{y_1^2+z_1^2}=3x_1$

Squaring, $y_1^2+z_1^2=9x_1^2$

Thus, path of $P(x_1, y_1, z_1)$ is got by putting x, y, z in place of x_1, y_1, z_1 as $y^2+z^2=9x^2$

56. (b) Let P(3, 4, – 1) and Q(– 1, 2, 3) be the end points of the diameter of a sphere.

$\therefore$ Length of diameter = PQ

$=\sqrt{(-1-3)^2+(2-4)^2+(3+1)^2}$

$=\sqrt{16+4+16}=\sqrt{36}=6$ units

$\therefore$ Radius $=\frac{6}{2}$ **= 3 units**

57. (d) Let A = (3, 4, 5), B = (– 2, – 1, 0) and P(x, y, z) be the required point. As P is three-fifth of the way from A to B, we have

$AP=\frac{3}{5}AB \Rightarrow AP=\frac{3}{5}(AP+PB)$

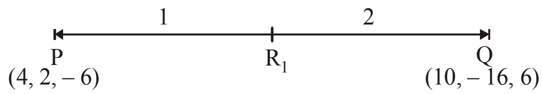
$\Rightarrow \quad 5AP=3AP+3PB \Rightarrow \frac{AP}{PB}=\frac{3}{2}$

$\Rightarrow$ P divides [AB] in the ratio 3 : 2

$\therefore \; P\equiv\left(\frac{3\times(-2)+2\times 3}{3+2},\frac{3\times(-1)+2\times 4}{3+2},\frac{3\times 0+2\times 5}{3+2}\right)$

$\Rightarrow$ P = (0, 1, 2)

58. (b) Let the points R_1 and R_2 trisects the line PQ i.e., R_1 divides the line in the ratio 1 : 2.

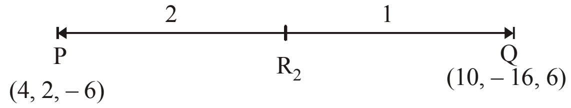


$$\Rightarrow R_1 = \left(\frac{1\times10+2\times4}{1+2}, \frac{1\times(-16)+2\times2}{1+2}, \frac{1\times6+2\times(-6)}{1+2}\right)$$

$$= \left(\frac{10+8}{3}, \frac{-16+4}{3}, \frac{6-12}{3}\right) = \left(\frac{18}{3}, \frac{-12}{3}, \frac{-6}{3}\right)$$

$$= (6, -4, -2)$$

Again, let the point R_2 divides PQ internally in the ratio 2 : 1. Then.



$$\Rightarrow R_2 = \left(\frac{2\times10+1\times4}{2+1}, \frac{2\times(-16)+1\times2}{2+1}, \frac{2\times6+1\times(-6)}{2+1}\right)$$

$$= \left(\frac{20+4}{3}, \frac{-32+2}{3}, \frac{12-6}{3}\right) = \left(\frac{24}{3}, \frac{-30}{3}, \frac{6}{3}\right)$$

$$= (8, -10, 2)$$

59. **(b)** $AB = \sqrt{(5-3)^2+(3-2)^2+(2-0)^2} = \sqrt{4+1+4} = 3$

$$AC = \sqrt{(-9-3)^2+(6-2)^2+(-3-0)^2}$$

$$= \sqrt{144+16+9} = 13$$

Since, AD is the bisector of $\angle BAC$, we have

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{13}$$

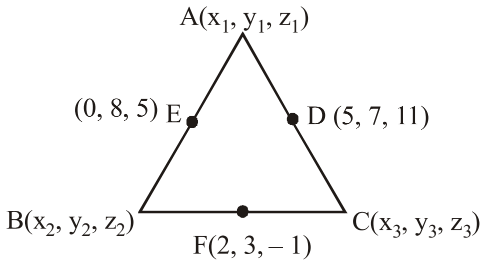
i.e., D divides BC in the ratio 3 : 13.

Hence, the coordinates of D are

$$\left(\frac{3(-9)+13(5)}{3+13}, \frac{3(6)+13(3)}{3+13}, \frac{3(-3)+13(2)}{3+13}\right)$$

$$= \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$$

60. (a) Let the vertices of a triangle be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$.



Since D, E and F are the mid-points of AC, BC and AB

$$\therefore \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = (0, 8, 5)$$

$\Rightarrow x_1 + x_2 = 0, y_1 + y_2 = 16, z_1 + z_2 = 10$ ...(i)

$$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right) = (2, 3, -1)$$

$\Rightarrow x_2 + x_3 = 4, y_2 + y_3 = 6, z_2 + z_3 = -2$ ...(ii)

and $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right) = (5, 7, 11)$

$\Rightarrow x_1 + x_3 = 10, y_1 + y_3 = 14, z_1 + z_3 = 22$ ...(iii)

On adding eqs. (i), (ii) and (iii), we get

$2(x_1 + x_2 + x_3) = 14$, $2(y_1 + y_2 + y_3) = 36$.

$2(z_1 + z_2 + z_3) = 30$,

$\Rightarrow x_1 + x_2 + x_3 = 7, y_1 + y_2 + y_3 = 18,$

$z_1 + z_2 + z_3 = 15$ ...(iv)

On solving eqs. (i), (ii), (iii) and (iv), we get

$x_3 = 7, x_1 = 3, x_2 = -3$

$y_3 = 2, y_1 = 12, y_2 = 4$

and $z_3 = 5, z_1 = 17, z_2 = -7$

Hence, vertices of a triangle are (7, 2, 5), (3, 12, 17) and (– 3, 4, – 7).

CHAPTER 13

LIMITS AND DERIVATIVE

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The limit of $f(x) = x^2$ as x tends to zero equals
(a) zero (b) one (c) two (d) three

2. Consider the function $f(x) = \begin{cases} 1, & x \le 0 \\ 2, & x > 0 \end{cases}$
Then, left hand limit and right hand limit of f (x) at x = 0, are respectively
(a) 1, 2 (b) 2, 1 (c) 1,1 (d) 2, 2

3. The value of $\lim_{x \to -1} \left[\frac{x^2 - 1}{x^2 + 3x + 2}\right]$ is
(a) 2 (b) –2 (c) 0 (d) –1

4. The value of $\lim_{x \to 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ is
(a) 2 (b) –2 (c) 1 (d) –1

5. Evaluate $\lim_{x \to 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$
(a) 1 (b) 2 (c) –1 (d) –2

6. Value of $\lim_{x \to 5} \frac{1 - \sqrt{x-4}}{x - 5}$ is
(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) does not exist

7. $\lim_{x \to 0} \frac{\sqrt{1+x+x^2} - 1}{x} =$
(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 0 (d) ∞

8. If $f(x) = \begin{cases} x^2 + 1, & x \ge 1 \\ 3x - 1, & x < 1 \end{cases}$, then the value of $\lim_{x \to 1} f(x)$ is
(a) 2 (b) –2 (c) 1 (d) –1

9. The value of $\lim_{x \to 0} \frac{\sqrt{(1+x^2)} - \sqrt{1-x^2}}{x^2}$ is
(a) 1 (b) –1 (c) 0 (d) does not exist

10. If $f(t) = \frac{1-t}{1+t}$, then the value of f ' (1/t) is
(a) $\frac{-2t^2}{(t+1)^2}$ (b) $\frac{2t}{(t+1)^2}$ (c) $\frac{2t^2}{(t-1)^2}$ (d) $\frac{-2t^2}{(t-1)^2}$

11. Let f and g be two functions such that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then, which of the following is incomplete?
(a) $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
(b) $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
(c) $\lim_{x \to a} [f(x) . g(x)] = \lim_{x \to a} f(x) . \lim_{x \to a} g(x)$
(d) $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

12. The derivative of the function f (x) = x is
(a) 0 (b) 1 (c) ∞ (d) None of these

13. The derivative of sin x at x = 0 is
(a) 0 (b) 2 (c) 1 (d) 3

14. The derivative of the function f (x) = 3x at x = 2 is
(a) 0 (b) 1 (c) 2 (d) 3

15. The derivative of f (x) = 3 at x = 0 and at x = 3 are
(a) negative (b) zero
(c) different (d) not defined

16. Derivative of f at x = a is denoted by
(a) $\left.\frac{d}{dx} f(x)\right|_a$ (b) $\left.\frac{df}{dx}\right|_a$
(c) $\left(\frac{df}{dx}\right)_{x=a}$ (d) All of these

17. If a is a non-zero constant, then the derivative of x + a is
(a) 1 (b) 0
(c) a (d) None of these

18. The derivative of $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$ is
(a) $\frac{2}{(1+x)^2}$ (b) $\frac{-2}{(1-x)^2}$
(c) $\frac{-1}{(1-x)^2}$ (d) $\frac{3}{(1-x)^2}$

19. The derivative of $4\sqrt{x} - 2$ is
(a) $\frac{1}{\sqrt{x}}$ (b) $2\sqrt{x}$ (c) $\frac{2}{\sqrt{x}}$ (d) $\sqrt{x}$

20. If a and b are fixed non-zero constants, then the derivative of $(ax+b)^n$ is
(a) $n(ax+b)^{n-1}$ (b) $na(ax+b)^{n-1}$
(c) $nb(ax+b)^{n-1}$ (d) $nab(ax+b)^{n-1}$

21. The derivative of $\sin^n x$ is
(a) $n \sin^{n-1} x$ (b) $n \cos^{n-1} x$
(c) $n \sin^{n-1} x \cos x$ (d) $n \cos^{n-1} x \sin x$

22. The derivative of $(x^2+1)\cos x$ is
(a) $-x^2 \sin x - \sin x - 2x \cos x$
(b) $-x^2 \sin x - \sin x + 2\cos x$
(c) $-x^2 \sin x - x \sin x + 2\cos x$
(d) $-x^2 \sin x - \sin x + 2x \cos x$

23. The derivative of $f(x) = \tan(ax+b)$ is
(a) $\sec^2(ax+b)$ (b) $b\sec^2(ax+b)$
(c) $a\sec^2(ax+b)$ (d) $ab\sec^2(ax+b)$

24. If $f(x) = x \sin x$, then $f'\left(\frac{\pi}{2}\right)$ is equal to
(a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$

25. The derivative of function $6x^{100} - x^{55} + x$ is
(a) $600x^{100} - 55x^{55} + x$ (b) $600x^{99} - 55x^{54} + 1$
(c) $99x^{99} - 54x^{54} + 1$ (d) $99x^{99} - 54x^{54}$

26. $\lim_{x\to0}\frac{x}{\tan x}$ is
(a) 0 (b) 1 (c) 4 (d) not defined

27. Derivative of $\log_x x$ is
(a) 0 (b) 1 (c) $\frac{1}{x}$ (d) x

28. Derivative of $e^{3\log x}$ is
(a) e^x (b) $3x^2$ (c) $3x$ (d) $\log x$

29. Derivative of $x^2 + \sin x + \frac{1}{x^2}$ is
(a) $2x + \cos x$ (b) $2x + \cos x + (-2)x^{-3}$
(c) $2x - 2x^{-3}$ (d) None of these

30. Derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ is
(a) $\frac{1}{x^2}$ (b) $1 - \frac{1}{x^2}$ (c) 1 (d) $1 + \frac{1}{x^2}$

31. If $f(x) = \alpha x^n$, then $\alpha =$
(a) $f'(1)$ (b) $\frac{f'(1)}{n}$ (c) $n \cdot f'(1)$ (d) $\frac{n}{f'(1)}$

32. Derivative of $x \sin x$
(a) $x \cos x$ (b) $x \sin x$
(c) $x \cos x + \sin x$ (d) $\sin x$

33. Value of $\lim_{x\to0}\frac{a^{\sin x} - 1}{\sin x}$ is
(a) $\log a$ (b) $\sin x$ (c) $\log(\sin x)$ (d) $\cos x$

34. $\lim_{x\to0}\frac{2\sin^2 3x}{x^2}$ is equal to :
(a) 12 (b) 18 (c) 0 (d) 6

35. $\lim_{\theta\to0}\frac{\sin m^2\theta}{\theta}$ is equal to :
(a) 0 (b) 1 (c) m (d) m^2

36. Derivative of the function $f(x) = 7x^{-3}$ is
(a) $21x^{-4}$ (b) $-21x^{-4}$ (c) $21x^4$ (d) $-21x^4$

37. If $f(x) = 2\sin x - 3x^4 + 8$, then $f'(x)$ is
(a) $2\sin x - 12x^3$ (b) $2\cos x - 12x^3$
(c) $2\cos x + 12x^3$ (d) $2\sin x + 12x^3$

38. Derivative of the function $f(x) = (x-1)(x-2)$ is
(a) $2x+3$ (b) $3x-2$
(c) $3x+2$ (d) $2x-3$

39. If $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]$ exists, then which one of the following correct ?
(a) Both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist
(b) $\lim_{x\to a} f(x)$ need not exist but $\lim_{x\to a} g(x)$ must exist
(c) Both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ need not exist
(d) $\lim_{x\to a} f(x)$ must exist but $\lim_{x\to a} g(x)$ need not exist

40. The value of $\lim_{x\to0}\frac{1+\frac{x}{3}-1+\frac{x}{3}}{x}$ is
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

41. The value of $\lim_{x\to0}\frac{\cos x}{\pi - x}$ is
(a) π (b) $-\pi$ (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

42. Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to
(a) $\frac{2}{7}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{7}{2}$

43. What is the derivative of
$f(x) = \frac{7x}{(2x-1)(x+3)}$?
(a) $-\frac{3}{(x+3)^2} - \frac{2}{(2x-1)^2}$ (b) $-\frac{3}{(x+3)^2} - \frac{1}{(2x-1)^2}$
(c) $\frac{3}{(x+3)^2} + \frac{1}{(2x-1)^2}$ (d) $\frac{3}{(x+3)^2} + \frac{2}{(2x-1)^2}$

44. As $x \to a$, $f(x) \to l$, then l is called .... of the function f(x),
(a) limit (b) value
(c) absolute value (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

45. Consider the function $g(x) = |x|, x \neq 0$.
Then
I. g (0) is not defined.
II. $\lim_{x\to0} g(x)$ is not defined.

Which of the following is/are true?
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

46. Consider the function $h(x)=\dfrac{x^2-4}{x-2}, x \neq 2$

Then,
I. h (2) is not defined.
II. $\lim\limits_{x\to 2} h(x)=4.$

Which of the following is/are true?
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

47. Which of the following is/are true?

I. $\lim\limits_{x\to 1}\left[\dfrac{x^{15}-1}{x^{10}-1}\right]=\dfrac{3}{2}$

II. $\lim\limits_{x\to 0}\left[\dfrac{\sqrt{1+x}-1}{x}\right]=\dfrac{1}{2}$

(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

48. Which of the following is/are true?

I. $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

II. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}=0$

(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

49. Which of the following is/are true?

I. $\lim\limits_{x\to 1}\dfrac{ax^2+bx+c}{cx^2+bx+a}$ (where $a+b+c\neq 0$) is 1.

II. $\lim\limits_{x\to -2}\dfrac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $\dfrac{1}{4}$

(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

50. $\lim\limits_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ is equal to

I. $a^2\sin a+2a\cos a$ II. $a^2\cos a+2a\sin a$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

51. Which of the following is/are true?
I. The derivative of $f(x)=\sin 2x$ is $2(\cos^2 x-\sin^2 x)$.
II. The derivative of $g(x)=\cot x$ is $-\text{cosec}^2 x$.
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

52. Which of the following is/are true?
I. The derivative of x^2-2 at $x=10$ is 18.
II. The derivative of $99x$ at $x=100$ is 99.
III. The derivative of x at $x=1$ is 1.
(a) I, II and III are true (b) I and II are true
(c) II and III are true (d) I and III are true

53. Which of the following is/are true?

I. The derivative of $y=2x-\dfrac{3}{4}$ is 2.

II. The derivative of $y=(5x^3+3x-1)(x-1)$ is $20x^3+15x^2+6x-4$

(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

54. Which of the following is/are true?
I. The derivative of $f(x)=x^3$ is x^2
II. The derivative of $f(x)=\dfrac{1}{x^3}$ is $\dfrac{-1}{x^2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

55. Which of the following is/are true?
I. The derivative of $-x$ is -1.
II. The derivative of $(-x)^{-1}$ is $\dfrac{1}{x^2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

56. Which of the following is/are true?
I. The derivative of $\sin(x+a)$ is $\cos(x+a)$, where *a* is a fixed non-zero constant.
II. The derivative of cosec x cot x is $\text{cosec}^3 x-\cot^2 x\,\text{cosec}\, x$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Borh I and II are false

57. Which of the following is/are true?
I. The derivative of $f(x)=1+x+x^2+...+x^{50}$ at $x=1$ is 1250.
II. The derivative of $f(x)=\dfrac{x+1}{x}$ is $\dfrac{1}{x^2}$.
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

58. Consider the following limits which holds for function *f* and *g*:

I. $\lim\limits_{x\to a}[f(x)\pm g(x)]=\lim\limits_{x\to a}f(x)\pm\lim\limits_{x\to a}g(x)$

II. $\lim\limits_{x\to a}[f(x).g(x)]=\lim\limits_{x\to a}f(x).\lim\limits_{x\to a}g(x)$

III. $\lim\limits_{x\to a}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{\lim\limits_{x\to a}f(x)}{\lim\limits_{x\to a}g(x)}$

Which of the above are true ?
(a) Only I (b) Only II
(c) Only III (d) All of the above

59. Consider the following derivatives which holds for function *u* and *v*.

I. $(u\pm v)'=u'\pm v'$ II. $(uv)'=uv'+vu'$

III. $\left(\dfrac{u}{v}\right)'=\dfrac{u'v-uv'}{v^2}$

Which of the above holds are true ?
(a) Only I (b) Only II
(c) Only III (d) All of these

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

60.

Column-I		Column-II	
A.	$\lim\limits_{x\to a^-} f(x)$	1.	left hand limit of *f* at *a*
B.	$\lim\limits_{x\to a^+} f(x)$	2.	limit of *f* at *a*
C.	$\lim\limits_{x\to a} f(x)$	4.	right hand limit of *f* at *a*

Codes

	A	B	C
(a)	3	1	2
(b)	1	3	2
(c)	1	2	3
(d)	2	3	1

61.

	Column-I (Limts)		**Column-II (Values)**
A.	$\lim\limits_{x\to 3} x+3$	1.	π
B.	$\lim\limits_{x\to \pi}\left(x-\dfrac{22}{7}\right)$	2.	6
C.	$\lim\limits_{r\to 1}\pi r^2$	3.	$\dfrac{19}{2}$
D.	$\lim\limits_{x\to 4}\left(\dfrac{4x+3}{x-2}\right)$	4.	$\dfrac{-1}{2}$
E.	$\lim\limits_{x\to -1}\left(\dfrac{x^{10}+x^5+1}{x-1}\right)$	5.	$\pi-\dfrac{22}{7}$

Codes

	A	B	C	D	E
(a)	5	2	1	4	3
(b)	2	5	1	3	4
(c)	5	2	1	3	4
(d)	2	5	3	1	4

62.

	Column-I (Limits)		**Column-II (Values)**
A.	$\lim\limits_{x\to \pi}\dfrac{\sin(\pi-x)}{\pi(\pi-x)}$	1.	4
B.	$\lim\limits_{x\to 0}\dfrac{\cos x}{\pi-x}$	2.	$\dfrac{1}{\pi}$
C.	$\lim\limits_{x\to 0}\dfrac{\cos 2x-1}{\cos x-1}$	3.	$\dfrac{a+1}{b}$
D.	$\lim\limits_{x\to 0}\dfrac{ax+x\cos x}{b\sin x}$	4.	0
E.	$\lim\limits_{x\to 0} x\sec x$	5.	1
F.	$\lim\limits_{x\to 0}\dfrac{\sin ax+bx}{ax+\sin bx}$ $(a, b, a+b\neq 0)$		

Codes

	A	B	C	D	E	F
(a)	2	2	1	3	5	4
(b)	2	2	3	1	4	5
(c)	2	2	1	4	3	5
(d)	2	2	1	3	4	5

63.

	Column-I (Functions)		**Column-II (Derivatives)**
A.	cosec x	1.	5 cos x + 6 sin x
B.	3 cot x + 5 cosec x	2.	$-3\,\text{cosec}^2 x - 5\,\text{cosec}\,x\cot x$
C.	5 sin x – 6 cos x + 7	3.	$2\sec^2 x - 7\sec x\tan x$
D.	2 tan x – 7 sec x	4.	– cot x cosec x

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	4	2	3	1
(c)	2	4	1	3
(d)	4	2	1	3

64.

	Column-I (Functions)		**Column-II (Derivatives)**
A.	$f(x)=10x$	1.	$2x$
B.	$f(x)=x^2$	2.	$-\dfrac{1}{x^2}$
C.	f(x) = a for fixed real no. a	3.	0
D.	$f(x)=\dfrac{1}{x}$	4.	10

Codes

	A	B	C	D
(a)	4	1	3	2
(b)	1	4	3	2
(c)	4	1	2	3
(d)	4	3	1	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

65. If value of $\lim\limits_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$ is equal to $\dfrac{1}{a\sqrt{2}}$ then 'a' equals

(a) 1 (b) 2 (c) 3 (d) 4

66. If value of $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ is equal to $\dfrac{2\sqrt{3}}{m}$, where m is equal to

(a) 2 (b) 8 (c) 9 (d) 3

67. $\lim\limits_{x\to \pi/2}(\sec x-\tan x)$ is equal to

(a) 0 (b) 2 (c) 1 (d) 3

68. Suppose $f(x)=\begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax, & x>1 \end{cases}$

and if $\lim\limits_{x\to 1} f(x)=f(1)$ then the value of $a+b$ is

(a) 0 (b) 2 (c) 4 (d) 8

69. If $\lim\limits_{x\to 0}\dfrac{\sin(2+x)-\sin(2-x)}{x}$ is equal to p cos q, then sum of p and q is

(a) 2 (b) 1 (c) 3 (d) 4

70. If $f(x)=|x|-5$, then the value of $\lim\limits_{x\to 5} f(x)$ is

(a) 9 (b) 1 (c) 0 (d) None of these

71. If value of $\lim\limits_{x\to 0}\dfrac{\sin x}{x(1+\cos x)}$ is equal to $\dfrac{a}{2}$ then the value of 'a' is

(a) 0 (b) 1 (c) 2 (d) 3

72. Value of $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\sin 2x}$ is

(a) 1 (b) 2 (c) 4 (d) None of these

73. If $f(x)=\begin{cases} 2x+3, & x\le 0 \\ 3(x+1), & x>0 \end{cases}$ then the value of $\lim\limits_{x\to 0} f(x)$ is

(a) 0 (b) 6 (c) 2 (d) 3

74. Let $f(x)=\begin{cases} x+2, & x\le -1 \\ cx^2, & x>-1 \end{cases}$

If $\lim\limits_{x\to -1} f(x)$ exists, then c is equal to

(a) 1 (b) 0 (c) 2 (d) 3

75. If value of $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$ is $a\sqrt{2}$, then the value of 'a' is

(a) 2 (b) 3 (c) 4 (d) 5

76. If $\lim\limits_{x\to 2} \dfrac{x^n-2^n}{x-2}=80$ and $n\in N$, then the value of 'n' is

(a) 2 (b) 3 (c) 4 (d) 5

77. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x}$ is equal to

(a) 2 (b) 0 (c) 1 (d) 3

78. If $f(x)=x^n$ and $f'(1)=10$, then the value of 'n' is

(a) 1 (b) 5 (c) 9 (d) 10

79. If $\lim\limits_{x\to 5} \dfrac{x^k-5^k}{x-5}=500$, then k is equal to :

(a) 3 (b) 4 (c) 5 (d) 6

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

80. **Assertion:** $\lim\limits_{x\to 0} \dfrac{\sin ax}{bx}=\dfrac{a}{b}$

Reason: $\lim\limits_{x\to 0} \dfrac{\sin ax}{\sin bx}=\dfrac{b}{a}$ $(a, b\ne 0)$

81. **Assertion:** $\lim\limits_{x\to 0} (\operatorname{cosec} x-\cot x)=0$

Reason: $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\tan 2x}{x-\frac{\pi}{2}}=1$

82. **Assertion:** If a and b are non-zero constants, then the derivative of $f(x)=ax+b$ is a.

Reason: If a, b and c are non-zero constants, then the derivative of $f(x)=ax^2+bx+c$ is $ax+b$.

83. Let $a_1, a_2, a_3, \ldots, a_n$ be fixed real numbers and define a function $f(x)=(x-a_1)(x-a_2)\ldots(x-a_n)$, then

Assertion: $\lim\limits_{x\to a_1} f(x)=0$.

Reason: $\lim\limits_{x\to a} f(x)=(a-a_1)(a-a_2)\ldots(a-a_n)$, for some $a\ne a_1, a_2, \ldots, a_n$.

84. **Assertion:** Suppose f is real valued function, the derivative of 'f' at x is given by $f'(x)=\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

Reason: If $y=f(x)$ is the function, then derivative of 'f' at any x is denoted by $f'(x)$.

85. **Assertion.** For the function

$f(x)=\dfrac{x^{100}}{100}+\dfrac{x^{99}}{99}+\ldots+\dfrac{x^2}{2}+x+1$, $f'(1)=100f'(0)$.

Reason: $\dfrac{d}{dx}(x^n)=n.x^{n-1}$.

86. **Assertion:** $\lim\limits_{x\to 0} (1+3x)^{1/x}=e^3$.

Reason: Since $\lim\limits_{x\to 0} (1+x)^{1/x}=e$.

87. **Assertion:** $\lim\limits_{x\to 0} \log_e\left(\dfrac{\sin x}{x}\right)=0$

Reason: $\lim\limits_{x\to 0} f(g(x))=f(\lim\limits_{x\to 0} g(x))$.

88. **Assertion:** $\lim\limits_{x\to 0} \dfrac{\tan x^0}{x^0}=1$ where x^0 means x degree.

Reason: If $\lim\limits_{x\to 0} f(x)=l$, $\lim\limits_{x\to 0} g(x)=m$, then $\lim\limits_{x\to 0} \{f(x)g(x)\}=lm$

89. **Assertion:** Derivative of $f(x)=x|x|$ is $2|x|$.

Reason: For function u and v, $(uv)'=uv'+vu'$.

90. **Assertion:** Let $\lim\limits_{x\to a} f(x)=l$ and $\lim\limits_{x\to a} g(x)=m$. If l and m both exist, then $\lim\limits_{x\to a} (fg)(x)=\lim\limits_{x\to a} f(x).\lim\limits_{x\to a} g(x)=lm$

Reason: Let f be a real valued function defined by $f(x)=x^2+1$, then $f'(2)=4$.

91. **Assertion:** Derivative of $f(x)=2$ is zero.

Reason: Differentiation of a constant function is zero.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

92. Evaluate $\lim\limits_{x\to 0} \dfrac{\sin^2 2x}{x^2}$.

(a) 4 (b) –4 (c) sin x (d) cos x

93. The value of $\lim\limits_{x\to 0} \dfrac{x^3 \cot x}{1-\cos x}$ is

(a) 1 (b) –2 (c) 2 (d) 0

94. The value of $\lim\limits_{x\to 0} \dfrac{x(e^x-1)}{1-\cos x}$ is

(a) 0 (b) 2 (c) –2 (d) does not exist

95. $\lim\limits_{x\to 0} \dfrac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) –1 (c) zero (d) does not exist

96. $\lim\limits_{x\to 0} \dfrac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is

(a) 2 (b) –2 (c) 1/2 (d) –1/2

97. $\lim\limits_{x\to 0} \dfrac{\sin(\pi\cos^2 x)}{x^2}$ equals

(a) –π (b) π (c) π/2 (d) 1

98. The value of $\lim\limits_{\theta\to -\frac{\pi}{4}} \dfrac{\cos\theta+\sin\theta}{\theta+\frac{\pi}{4}}$ is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{-\pi}{4}$ (c) $-\sqrt{2}$ (d) $\sqrt{2}$

99. If $f(x) = \frac{x + |x|}{x}$, then the value of $\lim_{x \to 0} f(x)$ is
(a) 0 (b) 2
(c) does not exist (d) None of these

100. f(x) is a function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$ and h (x) is a function such that $h(x) = [f(x)]^2 + [g(x)]^2$ and h (5) = 11, then the value of h (10) is
(a) 5 (b) –5 (c) –11 (d) 11

101. Let α and β be the distinct roots of $ax^2 + bx = c = 0$, then
$$\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$$ is equal to
(a) $\frac{a^2}{2}(\alpha - \beta)^2$ (b) 0
(c) $\frac{-a^2}{2}(\alpha - \beta)^2$ (d) $\frac{1}{2}(\alpha - \beta)^2$

102. If $\lim_{x \to 0} \frac{((a - n)nx - \tan x)\sin nx}{x^2} = 0$, where n is non-zero real number, then a is equal to
(a) 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$

103. If sin y = x sin (a + y), then value of dy/dx is
(a) $\frac{\sin(a + y)}{\sin a}$ (b) $\frac{\sin^2(a + y)}{\sin a}$
(c) $\sin^2(a + y)$ (d) $\frac{\cos^2(a + y)}{\sin a}$

104. If $y = x \tan \frac{x}{2}$, then value of $(1 + \cos x)\frac{dy}{dx} - \sin x$ is
(a) –x (b) x^2 (c) x (d) None

105. Differential coefficient of $\frac{x \sin x}{1 + \cos x}$ is
(a) $\frac{-x - \sin x}{1 + \cos x}$ (b) $\frac{x - \sin x}{1 + \cos x}$ (c) $\frac{x + \sin x}{1 - \cos x}$ (d) $\frac{x + \sin x}{1 + \cos x}$

106. If $x^3 + y^3 = 3xy$, then value of $\frac{dy}{dx}$ is
(a) $\frac{x - y^2}{x^2 - y}$ (b) $\frac{x^2 - y}{x - y^2}$ (c) $\frac{x^2 - y^2}{x - y}$ (d) $\frac{x - y}{x^2 - y^2}$

107. If $y = ax^{n+1} + bx^{-n}$, then value of $x^2 \frac{d^2y}{dx^2}$ is
(a) n (n + 1) y (b) $n^2 (n + 1) y$
(c) 2n (n – 1) y (d) None of these

108. If $y = f\left(\frac{2x - 1}{x^2 + 1}\right)$ and $f'(x) = \sin x^2$, then value of $\frac{dy}{dx}$ is
(a) $\sin\left(\frac{2x + 1}{x^2 + 1}\right)^2 \left[\frac{2 + 2x - 2x^2}{x^2 + 1}\right]$
(b) $\sin\left(\frac{2x - 1}{x^2 - 1}\right)^2 \left[\frac{2 + 2x - 2x^2}{x^2 + 1}\right]$
(c) $\sin\left(\frac{2x - 1}{x^2 + 1}\right)^2 \left[\frac{2 + 2x - 2x^2}{x^2 + 1}\right]$
(d) $\sin\left(\frac{2x + 1}{x^2 - 1}\right)^2 \left[\frac{2 + 2x - 2x^2}{x^2 + 1}\right]$

109. $\lim_{x \to 2} \left[\frac{1}{x - 2} - \frac{2(2x - 3)}{x^3 - 3x^2 + 2x}\right]$ is equal to
(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) 1 (d) – 2

110. If $\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$, the positive integer n is equal to
(a) 3 (b) 5 (c) 2 (d) 4

111. $\lim_{x \to 0} \frac{\cos ax - \cos bx}{\cos cx - 1}$ is equal to $\frac{m}{n}$, where m and n are respectively
(a) $a^2 + b^2, c^2$ (b) $c^2, a^2 + b^2$
(c) $a^2 - b^2, c^2$ (d) $c^2, a^2 - b^2$

112. $\lim_{x \to 1} [x - 1]$, where [.] is greatest integer function, is equal to
(a) 1 (b) 2 (c) 0 (d) does not exist

113. $\lim_{x \to 0} \frac{\tan x - \sin x}{\sin^3 x}$ is equal to
(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) Not defined

114. If a, b are fixed non-zero constant, then the derivative of $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$ is ma + nb – p, where
(a) $m = 4x^3, n = \frac{-2}{x^3}, p = \sin x$
(b) $m = \frac{-4}{x^5}, n = \frac{2}{x^3}, p = \sin x$
(c) $m = \frac{-4}{x^5}, n = \frac{-2}{x^3}, p = -\sin x$
(d) $m = 4x^3, n = \frac{2}{x^3}, p = -\sin x$

115. If a is a fixed non-zero constant, then the derivative of $\frac{\sin(x + a)}{\cos x}$ is
(a) $\frac{\cos a}{\cos^2 x}$ (b) $\frac{-\cos a}{\cos^2 x}$ (c) $\frac{\sin a}{\cos^2 x}$ (d) $\frac{-\sin a}{\cos^2 x}$

116. $\lim_{x \to 4} \frac{|x - 4|}{x - 4}$ is equal to
(a) 1 (b) 0 (c) – 1 (d) does not exist

117. Let $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{when } x \neq \frac{\pi}{2} \\ 3, & \text{when } x = \frac{\pi}{2} \end{cases}$

If $\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$, then k is equal to
(a) 2 (b) 4 (c) 6 (d) 8

118. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 0 (d) None of these

119. If $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R$ $(xy \neq 1)$ and $\lim\limits_{x\to 0}\frac{f(x)}{x} = 2$. Then, $f'\left(\frac{1}{\sqrt{3}}\right)$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{6}$ (d) $\frac{3}{2}$

120. If f be a function given by $f(x) = 2x^2 + 3x - 5$. Then, $f'(0) = mf'(-1)$, where m is equal to

(a) -1 (b) -2 (c) -3 (d) -4

121. For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots \frac{x^2}{2} + x + 1,$$

$f'(1) = mf'(0)$, where m is equal to

(a) 50 (b) 0 (c) 100 (d) 200

122. Evaluate: $\lim\limits_{x\to\pi/6}\frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

(a) 3 (b) -3
(c) 1 (d) -1

123. The function $u = e^x \sin x$, $v = e^x \cos x$ satisfy the equation

(a) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ (b) $\frac{d^2u}{dx^2} = 2v$

(c) $\frac{d^2v}{dx^2} = -2u$ (d) All of these

124. If $f(x) = \begin{cases} |x|+1, & x<0 \\ 0, & x=0 \\ |x|-1, & x>0 \end{cases}$ then $\lim\limits_{x\to a} f(x)$ exists for all

(a) $a \neq 1$ (b) $a \neq 0$ (c) $a \neq -1$ (d) $a \neq 2$

125. Evaluate : $\lim\limits_{x\to a}\frac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a}$.

(a) $\frac{-5}{3}(a+2)^{2/3}$ (b) $\frac{5}{3}(a-2)^{2/3}$

(c) $\frac{5}{3}(a+2)^{-2/3}$ (d) $\frac{5}{3}(a+2)^{2/3}$

126. $\lim\limits_{x\to 0}\sqrt{\frac{x-\sin x}{x+\sin^2 x}}$ is equal to

(a) 1 (b) 0 (c) ∞ (d) None of these

127. What is the value of $\lim\limits_{x\to 0}\frac{x\sin 5x}{\sin^2 4x}$?

(a) 0 (b) $\frac{5}{4}$ (c) $\frac{5}{16}$ (d) $\frac{25}{4}$

128. If $\lim\limits_{x\to 0}\frac{a^x - x^a}{x^a - a^a} = -1$, then a is equal to:

(a) -1 (b) 0 (c) 1 (d) 2

129. The value of $\lim\limits_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is

(a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{4\sqrt{3}}$ (c) 0 (d) None of these

130. The value of $\lim\limits_{x\to 2a}\frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ is

(a) $\frac{1}{\sqrt{a}}$ (b) $\frac{1}{2\sqrt{a}}$ (c) $\frac{\sqrt{a}}{2}$ (d) $2\sqrt{a}$

131. $\lim\limits_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

132. $\lim\limits_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ is equal to

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$

(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

133. Let $f : R \to [0, \infty)$ be such that $\lim\limits_{x\to 5} f(x)$ exists and $\lim\limits_{x\to 5}\frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim\limits_{x\to 5} f(x)$ equals :

(a) 0 (b) 1 (c) 2 (d) 3

134. The value of $\lim\limits_{x\to 0}\frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$ is at $\tan x = 3$, is

(a) 0 (b) 1 (c) 2 (d) 3

135. $\lim\limits_{x\to 0}\frac{x\sqrt[3]{z^2 - (z-x)^2}}{\left(\sqrt[3]{8xz - 4x^2} + \sqrt[3]{8xz}\right)^4}$ is equal to

(a) $\frac{z}{2^{11/3}}$ (b) $\frac{1}{2^{23/3}.z}$ (c) $2^{21/3}z$ (d) None of these

136. $\lim\limits_{h\to 0}\left(\frac{1}{h\sqrt[3]{8+h}} - \frac{1}{2h}\right)$ equals to

(a) $-\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{48}$ (d) $-\frac{1}{48}$

137. The value of $\lim\limits_{x\to 0}\frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6 (c) 1/4 (d) 1/2

138. The value of $\lim\limits_{x\to 0}\frac{1-\cos x + 2\sin x - \sin^3 x - x^2 + 3x^4}{\tan^3 x - 6\sin^2 x + x - 5x^3}$ is

(a) 1 (b) 2 (c) -1 (d) -2

139. A function f is said to be a rational function, if $f(x) = \frac{g(x)}{h(x)}$, where g (x) and h (x) are polynomials such that $h(x) \neq 0$, then

(a) $h(a) \neq 0 \Rightarrow \lim\limits_{x\to a} f(x) = \frac{g(a)}{h(a)}$

(b) $h(a) = 0$ and $g(a) \neq 0 \Rightarrow \lim\limits_{x\to a} f(x)$ does not exist

(c) Both (a) and (b) are true
(d) Both (a) and (b) are false.

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** Given function $f(x) = x^2$. Observe that as x takes values very close to 0, the value of f (x) also approaches towards 0.

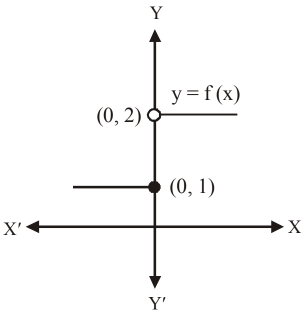
We say $\lim_{x \to 0} f(x) = 0$

(i.e, the limit of f (x) as x tends to zero equals zero).

2. **(a)** Given function $f(x) = \begin{cases} 1, & x \le 0 \\ 2, & x > 0 \end{cases}$

Graph of this function is shown below. It is clear that the value of f at 0 dictated by values of f (x) with $x \le 0$ equals 1, i.e. the left hand limit of f (x) at x = 0 is

$\lim_{x \to 0^-} f(x) = 1$



Similarly, the value of f at x = 0 dictated by values of f (x) with x > 0 equals 2, i.e., the right hand limit of f (x) at x = 0 is

$$\lim_{x \to 0^+} f(x) = 2$$

In this case the right and left hand limits are different, and hence we say that the limit of f (x) as x tends to zero does not exist (even though the function is defined at 0).

3. **(b)** Limit $= \lim_{x \to -1} \frac{(x-1)(x+1)}{(x+2)(x+1)} = \frac{-1-1}{-1+2} = -2$

4. **(a)** From direct substitution $\frac{\sqrt{1+0}+\sqrt{1-0}}{1+0} = \frac{2}{1} = 2$

5. **(a)** Limit $= \lim_{x \to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)} = \lim_{x \to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2} = 1$

6. **(c)** $\lim_{x \to 5} \frac{1-\sqrt{x-4}}{x-5} = \lim_{x \to 5} \frac{1-\sqrt{x-4}}{x-5} \cdot \frac{1+\sqrt{x-4}}{1+\sqrt{x-4}}$

$$= \lim_{x \to 5} \frac{1-x+4}{(x-5)(1+\sqrt{x-4})} = \lim_{x \to 5} \frac{-(x-5)}{(x-5)(1+\sqrt{x-4})}$$

$$= \lim_{x \to 5} \frac{-1}{(1+\sqrt{x-4})} = \frac{-1}{(1+\sqrt{5-4})} = \frac{-1}{2}$$

7. **(a)** By rationalisation of numerator, given expression

$$= \lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x} \cdot \frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}$$

$$= \lim_{x \to 0} \frac{1+x+x^2-1}{x\left(\sqrt{1+x+x^2}+1\right)} = \lim_{x \to 0} \frac{x(1+x)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x \to 0} \frac{1+x}{\sqrt{1+x+x^2}+1} = \frac{1}{2}$$

8. **(a)** Left hand limit $= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3x-1) = 3.1 - 1 = 2$

and Right hand limit $= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2+1)$

$= 1^2 + 1 = 2$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 2$

So $\lim_{x \to 1} f(x) = 2$

9. **(a)** $\lim_{x \to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x^2} \cdot \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$

$$= \lim_{x \to 0} \frac{1+x^2-1+x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$= \lim_{x \to 0} \frac{2x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)} = \frac{2}{\sqrt{1}+\sqrt{1}} = \frac{2}{2} = 1$$

10. **(a)** $f'(t) = \frac{d}{dt}\left[\frac{1-t}{1+t}\right] = \frac{(1+t)(-1)-(1-t)\times(1)}{(1+t)^2}$

$$= \frac{-1-t-1+t}{(1+t)^2} = \frac{-2}{(1+t)^2}$$

$$f'[1/t] = \frac{-2}{\left(1+\frac{1}{t}\right)^2} = \frac{-2t^2}{(t+1)^2}$$

11. **(d)** Let f and g be two functions such that both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then,

(i) Limit of sum of two functions is sum of the limits of the functions i.e.,

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x).$$

(ii) Limit of difference of two functions is difference of the limits of the functions, i.e.,

$$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x).$$

(iii) Limit of product of two functions is product of the limits of the functions, i.e.,

$$\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x).\lim_{x\to a} g(x).$$

(iv) Limit of quotient of two functions is quotient of the limits of the functions (whenever the denominator is non-zero), i.e.,

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{ if } \lim_{x\to a} g(x) \neq 0$$

12. (b) It is easy to see that the derivative of the function f (x) = x is the constant function 1. This is because

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{x+h-x}{h} = \lim_{h\to 0} 1 = 1$$

13. (c) Let f (x) = sin x. Then,

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(0+h)-\sin(0)}{h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$$

14. (d) We have,

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{3(2+h)-3(2)}{h}$$

$$= \lim_{h\to 0}\frac{6+3h-6}{h} = \lim_{h\to 0}\frac{3h}{h} = \lim_{h\to 0} 3 = 3.$$

The derivative of the function f (x) = 3x at x = 2 is 3.

15. (b) Since, the derivative measures the change in the function, intuitively it is clear that the derivative of the constant function must be zero at every point.

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{3-3}{h} = \lim_{h\to 0}\frac{0}{h} = 0$$

Similarly, $f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{3-3}{h} = 0$

16. (d) The derivative of f at x = a is denoted by

$$\left.\frac{d}{dx}f(x)\right|_a \text{ or } \left.\frac{df}{dx}\right|_a \text{ or even } \left(\frac{df}{dx}\right)_{x=a}$$

17. (a) Let y = x + a

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 1 + 0 = 1$$

18. (b) Let $y = \frac{1+\frac{1}{x}}{1-\frac{1}{x}} \Rightarrow y = \frac{x+1}{x-1}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(x-1)\frac{d}{dx}(x+1)-(x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{(x-1)(1+0)-(x+1)(1-0)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{(x-1)^2} = \frac{-2}{(1-x)^2}$$

19. (c) Let $y = 4\sqrt{x}-2 \Rightarrow y = 4x^{1/2}-2$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 4.\frac{1}{2}x^{\frac{1}{2}-1} - 0 = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$$

20. (b) Let $y = (ax+b)^n$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = n(ax+b)^{n-1}\frac{d}{dx}(ax+b) = n(ax+b)^{n-1}a$$

$$\Rightarrow \frac{dy}{dx} = na(ax+b)^{n-1}$$

21. (b) Let $y = \sin^n x \Rightarrow y = (\sin x)^n$

Differentiating y w.r.t.x, we get

$$\frac{dy}{dx} = n(\sin x)^{n-1}\frac{d}{dx}(\sin x) \Rightarrow \frac{dy}{dx} = n(\sin x)^{n-1}\cos x$$

22. (d) Let $y = (x^2+1)\cos x$,

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = (x^2+1)\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(x^2+1)$$

(by product rule)

$$= (x^2+1)(-\sin x) + \cos x(2x) = -x^2\sin x - \sin x + 2x\cos x$$

23. (c) We have, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{\tan[a(x+h)+b]-\tan(ax+b)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin(ax+ah+b)}{\cos(ax+ah+b)}-\frac{\sin(ax+b)}{\cos(ax+b)}}{h}$$

$$= \lim_{h\to 0}\frac{\sin(ax+ah+b)\cos(ax+b)-\sin(ax+b)\cos(ax+ah+b)}{h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0}\frac{a\sin(ah)}{a.h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0}\frac{a}{\cos(ax+b)\cos(ax+ah+b)}\lim_{h\to 0}\frac{\sin ah}{ah}$$

[as $h\to 0$, $ah\to 0$]

$$= \frac{a}{\cos^2(ax+b)} = a\sec^2(ax+b)$$

24. **(b)** $\because f(x) = x\sin x$

$$\Rightarrow f'(x) = \frac{d}{dx}(x\sin x)$$

$$= \sin x\frac{d}{dx}x + x\frac{d}{dx}\sin x = \sin x + x\cos x$$

$$\Rightarrow f'\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} = 1$$

25. **(b)** If given function is $6x^{100} - x^{55} + x$. Then, the derivative of function is $6.100.x^{99} - 55.x^{54} + 1$ or $600x^{99} - 55x^{54} + 1$

26. **(b)** $\lim_{x\to 0}\frac{x}{\tan x} = 1$

27. **(a)** We have,

$$\frac{d}{dx}(\log_x x) = \frac{d}{dx}(1) = 0 \quad [\because \log_x x = 1]$$

28. **(b)** We have,

$$\frac{d}{dx}(e^{3\log x}) = \frac{d}{dx}(e^{\log x^3}) = \frac{d}{dx}(x^3) = 3x^2 \quad [\because e^{\log k} = k]$$

29. **(b)** We have,

$$\frac{d}{dx}\left\{x^2 + \sin x + \frac{1}{x^2}\right\} = \frac{d}{dx}(x^2 + \sin x + x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin x) + \frac{d}{dx}(x^{-2})$$

$$= 2x + \cos x + (-2)x^{-3}$$

30. **(b)** We have,

$$\frac{d}{dx}\left\{\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2\right\} = \frac{d}{dx}\left\{x + \frac{1}{x} + 2\right\}$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(2) = 1 + (-1)x^{-2} + 0 = 1 - \frac{1}{x^2}$$

31. **(b)** We have, $f(x) = \alpha x^n$

Differentiating both sides w.r.t. x, we obtain

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(\alpha x^n)$$

$$\Rightarrow f'(x) = \alpha\frac{d}{dx}(x^n) \Rightarrow f'(x) = \alpha n.x^{n-1}$$

Putting x = 1 on both sides, we get

$$f'(1) = \alpha.n \Rightarrow \alpha = \frac{f'(1)}{n}$$

32. **(c)** We have,

$$\frac{d}{dx}(x\sin x) = x.\frac{d}{dx}(\sin x) + \sin x.\frac{d}{dx}(x)$$

$$= x\cos x + \sin x \cdot 1 = x\cos x + \sin x.$$

33. **(a)** We have,

$$\lim_{x\to 0}\frac{a^{\sin x} - 1}{\sin x} = \lim_{y\to 0}\frac{a^y - 1}{y} = \log a, \text{ where } y = \sin x$$

$$[\because x \to 0 \Rightarrow y = \sin x \to 0]$$

34. **(b)** Consider $\lim_{x\to 0}\frac{2\sin^2 3x}{x^2}$

$$= 2.\lim_{x\to 0}\left[\frac{\sin 3x}{x}\right]^2 = 2.\lim_{x\to 0}\left[3\frac{\sin 3x}{3x}\right]^2$$

$$= 2.9.\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2 = 18 \times 1 = 18$$

35. **(d)** Consider $\lim_{\theta\to 0}\frac{\sin m^2\theta}{\theta} = \lim_{\theta\to 0}\left(\frac{\sin m^2\theta}{m^2\theta}\right).m^2 = 1 \times m^2 = m^2$

36. **(b)** $f(x) = 7(-3)x^{-3-1} = -21x^{-4}$.

37. **(b)** $f'(x) = 2\cos x - 12x^3$

38. **(d)** Applying product rule,

$$f'(x) = (x-1)\frac{d}{dx}(x-2) + (x-2)\frac{d}{dx}(x-1)$$

$$= x - 1 + x - 2 = 2x - 3$$

39. **(a)** For $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]$ to exist, then both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist.

40. **(a)** $\lim_{x\to 0}\frac{1 + \frac{x}{3} - 1 + \frac{x}{3}}{x} = \lim_{x\to 0}\frac{2x}{3x} = \frac{2}{3}$

41. **(c)** $\lim_{x\to 0}\frac{\cos x}{\pi - x} = \frac{1}{\pi - 0} = \frac{1}{\pi}$

42. **(b)** $3f(x) - 2f\left(\frac{1}{x}\right) = x$ ...(i)

Put $x = \frac{1}{x}$, then $3f\left(\frac{1}{x}\right) - 2f(x) = \frac{1}{x}$ ...(ii)

Solving (i) and (ii), we get

$$5f(x) = 3x + \frac{2}{x} \Rightarrow f'(x) = \frac{3}{5} - \frac{2}{5x^2}$$

$$\therefore f'(2) = \frac{3}{5} - \frac{2}{20} = \frac{1}{2}$$

43. **(a)** Given function is $f(x)=\dfrac{7x}{(2x-1)(x+3)}$

Breaking into partial fraction

We get, $f(x)=\dfrac{1}{2x-1}+\dfrac{3}{x+3}$

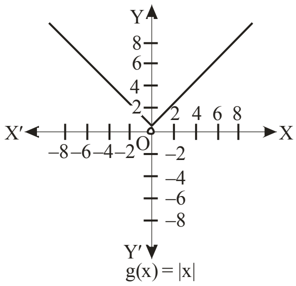
Differentiating w.r.t. x, we get

$$f'(x)=-\frac{2}{(2x-1)^2}-\frac{3}{(x+3)^2}$$

44. **(a)**

STATEMENT TYPE QUESTIONS

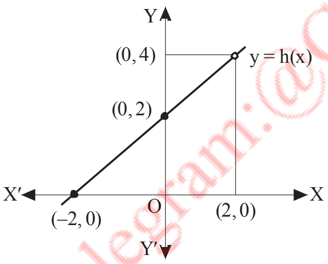
45. **(b)** Given function $g(x)=|x|, x\neq 0$. Observe that g (0) is not defined. Now, on computing the value of g (x) for values of x very near to 0, we see that the value of g(x) moves towards 0. So, $\lim_{x\to 0} g(x)=0$. This is intuitively clear from the graph of $y=|x|$ for $x\neq 0$.



46. **(a)** Given, the following function.

$h(x)=\dfrac{x^2-4}{x-2}, x\neq 2$

Now, on computing the value of h (x) for values of x very near to 2 (but not at x = 2), we get all these values are near to 4.



This is somewhat strengthened by considering the graph of the function y = h (x).

47. **(a)** I. Given,

$$\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}=\lim_{x\to 1}\left[\frac{x^{15}-1}{x-1}\div\frac{x^{10}-1}{x-1}\right]$$

$$=\lim_{x\to 1}\left[\frac{x^{15}-1}{x-1}\right]\div\lim_{x\to 1}\left[\frac{x^{10}-1}{x-1}\right]$$

$$=15(1)^{14}\div 10(1)^{9}=15\div 10=\frac{3}{2}$$

II. Put $y=1+x$, so that $y\to 1$ as $x\to 0$.

$$\text{Then, }\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{y\to 1}\frac{\sqrt{y}-1}{y-1}$$

$$=\lim_{y\to 1}\frac{y^{\frac{1}{2}}-1^{\frac{1}{2}}}{y-1}=\frac{1}{2}(1)^{\frac{1}{2}-1}=\frac{1}{2}$$

48. **(a)** I. $\lim_{x\to 0}\dfrac{\sin x}{x}=1$ (Standard Result)

II. Let us recall the trigonometric identity

$1-\cos x=2\sin^2\left(\dfrac{x}{2}\right)$.

$$\text{Then, }\lim_{x\to 0}\frac{1-\cos x}{x}=\lim_{x\to 0}\frac{2\sin^2\left(\frac{x}{2}\right)}{x}$$

$$=\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}.\sin\left(\frac{x}{2}\right)=\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}.\lim_{x\to 0}\sin\left(\frac{x}{2}\right)$$

$=1.0=0$

Observe that, we have implicity used the fact that $x\to 0$ is equivalent to $\dfrac{x}{2}\to 0$. This may be justified by putting $y=\dfrac{x}{2}$.

49. **(b)** I. Given, $\lim_{x\to 1}\dfrac{ax^2+bx+c}{cx^2+bx+a}=\dfrac{a\times(1)^2+b\times 1+c}{c\times(1)^2+b\times 1+a}$

$$=\frac{a+b+c}{c+b+a}=1$$

II. $$\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim_{x\to -2}\frac{(2+x)}{2x(x+2)}$$

$$=\lim_{x\to -2}\frac{1}{2x}=\frac{1}{2(-2)}=-\frac{1}{4}$$

50. **(c)** We have $\lim_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

$$=\lim_{h\to 0}\frac{(a^2+h^2+2ah)[\sin a\cos h+\cos a\sin h]-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\left[\frac{a^2\sin a(\cos h-1)}{h}+\frac{a^2\cos a\sin h}{h}+(h+2a)(\sin a\cos h+\cos a\sin h)\right]$$

$$=\lim_{h\to 0}\left[\frac{a^2\sin a\left(-2\sin^2\frac{h}{2}\right)}{\frac{h^2}{2}}.\frac{h}{2}\right]+\lim_{h\to 0}\frac{a^2\cos a\sin h}{h}+\lim_{h\to 0}(h+2a)\sin(a+h)$$

$=a^2\sin a\times 0+a^2\cos a(1)+2a\sin a=a^2\cos a+2a\sin a.$

51. **(a)** I. Recall the trigonometric rule $\sin 2x=2\sin x\cos x$. Thus,

$$\frac{df(x)}{dx}=\frac{d}{dx}(2\sin x\cos x)=2\frac{d}{dx}(\sin x\cos x)$$

$=2[(\sin x)'\cos x+\sin x(\cos x)']$

$=2[(\cos x)\cos x+\sin x(-\sin x)]$

$=2(\cos^2 x-\sin^2 x)$

II. $g(x) = \cot x = \dfrac{\cos x}{\sin x}$

$$\Rightarrow \frac{d}{dx}(g(x)) = \frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{(\cos x)'(\sin x) - (\cos x)(\sin x)'}{(\sin x)^2}$$

$$= \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\text{cosec}^2 x$$

52. (c) I. Let $f(x) = x^2 - 2$, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{[(x+h)^2 - 2] - (x^2 - 2)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + h^2 + 2xh - 2 - x^2 + 2}{h}$$

$$= \lim_{h \to 0} \frac{h(h+2x)}{h} = 0 + 2x = 2x$$

At $x = 10$, $f'(10) = 2 \times 10 = 20$

II. Let $f(x) = 99x$

We have $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{99(x+h) - 99x}{h}$$

$$= \lim_{h \to 0} \frac{99x + 99h - 99x}{h} = \lim_{h \to 0} \frac{99h}{h} = 99$$

At $x = 100$, $f'(100) = 99$

III. Let $f(x) = x$

We have, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{x + h - x}{h} \Rightarrow f'(x) = \lim_{h \to 0} \frac{h}{h} = 1$$

At $x = 1$, $f'(1) = 1$

53. (b) I. We have, $y = 2x - \dfrac{3}{4}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 2 \times 1 - 0 = 2$$

II. We have, $y = (5x^3 + 3x - 1)(x - 1)$

Differentiating y w.r.t x, we get

$$\frac{dy}{dx} = (5x^3 + 3x - 1)\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(5x^3 + 3x - 1)$$

$= (5x^3 + 3x - 1)(1 - 0) + (x - 1)(5 \times 3x^2 + 3 \times 1 - 0)$

$= (5x^3 + 3x - 1) + (x - 1)(15x^2 + 3)$

$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3$

$= 20x^3 - 15x^2 + 6x - 4$

54. (d) I. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + h^3 + 3xh(x+h) - x^3}{h}$$

$$= \lim_{h \to 0} (h^2 + 3x(x+h)) = 3x^2$$

II. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h} \quad \left[\because f(x) = \frac{1}{x^3}\right]$$

$$= \lim_{h \to 0} \frac{x^3 - (x+h)^3}{(x+h)^3 x^3 h}$$

$$= \lim_{h \to 0} \frac{x^3 - [x^3 + h^3 + 3xh(x+h)]}{(x+h)^3 x^3 h}$$

$$= \lim_{h \to 0} \frac{-h^3 - 3xh(x+h)}{(x+h)^3 x^3 h}$$

$$= \lim_{h \to 0} \frac{-h[h^2 + 3x(x+h)]}{(x+h)^3 x^3 h} = \frac{-3}{x^4}$$

55. (a) I. Let $f(x) = -x$

We have, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(by first principle)

$$= \lim_{h \to 0} \frac{-(x+h) - (-x)}{h} = \lim_{h \to 0} \frac{-x - h + x}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{-h}{h} = -1$$

II. Let $f(x) = (-x)^{-1}$

$$\Rightarrow f(x) = -\frac{1}{x}$$

We have, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ (by first principle)

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{-\frac{1}{x+h} + \frac{1}{x}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{1}{x} - \frac{1}{x+h}}{h} = \lim_{h \to 0} \frac{x + h - x}{x(x+h)h}$$

$$= \lim_{h \to 0} \frac{h}{x(x+h)h} = \frac{1}{x(x+0)} = \frac{1}{x^2}$$

56. (b) I. Let $y = \sin(x + a)$

$y = \sin x \cos a + \cos x \sin a$

$[\because \sin(A + B) = \sin A \cos B + \cos A \sin B]$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \cos a \frac{d}{dx}(\sin x) + \sin a \frac{d}{dx}(\cos x)$$

$= \cos a \cos x - \sin a \sin x = \cos(x + a)$

II. Let $y = \text{cosec}\, x \cot x$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \text{cosec}\, x \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(\text{cosec}\, x)$$

$= -\text{cosec}\, x\, \text{cosec}^2 x + \cot x\, (-\text{cosec}\, x \cot x)$

$= -\text{cosec}^3 x - \cot^2 x\, \text{cosec}\, x$

57. **(d)** I. The derivative of the function is $1 + 2x + 3x^2 + ... + 50x^{49}$. At x = 1 the value of this function equals to

$1 + 2(1) + 3(1)^2 + ... + 50(1)^{49} = 1 + 2 + 3 + ... + 50$

$$= \frac{(50)(51)}{2} = 1275$$

II. Clearly, this function is defined everywhere except at x = 0. We use the quotient rule with u = x + 1 and v = x. Hence, u′ = 1 and v′ = 1. Therefore,

$$\frac{df(x)}{dx} = \frac{d}{dx}\left(\frac{x+1}{x}\right) = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

$$= \frac{1(x)-(x+1)1}{x^2} = -\frac{1}{x^2}$$

58. **(d)** **59. (d)**

MATCHING TYPE QUESTIONS

60. **(b)** We say $\lim_{x\to a^-} f(x)$ is the expected value of *f* at *x* = *a* given the values of *f* near *x* to the left of *a*. This value is called the left hand limit of *f* at *a*.

Now, $\lim_{x\to a^+} f(x)$ is the expected value of *f* at *x* = *a* given the values of *f* near *x* to the right of *a*. This value is called the right hand limit of f (x) at *a* and if the right and left hand limits coincide, we call that common value as the limit of f(x) at *x* = *a* and denote it by $\lim_{x\to a} f(x)$.

61. **(b)** A. $\lim_{x\to 3} x + 3 = 3 + 3 = 6$

B. $\lim_{x\to\pi}\left(x - \frac{22}{7}\right) = \pi - \frac{22}{7}$

C. $\lim_{r\to 1} \pi r^2 = \pi \times (1)^2 = \pi$

D. $\lim_{x\to 4} \frac{4x+3}{x-2} = \frac{4\times 4+3}{4-2} = \frac{19}{2}$

E. $\lim_{x\to -1} \frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1} = \frac{1-1+1}{-2} = \frac{-1}{2}$

62. **(d)** A. Given, $\lim_{x\to\pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Let π – x = h, As x → π, then h → 0

$$\therefore \lim_{x\to\pi} \frac{\sin(\pi-x)}{\pi(\pi-x)} = \lim_{h\to 0}\frac{\sin h}{\pi h} = \lim_{h\to 0}\frac{1}{\pi}\times\frac{\sin h}{h}$$

$$= \frac{1}{\pi}\times 1 = \frac{1}{\pi} \quad \left(\because \lim_{h\to 0}\frac{\sin h}{h} = 1\right)$$

B. Given $\lim_{x\to 0}\frac{\cos x}{\pi - x}$

Put the limit directly, we get $\frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

C. Given, $\lim_{x\to 0}\frac{\cos 2x - 1}{\cos x - 1} = \lim_{x\to 0}\frac{1-\cos 2x}{1-\cos x} = \lim_{x\to 0}\frac{2\sin^2 x}{2\sin^2\frac{x}{2}}$

$$\left(\because 1 - \cos 2x = 2\sin^2 x \text{ and } 1 - \cos x = 2\sin^2\frac{x}{2}\right)$$

Multiplying and dividing by x^2 and then multiplying by $\frac{4}{4}$ in the numerator,

$$= \lim_{x\to 0}\frac{\sin^2 x}{x^2}\times\frac{4\times\frac{x^2}{4}}{\sin^2\frac{x}{2}} = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \times \left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2 \times 4$$

$= 1 \times 1 \times 4 = 4$

D. Given, $\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x}$

Dividing each term by x, we get

$$= \lim_{x\to 0}\frac{\frac{ax}{x}+\frac{x\cos x}{x}}{\frac{b\sin x}{x}} = \lim_{x\to 0}\frac{a+\cos x}{b\left(\frac{\sin x}{x}\right)}$$

$$= \frac{a+\cos 0}{b\times 1} = \frac{a+1}{b} \quad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

E. $\lim_{x\to 0} x \sec x = 0 \times \sec 0 = 0 \times 1 = 0$

F. $\lim_{x\to 0}\frac{\sin ax + bx}{ax + \sin bx}$

Dividing each term by x,

$$= \lim_{x\to 0}\frac{\frac{\sin ax}{x}+\frac{bx}{x}}{\frac{ax}{x}+\frac{\sin bx}{x}} = \lim_{x\to 0}\frac{\frac{a\sin ax}{ax}+b}{a+\frac{b\sin bx}{bx}}$$

$$= \frac{a\times 1+b}{a+b\times 1} = \frac{a+b}{a+b} = 1 \quad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

63. **(d)** A. Let $y = \text{cosec } x = \frac{1}{\sin x}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\sin x\frac{d}{dx}(1) - (1)\frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{\sin x \times 0 - 1\times\cos x}{\sin^2 x}$$

$$= \frac{0-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x}\times\frac{1}{\sin x}$$

$$\Rightarrow \frac{dy}{dx} = -\cot x \text{ cosec } x$$

B. Let y = 3 cot x + 5 cosec x

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = -3\text{ cosec}^2 x - 5\text{ cosec } x \cot x$$

C. Let y = 5 sin x – 6 cos x + 7

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 5\cos x - 6(-\sin x) + 0 = 5\cos x + 6\sin x$$

D. Let y = 2 tan x – 7 sec x

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 2\sec^2 x - 7\sec x\tan x$$

64. (a) A. Since, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{10(x+h)-10(x)}{h}=\lim_{h\to 0}\frac{10h}{h}$

$=\lim_{h\to 0}(10)=10$

B. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$

$=\lim_{h\to 0}(h+2x)=2x$

C. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{a-a}{h}=\lim_{h\to 0}\frac{0}{h}=0$ (as $h\neq 0$)

D. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$

$=\lim_{h\to 0}\frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right]=\lim_{h\to 0}\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]$

$=\lim_{h\to 0}\frac{-1}{x(x+h)}=\frac{-1}{x^2}$

INTEGER TYPE QUESTIONS

65. (b) $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\times\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}=\lim_{x\to 0}\frac{(2+x)-2}{x\left[\sqrt{2+x}+\sqrt{2}\right]}$

$=\lim_{x\to 0}\frac{1}{\sqrt{2+x}+\sqrt{2}}=\frac{1}{2\sqrt{2}}$

66. (c) $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}$

$=\lim_{x\to a}\frac{(a+2x)-3x}{\left(\sqrt{3a+x}-2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

Again rationalizing, we get

$=\lim_{x\to a}\frac{(a-x)\left[\sqrt{3a+x}+2\sqrt{x}\right]}{\left(\sqrt{a+2x}+\sqrt{3x}\right)(3a-3x)}=\frac{4\sqrt{a}}{6\sqrt{3a}}$

$=\frac{2\sqrt{3}}{9}$

67. (a) Put $y=\frac{\pi}{2}-x$

$\therefore \lim_{x\to\pi/2}(\sec x-\tan x)=\lim_{y\to 0}\left[\sec\left(\frac{\pi}{2}-y\right)-\tan\left(\frac{\pi}{2}-y\right)\right]$

$=\lim_{y\to 0}[\operatorname{cosec} y-\cot y]=\lim_{y\to 0}\left[\frac{1-\cos y}{\sin y}\right]$

$=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{2\sin\frac{y}{2}\cos\frac{y}{2}}=\lim_{\frac{y}{2}\to 0}\tan\frac{y}{2}=0$

68. (c) $\lim_{x\to 1}f(x)=f(1)$

i.e. RHL = LHL = f (1)

$\Rightarrow \lim_{x\to 1^+}f(x)=\lim_{x\to 1^{-1}}f(x)=4$

$\lim_{h\to 0}f(1+h)=\lim_{h\to 0}f(1-h)=4$

$\Rightarrow \lim_{h\to 0}b-a(1+h)=\lim_{h\to 0}a+b(1-h)=4$

$\Rightarrow b-a(1+0)=a+b(1-0)=4$

$\Rightarrow b-a=4$ and $b+a=4$

On solving, we get $a=0, b=4$

69. (d) $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$

$=\lim_{x\to 0}\frac{2\cos\frac{(2+x+2-x)}{2}\sin\frac{(2+x-2+x)}{2}}{x}$

$=\lim_{x\to 0}\frac{(2\cos 2)\sin x}{x}=2\cos 2$

$\Rightarrow p=2$ and $q=2$.

70. (c) At $x=5$, RHL $=\lim_{x\to 5^+}f(x)$

$=\lim_{h\to 0}f(5+h)=\lim_{h\to 0}|5+h|-5=0$

L.H.L $=\lim_{x\to 5}f(x)=\lim_{h\to 0}f(5-h)$

$=\lim_{h\to 0}|5-h|-5=0$

Hence, RHL = LHL = $\lim_{x\to 5}f(x)=0$

71. (b) $\lim_{x\to 0}\frac{\sin x}{x(1+\cos x)}=\lim_{x\to 0}\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{x\left[2\cos^2\frac{x}{2}\right]}$

$=\lim_{x\to 0}\frac{\tan x/2}{2.\frac{x}{2}}=\frac{1}{2}.\lim_{\frac{x}{2}\to 0}\frac{\tan\frac{x}{2}}{\frac{x}{2}}=\frac{1}{2}$

72. (b) $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}=\lim_{x\to 0}\frac{\sin 4x}{4x}\times\frac{4x}{\sin 2x}\times\frac{2x}{2x}$

$=\lim_{x\to 0}\frac{\sin 4x}{4x}\times\frac{2x}{\sin 2x}\times\frac{4x}{2x}=\frac{4}{2}=2$

$(\because x\to 0\Rightarrow 4x\to 0 \text{ and } 2x\to 0)$

73. (d) At $x=0$,

RHL $=\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)=\lim_{h\to 0}3(0+h+1)=3$

LHL $=\lim_{x\to 0^-}f(x)=\lim_{h\to 0}f(0-h)=\lim_{h\to 0}2(0-h)+3=3$

Hence, RHL = LHL = $\lim_{x\to 0}f(x)=3$

74. (a) At $x=-1$, limit exists.

$\therefore$ RHL = LHL

$\Rightarrow \lim_{x\to -1^+}f(x)=\lim_{x\to -1^-}f(x)$

$\Rightarrow \lim_{h\to 0}f(-1+h)=\lim_{h\to 0}f(-1-h)$

$\Rightarrow \lim_{h\to 0}c(-1+h)^2=\lim_{h\to 0}(-1-h+2)$

$\Rightarrow c(-1+0)^2=1-0\Rightarrow c=1$

75. (d) We have

$$\lim_{x\to\frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{2^{\frac{5}{2}}-[(\cos x+\sin x)^2]^{\frac{5}{2}}}{2-(1+\sin 2x)} = \lim_{x\to\frac{\pi}{4}} \frac{(1+\sin 2x)^{\frac{5}{2}}-2^{\frac{5}{2}}}{(1+\sin 2x)-2}$$

$$= \lim_{t\to 2} \frac{t^{\frac{5}{2}}-2^{\frac{5}{2}}}{t-2}, \text{ where } t=1+\sin 2x = \frac{5}{2}\times(2)^{\frac{5}{2}-1} = 5\sqrt{2}$$

76. (d) $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$

$\Rightarrow n.2^{n-1}=80 \Rightarrow n.2^{n-1}=5.2^{5-1} \Rightarrow n=5$

77. (a) $\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2 = 2.\lim_{x\to 0}\frac{\sin 2x}{2x}=2\times 1=2.$

78. (d) Let $f(x)=x^n$

$f'(x)=n.x^{n-1}$

$f'(1)=n.1^{n-1}=n$

$10=n$

79. (b) Let $\lim_{x\to 5}\frac{x^k-5^k}{x-5}=500$

By using $\lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}$, we have

$k.5^{k-1}=500$

Now, put $k=4$, we get

$4.5^{4-1}=500 \Rightarrow 4.5^3=500$

which is true.

$\therefore\ k=4$

ASSERTION- REASON TYPE QUESTIONS

80. (c) Assertion is correct but Reason is incorrect.

81. (c) Assertion is correct

$$\lim_{x\to 0}\left[\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right]=\lim_{x\to 0}\frac{1-\cos x}{\sin x}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\lim_{x\to 0}\tan\frac{x}{2}=0$$

82. (c) **Assertion** is correct but Reason is incorrect.

Reason: $f(x)=ax^2+bx+c$

$f'(x)=2ax+b$

83. (b) Both Assertion and Reason are correct but reason is not the correct explanation.

84. (b) Both Assertion and Reason are correct.

85. (a) We know that $\frac{d}{dx}(x^n)=nx^{n-1}$

$$\therefore \text{ For } f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+.....+\frac{x^2}{2}+x+1$$

$$f'(x)=\frac{100x^{99}}{100}+99\frac{x^{98}}{99}+.....+\frac{2x}{2}+1$$

$=x^{99}+x^{98}+..........+x+1$

Now, $f'(1)=1+1+.........\text{to } 100 \text{ term}=100$

$f'(0)=1$

$\therefore f'(1)=100\times 1=100f'(0)$

Hence, $f'(1)=100f'(0)$

86. (a) $\lim_{x\to 0}(1+3x)^{1/x}=\lim_{x\to 0}\left[\left(1+3x^{1/3x}\right)\right]^3=e^3$

because $\lim_{x\to 0}(1+x)^{1/x}=e$

87. (c) Obviously Assertion is true, but Reason is not always true.

Consider, $f(x)=[x]$ and $g(x)=\sin x$.

88. (b) $\because \lim_{x\to 0}\frac{\tan x^0}{x^0}=\lim_{x\to 0}\frac{\tan\left(\frac{\pi x}{180}\right)}{\left(\frac{\pi x}{180}\right)}=1$

and $\lim_{x\to 0}\{f(x)g(x)\}=\left(\lim_{x\to 0}f(x)\right)\left(\lim_{x\to 0}g(x)\right)=lm$

89. (a) **Assertion:** Let $u=x$, $v=|x|$

90. (b) Both Assertion and Reason are correct.

Reason: $f'(2)=\lim_{h\to 0}\frac{\{(2+h)^2+1\}-\{2^2+1\}}{h}$

$$=\lim_{h\to 0}\frac{h^2+4h}{h}=\lim_{h\to 0}h+4=4 \Rightarrow f'(2)=4$$

91. (a)

CRITICALTHINKING TYPE QUESTIONS

92. (a) $\lim_{x\to 0}\frac{\sin^2 2x}{x^2}$

$$=\lim_{x\to 0}\frac{(2\sin x\cos x)^2}{x^2}=4\lim_{x\to 0}\frac{\sin^2 x}{x^2}.\cos^2 x=4$$

93. (c) $\lim_{x\to 0}\frac{x^3\cot x}{1-\cos x}=\lim_{x\to 0}\left(\frac{x^3\cot x}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}\right)$

$$=\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3\times\lim_{x\to 0}\cos x\times\lim_{x\to 0}(1+\cos x)=2$$

94. (b) $\lim_{x\to 0}\frac{x(e^x-1)}{1-\cos x}=\lim_{x\to 0}\frac{2x^2(e^x-1)}{4\sin^2\frac{x}{2}}$

$$=2\lim_{x\to 0}\left[\frac{(x/2)^2}{\sin^2(x/2)}\right]\left(\frac{e^x-1}{x}\right)=2$$

95. (d) $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}=\lim_{x\to 0}\frac{\sqrt{1-(1-2\sin^2 x)}}{\sqrt{2}x}$;

$$=\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}=\lim_{x\to 0}\frac{|\sin x|}{x}$$

The limit of above does not exist as $LHS=-1\neq RHL=1$

96. (c) Given expression can be written as

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{4\sin^4 x}$$

$$= \lim_{x\to 0} \frac{x}{4\sin^4 x}\left[\frac{2\tan x}{1-\tan^2 x} - 2\tan x\right]$$

$$= \lim_{x\to 0} \frac{2x\tan x}{4\sin^4 x}\left[\frac{1-1+\tan^2 x}{1-\tan^2 x}\right]$$

$$= \lim_{x\to 0} \frac{2x\tan^3 x}{4\sin x^4(1-\tan^2 x)}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{x}{\sin x}.\frac{1}{\cos^3 x}.\frac{1}{1-\tan^2 x} = \frac{1}{2}.1.\frac{1}{1^3}.\frac{1}{1-0} = \frac{1}{2}$$

97. (b) $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin(\pi - \pi\sin^2 x)}{x^2}$ $[\because \sin(\pi-\theta) = \sin\theta]$

$$= \lim_{x\to 0} \frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x} \times \frac{(\pi\sin^2 x)}{x^2} = \pi$$

98. (d) Put $\theta + \frac{\pi}{4} = h$ or $\theta = -\frac{\pi}{4} + h$

$$\text{Limit} = \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{4}-h\right) - \sin\left(\frac{\pi}{4}-h\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{4}-h\right) - \cos\left(\frac{\pi}{4}+h\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\sin\frac{\pi}{4}.\sin h}{h} = \sqrt{2}$$

99. (c) $\text{LHL} = \lim_{h\to 0} \frac{-h+|h|}{-h} = \lim_{h\to 0} (0) = 0$

$\text{RHL} = \lim_{h\to 0} \frac{h+|h|}{h} = 2$

LHL $\neq$ RHL $\Rightarrow$ limit does not exist

100. (d) $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$
$= 2f(x)g(x) + 2g(x)f''(x)$
$= 2f(x)g(x) - 2f(x)g(x)$
$= 0 \quad [\because f''(x) = -f(x)]$
$\Rightarrow \quad h(x) = c \Rightarrow h(10) = h(5) = 11$

101. (a) Given limit $= \lim_{x\to\alpha} \frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2}$

$$= \lim_{x\to\alpha} \frac{2\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha} \frac{2}{(x-\alpha)^2} \times \frac{\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}} \times \frac{a^2(x-\alpha)^2(x-\beta)^2}{4}$$

$$= \frac{a^2(\alpha-\beta)^2}{2}.$$

102. (d) We are given that

$$\lim_{x\to 0} \frac{[(a-n)nx - \tan x]\sin nx}{x^2} = 0$$

where n is non zero real number

$$\Rightarrow \lim_{x\to 0} n.\frac{\sin nx}{nx}\left[\left\{(a-n)n - \frac{\tan x}{x}\right\}\right] = 0$$

$$\Rightarrow 1.n[(a-n)n-1] = 0 \Rightarrow a = \frac{1}{n} + n$$

103. (b) $\sin y = x\sin(a+y)$

$\therefore x = \frac{\sin y}{\sin(a+y)}$

Differentiating the function with respect to y

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

104. (c) Let $y = x\tan\frac{x}{2} \Rightarrow \frac{dy}{dx} = 1.\tan\frac{x}{2} + x.\sec^2\frac{x}{2}.\frac{1}{2}$

$$= \tan\frac{x}{2} + \frac{x}{2}\sec^2\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} + \frac{x}{2\cos^2\frac{x}{2}}$$

$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2} + x}{2\cos^2\frac{x}{2}} = \frac{\sin x + x}{1+\cos x}$$

$$\Rightarrow (1+\cos x)\frac{dy}{dx} - \sin x = x$$

105. (d) $\frac{d}{dx}\left(\frac{x\sin x}{1+\cos x}\right)$

$$= \frac{(1+\cos x)(\sin x + x\cos x) - (x\sin x)(0-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\sin x(1+\cos x) + x\cos x + x(\cos^2 x + \sin^2 x)}{(1+\cos x)^2}$$

$$= \frac{(x+\sin x)(1+\cos x)}{(1+\cos x)^2} = \frac{x+\sin x}{1+\cos x}$$

106. (b) Differentiating w.r.t. x,

$$3x^2 + 3y^2\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$$

$$\Rightarrow 3(x^2 - y) = 3\frac{dy}{dx}(x - y^2) \Rightarrow \frac{dy}{dx} = \frac{x^2-y}{x-y^2}$$

107. (a) $y = ax^{n+1} + bx^{-n}$

$$\frac{dy}{dx} = (n+1)ax^n - n\,bx^{-n-1}$$

$$\frac{d^2y}{dx^2} = (n+1)n\,ax^{n-1} + n(n+1)\,bx^{-n-2}$$

$$\therefore x^2\frac{d^2y}{dx^2} = (n+1)\,na.\,x^{n+1} + n(n+1)\,b\,x^{-n}$$

$$= n(n+1)[ax^{n+1} + bx^{-n}] = n(n+1)y$$

108. (c) We have, $y = f\left(\frac{2x-1}{x^2+1}\right)$

$$\Rightarrow \frac{dy}{dx} = f'\left(\frac{2x-1}{x^2+1}\right).\left[\frac{(x^2+1)2-(2x-1).2x}{(x^2+1)^2}\right]$$

$$= \sin\left(\frac{2x-1}{x^2+1}\right)^2.\left[\frac{2+2x-2x^2}{(x^2+1)}\right]$$

$$\left[\because f'(x) = \sin x^2, \therefore f'\left(\frac{2x-1}{x^2+1}\right) = \sin\left(\frac{2x-1}{x^2+1}\right)^2\right]$$

109. (b) We have,

$$\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right]$$

$$= \lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to 2}\left[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to 2}\left[\frac{x^2-5x+6}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to 2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right] \quad (x-2\neq 0)$$

$$= \lim_{x\to 2}\left[\frac{x-3}{x(x-1)}\right] = \frac{-1}{2}$$

110. (d) We have, $\lim_{x\to 3}\frac{x^n-3^n}{x-3} = n(3)^{n-1}$

Therefore, $n(3)^{n-1} = 108 = 4(27) = 4(3)^{4-1}$

On comparing, we get $n = 4$

111. (c) We have, $\lim_{x\to 0}\frac{\cos ax-\cos bx}{\cos cx-1}$

$$= \lim_{x\to 0}\frac{2\sin\left[\frac{(a+b)}{2}x\right]\sin\left(\frac{(a-b)x}{2}\right)}{2\sin^2\left(\frac{cx}{2}\right)}$$

$$= \lim_{x\to 0}\frac{\sin\frac{(a+b)x}{2}.\sin\frac{(a-b)x}{2}}{x^2}.\frac{x^2}{\sin^2\frac{cx}{2}}$$

$$= \lim_{x\to 0}\frac{\sin\frac{(a+b)x}{2}}{\frac{(a+b)x}{2}.\left(\frac{2}{a+b}\right)}.\frac{\sin\frac{(a-b)x}{2}}{\frac{(a-b)x}{2}.\frac{2}{a-b}}.\frac{\left(\frac{cx}{2}\right)^2\times\frac{4}{c^2}}{\sin^2\frac{cx}{2}}$$

$$= \left(\frac{a+b}{2}\times\frac{a-b}{2}\times\frac{4}{c^2}\right) = \frac{a^2-b^2}{c^2}$$. Hence m and n are $a^2 - b^2$ and c^2 respectively.

112. (d) Since, RHL $= \lim_{x\to 1^+}[x-1] = 0$

and LHL $= \lim_{x\to 1^-}[x-1] = -1$

Hence, at $x = 1$ limit does not exist.

113. (a) We have, $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x} = \lim_{x\to 0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{\sin^3 x}$

$$= \lim_{x\to 0}\frac{1-\cos x}{\cos x\sin^2 x} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\left(4\sin^2\frac{x}{2}.\cos^2\frac{x}{2}\right)} = \frac{1}{2}$$

114. (b) Let $y = \frac{a}{x^4}-\frac{b}{x^2}+\cos x$

$\Rightarrow \quad y = ax^{-4} - bx^{-2} + \cos x$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = a\frac{d}{dx}(x^{-4}) - b\frac{d}{dx}(x^{-2}) + \frac{d}{dx}(\cos x)$$

$$= a(-4)x^{-4-1} - b(-2)x^{-2-1}(-\sin x)$$

$$= -\frac{4a}{x^5}+\frac{2b}{x^3}-\sin x \qquad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

115. (a) Let $y = \frac{\sin(x+a)}{\cos x} = \frac{\sin x\cos a+\cos x\sin a}{\cos x}$

[$\because$ sin (A + B) = sin A cos B + cos A sin B]

$$= \frac{\sin x\cos a}{\cos x}+\frac{\cos x\sin a}{\cos x} = \cos a\tan x + \sin a$$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \cos a\frac{d}{dx}(\tan x)+\frac{d}{dx}(\sin a)$$

$$= \cos a\sec^2 x + 0 = \frac{\cos a}{\cos^2 x}$$

116. (d) Let $f(x) = \frac{|x-4|}{x-4}$

At $x=4$, RHL $= \lim_{x\to 4^+}f(x) = \lim_{h\to 0}f(4+h) = \lim_{h\to 0}\frac{|4+h-4|}{(4+h-4)}$

$$= \lim_{h\to 0}\left(\frac{4+h-4}{4+h-4}\right) = 1$$

At $x = 4$, LHL $= \lim_{x\to 4^-}f(x) = \lim_{h\to 0}f(4-h)$

$$= \lim_{h\to 0}\frac{|4-h-4|}{(4-h-4)} = \lim_{h\to 0}\frac{-(4-h-4)}{(4-h-4)} = -1$$

$\because$ RHL $\neq$ LHL

$\therefore$ Hence, at $x = 4$, limit does not exist.

117. (c) Given, $f(x) = \begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq\frac{\pi}{2}\\ 3, & x=\frac{\pi}{2}\end{cases}$

Since, $\lim_{x\to\frac{\pi}{2}}f(x) = f\left(\frac{\pi}{2}\right) \Rightarrow \lim_{x\to\frac{\pi}{2}}f(x) = 3$

$$\Rightarrow \lim_{h\to 0}f\left(\frac{\pi}{2}+h\right) = 3 \Rightarrow \lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)} = 3$$

$\Rightarrow \quad \lim_{h\to 0}\frac{-k\sin h}{\pi-\pi-2h}=3 \Rightarrow \lim_{h\to 0}\frac{-k\sin h}{-2h}=3$

$\Rightarrow \quad \frac{k}{2}\times\lim_{h\to 0}\frac{\sin h}{h}=3 \Rightarrow \frac{k}{2}\times 1=3$

$\Rightarrow \quad k=6 \quad \left(\because \lim_{h\to 0}\frac{\sin h}{h}=1\right)$

118. (d) We have, $f(x)=|\cos x-\sin x|$

$$\Rightarrow \quad f(x)=\begin{cases}\cos x-\sin x, & \text{for } 0<x\le\frac{\pi}{4}\\ \sin x-\cos x, & \text{for } \frac{\pi}{4}<x<\frac{\pi}{2}\end{cases}$$

Clearly, $Lf'\left(\frac{\pi}{4}\right)=\left\{\frac{d}{dx}(\cos x-\sin x)\right\}_{\text{at } x=\frac{\pi}{4}}$

$=(-\sin x-\cos x)_{x=\frac{\pi}{4}}=-\sqrt{2}$

and $Rf'\left(\frac{\pi}{4}\right)=\left\{\frac{d}{dx}(\sin x-\cos x)\right\}_{\text{at } x=\frac{\pi}{4}}$

$=(\cos x+\sin x)_{x=\frac{\pi}{4}}=\sqrt{2}$

$\because \quad Lf'\left(\frac{\pi}{4}\right)\neq Rf'\left(\frac{\pi}{4}\right)$

$\therefore \quad f'\left(\frac{\pi}{4}\right)$ doesn't exist.

119. (d) $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$ ...(i)

Putting $x=y=0$, we get $f(0)=0$

Putting $y=-x$, we get $f(x)+f(-x)=f(0)=0$

$\Rightarrow \quad f(-x)=-f(x)$ ...(ii)

Also, $\lim_{x\to 0}\frac{f(x)}{x}=2$ ...(iii)

Now, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

[using eq. (ii) $-f(x)=f(-x)$]

$$f'(x)=\lim_{h\to 0}\frac{f\left(\frac{x+h-x}{1-(x+h)(-x)}\right)}{h}$$

[using eq. (i)]

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\left[\frac{f\left(\frac{h}{1+x(x+h)}\right)}{h}\right]$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)}\times\left[\frac{1}{1+xh+x^2}\right]$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)}\times\lim_{h\to 0}\frac{1}{1+xh+x^2}$$

$\left[\text{using } \lim_{x\to 0}\frac{f(x)}{x}=2\right]$

$\Rightarrow \quad f'(x)=2\times\frac{1}{1+x^2}=\frac{2}{1+x^2}$

$\Rightarrow \quad f'\left(\frac{1}{\sqrt{3}}\right)=\frac{2}{1+\frac{1}{3}}=\frac{2}{4/3}=\frac{6}{4}=\frac{3}{2}$

120. (c) We first find the derivatives of $f(x)$ at $x=-1$ and at $x=0$.

We have,

$f'(-1)=\lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h}$

$=\lim_{h\to 0}\frac{[2(-1+h)^2+3(-1+h)-5]-[2(-1)^2+3(-1)-5]}{h}$

$=\lim_{h\to 0}\frac{2h^2-h}{h}=\lim_{h\to 0}(2h-1)=2(0)-1=-1$

and $f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{[2(0+h)^2+3(0+h)-5]-[2(0)^2+3(0)-5]}{h}$

$=\lim_{h\to 0}\frac{2h^2+3h}{h}=\lim_{h\to 0}(2h+3)=2(0)+3=3$

Clearly, $f'(0)=-3f'(-1)$

121. (c) Given, $f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+...+\frac{x^2}{2}+x+1$

$\Rightarrow \quad f'(x)=\frac{100x^{99}}{100}+\frac{99x^{98}}{99}+...+\frac{2x}{2}+1+0$

$[\because f(x)=x^n \Rightarrow f'(x)=nx^{n-1}]$

$\Rightarrow \quad f'(x)=x^{99}+x^{98}+...+x+1$ ...(i)

Putting $x=1$, we get

$f'(1)=\underbrace{(1)^{99}+1^{98}+...+1+1}_{100\text{ times}}=\underbrace{1+1+1...+1+1}_{100\text{ times}}$

$\Rightarrow \quad f'(1)=100$ ...(ii)

Again, putting $x=0$, we get

$f'(0)=0+0+...+0+1$

$\Rightarrow \quad f'(0)=1$ ...(iii)

From eqs. (ii) and (iii), we get

$f'(1)=100f'(0)$

Hence, $m=100$

122. (b) We have,

$\lim_{x\to\pi/6}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}=\lim_{x\to\pi/6}\frac{(2\sin x-1)(\sin x+1)}{(2\sin x-1)(\sin x-1)}$

$=\lim_{x\to\pi/6}\frac{\sin x+1}{\sin x-1}=\frac{\frac{1}{2}+1}{\frac{1}{2}-1}=-3$

123. (d) We have, $u=e^x\sin x$

$\Rightarrow \quad \frac{du}{dx}=e^x\sin x+e^x\cos x=u+v$

$v=e^x\cos x$

$\Rightarrow \quad \frac{dv}{dx}=e^x\cos x-e^x\sin x=v-u$

$\therefore$ Consider $v\frac{du}{dx}-u\frac{dv}{dx}=v(u+v)-u(v-u)=u^2+v^2$

$\frac{d^2u}{dx^2}=\frac{du}{dx}+\frac{dv}{dx}=u+v+v-u=2v$

and $\frac{d^2v}{dx^2}=\frac{dv}{dx}-\frac{du}{dx}=(v-u)-(v+u)=-2u$

124. **(b)** Given, $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}=\begin{cases}-x+1, & x<0\\ 0, & x=0\\ x-1, & x>0\end{cases}$

Let us first check the existence of limit of f (x) at x = 0.
At x = 0,

$$\text{RHL}=\lim_{x\to0^+}f(x)=\lim_{h\to0}f(0+h)=\lim_{h\to0}(0+h)-1$$
$$=\lim_{h\to0}h-1=0-1=-1$$
$$\text{LHL}=\lim_{x\to0^-}f(x)=\lim_{h\to0}f(0-h)$$
$$=\lim_{h\to0}-(0-h)+1$$
$$=\lim_{h\to0}h+1=0+1=1$$

$\Rightarrow$ RHL $\neq$ LHL
$\Rightarrow$ At x = 0, limit does not exist.
Note that for any a < 0 or a > 0, $\lim_{x\to a}f(x)$ exists,
as for a < 0, $\lim_{x\to a}f(x)=\lim_{x\to a}-x+1=-a+1$ exists and
for a > 0, $\lim_{x\to a}f(x)=\lim_{x\to a}x-1=a-1$ exists. Hence,
$\lim_{x\to a}f(x)$ exists for all $a\neq0$.

125. **(d)** $\lim_{x\to a}\dfrac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}=\lim_{x\to a}\dfrac{(x+2)^{5/3}-(a+2)^{5/3}}{(x+2)-(a+2)}$

$$=\lim_{y\to b}\frac{y^{5/3}-b^{5/3}}{y-b},$$

where $x+2=y$, $a+2=b$. and
when $x\to a, y\to b$

$$=\frac{5}{3}b^{5/3-1}=\frac{5}{3}b^{2/3}=\frac{5}{3}(a+2)^{2/3}.$$

126. **(b)** $\lim_{x\to0}\sqrt{\dfrac{x-\sin x}{x+\sin^2 x}}=\lim_{x\to0}\sqrt{\dfrac{1-\dfrac{\sin x}{x}}{1+\dfrac{\sin^2 x}{x}}}$

$$=\lim_{x\to0}\sqrt{\frac{1-\dfrac{\sin x}{x}}{1+\left(\dfrac{\sin x}{x}\right)\sin x}}=\sqrt{\frac{1-1}{1+1\times0}}=0$$

127. **(c)** $\lim_{x\to0}\dfrac{x\sin5x}{\sin^2 4x}$

[multiply denominator and numerator with x]
We get,

$$\lim_{x\to0}\frac{x^2\sin5x}{x\sin^2 4x}=\lim_{x\to0}\frac{\sin5x}{x}.\frac{x^2}{\sin^2 4x}$$

Rearranging to bring a standard form, we get,

$$\lim_{x\to0}\frac{5\sin5x}{5x}.\frac{(4x)^2}{16\sin^2 4x}$$
$$=\frac{5}{16}\left(\lim_{x\to0}\frac{\sin5x}{5x}\right).\frac{1}{\lim_{x\to0}\left(\dfrac{\sin4x}{4x}\right)^2}=\frac{5}{16}$$

128. **(c)** As given $\lim_{x\to0}\dfrac{a^x-x^a}{x^a-a^a}=-1$

Applying limit, we have

$$\frac{1-0}{0-a^a}=-1\quad(\because\ a^0=1)$$
$$\Rightarrow\quad\frac{1}{-a^a}=-1\Rightarrow a^a=1$$

Taking log on both the sides
a log a = 0 $\Rightarrow$ a = 0 or log a = 0
a $\neq$ 0 $\Rightarrow$ log a = 0 $\Rightarrow$ a = 1

129. **(a)** The required limit

$$=\lim_{x\to2}\frac{[1+\sqrt{2+x}-3]}{(x-2)\left[\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right]}\text{ (on rationalizing)}$$
$$=\lim_{x\to2}\frac{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\lim_{x\to2}\frac{(x+2)-4}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\lim_{x\to2}\frac{1}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\frac{1}{2\sqrt{3}}.\frac{1}{4}=\frac{1}{8\sqrt{3}}$$

130. **(b)** $\lim_{x\to2a}\dfrac{\sqrt{x-2a}}{\sqrt{x^2-4a^2}}+\dfrac{\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}$

$$=\lim_{x\to2a}\frac{1}{\sqrt{x+2a}}+\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{(x-2a)(x+2a)}}$$
$$=\frac{1}{2\sqrt{a}}+\lim_{x\to2a}\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{\left(\sqrt{x}-\sqrt{2a}\right)\left(\sqrt{x}+\sqrt{2a}\right)(x+2a)}}$$
$$=\frac{1}{2\sqrt{a}}+\lim_{x\to2a}\frac{\sqrt{\sqrt{x}-\sqrt{2a}}}{\sqrt{\left(\sqrt{x}+\sqrt{2a}\right)(x+2a)}}=\frac{1}{2\sqrt{a}}+0$$

131. **(d)** $\lim_{x\to\frac{\pi}{2}}\dfrac{\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right).(1-\sin x)}{(\pi-2x)^3}$

Let $x=\dfrac{\pi}{2}+y;\ y\to0$

$$=\lim_{y\to0}\frac{\tan\left(-\dfrac{y}{2}\right).(1-\cos y)}{(-2y)^3}=\lim_{y\to0}\frac{-\tan\dfrac{y}{2}\left(2\sin^2\dfrac{y}{2}\right)}{(-8).\dfrac{y^3}{8}.8}$$
$$=\lim_{y\to0}\frac{1}{32}\frac{\tan\dfrac{y}{2}}{\left(\dfrac{y}{2}\right)}.\left[\frac{\sin y/2}{y/2}\right]^2=\frac{1}{32}$$

132. (d) $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}=\lim_{x\to 2}\frac{\sqrt{2}|\sin(x-2)|}{x-2}$

$\underset{(\text{at } x=2)}{\text{L.H.L}}=-\lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=-1$

$\underset{(\text{at } x=2)}{\text{R.H.L}}=\lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=1$

Thus $\underset{(\text{at } x=2)}{\text{L.H.L}}\neq\underset{(\text{at } x=2)}{\text{R.H.L}}$

Hence, $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ does not exist.

133. (d) $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$

$\Rightarrow \lim_{x\to 5}[(f(x))^2-9]=0 \Rightarrow \lim_{x\to 5}f(x)=3$

134. (c) Consider $\lim_{x\to 0}\frac{\tan^2 x-2\tan x-3}{\tan^2 x-4\tan x+3}$

$=\lim_{x\to 0}\frac{\tan^2 x-3\tan x+\tan x-3}{\tan^2 x-3\tan x-\tan x+3}$

$=\lim_{x\to 0}\frac{(\tan x+1)(\tan x-3)}{(\tan x-1)(\tan x-3)}=\lim_{x\to 0}\frac{\tan x+1}{\tan x-1}$

Now, at $\tan x=3$, we have

$\lim_{\tan x\to 3}\frac{\tan x+1}{\tan x-1}=\frac{3+1}{3-1}=\frac{4}{2}=2$

135. (b) $\lim_{x\to 0}\frac{x\sqrt[3]{z^2-(z-x)^2}}{\left(\sqrt[3]{8xz-4x^2}+\sqrt[3]{8xz}\right)^4}$

$=\lim_{x\to 0}\frac{x\sqrt[3]{2xz-x^2}}{(\sqrt[3]{x}\sqrt[3]{8z-4x}+\sqrt[3]{8z}\sqrt[3]{x})^4}$

$=\lim_{x\to 0}\frac{x^{4/3}\sqrt[3]{2z-x}}{x^{4/3}[\sqrt[3]{8z-4x}+\sqrt[3]{8z}]^4}$

$=\frac{\sqrt[3]{2z}}{\left[2\sqrt[3]{8z}\right]^4}=\frac{1}{2^{23/3}.z}$

136. (d) $\lim_{h\to 0}\frac{2-\sqrt[3]{8+h}}{2h.\sqrt[3]{8+h}}$

$\lim_{h\to 0}\frac{8-(8+h)}{2h.\sqrt[3]{8+h}\{8^{2/3}+8^{1/3}.(8+h)^{1/3}+(8+h)^{2/3}\}}=-\frac{1}{48}$

137. (b) $\frac{2\cos\left(\frac{\sin x+x}{2}\right).\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$

$=\lim_{x\to 0}2\left[\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\right]\left[\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right]$

$\times\left[\frac{1}{2\left(\frac{1}{\frac{\sin x}{x}+1}\right).3\frac{x^3}{(x-\sin x)}}\right]=\frac{1}{6}$

138. (b)

139. (c) A function f is said to be a rational function, if

$f(x)=\frac{g(x)}{h(x)}$, where g (x) and h (x) are polynomials such that $h(x)\neq 0$. Then,

$\lim_{x\to a}f(x)=\lim_{x\to a}\frac{g(x)}{h(x)}=\frac{\lim_{x\to a}g(x)}{\lim_{x\to a}h(x)}=\frac{g(a)}{h(a)}$

If $h(a)=0$, there are two scenarios - (i) when $g(a)\neq 0$ and (ii) when $g(a)=0$. In case I, the limit does not exist. In case II, we can write $g(x)=(x-a)^k g_1(x)$, where k is the maximum of powers of $(x-a)$ in g (x). Similarly, $h(x)=(x-a)^l h_1(x)$ as $h(a)=0$. Now, if $k>l$, then

$\lim_{x\to a}f(x)=\frac{\lim_{x\to a}g(x)}{\lim_{x\to a}h(x)}=\frac{\lim_{x\to a}(x-a)^k g_1(x)}{\lim_{x\to a}(x-a)^l h_1(x)}$

$=\frac{\lim_{x\to a}(x-a)^{k-l}g_1(x)}{\lim_{x\to a}h_1(x)}=\frac{0.g_1(a)}{h_1(a)}=0$

If $k<l$, the limit is not defined.

CHAPTER

14

MATHEMATICAL REASONING

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Which of the following is a statement?
(a) Open the door. (b) Do your home work.
(c) Switch on the fan. (d) Two plus two is four.

2. Which of the following is a statement?
(a) May you live long!
(b) May God bless you!
(c) The sun is a star.
(d) Hurrah! we have won the match.

3. Which of the following is not a statement?
(a) Please do me a favour. (b) 2 is an even integer.
(c) $2+1=3$. (d) The number 17 is prime.

4. Which of the following is not a statement?
(a) 2 is an even integer.
(b) $2+1=3$.
(c) The number 17 is prime.
(d) $x+3=10, x \in R$.

5. Which of the following is the converse of the statement? "If Billu secure good marks, then he will get a bicycle."
(a) If Billu will not get bicycle, then he will not secure good marks.
(b) If Billu will get a bicycle, then he will secure good marks.
(c) If Billu will get a bicycle, then he will not secure good marks.
(d) If Billu will not get a bicycle, then he will secure good marks.

6. The connective in the statement :
"$2+7>9$ or $2+7<9$" is
(a) and (b) or (c) $>$ (d) $<$

7. The connective in the statement :
"Earth revolves round the Sun and Moon is a satellite of earth" is
(a) or (b) Earth (c) Sun (d) and

8. The negation of the statement
"A circle is an ellipse" is
(a) An ellipse is a circle.
(b) An ellipse is not a circle.
(c) A circle is not an ellipse.
(d) A circle is an ellipse.

9. The contrapositive of the statement
"If 7 is greater than 5, then 8 is greater than 6" is
(a) If 8 is greater than 6, then 7 is greater than 5.
(b) If 8 is not greater than 6, then 7 is greater than 5.
(c) If 8 is not greater than 6, then 7 is not greater than 5.
(d) If 8 is greater than 6, then 7 is not greater than 5.

10. The negation of the statement :
"Rajesh or Rajni lived in Bangalore" is
(a) Rajesh did not live in Bangalore or Rajni lives in Bangalore.
(b) Rajesh lives in Bangalore and Rajni did not live in Bangalore.
(c) Rajesh did not live in Bangalore and Rajni did not live in Bangalore.
(d) Rajesh did not live in Bangalore or Rajni did not live in Bangalore.

11. The statement
"If x^2 is not even, then x is not even" is converse of the statement
(a) If x^2 is odd, then x is even.
(b) If x is not even, then x^2 is not even.
(c) If x is even, then x^2 is even.
(d) If x is odd, then x^2 is even.

12. Which of the following is the conditional $p \rightarrow q$?
(a) q is sufficient for p. (b) p is necessary for q.
(c) p only if q. (d) if q, then p.

13. Which of the following statement is a conjunction?
(a) Ram and Shyam are friends.
(b) Both Ram and Shyam are tall.
(c) Both Ram and Shyam are enemies.
(d) None of the above.

14. The false statement in the following is
(a) $p \wedge (\sim p)$ is contradiction
(b) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction
(c) $\sim(\sim p) \Leftrightarrow p$ is a tautology
(d) $p \vee (\sim p) \Leftrightarrow$ is a tautology

15. $\sim(p \vee (\sim p))$ is equal to
(a) $\sim p \vee q$ (b) $(\sim p) \wedge q$
(c) $\sim p \vee \sim p$ (d) $\sim p \wedge \sim p$

16. If $(p \wedge \sim r) \Rightarrow (q \vee r)$ is false and q and r are both false, then p is
(a) True (b) False
(c) May be true or false (d) Data insufficient

17. $\sim((\sim p)\wedge q)$ is equal to
(a) $p\vee(\sim q)$ (b) $p\vee q$
(c) $p\wedge(\sim q)$ (d) $\sim p\wedge\sim q$

18. Which of the following is true?
(a) $p\Rightarrow q\equiv\sim p\Rightarrow\sim q$
(b) $\sim(p\Rightarrow\sim q)\equiv\sim p\wedge q$
(c) $\sim(\sim p\Rightarrow\sim q)\equiv\sim p\wedge q$
(d) $\sim(\sim p\Leftrightarrow q)\equiv[\sim(p\Rightarrow q)\wedge\sim(q\Rightarrow p)]$

19. Which of the following is not a statement?
(a) Please do me a favour (b) 2 is an even integer
(c) $2+1=3$ (d) The number 17 is prime

20. Which of the following is not a statement?
(a) Roses are red
(b) New Delhi is in India
(c) Every square is a rectangle
(d) Alas! I have failed

21. The inverse of the statement $(p\wedge\sim q)\to r$ is
(a) $\sim(p\vee\sim q)\to\sim r$ (b) $(\sim p\wedge q)\to\sim r$
(c) $(\sim p\vee q)\to\sim r$ (d) None of these

22. Negation of the statement $(p\wedge r)\to(r\vee q)$ is
(a) $\sim(p\wedge r)\to\sim(r\vee q)$ (b) $(\sim p\vee\sim r)\vee(r\vee q)$
(c) $(p\wedge r)\wedge(r\wedge q)$ (d) $(p\wedge r)\wedge(\sim r\wedge\sim q)$

23. The sentence "There are 35 days in a month" is
(a) a statement (b) not a statement
(c) may be statement or not (d) None of these

24. Which of the following is a statement?
(a) Everyone in this room is bold
(b) She is an engineering student
(c) $\sin^2\theta$ is greater than 1/2
(d) Three plus three equals six

25. The sentence "New Delhi is in India", is
(a) a statement (b) not a statement
(c) may be statement or not (d) None of the above

26. The negation of the statement "$\sqrt{2}$ is not a complex number" is
(a) $\sqrt{2}$ is a rational number
(b) $\sqrt{2}$ is an irrational number
(c) $\sqrt{2}$ is a complex number
(d) None of the above

27. Which of the following is/are connectives?
(a) Today (b) Yesterday
(c) Tomorrow (d) "And", "or"

28. The contrapositive of the statement
"If p, then q", is
(a) If q, then p (b) If p, then ~ q
(c) If ~ q, then ~ p (d) If ~ p, then ~ q

29. The contrapositive of the statement, ' If I do not secure good marks then I cannot go for engineering', is
(a) If I secure good marks, then I go for engineering.
(b) If I go for engineering then I secure good marks.
(c) If I cannot go for engineering then I donot secure good marks.
(d) None.

30. Which of the following is not a statement?
(a) Every set is a finite set
(b) 8 is less than 6
(c) Where are you going?
(d) The sum of interior angles of a triangle is 180 degrees

31. If $p\Rightarrow(\sim p\vee q)$ is false, the truth values of p and q are respectively
(a) F, T (b) F, F (c) T, T (d) T, F

32. Which of the following statement is a conjunction?
(a) Ram and Shyam are friends.
(b) Both Ram and Shyam are tall.
(c) Both Ram and Shyam are enemies.
(d) None of the above.

33. $p\Rightarrow q$ can also be written as
(a) $p\Rightarrow\sim q$ (b) $\sim p\vee q$
(c) $\sim q\Rightarrow\sim p$ (d) None of these

34. Which of the following is an open statement?
(a) Good morning to all (b) Please do me a favour
(c) Give me a glass of water (d) x is a natural number

35. If p, q, r are statement with truth vales F, T, F respectively, then the truth value of $p\to(q\to r)$ is
(a) false (b) true
(c) true if p is true (d) none

36. If $p\Rightarrow(q\vee r)$ is false, then the truth values of p, q, r are respectively
(a) T, F, F (b) F, F, F (c) F, T, T (d) T, T, F

37. A compound statement p or q is false only when
(a) p is false
(b) q is false
(c) both p and q are false
(d) depends on p and q

38. A compound statement p and q is true only when
(a) p is true (b) q is true
(c) both p and q are true (d) none of p and q is true

39. A compound statement $p\to q$ is false only when
(a) p is true and q is false
(b) p is false but q is true
(c) at least one of p or q is false
(d) both p and q are false

40. If p : Pappu passes the exam,
q : Papa will give him a bicycle.
Then, the statement 'Pappu passing the exam, implies that his papa will give him a bicycle' can be symbolically written as
(a) $p\to q$ (b) $p\leftrightarrow q$ (c) $p\wedge q$ (d) $p\vee q$

41. If Ram secures 100 marks in maths, then he will get a mobile. The converse is
(a) If Ram gets a mobile, then he will not secure 100 marks
(b) If Ram does not get a mobile, then he will secure 100 marks
(c) If Ram will get a mobile, then he secures 100 marks in maths
(d) None of these

42. In mathematical language, the reasoning is of ———— types.
(a) one (b) two (c) three (d) four

43. "Paris is in England" is a ________
(a) statement (b) sentence
(c) both 'a' and 'b' (d) neither 'a' nor 'b'

44. "The sun is a star" is a __________
(a) statement (b) sentence
(c) both 'a' and 'b' (d) neither 'a' nor 'b'

45. The negation of a statement is said to be a __________
(a) statement (b) sentence
(c) negation (d) ambiguous

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

46. Consider the following statements
Statement-I: The negation of the statement "The number 2 is greater than 7" is "The number 2 is not greater than 7".
Statement-II: The negation of the statement "Every natural number is an integer" is "every natural number is not an integer".
Choose the correct option.
(a) Only Statement I is true (b) Only Statement II is true
(c) Both Statement are true (d) Both Statement are false

47. Consider the following statements
Statement- I: The words "And" and "or" are connectives.
Statement-II: "There exists" and "For all" are called quantifiers.
(a) Only Statement I is true (b) Only Statement II is true
(c) Both Statement are true (d) Both Statement are false

48. Consider the following statements.
I. $x + y = y + x$ is true for every real numbers x and y.
II. There exists real numbers x and y for which $x + y = y + x$.
Choose the correct option.
(a) I and II are the negation of each other.
(b) I and II are not the negation of each other.
(c) I and II are the contrapositive of each other.
(d) I is the converse of II.

49. Read the following statements.
Statement: If x is a prime number, then x is odd.
I. **Contrapositive form :** If a number x is not odd, then x is not a prime number.
II. **Converse form :** If a number x is odd, then it is a prime number.
Choose the correct option.
(a) Both I and II are true. (b) Only I is true.
(c) Only II is true. (d) Neither I nor II true.

50. Consider the following statements.
I. A sentence is called a statement, if it is either true or false.
II. The sentence, "Today is a windy day", is not a statement.
Choose the correct option.
(a) Both I and II are true. (b) Only I is true.
(c) Only II is true. (d) Both I and II are false.

51. Consider the following statements
I. "Every rectangle is a square" is a statement.
II. "Close the door" is not a statement.
Choose the correct option.
(a) Only I is false. (b) Only II is false.
(c) Both are true. (d) Both are false.

52. Consider the following statements.
I. If a number is divisible by 10, then it is divisible by 5.
II. If a number is divisible by 5, then it is divisible by 10.
Choose the correct option.
(a) I is converse of II.
(b) II is converse of I.
(c) I is not converse of II.
(d) Both 'a' and 'b' are true.

53. Consider the following sentences.
I. She is a Mathematics graduate.
II. There are 40 days in a month.
Choose the correct option.
(a) Only I is a statement.
(b) Only II is a statement.
(c) Both are the statements.
(d) Neither I nor II is statement.

54. Consider the following sentences.
I. "Two plus three is five" is not a statement.
II. "Every square is a rectangle." is a statement.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

55. Consider the following
I. New Delhi is in Nepal.
II. Every relation is a function.
III. Do your homework.
Choose the correct option.
(a) I and II are statements.
(b) I and III are statements.
(c) II and III are statements.
(d) I, II and III are statements.

56. Consider the following sentences.
I. "Two plus two equals four" is a true statement.
II. "The sum of two positive numbers is positive" is a true statement.
III. "All prime numbers are odd numbers" is a true statement.
Choose the correct option.
(a) Only I is false.
(b) Only II is false.
(c) All I, II and III are false
(d) Only III is false.

57. The component statements of the statement "The sky is blue or the grass is green" are
I. p : The sky is blue.
q : The sky is not blue.
II. p : The sky is blue.
q : The grass is green.
Choose the correct option.
(a) I and II are component statements.
(b) Only I is component statement.
(c) Only II is component statement.
(d) Neither I nor II is component statement.

58. Consider the following statements.
I. If a statement is always true, then the statement is called "tautology".
II. If a statement is always false, then the statement is called "contradiction".

Choose the correct option.
(a) Both the statements are false.
(b) Only I is false.
(c) Only II is false.
(d) Both the statements are true.

59. If $p \to q$ is a conditional statement, then its
I. Converse : $q \to p$
II. Contrapositive : $\sim q \to \sim p$
III. Inverse : $\sim p \to \sim q$
Choose the correct option.
(a) Only I and II are true.
(b) Only II and III are true.
(c) Only I and III are true.
(d) All I, II and III are true.

60. Consider the following statement.
"If a triangle is equiangular, then it is an obtuse angled triangle."
This is equivalent to
I. a triangle is equiangular implies that it is an obtuse angled triangle.
II. for a triangle to be obtuse angled triangle it is sufficient that it is equiangular.
Choose the correct option.
(a) Both are correct. (b) Both are incorrect.
(c) Only I is correct. (d) Only II is correct.

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

61. **Assertion:** The compound statement with 'And' is true if all its component statements are true.
Reason: The compound statement with 'And' is false if any of its component statements is false.

62. **Assertion:** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Reason: $\sim(p \leftrightarrow \sim q)$ is a tautology

63. **Assertion:** "Mathematics is difficult", is a statement.
Reason: A sentence is a statement, if it is either true or false but not both.

64. **Assertion:** The sentence "8 is less than 6" is a statement.
Reason: A sentence is called a statement, if it is either true or false but not both.

65. **Assertion:** $\sim(p \vee q) \equiv \sim p \wedge \sim q$
Reason: $\sim(p \wedge q) \equiv \sim p \vee \sim q$

66. **Assertion:** $\sim(p \to q) \equiv p \wedge \sim q$
Reason: $\sim(p \leftrightarrow q) \equiv (p \vee \sim q) \wedge (q \wedge \sim p)$

67. **Assertion:** The contrapositive of $(p \vee q) \to r$ is $\sim r \to \sim p \wedge \sim q$.
Reason: If $(p \wedge \sim q) \to (\sim p \vee r)$ is a false statement, then respective truth values of p, q and r are F, T, T.

68. **Assertion:** If $p \to (\sim p \vee q)$ is false, the truth values of p and q are respectively F, T.
Reason: The negation of $p \to (\sim p \vee q)$ is $p \wedge \sim q$.

69. **Assertion :** The negation of $(p \vee \sim q) \wedge q$ is $(\sim p \wedge q) \vee \sim q$.
Reason : $\sim(p \to q) \equiv p \wedge \sim q$

70. **Assertion :** The denial of a statement is called negation of the statement.
Reason : A compound statement is a statement which can not be broken down into two or more statements.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

71. Which of the following is the conditional $p \to q$?
(a) q is sufficient for p (b) p is necessary for q
(c) p only if q (d) if q, then p

72. The converse of the statement "If $x > y$, then $x + a > y + a$" is
(a) If $x < y$, then $x + a < y + a$ (b) If $x + a > y + a$, then $x > y$
(c) If $x < y$, then $x + a > y + a$ (d) If $x > y$, then $x + a < y + a$

73. The statement "If x^2 is not even, then x is not even" is converse of the statement
(a) If x^2 is odd, then x is even
(b) If x is not even, then x^2 is not even
(c) If x is even, then x^2 is even
(d) If x is odd, then x^2 is even

74. The statement p: For any real numbers x, y if x = y, then $2x + a = 2y + a$ when $a \in Z$.
(a) is true
(b) is false
(c) its contrapositive is not true
(d) None of these

75. Which of the following is a statement?
(a) x is a real number (b) Switch off the fan
(c) 6 is a natural number (d) Let me go

76. Which of the following statement is false?
(a) A quadratic equation has always a real root
(b) The number of ways of seating 2 persons in two chairs out of n persons is P(n, 2)
(c) The cube roots of unity are in GP
(d) None of the above

77. The negation of the statement "A circle is an ellipse" is
(a) an ellipse is a circle (b) an ellipse is not a circle
(c) a circle is not an ellipse (d) a circle is an ellipse

78. Which of the following is not a negation of the statement "A natural number is greater than zero".
(a) A natural number is not greater than zero.
(b) It is false that a natural number is greater than zero.
(c) It is false that a natural number is not greater than zero.
(d) None of the above

79. For the statement "17 is a real number or a positive integer", the "or" is
(a) inclusive (b) exclusive
(c) Only (a) (d) None of these

80. The contrapositive of statement "If Chandigarh is capital of Punjab, then Chandigarh is in India" is
(a) "If Chandigarh is not in India, then Chandigarh is not the capital of Punjab".
(b) "If Chandigarh is in India, then Chandigarh is capital of Punjab".
(c) "If Chandigarh is not capital of Punjab, then Chandigarh is not the capital of India".
(d) "If Chandigarh is capital of Punjab, then Chandigarh is not in India".

81. Let p : I am brave,
q : I will climb the Mount Everest.
The symbolic form of a statement,
'I am neither brave nor I will climb the mount Everest' is
(a) $p \wedge q$ (b) $\sim(p \wedge q)$ (c) $\sim p \wedge \sim q$ (d) $\sim p \wedge q$

82. Let p : A quadrilateral is a parallelogram
q : The opposite side are parallel
Then the compound proposition
'A quadrilateral is a parallelogram if and only if the opposite sides are parallel' is represented by
(a) $p \vee q$ (b) $p \rightarrow q$ (c) $p \wedge q$ (d) $p \leftrightarrow q$

83. Let p: Kiran passed the examination,
q: Kiran is sad
The symbolic form of a statement "It is not true that Kiran passed therefore he is said" is
(a) $(\sim p \rightarrow q)$ (b) $(p \rightarrow q)$
(c) $\sim(p \rightarrow \sim q)$ (d) $\sim(p \leftrightarrow q)$

84. Which of the following is true?
(a) $p \Rightarrow q \equiv \sim p \Rightarrow \sim q$
(b) $\sim(p \Rightarrow \sim q) \equiv \sim p \wedge q$
(c) $\sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$
(d) $\sim(\sim p \Leftrightarrow q) \equiv [\sim(p \Rightarrow q) \wedge \sim(q \Rightarrow p)]$

85. If p and q are true statement and r, s are false statements, then the truth value of $\sim[(p \wedge \sim r) \vee (\sim q \vee s)]$ is
(a) true (b) false
(c) false if p is true (d) none

86. Consider the following statements
P **:** Suman is brilliant
Q **:** Suman is rich
R **:** Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as
(a) $\sim(Q \leftrightarrow (P \wedge \sim R))$ (b) $\sim Q \leftrightarrow \sim P \wedge R$
(c) $\sim(P \wedge \sim R) \leftrightarrow Q$ (d) $\sim P \wedge (Q \leftrightarrow \sim R)$

87. Let S be a non-empty subset of R. Consider the following statement :
P : There is a rational number $x \in$ S such that $x > 0$.
Which of the following statement is the negation of the statement P ?
(a) There is no rational number $x \in$ S such than $x \leq 0$.
(b) Every rational number $x \in$ S satisfies $x \leq 0$.
(c) $x \in$ S and $x \leq 0 \Rightarrow x$ is not rational.
(d) There is a rational number $x \in$ S such that $x \leq 0$.

88. The false statement in the following is
(a) $p \wedge (\sim p)$ is contradiction
(b) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction
(c) $\sim(\sim p) \Leftrightarrow p$ is a tautology
(d) $p \vee (\sim p) \Leftrightarrow$ is a tautology

89. The propositions $(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$ is
(a) Tautology and contradiction
(b) Neither tautology nor contradiction
(c) Contradiction
(d) Tautology

90. Truth value of the statement 'It is false that 3 + 3 = 33 or 1 + 2 = 12' is
(a) T (b) F
(c) both T and F (d) 54

91. If $\sim q \vee p$ is F, then which of the following is correct?
(a) $p \leftrightarrow q$ is T (b) $p \rightarrow q$ is T
(c) $q \rightarrow p$ is T (d) $p \rightarrow q$ is F

92. If p, q are true and r is false statement, then which of the following is true statement?
(a) $(p \wedge q) \vee r$ is F
(b) $(p \wedge q) \rightarrow r$ is T
(c) $(p \vee q) \wedge (p \vee r)$ is T
(d) $(p \rightarrow q) \leftrightarrow (p \rightarrow r)$ is T

93. Which of the following is true?
(a) $p \wedge \sim p \equiv T$ (b) $p \vee \sim p \equiv F$
(c) $p \rightarrow q \equiv q \rightarrow p$ (d) $p \rightarrow q \equiv (\sim q) \rightarrow (\sim p)$

94. Consider the following statements
p : x, y $\in$ Z such that x and y are odd.
q : xy is odd. Then,
(a) $p \Rightarrow q$ is true (b) $\sim q \Rightarrow p$ is true
(c) Both (a) and (b) (d) None of these

95. Consider the following statements
p : A tumbler is half empty.
q : A tumbler is half full.
Then, the combination form of "p if and only if q" is
(a) a tumbler is half empty and half full
(b) a tumbler is half empty if and only if it is half full
(c) Both (a) and (b)
(d) None of the above

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(d)** 'Two plus two is four', is a statement.
2. **(c)** "The sun is a star" is a statement.
3. **(a)** "Please do me a favour" is not a statement.
4. **(d)** $x + 3 = 10, x \in R$ is not a statement.
5. **(b)** Since the statement $q \to p$ is the converse of the statement $p \to q$.
6. **(b)** Connective word is 'or'.
7. **(d)** Connective word is 'and'.
8. **(c)** Negation : A circle is not an ellipse.
9. **(c)** If 8 is not greater than 6, then 7 is not greater than 5.
10. **(c)** Rajesh did not live in Bangalore and Rajni did not live in Bangalore.
11. **(b)** If x is not even, then x^2 is not even.
12. **(c)** p only if q.
13. **(d)**
14. **(b)** $p \Rightarrow q$ is logically equivalent to $\sim q \Rightarrow \sim p$

    $\therefore (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology but not a contradiction.
15. **(b)** $\sim (p \vee (\sim q)) \equiv \sim p \wedge \sim (\sim q) \equiv (\sim p) \wedge q$
16. **(a)** Given result means $p \wedge \sim r$ is true, $q \vee r$ is false.
17. **(a)** $\sim ((\sim p) \wedge q) \equiv \sim (\sim p) \vee \sim q \equiv p \vee (\sim q)$
18. **(c)** $\sim (p \Rightarrow q) \equiv p \wedge \sim q$

    $\therefore \sim (\sim p \Rightarrow \sim q) \equiv \sim p \wedge \sim (\sim q) \equiv \sim p \wedge q$

    Thus $\sim (\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$
19. **(a)** "Please do me a favour" is not a statement.
20. **(d)** "Alas! I have failed" is not a statement.
21. **(c)** The inverse of the proposition $(p \wedge \sim q) \to r$ is

    $\sim (p \wedge \sim q) \to \sim r$

    $\equiv \sim p \vee \sim (\sim q) \to \sim r$

    $\equiv \sim p \vee q \to \sim r$
22. **(d)** We know that $\sim (p \to q) \equiv p \wedge \sim q$

    $\therefore \sim ((p \wedge r) \to (r \vee q)) \equiv (p \wedge r) \wedge [\sim (r \vee q)]$

    $\equiv (p \wedge r) \wedge (\sim r \wedge \sim q)$
23. **(a)** We know that a month has 30 or 31 days. It is false to say that a month has 35 days. Hence, it is a statement.
24. **(d)** (a) Everyone in this room is bold. This is not a statement because from the context, it is not clear which room is referred here and the term 'bold' is not clearly defined.

    (b) She is an engineering student. This is also not a statement because it is not clear, who she is.

    (c) $\sin^2\theta$ is greater than 1/2. This is not a statement because we cannot say whether the sentence is true or not.

    (d) We know that, 3 + 3 = 6. It is true. Hence, the sentence is a statement.
25. **(a)** "New Delhi is in India" is true. So, it is a statement.
26. **(c)** In negative statement, if word not is not given in the statement, then insert word 'not' in the statement. If word 'not' is given in the statement, then remove word 'not' from the statement.

    $\therefore$ The negation of the given statement is "$\sqrt{2}$ is a complex number".
27. **(d)** Some of the connecting words which are found in compound statement like "And", "or", etc, are often used in mathematical statements. These are called connectives.
28. **(c)** Contrapositive statement is

    "If $\sim q$, then $\sim p$."
29. **(b)**
30. **(c)** "Where are you going?" is not a statement.
31. **(d)** $p \Rightarrow (\sim p \vee q)$ is false means p is true and $\sim p \vee q$ is false.

    $\Rightarrow$ p is true and both ~p and q are false

    $\Rightarrow$ p is true and q is false.
32. **(d)**
33. **(b)** $p \Rightarrow q \equiv \sim p \vee q$
34. **(d)** 35. **(b)**
36. **(a)** $p \Rightarrow q$ is false only when p is true and q is false.

    $\therefore p \Rightarrow q$ is false when p is true and $q \vee r$ is false, and $q \vee r$ is false when both q and r are false.
37. **(c)** It is a property.
38. **(c)** It is a property.
39. **(a)**
40. **(a)** Let p : Pappu pass the exam

    q : Papa will give him a bicycle.

    $\therefore$ Symbolic form is $p \to q$.
41. **(c)** Let p : Ram secures 100 marks in maths

    q : Ram will get a mobile

    Converse of $p \to q$ is $q \to p$

    i.e. If Ram will get a mobile, then he secures 100 marks in maths.
42. **(b)** In mathematical language, the reasoning is of two types.
43. **(a)** "Paris is in England" is a statement.
44. **(c)**
45. **(a)**

STATEMENT TYPE QUESTIONS

46. **(c)** I. The given statement is "The number 2 is greater than 7". Its negation is "The number 2 is not greater than 7".

    II. The given statement is "Every natural number is an integer". Its negation is "Every natural number is not an integer".
47. **(c)** The words "And" and "or" are called connectives and "There exists" and "For all" are called quantifiers.
48. **(b)** Statement I and II are not the negation of each other.

49. (a) Both contrapositive and converse statements are true.

50. (a) Today is a windy day. It is not clear that about which day it is said. Thus, it cannot be concluded whether it is true or false. Hence, it is not a statement.

51. (c) Both the statements I and II are true. "Every rectangle is a square" is false. So, it is a statement. "Close the door" is an order. So, it is not a statement.

52. (d) I and II are converse of each other.

53. (b) Only II is a statement.

54. (a) "Two plus three is five" is not a statement.

55. (a) Only I and II are statements.

56. (d) Statement I and II are correct but III is not correct.

57. (c) Only II is component statement.

58. (d) By definition, I and II both are true.

59. (d) All the statements are true.

60. (a) Given statement is equivalent to I and II both.

ASSERTION - REASON TYPE QUESTIONS

61. (b) Consider the following compound statements

p : A point occupies a position and its location can be determined.

The statement can be broken into two component statements as

q : A point occupies a position.

r : Its location can be determined.

Here, we observe that both statements are true.

Let us look at another statement.

p : 42 is divisible by 5, 6 and 7.

This statement has following component statements

q : 42 is divisible by 5.

r : 42 is divisible by 6.

s : 42 is divisible by 7.

Here, we know that the first is false, while the other two are true.

$\therefore$ p is false in this case.

Thus we can conclude that

1. The compound statement with 'and' is true, if all its component statements are true.
2. The compound statement with 'and' is false, if any of its component statement is false (this includes the case that some of its component statements are false or all of its component statements are false.)

62. (c) The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns for $\sim (p \leftrightarrow \sim q)$ and $p \leftrightarrow q$ are identical, therefore

$\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

But $\sim (p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.

$\therefore$ Statement-1 is true but statement-2 is false.

63. (d) Reason is correct but Assertion is not correct.

64. (a) Both are correct. Reason is correct explanation. We know that 8 is greater than 6.

65. (b) Assertion and Reason, both are correct but Reason is not the correct explanation for the Reason.

66. (c) Assertion is correct but Reason is not correct.

Reason: $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

67. (c) Assertion is correct. Reason is incorrect.

Reason : Truth values are T, F, F.

68. (d) Assertion is incorrect. Reason is correct.

69. (b) Both are correct but Reason is not the correct explanation for the Assertion.

70. (c) Assertion is correct but Reason is incorrect.

"A compound statement is a statement which is made up of two or more simple statements."

CRITICALTHINKING TYPE QUESTIONS

71. (c) $p \rightarrow q$ is the same as "p only if q".

72. (b) Converse statement is

"If $x + a > y + a$, then $x > y$".

73. (b) Converse statement is

"If x is not even, then x^2 is not even".

74. (a) We prove the statement p is true by contrapositive method and by direct method.

Direct method: For any real number x and y,

$$x = y$$
$$\Rightarrow \quad 2x = 2y$$
$$\Rightarrow \quad 2x + a = 2y + a \text{ for some } a \in Z$$

Contrapositive method: The contrapositive statement of p is "For any real numbers x, y if $2x + a \neq 2y + a$, where $a \in Z$, then $x \neq y$."

Given, $2x + a \neq 2y + a$

$$\Rightarrow \quad 2x \neq 2y$$
$$\Rightarrow \quad x \neq y$$

Hence, the given statement is true.

75. (c) (a) "x is a real number" is an open statement. So, this is not a statement.

(b) "Switch off the fan" is not a statement, it is an order.

(c) "6 is a natural number" is a true sentence. So, it is a statement

(d) "Let me go!" (optative sentence). So, it is not a statement.

76. (a) (a) It is false. (b) It is true. (c) It is true.

77. (c) The negation of statement "A circle is an ellipse" is "A circle is not an ellipse".

78. (c) The negation of given statement can be

(i) A natural number is not greater than zero.

(ii) It is false that a natural number is greater than zero.

$\therefore$ "It is false that a natural number is not greater than zero" is not a negation of the given statement.

79. (a) Inclusive "or". 17 is a real number or a positive integer or both.

80. (a) The contrapositive statement is "If Chandigarh is not in India, then Chandigarh is not the capital of Punjab".

81. (c) **82. (d)** **83. (b)**

84. (c) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$

$\therefore \sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge \sim(\sim q) \equiv \sim p \wedge q$

Thus $\sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$

85. (b)

86. (a) Suman is brilliant and dishonest if and only if Suman is rich is expressed as

$Q \leftrightarrow (P \wedge \sim R)$

Negation of it will be $\sim(Q \leftrightarrow (P \wedge \sim R))$

87. (b) P : there is a rational number $x \in S$ such that $x > 0$

$\sim$P : Every rational number $x \in S$ satisfies $x \leq 0$

88. (b) $p \Rightarrow q$ is logically equivalent to

$\sim q \Rightarrow \sim p$

$\therefore (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology but not a contradiction.

89. (c)

p	$\sim p$	$p \Rightarrow \sim p$	$\sim p \Rightarrow p$	$(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$
T	F	F	T	F
F	T	T	F	F

90. (a) p : 3 + 3 = 33, q : 1 + 2 = 12

Truth values of both p and q is F.

$\therefore \sim(F \vee F) \equiv \sim F \equiv T$

91. (b)

p	q	$\sim q$	$\sim q \vee p$	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$
T	T	F	T	T	T	T
T	F	T	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T

Alternate Method:

$\sim q \vee p$: F

$\therefore$ $\sim q$ is F, p is F

i.e. q is T, p is F

$\therefore p \rightarrow q \equiv F \rightarrow T \equiv T$

92. (c) $(p \vee q) \wedge (p \vee r)$

$\equiv (T \vee T) \wedge (T \vee F)$

$\equiv T \wedge T$

$\equiv T$

93. (d) $(\sim q) \rightarrow (\sim p)$ is contrapositive of $p \rightarrow q$ and both convey the same meaning.

94. (a) Let p : x, y $\in$ Z such that x and y are odd.

q : xy is odd.

To check the validity of the given statement, assume that if p is true, then q is true.

p is true means that x and y are odd integers. Then,

x = 2m + 1, for some integer m.

y = 2n + 1, for some integer n.

Thus, $xy = (2m + 1)(2n + 1)$

$= 2(2mn + m + n) + 1$

This shows that xy is odd. Therefore, the given statement is true.

Also, if we assume that q is not true. This implies that we need to consider the negation of the statement q. This gives the statement.

$\sim q$: product xy is even.

This is possible only, if either x or y is even. This shows that p is not true. Thus, we have shown that

$\sim q \Rightarrow \sim p$

Note: The above problem illustrates that to prove $p \Rightarrow q$. It is enough to show $\sim q \Rightarrow \sim p$ which is the contrapositive of the statement $p \Rightarrow q$.

95. (b) The given statements are

p : A tumbler is half empty.

q : A tumbler is half full.

We know that, if the first statement happens, then the second happens and also if the second happens, then the first happens. We can express this fact as

If a tumbler is half empty, then it is half full.

If a tumbler is half full, then it is half empty.

We combine these two statements and get the following. A tumbler is half empty, if and only if it is half full.

CHAPTER 15

STATISTICS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The measure of dispersion is:
(a) mean deviation (b) standard deviation
(c) quartile deviation (d) all (a) (b) and (c)

2. The observation which occur most frequently is known as :
(a) mode (b) median
(c) weighted mean (d) mean

3. The reciprocal of the mean of the reciprocals of n observation is the :
(a) geometric mean (b) median
(c) harmonic mean (d) average

4. The median of 18, 35, 10, 42, 21 is
(a) 20 (b) 19 (c) 21 (d) 22

5. While dividing each entry in a data by a non-zero number a, the arithmetic mean of the new data:
(a) is multiplied by a (b) does not change
(c) is divided by a (d) is diminished by a

6. The mode of the following series 3, 4, 2, 1, 7, 6, 7, 6, 8, 6, 5 is
(a) 5 (b) 6
(c) 7 (d) 8

7. The coefficient of variation is computed by:
(a) $\frac{\text{mean}}{\text{standard deviation}}$ (b) $\frac{\text{standard deviation}}{\text{mean}}$
(c) $\frac{\text{mean}}{\text{standard deviation}} \times 100$ (d) $\frac{\text{standard deviation}}{\text{mean}} \times 100$

8. If you want to measure the intelligence of a group of students,which one of the following measures will be more suitable?
(a) Arithmetic mean (b) Mode
(c) Median (d) Geometric mean

9. In computing a measure of the central tendency for any set of 51 numbers, which one of the following measures is well-defined but uses only very few of the numbers of the set?
(a) Arithmetic mean (b) Geometric mean
(c) Median (d) Mode

10. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is
(a) 6 (b) 7 (c) 8 (d) 10

11. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ?
(a) $a=0, b=7$ (b) $a=5, b=2$
(c) $a=1, b=6$ (d) $a=3, b=4$

12. Find the mean deviation about the mean for the data
4, 7, 8, 9, 10, 12, 13, 17
(a) 3 (b) 24 (c) 10 (d) 8

13. Find the mean deviation about the mean for the data.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

(a) 6 (b) 7.3 (c) 8 (d) 6.32

14. Find the mean and variance for the following data
6, 7, 10, 12, 13, 4, 8, 12
(a) mean = 9, variance = 9.25
(b) mean = 3, variance = 7.5
(c) mean = 7, variance = 12
(d) mean = 9, variance = 12.5

15. The method used in Statistics to find a representative value for the given data is called
(a) measure of skewness
(b) measure of central tendency
(c) measure of dispersion
(d) None of the above

16. The value which represents the measure of central tendency, is/are
(a) mean (b) median (c) mode (d) All of these

17. The number which indicates variability of data or observations, is called
(a) measure of central tendency
(b) mean
(c) median
(d) measure of dispersion

18. Which of the following is/are used for the measures of dispersion?
(a) Range (b) Quartile deviation
(c) Standard deviation (d) All of these

19. We can grouped data into .... ways.
(a) three (b) four (c) two (d) None of these

20. When tested, the lives (in hours) of 5 bulbs were noted as follows
1357, 1090, 1666, 1494, 1623
The mean deviations (in hours) from their mean is
(a) 178 (b) 179 (c) 220 (d) 356

21. Number which is mean of the squares of deviations from mean, is called ....
(a) standard deviation (b) variance
(c) median (d) None of these

22. The variance of n observations $x_1, x_2, ..., x_n$ is given by
(a) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$ (b) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$
(c) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + \bar{x})$ (d) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + \bar{x})^2$

23. The measure of variability which is independent of units, is called
(a) mean deviation (b) variance
(c) standard deviation (d) coefficient of variation

24. If $x_1, x_2, ..., x_n$ are n values of a variable X and $y_1, y_2,...,y_n$ are n values of variable Y such that $y_i = ax_i + b$; i = 1, 2, ..., n, then write Var(Y) in terms of Var(X).
(a) var (Y) = var (X) (b) var (Y) = a var (X)
(c) var (Y) = a^2 var (X) (d) var (X) = a^2 var (Y)

25. If X and Y are two variates connected by the relation $Y = \frac{aX+b}{c}$ and Var (X) = σ^2, then write the expression for the standard deviation of Y.
(a) $\left|\frac{a}{c}\right|$ (b) $\left|\frac{a}{c}\right|\sigma$ (c) $|a \cdot c|$ (d) $|a \cdot c|\sigma$

26. Variance of the numbers 3, 7, 10,18, 22 is equal to
(a) 12 (b) 6.4 (c) $\sqrt{49.2}$ (d) 49.2

27. The mean deviation from the mean of the following data :

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	15	16	6

is
(a) 10 (b) 10.22 (c) 9.86 (d) 9.44

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

28. Consider the following data which represents the runs scored by two batsmen in their last ten matches as
Batsman A : 30, 91, 0, 64, 42, 80, 30, 5, 117, 71
Batsman B : 53, 46, 48, 50, 53, 53, 58, 60, 57, 52
Which of the following is/are true about the data?
I. Mean of batsman A runs is 53.
II. Median of batsman A runs is 42.
III. Mean of batsman B runs is 53.
IV. Median of batsman B runs is 53.
(a) Only I is true (b) I and III are true
(c) I, III and IV are true (d) All are true

29. Which of the following is/are true about the range of the data?
I. It helps to find the variability in the observations on the basis of maximum and minimum value of observations.
II. Range of series = Minimum value – Maximum value.
III. It tells us about the dispersion of the data from a measure of central tendency.
(a) Only I is true (b) II and III are true
(c) I and II are true (d) All are true

30. **Statement I :** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3.5.
Statement II : The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

31. Consider the following data

Size	20	21	22	23	24
Frequency	6	4	5	1	4

I. Mean of the data is 22.65.
II. Mean deviation of the data is 1.25.
III. Mean of the data is 21.65.
IV. Mean deviation of the data is 2.25.
(a) I and II are true (b) II and III are true
(c) I and IV are true (d) III and IV are true

32. Consider the following data

Marks obtained	10	11	12	14	15
Number of students	2	3	8	3	4

I. Median of the data is 11.
II. Median of the data is 12.
III. Mean deviation about the median is 2.25.
IV. Mean deviation about the median is 1.25.
(a) I and III are true (b) I and IV are true
(c) II and III are true (d) II and IV are true

33. Consider the following data
6, 8, 10, 12, 14, 16, 18, 20, 22, 24
I. The variance of the data is 33.
II. The standard deviation of the data is 4.74.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

34. **Statement-I :** The mean and variance for first n natural numbers are $\frac{n+1}{2}$ and $\frac{n^2+1}{12}$, respectively.
Statement-II : The mean and variance for first 10 positive multiples of 3 are 16.5 and 74.25. respectively.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

35. Consider the following frequency distribution

Class	0–10	10–20	20–30	30–40	40–50
Frequency	5	8	15	16	6

I. Mean of the data is 27.
II. Mean of the data is 32.
III. Variance of the data is 132
IV. Variance of the data is 164
(a) II and IV are true (b) I and IV are true
(c) II and III are true (d) I and III are true

36. **Statement-I:** The series having greater CV is said to be less variable than the other.
Statement-II: The series having lesser CV is said to be more consistent than the other.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

37. If $\bar{x}_1$ and σ_1 are the mean and standard deviation of the first distribution and $\bar{x}_2$ and σ_2 are the mean and standard deviation of the second distribution.
I. CV (1st distribution) = $\frac{\sigma_1}{\bar{x}_1} \times 100$
II. CV (2nd distribution) = $\frac{\bar{x}_2}{\sigma_2} \times 100$
III. For $\bar{x}_1 = \bar{x}_2$, the series with lesser value of standard deviation is said to be more variable than the other.
IV. For $\bar{x}_1 = \bar{x}_2$, the series with greater value of standard deviation is said to be more consistent than the other.
(a) Only I is true (b) III and IV are true
(c) I, III and IV are true (d) All are true

38. If $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, ..., x_n$, then which of the following are true for the observations $ax_1, ax_2, ax_3, ..., ax_n$?
I. Mean of the observations is $\frac{\bar{x}}{a}$.
II. Variance of the observations is $\frac{\sigma^2}{a^2}$.
III. Mean of the observations is $a\bar{x}$.
IV. Variance of the observations is $a^2\sigma^2$.
(a) I and II are true (b) I and IV are true
(c) II and III are true (d) III and IV are true

39. Following are the marks obtained, out of 100 by two students Raju and Sita in 10 tests.

Raju	25	50	45	30	70	42	36	48	35	60
Sita	10	70	50	20	95	55	42	60	48	80

I. Raju is more intelligent.
II. Sita is more intelligent.
III. Raju is more consistent.
IV. Sita is more consistent.
(a) I and IV are true (b) II and III are true
(c) I and III are true (d) II and IV are true

40. If for a distribution $\sum(x-5) = 3$, $\sum(x-5)^2 = 43$ and the total number of items is 18.
Statement-I: Mean of the distribution is 4.1666.
Statement-II: Standard deviation of the distribution is 1.54.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

41. Consider the following statements :
I. Measures of dispersion Range, Quartile deviation, mean deviation, variance, standard deviation are measures of dispersion
Range = Maximum value – minimum values
II. Mean deviation for ungrouped data
$$\text{M.D.}(\bar{x}) = \frac{\Sigma|x_i - \bar{x}|}{n}$$
$$\text{M.D.}(M) = \frac{\Sigma|x_i - M|}{n}$$
III. Mean deviation for grouped data
$$\text{M.D.}(\bar{x}) = \frac{\Sigma f_i|x_i - \bar{x}|}{N}$$
$$\text{M.D.}(M) = \frac{\Sigma f_i|x_i - M|}{N}$$
where $N = \Sigma f_i$
Which of the above statements are true?
(a) Only (I) (b) Only (II)
(c) Only (III) (d) All of the abvoe

42. Consider the following statements :
I. Mode can be computed from histogram
II. Median is not independent of change of scale
III. Variance is independent of change of origin and scale.
Which of these is / are correct ?
(a) (I), (II) and (III) (b) Only (II)
(c) Only (I) and (II) (d) Only (I)

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

43.

Column - I	**Column - II**
(A) Mean deviation about the median for the data 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21, is	(1) $\frac{\sigma}{\bar{x}} \times 100$
(B) Mean deviation about the median for the data 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17, is	(2) $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$
(C) The standard deviation of n observations $x_1, x_2, \ldots, x_n$ is given by	(3) 2.33
(D) The coefficient of variation (CV) is defined as	(4) 5.27

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	4	3	1	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

44. The average of 5 quantities is 6, the average of three of them is 4, then the average of remaining two numbers is :
(a) 9 (b) 6
(c) 10 (d) 5

45. The mean deviation from the median of the following data is

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	5	3	6	2

(a) 14 (b) 10
(c) 5 (d) 7

46. Consider the following frequency distribution

x	A	2A	3A	4A	5A	6A
f	2	1	1	1	1	1

where, A is a positive integer and has variance 160. Then the value of A is.
(a) 5 (b) 6 (c) 7 (d) 8

47. Coefficient of variation of two distribution are 50% and 60% and their standard deviation are 10 and 15, respectively. Then, difference of their arithmetic means is
(a) 3 (b) 4 (c) 5 (d) 6

48. The mean of 5 observation is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6, then difference of the other two observations is
(a) 5 (b) 4 (c) 6 (d) 9

49. Consider the following data.
36, 72, 46, 42, 60, 45, 53, 46, 51, 49
Then the mean deviation about the median for the data is
(a) 6 (b) 8 (c) 7 (d) None of these

50. Given N = 10, $\Sigma x = 60$ and $\Sigma x^2 = 1000$. The standard deviation is
(a) 6 (b) 7 (c) 8 (d) 9

51. The standard deviation of 5 scores 1, 2, 3, 4, 5 is $\sqrt{a}$. The value of 'a' is
(a) 2 (b) 3 (c) 5 (d) 1

52. The variance of the data 2, 4, 6, 8, 10 is
(a) 8 (b) 7 (c) 6 (d) None of these

53. The range of set of observations 2, 3, 5, 9, 8, 7, 6,5, 7, 4, 3 is
(a) 6 (b) 7 (c) 4 (d) 5

54. The mean deviation from the mean for the set of observations –1, 0, 4 is
(a) 3 (b) 2 (c) 1 (d) None of these

55. The S. D of 15 items is 6 and if each item is decreased or increased by 1, then standard deviation will be
(a) 5 (b) 6 (c) 7 (d) None of these

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

56. **Assertion :** Sum of absolute values of

$$\text{Mean of deviations} = \frac{\text{Deviations}}{\text{Number of observations}}$$

Reason : Sum of the deviations from mean $(\bar{x})$ is 1.

57. **Assertion :** $\text{Mean of deviations} = \dfrac{\text{Product of deviations}}{\text{No. of observations}}$

Reason : To find the dispersion of values of x from mean $\bar{x}$, we take absolute measure of dispersion.

58. Let x_1 , x_2,...., x_n be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.
Assertion : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.
Reason : Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

59. **Assertion :** The range is the difference between two extreme observations of the distribution.
Reason : The variance of a variate X is the arithmetic mean of the squares of all deviations of X from the arithmetic mean of the observations.

60. **Assertion :** The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is 2.57
Reason : For individual observation,

$$\text{Mean deviation } (\bar{X}) = \frac{\sum |x_i - \bar{x}|}{n}$$

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

61. The mean of six numbers is 30. If one number is excluded, the mean of the remaining numbers is 29. The excluded number is
(a) 29 (b) 30 (c) 35 (d) 45

62. Mean of 20 observations is 15.5 Later it was found that the observation 24 was misread as 42. The corrected mean is:
(a) 14.2 (b) 14.8 (c) 14.0 (d) 14.6

63. The mean of a set of 20 observation is 19.3. The mean is reduced by 0.5 when a new observation is added to the set. The new observation is
(a) 19.8 (b) 8.8 (c) 9.5 (d) 30.8

64. The observations 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95 are arranged in ascending order. What is the value of x if the median of the data is 63?
(a) 61 (b) 62 (c) 62.5 (d) 63

65. The mean of 13 observations is 14. If the mean of the first 7 observations is 12 and that of the last 7 observations is 16, what is the value of the 7th observation ?
(a) 12 (b) 13 (c) 14 (d) 15

66. The mean and variance for first n natural numbers are respectively

(a) mean $= \dfrac{n+1}{2}$, variance $= \dfrac{n^2-1}{12}$

(b) mean $= \dfrac{n-1}{2}$, variance $= \dfrac{n^2+1}{12}$

(c) mean $= \dfrac{n^2-1}{12}$, variance $= \dfrac{n+1}{2}$

(d) mean $= \dfrac{n^2+1}{2}$, variance $= \dfrac{n-1}{2}$

67. Find the mean and standard deviation for the following data :

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

(a) mean = 6.59, S.D = 19 (b) mean = 8, S.D = 19
(c) mean = 19, S.D = 6.59 (d) mean = 19, S.D = 6

68. The median of a set of 9 distinct observation is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set is
(a) increased by 2
(b) decreased by 2
(c) two times the original median
(d) remains the same as that of original set

69. The mean deviation from the mean of the set of observations – 1, 0 and 4 is
(a) 3 (b) 1 (c) – 2 (d) 2

70. Variance of the data 2, 4, 5, 6, 8, 17 is 23.33. Then, variance of 4, 8, 10, 12, 16, 34 will be
(a) 23.33 (b) 25.33 (c) 93.32 (d) 98.32

71. The mean of 100 observations is 50 and their standard deviation is 5. The sum of squares of all observation is
(a) 50000 (b) 250000 (c) 252500 (d) 255000

72. Consider the following data
1, 2, 3, 4, 5, 6, 7, 8, 9, 10
If 1 is added to each number, then variance of the numbers so obtained is
(a) 6.5 (b) 2.87 (c) 3.87 (d) 8.25

73. Consider the first 10 positive integers. If we multiply each number by (– 1) and then add 1 to each number, the variance of the numbers so obtained is
(a) 8.25 (b) 6.5 (c) 3.87 (d) 2.87

74. Coefficient of variation of two distributions are 50 and 60 and their arithmetic means are 30 and 25, respectively. Then, difference of their standard deviations is
(a) 0 (b) 1 (c) 1.5 (d) 2.5

75. The sum of the squares of deviations for 10 observations taken from their mean 50 is 250. Then, the coefficient of variation is
(a) 10% (b) 40%
(c) 50% (d) None of these

76. If $n = 10$, $\bar{x} = 12$ and $\sum x_i^2 = 1530$, then the coefficient of variation is
(a) 35% (b) 42% (c) 30% (d) 25%

77. The variance of 20 observations is 5. If each observation is multiplied by 2, then the new variance of the resulting observation is
(a) $2^3 \times 5$ (b) $2^2 \times 5$
(c) 2×5 (d) $2^4 \times 5$

78. Let a, b, c, d and e be the observations with mean m and standard deviation s. The standard deviation of the observations a + k, b + k, c + k, d + k and e + k is
(a) s (b) ks (c) s + k (d) s/k

79. Let x_1, x_2, x_3, x_4 and x_5 be the observations with mean m and standard deviations. Then, standard deviation of the observations kx_1, kx_2, kx_3, kx_4 and kx_5 is
(a) k + 5 (b) π / k (c) ks (d) s

80. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ?
(a) $a = 0, b = 7$ (b) $a = 5, b = 2$
(c) $a = 1, b = 6$ (d) $a = 3, b = 4$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
(a) $\frac{11}{2}$ (b) 6 (c) $\frac{13}{2}$ (d) $\frac{5}{2}$

82. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?
(a) mean (b) median
(c) mode (d) variance

83. If mean of the n observations $x_1, x_2, x_3, ... x_n$ be $\bar{x}$, then the mean of n observations $2x_1 + 3, 2x_2 + 3, 2x_3 + 3, ..., 2x_n + 3$ is
(a) $3\bar{x} + 2$ (b) $2\bar{x} + 3$ (c) $\bar{x} + 3$ (d) $2\bar{x}$

84. If the mean of n observations $1^2, 2^2, 3^2, ..., n^2$ is $\frac{46n}{11}$, then n is equal to
(a) 11 (b) 12 (c) 23 (d) 22

85. If the mean of four observations is 20 and when a constant c is added to each observation, the mean becomes 22. The value of c is :
(a) – 2 (b) 2 (c) 4 (d) 6

86. The arithmetic mean of a set of observations is $\bar{x}$. If each observation is divided by α then it is increased by 10, then the mean of the new series is:
(a) $\frac{\bar{x}}{\alpha}$ (b) $\frac{\bar{x} + 10}{\alpha}$
(c) $\frac{\bar{x} + 10\alpha}{\alpha}$ (d) $\alpha\bar{x} + 10$

87. The mean of n items is $\bar{X}$. If the first item is increased by 1, second by 2 and so on, the new mean is :
(a) $\bar{X} + \frac{x}{2}$ (b) $\bar{X} + x$
(c) $\bar{X} + \frac{n+1}{2}$ (d) none of these

88. The coefficient of variation from the given data

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	10	8	4	6

is :
(a) 50 (b) 51.9 (c) 48 (d) 51.8

89. Coefficient of variation of two distribution are 60 and 70, and their standard deviations are 21 and 16, respectively. What are their arithmetic means?
(a) 35, 22.85 (b) 22.85, 35.28
(c) 36, 22.85 (d) 35.28, 23.85

90. Standard deviation for first 10 natural numbers is
(a) 5.5 (b) 3.87
(c) 2.97 (d) 2.87

91. In a batch of 15 students, if the marks of 10 students who passed are 70, 50, 95, 40, 60, 70, 80, 90, 75, 80 then the median marks of all the 15 students is:
(a) 40 (b) 50 (c) 60 (d) 70

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(d)** The measure of dispersion is mean deviation, standard deviation and quartile deviation.
2. **(a)** We know that the observation which occur most frequently is known as mode.
3. **(c)** Let $x_1, x_2, \ldots\ldots\ldots x_n$ be n observation.
Now, reciprocals of n observations are
$$\frac{1}{x_1}, \frac{1}{x_2}, \ldots\ldots, \frac{1}{x_n}$$
Now, mean of the reciprocals of n observation.
$$= \frac{\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots + \frac{1}{x_n}}{n}$$
Now, reciprocal of mean of the reciprocals of n observations
$$= \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots + \frac{1}{x_n}} = \text{Harmonic mean}$$
4. **(c)** First we write all the observation as 10, 18, 21, 35, 42.
Since, number of observation = 5 (odd)
$$\therefore \text{ Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{observation}$$
$$= \left(\frac{6}{2}\right) = \text{3rd observation} = 21$$
5. **(c)** While dividing each entry in a data by a nonzero number a, the arithmetic mean of the new data is divided by a.
6. **(b)** Since 6 occurs most of the times in the given series.
$\therefore$ Mode of the given series = 6
7. **(d)** Coefficient of variation $= \frac{\text{standard deviation}}{\text{Mean}} \times 100$
8. **(b)** To measure the intelligence of a group of students mode will be more suitable.
9. **(d)** Mode is the required measure.
10. **(c)** Mode of the data is 8 as it is repeated maximum number of times.
11. **(d)** Mean of $a, b, 8, 5, 10$ is 6
$$\Rightarrow \frac{a+b+8+5+10}{5} = 10 \Rightarrow a+b=7 \qquad \ldots\text{(i)}$$
Variance of $a, b, 8, 5, 10$ is 6.80
$$\Rightarrow \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5} = 6.80$$
$$\Rightarrow a^2 - 12a + 36 + (1-a)^2 + 21 = 34 \quad \text{[using eq. (i)]}$$
$$\Rightarrow 2a^2 - 14a + 24 = 0 \Rightarrow a^2 - 7a + 12 = 0$$
$$\Rightarrow a = 3 \text{ or } 4 \Rightarrow b = 4 \text{ or } 3$$
$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$ or, $a = 4$ and $b = 3$
12. **(a)** Arithmetic mean $\bar{x}$ of 4, 7, 8, 9, 10, 12, 13, 17 is
$$\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$$
$$\sum|x_i - \bar{x}| = 6+3+2+1+0+2+3+7 = 24$$
$\therefore$ Mean deviation about mean
$$= \text{M.D. } (\bar{x}) = \frac{\sum|x_i - \bar{x}|}{n} = \frac{24}{8} = 3$$
13. **(d)**

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
Total	**25**	**350**		**158**

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{350}{25} = 14$$
Mean deviation from the mean
$$= \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{158}{25} = 6.32$$
14. **(a)** Mean $\bar{x} = \frac{\sum x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	6 – 9	$(-3)^2$
7	7 – 9	$(-2)^2$
10	10 – 9	1^2
12	12 – 9	3^2
13	13 – 9	4^2
4	4 – 9	$(-5)^2$
8	8 – 9	$(-1)^2$
12	12 – 9	$(-3)^2$

$$\Sigma(x_i - \bar{x})^2 = 9+4+1+9+16+25+1+9 = 74$$
$$\text{Variance} = \frac{\Sigma(x_i - \bar{x})^2}{\Sigma f_i} = \frac{74}{8} = 9.25$$
15. **(b)** We know that, Statistics deals with data collected for specific purposes. We can make decisions about the data by analysing and interpreting it. We have studied methods of representing data graphically and in tabular form. This representation reveals certain salient features or characteristics of the data. We have also studied the methods of finding a representative value for the given data. This value is called the measure of central tendency.
16. **(d)** Mean (arithmetic mean), median and mode are three measures of central tendency. A measure of central tendency gives us a rough idea, where data points are centred.

17. (d) Variability is another factor which is required to be studied under Statistics. Like 'measure of central tendency' we want to have a single number to describe variability. This single number is called a 'measure of dispersion'.

18. (d) The dispersion or scatter in a data is measured on the basis of the observations and the types of the measure of central tendency, used there. There are following measure of dispersion.
(i) Range; (ii) Quartile deviation; (iii) Mean deviation; (iv) Standard deviation

19. (c) We know that, data can be grouped into two ways
(i) Discrete frequency distribution.
(ii) Continuous frequency distribution.

20. (a) $\text{Mean}(\bar{x}) = \frac{1357+1090+1666+1494+1623}{5} = \frac{7230}{5}$
$= 1446$

$$\text{Mean deviation} = \sum_{i=1}^{5} |x_i - \bar{x}|$$

$$= \frac{\left[|1357-1446| + |1090-1446| + |1666-1446| + |1494-1446| + |1623-1446|\right]}{5}$$

$$= \frac{89+356+220+48+177}{5} = \frac{890}{5} = 178$$

21. (b) This number, i.e., means of the squares of the deviations from mean is called the variance and is denoted by σ^2 (read as sigma square).

22. (b) The variance of n observations $x_1, x_2, \ldots x_n$ is given by

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

23. (d) We have studied about some types of measures of dispersion. The mean deviation and the standard deviation have the same units in which the data are given. Whenever we want to compare the variability of two series with same mean, which are measured in different units, we do not merely calculate the measures of dispersion but we require such measures which are independent of the units. The measure of variability which is independent of units, is called coefficient of variation (denoted as CV).

24. (c) $\text{Var}(Y) = a^2 \text{Var}(X)$

25. (b) $\left|\frac{a}{c}\right|\sigma$

26. (d) The mean of the given items

$$\bar{x} = \frac{3+7+10+18+22}{5} = 12$$

$$\text{Hence, variance} = \frac{1}{n}\sum (x_i - \bar{x})^2$$

$$= \frac{1}{5}\left\{(3-12)^2 + (7-12)^2 + (10-12)^2 + (18-12)^2 + (22-12)^2\right\}$$

$$= \frac{1}{5}\{81+25+4+36+100\} = \frac{246}{5} = 49.2$$

27. (d) Construct the following table taking assumed mean $a = 25$.

Class	x_i	f_i	$u_i = \frac{x_i - a}{10}$	$f_i u_i$	$\|x_i - 27\|$	$f_i \|x_i - 27\|$
0–10	5	5	–2	–10	22	110
10–20	15	8	–1	–8	12	96
20–30	25	15	0	0	2	30
30–40	35	16	1	16	8	128
40–50	45	6	2	12	18	108
Total	50			10		472

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times c = 25 + \frac{10}{50} \times 10 = 27,$$

and mean deviation (about mean)

$$= \frac{\sum f_i |x_i - 27|}{\sum f_i} = \frac{472}{50} = 9.44.$$

STATEMENT TYPE QUESTIONS

28. (c) The runs scored by two batsmen in their last ten matches are as follows
Batsman A: 30, 91, 0, 64, 42, 80, 30, 5, 117, 71
Batsman B: 53, 46, 48, 50, 53, 53, 58, 60, 57, 52
Clearly, the mean and median of the data are

	Batsman A	Batsman B
Mean	53	53
Median	53	53

We calculate the mean of a data (denoted by $\bar{x}$), i.e.,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Also, the median is obtained by first arranging the data in ascending or descending order and applying the rules
Mean for batsman A

$$= \frac{30+91+0+64+42+80+30+5+117+71}{10} = \frac{530}{10} = 53$$

Mean for batsman B

$$= \frac{53+46+48+50+53+53+58+60+57+52}{10} = \frac{530}{10} = 53$$

To apply the formula to obtain median first arrange the data in ascending order

For batsman A	0	5	30	30	42	64	71	80	91	117
For batsman B	46	48	50	52	53	53	53	57	58	60

Here, we have n = 10 which is even number. So median is the mean of 5[th] and 6[th] observations.

$$\text{Median for batsman A} = \frac{42+64}{2} = \frac{106}{2} = 53$$

$$\text{Median for batsman B} = \frac{53+53}{2} = \frac{106}{2} = 53$$

29. (a) Consider the data [given in above question] of runs scored by two batsmen A and B, we had some idea of variability in the scores on the basis of minimum and maximum runs in each series.

To obtain a single number for this, we find the difference of maximum and minimum values of each series. This difference is called the range of the data.
In case of batsman A, Range = 117 – 0 = 117
and for batsman B, Range = 60 – 46 = 14
Clearly, range of A > range of B. Therefore, the scores are scattered or dispersed in case of A, while for B these are close to each other.
Thus, range of a series

= Maximum value – Minimum value

The range of data gives us a rough idea of variability or scatter but does not tell about the dispersion of the data from a measure of central tendency.

30. (d) I. Mean of the given series

$$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$$

$$= \frac{4+7+8+9+10+12+13+17}{8} = 10$$

x_i	$\lvert x_i - \bar{x}\rvert$
4	$\lvert 4-10\rvert = 6$
7	$\lvert 7-10\rvert = 3$
8	$\lvert 8-10\rvert = 2$
9	$\lvert 9-10\rvert = 1$
10	$\lvert 10-10\rvert = 0$
12	$\lvert 12-10\rvert = 2$
13	$\lvert 13-10\rvert = 3$
17	$\lvert 17-10\rvert = 7$
$\sum x_i = 80$	$\sum \lvert x_i - \bar{x}\rvert = 24$

$\therefore$ Mean deviation about mean $= \frac{\sum \lvert x_i - \bar{x}\rvert}{n} = \frac{24}{8} = 3$

II. Mean of the given series

$$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$$

$$= \frac{38+70+48+40+42+55+63+46+54+44}{10} = 50$$

x_i	$\lvert x_i - \bar{x}\rvert$
38	$\lvert 38-50\rvert = 12$
70	$\lvert 70-50\rvert = 20$
48	$\lvert 48-50\rvert = 02$
40	$\lvert 40-50\rvert = 10$
42	$\lvert 42-50\rvert = 08$
55	$\lvert 55-50\rvert = 05$
63	$\lvert 63-50\rvert = 13$
46	$\lvert 46-50\rvert = 04$
54	$\lvert 54-50\rvert = 04$
44	$\lvert 44-50\rvert = 06$
$\sum x_i = 500$	$\sum \lvert x_i - \bar{x}\rvert = 84$

$\therefore$ Mean deviation about mean $= \frac{\sum \lvert x_i - \bar{x}\rvert}{n} = \frac{84}{10} = 8.4$

31. (b) Let us write the given data in tabular form and calculate the required values to find mean deviation about the mean as

x_i	f_i	$f_i x_i$	$\lvert d_i\rvert = \lvert x_i - \bar{x}\rvert = \lvert x_i - 21.65\rvert$	$f_i \lvert d_i\rvert$
20	6	120	1.65	9.90
21	4	84	0.65	2.60
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
Total	20	433		25.00

Mean $(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = 21.65$

Hence, mean of the data is 21.65

Mean deviation $= \frac{\sum f_i \lvert x_i - \bar{x}\rvert}{\sum f_i} = \frac{25}{20} = 1.25$

Hence, mean deviation of the data is 1.25

32. (d) Total number of students (n) = 2 + 3 + 8 + 3 + 4 = 20

$$\text{Median of numbers} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[\left(\frac{20}{2}\right)^{th} \text{term} + \left(\frac{20}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[10^{th} \text{term} + 11^{th} \text{term}\right]$$

Marks obtained	f_i	cf	$\lvert d_i\rvert = \lvert x_i - 12\rvert$	$f_i \lvert d_i\rvert$
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
Total	**20**		$\sum f_i \lvert d_i\rvert = 25$	**25**

Median $= \frac{12+12}{2} = 12$

Mean deviation $= \frac{\sum f_i \lvert d_i\rvert}{\sum f_i} = \frac{25}{20} = 1.25$

33. (a) From the given data we can form the following table. The mean is calculated by step-deviation method taking 14 as assumed mean. The number of observations is n = 10.

x_i	$d_i = \frac{x_i - 14}{2}$	Deviations from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	–4	–9	81
8	–3	–7	49
10	–2	–5	25
12	–1	–3	9
14	0	–1	1
16	1	1	1
18	2	3	9
20	3	5	25
22	4	7	49
24	5	9	81
	5		330

$$\therefore \quad \text{Mean } (\bar{x}) = \text{Assumed mean} + \frac{\sum_{i=1}^{n} d_i}{n} \times h$$

$$= 14 + \frac{5}{10} \times 2 = 15$$

$$\text{Variance } (\sigma^2) = \frac{1}{n}\sum_{i=1}^{10}(x_i - \bar{x})^2 = \frac{1}{10} \times 330 = 33$$

$$\therefore \quad \text{Standard deviation } (\sigma) = \sqrt{33} = 5.74$$

34. **(b)** I. First n natural numbers are 1, 2, 3, 4, ..., n.

x_i	x_i^2
1	1^2
2	2^2
3	3^2
4	4^2
.	.
.	.
.	.
n	n^2
Total $= \frac{n(n+1)}{2}$	$\frac{n(n+1)(2n+1)}{6}$

$$\therefore \quad \text{Mean} = \frac{\sum x_i}{n}$$

$$\therefore \quad \bar{x} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\text{Variance} = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left[\frac{n(n+1)}{2n}\right]^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{(n+1)}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \left(\frac{n+1}{2}\right)\left[\frac{n-1}{6}\right] = \frac{n^2-1}{12}$$

II. First 10 positive multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

x_i	x_i^2
3	9
6	36
9	81
12	144
15	225
18	324
21	441
24	576
27	729
30	900
Total = 165	**3465**

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n} = \frac{165}{10} = 16.5$$

$$\text{Variance} = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{3465}{10} - \left(\frac{165}{10}\right)^2$$

$$= 346.5 - (16.5)^2 = 346.5 - 272.25 = 74.25$$

35. **(d)**

Class	Frequency (f_i)	Mid value (x_i)	Deviation from mean $d_i = \frac{x_i - 25}{10}$	d_i^2	$f_i d_i$	$f_i d_i^2$
0 – 10	5	5	–2	4	–10	20
10 – 20	8	15	–1	1	–8	8
20 – 30	15	25	0	0	0	0
30 – 40	16	35	1	1	16	16
40 – 50	6	45	2	4	12	24
Total	**50**				**10**	**68**

$$\text{Mean}(\bar{x}) = A + \frac{\sum f_i d_i}{\sum f_i} \times h = 25 + \frac{10}{50} \times 10$$

$$= 25 + \frac{100}{50} = 25 + 2 = 27$$

$$\text{Variance} = \left[\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2\right] \times h^2$$

$$= \left[\frac{68}{50} - \left(\frac{10}{50}\right)^2\right] \times (10)^2 = \frac{[68 \times 50 - 100]}{50 \times 50} \times 100$$

$$= \frac{(3400 - 100)}{50} \times 2 = \frac{3300 \times 2}{50} = \frac{6600}{50} = 132$$

36. **(b)** For comparing the variability or dispersion of two series, we calculate the coefficient of variance for each series. The series having greater CV is said to be more variable than the other. The series having lesser CV is said to be more consistent than the other.

37. **(a)** If $\bar{x}_1$ and σ_1 are the mean and standard deviation of the first distribution, and $\bar{x}_2$ and σ_2 are the mean and standard deviation of the second distribution.

Then, CV (1st distribution) $= \frac{\sigma_1}{\bar{x}_1} \times 100$

and CV (2nd distribution) $= \frac{\sigma_2}{\bar{x}_2} \times 100$

Given, $\bar{x}_1 = \bar{x}_2 = \bar{x}$ (say)

Therefore, CV (1st distribution) $= \frac{\sigma_1}{\bar{x}} \times 100$ ...(i)

and CV (2nd distribution) $= \frac{\sigma_2}{\bar{x}} \times 100$ ...(ii)

It is clear from Eqs. (i) and (ii) that the two CVs can be compared on the basis of values of σ_1 and σ_2 only. Thus, we say that for two series with equal means, the series with greater standard deviation (or variance) is called more variable or dispersed than the other. Also, the series with lesser value of standard deviation (or variance) is said to be more consistent than the other.

38. **(d)** Mean of $ax_1, ax_2, ..., ax_n$

$$= \frac{ax_1 + ax_2 + \ldots + ax_n}{n} = a\left(\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}\right)$$

$= a\bar{x} \quad \left(\because \quad \bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}\right)$

Variance of $ax_1, ax_2, \ldots, ax_n$

$= \frac{\sum (ax_i - a\bar{x})^2}{n}$

$= \frac{(ax_1 - a\bar{x})^2 + (ax_2 - a\bar{x})^2 + ... + (ax_n - a\bar{x})^2}{n}$

$= \frac{a^2\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_n - \bar{x})^2\right]}{n}$

$= a^2 \frac{\sum (x_i - \bar{x})^2}{n} = a^2\sigma^2 \left[\because \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}\right]$

39. (b) For Raju

x_i	$d_i = x_i - 45$	d_i^2
25	–20	400
50	5	25
45	0	0
30	–15	225
70	25	625
42	–3	9
36	–9	81
48	3	9
35	–10	100
60	15	225
	–9	**1699**

Standard deviation$(\sigma) = \sqrt{\frac{169.9}{10} - \left(\frac{-9}{10}\right)^2} = \sqrt{169.9 - 0.81}$

$= \sqrt{169.09} = 13$

Mean $(\bar{x}) = 45 - \left(\frac{9}{10}\right) = 45 - 0.9 = 44.1$

For Sita

x_i	$d_i = x_i - 55$	d_i^2
10	–45	2025
70	15	225
50	–5	25
20	–35	1225
95	40	1600
55	0	0
42	–13	169
60	5	25
48	–7	49
80	25	625
	–20	**5968**

Mean$(\bar{x}) = 55 - \frac{20}{10} = 53$

Standard deviation$(\sigma) = \sqrt{\frac{5968}{10} - (-10)^2} = \sqrt{496.8} = 22.28$

Coefficient of variation of both Raju and Sita are

For Raju

$= \frac{\sigma}{\bar{x}} \times 100 = \frac{13}{44.1} \times 100 = \frac{1300}{44.1} = 29.47$

For Sita

$= \frac{\sigma}{\bar{x}} \times 100 = \frac{22.28}{53} \times 100 = \frac{2228}{53} = 42.04$

Since, CV of Sita > CV of Raju

Also, mean of Sita > mean of Raju

Hence, Raju is more consistent, but Sita is more intelligent.

40. (b) Given, $\sum (x - 5) = 3$

$\therefore \sum x - \sum 5 = 3$

$\Rightarrow \sum x - 5 \times 18 = 3 \quad (\because n = 18)$

$\Rightarrow \sum x = 3 + 90 \Rightarrow \sum x = 93$

Now, $\sum (x - 5)^2 = 43$

$\Rightarrow \sum (x^2 + 25 - 10x) = 43$

$\Rightarrow \sum x^2 + \sum 25 - 10\sum x = 43$

$\Rightarrow \sum x^2 + 25 \times 18 - 10 \times 93 = 43$

$\Rightarrow \sum x^2 = 43 + 930 - 450$

$\Rightarrow \sum x^2 = 973 - 450 \Rightarrow \sum x^2 = 523$

Now, mean $= \frac{\sum x}{n} = \frac{93}{18} = 5.16$

and $SD(\sigma) = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2}$

$= \sqrt{\frac{523 \times 18 - 93 \times 93}{18 \times 18}} = \frac{1}{18}\sqrt{9414 - 8649}$

$= \frac{1}{18}\sqrt{765} = \frac{27.66}{18} = 1.54$

41. (d)

42. (c) Only first (I) and second (II) statements are correct.

MATCHING TYPE QUESTIONS

43. (b) (A) Given data is

3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21.

Median (M) = 6^{th} obs = 9

$|x_i - M|$ are 6, 6, 5, 4, 2, 0, 1, 3, 9, 10, 12

$\therefore \sum_{i=1}^{11} |x_i - M| = 58$

$M.D(M) = \frac{1}{11} \times 58 = 5.27$

(B) Data in ascending order is

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$\text{Median} = \frac{6^{th}\text{ obs} + 7^{th}\text{ obs}}{2} = \frac{13 + 14}{2} = \frac{27}{2}$

$= 13.5$

Now, $\sum |x_i - M| = 28$

$\therefore \quad M.D(M) = \frac{28}{12} = 2.33$

INTEGER TYPE QUESTIONS

44. (a) Let a_1, a_2, a_3, a_4 and a_5 be five quantities

Then $a_1 + a_2 + a_3 + a_4 + a_5 = 30$ (given)

Also given that $a_1 + a_2 + a_3 = 12$
Now $a_4 + a_5 = 18$
Thus the average of a_4 and a_5 will be

$$\frac{a_4 + a_5}{2} = \frac{18}{2} = 9.$$

45. (d)

Class interval	f_i	x_i	cf	$\lvert d_i\rvert = \lvert x_i - 14\rvert$	$f_i\lvert d_i\rvert$
0 – 6	4	3	4	11	44
6 – 12	5	9	9	5	25
12 – 18	3	15	12	1	3
18 – 24	6	21	18	7	42
24 – 30	2	27	20	13	26
	$\Sigma f_i = 20$				$\Sigma f_i \lvert d_i\rvert = 140$

Median number $= \frac{n}{2} = \frac{20}{2} = 10$ $(\because$ n is even number)

$\therefore$ Median class = 12 – 18

$$\text{Median } = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 12 + \frac{6}{3}(10 - 9) = 12 + 2 = 14$$

$$\text{Mean deviation } = \frac{\sum f_i \lvert d_i \rvert}{\sum f_i} = \frac{140}{20} = 7$$

Hence, mean deviation about the median is 7.

46. (c)

x_i	f_i	$f_i x_i$	$f_i x_i^2$
A	2	2A	$2A^2$
2A	1	2A	$4A^2$
3A	1	3A	$9A^2$
4A	1	4A	$16A^2$
5A	1	5A	$25A^2$
6A	1	6A	$36A^2$
Total	**7**	**22A**	$\mathbf{92A^2}$

$$\because \ \sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$\Rightarrow 160 = \frac{92A^2}{7} - \left(\frac{22A}{7}\right)^2$$

$$\Rightarrow 160 = \frac{92A^2}{7} - \frac{484A^2}{49} \Rightarrow 160 = \frac{92 \times 7A^2 - 484A^2}{49}$$

$$\Rightarrow 160 \times 49 = 644A^2 - 484A^2 \Rightarrow 160A^2 = 7840$$

$$\Rightarrow A^2 = \frac{7840}{160} \Rightarrow A^2 = 49 \Rightarrow A = \pm 7$$

A = 7 as A is a positive integer.

47. (c) We have,
CV of 1st distribution $(CV_1) = 50$
CV of 2nd distribution $(CV_2) = 60$
$\sigma_1 = 10$ and $\sigma_2 = 15$

We know that, $CV = \frac{\sigma}{\bar{x}} \times 100$

$$\therefore CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100 \Rightarrow 50 = \frac{10}{\bar{x}_1} \times 100$$

$$\Rightarrow \bar{x}_1 = \frac{10 \times 100}{50} = 20$$

Also, $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$$\Rightarrow 60 = \frac{15 \times 100}{\bar{x}_2} \Rightarrow \bar{x}_2 = \frac{15 \times 100}{60} \Rightarrow \bar{x}_2 = 25$$

Thus, $\bar{x}_2 - \bar{x}_1 = 25 - 20 = 5$

48. (a) Let the other two observations be x and y.
Therefore, the series is 1, 2, 6, x, y.

Now, mean $(\bar{x}) = 4.4 = \frac{1 + 2 + 6 + x + y}{5}$

or $22 = 9 + x + y$

Therefore, $x + y = 13$ ...(i)

Also, variance $(\sigma^2) = 8.24 = \frac{1}{n}\sum_{i=1}^{5}(x_i - \bar{x})^2$

i.e., $8.24 = \frac{1}{5}[(3.4)^2 + (2.4)^2 + (1.6)^2 + x^2 + y^2$
$- 2 \times 4.4(x + y) + 2 \times (4.4)^2]$

or $41.20 = 11.56 + 5.76 + 2.56 + x^2 + y^2 - 8.8 \times 13 + 38.72$

Therefore, $x^2 + y^2 = 97$ ...(ii)

But from eq. (i), we have
$x^2 + y^2 + 2xy = 169$ ...(iii)

From eqs. (ii) and (iii), we have
$2xy = 72$ ...(iv)

On subtracting eq, (iv) from eq. (ii), we get
$x^2 + y^2 - 2xy = 97 - 72$

i.e. $(x - y)^2 = 25$ or $x - y = \pm 5$ ...(v)

So, from eqs. (i) and (v), we get
$x = 9, y = 4$ when $x - y = 5$
or $x = 4, y = 9$ when $x - y = -5$
Thus, the remaining observations are 4 and 9.
Required difference = 5

49. (c) The given data is 36, 72, 46, 42, 60, 45, 53, 46, 51, 49
Arranging the data in ascending order,
36, 42, 45, 46, 46, 49, 51, 53, 60, 72
Number of observation = 10 (even)
Median (M)

$$= \frac{\left(\frac{N}{2}\right)^{th} \text{observation} + \left(\frac{N}{2} + 1\right)^{th} \text{observation}}{2}$$

$$= \frac{\left(\frac{10}{2}\right)^{th} \text{observation} + \left(\frac{10}{2} + 1\right)^{th} \text{observation}}{2}$$

$$= \frac{5^{th} \text{observation} + 6^{th} \text{observation}}{2} = \frac{46 + 49}{2} = 47.5$$

x_i	$\lvert x_i - M\rvert$
36	$\lvert 36 - 47.5\rvert = 11.5$
42	$\lvert 42 - 47.5\rvert = 5.5$
45	$\lvert 45 - 47.5\rvert = 2.5$
46	$\lvert 46 - 47.5\rvert = 1.5$
46	$\lvert 46 - 47.5\rvert = 1.5$
49	$\lvert 49 - 47.5\rvert = 1.5$
51	$\lvert 51 - 47.5\rvert = 3.5$
53	$\lvert 53 - 47.5\rvert = 5.5$
60	$\lvert 60 - 47.5\rvert = 12.5$
72	$\lvert 72 - 47.5\rvert = 24.5$
	$\mathbf{\Sigma\lvert x_i - M\rvert = 70}$

$\therefore$ Mean deviation about median

$$= \frac{\sum \lvert x_i - M \rvert}{n} = \frac{70}{10} = 7$$

50. **(c)** $\sigma^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2 = \frac{1000}{10} - \left(\frac{60}{10}\right)^2 = 100 - 36 = 64$

$\sigma = \sqrt{64} = 8$

51. **(a)** Mean $(\bar{x}) = \frac{1+2+3+4+5}{5} = 3$

S.D $= \sigma = \sqrt{\frac{1}{5}(1+4+9+16+25) - 9} = \sqrt{11-9} = \sqrt{2}$

52. **(a)** $\bar{x} = \frac{2+4+6+8+10}{5} = 6$

Hence, variance $= \frac{1}{n}\sum(x_i - \bar{x})^2$

$= \frac{1}{5}\{(16)+4+0+4+16\} = \frac{40}{5} = 8$

53. **(b)** Range = Maximum observation – Minimum observation

$= 9 - 2 = 7$

54. **(b)** Mean $= \frac{-1+0+4}{3} = 1$

M. D (about mean) $= \frac{|-1-1| + |0-1| + |4-1|}{3} = 2$

55. **(b)** If each item of a data is increased or decreased by the same constant, the standard deviation of the data remains unchanged.

ASSERTION - REASON TYPE QUESTIONS

56. **(c)** Assertion is correct. It is a formula.

Reason is incorrect.

Sum of the deviations from mean $(\bar{x})$ is zero.

57. **(d)** Assertion is incorrect but Reason is correct.

58. **(c)** If each observation is multiplied by k, mean gets multiplied by k and variance gets multiplied by k^2.

Hence the new mean should be $2\bar{x}$ and new variance should be $k^2\sigma^2$.

So Assertion is true and Reason is false.

59. **(b)** Both Assertion and Reason are correct but Reason is not the correct explanation for Assertion.

60. **(a)** Mean $(\bar{X}) = \frac{2+9+9+3+6+9+4}{7} = \frac{42}{7} = 6$

MD $(\bar{X}) = \frac{\sum|x_i - \bar{x}|}{n} = \frac{4+3+3+3+0+3+2}{7} = \frac{18}{7} = 2.57$

CRITICALTHINKING TYPE QUESTIONS

61. **(c)** Sum of 6 numbers $= 30 \times 6 = 180$

Sum of remaining 5 numbers $= 29 \times 5 = 145$

$\therefore$ Excluded number $= 180 - 145 = 35$.

62. **(d)** Sum of 20 observations $= 20 \times 15.5 = 310$

Corrected sum $= 310 - 42 + 24 = 292$

So, corrected Mean $= \frac{292}{20} = 14.6$

63. **(b)**

64. **(b)** Given observations are 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95.

Number of observations = 10

As per definition

$$\text{median} = \frac{\text{value of } \frac{10}{2} \text{th term} + \text{value of } \left(\frac{10}{2}+1\right) \text{th term}}{2}$$

$$= \frac{\text{value of 5th term} + \text{value of 6th term}}{2}$$

$$= \frac{x + x + 2}{2} = \frac{2(x+1)}{2} = x + 1$$

But Median = 63, is given.

So, $63 = x + 1 \Rightarrow x = 62$

65. **(c)** Total sum of 13 observations $= 14 \times 13 = 182$

Sum of 14 observation $= 7 \times 12 + 7 \times 16$

$= 84 \times 112 = 196$

So, the 7th observation $= 196 - 182 = 14$

66. **(a)** The first n natural numbers are 1, 2, 3, ...n

Their mean, $\bar{x} = \frac{1+2+3+4+...+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$

[$\because$ The sum of Ist n natural numbers is $\frac{n(n+1)}{2}$]

Now, Variance $= \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$

$= \frac{1}{n}\left[\sum(x_i^2 - 2\bar{x}x_i + \bar{x}^2)\right] = \frac{\sum x_i^2}{n} - 2\bar{x}\frac{\sum x_i}{n} + \frac{\bar{x}^2.n}{n}$

$= \frac{\sum x_i^2}{n} - 2\bar{x}^2 + \bar{x}^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

[Since frequency of each variate is one]

$\because \sum x_i^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore$ Variance $= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{(n+1)}{2}\right)^2$

$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$

$= (n+1)\left(\frac{2n+1}{6} - \frac{n+1}{4}\right) = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$

67. **(c)** Calculation for Mean and Standard Deviation

x_i	f_i	$f_i x_i$	$f_i x_i^2$
6	2	12	72
10	4	40	400
14	7	98	1372
18	12	216	3888
24	8	192	4608
28	4	112	3136
30	3	90	2700
130	$\Sigma f_i = 40$	$\Sigma f_i x_i = 760$	$\Sigma f_i x_i^2 = 16176$

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{760}{40} = 19$

S.D. $= \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2} = \sqrt{\frac{16176}{40} - \left(\frac{760}{40}\right)^2}$

$= \sqrt{404.4 - 361} = \sqrt{43.4} = 6.59$.

Hence, Mean = 19, S.D. = 6.59.

68. **(d)** After arranging the terms in ascending order median is the $\left(\frac{n+1}{2}\right)^{th}$ term i.e., 5th term.

Here, we increase largest four observations of the set which will come after 5th term.

Hence, median remains the same as that of original set.

69. (d) Mean $\left(\overline{x}\right) = \dfrac{-1+0+4}{3} = 1$

$\therefore$ MD $= \dfrac{\sum\left|x_i - \overline{x}\right|}{n} = \dfrac{|-1-1|+|0-1|+|4-1|}{3} = 2$

70. (c) When each observation is multiplied by 2, then variance is also multiplied by 2.
We are given, 2, 4, 5, 6, 8, 17.
When each observation multiplied by 2, we get 4, 8, 10, 12, 16, 34.
$\therefore$ Variance of new series $= 2^2 \times$ Variance of given data $= 4 \times 23.33 = 93.32$

71. (c) We have $n = 100$, $\overline{x} = 50$, $\sigma = 5$, $\sigma^2 = 25$
We know that

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{1}{n}\sum x_i\right)^2$$

$$\Rightarrow \quad 25 = \frac{\sum x_i^2}{100} - (50)^2 \Rightarrow 2500 = \sum x_i^2 - 250000$$

$$\Rightarrow \quad \sum x_i^2 = 252500$$

72. (d) We have the following numbers
1, 2, 3, 4, 5, 6, 7, 8, 9, 10
If 1 is added to each number, we get
2, 3, 4, 5, 6, 7, 8, 9, 10, 11
Sum of these numbers, $\sum x_i = 2+3+...+11 = 65$
Sum of squares of these numbers.
$\sum x_i^2 = 2^2 + 3^2 + ... + 11^2 = 505$

$$\text{Variance } \left(\sigma^2\right) = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$= \frac{505}{10} - (6.5)^2 = 50.5 - 42.25 = 8.25$$

73. (a) First ten positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.
Sum of these numbers $\left(\sum x_i\right) = 1 + 2 + ... + 10 = 55$
Sum of squares of these numbers $\left(\sum x_i^2\right)$
$= 1^2 + 2^2 + \ldots + 10^2 = 385$

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{1}{n}\sum x_i\right)^2}$$

$$= \sqrt{\frac{385}{10} - (5.5)^2} = \sqrt{38.5 - 30.25} = \sqrt{8.25}$$

$\therefore$ Variance $(\sigma^2) = 8.25$

74. (a) We know that,
Coefficient of variation $= \dfrac{\sigma}{\overline{x}} \times 100$

$\therefore$ CV of 1st distribution $= \dfrac{\sigma_1}{30} \times 100$

$\Rightarrow \quad 50 = \dfrac{\sigma_1}{30} \times 100$ [CV of 1st distribution = 50 (given)]

$\Rightarrow \quad s_1 = 15$

Also, CV of 2nd distribution $= \dfrac{\sigma_2}{25} \times 100$

$\Rightarrow \quad 60 = \dfrac{\sigma_2}{25} \times 100 \Rightarrow \sigma_2 = \dfrac{60 \times 25}{100} \Rightarrow \quad \sigma_2 = 15$

Thus, $\sigma_1 - \sigma_2 = 15 - 15 = 0$

75. (a) We have

$\sum\left(x_i - \overline{x}\right)^2 = 250$

$n = 10$ and $\overline{x} = 50$

$$\therefore \quad \sigma^2 = \left[\frac{\sum_{i=1}^{10}\left(x_i - \overline{x}\right)^2}{n}\right] \Rightarrow \sigma^2 = \left[\frac{250}{10}\right]$$

$$\Rightarrow \quad \sigma = \sqrt{25} = 5$$

Coefficient of variation $= \dfrac{\sigma}{\overline{x}} \times 100 = \dfrac{5}{50} \times 100 = 10\%$

76. (d) We have, $n = 10$, $\overline{x} = 12$ and $\sum x_i^2 = 1530$

$$\therefore \quad \sigma^2 = \frac{1}{10}\left(\sum_{i=1}^{10} x_i^2\right) - \left(\frac{1}{10}\sum_{i=1}^{10} x_i\right)^2$$

$$\Rightarrow \quad \sigma^2 = \frac{1530}{10} - (12)^2 \Rightarrow \sigma^2 = 153 - 144$$

$$\Rightarrow \quad \sigma^2 = 9 \Rightarrow \sigma = 3$$

Coefficient of variation $= \dfrac{\sigma}{\overline{x}} \times 100 = \dfrac{3}{12} \times 100 = 25\%$

77. (b) Let the observations be $x_1, x_2, ..., x_{20}$ and $\overline{x}$ be their mean. Given that, variance = 5 and n = 20. We know that,

$$\text{Variance } \left(\sigma^2\right) = \frac{1}{n}\sum_{i=1}^{20}(x_i - \overline{x})^2$$

i.e. $5 = \dfrac{1}{20}\sum_{i=1}^{20}(x_i - \overline{x})^2$ or $\sum_{i=1}^{20}(x_i - \overline{x})^2 = 100$ ...(i)

If each observation is multiplied by 2 and the new resulting observations are y_i, then

$y_i = 2x_i$ i.e., $x_i = \dfrac{1}{2}y_i$

Therefore, $\overline{y} = \dfrac{1}{n}\sum_{i=1}^{20} y_i = \dfrac{1}{20}\sum_{i=1}^{20} 2x_i = 2.\dfrac{1}{20}\sum_{i=1}^{20} x_i$

i.e., $\overline{y} = 2\overline{x}$ or $\overline{x} = \dfrac{1}{2}\overline{y}$

On substituting the values of x_i and $\overline{x}$ in eq. (i), we get

$$\sum_{i=1}^{20}\left(\frac{1}{2}y_i - \frac{1}{2}\overline{y}\right)^2 = 100 \text{ i.e., } \sum_{i=1}^{20}\left(y_i - \overline{y}\right)^2 = 400$$

Thus, the variance of new observations

$$= \frac{1}{20} \times 400 = 20 = 2^2 \times 5$$

78. (a) We know that, if any constant is added in each observation, then standard deviation remains same.
$\therefore$ The standard deviation of the observations $a + k, b + k, c + k, d + k, e + k$ is s.

79. (c) Standard deviation is dependent on change of scale. Therefore, the standard deviation of $kx_1, kx_2, kx_3, kx_4, kx_5$ is ks.

80. (d) Mean of *a, b*, 8, 5, 10 is 6

$\Rightarrow \quad \dfrac{a+b+8+5+10}{5} = 6 \Rightarrow a + b = 7$ ...(i)

Variance of *a, b*, 8, 5, 10 is 6.80

$$\Rightarrow \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5} = 6.80$$

$\Rightarrow a^2 - 12a + 36 + (1-a)^2 + 21 = 34$ [using eq. (i)]

$\Rightarrow 2a^2 - 14a + 24 = 0 \Rightarrow a^2 - 7a + 12 = 0$

$\Rightarrow a = 3$ or $4 \quad \Rightarrow b = 4$ or 3

$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$ or, $a = 4$ and $b = 3$

81. (a) $\sigma_x^2 = 4, \sigma_y^2 = 5, x = 2, y = 4$

$\frac{1}{5}\sum x_i^2 - (2)^2 = 4; \ \frac{1}{5}\sum y_i^2 - (4)^2 = 5$

$\sum x_i^2 = 40; \sum y_i^2 = 105 \Rightarrow \sum\left(x_i^2 + y_i^2\right) = 145$

$\Rightarrow \sum(x_i + y_i) = 5(2) + 5(4) = 30$

Variance of combined data

$$= \frac{1}{10}\sum\left(x_i^2 + y_i^2\right) - \left(\frac{1}{10}\sum(x_i + y_i)\right)^2 = \frac{145}{10} - 9 = \frac{11}{2}$$

82. (d) If initially all marks were x_i then $\sigma_i^2 = \frac{\sum_i (x_i - \bar{x})^2}{N}$

Now each is increased by 10

$$\sigma_i^2 = \frac{\sum_i [(x_i+10) - (\bar{x}+10)]^2}{N} = \frac{\sum_i (x_i - \bar{x})^2}{N} = \sigma_i^2$$

Hence, variance will not change even after the grace marks were given.

83. (b) Required mean $= \frac{1}{n}\sum_{i=1}^{n}(2x_i + 3)$

$$= \frac{2}{n}\left(\sum_{i=1}^{n} x_i\right) + \frac{3n}{n} = 2\left\{\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\right\} + 3 = 2\bar{x} + 3$$

84. (a) Mean of n observations is

$$\frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{n} = \frac{n(n+1)(2n+1)}{6n}$$

From the description of the problem:

$$\frac{(n+1)(2n+1)}{6} = \frac{46n}{11}$$

$\Rightarrow 11 \times (2n^2 + 3n + 1) = 6 \times 46n$

$\Rightarrow 22n^2 + 33n + 11 = 276n \Rightarrow 22n^2 + 243n + 11 = 0$

$\Rightarrow 22n^2 - 242n - n + 11 = 0$

$\Rightarrow 22n(n - 11) - 1(n - 11) = 0$

$\Rightarrow (n - 11)(22n - 1) = 0$

Now, $22n - 1 = 0 \Rightarrow n = \frac{1}{22}$

which is discarded as n cannot be a fraction.

$\therefore n - 11 = 0 \Rightarrow n = 11$

85. (b) Mean of four observations = 20

$\therefore$ total observations $= 20 \times 4 = 80$

When add c in each observation total observation will be $80 + 4c$, then new mean = 22

$\therefore$ According to the question,

$80 + 4c = 22 \times 4 \Rightarrow 80 + 4c = 88 \Rightarrow 4c = 8 \Rightarrow c = 2$

86. (c) Let x be a set of observations given as

$x = a_1, a_2, \ldots\ldots, a_n$

Then $\bar{x} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$

If now each observation is divided by α, then

$$\frac{\frac{a_1}{\alpha} + \frac{a_2}{\alpha} + \ldots\ldots + \frac{a_n}{\alpha} + 10n}{n} = \frac{\bar{x}}{\alpha} + 10 = \frac{\bar{x} + 10\alpha}{\alpha}$$

87. (c) Let the items be $a_1, a_2, \ldots\ldots, a_n$.

then $\bar{X} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$

Now, according to the given condition:

$$\bar{X} = \frac{(a_1 + 1) + (a_2 + 2) + \ldots\ldots + (a_n + n)}{n}$$

$$= \bar{X} + \frac{1 + 2 + 3 + \ldots\ldots + n}{n} = \bar{X} + \frac{n(n+1)}{2n}$$

(using sum of n natural nos.)

$$= \bar{X} + \frac{n+1}{2}.$$

88. (c)

Class	mid-value (x)	f	f x	d = x–M	d^2	fd^2
0 – 10	5	2	10	–20.7	428.49	856.98
10 – 20	15	10	150	– 10.7	114.49	1144.9
20 – 30	25	8	200	– 0.7	0.49	3.92
30 – 40	35	4	140	9.3	86.49	345.96
40 – 50	45	6	270	19.3	372.49	2234.94
		Σf = 30	Σfx = 770			Σfd^2 = 4586.7

Now, M (A.M) $= \frac{\Sigma fx}{\Sigma f} = \frac{770}{30} = 25.7$

Now, standard deviation (S.D)

$$= \sqrt{\frac{\Sigma fd^2}{\Sigma f}} = \sqrt{\frac{4586.70}{30}} = 12.36$$

$\therefore$ Coeff of SD $= \frac{SD}{M} = \frac{12.36}{25.7} = 0.480$

$\therefore$ Coeff of variation = Coeff of S.D × 100 $= 0.480 \times 100 = 48.$

89. (a) C.V. (1st distribution) = 60, $\sigma_1 = 21$

C.V. (2nd distribution) = 70, $\sigma_2 = 16$

Let $\bar{x}_1$ and $\bar{x}_2$ be the means of 1st and 2nd distribution, respectively, Then

C.V. (1st distribution) $= \frac{\sigma_1}{\bar{x}_1} \times 100$

$\therefore 60 = \frac{21}{\bar{x}_1} \times 100$ or $\bar{x}_1 = \frac{21}{60} \times 100 = 35$

and C.V. (2nd distribution) $= \frac{\sigma_2}{\bar{x}_2} \times 100$

i.e., $70 = \frac{16}{\bar{x}_2} \times 100$ or, $\bar{x}_2 = \frac{16}{70} \times 100 = 22.85$

90. (d) Variance $= \frac{(10)^2 - 1}{12} = \frac{99}{12}$

$\therefore$ S.D $= \sqrt{\frac{99}{12}} = \sqrt{8.25} = 2.87$

91. (c) As given : marks of 10 students out of 15 in the ascending order are 40, 50, 60, 70, 70, 75, 80, 80, 90, 95

Total number of terms = 15 and 5 students who failed are below 40 marks, median $= \left(\frac{n+1}{2}\right)$th term

$$= \left(\frac{15+1}{2}\right)^{th} \text{ term} = 8^{th} \text{ term} = 60$$

CHAPTER

16

PROBABILITY-I

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If $\frac{1+4p}{4}, \frac{1-p}{2}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then value of p is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

2. Which of the following cannot be the probability of an event?

(a) 2/3 (b) – 1/5 (c) 15% (d) 0.7

3. Probability of an event can be

(a) – 0.7 (b) $\frac{11}{9}$ (c) 1.001 (d) 0.6

4. In an experiment, the sum of probabilities of different events is

(a) 1 (b) 0.5 (c) – 2 (d) 0

5. In rolling a dice, the probability of getting number 8 is

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

6. In a simultaneous throw of 2 coins, the probability of having 2 heads is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) $\frac{1}{6}$

7. The probability of getting sum more than 7 when a pair of dice are thrown is:

(a) $\frac{7}{36}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) None of these

8. The probability of raining on day 1 is 0.2 and on day 2 is 0.3. The probability of raining on both the days is

(a) 0.2 (b) 0.1 (c) 0.06 (d) 0.25

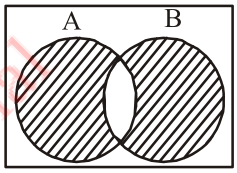
9. If A and B are two events, such that

$$P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(A^c) = \frac{2}{3}$$

where A^c stands for the complementary event of A, then P(B) is given by:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

10. In the following Venn diagram circles A and B represent two events:



The probability of the union of shaded region will be

(a) $P(A) + P(B) - 2P(A \cap B)$

(b) $P(A) + P(B) - P(A \cap B)$

(c) $P(A) + P(B)$

(d) $2P(A) + 2P(B) - P(A \cap B)$

11. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$ (c) $\frac{4}{11}$ (d) 0

12. A bag contains 10 balls, out of which 4 balls are white and the others are non-white. The probability of getting a non-white ball is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

13. The dice are thrown together. The probability of getting the sum of digits as a multiple of 4 is:

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{5}{9}$

14. If the probabilities for *A* to fail in an examination is 0.2 and that for *B* is 0.3, then the probability that either *A* or *B* fails as

(a) $> .5$ (b) 0.5 (c) $\leq .5$ (d) 0

15. If $\frac{2}{11}$ is the probability of an event, then the probability of the event 'not A', is

(a) $\frac{9}{11}$ (b) $\frac{11}{2}$ (c) $\frac{11}{9}$ (d) $\frac{2}{11}$

16. An experiment is called random experiment, if it

(a) has more than one possible outcome

(b) is not possible to predict the outcome in advance

(c) Both (a) and (b)

(d) None of the above

17. An event can be classified into various types on the basis of the
(a) experiment (b) sample space
(c) elements (d) None of the above

18. An event which has only ...... sample point of a sample space, is called simple event.
(a) two (b) three (c) one (d) zero

19. If an event has more than one sample point, then it is called a/an
(a) simple event (b) elementary event
(c) compound event (d) None of these

20. When the sets A and B are two events associated with a sample space. Then, event '$A \cup B$' denotes
(a) A and B (b) Only A (c) A or B (d) Only B

21. If A and B are two events, then the set $A \cap B$ denotes the event
(a) A or B (b) A and B (c) Only A (d) Only B

22. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number", Then, E and F are
(a) mutually exclusive
(b) exhaustive
(c) mutually exclusive and exhaustive
(d) None of the above

23. Let $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$, then $\overline{E}$ is
(a) $\{2,4\}$ (b) $\{3,6\}$ (c) $\{1,2,4\}$ (d) $\{2,4,6\}$

24. If A and B are two events, then which of the following is true?
(a) $P(A \cup B) = P(A) + P(B)$
(b) $P(A \cup B) = P(A) + P(B) - \sum P(\omega_i), \forall \omega_i \in A \cap B$
(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(d) Both (b) and (c)

25. A coin is tossed once, then the sample space is
(a) {H} (b) {T} (c) {H, T} (d) None of these

26. A set containing the numbers from 1 to 25. Then, the set of event getting a prime number, when each of the given number is equally likely to be selected, is
(a) $\{2, 3, 7, 11, 13, 17\}$
(b) $\{1, 2, 3, 7, 11, 19\}$
(c) $\{2, 5, 7, 9, 11, 13, 17, 19, 23\}$
(d) $\{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

27. Let S be a sample space containing outcomes $\omega_1, \omega_2, \omega_3, ..., \omega_n$ i.e., $S = \{\omega_1, \omega_2, ..., \omega_n\}$.
Then, which of the following is true?
I. $0 \le P(\omega_i) \le 1$ for each $\omega_i \in S$
II. $P(\omega_1) + P(\omega_2) + ... P(\omega_n) = 1$
III. For any event A, $P(A) = \sum P(\omega_i)$, $\omega_i \in A$
(a) Only I (b) Only II (c) Only III (d) All of these

28. Which of the following is true?
I. If the empty set ϕ and the sample space describe events, then ϕ is an impossible event.
II. In the above statement, the whole sample space S is called the sure event.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

29. Consider the experiment of rolling a die. Let A be the event 'getting a prime number' and B be the event 'getting an odd number'.
Then, which of the following is true?
I. A or B = $A \cup B = \{1, 2, 3\}$
II. A and B = $A \cap B = \{3, 5\}$
III. A but not B = $A - B = \{2\}$
IV. Not A = $A' = \{1, 5, 6\}$
(a) Only I is true (b) Only II is true
(c) II and III is true (d) Only IV is true

30. A letter is chosen at random from the word 'ASSASSINATION'.
I. The probability that letter is a vowel is $\frac{6}{13}$.
II. The probability that letter is a consonant is $\frac{7}{13}$.
(a) Only I is correct.
(b) Both I and II are correct.
(c) Only II is correct.
(d) Both are incorrect.

31. A die is thrown.
I. The probability of a prime number will appear is $\frac{1}{2}$.
II. The probability of a number more than 6 will appear is 1.
(a) Only I is correct.
(b) Only II is correct.
(c) Both I and II are correct.
(d) Both I and II are incorrect.

32. A card is selected from a pack of 52 cards.
I. The probability that card is an ace of spades, is $\frac{2}{52}$.
II. The probability that the card is black card, is $\frac{26}{52}$.
(a) Only I is false. (b) Only II is false.
(c) Both I and II are false. (d) Both I and II are true.

33. A die is rolled, let E be the event "die shows 4" and F be the event "die shows even number". Then
I. E and F are mutually exclusive.
II. E and F are not mutually exclusive.
(a) Only I is true. (b) Only II is true.
(c) Neither I nor II is true. (d) Both I and II are true.

34. Consider the following statements.

I. If an event has only one sample point of the sample space is called a simple event.

II. A sample space is the set of all possible outcomes of an experiment.

(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Both I and II are false.

35. Consider the following statements.

I. If an event has more than one sample point it is called a compound event.

II. A set of events is said to be mutually exclusive if the happening of one excludes the happening of the other i.e. $A \cap B = \phi$.

III. An event having no sample point is called null or impossible event.

(a) I and II are true (b) II and III are true.
(c) I, II and III are true. (d) None of them are true.

36. Consider the following statements.

I. P(A or B) = $P(A \cup B) = P(A) + P(B)$, where A and B are two mutually exclusive events.

II. P(not 'A') = $1 - P(A) = P(\bar{A})$, where $P(\bar{A})$ denotes the probability of not happening the event A.

III. $P(A \cap B)$ = Probability of simultaneous occurrence of A and B.

(a) I, II are true but III is false.
(b) I, III are true but II is false.
(c) II, III are true but I is false.
(d) All three statements are true.

37. Two dice are thrown. The events *A*, *B* and *C* are as follows:

A : getting an even number on the first die.
B : getting an odd number on the first die.
C : getting the sum of the numbers on the dice ≤ 5.

Then,

I. A' : getting an odd number on the first die

II. A and B = $A \cap B = \phi$

III. B and C = $B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(a) Only I and II is false.
(b) Only II and III is false.
(c) All I, II and III are false.
(d) All I, II and III are true.

38. If *A* and *B* are events such that *P*(*A*) = 0.42, *P*(*B*) = 0.48 and *P*(*A* and *B*) = 0.16. then,

I. P(not A) = 0.58

II. P(not B) = 0.52

III. P(A or B) = 0.47

(a) Only I and II are correct.
(b) Only II and III are correct.
(c) Only I and III are true.
(d) All three statements are correct.

39. If *E* and *F* are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, then,

I. $P(\text{E or F}) = \frac{5}{8}$

II. $P(\text{not E and not F}) = \frac{3}{8}$

(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Neither I nor II is true.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

40. A die is thrown. Then, match the events of column-I with their respective sample points in column-II.

Column - I	Column - II
A. a number less than 7.	1. $\{3,4,5,6\}$
B. a number greater than 7.	2. $\{6\}$
C. a multiple of 3.	3. $\{1,2,3\}$
D. a number less than 4.	4. $\{3,6\}$
E. an even number greater than 4.	5. $\{\ \}$
F. a number not less than 3.	6. $\{1,2,3,4,5,6\}$

Codes

	A	B	C	D	E	F
(a)	6	5	4	3	2	1
(b)	1	2	3	4	5	6
(c)	5	6	4	3	2	1
(d)	3	4	5	6	2	1

41. A die is thrown. If A, B, C, D, E and F are events described in above question. Then, match the events of column-I with their respective sample points in column-II.

Column - I	Column - II
A. $A \cup B$	1. $\{1,2\}$
B. $A \cap B$	2. ϕ
C. $B \cup C$	3. $\{1,2,3\}$
D. $E \cap F$	4. $\{1,2,4,5\}$
E. $D \cap E$	5. $\{6\}$
F. $A - C$	6. $\{3,6\}$
G. $D - E$	7. $\{1,2,3,4,5,6\}$
H. $E \cap F'$	
I. F'	

Codes

	A	B	C	D	E	F	G	H	I
(a)	1	2	7	3	4	5	6	4	2
(b)	7	2	6	5	2	4	3	2	1
(c)	1	2	4	7	5	3	4	2	1
(d)	6	5	4	1	2	3	6	5	3

42.

Column-I	Column-II
A. If E_1 and E_2 are the two mutually exclusive events, then	1. $E_1 \cap E_2 = E_1$
B. If E_1 and E_2 are the mutually exclusive and exhaustive events, then	2. $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$
C. If E_1 and E_2 have common outcomes, then	3. $E_1 \cap E_2 = \phi$, $E_1 \cup E_2 = S$
D. If E_1 and E_2 are two events such that $E_1 \subset E_2$, then	4. $E_1 \cap E_2 = \phi$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	2	1
(c)	2	3	4	1
(d)	1	4	2	3

43. Match the proposed probability under column I with the appropriate written description under column II.

Column-I (Probability)	Column-II (Written description)
A. 0.95	1. An incorrect assignment
B. 0.02	2. No chance of happening
C. – 0.3	3. As much chance of happening as not
D. 0.5	4. Very likely to happen
E. 0	5. Very little chance of happening

Codes

	A	B	C	D	E
(a)	4	5	1	3	2
(b)	1	2	3	4	5
(c)	3	2	4	5	1
(d)	5	2	3	4	1

44. A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$.

Then, match the terms of column-I with terms of column-II.

Column-I	Column-II
A. $P(A \cup B)$	1. 0.34
B. $P(A' \cap B')$	2. 0.19
C. $P(A \cap B')$	3. 0.12
D. $P(B \cap A')$	4. 0.88

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	1	2	3	4
(c)	2	3	4	1
(d)	3	2	1	4

45.

Column -I	Column - II
A. Three coins are tossed once. The probability of getting all heads, is	1. $\frac{5}{12}$
B. Two coins are tossed simultaneously. The probability of getting exactly one head, is	2. $\frac{2}{3}$
C. A die is thrown. The probability of getting a number less than or equal to 4, is	3. $\frac{1}{8}$
D. Two dice are thrown simultaneously. The probability of getting the sum as a prime number is	4. $\frac{1}{2}$

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	4	3	2	1
(c)	4	3	1	2
(d)	3	4	2	1

46. Let A, B and C are three arbitrary events, then match the columns and choose the correct option from the codes given below.

Column -I (events)	Column - II (Symbolic form)
A. Only A occurs	1. $\bar{A} \cap \bar{B} \cap \bar{C}$
B. Both A and B, but no C occur	2. $A \cap B \cap C$
C. All three events occur	3. $A \cap \bar{B} \cap \bar{C}$
D. At least one occur	4. $A \cup B \cup C$
E. None occurs	5. $A \cap B \cap \bar{C}$

Codes

	A	B	C	D	E
(a)	3	2	5	1	4
(b)	3	5	2	4	1
(c)	3	5	4	2	1
(d)	1	5	4	2	3

47.

Column -I	Column - II
A. A possible result of a random experiment is called	1. Complementary event
B. The set of outcomes is called the	2. An event
C. Any subset E of a sample space S is called	3. Sample space

D.	For every event A, there corresponds another event A′ called the ______ to A	4.	Outcome

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	4	2	3	1
(c)	1	3	2	4
(d)	1	2	3	4

48. Let A and B be two events related to a random experiment.

Column -I		**Column - II**	
A.	$P(A \cup B)$	1.	Probability of non-occurrence of A.
B.	$P(A \cap B)$	2.	$\frac{\text{No. of fav. outcome}}{\text{Total outcome}}$
C.	$P(\bar{A})$	3.	Probability that at least one of the events occur.
D.	$P(A)$	4.	Probability of simultaneous occurrence of A and B.

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	1	2	3	4
(c)	3	4	2	1
(d)	3	1	4	2

49.

Column -I (Experiment)		**Column - II (Sample space)**	
A.	A coin is tossed three times.	1.	{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}
B.	A coin is tossed two times.	2.	{HH, HT, T1, T2, T3, T4, T5, T6}
C.	A coin is tossed and a die is thrown.	3.	{HHH, HHT, HTH, THH, THT, TTH, HTT, TTT}
D.	Toss a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once.	4.	{HH, TT, HT, TH}

Codes

	A	B	C	D
(a)	3	4	2	1
(b)	4	3	1	2
(c)	3	4	1	2
(d)	4	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

50. Two dice are thrown simultaneously. The probability of obtaining a total score of seven is $\frac{1}{m}$. The value of 'm' is

(a) 3 (b) 2 (c) 6 (d) 9

51. A coin is tossed 3 times, the probability of getting exactly two heads is $\frac{m}{8}$. The value of 'm' is

(a) 1 (b) 2 (c) 3 (d) 4

52. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{3}{5}$ (b) 0 (c) 1 (d) $\frac{2}{5}$

53. In a simultaneous toss of two coins, the probability of getting exactly 2 tails is $\frac{m}{n}$. The value of m + n is

(a) 1 (b) 4 (c) 5 (d) 2

54. A die is thrown. The probability of getting a number less than or equal to 6 is

(a) 6 (b) 1 (c) 2 (d) 5

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

55. **Assertion :** Probability of getting a head in a toss of an unbiased coin is $\frac{1}{2}$.

Reason : In a simultaneous toss of two coins, the probability of getting 'no tails' is $\frac{1}{4}$.

56. **Assertion :** In tossing a coin, the exhaustive number of cases is 2.

Reason : If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$.

57. **Assertion :** A letter is chosen at random from the word NAGATATION. Then, the total number of outcomes is 10.

Reason : A letter is chosen at random from the word 'ASSASSINATION' Then, the total number of outcomes is 13.

58. Consider a single throw of die and two events.

A = the number is even = {2, 4, 6}

B = the number is a multiple of 3 = {3, 6}

Assertion : $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$

Reason : $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - \frac{2}{3} = \frac{1}{3}$

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

59. In a school there are 40% science students and the remaining 60% are arts students. It is known that 5% of the science students are girls and 10% of the arts students are girls. One student selected at random is a girl. What is the probability that she is an arts student?

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$

60. A bag contains 10 balls, out of which 4 balls are white and the others are non-white. The probability of getting a non-white ball is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

61. In a leap year the probability of having 53 Sundays or 53 Mondays is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

62. A fair die is thrown once. The probability of getting a composite number less than 5 is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) 0

63. The probability that a two digit number selected at random will be a multiple of '3' and not a multiple of '5' is

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$ (c) $\frac{1}{15}$ (d) $\frac{4}{90}$

64. Three identical dice are rolled. The probability that the same number will appear on each of them is:

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) $\frac{3}{28}$

65. The probability that a card drawn from a pack of 52 cards will be a diamond or king is:

(a) $\frac{1}{52}$ (b) $\frac{2}{13}$ (c) $\frac{4}{13}$ (d) $\frac{1}{13}$

66. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$

The set of possible values of x are in the interval is

(a) $[0, 1]$ (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$

(c) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

67. A coin is tossed repeatedly until a tail comes up for the first time. Then, the sample space for this experiment is

(a) {T, HT, HTT}
(b) {TT, TTT, HTT, THH}
(c) {T, HT, HHT, HHHT, HHHHT, ...}
(d) None of the above

68. The probability that a randomly chosen two-digit positive integer is a multiple of 3, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

69. If M and N are any two events, the probability that atleast one of them occurs is ...

(a) $P(M)+P(N)-2P(M\cap N)$
(b) $P(M)+P(N)-P(M\cap N)$
(c) $P(M)+P(N)+P(M\cap N)$
(d) $P(M)+P(N)+2P(M\cap N)$

70. If $P(A\cup B)=P(A\cap B)$ for any two events A and B, then

(a) $P(A)=P(B)$ (b) $P(A)>P(B)$
(c) $P(A)<P(B)$ (d) None of these

71. If A and B are mutually exclusive events, then

(a) $P(A)\le P(\overline{B})$ (b) $P(A)\ge P(\overline{B})$
(c) $P(A)<P(B)$ (d) None of these

72. If A, B and C are three mutually exclusive and exhaustive events of an experiment such that $3P(A)=2P(B)=P(C)$, then P(A) is equal to ...

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{5}{11}$ (d) $\frac{6}{11}$

73. A coin is tossed twice. Then, the probability that atleast one tail occurs is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

74. While shuffling a pack of 52 playing cards, 2 are accidentally dropped. The probability that the missing cards to be of different colours is

(a) $\frac{29}{52}$ (b) $\frac{1}{2}$ (c) $\frac{26}{51}$ (d) $\frac{27}{51}$

75. In a leap year, the probability of having 53 Sundays or 53 Mondays is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

76. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is

(a) 0.39 (b) 0.25
(c) 0.11 (d) None of these

77. If, $P(B)=\frac{3}{4}, P(A\cap B\cap \overline{C})=\frac{1}{3}$

and $P(\overline{A}\cap B\cap \overline{C})=\frac{1}{3}$, then $P(B\cap C)$ is

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** $\frac{1+4p}{4}, \frac{1-p}{2}, \frac{1-2p}{2}$ are probabilities of the three mutually exclusive events, then

$0 \le \frac{1+4p}{4} \le 1,\ 0 \le \frac{1-p}{2} \le 1,\ 0 \le \frac{1-2p}{2} \le 1$

and $0 \le \frac{1+4p}{4} + \frac{1-p}{2} + \frac{1-2p}{2} \le 1$

$\therefore -\frac{1}{4} \le p \le \frac{3}{4}, -1 \le p \le 1, -\frac{1}{2} \le p \le \frac{1}{2}, \frac{1}{2} \le p \le \frac{5}{2}$

$\therefore \frac{1}{2} \le p \le \frac{1}{2}$

[The intersection of above four intervals]

$\therefore p = \frac{1}{2}$

2. **(b)**

3. **(d)** Probability of an event always lies between 0 and 1. (both inclusive)

4. **(a)**

5. **(a)** Number 8 does not represent on dice.

6. **(a)** Let S be the sample space.

Since, simultaneously we throw 2 coins

S = {HH, HT, TH, TT}

$\therefore n(S) = 2^2$

Now, Let E be the event getting 2 heads i.e. HH

$\therefore n(E) = 1$

Thus, required prob $= \frac{n(E)}{n(S)} = \frac{1}{4}$

7. **(b)** Here $n(S) = 6^2 = 36$

Let E be the event "getting sum more than 7" i.e. sum of pair of dice = 8, 9, 10, 11, 12

i.e. $E = \begin{Bmatrix} (2,6) & (3,5) & (4,4) & (5,3) & (6,2) \\ (3,6) & (4,5) & (5,4) & (6,3) & \\ (4,6) & (5,5) & (6,4) & & \\ (5,6) & (6,5) & (6,6) & & \end{Bmatrix}$

$\therefore \quad n(E) = 15$

$\therefore \quad$ Required prob $= \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

8. **(d)**

9. **(b)** From the given problem :

$P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}$

$P(A^c) = \frac{2}{3} = 1 - P(A)$

$\Rightarrow \quad P(A) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$= \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = 1 - \frac{1}{3} = \frac{2}{3}$

10. **(b)** From the given Venn diagram the shadow region is $n(A) + n(B) - n(A \cap B)$.

Probability of the union of the shaded region is $P(A) + P(B) - P(A \cap B)$

11. **(c)** Required probability $= \frac{1+2+1}{11} = \frac{4}{11}$

12. **(b)** Total no. of balls = 10

No. of white balls = 4

No. of non-white balls = 10 – 4 = 6

So, Required prob $= \frac{6}{10} = \frac{3}{5}$

13. **(c)** Total exhaustive cases $= 6^2 = 36$

Following 9 pairs are favourable as the sum of their digits are multiple of 4

i.e., 4 or 8 or 12

(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)

$\therefore$ Required probability $= \frac{9}{36} = \frac{1}{4}$

14. **(c)**

15. **(a)** Let $P(A) = \frac{2}{11}$;

$P(\text{not } A) = 1 - P(A) = 1 - \frac{2}{11} = \frac{9}{11}$

16. **(c)** In our day-to-day life, we perform many activities which have a fixed result no matter any number of times they are repeated. Such as, given any triangle, without knowing the three angles, we can definitely say that the sum of measure of angles is 180°.

When a coin is tossed it may turn up a head or a tail, but we are not sure which one of these results will actually be obtained. Such experiments are called random experiments.

17. **(c)** Events can be classified into various types on the basis of the elements they have.

18. **(c)** If an event E has only one sample point of a sample space, then it is called a simple (or elementary) event.

In a sample space containing n distinct elements, there are exactly n simple events.

For example, in the experiment of tossing two coins, a sample space is

S = {HH, HT, TH, TT}

There are four simple event corresponding to this sample space. There are E_1 = {HH}, E_2 = {HT}, E_3 = {TH} and E_4 = {TT}

19. **(c)** If an event has more than one sample point, then it is called a compound event.
For example, in the experiment of "tossing a coin thrice" the events
E: 'exactly one head appeared'
F: 'atleast one head appeared'
G: 'atmost one head appeared' etc.
are all compound events. The subsets of associated with these events are
E = {HTT, THT, TTH}
F = {HTT, THT, TTH, HHT, HTH, THH, HHH]
G = {TTT, THT, HTT, TTH}
Each of the above subsets contain more than one sample point, hence they are all compound events.

20. **(c)** Recall that union of two sets A and B denoted by $A \cup B$ contains all those elements which are either in A or in B or in both. When the sets A and B are two events associated with a sample space, then $A' \cup B'$ is the event 'either A or B' or both'. This event $A' \cup B'$ is also called 'A or B'.
Therefore, event 'A or B' = $A \cup B$
$= \{\omega : \omega \in A \text{ or } \omega \in B\}$

21. **(b)** We know that, intersection of two sets $A \cap B$ is the set of those elements which are common to both A and B, i.e., which belong to both 'A and B'.
Thus, $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$
For example, in the experiment of 'throwing a die twice' Let A be the event 'score on the first throw is 6' and B is the event 'sum of two scores is atleast 11'. Then,
A = {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.
and B = {(5, 6), (6, 5), (6, 6)}
So, $A \cap B = \{(6,5),(6,6)\}$

22. **(d)** Let E = The die shows 4 = {4}
F = The die shows even number
= {2, 4, 6}
$\therefore\ E \cap F = \{4\} \neq \phi$
Hence, E and F are not mutually exclusive.

23. **(d)** Given that S = {1, 2, 3, 4, 5, 6} and E = {1, 3, 5}
Then, $\overline{E} = S - E = \{2,4,6\}$

24. **(d)** To find the probability of event 'A or B', i.e., $P(A \cup B)$. If S is sample space for tossing of three coins, then
S = {HHT, HHH, HTH, HTT, THH, THT, TTH, TTT}
Let A = {HHT, HTH, THH} and B = {HTH, THH, HHH}
be two events associated with 'tossing of a coin thrice'.
Clearly, $A \cup B$ = {HHT, HTH, THH, HHH}
Now, $P(A \cup B)$ = P(HHT) + P(HTH) + P(THH) + P(HHH)
If all the outcomes are equally likely, then
$$P(A \cup B) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$
Also, P(A) = P(HHT) + P(HTH) + P(THH) = $\frac{3}{8}$
and P(B) = P(HTH) + P(THH) + P(HHH) = $\frac{3}{8}$
Therefore, $P(A) + P(B) = \frac{3}{8} + \frac{3}{8} = \frac{6}{8}$
It is clear that $P(A \cup B) \neq P(A) + P(B)$
The points HTH and THH are common to both A and B. In the computation of P(A) + P(B) the probabilities of points HTH and THH, i.e., the elements of $A \cap B$ are included twice. Thus, to get the probability of $P(A \cup B)$ we have to subtract the probabilities of the sample points in $A \cap B$ from P(A) + P(B).
i.e., $P(A \cup B) = P(A) + P(B) - \sum P(\omega_i), \forall \omega_i \in A \cap B$
$= P(A) + P(B) - P(A \cap B)$
Thus, we observe that,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

25. **(c)** A coin is tossed once, then the sample space is
S = {H, T}

26. **(d)** Let the set be S
Then, S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}.
Now, let the event E = Getting a prime number when each of the given number is equally likely to be selected
E = {2, 3, 5, 7, 11, 13, 17, 19, 23}

STATEMENT TYPE QUESTIONS

27. **(d)** Let S be the sample space containing outcomes $\omega_1, \omega_2, \ldots, \omega_n$ i.e., S = $\{\omega_1, \omega_2, \ldots, \omega_n\}$
It follows from the axiomatic definition of probability that
I. $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$
II. $P(\omega_1) + P(\omega_2) + \ldots. + P(\omega_n) = 1$
III. For any event A, P(A) = $\sum P(\omega_i), \omega_i \in A$.

28. **(c)** The empty set ϕ and the sample space S describe events. Infact ϕ is called and impossible event and S, i.e., the whole sample space is called the sure event.

29. **(c)** Here, S = {1, 2, 3, 4, 5, 6}, A = {2, 3, 5} and B = {1, 3, 5}
Obviously
I. 'A or B' = $A \cup B$ = {1, 2, 3, 5}
II. 'A and B' = $A \cap B$ = {3, 5}
III. 'A but not B' = A – B = {2}
IV. 'not A' = A′ = {1, 4, 6}

30. **(b)** The word '*ASSASSINATION*' has 13 letters in which there are 6 vowels viz. *AAAIIO* and 7 consonants *SSSSNNT*.
∴ $n(S) = 13$, No. of vowels = 6
∴ Probability of chosing a vowel = $\frac{6}{13}$
No. of consonants = 7
∴ Probability of chosing a consonant = $\frac{7}{13}$

31. **(a)** In this case, the possible outcomes are 1, 2, 3, 4, 5 and 6.
Total number of possible outcomes = 6.
I. Number of outcomes favourable to the event "a prime number" = 3 (*i.e.*, 2, 3, 5)

$P(\text{prime number}) = \frac{3}{6} = \frac{1}{2}$

II. Number of outcomes favourable to the event "a number more than 6" = 0

$P(\text{a number more than 6}) = \frac{0}{6} = 0$

32. (a) I. There is 1 ace of spade.

$\therefore n(A) = 1, n(S) = 52$

Probability that the card drawn is an ace of spade $= \frac{n(A)}{n(S)} = \frac{1}{52}$

II. There are 26 black cards.

$n(A) = 26, n(S) = 52$

Probability of getting a black card

$= \frac{26}{52} = \frac{1}{2}$

33. (b) When we throw a die, it can result in any one of the six number 1, 2, 3, 4, 5, 6

and $S = \{1, 2, 3, 4, 5, 6\}$

E (die shows 4) = $\{4\}$

F (die shows even number) = $\{2, 4, 6\}$

$\therefore \quad E \cap F = \{4\} \Rightarrow E \cap F \neq \phi$

$\Rightarrow$ E and F are not mutually exclusive.

34. (c) By Definition, both the given statements are correct.

35. (c) By Definition, all the three statements are correct.

36. (d) By definition, All the three statements are true.

37. (d) B : getting an odd number on the first die.

$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

C : $\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

38. (a) I. $P(\text{not } A) = 1 - 0.42 = 0.58$

II. $P(\text{not } B) = 1 - P(B) = 1 - 0.48 = 0.52$

III. $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16 = 0.74$

39. (c) I. $P(E \text{ or } F) = P(E \cup F)$

$= P(E) + P(F) - P(E \cap F)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$

II. not E and not F = $E' \cap F' = (E \cup F)'$

$\therefore P$ (not E and not F)

$= P(E \cup F)' = 1 - P(E \cup F) = 1 - \frac{5}{8} = \frac{3}{8}$

MATCHING TYPE QUESTIONS

40. (a) A. a number less than 7 = $\{1, 2, 3, 4, 5, 6\}$

B. a number greater than 7 = $\{\}$ = f($\because$ the maximum number on a die is 6, so there cannot be a number on die greater than 7).

C. a multiple of 3 = $\{3, 6\}$.

D. a number less than 4 = $\{1, 2, 3\}$

E. an even number greater than 4 = $\{6\}$

F. a number not less than 3 = $\{3, 4, 5, 6\}$

41. (b) A. Now, $A \cup B$ = The elements which are in A or B

$= \{1, 2, 3, 4, 5, 6\} \cup \phi = \{1, 2, 3, 4, 5, 6\}$

B. $A \cap B$ = The elements which are common in both A and B.

$= \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$

C. $B \cup C$ = The elements which are in both B and C.

$= \{\} \cup \{3, 6\} = \{3, 6\}$

D. $E \cap F$ = The elements which are common in both E and F.

$= \{6\} \cap \{3, 4, 5, 6\} = \{6\}$

E. $D \cap E$ = The elements which are common in both D and E.

$= \{1, 2, 3\} \cap \{6\} = \phi$

F. $A - C$ = The elements which are in A but not in C

$= \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$

G. $D - E$ = The elements which are in D but not in E

$= \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$

H. $E \cap F' = E \cap (U - F) = E \cap [\{1, 2, 3, 4, 5, 6\} - (3, 4, 5, 6\}]$

$[\because U = \{1, 2, 3, 4, 5, 6\}] = \{6\} \cap \{1, 2\}] = \phi$

I. and $F' = (U - F) = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$

42. (b) A. If E_1 and E_2 are two mutually exclusive events, then $E_1 \cap E_2 = \phi$

B. If E_1 and E_2 are the mutually exclusive and exhaustive events, then $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$ where, S is the sample space for the events E_1 and E_2.

C. If E_1 and E_2 have common outcomes, then $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$

D. If E_1 and E_2 are two events such that $E_1 \subset E_2$ and $E_1 \cap E_2 = E_1$

43. (a) A. Probability = 0.95

That means it is very likely to happen.

B. Probability = 0.02

That mean it is very little chance of happening.

C. Proabability = – 0.3

We know that, $0 \le P(E) \le 1$

So, it is an incorrect assignment.

D. Probability = 0.5

That means as much chance of happening as not

E. Probability = 0

That means no chance of happening.

44. (a) Using the relation,

A. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.54 + 0.69 - 0.35$

$= 1.23 - 0.35 = 0.88$

B. $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - 0.88 = 0.12$

C. $P(A \cap B') = P(A \text{ only})$

$= P(A) - P(A \cap B)$

$= 0.54 - 0.35 = 0.19$

D. $P(B \cap A') = P(B \text{ only})$

$= P(B) - P(B \cap A) = 0.69 - 0.35 = 0.34$

45. **(d)** A. $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

$E = \{HHH\}, n(E) = 1$

$\therefore$ Required Prob $= \frac{1}{8}$

B. $S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$

$E = \{HT, TH\} \Rightarrow n(E) = 2$

$\therefore$ Required prob $= \frac{2}{4} = \frac{1}{2}$

C. $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

$E = \{1, 2, 3, 4\} \Rightarrow n(E) = 4$

$\therefore$ Required prob $= \frac{4}{6} = \frac{2}{3}$

D. $$S = \begin{Bmatrix} (1,1),(1,2),\ldots,(1,6) \\ (2,1),(2,2),\ldots,(2,6) \\ (3,1),(3,2),\ldots,(3,6) \\ (4,1),(4,2),\ldots,(4,6) \\ (5,1),(5,2),\ldots,(5,6) \\ (6,1),(6,2),\ldots,(6,6) \end{Bmatrix}$$

E = event "getting sum as 2, 3, 5, 7, 11

E= $\{(1,1),(1,2),(2,1),(1,4),(4,1),(2,3),(3,2),(1,6),$
$(2,5),(3,4),(6,1),(5,2),(4,3),(5,6),(6,5)\}$

$n(S) = 36, n(E) = 15$

$\therefore$ Required Prob $= \frac{15}{36} = \frac{5}{12}$

46. **(b)** By Algebra of Events

47. **(a)** By the definitions.

48. **(a)** **49.** **(c)**

INTEGER TYPE QUESTIONS

50. **(c)** When two dice are thrown then there are 6 × 6 exhaustive cases $\therefore$ n = 36. Let A denote the event "total score of 7" when 2 dice are thrown then
$A = \{(1, 6), (2, 5), (3, 4), (4,3), (5, 2), (6, 1)\}$.

Thus there are 6 favourable cases.
$\therefore m = 6$

By definition $P(A) = \frac{m}{n}$

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}$.

51. **(c)** The sample space (S) of toss of 3 coins will be given as:

H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$n(S) = 2^3 = 8$

Let E be the event of getting exactly 2 heads

$\therefore$ $n(E) = 3$

Thus the probability of getting exactly 2 heads

$= \frac{n(E)}{n(S)} = \frac{3}{8}$.

52. **(c)** $A \equiv$ number is greater than 3

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

$B \equiv$ number is less than 5 $\Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

$A \cap B \equiv$ number is greater than 3 but less than 5.

$\Rightarrow P(A \cap B) = \frac{1}{6}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{3+4-1}{6} = 1$

53. **(c)** Exactly 2 tails can be obtained in one way i.e. TT. So, favourable number of elementary events = 1

Hence, required probability $= \frac{1}{4}$

$\Rightarrow m = 1, n = 4$ and $m + n = 5$.

54. **(b)** Since every face of a die is marked with a number less than or equal to 6. So, favourable number of elementary events = 6

$\therefore$ Prob $= \frac{6}{6} = 1$

ASSERTION - REASON TYPE QUESTIONS

55. **(b)** Assertion : $S = \{H, T\}$

number of favourable event = 1

$\therefore$ Probability $= \frac{1}{2}$ (i.e., H)

Reason : S = {HH, HT, TH, TT}
E = {HH}

$$\text{Probability} = \frac{n(E)}{n(S)} = \frac{1}{4}$$

56. **(b)** Both Assertion and Reason is correct.
57. **(b)** Both Assertion and Reason are correct.
58. **(b)** Both Assertion and Reason are correct but Reason is not the correct explanation.

CRITICALTHINKING TYPE QUESTIONS

59. **(b)** Let there be 100 students.
So, there are 40 students of science and 60 students of arts.
5% of 40 = 2 science students (girls)
10% of 60 = 6 science students (girls)
Total girls students = 8
If a girl is chosed then

$$\text{P (arts)} = \frac{6}{8} = \frac{3}{4}$$

60. **(b)** Total no. of balls = 10
No. of white balls = 4
No. of non-white balls = 10 – 4 = 6

$$\text{So, Required prob} = \frac{6}{10} = \frac{3}{5}$$

61. **(b)** Since a leap year has 366 days and hence 52 weeks and 2 days. The 2 days can be SM, MT, TW, WTh, ThF, FSt, St.S.

$$\text{Therefore, P(53 Sundays or 53 Mondays)} = \frac{3}{7}$$

62. **(b)** [**Hint:** The outcomes are 1, 2, 3, 4, 5, 6. Out of these, 4 is the only composite number which is less than 5].

63. **(b)** 24 out of the 90 are two digit numbers which are divisible by '3' and not by '5'.
The required probability is therefore,

$$\frac{24}{90} = \frac{4}{15}.$$

64. **(b)** Total out comes

$$= \begin{Bmatrix} (1,1,1),(1,1,2)........(1,1,6) \\ .... \\ .... \\ (6,6,1)................(6,6,6) \end{Bmatrix}$$

i.e. n (S) = $6^3 = 6 \times 6 \times 6$
E = {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)}
$\Rightarrow$ n (E) = 6

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{6}{6 \times 6 \times 6} = \frac{1}{36}$$

65. **(c)** Total no. of cards = 52
13 cards are diamonds and 4 cards are king.
There is only one card which is a king of diamond.

$$\therefore \text{P (card is diamond)} = \frac{13}{52}$$

$$\text{P (card is king)} = \frac{4}{52}$$

$$\text{P (card is king of diamond)} = \frac{1}{52}$$

$\therefore$ P (card is diamond or king)

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

66. **(b)** $P(A) = \dfrac{3x+1}{3},\ P(B) = \dfrac{1-x}{4}, P(C) = \dfrac{1-2x}{2}$

$\because$ For any event $E, 0 \le P(E) \le 1$

$$\Rightarrow 0 \le \frac{3x+1}{3} \le 1,\quad 0 \le \frac{1-x}{4} \le 1 \text{ and } 0 \le \frac{1-2x}{2} \le 1$$

$$\Rightarrow -1 \le 3x \le 2, -3 \le x \le 1 \text{ and } -1 \le 2x \le 1$$

$$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3} \text{ and } -3 \le x \le 1,$$

$$\text{and } -\frac{1}{2} \le x \le \frac{1}{2}$$

Also for mutually exclusive events $A, B, C,$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$\Rightarrow P(A \cup B \cup C) = \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2}$$

$$\therefore\ 0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$$

$$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13$$

$$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$$

Considering all inequations, we get

$$\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le x \le \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$$

$$\Rightarrow \frac{1}{3} \le x \le \frac{1}{2} \quad \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$$

67. **(c)** The sample space is
S = {T, HT, HHT, HHHT, HHHHT, ...}

68. **(b)** 2-digit positive integers are 10, 11, 12, ... 99. Thus, there are 90 such numbers. Since, out of these, 30 numbers are multiple of 3, therefore, the probability that a randomly chosen positive 2-digit ingeter is a multiple of 3, is $\dfrac{30}{90} = \dfrac{1}{3}$.

69. **(b)** Given that, M and N are two events, then the probability that atleast one of them occurs is

$$P(M \cup N) = P(M) + P(N) - P(M \cap N)$$

70. **(a)** Given that, $P(A \cup B) = P(A \cap B)$

$$\Rightarrow A = B \Rightarrow P(A) = P(B)$$

71. **(a)** Given that A and B are two mutually exclusively events Then,

$P(A \cup B) = P(A) + P(B) \quad [\because (A \cap B) = \phi]$

since, $P(A \cup B) \le 1$

$\therefore P(A) + P(B) \le 1$

$\Rightarrow P(A) + 1 - P(\overline{B}) \le 1$

$\Rightarrow P(A) \le P(\overline{B})$

72. **(b)** Let $3P(A) = 2P(B) = P(C) = p$ which gives

$P(A) = \frac{p}{3}, P(B) = \frac{p}{2}$ and $P(C) = p$

Now, since A, B, C are mutually exclusive and exhaustive events, we have

$P(A) + P(B) + P(C) = 1$

$\Rightarrow \frac{p}{3} + \frac{p}{2} + p = 1 \Rightarrow p = \frac{6}{11}$

Hence, $P(A) = \frac{p}{3} = \frac{2}{11}$

73. **(d)** The sample space is S = {HH, HT, TH, TT}

Let E be the event of getting atleast one tail

$\therefore$ E = {HT, TH, TT}

$\therefore$ Required probability p

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)} = \frac{3}{4}$

74. **(c)** There are 26 red cards and 26 black cards i.e., total number of cards = 52

P(both cards of different colours)

$= P(B)P(R) + P(R)P(B)$

$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51} = 2\left(\frac{26}{52} \times \frac{26}{51}\right) = \frac{26}{51}$

75. **(b)** Since, a leap year has 366 days and hence 52 weeks and 2 days. The 2 days can be SM, MT, TW, WTh, ThF, FSt, StS.

Therefore, P(53 Sundays or 53 Mondays) = $\frac{3}{7}$

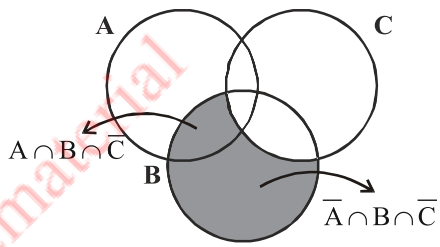
76. **(a)** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.50 - 0.14 = 0.61$

$\therefore P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$

$= 1 - 0.61 = 0.39$

77. **(a)** From venn diagram, we can see that



$P(B \cap C) = P(B) - P(A \cap B \cap \overline{C}) - P(\overline{A} \cap B \cap \overline{C})$

$= \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{1}{12}$.

CHAPTER

17

RELATIONS AND FUNCTIONS-II

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Which of the following functions from I to itself is a bijection?
(a) $f(x) = x^3$ (b) $f(x) = x + 2$
(c) $f(x) = 2x + 1$ (d) $f(x) = x^2 + x$

2. Which of the following function is an odd function ?
(a) $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$
(b) $f(x) = x\left(\dfrac{a^x+1}{a^x-1}\right)$
(c) $f(x) = \log\left(\dfrac{1-x^2}{1+x^2}\right)$
(d) $f(x) = k$, k is a constant

3. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{when n is odd} \\ -\dfrac{n}{2}, & \text{when n is even} \end{cases}$ is
(a) neither one-one nor onto (b) one-one but not onto
(c) onto but not one-one (d) one-one and onto both

4. The relation R is defined on the set of natural numbers as {(a, b) : a = 2b}. Then, R^{-1} is given by
(a) {(2, 1), (4, 2), (6, 3),...}
(b) {(1, 2), (2, 4), (3, 6),...}
(c) R^{-1} is not defined
(d) None of these

5. The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is
(a) Reflexive but not symmetric
(b) Reflexive but not transitive
(c) Symmetric and transitive
(d) Neither symmetric nor transitive

6. Let P = {(x, y) | $x^2 + y^2 = 1$, x, y $\in$ R}. Then, P is
(a) Reflexive (b) Symmetric
(c) Transitive (d) Anti-symmetric

7. For real numbers x and y, we write $x\,R\,y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then, the relation R is
(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

8. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. Then, R is
(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

9. Let S be the set of all real numbers. Then, the relation R = {(a, b) : 1 + ab > 0} on S is
(a) Reflexive and symmetric but not transitive
(b) Reflexive and transitive but not symmetric
(c) Symmetric, transitive but not reflexive
(d) Reflexive, transitive and symmetric

10. Let R be a relation on the set N be defined by {(x, y) | x, y $\in$ N, 2x + y = 41}. Then, R is
(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

11. Let X = {– 1, 0, 1}, Y = {0, 2} and a function f : X $\to$ Y defined by $y = 2x^4$, is
(a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

12. Let X = {0, 1, 2, 3} and Y = {–1, 0, 1, 4, 9} and a function f :X $\to$ Y defined by $y = x^2$, is
(a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

13. Let $g(x) = x^2 - 4x - 5$, then
(a) g is one-one on R (b) g is not one-one on R
(c) g is bijective on R (d) None of these

14. The mapping f : N $\to$ N given by $f(n) = 1 + n^2$, n $\in$ N when N is the set of natural numbers, is
(a) one-one and onto (b) onto but not one-one
(c) one-one but not onto (d) neither one-one nor onto

15. The function f: R $\to$ R given by $f(x) = x^3 - 1$ is
(a) a one-one function (b) an onto function
(c) a bijection (d) neither one-one nor onto

16. If N be the set of all natural numbers, consider f : N $\to$ N such that f(x) = 2x, $\forall$ x $\in$ N, then f is
(a) one-one onto (b) one-one into
(c) many-one onto (d) None of these

17. Let A = {1, 2, 3} and B = {2, 4, 6, 8}.
Consider the rule f : A $\to$ B, f(x) = 2x $\forall$ x $\in$ A. The domain, codomain and range of *f* respectively are
(a) {1, 2, 3}, {2, 4, 6}, {2, 4, 6, 8}
(b) {1, 2, 3}, {2, 4, 6, 8}, {2, 4, 6}
(c) {2, 4, 6, 8}, {2, 4, 6, 7}, {1, 2, 3}
(d) {2, 4, 6}, {2, 4, 6, 8}, {1, 2, 3}

18. The function $f : A \to B$ defined by $f(x) = 4x + 7,\ x \in R$ is
(a) one-one (b) many-one (c) odd (d) even

19. The smallest integer function $f(x) = [x]$ is
(a) one-one (b) many-one
(c) Both (a) & (b) (d) None of these

20. The signum function, $f : R \to R$ is given by
$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \text{ is}$$
(a) one-one (b) onto
(c) many-one (d) None of these

21. If $f : R \to R$ and $g : R \to R$ defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the value of x for which $f(g(x)) = 25$ is
(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

22. If $f: R \to R$ is given by
$$f(x) = \begin{cases} -1, \text{ when } x \text{ is rational} \\ 1, \text{ when } x \text{ is irrational,} \end{cases}$$
then (fof) $\left(1-\sqrt{3}\right)$ is equal to
(a) 1 (b) – 1 (c) $\sqrt{3}$ (d) 0

23. Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x + x^3}{1+3x^2}$, then fog(x) equals
(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) None of these

24. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ are such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of (go (foh)) (x), if x = 1 will be
(a) 0 (b) 1 (c) –1 (d) π

25. Let $f : R \to R$, $g : R \to R$ be two functions such that $f(x) = 2x - 3$, $g(x) = x^3 + 5$. The function $(fog)^{-1}(x)$ is equal to
(a) $\left(\frac{x+7}{2}\right)^{1/3}$ (b) $\left(x-\frac{7}{2}\right)^{1/3}$
(c) $\left(\frac{x-2}{7}\right)^{1/3}$ (d) $\left(\frac{x-7}{2}\right)^{1/3}$

26. If $f : R \to R$ defined by $f(x) = \frac{2x-7}{4}$ is an invertible function, then f^{-1} is equal to
(a) $\frac{(4x+5)}{2}$ (b) $\frac{(4x+7)}{2}$
(c) $\frac{3x+2}{2}$ (d) $\frac{9x+3}{5}$

27. Consider the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$, then f^{-1} is equal to
(a) $\frac{3+4x}{6x-4}$ (b) $\frac{6x-4}{3+4x}$
(c) $\frac{3-4x}{6x-4}$ (d) $\frac{9+2x}{6x-4}$

28. If the binary operation * on the set of integers Z, is defined by $a * b = a + 3b^2$, then the value of 8 * 3 is
(a) 32 (b) 40 (c) 36 (d) 35

29. For binary operation * defined on R – {1} such that $a * b = \frac{a}{b+1}$ is
(a) not associative (b) not commutative
(c) commutative (d) both (a) and (b)

30. The binary operation * defined on N by $a * b = a + b + ab$ for all a, b $\in$ N is
(a) commutative only
(b) associative only
(c) both commutative and associative
(d) None of these

31. If a binary operation * is defined by $a * b = a^2 + b^2 + ab + 1$, then (2 * 3) * 2 is equal to
(a) 20 (b) 40 (c) 400 (d) 445

32. If a * b denote the bigger among *a* and *b* and if a . b = (a * b) + 3, then 4.7 =
(a) 14 (b) 31 (c) 10 (d) 8

33. Consider the non-empty set consisting of children in a family and a relation *R* defined as *a R b* if *a* is brother of *b*. Then *R* is
(a) symmetric but not transitive
(b) transitive but not symmetric
(c) neither symmetric nor transitive
(d) both symmetric and transitive

34. Let us define a relation R in R as aRb if $a \geq b$. Then R is
(a) an equivalence relation
(b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive
(d) neither transitive nor reflexive but symmetric

35. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x} \forall x \in R$. Then f is
(a) one-one (b) onto
(c) bijective (d) f is not defined

36. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$. Then gof is
(a) $\frac{3x^2-5}{9x^4-30x^2+26}$ (b) $\frac{3x^2-5}{9x^4-6x^2+26}$
(c) $\frac{3x^2}{x^4+2x^2-4}$ (d) $\frac{3x^2}{9x^4+30x^2-2}$

37. Let $f : R - \left\{\frac{3}{5}\right\} \to R$ be defined by $f(x) = \frac{3x+2}{5x-3}$. Then
(a) $f^{-1}(x) = f(x)$ (b) $f^{-1}(x) = -f(x)$
(c) $(fof)x = -x$ (d) $f^{-1}(x) = \frac{1}{19}f(x)$

38. Let $f : R \to R$ be defined as $f(x) = 2x^3$. then
(a) f is one-one onto
(b) f is one-one but not onto
(c) f is onto but not one-one
(d) f is neither one-one nor onto

39. If $f : R \to R$ is given by $f(x) = \sqrt{1-x^2}$, then fof is
(a) $\sqrt{x}$ (b) x^2
(c) x (d) $1-x^2$

40. If *f* is an even function and *g* is an odd function, then the function *fog* is
(a) an even function (b) an odd function
(c) neither even nor odd (d) a periodic function

41. If f : R® R be a mapping defined by $f(x) = x^3 + 5$, then $f^{-1}(x)$ is equal to:
(a) $(x + 3)^{1/3}$ (b) $(x - 5)^{1/3}$
(c) $(5 - x)^{1/3}$ (d) $(5 - x)$

42. The relation "less than" in the set of natural numbers is :
(a) only symmetric (b) only transitive
(c) only reflexive (d) equivalence relation

43. The relation R = { (1, 1), (2, 2), (3, 3)} on the set {1, 2, 3} is :
(a) symmetric only (b) reflexive only
(c) an equivalence relation (d) transitive only

44. Let A be the non-empty set of children in a family. The relation 'x is brother of y' in A is:
(a) reflexive (b) symmetric
(c) transitive (d) None of these

45. Let f : R → R be defined as $f(x) = \frac{x^2+1}{2}$, then
(a) f is one-one onto
(b) f is one-one but not onto
(c) f is onto but not one-one
(d) f is neither one-one nor onto

46. If f (x) is defined on [0, 1] by the rule
$$f(x) = \begin{cases} x & : x \text{ is rational} \\ 1-x & : x \text{ is irrational} \end{cases}$$
then for all x ∈ R, f (f(x)) is
(a) constant (b) 1 + x
(c) x (d) None of these

47. Let A = {1, 2, 3, 4} and let R = {(2, 2), (3, 3), (4, 4), (1, 2)} be a relation on A. Then R is:
(a) reflexive (b) symmetric
(c) transitive (d) None of these

48. If R is a relation in a set A such that (a, a) ∈ R for every a ∈ A, then the relation R is called
(a) symmetric (b) reflexive
(c) transitive (d) symmetric or transitive

49. A relation R in a set A is called empty relation, if
(a) no element of A is related to any element of A
(b) every element of A is related to every element of A
(c) some elements of A are related to some elements of A
(d) None of the above

50. A relation R in a set A is called universal relation, if
(a) each element of A is not related to every element of A
(b) no element of A is related to any element of A
(c) each element of A is related to every element of A
(d) None of the above

51. A relation R in a set A is said to be an equivalence relation, if R is
(a) symmetric only (b) reflexive only
(c) transitive only (d) All of these

52. A relation R in a set A is called transitive, if for all a_1, a_2, a_3 ∈ A, (a_1, a_2) ∈ R and (a_2, a_3) ∈ R implies
(a) $(a_2, a_1) \in R$ (b) $(a_1, a_3) \in R$
(c) $(a_3, a_1) \in R$ (d) $(a_3, a_2) \in R$

53. A relation R in a set A is called symmetric, if for all a_1, a_2 ∈ A and (a_2, a_3) ∈ R implies
(a) $(a_1, a_2) \in R \in (a_2, a_1) \in R$
(b) $(a_1, a_2) \in R \in (a_1, a_1) \in R$
(c) $(a_1, a_2) \in R \in (a_2, a_2) \in R$
(d) None of these

54. If R = {(x, y) : x is father of y}, then R is
(a) reflexive but not symmetric
(b) symmetric and transitive
(c) neither reflexive nor symmetric nor transitive
(d) Symmetric but not reflexive

55. If R = {(x, y) : x is exactly 7 cm taller than y}, then R is
(a) not symmetric
(b) reflexive
(c) symmetric but not transitive
(d) an equivalence relation

56. If R = {(x, y) : x is wife of y}, then R is
(a) reflexive (b) symmetric
(c) transitive (d) an equivalence relation

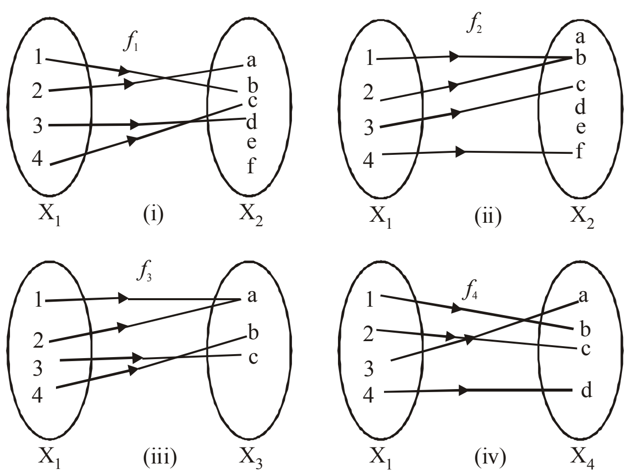
57. Let R be the relation in the set Z of all integers defined by R = {(x, y) : x – y is an integer}. Then R is
(a) reflexive (b) symmetric
(c) transitive (d) an equivalence relation

58. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}.
(a) R is reflexive and symmetric but not transitive
(b) R is reflexive and transitive but not symmetric
(c) R is symmetric and transitive but not reflexive
(d) R is equivalence relation

59. A function f : X → Y is said to be onto, if for every y ∈ Y, there exists an element x in X such that
(a) f(x) = y (b) f(y) = x
(c) f(x) + y = 0 (d) f(y) + x = 0

60. f : X →Y is onto, if and only if
(a) range of f = Y (b) range of f ≠ Y
(c) range of f < Y (d) range of f ≥ Y

61. Consider the four functions f_1, f_2, f_3 and f_4 as follows



(a) f_1 and f_2 are onto (b) f_2 and f_4 are onto
(c) f_2 and f_3 are onto (d) f_3 and f_4 are onto

62. Let f : R → R be defined as $f(x) = x^4$, then
(a) f is one-one onto
(b) f is many-one onto
(c) f is one-one but not onto
(d) f is neither one-one nor onto

63. The function f : R → R defined by $f(x) = x^2 + x$ is.
(a) one-one (b) onto
(c) many-one (d) None of the above

64. Which of the following functions from Z into Z are bijective?
(a) $f(x) = x^3$ (b) f(x) = x + 2
(c) f(x) = 2x + 1 (d) $f(x) = x^2 + 1$

65. If $f : X \to Y$ is a function such that there exists a function $g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$, then f must be
(a) one-one (b) onto
(c) one-one and onto (d) None of these

66. Which of the following option is correct?
(a) gof is one-one $\Rightarrow$ g is one-one
(b) gof is one-one $\Rightarrow$ f is one-one
(c) gof is onto $\Rightarrow$ g is not onto
(d) gof is onto $\Rightarrow$ I is onto

67. If $f = \{(5, 2), (6, 3)\}$ and $g = \{(2, 5), (3, 6)\}$, then fog is
(a) $\{(2, 2), (3, 3)\}$ (b) $\{(5, 3), (6, 2)\}$
(c) $\{(2, 2), (5, 5)\}$ (d) $\{(6, 6), (3, 3)\}$

68. Which of the following is not a binary operation on the indicated set?
(a) On Z^+, * defined by $a * b = a - b$
(b) On Z^+, * defined by $a * b = ab$
(c) On R, * defined by $a * b = ab^2$
(d) None of the above

69. Consider a binary operation * on N defined as $a * b = a^3 + b^3$
(a) * is both associative and commutative
(b) * is commutative but not associative
(c) * is associative but not commutative
(d) * is neither commutative nor associative

70. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Then, R is
(a) reflexive (b) symmetric
(c) transitive (d) an equivalence relation

71. Let $f(x) = \dfrac{ax+b}{cx+d}$. Then fof (x) = x provided that
(a) $d = -a$ (b) $d = a$
(c) $a = b = c = d = 1$ (d) $a = b = 1$

72. Let $f : (2, 3) \to (0, 1)$ be defined by $f(x) = x - [x]$. Then, $f^{-1}(x)$ equals to
(a) $x - 2$ (b) $x + 1$ (c) $x - 1$ (d) $x + 2$

73. Let $A = R - \{3\}$ and $B = R - \{1\}$. If $f : A \to B$ defined by $f(x) = \dfrac{x-2}{x-3}$ is invertible, then the inverse of f is
(a) $\dfrac{3y+2}{y-1}$ (b) $\dfrac{3y-2}{y+1}$
(c) $\dfrac{3y-2}{y-1}$ (d) None of these

74. If $f : R \to R$, $f(x) = x^3 + 2$, then $f^{-1}(x)$ is
(a) $(x-1)^{1/2}$ (b) $x - 2$
(c) $(x-2)^{1/3}$ (d) $(x-2)^{1/2}$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

75. Consider the following statements on a set $A = \{1, 2, 3\}$
I. $R = \{(1, 1), (2, 2)\}$ is reflexive relation on A
II. $R = \{(3, 3)\}$ is symmetric and transitive but not a reflexive relation on A

Which of the statements given above is/are correct ?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

76. Consider the following statements
Statement - I : An onto function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ is always one-one.
Statement - II : A one-one function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ must be onto.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Neither I nor II is true

77. Consider the following statements
I. Addition, subtraction and multiplication are binary operations on R.
II. Division is a binary operation on R but not a binary operation on non-zero real numbers.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Neither I nor II is true

78. Consider the following statements
I. The operation * defined on Z^+ by $a * b = |a - b|$ is a binary operation.
II. The operation * defined on Z^+ by $a * b = a$ is not a binary operation.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Neither I nor II is true

79. In the set N of natural numbers, define the binary operation * by $m * n = GCD(m, n)$, $m, n \in N$. Then, which of the following is true?
I. * is not a binary operation
II. * is a binary operation
III. Inverse of each element of N exist
IV. Inverse of each element of N does not exist
(a) I and IV are true (b) II and III are true
(c) Only I is true (d) II and IV are true

80. Consider the following statements
I. For an arbitrary binary operation * on a set N, $a * a = a \quad \forall a \in N$.
II. If * is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Neither I nor II is true

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

81. A binary operation * on the set {0, 1, 2, 3, 4, 5} is defined as

$$a * b = \begin{cases} a+b & , \text{ if } a+b < 6 \\ a+b-6 & , \text{ if } a+b \geq 6 \end{cases}$$

the identity element is
(a) 0 (b) 1 (c) 2 (d) 3

82. Let * be a binary operation on set Q of rational numbers defined as $a * b = \frac{ab}{5}$. The identity for * is

(a) 5 (b) 3 (c) 1 (d) 6

83. Let * be the binary operation on N given by a * b = HCF (a, b) where, a, b $\in$ N. The value of 22 * 4 is

(a) 1 (b) 2 (c) 3 (d) 4

84. Let $f : N \to R$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g : Q \to R$ be another function defined by g(x) = x + 2. Then (gof) $\frac{3}{2}$ is

(a) 1 (b) 0 (c) $\frac{7}{2}$ (d) 3

85. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 2x & : \quad x > 3 \\ x^2 & : \quad 1 < x \le 3 \\ 3x & : \quad x \le 1 \end{cases}$$

Then f(– 1) + f(2) + f(4) is

(a) 9 (b) 14

(c) 5 (d) None of these

86. If $f : Q \to Q$, f(x) = 2x ; $g : Q \to Q$, g(x) = x + 2, then value of $(fog)^{-1}$ (20) is

(a) 5 (b) – 8

(c) 4 (d) 8

87. The relation R on the set Z defined by R = {(a, b) : (a – b) is divisible by 5} divides the set Z into how many disjoint equivalence classes ?

(a) 5 (b) 2 (c) 3 (d) 4

88. Let $f(x) = 2x^2$, g(x) = 3x + 2 and fog (x) = $18x^2 + 24x + c$, then c =

(a) 2 (b) 8 (c) 6 (d) 4

89. Let $f(x) = \frac{2}{x-3}, x \neq 3$ The inverse of f(x) is $g(x) = \frac{2+ax}{x}, x \neq 0$. Then a =

(a) 5 (b) 2 (c) 3 (d) 4

90. Let A = {1, 2, 3} and B = {a, b, c}, then the number of bijective functions from A to B are

(a) 2 (b) 8 (c) 6 (d) 4

91. If g(x) = x – 2 is the inverse of the function f(x) = x + 2, then graph of g(x) is the image of graph of f(x) about the line y = kx. Here k =

(a) 1 (b) 2 (c) 3 (d) 4

92. Let A = {1, 2, 3} and B = {a, b, c}, and let f = {(1, a), (2, b), (P, c)}be a function from A to B. For the function f to be one-one and onto, the value of P =

(a) 1 (b) 2 (c) 3 (d) 4

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

93. Assertion : $f : R \to R$ defined by f(x) = sin x is a bijection.
Reason : If f is both one-one and onto it is bijection.

94. Assertion : $f : R \to R$ is a function defined by $f(x) = \frac{2x+1}{3}$. Then $f^{-1}(x) = \frac{3x-1}{2}$.
Reason : f(x) is not a bijection.

95. Assertion : If f is even function, g is odd function, then $\frac{f}{g}$, $(g \neq 0)$ is an odd function.
Reason : If f(–x) = –f(x) for every x of its domain, then f(x) is called an odd function and if f(–x) = f(x) for every x of its domain, then f(x) is called an even function.

96. Assertion : Let L be the set of all lines in a plane and R be the relations in L defined as R = $\{(L_1, L_2) : L_1$ is perpendicular to $L_2\}$. This relation is not equivalence relation.
Reason : A relation is said to be equivalence relation if it is reflexive, symmetric and transitive.

97. Assertion : If f(x) is odd function and g(x) is even function, then f(x) + g(x) is neither even nor odd.
Reason : $f(x) = \begin{cases} f(x) & , \quad f(x) \text{ is even} \\ -f(x) & , \quad f(x) \text{ is odd} \end{cases}$

98. Assertion : If $f : R \to R$ and $g : R \to R$ be two mappings such that f(x) = sin x and g(x) = x^2, then fog $\neq$ gof.
Reason : (fog) x = f(x) g(x) = (gof) x

99. Assertion : If the relation R defined in A = {1, 2, 3} by aRb, if $|a^2 - b^2| \le 5$, then $R^{-1} = R$
Reason : For above relation, domain of R^{-1} = Range of R.

100. Assertion : Let A = {–1, 1, 2, 3} and B = {1, 4, 9}, where $f : A \to B$ given by f(x) = x^2, then f is a many-one function.
Reason : If $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in$ domain, then f is one-one or else many-one.

101. Assertion : The function $f : R \to R$ given by f(x) = x^3 is injective.
Reason : The function $f : X \to Y$ is injective, if f(x) = f(y) $\Rightarrow$ x = y for all x, y $\in$ X.

102. Assertion : The binary operation $* : R \times R \to R$ given by $a * b \to a + 2b$ is associative.
Reason : A binary operation*: $A \times A \to A$ is said to be associative, if
(a * b) * c = a * (b * c) for all a, b, c $\in$ A.

103. Let $f(x) = (x+1)^2 - 1, x \geq -1$

Assertion : The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}$

Reason : f is a bijection.

104. **Assertion :** Let f : R → R be defined by $f(x) = \frac{1}{x}$, then f is one-one and onto.

Reason : x = 0 does not belong to the domain of f.

105. **Assertion :** Division is a binary operation on the set of natural numbers.

Reason : 5 ÷ 4 = 1.25 is not a natural number.

106. **Assertion :** The binary operation subtraction on the set of real numbers is not commutative.

Reason : If a and b are two real numbers, then in general $a - b \neq b - a$

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

107. Let R be the relation on the set of all real numbers defined by a R b iff $|a - b| \leq 1$. Then, R is
(a) Reflexive and symmetric (b) Symmetric only
(c) Transitive only (d) Anti-symmetric only

108. Let A = N × N and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Then * is
(a) commutative (b) associative
(c) Both (a) and (b) (d) None of these

109. If the binary operation * is defined on the set Q^+ of all positive rational numbers by $a * b = \frac{ab}{4}$. Then $3 * \left(\frac{1}{5} * \frac{1}{2}\right)$ is equal to
(a) $\frac{3}{160}$ (b) $\frac{5}{160}$ (c) $\frac{3}{10}$ (d) $\frac{3}{40}$

110. Let f : R → R be given by f(x) = tan x. Then $f^{-1}(1)$ is
(a) $\frac{\pi}{4}$ (b) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$
(c) does not exist (d) None of these

111. Let S be a finite set containing n elements. Then the total number of binary operations on S is:
(a) n^{n^2} (b) n^n (c) 2^{n^2} (d) n^2

112. The function f : R → R defined by f (x) = sin x is :
(a) into (b) onto
(c) one-one (d) many one

113. The domain of $y = \frac{1}{\sqrt{|x| - x}}$ is
(a) $[0, \infty)$ (b) $(-\infty, 0)$
(c) $(-\infty, 0]$ (d) $[1, \infty)$

114. Which one of the following relations on the set of real numbers R is an equivalence relation ?
(a) $aR_1b \Leftrightarrow |a| = |b|$ (b) $aR_2b \Leftrightarrow a \geq b$
(c) $aR_3b \Leftrightarrow$ a divides b (d) $aR_4b \Leftrightarrow a < b$

115. A function f : R → [–1, 1] defined by f(x) = sin x, ∀x ∈ R, where R is the subset of real numbers is one-one and onto if R is the interval:
(a) $[0, 2\pi]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(c) $[-\pi, \pi]$ (d) $[0, \pi]$

116. Let f : R → R be function defined by $f(x) = \sin(2x - 3)$, then f is
(a) injective (b) surjective
(c) bijective (d) None of these

117. Let f : R → R be a function defined by $f(x) = x^3 + 4$, then f is
(a) injective (b) surjective
(c) bijective (d) None of these

118. Let R be the relation defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}. If R is an equivalence relation and there are three right angled triangles T_1 with sides 3, 4, 5; T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Then,
(a) T_1 is related to T_2 (b) T_2 is related to T_3
(c) T_1 is related to T_3 (d) None of these

119. For the set A = {1, 2, 3}, define a relation R in the set A as follows
R = {(1, 1), (2, 2), (3, 3), (1, 3)}
Then, the ordered pair to be added to R to make it the smallest equivalence relation is
(a) (1, 3) (b) (3, 1) (c) (2, 1) (d) (1, 2)

120. On the set N of all natural numbers, define the relation R by a R b, iff GCD of a and b is 2. Then, R is
(a) reflexive, but not symmetric
(b) symmetric only
(c) reflexive and transitive
(d) not reflexive, not symmetric, not transitive

121. Let A = {1, 2, 3}and R = {(1, 2), (2, 3)} be a relation in A. Then, the minimum number of ordered pairs may be added, so that R becomes an equivalence relation, is
(a) 7 (b) 5 (c) 1 (d) 4

122. Let A = {1, 2, 3}. Then, the number of relations containing (1, 2) and (1, 3), which are reflexive and symmetric but not transitive, is
(a) 1 (b) 2 (c) 3 (d) 4

123. The number of all one-one functions from set A = {1, 2, 3} to itself is
(a) 2 (b) 6 (c) 3 (d) 1

124. If the function gof is defined and is one-one then
(a) neither f nor g is one-one
(b) f and g both are necessarily one-one
(c) g must be one-one
(d) None of the above

125. If $f: B \rightarrow A$ is defined by $f(x)=\frac{3x+4}{5x-7}$ and $g: A \rightarrow B$ is defined by $g(x)=\frac{7x+4}{5x-3}$, where $A = R - \left\{\frac{3}{5}\right\}$ and $B = R - \left\{\frac{7}{5}\right\}$ and I_A is an identity function on A and I_B is identity function on B, then
(a) $fog = I_A$ and $gof = I_A$
(b) $fog = I_A$ and $gof = I_B$
(c) $fog = I_B$ and $gof = I_B$
(d) $fog = I_B$ and $gof = I_A$

126. If $f: R \rightarrow R$ be given by $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$, then fof (x) is
(a) $x^{\frac{1}{3}}$
(b) x^3
(c) x
(d) $(3 - x^3)$

127. If $f(x) = |x|$ and $g(x) = |5x - 2|$, then
(a) $gof(x) = |5x - 2|$
(b) $gof(x) = |5|x| - 2|$
(c) $fog(x) = |5|x| - 2|$
(d) $fog(x) = |5x + 2|$

128. If $f(x) = e^x$ and $g(x) = \log_e x$, then which of the following is true?
(a) $f\{g(x)\} \neq g\{f(x)\}$
(b) $f\{g(x)\} = g\{f(x)\}$
(c) $f\{g(x)\} + g\{f(x)\} = 0$
(d) $f\{g(x)\} - g\{f(x)\} = 1$

129. For a binary operation * on the set {1, 2, 3, 4, 5}, consider the following multiplication table.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Which of the following is correct?
(a) (2 * 3) * 4 = 1
(b) 2 * (3 * 4) = 2
(c) * is not commutative
(d) (2 * 3) * (4 * 5) = 2

130. The number of equivalence relations in the set {1, 2, 3} containing (1, 2) and (2, 1) is
(a) 2 (b) 3 (c) 1 (d) 4

131. The function $f: [0, \pi] \rightarrow R$, $f(x) = \cos x$ is
(a) one-one function
(b) onto function
(c) a many one function
(d) None of these

132. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain
(a) $\left[0, \frac{\pi}{2}\right]$
(b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(d) $[0, \pi]$

133. The function $f: R \rightarrow R$ defined by
$f(x) = (x - 1)(x - 2)(x - 3)$ is
(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

134. The number of surjective functions from A to B where A = {1, 2, 3, 4} and B = {a, b} is
(a) 14 (b) 12 (c) 2 (d) 15

135. The inverse of the function
$f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}+2$ is
(a) $\log_e\left(\frac{x-3}{x-1}\right)^{1/2}$
(b) $\log_e\left(\frac{x-1}{3-x}\right)^{1/2}$
(c) $\log_e\left(\frac{x+2}{x-3}\right)^{1/2}$
(d) $\log_e\left(\frac{x+1}{x-2}\right)^{1/2}$

136. If $f: R \rightarrow S$, defined by $f(x)=\sin x-\sqrt{3}\cos x+1$, is onto, then the interval of S is
(a) [−1, 3] (b) [−1, 1] (c) [0, 1] (d) [0, 3]

137. Let function $f: R \rightarrow R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then f is
(a) one-one and onto
(b) one-one but NOT onto
(c) onto but NOT one-one
(d) neither one-one nor onto

138. Range of the function $f(x)=\frac{x^2+x+2}{x^2+x+1}; x \in R$ is
(a) $(1, \infty)$
(b) (1, 11/7]
(c) (1, 7/3]
(d) (1, 7/5]

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** (a) $f(x) = x^3$ is one-one as the cube of every integer can be found but it is not onto, because many integers have no integral cuberoots. For example, 2, 3, 4,...... do not have pre-images.

[Let $y = x^3 \Rightarrow x = (y)^{1/3}$, which is not an integer if y = 2, 3, 4,.......]

(b) f(x) = x + 2 is a bijection on **I** as it is one-one as well as onto on **I**.

(c) f(x) = 2x + 1 is one-one but not onto.

$$\text{if } y = 2x + 1 \Rightarrow x = \frac{y-1}{2}$$

That is for many values of y, x will not be integer, e.g. y = 2, 4, 6... or no even number has its pre-image.

(d) $f(x) = x^2 + x$ is not one-one (quadratic function can never be one-one), hence not bijective.

2. **(a)** (a) $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$

$$f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2} = -f(x)$$

∴ f(x) is an odd function

(b) $f(x) = x\left(\dfrac{a^x+1}{a^x-1}\right)$

$$\Rightarrow f(-x) = (-x)\left(\frac{a^{-x}+1}{a^{-x}-1}\right)$$

$$= (-x)\left(\frac{1+a^x}{1-a^x}\right) = x\left(\frac{a^x+1}{a^x-1}\right) = f(x)$$

∴ It is an even function

(c) $f(x) = \log\left(\dfrac{1-x^2}{1+x^2}\right)$

$$\Rightarrow f(-x) = \log\left(\frac{1-x^2}{1+x^2}\right) = f(x)$$

∴ It is an even function

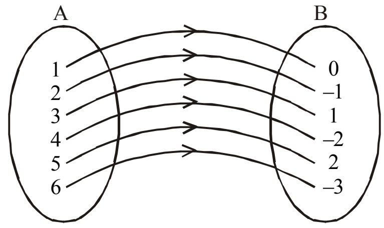
(d) $f(x) = k \Rightarrow f(-x) = k = f(x)$

∴ It is an even function

3. **(d)** $f : N \to I$

$f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2,$

$f(5) = 2, \text{and } f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B.

Hence f is one-one and onto function.

4. **(b)** R = {(2, 1), (4, 2), (6, 3)....}

So, R^{-1} = {(1, 2), (2, 4), (3, 6), ...}

5. **(a)** Reflexive: (1, 1), (2, 2), (3, 3) ∈ R, R is reflexive.

Symmetric : (1, 2) ∈ R but (2, 1) ∉ R, R is not symmetric.

Transitive: (1, 2) ∈ R and (2, 3) ∈ R ⇒ (1, 3) ∈ R, R is Transitive.

6. **(b)** The relation is not reflexive and transitive but it is symmetric, because

$x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$

7. **(a)** Reflexive: For any x ∈ R, we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number.

⇒ x R x for all x. So, R is reflexive.

Symmetric: R is not symmetric, because $\sqrt{2}$ R 1 but $1 \not R \sqrt{2}$,

Transitive: R is not transitive also because $\sqrt{2}$ R 1 and 1 R $2\sqrt{2}$ but $\sqrt{2} \not R 2\sqrt{2}$.

8. **(b)** Given $\alpha R \beta \Leftrightarrow \alpha \perp \beta \therefore \alpha \perp \beta \Leftrightarrow \beta \perp \alpha \Rightarrow \beta R \alpha$

Hence, R is symmetric.

9. **(a)** Reflexive: As $1 + a.a = 1 + a^2 > 0,\ a \in S$

$\therefore (a, a) \in R$

∴ R is reflexive.

Symmetric: $(a, b) \in R \Rightarrow 1 + ab > 0$

$\Rightarrow 1 + ba > 0 \Rightarrow (b, a) \in R$,

∴ R is symmetric.

Transitive: $(a, b) \in R$ and $(b, c) \in R$ need not imply $(a, c) \in R$.

Hence, R is not transitive.

10. **(d)** On the set N of natural numbers.

$R = \{(x, y); x, y \in N, 2x + y = 41\}$

Reflexive: $(1, 1) \notin R$ as $2.1 + 1 = 3 \neq 41$. So, R is not reflexive.

Symmetric: $(1,39) \in R$ but $(39,1) \notin R$. So R is not symmetric.
Transitive: $(20,1) \in R$ and $(1,39) \in R$. But $(20,39) \in R$, so R is not transitive.

11. (c) We have, $y = 2x^4$
$\therefore$ $y(-1) = y(1) = 2$, $y(0) = 0$ (many-one onto)
Here, we see that for two different values of x, we will get a same image and no element of y is left, which do not have pre-image.
$\therefore$ Function is many-one onto.

12. (b) $y(0) = 0$, $y(1) = 1$, $y(2) = 4$, $y(3) = 9$. No two different values of x (where $x \in X$) gives same image. Also -1 is element of set Y, which does not have its pre-image in set X. So, function is one-one into.

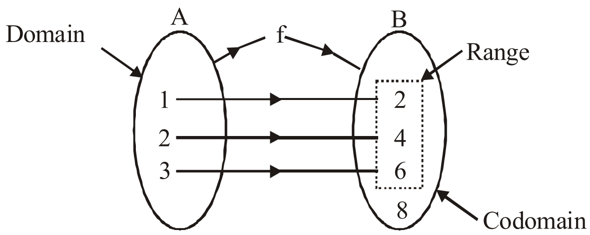
13. (b) Let $g(x_1) = g(x_2)$
$\Rightarrow\ x_1^2 - 4x_1 - 5 = x_2^2 - 4x_2 - 5$
$\Rightarrow\ x_1^2 - x_2^2 = 4(x_1 - x_2)$
$\Rightarrow\ (x_1 - x_2)(x_1 + x_2 - 4) = 0$
Either $x_1 = x_2$ or $x_1 + x_2 = 4$
Either $x_1 = x_2$ or $x_1 = 4 - x_2$
$\therefore$ There are two values of x_1, for which $g(x_1) = g(x_2)$
$\therefore$ g(x) is not one-one $\forall\ x \in R$

14. (c) Since, $f(n) = 1 + n^2$
For one-one, $1 + n_1^2 = 1 + n_2^2$
$\Rightarrow n_1^2 - n_2^2 = 0 \Rightarrow n_1 = n_2 \quad (\because n_1 + n_2 \neq 0)$
$\therefore$ f(n) is one-one.
f(n) is not onto.
Hence, f(n) is one-one but not onto.

15. (c) Given, $f(x) = x^3 - 1$
Let $x_1, x_2 \in R$.
Now, $f(x_1) = f(x_2)$
$\Rightarrow x_1^3 - 1 = x_2^3 - 1 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$
$\therefore$ f(x) is one-one. Also, it is onto.
Hence, it is a bijection.

16. (b) Let $x_1, x_2 \in N$, then $f(x_1) = f(x_2)$
$\Rightarrow x_1 = x_2$
Let $y = 2x \Rightarrow x = \frac{y}{2} \notin N$
Thus, f is into.
Hence, f(x) is one-one into.

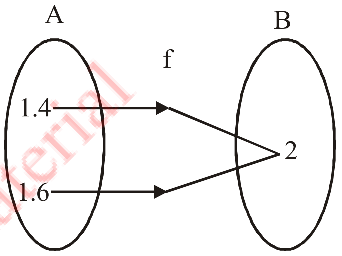
17. (b) Given, $f(x) = 2x,\ \forall\ x \in A$



Value of function at x = 1, f(1) = 2(1) = 2
Value of function at x = 2, f(2) = 2(2) = 4
Value of function at x = 3, f(3) = 2(3) = 6
Domain of f = {1, 2, 3}
Codomain of f = {2, 4, 6, 8}
Range of f = {2, 4, 6}

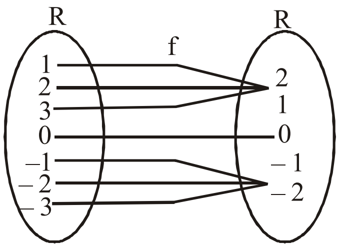
18. (a) We have, $f(x) = 4x + 7,\ x \in R$
Let $x_1, x_2 \in R$, such that $f(x_1) = f(x_2)$
$\Rightarrow 4x_1 + 7 = 4x_2 + 7 \Rightarrow 4x_1 = 4x_2$
$\Rightarrow x_1 = x_2$
So, f is one-one.

19. (b) We have, [1. 4] = [1.6] = 2
Here, two elements in A, 1.4 and 1.6 have the same image i.e., 2 in B.



Thus f(x)= [x] is a many-one function.

20. (c) We have, $f(1) = f(2) = f(3) = 1$, $f(0) = 0$
$f(-1) = f(-2) = f(-3) = -1$
Hence, function *f* is not one-one, so signum function is many-one function.



21. (b) $f(g(x)) = f(x^2 + 7) = 2(x^2 + 7) + 3 = 25$
$\Rightarrow\ 2x^2 = 8\ \Rightarrow\ x = \pm 2$

22. (b) Given, $f(x) = \begin{cases} -1, \text{when } x \text{ is rational} \\ 1, \text{when } x \text{ is irrational} \end{cases}$

Now, $(fof)(1-\sqrt{3}) = f\left[f(1-\sqrt{3})\right] = f(1) = -1$

23. (b) Since, $g(x) = \frac{3x + x^3}{1 + 3x^2} = y$, (say) ...(i)

$\therefore\ f[g(x)] = f(y) = \log\left(\frac{1+y}{1-y}\right)$

$\Rightarrow f(g(x)) = \log\left\{\frac{1 + \frac{3x + x^3}{1 + 3x^2}}{1 - \frac{3x + x^3}{1 + 3x^2}}\right\}$

$\Rightarrow f(g(x)) = \log\left(\frac{1+x}{1-x}\right)^3$

$\therefore f(g(x)) = 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$

24. (a) (go (foh)) (x) = go(f(h)) (x)
$= g((\log x)^2) = (\tan(\log x)^2) = \tan(\log 1)^2$
$= \tan(0) = 0$

25. (d) We have, $f(x) = 2x - 3$, $g(x) = x^3 + 5$
$(fog)x = f(x^3 + 5) = 2(x^3 + 5) - 3 = 2x^3 + 7$
Let $y = (fog)x = 2x^3 + 7$

$\Rightarrow x = \left(\frac{y-7}{2}\right)^{1/3}$

$\Rightarrow (fog)^{-1} x = \left(\frac{x-7}{2}\right)^{1/3}$

26. (b) Given, that, $f(x) = \frac{2x-7}{4}$

Let $y = \frac{2x-7}{4} \Rightarrow 4y = 2x - 7$

$\Rightarrow 2x = 4y + 7 \Rightarrow x = \frac{4y+7}{2}$

$\therefore f^{-1}(y) = \frac{4y+7}{2} \Rightarrow f^{-1}(x) = \frac{4x+7}{2}$

27. (a) Given $f(x) = \frac{4x+3}{6x-4}$

Let $y = \frac{4x+3}{6x-4}$,

$\Rightarrow 6xy - 4y = 4x + 3 \Rightarrow x(6y - 4) = 3 + 4y$

$\Rightarrow x = \frac{3+4y}{6y-4}$

$f^{-1}(x) = \frac{3+4x}{6x-4}$

28. (d) Given that, $a * b = a + 3b^2, \forall a, b \in Z$
On putting a = 8 and b = 3, we have
$8 * 3 = 8 + 3.3^2 = 8 + 27 = 35$

29. (d) Commutative: $a * b = \frac{a}{b+1}$ and $b * a = \frac{b}{a+1}$

$a * b \neq b * a$
$\Rightarrow$ * is not commutative.
Associative:

Now, $(a*b)*c = \left(\frac{a}{b+1}\right)*c = \frac{\left(\frac{a}{b+1}\right)}{c+1} = \frac{a}{(b+1)(c+1)}$

and $a*(b*c) = a*\left(\frac{b}{c+1}\right) = \frac{a}{\left(\frac{b}{c+1}\right)+1} = \frac{a(c+1)}{b+c+1}$

So, clearly $(a * b) * c \neq a * (b * c)$
Hence, * is not associative.

30. (c) Commutative:
$a * b = a + b + ab = b + a + ba = b * a$
{$\because$ addition and multiplication are commutative}
Hence, * is commutative.
Associative.
$(a * b) * c = (a + b + ab) * c$
$= a + b + ab + c + ac + bc + abc$
$= a + b + c + bc + ab + ac + abc$
$= a + (b + c + bc) + a(b + c + bc)$
$= a * (b + c + bc)$
$= a * (b * c)$
Hence, * is associative.

31. (d) Given binary operation is $a * b = a^2 + b^2 + ab + 1$
$\therefore (2 * 3) * 2 = \{(2)^2 + (3)^2 + (2)(3) + 1\} * 2$
$= (4 + 9 + 6 + 1) * 2$
$= 20 * 2 = (20)^2 + (2)^2 + 20 \times 2 + 1$
$= 400 + 4 + 40 + 1 = 445$

32. (c) We have, a * b = Bigger among a and b and
a.b = (a * b) + 3
$\therefore$ 4.7 = (4 * 7) + 3
= 7 + 3 {$\because$ 7 is greater than 4}.
= 10

33. (b) Given aRb $\in$ R $\Rightarrow$ a is brother of b.
But bRa $\notin$ R
$\because$ b may or may not be brother of a.
$\therefore$ R is not symmetric.
Let aRb $\in$ R and bRc $\in$ R
$\Rightarrow$ a is brother of b and b is brother of c.
$\therefore$ a is brother of c $\Rightarrow$ (a, c) $\in$ R.
$\therefore$ It is transitive.

34. (b) Given aRb, $a \geq b$
(i) Now $a \geq a$ is true for all real numbers
$\therefore$ R is reflexive.
(ii) Let $(a, b) \in R, a \geq b$
Now $a \geq b$ but does not imply $b \geq a$.
$\therefore (b, a) \notin R$
$\therefore$ R is not symmetric.
(iii) Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow a \geq b$ and $b \geq c$
$\therefore a \geq c \Rightarrow (a, c) \in R$
$\therefore$ It is transitive.

35. (d) Since, $\frac{1}{x}$ is not defined for $x = 0$
$\therefore f: R \to R$ can not be defined.

36. (a) $f(x) = 3x^2 - 5$, $g(x) = \frac{x}{x^2+1}$
$gof(x) = g(f(x)) = g(3x^2 - 5)$

$$= \frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4-30x^2+26}$$

37. (a) $f(x) = \frac{3x+2}{5x-3}$

Let $f(x) = y = \frac{3x+2}{5x-3}$

$\Rightarrow 5xy - 3y = 3x + 2 \Rightarrow x(5y-3) = 2 + 3y$

$\Rightarrow x = \frac{2+3y}{5y-3}$

$\therefore f^{-1}(x) = \frac{2+3x}{5x-3} = f(x)$

38. (a) f is one-one onto

39. (c) $f[f(x)] = \sqrt{1-\{f(x)\}^2} = \sqrt{1-(1-x^2)} = \sqrt{x^2} = x$

40. (a) We have, $fog(-x) = f[g(-x)]$

$= f[-g(x)]$ ($\because$ g is odd)

$= f[g(x)]$ ($\because$ f is even)

$= fog(x)\ \forall x \in R$

$\therefore$ *fog* is an even function

41. (b) Let $f : R \to R$ defined as

$f(x) = x^3 + 5$

Let $f(x) = y \Rightarrow x = f^{-1}(y)$ ....(i)

Given : $f(x) = x^3 + 5$

$\Rightarrow y = x^3 + 5$

$\Rightarrow y - 5 = x^3$

$\Rightarrow (y-5)^{\frac{1}{3}} = x = f^{-1}(y)$ (from (i))

$\therefore f^{-1}(x) = (x-5)^{\frac{1}{3}}$

42. (b) Only transitive

Since for three numbers a, b, c

a is less than b and b is less than c

$\Rightarrow$ a is less than c.

not reflexive and symmetric

Since a is not less than a and a is less than b does not implies b is less than a.

43. (b) Given relation is R = {(1, 1), (2, 2), (3, 3)} on the set {1, 2, 3}.

This relation is not symmetric, not transitive. only reflexive. ($\because$ aRa, bRb, cRc).

44. (c) Let x_1, x_2, x_3, are three brothers $x_1 R x_2$, $x_2 R x_3$

$\Rightarrow x_1 R x_3$ are element of relation 'x is brother of y'. So, transitive.

45. (d) f is neither one-one nor onto.

46. (c) $f(f(x)) = f(x) = x$ if x is rational

$= f(1-x) = 1-(1-x) = x$ if x is irrational.

Hence, $f(f(x)) = x\ \forall\ x \in R$

47. (c) The relation R is not reflexive as

for $1 \in A$, $(1, 1) \notin R$

Similarly, R is not symmetric as

$(1, 2) \in R$ but $(2, 1) \notin R$

But R is transitive as:

$(1,2) \in R$ and $(2, 2) \in R$ imply $(1, 2) \in R$.

48. (b) A relation R in a set A is called reflexive, if $(a, a) \in R$ for every $a \in A$.

49. (a) A relation R in a set A is called empty relation, if no element of A is a related to any element of A, i.e., $R = \phi \subset A \times A$.

50. (c) A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

51. (d) A relation R in a set A is said to be an equivalence relation, if R is reflexive, symmetric and transitive.

52. (b) A relation R in a set A is called transitive, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$, implies that $(a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$.

53. (a) A relation R in a set A is called symmetric, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$.

54. (c) Here, R is not reflexive; as x cannot be father of x, for any x, R is not symmetric as if x is father of y, then y cannot be father of x. R is not transitive as if x is father of y and y is father of z, then x is grandfather (not father) of z.

55. (a) Here, R is not reflexive as x is not 7 cm taller than x. R is not symmetric as if x is exactly 7 cm taller than y, then y cannot be 7 cm taller than x and R is not transitive as if x is exactly 7 cm taller than y and y is exactly 7 cm taller than z, then x is exactly 14 cm taller than z.

56. (c) Here, R is not reflexive; as x cannot wife of x, R is not symmetric, as if x is wife of y, then y is husband (not wife) of x and R is transitive as transitivity is not contradicted in this case. Whenever $(x, y) \in R$, then $(y, z) \notin R$ for any z as if x is wife of y, then y is a male and a male cannot be a wife.

57. (d) Here, $R = \{(x, y) : x - y$ is an integer$\}$ is a relation in the set of integers.

For reflexivity, put $y = x$, $x - x = 0$ which is an integer for all $x \in Z$. So, R is reflexive in Z.

For symmetry, let $(x, y) \in R$, then $(x - y)$ is an integer λ (say) and also $y - x = -\lambda$. $(\because \lambda \in Z \Rightarrow -\lambda \in Z)$

$\therefore y - x$ is an integer $\Rightarrow (y,x) \in R$. So, R is symmetric.

For transitivity, let $(x, y) \in R$ and $(y, z) \in R$, so $x - y$ = integer and $y - z$ = integers, then $x - z$ is also an integer.

$\therefore (x, z) \in R$. So, R is transitive.

58. (b) Here, R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}
Since, $(a, a) \in R$, for every $a \in \{1, 2, 3, 4\}$. Therefore, R is reflexive.
Now, since $(1, 2) \in R$ but $(2, 1) \notin R$. Therefore, R is not symmetric.
Also, it is observed that $(a, b), (b, c) \in R$
$\Rightarrow (a, c) \in R$ for all $a, b, c \in \{1, 2, 3, 4\}$
Therefore, R is transitive. Hence, R is reflexive and transitive but not symmetric.

59. (a) A function $f: X \to Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under f i.e., for every $y \in Y$, there exists an element x in X such that $f(x) = y$.

60. (a) $f: X \to Y$ is onto, if and only if range of f = Y.

61. (d) The function f_3 and f_4 in (iii) and (iv) are onto and the function f_1 in (i) is not onto as elements e, f in X_2 are not the image of any element in X_1 under f_1. Similarly, f_2 is not onto.

62. (d) Function $f: R \to R$ is defined as $f(x) = x^4$
Let $x, y \in R$ such that $f(x) = f(y)$
$\Rightarrow x^4 = y^4$
$\Rightarrow x = \pm y$ (considering only real values)
Therefore, $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$
For instance, $f(1) = f(-1) = 1$
Therefore, f is not one-one.
Consider an element – 2 in codomain R. It is clear that there does not exist any x in domain R such that $f(x) = -2$.
Therefore, f is not onto. Hence, function f is neither one-one nor onto.

63. (c) The given function $f: R \to R$ defined by $f(x) = x^2 + x$
Now, for x = 0 and – 1
We have, $f(0) = 0$ and $f(-1) = 0$
Hence, $f(0) = f(-1)$ but $0 \neq -1$
$\Rightarrow$ f is not one-one
$\Rightarrow$ f is many-one.

64. (b) The function $f(x) = x + 2$ is one-one as for $x_1, x_2 \in Z$.
Consider, $f(x_1) = f(x_2)$
$\Rightarrow x_1 + 2 = x_2 + 2$
$\Rightarrow x_1 = x_2$
Also, let $y \in$ codomain of f = Z such that $y = f(x)$
$\Rightarrow y = x + 2$
$\Rightarrow x = y - 2 \in Z$ for all $y \in Z$
$\therefore$ f is onto.
Hence, $f(x) = x + 2$ is bijective.

65. (c) If $f: X \to Y$ is a function such that there exists a function $g: Y \to X$ such that $gof = I_x$ and $fog = I_y$, then f must be one-one and onto.

66. (b) In general, gof is one-one implies that f is one-one. Similarly, gof is onto implies that g is onto.

67. (a) The given functions are f = {(5, 2), (6, 3)} and g = {(2, 5), (3, 6)}
$\Rightarrow f(5) = 2, f(6) = 3$ and $g(2) = 5$ and $g(3) = 6$
Now, $fog(2) = f[g(2)]$
$= f(5) = 2$
$fog(3) = f(g(3)) = f(6) = 3$
$\therefore$ fog = {(2, 2), (3, 3)}

68. (a) On Z^+, * is defined by $a * b = a - b$
It is not a binary operation as the image of (1, 2) under * is $1 * 2 = 1 - 2 = -1 \notin Z^+$,
On Z^+, * is defined by $a * b = ab$
It is seen that for each $a, b \in Z^+$, there is a unique element ab in Z^+.
This means that * carries each pair (a, b) to a unique element $a * b = ab$ in Z^+.
Therefore, * is a binary operation.
On R, * is defined by $a * b = ab^2$.
It is seen that for each $a, b \in R$, there is a unique element ab^2 in R.
This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in R. Therefore, * is binary operation.

69. (b) On N, the operation * is defined as $a * b = a^3 + b^3$.
For $a, b \in N$, we have
$a * b = a^3 + b^3 = b^3 + a^3 = b * a$
($\because$ addition is commutative in N)
Therefore, the operation * is commutative.
It can be observed that
$(1 * 2) * 3 = (1^3 + 2^3) * 3 = 9 * 3 = 9^3 + 3^3$
$= 729 + 27 = 756$
$1 * (2 * 3) = 1 * (2^3 + 3^3) = 1 * (8 + 27) = 1 * 35$
$= 1^3 + 35^3 = 1 + (35)^3 = 1 + 42875 = 42876$
Therefore, $(1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in N$
Therefore, the operation * is not associative.
Hence, the operation * is commutative but not associative.

70. (d) Clearly, (x, y) R (x, y) $\forall$ $(x, y) \in A$, since xy = yx. This shows that R is reflexive. Further (x, y) R (u, v)
$\Rightarrow xv = yu$
$\Rightarrow uy = vx$ and hence (u, v) R (x, y). This shows that R is symmetric. Similarly, (x, y) R (u, v) and (u, v) R (a, b).
$\Rightarrow xv = yu$ and $ub = va \Rightarrow xv\frac{a}{u} = yu\frac{a}{u}$
$\Rightarrow xv\frac{b}{v} = yu\frac{a}{u}$
$\Rightarrow xb = ya$ and hence (x, y) R (a, b). Therefore, R is transitive.
Thus, R is an equivalence relation.

71. (a) Given, $f(x) = \frac{ax + b}{cx + d}$ and $fof(x) = x$
$\Rightarrow f\left(\frac{ax + b}{cx + d}\right) = x$

$$\Rightarrow \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d} = x$$

$$\Rightarrow \frac{x\left(a^2+bc\right)+ab+bd}{x\left(ac+cd\right)+bc+d^2} = x$$

$\Rightarrow a^2 + bc = bc + d^2$, $ab + bd = 0$ and $ac + cd = 0$

$\Rightarrow d = -a$

72. (d) The given function is $f: (2, 3) \to (0, 1)$ defined by $f(x) = x - [x]$

Let $y \in (0, 1)$ such that

$y = f(x)$

$\therefore y = x - 2$ $\{\because 2 < x < 3 \Rightarrow [x] = 2\}$

$\Rightarrow x = y + 2$

$\therefore f^{-1}(y) = y + 2$ $\left[\because x = f^{-1}(y)\right]$

$\Rightarrow f^{-1}(x) = x + 2$

73. (c) Let $y \in B$ such that $f(x) = y$

$\Rightarrow \frac{x-2}{x-3} = y$

$\Rightarrow x - 2 = xy - 3y$

$\Rightarrow 3y - 2 = xy - x$

$\Rightarrow 3y - 2 = x(y - 1)$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\Rightarrow f^{-1}(y) = \frac{3y-2}{y-1}$ $\left[\because f(x) = y \Rightarrow x = f^{-1}(y)\right]$

74. (c) $f(x) = x^3 + 2, x \in R$

Since this is a one-one onto function. Therefore inverse of this function (f^{-1}) exists.

Let $f^{-1}(x) = y$

$\therefore x = f(y) \Rightarrow x = y^3 + 2 \Rightarrow y = (x - 2)^{1/3}$

$\therefore f^{-1}(x) = (x - 2)^{1/3}$

STATEMENT TYPE QUESTIONS

75. (a) Only (I) statement is correct. Since, $R = \{(1,1), (2,2)\}$ is reflexive.

76. (c) I. Suppose f is not one-one. Then, there exists two elements, say 1 and 2 in the domain whose image in the codomain is same. Also, the image of 3 under f can be only one element. Therefore, the range set can have atmost two elements of the codomain $\{1, 2, 3\}$ showing that f is not onto, a contradiction. Hence, f must one-one.

II. Since, f is one-one, three elements of $\{1, 2, 3\}$ must be taken to 3 different elements of the codomain $\{1, 2, 3\}$ under f. Hence, f has to be onto.

77. (a) I. $+ : R \times R \to R$ is given by $(a, b) \to a + b$

$- : R \times R \to R$ is given by $(a, b) \to a - b$

$\times : R \times R \to R$ is given by $(a, b) \to ab$

Since, '+', '–' and '×' are functions, they are binary operations on R.

II. But $\div : R \times R \to R$, given by $(a, b) \to \frac{a}{b}$, is not a function and hence not a binary operation, as for $b = 0$, $\frac{a}{b}$ is not defined.

However, $\div : R \times R \to R$, given by $(a, b) \to \frac{a}{b}$ is a function and hence a binary operation on R, where R is the set of non-zero real numbers.

78. (a) I. On Z^+ *, is defined by $a * b = |a - b|$.

It is seen that for each, $a, b \in Z^+$, there is a unique element $|a - b|$ in Z^+.

This means that * carries each pair (a, b) to a unique element $a * b = |a - b|$ in Z^+. Therefore, * is a binary operation.

On Z^+, * is defined by $a * b = a$.

It is seen that for each $a, b \in Z^+$, there is a unique element $a \in Z^+$.

This means that * carries each pair (a, b) to a unique element $a * b = a$ in Z^+.

Therefore, * is a binary operation.

79. (d) Clearly * is a binary operation, as * carries each pair $(m, n) \in N \times N$ to a unique element GCD (m, n) in N.

Now, in order to find the inverse of elements of N, let us first find the identity element if exist.

Let $e \in N$ be the identity element for *

i.e., $a * e = a = e * a \ \forall \ a \in N$.

$\Rightarrow$ GCD (a, e) = a = GCD (e, a)

Note that GCD (a, e) = a iff e = a or e is a multiple of a. Thus, identity element is not unique.

$\therefore$ Identity element for * does not exist.

Hence inverse of elements of N does not exist.

80. (b) I. Define an operation * on N as $a*b=a+b, \forall a,b \in N$

Then, in particular, for $b = a = 3$, we have

$3 * 3 = 3 + 3 = 6 \neq 3$.

Therefore, statement I is false.

II. RHS = (c * b) * a = (b * c) * a (* is commutative)

= a * (b * c) (again, as * is commutative)

= LHS

Therefore, a * (b * c) = (c * b) * a

Thus, statement II is true.

INTEGER TYPE QUESTIONS

81. (a) The operation table for * is given as

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

From the table, we note that

$a * 0 = 0 * a = a,\ \forall\ a \in \{0,1,2,3,4,5\}$

Hence, 0 is the identity for operation.

82. (a) Given binary operation is $a * b = \frac{ab}{5}$.

Let *e* be an identity element of * on Q.

$\therefore$ a * e = a $\forall a \in Q$ (by definition of identity element)

$\Rightarrow \frac{ae}{5} = a \left[\because a * b = \frac{ab}{5}\right]$

$\Rightarrow e = 5$

Hence, 5 is the identity element of *.

83. (b) Given binary operation is a * b = HCF (a, b), where a and b $\in$ N

$\therefore$ 22 * 4 = HCF (22, 4) = 2

Hence, 22 * 4 = 2

84. (d) $f(x) = \frac{2x-1}{2}$, $g(x) = x + 2$

$gof(x) = g(f(x)) = g\left(\frac{2x-1}{2}\right) = \frac{2x-1}{2} + 2 = \frac{2x+3}{2}$

$gof\left(\frac{3}{2}\right) = \frac{2 \times \frac{3}{2} + 3}{2} = 3$

85. (a) For f(– 1) = 3(– 1) = – 3

$f(2) = (2)^2 = 4$, f(4) = 2(4) = 8

$\therefore$ f (–1) + f(2) + f(4) = – 3 + 4 + 8 = 9

86. (d) Since $f^{-1}(x) = x/2$, $g^{-1}(x) = x - 2$

$\therefore (fog)^{-1}(20) = (g^{-1} \circ f^{-1})(20)$

$= g^{-1}[f^{-1}(20)] = g^{-1}(10)$

$= 10 - 2 = 8$

87. (a) Five disjoint equivalence classes which are

{..... –15, –10, –5, 0, 5, 10, 15 .....},
{..... –14, –9, –4, 1, 6, 11, 16 .....},
{..... –13, –8, –3, 2, 7, 12, 17 .....},
{..... –12, –7, –2, 3, 8, 13, 18 .....},
{..... –11, –6, –1, 4, 9, 14, 19 .....},

88. (b) fog (x) = f{g(x)}

= f (3x + 2)

$= 2(3x + 2)^2$

$= 2(9x^2 + 4 + 12x)$

$= 18x^2 + 8 + 24x$

$= 18x^2 + 24x + 8$

$\therefore$ c = 8

89. (c) Let $f(x) = y = \frac{2}{x-3}$, $x \neq 3$.

Then $x - 3 = \frac{2}{y}$

or $x = \frac{2}{y} + 3$

or $x = \frac{2+3y}{y}$

Replacing x by y and y by x, we get

$y = \frac{2+3x}{x}$

Let $y = g(x) = \frac{2+3x}{x}$, then g(x) is the inverse of f(x).

Hence, a = 3.

90. (c) Initially when no element of A is mapped with any element of B, the element 1 of set A can be mapped with any of the elements a, b and c of set B. Therefore 1 can be mapped in 'three' ways. Having mapped 1 with one element of B, now we have 'two' ways in which element 2 can be mapped with the remaining two elements of B. Having mapped 1 and 2 we have one element left in the set B so there is only 'one' way in which the element 3 can be mapped. Therefore the total number of ways in which the elements of A can be mapped with elements of B in this way are 3 × 2 × 1 = 6. Hence the number of bijective functions from A to B are 6.

91. (a) The graph of inverse of a function is the image of graph of the function about the line y = x. Therefore k = 1

92. (c) The value of P is 3.

ASSERTION - REASON TYPE QUESTIONS

93. (d) Range of sin x is [– 1, 1]

$\Rightarrow f : R \to R$ defined by f (x) = sin x is not onto

$\Rightarrow$ it is not bijected.

If f is both one-one and onto then f is bijection.

Assertion is false and Reason is true.

94. (c) $f : R \to R$ defined as $f(x) = \frac{2x+1}{3}$ is a bijection

$\Rightarrow f^{-1} = \frac{3x-1}{2}$.

95. (a) Let $h(x) = \frac{f(x)}{g(x)}$ then,

$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -h(x)$

$\therefore h(x) = \frac{f}{g}$ is an odd function

96. (a)

97. (a) $\because$ f(x) is odd

$\Rightarrow$ f(–x) = – f(x)

and g(x) is even $\Rightarrow$ g(– x) = g(x)

let F(x) = f(x) + g(x)

$\therefore F(-x) = f(-x) + g(-x) = -f(x) + g(x) \neq \pm F(x)$

$\therefore$ F(x) is neither even nor odd.

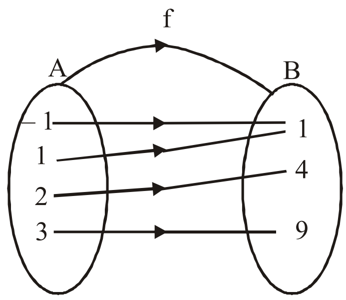
Hence, Assertion is true.

Reason is also true and is the correct explanation of Assertion.

98. (c) Since, $x = f\{g(x)\} = f(x^2) = \sin x^2$
and $(gof)x = g\{f(x)\} = g(\sin x) = \sin^2 x$
$\Rightarrow fog \neq gof$.

99. (b) Assertion:
$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)\}$
$R^{-1} = \{(y, x) : (x, y) \in R\}$
$= \{(1, 1), (2, 1), (1, 2), (2, 2), (3, 2), (2, 3), (3, 3)\} = R$
Reason: Domain of $R^{-1} = \{1, 2, 3\}$
Range of $R = \{1, 2, 3\}$
Assertion and Reason are true but Reason is not the correct explanation of Assertion.

100. (a)



Here $f(-1) = 1, f(1) = 1, f(2) = 4, f(3) = 9$
Two elements 1 and – 1 have the same image $1 \in B$..
So, f is a many-one function.
Assertion and Reason are true and Reason is the correct explanation of Assertion.

101. (a) Here, $f : R \to R$ is given as $f(x) = x^2$.
Suppose $f(x) = f(y)$
where $x, y \in R \Rightarrow x^3 = y^3$ ...(i)
Now, we try to show that $x = y$.
Suppose $x \neq y$, their cubes will also be not equal.
$x^3 \neq y^3$
However, this will be a contradiction to eq. (i)
Therefore, $x = y$. Hence, f is injective.

102. (d) The operation * is not associative, since
$(8 * 5) * 3 = (8 + 10) * 3 = (8 + 10) + 6 = 24$.
while $8 * (5 * 3) = 8 * (5 + 6) = 8 * 11 = 8 + 22 = 30$

103. (c) Given that $f(x) = (x + 1)^2 - 1,\ x \geq -1$
Clearly $D_f = [-1, \infty)$ but co-domain is not given. Therefore $f(x)$ need not be necessarily onto.
But if $f(x)$ is onto then as $f(x)$ is one-one also, $(x + 1)$ being something +ve, $f^{-1}(x)$ will exist where
$(x + 1)^2 - 1 = y$
$\Rightarrow x + 1 = \sqrt{y+1}$ (+ve square root as $x + 1 \geq 0$)
$\Rightarrow x = -1 + \sqrt{y+1}$
$\Rightarrow f^{-1}(x) = \sqrt{x+1} - 1$
Then $f(x) = f^{-1}(x)$
$\Rightarrow (x + 1)^2 - 1 = \sqrt{x+1} - 1$
$\Rightarrow (x + 1)^2 = \sqrt{x+1} \Rightarrow (x + 1)^4 = (x + 1)$
$\Rightarrow (x + 1)[(x + 1)^3 - 1] = 0 \Rightarrow x = -1, 0$
$\therefore$ The Assertion is correct but Reason is false.

104. (d) Let $f : R - \{0\} \to R - \{0\}$ defined by
$f(x) = \frac{1}{x}$ then f is one-one and onto.

105. (d)

106. (a)

CRITICALTHINKING TYPE QUESTIONS

107. (a) Reflexive: $|a - a| = 0 < 1 \therefore a R a\ \forall\ a \in R$
$\therefore$ R is reflexive.
Symmetric: $a R b \Rightarrow |a - b| \leq 1 \Rightarrow |b - a| \leq 1 \Rightarrow bRa$
$\therefore$ R is symmetric.
Anti-symmetric: $1\ R\frac{1}{2}$ and $\frac{1}{2}R\,1$ but $\frac{1}{2} \neq 1$
$\therefore$ R is not anti-symmetric.
Transitive: 1R2 and 2R3 but $1 \not R 3$, $[\because |1-3| = 2 > 1]$
$\therefore$ R is not transitive.

108. (c) Commutative: Let $(a, b), (c, d), (e, f) \in A = N \times N$ be arbitrary elements, then
$(a, b) * (c, d) = (a + c, b + d) = (c + a, d + b) = (c, d) * (a, b)$
($\because$ addition is commutative in the set of natural numbers)
and $((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f)$
$= ((a + c) + e, (b + d) + f) = (a + (c + e), b + (d + f))$
$= (a, b) * (c + e, d + f) = (a, b) * ((c, d) * (e, f))$
Hence, * is commutative as well as associative.

109. (a) Given that $a * b = \frac{ab}{4} \forall a, b \in Q^+$
$$\therefore \quad 3*\left(\frac{1}{5}*\frac{1}{2}\right) = 3*\left\{\frac{\frac{1}{5}\times\frac{1}{2}}{4}\right\} = 3*\frac{1}{40} = \frac{\left(3\times\frac{1}{40}\right)}{4} = \frac{3}{160}$$

110. (b) $f(x) = \tan x$
Let $f(x) = y = \tan x$
$\Rightarrow x = \tan^{-1} y$
$\therefore\ f^{-1}(x) = \tan^{-1}(x)$
$f^{-1}(1) = \tan^{-1}(1) = \left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$

111. (a) Let S be a finite set containing n elements. Then total number of binary operation on S is n^{n^2}.

112. (d) We have $f(x) = \sin x$
clearly domain of $f(x)$ is R
but its Range is $[-1, 1]$
further $\sin 0 = \sin \pi = \sin 2\pi = ...\sin n\pi = 0$.
Hence, it is many one function

113. (b) $(-\infty, 0)$

Hint : The given function is defined for those values of x for which $|x| - x > 0$ i.e. $|x| > x$. This inequality is satisfied only if $x < 0$. Hence the domain of the given function is $(-\infty, 0)$.

114. (a) The relation R_1 is an equivalence relation

$\forall a \in R, |a| = |a|$, i.e. $aR_1a \; \forall a \in R$

$\therefore R_1$ is reflexive.

Again $\forall \; a, b \in R, |a| = |b| \Rightarrow |b| = |a|$

$\therefore aR_1b \Rightarrow bR_1a$. Therefore R is symmetric.

Also, $\forall \; a, b, c \in R, |a| = |b|$ and $|b| = |c|$

$\Rightarrow |a| = |c|$

$\therefore aR_1b$ and $bR_1c \Rightarrow aR_1c$

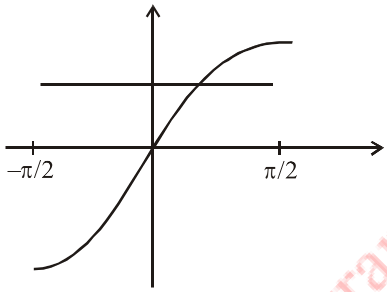
$\Rightarrow R_1$ is transitive

R_2 and R_3 are not symmetric.

R_4 is neither reflexive nor symmetric.

115. (b) We know that f (x) is said to be one-one

If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$



f (x) is said to be onto if f (x) is always increasing.

$\therefore \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \qquad (\because f(x) = \sin x)$

116. (d) Since sin (2x – 3) is a periodic function with period π, therefore *f* is not injective. Also, *f* is not surjective as its range [–1, 1] is a proper subset of its co-domain R.

117. (c) Let $f(x_1) = f(x_2)$ for $x_1, x_2 \in R$.

$\Rightarrow \; x_1^3 + 4 = x_2^3 + 4 \Rightarrow x_1^3 - x_2^3 = 0$

$\Rightarrow \; (x_1 + x_2)(x_1^2 + x_2^2 + x_1x_2) = 0$

$\Rightarrow \; (x_1 - x_2)\left(\left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3}{4}x^2\right) = 0$

$x_1 - x_2 = 0 \Rightarrow x_1 = x_2 \qquad \therefore$ f is one-one.

Let $k \in R$.

$f(x) = k \Rightarrow x^3 + 4 = k \Rightarrow x = (k-4)^{1/3} \in R$

$\therefore$ f is onto

118. (c) Here, $R = \{(T_1, T_2) : T_1$ is similar to $T_2\}$

R is reflexive; since every triangle is similar to itself.

Further, if $(T_1, T_2) \in R$, then T_1 is similar to T_2.

$\Rightarrow T_2$ is similar to $T_1 \Rightarrow (T_2, T_1) \in R$

Therefore, R is symmetric.

Now, let $(T_1, T_2), (T_2, T_3) \in R$

$\Rightarrow T_1$ is similar to T_2 and T_2 is similar to T_3.

$\Rightarrow T_1$ is similar to $T_3 \Rightarrow (T_1, T_3) \in R$

Therefore, R is transitive. Thus, R is an equivalence relation (which is already given).

Now, we can observe that $\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \left(\frac{1}{2}\right)$

i.e., the corresponding sides of triangles T_1 and T_3 are in the same ratio. Therefore, triangle T_1 is similar to triangle T_3.

Hence, T_1 is related to T_3.

119. (b) The given relation is R = {(1, 1), (2, 2),(3, 3), (1, 3)} on the set A = {1, 2, 3}.

Clearly, R is reflexive and transitive.

To make R symmetric, we need (3, 1) as $(1, 3) \in R$.

$\therefore$ If $(3, 1) \in R$, then R will be an equivalence relation.

Hence, (3, 1) is the single ordered pair which needs to be added to R to make it the smallest equivalence relation.

120. (b) a R a, then GCD of a and a is a.

$\therefore$ R is not reflexive.

Now, $aRb \Rightarrow bRa$

If GCD of a and b is 2, then GCD of b and a is 2.

$\therefore$ R is symmetric.

Now, aRb, bRc $\not\Rightarrow$ aRc

If GCD of a and b is 2 and GCD of b and c is 2, then it need not be GCD of a and c is 2.

$\therefore$ R is not transitive.

e.g., 6R2, 2R12 but $6 \not R 12$.

121. (a) The given relation is R = {(1, 2), (2, 3)} in the set A = {1, 2, 3}.

Now, R is reflexive, if $(1, 1), (2, 2),(3, 3) \in R$.

R is symmetric, if $(2, 1), (3, 2) \in R$.

R is transitive, if (1, 3) and $(3, 1) \in R$.

Thus, the minimum number of ordered pairs which are to be added, so that R becomes an equivalence relation, is 7.

122. (a) Let R be a relation containing (1, 2) and (1, 3) R is reflexive, if $(1, 1), (2, 2), (3, 3) \in R$.

Relation R is symmetric, if $(2, 1) \in R$ but $(3, 1) \notin R$.

But relation R is not transitive as $(3,1), (1,2) \in R$ but $(3,2) \notin R$.

Now, if we add the pair (3, 2) and (2, 3) to relation R, then relation R will become transitive.

Hence, the total number of desired relations is one.

123. (b) One-one function from {1, 2, 3} to itself is simply a permutation on three symbols 1, 2, 3. Therefore, total number of one-one maps from {1, 2, 3} to itself is same as total number of permutations on symbols 1, 2, 3, which is 3! = 6.

124. (d) Consider f : {1, 2, 3, 4} → {1, 2, 3, 4, 5, 6} defined as f(x) = x, ∀x and g : {1, 2, 3, 4, 5, 6} → {1, 2, 3, 4, 5, 6} as g(x) = x, for x = 1, 2, 3, 4 and g(5) = g(6) = 5. Then, gof (x) = x ∀ x which shows that gof is one-one. But g is clearly not one-one.

125. (b) We have, $gof(x) = g\left(\frac{3x+4}{5x-7}\right) = \frac{7\left(\frac{(3x+4)}{(5x-7)}\right)+4}{5\left(\frac{(3x+4)}{(5x-7)}\right)-3}$

$= \frac{21x+28+20x-28}{15x+20-15x+21} = \frac{41x}{41} = x$

Similarly, $fog(x) = f\left(\frac{7x+4}{5x-3}\right)$

$= \frac{3\left(\frac{(7x+4)}{(5x-3)}\right)+4}{5\left(\frac{(7x+4)}{(5x-3)}\right)-7}$

$= \frac{21x+12+20x-12}{35x+20-35x+21} = \frac{41x}{41} = x$

Thus, gof (x) = x, ∀ x ∈ B and fog (x) = x, ∀ x ∈ A, which implies that gof = I_B and fog = I_A.

126. (c) Here, function f : R → R is given as $f(x) = (3 - x^3)^{1/3}$

∴ $fof(x) = f(f(x)) = f((3 - x^3)^{1/3})$

$= [3 - ((3 - x^3)^{1/3})^3]^{1/3}$

$= [3 - (3 - x^3)]^{1/3} = (x^3)^{1/3} = x$

∴ fof (x) = x.

127. (b) f(x) = |x| and g(x) = |5x – 2|

∴ fog (x) = f (g(x))

= f (|5x – 2|) = ||5x – 2|| = |5x – 2|

and (gof) (x) = g(f(x)) = g (|x|) = |5| x| – 2|

128. (b) Given, $f(x) = e^x$ and $g(x) = \log_e x$

Now, $f\{g(x)\} = e^{\log_e^x} = x$

and $g\{f(x)\} = \log_e e^x = x \quad \therefore f\{g(x)\} = g\{f(x)\}$

129. (a) We have, (2 * 3) * 4 = 1 * 4 = 1

and 2 * (3 * 4) = 2 * 1 = 1

Since, each row is same as their corresponding column.

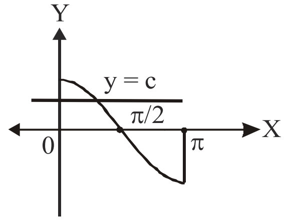
Therefore, the operation * is commutative.

We have (2 * 3) = 1 and (4 * 5) = 1

Therefore, (2 * 3) * (4 * 5) = 1 * 1 = 1

130. (a) The smallest equivalence relation R_1 containing (1, 2) and (2, 1) is {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}. Now, we are left with only 4 pairs namely (2, 3), (3, 2), (1, 3) and (3, 1). If we add any one, say (2, 3) to R_1, then for symmetry we must add (3, 2) also and now for transitivity we are forced to add (1, 3) and (3, 1). Thus, the only equivalence relation bigger than R_1 is the universal relation. This shows that the total number of equivalence relations containing (1, 2) and (2, 1) is two.

131. (a)



Since line parallel to x-axis cuts the graph at one point. So function is one-one.

132. (b) $f(x) = \sin x + \cos x, \; g(x) = x^2 - 1$

$\Rightarrow g(f(x)) = (\sin x + \cos x)^2 - 1 = \sin 2x$

Clearly $g(f(x))$ is invertible in $-\frac{\pi}{2} \le 2x \le \frac{\pi}{2}$

[∵ sin θ is invertible when $-\pi/2 \le \theta \le \pi/2$]

$\Rightarrow -\frac{\pi}{4} \le x \le \frac{\pi}{4}$

133. (b) In the function f (x) = (x – 1) (x – 2) (x – 3) for more than one value of x, i.e. x = 1, x = 2 and x = 3, value of the function is zero.

So, the function is not one-one.

Range of the function is the set of all real number i.e. R.

Since Range = Co-domain = R, the function is onto.

Thus the given function f(x) is onto but not one-one.

134. (a) If A and B are two sets having m and n elements such that

$1 \le n \le m = \sum_{r=1}^{n} (-1)^{n-r} \, {}^nC_r r^m$

Number of surjection from A to B

$= \sum_{r=1}^{n} (-1)^{2-r} \, {}^2C_r (r)^4$

$= (-1)^{2-1} \, {}^2C_1 (1)^4 + (-1)^{2-2} \, {}^2C_2 (2)^4 = -2 + 16 = 14$

135. (b) $\frac{y-2}{1} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Applying comp. and dividendo.

$\frac{y-1}{3-y} = \frac{2e^x}{2e^{-x}} = e^{2x}$

$\therefore \; x = \frac{1}{2}\log\left(\frac{y-1}{3-y}\right) = \log\left(\frac{y-1}{3-y}\right)^{1/2}$

Hence, the inverse of the function

$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is $\log_e\left(\frac{x-1}{3-x}\right)^{1/2}$

136. (a) $f(x)$ is onto

∴ S = range of $f(x)$

Now $f(x) = \sin x - \sqrt{3}\cos x + 1 = 2\sin\left(x - \frac{\pi}{3}\right) + 1$

$\because -1 \le \sin\left(x - \frac{\pi}{3}\right) \le 1$

$-1 \le 2\sin\left(x - \frac{\pi}{3}\right) + 1 \le 3$

$\therefore f(x) \in [-1, 3] = S$

137. (a) Given that

$f(x) = 2x + \sin x, \quad x \in R$

$\Rightarrow f'(x) = 2 + \cos x$

But $-1 \le \cos x \le 1$

$\Rightarrow \quad 1 \le 2 + \cos x \le 3$

$\Rightarrow \quad 1 \le 2 + \cos x \le 3$

$\therefore \quad f'(x) > 0, \ \forall x \in R$

$\Rightarrow \quad f(x)$ is strictly increasing and hence one-one

Also as $x \to \infty$, $f(x) \to \infty$ and $x \to -\infty$, $f(x) \to -\infty$

$\therefore$ Range of $f(x) = R =$ domain of $f(x) \Rightarrow f(x)$ is onto.

Thus, $f(x)$ is one-one and onto.

138. (c) We have

$$f(x) = \frac{x^2 + x + 2}{x^2 + x + 1} = \frac{(x^2 + x + 1) + 1}{x^2 + x + 1}$$

$$= 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

We can see here that as $x \to \infty$, $f(x) \to 1$ which is the min value of $f(x)$. i.e. $f_{\min} = 1$. Also $f(x)$ is max when

$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$ is min which is so when $x = -1/2$

i.e. when $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4}$.

$\therefore f_{\max} = 1 + \frac{1}{3/4} = 7/3 \qquad \therefore R_f = (1, 7/3]$

CHAPTER 18

INVERSE TRIGONOMETRIC FUNCTION

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The value of $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ is

(a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3-\sqrt{5}}{2}$

(c) $\frac{\sqrt{5}}{6}$ (d) None of these

2. The number of roots of equation $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$ is

(a) 0 (b) 1 (c) 2 (d) infinite

3. The value of x satisfying the equation

$3\tan^{-1}\frac{1}{2+\sqrt{3}}-\tan^{-1}\frac{1}{x}=\tan^{-1}\frac{1}{3}$ is

(a) $x=2$ (b) $x=\frac{1}{2}$

(c) $x=\frac{1}{2-\sqrt{3}}$ (d) None of these

4. If $\tan^{-1}(x+1)+\cot^{-1}(x-1)=\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{3}{5}$, then x has the value :

(a) $4\sqrt{\frac{3}{7}}$ (b) $4\sqrt{\frac{7}{3}}$ (c) $14\sqrt{3}$ (d) $6\sqrt{7}$

5. The value of $\cos\left(2\cos^{-1}x+\sin^{-1}x\right)$ at $x=\frac{1}{5}$ is

(a) $-\frac{2\sqrt{6}}{5}$ (b) $-2\sqrt{6}$ (c) $-\frac{\sqrt{6}}{5}$ (d) None of these

6. Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is equal to

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

7. Principal value of $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is equal to

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

8. Principal value of $\sec^{-1}(2)$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

9. Prinicpal value of $\tan^{-1}(\sqrt{3})$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

10. Principal value of $\tan^{-1}1+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 6π

11. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

12. The value of $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{7}{8}\right)$ is

(a) $\tan^{-1}\left(\frac{7}{8}\right)$ (b) $\cot^{-1}(15)$

(c) $\tan^{-1}(15)$ (d) $\tan^{-1}\left(\frac{25}{24}\right)$

13. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is

(a) $\frac{5}{17}$ (b) $\frac{6}{17}$ (c) $\frac{3}{17}$ (d) $\frac{4}{17}$

14. If $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$, then the value of x are

(a) $\pm\frac{1}{2}$ (b) $0,\frac{1}{2}$ (c) $0,-\frac{1}{2}$ (d) $0,\pm\frac{1}{2}$

15. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then the value of x is

(a) -1 (b) $\frac{2}{5}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

16. $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\frac{1}{\sqrt{3}}$ is equal to

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{3\pi}{4}$

17. The value of $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to

(a) π (b) $\frac{5\pi}{4}$ (c) $\frac{\pi}{2}$ (d) None of these

18. If $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then x is equal to

(a) $\sqrt{ab}$ (b) $\sqrt{2ab}$ (c) 2ab (d) ab

19. If $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$, then A is equal to

(a) $x - y$ (b) $x + y$ (c) $\frac{x-y}{1+xy}$ (b) $\frac{x+y}{1-xy}$

20. If $\tan^{-1}\left(\frac{x-1}{x+2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$, then x is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\pm\sqrt{\frac{5}{2}}$ (d) $\pm\frac{1}{2}$

21. $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$ is equal to…

(a) 0 (b) $\frac{\pi}{4}$ (c) 1 (d) 5

22. The value of $\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right) =$

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$ (c) $\frac{16}{7}$ (d) None of these

23. $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} =$

(a) π (b) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

24. If $\sin^{-1}(x^2 - 7x + 12) = n\pi,\ \forall\ n \in I$, then x =

(a) –2 (b) 4 (c) –3 (d) 5

25. If $A = \tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ and $B = \tan^{-1}\left(\frac{2k-k}{k\sqrt{3}}\right)$, then the value of A–B is

(a) 10° (b) 45° (c) 60° (d) 30°

26. If $4\cos^{-1}x + \sin^{-1}x = \pi$, then the value of x is

(a) $\frac{3}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

27. If $\tan^{-1}(\cot\theta) = 2\theta$, then θ is equal to

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) None of these

28. If $\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) + \sin^{-1}\left(\frac{2\beta}{1+\beta^2}\right) = 2\tan^{-1}x$, then x =

(a) α/β (b) β/α

(c) $\frac{\alpha+\beta}{1+\alpha\beta}$ (d) $\frac{\alpha+\beta}{1-\alpha\beta}$

29. If $\tan^{-1}3 + \tan^{-1}x = \tan^{-1}8$, then x =

(a) 5 (b) $\frac{1}{5}$ (c) $\frac{5}{14}$ (d) $\frac{14}{5}$

30. $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\pm\frac{1}{\sqrt{2}}$ (d) 0

31. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ then value of x is

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

32. Which of the following is the principal value branch of $\cos^{-1}x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $(0, \pi)$

(c) $[0, \pi]$ (d) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

33. Which of the following is the principal value branch of $\operatorname{cosec}^{-1}x$?

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (b) $(0, \pi) - \left[\frac{\pi}{2}\right]$

(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

34. If $3\tan^{-1}x + \cot^{-1}x = \pi$, then x equals

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

35. The value of $\cos^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$ is

(a) $\frac{3\pi}{5}$ (b) $\frac{-3\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$

36. If $\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = 0$, then x is equal to

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) 0 (d) 1

37. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

38. The value of expression $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

39. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$, then $\cot^{-1}x + \cot^{-1}y$ equals

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) π

40. If, $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where a, x ∈]0,1[, then the value of x is

(a) 0 (b) $\frac{a}{2}$ (c) a (d) $\frac{2a}{1-a^2}$

41. If $|x| \le 1$, then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $4\tan^{-1}x$ (b) $\pi/2$ (c) 0 (d) π

42. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is:

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $-\frac{2\pi}{3}$ (d) $\frac{2\pi}{3}$

43. If $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)=\theta$, then the value of $\operatorname{cosec}^{-1}(\sqrt{5})$ is

(a) $\left(\frac{\pi}{2}\right)+\theta$ (b) $\left(\frac{\pi}{2}\right)-\theta$ (c) $\frac{\pi}{2}$ (d) $-\theta$

44. The value of $\cos\left[\tan^{-1}\left\{\tan\left(\frac{15\pi}{4}\right)\right\}\right]$ is

(a) $-\frac{1}{\sqrt{2}}$ (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

45. If $k \le \sin^{-1}x+\cos^{-1}x+\tan^{-1}x \le K$, then

(a) $k=-\pi,\ K=\pi$ (b) $k=0,\ K=\frac{\pi}{2}$

(c) $k=\frac{\pi}{4},\ K=\frac{3\pi}{4}$ (d) $k=0, K=\pi$

46. The value of $\tan\left[2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right]$ is

(a) $\frac{5}{18}$ (b) $-\frac{3}{2}$ (c) $-\frac{7}{17}$ (d) $\frac{3}{8}$

47. The value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $-\frac{3}{4}$

48. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(a) $-\frac{5\pi}{3}$ (b) $\frac{5\pi}{3}$ (c) $-\frac{p}{3}$ (d) $\frac{4p}{3}$

49. If $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, then x equals

(a) $0,-\frac{1}{2}$ (b) $0,\frac{1}{2}$ (c) 0 (d) None of these

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

50. If $6\sin^{-1}(x^2-6x+8.5)=\pi$, then x is equal to

(a) 1 (b) 2 (c) 3 (d) 8

51. $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=$

(a) 7 (b) 6 (c) 5 (d) None of these

52. The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$

(c) $\frac{10\pi}{3}$ (d) 0

53. If $\sin^{-1}x=\tan^{-1}y$, what is the value of $\frac{1}{x^2}-\frac{1}{y^2}$?

(a) 1 (b) –1 (c) 0 (d) 2

54. If $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\operatorname{cosec}^{-1}(2)$ has the value $k\frac{\pi}{12}$, then value of k is

(a) 1 (b) 2 (c) 4 (d) 5

55. The value of $\frac{\sin(\tan^{-1}x+\cot^{-1}x)}{\sin(\sin^{-1}x+\cos^{-1}x)}$ is

(a) 1 (b) 2 (c) 4 (d) 5

56. If $\sin^{-1}\left(\frac{6x}{1+9x^2}\right)=2\tan^{-1}(ax)$, then a =

(a) 3 (b) 8 (c) 6 (d) 9

57. If $\tan^{-1}k-\tan^{-1}3=\tan^{-1}\frac{1}{13}$, then k =

(a) 1 (b) 2 (c) 4 (d) 5

58. Given that $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)=\frac{2\pi}{k}$, then k =

(a) 3 (b) 8 (c) 6 (d) 9

59. If $\sin^{-1}\left(\frac{1}{5}\right)+\sec^{-1}(2)+2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}(5)$

$+\sin^{-1}\left(\frac{1}{2}\right)+2\tan^{-1}(\sqrt{3})=k\pi$, then k =

(a) 1 (b) 2 (c) 4 (d) 5

ASSERTION- REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

60. Assertion : The value of $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)$ is $\frac{\pi}{4}$

Reason : If $x>0, y>0$ then

$\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y-x}{y+x}\right)=\frac{\pi}{4}$

61. **Assertion:** $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{2}{9}+\tan^{-1}\frac{4}{33}+....\infty=\frac{\pi}{4}$

Reason : If $xy<1$ then $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

62. **Assertion:** The value of $\sin\left[\tan^{-1}(-\sqrt{3})+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is 1.

Reason: $\tan^{-1}(-x)=\tan x$ and $\cos^{-1}(-x)=\cos^{-1}x$.

63. **Assertion:** $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

Reason: $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ if $-1<x<1$.

64. **Assertion:**

$$\operatorname{cosec}^{-1}\left(\frac{3}{2}\right)+\cos^{-1}\left(\frac{2}{3}\right)-2\cot^{-1}\left(\frac{1}{7}\right)-\cot^{-1}(7)$$

is equal to $\cot^{-1}7$.

Reason: $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$,

$\operatorname{cosec}^{-1}x=\sin^{-1}\left(\frac{1}{x}\right)$, $\cot^{-1}(x)=\tan^{-1}\left(\frac{1}{x}\right)$

65. **Assertion:** $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{77}{85}$

Reason: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

66. **Assertion:** The value of

$\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ is $\pi-\frac{x}{2}$.

Reason: $\tan^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

67. **Assertion:** If $2(\sin^{-1}x)^2-5(\sin^{-1}x)+2=0$, then x has 2 solutions.

Reason: $\sin^{-1}(\sin x)=x$ if $x\in R$.

68. **Assertion:** If $x<0$, $\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)=\frac{\pi}{2}$

Reason: $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2},\forall x\in R$

69. **Assertion:** The value of $\tan\left\{\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right\}$ is $\frac{17}{6}$.

Reason: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$.

70. **Assertion:** The function $f(x)=\sin x$ does not possess inverse if $x\in R$.

Reason: The function $f(x)=\sin x$ is not one-one onto if $x\in R$.

71. **Assertion:** To define the inverse of the function $f(x)=\tan x$ any of the intervals $\left(-\frac{3\pi}{2},\frac{-\pi}{2}\right),\left(-\frac{\pi}{2},\frac{\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ etc. can be chosen.

Reason: The branch having range $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is called principal value branch of the function $g(x)=\tan^{-1}x$.

72. **Assertion:** The domain of the function $\sec^{-1}x$ is the set of all real numbers.

Reason: For the function $\sec^{-1}x$, x can take all real values except in the interval (–1, 1).

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

73. If $x, a\in\mathbf{R}$, where $\frac{-a}{\sqrt{3}}<x<\frac{a}{\sqrt{3}}$, then $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ is equal to

(a) $3\tan^{-1}\left(\frac{x}{a}\right)$ (b) $-3\tan^{-1}\left(\frac{x}{a}\right)$

(c) $3\tan^{-1}\left(\frac{a}{x}\right)$ (d) None of these

74. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then the value of x is

(a) 4 (b) 5 (c) 1 (d) 3

75. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ is equal to (Where $x>y>0$)

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{3\pi}{4}$ (d) None of these

76. If $\sin(\cot^{-1}(1+x))=\cos(\tan^{-1}x)$, then x =

(a) $\frac{1}{2}$ (b) 1 (c) 0 (d) $-\frac{1}{2}$

77. Domain of $\cos^{-1}[x]$ is

(a) [–1, 2] (b) [–1, 2)

(c) (–1, 2] (d) None of these

78. The value of $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)$ is

(a) 12 (b) 5 (c) 15 (d) 9

79. $\tan\left[\cos^{-1}\frac{1}{\sqrt{82}}-\sin^{-1}\left(\frac{5}{\sqrt{26}}\right)\right]$ is equal to

(a) $\frac{2}{23}$ (b) $\frac{4}{31}$

(c) $\frac{29}{3}$ (d) $\frac{6}{13}$

80. The equation $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has

(a) unique solution
(b) no solution
(c) infinitely many solutions
(d) None of these

81. Solve for x : $\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 = \frac{51}{50}$,

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{5\sqrt{2}}$
(c) $2\sqrt{2}$ (d) $5\sqrt{2}$

82. $\sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\}$ is equal to

(a) $\frac{6}{25}$ (b) $\frac{24}{25}$ (c) $\frac{4}{5}$ (d) $-\frac{24}{25}$

83. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) [1, 2] (b) [–1, 1] (c) [0, 1] (d) None of these

84. The value of expression $\tan^{-1}\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

(a) $2+\sqrt{5}$ (b) $\sqrt{5}-2$ (c) $\frac{\sqrt{5}+2}{2}$ (d) $5+\sqrt{2}$

85. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$ then $xy + yz + zx$ is equal to :

(a) 1 (b) 0
(c) –3 (d) 3

86. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then what is the value of $x + y + z$?

(a) –3 (b) 3 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

87. If $xy + yz + zx = 1$, then :

(a) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = 0$
(b) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$
(c) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{4}$
(d) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

88. The value of $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1}(3)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

89. The solution of $\sin^{-1}x - \sin^{-1}2x = \pm\frac{\pi}{3}$ is

(a) $\pm\frac{1}{3}$ (b) $\pm\frac{1}{4}$
(c) $\pm\frac{\sqrt{3}}{2}$ (d) $\pm\frac{1}{2}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** Let $\cos^{-1}\frac{\sqrt{5}}{3}=\theta$, then $0<\theta<\frac{\pi}{2}$ and $\cos\theta=\frac{\sqrt{5}}{3}$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta}=\frac{2}{3}$

So, $\tan\left[\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right]=\tan\frac{\theta}{2}=\frac{1-\cos\theta}{\sin\theta}=\frac{3-\sqrt{5}}{2}$

2. **(b)** $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{2x+3x}{1-2x.3x}=\tan^{-1}1$

$\Rightarrow \frac{5x}{1-6x^2}=1 \quad \Rightarrow 6x^2+5x-1=0$

$\Rightarrow (6x-1)(x+1)=0 \Rightarrow x=\frac{1}{6}$ or -1

Now for $x=-1$, LHS of equation becomes negative, so $x=\frac{1}{6}$.

3. **(a)** $3\tan^{-1}\frac{1}{2+\sqrt{3}}-\tan^{-1}\frac{1}{x}=\tan^{-1}\frac{1}{3}$

$\Rightarrow\ 2\tan^{-1}\frac{1}{2+\sqrt{3}}+\tan^{-1}\frac{1}{2+\sqrt{3}}-\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{1}{x}$

$\Rightarrow \tan^{-1}\frac{2\times\frac{1}{2+\sqrt{3}}}{1-\frac{1}{(2+\sqrt{3})^2}}+\tan^{-1}\frac{\frac{1}{2+\sqrt{3}}-\frac{1}{3}}{1+\frac{1}{3}\times\frac{1}{2+\sqrt{3}}}=\tan^{-1}\frac{1}{x}$

$\Rightarrow \tan^{-1}\frac{1}{\sqrt{3}}+\tan^{-1}\frac{1-\sqrt{3}}{7+3\sqrt{3}}=\tan^{-1}\frac{1}{x}$

$\Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{3}}+\frac{1-\sqrt{3}}{7+3\sqrt{3}}}{1-\frac{1}{\sqrt{3}}\times\frac{1-\sqrt{3}}{7+3\sqrt{3}}}=\tan^{-1}\frac{1}{x}$

$\Rightarrow \frac{7+3\sqrt{3}+\sqrt{3}-3}{\sqrt{3}(7+3\sqrt{3})-(1-\sqrt{3})}=\frac{1}{x} \quad \Rightarrow x=2$

4. **(a)** Given $\tan^{-1}(x+1)+\cot^{-1}(x-1)$

$=\sin^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)$

$\Rightarrow \tan^{-1}(x+1)+\tan^{-1}\left(\frac{1}{x-1}\right)$

$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{4}{3}=2\tan^{-1}\frac{4}{3}$

$\Rightarrow \tan^{-1}\left[\frac{x+1+\frac{1}{x-1}}{1-(x+1)\left(\frac{1}{x-1}\right)}\right]=\tan^{-1}\left[\frac{2\times\frac{4}{3}}{1-\frac{16}{9}}\right]$

$\left[\text{using } 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$\Rightarrow \tan^{-1}\left(-\frac{x^2}{2}\right)=\tan^{-1}\left(-\frac{24}{7}\right)$

$\Rightarrow \frac{x^2}{2}=\frac{24}{7} \Rightarrow x^2=\frac{48}{7} \Rightarrow x=4\sqrt{\frac{3}{7}}$

5. **(a)** $\cos\left[2\cos^{-1}x+\sin^{-1}x\right]$

$=\cos\left[\cos^{-1}x+\cos^{-1}x+\sin^{-1}x\right]$

$=\cos\left[\cos^{-1}x+\pi/2\right]=-\sin[\cos^{-1}x]$

$=-\sin\left[\sin^{-1}\sqrt{1-x^2}\right]=-\sqrt{1-x^2}$

$=-\sqrt{1-\left(\frac{1}{5}\right)^2}=-\sqrt{\frac{24}{25}}=-\frac{2\sqrt{6}}{5}$

6. **(d)** Let $\sin^{-1}\left(\frac{-1}{2}\right)=\theta$

$\Rightarrow\ \sin\theta=\frac{-1}{2}=-\sin\frac{\pi}{6}=\sin\left(\frac{-\pi}{6}\right)$

$\Rightarrow\ \theta=\frac{-\pi}{6}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

$\therefore$ Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $\left(\frac{-\pi}{6}\right)$

7. **(a)** Let $\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)=\theta$

$\Rightarrow\ \operatorname{cosec}\theta=\frac{-2}{\sqrt{3}}=-\operatorname{cosec}\frac{\pi}{3}=\operatorname{cosec}\left(\frac{-\pi}{3}\right)$

$\Rightarrow\ \theta=\frac{-\pi}{3}\in\left[\frac{-\pi}{3},\frac{\pi}{2}\right]-\{0\}$

$\therefore$ Principal value of $\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is $\left(\frac{-\pi}{3}\right)$

8. **(b)** Let $\sec^{-1}(2)=\theta$

$\Rightarrow\ \sec\theta=2=\sec\frac{\pi}{3}$

$\Rightarrow\ \theta=\frac{\pi}{3}\in[0,\pi]-\left\{\frac{\pi}{2}\right\}$

$\therefore$ Principal value of $\sec^{-1}(2)$ is $\frac{\pi}{3}$

9. **(b)** Let $\tan^{-1}(\sqrt{3}) = \theta$

$\Rightarrow \tan\theta = \sqrt{3} = \tan\dfrac{\pi}{3}$

$\therefore$ Principal value of $\tan^{-1}\sqrt{3}$ is $\dfrac{\pi}{3}$

10. **(b)** Let $\tan^{-1}(1) = \theta$

$\Rightarrow \tan\theta = 1 = \tan\dfrac{\pi}{4}$

$\Rightarrow \theta = \dfrac{\pi}{4} \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

$\therefore$ Principal value of $\tan^{-1}(1)$ is $\dfrac{\pi}{4}$

Let $\cos^{-1}\left(\dfrac{-1}{2}\right) = \phi \Rightarrow \cos\phi = \dfrac{-1}{2} = -\cos\dfrac{\pi}{3}$

$= \cos\left(\pi - \dfrac{\pi}{3}\right) = \cos\dfrac{2\pi}{3}$

$\Rightarrow \phi = \dfrac{2\pi}{3} \in [0, \pi]$

$\therefore$ Principal value of $\cos^{-1}\left(\dfrac{-1}{2}\right)$ is $\dfrac{2\pi}{3}$

Also, principal value of $\sin^{-1}\left(\dfrac{-1}{2}\right)$ is $\left(\dfrac{-\pi}{6}\right)$

Principal value of $\tan^{-1}(1) + \cos^{-1}\left(\dfrac{-1}{2}\right) + \sin^{-1}\left(\dfrac{-1}{2}\right)$

$= \dfrac{\pi}{4} + \dfrac{2\pi}{3} - \dfrac{\pi}{6} = \dfrac{3\pi}{4}$

11. **(d)** $\tan^{-1}\left(\dfrac{1}{4}\right) + \tan^{-1}\left(\dfrac{2}{9}\right) = \tan^{-1}\left\{\dfrac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right\}$

$= \tan^{-1}\left\{\dfrac{9+8}{36-2}\right\} = \tan^{-1}\left(\dfrac{1}{2}\right)$

12. **(c)** $\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{3}\right) + \tan^{-1}\left(\dfrac{7}{8}\right)$

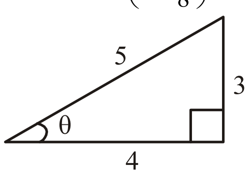
$= \tan^{-1}\left(\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right) + \tan^{-1}\left(\dfrac{7}{8}\right)$

$= \tan^{-1}\left(\dfrac{5}{5}\right) + \tan^{-1}\left(\dfrac{7}{8}\right)$

$= \tan^{-1}(1) + \tan^{-1}\left(\dfrac{7}{8}\right)$

$= \tan^{-1}\left(\dfrac{1+\frac{7}{8}}{1-\frac{7}{8}}\right) = \tan^{-1}(15)$

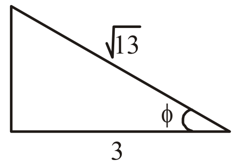
13. **(b)**



Let $\text{cosec}^{-1}\left(\dfrac{5}{3}\right) = \theta, \tan^{-1}\left(\dfrac{2}{3}\right) = \phi$

$\Rightarrow \text{cosec}\,\theta = \dfrac{5}{3}, \tan\phi = \dfrac{2}{3}$

$\cot\theta = \dfrac{4}{3}, \cot\phi = \dfrac{3}{2}$



Now, $\cot(\theta+\phi) = \dfrac{\cot\theta\cot\phi - 1}{\cot\theta + \cot\phi}$

$= \dfrac{\left(\frac{4}{3}\times\frac{3}{2}\right)-1}{\frac{4}{3}+\frac{3}{2}} = \dfrac{6(2-1)}{9+8} = \dfrac{6}{17}$

14. **(d)** We have,

$\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$

$\Rightarrow \tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}x$

$\Rightarrow \tan^{-1}\left\{\dfrac{(x-1)+(x+1)}{1-(x^2-1)}\right\} = \tan^{-1}\left\{\dfrac{3x-x}{1+3x^2}\right\}$

$\Rightarrow \dfrac{2x}{2-x^2} = \dfrac{2x}{1+3x^2}$

$\Rightarrow 2x(1+3x^2) = 2x(2-x^2)$

$\Rightarrow 2x(4x^2-1) = 0$

$\Rightarrow x = 0$ or $4x^2 - 1 = 0$

$\Rightarrow x = 0$ or $x = \pm\dfrac{1}{2}$

15. **(d)** We have, $\sin\left(\sin^{-1}\dfrac{1}{5} + \cos^{-1}x\right) = 1$

$\Rightarrow \sin^{-1}\dfrac{1}{5} + \cos^{-1}x = \sin^{-1}(1)$

$\Rightarrow \sin^{-1}\dfrac{1}{5} + \dfrac{\pi}{2} - \sin^{-1}x = \dfrac{\pi}{2}$

$\Rightarrow \sin^{-1}\dfrac{1}{5} - \sin^{-1}x = 0$

$\Rightarrow \sin^{-1}\dfrac{1}{5} = \sin^{-1}x$

$\Rightarrow x = \sin\left(\sin^{-1}\dfrac{1}{5}\right)$

$\Rightarrow x = \dfrac{1}{5}$

16. **(a)** $\cos^{-1}\left(\dfrac{1}{2}\right) + \sin^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\dfrac{1}{\sqrt{3}}$

$= \dfrac{\pi}{3} + \dfrac{\pi}{2} + \dfrac{\pi}{6} = \dfrac{6\pi}{6} = \pi$

17. **(a)** $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$

$= \dfrac{\pi}{4} + \pi + \tan^{-1}\left(\dfrac{2+3}{1-2.3}\right)$ (as $2.3 > 1$)

$= \dfrac{5\pi}{4} + \tan^{-1}(-1) = \dfrac{5\pi}{4} - \dfrac{\pi}{4} = \pi$

18. **(a)** Let $\tan^{-1}\left(\dfrac{a}{x}\right) + \tan^{-1}\left(\dfrac{b}{x}\right) = \dfrac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\dfrac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right) = \dfrac{\pi}{2}$

$$\Rightarrow \frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}} = \tan\frac{\pi}{2}$$

$$\Rightarrow 1-\frac{ab}{x^2} = 0 \Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$$

19. (c) Given that, $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$

$$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}A$$

Hence, $A = \dfrac{x-y}{1+xy}$

20. (c) We have, $\tan^{-1}\dfrac{x-1}{x+2} + \tan^{-1}\dfrac{x+1}{x+2} = \dfrac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x+2}+\frac{x+1}{x+2}}{\left(\frac{x-1}{x+2}\right)+\left(\frac{x+1}{x+2}\right)}\right] = \tan\frac{\pi}{4}$$

$$\Rightarrow \left[\frac{2x(x+2)}{x^2+4+4x-x^2+1}\right] = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2x(x+2)}{4x+5} = 1$$

$$\Rightarrow 2x^2+4x = 4x+5$$

$$\Rightarrow x = \pm\sqrt{\frac{5}{2}}$$

21. (a) $\cot^{-1}\left(\dfrac{ab+1}{a-b}\right)+\cot^{-1}\left(\dfrac{bc+1}{b-c}\right)+\cot^{-1}\left(\dfrac{ac+1}{c-a}\right)$

$= \cot^{-1}a - \cot^{-1}b + \cot^{-1}b - \cot^{-1}c + \cot^{-1}c - \cot^{-1}a$

$= 0$

22. (d) $\tan\left(\tan^{-1}\dfrac{3}{4}+\tan^{-1}\dfrac{2}{3}\right)$

$$= \tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right] = \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}} = \frac{17}{6}$$

23. (b) Given expression is equal to

$$\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{15}}\right)+\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{56}}\right)$$

$$= \tan^{-1}\frac{4}{7}+\tan^{-1}\frac{3}{11} = \tan^{-1}\left(\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{12}{77}}\right) = \frac{\pi}{4}$$

24. (b) $\sin^{-1}(x^2-7x+12) = n\pi$

$\Rightarrow x^2-7x+12 = \sin n\pi$

$\Rightarrow x^2-7x+12 = 0$ $\quad (\because \sin n\pi = 0 \ \forall \ n \in I)$

$\Rightarrow (x-4)(x-3) = 0$

$\Rightarrow x = 4, 3$

25. (d) $(A-B) = \tan^{-1}\left(\dfrac{x\sqrt{3}}{2k-x}\right) - \tan^{-1}\left(\dfrac{2x-k}{k\sqrt{3}}\right)$

$$= \tan^{-1}\frac{\frac{x\sqrt{3}}{2k-x}-\frac{2k-k}{k\sqrt{3}}}{1+\frac{x\sqrt{3}}{2k-x}\cdot\frac{2x-k}{k\sqrt{3}}} = \tan^{-1}\frac{1}{\sqrt{3}}$$

$\Rightarrow A-B = 30°$

26. (c) We have, $4\cos^{-1}x+\sin^{-1}x = \pi$

$$\Rightarrow 4\left\{\frac{\pi}{2}-\sin^{-1}x\right\}+\sin^{-1}x = \pi$$

$\Rightarrow 2\pi - 4\sin^{-1}x + \sin^{-1}x = \pi$

$\Rightarrow 3\sin^{-1}x = \pi$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{3}$$

$$\Rightarrow x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

27. (c) $\tan^{-1}(\cot\theta) = 2\theta$

$\Rightarrow \cot\theta = \tan 2\theta$

$$\Rightarrow \cot\theta = \cot\left(\frac{\pi}{2}-2\theta\right)$$

$$\Rightarrow \theta = \frac{\pi}{2}-2\theta$$

$$\Rightarrow 3\theta = \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

28. (d) $\sin^{-1}\left(\dfrac{2\alpha}{1+\alpha^2}\right)+\sin^{-1}\left(\dfrac{2\beta}{1+\beta^2}\right) = 2\tan^{-1}x$

$\Rightarrow 2\tan^{-1}\alpha + 2\tan^{-1}\beta = 2\tan^{-1}x$

$$\left[\because 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]$$

$\Rightarrow \tan^{-1}\alpha + \tan^{-1}\beta = 2\tan^{-1}x$

$$\left[\because 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]$$

$\tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}x$

$$\tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right) = \tan^{-1}x$$

$$\left[\because \tan^{-1}x+\tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow x = \frac{\alpha+\beta}{1-\alpha\beta}$$

29. (b) We have,

$\tan^{-1}3+\tan^{-1}x = \tan^{-1}8$

$$\Rightarrow \tan^{-1}x = \tan^{-1}\left\{\frac{8-3}{1+24}\right\}$$

$$\Rightarrow \tan^{-1}x = \tan^{-1}\left\{\frac{5}{25}\right\}$$

$$\Rightarrow x = \frac{1}{5}$$

30. (c) We have,

$$\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$$

$\Rightarrow \tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\tan^{-1}1$

$\Rightarrow \tan^{-1}\frac{x-1}{x-2}=\tan^{-1}1-\tan^{-1}\frac{x+1}{x+2}$

$\Rightarrow \tan^{-1}\frac{x-1}{x-2}=\tan^{-1}\left(\frac{1-\frac{x+1}{x+2}}{1+\frac{x+1}{x+2}}\right)$

$\Rightarrow \tan^{-1}\frac{x-1}{x-2}=\tan^{-1}\left(\frac{x+2-x-1}{x+2+x+1}\right)$

$\Rightarrow \tan^{-1}\frac{x-1}{x-2}=\tan^{-1}\left(\frac{1}{2x+3}\right)$

$\Rightarrow \frac{x-1}{x-2}=\frac{1}{2x+3}$

$\Rightarrow (2x+3)(x-1)=x-2$

$\Rightarrow 2x^2-1=0$

$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$

31. (c) We have,

$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \tan^{-1}\frac{2\cos x}{1-\cos^2 x}=\tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \frac{2\cos x}{\sin^2 x}=2(\operatorname{cosec} x)$

$\Rightarrow \cos x=\sin x$

$\Rightarrow \tan x=1$

$\Rightarrow x=\frac{\pi}{4}$

32. (c) Principal value branch of $\cos^{-1}x=[0,\pi]$

33. (d) Principal value branch of $\operatorname{cosec}^{-1}x$

$=\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$

34. (b) $3\tan^{-1}x+\cot^{-1}x=\pi$

$\Rightarrow 2\tan^{-1}x+\tan^{-1}x+\cot^{-1}x=\pi$

$\Rightarrow 2\tan^{-1}x+\frac{\pi}{2}=\pi$

$\Rightarrow 2\tan^{-1}x=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

$\Rightarrow \tan^{-1}x=\frac{\pi}{4}$

$\Rightarrow x=1$

35. (a) $\cos^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)=\cos^{-1}\left(\cos\left(6\pi+\frac{3\pi}{5}\right)\right)$

$=\cos^{-1}\left(\cos\frac{3\pi}{5}\right)=\frac{3\pi}{5}$

($\because$ Principal value branch of $\cos^{-1}x=[0,\pi]$)

36. (b) $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$

$\Rightarrow \sin^{-1}\frac{2}{5}+\cos^{-1}x=\cos^{-1}0=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{2}{5}+\frac{\pi}{2}-\sin^{-1}x=\frac{\pi}{2}$

$\left(\because \cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}\right)$

$\Rightarrow \sin^{-1}x=\sin^{-1}\left(\frac{2}{5}\right)$

$\therefore\ x=\frac{2}{5}$

37. (a) $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$

$=\cos^{-1}\left(\cos\left(\pi+\frac{\pi}{2}\right)\right)=\cos^{-1}\left(-\cos\frac{\pi}{2}\right)$

$=\pi-\cos^{-1}\left(\cos\frac{\pi}{2}\right)$ $\quad(\because \cos^{-1}(-\theta)=\pi-\cos^{-1}\theta)$

$=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

38. (b) $2\sec^{-1}(2)+\sin^{-1}\left(\frac{1}{2}\right)$

$=2\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$

$=\cos^{-1}\left(\frac{1}{2}\right)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$

$=\cos^{-1}\left(\frac{1}{2}\right)+\frac{\pi}{2}$ $\quad(\because \cos^{-1}\theta+\sin^{-1}\theta=\frac{\pi}{2})$

$=\frac{\pi}{3}+\frac{\pi}{2}=\frac{5\pi}{6}$

39. (a) $\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$

$\Rightarrow \frac{\pi}{2}-\cot^{-1}x+\frac{\pi}{2}-\cot^{-1}y=\frac{4\pi}{5}$

$\left\{\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right\}$

$\Rightarrow \pi-\frac{4\pi}{5}=\cot^{-1}x+\cot^{-1}y$

$\Rightarrow \cot^{-1}x+\cot^{-1}y=\frac{\pi}{5}$

40. (d) $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\Rightarrow 2\tan^{-1}a+2\tan^{-1}a=2\tan^{-1}x$

$\Rightarrow 4\tan^{-1}(a)=2\tan^{-1}x$

$\Rightarrow 2\tan^{-1}a=\tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\frac{2a}{1-a^2}\right)=\tan^{-1}x$

$\Rightarrow x=\frac{2a}{1-a^2}$

41. (a) $2\tan^{-1}x+\sin^{-1}\frac{2x}{1+x^2}=2\tan^{-1}x+2\tan^{-1}x=4\tan^{-1}x$

42. **(a)** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\left[\sin^{-1}\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$

43. **(b)** Let, $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)=\theta \Rightarrow \cos\theta=\frac{1}{\sqrt{5}}$

$\Rightarrow \sec\theta=\sqrt{5} \Rightarrow \sec^{-1}(\sqrt{5})=\theta$

$\Rightarrow \frac{\pi}{2}-\operatorname{cosec}^{-1}(\sqrt{5})=\theta$

$(\because \sec^{-1}x+\operatorname{cosec}^{-1}x=\frac{\pi}{2})$

$\Rightarrow \operatorname{cosec}^{-1}(\sqrt{5})=\frac{\pi}{2}-\theta$

44. **(c)** The given trigonometric expression is :

$\cos\left[\tan^{-1}\left\{\tan\left(\frac{15\pi}{4}\right)\right\}\right] = \cos\left[\tan^{-1}\left\{\tan\left(4\pi-\frac{\pi}{4}\right)\right\}\right]$

$= \cos\left[\tan^{-1}\left\{-\tan\frac{\pi}{4}\right\}\right] = \cos\left[\tan^{-1}\tan\left(\frac{-\pi}{4}\right)\right]$

Since $\tan^{-1}\theta$ is defined for $\frac{-\pi}{2}<\theta<\frac{-\pi}{2}$

$= \cos\left(\frac{-\pi}{4}\right) = \cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ [since $\cos(-\theta)=\cos\theta$]

45. **(c)** We have,

$\sin^{-1}x+\cos^{-1}x+\tan^{-1}x = \frac{\pi}{2}+\tan^{-1}x$

Now $\sin^{-1}x$ and $\cos^{-1}x$ are defined only if $-1\le x\le 1$

So, $-\frac{\pi}{4}\le\tan^{-1}x\le\frac{\pi}{4} \Rightarrow \frac{\pi}{4}\le\frac{\pi}{2}+\tan^{-1}x\le\frac{3\pi}{4}$

$\therefore\ k=\frac{\pi}{4}$ and $K=\frac{3\pi}{4}$

46. **(c)** $2\tan^{-1}\frac{1}{5}=\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{5}$

$=\tan^{-1}\frac{\frac{1}{5}+\frac{1}{5}}{1-\frac{1}{5}\cdot\frac{1}{5}}=\tan^{-1}\frac{\frac{2}{5}}{\frac{24}{25}}=\tan^{-1}\frac{5}{12}$

$\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)=\tan\left(\tan^{-1}\frac{5}{12}-\frac{\pi}{4}\right)$

$=\frac{\tan\left(\tan^{-1}\frac{5}{12}\right)-\tan\frac{\pi}{4}}{1+\tan\left(\tan^{-1}\frac{5}{12}\right)\tan\frac{\pi}{4}}$

$=\frac{\frac{5}{12}-1}{1+\frac{5}{12}.1}=\frac{-\frac{7}{12}}{\frac{17}{12}}=-\frac{7}{17}$

47. **(a)** Let $\cos^{-1}\frac{1}{8}=\theta$, where $0<\theta<\frac{\pi}{2}$. Then

$\frac{1}{2}\cos^{-1}\frac{1}{8}=\frac{1}{2}\theta \Rightarrow \cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)=\cos\frac{1}{2}\theta$

Now, $\cos^{-1}\frac{1}{8}=\theta$

$\Rightarrow \cos\theta=\frac{1}{8} \Rightarrow 2\cos^2\frac{\theta}{2}-1=\frac{1}{8} \Rightarrow \cos^2\frac{\theta}{2}=\frac{9}{16}$

$\Rightarrow \cos\frac{\theta}{2}=\frac{3}{4}$ $[\because 0<\frac{\theta}{2}<\frac{\pi}{4}$, so $\cos\frac{\theta}{2}\ne-\frac{3}{4}]$

48. **(c)** Let $\theta=\sin^{-1}\left[\sin\frac{5\pi}{3}\right]$

$\Rightarrow \sin\theta=\sin\frac{5\pi}{3}=\sin\left[2\pi-\frac{\pi}{3}\right]$

$\Rightarrow \sin\theta=-\sin\frac{\pi}{3}=\sin\left(\frac{-\pi}{3}\right)$ $(\because \sin(-\theta)=-\sin\theta)$

Therefore, principal value of $\sin^{-1}\left[\sin\frac{5\pi}{3}\right]$ is $\frac{-\pi}{3}$,

as principal value of $\sin^{-1}x$ lies between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$.

49. **(b)** Let $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-x)=\frac{\pi}{2}+2\sin^{-1}x$

$\Rightarrow (1-x)=\sin\left(\frac{\pi}{2}+2\sin^{-1}x\right)$

$\Rightarrow 1-x=\cos(2\sin^{-1}x)$ $(\because \sin(90°+\theta)=\cos\theta)$

$\Rightarrow 1-x=\cos[\cos^{-1}(1-2x^2)] \Rightarrow 1-x=1-2x^2$

$\Rightarrow 2x^2+1-x-1=0 \Rightarrow 2x^2-x=0 \Rightarrow x(2x-1)=0$

$\Rightarrow x=0,\ \frac{1}{2}$

INTEGER TYPE QUESTIONS

50. **(b)** We have. $6\sin^{-1}(x^2-6x+8.5)=\pi$

$\Rightarrow \sin^{-1}(x^2-6x+8.5)=\frac{\pi}{6}$

$\Rightarrow x^2-6x+8.5=\sin\frac{\pi}{6}=\frac{1}{2}$

$\Rightarrow x^2-6x+8.5-0.5=0$

$\Rightarrow x^2-6x+8=0$

$\Rightarrow (x-4)(x-2)=0$

$\Rightarrow x=4$ or $x=2$

51. **(a)** $\cot\left\{\frac{\pi}{4}-2\cot^{-1}3\right\}=\cot\left\{\frac{\pi}{4}-2\tan^{-1}\frac{1}{3}\right\}$

$=\cot\left\{\frac{\pi}{4}-\tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)\right\}$

$=\cot\left\{\frac{\pi}{4}-\tan^{-1}\left(\frac{3}{4}\right)\right\}=\frac{1}{\tan\left\{\frac{\pi}{4}-\tan^{-1}\left(\frac{3}{4}\right)\right\}}$

$=\frac{1+\frac{3}{4}}{1-\frac{3}{4}}=7$

52. **(d)** $\cos^{-1}\left[\cos\left(\frac{5\pi}{3}\right)\right]+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

$= \cos^{-1}\left\{\cos\left(2\pi-\frac{\pi}{3}\right)\right\}+\sin^{-1}\left\{\sin\left(2\pi-\frac{\pi}{3}\right)\right\}$

$= \cos^{-1}\left\{\cos\frac{\pi}{3}\right\}+\sin^{-1}\left(-\sin\frac{\pi}{3}\right)$

$= \frac{\pi}{3}-\frac{\pi}{3}=0$

53. **(a)** Let, $\sin^{-1}x=\tan^{-1}y=\theta$

$\Rightarrow x=\sin\theta$ and $y=\tan\theta$

$\frac{1}{x^2}=\frac{1}{\sin^2\theta}=\text{cosec}^2\theta$

and $\frac{1}{y^2}=\frac{1}{\tan^2\theta}=\cot^2\theta.$

$\Rightarrow \frac{1}{x^2}-\frac{1}{y^2}=\text{cosec}^2\theta-\cot^2\theta=1$

54. **(d)** $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\text{cosec}^{-1}(2)=k\frac{\pi}{12}$

$\frac{\pi}{4}+\frac{\pi}{6}=\frac{k\pi}{12}$

$\frac{5\pi}{12}=\frac{k\pi}{12}$

$\therefore$ k = 5

55. **(a)** $\frac{\sin(\tan^{-1}x+\cot^{-1}x)}{\sin(\sin^{-1}x+\cos^{-1}x)}=\frac{\sin\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)}=1$

56. **(a)** $\sin^{-1}\left(\frac{6x}{1+9x^2}\right)=\sin^{-1}\left(\frac{2(3x)}{1+(3x)^2}\right)$

$=2\tan^{-1}3x=2\tan^{-1}(ax)$

$\therefore$ a = 3

57. **(c)** Given that

$\tan^{-1}k-\tan^{-1}3=\tan^{-1}\frac{1}{13}$

or $\tan^{-1}\left(\frac{k-3}{1+3k}\right)=\tan^{-1}\frac{1}{13}$

or $\frac{k-3}{1+3k}=\frac{1}{13}$

or 13k – 39 = 1 + 3k

or 13k – 3k = 1 + 39

or 10k = 40

or k = 4

58. **(b)** $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)=\sin^{-1}\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}$

$=\sin^{-1}\left\{\sin\left(\frac{\pi}{4}\right)\right\}$

$=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}=\frac{2\pi}{k}$

$\therefore$ k = 8

59. **(b)** The given question can be written as

$\sin^{-1}\left(\frac{1}{5}\right)+\sec^{-1}(5)+\sec^{-1}(2)+\sin^{-1}\left(\frac{1}{2}\right)$

$+2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+2\tan^{-1}(\sqrt{3})=k\pi$

or $\left\{\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}\left(\frac{1}{5}\right)\right\}+\left\{\sec^{-1}(2)+\text{cosec}^{-1}(2)\right\}$

$+\left\{2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+2\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)\right\}=k\pi$

or $\frac{\pi}{2}+\frac{\pi}{2}+2\left\{\tan^{-1}\frac{1}{\sqrt{3}}+\cot^{-1}\frac{1}{\sqrt{3}}\right\}=k\pi$

or $\pi+2\left(\frac{\pi}{2}\right)=k\pi$

or $\pi+\pi=k\pi$

or $2\pi=k\pi$

or k = 2

ASSERTION- REASON TYPE QUESTIONS

60. **(a)** $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}.\frac{1}{7}}\right)=\tan^{-1}(1)=\frac{\pi}{4}$

$=\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y-x}{y+x}\right)=\frac{\pi}{4}$

Both Assertion and Reason are correct and Reason is correct explanation of Assertion.

61. **(b)** Reason is clearly correct.

Now any general term of the series

$t_k=\tan^{-1}\frac{2^{k-1}}{1+2^{2k-1}}=\tan^{-1}\frac{2^k-2^{k-1}}{1+2^k.2^{k-1}}$

$=\tan^{-1}2^k-\tan^{-1}2^{k-1}$

$\therefore$ Sum to n terms $=S_n=\tan^{-1}2^n-\tan^{-1}1$

Sum to infinite terms

$=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\tan^{-1}2^n-\tan^{-1}1=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

62. **(c)** $\sin\left[\tan^{-1}(-\sqrt{3})+\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

$=\sin\left[-\tan^{-1}\sqrt{3}+\pi-\cos^{-1}\frac{\sqrt{3}}{2}\right]$

$=\sin\left[-\frac{\pi}{3}+\pi-\frac{\pi}{6}\right]=\sin\frac{\pi}{2}=1$

Hence, Assertion is correct but Reason is incorrect.

63. **(a)** We have

$2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\left\{\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right\}+\tan^{-1}\frac{1}{7}$$

$$\left(\because\ 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right),\ \text{if} -1<x<1\right)$$

$$= \tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}=\tan^{-1}\left\{\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right\}=\tan^{-1}\frac{31}{17}$$

64. (d) $\sin^{-1}\left(\frac{2}{3}\right)+\cos^{-1}\left(\frac{2}{3}\right)-\tan^{-1}7-\cot^{-1}7-\cot^{-1}\left(\frac{1}{7}\right)$

$$=\frac{\pi}{2}-\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{7}\right)=-\tan^{-1}7$$

Hence, Assertion is incorrect and Reason is correct.

65. (b) We have,

$$\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}$$

$$=\sin^{-1}\left\{\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}+\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2}\right\}$$

$$=\sin^{-1}\left\{\frac{8}{17}\times\frac{4}{5}+\frac{3}{5}\times\frac{15}{17}\right\}=\sin^{-1}\left\{\frac{77}{85}\right\}$$

Hence, Assertion is correct and Reason is correct but Reason is not the correct explanation of Assertion.

66. (b) $\cot^{-1}\left[\dfrac{\left(\cos\frac{1}{2}x-\sin\frac{1}{2}x\right)+\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}x+\sin\frac{x}{2}\right)}\right]$

$$=\cot^{-1}\left(-\cot\frac{x}{2}\right)=\cot^{-1}\left(\cot\left(\pi-\frac{x}{2}\right)\right)=\pi-\frac{x}{2}.$$

Hence, both Assertion and Reason are correct but Reason is not correct explanation of Assertion.

67. (d) $2(\sin^{-1}x)^2-5(\sin^{-1}x)+2=0$

$$\Rightarrow\ \sin^{-1}x=\frac{5\pm\sqrt{25-16}}{4}=2,\frac{1}{2}$$

$\therefore\ \sin^{-1}x=\frac{1}{2},\ \sin^{-1}x=2$

$\therefore\ x=\sin\left(\frac{1}{2}\right)$ & $\sin^{-1}2$ is not possible

$\therefore\ x=\sin\left(\frac{1}{2}\right)$ is only solution

$\therefore$ Assertion is incorrect.

68. (d) If $x<0,\ \tan^{-1}\left(\frac{1}{x}\right)=-\pi+\cot^{-1}x$

$$\tan^{-1}x+\tan^{-1}\frac{1}{x}=\tan^{-1}x-\pi+\cot^{-1}x$$

$$=-\pi+\frac{\pi}{2}=-\frac{\pi}{2}$$

Assertion is incorrect but Reason is correct.

69. (c) We have,

$$\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left[\tan^{-1}\left(\frac{\sqrt{1-\frac{16}{25}}}{\frac{4}{5}}\right)+\tan^{-1}\frac{2}{3}\right]$$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}.\frac{2}{3}}\right]$$

$$=\tan\left(\tan^{-1}\frac{17}{6}\right)=\frac{17}{6}$$

Hence, Assertion is true and Reason is false.

70. (a) To possess inverse, the function must be one-one onto in the given domain.

71. (b) The tangent function for the intervals $\left(-\frac{3\pi}{2},-\frac{\pi}{2}\right),\left(-\frac{\pi}{2},\frac{\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ etc. is bijective and thus possesses inverse.

72. (d) The domain of the function $\sec^{-1}x$ is R – (–1, 1).

CRITICALTHINKING TYPE QUESTIONS

73. (a) $y=\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right)=\tan^{-1}\left\{\dfrac{3\frac{x}{a}-\frac{x^3}{a^3}}{1-3\frac{x^2}{a^2}}\right\}$

Let $\frac{x}{a}=\tan\theta$, then $y=\tan^{-1}\tan3\theta$

If $-\frac{\pi}{2}<3\theta<\frac{\pi}{2}\Rightarrow-\frac{\pi}{6}<\theta<\frac{\pi}{6}$, then

$$y=3\theta=3\tan^{-1}\frac{x}{a}$$

$$\therefore\ -\frac{a}{\sqrt{3}}<x<\frac{a}{\sqrt{3}}\Rightarrow y=3\tan^{-1}\frac{x}{a}$$

74. (d) Let $\sin^{-1}\left(\frac{x}{5}\right)+\text{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$

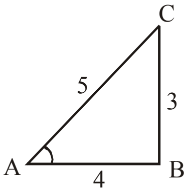
$$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right)=\frac{\pi}{2}-\text{cosec}^{-1}\left(\frac{5}{4}\right)$$

$$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right)=\frac{\pi}{2}-\sin^{-1}\left(\frac{4}{5}\right)$$

$[\because \sin^{-1}x+\cos^{-1}x=\pi/2]$

$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right)=\cos^{-1}\left(\frac{4}{5}\right)$ ...(i)

Let $\cos^{-1}\frac{4}{5}=A\Rightarrow\cos A=\frac{4}{5}$



$\Rightarrow\ \sin A=\frac{3}{5}$

$\Rightarrow\ A=\sin^{-1}\frac{3}{5}$

$\therefore$ $\cos^{-1}(4/5)=\sin^{-1}(3/5)$

$\therefore$ equation (i) become,

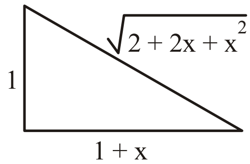
$$\sin^{-1}\frac{x}{5}=\sin^{-1}\frac{3}{5}\Rightarrow\frac{x}{5}=\frac{3}{5}\Rightarrow x=3$$

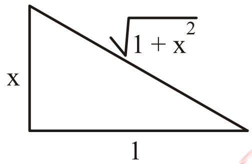
75. (b) $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=\tan^{-1}\frac{x}{y}-\tan^{-1}\left[\frac{1-\frac{y}{x}}{1+\frac{y}{x}}\right]$

$=\tan^{-1}\frac{x}{y}-\tan^{-1}1+\tan^{-1}\frac{y}{x}$

$=\tan^{-1}\frac{x}{y}+\cot^{-1}\frac{x}{y}-\tan^{-1}1=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

76. (d)





$\sin\left(\cot^{-1}(1+x)\right)=\frac{1}{\sqrt{2+2x+x^2}}$

$\cos(\tan^{-1}x)=\frac{1}{\sqrt{1+x^2}}$

So, $\frac{1}{\sqrt{2+2x+x^2}}=\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow\ 1+2x=0$

$\Rightarrow\ x=-\frac{1}{2}$

77. (b) $0\le\cos^{-1}[x]\le\pi,-1\le[x]\le1$

$\Rightarrow\ -1\le x<2\ \Rightarrow x\in[-1,2)$

78. (c) $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)$

$=[\sec(\tan^{-1}2)]^2+[\operatorname{cosec}(\cot^{-1}3)]^2$

$=\left[\sec\left(\sec^{-1}\sqrt{5}\right)\right]^2+\left[\operatorname{cosec}\left(\operatorname{cosec}^{-1}\sqrt{10}\right)\right]^2$

$=\left(\sqrt{5}\right)^2+\left(\sqrt{10}\right)^2=5+10=15$

79. (a) $\tan\left[\cos^{-1}\left(\frac{1}{\sqrt{82}}\right)-\sin^{-1}\left(\frac{5}{\sqrt{26}}\right)\right]$

$=\tan(\tan^{-1}9-\tan^{-1}5)$

$=\tan\left\{\tan^{-1}\left(\frac{9-5}{1+9\times5}\right)\right\}=\frac{2}{23}$

80. (a) Given, $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$ ...(i)

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ ...(ii)

Adding equations (i) and (ii), we get

$2\sin^{-1}x=\frac{2\pi}{3}\ \Rightarrow\ \sin^{-1}x=\frac{\pi}{3}$

$\Rightarrow\ x=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\therefore$ Given equation has unique solution.

81. (b) $\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2=\frac{51}{50}$

$\Rightarrow\left\{x\cos\left(\tan^{-1}\frac{1}{x}\right)+\sin\left(\tan^{-1}\frac{1}{x}\right)\right\}^2=\frac{51}{50}$

$\Rightarrow\left\{x\cos\left[\cos^{-1}\left(\frac{1}{\sqrt{1+\frac{1}{x^2}}}\right)\right]+\sin\left[\sin^{-1}\left(\frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}}\right)\right]\right\}^2=\frac{51}{50}$

$\Rightarrow\left(\frac{x^2}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right)^2=\frac{51}{50}$

$\Rightarrow\ \frac{(x^2+1)^2}{(x^2+1)}=\frac{51}{50}$

$\Rightarrow\ x^2+1=\frac{51}{50}$

$\Rightarrow\ x=\pm\frac{1}{5\sqrt{2}}$

82. (d) $\sin\left\{2\cos^{-1}\left(-\frac{3}{5}\right)\right\}$

$=2\sin\left\{\cos^{-1}\left(-\frac{3}{5}\right)\right\}\cos\left\{\cos^{-1}\left(-\frac{3}{5}\right)\right\}$

$=2\sin\left\{\pi-\cos^{-1}\frac{3}{5}\right\}\times\left(-\frac{3}{5}\right)$

$=\frac{-6}{5}\sin\left\{\cos^{-1}\frac{3}{5}\right\}=\frac{-6}{5}\sin\left(\sin^{-1}\sqrt{1-\frac{9}{25}}\right)$

$=\frac{-6}{5}\times\frac{4}{5}=\frac{-24}{25}$

83. (a) $\frac{-\pi}{2}\le\sin^{-1}\sqrt{x-1}\le\frac{\pi}{2}$

$\Rightarrow\ -1\le\sqrt{x-1}\le1$

$\Rightarrow\ 0\le x-1<1$

$\Rightarrow\ 1\le x\le2$

$\therefore$ Domain of $f(x)$ is $[1,2]$

84. **(b)** Let $\tan^{-1}\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)=\theta$

$\Rightarrow \cos^{-1}\frac{2}{\sqrt{5}}=2\tan\theta$

$\Rightarrow \cos^{-1}\frac{2}{\sqrt{5}}=\cos^{-1}\left(\frac{1-\theta^2}{1+\theta^2}\right) \Rightarrow \frac{2}{\sqrt{5}}=\frac{1-\theta^2}{1+\theta^2}$

$\Rightarrow 2+2\theta^2=\sqrt{5}-\sqrt{5}\theta^2$

$\Rightarrow \theta^2\left(\sqrt{5}+2\right)=\sqrt{5}-2$

$\Rightarrow \theta^2=\frac{\sqrt{5}-2}{\sqrt{5}+2}$

$\Rightarrow \theta^2=\left(\sqrt{5}-2\right)^2 \Rightarrow \theta=\sqrt{5}-2$

85. **(d)** As given: $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$

and we know that $0\le\cos^{-1}x\le\pi$

$\therefore \cos^{-1}x=\pi, \cos^{-1}y=\pi, \cos^{-1}z=\pi$

$\therefore x=y=z=\cos\pi=-1.$

$\therefore xy+yz+zx=(-1)(-1)+(-1)(-1)+(-1)(-1)$
$=1+1+1=3$

86. **(b)** Given equation is

$\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=3\pi/2$

$\Rightarrow \sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\pi/2$

$\Rightarrow x=y=z=\sin\pi/2=1$

$\Rightarrow x=y=z=1$

$\Rightarrow x+y+z=1+1+1=3$

87. **(d)** Given: $xy+yz+zx=1$ ....(i)

Now, we know $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z$

$=\tan^{-1}\left[\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right]$

using equation (i), we have

$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left(\frac{1}{0}\right)=\tan\infty$

$\Rightarrow \tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$

88. **(b)** Consider $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)+\cot^{-1}3$ ...(i)

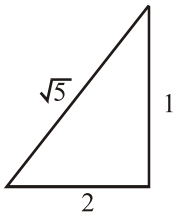
We have, $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)=\cot^{-1}2$

$\therefore$ From equation (i), we have

$\cos^{-1}2+\cot^{-1}3=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$

$=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)$

$=\tan^{-1}\left(\frac{5/6}{\frac{6-1}{6}}\right)=\tan^{-1}1=\frac{\pi}{4}$



89. **(d)** Given, $\sin^{-1}x-\sin^{-1}2x=\pm\frac{\pi}{3}$

$\sin^{-1}x-\sin^{-1}2x=\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \sin^{-1}x-\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)=\sin^{-1}2x$

$\Rightarrow \sin^{-1}\left[x\sqrt{1-\frac{3}{4}}-\left(\pm\frac{\sqrt{3}}{2}\right)\sqrt{1-x^2}\right]=\sin^{-1}2x$

$\Rightarrow \frac{x}{2}-\left(\pm\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)=2x$

$\Rightarrow -(\pm\sqrt{3}\sqrt{1-x^2})=3x$

On squaring both sides, we get

$3(1-x^2)=9x^2$

$\Rightarrow 1-x^2=3x^2 \Rightarrow 4x^2=1 \Rightarrow x=\pm\frac{1}{2}$

CHAPTER 19
MATRICES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Let $F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$ where $\alpha \in \mathbf{R}$. Then $[F(\alpha)]^{-1}$ is equal to

(a) $F(-\alpha)$ (b) $F(\alpha^{-1})$
(c) $F(2\alpha)$ (d) None of these

2. Let $A=\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}$, then A^n is equal to

(a) $\begin{bmatrix}a^n & 0 & 0\\ 0 & a^n & 0\\ 0 & 0 & a\end{bmatrix}$ (b) $\begin{bmatrix}a^n & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}$

(c) $\begin{bmatrix}a^n & 0 & 0\\ 0 & a^n & 0\\ 0 & 0 & a^n\end{bmatrix}$ (d) $\begin{bmatrix}na & 0 & 0\\ 0 & na & 0\\ 0 & 0 & na\end{bmatrix}$

3. If $A=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}$ and $B=\begin{bmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{bmatrix}$, then AB is equal to

(a) B (b) A (c) O (d) I

4. If A is a square matrix such that $(A-2I)(A+I)=0$, then $A^{-1}=$

(a) $\frac{A-I}{2}$ (b) $\frac{A+I}{2}$ (c) $2(A-I)$ (d) $2A+I$

5. If $[1\ x\ 1]\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 1\\ 0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix}=0$, then x is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) -1

6. If $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, then A^2+2A equals

(a) 4A (b) 3A (c) 2A (d) A

7. If $\begin{bmatrix}x+y & 2x+z\\ x-y & 2z+w\end{bmatrix}=\begin{bmatrix}4 & 7\\ 0 & 10\end{bmatrix}$, then the values of x, y, z and w respectively are

(a) 2, 2, 3, 4 (b) 2, 3, 1, 2
(c) 3, 3, 0, 1 (d) None of these

8. If $A=[a_{ij}]_{2\times 2}$, where $a_{ij}=\frac{(i+2j)^2}{2}$, then A is equal to

(a) $\begin{bmatrix}9 & 25\\ 8 & 18\end{bmatrix}$ (b) $\begin{bmatrix}9/2 & 25/2\\ 8 & 18\end{bmatrix}$

(c) $\begin{bmatrix}9 & 25\\ 4 & 9\end{bmatrix}$ (d) $\begin{bmatrix}9/2 & 15/2\\ 4 & 9\end{bmatrix}$

9. A square matrix $A=[a_{ij}]_{n\times n}$ is called a lower triangular matrix if $a_{ij}=0$ for

(a) $i=j$ (b) $i<j$ (c) $i>j$ (d) None of these

10. For what values of x and y are the following matrices equal

$A=\begin{bmatrix}2x+1 & 3y\\ 0 & y^2-5y\end{bmatrix}$, $B=\begin{bmatrix}x+3 & y^2+2\\ 0 & -6\end{bmatrix}$

(a) 2, 3 (b) 3, 4 (c) 2, 2 (d) 3, 3

11. The order of the single matrix obtained from

$\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 3\end{bmatrix}\left\{\begin{bmatrix}-1 & 0 & 2\\ 2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 23\\ 1 & 0 & 21\end{bmatrix}\right\}$ is

(a) 2×3 (b) 2×2 (c) 3×2 (d) 3×3

12. A square matrix $A=[a_{ij}]_{n\times n}$ is called a diagonal matrix if $a_{ij}=0$ for

(a) $i=j$ (b) $i<j$ (c) $i>j$ (d) $i\neq j$

13. If $\begin{bmatrix}x+3 & z+4 & 2y-7\\ 4x+6 & a-1 & 0\\ b-3 & 3b & z+2c\end{bmatrix}=\begin{bmatrix}0 & 6 & 3y-2\\ 2x & -3 & 2c+2\\ 2b+4 & -21 & 0\end{bmatrix}$

then, the values of a, b, c, x, y and z respectively are

(a) −2, −7, −1, −3, −5, −2 (b) 2, 7, 1, 3, 5, −2
(c) 1, 3, 4, 2, 8, 9 (d) −1, 3, −2, −7, 4, 5

14. If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then PQ is equal to

(a) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

15. If $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & x \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $D = \begin{bmatrix} 15+x \\ 1 \end{bmatrix}$ such that $(2A - 3B)\,C = D$, then $x =$

(a) 3 (b) -4 (c) -6 (d) 6

16. For any square matrix A, AA^T is a

(a) unit matrix (b) symmetric matrix
(c) skew-symmetric matrix (d) diagonal matrix

17. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then AB is equal to

(a) $\begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 11 & 4 & 3 \\ 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 8 & 4 \\ 2 & 9 & 6 \\ 0 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 3 \\ 1 & 8 & 2 \end{bmatrix}$

18. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying $AA^T = 9I_3$, then the values of a and b respectively are

(a) 1, 2 (b) $-2, -1$ (c) $-1, 2$ (d) $-2, 1$

19. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) I (d) 3 A

20. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then AA^T is

(a) Zero matrix (b) I_2

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) None of these

21. If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$, then A is a

(a) symmetric matrix
(b) skew-symmetric matrix
(c) diagonal matrix
(d) none of these

22. If $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$, then $(AB)'$ is equal to,

(a) $\begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 4 & -3 \\ 2 & 8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

23. Each diagonal element of a skew-symmetric matrix is

(a) zero (b) positive (c) non-real (d) negative

24. If $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ is the sum of a symmetric matrix B and a skew-symmetric matrix C, then C is

(a) $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -3/2 \\ 5/2 & 1 \end{bmatrix}$

25. If a matrix A is both symmetric and skew-symmetric, then

(a) A is a diagonal matrix (b) A is zero matrix
(c) A is a scalar matrix (d) A is square matrix

26. If ω is a complex cube root of unity, then the matrix

$A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$ is

(a) symmetric matrix (b) diagonal matrix
(c) skew-symmetric matrix (d) None of these

27. Using elementary transformation, the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ is

(a) $\begin{bmatrix} 3 & -3 & 3 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -3 & 3 \\ -4 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -2 & 1 \\ -4 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

28. The inverse of the matrix $A = \begin{bmatrix} 0 & 2 & -2 \\ -1 & 3 & 0 \\ 1 & -2 & 1 \end{bmatrix}$ by using elementary row transformations, is equal to

(a) $\frac{1}{4}\begin{bmatrix} 3 & 2 & 6 \\ 2 & 2 & 2 \\ -1 & 2 & 2 \end{bmatrix}$ (b) $\frac{1}{4}\begin{bmatrix} 4 & 2 & 6 \\ 1 & 2 & 2 \\ -1 & 2 & 2 \end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix} 4 & 2 & 6 \\ 1 & 2 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ (d) $\frac{1}{4}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 2 & 2 \\ -1 & 2 & 2 \end{bmatrix}$

29. If $A=\begin{bmatrix}1 & 2\\ 3 & -5\end{bmatrix}$, then A^{-1} is equal to

(a) $\frac{1}{11}\begin{bmatrix}-5 & -2\\ -3 & 1\end{bmatrix}$ (b) $\frac{1}{11}\begin{bmatrix}5 & 2\\ 3 & -1\end{bmatrix}$

(c) $\frac{1}{11}\begin{bmatrix}-5 & -2\\ -3 & -1\end{bmatrix}$ (d) $\frac{1}{11}\begin{bmatrix}5 & -2\\ 3 & -1\end{bmatrix}$

30. If $A=\begin{bmatrix}2x & 0\\ x & x\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$, then x equals

(a) 2 (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{1}{2}$

31. If $A^2-A+I=O$, then the inverse of A is

(a) $I-A$ (b) $A-I$ (c) A (d) $A+I$

32. The inverse of the matrix $A=\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$, using elementary row transformation, is equal to

(a) $\begin{bmatrix}7 & -3\\ -2 & 1\end{bmatrix}$ (b) $\begin{bmatrix}5 & -3\\ -2 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & -3\\ -2 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -3\\ 2 & 1\end{bmatrix}$

33. If A and B are matrices of same order, then $(AB'-BA')$ is a

(a) skew symmetric matrix (b) null matrix

(c) symmetric matrix (d) unit matrix

34. For any two matrices A and B, we have

(a) $AB=BA$ (b) $AB\neq BA$

(c) $AB=O$ (d) None of these

35. If matrix $A=[a_{ij}]_{2\times 2}$, where $a_{ij}=\begin{cases}1 & \text{if } i\neq j\\ 0 & \text{if } i=j\end{cases}$, then A^2 is equal to

(a) I (b) A

(c) O (d) None of these

36. If A is a square matrix such that $A^2=I$, then $(A-I)^3+(A+I)^3-7A$ is equal to

(a) A (b) $I-A$ (c) $I+A$ (d) 3A

37. Let A and B be two matrices then (AB)' equals:

(a) B'A' (b) A'B (c) $-AB$ (d) 1

38. The matrix product

$$\begin{bmatrix}a\\ b\\ c\end{bmatrix}\times[x\ y\ z]\times\begin{bmatrix}p\\ q\\ r\end{bmatrix} \text{ equals :}$$

(a) $\frac{pqr-abc}{xyz}$ (b) $\frac{xyz\,.\,pqr}{abc}$

(c) $\frac{pqr\,.\,abc}{xyz}$ (d) None of these

39. If $A=\begin{bmatrix}a & b\\ b & a\end{bmatrix}$ and $A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}$, then:

(a) $\alpha=a^2+b^2,\ \beta=ab$

(b) $\alpha=a^2+b^2,\ \beta=2ab$

(c) $\alpha=a^2+b^2,\ \beta=a^2-b^2$

(d) $\alpha=2ab,\ \beta=a^2+b^2$

40. The matrix $\begin{bmatrix}2 & 5 & -7\\ 0 & 3 & 11\\ 0 & 0 & 9\end{bmatrix}$ is :

(a) symmetric (b) diagonal

(c) upper triangular (d) skew symmetric

41. For the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & m & -1\end{bmatrix}$, A^2 is equal to

(a) 0 (b) A (c) I (d) None of these

42. The construction of 3×4 matrix A whose elements a_{ij} is given by $\frac{(i+j)^2}{2}$ is

(a) $\begin{bmatrix}2 & 9/2 & 8 & 25\\ 9 & 4 & 5 & 18\\ 8 & 25 & 18 & 49\end{bmatrix}$

(b) $\begin{bmatrix}2 & 9/2 & 25/2 & 9\\ 9/2 & 5/2 & 5 & 45/2\\ 25 & 18 & 25 & 9/2\end{bmatrix}$

(c) $\begin{bmatrix}2 & 9/2 & 8 & 25/2\\ 9/2 & 8 & 25/2 & 18\\ 8 & 25/2 & 18 & 49/2\end{bmatrix}$

(d) None of these

43. If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$, then

(a) AB, BA exist and are equal

(b) AB, BA exist and are not equal

(c) AB exists and BA does not exist

(d) AB does not exist and BA exists

44. If A is a square matrix, then $A+A^T$ is

(a) Non-singular matrix (b) Symmetric matrix

(c) Skew-symmetric matrix (d) Unit matrix

45. For a matrix A, $AI=A$ and $AA^T=I$ is true for

(a) If A is a square matrix. (b) If A is a non singular matrix.

(c) If A is symmetric matrix. (d) If A is any matrix.

46. What is true about matrix multiplication ?

(a) It is commutative. (b) It is associative.

(c) Both of the above. (d) None of the above.

47. If $\begin{bmatrix}x+y+z\\ x+y\\ y+z\end{bmatrix}=\begin{bmatrix}9\\ 5\\ 7\end{bmatrix}$ then the value of (x, y, z) is:

(a) (4, 3, 2) (b) (3, 2, 4)

(c) (2, 3, 4) (d) None of these

48. $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$ is equal to:

(a) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

49. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then A^{16} is equal to :

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

50. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then $f(A)$ is equal to

(a) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

51. If $R(t) = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$, then R(s) R(t) equals

(a) $R(s+t)$ (b) $R(s-t)$

(c) $R(s) + R(t)$ (d) None of these

52. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, then the value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$.

53. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then $A^2 - 5A + 6I =$

(a) $\begin{bmatrix} 1 & -1 & -5 \\ -1 & -1 & 4 \\ -3 & -10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

(c) 0 (d) I

54. If A is a $m \times n$ matrix with entries a_{ij}, then the matrix A can be represented as

(a) $A = [a_{ij}]_{m \times n}$ (b) $A = [a_{ji}]_{m \times n}$

(c) $A = [a_{ij}]_{n \times m}$ (d) $A = [a_{ji}]_{n \times m}$

55. If A is a 3×2 matrix, B is a 3×3 matrix and C is a 2×3 matrix, then the elements in A, B and C are respectively

(a) 6, 9, 8 (b) 6, 9, 6 (c) 9, 6, 6 (d) 6, 6, 9

56. If A is a matrix having m rows and n columns, then the matrix A is called a matrix of order

(a) $m \times n$ (b) $n \times m$ (c) m^n (d) n^m

57. If $A = [a_{ij}]_{3 \times 4}$ is matrix given by

$$A = \begin{bmatrix} 4 & -2 & 1 & 3 \\ 5 & 7 & 9 & 6 \\ 21 & 15 & 18 & -25 \end{bmatrix}$$

Then, $a_{23} + a_{24}$ will be equal to the element

(a) a_{14} (b) a_{44} (c) a_{13} (d) a_{32}

58. If A is a square matrix of order m with elements a_{ij}, then

(a) $A = [a_{ij}]_{n \times n}$ (b) $A = [a_{ji}]_{m \times n}$

(c) $A = [a_{ij}]_{m \times m}$ (d) $A = [a_{ji}]_{n \times n}$

59. A square matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix, if

(a) all its non-diagonal elements are non-zero i.e., $b_{ji} \neq 0$; $i \neq j$

(b) all its diagonal elements are zero, i.e., $b_{ij} = 0$, $i = j$

(c) all its non-diagonal elements are zero i.e, $b_{ij} = 0$ when $i \neq j$

(d) None of the above

60. Choose the incorrect statement.

(a) A matrix A = [3] is a scalar matrix of order 1

(b) A matrix $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is a scalar matrix of order 2

(c) A matrix $C = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ of order 3 is not a scalar matrix

(d) None of the above

61. A square matrix $B = [b_{ij}]_{n \times n}$ is said to be a scalar matrix, if

(a) $b_{ij} = 0$ for $i \neq j$ and $b_{ij} = k$ for $i = j$, for some constant k

(b) $b_{ij} = 0$ for $i = j$

(c) $b_{ij} \neq 0$ for $i = j$ and $b_{ij} = 0$ for $i = j$

(d) None of the above

62. If the diagonal elements of a diagonal matrix are all equal, then the matrix is called

(a) row matrix (b) scalar matrix

(c) rectangular matrix (d) None of the above

63. Which of the following is correct statement?

(a) Diagonal matrix is also a scalar matrix

(b) Identity matrix is a particular case of scalar matrix

(c) Scalar matrix is not a diagonal matrix

(d) Null matrix cannot be a square matrix

64. If $A = [a_{ij}]$ is a matrix of order 4×5, then the diagonal elements of A are

(a) $a_{11}, a_{22}, a_{33}, a_{44}$ (b) $a_{55}, a_{44}, a_{33}, a_{22}, a_{11}$

(c) a_{11}, a_{22}, a_{33} (d) do not exist

65. If the matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ and $C = [c_{ij}]$ are of the same order, say $m \times n$, satisfy Associative law, then

(a) $(A + B) + C = A + (B + C)$

(b) $A + B = B + C$

(c) $A + C = B + C$

(d) $A + B + C = A - B - C$

66. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b are respectively.

(a) –6, –12, –18 (b) –6, 4, 9

(c) –6, –4, –9 (d) –6, 12, 18

67. Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{jk}]$ be an $n \times p$ matrix. Then, the product of the matrices A and B is the matrix C of order.

(a) $n \times m$ (b) $m \times n$

(c) $p \times m$ (d) $m \times p$

68. The product of two matrices A and B is defined, if the number of columns of A is

(a) greater than the number of rows of B

(b) equal to the number of rows of B

(c) less than the number of rows of B

(d) None of the above

69. The matrix X such that

$X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$ is

(a) $\begin{bmatrix}1 & 2\\ 2 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\ -2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2\\ 0 & -2\end{bmatrix}$

70. If $A=[a_{ij}]_{m\times n}$, then A' is equal to

(a) $[a_{ji}]_{n\times m}$ (b) $[a_{ij}]_{m\times n}$

(c) $[a_{ji}]_{m\times n}$ (d) $[a_{ij}]_{n\times m}$

71. After applying $R_2\to R_2-2R_1$ to $C=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$, we get

(a) $\begin{bmatrix}1 & 2\\ 2 & -5\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\ 0 & -5\end{bmatrix}$

(c) $\begin{bmatrix}1 & 4\\ 2 & -3\end{bmatrix}$ (d) $\begin{bmatrix}2 & -1\\ 1 & 2\end{bmatrix}$

72. If A is a square matrix of order m, then the matrix B of same order is called the inverse of the matrix A, if

(a) $AB=A$ (b) $BA=A$

(c) $AB=BA=I$ (d) $AB=-BA$

73. If X, A and B are matrices of the same order such that $X=AB$, then we apply elementary column transformation simultaneously on X and on the matrix

(a) B (b) A

(c) AB (d) Both A and B

74. If $A=\begin{pmatrix}2 & -1\\ -7 & 4\end{pmatrix}$ and $B=\begin{pmatrix}4 & 1\\ 7 & 2\end{pmatrix}$ then which statement is true ?

(a) $AA^T=I$ (b) $BB^T=I$

(c) $AB\neq BA$ (d) $(AB)^T=I$

75. If $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $J=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then value of B in terms of I and J is

(a) $I\sin\theta+J\cos\theta$ (b) $I\sin\theta-J\cos\theta$

(c) $I\cos\theta+J\sin\theta$ (d) $-I\sin\theta+J\cos\theta$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

76. Consider the following statements

I. For multiplication of two matrices A and B, the number of columns in A should be less than the number of rows in B.

II. For getting the elements of the product matrix, we take rows of A and column of B, multiply them elementwise and take the sum.

Choose the correct option.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Neither I nor II is true

77. Consider the matrix $A=\begin{bmatrix}2 & 5 & 19 & -7\\ 35 & -2 & 5/2 & 12\\ \sqrt{3} & 1 & -5 & 17\end{bmatrix}$

Now, consider the following statements

I. The order of the matrix is 4×3 and number of elements is 12.

II. The elements a_{13}, a_{21}, a_{33} are respectively 19, 35, –5.

Choose the correct option.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Neither I nor II is true

78. Consider the following statements.

I. If a matrix has 24 elements, then all the possible orders it can have are 24×1, 1×24, 2×4, 4×2, 2×12, 12×2, 3×8, 8×3, 4×6 and 6×4.

II. For a matrix having 13 elements, its all possible orders are 1×13 and 13×1.

III. For a matrix having 18 elements, its all possible orders are 18×1, 1×18, 2×9, 9×2, 3×6, 6×3.

IV. For a matrix having 5 elements, its all possible orders are 1×5 and 5×1.

Choose the correct option

(a) Only I is false (b) Only II is a false

(c) Only III is false (d) All are true

79. Let $A=[4]$, $B=\begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}$ and $C=\begin{bmatrix}-1.1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$ are the matrices.

Consider the following statements

I. The matrices A, B and C are diagonal matrices.

II. The matrices A, B and C are of order 1, 3 and 2, respectively.

Choose the correct option.

(a) I is true and II is false (b) I is false and II is true

(c) Both I and II are true (d) Both I and II are false

80. Consider the following statements

I. Scalar matrix $A=[a_{ij}]=\begin{cases}k; & i=j\\ 0; & i\neq j\end{cases}$ where k is a scalar, in an identity matrix when $k=1$.

II. Every identity matrix is not a scalar matrix.

Choose the correct option.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Both I and II are false

81. Consider the following statements

I. If AB and BA are both defined, then they must be equal i.e., $AB=BA$.

II. If AB and BA are both defined, it is not necessary that $AB=BA$.

Choose the correct option.

(a) Only I is true (b) Only II is true

(c) both I and II are true (d) None of these

82. Let A, B and C are three matrices of same order. Now, consider the following statements

I. If $A=B$, then $AC=BC$

II. If $AC=BC$, then $A=B$

Choose the correct option

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Neither I nor II is true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

83. **Column -I** | **Column- II**

A. If $A=[a_{ij}]_{2\times 2}$ is a matrix, where $a_{ij}=\frac{(i+j)^2}{2}$, then a_{21} is — 1. $\frac{49}{2}$

B. If $B=[b_{ij}]_{2\times 3}$ is a matrix, where $b_{ij}=\frac{(i+2j)^2}{2}$, then b_{13} is — 2. 1

C. If $C=[c_{ij}]_{3\times 4}$ is a matrix, where $c_{ij}=\frac{1}{2}|-3i+j|$, then c_{11} is — 3. 2

D. If $D=[d_{ij}]_{3\times 4}$ is a matrix, where $d_{ij}=2i-j$, then d_{34} is — 4. $\frac{9}{2}$

Codes

	A	B	C	D
(a)	1	4	2	3
(b)	2	4	3	1
(c)	4	2	1	3
(d)	4	1	2	3

84. **Column - I (Equal matrices)** | **Column - II (Values of x, y, z)**

A. $\begin{bmatrix}4 & 3\\ x & 5\end{bmatrix}=\begin{bmatrix}y & z\\ 1 & 5\end{bmatrix}$ — 1. $x=2, y=4, z=0$

B. $\begin{bmatrix}x+y & 2\\ 5+z & xy\end{bmatrix}=\begin{bmatrix}6 & 2\\ 5 & 8\end{bmatrix}$ — 2. $x=2, y=4, z=3$

C. $\begin{bmatrix}x+y+z\\ x+z\\ y+z\end{bmatrix}=\begin{bmatrix}9\\ 5\\ 7\end{bmatrix}$ — 3. $x=1, y=4, z=3$

Codes

	A	B	C
(a)	1	2	3
(b)	3	2	1
(c)	2	1	3
(d)	3	1	2

85. **Column - I** | **Column - II**

A. $\begin{bmatrix}a & b\\ -b & a\end{bmatrix}+\begin{bmatrix}a & b\\ b & a\end{bmatrix}$ — 1. $\begin{bmatrix}(a+b)^2 & (b+c)^2\\ (a-c)^2 & (a-b)^2\end{bmatrix}$

B. $\begin{bmatrix}a^2+b^2 & b^2+c^2\\ a^2+c^2 & a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab & 2bc\\ -2ac & -2ab\end{bmatrix}$ — 2. $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

C. $\begin{bmatrix}\cos^2 x & \sin^2 x\\ \sin^2 x & \cos^2 x\end{bmatrix}+\begin{bmatrix}\sin^2 x & \cos^2 x\\ \cos^2 x & \sin^2 x\end{bmatrix}$ — 3. $\begin{bmatrix}2a & 2b\\ 0 & 2a\end{bmatrix}$

Codes

	A	B	C
(a)	3	2	1
(b)	1	3	2
(c)	2	1	3
(d)	3	1	2

86. **Column - I Matrices** | **Column - II Their product**

A. $\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$ — 1. $\begin{bmatrix}11 & 10\\ 11 & 2\end{bmatrix}$

B. $\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$ — 2. $\begin{bmatrix}2 & 3 & 4\\ 4 & 6 & 8\\ 6 & 9 & 12\end{bmatrix}$

C. $\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}$ — 3. $\begin{bmatrix}-6 & 26\\ -1 & 19\end{bmatrix}$

Codes

	A	B	C
(a)	2	1	3
(b)	3	2	1
(c)	3	1	2
(d)	1	3	2

87. **Column - I** | **Column - II**

A. $(A')'$ — 1. $B'A'$

B. $(kA)'$, where k is any constant — 2. A

C. $(A+B)'$ — 3. kA'

D. $(AB)'$ — 4. $A'+B'$

5. $A'B'$

Codes

	A	B	C	D
(a)	2	4	3	5
(b)	1	4	3	2
(c)	3	1	4	2
(d)	2	3	4	1

88. **Column - I (Matrices)** | **Column - II (Inverse of matrices)**

A. $\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ — 1. $\begin{bmatrix}-7 & 3\\ 5 & -2\end{bmatrix}$

B. $\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$ — 2. $\begin{bmatrix}7 & -3\\ -2 & 1\end{bmatrix}$

C. $\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$ — 3. $\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$

D. $\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}$ — 4. $\begin{bmatrix}3/5 & 1/5\\ -2/5 & 1/5\end{bmatrix}$

Codes

	A	B	C	D
(a)	2	3	1	4
(b)	4	3	2	1
(c)	3	4	1	2
(d)	3	1	4	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

89. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}x & 1\\ y & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then $x+y=$

(a) 2 (b) 3 (c) 4 (d) 5

90. Given that $\begin{bmatrix}x+y\\ x+y+z\\ y+z\end{bmatrix}=\begin{bmatrix}8\\ 9\\ 4\end{bmatrix}$,

then x =

(a) 2 (b) 3 (c) 4 (d) 5

91. Given that $A=\begin{bmatrix}3 & 2\\ 5 & 7\\ 8 & 9\end{bmatrix}$, $B=\begin{bmatrix}1 & 9\\ 0 & 3\\ 4 & 10\end{bmatrix}$ and $X=\begin{bmatrix}6 & 29\\ 5 & 16\\ 20 & 39\end{bmatrix}$

and if 2A + 6B = kX, then the value of k is

(a) 2 (b) 3 (c) 4 (d) 5

92. If $\begin{bmatrix}x & y\\ 2 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}23 & 34\\ 14 & 20\end{bmatrix}$,

then y =

(a) 6 (b) 1 (c) 8 (d) 9

93. If $B^n - A = I$

and $A=\begin{bmatrix}26 & 26 & 18\\ 25 & 37 & 17\\ 52 & 39 & 50\end{bmatrix}$, $B=\begin{bmatrix}1 & 4 & 2\\ 3 & 5 & 1\\ 7 & 1 & 6\end{bmatrix}$,

then n =

(a) 2 (b) 3 (c) 4 (d) 5

94. Given that

$$\begin{bmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{bmatrix}\begin{bmatrix}k & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

then k =

(a) 6 (b) 1 (c) 8 (d) 9

95. Consider the matrix

$A=\begin{bmatrix}4 & 3\\ 1 & 5\end{bmatrix}$

On applying elementary row operation $R_2 \to R_2 - nR_1$, it becomes $\begin{bmatrix}4 & 3\\ -11 & -4\end{bmatrix}$, then the value of n =

(a) 2 (b) 3 (c) 4 (d) 5

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

96. **Assertion :** The possible dimensions of a matrix containing 32 elements is 6.

Reason : The No. of ways of expressing 32 as a product of two positive integers is 6.

97. **Assertion :** The order of the matrix A is 3 × 5 and that of B is 2 × 3. Then the matrix AB is not possible.

Reason : No. of columns in A is not equal to no. of rows in B.

98. **Assertion :** Addition of matrices is an example of binary operation on the set of matrices of the same order.

Reason: Addition of matrix is commutative.

99. **Assertion :** If $A=\frac{1}{3}\begin{bmatrix}1 & -2 & 2\\ -2 & 1 & 2\\ -2 & -2 & -1\end{bmatrix}$, then $\left(A^T\right)A=I$

Reason: For any square matrix, $A\,(A^T)^T = A$

100. For any square matrix A with real number entries, consider the following statements.

Assertion : $A + A'$ is a symmetric matrix.

Reason: $A - A'$ is a skew-symmetric matrix.

101. **Assertion :** $\begin{bmatrix}3 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 7\end{bmatrix}$ is a diagonal matrix.

Reason: $A=[a_{ij}]$ is a square matrix such that $a_{ij}=0,\ \forall\ i \neq j$, then A is called diagonal matrix.

102. **Assertion** : If $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$, then B is the inverse of A.

Reason : If A is a square matrix of order m and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse of A.

103. **Assertion :** Let $A=\begin{bmatrix}1 & 4\\ 2 & 5\\ 4 & 7\end{bmatrix}$ and $B=\begin{bmatrix}4 & 3 & 6\\ 7 & 8 & 9\\ 5 & 1 & 2\end{bmatrix}$, then the product of the matrices A and B is not defined.

Reason : The number of rows in B is not equal to number of columns in A.

104. **Assertion :** The matrix $A=\begin{bmatrix}0 & -1 & -2\\ 1 & 0 & -3\\ 2 & 3 & 0\end{bmatrix}$ is a skew symmetric matrix.

Reason : For the given matrix A we have $A' = A$.

105. **Assertion :** The matrix $A=\begin{bmatrix}9 & 1 & 2\\ 3 & 7 & 4\end{bmatrix}$ does not possesses any inverse.

Reason : A is not a square matrix.

CRITICAL THINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

106. If $A=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2+2AB$, then values of a and b are
(a) $a=1, b=-2$ (b) $a=1, b=2$
(c) $a=-1, b=2$ (d) $a=-1, b=-2$

107. If A is a square matrix, then AA' is a
(a) skew-symmetric matrix (b) symmetric matrix
(c) diagonal matrix (d) None of these

108. If $A=\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$, then value of α for which $A^2=B$, is
(a) 1 (b) –1
(c) 4 (d) no real values

109. The number of all possible matrices of order 3×3 with each entry 0 or 1 is
(a) 18 (b) 512
(c) 81 (d) None of these

110. Let $A=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and
$B=\begin{bmatrix}\cos^2\phi & \sin\phi\cos\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$, then $AB=O$, if
(a) $\theta=n\phi, n=0,1,2,......$
(b) $\theta+\phi=n\pi, n=0,1,2,.....$
(c) $\theta=\phi+(2n+1)\dfrac{\pi}{2}, n=0,1,2,...$
(d) $\theta=\phi+\dfrac{n\pi}{2}, n=0,1,2,...$

111. If A and B are 2×2 matrices, then which of the following is true?
(a) $(A+B)^2=A^2+B^2+2AB$
(b) $(A-B)^2=A^2+B^2-2AB$
(c) $(A-B)(A+B)=A^2+AB-BA-B^2$
(d) $(A+B)(A-B)=A^2-B^2$

112. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then $A^T+A=I_2$, if
(a) $\theta=n\pi, n\in Z$ (b) $\theta=(2n+1)\dfrac{\pi}{2}, n\in Z$
(c) $\theta=2n\pi+\dfrac{\pi}{3}, n\in Z$ (d) None of these

113. If A is any square matrix, then which of the following is skew-symmetric?
(a) $A+A^T$ (b) $A-A^T$ (c) AA^T (d) A^TA

114. If A is matrix of order $m\times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is
(a) $m\times m$ (b) $n\times n$ (c) $n\times m$ (d) $m\times n$

115. If A and B are two square matrices such that $B=-A^{-1}BA$, then $(A+B)^2=$
(a) O (b) A^2+B^2
(c) $A^2+2AB+B^2$ (d) $A+B$

116. If A, B are two square matrices such that AB = A and BA = B, then
(a) only B is idempotent (b) A, B are idempotent
(c) only A is idempotent (d) None of these

117. If $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then the matrix A equals
(a) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$

118. If A and B are two matrices such that A + B and AB are both defined, then
(a) A and B are two matrices not necessarily of same order.
(b) A and B are square matrices of same order.
(c) Number of columns of A = Number of rows of B.
(d) None of these.

119. If $A=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$ then A^{4n} where n is a natural number, equals :
(a) I (b) $-A$ (c) $-I$ (d) A

120. If A is symmetric as well as skew-symmetric matrix, then A is
(a) Diagonal (b) Null
(c) Triangular (d) None of these

121. If a matrix has 8 elements, then which of the following will not be a possible order of the matrix?
(a) 1×8 (b) 2×4 (c) 4×2 (d) 4×4

122. The matrix $C=[c_{ik}]_{m\times p}$ is the product of $A=[a_{ij}]_{m\times n}$ and $B=[b_{jk}]_{n\times p}$ where c_{ik} is
(a) $c_{ik}=\sum_{j=1}^{n} b_{jk}\,a_{ij}$ (b) $c_{ik}=\sum_{k=1}^{p} a_{ij}\,b_{jk}$
(c) $c_{ik}=\sum_{j=1}^{n} a_{ij}\,b_{jk}$ (d) $c_{ik}=\sum_{j=1}^{n} a_{ij}\,b_{j}$

123. If $A=\begin{bmatrix}0 & -\tan\dfrac{\alpha}{2}\\ \tan\dfrac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity matrix of order 2, then $(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ is equal to
(a) $I+A$ (b) $I-A$ (c) $A-I$ (d) A

124. Let $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}$, $a,b\in N$. Then
(a) there cannot exist any B such that AB = BA
(b) there exist more than one but finite number of B′s such that AB = BA
(c) there exists exactly one B such that AB = BA
(d) there exist infinitely many B′s such that AB = BA

125. If $A=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ &
$B=\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$, and $AB=0$, then
(a) $(\theta-\phi)$ is a multiple of $\dfrac{\pi}{2}$
(b) $(\theta-\phi)$ is an even multiple of $\dfrac{\pi}{2}$
(c) $(\theta-\phi)$ is a multiple of $\dfrac{\pi}{3}$
(d) $(\theta-\phi)$ is an odd multiple of $\dfrac{\pi}{2}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a)

$$F(\alpha).F(-\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(-\alpha) & -\sin(-\alpha) & 0\\ \sin(-\alpha) & \cos(-\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(\alpha).F(-\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha+0 & \cos\alpha\sin\alpha-\cos\alpha\sin\alpha+0 & 0+0+0\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha+0 & \sin^2\alpha+\cos^2\alpha+0 & 0+0+0\\ 0+0+0 & 0+0+0 & 0+0+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I \qquad [\because \cos^2\alpha+\sin^2\alpha=1]$$

$$F(\alpha)\,.\,F(-\alpha)=1 \quad \therefore [F(\alpha)]^{-1}=F(-\alpha)$$

2. (c) $$A^2=\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}=\begin{bmatrix}a^2 & 0 & 0\\ 0 & a^2 & 0\\ 0 & 0 & a^2\end{bmatrix}$$

$$A^3=\begin{bmatrix}a^2 & 0 & 0\\ 0 & a^2 & 0\\ 0 & 0 & a^2\end{bmatrix}\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}=\begin{bmatrix}a^3 & 0 & 0\\ 0 & a^3 & 0\\ 0 & 0 & a^3\end{bmatrix}$$

3. (c) $$AB=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}\begin{bmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{bmatrix}$$

$$AB=\begin{bmatrix}abc-abc & b^2c-b^2c & bc^2-bc^2\\ -a^2c+a^2c & -abc+abc & -ac+ac\\ a^2b-a^2b & ab^2-ab^2 & abc-abc\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$$

4. (a) Let $(A-2I)(A+I)=0$

$\Rightarrow AA-A-2I=0 \qquad (\because AI=A)$

$\Rightarrow A\left(\dfrac{A-I}{2}\right)=I \qquad \therefore \dfrac{A-I}{2}=A^{-1}$

5. (b) We have $$\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 1\\ 0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}1 & 5x+6 & x+4\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix}=0$$

$\Rightarrow x+5x+6-2x-8=0$

$\Rightarrow 4x-2=0 \ \Rightarrow x=\dfrac{1}{2}$

6. (b) $$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=A$$

$\therefore A^2+2A=A+2A=3A$

7. (a) Since, $\begin{bmatrix}x+y & 2x+z\\ x-y & 2z+w\end{bmatrix}=\begin{bmatrix}4 & 7\\ 0 & 10\end{bmatrix}$

$\Rightarrow x+y=4$ ...(i)

$x-y=0$ ...(ii)

$2x+z=7$ ...(iii)

and $2z+w=10$ ...(iv)

On solving these equations, we get

$x=2, y=2, z=3, w=4$

8. (b) Here, $a_{ij}=\dfrac{(i+2j)^2}{2}$

Therefore,

$$a_{11}=\frac{(1+2\times1)^2}{2}=\frac{(1+2)^2}{2}=\frac{9}{2},\ a_{12}=\frac{(1+2\times2)^2}{2}=\frac{25}{2}$$

$$a_{21}=\frac{(2+2\times1)^2}{2}=8 \text{ and } a_{22}=\frac{(2+2\times2)^2}{2}=18$$

So, the required matrix A is $\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}=\begin{bmatrix}9/2 & 25/2\\ 8 & 18\end{bmatrix}$

9. (b) $A=[a_{ij}]_{n\times n}$ is lower triangular matrix iff all entries above the diagonal vanish, i.e., $a_{ij}=0$ for $i<j$.

10. (c) Since the corresponding elements of two equal matrices are equal, therefore

$A=B \Rightarrow 2x+1=x+3,\ 3y=y^2+2$ and $y^2-5y=-6$

Now, $2x+1=x+3 \Rightarrow x=2$,

$3y=y^2+2 \Rightarrow y^2-3y+2=0 \ \Rightarrow y=1, 2$

and $y^2-5y=-6 \Rightarrow y^2-5y+6=0 \Rightarrow y=2,3$

since, $3y=y^2+2$ and $y^2-5y=-6$

must hold good simultaneously so, we take the common solution of these two equations. Therefore $y=2$.

Hence, $A=B$ if $x=2, y=2$

11. (d) When a 3×2 matrix is post multiplied by a 2×3 matrix, the product is a 3×3 matrix.

12. (d) $A=\left[a_{ij}\right]_{n\times n}$ is a diagonal matrix iff all non-diagonal entries are 0, i.e., $a_{ijs}=0$ for $i\neq j$.

13. (a) Since $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix}$

$= \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$

$\therefore x+3=0 \Rightarrow x=-3$
$b-3=2b+4 \Rightarrow b=-7$
$z+4=6 \Rightarrow z=2$
$a-1=-3 \Rightarrow a=-2$
$2y-7=3y-2 \Rightarrow y=-5,$
$2c+2=0 \Rightarrow c=-1$
$\therefore x=-3, y=-5, z=2, a=-2, b=-7, c=-1$

14. (b) Since, $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$\therefore PQ = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$

15. (c) $(2A-3B)C = D$

$\Rightarrow \left(2\begin{bmatrix} 4 & 1 & 0 \\ 1 & -2 & 2 \end{bmatrix} - 3\begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & x \end{bmatrix}\right)\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 15+x \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 2 & 3 \\ -7 & -7 & 4-3x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 15+x \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9 \\ -17-3x \end{bmatrix} = \begin{bmatrix} 15+x \\ 1 \end{bmatrix} \Rightarrow x = -6$

16. (b) We have, $(AA^T)^T = (A^T)^T A^T = AA^T$
$\therefore$ AA^T is a symmetric matrix.

17. (a) Since, $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+4+0 & 0+2-1 & 0+0-3 \\ 3+0+0 & 0+0+2 & 0+0+6 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{bmatrix}$

$= \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$

18. (b) we have

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$

$\because AA^T = 9I_3$

$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9 & 0 & a+2b+4 \\ 0 & 9 & 2a+2-2b \\ a+2b+4 & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$a+2b+4=0$ and $2a+2-2b=0 \Rightarrow a-b=-1$
on solving these, we get, $a=-2$ and $b=-1$

19. (c) We have, $A^2 = A$ ...(i)
Now, $(I+A)^3 - 7A = I^3 + A^3 + 3A^2I + 3AI^2 - 7A$
$= I + A^2A + 3A^2I + 3AI - 7A$
$= I + AA + 3A + 3A - 7A$ {using (i)}
$= I + A^2 - A = I + A - A$ {using (i)}

20. (b) We have $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

Now $AA^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$= \begin{bmatrix} \cos^2 x + \sin^2 x & \cos x \sin x - \sin x \cos x \\ \sin x \cos x - \cos x \sin x & \sin^2 x + \cos^2 x \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

21. (b) $A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = -A$

Since $A^T = -A$, therefore, A is a skew symmetric matrix.

22. (a) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

23. (a) Each diagonal entry of a skew symmetric matrix is 0.
As for a skew symmetric matrix
$A = [a_{ij}]_{n \times n}$
$a_{ij} = -a_{ji}$
$\Rightarrow a_{ii} = -a_{ii}$ for $j = i \Rightarrow a_{ii} = 0$

24. (c) $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$A = \left(\frac{A+A'}{2}\right) + \left(\frac{A-A'}{2}\right)$

[where B and C are symmetric and skew-symmetric matrices respectively]

Now, $C = \frac{A-A'}{2} = \frac{1}{2}\left\{\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right\}$

$= \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

25. (b) A is a symmetric matrix
$\therefore A^T = A$ ...(i)
A is also a skew-symmetric matrix
$\therefore A^T = -A$ ...(ii)
From eq. (i) and (ii)
$A = -A$
$\Rightarrow A = 0$
Hence, A is zero matrix.

26. (a) $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix} = A$

$\therefore A^T = A$
Hence, A is symmetric matrix.

27. (c) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

Consider A = IA

$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 2R_1$, we get

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - 3R_3$, $R_2 \to R_2 - R_3$, we get

$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -3 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2$, we get

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

28. (d) We have, A = IA

$\begin{bmatrix} 0 & 2 & -2 \\ -1 & 3 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$R_1 \leftrightarrow R_3$

$\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

$R_2 \to R_2 + R_1$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} A$

$R_1 \to R_1 + 2R_2$, $R_3 \to R_3 + (-2)R_2$

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & -2 & -2 \end{bmatrix} A$

$R_3 \to \left(-\frac{1}{4}\right) R_3$

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 1 & 1 \\ -1/4 & 1/2 & 1/2 \end{bmatrix} A$

$R_1 \to R_1 + (-3)R_3$, $R_2 \to R_2 - R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 & 3/2 \\ 1/4 & 1/2 & 1/2 \\ -1/4 & 1/2 & 1/2 \end{bmatrix} A$

$\therefore A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 2 & 2 \\ -1 & 2 & 2 \end{bmatrix}$

29. (b) We have, A = IA

$\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 + (-3)R_1$

$\begin{bmatrix} 1 & 2 \\ 0 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A$

$R_2 \to \left(\frac{-1}{11}\right) R_2$

$$\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\3/11 & -1/11\end{bmatrix}A$$

$R_1 \to R_1 + (-2)R_2$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}5/11 & 2/11\\3/11 & -1/11\end{bmatrix}A$$

$$\therefore\ A^{-1}=\frac{1}{11}\begin{bmatrix}5 & 2\\3 & -1\end{bmatrix}$$

30. (d) $A=\begin{bmatrix}2x & 0\\x & x\end{bmatrix}, A^{-1}=\begin{bmatrix}1 & 0\\-1 & 2\end{bmatrix}$

We know that

$AA^{-1} = I$

$$\begin{bmatrix}2x & 0\\x & x\end{bmatrix}\begin{bmatrix}1 & 0\\-1 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x & 0\\0 & 2x\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

On comparing, we get

$$2x = 1 \Rightarrow x = \frac{1}{2}$$

31. (a) If A is any square matrix, then

$AA^{-1} = I$ and $A^{-1}I = A^{-1}$

Since $A^2 - A + I = O$

$\Rightarrow\ A^{-1}A^2 - A^{-1}A + A^{-1}I = O$

$\Rightarrow\ (A^{-1}A)A - (A^{-1}A) + A^{-1} = O$

$\Rightarrow\ IA - I + A^{-1} = O$

$\Rightarrow\ A - I + A^{-1} = O$

$\Rightarrow\ A^{-1} = I - A$

32. (a) We have A = IA

$$\text{or } \begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 + (-2)\,R_1$

$$\Rightarrow \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1 + (-3)\,R_2$

$$\Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & -3\\-2 & 1\end{bmatrix}A$$

$$\therefore\ A^{-1}=\begin{bmatrix}7 & -3\\-2 & 1\end{bmatrix}$$

33. (a) $(AB' - BA')' = (AB')' - (BA')'$

$$= (B')'A' - (A')'B' = BA' - AB' = -(AB' - BA')$$

Hence, $(AB' - BA')$ is a skew-symmetric matrix.

34. (d)

35. (a) $a_{11} = 0, a_{12} = 1, a_{21} = 1, a_{22} = 0$

$$\therefore\ A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$\therefore\ A^2=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}0+1 & 0+0\\0+0 & 1+0\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$$

36. (a) $A^2 = I$

Now, $(A - I)^3 + (A + I)^3 - 7A$

$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$

$= 2A^3 + 6AI^2 - 7A = 2A^2A + 6AI - 7A$

$= 2IA + 6A - 7A = 2A + 6A - 7A = A \quad [\because A^2 = I]$

37. (a) We know that if A and B be two matrices then (AB)' = B' A'

38. (d) Matrix product is shown below.

$$\begin{bmatrix}a\\b\\c\end{bmatrix}\times[x\ y\ z]\times\begin{bmatrix}p\\q\\r\end{bmatrix}=\begin{bmatrix}ax & ay & az\\bx & by & bz\\cx & cy & cz\end{bmatrix}\times\begin{bmatrix}p\\q\\r\end{bmatrix}$$

$$=\begin{bmatrix}a(xp+yq+zr)\\b(xp+yq+zr)\\c(xp+yq+zr)\end{bmatrix}=(xp+yq+zr)\begin{bmatrix}a\\b\\c\end{bmatrix}$$

39. (b) Let $A=\begin{bmatrix}a & b\\b & a\end{bmatrix}$, then

$$A^2=\begin{bmatrix}a & b\\b & a\end{bmatrix}\begin{bmatrix}a & b\\b & a\end{bmatrix}=\begin{bmatrix}a^2+b^2 & 2ab\\2ab & b^2+a^2\end{bmatrix}$$

But given $A^2=\begin{bmatrix}\alpha & \beta\\\beta & \alpha\end{bmatrix}$

$\therefore\ a^2 + b^2 = \alpha$ and $2ab = \beta$

40. (c) From the given matrix, we observe that the all the elements below the main diagonal are zero. Hence, the given matrix is upper triangular matrix.

41. (c) We have $A^2=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & m & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & m & -1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=I$

42. (c) $A_{3\times4}$ is matrix having 3 rows and 4 columns

$\therefore\ a_{ij}=\frac{1}{2}(i+j)^2$ where i varies from 1 to 3 and j varies from 1 to 4.

Thus $a_{11}=\frac{1}{2}(1+1)^2=2$,

$a_{12}=\frac{1}{2}(1+2)^2=\frac{9}{2}$, etc.

We note that in the given options a_{22} is different in each. So we check for a_{22}.

We have, $a_{22}=\frac{1}{2}(2+2)^2=8$

43. (b) A is 2×3 matrix and B is 3×2 matrix

$\therefore$ both AB and BA exist and AB is a 2×2 matrix, while BA is a 3×3 matrix

$\therefore\ AB \neq BA$

44. (b) $A + A^T$ is a square matrix.

$(A + A^T)^T = A^T + (A^T)^T = A^T + A$

Hence, A is a symmetric matrix.

45. (a) It is obvious.

46. (b) Matrix multiplication is not commutative i.e. $AB \neq BA$

But it is associative i.e. $(AB)C = A(BC)$

47. **(c)** Given:

$x+y=5$ ...(i)

$y+z=7$ ...(ii)

and $x+y+z=9$ ...(iii)

Putting the value of $x+y=5$ in equation (iii) we get

$z=9-5=4$

So, $y=7-4=3$ and $x=5-3=2$

thus $x=2$, $y=3$ and $z=4$.

48. **(d)** $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\cos\theta\sin\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\cos\theta\sin\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.

49. **(d)** We have $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

Now, $A^2=A.A=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$

$=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}=-I$

where $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ is identity matrix

$(A^2)^8=(-I)^8=I$

Hence, $A^{16}=I$

50. **(d)** Given : $A=\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}$

$\therefore\ A^2=A.\ A=\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}$

$=\begin{bmatrix}1+8 & 2-6\\ 4-12 & 8+9\end{bmatrix}=\begin{bmatrix}9 & -4\\ -8 & 17\end{bmatrix}$

Now, $f(x)=x^2+4x-5$

$\therefore\ f(A)=A^2+4A-5$

$=A^2+4A-5I$ (I is a 2×2 unit matrix)

$=\begin{bmatrix}9 & -4\\ -8 & 17\end{bmatrix}+4\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}-5\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}9 & -4\\ -8 & 17\end{bmatrix}+\begin{bmatrix}4 & 8\\ 16 & -12\end{bmatrix}+\begin{bmatrix}-5 & 0\\ 0 & -5\end{bmatrix}$

$=\begin{bmatrix}8 & 4\\ 8 & 0\end{bmatrix}$

51. **(a)** $R(s)R(t)=\begin{bmatrix}\cos s & \sin s\\ -\sin s & \cos s\end{bmatrix}\times\begin{bmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{bmatrix}$

$=\begin{bmatrix}\cos s\cos t-\sin s\sin t & \cos s\sin t+\sin s\cos t\\ -\sin s\cos t-\cos s\sin t & -\sin s\sin t+\cos s\cos t\end{bmatrix}$

$=\begin{bmatrix}\cos(s+t) & \sin(s+t)\\ -\sin(s+t) & \cos(s+t)\end{bmatrix}=R(s+t)$

52. **(b)** $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, $A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$A+A'=\begin{bmatrix}\cos\alpha+\cos\alpha & -\sin\alpha+\sin\alpha\\ \sin\alpha-\sin\alpha & \cos\alpha+\cos\alpha\end{bmatrix}$

$=\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (given)

$\Rightarrow\ 2\cos\alpha=1,\ \Rightarrow\cos\alpha=\frac{1}{2}$

$\therefore\ \alpha=\frac{\pi}{3}$.

53. **(b)** $A^2-5A+6I=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

$-\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & 15\\ 5 & -5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$

$=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & 15\\ 5 & -5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$

$=\begin{bmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}=\begin{bmatrix}1 & -1 & -3\\ -1 & -1 & -10\\ -5 & 4 & 4\end{bmatrix}$

54. **(a)** Since, A is a $m\times n$ matrix with entries a_{ij}.

$\therefore$ A can be represented as

$A=[a_{ij}]_{m\times n}$

55. **(b)** If A is a 3×2 matrix, then A has $3\times2=6$ elements. Similarly, if B is a 3×3 matrix, then B has $3\times3=9$ elements and C has $2\times3=6$ elements.

56. **(a)** A matrix having m rows and n columns is called a matrix of order $m\times n$ or simply $m\times n$ matrix.

57. **(d)** The given matrix is $A=\begin{bmatrix}4 & -2 & 1 & 3\\ 5 & 7 & 9 & 6\\ 21 & 15 & 18 & -25\end{bmatrix}$.

Here, $a_{23}=9$ and $a_{24}=6$

$\therefore\ a_{23}+a_{24}=15$

Also, 15 lies in 3rd row and 2nd column.

$\therefore\ 15=a_{32}$

58. **(c)** In general, $A=[a_{ij}]_{m\times m}$ is a square matrix of order m.

59. **(c)** A square matrix $B=[b_{ij}]_{m\times m}$ is said to be a diagonal matrix, if all its non-diagonal elements are zero, that is a matrix $B=[b_{ij}]_{m\times m}$ is said to be a diagonal matrix if $b_{ij}=0$, when $i\neq j$.

60. **(c)** $A=[3]$, $B=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$, $C=\begin{bmatrix}\sqrt{3} & 0 & 0\\ 0 & \sqrt{3} & 0\\ 0 & 0 & \sqrt{3}\end{bmatrix}$ are scalar matrices of order 1, 2 and 3, respectively.

61. (a) A square matrix $B = [b_{ij}]_{n \times n}$ is said to be a scalar matrix, if
$b_{ij} = 0$ when $i \neq j$
$b_{ij} = k$ when $i = j$, for some constant k

62. (b) A diagonal matrix is said to be a scalar matrix, if its diagonal elements are equal.

63. (b) Scalar matrix is a particular case of a diagonal matrix, where all the diagonal elements are same.
Thus, every diagonal matrix is not a scalar matrix. Identity matrix is a particular case of scalar matrix, since all diagonal elements are same and have the value 1.
By definition of scalar matrix, it is a diagonal matrix.
Null matrix is a matrix in which all elements are zero. Such a matrix can be of any order and any type.

64. (d) The given matrix $A = [a_{ij}]$ is a matrix of order 4 × 5, which is not a square matrix.
∴ The diagonal elements of A do not exist.

65. (a) Associative law: For any three matrices $A = [a_{ij}]$, $B = [b_{ij}]$ and $C = [c_{ij}]$ of the same order, say m × n, $(A + B) + C = A + (B + C)$.
Now, $(A + B) + C = ([a_{ij}] + [b_{ij}]) + [c_{ij}]$
$= [a_{ij} + b_{ij}] + [c_{ij}] = [(a_{ij} + b_{ij}) + c_{ij}]$
$= [a_{ij}] + [(b_{ij}) + (c_{ij})]$
$= [a_{ij}] + ([b_{ij}] + [c_{ij}])$
$= A + (B + C)$

66. (c) The given matrix is $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$

Now, $kA = k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

Also, it is given that $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\therefore \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

On equating corresponding elements, we get
$2k = 3a$, $3k = 2b$ and $-4k = 24$
$\Rightarrow k = -6, a = -4, b = -9$

67. (d) Let $A = [a_{ij}]$ be an m × n matrix and $B = [b_{jk}]$ be an n × p matrix. Then, the product of the matrices A and B is the matrix C of order m × p.

68. (b) The product of two matrices A and B is defined, if the number of columns of A is equal to the number of rows of B.

69. (b) Here, $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$.

Therefore, we have

$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

On equating the corresponding elements of the two matrices, we have
$a + 4c = -7$, $2a + 5c = -8$ $3a + 6c = -9$
$b + 4d = 2$, $2b + 5d = 4$, $3b + 6d = 6$
Now, $a + 4c = -7$
$\Rightarrow a = -7 - 4c$
$2a + 5c = -8$
$\Rightarrow -14 - 8c + 5c = -8$
$\Rightarrow -3c = 6$
$\Rightarrow c = -2$
$\therefore a = -7 - 4(-2) = -7 + 8 = 1$
Now, $b + 4d = 2$
$\Rightarrow b = 2 - 4d$
$2b + 5d = 4$
and $4 - 8d + 5d = 4$
$\Rightarrow -3d = 0 \Rightarrow d = 0$
$\therefore b = 2 - 4(0) = 2$
Thus $a = 1, b = 2, c = -2, d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

70. (a) If $A = [a_{ij}]_{m \times n}$, then $A' = [a_{ji}]_{n \times m}$

71. (b) After applying $R_2 \to R_2 - 2R_1$ to $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, we get

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}$ (first multiply all elements of R_1 by 2 and then subtract these elements from R_2)

72. (c) If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse matrix of A. In this case A is said to be invertible.

73. (a) In order to apply a sequence of elementary column operations on the matrix equation X = AB, we will apply these operations simultaneously on X and on the second matrix B of the product AB on RHS.

74. (d) Here $AA^T = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}\begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$(BB^T)_{11} = (4)^2 + (1)^2 \neq 1$
$(AB)_{11} = 8 - 7 = 1$, $(BA)_{11} = 8 - 7 = 1$
$\therefore$ AB ≠ BA may be not true.

Now, $AB = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 7 & 2 \end{pmatrix}$

$= \begin{pmatrix} 8-7 & 2-2 \\ -28+28 & -7+8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $(AB)^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

75. (c) Here $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

$= \cos\theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = I\cos\theta + J\sin\theta$

STATEMENT TYPE QUESTIONS

76. (b) For multiplication of two matrices A and B, the number of columns in A should be equal to the number of rows in B.

77. (b) The given matrix is

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

Since, A has 3 rows and 4 columns.

$\therefore$ The order of A is 3×4 and the number of elements in A is 12. Also a_{13} is the element lying in the first row and third column

$\Rightarrow a_{13} = 19$

Similarly, $a_{21} = 35$ and $a_{33} = -5$

78. (a) If a matrix is of order $m \times n$, then it has mn elements.

I. Thus, to find the all possible orders of a matrix with 24 elements, we will find all ordered pairs of natural numbers, whose product is 24.
Thus all possible order pairs are (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), (6, 4).

$\therefore$ All possible orders are

$1 \times 24, 24 \times 1, 2 \times 12, 12 \times 2, 3 \times 8, 8 \times 3, 4 \times 6, 6 \times 4$

II. Similarly, if a matrix has 13 elements, then its all possible orders are 1×13 and 13×1.

III. If a matrix has 18 elements, the all its possible orders are
$18 \times 1, 1 \times 18, 2 \times 9, 9 \times 2, 3 \times 6, 6 \times 3$

IV. A matrix have 5 elements, then its possible orders are 1×5 and 5×1.

79. (a) $A = [4], B = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} -1.1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, are diagonal matrices of order 1, 2 and 3, respectively.

80. (a) A scalar matrix $A = [a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$ is an identity matrix when k = 1. But every identity matrix is clearly a scalar matrix.

81. (b) Non-commutativity of multiplication of matrices. If AB and BA are both defined, it is not necessary that AB = BA.

82. (a) For three matrices A, B and C of the same order, if A = B, then AC = BC but the converse is not true.

MATCHING TYPE QUESTIONS

83. (d) A. Here, $A = [a_{ij}]_{2 \times 2}$ is a matrix with $a_{ij} = \frac{(i+j)^2}{2}$.

$$\therefore a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

B. Here, $B = [b_{ij}]_{2 \times 3}$ is a matrix with $b_{ij} = \frac{(i+2j)^2}{2}$.

$$\therefore b_{13} = \frac{(1+2\times 3)^2}{2} = \frac{49}{2}$$

C. Given that, $C = [c_{ij}]_{3 \times 4}$ is a matrix with $c_{ij} = \frac{1}{2}|-3i + j|$.

$$\therefore c_{11} = \frac{1}{2}|-3\times 1 + 1| = 1$$

D. Here, $D = [d_{ij}]_{3 \times 4}$ is a matrix with $d_{ij} = 2i - j$.

$\therefore d_{34} = 2 \times 3 - 4 = 2$

84. (d) A. $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

On equating the corresponding elements, we get
x = 1, y = 4 and z = 3

B. $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

On equating the corresponding elements, we get
$5 + z = 5 \Rightarrow z = 0$
$x + y = 6$ and $xy = 8$

$\Rightarrow x + \frac{8}{x} = 6 \Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow (x-4)(x-2) = 0$

$\Rightarrow x = 4$ or $x = 2$

$\Rightarrow y = 2$ or $y = 4$

Thus, we have
x = 4, y = 2, z = 0
or x = 2, y = 4 and z = 0

C. $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

On equating the corresponding elements, we get
$x + y + z = 9$
$\Rightarrow x + z = 5$
$\Rightarrow y + z = 7$
$\Rightarrow x + 7 = 9 \Rightarrow x = 2$
Also, $2 + z = 5 \Rightarrow z = 3$
$y + 3 = 7 \Rightarrow y = 4$
$\therefore x = 2, y = 4$ and $z = 3$

85. (d) A. $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix}$

$$= \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

B. $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

C. $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\quad (\because \sin^2 x + \cos^2 x = 1)$

86. (c) A. $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2\times1+4\times(-2) & 2\times3+4\times5 \\ 3\times1+2\times(-2) & 3\times3+2\times5 \end{bmatrix}$

$= \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times2+3\times3 & 1\times4+3\times2 \\ (-2)\times2+5\times3 & (-2)\times4+5\times2 \end{bmatrix}$

$= \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{(3\times1)} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}_{1\times3}$

$= \begin{bmatrix} 1\times2 & 1\times3 & 1\times4 \\ 2\times2 & 2\times3 & 2\times4 \\ 3\times2 & 3\times3 & 3\times4 \end{bmatrix}_{3\times3}$

$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

87. (d) For any matrices A and B of suitable orders, we have

A. $(A')' = A$,

B. $(kA)' = kA'$ where k is any constant

C. $(A+B)' = A' + B'$

D. $(AB)' = A'B'$

88. (b) A. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ We know that, A = IA

$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ $\quad$ (using $R_2 \to R_2 - 2R_1$)

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} A$ $\quad \left(\text{using } R_2 \to \frac{1}{5}R_2\right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} A$ (using $R_1 \to R_1 + R_2$)

$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$ $\quad (\because AA^{-1} = I)$

B. Let $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

We know that, B = IB

$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} B$

(using $R_1 \leftrightarrow R_2$)

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} B$ (using $R_2 \to R_2 - 2R_1$)

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} B$ (using $R_2 \to (-1)R_2$)

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} B$ (using $R_1 \to R_1 - R_2$)

$\therefore B^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

C. Let $C = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$. We know that, C = IC

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} C \Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} C$

(using $R_2 \to R_2 - 2R_1$)

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} C$ (using $R_1 \to R_1 - 3R_2$)

$\Rightarrow C^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

D. Let $D = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$. We know that, D = ID

$\Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} D$

$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} D \left(R_1 \to \frac{1}{2}R_1\right)$

$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{-5}{2} & 1 \end{bmatrix} D$ (using $R_2 \to R_2 - 5R_1$)

$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 5 & -2 \end{bmatrix} D$ $\quad$ using $R_2 \to (-2)R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} D \left(\text{using } R_1 \to R_1 - \frac{3}{2}R_2\right)$

$\therefore D^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

INTEGER TYPE QUESTIONS

89. (d) $(A+B)^2 = A^2 + B^2$

$\Rightarrow AB + BA = O$

$$\Rightarrow \begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}\begin{bmatrix}x & 1\\ y & -1\end{bmatrix} + \begin{bmatrix}x & 1\\ y & -1\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix}x-y & 2\\ 2x-y & 3\end{bmatrix} + \begin{bmatrix}x+2 & -x-1\\ y-2 & -y+1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$\Rightarrow 2x - y + 2 = 0$ ...(i)

$-x + 1 = 0$ ...(ii)

$2x - 2 = 0$ ...(iii)

$-y + 4 = 0$ ...(iv)

from (ii), $x = 1$ and from (iv), $y = 4$

Now, $x + y = 1 + 4 = 5$

90. (d) We have $x + y = 8$ ...... (i)

$x + y + z = 9$ ...... (ii)

$y + z = 4$ ...... (iii)

On solving these equations, we get $x = 5$.

91. (a) $\because$ $2A + 6B = kX$

$$\therefore\ 2\begin{bmatrix}3 & 2\\ 5 & 7\\ 8 & 9\end{bmatrix} + 6\begin{bmatrix}1 & 9\\ 0 & 3\\ 4 & 10\end{bmatrix} = k\begin{bmatrix}6 & 29\\ 5 & 16\\ 20 & 39\end{bmatrix}$$

$$\text{or}\ \begin{bmatrix}6 & 4\\ 10 & 14\\ 16 & 18\end{bmatrix} + \begin{bmatrix}6 & 54\\ 0 & 18\\ 24 & 60\end{bmatrix} = k\begin{bmatrix}6 & 29\\ 5 & 16\\ 20 & 39\end{bmatrix}$$

$$\text{or}\ \begin{bmatrix}12 & 58\\ 10 & 32\\ 40 & 78\end{bmatrix} = k\begin{bmatrix}6 & 29\\ 5 & 16\\ 20 & 39\end{bmatrix}$$

$$\text{or}\ 2\begin{bmatrix}6 & 29\\ 5 & 16\\ 20 & 39\end{bmatrix} = k\begin{bmatrix}6 & 29\\ 5 & 16\\ 20 & 39\end{bmatrix}$$

$\therefore$ k = 2

92. (a) $\because$ $\begin{bmatrix}x & y\\ 2 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \begin{bmatrix}23 & 34\\ 14 & 20\end{bmatrix}$

$$\therefore\ \begin{bmatrix}x+3y & 2x+4y\\ 2+12 & 4+16\end{bmatrix} = \begin{bmatrix}23 & 34\\ 14 & 20\end{bmatrix}$$

$\therefore$ $x + 3y = 23$ .... (i)

$2x + 4y = 34$ .... (ii)

On solving eqs. (i) and (ii), we get $y = 6$.

93. (a) $\because$ $B^n - A = I$

$\therefore$ $B^n = I + A$

$$B^n = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} + \begin{bmatrix}26 & 26 & 18\\ 25 & 37 & 17\\ 52 & 39 & 50\end{bmatrix}$$

$$B^n = \begin{bmatrix}27 & 26 & 18\\ 25 & 38 & 17\\ 52 & 39 & 51\end{bmatrix}$$

$$\text{or}\ \begin{bmatrix}1 & 4 & 2\\ 3 & 5 & 1\\ 7 & 1 & 6\end{bmatrix}^n = \begin{bmatrix}27 & 26 & 18\\ 25 & 38 & 17\\ 52 & 39 & 51\end{bmatrix} \quad \text{....... (i)}$$

$\therefore$ $n \neq 1$

Now put n = 2, then

$$B^2 = \begin{bmatrix}1 & 4 & 2\\ 3 & 5 & 1\\ 7 & 1 & 6\end{bmatrix}^2 = \begin{bmatrix}1 & 4 & 2\\ 3 & 5 & 1\\ 7 & 1 & 6\end{bmatrix}\begin{bmatrix}1 & 4 & 2\\ 3 & 5 & 1\\ 7 & 1 & 6\end{bmatrix}$$

$$= \begin{bmatrix}1+12+14 & 4+20+2 & 2+4+12\\ 3+15+7 & 12+25+1 & 6+5+6\\ 7+3+42 & 28+5+6 & 14+1+36\end{bmatrix}$$

$$= \begin{bmatrix}27 & 26 & 18\\ 25 & 38 & 17\\ 52 & 39 & 51\end{bmatrix}$$

Which is equal to R.H.S. of eq. (i).

$\therefore$ n = 2

94. (b) $\begin{bmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{bmatrix}\begin{bmatrix}k & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}k+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2\\ k\omega+\omega^2+1 & \omega+\omega^2+1 & \omega+\omega^2+1\\ k\omega^2+1+\omega & \omega^2+1+\omega & \omega^2+1+\omega\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1+\omega+\omega^2+k-1 & 0 & 0\\ 1+\omega+\omega^2+k\omega-\omega & 0 & 0\\ 1+\omega+\omega^2+k\omega^2-\omega^2 & 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}k-1 & 0 & 0\\ (k-1)\omega & 0 & 0\\ (k-1)\omega^2 & 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Which gives $k - 1 = 0$ or $k = 1$

95. (b) We have $A = \begin{bmatrix}4 & 3\\ 1 & 5\end{bmatrix}$

On applying $R_2 \to R_2 - 3R_1$

we get $\begin{bmatrix}4 & 3\\ 1-3\times 4 & 5-3\times 3\end{bmatrix}$

$$= \begin{bmatrix}4 & 3\\ -11 & -4\end{bmatrix}$$

$\therefore$ n = 3

ASSERTION - REASON TYPE QUESTIONS

96. (c) $32 = 2^5$
Number of ways of expressing 32 as product of two positive integers $= \frac{5+1}{2} = 3$.
Possible dimensions of a matrix are
$\{1\times 32, 32\times 1, 2\times 16, 16\times 2, 4\times 8, 8\times 4\} = 6$
$\Rightarrow$ Assertion is true and Reason is false

97. (a) Matrix AB is possible only when number of columns in A is equal to number of rows in B.

98. (b) Addition of matrices is an example of binary operation on the set of matrices of the same order.
And Reason is true but not a correct explanation of Assertion.

99. (b) $\because AA' = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

100. (b) Let $B = A + A'$, then
$B' = (A + A')'$
$= A' + (A')'$ $\quad \left[\text{as } (A+B)' = A' + B'\right]$
$= A' + A$ $\quad \left[\text{as } (A')' = A\right]$
$= A + A'$ $\quad (\text{as } A + B = B + A)$
$= B$
Therefore, $B = A + A'$ is a symmetric matrix.
Now let $C = A - A'$
$C' = (A - A')' = A' - (A')'$
$= A' - A = -(A - A') = -C$
Therefore $C = A - A'$ is a skew-skymmetric matrix

101. (a) If $A = [a_{ij}]_{n\times n}$ is a square matrix such that $a_{ij} = 0$ for $i \neq j$; then A is called diagonal matrix. Thus the given statement is true and $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix.

102. (a) Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ be two matrices.
Now, $AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$
$= \begin{bmatrix} 4-3 & -6+6 \\ 2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
Also, $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
Thus, B is the inverse of A, in other words $B = A^{-1}$ and A is inverse of B, i. e., $A = B^{-1}$.

103. (a)

104. (c) For the given matrix A we have $A' = -A$.

105. (a) The matrices which are not square matrices do not possess inverse. The order of the matrix A is 2 × 3, hence it is not a square matrix.

CRITICALTHINKING TYPE QUESTIONS

106. (d) Given $(A+B)^2 = A^2 + B^2 + 2AB$
$\Rightarrow (A+B)(A+B) = A^2 + B^2 + 2AB$
$\Rightarrow A^2 + AB + BA + B^2 = A^2 + B^2 + 2AB \Rightarrow BA = AB$
$\Rightarrow \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} a+2 & -a+1 \\ b-2 & -b-1 \end{bmatrix} = \begin{bmatrix} a-b & 1+1 \\ 2a+b & 2-1 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} a+2 & -a+1 \\ b-2 & -b-1 \end{bmatrix} = \begin{bmatrix} a-b & 2 \\ 2a+b & 1 \end{bmatrix}$
The corresponding elements of equal matrices are equal.
$a + 2 = a - b, \quad -a + 1 = 2 \Rightarrow a = -1$
$b - 2 = 2a + b, \; -b - 1 = 1 \Rightarrow b = -2$
$\Rightarrow a = -1, b = -2$

107. (b) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$
$\therefore \quad AA' = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 1 \\ 4 & 1 & 4 \end{bmatrix}$

108. (d) $A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$;
Clearly, no real value of α.

109. (b) There are in total 9 entries and each entry can be selected in exactly 2 ways. Hence, the total number of all possible matrices of the given type is 2^9.

110. (c) $AB = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \sin\theta\cos\phi\cos\phi\sin\phi & \cos^2\theta\sin\phi\cos\phi + \sin^2\phi\sin\theta\cos\theta \\ \cos^2\phi\cos\theta\sin\theta + \sin^2\theta\sin\phi\cos\phi & \cos\theta\sin\theta\sin\phi\cos\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$

$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \sin\phi\cos\theta\cos(\theta-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$

$\because AB = O$

$\Rightarrow \quad \cos(\theta-\phi)=0 \Rightarrow \cos(\theta-\phi)=\cos(2n+1)\frac{\pi}{2}$

$\Rightarrow \quad \theta=(2n+1)\frac{\pi}{2}+\phi$, where n = 0, 1, 2, ...

111. (c) Given that, A and B are 2 × 2 matrices.

$\therefore (A-B)\times(A+B)=A\times A+A\times B-B\times A-B\times B$
$=A^2-B^2+AB-BA$

$\Rightarrow (A-B)(A+B)=A^2+AB-BA+B^2$

112. (c) We have, $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$\therefore A^T=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

Now, $A^T+A=I_2$ (given)

$\Rightarrow \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2\cos\theta & 0\\ 0 & 2\cos\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow 2\cos\theta=1 \Rightarrow \cos\theta=\frac{1}{2}$

$\Rightarrow \theta=2n\pi+\frac{\pi}{3}, n\in Z$

113. (b) $(A-A^T)^T=A^T-(A^T)^T=A^T-A=-(A-A^T)$

Hence, $(A-A^T)$ is skew-symmetric.

114. (d) Let matrix B is of order p × q.

$\therefore$ matrix B′ is of order q × p.

matrix A is of order m × n

Since, AB′ is defined

$\therefore$ number of columns of A = number of rows of B′

$\Rightarrow n=q$

Also, B′A is defined

$\therefore$ number of column of B′ = number of rows of A

$\Rightarrow p=m$

Hence, B is of order p × q i.e m × n

115. (b) $B=-A^{-1}BA \Rightarrow AB=-BA \Rightarrow AB+BA=0$

$\therefore (A+B)^2=A^2+AB+BA+B^2=A^2+B^2$

116. (b) We have, AB = A and BA=B.

Now, $AB=A \Rightarrow (AB)A=A.A$

$\Rightarrow A(BA)=A^2$

$\Rightarrow AB=A^2 (\because BA=B)$

$\Rightarrow A=A^2 (\because AB=A)$

Again, $BA=B \Rightarrow (BA)B=B^2$

$\Rightarrow B(AB)=B^2$

$\Rightarrow BA=B^2 (\because AB=A)$

$\Rightarrow B=B^2 (\because BA=B)$

$\therefore$ A and B are idempotent matrices.

117. (a) Let $B=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$ and $C=\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}$

Given $BAC=I \Rightarrow B^{-1}(BAC)=B^{-1}I$

$\Rightarrow I(AC)=B^{-1} \Rightarrow AC=B^{-1}$

$\Rightarrow ACC^{-1}=B^{-1}C^{-1} \Rightarrow AI=B^{-1}C^{-1}$

$\therefore A=(B^{-1})(C^{-1})$

Now $B^{-1}=\frac{1}{4-3}\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$

$C^{-1}=\frac{1}{9-10}\begin{bmatrix}-3 & -2\\ -5 & -3\end{bmatrix}=\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}$

$\therefore (B^{-1})(C^{-1})=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

$\therefore A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

118. (b) A + B is defined ⇒ A and B are of same order.

Also AB is defined ⇒

Number of columns in A = Number of rows in B

Obviously, both simultaneously mean that the matrices A and B are square matrices of same order.

119. (c) Given : $A=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$

$A^2=AA=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}=\begin{bmatrix}i^2 & 0\\ 0 & i^2\end{bmatrix}$

$=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}=-I$

$\therefore \quad A^{4n}=-I$

120. (b) Let $A=[a_{ij}]_{n\times m}$. Since A is skew–symmetric $a_{ii}=0$ (i = 1, 2, ..., n) and $a_{ji}=-a_{ji} (i\neq j)$

Also, A is symmetric so $a_{ji}=a_{ji}$ ∀ i and j

$\therefore \quad a_{ji}=0 \quad \forall \quad i\neq j$

Hence $a_{ij}=0$ ∀ i and j ⇒ A is a null zero matrix

121. (d) We know that, if a matrix is of order m × n, then it has mn elements. Thus, to find all possible orders of a matrix with 8 elements, we will find all ordered pairs of natural numbers, whose product is 8. Thus, all possible ordered pair are (1, 8), (8, 1), (2, 4), (4, 2).

122. (c) If $A=[a_{ij}]_{m\times n}$, $B=[b_{jk}]_{n\times p}$, then the i^{th} row of A is $[a_{i1}\ a_{i2} \ldots a_{in}]$ and the k^{th} column of B is

$\begin{bmatrix}b_{ik}\\ b_{2k}\\ \vdots\\ b_{nk}\end{bmatrix}$, then

$c_{ik}=a_{i1}b_{1k}+a_{i2}b_{2k}+a_{i3}b_{3k}+\ldots+a_{in}b_{nk}$

$=\sum_{j=1}^{n}a_{ij}b_{jk}$

The matrix $C=[c_{ik}]_{m\times p}$ is the product of A and B.

123. (a) Here, $A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$, where $t = \tan\left(\frac{\alpha}{2}\right)$

Now, $\cos\alpha = \dfrac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \dfrac{1-t^2}{1+t^2}$

and $\sin\alpha = \dfrac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \dfrac{2t}{1+t^2}$

$$= (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ +t & 0 \end{bmatrix}\right)\begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

Also, $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0+1 & -t+0 \\ t+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

124. (d) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$$

Hence, $AB = BA$ only when $a = b$

$\therefore$ There can be infinitely many B's for which $AB = BA$

125. (d) Here

AB =

$$\begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\sin\theta\cos\phi\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \text{ if } \cos(\theta-\phi) = 0$$

Now, $\cos(\theta-\phi) = 0 \Rightarrow \theta-\phi$, is an odd multiple of $\pi/2$.

CHAPTER 20

DETERMINANT

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If the system of equations $x + \lambda y + 2 = 0$, $\lambda x + y - 2 = 0$, $\lambda x + \lambda y + 3 = 0$ is consistent, then

(a) $\lambda = \pm 1$ (b) $\lambda = \pm 2$ (c) $\lambda = 1, -2$ (d) $\lambda = -1, 2$

2. If x, y, z are all distinct and

$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then the value of xyz is

(a) –2 (b) –1 (c) –3 (d) None of these

3. The equations $2x + 3y + 4 = 0$; $3x + 4y + 6 = 0$ and $4x + 5y + 8 = 0$ are

(a) consistent with unique solution
(b) inconsistent
(c) consistent with infinitely many solutions
(d) None of the above

4. Given : $2x - y - 4z = 2$, $x - 2y - z = -4$, $x + y + \lambda z = 4$, then the value of λ such that the given system of equation has NO solution, is

(a) 3 (b) 1 (c) 0 (d) – 3

5. $\begin{vmatrix} 2xy & x^2 & y^2 \\ x^2 & y^2 & 2xy \\ y^2 & 2xy & x^2 \end{vmatrix} =$

(a) $(x^3 + y^3)^2$ (b) $(x^2 + y^2)^3$
(c) $-(x^2 + y^2)^3$ (d) $-(x^3 + y^3)^2$

6. $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} =$

(a) $(a + b)^2$ (b) $(a + b + c + d)^2$
(c) $(a^2 + b^2 - c^2 - d^2)$ (d) $a^2 + b^2 + c^2 + d^2$

7. $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix} =$

(a) 0 (b) 5 (c) 3 (d) 7

8. If x is positive integer, then

$\begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix}$ is equal to

(a) $2x!(x+1)!$ (b) $2x!(x+1)!(x+2)!$
(c) $2x!(x+3)!$ (d) $2(x+1)!(x+2)!(x+3)!$

9. If the area of a triangle ABC, with vertices A(1, 3), B(0, 0) and C(k, 0) is 3 sq. units, then the value of k is

(a) 2 (b) 3 (c) 4 (d) 5

10. Find the cofactors of elements a_{12}, a_{22}, a_{32}, respectively of the matrix $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

(a) $0, 2, -2\sin\theta$ (b) $2, 0, 2\sin\theta$
(c) $2, 0, -2\sin\theta$ (d) $-2\sin\theta, 2, 0$

11. If A_{ij} denotes the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then value of $a_{11}A_{31} + a_{13}A_{32} + a_{13}A_{33}$ is

(a) 0 (b) 5 (c) 10 (d) – 5

12. If c_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32}.c_{32}$

(a) 110 (b) 22 (c) – 110 (d) – 22

13. If the equations $x + ay - z = 0$, $2x - y + az = 0$, $ax + y + 2z = 0$ have non-trivial solutions, then $a =$

(a) 2 (b) – 2 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

14. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then

(a) $f(a) = 0$ (b) $f(b) = 0$ (c) $f(0) = 0$ (d) $f(1) = 0$

15. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is:

(a) {0, 1} (b) {1, 2} (c) {1, 5} (d) {2, – 1}

16. Consider the system of linear equations;

$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(a) exactly 3 solutions (b) a unique solution
(c) no solution (d) infinite number of solutions

17. The system of linear equations : $x+y+z=0$, $2x+y-z=0$, $3x+2y=0$ has :
(a) no solution
(b) a unique solution
(c) an infinitely many solution
(d) None of these

18. The roots of the equation

$$\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0 \text{ are :}$$

(a) 0, 12 and 12 (b) 0 and ± 12
(c) 0, 12 and 16 (d) 0, 9 and 16

19. If $A=\begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB|$ is equal to :
(a) 80 (b) 100 (c) -110 (d) 92

20. If A and B are two matrices such that A + B and AB are both defined, then
(a) A and B are two matrices not necessarily of same order.
(b) A and B are square matrices of same order.
(c) Number of columns of A = Number of rows of B.
(d) None of these.

21. If B is a non-singular matrix and A is a square matrix, then det $(B^{-1}AB)$ is equal to
(a) det (A^{-1}) (b) det (B^{-1})
(c) det (A) (d) det (B)

22. If $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|\text{adj } A|$ is
(a) a^{27} (b) a^9 (c) a^6 (d) a^2

23. For any 2×2 matrix A, if A (adj. A) $=\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ is equal to :
(a) 0 (b) 10 (c) 20 (d) 100

24. The factors of $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$ are:
(a) $x-a$, $x-b$ and $x+a+b$
(b) $x+a$, $x+b$ and $x+a+b$
(c) $x+a$, $x+b$ and $x-a-b$
(d) $x-a$, $x-b$ and $x-a-b$

25. $\begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bc \\ 1 & 1+ac & 1+bc \end{vmatrix}$ is equal to:
(a) $a+b+c$ (b) 1 (c) 0 (d) 3

26. Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is :
(a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$
(c) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ (d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

27. The system of simultaneous linear equations $kx+2y-z=1$, $(k-1)y-2z=2$ and $(k+2)z=3$ have a unique solution if k equals:
(a) -1 (b) -2 (c) 0 (d) 1

28. If $\Delta=\begin{vmatrix} 3 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & x & 5 \end{vmatrix}$, then $\begin{vmatrix} x & 10 & 5 \\ 5 & 3 & 6 \\ 8 & 7 & 9 \end{vmatrix}$ equal to:
(a) Δ (b) $-\Delta$ (c) Δx (d) 0

29. If $(x+9)=0$ is a factor of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$, then the other factor is:
(a) $(x-2)(x-7)$ (b) $(x-2)(x-a)$
(c) $(x+9)(x-a)$ (d) $(x+2)(x+a)$

30. The value of

$$\begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

is independent of :
(a) n (b) a (c) x (d) None of these

31. If matrix $A=\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$ and its inverse is denoted by $A^{-1}=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ then the value of a_{23} is equal to :
(a) 21/20 (b) 1/5 (c) $-2/5$ (d) 2/5

32. If $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix}=kxyz$, then the value of k is :
(a) 2 (b) 4 (c) 6 (d) 8

33. The coefficient of x in

$$f(x)=\begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}, -1 < x \le 1, \text{ is}$$

(a) 1 (b) -2 (c) -1 (d) 0

34. If matrix $A=\begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1}=\frac{1}{k}(\text{adj } A)$, then k is :
(a) 7 (b) -7 (c) 15 (d) -11

35. If $A=\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}$, then adj. (A) equals:
(a) $\begin{bmatrix} 36 & -36 & 18 \\ 36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) None of these

36. If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & \alpha\end{bmatrix}$, then Adj. A is equal to :

(a) $\begin{bmatrix}\delta & -\gamma\\ -\beta & \alpha\end{bmatrix}$ (b) $\begin{bmatrix}\delta & -\beta\\ -\gamma & \alpha\end{bmatrix}$

(c) $\begin{bmatrix}-\delta & \beta\\ \gamma & -\alpha\end{bmatrix}$ (d) $\begin{bmatrix}-\delta & -\beta\\ \gamma & \alpha\end{bmatrix}$

37. If A and B are square matrices and A^{-1} and B^{-1} of the same order exist, then $(AB)^{-1}$ is equal to :

(a) AB^{-1} (b) $A^{-1}B$ (c) $A^{-1}B^{-1}$ (d) $B^{-1}A^{-1}$

38. If A is a square matrix such that $(A-2I)(A+I)=0$, then $A^{-1}=$

(a) $\frac{A-I}{2}$ (b) $\frac{A+I}{2}$ (c) $2(A-I)$ (d) $2A+I$

39. If A and B are two square matrices such that $B=-A^{-1}BA$, then $(A+B)^2=$

(a) 0 (b) A^2+B^2

(c) $A^2+2AB+B^2$ (d) $A+B$

40. If I_3 is the identity matrix of order 3, then I_3^{-1} is

(a) 0 (b) $3I_3$ (c) I_3 (d) Does not exist

41. If a square matrix satisfies the relation $A^2+A-I=0$ then A^{-1}:

(a) exists and equals $I+A$ (b) exists and equals $I-A$

(c) exists and equals A^2 (d) None of these

42. If for $AX=B$, $B=\begin{bmatrix}9\\52\\0\end{bmatrix}$ and

$A^{-1}=\begin{bmatrix}3 & -1/2 & -1/2\\ -4 & 3/4 & 5/4\\ 2 & -3/4 & -3/4\end{bmatrix}$, then X is equal to:

(a) $\begin{bmatrix}3\\3/4\\-3/4\end{bmatrix}$ (b) $\begin{bmatrix}-1/2\\1/2\\2\end{bmatrix}$

(c) $\begin{bmatrix}-4\\2\\3\end{bmatrix}$ (d) $\begin{bmatrix}1\\3\\-21\end{bmatrix}$

43. Let $A=\begin{pmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{pmatrix}$. The only correct statement about the matrix A is

(a) $A^2=I$

(b) $A=(-1)I$, where I is a unit matrix

(c) A^{-1} does not exist

(d) A is a zero matrix

44. Let $A=\begin{pmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{pmatrix}$ and $10\,B=\begin{pmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{pmatrix}$.

If B is the inverse of matrix A, then α is

(a) 5 (b) –1

(c) 2 (d) –2

45. The parameter on which the value of the determinant $\begin{vmatrix}1 & a & a^2\\ \cos(p-d)x & \cos px & \cos(p+d)x\\ \sin(p-d)x & \sin px & \sin(p+d)x\end{vmatrix}$ does not depend upon is

(a) a (b) p (c) d (d) x

46. A system of linear equations

$a_1x+b_1y=c_1$

$a_2x+b_2y=c_2$

can be represented in matrix form as

(a) $\begin{bmatrix}a_1 & a_2\\ b_1 & b_2\end{bmatrix}=\begin{bmatrix}c_1\\c_2\end{bmatrix}$ (b) $\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}c_1\\c_2\end{bmatrix}$

(c) $\begin{bmatrix}a_1 & a_2\\ b_1 & b_2\end{bmatrix}\begin{bmatrix}y\\x\end{bmatrix}=\begin{bmatrix}c_1\\c_2\end{bmatrix}$ (d) $\begin{bmatrix}a_1 & a_2\\ b_1 & b_2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}c_2\\c_1\end{bmatrix}$

47. If $A=\begin{bmatrix}1 & 2 & 3\\ -4 & -5 & 6\end{bmatrix}$, then $|A|$ is

(a) 2 (b) 0 (c) –2 (d) Does not exist

48. The determinant $\begin{vmatrix}x & \sin\theta & \cos\theta\\ -\sin\theta & -x & 1\\ \cos\theta & 1 & x\end{vmatrix}$ is

(a) independent of θ only

(b) independent of x only

(c) independent of both θ and x

(d) None of the above

49. If rows and columns of the determinant are interchanged, then its value

(a) remains unchanged (b) becomes change

(c) is doubled (d) is zero

50. If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then the value of determinant is

(a) 1 (b) –1 (c) 0 (d) 2

51. Area of the triangle whose vertices are (a, b + c), (b, c + a) and (c, a + b), is

(a) 2 sq units (b) 3 sq units

(c) 0 sq unit (d) None of these

52. If area of triangle is 4 sq units with vertices (–2, 0), (0, 4) and (0, k), then k is equal to

(a) 0, –8 (b) 8 (c) – 8 (d) 0, 8

53. If $A=\begin{bmatrix}0 & -1 & 2\\ 2 & -2 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\\ 1 & 1\end{bmatrix}$ and $M=AB$, then the value of M^{-1} is

(a) $\begin{bmatrix}\frac{2}{3} & \frac{-1}{3}\\ \frac{1}{3} & \frac{4}{5}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{1}{3} & \frac{-1}{3}\\ \frac{1}{3} & \frac{1}{6}\end{bmatrix}$

(c) $\begin{bmatrix}2 & \frac{-1}{3}\\ \frac{2}{3} & 0\end{bmatrix}$ (d) None of these

54. If $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$. for $x\neq 0, y\neq 0$, then D is

(a) divisible by x but not y

(b) divisible by y but not x

(c) divisible by neither x nor y

(d) divisible by both x and y

55. If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ and $|A^3|=125$, then the value of α is

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 5

56. If $M(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$; $M(\beta)=\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix}$,

then value of $[M(\alpha)\,M(\beta)]^{-1}$ is

(a) $M(\beta).M(\alpha)$ (b) $M(-\alpha).M(\beta)$

(c) $M(-\beta)M(\alpha)$ (d) $M(-\beta)M(-\alpha)$

57. The value of x obtained from the equation

$$\begin{vmatrix}x+\alpha & \beta & \gamma\\ \gamma & x+\beta & \alpha\\ \alpha & \beta & x+\gamma\end{vmatrix}=0 \text{ will be}$$

(a) 0 and $-(\alpha+\beta+\gamma)$ (b) 0 and $\alpha+\beta+\gamma$

(c) 1 and $(\alpha-\beta-\gamma)$ (d) 0 and $\alpha^2+\beta^2+\gamma^2$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

58. Consider the following statements

I. To every rectangular matrix $A=[a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of A.

II. Determinant is a function which associates each square matrix with a unique number (real or complex).

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Neither I nor II is true

59. Consider the following statements

I. $|A|$ is also called modulus of square matrix A.

II. Every matrix has determinant.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Neither I nor II is true

60. Consider the following statements

I. Matrix cannot be reduced to a number.

II. Determinant can be reduced to a number.

(a) Only I is true

(b) Only II is true

(c) Both I and II are true

(d) Neither I nor II is true

61. Consider the following statements

I. If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

II. If any two rows (or columns) of a determinant are interchanged, then the value of the determinant remains same.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Neither I nor II is true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

62.

Column I (Vertices)		**Column II (Area of triangle)**	
A.	(1, 0), (6, 0), (4, 3)	1.	15
B.	(2, 7), (1, 1), (10, 8)	2.	0
C.	(–2, –3), (3, 2), (–1, –8)	3.	$\frac{47}{2}$
		4.	$\frac{15}{2}$

Codes

	A	B	C
(a)	2	1	3
(b)	4	3	1
(c)	4	1	3
(d)	3	1	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

63. The number of distinct real roots of $\begin{vmatrix}\sin x & \cos x & \cos x\\ \cos x & \sin x & \cos x\\ \cos x & \cos x & \sin x\end{vmatrix}=0$

in the interval $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$ is

(a) 0 (b) 2 (c) 1 (d) 3

64. If $\begin{vmatrix}p & q-y & r-z\\ p-x & q & r-z\\ p-x & q-y & r\end{vmatrix}=0$, then the value of $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}$ is

(a) 0 (b) 1

(c) 2 (d) 4pqr

65. If A, B, C are the angles of a triangle, then the value of

determinant $\begin{vmatrix}\sin 2A & \sin C & \sin B\\ \sin C & \sin 2B & \sin A\\ \sin B & \sin A & \sin 2C\end{vmatrix}$ is

(a) π (b) 0

(c) 2π (d) None of these

66. For positive numbers x, y, z the numerical value of the

determinant $\begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y x & 3 & \log_y z\\ \log_z x & \log_z y & 5\end{vmatrix}$ is

(a) 0 (b) log x log y log z

(c) 1 (d) 8

67. The number of values of k for which the system of equations $(k+1)x+8y=4k$; $kx+(k+3)y=3k-1$ has infinitely many solutions is

(a) 0 (b) 1 (c) 2 (d) infinite

68. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = k(a+b+c)^3$, then k is
(a) 0 (b) 1 (c) 2 (d) 3

69. If a, b, c are cube roots of unity, then $\begin{vmatrix} e^a & e^{2a} & e^{3a}-1 \\ e^b & e^{2b} & e^{3b}-1 \\ e^c & e^{2c} & e^{3c}-1 \end{vmatrix} =$
(a) 0 (b) e (c) e^2 (d) e^3

70. If the matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then λ =
(a) –2 (b) 4 (c) 2 (d) – 4

71. $\begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix}$ is equal to:
(a) 57 (b) – 39 (c) 96 (d) 0

72. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$ is equal to:
(a) 0 (b) $12\cos^2 x - 10\sin^2 x$
(c) $12\cos^2 x - 10\sin^2 x - 2$ (d) $10\sin 2x$

73. If a, b, c are in A. P., then the value of
$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is:
(a) 3 (b) – 3 (c) 0 (d) None of these

74. In how many ways, the determinant of order 3 can be expanded?
(a) 5 (b) 4 (c) 3 (d) 6

75. The value of $\begin{vmatrix} \sin 10° & -\cos 10° \\ \sin 80° & \cos 80° \end{vmatrix}$ is
(a) –1 (b) 1 (c) 2 (d) 0

76. For positive numbers x, y, z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is
(a) 0 (b) 1 (c) 2 (d) None of these

77. The value of the determinant $\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 0 & 12 & 9 \\ 1 & 2 & 2 \end{vmatrix}$ is
(a) 2 (b) 4 (c) 6 (d) 8

78. The area of the triangle formed by the points (1, 2), (k, 5) and (7, 11) is zero then the value of k is
(a) 0 (b) 3 (c) 5 (d) 7

79. The minor of the element a_{11} in the determinant
$\begin{vmatrix} 2 & 6 & 9 \\ 1 & 7 & 8 \\ 1 & 4 & 5 \end{vmatrix}$ is
(a) 0 (b) 3 (c) 5 (d) 7

80. If $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ Then $|\text{adj } A| =$
(a) 0 (b) 3
(c) 5 (d) 7

81. If A is a non-singular matrix of order 3, then $|\text{adj } A| = |A|^n$. Here the value of n is
(a) 2 (b) 4
(c) 6 (d) 8

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

82. **Assertion :** If three lines $L_1 : a_1x + b_1y + c_1 = 0$, $L_2 : a_2x + b_2y + c_2 = 0$ and $L_3 : a_3x + b_3y + c_3 = 0$ are concurrent lines, then
$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Reason : If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the lines L_1, L_2, L_3 must be concurrent.

83. Consider the system of equations
$x - 2y + 3z = -1$
$-x + y - 2z = k$
$x - 3y + 4z = 1$

Assertion: The system of equations has no solution for $k \neq 3$.

Reason: The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

84. Let $A = \begin{bmatrix} 1 & 0 & a \\ 2 & 3 & b \\ -3 & 1 & c \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & x \\ 2 & 3 & y \\ -3 & 1 & z \end{bmatrix}$

and $C = \begin{bmatrix} 1 & 0 & a+x \\ 2 & 3 & b+y \\ -3 & 1 & c+z \end{bmatrix}$

Assertion: det A + det B = det C.
Reason: A + B = C.

85. **Assertion:** If a, b, c are even natural numbers, then

$$\Delta = \begin{vmatrix} a-1 & a & a+1 \\ b-1 & b & b+1 \\ c-1 & c & c+1 \end{vmatrix}$$ is an even natural number.

Reason: Sum and product of two even natural number is also an even natural number.

86. **Assertion:** If the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, then

$5A^{-1} = A^2 - 7A + 10I.$

Reason: If $\det(A - \lambda I) = \sum_{r=0}^{3} C_r \lambda^r$, then

$C_0 I + C_1 A + C_2 A^2 + C_3 A^3 = 0.$

87. Consider the system

$x + y + z = 1$
$2x + 2y + 2z = 2$
$4x + 4y + 4z = 3$

Assertion: The above system has infinitely many solutions.

Reason: For the above system det A = 0 and (adj A) B = O, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

88. **Assertion:** $\begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ is independent of θ

Reason: If $f(\theta) = c$, then $f(\theta)$ is independent of θ.

89. Consider the system

$2x + 3y + 6z = 8$
$x + 2y + 3z = 5$
$x + y + 3z = 4$

Assertion: The above system of equations has no solution.

Reason: det A = 0 and (adj A)B = 0, where

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}$$

90. Let $A = [a_{ij}]$ be a matrix of order 3 × 3.

Assertion: Expansion of determinant of A along second row and first column gives the same value.

Reason: Expanding a determinant along any row or column gives the same value.

91. **Assertion:** $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ ka_1 & ka_2 & ka_3 \end{vmatrix} = 0$

Reason: If corresponding elements of any two rows of a determinant are proportional, then its value is zero.

92. **Assertion:** The points A (a, b + c), B (b, c + a) and C (c, a + b) are collinear.

Reason: Area of a triangle with three collinear points is zero.

93. **Assertion :** $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where, A_{ij} is cofactor of a_{ij}.

Reason : Δ = Sum of the products of elements of any row (or column) with their corresponding cofactors.

94. Let A be a 2 × 2 matrix

Assertion: adj (adj A) = A

Reason: |adj A |= |A|

95. **Assertion:** The matrix $A = \begin{bmatrix} 2 & 3 & -\frac{1}{2} \\ 7 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ is singular.

Reason: The value of determinant of matrix A is zero.

96. **Assertion:** The value of determinant of a matrix and the value of determinant of its transpose are equal.

Reason: The value of determinant remains unchanged if its rows and columns are interchanged.

97. **Assertion:** The matrix $A = \begin{bmatrix} 2 & 5 & 7 \\ 5 & 4 & 9 \\ 7 & 9 & 3 \end{bmatrix}$ is a symmetric matrix.

Reason: For the given matrix A' = A

98. **Assertion:** The matrix $A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 0 & 8 \\ 4 & 8 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Reason: For the given matrix A' = A.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

99. If $\begin{vmatrix} 1+a & 1 & 1 \\ 1+b & 1+2b & 1 \\ 1+c & 1+c & 1+3c \end{vmatrix} = 0$ where

$a \neq 0, b \neq 0, c \neq 0$, then $a^{-1} + b^{-1} + c^{-1}$ is

(a) 4 (b) –3 (c) –2 (d) –1

100. If the system of linear equations

$x + 2ay + az = 0$
$x + 3by + bz = 0$
$x + 4cy + cz = 0$

has a non - zero solution, then a, b, c

(a) satisfy a + 2b + 3c = 0 (b) are in A.P
(c) are in G.P (d) are in H.P.

101. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ is (θ is real number)

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\frac{2\sqrt{3}}{4}$

102. If each of third order determinant of value Δ is multiplied by 4, then value of the new determinant is:

(a) Δ (b) 21Δ (c) 64Δ (d) 128Δ

103. Let A be a matrix of order 3 and let Δ denotes the value of determinant A. Then determinant (– 2A) =

(a) – 8 Δ (b) – 2Δ (c) 2 Δ (d) 8 Δ

104. If $a^{-1}+b^{-1}+c^{-1}=0$ such that

$$\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=\lambda$$

then the value of λ is :

(a) 0 (b) – abc (c) abc (d) None of these

105. If 1, ω, ω^2 the cube roots of unity, then the value of

$$\begin{vmatrix}1 & \omega^n & \omega^{2n}\\ \omega^{2n} & 1 & \omega^n\\ \omega^n & \omega^{2n} & 1\end{vmatrix}$$ is equal to

(a) 1 (b) ω (c) ω^2 (d) 0

106. The value of $\begin{vmatrix}-a^2 & ab & ac\\ ab & -b^2 & bc\\ ac & bc & -c^2\end{vmatrix}$ is :

(a) 0 (b) abc (c) $4a^2b^2c^2$ (d) None of these

107. If $x \neq y \neq z$ and $\begin{vmatrix}x & x^2 & x^3\\ y & y^2 & y^3\\ z & z^2 & z^3\end{vmatrix}=0$, then xyz is equal to:

(a) 1 (b) – 1 (c) 0 (d) $x+y+z$

108. The value of the determinant

$$\begin{vmatrix}\cos\alpha & -\sin\alpha & 1\\ \sin\alpha & \cos\alpha & 1\\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1\end{vmatrix}$$ is

(a) independent of α (b) independent of β

(c) independent of α and β (d) None of the above

109. If $a^2+b^2+c^2=-2$ and

$$f(x)=\begin{vmatrix}1+a^2x & (1+b^2)x & (1+c^2)x\\ (1+a^2)x & 1+b^2x & (1+c^2)x\\ (1+a^2)x & (1+b^2)x & 1+c^2x\end{vmatrix},$$

then f (x) is a polynomial of degree

(a) 1 (b) 0

(c) 3 (d) 2

110. If p, q, r are in A.P., then the value of

$$\begin{vmatrix}x+4 & x+9 & x+p\\ x+5 & x+10 & x+q\\ x+6 & x+11 & x+r\end{vmatrix}$$ is

(a) $x+15$ (b) $x+20$

(c) $x+p+q+r$ (d) None of these

111. Suppose $\alpha, \beta, \gamma \in R$ are such that $\sin\alpha, \sin\beta, \sin\gamma \neq 0$ and

$$\Delta=\begin{vmatrix}\sin^2\alpha & \sin\alpha\cos\alpha & \cos^2\alpha\\ \sin^2\beta & \sin\beta\cos\beta & \cos^2\beta\\ \sin^2\gamma & \sin\gamma\cos\gamma & \cos^2\gamma\end{vmatrix}$$ then Δ cannot exceed

(a) 1 (b) 0 (c) $-\frac{1}{2}$ (d) None of these

112. If $x, y \in R$, then the determinant

$$\begin{vmatrix}\cos x & -\sin x & 1\\ \sin x & \cos x & 1\\ \cos(x+y) & -\sin(x+y) & 0\end{vmatrix}$$ lies in the interval

(a) $\left[-\sqrt{2},\sqrt{2}\right]$ (b) [–1, 1]

(c) $\left[-\sqrt{2},1\right]$ (d) $\left[-1,-\sqrt{2}\right]$

113. Let $A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$. If $|A^2|=25$, then $|\alpha|$ equals to

(a) 5^2 (b) 1 (c) $\frac{1}{5}$ (d) 5

114. If $A=\begin{bmatrix}1 & 0 & 3\\ 2 & 1 & 1\\ 0 & 0 & 2\end{bmatrix}$, then the value of $|\text{adj (adj A)}|$ is

(a) 14 (b) 16 (c) 15 (d) 12

115. If $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, then value of A^{-n} is

(a) $\begin{bmatrix}-1 & 0\\ n & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\ 2 & n\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\ -n & 1\end{bmatrix}$ (d) None of these

116. If $\Delta=\begin{vmatrix}a & b & c\\ x & y & z\\ p & q & r\end{vmatrix}$, then value of $\begin{vmatrix}ka & kb & kc\\ kx & ky & kz\\ kp & kq & kr\end{vmatrix}$ is

(a) $k^2\Delta$ (b) $k^3\Delta$ (c) $k\Delta$ (d) $k^4\Delta$

117. If $\begin{vmatrix}3^2+k & 4^2 & 3^2+3+k\\ 4^2+k & 5^2 & 4^2+4+k\\ 5^2+k & 6^2 & 5^2+5+k\end{vmatrix}=0$, then the value of k is

(a) 0 (b) –1 (c) 2 (d) 1

118. $$\begin{vmatrix}{}^xC_1 & {}^xC_2 & {}^xC_3\\ {}^yC_1 & {}^yC_2 & {}^yC_3\\ {}^zC_1 & {}^zC_2 & {}^zC_3\end{vmatrix}=$$

(a) $xyz(x-y)(y-z)(z-x)$ (b) $\frac{xyz}{6}(x-y)(y-z)(z-x)$

(c) $\frac{xyz}{12}(x-y)(y-z)(z-x)$ (d) None of these

119. Value of the determinant (when $n \in N$)

$$D=\begin{vmatrix}n! & (n+1)! & (n+2)!\\ (n+1)! & (n+2)! & (n+3)!\\ (n+2)! & (n+3)! & (n+4)!\end{vmatrix}$$ is

(a) $(n!)^2\left(2n^3-8n^2\right)$

(b) $(2n!)^3\left(3n^2+4n-5\right)$

(c) $(n!)^3\left(2n^3+8n^2+10n+4\right)$

(d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) The system of equations will be consistent if

$$\Delta = \begin{vmatrix} 1 & \lambda & 2 \\ \lambda & 1 & -2 \\ \lambda & \lambda & 3 \end{vmatrix} = 0$$

To evaluate Δ we use $R_1 \to R_1 + R_2$ followed by $C_2 \to C_2 - C_1$ to obtain

$$\Delta = \begin{vmatrix} \lambda+1 & \lambda+1 & 0 \\ \lambda & 1 & -2 \\ \lambda & \lambda & 3 \end{vmatrix} = \begin{vmatrix} \lambda+1 & 0 & 0 \\ \lambda & 1-\lambda & -2 \\ \lambda & 0 & 3 \end{vmatrix}$$

$= 3(\lambda+1)(1-\lambda) = 3(1-\lambda^2)$

For the system to be consistent, we must have

$1-\lambda^2 = 0$ or $\lambda = \pm 1$.

2. (b) We have $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} (1+xyz) = 0$$

$\Rightarrow (x-y)(y-z)(z-x)(1+xyz) = 0$

$\Rightarrow xyz + 1 = 0 \qquad [\because x \neq y \neq z]$

$\Rightarrow xyz = -1$

3. (a) Consider first two equations :

$2x + 3y = -4$ and $3x + 4y = -6$

We have $\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \neq 0$

$\Delta_x = \begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} = 2$ and $\Delta_y = \begin{vmatrix} 2 & -4 \\ 3 & -6 \end{vmatrix} = 0$

$\therefore$ $x = -2$ and $y = 0$

Now this solution satisfies all the equations, so the equations are consistent with unique solution.

4. (d) Since the system has no solution

$$\begin{vmatrix} 2 & -1 & -4 \\ 1 & -2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow 2(-2\lambda+1) + 1(\lambda+1) - 4(3) = 0$$

$\Rightarrow -4\lambda + 2 + \lambda + 1 - 12 = 0 \Rightarrow -3\lambda = 9 \Rightarrow \lambda = -3$

5. (d) $C_1 \to C_1 + C_2 + C_3$, gives

$$\Delta = (x+y)^2 \begin{vmatrix} 1 & x^2 & y^2 \\ 1 & y^2 & 2xy \\ 1 & 2xy & x^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (x+y)^2 \begin{vmatrix} 1 & x^2 & y^2 \\ 0 & y^2 - x^2 & 2xy - y^2 \\ 0 & 2xy - x^2 & x^2 - y^2 \end{vmatrix}$$

$= (x+y)^2 [-(x^2 - y^2)^2 - (2xy - x^2)(2xy - y^2)]$

$= -(x+y)^2 [x^2 - xy + y^2]^2 = -(x^3 + y^3)^2$

6. (d) We have

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = (a+ib)(a-ib) - (c+id)(-c+id)$$

$= (a^2 + b^2) - (-c^2 - d^2)$

$= a^2 + b^2 + c^2 + d^2$

7. (a) We have $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$

$= \cos 75° \cos 15° - \sin 75° \sin 15°$

$= \cos(75° + 15°) = \cos 90° = 0$

8. (b) Let $\Delta = \begin{vmatrix} x! & (x+1)x! & (x+2)(x+1)x! \\ (x+1)! & (x+2)(x+1)! & (x+3)(x+2)(x+1)! \\ (x+2)! & (x+3)(x+2)! & (x+4)(x+3)(x+2)! \end{vmatrix}$

Taking common $x!$, $(x+1)!$ and $(x+2)!$ form R_1, R_2 and R_3 respectively, we get

$$\Delta = x!(x+1)!(x+2)! \begin{vmatrix} 1 & (x+1) & (x+2) & (x+1) \\ 1 & (x+2) & (x+3) & (x+2) \\ 1 & (x+3) & (x+4) & (x+3) \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= x!(x+1)!(x+2)! \begin{vmatrix} 1 & (x+1) & (x+2)(x+1) \\ 0 & 1 & 2(x+2) \\ 0 & 2 & 2(2x+5) \end{vmatrix}$$

$= x!(x+1)!(x+2)![4x + 10 - 4x - 8]$

$= 2(x!)(x+1)!(x+2)!$

9. (a) Area of $\Delta ABC = 3$ sq. units

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 6$$

$\Rightarrow 1(0-0) - 3(0-k) + 1(0-0) = \pm 6$

$\Rightarrow 3k = \pm 6 \Rightarrow k = \pm 2$

10. (a) $M_{12} = \begin{vmatrix} -\sin\theta & \sin\theta \\ -1 & 1 \end{vmatrix} = -\sin\theta + \sin\theta = 0$

$\Rightarrow c_{12} = -M_{12} = 0$

$$M_{22} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1 + 1 = 2 \Rightarrow c_{22} = M_{22} = 2$$

$$M_{32} = \begin{vmatrix} 1 & 1 \\ -\sin\theta & \sin\theta \end{vmatrix} = \sin\theta + \sin\theta = 2\sin\theta$$

$c_{32} = -M_{32} = -2\sin\theta$

11. (a) Given determinant is $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

We have $M_{31} = \begin{vmatrix} -3 & 5 \\ 0 & 4 \end{vmatrix} = -12 - 0 = -12$

$\Rightarrow A_{31} = M_{31} = -12$

$M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = 8 - 30 = -22 \Rightarrow A_{32} = -M_{32} = 22$

$M_{33} = \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 0 + 18 = 18 \Rightarrow A_{33} = M_{33} = 18$

$\therefore a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

$= (2)(-12) + (-3)(22) + (5)(18)$

$= -24 - 66 + 90 = -90 + 90 = 0$

12. (a) Let $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Here, $a_{32} = 5$

Then,

$c_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = (-1)^5(8-30) = -(-22) = 22$

$\therefore a_{32}.c_{32} = 5 \times 22 = 110$

13. (b) $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 \end{vmatrix} = 0$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - aR_1$, we get

$\begin{vmatrix} 1 & a & -1 \\ 0 & -1-2a & a+2 \\ 0 & 1-a^2 & 2+a \end{vmatrix} = 0$

Expanding along C_1, we get

$(a+2)(a^2 - 2a - 2) = 0$

$\Rightarrow a = -2, 1 \pm \sqrt{3}$

14. (c) $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$

Expanding along R_1

$f(x) = 0 - (x-a)[0 - (x-c)(x+b)]$

$+ (x-b)[(x+a)(x+c) - 0]$

$f(x) = (x-a)(x-c)(x+b) + (x-b)(x+a)(x+c)$

Now, $f(0) = (0-a)(0-c)(0+b) + (0-b)(0+a)(0+c)$

$= (-a)(-c)(b) + (-b)(a)(c) = abc - abc = 0$

15. (d) Given $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

Operate, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix} = 0$

$\Rightarrow 1[-30x^2 + 120 + 30x - 60] = 0$

$\Rightarrow 30x^2 - 30x - 60 = 0 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$

Thus, solution set is $\{2, -1\}$.

16. (c) $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0$

$D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$ No solution

17. (c) The system is homogenuous system.

$\therefore$ it has either unique solution or infinite many solution depend on $|A|$

$\therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 2 \times 1 + 1(-3) + (4-3)$

$= 2 - 3 + 1 = 0$

Hence, system has infinitely many solution.

18. (b) Given $\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0$

$\Rightarrow 0(5x - 63) - x(x^2 - 0) + 16(9x - 0) = 0$

$\Rightarrow -x^3 + 144x = 0 \Rightarrow x(144 - x^2) = 0 \Rightarrow x = 0, \pm 12.$

19. (b) Let $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$

$= \begin{bmatrix} 3+0 & 51-50 \\ 2+0 & 34-0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 34 \end{bmatrix}$

$\Rightarrow |AB| = \begin{vmatrix} 3 & 1 \\ 2 & 34 \end{vmatrix} = 102 - 2 = 100$

20. (b)

21. (c) $\det(B^{-1}AB) = \det(B^{-1})\det A \det B$

$= \det(B^{-1}).\det B.\ \det A = \det(B^{-1}B).\det A$

$= \det(I).\ \det A = 1.\ \det A = \det A.$

22. (c) Cofactor matrix $= \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$

$\therefore \text{adj } A = (\text{Cofactor matrix})^T = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$

$\therefore \quad |\text{adj } A| = \begin{vmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{vmatrix} = a^6.$

23. (b) Let A be any 2×2 matrix.

Given $A(\text{adj}A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$\Rightarrow \quad A(\text{adj}A) = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 10I$ ...(i)

where I = identity matrix of order 2×2.

We know $A^{-1} = \frac{1}{|A|}(\text{Adj}.A)$

Pre multiplied by 'A', we get

$AA^{-1} = \frac{A}{|A|}.(\text{Adj } A)$

$\Rightarrow \quad I = \frac{A.\text{Adj }(A)}{|A|}$

$\Rightarrow \quad A(\text{adj}A) = |A|\,I$ ...(ii)

$\therefore$ From equations (i) and (ii), we have

$|A| = 10$

24. (a) Given $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = \begin{vmatrix} x+a+b & a & b \\ x+a+b & x & b \\ x+a+b & b & x \end{vmatrix}$

$(C_1 \to C_1 + C_2 + C_3)$

Take out common $(x+a+b)$ from C_1

$= (x+a+b)\begin{vmatrix} 1 & a & b \\ 1 & x & b \\ 1 & b & x \end{vmatrix}$

$= (x+a+b)\begin{vmatrix} 1 & a & b \\ 0 & x-a & 0 \\ 0 & -(x-b) & x-b \end{vmatrix}$

$(R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_2)$

$= (x+a+b)(x-a)(x-b)\begin{vmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$

$= (x+a+b)(x-a)(x-b)$ is a factor of given determinant.

25. (c) Let $A = \begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bc \\ 1 & 1+ac & 1+bc \end{vmatrix}$

Since two rows of given determinant are equal

$\therefore \ A = 0$

26. (d) Let,

$A = \begin{bmatrix} \cos 2q & -\sin 2q \\ \sin 2q & \cos 2q \end{bmatrix}$

And $A^{-1} = \frac{\text{Adj.}A}{|A|}$

Here $|A| = \cos^2 2\theta - (-\sin^2 2\theta)$

$= \cos^2 2\theta + \sin^2 2\theta$

$= 1 \quad (\because \sin^2\theta + \cos^2\theta = 1)$

And, Adj $A = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}'$

Where, A_{11} = cofactor

and, $A_{11} = (-1)^{1+1}.\cos 2\theta = \cos 2\theta$

$A_{12} = (-1)^{1+2}.\sin 2\theta = -\sin 2\theta$

$A_{21} = (-1)^{2+1}.(-\sin 2\theta) = +\sin 2\theta$

$A_{22} = (-1)^{2+2}\cos 2\theta = \cos 2\theta$

Hence, Adj $A = \begin{bmatrix} \cos 2q & -\sin 2q \\ \sin 2q & \cos 2q \end{bmatrix}^T$

Thus, Adj(A) $= \begin{bmatrix} \cos 2q & \sin 2q \\ -\sin 2q & \cos 2q \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}.$

27. (a) The system of equations $kx + 2y - z = 1$, $(k-1)y - 2z = 2, (k+2)z = 3$ will have a unique solution if $\Delta \neq 0$

$\therefore \quad \begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0$

$\Rightarrow \quad k(k-1)(k+2) \neq 0$

$\therefore \quad k \neq 0, k \neq 1, k \neq -2,$

$\therefore$ From the choices, we have $k = -1$

28. (b) Given: $\Delta = \begin{vmatrix} 3 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & x & 5 \end{vmatrix}$

using properties of determinants

$\Rightarrow \quad \Delta = (-1)\begin{vmatrix} 5 & 3 & 6 \\ 8 & 7 & 9 \\ x & 10 & 5 \end{vmatrix}$

$\Rightarrow \quad \Delta = (-1)^2\begin{vmatrix} 5 & 3 & 6 \\ x & 10 & 5 \\ 8 & 7 & 9 \end{vmatrix}$

$\Rightarrow \quad \Delta = (-1)^3\begin{vmatrix} x & 10 & 5 \\ 5 & 3 & 6 \\ 8 & 7 & 9 \end{vmatrix} = -\Delta$

29. (a) Let $A = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$\Rightarrow \quad x(x^2 - 12) - 3(2x - 14) + 7(12 - 7x) = 0$

$\Rightarrow \quad x^3 - 12x - 6x + 42 + 84 - 49x = 0$

$\Rightarrow \quad x^3 - 67x + 126 = 0$

If $(x + 9)$ is a factor of the given equation then

$(x+9)(x^2 - 9x + 14) = 0$

$\Rightarrow \quad x^2 - 9x + 14 = 0$

Thus $(x - 7)(x - 2) = 0$ is the other factor.

30. (a) Let $A = \begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$

$= a^2[\sin(n+2)x\cos(n+1)x - \cos(n+2)x\sin(n+1)x]$

$-a[\sin(n+2)x\cos nx - \cos(n+2)x\sin nx]$

$+1[\sin(n+1)x\cos nx - \cos(n+1)x\sin nx]$

$= a^2 [\sin (n+2-n-1)x] - a [\sin (n+2-n)x] + [\sin (n+1-n)x]$

$= a^2 \sin x - a \sin 2x + \sin x$

Thus the value of the determinant is independent of n.

31. (c) Given : $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$

$\therefore |A| = 1(-2) - 1(18) = -20$

we know that $A^{-1} = \dfrac{\text{Adj. } A}{|A|}$

The element a_{23} will be $\dfrac{A_{32}}{|A|}$, because Adj. A is the transpose of the respective co-factors founded.

Now, $A_{32} = 5 - (-3) = 8$

Thus $a_{23} = \dfrac{8}{-20} = \dfrac{-2}{5}$.

32. (d) Let $A = \begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} 2x & x-z & x-y \\ 2y & z+x & y-x \\ 2z & z-x & x+y \end{vmatrix}$

Applying $R_1 \to R_2 + R_1, R_3 \to R_3 + R_1$

$= \begin{vmatrix} 2x & x-z & x-y \\ 2(x+y) & 2x & 0 \\ 2(z+z) & 0 & 2x \end{vmatrix}$

On expanding, we get

$= 2x(4x^2) - (x-z)[4x(x+y)] + (x-y)[-4x(x+z)]$

$= 8x^2 - (x-z)(4x^2+4xy) - (x-y)(4x^2+4xz)$

$= 8x^3 - 4x^3 - 4x^2y + 4zx^2 + 4xyz - 4x^3 - 4x^2z + 4yx^2 + 4xyz$

$= 8xyz$

Given : $A = kxyz \Rightarrow 8xyz = kxyz \Rightarrow k = 8$

33. (b) Given : $f(x) = \begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}$

Applying $C_1 \to 2C_1$ and dividing whole the determinant by 2 ; and applying $C_1 \to C_1 - C_3$, we get

$f(x) = \dfrac{1}{2}\begin{vmatrix} 2x-\cos x & 1+\sin x & \cos x \\ 0 & \log(1+x) & 2 \\ 2x^2 & 1+x^2 & 0 \end{vmatrix}$

Expanding along C_1, we get

$f(x) = \dfrac{1}{2}[(2x - \cos x)(-2-2x^2) + 2x^2 \{2 + 2\sin x - \cos\log(1+x)\}]$

$= \dfrac{1}{2}[-2x^2(2x - \cos x) - 4x + 2\cos x + 2x^2 \{2 + 2\sin x - \cos\log(1+x)\}]$

$\therefore$ Coefficient of x in $f(x) = \dfrac{1}{2}(-4) = -2$.

34. (c) If $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

and $A^{-1} = \dfrac{1}{k}\text{adj}(A)$ ......(i)

Also, we know $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$ ....(ii)

$\therefore$ By comparing (i) and (ii)

$|A| = k \Rightarrow |A| = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$

$= 3(2+1) + 2(1+0) + 4(1-0)$

$= 9 + 2 + 4 = 15$

35. (b) $A = \begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}$

$C_{11} = (-1)^2 \begin{vmatrix} -4 & 2 \\ 2 & 8 \end{vmatrix} = -32 - 4 = -36$

$C_{12} = (-1)^3 \begin{vmatrix} 5 & 2 \\ 2 & 8 \end{vmatrix} = -(40-4) = -36$

$C_{13} = (-1)^4 \begin{vmatrix} 5 & -4 \\ 2 & 2 \end{vmatrix} = 10 + 8 = 18$

$C_{21} = (-1)^3 \begin{vmatrix} -5 & -2 \\ 2 & 8 \end{vmatrix} = -(-40+4) = 36$

$C_{22} = (-1)^4 \begin{vmatrix} 4 & -2 \\ 2 & 8 \end{vmatrix} = (32+4) = 36$

$C_{23} = (-1)^5 \begin{vmatrix} 4 & -5 \\ 2 & 2 \end{vmatrix} = -(8+10) = -18$

$C_{31} = (-1)^4 \begin{vmatrix} -5 & -2 \\ -4 & 2 \end{vmatrix} = -10 - 8 = -18$

$C_{32} = (-1)^5 \begin{vmatrix} 4 & -2 \\ 5 & 2 \end{vmatrix} = -(8+10) = -18$

$C_{33} = (-1)^6 \begin{vmatrix} 4 & -5 \\ 5 & -4 \end{vmatrix} = -16 + 25 = 9$

$\therefore \text{adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

$= \begin{bmatrix} -36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}'$

$= \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$

36. (b) Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$C_{11} = \delta,\ C_{12} = -\gamma, C_{21} = -\beta,\ C_{22} = \alpha$

$\therefore \text{adj } A = \begin{bmatrix} \delta & -\gamma \\ -\beta & \alpha \end{bmatrix}' = \begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix}$.

37. (d) Let A and B be square matrices and A^{-1}, B^{-1} be of same order.
then $(AB)^{-1} = B^{-1}.A^{-1}$.

38. (a) $(A-2I)(A+I)=0$
$\Rightarrow AA - A - 2I = 0 \quad (\because AI = A)$
$$\Rightarrow A\left(\frac{A-I}{2}\right) = I \quad \therefore \frac{A-I}{2} = A^{-1}$$

39. (b) $B = -A^{-1}BA \Rightarrow AB = -BA \Rightarrow AB + BA = 0$
$\therefore (A+B)^2 = A^2 + AB + BA + B^2 = A^2 + B^2$

40. (c) Inverse of an identity matrix is the matrix itself.

41. (a) Given, $A^2 + A - I = 0$
Pre Multiply it with A^{-1}, we get
$A + I - A^{-1} = 0 \Rightarrow A^{-1} = I + A$
Hence, A^{-1} exists and equals $I + A$.

42. (d) Here $X = A^{-1}B$
$$\text{i.e., } X = \begin{bmatrix} 3 & -1/2 & -1/2 \\ -4 & 3/4 & 5/4 \\ 2 & -3/4 & -3/4 \end{bmatrix}_{3\times 3}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 1 \\ 3 \\ -21 \end{bmatrix}_{3\times 1}.$$

43. (a) $$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$
clearly $A \neq 0$. Also $|A| = -1 \neq 0$
$$\therefore A^{-1} \text{ exists, further } (-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$$
$$\text{Also } A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

44. (a) Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$
$$\Rightarrow B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$
Also since, $B = A^{-1} \Rightarrow AB = I$
$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$$

45. (b) $$\Delta = \begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$
Expanding along first row, the determinant is
$$\Delta = [\cos px \sin(p+d)x - \sin px \cos(p+d)x]$$
$$-a[\cos(p-d)x \sin(p+d)x - \sin(p-d)x \cos(p+d)x]$$
$$+a^2[\cos(p-d)x \sin px - \sin(p-d)x \cos px]$$
$$= \sin(p+d-p)x - a\sin(p+d-p+d)x + a^2 \sin(p-p+d)x$$
$$= \sin dx - a \sin 2dx + a^2 \sin dx$$
Clearly Δ is independent of p.

46. (b) A system of linear equations like
$a_1x + b_1y = c_1$.
$a_2x + b_2y = c_2$.
can be represented in matrix form as
$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

47. (d) The given matrix is $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -5 & 6 \end{bmatrix}$
Since, this matrix is not a square matrix.
Therefore, its determinant does not exist.

48. (a) Let $\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$
By expanding along first row, we get
$$\Delta = \begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta \begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta \begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$$
$$= x\left(-x^2 - 1\right) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$$
$$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$$
$$= -x^3 - x + x\left(\sin^2\theta + \cos^2\theta\right) \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$
$$= -x^3 - x + x = -x^3$$
Hence A is independent of θ.

49. (a) The value of the determinant remains unchanged, if its rows and columns are interchanged.

50. (c) If we interchange the identical rows (or columns) of the determinant Δ, then numerical value of Δ does not change but due to interchange, the sign of Δ will change.
$\therefore \quad \Delta = -\Delta$
$\Rightarrow \quad \Delta = 0$

51. (c) The given points are (a, b + c), (b, c + a) and (c, a + b).
$$\text{Area of triangle, } \Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$
Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we get

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} b-a & a-b \\ c-a & a-c \end{vmatrix}$$

$$= \frac{1}{2}\left((b-a)(a-c)-(a-b)(c-a)\right)$$

$$= \frac{1}{2}\left(-(a-b)(a-c)+(a-b)(a-c)\right)$$

$$= \frac{1}{2}\times 0 = 0$$

$\therefore\ \Delta = 0$

Thus, area of triangle is zero. Hence, the three given points are collinear.

52. (d) Given $\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4$

$\Rightarrow\ |-2(4-k)+1(0-0)| = 8$

$\Rightarrow\ -2(4-k)+1(0-0) = \pm 8$

$\Rightarrow\ (-8+2k) = \pm 8$

Taking positive sign,

$2k-8=8$

$\Rightarrow\ 2k = 16 \Rightarrow k = 8$

Taking negative sign,

$2k-8=-8$

$\Rightarrow\ 2k = 0 \Rightarrow k = 0$

$\therefore\ k = 0, 8$

53. (b) $M = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$

$|M| = 6$, adj $M = \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$

$M^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$

54. (d) Given, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Apply $R_2 \to R_2 - R_1$ and $R \to R_3 - R_1$

$\therefore\ D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$

Hence, D is divisible by both x and y

55. (c) $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125 \Rightarrow |A|^3 = 125$

Now, $|A| = \alpha^2 - 4$

$\Rightarrow (\alpha^2-4)^3 = 125 = 5^3$

$\Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha = \pm 3$

56. (d) $[M(\alpha)\,M(\beta)]^{-1} = M(\beta)^{-1}\,M(\alpha)^{-1}$

Now $M(\alpha)^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} = M(-\alpha)$

$M(\beta)^{-1} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$

$= M(-\beta)\,[M(\alpha)\,M(\beta)]^{-1}$

$= M(-\beta)\,M(-\alpha)$

57. (a) Given $\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+\gamma \end{vmatrix} = 0$

Operate $C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} x+\alpha+\beta+\gamma & \beta & \gamma \\ x+\alpha+\beta+\gamma & x+\beta & \alpha \\ x+\alpha+\beta+\gamma & \beta & x+\gamma \end{vmatrix} = 0$

$= (x+\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta & \gamma \\ 1 & x+\beta & \alpha \\ 1 & \beta & x+\gamma \end{vmatrix} = 0$

$\Rightarrow x+\alpha+\beta+\gamma = 0 \Rightarrow x = -(\alpha+\beta+\gamma)$

Again if

$\begin{vmatrix} 1 & \beta & \gamma \\ 1 & x+\beta & \alpha \\ 1 & \beta & \gamma \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & \beta & \gamma \\ 0 & x & \alpha-\gamma \\ 0 & 0 & x \end{vmatrix} = 0$

$\Rightarrow x^2 = 0 \Rightarrow x = 0$

$\therefore$ Solutions of the equation are $x = 0, -(\alpha+\beta+\gamma)$

STATEMENT TYPE QUESTIONS

58. (b) To every square matrix $A = [a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of the square matrix A, where $a_{ij} = (i, j)^{th}$ element of A.
Determinant is a function which associates each square matrix with a unique number.

59. (d) For matrix A, |A| is read as determinant of A and not modulus of A. Also, only square matrices have determinant.

60. (c) Matrix cannot be reduced to a number, because it is just an arrangement of numbers, while if $A = [a_{ij}]_{n\times n}$ be a square matrix of order n, then the expression $|A| = |a_{ij}|$ is called determinant of A which can be reduced to a number.

61. **(a)** If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

MATCHING TYPE QUESTIONS

62. **(b)** We know that, the area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Thus,

A. Required area $= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}|1(0-3)-0(6-4)+1(18-0)|$

$= \frac{1}{2}|-3+18| = \frac{15}{2}$ sq units.

B. Required area $= \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

$= \frac{1}{2}|2(1-8)-7(1-10)+1(8-10)|$

$= \frac{1}{2}|2(-7)-7(-9)+1(-2)|$

$= \frac{1}{2}|-14+63-2| = \frac{47}{2}$ sq units.

C. Required area $= \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$

$= \frac{1}{2}|-2(2+8)+3(3+1)+(-24+2)|$

$= \frac{1}{2}|-20+12-22|$

$= \frac{1}{2}|-30| = 15$ sq units.

INTEGER TYPE QUESTIONS

63. **(c)** $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ 2\cos x + \sin x & \cos x & \sin x \end{vmatrix} = 0$$

taking $(\sin x + 2\cos x)$ common from C_1

$$(\sin x + 2\cos x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$(\sin x + 2\cos x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$(\sin x + 2\cos x)(\sin x - \cos x)^2 = 0$

$\Rightarrow \sin x + 2\cos x = 0$ or $(\sin x - \cos x)^2 = 0$

$\Rightarrow \tan x = -2$ or $\tan x = 1$

$\Rightarrow x = -\tan^{-1}(2)$ or $x = \frac{\pi}{4}$

$\because -\tan^{-1}(2) \notin \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

So, $x = \frac{\pi}{4}$

Hence, number of roots = 1

64. **(c)** $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$

Apply $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} x & 0 & -z \\ 0 & y & -z \\ p-x & q-y & r \end{vmatrix} = 0$$

$\Rightarrow x[yr + z(q-y)] - z[0 - y(p-x)] = 0$

[Expansion along first row]

$\Rightarrow xyr + xzq - xzy + yzp - zyx = 0$

$\Rightarrow xyr + zxq + yzp = 2xyz \Rightarrow \frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$

65. **(b)** Let $\Delta = \begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix}$

$$= \begin{vmatrix} 2\sin A\cos A & \sin C & \sin B \\ \sin C & 2\sin B\cos B & \sin A \\ \sin B & \sin A & 2\sin C\cos C \end{vmatrix}$$

The above determinant is the product of two determinants,

$$\Delta = \begin{vmatrix} \sin A & \cos A & 0 \\ \sin B & \cos B & 0 \\ \sin C & \cos C & 0 \end{vmatrix} \times \begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix} = 0$$

66. **(d)** Replace $\log_b a$ by $\frac{\log a}{\log b}$

$$\therefore \Delta = \frac{1}{\log x \log y \log z} \times \begin{vmatrix} \log x & \log y & \log z \\ \log x & 3\log y & \log z \\ \log x & \log y & 5\log z \end{vmatrix}$$

Take log x, log y, log z common from C_1, C_2, C_3 respectively.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 4 \end{vmatrix} = 1 \times 2 \times 4 = 8$$

67. **(b)** Here $\Delta = 0$ for $k = 3, 1$, $\Delta_x = 0$ for $k = 2, 1$, $\Delta_y = 0$ for $k = 1$. Hence $k = 1$.

Alternatively, for infinitely many solutions the two equations become identical

$\Rightarrow \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1} \Rightarrow k = 1$

68. **(b)** Applying $R_1 \to R_1 + R_2 + R_3$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$= (a + b + c)^3$.

$\therefore \quad k = 1$

69. **(a)** $\Delta = \begin{vmatrix} e^a & e^{2a} & e^{3a} \\ e^b & e^{2b} & e^{3b} \\ e^c & e^{2c} & e^{3c} \end{vmatrix} - \begin{vmatrix} e^a & e^{2a} & 1 \\ e^b & e^{2b} & 1 \\ e^c & e^{2c} & 1 \end{vmatrix}$

$$= e^a.e^b.e^c \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} + \begin{vmatrix} e^a & 1 & e^{2a} \\ e^b & 1 & e^{2b} \\ e^c & 1 & e^{2c} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} - \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} = 0$$

{Since, $a + b + c = 0$. So, $e^a.e^b.e^c = 1$}

70. **(b)** $|A| = 0$ as the matrix A is singular

$$\therefore \ |A| = \begin{vmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$$

Apply $R_2 \to R_2 - 2R_1$ and $R_3 \to R_2 - 3R_1$ and expand.

$-2(4 - 3\lambda) + 4(4 - 2\lambda) = 0$

$\Rightarrow 8 - 2\lambda = 0 \Rightarrow \lambda = 4$

71. **(d)** Let $A = \begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix}$

Operate R_2 ® R_2 - R_1, R_3 ® R_3 - R_2

Then $A = \begin{vmatrix} 13 & 16 & 19 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

($\because$ entry of two rows are same)

72. **(a)** Let $A = \begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2$, we get

$$A = \begin{vmatrix} \sin^2 x + \cos^2 x & \cos^2 x & 1 \\ \cos^2 x + \sin^2 x & \sin^2 x & 1 \\ -10+12 & 12 & 2 \end{vmatrix} = \begin{vmatrix} 1 & \cos^2 x & 1 \\ 1 & \sin^2 x & 1 \\ 2 & 12 & 2 \end{vmatrix}$$

Since, two columns are identical.

$\therefore \quad A = 0$

73. **(c)** Given a, b, c are in A.P.

$\therefore \quad 2b = a + c$ ....(i)

Now, $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ [Applying $R_2 \to 2R_2$]

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+(a+c) \\ x+3 & x+4 & x+c \end{vmatrix} \quad \text{[using equation (i)]}$$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+3 & x+4 & x+c \end{vmatrix}$$

[Applying $R_2 \to R_2 - (R_1 + R_3)$]

$= \frac{1}{2} . 0 = 0$

74. **(d)** There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1, C_2 and C_3).

75. **(b)** Consider, $\begin{vmatrix} \sin 10° & -\cos 10° \\ \sin 80° & \cos 80° \end{vmatrix}$

$= \sin 10° \cos 80° + \sin 80° \cos 10°$

$= \sin (10° + 80°)$

[$\because$ $\sin (A + B) = \sin A \cos B + \cos A \sin B$]

$= \sin (90°) = 1$ ($\because$ $\sin 90° = 1$)

76. **(a)** We have,

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = \begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1 \end{vmatrix}$$

$$= \frac{1}{\log x.\log y.\log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

[$\because$ all rows are identical]

77. **(c)** $\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 0 & 12 & 9 \\ 1 & 2 & 2 \end{vmatrix}$

$= 1(12 \times 2 - 2 \times 9) - 4(0 \times 2 - 1 \times 9) + 3(0 \times 2 - 1 \times 12)$

$= 1(24 - 18) - 4(0 - 9) + 3(0 - 12)$

$= 6 + 36 - 36 = 6$

78. **(b)** The area of triangle formed by the points (1, 2), (k, 5) and (7, 11) is given by

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ k & 5 & 1 \\ 7 & 11 & 1 \end{vmatrix} = 0$$

or $\quad 1(5 - 11) - (k - 7) + 1(11k - 35) = 0$

or $\quad -6 - 2k + 14 + 11k - 35 = 0$

or $\quad 9k - 27 = 0$

or $\quad k = 3$

79. (b) The element $a_{11} = 2$. Its minor is given by determinant of the matrix obtained by deleting the rows and column which contain element $a_{11} = 2$

i.e., minor of $a_{11} = \begin{vmatrix} 7 & 8 \\ 4 & 5 \end{vmatrix} = 35 - 32 = 3$

80. (a) $A_{11} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{12} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{13} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{21} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{23} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{31} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{32} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1\times1 - 1\times1 = 0$

$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

or $\text{adj } A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$|\text{adj } A| = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

81. (a) If A is a non singular matrix of order m, then $|\text{adj } (A)| = |A|^{m-1}$

Here m = 3

$\therefore \quad |\text{adj } (A)| = |A|^{3-1} = |A|^2$

$\therefore \quad n = 2$

ASSERTION- REASON TYPE QUESTIONS

82. (c) If $\Delta = 0$, then two of rows or column are proportional which is possible even if three lines are parallel or two of them are coincident.

83. (a)

84. (c) Clearly, $A + B \neq C$. Hence Reason is false. However, by a property of determinants, det C = det A + det B.

85. (d) $\Delta = \begin{vmatrix} a-1 & a & a+1 \\ b-1 & b & b+1 \\ c-1 & c & c+1 \end{vmatrix} = \begin{vmatrix} 0 & a & a+1 \\ 0 & b & b+1 \\ 0 & c & c+1 \end{vmatrix}$

$(C_1 \to C_1 + C_3 - 2C_2)$

$\therefore \Delta = 0$, which is not a natural number.

86. (d) $\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0 \Rightarrow 5 - 11\lambda + 7\lambda^2 - \lambda^3 = 0$

$\therefore \quad 5I - 11A + 7A^2 - A^3 = 0$

Multiplying by A^{-1}, we get $5A^{-1} = A^2 - 7A + 11I$.

87. (d) The given system of linear equations can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

i.e., AX = B where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Here, $\det A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix} = (2\times4)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Also, $(\text{adj } A)B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

Thus, Reason is true.

However, the Assertion is not true as the given system is inconsistent. Here, the third equation contradicts the first and second which are identical. So, the given system has no solution.

88. (a) Let $f(\theta) = \begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$

$$\therefore f'(\theta) = \begin{vmatrix} -\sin(\theta+\alpha) & -\sin(\theta+\beta) & -\sin(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$$

$$+ \begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$$

$$+ \begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ 0 & 0 & 0 \end{vmatrix}$$

$= 0 + 0 + 0 = 0$

$\Rightarrow \quad f'(\theta) = 0 \Rightarrow f(\theta) = c$

89. (c) The given system can be written as AX = B, where

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}$$

$$\text{Here, }(\text{adj A})B=\begin{bmatrix}3 & 0 & -1\\ -3 & 0 & 1\\ -3 & 0 & 1\end{bmatrix}^T\begin{bmatrix}8\\5\\4\end{bmatrix}$$

$$=\begin{bmatrix}3 & -3 & -3\\ 0 & 0 & 0\\ -1 & 1 & 1\end{bmatrix}\begin{bmatrix}8\\5\\4\end{bmatrix}=\begin{bmatrix}24-15-12\\ 0+0+0\\ -8+5+4\end{bmatrix}=\begin{bmatrix}-3\\0\\1\end{bmatrix}\neq 0$$

$$\text{and }|A|=\begin{vmatrix}2 & 3 & 6\\ 1 & 2 & 3\\ 1 & 1 & 3\end{vmatrix}$$

$= 2(6 - 3) - 3(3 - 3) + 6(1 - 2) = 6 - 0 - 6 = 0$

So, the Assertion is true but Reason is false.

($\because$ |A| = 0,(adj A) B ≠ 0, $\therefore$ the given system is inconsistent and has no solution)

90. (a) Expansion along second row (R_2)

$$|A|=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$

By expanding along R_2, we get

$$|A|=(-1)^{2+1}a_{21}\begin{vmatrix}a_{12} & a_{13}\\ a_{32} & a_{33}\end{vmatrix}+(-1)^{2+2}a_{22}\begin{vmatrix}a_{11} & a_{13}\\ a_{31} & a_{33}\end{vmatrix}$$

$$+(-1)^{2+3}a_{23}\begin{vmatrix}a_{11} & a_{12}\\ a_{31} & a_{32}\end{vmatrix}\text{ (applying all steps together)}$$

$= - a_{21}(a_{12}a_{33} - a_{32}a_{13}) + a_{22}(a_{11}a_{33} - a_{31}a_{13}) - a_{23}(a_{11}a_{32} - a_{31}a_{12})$

$|A| = -a_{21}a_{12}a_{33} + a_{21}a_{32}a_{13} + a_{22}a_{11}a_{33} - a_{22}a_{31}a_{13} - a_{23}a_{11}a_{32} + a_{23}a_{31}a_{12}$

$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22}$ ....(i)

Expansion along first column (C_1)

$$|A|=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{33} & a_{33}\end{vmatrix}$$

By expanding along C_1, we get

$$|A|=a_{11}(-1)^{1+1}\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix}+a_{21}(-1)^{2+1}\begin{vmatrix}a_{12} & a_{13}\\ a_{32} & a_{33}\end{vmatrix}$$

$$+a_{31}(-1)^{3+1}\begin{vmatrix}a_{12} & a_{13}\\ a_{22} & a_{23}\end{vmatrix}$$

$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{31}(a_{12}a_{23} - a_{13}a_{22})$

$|A| = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{21}a_{12}a_3 + a_{21}a_{13}a_{32} + a_{31}a_{12}a_{23} - a_{31}a_{13}a_{22}$

$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22}$ ... (ii)

Clearly, the values of |A| in eqs. (i) and (ii) are equal. Also, it can be easily verified that the values of |A| by expanding along R_3, C_2 and C_3 are equal to the value of |A| obtained in eqs. (i) and (ii).

Hence, expanding a determinant along any row or column gives same value.

91. (a) if corresponding elements of any two rows (or columns) of a determinant are proportional (in the same ratio), then its value is zero.

$$\text{Thus, }\Delta=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ ka_1 & ka_2 & ka_3\end{vmatrix}=0$$

($\because$ R_1 and R_3 are proportional)

92. (a) We know that, the area of triangle with three collinear points is zero.

Now, consider

$$\text{Area of }\Delta ABC=\frac{1}{2}\begin{vmatrix}a & b+c & 1\\ b & c+a & 1\\ c & a+b & 1\end{vmatrix}$$

$$=\frac{1}{2}|a\{(c+a)\times 1-(a+b)\times 1\}-(b+c)\{b\times 1-1\times c\}+1\{b\times(a+b)-(c+a)\times c\}|$$

$$=\frac{1}{2}|a(c+a-a-b)-(b+c)(b-c)+1(ab+b^2-c^2-ac)|$$

$$=\frac{1}{2}|ac-ab-b^2+c^2+ab+b^2-c^2-ac|=\frac{1}{2}\times 0=0$$

Since, area of ΔABC = 0

Hence, points A(a, b + c), B(b, c + a) C(c, a + b) are collinear.

93. (a) By expanding the determinant $\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$

along R_1, we have

$$\Delta=(-1)^{1+1}a_{11}\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix}+(-1)^{1+2}a_{12}\begin{vmatrix}a_{21} & a_{23}\\ a_{31} & a_{33}\end{vmatrix}+(-1)^{1+3}a_{13}\begin{vmatrix}a_{21} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}$$

$= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where, A_{ij} is cofactor of a_{ij}

= Sum of products of elements of R_1 with their corresponding cofactors.

Similarly, Δ can be calculated by other five ways of expansion that is along R_2, R_3, C_1, C_2 and C_3.

Hence Δ = sum of the products of elements of any row (or column) with their corresponding cofactors.

94. (b) We know that $|adj\,(adj\,A)| = |A|^{n-2}A$.

$= |A|^0 A = A$

Also $|adj\,A| = |A|^{n-1} = |A|^{2-1} = |A|$

$\therefore$ Both the statements are true but Reason is not a correct explantion for Assertion .

95. (a)

96. (a) **97. (a)**

98. (d) The given matrix is a symmetric matrix.

CRITICALTHINKING TYPE QUESTIONS

99. (b) Take a, b, c common from R_1, R_2, R_3 respectively.

$$\therefore\Delta=abc\begin{vmatrix}\frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a}\\ \frac{1}{b}+1 & \frac{1}{b}+2 & \frac{1}{b}\\ \frac{1}{c}+1 & \frac{1}{c}+1 & \frac{1}{c}+3\end{vmatrix}$$

Apply $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Delta = abc\left(3+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ 1+\frac{1}{b} & 2+\frac{1}{b} & \frac{1}{b} \\ 1+\frac{1}{c} & 1+\frac{1}{c} & 3+\frac{1}{c} \end{vmatrix}$$

Now apply

$C_3 \rightarrow C_3 - C_2$ and $C_2 \rightarrow C_2 - C_1$ & expand

$$\Delta = 2abc\left[3+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right] = 0$$

As $a \neq 0, b \neq 0, c \neq 0$

$\therefore\ a^{-1}+b^{-1}+c^{-1} = -3$

100. (d) $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$C_2 \rightarrow C_2 - 2C_3$

$$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0$$

$R_3 \rightarrow R_3 - R_2, R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0$$

$b(c-b) - (b-a)(2c-b) = 0$

$\Rightarrow$ On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore a, b, c$ are in Harmonic Progression.

101. (a) $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$, we get

$$\Delta = \begin{vmatrix} -\cos\theta & 0 & 0 \\ -\cos\theta & \sin\theta & 0 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$$

Expanding along C_3

$$\Delta = 1\,(-\cos\theta\sin\theta - 0) = -\cos\theta\sin\theta = \frac{-1}{2}\sin 2\theta$$

We know, $-1 \leq \sin 2\theta \leq 1$

$$\Rightarrow \frac{-1}{2} \leq \frac{1}{2}\sin 2\theta \leq \frac{1}{2} \Rightarrow \frac{1}{2} \geq -\frac{1}{2}\sin 2\theta \geq \frac{-1}{2}$$

i.e., $-\frac{1}{2} \leq -\frac{1}{2}\sin 2\theta \leq \frac{1}{2} \Rightarrow \frac{-1}{2} \leq \Delta \leq \frac{1}{2}$

Hence, maximum value of Δ is $\frac{1}{2}$

102. (c) Value of the new determinant

$= (4)^{\text{order of det.}}\ \Delta = 4^3\Delta = 64\,\Delta.$

103. (a) $|-2A| = (-2)^3|A| = -8\,\Delta$

104. (c) Given: $a^{-1}+b^{-1}+c^{-1} = 0$

and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda,$

$$\Rightarrow abc\begin{vmatrix} \frac{1+a}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1+b}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1+c}{c} \end{vmatrix} = \lambda$$

$$\Rightarrow abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

$(R_1 \rightarrow R_1 + R_2 + R_3)$

$$\Rightarrow abc\begin{vmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

$$\Rightarrow abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

$$\Rightarrow abc\,(1)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

$R_2 \rightarrow \frac{1}{b}R_1 - R_2,\ R_3 \rightarrow \frac{1}{c}R_1 - R_3$

$$\Rightarrow abc\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = \lambda$$

$\Rightarrow abc = \lambda$

105. (d) Let $1, \omega, \omega^2$ be the cube roots of unity.

$\because\quad \omega^3 = 1,\ 1+\omega+\omega^2 = 0$

Let $A = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$

$= 1\,(1-\omega^{3n}) - \omega^n(\omega^{2n}-\omega^{2n}) + \omega^{2n}(\omega^{4n}-\omega^n)$

$= 1 - \omega^{3n} + \omega^{6n} - \omega^{3n}$

$= 1 - 2\,(\omega^3)^n + [\omega^3]^{2n}$

$= 1 - 2 + 1 \qquad (\because\ \omega^3 = 1)$

$= 0$

106. (c) Let $\Delta = \begin{bmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{bmatrix}$

Taking a, b, c common from R_1, R_2 and R_3 respectively, we get.

$$\Delta = abc \begin{bmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{bmatrix} = a^2b^2c^2 \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

[taking a, b, c common from C_1, C_2, C_3 respectively]

$$= a^2b^2c^2 \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

(applying $C_2 \to C_2 + C_1$, $C_3 \to C_3 + C_1$)

$$= a^2b^2c^2.(-1) \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = a^2b^2c^2 (-1)(0-4)$$

$\Rightarrow \quad \Delta = 4a^2b^2c^2$

107. (c) Let $A = \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$

By taking x, y, z common from the rows R_1, R_2 and R_3 respectively. So,

$$A = xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

Operate $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow A = xyz \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$$

Now take common $y-x$ and $z-x$ from the rows R_2 and R_3 respectively. Thus

$$A = xyz\,(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$$

$= xyz\,(y-x)(z-x)(z-y)$

$= xyz\,(x-y)(y-z)(z-x)$

Given $|A| = 0$

So, $xyz = 0$ $\quad \because x \neq y \neq z$ (given)

108. (a) $\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$

Applying $R_1 \to R_1 \cos\beta$ and $R_2 \to R_2 \sin\beta$ as below

$$= \frac{1}{\cos\beta\sin\beta} \begin{vmatrix} \cos\alpha\cos\beta & -\sin\alpha\cos\beta & \cos\beta \\ \sin\alpha\sin\beta & \cos\alpha\sin\beta & \sin\beta \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1 + R_2$

$$= \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1+\sin\beta-\cos\beta \end{vmatrix}$$

$= (1+\sin\beta - \cos\beta)(\cos^2\alpha + \sin^2\alpha)$

$= 1 + \sin\beta - \cos\beta$

which is independent of α.

109. (d) Applying, $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2x) \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2x) \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

[As given that $a^2+b^2+c^2 = -2 \therefore a^2+b^2+c^2+2 = 0$]

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\therefore f(x) = \begin{vmatrix} 0 & x-1 & 0 \\ 0 & 1-x & x-1 \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$f(x) = (x-1)^2$

Hence degree = 2.

110. (d) $\begin{vmatrix} x+4 & x+9 & x+p \\ x+5 & x+10 & x+q \\ x+6 & x+11 & x+r \end{vmatrix} = \begin{vmatrix} 0 & 0 & p-2q+r \\ x+5 & x+10 & x+q \\ x+6 & x+11 & x+r \end{vmatrix}$

$[R_1 \to R_1 - 2R_2 + R_3]$

$= 0$

[since $p + r = 2q$, hence all entries in first row become 0.]

111. (a) We can write Δ as,

$$\Delta = \sin^2\alpha\sin^2\beta\sin^2\gamma \begin{vmatrix} 1 & \cot\alpha & \cot^2\alpha \\ 1 & \cot\beta & \cot^2\beta \\ 1 & \cot\gamma & \cot^2\gamma \end{vmatrix}$$

$= \sin^2\alpha\sin^2\beta\sin^2\gamma(\cot\beta - \cot\alpha)$
$(\cot\gamma - \cot\alpha)(\cot\gamma - \cot\beta)$

$= \sin(\alpha-\beta)\sin(\alpha-\gamma)\sin(\beta-\gamma)$

It is clear from here that Δ cannot exceed 1.

$[\because \sin\theta \ngtr 1$, for any $\theta \in \mathbf{R}]$

112. (a) The given determinant is

$$\begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$$

Applying $R_3 \to R_3 - \cos y\, R_1 + \sin y\, R_2$, we get

$$\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ 0 & 0 & \sin y - \cos y \end{vmatrix}$$

By expanding along R_3, we have

$\Delta = (\sin y - \cos y)(\cos^2 x + \sin^2 x)$

$$= (\sin y - \cos y) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin y - \frac{1}{\sqrt{2}}\cos y\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4}\sin y - \sin\frac{\pi}{4}\cos y\right] = \sqrt{2}\sin\left(y - \frac{\pi}{4}\right)$$

Hence, $-\sqrt{2} \le \Delta \le \sqrt{2}$.

113. (c) Since, $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$

$$\therefore\ A^2 = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}\begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 & 10\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{vmatrix}$$

$$\Rightarrow \quad |A^2| = \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 & 10\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{vmatrix}$$

$$= 25\begin{vmatrix} \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 25 \end{vmatrix} = 625\alpha^2$$

$\therefore$ $625\alpha^2 = 25$ (given)

$\Rightarrow$ $|\alpha| = 1/5$

114. (b) $|A| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 2$

$\therefore |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = |A|^{2^2}$ [$\because$ Here $n = 3$]

$= 2^4 = 16$

115. (c) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$$

116. (b) We know that if any row of a determinant is multiplied by k, then the value of the determinant is also multiplied by k, Here all the three rows are multiplied by k, therefore the value of new determinant will be $k^3\,\Delta$.

117. (d) Breaking the given determinant into two determinants, we get

$$\begin{vmatrix} 3^2+k & 4^2 & 3^2+k \\ 4^2+k & 5^2 & 4^2+k \\ 5^2+k & 6^2 & 5^2+k \end{vmatrix} + \begin{vmatrix} 3^2+k & 4^2 & 3 \\ 4^2+k & 5^2 & 4 \\ 5^2+k & 6^2 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 0 + \begin{vmatrix} 9+k & 16 & 3 \\ 7 & 9 & 1 \\ 9 & 11 & 1 \end{vmatrix} = 0$$

[Applying $R_3 - R_2$ and $R_2 - R_1$ in second det.]

$$\Rightarrow \begin{vmatrix} 9+k & 16 & 3 \\ 7 & 9 & 1 \\ 2 & 2 & 0 \end{vmatrix} = 0 \quad [\text{Applying } R_3 - R_2]$$

$$\Rightarrow \begin{vmatrix} 9+k & 7-k & 3 \\ 7 & 2 & 1 \\ 2 & 0 & 0 \end{vmatrix} = 0 \quad [\text{Applying } C_2 - C_1]$$

$\Rightarrow 2(7 - k - 6) = 0 \qquad \Rightarrow k = 1$

118. (c) $\Delta = \begin{vmatrix} x & \dfrac{x(x-1)}{2} & \dfrac{x(x-1)(x-2)}{6} \\ y & \dfrac{y(y-1)}{2} & \dfrac{y(y-1)(y-2)}{6} \\ z & \dfrac{z(z-1)}{2} & \dfrac{z(z-1)(z-2)}{6} \end{vmatrix} = \dfrac{xyz}{12}$

$$\begin{vmatrix} x & x-1 & (x-1)(x-2) \\ y & y-1 & (y-1)(y-2) \\ z & z-1 & (z-1)(z-2) \end{vmatrix}$$

$$= \frac{xyz}{12}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad (\text{by } C_2 + C_1,\ C_3 + C_1 + 3C_2)$$

$$= \frac{xyz}{12}(x-y)(y-z)(z-x)$$

119. (c) Here

$$D = (n!)^3\begin{vmatrix} 1 & (n+1) & (n+2)(n+1) \\ (n+1) & (n+2)(n+1) & (n+3)(n+2)(n+1) \\ (n+2)(n+1) & (n+3)(n+2)(n+1) & (n+4)(n+3)(n+2)(n+1) \end{vmatrix}$$

$$= (n!)^3 (n+1)^2 (n+2)$$

$$\times \begin{vmatrix} 1 & 1 & 1 \\ n+1 & n+2 & n+3 \\ (n+2)(n+1) & (n+3)(n+2) & (n+4)(n+3) \end{vmatrix}$$

Operating $C_2 - C_1$, $C_3 - C_2$ and expanding

$= (n!)^3 (n+1)^2 (n+2).\ 2$

$= (n!)^3 (2n^3 + 8n^2 + 10n + 4)$ as on simplification.

CHAPTER 21

CONTINUITY AND DIFFERENTIABILITY

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ is a continuous function:

(a) for x = 2 only.
(b) for all real value of x such that $x \neq 2$.
(c) for all real value of x.
(d) for all integral value of x only.

2. If $y = 2^{-x}$, then $\frac{dy}{dx}$ is equal to :

(a) $-\frac{x}{2^{x+1}}$ (b) $2^x \log 2$
(c) $2^{-x} \log 2$ (d) $\frac{\log \frac{1}{2}}{2^x}$

3. If $y = \sec x°$, then $\frac{dy}{dx}$ is equal to :

(a) sec x tan x (b) sec x° tan x°
(c) $\frac{\pi}{180} \sec x° \tan x°$ (d) None of these

4. If y = log (log x), then the value of $e^y \frac{dy}{dx}$ is :

(a) e^y (b) $\frac{1}{x}$
(c) $\frac{1}{(\log x)}$ (d) $\frac{1}{(x \log x)}$

5. If $y = \cot^{-1}(x^2)$, then the value of $\frac{dy}{dx}$ is equal to:

(a) $\frac{2x}{1+x^4}$ (b) $\frac{2x}{\sqrt{1+4x}}$
(c) $\frac{-2x}{1+x^4}$ (d) $\frac{-2x}{\sqrt{1+x^2}}$

6. If $y = \log \tan \sqrt{x}$ then the value of $\frac{dy}{dx}$ is :

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\sec^2 \sqrt{x}}{\sqrt{x} \tan x}$
(c) $2\sec^2 \sqrt{x}$ (d) $\frac{\sec^2 \sqrt{x}}{2\sqrt{x} \tan \sqrt{x}}$

7. If $y = e^{(1+\log_e x)}$, then $\frac{dy}{dx}$ is equal to :

(a) e (b) 1
(c) 0 (d) $\log_e x \,.\, x$

8. If $y = (\cos x^2)^2$, then $\frac{dy}{dx}$ is equal to :

(a) $-4x \sin 2x^2$ (b) $-x \sin x^2$
(c) $-2x \sin 2x^2$ (d) $-x \cos 2x^2$

9. The differential equation satisfied by the function

$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$ is

(a) $(2y-1)\frac{dy}{dx} - \sin x = 0$ (b) $(2y-1)\cos x + \frac{dy}{dx} = 0$
(c) $(2y-1)\cos x - \frac{dy}{dx} = 0$ (d) $(2y-1)\frac{dy}{dx} - \cos x = 0$

10. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx} =$

(a) $2^{x-y}\frac{2^y - 1}{2^x - 1}$ (b) $2^{x-y}\frac{2^y - 1}{1 - 2^x}$
(c) $\frac{2^x + 2^y}{2^x - 2^y}$ (d) None of these

11. $f(x) = \frac{1}{1 + \tan x}$

(a) is a continuous, real-valued function for all $x \in (-\infty, \infty)$
(b) is discontinuous only at $x = \frac{3\pi}{4}$
(c) has only finitely many discontinuities on $(-\infty, \infty)$
(d) has infinitely many discontinuities on $(-\infty, \infty)$

12. Let $f(x)=\begin{cases}3x-4, & 0\le x\le 2\\ 2x+\ell, & 2<x\le 9\end{cases}$

If f is continuous at x = 2, then what is the value of ℓ ?

(a) 0 (b) 2
(c) −2 (d) −1

13. If a function f(x) is defined as

$$f(x)=\begin{cases}\dfrac{x}{\sqrt{x^2}}, & x\ne 0\\ 0, & x=0\end{cases}$$ then :

(a) f(x) is continuous at x = 0 but not differentiable at x = 0
(b) f(x) is continuous as well as differentiable at x = 0
(c) f(x) is discontinuous at x = 0
(d) None of these.

14. The value of λ, for which the function

$f(x)=\begin{cases}\lambda(x^2-2x) & \text{if } x\le 0\\ 4x+1 & \text{if } x>0\end{cases}$ is continuous at x = 0, is :

(a) 1 (b) −1
(c) 0 (d) None of these

15. If $y=\sqrt{\left(\dfrac{1+\cos 2\theta}{1-\cos 2\theta}\right)}$, then $\dfrac{dy}{d\theta}$ at $\theta=\dfrac{3\pi}{4}$ is :

(a) −2 (b) 2
(c) ±2 (d) None of these

16. If x = sin t cos 2t and y = cos t sin 2t, then at $t=\dfrac{\pi}{4}$, the value of $\dfrac{dy}{dx}$ is equal to :

(a) −2 (b) 2 (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

17. The value of $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{a-x}{1+ax}\right)\right]$ is :

(a) $-\dfrac{1}{1+x^2}$ (b) $\dfrac{1}{1+a^2}-\dfrac{1}{1+x^2}$
(c) $\dfrac{1}{1+x^2}$ (d) None of these

18. If $x=\dfrac{1-t^2}{1+t^2}$ and $y=\dfrac{2t}{1+t^2}$, then $\dfrac{dy}{dx}$ is equal to :

(a) $-\dfrac{y}{x}$ (b) $\dfrac{y}{x}$ (c) $-\dfrac{x}{y}$ (d) $\dfrac{x}{y}$

19. The derivative of

$\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ w.r.t. $\cot^{-1}\left(\dfrac{1-3x^2}{3x-x^3}\right)$ is:

(a) $\dfrac{3}{2}$ (b) 1 (c) $\dfrac{1}{2}$ (d) $\dfrac{2}{3}$

20. If $f(x)=\dfrac{\sqrt{4+x}-2}{x}$, $x\ne 0$ be continuous at x = 0, then f(0) =

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$ (c) 2 (d) $\dfrac{3}{2}$

21. Let $f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & x>0\end{cases}$

If f(x) is continuous at x = 0, then

(a) a + c = 0, b = 1 (b) a + c = 1, b ∈ R
(c) a + c = −1, b ∈ R (d) a + c = −1, b = −1

22. Let $f(x)=\dfrac{\ln(1+ax)-\ln(1-bx)}{x}$, $x\ne 0$

If f(x) is continuous at x = 0, then f(0) =

(a) a − b (b) a + b
(c) b − a (d) ln a + ln b

23. If $f(x)=\sqrt{1+\cos^2(x^2)}$, then the value of $f'\left(\dfrac{\sqrt{\pi}}{2}\right)$ is

(a) $\dfrac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\dfrac{\pi}{6}}$ (c) $\dfrac{1}{\sqrt{6}}$ (d) $\dfrac{\pi}{\sqrt{6}}$

24. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{x+1}{x}$ (b) $\dfrac{1}{1+x}$ (c) $\dfrac{-1}{(1+x)^2}$ (d) $\dfrac{x}{1+x}$

25. If $y=\tan^{-1}\left(\dfrac{\sqrt{x}-x}{1+x^{3/2}}\right)$, then y′(1) is equal to

(a) 0 (b) $\dfrac{1}{2}$ (c) −1 (d) $-\dfrac{1}{4}$

26. $\dfrac{d}{dx}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\dfrac{x}{a}\right)\right)$ is equal to

(a) $\sqrt{a^2-x^2}$ (b) $2\sqrt{a^2-x^2}$
(c) $\dfrac{1}{\sqrt{a^2-x^2}}$ (d) None of these

27. If $\sec\left(\dfrac{x-y}{x+y}\right)=a$, then $\dfrac{dy}{dx}$ is

(a) $-\dfrac{y}{x}$ (b) $\dfrac{x}{y}$ (c) $-\dfrac{x}{y}$ (d) $\dfrac{y}{x}$

28. $\dfrac{d}{dx}\left[\sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)\right]$ is equal to

(a) $\dfrac{1}{2\sqrt{x(1-x)}}-\dfrac{1}{\sqrt{1-x^2}}$

(b) $\dfrac{1}{\sqrt{1-\left\{x\sqrt{1-x}-\sqrt{x(1-x^2)}\right\}^2}}$

(c) $\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{2\sqrt{x(1-x)}}$

(d) $\dfrac{1}{\sqrt{x(1-x)(1-x)^2}}$

29. If $\sin y + e^{-x\cos y} = e$, then $\frac{dy}{dx}$ at $(1, \pi)$ is equal to

(a) $\sin y$ (b) $-x\cos y$
(c) e (d) $\sin y - x\cos y$

30. If, $y = e^{3x+7}$, then the value of $\left.\frac{dy}{dx}\right|_{x=0}$ is

(a) 1 (b) 0 (c) -1 (d) $3e^7$

31. If $y = e^{\frac{1}{2}\log\left(1+\tan^2 x\right)}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2}\sec^2 x$ (b) $\sec^2 x$
(c) $\sec x \tan x$ (d) $e^{\frac{1}{2}\log\left(1+\tan^2 x\right)}$

32. If $f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1} + \tan^{-1}\frac{4x}{4-x^2}$, then $f'(2)$ is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) -1

33. If $y = \log_a x + \log_x a + \log_x x + \log_a a$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x} + x\log a$ (b) $\frac{\log a}{x} + \frac{x}{\log a}$
(c) $\frac{1}{x\log a} + x\log a$ (d) $\frac{1}{x\log a} - \frac{\log a}{x(\log x)^2}$

34. Let $y = t^{10} + 1$ and $x = t^8 + 1$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{5}{2}t$ (b) $20t^8$
(c) $\frac{5}{16t^6}$ (d) None of these

35. If $x^x = y^y$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y}{x}$ (b) $-\frac{x}{y}$
(c) $1+\log\left(\frac{x}{y}\right)$ (d) $\frac{1+\log x}{1+\log y}$

36. If $y = e^{x+e^{x+e^{x+\ldots \text{to } \infty}}}$, then $\frac{dy}{dx} =$

(a) $\frac{y^2}{1-y}$ (b) $\frac{y^2}{y-1}$
(c) $\frac{y}{1-y}$ (d) $\frac{-y}{1-y}$

37. If $y = (\tan x)^{\sin x}$, then $\frac{dy}{dx}$ is equal to

(a) $\sec x + \cos x$ (b) $\sec x + \log \tan x$
(c) $(\tan x)^{\sin x}$ (d) None of these

38. If $x = a\cos^4\theta$, $y = a\sin^4\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{3\pi}{4}$ is

(a) -1 (b) 1 (c) $-a^2$ (d) a^2

39. If $y = a^x \cdot b^{2x-1}$, then $\frac{d^2y}{dx^2}$ is

(a) $y^2.\log ab^2$ (b) $y.\log ab^2$
(c) $y.\left(\log ab^2\right)^2$ (d) $y.\left(\log a^2 b\right)^2$

40. If $x = f(t)$ and $y = g(t)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{g''(t)}{f''(t)}$

(b) $\frac{g''(t)f'(t) - g'(t)f''(t)}{(f'(t))^3}$

(c) $\frac{g''(t)f'(t) - g'(t)f''(t)}{(f'(t))^2}$

(d) None of these

41. A value of c for which the Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is

(a) $2\log_3 e$ (b) $\frac{1}{2}\log_e 3$ (c) $\log_3 e$ (d) $\log_e 3$

42. Rolle's Theorem holds for the function $x^3 + bx^2 + cx$, $1 \le x \le 2$ at the point $\frac{4}{3}$, the value of b and c are

(a) $b = 8, c = -5$ (b) $b = -5, c = 8$
(c) $b = 5, c = -8$ (d) $b = -5, c = -8$

43. If $y = \cos^2\left(\frac{3x}{2}\right) - \sin^2\left(\frac{3x}{2}\right)$, then $\frac{d^2y}{dx^2}$ is

(a) $-3\sqrt{1-y^2}$ (b) $9y$
(c) $-9y$ (d) $3\sqrt{1-y^2}$

44. If we can draw the graph of the function around a point without lifting the pen from the plane of the paper, then the function is said to be

(a) not continuous (b) continuous
(c) not defined (d) None of these

45. A real function *f* is said to be continuous, if it is continuous at every point in the

(a) domain of *f* (b) codomain of *f*
(c) range of *f* (d) None of these

46. All the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} 3, & \text{if } 0 \le x \le 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \le x \le 10 \end{cases} \text{ are}$$

(a) 1,3 (b) 3,10
(c) 1,3,10 (d) 0,1,3

47. If $f(x) = x^2 - \sin x + 5$, then
(a) $f(x)$ is continuous at all points
(b) $f(x)$ is discontinuous at $x = \pi$.
(c) It is discontinuous at $x = \frac{\pi}{2}$
(d) None of the above

48. The relationship between a and b, so that the function f defined by $f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$, is
(a) $a = b + \frac{2}{3}$ (b) $a - b = \frac{3}{2}$
(c) $a + b = \frac{2}{3}$ (d) $a + b = 2$

49. The number of points at which the function $f(x) = \frac{1}{x - [x]}$, [.] denotes the greatest integer function is not continuous is
(a) 1 (b) 2
(c) 3 (d) None of these

50. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?
(a) $f(x) + g(x)$ (b) $f(x) - g(x)$
(c) $f(x).g(x)$ (d) $\frac{g(x)}{f(x)}$

51. If $f(x) = \begin{cases} \frac{1 - \sqrt{2}\sin x}{\pi - 4x}, & \text{if } x \ne \frac{\pi}{4} \\ a, & \text{if } x = \frac{\pi}{4} \end{cases}$ is continuous at $\frac{\pi}{4}$, then a is equal to
(a) 4 (b) 2 (c) 1 (d) $\frac{1}{4}$

52. If $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{for } -1 \le x < 0 \\ 2x^2 + 3x - 2, & \text{for } 0 \le x \le 1 \end{cases}$ is continuous at $x = 0$, then k is equal to
(a) -4 (b) -3
(c) -2 (d) -1

53. Suppose f is a real function and c is a point in its domain. The derivative of f at c is defined by (if limit exist)
(a) $\lim_{h \to 0} \frac{f(c-h) - f(c)}{h}$ (b) $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$
(c) $\lim_{h \to 0} \frac{f(c+h) + f(c)}{h}$ (d) $\lim_{h \to 0} \frac{f(c-h) + f(c)}{h}$

54. If both $\lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$ and $\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$ are finite and equal, then
(a) f is continuous at a point c
(b) f is not continuous at c
(c) f is differentiable at a point c in its domain
(d) None of the above

55. If $y = \log x . e^{(\tan x + x^2)}$, then $\frac{dy}{dx}$ is equal to
(a) $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x + x)\log x\right]$
(b) $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x - x)\log x\right]$
(c) $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x + 2x)\log x\right]$
(d) $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x - 2x)\log x\right]$

56. If $y = \log\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\frac{dy}{dx}$, is equal to
(a) $\frac{4x^3}{1 - x^4}$ (b) $\frac{-4x}{1 - x^4}$
(c) $\frac{1}{4 - x^4}$ (d) $\frac{-4x^3}{1 - x^4}$

57. If $y = 5^x . x^5$, then $\frac{dy}{dx}$ is
(a) $5^x\left(x^5 \log 5 - 5x^4\right)$ (b) $x^5 \log 5 - 5x^4$
(c) $x^5 \log 5 + 5x^4$ (d) $5^x\left(x^5 \log 5 + 5x^4\right)$

58. If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, then
(a) $x\frac{dy}{dx} + y = 0$ (b) $x\frac{dy}{dx} = y$
(c) $y\frac{dy}{dx} = x$ (d) None of these

59. If $y = e^{x^x}$, then $\cdot \frac{dy}{dx} =$
(a) $y(1 + \log_e x)$ (b) $yx^x(1 + \log_e x)$
(c) $ye^x(1 + \log_e x)$ (d) None of these

60. If the function $f(x) = \begin{cases} 1, & x \le 2 \\ ax + b, & 2 < x < 4 \\ 7, & x \ge 4 \end{cases}$ is continuous at $x = 2$ and 4, then the values of a and b are.
(a) $a = 3, b = -5$ (b) $a = -5, b = 3$
(c) $a = -3, b = 5$ (d) $a = 5, b = -3$

61. If $f(x) = \begin{cases} \frac{[x] - 1}{x - 1}, & x \ne 1 \\ 0, & x = 1 \end{cases}$ then $f(x)$ is
(a) continuous as well as differentiable at $x = 1$
(b) differentiable but not continuous at $x = 1$
(c) continuous but not differentiable at $x = 1$
(d) neither continuous nor differentiable at $x = 1$

62. If $f(x)=\begin{cases} x^k \cos(1/x), & x \neq 0 \\ 0 \quad , & x = 0 \end{cases}$ is continuous at $x = 0$, then

(a) $k<0$ (b) $k>0$ (c) $k=0$ (d) $k \geq 0$

63. If $f(x)=\dfrac{1}{1-x}$, then the points of discontinuity of the function $f[f\{f(x)\}]$ are

(a) $\{0,-1\}$ (b) $\{0,1\}$ (c) $\{1,-1\}$ (d) None

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

64. Let $f(x)$ be a differentiable even function. Consider the following statements:

(I) $f'(x)$ is an even function.
(II) $f'(x)$ is an odd function.
(III) $f'(x)$ may be even or odd.

Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) I and III (d) II and III

65. Which of the following statements is/are true?

Statement I : The function $f(x) = |\cos x|$ is continuous function.

Statement II : The function $f(x) = \sin |x|$ is continuous function.

(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Neither I nor II is true

66. Consider the following statements:

I. The function $f(x)=$ greatest integer $\leq x, x \in R$ is a continuous function.
II. All trigonometric functions are continuous on R.

Which of the statements given above is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

67. Suppose f and g be two real functions continuous at a real number c. Then, which of the following statements is/are true?

I. $f+g$ is continuous at $x=c$.
II. $f-g$ is continuous at $x=c$.
III. $f \cdot g$ is discontinuous at $x=c$.
IV. $\left(\dfrac{f}{g}\right)$ is continuous at $x = c$ (provided $g(c) = 0$)

(a) II and III are true (b) III and IV are true
(c) I and II are true (d) All are true

68. The function f defined by $f(x)=\begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ is

I. continuous at $x = 0$.
II. discontinuous at $x = 1$.
III. continuous at $x = 2$.

Then, which of the following is/are true?

(a) Only I is true (b) Only II is true
(c) I and II are true (d) All are true

69. Which of the following functions is/are continuous?

I. Every rational function in its domain.
II. Sine function.
III. Cosine function.
IV. Tangent function is continuous in their domain.

(a) Only I is continuous (b) Only II is continuous
(c) I and II are continuous (d) All are continuous

70. Which of the following is/are true?

Statement I: If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, then $\dfrac{dy}{dx} = -\cot\dfrac{\theta}{2}$.

Statement II : If $x=\dfrac{\sin^3 t}{\sqrt{\cos 2t}}$, $y=\dfrac{\cos^3 t}{\sqrt{\cos 2t}}$, then derivative of y with respect to x is $-\cot 3t$.

(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Neither I nor II is true.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

71.

	Column-I		**Column-II**
A.	$f(x) = \cos x$	1.	f(x) is continuous of all points except $x = n\pi$, $n \in Z$.
B.	$f(x) = \operatorname{cosec} x$	2.	f(x) is continuous at all points except $x=(2n+1)\dfrac{\pi}{2}, n \in Z$.
C.	$f(x) = \sec x$	3.	f(x) is continuous at all points.
D.	$f(x) = \cot x$		

Codes

	A	B	C	D
(a)	2	3	1	1
(b)	1	2	3	1
(c)	3	1	2	1
(d)	1	3	1	2

72. Match the functions given in column - I with their derivatives in column - II.

	Column - I		**Column - II**
A.	$f(x)=\sin^{-1} x$	1.	$\dfrac{1}{1+x^2}, x \in R$
B.	$f(x)=\tan^{-1} x$	2.	$\dfrac{1}{\sqrt{1-x^2}}, x \in (-1,1)$
C.	$f(x)=\cos^{-1} x$	3.	$-\dfrac{1}{\sqrt{1-x^2}}, x \in (-1,1)$
D.	$f(x)=\cot^{-1} x$	4.	$-\dfrac{1}{1+x^2}, x \in R$

Codes

	A	B	C	D
(a)	3	1	2	4
(b)	2	1	3	4
(c)	1	4	3	2
(d)	1	2	3	4

73.

Column - I	**Column - II**
A. $x = 2at^2, y = at^4$	1. $\frac{dy}{dx} = -\frac{1}{t^2}$
B. $x = a\cos\theta, y = b\cos\theta$	2. $\frac{dy}{dx} = t^2$
C. $x = \sin t, y = \cos 2t$	3. $\frac{dy}{dx} = \frac{b}{a}$
D. $x = 4t,\ y = \frac{4}{t}$	4. $\frac{dy}{dx} = -4\sin t$
E. $x = \cos\theta - \cos 2\theta,$ $y = \sin\theta - \sin 2\theta$	5. $\frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$

Codes

	A	B	C	D	E
(a)	2	3	4	1	5
(b)	1	2	4	3	5
(c)	3	1	4	2	5
(d)	4	5	1	2	3

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

74. If $f(x) = \begin{cases} x + \lambda, & x < 3 \\ 4, & x = 3 \\ 3x - 5, & x > 3 \end{cases}$ is continuous at x = 3, then the value of λ is equal to :

(a) 1 (b) – 1
(c) 0 (d) does not exist

75. The value of the derivative of |x – 1| + |x – 3| at x = 2 is :

(a) –2 (b) 0
(c) 2 (d) not defined

76. If $f(x) = x^{1/x} - 1$ for all positive $x \neq 1$ and if f is continuous at 1, then x equals:

(a) 0 (b) $\frac{1}{e}$ (c) e (d) e^2

77. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$ is equal to :

(a) 1 (b) – 1
(c) 2 (d) None of these

78. Let $f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$

If f(x) is continuous for all x, then k =

(a) 3 (b) 5 (c) 7 (d) 9

79. If $f(x) = x^2 \sin\frac{1}{x}$, where $x \neq 0$, then the value of the function f at x = 0, so that the function is continuous at x = 0, is

(a) 0 (b) – 1
(c) 1 (d) None of these

80. The function $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, when k equals

(a) – 6 (b) 6 (c) 5 (d) – 5

81. If $f : R \to R$ is defined by

$f(x) = \begin{cases} \frac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$ then the value of *a*, so that *f* is continuous at 0, is

(a) 2 (b) 1 (c) – 1 (d) 0

82. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at x = 0, then the value of k is

(a) 1 (b) – 2 (c) 2 (d) $\frac{1}{2}$

83. In the interval [7, 9] the function f(x) = [x] is discontinuous at ______, where [x] denotes the greatest integer function

(a) 2 (b) 4 (c) 6 (d) 8

84. At how many points between the interval (–∞, ∞) is the function f (x) = sin x is not differentiable.

(a) 0 (b) 7 (c) 9 (d) 3

85. The no. of points of discontinuity of the function f (x) = x – [x] in the interval (0, 7) are

(a) 2 (b) 4 (c) 6 (d) 8

86. If y = a cos x – b sin x and $\frac{d^n y}{dx^n} = -a\cos x + b\sin x$, then n =

(a) 2 (b) 4 (c) 6 (d) 8

87. If the function f (x) = $\begin{cases} x^2, & \text{if } x \leq 4 \\ ax, & \text{if } x > 4 \end{cases}$ is continuous at x = 4, then a =

(a) 2 (b) 4 (c) 6 (d) 8

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

88. **Assertion :** For $x < 0$, $\frac{d}{dx}(\ell n|x|) = -\frac{1}{x}$

Reason : For $x < 0$, $|x| = -x$

89. Consider the function
$f(x) = [\sin x], x \in [0, \pi]$

Assertion: f(x) is not continuous at $x = \frac{\pi}{2}$

Reason : $\lim_{x \to \frac{\pi}{2}} f(x)$ does not exist

90. **Assertion :** If $y = \log_{10}x + \log_e y$, then
$\frac{dy}{dx} = \frac{\log_{10} e}{x}\left(\frac{y}{y-1}\right)$

Reason : $\frac{d}{dx}(\log_{10} x) = \frac{\log x}{\log 10}$

and $\frac{d}{dx}(\log_e x) = \frac{\log x}{\log e}$

91. **Assertion :** If $x = at^2$ and $y = 2at$, then $\left.\frac{d^2y}{dx^2}\right|_{t=2} = \frac{-1}{6a}$

Reason : $\frac{d^2y}{dx^2} = \left(\frac{dy}{dt}\right)^2 \times \left(\frac{dt}{dx}\right)^2$

92. **Assertion :** If $u = f(\tan x)$, $v = g(\sec x)$ and $f'(1) = 2$, $g(\sqrt{2}) = 4$, then $\left(\frac{du}{dv}\right)_{x=\pi/4} = \frac{1}{\sqrt{2}}$

Reason : If $u = f(x)$, $v = g(x)$, then the derivative of f with respect to g is $\frac{du}{dv} = \frac{du/dx}{dv/dx}$

93. **Assertion :** $f(x) = x^n \sin\left(\frac{1}{x}\right)$ is differentiable for all real values of x $(n \ge 2)$.

Reason : For $n \ge 2$, $\lim_{x \to 0} f(x) = 0$

94. **Assertion :** If a function f is discontinuous at c, then c is called a point of discontinuity.

Reason : A function is continuous at x = c, if the function is defined at x = c and the value of the function at x = c equals the limit of the function at x = c.

95. **Assertion :** The function defined by $f(x) = \cos(x^2)$ is a continuous function.

Reason : The cosine function is continuous in its domain i.e., $x \in R$.

96. **Assertion :** Every differentiable function is continuous but converse is not true.

Reason : Function $f(x) = |x|$ is continuous.

97. **Assertion :** $\frac{d}{dx}e^{\cos x} = e^{\cos x}(-\sin x)$

Reason : $\frac{d}{dx}e^x = e^x$

98. **Assertion :** If $xy = e^{x-y}$, then $\frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$.

Reason : $\frac{d}{dx}(u.v) = u\frac{d}{dx}v + v\frac{d}{dx}u$

99. **Assertion :** $f(x) = |x| \sin x$, is differentiable at $x = 0$.

Reason : If f (x) is not differentiable and g (x) is differentiable at x = a, then f (x) . g (x) can still be differentiable at x = a.

100. **Assertion :** $f(x) = |[x]x|$ in $x \in [-1, 2]$, where [.] represents greatest integer function, is non-differentiable at x = 2.

Reason : Discontinuous function is always non differentiable.

101. **Assertion :** The function $f(x) = |\sin x|$ is not differentiable at points $x = n\pi$.

Reason : The left hand derivative and right hand derivative of the function $f(x) = |\sin x|$ are not equal at points $x = n\pi$.

102. **Assertion :** Rolle's theorem can not be verified for the function $f(x) = |x|$ in the interval [–1, 1].

Reason : The function $f(x) = |x|$ is differentiable in the interval (–1, 1) everywhere.

103. **Assertion :** The function $f(x) = \frac{|x|}{x}$ is continuous at x = 0.

Reason : The left hand limit and right hand limit of the function $f(x) = \frac{|x|}{x}$ are not equal at x = 0.

CRITICAL THINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

104. Let $f(x) = e^x$, $g(x) = \sin^{-1}x$ and $h(x) = f(g(x))$, then $h'(x)/h(x) =$

(a) $e^{\sin^{-1} x}$ (b) $1/\sqrt{1-x^2}$
(c) $\sin^{-1} x$ (d) $1/(1-x^2)$

105. The number of points at which the function $f(x) = \frac{1}{\log|x|}$ is discontinuous is

(a) 1 (b) 2 (c) 3 (d) 4

106. If the function,

$$f(x)=\begin{cases} x+a^2\sqrt{2}\sin x &, 0\le x\le \pi/4 \\ x\cot x+b &, \pi/4\le x\le \pi/2 \\ b\sin 2x-a\cos 2x &, \pi/2\le x\le \pi \end{cases}$$

is continuous in the interval $[0,\pi]$ then the value of (a, b) are :

(a) $(-1,-1)$ (b) $(0,0)$ (c) $(-1,1)$ (d) $(1,0)$

107. Let $f(x)=\begin{cases}(x-1)\sin\dfrac{1}{x-1} & \text{if } x\ne 1\\ 0 & \text{if } x=1\end{cases}$

Then which one of the following is true?

(a) f is neither differentiable at $x=0$ nor at $x=1$

(b) f is differentiable at $x=0$ and at $x=1$

(c) f is differentiable at $x=0$ but not at $x=1$

(d) f is differentiable at $x=1$ but not at $x=0$

108. The point of discontinuity of $f(x)=\tan\left(\dfrac{\pi x}{x+1}\right)$ other than $x=-1$ are :

(a) $x=0$ (b) $x=\pi$

(c) $x=\dfrac{2m+1}{1-2m}$ (d) $x=\dfrac{2m-1}{2m+1}$

109. If $\sin y = x\sin(a+y)$, then $\dfrac{dy}{dx}$ is equal to :

(a) $\dfrac{\sin\sqrt{a}}{\sin(a+y)}$ (b) $\dfrac{\sin^2(a+y)}{\sin a}$

(c) $\sin(a+y)$ (d) None of these

110. Let $f(x)=\dfrac{1-\tan x}{4x-\pi}$, $x\ne\dfrac{\pi}{4}$, $x\in\left(0,\dfrac{\pi}{2}\right)$.

If f(x) is continuous in $\left(0,\dfrac{\pi}{2}\right)$, then $f\left(\dfrac{\pi}{4}\right)=$

(a) 1 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) -1

111. The number of discontinuous functions y(x) on $[-2, 2]$ satisfying $x^2+y^2=4$ is

(a) 0 (b) 1 (c) 2 (d) >2

112. Let $f(x)=\begin{cases}\sin x, & \text{for } x\ge 0\\ 1-\cos x, & \text{for } x\le 0\end{cases}$ and $g(x)=e^x$. Then the value of $(gof)'(0)$ is

(a) 1 (b) -1

(c) 0 (d) None of these

113. The derivative of e^{x^3} with respect to log x is

(a) e^{x^3} (b) $3x^2 2e^{x^3}$

(c) $3x^3e^{x^3}$ (d) $3x^3e^{x^3}+3x^2$

114. The 2nd derivative of a $\sin^3 t$ with respect to a $\cos^3 t$ at $t=\dfrac{\pi}{4}$ is

(a) $\dfrac{4\sqrt{2}}{3a}$ (b) 2

(c) $\dfrac{1}{12a}$ (d) None of these

115. Let $f(x)=\sin x$, $g(x)=x^2$ and $h(x)=\log_e x$.

If $F(x)=(hogof)(x)$, then $F''(x)$ is equal to

(a) a $\text{cosec}^3 x$ (b) $2\cot x^2-4x^2\,\text{cosec}^2 x^2$

(c) $2x\cot x^2$ (d) $-2\,\text{cosec}^2 x$

116. If $u=x^2+y^2$ and $x=s+3t$, $y=2s-t$, then $\dfrac{d^2u}{ds^2}$ is equal to

(a) 12 (b) 32 (c) 36 (d) 10

117. If $x^2+y^2=1$, then

(a) $yy''-(2y')^2+1=0$ (b) $yy''-(y')^2+1=0$

(c) $yy''-(y')^2-1=0$ (d) $yy''-2(y')^2+1=0$

118. The value of c in Rolle's Theorem for the function $f(x)=e^x\sin x$, $x\in[0,\pi]$ is

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{3\pi}{4}$

119. Let f(x) satisfy the requirements of Lagrange's mean value theorem in [0, 2]. If $f(0)=0$ and $f'(x)\le\dfrac{1}{2}$ for all x in [0, 2], then

(a) $|f(x)|\le 2$

(b) $f(x)\le 1$

(c) $f(x)=2x$

(d) $f(x)=3$ for atleast one x in [0, 2]

120. Let $f(x)=|\sin x|$. Then

(a) f is everywhere differentiable

(b) f is everywhere continuous but not differentiable at $x=n\pi, n\in Z$

(c) f is everywhere continuous but not differentiable at $x=(2n+1)\dfrac{\pi}{2}, n\in Z$.

(d) None of these

121. The function $f(x)=\cot x$ is discontinuous on the set

(a) $\{x=n\pi, n\in Z\}$ (b) $\{x=2n\pi, n\in Z\}$

(c) $\left\{x=(2n+1)\dfrac{\pi}{2}; n\in Z\right\}$ (d) $\left\{x=\dfrac{n\pi}{2}; n\in Z\right\}$

122. If $y=x-x^2$, then the derivative of y^2 with respect to x^2 is

(a) $1-2x$ (b) $2-4x$ (c) $3x-2x^2$ (d) $1-3x+2x^2$

123. $f(x)=\begin{cases}|x|\cos\left(\frac{1}{x}\right), & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$ is

(a) discontinuous at $x = 0$
(b) continuous at $x = 0$
(c) Does not exist
(d) None of the above

124. If $f: R \to R$ is defined by

$$f(x)=\begin{cases}\dfrac{x+2}{x^2+3x+2}, & \text{if } x\in R-\{-1,-2\}\\ -1, & \text{if } x=-2\\ 0, & \text{if } x=-1\end{cases}$$

then f is continuous on the set

(a) R (b) $R-\{-2\}$
(c) $R-\{-1\}$ (d) $R-\{-1,-2\}$

125. The function $f(x)=\begin{cases}x[x], & \text{if } 0\le x<2\\ (x-1)x, & \text{if } 2\le x<3\end{cases}$ is

(a) differentiable at $x = 2$ (b) not differentiable at $x = 2$
(c) continuous at $x = 2$ (d) None of these

126. If $f(x)=ae^{|x|}+b|x|^2$, a $b\in R$ and f(x) is differentiable at $x = 0$. Then, a and b are

(a) $a = 0, b \in R$ (b) $a = 1, b = 2$
(c) $b = 0, a \in R$ (d) $a = 4, b = 5$

127. $\dfrac{d}{dx}\left[\log\left\{e^x\left(\dfrac{x-2}{x+2}\right)\right\}^{3/4}\right]$ is equal to

(a) 1 (b) $\dfrac{x^2+1}{x^2-4}$
(c) $\dfrac{x^2-1}{x^2-4}$ (d) $e^x\dfrac{x^2-1}{x^2-4}$

128. If $y^x = e^{y-x}$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{1+\log y}{y\log y}$ (b) $\dfrac{(1+\log y)^2}{y\log y}$
(c) $\dfrac{1+\log y}{(\log y)^2}$ (d) $\dfrac{(1+\log y)^2}{\log y}$

129. If $y=|\sin x|^{|x|}$, then the value of $\dfrac{dy}{dx}$ at $x=-\dfrac{\pi}{6}$ is

(a) $\dfrac{2^{-\frac{\pi}{6}}}{6}\left[6\log 2-\sqrt{3}\pi\right]$ (b) $\dfrac{2^{\frac{\pi}{6}}}{6}\left[6\log 2+\sqrt{3}\pi\right]$
(c) $\dfrac{2^{-\frac{\pi}{6}}}{6}\left[6\log 2+\sqrt{3}\pi\right]$ (d) None of these

130. If $y = 3\cos(\log x) + 4\sin(\log x)$, then

(a) $xy_2+y_1+y=0$ (b) $xy_2+y_1-y=0$
(c) $x^2y_2+xy_1+y=0$ (d) None of these

131. Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to

(a) $\dfrac{2}{7}$ (b) $\dfrac{1}{2}$ (c) 2 (d) $\dfrac{7}{2}$

132. If $2f(\sin x)+f(\cos x)=x$, then $\dfrac{d}{dx}f(x)$ is

(a) $\sin x + \cos x$ (b) 2
(c) $\dfrac{1}{\sqrt{1-x^2}}$ (d) None of these

133. Let $f: R\to R$ be a function defined by

$f(x)=\min\{x+1,|x|+1\}$, Then which of the following is true?

(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x=0$
(c) $f(x)\ge 1$ for all $x\in R$
(d) $f(x)$ is not differentiable at $x=1$

134. Let $f: R\to R$ be a function defined by $f(x)=\max\{x,x^3\}$. The set of all points where $f(x)$ is NOT differentiable is

(a) $\{-1,1\}$ (b) $\{-1,0\}$ (c) $\{0,1\}$ (d) $\{-1,0,1\}$

135. Let f(x) be a twice differentiable function and $f''(0) = 5$, then the value of $\lim\limits_{x\to 0}\dfrac{3f(x)-4f(3x)+f(9x)}{x^2}$ is

(a) 0 (b) 120
(c) –120 (d) does not exist

136. $f(x)=\begin{cases}x\sin 1/x, & x\neq 0\\ 0, & x=0\end{cases}$ at $x = 0$ is

(a) continuous as well as differentiable
(b) differentiable but not continuous
(c) continuous but not differentiable
(d) neither continuous nor differentiable

137. The set of the points where $f(x) = x\,|x|$ is twice differentiable, will be

(a) R (b) R_0
(c) R^+ (d) R^-

138. If $f(x) = (x+1)^{\cot x}$ be continuous at $x = 0$ then $f(0)$ is equal to:

(a) 0 (b) – e
(c) e (d) None

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (c) **Note:** A polynomial function is always a continuous function.
And the given question is a polynomial function with degree 1.
Thus the continuity is for all real value of x.

2. (d) Let $y = 2^{-x}$

$$\therefore \quad \frac{dy}{dx} = \frac{2^{-x}}{\log 2}(-1) = \frac{-1}{2^x \log 2} = \frac{\log\frac{1}{2}}{2^x}$$

3. (c) Let $y = \sec x°$

Now, $x° = \frac{\pi}{180}.x \quad \therefore \quad y = \sec\frac{\pi}{180}x$

Now, $\frac{dy}{dx} = \frac{\pi}{180}\sec\frac{x\pi}{180}\tan\frac{\pi}{180}x$

$$\Rightarrow \frac{dy}{dx} = \frac{\pi}{180}\sec x°.\tan x°$$

4. (b) Let y = log (log x)
Diff both side w.r.t 'x', we get

$$\frac{dy}{dx} = \frac{1}{\log x}\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log x} \times \frac{1}{x}$$

$$\Rightarrow \log x . \frac{dy}{dx} = \frac{1}{x}$$

$$\Rightarrow e^y\frac{dy}{dx} = \frac{1}{x} \quad (\because e^y = e^{\log(\log x)} = \log x)$$

5. (c) Let $y = \cot^{-1}(x^2)$
$\Rightarrow \cot y = x^2$
Diff both side, w.r.t. 'x'

$$-\text{cosec}^2 y . \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{-\text{cosec}^2 y}$$

$$= \frac{2x}{-(1+\cot^2 y)} = \frac{2x}{-(x^4+1)} = \frac{-2x}{(x^4+1)}$$

6. (d) Let $y = \log\tan\sqrt{x}$
Diff. both side w.r.t 'x'

$$\frac{dy}{dx} = \frac{1}{\tan\sqrt{x}} . \frac{d}{dx}(\tan\sqrt{x})$$

$$= \frac{1}{\tan\sqrt{x}}.\sec^2\sqrt{x}.\frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1\sec^2\sqrt{x}}{2\sqrt{x}\tan\sqrt{x}}$$

7. (a) We have, $y = e^{(1+\log x)}$

$\Rightarrow y = e^1 . e^{\log x}$

$\Rightarrow y = ex \quad [\because e^{\log x} = x]$

On differentiating, w. r. to x we get

$$\frac{dy}{dx} = \frac{d}{dx}(ex)$$

$$\Rightarrow \frac{dy}{dx} = e$$

8. (c) As given : $y = (\cos x^2)^2$
Diff both side w.r.t 'x'

$$\frac{dy}{dx} = 2\cos x^2(-\sin x^2)\,2x$$
$$= -4x\cos x^2\sin x^2$$
$$= -2x(2\sin x^2\cos x^2)$$
$$(\because \sin 2\theta = 2\sin\theta\cos\theta)$$
$$= -2x\sin 2x^2$$

9. (d) $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$

$\Rightarrow y = \sqrt{\sin x + y} \Rightarrow y^2 = \sin x + y$

On differentiating both sides, we get

$$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx}(2y-1) = \cos x.$$

10. (d) On differentiating

$$2^x\log 2 + 2^y\log 2.\frac{dy}{dx}$$

$$= 2^x.2^y\frac{dy}{dx}.\log 2 + 2^y.2^x\log 2$$

$$\Rightarrow 2^x + 2^y\frac{dy}{dx} = 2^{x+y}\frac{dy}{dx} + 2^{x+y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y - 2^{x+y}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^x + 2^y - 2^x}{2^y - 2^x - 2^y} = -2^{y-x}$$

11. (d) tan x is not continuous at

$x = \frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}$ etc...

So, tan x has infinitely many discontinuities on $(-\infty, \infty)$

$\Rightarrow f(x) = \frac{1}{1+\tan x}$ has infinitely many discontinuities on $(-\infty, \infty)$.

12. (c) Given function is :

$$f(x) = \begin{cases} 3x-4, & 0 \le x \le 2 \\ 2x+\ell, & 2 < x \le 9 \end{cases}$$

and also given that f (x) is continuous at x = 2

For a function to be continuous at a point

LHL = RHL = Value of a function at that point.

$f(2) = 2$

$\Rightarrow$ RHL : $\lim_{x\to 2}(2x+\ell) = 3(2) - 4$

$\Rightarrow \lim_{h\to 0}\{2(2+h)+\ell\} = 6-4$

$\Rightarrow 4+\ell = 2$

$\Rightarrow \ell = -2$

13. (c) Given : $f(x) = \begin{cases} \dfrac{x}{\sqrt{x^2}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

$\therefore\ f(x) = \begin{cases} \dfrac{x}{|x|} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

$\therefore\ f(0) = 0$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0}\frac{0+h}{|0+h|} = \lim_{h\to 0}\frac{h}{h} = 1$$

$$\text{L.H.L.} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0}\frac{(0-h)}{|(0-h)|}$$

$$= \lim_{h\to 0}\frac{-h}{h} = -1$$

R.H.L $\neq$ L.H.L

i.e. $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\therefore$ f (x) is discontinuous at x = 0

14. (d) Given : $f(x) = \begin{cases} \lambda(x^2 - 2x) & \text{if } x \le 0 \\ 4x+1 & \text{if } x > 0 \end{cases}$ is continuous at x = 0

$\therefore \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0).$

But R.H.L = $\lim_{h\to 0}$ 4h + 1 = 1,

L.H.L = $\lim_{h\to 0} \lambda(h^2 - 2h) = 0$ and f (0) = 0.

$\therefore$ There is no value of 'λ' for which the f(x) is continuous at x = 0.

15. (a) $y = \sqrt{\dfrac{1+\cos 2\theta}{1-\cos 2\theta}}$

$\Rightarrow\ y = \sqrt{\dfrac{2\cos^2\theta}{2\sin^2\theta}} = \sqrt{\cot^2\theta}$

$\Rightarrow\ y = \cot\theta$

Differentiate w.r.t. 'θ', we get

$$\frac{dy}{d\theta} = -\text{cosec}^2\theta$$

Now, $\left(\dfrac{dy}{d\theta}\right)_{\theta=\frac{3\pi}{4}} = -\text{cosec}^2\left(\dfrac{3\pi}{4}\right)$

$= -\text{cosec}^2\left(\pi - \dfrac{\pi}{4}\right) = -\text{cosec}^2\dfrac{\pi}{4}$

$= -2 \qquad \left(\because \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}\right)$

16. (c) Let x = sin t cos2 t and y = cos t. sin 2t

Differentiate both w.r.t 't'

$$\frac{dx}{dt} = \cos t\cos 2t - 2\sin t.\sin 2t$$

and $\dfrac{dy}{dt} = 2\cos t.\cos 2t - \sin 2t.\sin t$

Now,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t.\cos 2t - \sin 2t.\sin t}{\cos t.\cos 2t - 2\sin t.\sin 2t}$$

Put $t = \dfrac{\pi}{4}$, $\dfrac{dy}{dx} = \dfrac{2\cos\frac{\pi}{4}.\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}\cos\frac{\pi}{2} - 2\sin\frac{\pi}{4}\sin\frac{\pi}{2}}$

$$= \frac{\frac{-1}{\sqrt{2}}}{-2\left(\frac{1}{\sqrt{2}}\right)} = \frac{1}{2}$$

17. (a) $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{a-x}{1+ax}\right)\right]$

$= \dfrac{d}{dx}(\tan^{-1}a - \tan^{-1}x)$

$= 0 - \dfrac{1}{1+x^2} = -\dfrac{1}{1+x^2}$

18. (c) Let $x = \dfrac{1-t^2}{1+t^2}$ and $y = \dfrac{2t}{1+t^2}$

Put $t = \tan\theta$, we get

$x = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ and $y = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\Rightarrow$ x = cos 2θ and y = sin 2θ

$\therefore\ \dfrac{dx}{d\theta} = -2\sin 2\theta$ and $\dfrac{dy}{d\theta} = 2\cos 2\theta$

Now, $\dfrac{dy}{dx} = \dfrac{dy}{d\theta}\times\dfrac{d\theta}{dx} = -\dfrac{\cos 2\theta}{\sin 2\theta} = -\dfrac{x}{y}$

19. (d) Let $u = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ and

$v = \cot^{-1}\left(\dfrac{1-3x^2}{3x-x^3}\right)$

Put x = tan θ

$u = \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$ and

$$v = \cot^{-1}\left(\frac{1-3\tan^2\theta}{3\tan\theta-\tan^3\theta}\right)$$

$u = \cos^{-1}[\cos 2\theta]$ and $v = \cot^{-1}[\cot 3\theta]$

$u = 2\theta$ and $v = 3\theta$

$$\frac{du}{d\theta} = 2 \text{ and } \frac{dv}{d\theta} = 3 \ \therefore \frac{du}{dv} = \frac{du}{d\theta}\times\frac{d\theta}{dv} = \frac{2}{3}$$

20. (b) $f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$

$$= \lim_{x\to 0}\left(\frac{\sqrt{4+x}-2}{x}\times\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right)$$

$$= \lim_{x\to 0}\frac{\sqrt{4+x}-4}{x\left(\sqrt{4+x}+2\right)} = \lim_{x\to 0}\frac{x}{x\left(\sqrt{4+x}+2\right)}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{4+x}+2} = \frac{1}{2+2} = \frac{1}{4}$$

21. (c) L.H.L. at x = 0: $\lim_{x\to 0}\frac{\sin(a+1)x+\sin x}{x}\left(\frac{0}{0}\text{form}\right)$

Using L Hospital's Rule

$$\lim_{x\to 0}(a+1)\cos(a+1)x+\cos x = a+2 \quad ...(i)$$

R.H.L. at x = 0 :

$$\lim_{x\to 0}\frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} = \lim_{x\to 0}\frac{\sqrt{1+bx}-1}{bx}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{1+bx}+1} = \frac{1}{2} \quad ...(ii)$$

From (i) and (ii), we get

$$a+2 = c = \frac{1}{2} \Rightarrow a = -\frac{3}{2} \text{ and } a + c = -1$$

22. (b) $f(0) = \lim_{x\to 0} f(x)$

$$= \lim_{x\to 0}\frac{\ln(1+ax)-\ln(1-bx)}{x}\left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x\to 0}\left[\frac{a}{1+ax}+\frac{b}{1-bx}\right] \text{(L's Hospital rule)}$$

$= a + b$

23. (b) We have, $f(x) = \sqrt{1+\cos^2(x^2)}$ ...(i)

On differentiating (i) w.r.t.x, we get

$$f'(x) = \frac{-2\sin x^2\cos x^2}{\sqrt{1+\cos^2 x^2}}(x)$$

$$\Rightarrow f'(x) = \frac{-\sin 2x^2}{\sqrt{1+\cos^2 x^2}}(x) \quad ...(ii)$$

Put, $x = \frac{\sqrt{\pi}}{2}$ in (ii), we get

$$f'\left(\frac{\sqrt{\pi}}{2}\right) = -\frac{\sqrt{\pi}}{2}\cdot\frac{\sin 2\left(\frac{\pi}{4}\right)}{\sqrt{1+\frac{1}{2}}}$$

$$= -\frac{\sqrt{\pi}}{2}\cdot\frac{\sin\frac{\pi}{2}}{\sqrt{\frac{3}{2}}} = -\sqrt{\frac{\pi}{6}}$$

24. (c) Given $x\sqrt{1+y}+y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides, we get

$x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 + x^2y - xy^2 = 0 \Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow y = x$ or $y(1+x) = -x \Rightarrow y = x$ or $y = -\frac{x}{1+x}$

$$\Rightarrow \frac{dy}{dx} = \frac{-(1+x).1+x.1}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

25. (d) $y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right) = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+\sqrt{x}.x}\right)$

$= \tan^{-1}\left(\sqrt{x}\right) - \tan^{-1}(x)$

On differentiating w.r.t. x, we get

$$y' = \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$

$$\Rightarrow y'(1) = \frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$$

26. (b) $\frac{d}{dx}\left\{x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\frac{x}{a}\right)\right\}$

$$= \frac{x\times 1\times(-2x)}{2\sqrt{a^2-x^2}} + \sqrt{a^2-x^2} + a^2\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\times\frac{1}{a}$$

$$= \frac{-x^2}{\sqrt{a^2-x^2}} + \sqrt{a^2-x^2} + \frac{a^2}{\sqrt{a^2-x^2}}$$

$$= \sqrt{a^2-x^2} + \frac{\left(a^2-x^2\right)}{\sqrt{a^2-x^2}} = 2\sqrt{a^2-x^2}$$

27. (d) Given $\sec\left(\frac{x-y}{x+y}\right) = a$

$$\Rightarrow \frac{x-y}{x+y} = \sec^{-1} a$$

Differentiating both sides, w.r.t. x, we get

$$\frac{(x+y)\left(1-\frac{dy}{dx}\right)-(x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2} = 0$$

$$\Rightarrow x+y-(x+y)\frac{dy}{dx}-(x-y)-(x-y)\frac{dy}{dx} = 0$$

$$\Rightarrow 2y = \frac{dy}{dx}(x+y+x-y) \Rightarrow 2y = 2x\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

28. (c) Let $y = \frac{d}{dx}\left[\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)\right]$

Put $x = \sin\alpha$ and $\sqrt{x} = \sin\beta$

$$\therefore y = \frac{d}{dx}\left[\sin^{-1}\left(\sin\alpha\sqrt{1-\sin^2\beta} - \sin\beta\sqrt{1-\sin^2\alpha}\right)\right]$$

$$= \frac{d}{dx}\left[\sin^{-1}(\sin(\alpha-\beta))\right] = \frac{d}{dx}(\alpha-\beta)$$

$$= \frac{d}{dx}\left[\sin^{-1}x - \sin^{-1}\sqrt{x}\right]$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x(1-x)}}$$

29. (c) We have,

$\sin y + e^{-x\cos y} = e$

Differentiating both sides, w.r.t. x, we get

$$\cos y\frac{dy}{dx} + e^{-x\cos y}\left\{x\sin y\frac{dy}{dx} - \cos y\right\} = 0$$

$$\Rightarrow \left(\cos y + e^{-x\cos y}.x\sin y\right)\frac{dy}{dx} = \left(e^{-x\cos y}\right)\cos y$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{-x\cos y}.\cos y}{\cos y + x.e^{-x\cos y}\sin y}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(1,\pi)} = \frac{e^{\cos\pi}.\cos\pi}{\cos\pi + e^{-\cos\pi}\sin\pi} = e$$

30. (d) $\because y = e^{3x+7}$

$$\therefore \frac{dy}{dx} = 3e^{3x+7}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 3e^{3\times0+7} = 3e^7$$

31. (c) $y = e^{\frac{1}{2}\log(1+\tan^2 x)} = \left(\sec^2 x\right)^{1/2} = \sec x$

$$\therefore \frac{dy}{dx} = \sec x\tan x$$

32. (a) $f(x) = (\log_{\cot x}\tan x)(\log_{\tan x}\cot x)^{-1} + \tan^{-1}\frac{4x}{4-x^2}$

$$= \frac{\log\tan x}{\log\cot x}.\frac{\log\tan x}{\log\cot x} + \tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$= \frac{(\log\tan x)^2}{(-\log\tan x)^2} + \tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$= 1 + \tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$\therefore f'(x) = \frac{1}{1+\left(\frac{4x}{4-x^2}\right)^2}.\frac{\left(4-x^2\right)4 - 4x(-2x)}{\left(4-x^2\right)^2}$$

$$= \frac{16-4x^2+8x^2}{\left(4-x^2\right)^2+16x^2} = \frac{4\left(4+x^2\right)}{\left(4-x^2\right)^2+(4x)^2}$$

Hence, $f'(2) = \frac{4(4+4)}{0+(8)^2} = \frac{32}{64} = \frac{1}{2}$

33. (d) $y = \log_a x + \frac{\log a}{\log x} + 1 + 1 \quad \{\because \log_x x = 1\}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}\log_a e - \log a\left(\frac{1}{\log x}\right)^2\frac{1}{x}$$

$$= \frac{1}{x\log a} - \frac{\log a}{x(\log x)^2}$$

34. (c) $y = t^{10} + 1,\ x = t^8 + 1$

$$\frac{dy}{dt} = 10t^9,\ \frac{dx}{dt} = 8t^7$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{10t^9}{8t^7} = \frac{5}{4}t^2$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{5}{4}(2t)\frac{dt}{dx}$$

$$= \frac{5}{4}\times 2t\times\frac{1}{8t^7}$$

$$= \frac{5}{16t^6}$$

35. (d) Given $x^x = y^y$, taking log on the both sides, we get $\log x^x = \log y^y \Rightarrow x\log x = y\log y$,

Differentiating w.r.t. x, we get

$$x\left(\frac{1}{x}\right) + \log x.1 = y\left(\frac{1}{y}\frac{dy}{dx}\right) + (\log y)\frac{dy}{dx}$$

$$\Rightarrow 1 + \log x = (1+\log y)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+\log x}{1+\log y}.$$

36. (c) We may write the given series as

$y = e^{x+y} \Rightarrow \log y = (x+y) \quad ...(i)$

On differentiating both sides of (i) w.r.t. x, we get

$$\frac{1}{y}.\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} - 1\right)\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{(1-y)}{y}.\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{(1-y)}.$$

37. (d) We have, $y = (\tan x)^{\sin x}$

Taking logarithm on both sides

$\log y = \sin x \log (\tan x)$

Differentiating w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{\tan x}.\sec^2 x + \cos x \log(\tan x)$$

$$= (\tan x)^{\sin x}\left[\sin x \frac{1}{\tan x}\sec^2 x + \cos x(\log \tan x)\right]$$

$$= (\tan x)^{\sin x}[\sec x + \cos x \log \tan x]$$

38. (a) Given that, $x = a\cos^4\theta$ and $y = a\sin^4\theta$

On differentiating w.r.t θ, we get

$$\frac{dx}{d\theta} = 4a\cos^3\theta(-\sin\theta)$$

and $\frac{dy}{d\theta} = 4a\sin^3\theta\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{4a\sin^3\theta\cos\theta}{4a\cos^3\theta\sin\theta} = -\frac{\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$$

Now, $\left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1$

39. (c) $\because y = a^x b^{2x-1}$

Taking log on both sides, we get

$\log y = x \log a + (2x - 1) \log b$

On differentiating w.r.t x, we get

$$\frac{1}{y}\frac{dy}{dx} = \log a + \log b^2$$

$$\frac{dy}{dx} = y\log ab^2$$

Again differentiating, we get

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}\log ab^2 = y(\log ab^2)^2$$

40. (b) $\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{g'(t)}{f'(t)}$,

Differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{f'(t)g''(t) - g'(t)f''(t)}{(f'(t))^2}.\frac{dt}{dx}$$

$$= \frac{f'(t)g''(t) - g'(t)f''(t)}{(f'(t))^3}$$

41. (a) Using Mean Value theorem

$$f'(c) = \frac{f(3) - f(1)}{3-1}$$

$$\Rightarrow \frac{1}{c} = \frac{\log_e 3 - \log_e 1}{2} \Rightarrow c = \frac{2}{\log_e 3} = 2\log_3 e$$

42. (b) Here, $f(1) = f(2)$ and $f'\left(\frac{4}{3}\right) = 0$

$\Rightarrow 1 + b + c = 8 + 4b + 2c \Rightarrow -7 = 3b + c$ ...(i)

and $3\left(\frac{4}{3}\right)^2 + 2b\left(\frac{4}{3}\right) + c = 0 \Rightarrow \frac{16}{3} + \frac{8b}{3} + \frac{3c}{3} = 0$

$\Rightarrow 8b + 3c = -16$ ...(ii)

Solving (i) and (ii), we get

$b = -5$ and $c = 8$.

43. (c) Given $y = \cos^2\left(\frac{3x}{2}\right) - \sin^2\left(\frac{3x}{2}\right)$

$\Rightarrow y = \cos 3x$

$$\Rightarrow \frac{dy}{dx} = -3\sin 3x$$

and $\frac{d^2y}{dx^2} = -3 \times 3\cos 3x = -9\cos 3x = -9y$

44. (b) We may say that a function is continuous at a fixed point, if we can draw the graph of the function around that point without lifting the pen from the plane of the paper.

45. (a) A real function f is said to be continuous, if it is continuous at every point in the domain of f.

46. (a)
$$f(x) = \begin{cases} 3, & \text{if } 0 \le x \le 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \le x \le 10 \end{cases}$$

For $0 \le x \le 1$, $f(x) = 3$; $1 < x < 3$; $f(x) = 4$ and $3 \le x \le 10$, $f(x) = 5$ are constant functions, so it is continuous in the above interval,

so we have to check the continuity at x = 1, 3

At x = 1, LHL = $\lim_{x\to1^-} f(x) = \lim_{x\to1^-}(3) = 3$

RHL = $\lim_{x\to1^+} f(x) = \lim_{x\to1^+}(4) = 4$

$\therefore$ LHL ≠ RHL

Thus, f(x) is discontinuous at x = 1

At x = 3, LHL = $\lim_{x\to3^-} f(x) = \lim_{x\to3^-}(4) = 4$

RHL = $\lim_{x\to3^+} f(x) = \lim_{x\to3^+}(5) = 5$

$\therefore$ LHL ≠ RHL

Thus, f(x) is continuous everywhere except at x = 1, 3.

47. (a) Since, f(x) is a sum of a polynomial function $(x^2 - 5)$ and sin x, both of which are continuous functions everywhere. Thus, f(x) is continuous everywhere.

48. (a) Here, $f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$

LHL = $\lim_{x\to3^-} f(x) = \lim_{x\to3^-}(ax+1)$

Putting $x = 3 - h$ as $x \to 3^-$, $h \to 0$

$$\therefore \lim_{h\to0}[a(3-h)+1] = \lim_{h\to0}(3a - ah + 1) = 3a + 1$$

RHL = $\lim_{x\to3^+} f(x) = \lim_{x\to3^+}(bx+3)$

Putting $x = 3 + h$ as $x \to 3^+$, $h \to 0$

$\therefore \lim_{h\to 0}[b(3+h)+3] = \lim_{h\to 0}(3b+bh+3) = 3b+3$

Also, f(3) = 3a + 1 $\quad [\because f(x) = ax+1]$

Since, f(x) is continuous at x = 3.

$\therefore$ LHL = RHL = f(3)

$\Rightarrow 3a + 1 = 3b + 3 \Rightarrow 3a = 3b + 2 \Rightarrow a = b + \frac{2}{3}$

49. (d) x – [x] = 0 when x is an integer, so that f(x) is discontinuous for all x ∈ I i.e., f(x) is discontinuous at infinite number of points.

50. (d) We know that, sum, product and difference of two polynomials is a polynomials, and polynomial function is everywhere continuous.

Now, we check the continuity of $\frac{g(x)}{f(x)}$

$$\frac{g(x)}{f(x)} = \frac{\frac{x^2}{2}+1}{2x}$$

Clearly, $\frac{g(x)}{f(x)}$ is not defined at x = 0

$\therefore$ It is discontinuous at x = 0

51. (d) $\lim_{x\to\frac{\pi}{4}} f(x) = \lim_{x\to\frac{\pi}{4}} \frac{1-\sqrt{2}\sin x}{\pi - 4x}$

$= \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\cos x}{-4} = \frac{1}{4}$ (by L Hospital's rule)

Since, f(x) is continuous at $x = \frac{\pi}{4}$

$\therefore \lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right) \Rightarrow \frac{1}{4} = a$

52. (c) LHL $= \lim_{x\to 0^-} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}$

$= \lim_{x\to 0^-} \frac{2kx}{x\left(\sqrt{1+kx}+\sqrt{1-kx}\right)} = k$

RHL $= \lim_{x\to 0^+} \left(2x^2+3x-2\right) = -2$

f(0) = – 2

$\because$ It is given that f(x) is continuous

$\therefore$ LHL = RHL = f(0)

$\Rightarrow$ k = – 2

53. (b) Suppose f is a real function and c is a point in its domain.

The derivative of f at c is defined by

$$\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

Provided this limit exist.

54. (c) We say that a function f is differentiable at a point *c* in its domain if both

$\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h}$ and

$\lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$ are finite and equal.

55. (c) Given, $y = \log x . e^{(\tan x + x^2)}$

$\therefore \frac{dy}{dx} = e^{(\tan x+x^2)} . \frac{1}{x} + \log x . e^{(\tan x + x^2)}\left(\sec^2 x + 2x\right)$

$= e^{(\tan x + x^2)}\left[\frac{1}{x} + \left(\sec^2 x + 2x\right)\log x\right]$

56. (b) $y = \log\left(\frac{1-x^2}{1+x^2}\right)$

$$\frac{dy}{dx} = \frac{1}{\frac{1-x^2}{1+x^2}} \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\frac{dy}{dx} = \frac{1+x^2}{1-x^2} \times \left[\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}\right]$$

$$= \frac{1+x^2}{1-x^2} \times \frac{2x\left(-1-x^2-1+x^2\right)}{(1+x^2)^2}$$

$$= \frac{1+x^2}{1-x^2} \times \frac{-4x}{(1+x^2)^2} = \frac{-4x}{1-x^4}$$

57. (d) Given, $y = 5^x . x^5 \Rightarrow \frac{dy}{dx} = 5^x \log 5 . x^5 + 5^x 5x^4$

$= 5^x(x^5 \log 5 + 5x^4)$

58. (a) Given, $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$

i.e., $x = a^{\frac{1}{2}\sin^{-1} t}$ and $y = a^{\frac{1}{2}\cos^{-1} t}$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt} = a^{\frac{1}{2}\sin^{-1} t} \log a \frac{d}{dt}\left(\frac{1}{2}\sin^{-1} t\right)$$

$$\left(\because \frac{d}{dx}a^x = a^x \log a\right)$$

$$= a^{\frac{1}{2}\sin^{-1} t} \log a\left(\frac{1}{2\sqrt{1-t^2}}\right) = \frac{a^{\frac{1}{2}\sin^{-1} t}\log a}{2\sqrt{1-t^2}}$$

and $\frac{dy}{dt} = a^{\frac{1}{2}\cos^{-1} t} \log a \frac{d}{dt}\left(\frac{1}{2}\cos^{-1} t\right)$

(using chain rule)

$$= a^{\frac{1}{2}\cos^{-1}t} \log a \left(\frac{-1}{2\sqrt{1-t^2}}\right) = \frac{-a^{\frac{1}{2}\cos^{-1}t}\log a}{2\sqrt{1-t^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-a^{\frac{1}{2}\cos^{-1}t}}{a^{\frac{1}{2}\sin^{-1}t}} = -\frac{\sqrt{a^{\cos^{-1}t}}}{\sqrt{a^{\sin^{-1}t}}} = -\frac{y}{x}$$

59. (b) Now, $y = e^{x^x}$

Taking logarithms with base e, we get

$\log_e y = \log_e e^{x^x}$

$\log_e y = x^x . \log_e e = x^x, \quad \{\because \log_e e = 1\}.$

Again taking logarithms with base e, we get,

$\log_e(\log_e y) = \log_e x^x$ or $\log_e(\log_e y) = x \log_e x.$

Differentiating both sides with respect to x, we get

$$\frac{1}{\log_e y}.\frac{1}{y}.\frac{dy}{dx} = 1.\log_e x + x.\frac{1}{x}$$

or $\dfrac{dy}{dx} = y \log_e y.(\log x + 1)$

$= e^{x^x}.x^x.(\log_e x + 1). = y.x^x(1 + \log_e x)$

60. (a) Since f (x) is continuous at x = 2

$\therefore f(2) = \lim_{x \to 2^+} f(x) = 1 = \lim_{x \to 2^+} (ax + b)$

$\therefore \; 1 = 2a + b \qquad .....(i)$

Again f (x) is continuous at x = 4,

$\therefore f(4) = \lim_{x \to 4^-} f(x) = 7 = \lim_{x \to 4^-} (ax + b)$

$\therefore \; 7 = 4a + b \qquad ....(ii)$

Solving (i) and (ii), we get a = 3, b = – 5

61. (d) We have $f(x) = \begin{cases} \dfrac{-1}{x-1}, & 0 < x < 1 \\ \dfrac{1-1}{x-1}, & 1 < x < 2 \\ 0 \quad , & x = 1 \end{cases}$

$$\lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{-1}{(1-h)-1} = \lim_{h \to 0} \frac{1}{h} = \infty$$

$\therefore$ f(x) is not continuous and hence not differentiable at x = 1.

62. (b) Since f (x) is continuous at x = 0

$\therefore \lim_{x \to 0} f(x) = f(0)$

but f (0) = 0 (given)

$\therefore \lim_{x \to 0} f(x) = \lim_{x \to 0} x^k \cos(1/x) = 0$, if k > 0

63. (b) We have, $f(x) = \dfrac{1}{1-x}$.

As at x = 1, f(x) is not defined, x = 1 is a point of discontinuity of f(x).

If $x \neq 1$, $f[f(x)] = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{1-1/(1-x)} = \dfrac{x-1}{x}$

$\therefore$ x = 0, 1 are points of discontinuity of f [f(x)] .

If $x \neq 0, x \neq 1$

$$f[f\{f(x)\}] = f\left(\frac{x-1}{x}\right) = \frac{1}{1-\frac{(x-1)}{x}} = x.$$

STATEMENT TYPE QUESTIONS

64. (b) Given that f (x) is an even function,

$\Rightarrow$ f (– x) = f (x) for all x

Since it is differentiable, so,

– f ' (–x) = f ' (x) for all x

$\Rightarrow$ f ' (–x) = – f ' (x) for all x

$\Rightarrow$ f ' (x) is an odd function.

65. (c) I. Let g(x) = cos x and h(x) = |x|

Now, g(x) is a cosine function, so it is continuous function in its domain i.e., $x \in R$.

h (x) = |x| is the absolute valued function, so it is continuous function for all $x \in R$.

$\therefore$ (hog) (x) = h[g(x)] = h(cos x) = |cos x|

Since g(x) and h(x) are both continuous function for all $x \in R$, so composition of g(x) and h(x) is also a continuous function for all $x \in R$.

Thus, f(x) = |cos x| is a continuous function for all $x \in R$.

II. Let g(x) = |x| and h(x) = sin x

Now, g(x) = |x| is the absolute valued function, so it is continuous function for all $x \in R$.

h(x) = sin x is the sine function, so it is a continuous function for all $x \in R$.

$\therefore$ (hog) (x) = h[g(x)] = h(|x|) = sin |x|

Since, g(x) and h(x) are both continuous functions for all $x \in R$, so composition of g(x) and h(x) is also a continuous function for all $x \in R$.

Thus, f(x) = sin |x| is a continuous function.

66. (d) Here, greatest integer function [x] is discontinuous at its integral value of x, cot x and cosec x are discontinuous at 0, π, 2π etc. and tan x and sec x are discontinuous at $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}$ etc. Therefore the greatest integer function and all trigonometric functions are not continuous for $x \in R$

Therefore, neither (I) nor (II) are true.

67. (c) I. We are investigating continuity of $(f + g)$ at $x = c$.

Clearly, it is defined at $x = c$. we have

$$\lim_{x \to c} (f+g)(x) = \lim_{x \to c} \left[f(x) + g(x)\right]$$

(by definition of $f + g$)

$$= \lim_{x \to c} f(x) = \lim_{x \to c} g(x)$$

(by the theorem on limits)

$= f(c) + g(c)$ (as f and g are continuous)

$= (f + g)(c)$ (by definition of $f + g$)

Hence, $f + g$ is continuous at $x = c$.

II. We have, $\lim_{x\to c}(f-g)x = \lim_{x\to c}[f(x)-g(x)]$

$= \lim_{x\to c} f(x) = \lim_{x\to c} g(x) = f(c) - g(c)$

$\Rightarrow \lim_{x\to c}(f-g)x = (f-g)c$

$\therefore f - g$ is continuous at $x = c$.

III. We have,

$\lim_{x\to c}(f.g)x = \lim_{x\to c}\{(f(x).g(x))\}$

$= \lim_{x\to c} f(x)\cdot \lim_{x\to c} g(x) = f(c).\ g(c)$

$\therefore f\,.\,g$ is continuous $x = c$

IV. $\lim_{x\to c}\frac{f}{g}(x) = \lim_{x\to c}\left(\frac{f}{g}\right)(x) = \lim_{x\to c}\left\{\frac{f(x)}{g(x)}\right\} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$

$= \frac{f(c)}{g(c)} = \frac{f}{g}(c)$

$\therefore \left(\frac{f}{g}\right)$ is continuous at x = c provided $g(c) \neq 0$

68. (d) Here, $f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$

At x = 0, LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x)$

Putting x = 0 – h and x → 0, h → 0

$\lim_{h\to 0}(0-h) = 0 - 0 = 0$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(x)$

Putting x = 0 + h as x → 0, h → 0

$\lim_{x\to 0^+}(0+h) = 0 + 0 = 0$

Also, f(0) = 0 $[\because f(x) = x]$

∴ LHL = RHL = f(0)

Thus, f(x) is continuous at x = 0

At x = 1, LHL = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x)$

Putting x = 1 – h as $x \to 1^-$, h → 0

$= \lim_{h\to 0}(1-h) = 1 - 0 = 1$

RHL = $\lim_{x\to 1^+} f(x) = 5$

∴ LHL ≠ RHL

Thus, $f(x)$ is discontinuous at $x = 1$

At $x = 2$, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = 5$

Also, f(2) = 5

∴ LHL = RHL = f(2).

Thus, f(x) is continuous at x = 2.

69. (d) (I) Every rational function f is given by

$f(x) = \frac{p(x)}{q(x)}, q(x) \neq 0$

where p and q are polynomial functions. The domain of f is all real numbers except those points at which q is zero. Since, polynomial functions are continuous, f is continuous.

(III) Let f(x) = cos x and let c be any real number.

Then, $\lim_{x\to c^+} f(x) = \lim_{h\to 0} f(c+h)$

$\Rightarrow \lim_{x\to c^+} f(x) = \lim_{h\to 0}\cos(c+h)$

$\Rightarrow \lim_{x\to c^+} f(x) = \lim_{h\to 0}(\cos c \cos h - \sin c \sin h)$

$\Rightarrow \lim_{x\to c^+} f(x) = \cos c \lim_{h\to 0}\cosh - \sin c \lim_{h\to 0}\sinh$

= cos c × 1 – sinc × 0

$\lim_{x\to c^+} f(x) = \cos c$ $\left(\because \lim_{h\to 0}\cos h = 1 \text{ and } \lim_{h\to 0}\sin h = 0\right)$

Similarly, we have

$\lim_{x\to c^-} f(x) = f(c)$

$\therefore \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$

f(x) is continuous at x = c.

∵ c is arbitrary real number, so f(x) is everywhere continuous.

(IV) Let f(x) = tan x

We have, $f(x) = \tan x = \frac{\sin x}{\cos x}$

∵ sin x and cos x are everywhere continuous.

Therefore, f(x) = tan x is continuous for all x ∈ R such that cos x ≠ 0

Hence, f(x) = tan x is continuous for all

$x \in R - \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$

70. (c) I. Given, x = a (θ – sin θ), y = a (1 + cos θ),

On differentiating w.r.t. θ, we get

$\frac{dx}{d\theta} = \frac{d}{d\theta}a(\theta - \sin\theta) = a(1-\cos\theta)$

and $\frac{dy}{d\theta} = \frac{d}{d\theta}a(1+\cos\theta) = a(0-\sin\theta)$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{d\theta}\times\frac{d\theta}{dx}$

$= \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\left(\frac{\theta}{2}\right)} = -\cot\left(\frac{\theta}{2}\right)$

II. Given, $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

On differentiating w.r.t *t*, we get

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{\sin^3 t}{\sqrt{\cos 2t}}\right)$$

$$= \frac{\sqrt{\cos 2t}\left(3\sin^2 t\cos t\right) - \sin^3 t\left(\frac{-2\sin 2t}{2\sqrt{\cos 2t}}\right)}{\left(\sqrt{\cos 2t}\right)^2}$$

$$= \frac{3(\cos 2t)\sin^2 t\cos t + \sin 2t\sin^3 t}{\cos 2t\sqrt{\cos 2t}}$$

$$= \frac{3\left(1 - 2\sin^2 t\right)\sin^2 t\cos t + (2\sin t\cos t)\sin^3 t}{\cos 2t\sqrt{\cos 2t}}$$

($\because$ $\cos 2t = 1 - 2\sin^2 t$ and $\sin 2t = 2\sin t\cos t$)

$$= \frac{3\sin^2 t\cos t - 4\sin^4 t\cos t}{\cos 2t\sqrt{\cos 2t}}$$

and $\frac{dy}{dt} = \frac{d}{dt}\left(\frac{\cos^3 t}{\sqrt{\cos 2t}}\right)$

$$= \frac{\sqrt{\cos 2t}\left(-3\cos^2 t\sin t\right) - \cos^3 t\left(\frac{-2\sin 2t}{2\sqrt{\cos 2t}}\right)}{\left(\sqrt{\cos 2t}\right)^2}$$

(using quotient rule)

$$= \frac{-3(\cos 2t)\cos^2 t\sin t + \sin 2t\cos^3 t}{\cos 2t\sqrt{\cos 2t}}$$

$$= \frac{-3\left(2\cos^2 t - 1\right)\cos^2 t\sin t + \cos^3 t(2\sin t\cos t)}{\cos 2t\sqrt{\cos 2t}}$$

$$\left(\begin{array}{l}\because \cos 2t = 2\cos^2 t - 1 \\ \sin 2t = 2\sin t\cos t\end{array}\right)$$

$$= \frac{3\cos^2 t\sin t - 4\cos^4 t\sin t}{\cos 2t\sqrt{\cos 2t}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3\cos^2 t\sin t - 4\cos^4 t\sin t}{3\sin^2 t\cos t - 4\sin^4 t\cos t}$$

$$= \frac{\cos^2 t\sin t\left(3 - 4\cos^2 t\right)}{\sin^2 t\cos t\left(3 - 4\sin^2 t\right)} = \frac{\cos t\left(3 - 4\cos^2 t\right)}{\sin t\left(3 - 4\sin^2 t\right)}$$

$$= \frac{3\cos t - 4\cos^3 t}{3\sin t - 4\sin^3 t}$$

$$= \frac{-\cos 3t}{\sin 3t} = -\cot 3t$$

($\because$ $\cos 3t = 4\cos^3 t - 3\cos t$ and $\sin 3t = 3\sin t - 4\sin^3 t$)

MATCHING TYPE QUESTIONS

71. (c) A. Here, f(x) = cosx

At x = a, where $a \in R$

$$\lim_{x\to a} f(x) = \lim_{x\to a} \cos x = \cos a \quad [\because f(x) = \cos x]$$

f(a) = cos a

$$\therefore \lim_{x\to a} f(x) = f(a)$$

Thus f(x) is continuous at x = a. But a is an arbitrary point so f(x) is continuous at all points.

B. Here, f(x) = cosec $x = \frac{1}{\sin x}$. Since, f(x) is not defined at $x = n\pi$, $n \in Z$.

Thus, f(x) is continuous at all points except $x = n\pi$, $n \in Z$ (as 1 and sinx are continuous functions)

C. Here, $f(x) = \sec x = \frac{1}{\cos x}$

Since, f(x) is not defined at $4x = (2n+1)\frac{\pi}{2}$, $n \in Z$.

Thus, f(x) is continuous at all points except

$x = (2n+1)\frac{\pi}{2}$, $n \in Z$.

(as 1 and cos x are continuous functions)

D. Here, $f(x) = \cot x = \frac{\cos x}{\sin x}$

Since, f(x) is not defined at $x = n\pi$, $n \in Z$.

Thus, f(x) is continuous at all points except $x = n\pi$, $n \in Z$.

(as cos x and sin x are continuous functions)

72. (b) A. Let $y = \sin^{-1}x$. Then, $x = \sin y$.

On differentiating both sides w.r.t. x, we get

$$1 = \cos y\frac{dy}{dx}$$

which implies that $\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\cos\left(\sin^{-1} x\right)}$

Observe that this is defined only for $\cos y \neq 0$,

i.e., $\sin^{-1} x \neq -\frac{\pi}{2}, \frac{\pi}{2}$, i.e., $x \neq -1, 1$, i.e., $x \in (-1, 1)$.

for $x \in (-1, 1)$, $\sin(\sin^{-1} x) = x$ and hence $\cos^2 y = 1 - (\sin y)^2 = 1 - (\sin(\sin^{-1}x))^2 = 1 - x^2$

Also, since, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, cos y is positive and

hence $\cos y = \sqrt{1 - x^2}$

Thus, for $x \in (-1, 1)$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - x^2}}$$

B. Let $y = \tan^{-1}x$. Then, $x = \tan y$

On differentiating both sides w.r.t. x, we get

$$1 = \sec^2 y\frac{dy}{dx}$$

which implies that

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y}$$

$$= \frac{1}{1+\left(\tan\left(\tan^{-1}x\right)\right)^2} = \frac{1}{1+x^2}$$

C. $\cos^{-1}x = y$, $\cos y = x$

On differentiating both sides w.r.t. x, we get

$$-\sin y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = -\frac{1}{\sin y} = \frac{1}{\sin\left(\cos^{-1}x\right)}$$

$$= \frac{-1}{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)}$$

$$= \frac{-1}{\sqrt{1-x^2}} \qquad \left[\because x \in (-1,1)\right]$$

D. Let $\cot^{-1}x = y$, $\cot y = x$

On differentiating both side w.r.t.x, we get

$$-\operatorname{cosec}^2 y\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{\operatorname{cosec}^2 y}$$

$$= -\frac{1}{1+\cot^2 y}$$

$$= -\frac{1}{1+\left\{\cot\left(\cot^{-1}x\right)\right\}^2}$$

$$= -\frac{1}{1+x^2}$$

73. (a) A. Given, $x = 2at^2$, $y = at^4$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt} = (2a)(2t) \text{ and } \frac{dy}{dt} = a\left(4t^3\right)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}\times\frac{dt}{dx} \quad \left(\because \frac{dy}{dx} = \frac{dy/dt}{dx/dt}\right)$$

$$= \frac{4at^3}{4at} = \frac{t^3}{t} = t^2$$

B. Given, $x = a\cos\theta$, $y = b\cos\theta$

On differentiating w.r.t. θ, we get $\frac{dx}{d\theta} = a(-\sin\theta)$

and $\frac{dy}{d\theta} = b(-\sin\theta)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{d\theta}\times\frac{d\theta}{dx}$$

$$= \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$$

C. Given, $x = \sin t$, $y = \cos 2t$

On differentiating w.r.t. t, we get

$$\therefore \frac{dx}{dt} = \cos t \text{ and } \frac{dy}{dt} = -(\sin 2t)2$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}\times\frac{dt}{dx} = \frac{-2\sin 2t}{\cos t}$$

$$= \frac{-2(2\sin t\cos t)}{\cos t} = -4\sin t$$

$$(\because \sin 2\theta = 2\sin\theta\cos\theta)$$

D. Given, $x = 4t$, $y = \frac{4}{t}$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt} = 4 \text{ and } \frac{dy}{dt} = 4(-1)t^{-2}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4t^{-2}}{4} = -\frac{1}{t^2}$$

E. Given, $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$

On differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(\cos\theta - \cos 2\theta) = -\sin\theta - (-\sin 2\theta)2$$

$$= -\sin\theta + 2\sin 2\theta$$

$$\text{and } \frac{dy}{d\theta} = \frac{d}{d\theta}(\sin\theta - \sin 2\theta) = \cos\theta$$

$$-(\cos 2\theta)2 = \cos\theta - 2\cos 2\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{d\theta}\times\frac{d\theta}{dx}$$

$$= \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$$

INTEGER TYPE QUESTIONS

74. (a) **Note:** A function f (x) is said to be continuous at x = a iff. R.H.L = L.H.L. = f (a)

i.e., $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = \lim_{x\to a} f(x)$

Let, $f(x) = \begin{cases} x+\lambda & , x<3 \\ 4 & , x=3 \\ 3x-5 & , x>3 \end{cases}$ is

continuous at x = 3

$\therefore \lim_{x\to 3^-}(x+\lambda) = f(3) = \lim_{x\to 3^+}(3x-5)$

$\Rightarrow 3+\lambda = 4$

$\Rightarrow \lambda = 4-3 = 1$

75. (b) Let f (x) = | x – 1| + | x – 3 |

At x = 2,

| x – 1 | = x – 1 and

| x – 3 | = – x + 3

$\Rightarrow$ f (x) = x – 1 – x + 3 = 2

which is constant function.

$\Rightarrow$ f ' (2) = 0

76. (a) Given: $f(x) = x^{1/x} - 1$

$\Rightarrow$ f (1) = 1 – 1 = 0

we know that a function f(x) be continuous at x = a if

$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$

Here $\lim_{x\to 1^-} f(x) = \lim_{h\to 0}(1-h)^{\frac{1}{1-h}} - 1 = 1-1 = 0$

and $\lim_{x\to 1^+} f(x) = \lim_{h\to 0}(1+h)^{\frac{1}{1+h}} - 1 = 1-1 = 0$

$\therefore f(1) = \lim_{x\to 1^-} f(x) = \lim_{h\to 1^+} f(x) = 0$

77. (a) Let $s = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $t = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

We have to find out $\frac{ds}{dt}$

Putting x = tan θ, we get

$s = \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} x$

$\therefore \frac{ds}{dx} = \frac{2}{1+x^2}$

and $t = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1} x$

$\therefore \frac{dt}{dx} = \frac{2}{1+x^2}$

$\therefore \frac{ds}{dt} = \frac{ds/dx}{dt/dx} = \frac{2}{1+x^2} \times \frac{1+x^2}{2} = 1$

78. (c) $k = f(2) = \lim_{x\to 2} f(x)$

$= \lim_{x\to 2} \frac{x^3+x^2-16x+20}{(x-2)^2} \left(\frac{0}{0}\text{form}\right)$

Using L's Hospital Rule

$= \lim_{x\to 2} \frac{3x^2+2x-16}{2(x-2)} \left(\frac{0}{0}\text{form}\right)$

Using L's Hospital Rule

$= \lim_{x\to 2} \frac{6x+2}{2} = 7$

79. (a) $f(x) = x^2 \sin\frac{1}{x}$ for $x \neq 0$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \sin\frac{1}{x}$

Since the value of $\sin\frac{1}{x}$ is between – 1 to 1.

$\therefore \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$

= $(0)^2$ × (value between – 1 to 1.)

= 0

So f(x) is continuous at x = 0

If $\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow$ f(0) = 0

80. (b) Here, $f(x) = \begin{cases} \frac{k\cos x}{\pi-2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$

$\therefore \text{LHL} = \lim_{x\to \frac{\pi^-}{2}} f(x) = \lim_{x\to\frac{\pi^-}{2}} \frac{k\cos x}{\pi-2x}$

Putting $x = \frac{\pi}{2} - h$ as $x \to \frac{\pi^-}{2}$ when $h \to 0$

$\therefore \lim_{h\to 0} \frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi - 2\left(\frac{\pi}{2}-h\right)} = \lim_{h\to 0}\frac{k\sin h}{2h}$

$= \lim_{h\to 0} \frac{k}{2} \times \frac{\sin h}{h} = \frac{k}{2} \times 1 = \frac{k}{2} \quad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$

$\text{RHL} = \lim_{x\to\frac{\pi^+}{2}} f(x) = \lim_{x\to\frac{\pi^+}{2}} \frac{k\cos x}{\pi-2x}$

Putting $x = \frac{\pi}{2} + h$ as $x \to \frac{\pi^+}{2}$ when $h \to 0$

$$\therefore \lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}=\lim_{h\to 0}\frac{-k\sin h}{-2h}$$

$$=\lim_{h\to 0}\frac{k}{2}\times\frac{\sin h}{h}=\frac{k}{2}\times 1=\frac{k}{2}\quad\left(\because \lim_{x\to 0}\frac{\sin x}{x}=1\right)$$

Also, $f\left(\frac{\pi}{2}\right)=3$. Since f(x) is continuous at $x=\frac{\pi}{2}$.

$\therefore LHL=RHL=f\left(\frac{\pi}{2}\right)\Rightarrow\frac{k}{2}=3\Rightarrow k=6$

81. (d) Given $f(x)=\begin{cases}\dfrac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0\\ a, & \text{if } x=0\end{cases}$

Now, $\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{2\sin x-\sin 2x}{2x\cos x}\left(\frac{0}{0}\text{form}\right)$

$$=\lim_{x\to 0}\frac{2\sin x(1-\cos x)}{2x\cos x}$$

$$=\lim_{x\to 0}\frac{\tan x}{x}\cdot\lim_{x\to 0}(1-\cos x)$$

$= 1.0$

$= 0$

82. (c) $LHL=\lim_{h\to 0}f(0-h)=\lim_{h\to 0}\frac{\sin 5(0-h)}{(0-h)^2+2(0-h)}$

$$=-\lim_{h\to 0}\frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)}=\frac{5}{2}$$

$$RHL=\lim_{x\to 0^+}\frac{\sin 5x}{x^2+2x}$$

$$=\lim_{x\to 0^+}\frac{\sin 5x}{5x}\cdot\lim_{x\to 0^+}\frac{1}{(x+2)}=\frac{5}{2}$$

$f(0)=k+\frac{1}{2}$

Since, it is continuous at x = 0

$\therefore$ LHL = RHL = f(0)

$\Rightarrow\frac{5}{2}=k+\frac{1}{2}$

$\Rightarrow k=2$

83. (d) At x = 8,

L.H.L = $\lim_{x\to 8^-}f(x)=\lim_{x\to 8^-}[x]$

Put x = 8 – h. Then as $x\to 8$, $h\to 0$

L.H.L = $\lim_{h\to 0}[8-h]=7$ ....(i)

R.H.L = $\lim_{x\to 8^+}f(x)=\lim_{x\to 8^+}[x]$

Put x = 8 + h. Then as $x\to 8$, $h\to 0$

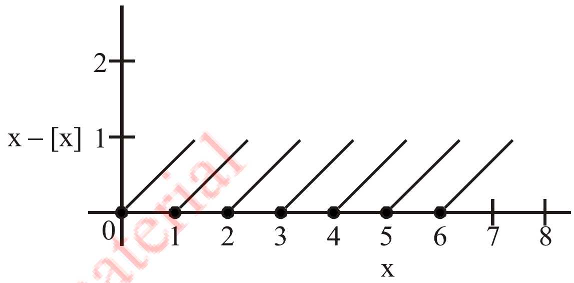
R.H.L = $\lim_{h\to 0}[8+h]=8$ ....(ii)

From (i) and (ii) L.H.L $\neq$ R.H.L

Therefore the function is discontinuous at x = 8, in the given interval.

84. (a) The function f (x) = sin x is differentiable for all $x\in R$. Therefore the number of points in the interval $(-\infty,\infty)$ where the function is not differentiable are zero.

85. (c) The graph of the function f (x) = x – [x] for the interval (0, 7) is shown below :



It is obvious from the above graph that the function x – [x] is discontinuous at the points x = 1, 2, 3, 4, 5, 6. Therefore no. of points of discontinuity of the given function in the given interval are 6.

86. (a) $\frac{dy}{dx}=-a\sin x-b\cos x$

and $\frac{d^2y}{dx^2}=-a\cos x+b\sin x$

$\therefore$ n = 2.

87. (b) L.H.L = $\lim_{x\to 4^-}f(x)=\lim_{x\to 4^-}x^2$

Put x = 4 – h. as $x\to 4$, $h\to 0$.

$\therefore$ L.H.L. $=\lim_{h\to 0}(4-h)^2=\lim_{h\to 0}(16+h^2-8h)$

= 16

R.H.L = $\lim_{x\to 4^+}f(x)=\lim_{x\to 4^+}ax$.

Put x = 4 + h. as $x\to 4$, $h\to 0$.

$\therefore$ R.H.L. $=\lim_{h\to 0}a(4+h)=\lim_{h\to 0}4a+ah$

= 4a

For the function to be continuous at x = 4, L.H.L. = R.H.L. Therefore 16 = 4a which gives a = 4.

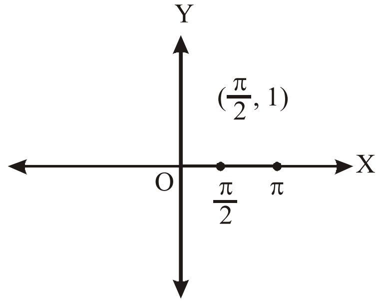
ASSERTION - REASON TYPE QUESTIONS

88. (d) For x < 0, $\frac{d}{dx}(\ell n|x|)=\frac{d}{dx}(\ell n(-x))$

$$=\frac{1}{(-x)}(-1)=\frac{1}{x}$$

89. (c) We know that for all

$x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$, $0 < \sin x < 1$



$\Rightarrow [\sin x] = 0$

$\therefore \lim_{x \to \frac{\pi}{2}} [\sin x] = 0$

Thus, we see that the Reason is not true.

Also, $f\left(\frac{\pi}{2}\right) = \left[\sin\frac{\pi}{2}\right] = 1$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} f(x) \neq f\left(\frac{\pi}{2}\right)$

$\therefore$ f is not continuous at $x = \frac{\pi}{2}$

90. (c) We have $y = \log_{10} x + \log_e y$

$$\frac{dy}{dx} = \frac{1}{x}\log_{10} e + \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{y-1}{y}\right) = \frac{\log_{10} e}{x} \Rightarrow \frac{dy}{dx} = \frac{\log_{10} e}{x}\left(\frac{y}{y-1}\right)$$

91. (c) We have, $x = at^2$ and, $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2at \text{ and } \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1}{t}$$

$$\text{Now } \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{t}\right)$$

$$= \frac{-1}{t^2} \times \frac{dt}{dx} = \frac{-1}{t^2} \times \frac{1}{2at} = \frac{-1}{2at^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=2} = \frac{-1}{2 \times a \times (2)^3} = \frac{-1}{16a}$$

92. (a) Given, $u = f(\tan x)$

$$\Rightarrow \frac{du}{dx} = f'(\tan x)\sec^2 x$$

and $v = g(\sec x)$

$$\Rightarrow \frac{dv}{dx} = g'(\sec x)\sec x \tan x$$

$$\therefore \frac{du}{dv} = \frac{(du/dx)}{(dv/dx)} = \frac{f'(\tan x)}{g'(\sec x)} \cdot \frac{1}{\sin x}$$

$$\therefore \left(\frac{du}{dv}\right)_{x=\pi/4} = \frac{f'(1)}{g'(\sqrt{2})} \cdot \sqrt{2}$$

$$= \frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}}$$

93. (d) $\lim_{x \to 0} f(x) = \lim_{x \to 0} x^n \sin\left(\frac{1}{x}\right) = 0$ for positive integer n

Now $f(0)$ does not exist, hence function is not continuous at $x = 0$

94. (b) A function is continuous at $x = c$, if the function is defined at $x = c$ and if the value of the function at $x = c$ equals the limit of the function at $x = c$. And if f is not continuous at c, we say f is discontinuous at c and c is called a point of discontinuity of f.

95. (b) Let $h(x) = x^2$ and $g(x) = \cos x$

Now, h(x) is a polynomial function, so it is continuous for all $x \in R$.

g(x) is a cosine function, so it is continuous function in its domain i.e., $x \in R$.

$\therefore$ $(goh)(x) = g[h(x)] = g(x^2) = \cos x^2$

Since g(x) and h(x) are both continuous functions for all $x \in R$, so composition of g(x) and h(x) is also a continuous function for all $x \in R$.

Thus, $f(x) = \cos(x^2)$ is a continuous function for all $x \in R$.

We know, if g is continuous at c and if f is continuous at g(c), then g is continuous at c.

Thus Reason itself is true but not correct explanation of Assertion.

96. (b) Every differentiable function is continuous.

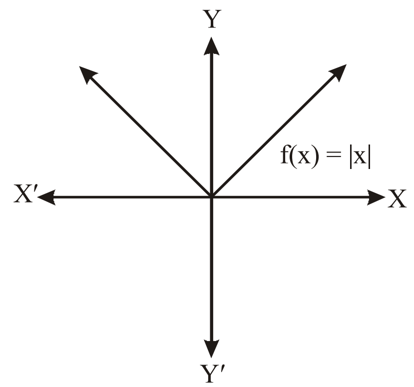
But the converse of the above statement is not true.

We know that, $f(x) = |x|$ is a continuous function. Consider the left hand limit

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \frac{-h}{h} = -1$$

The right hand limit

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \frac{h}{h} = 1$$



Since, the above left and right hand limits at 0 are not equal $\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$ does not exist and hence f is not differentiable at 0. Thus, f is not a differentiable function.

97. (b) Let, $y = e^{\cos x}$. Using chain rule, we have

$$\frac{dy}{dx} = e^{\cos x}.(-\sin x) = -(\sin x)e^{\cos x}$$

Also, $\frac{d}{dx}(e^x) = e^x$

98. (b) Given, $xy = e^{(x-y)}$

On differentiating both sides w.r.t.x, we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}\left(e^{x-y}\right)$$

$$\Rightarrow x\frac{dy}{dx} + y.1 = e^{x-y}\frac{d}{dx}(x-y)$$

(using product rule in LHS and chain rule in RHS)

$$\Rightarrow x\frac{dy}{dx} + y = e^{x-y}\left(1-\frac{dy}{dx}\right)$$

$$\Rightarrow x\frac{dy}{dx} + e^{x-y}\frac{dy}{dx} = e^{x-y} - y$$

$$\Rightarrow \left(x+e^{x-y}\right)\frac{dy}{dx} = e^{x-y} - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{x-y}-y}{x+e^{x-y}} = \frac{xy-y}{x+xy}$$

$$\left(\because e^{x-y} = xy \text{ is given}\right)$$

$$= \frac{y(x-1)}{x(1+y)}$$

Also, $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

99. (a) $f(x) = |x| \sin x$

$$\text{L.H.D.} = \lim_{h\to 0}\frac{|0-h|\sin(0-h)-0}{h} = \lim_{h\to 0}\frac{-h\sin h}{h} = 0$$

$$\text{R.H.D.} = \lim_{h\to 0}\frac{|0+h|\sin(0+h)-0}{h}$$

$$= \lim_{h\to 0}\frac{h\sin h}{h} = 0$$

f (x) is differentiable at x = 0

100. (a) $f(2) = 4$

$$f(2^-) = \lim_{x\to 2^-} |[x]\,x| = 2$$

L. H. L ≠ R. H. L

Discontinuous ⇒ non-differentiable

101. (d)

102. (c) The function f (x) = |x| is not differentiable in the interval at x = 0. Hence Rolle's theorem can not be verified.

103. (d) The function $f(x) = \frac{|x|}{x}$ is not continuous at x = 0

CRITICAL THINKING TYPE QUESTIONS

104. (b) $f(x) = e^x$ and $g(x) = \sin^{-1}x$ and $h(x) = f(g(x))$

$$\Rightarrow h(x) = f(\sin^{-1}x) = e^{\sin^{-1}x}$$

$$\Rightarrow h'(x) = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{h'(x)}{h(x)} = \frac{1}{\sqrt{1-x^2}}$$

105. (c) The function log | x | is not defined at x = 0 so, x = 0 is a point of discontinuity.

Also, for f (x) to defined, log | x | = 0 that is x ≠ ± 1. Hence 1 and –1 are also points of discontinuity. Clearly f (x) is continuous for x ∈ R – {0, 1, –1}. Thus there are three points of discontinuity.

106. (b) Since f (x) is continuous at x = π/4

$$\therefore f(\pi/4) = \lim_{x\to\frac{\pi^-}{4}} f(\pi/4-h) = \lim_{x\to\frac{\pi^+}{4}} f(\pi/4+h)$$

$$\text{L.H.L at } \pi/4 = \lim_{h\to 0}\left(\frac{\pi}{4}-h\right) + a^2\sqrt{2}\sin(\pi/4-h)$$

$$= \frac{\pi}{4} + a^2 \quad .....(i)$$

$$\text{R.H.L at } \pi/4 = \lim_{h\to 0}(\pi/4+h)\cot(\pi/4+h) + b$$

$$= \frac{\pi}{4} + b \quad ......(ii)$$

From equation (i) & (ii) we get

$$\frac{\pi}{4} + a^2 = \frac{\pi}{4} + b \Rightarrow a^2 = b$$

Also, f (x) is continuous at x = π/2

$$\therefore f(\pi/2) = \lim_{x\to\frac{\pi^-}{2}} f(\pi/2-h) = \lim_{x\to\frac{\pi^+}{2}} f(\pi/2+h)$$

Now, LHL at (π/2)

$$= \lim_{h\to 0}[(\pi/2-h)\cot(\pi/2-h) + b] = b \quad .....(iii)$$

RHL at π/2

$$= \lim_{h\to 0}[\,b\sin 2(\pi/2+h) - a\cos 2(\pi/2+h)]$$

$$= \lim_{h\to 0}[b\sin(\pi+2h) - a\cos(\pi+2h)]$$

$$= [b\,.\,0 - a(-1)] = a \quad ....(iv)$$

From (iii) & (iv) we have, a = b

Hence, (0, 0) and (1, 1) satisfies the both equation a = b and $a^2 = b$

But we have only (0, 0) in the options

∴ (0, 0) = (a, b).

107. (c) We have

$$f(x)=\begin{cases}(x-1)\sin\left(\frac{1}{x-1}\right), & \text{if } x\neq 1\\ 0, & \text{if } x=1\end{cases}$$

$$Rf'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{h\sin\frac{1}{h}-0}{h}=\lim_{h\to 0}\sin\frac{1}{h}=\text{a finite number}$$

Let this finite number be l

$$L\ f'(1)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$=\lim_{h\to 0}\frac{-h\sin\left(\frac{1}{-h}\right)}{-h}$$

$$=\lim_{h\to 0}\sin\left(\frac{1}{-h}\right)=-\lim_{h\to 0}\sin\left(\frac{1}{h}\right)$$

$= -$ (a finite number) $= -l$

Thus $Rf'(1)\neq Lf'(1)$

$\therefore$ f is not differentiable at x = 1

Also, $$f'(0)=\left[\sin\frac{1}{(x-1)}-\frac{x-1}{(x-1)^2}\cos\left(\frac{1}{x-1}\right)\right]_{x=0}$$

$= -\sin 1 + \cos 1$

$\therefore$ f is differentiable at x = 0

108. (c) We have, function $f(x)=\tan\left(\frac{\pi x}{x+1}\right)$ and we know that function f (x) is discontinuous at those points, where $\tan\left(\frac{\pi x}{x+1}\right)=\tan\frac{\pi}{2}$ ($\because \tan\frac{\pi}{2}$ is not defined)

By using $\tan\theta=\tan\alpha$, we have $\theta = m\pi+\alpha$

$$\Rightarrow \frac{\pi x}{x+1}=m\pi+\frac{\pi}{2}$$

$$\Rightarrow \pi\left(\frac{x}{x+1}\right)=\pi\left(m+\frac{1}{2}\right)$$

$$\Rightarrow \left(\frac{x}{x+1}\right)=m+\frac{1}{2}$$

$$\Rightarrow \frac{x}{x+1}=\frac{2m+1}{2}$$

$\Rightarrow$ 2x = (2m + 1) x + (2m + 1)

$\Rightarrow$ (2 – 2m – 1) x = 2m + 1 $\Rightarrow$ $x=\frac{2m+1}{1-2m}$

109. (b) Given, sin y = x sin (a + y)

$$\Rightarrow x=\frac{\sin y}{\sin(a+y)}$$

On differentiating w.r. to y, we get

$$\frac{dx}{dy}=\frac{d}{dy}\left[\frac{\sin y}{\sin(a+y)}\right]$$

$$=\frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{\sin^2(a+y)}$$

$$=\frac{\sin a}{\sin^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}.$$

110. (c) $$f\left(\frac{\pi}{4}\right)=\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{4x-\pi},\ \left(\frac{0}{0}\text{form}\right)$$

Using L' Hospitals rule

$$\lim_{x\to\frac{\pi}{4}}\frac{-\sec^2 x}{4}=-\frac{1}{2}$$

111. (a) Functions which satisfy the relation $x^2 + y^2 = 4$ and $y(x)=\sqrt{4-x^2}$ and $y(x)=-\sqrt{4-x^2}$.

And both functions are continuous in [– 2, 2]

112. (c) Given, $f(x)=\begin{cases}\sin x, & \text{for } x\geq 0\\ 1-\cos x, & \text{for } x\leq 0\end{cases}$

and $g(x) = e^x$

$$\therefore\ gof=\begin{cases}e^{\sin x}, & x\geq 0\\ e^{1-\cos x}, & x\leq 0\end{cases}$$

$$\therefore LHD=(gof)'(0-h)=\lim_{h\to 0}\frac{gof(0-h)-gof(h)}{-h}$$

$$=\lim_{h\to 0}\frac{e^{1-\cos(0-h)}-e^{1-\cos h}}{-h}=0$$

RHD = (gof)′ (0 + h)

$$=\lim_{h\to 0}\frac{gof(0+h)-gof(h)}{h}$$

$$=\lim_{h\to 0}\frac{e^{\sin h}-e^{\sin h}}{h}=0$$

$\therefore$ RHD = LHD = 0

$\Rightarrow$ (gof)′(0) = 0

113. (c) Let $y=e^{x^3}$, z = log x

On differentiating w.r.t.x, we get

$$\frac{dy}{dx}=e^{x^3}\left(3x^2\right)=3x^2e^{x^3} \text{ and } \frac{dz}{dx}=\frac{1}{x}$$

$$\therefore \frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\frac{3x^2e^{x^3}}{\left(\frac{1}{x}\right)}=3x^3e^{x^3}$$

114. (a) Let $y = a\sin^3 t$ and $x = a\cos^3 t$, then

On differentiating w.r.t. t, we get

$$\frac{dy}{dt}=3a\sin^2 t\cos t$$

and $$\frac{dx}{dt}=3a\cos^2 t(-\sin t)$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)} = -\tan t$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -\sec^2 t\frac{dt}{dx} = \frac{-\sec^2 t}{3a\cos^2 t(-\sin t)}$$

$$= \frac{1}{3a}\left(\frac{\sec^4 t}{\sin t}\right)$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a}.\frac{4}{\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{3a}$$

115. (d) Given, $f(x) = \sin x$, $g(x) = x^2$
and $h(x) = \log_e x$
Also, $F(x) = (hogof)(x) = (hog)(\sin x) = h(\sin x)^2$
$\Rightarrow F(x) = 2\log \sin x$
On differentiating, we get
$F'(x) = 2\cot x$
Again differentiating, we get
$F''(x) = -2\text{cosec}^2 x$

116. (d) Given, $u = x^2 + y^2$, $x = s + 3t$, $y = 2s - t$

Now, $\frac{dx}{ds} = 1$, $\frac{dy}{ds} = 2$

$\frac{d^2x}{ds^2} = 0$, $\frac{d^2y}{ds^2} = 0$

Now, $u = x^2 + y^2$

$$\frac{du}{ds} = 2x\frac{dx}{ds} + 2y\frac{dy}{ds}$$

$$= \frac{d^2u}{ds^2} = 2\left(\frac{dx}{ds}\right)^2 + 2x\frac{d^2x}{ds^2} + 2\left(\frac{dy}{ds}\right)^2 + 2y\left(\frac{d^2y}{ds^2}\right)$$

$$\Rightarrow \frac{d^2u}{ds^2} = 2(1)^2 + 2x(0) + 2(2)^2 + 2y(0) = 2 + 8 = 10$$

117. (b) Here, $y^2 = 1 - x^2 \Rightarrow \frac{d}{dx}(y^2) = 0 - 2x$

$\Rightarrow 2yy' = -2x,$
Again differentiating w.r.t. x, we get

$$2\frac{d}{dx}(yy') = -2 \Rightarrow 2(yy'' + y'y') = -2$$

$$\Rightarrow yy'' + (y')^2 + 1 = 0$$

118. (d) Since Rolle's theorem is satisfied
$\therefore f'(c) = 0$

$$\Rightarrow e^c \sin c + \cos e^c = 0$$

$$\Rightarrow e^c\{\sin c + \cos c\} = 0$$

$\therefore \sin c + \cos c = 0 \qquad (\because e^c \neq 0)$

$\Rightarrow \tan c = -1$

$$\Rightarrow c = \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

119. (b) Applying Lagrange's mean value theorem to f(x) in [0, x], $0 < x \le 2$, we get

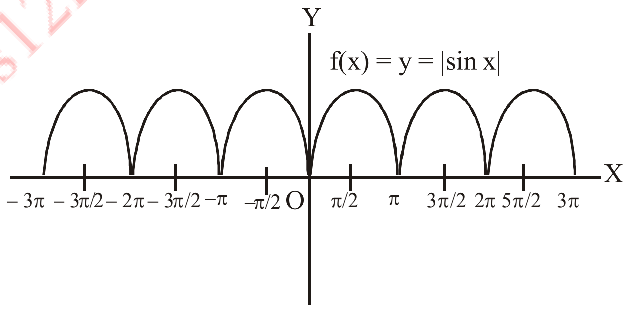
$$\frac{f(x) - f(0)}{x - 0} = f'(c) \text{ for some } c \in (0, 2)$$

$$\Rightarrow \frac{f(x)}{x} = f'(c) \le \frac{1}{2}$$

$$\Rightarrow f(x) \le \frac{1}{2}x \le \frac{1}{2}.2 \qquad (\because x \le 2)$$

$\Rightarrow f(x) \le 1$

120. (b) From the graph of $f(x) = |\sin x|$, it is clear that f(x) is continuous everywhere but not differentiable at $x = n\pi$, $n \in Z$.



121. (a) $f(x) = \cot x$ is discontinuous if $\cot x \to \infty$
$\Rightarrow \cot x = \cot 0$
$\Rightarrow x = n\pi$ and $\forall n \in Z$

122. (d) Let $u = y^2$ and $v = x^2$

$$\therefore \frac{du}{dx} = \frac{d}{dx}y^2$$

$$= 2y.(1-2x) = 2(x - x^2)(1-2x)$$

$$= 2x(1-x)(1-2x) \qquad ....(i)$$

$$\text{and } \frac{dv}{dx} = 2x \qquad .....(ii)$$

$$\text{Hence, } \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = \frac{2x(1-x)(1-2x)}{2x}$$

(from (i) and (ii))

$$= (1-x)(1-2x) = 1 - 3x + 2x^2$$

123. (b) To check the continuity at x = 0

$$LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} f(0-h) = \lim_{h\to 0}|-h|\cos\left(\frac{1}{-h}\right)$$

$$= \lim_{h\to 0} h\cos\left(\frac{1}{h}\right) \qquad (\because -1\le \cos x\le 1\,\forall x\in R)$$

= 0 (as oscillating value between – 1 and 1) = 0

$$RHL = \lim_{x\to 0^+} f(x) = \lim_{h\to 0}(0+h) = \lim_{h\to 0}(h)\lim_{h\to 0} h\cos\frac{1}{h}$$

= 0 (an oscillating number between – 1 and 1) = 0 and f(0) = 0

Thus, LHL = RHL = f(0) = 0. Hence, function is continuous at x = 0.

124. (c) Since, f(x) is continuous for every value of R except {– 1, – 2}. Now, we have to check that points

At x = – 2

$$LHL = \lim_{h\to 0}\frac{(-2-h)+2}{(-2-h)^2+3(-2-h)+2}$$

$$= \lim_{h\to 0}\frac{-h}{h^2+h} = -1$$

$$RHL = \lim_{h\to 0}\frac{(-2+h)+2}{(-2+h)^2+3(-2+h)+2}$$

$$= \lim_{h\to 0}\frac{h}{h^2-h} = -1$$

⇒ LHL = RHL = f(– 2)

∴ It is continuous at x = – 2

Now, check for x = – 1

$$LHL = \lim_{h\to 0}\frac{(-1-h)+2}{(-1-h)^2+3(-1-h)+2}$$

$$= \lim_{h\to 0}\frac{1-h}{h^2-h} = -\infty$$

$$RHL = \lim_{h\to 0}\frac{(-1+h)+2}{(-1+h)^2+3(-1+h)+2}$$

$$= \lim_{h\to 0}\frac{1+h}{h^2+h} = \infty$$

f(– 1) = 0

⇒ LHL ≠ RHL ≠ f(– 1)

∴ It is not continuous at x = – 1

The required function is continuous in R – {– 1}

125. (b) Here, $f(x) = \begin{cases} x[x], & \text{if } 0\le x<2 \\ (x-1)x, & \text{if } 2\le x<3\end{cases}$ at x = 2

$$LHD = Lf'(2) = \lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}$$

$$= \lim_{h\to 0}\frac{(2-h)[2-h]-(2-1)2}{-h}$$

$$= \lim_{h\to 0}\frac{(2-h)(1)-2}{-h} \qquad [\because [2-h]=1]$$

$$= \lim_{h\to 0}\frac{2-h-2}{-h} = \lim_{h\to 0}\frac{-h}{-h} = 1$$

$$RHD = Rf'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0}\frac{(2+h-1)(2+h)-(2-1)(2)}{h}$$

$$= \lim_{h\to 0}\frac{(h+1)(2+h)-2}{h}$$

$$= \lim_{h\to 0}\frac{h^2+3h+2-2}{h}$$

$$= \lim_{h\to 0}\frac{h^2+3h}{h} = \lim_{h\to 0}\frac{h(h+3)}{h} = 3$$

∵ LHD ≠ RHD

∴ f(x) is not differentiable at x = 2.

126. (a) Given, $f(x) = ae^{|x|} + b|x|^2$

We know, $e^{|x|}$ is not differentiable at x = 0 and $|x|^2$ is differentiable at x = 0

∴ f(x) is differentiable at x = 0, if a = 0 and b ∈ R.

127. (c) Let $y = \left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$

$$= \log e^x + \log\left(\frac{x-2}{x+2}\right)^{3/4}$$

$$\Rightarrow y = x + \frac{3}{4}[\log(x-2)-\log(x+2)]$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[x+\frac{3}{4}\{\log(x-2)-\log(x+2)\}\right]$$

$$= 1+\frac{3}{4}\left[\frac{1}{x-2}-\frac{1}{x+2}\right] = 1+\frac{3}{x^2-4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2-1}{x^2-4}$$

128. (d) Here, $y^x = e^{y-x}$

Taking log on both sides, we get

$$\log y^x = \log e^{y-x}$$

$$(\because \log a^b = b\log a \text{ and } \log e = 1)$$

⇒ x logy = (y – x) log e ⇒ x log y = y – x ...(i)

On differentiating w.r.t. x, we get

$$\frac{d}{dx}(x\log y) = \frac{d}{dx}(y-x) \qquad \text{(using product rule)}$$

$$\Rightarrow x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y(1) = \frac{dy}{dx} - 1$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x}{y}-1\right)=-1-\log y$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{y}{(1+\log y)y}-1\right]=-(1+\log y)$$

$$\left[\because \text{ from eq.}(i), x=\frac{y}{(1+\log y)}\right]$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1-1-\log y}{1+\log y}\right]=-(1+\log y)$$

$$\Rightarrow \frac{dy}{dx}=-\frac{(1+\log y)^2}{-\log y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(1+\log y)^2}{\log y}$$

129. (a) Given, $y = |\sin x|^{|x|}$

In the neighbourhood of $-\frac{\pi}{6}$, |x| and |sin x| both are negative

i.e., $y=(-\sin x)^{(-x)}$

Taking log on both sides, we get

log y = (– x). log (– sin x)

On differentiating w.r.t x, we get

$$\frac{1}{y}\frac{dy}{dx}=(-x)\left(\frac{1}{-\sin x}\right).(-\cos x)+\log(-\sin x).(-1)$$

= – x cot x – log (– sin x)

= – [x cot x + log (– sin x)]

$$\Rightarrow \frac{dy}{dx}=-y[x\cot x+\log(-\sin x)]$$

$$=\frac{(2)^{\frac{-\pi}{6}}\left[6\log 2-\sqrt{3}\pi\right]}{6}$$

130. (c) Given, y = 3 cos (log x) + 4 sin (log x) ...(i)

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=y_1=-3\sin(\log x)\frac{d}{dx}(\log x)+4\cos(\log x)\frac{d}{dx}\log x$$

$$=-3\sin(\log x)\frac{1}{x}+4\cos(\log x)\frac{1}{x}$$

Multiplying by x, we get

xy_1 = – 3 sin(log x) + 4cos(log x) ...(ii)

Again differentiating w.r.t. x, we obtain

$$xy_2+y_1.1=-3\cos(\log x)\frac{d}{dx}(\log x)-4\sin(\log x)\frac{d}{dx}\log x$$

$$=-3\cos(\log x)\frac{1}{x}-4\sin(\log x)\frac{1}{x}$$

Multiplying throughout by x, we have

$x^2y_2 + xy_1$ = – (3cos(log x) + 4 sin(logx)) [from eq. (i)]

$\Rightarrow x^2y_2 + xy_1 = -y \Rightarrow x^2y_2 + xy_1 + y = 0$

131. (b) $3f(x)-2f\left(\frac{1}{x}\right)=x$ ...(i)

Put $x=\frac{1}{x}$, then $3f\left(\frac{1}{x}\right)-2f(x)=\frac{1}{x}$ ...(ii)

Solving (i) and (ii) , we get

$$5f(x)=3x+\frac{2}{x}\Rightarrow f'(x)=\frac{3}{5}-\frac{2}{5x^2}$$

$$\therefore f'(2)=\frac{3}{5}-\frac{2}{20}=\frac{1}{2}$$

132. (c) $2f(\sin x)+f(\cos x)=x$ ...(i)

Replace x by $\frac{\pi}{2}-x$

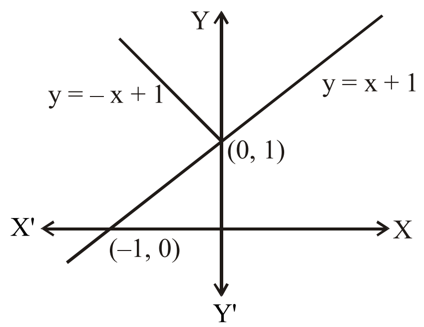
$2f(\cos x)+f(\sin x)=\frac{\pi}{2}-x$ ...(ii)

Solving we get, $3f(\sin x)=\frac{\pi}{2}+3x$

$$\therefore f(x)=\frac{\pi}{6}+\sin^{-1}x \qquad \therefore \frac{d}{dx}f(x)=\frac{1}{\sqrt{1-x^2}}$$

133. (a) $f(x) = \min\{x+1, |x|+1\} \Rightarrow f(x)$

$= x + 1 \;\forall\; x \in R$



Hence, $f(x)$ is differentiable everywhere for all $x \in R$.

134. (d) $f(x)$ = max. $\{x, x^3\}$

$$=\begin{cases} x\;; & x<-1 \\ x^3; & -1\le x\le 0 \\ x\;; & 0\le x\le 1 \\ x^3; & x\ge 1\end{cases}$$

$$\therefore f'(x)=\begin{cases} 1\;; & x<-1 \\ 3x^2; & -1\le x\le 0 \\ 1\;; & 0\le x\le 1 \\ 3x^2; & x\ge 1\end{cases}$$

Clearly f is not differentiable at – 1, 0 and 1.

135. (b) $\lim_{x\to 0}\frac{3f(x)-4f(3x)+f(9x)}{x^2}$ $\left(\frac{0}{0}\text{form}\right)$

$= \lim_{x\to 0}\frac{3f'(x)-12f'(3x)+9f'(9x)}{2x}$ $\left(\frac{0}{0}\text{form}\right)$

$= \lim_{x\to 0}\frac{3f''(x)-36f''(3x)+81f''(9x)}{2}$

$= \frac{3f''(0)-36f''(0)+81f''(0)}{2}$

$= 24\, f''(0) = 24 .5 = 120$

136. (c) For function to be continuous :

$f(0 + h) = f(0 - h) = f(0)$

$f(0 + h) = \lim_{h\to 0} h \sin 1/h = 0 \times$ (a finite quantity) $= 0$

$f(0 - h) = \lim_{h\to 0} -h \sin(1/-h) = 0 \times$ (a finite quantity) $= 0$

Also, $\lim_{x\to 0} x \sin 1/x = 0 \times$ (a finite quantity) $= 0$

$\Rightarrow$ function is continuous at $x = 0$

For function to be differentiable :

$f'(0 + h) = f'(0 - h)$

$f'(0+h) = \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{h\sin\frac{1}{h}-0}{h} = \lim_{h\to 0}\sin\left(\frac{1}{h}\right)$

which does not exist.

$f'(0-h) = \lim_{h\to 0}\frac{(-h)\sin\left(-\frac{1}{h}\right)-0}{-h} = \lim_{h\to 0}\sin\left(-\frac{1}{h}\right)$

which does not exist. So function is not differentiable at $x = 0$

137. (b) $\because f(x) = \begin{cases} x^2 , & x \ge 0 \\ -x^2 , & x < 0 \end{cases}$

$\Rightarrow f'(x) = 2x$, when $x > 0$ and $f'(x) = -2x$, when $x < 0$

Also $f'(0 + 0) = 0$, $f'(0 - 0) = 0 \Rightarrow f'(0) = 0$

$\therefore f'(x) = \begin{cases} 2x , & x > 0 \\ 0 , & x = 0 \\ -2x , & x < 0 \end{cases}$

$\Rightarrow f''(x) = \begin{cases} 2 , & x > 0 \\ -2 , & x < 0 \end{cases}$

Also $f''(0 + 0) = 2$, $f''(0 - 0) = -2 \Rightarrow f''(0)$ does not exist.

Hence $f(x)$ is twice differentiable in R_0.

138. (c) Given $f(x) = (x + 1)^{\cot x}$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

Now, $\lim_{x\to 0}(1+x)^{\cot x} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{x\cot x}$

$= \lim_{x\to 0} e^{x\cot x}$ $\left(\because \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right)$

$= e^{\lim_{x\to 0}\frac{x}{\tan x}} = e$ $\left[\because \lim_{x\to 0}\frac{x}{\tan x} = 1\right]$

$\therefore f(0) = e$

CHAPTER

22

APPLICATION OF DERIVATIVES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$ (c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) $(2, 4)$

2. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$. If its graph passes through the point (2,1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

(a) $(x+1)^2$ (b) $(x-1)^3$ (c) $(x+1)^3$ (d) $(x-1)^2$

3. A stone is dropped into a quiet lake and waves moves in circles at the speed of 5 cm/s. If at a instant, the radius of the circular wave is 8 cm, then the rate at which enclosed area is increasing, is

(a) 20π cm²/s (b) 40π cm²/s

(c) 60π cm²/s (d) 80π cm²/s

4. A particle moves along the curve $6y = x^3 + 2$. The point 'P' on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate, are (4, 11) and $\left(-4, -\frac{31}{3}\right)$.

(a) x-coordinates at the point P are ± 4

(b) y-coordinates at the point P are 11 and $\frac{-31}{3}$

(c) Both (a) and (b)

(d) None of the above

5. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then the rate at which the slope of curve is changing when x = 3, is

(a) – 78 units/s (b) – 72 units/s

(c) – 36 units/s (d) – 18 units/s

6. The radius of a cylinder is increasing at the rate of 3 m/s and its altitude is decreasing at the rate of 4 m/s. The rate of change of volume when radius is 4 m and altitude is 6m, is

(a) 20π m³/s (b) 40π m³/s

(c) 60π m³/s (d) None of these

7. If I be an open interval contained in the domain of a real valued function f and if $x_1 < x_2$ in I, then which of the following statements is true?

(a) f is said to be increasing on I, if $f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in I$

(b) f is said to be strictly increasing on I, if $f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$

(c) Both (a) and (b) are true

(d) Both (a) and (b) are false

8. If $f(x) = \cos x$, then

(a) f(x) is strictly decreasing in $(0, \pi)$

(b) f(x) is strictly increasing in $(0, 2\pi)$

(c) f(x) is neither increasing nor decreasing in $(\pi, 2\pi)$

(d) All the above are correct

9. The function $f(x) = \tan x - 4x$ is strictly decreasing on

(a) $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ (b) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

(c) $\left(-\frac{\pi}{3}, \frac{\pi}{2}\right)$ (d) $\left(\frac{\pi}{2}, \pi\right)$

10. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $[-1, \infty)$ (b) $[-2, -1]$

(c) $(-\infty, -2]$ (d) $[-1, 1]$

11. If a tangent line to the curve $y = f(x)$ makes an angle θ with X-axis in the positive direction, then

(a) $\frac{dy}{dx}$ = slope of the tangent

(b) $\frac{dy}{dx} = \tan\theta$

(c) Both (a) and (b) are true

(d) Both (a) and (b) are false

12. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$?

(a) sin 2x (b) tan x

(c) cos x (d) cos 3x

13. The equation of all lines having slope 2 which are tangent to the curve $y = \frac{1}{x-3}, x \neq 3$, is

(a) y = 2 (b) y = 2x

(c) y = 2x + 3 (d) None of these

14. The slope of the normal to the curve

(a) $x = a\cos^3\theta,\ y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is 0

(b) $x = 1 - a\sin\theta,\ y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$ is $\frac{a}{2b}$

(c) Both (a) and (b) are true

(d) Both (a) and (b) are not true

15. The curve given by $x + y = e^{xy}$ has a tangent parallel to the Y-axis at the point

(a) (0, 1) (b) (1, 0) (c) (1, 1) (d) None of these

16. If $f(x) = x^3 - 7x^2 + 15$, then the approximate value of f(5.001) is

(a) 34.995 (b) – 30.995 (c) 24.875 (d) None of these

17. If the error committed in measuring the radius of sphere, then ... will be the percentage error in the surface area.

(a) 1% (b) 2% (c) 3% (d) 4%

18. If f be a function defined on an interval I and there exists a point c in I such that $f(c) > f(x)$, for all $x \in I$, then

(a) function 'f' is said to have a maximum value in I

(b) the number f(c) is called the maximum value of f in I

(c) the point c is called a point of maximum value of f in I

(d) All the above are true

19. A monotonic function f in an interval I means that f is

(a) increasing in I

(b) dereasing in I

(c) either increasing in I or decreasing in I

(d) neither increasing in I nor decreasing in I

20. If the function f be given by $f(x) = |x|,\ x \in R$, then

(a) point of minimum value of f is x = 1

(b) f has no point of maximum value in R

(c) Both (a) and (b) are true

(d) Both (a) and (b) are not true

21. Test to examine local maxima and local minima of a given function is/are

(a) first derivative test (b) second derivative test

(c) Both (a) and (b) (d) None of these

22. If at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2], then the value of a is

(a) 110 (b) 10 (c) 55 (d) None of these

23. The maximum value of $\frac{\ln x}{x}$ in $(2, \infty)$ is

(a) 1 (b) e (c) 2/e (d) 1/e

24. The difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{4}$

25. If for a function f(x), $f'(a) = 0$, $f''(a) = 0$, $f'''(a) > 0$, then at x = a, f(x) is

(a) Minimum (b) Maximum

(c) Not an extreme point (d) Extreme point

26. The normal to a given curve is parallel to x-axis if

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$

(c) $\frac{dx}{dy} = 0$ (d) $\frac{dx}{dy} = 1$

27. If y = (4x – 5) is a tangent to the curve $y^2 = px^3 + q$ at (2, 3), then

(a) p = – 2, q = – 7 (b) p = – 2, q = 7

(c) p = 2, q = – 7 (d) p = 2, q = 7

28. The radius of a sphere initially at zero increases at the rate of 5 cm/sec. Then its volume after 1 sec is increasing at the rate of :

(a) 50π (b) 5π (c) 500π (d) None of these

29. The interval in which the function $f(x) = \frac{4x^2+1}{x}$ is decreasing is :

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(c) (–1, 1) (d) [–1, 1]

30. The function $f(x) = x^2 - 2x$ is strictly increasing in the interval:

(a) (–2, –1) (b) (–1, 0) (c) (0, 1) (d) (1, 2)

31. The slope of the tangent to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at x = 1 is:

(a) $\frac{1}{2}$ (b) 0 (c) –2 (d) ∞

32. The volume V and depth x of water in a vessel are connected by the relation $V = 5x - \frac{x^2}{6}$ and the volume of water is increasing, at the rate of 5 cm^3/sec, when x = 2 cm. The rate at which the depth of water is increasing, is

(a) $\frac{5}{18}$ cm / sec (b) $\frac{1}{4}$ cm / sec

(c) $\frac{5}{16}$ cm / sec (d) None of these

33. The straight line $\frac{x}{a} + \frac{y}{b} = 2$ touches the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at the point (a, b) for

(a) n = 1, 2 (b) n = 3, 4, –5

(c) n = 1, 2, 3 (d) any value of n

34. A ladder is resting with the wall at an angle of 30°. A man is ascending the ladder at the rate of 3 ft/sec. His rate of approaching the wall is

(a) 3 ft / sec (b) $\frac{3}{2}$ ft / sec

(c) $\frac{3}{4}$ ft / sec (d) $\frac{3}{\sqrt{2}}$ ft / sec

35. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

36. The angle of intersection to the curve $y = x^2$, $6y = 7 - x^3$ at (1, 1) is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) π

37. The maximum area of rectangle inscribed in a circle of diameter R is

(a) R^2 (b) $\frac{R^2}{2}$ (c) $\frac{R^2}{4}$ (d) $\frac{R^2}{8}$

38. If sum of two numbers is 3, the maximum value of the product of first and the square of second is
(a) 4 (b) 3
(c) 2 (d) 1

39. A right circular cylinder which is open at the top and has a given surface area, will have the greatest volume if its height h and radius r are related by
(a) $2h = r$ (b) $h = 4r$
(c) $h = 2r$ (d) $h = r$

40. If tangent to the curve $x = at^2$, $y = 2at$ is perpendicular to x-axis, then its point of contact is
(a) (a, a) (b) (0, a)
(c) (0, 0) (d) (a, 0)

41. What is the slope of the normal at the point $(at^2, 2at)$ of the parabola $y^2 = 4ax$?
(a) $\frac{1}{t}$ (b) t
(c) $-t$ (d) $-\frac{1}{t}$

42. What is the interval in which the function $f(x) = \sqrt{9 - x^2}$ is increasing? $(f(x) > 0)$
(a) $0 < x < 3$ (b) $-3 < x < 0$
(c) $0 < x < 9$ (d) $-3 < x < 3$

43. A wire 34 cm long is to be bent in the form of a quadrilateral of which each angle is 90°. What is the maximum area which can be enclosed inside the quadrilateral?
(a) $68\ cm^2$ (b) $70\ cm^2$
(c) $71.25\ cm^2$ (d) $72.25\ cm^2$

44. What is the x-coordinate of the point on the curve $f(x) = \sqrt{x}\,(7x - 6)$, where the tangent is parallel to x-axis?
(a) $-\frac{1}{3}$ (b) $\frac{2}{7}$
(c) $\frac{6}{7}$ (d) $\frac{1}{2}$

45. If $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$, then f (x) is
(a) increasing in $(-\infty, -2)$ and in (0, 1)
(b) increasing in $(-2, 0)$ and in $(1, \infty)$
(c) decreasing in $(-2, 0)$ and in (0,1)
(d) decreasing in $(-\infty, -2)$ and in $(1, \infty)$

46. $f(x) = \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)$ is
(a) an increasing function (b) a decreasing function
(c) an even function (d) None of these

47. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in
(a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$
(c) $\left(0, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

48. The function $f(x) = \cot^{-1}x + x$ increases in the interval
(a) $(1, \infty)$ (b) $(-1, \infty)$
(c) $(0, \infty)$ (d) $(-\infty, \infty)$

49. The distance between the point (1, 1) and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$ is
(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{-1}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{-2}{\sqrt{5}}$

50. At what point, the slope of the tangent to the curve $x^2 + y^2 - 2x - 3 = 0$ is zero?
(a) (3, 0), (–1, 0) (b) (3, 0), (1, 2)
(c) (–1, 0), (1, 2) (d) (1, 2), (1, –2)

51. The approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%, is
(a) $1.06x^3m^3$ (b) $1.26x^3m^3$
(c) $2.50x^3m^3$ (d) $0.06x^3m^3$

52. $f(x) = \sin(\sin x)$ for all $x \in R$
(a) $-\sin 1$ (b) $\sin 6$
(c) $\sin 1$ (d) $-\sin 3$

53. Let AP and BQ be two vertical poles at points A and B respectively. If AP = 16 m, BQ = 22 m and AB = 20 m, then the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum, is
(a) 5m (b) 6m
(c) 10m (d) 14m

54. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has
(a) two points of local maximum
(b) two points of local minimum
(c) one maxima and one minima
(d) no maxima or minima

55. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$?
(a) sin 2x (b) tan x
(c) cos x (d) cos 3x

56. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

57. The curve $y = x^{\frac{1}{5}}$ at (0, 0) has
(a) a vertical tangent (parallel to y-axis)
(b) a horizontal tangent (parallel to x-axis)
(c) no oblique tangent
(d) no tangent

58. The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, – 1) is
(a) $\frac{22}{7}$ (b) $\frac{6}{7}$
(c) $\frac{-6}{7}$ (d) -6

59. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0, 9] is
(a) 126 (b) 0
(c) 135 (d) 160

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

60. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y is increasing at the rate of 4 cm/min. If x = 8 cm and y = 6 cm, then which of the following is correct?
I. The rate of change of the perimeter is – 2 cm/min.
II. The rate of change of the area of the rectangle is 12 cm^2/min.
(a) Only I is correct
(b) Only II is correct
(c) Both I and II are correct
(d) Both I and II are incorrect

61. Statement I: A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Then, the radius is decreasing at a constant rate.
Statement II: If the area of a circle increases at a uniform rate, then its perimeter varies inversely as the radius.
(a) Only statement I is true
(b) Only statement II is true
(c) Both the statements are true
(d) Both the statements are false

62. Two men A and B start with velocities v at the same time from the junction of two roads inclined at 45° to each other.
Statement I: If they travel by different roads, then the rate at which they are being separated, is $\left(\sqrt{2-\sqrt{2}}\right)v$ unit / s.
Statement II: If they travel by different roads, then the rate at which they are being separated, is $2v \sin \pi/8$ unit/s.
(a) Only statement I is true
(b) Only statement II is true
(c) Both the statements are true
(d) Both the statements are false

63. The function $f(x) = \sin x$ is
I. strictly increasing in $\left[0, \frac{\pi}{2}\right]$.
II. strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$
III. neither increasing nor decreasing in $[0, \pi]$
(a) I and II are true (b) II and III are true
(c) Only II is true (d) Only III is true

64. Statement I: The logarithm function is strictly increasing on $(0, \infty)$.
Statement II: The function f given by $f(x) = x^2 - x + 1$ is neither increasing nor decreasing strictly on $(-1, 1)$
(a) Only statement I is true
(b) Only statement II is true
(c) Both the statements are true
(d) Both the statements are false

65. Statement I: If slope of the tangent line is zero, then tangent line is perpendicular to the X-axis.
Statement II: If $\theta \to \frac{\pi}{2}$, then tangent line is parallel to the Y-axis.
(a) Only statement I is true
(b) Only statement II is true
(c) Both the statements are true
(d) Both the statements are false

66. If f be a function defined on an open interval I. Suppose $c \in I$ be any point. If f has a local maxima or a local minima at x = c, then
Statement I: $f'(c) = 0$
Statement II: f is not differentiable at c.
(a) Only statement I is true
(b) Only statement II is true
(c) Both the statements I and II are true
(d) Both the statements I or II are false

67. A point c in the domain of a function f is called a critical point of f if
I. $f'(c) = 0$
II. f is not differentiable at c.
Choose the correct option
(a) Either I or II are true (b) Only I is true
(c) Only II is true (d) Neither I nor II is true

68. If the function f be given by
$f(x) = x^3 - 3x + 3$, then
I. $x = \pm 2$ are the only critical points for local maxima or local minima.
II. x = 1 is a point of local minima.
III. local minimum value is 2.
IV. local maximum value is 5.
(a) Only I and II are true (b) Only II and III are true
(c) Only I, II and III are true (d) Only II and IV are true

69. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a.
Statement I: The area of triangle is maximum when $\theta = \frac{\pi}{6}$
Statement II: The area of triangle is minimum when $\theta = \frac{\pi}{6}$
(a) Only statement I is true
(b) Only statement II is true
(c) Both the statements are true
(d) Both the statements are false

70. A window is in the form of rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m.
Statement I: One of the dimension of the window to admit maximum light through the whole opening is $\frac{20}{\pi+4}$ m.
Statement II: One of the dimension of the window to admit maximum light through the whole opening is $\frac{10}{\pi+4}$ m.
(a) Only statement I is true
(b) Only statement II is true
(c) Both the statements are true
(d) None of the above

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

71.

	Column-I		Column-II
A.	$f(x) = x^2 - 2x + 5$ is	1.	strictly decreasing in $(-\infty, -1)$ and strictly increasing in $(-1, -\infty)$.
B.	$f(x) = 10 - 6x - 2x^2$ is	2.	strictly increasing in $(-\infty, -9/2)$ and strictly decreasing in $\left(-\frac{9}{2}, \infty\right)$.
C.	$f(x) = -2x^3 - 9x^2 - 12x + 1$ is	3.	strictly decreasing in $(-\infty, -2)$ and $(-1, \infty)$ and strictly increasing in $(-2, -1)$
D.	$f(x) = 6 - 9x - x^2$ is	4.	strictly increasing in $(-\infty, -9/2)$ and strictly decreasing in $\left(-\frac{3}{2}, \infty\right)$
E.	$(x + 1)^3 (x - 3)^3$ is	5.	strictly increasing in $(1, 3)$ and $(3, \infty)$ and strictly decreasing in $(-\infty, -1)$ and $(-1, 1)$

Codes

	A	B	C	D	E
(a)	1	2	3	4	5
(b)	2	3	4	1	5
(c)	1	4	3	2	5
(d)	5	4	3	2	1

72.

	Column-I (Function)		Column-II (Maximum and minimum values respectively)
A.	$f(x) = \|x + 2\| - 1$ has	1.	no maximum value and -1 as minimum value
B.	$g(x) = -\|x + 1\| + 3$ has	2.	3 as maximum value, but it has no minimum value
C.	$h(x) = \sin(2x) + 5$ has	3.	6 as maximum value, and 4 as minimum value
D.	$F(x) = \|\sin 4x + 3\|$ has	4.	4 as maximum value, and 2 as minimum value
E.	$h(x) = x + 1, x \in (-1, 1)$ has	5.	neither a maximum value nor a minimum value

Codes

	A	B	C	D	E
(a)	1	2	3	4	5
(b)	2	3	4	1	5
(c)	1	3	2	4	5
(d)	3	1	2	5	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

73. The local minimum value of the function f given by $f(x) = 3 + |x|, x \in R$ is

(a) 1 (b) 2 (c) 3 (d) 0

74. The velocity of an object at any time t is given by $v = 2t^2 + t + 1$. At $t = 2$ the velocity is changing at the rate of ______ m/s^2.

(a) 0 (b) 2
(c) 8 (d) 9

75. A football is inflated by pumping air in it. When it acquires spherical shape its radius increases at the rate of 0.02 cm/s. The rate of increase of its volume when the radius is 10 cm is __________ π cm/s

(a) 0 (b) 2 (c) 8 (d) 9

76. The minimum value of the function $y = x^4 - 2x^2 + 1$ in the interval $\left[\frac{1}{2}, 2\right]$ is

(a) 0 (b) 2 (c) 8 (d) 9

77. The maximum value of the function $y = -x^2$ in the interval $[-1, 1]$ is

(a) 0 (b) 2 (c) 8 (d) 9

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

78. **Assertion :** Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is one-one.

Reason : f(x) neither increasing nor decreasing function.

79. **Assertion** : $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval $(1, 2)$.

Reason : $f'(x) < 0$ for $x \in (1, 2)$.

80. **Assertion:** The curves $x = y^2$ and $xy = k$ cut at right angle, if $8k^2 = 1$.

Reason: Two curves intersect at right angle, if the tangents to the curves at the point of intersection are perpendicular to each other i.e., product of their slope is -1.

81. **Assertion:** If the radius of a sphere is measure as 9 m with an error of 0.03 m, then the approximate error in calculating its surface area is $2.16\ \pi m^2$.

Reason: We have, $\Delta S = \left(\frac{ds}{dr}\right)\Delta r$ where, ΔS = Approximate error in calculating the surface area, Δr = Error in measuring radius r.

82. **Assertion:** If the length of three sides of a trapezium other than base are equal to 10 cm, then the area of trapezium when it is maximum, is $75\sqrt{3}$ cm^2.
Reason: Area of trapezium is maximum at x = 5.

83. **Assertion:** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8.
Reason: If f be a function defined on an interval I and $c \in I$ and let f be twice differentiable at c, then x = c is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$ and f(c) is local minimum value of f.

84. **Assertion :** Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is one-one.
Reason : f(x) neither increasing nor decreasing function.

85. **Assertion :** $f(x) = \cos^2 x + \cos^3\left(x + \frac{\pi}{3}\right) - \cos x \cos^3\left(x + \frac{\pi}{3}\right)$ then $f'(x) = 0$
Reason : Derivative of constant function is zero.

86. **Assertion:** The function $f(x) = \frac{ae^x + be^{-x}}{ce^x + de^{-x}}$ is increasing function of x, then bc > ad.
Reason: f(x) is increasing if $f'(x) > 0$ for all x.

87. **Assertion:** The ordinate of a point describing the circle $x^2 + y^2 = 25$ decreases at the rate of 1.5 cm/s. The rate of change of the abscissa of the point when ordinate equals 4 cm is 2 cm/s.
Reason: xdx + ydy = 0.

88. **Assertion:** If $f'(x) = (x-1)^3 (x-2)^8$, then f(x) has neither maximum nor minimum at x = 2.
Reason: f′(x) changes sign from negative to positive at x = 2.

89. Consider the function

$$f(x) = \begin{cases} |\sin x| & \text{for } 0 < |x| \le \frac{\pi}{2} \\ \frac{1}{2} & \text{for } x = 0 \end{cases}$$

Assertion: f has a local maximum value at x = 0.
Reason: $f'(0) = 0$ and $f''(0) < 0$

90. **Assertion:** The maximum value of the function y = sin x in [0, 2π] is at $x = \frac{\pi}{2}$.
Reason: The first derivative of the function is zero at $x = \frac{\pi}{2}$ and second derivative is negative at $x = \frac{\pi}{2}$.

91. **Assertion:** The minimum value of the function y = cos x in [0, 2π] is at x = π.
Reason: The first derivative of the function is zero at x = π and second derivative is negative at x = π.

92. **Assertion:** The function $y^2 = 4x$ has no absolute maximum or minimum.
Reason: In the graph of the function the value of increases unboundedly and decreases unboundedly as x increases.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

93. The largest distance of the point (a, 0) from the curve $2x^2 + y^2 - 2x = 0$, is given by
(a) $\sqrt{(1 - 2a + a^2)}$ (b) $\sqrt{(1 + 2a + 2a^2)}$
(c) $\sqrt{(1 + 2a - a^2)}$ (d) $\sqrt{(1 - 2a + 2a^2)}$

94. The equation of one of the tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$ that is parallel to the line x + 2y = 0, is
(a) x + 2y = 1 (b) x + 2y = π/2
(c) x + 2y = π/4 (d) None of these

95. If the parabola y = f (x), having axis parallel to the y-axis, touches the line y = x at (1, 1), then
(a) $2f'(0) + f(0) = 1$ (b) $2f(0) + f'(0) = 1$
(c) $2f(0) - f'(0) = 1$ (d) $2f'(0) - f(0) = 1$

96. Angle formed by the positive Y-axis and the tangent to $y = x^2 + 4x - 17$ at $\left(\frac{5}{2}, \frac{-3}{4}\right)$ is
(a) $\tan^{-1} 9$ (b) $\frac{\pi}{2} - \tan^{-1} 9$
(c) $\frac{\pi}{2} + \tan^{-1} 9$ (d) $\frac{\pi}{2}$

97. A kite is moving horizontally at a height of 151.5. If the speed of kite is 10 m/s, then the rate at which the string is being let out; when the kite is 250 m away from the boy who is flying the kite and the height of the boy is 1.5 m, is
(a) 4 m/s (b) 6 m/s (c) 7 m/s (d) 8 m/s

98. Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/s is the surface area through a tiny hole at the vertex of the bottom. When the slant height of cone is 4 cm, then rate of decrease of the slant height of water is
(a) $\frac{\sqrt{2}}{3\pi}$ cm/s (b) $\frac{\sqrt{2}}{\pi}$ cm/s
(c) $\frac{\sqrt{2}}{4\pi}$ cm/s (d) None of these

99. If f(x) = cosx, g(x) = cos 2x, h(x) = cos3x and I(x) = tanx, then which of the following option is correct?
(a) f(x) and g(x) are strictly decreasing in (0, π/2)
(b) h(x) is neither increasing nor decreasing in (0, π/2)
(c) I(x) is strictly increasing in (0, π/2)
(d) All are correct

100. The points at which the tangent passes through the origin for the curve $y = 4x^3 - 2x^5$ are
(a) (0, 0), (2, 1) and (– 1, – 2)
(b) (0, 0), (2, 1) and (– 2, – 1)
(c) (2, 0), (2, 1) and (– 3, 1)
(d) (0, 0), (1, 2) and (– 1, – 2)

101. The angle of intersection of the curve $y^2 = x$ and $x^2 = y$ is

(a) $\tan^{-1}\left(\frac{3}{2}\right)$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$

(c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $\tan^{-1}\left(\frac{1}{5}\right)$

102. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

(a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{2\sqrt{3}}{8}$ (c) $\frac{3\sqrt{2}}{5}$ (d) $\frac{\sqrt{3}}{4}$

103. There is an error of 0.04 cm in the measurement of the diameter of a sphere. When the radius is 10 cm, the percentage error in the volume of the sphere is

(a) ± 1.2 (b) ± 1.0 (c) ± 0.6 (d) ± 0.8

104. The maximum value of the function $\sin x + \cos x$ is

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) None of these

105. The maximum value of $[x(x-1)+1]^{1/3}$, $0 \le x \le 1$ is

(a) $\left(\frac{1}{3}\right)^{1/3}$ (b) $\frac{1}{2}$ (c) 1 (d) zero

106. The minimum value of $e^{(2x^2-2x+1)\sin^2 x}$ is

(a) 0 (b) 1 (c) 2 (d) 3

107. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. The rate, at which its area is increasing when its radius is 3.2 cm, is

(a) $0.320\ \pi cm^2/s$ (b) $0.160\ \pi cm^2/s$

(c) $0.260\ \pi cm^2/s$ (d) $1.2\ \pi cm^2/s$

108. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/s. If the two equal sides are equal to the base then the rate at which its area is decreasing, is

(a) $\frac{b}{3} cm^2/s$ (b) $b^2\ cm^2/s$

(c) $\frac{b}{\sqrt{3}} cm^2/s$ (d) $b\sqrt{3} cm^2/s$

109. If a point on the hypotenuse of a triangle is at distance a and b from the sides of triangle, then the minimum length of the hypotenuse is

(a) $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)$ (b) $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$

(c) $\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)^{\frac{3}{2}}$ (d) None of these

110. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point:

(a) (1, 1) (b) at no point (c) (0, 1) (d) (1, 0)

111. If the radius of a spherical balloon increases by 0.2%. Find the percentage increase in its volume

(a) 0.8% (b) 0.12% (c) 0.6% (d) 0.3%

112. The function $f(x) = x^2 \log x$ in the interval [1, e] has

(a) a point of maximum and minimum

(b) a point of maximum only

(c) no point of maximum and minimum in [1, e]

(d) no point of maximum and minimum

113. Each side of an equilateral triangle expands at the rate of 2 cm/s. What is the rate of increase of area of the triangle when each side is 10 cm?

(a) $10\sqrt{2}\ cm^2/s$ (b) $10\sqrt{3}\ cm^2/s$

(c) $10\ cm^2/s$ (d) $5\sqrt{3}\ cm^2/s$

114. If the curves $x^2 = 9A(9 - y)$ and $x^2 = A(y + 1)$ intersect orthogonally, then the value of A is

(a) 3 (b) 4 (c) 5 (d) 7

115. The equation of the tangent to $4x^2 - 9y^2 = 36$ which is perpendicular to the straight line $5x + 2y - 10 = 0$ is

(a) $5(y-3) = 4\left(x - \frac{\sqrt{11}}{2}\right)$

(b) $2x - 5y + 10 - 12\sqrt{3} = 0$

(c) $2x - 5y + 10 + 12\sqrt{3} = 0$

(d) None of these

116. If the tangent at P(1, 1) on $y^2 = x(2 - x)^2$ meets the curve again at Q, then Q is

(a) (2, 2) (b) (– 1, – 2) (c) $\left(\frac{9}{4}, \frac{3}{8}\right)$ (d) None of these

117. If $y = \frac{ax - b}{(x-1)(x-4)}$ has a turning point P(2, – 1), then the value of a and b respectively, are

(a) 1, 2 (b) 2, 1 (c) 0, 1 (d) 1, 0

118. If the error k% is made in measuring the radius of a sphere, then percentage error in its volume is

(a) k% (b) 3k% (c) 2k% (d) $\frac{k}{3}\%$

119. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximating error in calculating its volume.

(a) $2.46\pi cm^3$ (b) $8.62\pi cm^3$

(c) $9.72\pi cm^3$ (d) $7.6\pi cm^3$

120. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. The cost of the material will be least when depth of the tank is

(a) twice of its width (b) half of the width

(c) equal to its width (d) None of these

121. Find the maximum profit that a company can make, if the profit function is given by $P(x) = 41 + 24x - 18x^2$.

(a) 25 (b) 43 (c) 62 (d) 49

122. The coordinates of the point on the parabola $y^2 = 8x$ which is at minimum distance from the circle $x^2 + (y + 6)^2 = 1$ are

(a) (2, – 4) (b) (18, – 12) (c) (2, 4) (d) None of these

123. Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a.

(a) 2a/3 (b) $\frac{2a}{\sqrt{3}}$ (c) a/3 (d) a/5

124. The maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) e (b) e^e (c) $e^{\frac{1}{e}}$ (d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** $y^2 = 18x \Rightarrow 2y\frac{dy}{dx} = 18 \Rightarrow \frac{dy}{dx} = \frac{9}{y}$

Given $\frac{dy}{dx} = 2 \Rightarrow \frac{9}{y} = 2 \Rightarrow y = \frac{9}{2}$

Putting in $y^2 = 18x \Rightarrow x = \frac{9}{8}$

$\therefore$ Required point is $\left(\frac{9}{8}, \frac{9}{2}\right)$

2. **(b)** $f''(x) = 6(x - 1)$. Integrating, we get

$f'(x) = 3x^2 - 6x + c$

Slope at (2, 1) $= f'(2) = c = 3$

[$\because$ slope of tangent at (2,1) is 3]

$\therefore \quad f'(x) = 3x^2 - 6x + 3 = 3(x - 1)^2$

Integrating again, we get , $f(x) = (x - 1)^3 + D$

The curve passes through (2, 1)

$\Rightarrow \quad 1 = (2 - 1)^3 + D \Rightarrow D = 0 \quad \therefore \quad f(x) = (x - 1)^3$

3. **(d)** Let r be the radius of the circular wave and A be the area, then $A = \pi r^2$

Therefore, the rate of change of area (A) with respect to time (t) is given by

$$\frac{dA}{dt} = \frac{d}{dt}\left(\pi r^2\right)$$

$$\Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt} \qquad \text{(by Chain rule)}$$

It is given that waves move in circles at the speed of 5 cm/s.

So, dr/dt = 5 cm/s

$$\therefore \frac{dA}{dt} = 2\pi r \times 5 = 10\pi r \text{ cm/s}$$

Thus, when r = 8 cm, $\frac{dA}{dt} = 10\pi(8) = 80\pi \text{ cm}^2/\text{s}$

Hence, when the radius of the circular wave is 8 cm, the enclosed area is increasing at the rate of 80 cm^2/s.

4. **(c)** Given, $6y = x^3 + 2$

On differentiating w.r.t. t, we get

$$6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

$$\Rightarrow \quad 6 \times 8\frac{dx}{dt} = 3x^2\frac{dx}{dt}$$

$\Rightarrow 3x^2 = 48$

$\Rightarrow x^2 = 16$

$\Rightarrow \quad x = \pm 4$

When x = 4, then $6y = (4)^3 + 2$

$\Rightarrow \; 6y = 64 + 2 \Rightarrow \; y = \frac{66}{6} = 11$

When x = – 4, then $6y = (-4)^3 + 2$

$\Rightarrow \; 6y = -64 + 2$

$\Rightarrow \; y = \frac{-62}{6} = \frac{-31}{3}$

Hence, the required points on the curve are (4, 11) and $\left(-4, \frac{-31}{3}\right)$

5. **(b)** Slope of curve $= \frac{dy}{dx} = 5 - 6x^2$

$$\Rightarrow \frac{d}{dt}\left(\frac{dy}{dt}\right) = -12x.\frac{dx}{dt}$$

Thus, slope of curve is decreasing at the rate of 72 units/s when x is increasing at the rate of 2 units/s.

6. **(d)** Let h and r be the height and radius of cylinder.

Given that, $\frac{dr}{dt} = 3\text{m/s}$, $\frac{dh}{dt} = -4\text{m/s}$

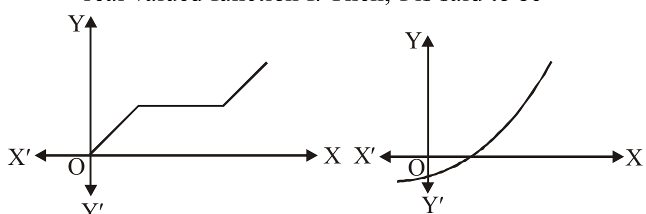
Let volume of cylinder, $V = \pi r^2 h$

$$\Rightarrow \frac{dV}{dt} = \pi\left[r^2\frac{dh}{dt} + h.2r\frac{dr}{dt}\right]$$

At r = 4 m and h = 6 m

$$\therefore \quad \frac{dV}{dt} = \pi[-64 + 144] = 80\ \pi\text{m}^3/\text{s}$$

7. **(c)** If I be an open interval contained in the domain of a real valued function f. Then, f is said to be



Increasing function (i) Strictly increasing function (ii)

(a) Increasing on I if $x_1 < x_2$ in I

$\Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in I$.

(b) Strictly increasing on I if $x_1 < x_2$ in I

$\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$.

8. **(a)** We have, $f(x) = \cos x$

$f'(x) = -\sin x$

(a) Since, for each $x \in (0, \pi)$, $\sin x > 0$ we have $f'(x) < 0$ and so f is strictly decreasing in $(0, \pi)$.

(b) Since, for each $x \in (\pi, 2\pi)$, $\sin x < 0$ we have $f'(x) > 0$ and so f is strictly decreasing in $(0, 2\pi)$.

(c) Clearly, by (a) and (b) above, f is neither increasing nor decreasing in $(0, 2\pi)$.

9. (a) $f(x) = \tan x - 4x \Rightarrow f'(x) = \sec^2 x - 4$

When $\frac{-\pi}{3} < x < \frac{\pi}{3}$, $1 < \sec x < 2$

Therefore, $1 < \sec^2 x < 4$

$\Rightarrow -3 < (\sec^2 x - 4) < 0$

Thus, for $\frac{-\pi}{3} < x < \frac{\pi}{3}$, $f'(x) < 0$

Hence, f is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$

10. (b) $\because f(x) = 2x^3 + 9x^2 + 12x - 1$

$\therefore f'(x) = 6x^2 + 18x + 12$

+ (left of -2), − (between -2 and -1), + (right of -1)

$= 6(x^2 + 3x + 2)$

For decreasing function

$f'(x) \le 0$

$\Rightarrow 6(x+1)(x+2) \le 0$

$\therefore$ f(x) is decreasing in $[-2, -1]$

11. (c) If a tangent line to the curve $y = f(x)$ makes an angle θ with X-axis in the positive direction, then $\frac{dy}{dx}$ = slope of the tangent = tan θ.

12. (c) $\because f(x) = \cos x$

$\Rightarrow f'(x) = -\sin x < 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$

So, $f(x) = \cos x$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

13. (d) The equation of the given curve is $y = \frac{1}{x-3}, x \ne 3$.

The slope of the tangent to the given curve at any point (x, y) is given by

$$\frac{dy}{dx} = \frac{-1}{(x-3)^2}$$

For tangent having slope 2, we must have

$$2 = \frac{-1}{(x-3)^2}$$

$\Rightarrow 2(x-3)^2 = -1$

$\Rightarrow (x-3)^2 = -\frac{1}{2}$

which is not possible as square of a real number cannot be negative.

Hence, there is no tangent to the given curve having slope 2.

14. (d) (a) Given, $x = a\cos^3\theta$ and $y = a\sin^3\theta$.

On differentiating x and y both w.r.t θ, we get

$$\frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta) = -3a\cos^2\theta\sin\theta$$

and $\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

$\therefore$ Slope of normal at the point $\theta = \frac{\pi}{4}$ is

$$-\left(\frac{dx}{dy}\right)_{\theta=\frac{\pi}{4}}$$

$$= -\left(\frac{1}{dy/dx}\right)_{\theta=\frac{\pi}{4}} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(\theta=\pi/4)}}$$

$$= \frac{-1}{-\tan(\pi/4)} = \frac{-1}{-1} = 1$$

(b) It is given that $x = 1 - a\sin\theta$ and $y = b\cos^2\theta$

On differentiating x and y w.r.t. θ, we get

$$\frac{dx}{d\theta} = \frac{d}{d\theta}[1 - a\sin\theta] = -a\cos\theta$$

and $\frac{dy}{d\theta} = \frac{d}{d\theta}\left[b\cos^2\theta\right]$

$= 2b\cos\theta(-\sin\theta) = -2b\cos\theta\sin\theta$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2b\cos\theta\sin\theta}{-a\cos\theta} = \frac{2b}{a}\sin\theta$$

$\therefore$ Slope of normal at the point $\theta = \frac{\pi}{2}$, is

$$= \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}} = \frac{-1}{\frac{2b}{a}\sin\left(\frac{\pi}{2}\right)} = \frac{-a}{2b}$$

So, both (a) and (b) are not true.

15. (b) $\because x + y = e^{xy}$

Differentiating w.r.t. x, we get

$$1 + \frac{dy}{dx} = e^{xy}\left[y + x\frac{dy}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx}\left(1 - xe^{xy}\right) = ye^{xy} - 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{ye^{xy} - 1}{1 - xe^{xy}}$$

$\because \frac{dy}{dx} = \infty$, as tangent is parallel to Y-axis

$\Rightarrow 1 - xe^{xy} = 0$

$\therefore xe^{xy} = 1$

This holds, when $x = 1$ and $y = 0$

16. **(b)** Consider $f(x) = x^3 - 7x^2 + 15$

$\Rightarrow f'(x) = 3x^2 - 14x$

Let $x = 5$ and $\Delta x = 0.001$

Also, $f(x + \Delta x) \approx f(x) + \Delta x\, f'(x)$

Therefore, $f(x + \Delta x) = (x^3 - 7x^2 + 15) + \Delta x\,(3x^2 - 14x)$

$\Rightarrow f(5.001) = (5^3 - 7 \times 5^2 + 15) + (3 \times 5^2 - 14 \times 5)(0.001)$

(as $x = 5$, $\Delta x = 0.001$)

$= 125 - 175 + 15 + (75 - 70)(0.001)$

$= -34.995$

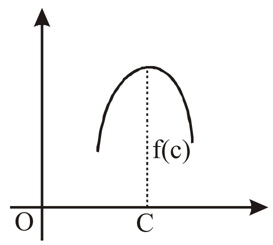
17. **(d)** $\because A = \pi r^2$

$\Rightarrow \log A = \log\pi + 2\log r$

$$\Rightarrow \frac{\Delta A}{A} \times 100 = 2 \times \frac{\Delta r}{r} \times 100$$

$2 \times 0.05 = 0.1\%$

18. **(d)** Let f be a function defined on an interval **I**. Then, f said to have a maximum value in **I**, if there exists a point c in **I** such that

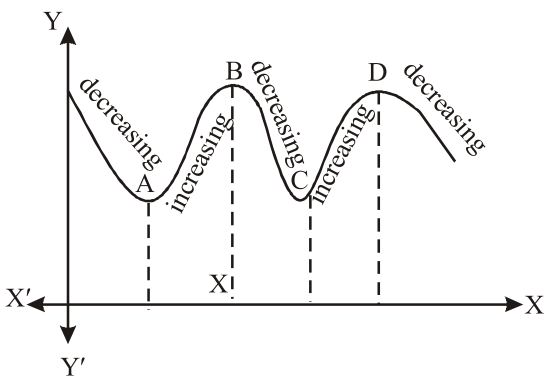


$f(c) > f(x)$, for all $x \in$ **I**.

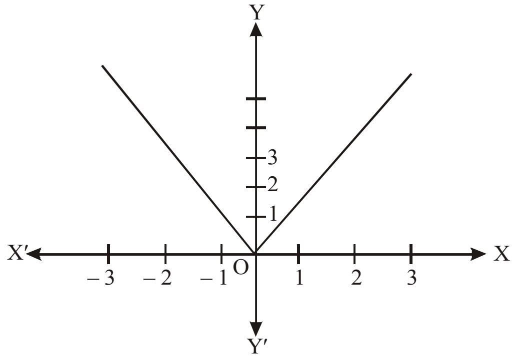
The number f(c) is called the maximum value of f in **I** and the point c is called a point of maximum value of f in **I**.

19. **(c)** By a monotonic function f in an interval **I**, we mean that f is either increasing in I or decreasing in **I**.

By examine the graph of given function as shown below, we see that at point A, B, C and D on the graph, the function changes its nature from decreasing to increasing or vice-versa. These points may be called turning points, the graph has either a little hill or a little valley.



20. **(b)** From the graph of the given function, note that $f(x) \geq 0$ for all $x \in R$ and $f(x) = 0$, if $x = 0$



Therefore, the function f has a minimum value 0 and the point of minimum value of f is $x = 0$. Also, the graph clearly shows that f has no maximum value in R and hence no point of maximum value in R.

21. **(c)** Test the examine local maxima and local minima of a given function are first derivative test and second derivative test. Second derivative test is often easier to apply that the first derivative test.

22. **(d)** Let $f(x) = x^4 - 62x^2 + ax + 9$

$\Rightarrow f'(x) = 4x^3 - 124x + a$

It is given that function f attains its maximum value on the interval [0, 2] at $x = 1$.

$\therefore f'(1) = 0$

$\Rightarrow 4 \times 1^3 - 124 \times 1 + 1 = 0$

$\Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$

Hence, the value of a is 120.

23. **(d)** Let $y = \frac{\ln x}{x}$

$$\frac{dy}{dx} = \frac{x.\frac{1}{x} - \ln x.1}{x^2} = \frac{1 - \log x}{x^2}$$

For maxima, put $\frac{dy}{dx} = 0$

$$\Rightarrow \frac{1 - \ln x}{x^2} = 0 \Rightarrow x = e$$

Now, $$\frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \ln x)2x}{\left(x^2\right)^2}$$

At $x = e$ we have $\frac{d^2y}{dx^2} < 0$

$\therefore$ The maximum value at $x = e$ is $y = \frac{1}{e}$

24. **(b)** $f(x) = \sin 2x - x \Rightarrow f'(x) = 2\cos 2x - 1$

Therefore, $f'(x) = 0 \Rightarrow \cos 2x = \frac{1}{2}$

$$\Rightarrow 2x = \frac{\pi}{3} \text{ or } -\frac{\pi}{3} \Rightarrow x = -\frac{\pi}{6} \text{ or } \frac{\pi}{6}$$

$$\Rightarrow f\left(-\frac{\pi}{2}\right) = \sin(-\pi) + \frac{\pi}{2} = \frac{\pi}{2}$$

$\Rightarrow f\left(-\frac{\pi}{6}\right)=\sin\left(-\frac{2\pi}{6}\right)+\frac{\pi}{6}=-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$

$\Rightarrow f\left(\frac{\pi}{6}\right)=\sin\left(\frac{2\pi}{6}\right)-\frac{\pi}{6}=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$

$\Rightarrow f\left(\frac{\pi}{2}\right)=\sin(\pi)-\frac{\pi}{2}=-\frac{\pi}{2}$

Clearly, $\frac{\pi}{2}$ is the greatest value and $-\frac{\pi}{2}$ is the least.

Therefore, difference $=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

25. **(c)** It is a fundamental property.

26. **(a)** We know that, the normal to a given curve is parallel to x-axis if $\frac{dy}{dx}=0$

27. **(c)** Given: tangent $y = 4x - 5$

$\therefore$ Slope $m = 4$ ... (i)

Curve $y^2 = px^3 + q$ ... (ii)

$\Rightarrow 2y.\frac{dy}{dx}=3px^2 \Rightarrow \frac{dy}{dx}=\frac{3px^2}{2y}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)}=\frac{3p(2)^2}{2(3)}$

$\Rightarrow 4=\frac{12p}{6}$ [using (i)]

$\Rightarrow p = 2$

On putting the value of $p = 2$, $x = 2$ and $y = 3$ in equation (ii), we get, $(3)^2 = 2 \times (2)^3 + q$

$\Rightarrow 16 + q = 9 \Rightarrow q = -7$,

So, $p = 2$ and $q = -7$

28. **(c)** Let 'r' be the radius and V be the volume of the sphere.

Given : Radius increases at the rate of 5cm/sec.

$\therefore \frac{dr}{dt}=5\text{cm/sec}$

Now, $V=\frac{4}{3}\pi r^3$

$\therefore \frac{dV}{dt}=\frac{4}{3}\pi(3r^2)\frac{dr}{dt}=4\pi r^2(5)=20\pi r^2$

Now, after one second, $r = 5$

$\therefore \frac{dV}{dt}$ after 1 sec $=20\pi(5)^2=500\pi$.

29. **(a)** Given $f(x)=\frac{4x^2+1}{x}$ Thus $f'(x)=4-\frac{1}{x^2}$

f(x) will be decreasing if $f'(x) < 0$

Thus $4-\frac{1}{x^2}<0 \Rightarrow \frac{1}{x^2}>4 \Rightarrow \frac{-1}{2}<x<\frac{1}{2}$

Thus interval in which f(x) is decreasing, is $\left(-\frac{1}{2},\frac{1}{2}\right)$.

30. **(d)** $f(x) = x^2 - 2x$

$f'(x) = 2x - 2 = 2(x - 1)$

For f (x) to be strictly increasing,

$f'(x) > 0$

$\Rightarrow 2(x - 1) > 0$

$\Rightarrow x - 1 > 0$

$\Rightarrow x > 1$

31. **(b)** Given curve is $x = 3t^2 + 1$ ...(i)

$\therefore \frac{dx}{dt}=6t$

Second curve is $y = t^3 - 1$ ...(ii)

$\therefore \frac{dy}{dt}=3t^2$

$\therefore \frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=3t^2\times\frac{1}{6t}=\frac{t}{2}$

But from (i) when $x = 1$

we have $1 = 3t^2 + 1 \Rightarrow 3t^2 = 0 \Rightarrow t = 0$

$\therefore$ When $x = 1$ then $t = 0$

$\therefore \frac{dy}{dx}=0$

Hence, slope of the tangent to the curve = 0

32. **(d)** $V=5x-\frac{x^2}{6}\Rightarrow\frac{dV}{dt}=5\frac{dx}{dt}-\frac{x}{3}\cdot\frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt}=\frac{\frac{dV}{dt}}{\left(5-\frac{x}{3}\right)}$

$\Rightarrow \left(\frac{dx}{dt}\right)_{x=2}=\frac{5}{5-\frac{2}{3}}=\frac{15}{13}\text{cm/sec}$.

33. **(d)** The point (a, b) lies on both the straight line and the given curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$.

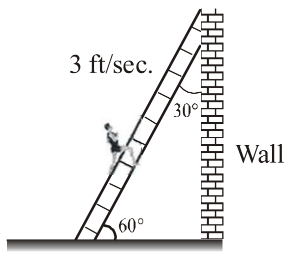
Differentiating the equation, we get

$\frac{dy}{dx}=-\frac{x^{n-1}}{a^n}\cdot\frac{b^n}{y^{n-1}}$

$\therefore \left(\frac{dy}{dx}\right)_{at\,(a,b)}=-\frac{b}{a}=$ the slope of $\frac{x}{a}+\frac{y}{b}=2$

Hence, it touches the curve at (a, b) whatever may be the value of n.

34. **(b)**



His rate of approaching the wall

$=3\times\cos 60°=\frac{3}{2}\text{ft/sec}$.

35. (b) Let $y = x^{25}(1-x)^{75}$

$\Rightarrow \frac{dy}{dx} = 25x^{24}(1-x)^{74}(1-4x)$

For maximum value of y, $\frac{dy}{dx} = 0$

$\Rightarrow x = 0, 1, 1/4$

$\Rightarrow x = 1/4 \in (0,1)$

Also at x = 0, y = 0, at x = 1, y = 0, and at x = 1/4, y > 0

$\therefore$ Max. value of y occurs at x = 1/4

36. (a) Let m_1 and m_2 be slope of curve $y = x^2$ and $6y = 7 - x^3$ respectively.

Now, $y = x^2$

$\Rightarrow \frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2$ i.e. $m_1 = 2$

and $6y = 7 - x^3 \Rightarrow 6\frac{dy}{dx} = -3x^2$

$\Rightarrow \frac{dy}{dx} = -\frac{3}{6}x^2 = -\frac{1}{2}x^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{2}(1)^2 = -\frac{1}{2}$

$\therefore m_2 = -\frac{1}{2}$

$\therefore m_1 m_2 = 2 \cdot -\frac{1}{2} = -1$

$\therefore$ Angle of intersection is 90° i.e. $\frac{\pi}{2}$

37. (b) The diagonal = R

Thus the area of rectangle

$= \frac{1}{2} \times R \times R = \frac{R^2}{2}$

38. (a) Let x, y be two numbers such that

$x + y = 3 \Rightarrow y = 3 - x$

and let product $P = xy^2$

thus $P = x(3-x)^2 = x^3 - 6x^2 + 9x$

For a maxima or minima

$\frac{dP}{dx} = 0$

Thus $\frac{dP}{dx} = 3x^2 - 12x + 9$ and $\frac{d^2P}{dx^2} = 6x - 12$

Now, $\frac{dP}{dx} = 0 \Rightarrow 3x^2 - 12x + 9 = 0 \Rightarrow x = 1, 3.$

Thus $\left(\frac{d^2P}{dx^2}\right)_{x=1} = -6$ and $\left(\frac{d^2P}{dx^2}\right)_{x=3} = 6$

Thus P is maximum when $x = 1 \Rightarrow y = 2$

So, $P = 1.2^2 = 4$.

39. (d) Volume of cylinder, $(V) = \pi r^2 h$;

Surface area, $(S) = 2\pi rh + \pi r^2$ ...(i)

$\Rightarrow h = \frac{S - \pi r^2}{2\pi r}$

$\therefore V = \pi r^2\left[\frac{S - \pi r^2}{2\pi r}\right] = \frac{r}{2}[S - \pi r^2] = \frac{1}{2}[Sr - \pi r^3]$

Now, Differentiate both sides, w.r.t 'r'

$\frac{dV}{dr} = \frac{1}{2}[S - 3\pi r^2]$

Now, circular cylinder will have the greatest volume, when $\frac{dV}{dr} = 0$

$\Rightarrow S = 3\pi r^2$

$\Rightarrow 2\pi rh + \pi r^2 = 3\pi r^2 \Rightarrow 2\pi rh = 2\pi r^2 \Rightarrow r = h.$

40. (c) Given $x = at^2$, $y = 2at$

Note: When tangent to the curve is perpendicular to x-axis, then $\frac{dy}{dx} = \infty$

Now, $\frac{dx}{dt} = 2at$ and $\frac{dy}{dt} = 2a$

So $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{2a}{2at} = \infty$

$\Rightarrow \frac{1}{t} = \infty$ so, $t = \frac{1}{\infty} = 0$

So, the point of contact will be

$x = a.(0)^2 = 0$ and $y = 2a.(0) = 0$

41. (c) Equation of parabola is

$y^2 = 4ax$

$\Rightarrow 2y\frac{dy}{dx} = 4a$ (On differentiating w.r.t 'x')

$\therefore \frac{dy}{dx} = \frac{2a}{y}$, [slope of tangent]

So, slope of normal $= -\left(\frac{dx}{dy}\right)_{(at^2, 2at)}$

$= -\left(\frac{y}{2a}\right) = -\frac{2at}{2a} = -t$

42. (b) $f(x) = \sqrt{9 - x^2}$

$f'(x) = \frac{1}{2\sqrt{9-x^2}} \times (-2x) = -\frac{x}{\sqrt{9-x^2}}$

For function to be increasing

$-\frac{x}{\sqrt{9-x^2}} > 0$ or $-x > 0$ or $x < 0$

but $\sqrt{9-x^2}$ is defined only when

$9 - x^2 > 0$ or $x^2 - 9 < 0$

$(x + 3)(x - 3) < 0$

i.e. $-3 < x < 3$

$-3 < x < 3 \cap x < 0$

$\Rightarrow -3 < x < 0$

43. (d) Let one side of quadrilateral be x and another side be y so, $2(x + y) = 34$

or, $(x + y) = 17$ … (i)

We know from the basic principle that for a given perimeter square has the maximum area, so, x = y and putting this value in equation (i)

$$x = y = \frac{17}{2}$$

$$\text{Area} = x \cdot y = \frac{17}{2}\times\frac{17}{2} = \frac{289}{4} = 72.25$$

44. (b) $f(x) = \sqrt{x}\,(7x - 6) = 7x^{3/2} - 6x^{1/2}$

$$f'(x) = 7\times\frac{3}{2}x^{1/2} - 6\times\frac{1}{2}x^{-1/2}$$

When tangent is parallel to x axis then $f'(x) = 0$

or, $\frac{21}{2}x^{1/2} - 3x^{-1/2} = 0$

or $\frac{21}{2}\sqrt{x} = \frac{3}{\sqrt{x}}$

or, $7x = 2 \Rightarrow x = \frac{2}{7}$

45. (b) Given : $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

Differentiating with respect to x, we get

$f'(x) = 12x^3 + 12x^2 - 24x$

For f (x) to be increasing

$f'(x) > 0$

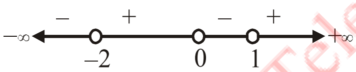
$\Rightarrow 12x^3 + 12x^2 - 24x > 0$

$\Rightarrow 12x(x^2 + x - 2) > 0$

$\Rightarrow 12x(x - 1)(x + 2) > 0$

$\Rightarrow x(x - 1)(x + 2) > 0$

$\Rightarrow -2 < x < 0$ or $x > 1$



It means $x \in (-2, 0) \cup (1, \infty)$.

Hence f (x) is increasing in $(-2, 0)$ and $(1, \infty)$

46. (a) $\because f(x) = \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)$

$$\therefore f(-x) = \frac{e^{-2x} - 1}{e^{-2x} + 1} = \frac{1 - e^{2x}}{1 + e^{2x}}$$

$$\Rightarrow f(-x) = \frac{-(e^{2x} - 1)}{e^{2x} + 1} = -f(x)$$

$\therefore$ f(x) is an odd function.

Again, $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$

$$f(x) = \frac{e^{2x}}{(1 + e^{2x})^2} > 0,\ \forall x \in R$$

$\Rightarrow$ f(x) is an increasing function.

47. (b) Since, $f(x) = \tan^{-1}(\sin x + \cos x)$

$$\therefore f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$$

$$= \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$$

f(x) is increasing if $f'(x) > 0$

$$\Rightarrow \cos\left(x + \frac{\pi}{4}\right) > 0$$

$$\Rightarrow -\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2} \Rightarrow -\frac{3\pi}{2} < x + \frac{\pi}{4}$$

Hence, f(x) is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

48. (d) $f(x) = \cot^{-1}x + x$

$$\Rightarrow f'(x) = \frac{-1}{1 + x^2} + 1 \Rightarrow f'(x) = \frac{x^2}{1 + x^2} \ge 0, \text{ for } x \in R$$

$\therefore$ f(x) is increasing on $(-\infty, \infty)$.

49. (c) Putting x = 0 in $y = e^{2x} + x^2$

we get y = 1

$\therefore$ The given point is P(0, 1)

$y = e^{2x} + x^2$ … (i)

$$\frac{dy}{dx} = 2e^{2x} + 2x \Rightarrow \left[\frac{dy}{dx}\right]_P = 2$$

$\therefore$ Equation of tangent at P to equation (i) is

$y - 1 = 2(x - 0) \Rightarrow 2x - y + 1 = 0$ … (ii)

$\therefore$ Required distance = Length of $\perp$ from (1, 1) to equation (ii).

$$= \frac{2 - 1 + 1}{\sqrt{4 + 1}} = \frac{2}{\sqrt{5}}.$$

50. (d) Slope of tangent = 0

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2 - 2x}{2y} = 0 \Rightarrow 2 - 2x = 0 \Rightarrow x = 1$$

$\therefore 1 + y^2 - 2(1) - 3 = 0 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

51. (d) Let Δx be the change in x and ΔV be the corresponding change in V.

It is given that $\frac{\Delta x}{x}\times 100 = 2$

$\therefore V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$

$\Delta V = \frac{dV}{dx}\times\Delta x \Rightarrow \Delta V = 3x^2\Delta x$

$\Rightarrow \Delta V = 3x^2\times\frac{2x}{100}$

$\Rightarrow \Delta V = 0.06x^3$

Thus, the approximate change in volume is $0.06x^3 m^3$.

52. (c) We have, $f(x) = \sin(\sin x),\ x \in R$

Now, $-1 \le \sin x \le 1$ for all $x \in R$

$\Rightarrow \sin(-1) \le \sin(\sin x) \le \sin 1$ for all $x \in R$

[$\because$ sinx is an increasing function on $[-1, 1]$]

$\Rightarrow -\sin 1 \le f(x) \le \sin 1$ for all $x \in R$

This shows that the maximum value of f(x) is sin 1.

53. (c) Let R be a point on AB such that AR = x m. Then, RB = (20 – x)m

In Δ's RAP and RBQ, we have

$PR^2 = x^2 + 16^2$ ... (i)

$RQ^2 = 22^2 + (20 - x)^2$

$\therefore\ PR^2 + RQ^2 = x^2 + 16^2 + 22^2 + (20 - x)^2$

$= 2x^2 - 40x + 1140$

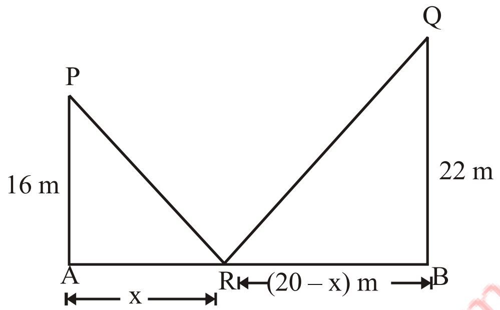
Let $Z = PR^2 + RQ^2$. Then,

$Z = 2x^2 - 40x + 1140.$

$$\frac{dZ}{dx} = 4x - 40 \text{ and } \frac{d^2Z}{dx^2} = 4$$

For maximum or minimum, we must have

$$\frac{dZ}{dx} = 0 \Rightarrow 4x - 40 = 0 \Rightarrow x = 10$$



Clearly, $\frac{d^2Z}{dx^2} = 4 > 0$ for all x

$\therefore$ Z is minimum when x = 10

54. (c) $f(x) = 2x^3 - 3x^2 - 12x + 4$

$\Rightarrow f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$= 6(x - 2)(x + 1)$

For maxima and minima $f'(x) = 0$

$\therefore\ 6(x - 2)(x + 1) = 0$

$\Rightarrow\ x = 2, -1$

Now, $f''(x) = 12x - 6$

At x = 2

$f''(x) = 24 - 6 = 18 > 0$

$\therefore\ x = 2$, local min. point

At x = – 1

$f''(x) = 12(-1) - 6 = -18 < 0$

$\therefore\ x = -1$ local max. point

55. (c) $f(x) = \cos x$

$f'(x) = -\sin x$

In interval $\left(0, \frac{\pi}{2}\right)$, sin x is positive

$f'(x) < 0\ \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore$ f(x) is decreasing $\left(0, \frac{\pi}{2}\right)$

56. (c) $x^3 - 3xy^2 + 2 = 0$

differentiating w.r.t. x

$$3x^2 - 3x(2y)\frac{dy}{dx} - 3y^2 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 3y^2}{6xy}$$

and $3x^2y - y^3 - 2 = 0$

differentiating w.r.t. x

$$\Rightarrow 3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{6xy}{3x^2 - 3y^2}\right)$$

Now, product of slope

$$= \frac{3x^2 - 3y^2}{6xy} \times -\left(\frac{6xy}{3x^2 - 3y^2}\right) = -1$$

$\therefore$ they are perpendicular.

Hence, angle $= \pi/2$

57. (b) Given $y = x^{\frac{1}{5}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{5}x^{-4/5} \Rightarrow \frac{dy}{dx} \text{ at } (0, 0) = \frac{1}{5}(0) \Rightarrow \frac{dy}{dx} = 0$$

Hence, tangent is parallel to x-axis.

58. (b) $x = t^2 + 3t - 8,\ y = 2t^2 - 2t - 5$ at $(2, -1)$

$\therefore\ t^2 + 3t - 8 = 2$ ...(i)

$2t^2 - 2t - 5 = -1$ ...(ii)

On solving eqs (i) and (ii)

we get t = 2

$$\text{Now } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t - 2}{2t + 3}$$

$$\therefore \left[\frac{dy}{dx}\right]_{t=2} = \frac{6}{7}$$

59. (b) $f(x) = x^3 - 18x^2 + 96x$

$\Rightarrow f'(x) = 3x^2 - 36x + 96$

$\therefore\ f'(x) = 0 \Rightarrow x^2 - 12x + 32 = 0$

$\Rightarrow x = 8, 4$

Now, f(0) = 0, f(4) = 160

f(8) = 128, f(9) = 135

So, smallest value of f(x) is 0 at x = 0.

STATEMENT TYPE QUESTIONS

60. (a) At any instant time t, let length, breadth, perimeter and area of the rectangle are x, y, P and A respectively, then

$P = 2(x + y)$ and $A = xy$

It is given that $\frac{dx}{dt} = -5$ cm/min and $\frac{dy}{dt} = 4$ cm/min

(– ve sign shows that the length is decreasing)

I. Now $P = 2(x + y)$.

On differentiating w.r.t t, we get

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$= 2\{-5 + 4\}$ cm/min $= -2$ cm/min

Hence, perimeter of the rectangle is decreasing at the rate of 2 cm/min.

II. Here, area of rectangle $A = xy$.

On differentiating w.r.t. t, we get

Rate of change of area

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} = 8 \times 4 + 6 \times (-5)$$

$= 32 - 30 = 2$ cm^2/min

Hence, area of the rectangle is increasing at the rate of 2cm^2/min.

61. (c) I. $\because \frac{dV}{dt} \propto S$

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) \propto 4\pi r^2 \quad \left(\because V = \frac{4}{3}\pi r^3 \text{ and } S = 4\pi r^2\right)$$

$$\Rightarrow \frac{4}{3}\pi \times 3r^2 . \frac{dr}{dt} \propto 4\pi r^2$$

$\Rightarrow \frac{dr}{dt} = k.1$, where k is proportional constant.

So, statement I is true.

II. Let A = Area of circle $= \pi r^2$ where r is the radius of circle.

$$\therefore \frac{dA}{dt} = 2\pi r . \frac{dr}{dt} = k \text{ (say)}$$

($\because$ A increases at constant rate)

$$\Rightarrow \frac{dr}{dt} = \frac{k}{2\pi r}$$

Now, let $P = 2\pi r$ and $\frac{dP}{dt} = 2\pi\frac{dr}{dt}$

$$\Rightarrow \frac{dP}{dt} = 2\pi . \frac{k}{2\pi r}$$

$$\Rightarrow \frac{dP}{dt} \propto \frac{1}{r}$$

So, statement II is also true.

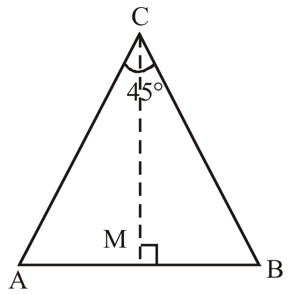
62. (c) Let the two men start from the point C moves with velocity v each at the same time.

Let angle between CA and CB be 45°.

Since, A and B are moving with the same velocity v.

$\therefore$ ΔABC is an isosceles triangle with CA = CB. Draw $CM \perp AB$.

At any instant t the distance between them is AB.



Let AC = BC = x, Let AB = y

Since, $\angle ACM = \frac{1}{2}\angle ACB$

$$\Rightarrow \angle ACM = \frac{1}{2} \times 45° = \frac{\pi}{8}$$

Now, AM = MB = AC $\sin\frac{\pi}{8}$

$$\Rightarrow \frac{y}{2} = x \sin\frac{\pi}{8}$$

On differentiating both sides w.r.t. t, we get

$$\frac{1}{2}\frac{dy}{dt} = \frac{dx}{dt}\sin\left(\frac{\pi}{8}\right)$$

$$\Rightarrow \frac{dy}{dt} = 2v\left(\sin\left(\frac{\pi}{8}\right)\right)\left(\because \frac{dx}{dt} = v\right) \text{ (statement II)}$$

$$= 2v \times \frac{\sqrt{2-\sqrt{2}}}{2} = \left(\sqrt{2-\sqrt{2}}\right) v \text{ unit/s}$$

63. (c) The given function is $f(x) = \sin x$

On differentiating w.r.t. x, we get $f'(x) = \cos x$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right)$, $\cos x > 0$, we have $f'(x) > 0$

($\because$ cosx in Ist quadrant is positive)

Hence, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since, for each $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$, we have

$f'(x) < 0$ ($\because$ cos x in IInd quadrant is negative)

Hence, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) When $x \in (0, \pi)$. We see that $f'(x) > 0$ in $\left(0, \frac{\pi}{2}\right)$

and $f'(x) < 0$ in $\left(\frac{\pi}{2}, \pi\right)$

So, $f'(x)$ is positive and negative in $(0, \pi)$.

Thus, $f(x)$ is neither increasing nor decreasing in $(0, \pi)$.

64. **(c)** I. Let $f(x) = \log x \Rightarrow f'(x) = \frac{1}{x}$

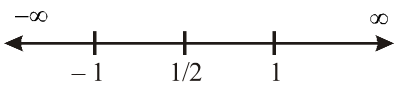
When $x \in (0, \infty)$, $f'(x) > 0$. Therefore, f (x) is strictly increasing in $(0, \infty)$

II. Given, $f(x) = x^2 - x + 1$

$\Rightarrow f'(x) = 2x - 1$

On putting $f'(x) = 0$, we get $x = 1/2$

$x = \frac{1}{2}$ divides the given interval into two intervals as $\left(-1, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-1, 1/2)$	–ve	Strictly decreasing
$(1/2, 1)$	+ve	Strictly increasing

$\therefore$ $f'(x)$ does not have same sign throughout the interval (– 1, 1).

Thus, f(x) is neither increasing nor decreasing strictly in the interval (– 1, 1)

65. **(b)** If slope of the tangent line is zero, then $\tan\theta = 0$ and so $\theta = 0$ which means the tangent line is parallel to the X-axis.

In this case, the equation of the tangent at the point (x_0, y_0) is given by $y = y_0$.

So, statement I is not true.

$\because \theta \to \frac{\pi}{2} \therefore \tan\theta \to \infty$

That means tangent is perpendicular to X-axis. Hence, equation of tangent at (x_0, y_0) is given by $x = x_0$.

So, statement II is true.

66. **(d)** Note that function f is increasing (i.e., $f'(x) > 0$) in the interval $(c - h, c)$ and decreasing (i.e., $f'(x) < 0$) in the interval $(c, c + h)$

This suggests that $f'(c)$ must be zero.

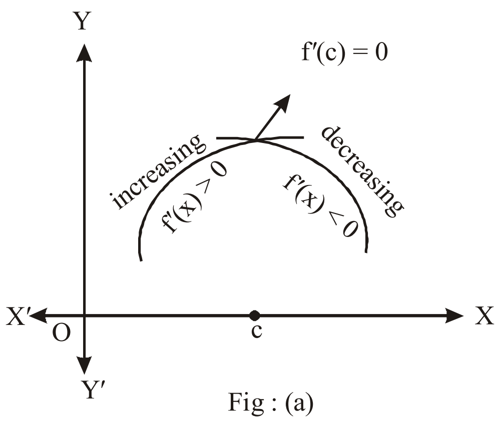


Fig : (a)

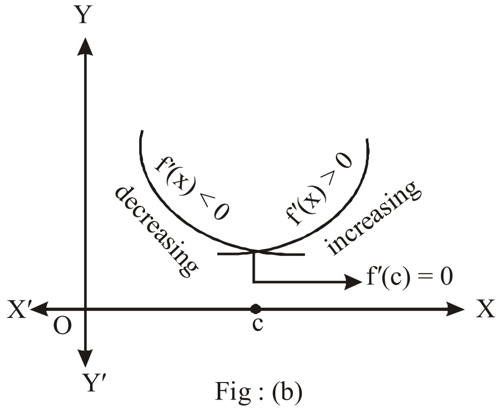


Fig : (b)

Similarly, if 'c' is a point of local minima of f, then the graph of f around 'c' will be as shown in Fig (b). Here, f is decreasing (i.e., $f'(x) < 0$) in the interval $(c - h, c)$ and increasing (i.e., $f'(x) > 0$) in the interval $(c, c + h)$. This again suggest that $f'(c)$ must zero.

The above discussion lead us to the following result

Let f be a function defined on an open interval I. Suppose $c \in I$ be any point. If f has a local maxima or local minima at $x = c$, then either $f'(c) = 0$ or f is not differentiable at c.

67. **(a)** A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a critical point of f.

68. **(d)** We have

$f(x) = x^3 - 3x + 3$

or $f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$

or $f'(x) = 0$ at $x = 1$ and $x = -1$

Thus, $x = \pm 1$ are the only critical points which could possibly be the points of local maxima and/or local minima of f. Let us first examine the point $x = 1$.

Note that for values close to 1 and to the right of 1. $f'(x) > 0$ and for values close to 1 and to the left of 1. $f'(x) < 0$. Therefore, by first derivative test, $x = 1$ is a point of local minima and local minimum value is $f(1) = 1$. In the case of $x = -1$, note that $f'(x) > 0$, for values close to and to the left of – 1 and $f'(x) < 0$, for values close to and to the right of – 1.

Therefore, by first derivative test, $x = -1$ is a point of local maxima and local maximum value is $f(-1) = 5$.

Values of x		**Sign of $f'(x) = 3(x-1)(x+1)$**
Close to 1	to the right (say 1.1 etc.)	> 0
	to the left (say 0.9 etc.)	< 0
Close to –1	to the right (say –0.9 etc.)	< 0
	to the left (say –1.1 etc.)	> 0

69. **(a)** Let ABC be an isosceles triangle inscribed in the circle with radius a such that AB = AC.

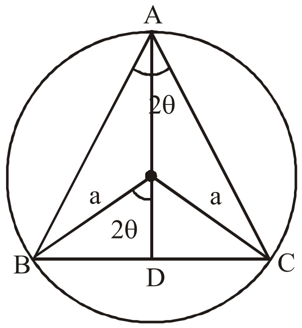
AD = AO + OD = a + a sin2θ and

BC = 2BD = 2a sin 2θ

Therefore, area of the ΔABC, i.e., $\Delta = \frac{1}{2} BC.AD$

$= \frac{1}{2} 2a \sin 2\theta (a + a\cos 2\theta) = a^2 \sin 2\theta (1 + \cos 2\theta)$

$\Rightarrow \ \Delta = a^2 \sin 2\theta + \frac{1}{2} a^2 \sin 4\theta$



Therefore, $\frac{d\Delta}{d\theta} = 2a^2 \cos 2\theta + 2a^2 \cos 4\theta$

= $2a^2$ (cos2θ + cos4θ)

$\frac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta = \cos(\pi - 4\theta)$

Therefore, $2\theta = \pi - 4\theta \Rightarrow \theta = \frac{\pi}{6}$

$\frac{d^2\Delta}{d\theta^2} = 2a^2 (-2\sin 2\theta - 4\sin 4\theta) < 0 \left(\text{at } \theta = \frac{\pi}{6}\right)$

70. **(c)** Let radius of semi-circle = r

∴ One side of rectangle = 2r. Let the other side = x.

∵ P = Perimeter = 10 (given)

$\Rightarrow \ 2x + 2r + \frac{1}{2}(2\pi r) = 10$

⇒ 2x = 10 – r (π + 2) … (i)

Let A be area of the window, then

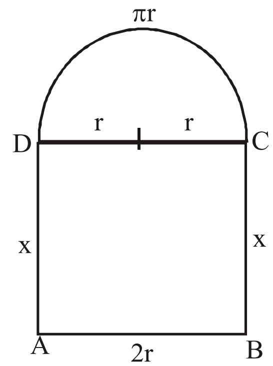
A = Area of semi-circle + Area of rectangle

$= \frac{1}{2}\pi r^2 + 2rx$

$\Rightarrow \ A = \frac{1}{2}(\pi r^2) + r[10 - r(\pi + 2)]$ [using Eq. (i)]

$= \frac{1}{2}(\pi r^2) + 10r - r^2\pi - 2r^2$

$= 10r - \frac{\pi r^2}{2} - 2r^2$



On differentiating twice w.r.t. r, we get

$\frac{dA}{dr} = 10 - \pi r - 4r$ … (ii)

and $\frac{d^2A}{dr^2} = -\pi - 4$ ...(iii)

For maxima or minima, put $\frac{dA}{dt} = 0$

$\Rightarrow \ r = \frac{10}{4 + \pi}$

On putting $r = \frac{10}{4+\pi}$ in eq. (iii), we get $\frac{d^2A}{dr^2} < 0$

Thus, A has local maximum when

$r = \frac{10}{4+\pi}$ … (iv)

∴ Radius of semi-circle = $\frac{10}{4+\pi}$

and one side of rectangle = $2r = \frac{2 \times 10}{4+\pi} = \frac{20}{4+\pi}$

and other side of rectangle i.e., x from eq. (i) is given by

$x = \frac{1}{2}[10 - r(\pi + 2)]$

$= \frac{1}{2}\left[10 - \left(\frac{10}{\pi+4}\right)(\pi + 2)\right]$ [from eq. (iv)]

$= \frac{10\pi + 40 - 10\pi - 20}{2(\pi + 4)} = \frac{10}{\pi + 4}$

Light is maximum when area is maximum.

So, dimensions of the window are length

$= 2r = \frac{20}{\pi+4}$ and breadth $x = \frac{10}{\pi+4}$

So, both the statements are true.

MATCHING TYPE QUESTIONS

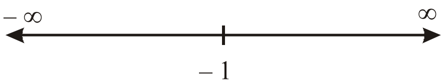
71. (c) A. Let $f(x) = x^2 + 2x - 5 \Rightarrow f'(x) = 2x + 2$

Putting $f'(x) = 0$, we get $2x + 2 = 0$

$\Rightarrow x = -1$

$x = -1$ divides real line into two intervals namely

$(-\infty, -1)$ and $(-1, \infty)$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty,-1)$	$-$ve	Strictly decreasing
$(-1,\infty)$	$+$ve	Strictly increasing

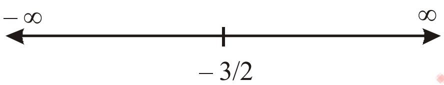
Therefore, f(x) is strictly increasing when $x > -1$ and strictly decreasing when $x < -1$.

B. Let $f(x) = 10 - 6x - 2x^2$

$\Rightarrow f'(x) = 0 - 6 - 2.\,2x = -6 - 4x$

On putting $f'(x) = 0$, we get $-6 - 4x = 0$

$\Rightarrow x = \frac{-3}{2}$ which divides real line into two intervals namely $\left(-\infty, \frac{-3}{2}\right)$ and $\left(\frac{-3}{2}, \infty\right)$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty,-3/2)$	$+$ve	Strictly increasing
$(-3/2,\infty)$	$-$ve	Strictly decreasing

Hence, f is strictly increasing for $x < -\frac{3}{2}$ and strictly decreasing for $x > -\frac{3}{2}$

C. Given, $f(x) = -2x^3 - 9x^2 - 12x + 1$,

$\Rightarrow f(x) = -2.3x^2 - 9.2x - 12$

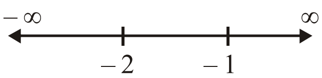
$= -6x^2 - 18x - 12$

On putting $f'(x) = 0$, we get $-6x^2 - 18x - 12 = 0$

$\Rightarrow -6(x + 2)(x + 1) = 0$

$\Rightarrow x = -2, -1$

which divides real line into three intervals $(-\infty, -2)$, $(-2, -1)$, and $(-1, \infty)$.



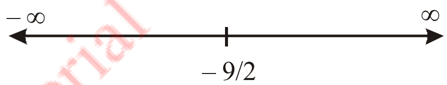
Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty,-2)$	$(-)(-)(-) = -$ve	Strictly decreasing
$(-2,-1)$	$(-)(-)(+) = +$ve	Strictly increasing
$(-1,\infty)$	$(-)(+)(+) = -$ve	Strictly decreasing

Therefore, f(x) is strictly increasing in $-2 < x < -1$ and strictly decreasing for $x < -2$ and $x > -1$

D. Given $f(x) = 6 - 9x - x^2 \Rightarrow f'(x) = -9 - 2x$.

On putting $f'(x) = 0$, we get $-9 - 2x = 0$

$\Rightarrow x = -\frac{9}{2}$ which divides the real line in two disjoint intervals $\left(-\infty, \frac{-9}{2}\right)$ and $\left(-\frac{9}{2}, \infty\right)$.



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty,-9/2)$	$+$ve	Strictly increasing
$(-9/2,\infty)$	$-$ve	Strictly decreasing

Therefore, f(x) is strictly increasing when $x < -\frac{9}{2}$ and strictly decreasing when $x > \frac{-9}{2}$.

E. Given, $f(x) = (x + 1)^3 (x - 3)^3$

On differentiating, we get

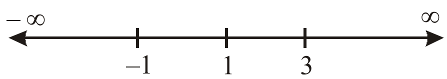
$f'(x) = (x + 1)^3 \cdot (x - 3)^2 \cdot 1 + (x - 3)^3 \cdot 3(x + 1)^2 \cdot 1$

$= 3(x - 3)^2 (x + 1)^2 \{(x + 1) + (x - 3)\}$

$= 3(x - 3)^2 (x + 1)^2 (2x - 2)$

$= 6(x - 3)^2 (x + 1)^2 (x - 1)$

On putting $f'(x) = 0$, we get $x = -1, 1, 3$. which divides real line into four disjoint intervals namely $(-\infty, -1)$, $(-1, 1)$, $(1, 3)$ and $(3, \infty)$.



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$-\infty < x < -1$	$(+)(+)(-) = -$ve	Strictly decreasing
$-1 < x < 1$	$(+)(+)(-) = -$ve	Strictly decreasing
$1 < x < 3$	$(+)(+)(+) = +$ve	Strictly increasing
$3 < x < \infty$	$(+)(+)(+) = +$ve	Strictly increasing

Therefore, f(x) is strictly increasing in (1, 3), (3, ∞) and strictly decreasing in (– ∞, – 1) and (– 1, 1).

72. **(a)** A. Given function is $f(x) = |x + 2| - 1$

We know that $|x + 2| \geq 0$ for all $x \in R$.

Therefore, $f(x) = |x + 2| - 1 \geq -1$ for every $x \in R$.

The minimum value of f is attained when $|x + 2| = 0$.

i.e, $|x + 2| = 0 \Rightarrow x = -2$

$\therefore$ Minimum value of $f = f(-2) = |-2 + 2| - 1$
$= 0 - 1 = -1$

Hence, f(x) has minimum value – 1 at x = 2, but f(x) has no maximum value.

Note The modulus value of a function is always ≥ 0.

B. Given function is $g(x) = -|x + 1| + 3$.

We know that, $|x + 1| \geq 0$ for all $x \in R$.

$\Rightarrow -|x + 1| \leq 0$ for all $x \in R$

$\Rightarrow -|x + 1| + 3 \leq 3$ for all $x \in R$.

The maximum value of g is attained when $|x + 1| = 0$.

i.e., $|x + 1| = 0 \Rightarrow x = -1$.

$\therefore$ Maximum value of $g = g(-1) = -|-1 + 1| + 3 = 3$.

Hence, g(x) has maximum value 3 at $x = -1$, but g(x) has no minimum value.

C. Given function is $h(x) = \sin 2x + 5$.

We know that, $-1 \leq \sin x \leq 1 \Rightarrow -1 \leq \sin 2x \leq 1$.

$\Rightarrow \; -1 + 5 \leq \sin 2x + 5 \leq 1 + 5$

$\Rightarrow \; 4 \leq \sin 2x + 5 \leq 6$

Hence, maximum value of h(x) is 6 and minimum value of h(x) is 4.

D. Given function is $f(x) = |\sin 4x + 3|$

We know that, $-1 \leq \sin 4x \leq 1$

$\Rightarrow \; 3 - 1 \leq \sin 4x + 3 \leq 1 + 3$

$\Rightarrow \; 2 \leq \sin 4x + 3 \leq 4$

$\Rightarrow \; 2 \leq |\sin 4x + 3| \leq 4$

Hence, maximum value of f(x) is 4 and minimum value of f(x) is 2.

E. Given function is $h(x) = x + 1, -1 < x < 1$.

Now, $-1 < x < 1 \Leftrightarrow -1 + 1 < x + 1 < 1 + 1$

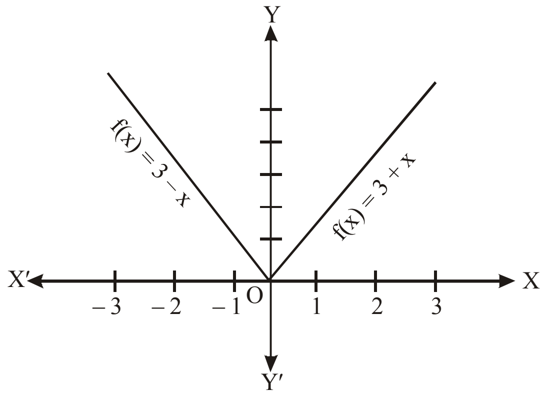
$\Leftrightarrow 0 < x + 1 < 2$.

Here, in the interval (0, 2), f has neither a maximum value nor a minimum value.

INTEGER TYPE QUESTIONS

73. **(c)** Note that the given function is not differentiable at $x = 0$. So, second derivative test fails. Let us try first derivative test.

Note that 0 is a critical point of f. Now to the left of 0, $f(x) = 3 - x$ and so $f'(x) = -1 < 0$. Also to the right of 0, $f(x) = 3 + x$ and so, $f'(x) = 1 > 0$. Therefore, by first derivative test, $x = 0$ is a point of local minima of f and local minimum value of f is $f(0) = 3$.



74. **(d)** $\frac{dv}{dt} = \frac{d}{dt}(2t^2 + t + 1) = 4t + 1$

$$\left(\frac{dv}{dt}\right)_{t=2} = 4(2) + 1 = 9\ m/s^2$$

75. **(c)** $v = \frac{4}{3}\pi r^3$, $\frac{dv}{dt} = \frac{4}{3}\pi\frac{d}{dt}r^3 = \frac{4}{3}\pi 3r^2.\frac{dr}{dt}$

$= 4\pi r^2.\frac{dr}{dt}$

when r = 10cm

$\frac{dv}{dt} = 4\pi(10)^2.(0.02) = 8\ \pi\ cm^3/s.$

76. **(a)** $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 1) = 4x^3 - 4x = 4x(x^2 - 1)$

For max. or min, $\frac{dy}{dx} = 0$

$4x(x^2 - 1) = 0$

either $x = 0$ or $x = \pm 1$

$x = 0$ and $x = -1$ does not belong to $\left[\frac{1}{2}, 2\right]$

$\frac{d^2y}{dx^2} = 12x^2 - 4$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 12(1)^2 - 4 = 8 > 0$$

$\therefore$ there is minimum value of function at $x = 1$

$\therefore$ minimum value is

$y(1) = 1^4 - 2(1)^2 + 1 = 1 - 2 + 1 = 0$

77. **(a)** $\because \; x^2 \geq 0$

$\therefore \; -x^2 \leq 0$

$\therefore$ the maximum value of $y = -x^2$ is 0. This value is attained when $x = 0 \in [-1, 1]$

ASSERTION - REASON TYPE QUESTIONS

78. (c) Every increasing or decreasing function is one-one

$$f'(x)=3x^2+2x+3+\cos x=3\left(x+\frac{1}{3}\right)^2+\frac{8}{3}+\cos x>0$$

$$[\because |\cos x|<1 \text{ and } 3\left(x+\frac{1}{3}\right)^2+\frac{8}{3}\ge\frac{8}{2}]$$

$\therefore$ f(x) is strictly increasing

79. (b) $f'(x)=6x^2-18x+12$

For increasing function, $f'(x)\ge 0$

$\therefore \quad 6(x^2-3x+2)\ge 0$

$\Rightarrow \quad 6(x-2)(x-1)\ge 0$

$\Rightarrow \quad x\le 1$ and $x\ge 2$

$\therefore$ f(x) is increasing outside the interval (1, 2). Therefore it is true statement

Now, $\quad f'(x)<0$

$\Rightarrow \quad 6(x-2)(x-1)<0$

$\Rightarrow \quad 1<x<2$

$\therefore$ Assertion and Reason are both correct but Reason is not the correct explanation of Assertion

80. (a) The equation of the given curves are

$x=y^2$ ... (i)

and $xy=k$ ... (ii)

The two curves meet where $\frac{k}{y}=y^2$

$\Rightarrow \quad y^3=k\Rightarrow y=k^{1/3}$

Substituting this value of y in eq. (i), we get

$x=(k^{1/3})^2=k^{2/3}$

So, eqs. (i) and (ii) are intersect at the point $(k^{2/3}, k^{1/3})$.

On differentiating eq. (i), w.r.t. x, we get

$$1=2y\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{2y}$$

$\therefore$ Slope of the tangent to the first curve eq. (i) at $(k^{2/3}, k^{1/3})$

$$=\frac{1}{2k^{1/3}} \quad \text{... (iii)}$$

From eq. (ii), $y=\frac{k}{x}\Rightarrow\frac{dy}{dx}=-\frac{k}{x^2}$

$\therefore$ Slope of the tangent to the second curve eq. (ii) at $(k^{2/3}, k^{1/3})$

$$=-\frac{k}{\left(k^{2/3}\right)^2}=-\frac{1}{k^{1/3}} \quad \text{... (iv)}$$

We know that, two curves intersect at right angles, if the tangents to the curves at the point of intersection i.e., at $(k^{2/3}, k^{1/3})$ are perpendicular to each other.

This implies that we should have the product of the tangents = – 1

$$\Rightarrow \left(\frac{1}{2k^{1/3}}\right)\left(-\frac{1}{k^{1/3}}\right)=-1$$

$\Rightarrow \quad 1=2k^{2/3}$

$\Rightarrow \quad 1^3=(2k^{2/3})3\Rightarrow 1=8k^2$

Hence, the given two curves cut at right angles, if $8k^2=1$, so both the statements are correct and Reason is a correct explanation of Assertion.

81. (a) Let r be the radius of the sphere and Δr be the error in measuring radius.

Thus, r = 9 m

and Δr = 0.03 m

Now, surface area of a sphere is given by $S=\pi r^2$

$$\Rightarrow \frac{dS}{dr}=8\pi r$$

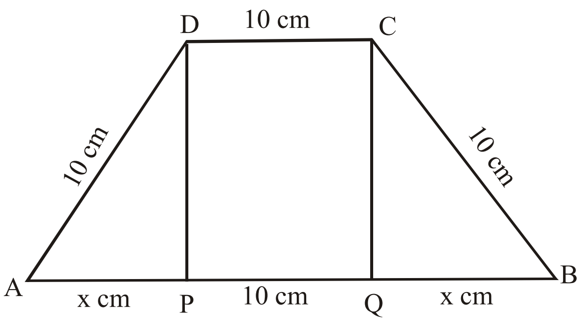
Hence, $\Delta S=\left(\frac{dS}{dr}\right)\Delta r=(8\pi r)\Delta r$

$=8\pi\times 9\times 0.03=2.16\,\pi m^2$

Hence, the approximate error in calculating the surface area is $2.16\,\pi\, m^2$.

$\therefore$ Reason is the correct explanation of Assertion.

82. (a) The required trapezium is as given in figure. Draw perpendicular DP and CQ on AB. Let AP = x cm. Note that $\Delta APD \sim \Delta BQC$.



Therefore, QB = x cm. Also, by Pythagoras theorem, $DP=QC-\sqrt{100-x^2}$. Let A be the area of the trapezium. Then,

$$A=A(x)=\frac{1}{2}\text{(Sum of parallel sides)}\times\text{(Height)}$$

$$=\frac{1}{2}(2x+10+10)\left(\sqrt{100-x^2}\right)$$

$$=(x+10)\left(\sqrt{100-x^2}\right)$$

or $A'(x) = (x+10)\dfrac{(-2x)}{2\sqrt{100-x^2}} + \sqrt{100-x^2}$

$$= \frac{-2x^2 - 10x + 100}{\sqrt{100-x^2}}$$

Now, $A'(x) = 0$ gives $2x^2 + 10x - 100x = 0$

i.e., x = 5 and x = – 10.

Since, x represents distance, it cannot be negative.

So, x = 5, Now

$$A''(x) = \frac{\left(\sqrt{100-x^2}\,(-4x-10) - \left(-2x^2-10x+100\right)\dfrac{(-2x)}{2\sqrt{100-x^2}}\right)}{100-x^2}$$

$$= \frac{2x^3 - 300x - 1000}{\left(100-x^2\right)^{3/2}} \text{ (on simplification)}$$

or $A''(5) = \dfrac{2(5)^3 - 300(5) - 1000}{\left(100-(5)^2\right)^{\frac{3}{2}}} = \dfrac{-2250}{75\sqrt{75}} = \dfrac{-30}{\sqrt{75}} < 0$

Thus, area of trapezium is maximum at x = 5 and the area is given by

$$A(5) = (5+10)\sqrt{100-(5)^2} = 15\sqrt{75} = 75\sqrt{3}\text{cm}^2$$

83. (a) Let one number is x. Then, the other number will be (16 – x)

Let the sum of the cubes of these numbers be denoted by S.

Then, $S = x^3 + (16 - x)^3$

On differentiating w.r.t. x we get

$$\frac{dS}{dx} = 3x^2 + 3(16-x)^2(-1) = 3x^2 - 3(16-x)^2$$

$$\Rightarrow \frac{d^2S}{dx^2} = 6x + 6(16-x) = 96$$

For minima put $\dfrac{dS}{dx} = 0$

$\therefore\ 3x^2 - 3(16-x)^2 = 0$

$\Rightarrow x^2 - (256 + x^2 - 32x) = 0$

$\Rightarrow 32x = 256 \Rightarrow x = 8$

At x = 8, $\left(\dfrac{d^2S}{dx^2}\right)_{x=8} = 96 > 0.$

By second derivative test, x = 8 is the point of local minima of S.

Thus, the sum of the cubes of the numbers is the minimum when the numbers are 8 and 16 – 8 = 8

Hence, the required numbers are 8 and 8.

84. (c) Every increasing or decreasing function is one-one

$f'(x) = 3x^2 + 2x + 3 + \cos x$

$$= 3\left(x+\frac{1}{3}\right)^2 + \frac{8}{3} + \cos x > 0$$

$$[\because |\cos x| < 1 \text{ and } 3\left(x+\frac{1}{3}\right)^2 + \frac{8}{3} \geq \frac{8}{2}]$$

$\therefore$ f(x) is strictly increasing

85. (a) $f(x) = \cos^2 x + \cos^3\left(x+\frac{\pi}{3}\right) - \cos x \cos^3\left(x+\frac{\pi}{3}\right)$

$$= 1 + \cos\left(2x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) - \frac{1}{2}\left[\cos\left(2x+\frac{\pi}{3}\right) + \cos\frac{\pi}{3}\right]$$

$$= 1 + \frac{1}{2}\cos\left(2x+\frac{\pi}{3}\right) - \frac{1}{2}\cos\left(2x+\frac{\pi}{3}\right) - \frac{1}{4} = \frac{3}{4}$$

$f'(x) = 0$

Derivative of constant function is zero.

86. (d) $f'(x) = \dfrac{2(ad-bc)}{\left(ce^x + de^{-x}\right)^2}$

and f(x) is an increasing function

$\therefore f'(x) > 0$

$$\Rightarrow \frac{2(ad-bc)}{\left(ce^x + de^{-x}\right)^2} > 0$$

$\therefore\ 2(ad - bc) > 0$

$\Rightarrow\ ad > bc \Rightarrow bc < ad$

87. (b) Given, $x^2 + y^2 = 25$

Differentiating we get,

$\Rightarrow\ 2xdx + 2ydy = 0$

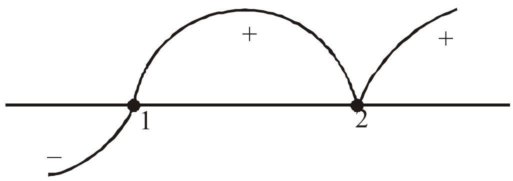
$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y} \Rightarrow \frac{dy/dt}{dx/dt} = -\frac{x}{y}$$

Now $\dfrac{dy}{dt} = -1.5$ and y = 4

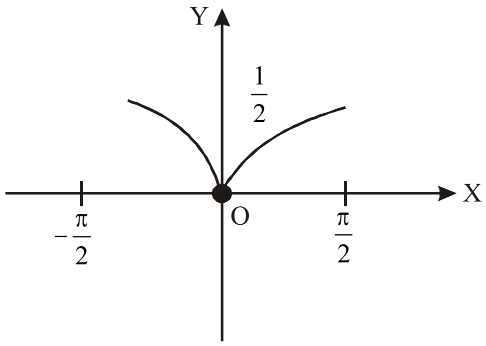
$\Rightarrow x^2 = \sqrt{25-16} \Rightarrow x = 4$

$$\Rightarrow -\frac{1.5}{dx/dt} = -\frac{3}{4} \Rightarrow \frac{dx}{dt} = \frac{1.5\times4}{3} = 2 \text{ cm/s}$$

88. (c) It is clear from figure $f'(x)$ has no sign change at $x = 2$. Hence, f(x) is neither maximum nor minimum at $x = 2$.



89. (c) We draw the graph of f(x) for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



i.e., for $|x| \le \frac{\pi}{2}$

Here, $f(0) = \frac{1}{2}$, whereas

$$\underset{x\to 0}{\text{Lt}}\ f(x) = \underset{x\to 0}{\text{Lt}}\ |\sin x|$$
$$= 0 \ne f(0)$$

$\therefore$ f is discontinuous at 0

$\Rightarrow$ f is not derivable at 0.

Thus, Reason is false.

However, we note that for all

$$x \in \left(\frac{-\pi}{6}, 0\right) \cup \left(0, \frac{\pi}{6}\right)$$

$|\sin x| < \frac{1}{2}$ i.e., $f(x) < f(0)$

$\Rightarrow$ f has a local maximum value at 0.

So, Assertion is true.

90. (a)

91. (c) There is minimum value of function at $x = \pi$. The first derivative is zero at $x = \pi$ but the second derivative is negative at $x = \pi$.

92. (a)

CRITICALTHINKING TYPE QUESTIONS

93. (d) Let (x, y) be the point on the curve $2x^2 + y^2 - 2x = 0$. Then its distance from (a, 0) is given by

$$S = \sqrt{\{(x-a)^2 + y^2\}} \quad \ldots \text{(i)}$$

$\Rightarrow S^2 = x^2 - 2ax + a^2 + 2x - 2x^2$ [Using $2x^2 + y^2 - 2x = 0$]

$\Rightarrow S^2 = -x^2 + 2x(1-a) + a^2 \Rightarrow 2S\frac{dS}{dx} = -2x + 2(1-a)$

For S to be maximum,

$\frac{dS}{dx} = 0 \Rightarrow -2x + 2(1-a) = 0 \Rightarrow x = 1-a$

It can easily checked that $\frac{d^2S}{dx^2} < 0$ for $x = 1 - a$.

Hence, S is maximum for $x = 1 - a$. Putting $x = 1 - a$ in (i).

We get, $S = \sqrt{(1 - 2a + 2a^2)}$

94. (b) $y = \cos(x+y)$ … (i)

$\therefore \frac{dy}{dx} = -\sin(x+y)\left\{1 + \frac{dy}{dx}\right\}$

$\therefore \frac{dy}{dx} = -\frac{\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$

$\Rightarrow \sin(x+y) = 1$, so $\cos(x+y) = 0$

$\therefore$ from (i), $y = 0$ and $(x+y) = 2n\pi + \frac{\pi}{2}$

Tangent at $\left(\frac{\pi}{2}, 0\right)$ is $x + 2y = \frac{\pi}{2}$

95. (b) Let $y = f(x) = ax^2 + bx + c$

$\therefore f'(x) = 2ax + b$

$f(0) = c$ and $f'(0) = b$,

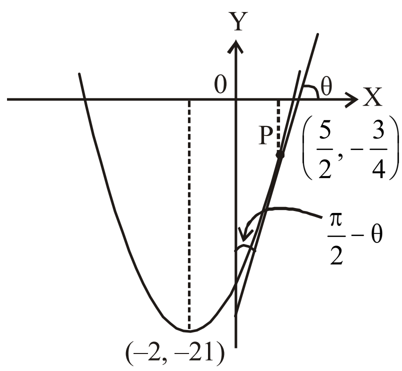
$f'(x)\{at (1,1)\} = 2a + b = 1$

$f(1) = a + b + c = 1$

Solving, we have $a - c = 0$ or $a = c$.

Now, $2f(0) + f'(0) = 2c + b = 2a + b = 1$

96. (b)



$y = x^2 + 4x + 4 - 4 - 17$

$y = (x+2)^2 - 21 \Rightarrow$ Vertex is $(-2, -21)$

Also $y = x^2 + 4x - 17 \Rightarrow \frac{dy}{dx} = 2x + 4$

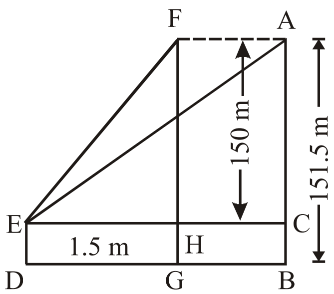
$\Rightarrow$ Slope of tangent at $\left(\frac{5}{2}, -\frac{3}{4}\right)$

$m = \frac{dy}{dx} = 2 \times \frac{5}{2} + 4 = 9$

$\theta = \tan^{-1} 9$

$\therefore$ angle by y-axis = $\frac{\pi}{2} - \tan^{-1} 9 = \cot^{-1} 9$

97. (d)



Let AB be the height of the kite and DE be the height of the boy.

Let DB = x = EC

$\therefore \frac{dx}{dt} = 10 m/s$

Let AE = y

$\because$ AB = 15.15 m

$\therefore$ AC = AB – BC

= 151.5 m – 1.5 m = 150 m

Also, $AC^2 + EC^2 = AE^2$ (by Pythagoras theorem)

$\Rightarrow 150^2 + x^2 = y^2$

Differentiating both sides w.r.t. t, we have

$0 + 2x\frac{dx}{dt} = 2y\frac{dy}{dt} \Rightarrow x\frac{dx}{dt} = y\frac{dy}{dt}$

Now, when y = 250 m

$x = \sqrt{y^2 - (150)^2}$

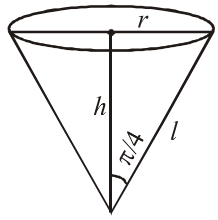
$= \sqrt{62500 - 22500} = 200$ m

$\therefore 200 \times 10 = 250 \times \frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{2000}{250} = 8 m/s$

98 (c) It represents the surface area, then

$\frac{ds}{dt} = 2 cm^2/s$



$s = \pi r.l = \pi l.\sin\frac{\pi}{4} l = \frac{\pi}{\sqrt{2}} l^2 \quad \left(\because r = l \sin\frac{\pi}{4}\right)$

Therefore, $\frac{ds}{dt} = \frac{2\pi}{\sqrt{2}} l.\frac{dl}{dt} = \sqrt{2}\pi l.\frac{dl}{dt}$

When $\frac{ds}{dt} = 2cm^2/s$, $l = 4$ cm

$\frac{dl}{dt} = \frac{1}{\sqrt{2}\pi.4}.2 = \frac{1}{2\sqrt{2}\pi} = \frac{\sqrt{2}}{4\pi}$ cm/s

99. (d) (a) Let f (x) = cos x, then f ′(x) = – sinx.

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) < 0$

Therefore, f(x) is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

(b) Let f(x) = cos 2x

$\Rightarrow f'(x) = -2 \sin 2x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) < 0$

Because sin 2x will either lie in the first or second quadrant which will give a positive value.

Therefore, f (x) is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

(c) Let f ′(x) = cos 3x

$\Rightarrow f'(x) = -3\sin 3x$. In Interval $\left(0, \frac{\pi}{3}\right)$, $f'(x) < 0$

Because sin 3x will either lie in the first or second quadrant which will give a positive value.

Therefore, f (x) is strictly decreasing on $\left(0, \frac{\pi}{3}\right)$.

When $x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$, then $f'(x) > 0$

Because sin3x will lie in the third quadrant. Therefore, f (x) is not strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

(d) Let f(x) = tanx

$\Rightarrow f'(x) = \sec^2 x$.

In Interval $x \in \left(0, \frac{\pi}{2}\right)$, $f'(x) > 0$

Therefore, f(x) is not strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

100. (d) The equation of the given curve is $y = 4x^3 - 2x^5$

$\frac{dy}{dx} = 12x^2 - 10x^4$

Therefore, the slope of the tangent at point (x, y) is $12x^2 - 10x^4$.

The equation of the tangent at (x, y) is given by

$Y - y = (12x^2 - 10x^4)(X - x)$ … (i)

When, the tangent passes through the origin (0, 0), then X = Y = 0

Therefore, eq. (i) reduce to

$-y = (12x^2 - 10x^4)(-x)$

$\Rightarrow y = 12x^3 - 10x^5$

Also, we have $y = 4x^3 - 2x^5$

$12x^3 - 10x^5 = 4x^3 - 2x^5$

$\therefore 12x^3 - 10x^5 = 4x^3 - 2x^5$

$\Rightarrow 8x^5 - 8x^3 = 0 \Rightarrow x^5 - x^3 = 0$

$\Rightarrow x^3(x^2 - 1) = 0 \Rightarrow x = 0, \pm 1$

When, x = 0, $y = 4(0)^3 - 2(0)^5 = 0$

When, x = 1, $y = 4(1)^3 - 2(1)^5 = 2$

When, x = – 1, $y = 4(-1)^3 - 2(-1)^5 = -2$

Hence, the require points are (0, 0), (1, 2) and (– 1, – 2).

101. (b) Solving the given equations, we have,

$y^2 = x$ and $x^2 = y \Rightarrow x^4 = x$.

or $x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0 \Rightarrow x = 0, x = 1$

Therefore, y = 0, y = 1

i.e., points of intersection are (0, 0) and (1, 1).

Further $y^2 = x \Rightarrow 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

and $x^2 = y \Rightarrow \frac{dy}{dx} = 2x$.

At (0, 0), the slope of the tangent to the curve $y^2 = x$ is parallel to Y-axis and the tangent to the curve $x^2 = y$ is parallel to X-axis.

$\Rightarrow$ Angle of intersection $= \frac{\pi}{2}$

At (1, 1) slope of the tangent to the curve $y^2 = x$ is equal to $\frac{1}{2}$ and that of $x^2 = y$ is 2.

$$\tan\theta = \left|\frac{2 - \frac{1}{2}}{1 + 1}\right| = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

102. (a) Given, $y - x = 1 \Rightarrow y = x + 1$

$\frac{dy}{dx} = 1$ and $y^2 = x$

$\Rightarrow 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

$\therefore 1 = \frac{1}{2y} \Rightarrow 2y = 1$

$\Rightarrow y = \frac{1}{2}$

$\therefore$ Point on the curve is $\left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ Required shortest distance

$$= \left|\frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

103. (c) Given error in diameter $= \pm 0.04$

$\therefore$ Error in radius, $\delta r = \pm 0.02$

$\therefore$ Percent error in the volume of sphere

$$= \frac{\delta r}{V} \times 100 = \frac{\delta\left(\frac{4}{3}\pi r^3\right)}{\frac{4}{3}\pi r^3} \times 100 = \frac{3\delta r}{r} \times 100$$

$$= \frac{3 \times (\pm 0.02)}{10} \times 100 = \pm 0.6$$

104. (b) Let $f(x) = \sin x + \cos x \Rightarrow f'(x) = \cos x - \sin x$

and $f''(x) = -\sin x - \cos x = -(\sin x + \cos x)$

$\therefore \cos x - \sin x = 0$

$\Rightarrow \sin x = \cos x$

$\Rightarrow \frac{\sin x}{\cos x} = 1$

$\Rightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots,$

Now, $f''(x)$ will be negative when (sin x + cos x) is positive i.e., when sinx and cos x both positive. Also, we know that sin x and cos x both are positive in the first quadrant.

Then, $f''(x)$ will be negative when $x \in \left(0, \frac{\pi}{2}\right)$

Thus, we consider $x = \frac{\pi}{4}$

$$f''\left(\frac{\pi}{4}\right) = -\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$$

$$= -\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} = -\sqrt{2} < 0$$

By second derivative test, f will be maximum at $x=\frac{\pi}{4}$ and the maximum value of f is

$$f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}+\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

105. (c) Let $f(x)=[x(x-1)+1]^{1/3}, 0\le x\le 1=\{x^2-x+1\}^{1/3}$

On differentiating w.r.t. x , we get

$$f'(x)=\frac{1}{3}\left(x^2-x+1\right)^{\frac{1}{3}-1}(2x-1)$$

$$=\frac{1(2x-1)}{3\left(x^2-x+1\right)^{2/3}}$$

Now put, $f'(x)=0$

$\Rightarrow\ 2x-1=0$

$\Rightarrow\ x=\frac{1}{2}\in[0,1]$

So, $x=\frac{1}{2}$ is a critical point.

Now, we evaluate the value of f at critical point $x=\frac{1}{2}$ and at the end points of the interval [0, 1]

At $x=0$, $f(0)=(0-0+1)^{1/3}=1$

At $x=1$, $f(1)=(1-1+1)^{1/3}=1$

At $x=\frac{1}{2}$, $f\left(\frac{1}{2}\right)=\left(\frac{1}{4}-\frac{1}{2}+1\right)^{1/3}=\left(\frac{3}{4}\right)^{1/3}$

$\therefore$ Maximum value of f(x) is 1 at x = 0, 1.

106. (b) Given, $y=e^{\left(2x^2-2x+1\right)\sin^2 x}$

For minimum or maximum, put $\frac{dy}{dx}=0$

$$\therefore\ e^{\left(2x^2-2x+1\right)\sin^2 x}[4x-2]\sin^2x$$
$$+2(2x^2-2x+1)\sin x\cos x]=0$$

$\Rightarrow (4x-2)\sin^2x+2(2x^2-2x+1)\sin x\cos x=0$

$\Rightarrow 2\sin x\,[(2x-1)\sin x+(2x^2-2x+1)\cos x]=0$

$\Rightarrow \sin x=0$

$\therefore$ y is minimum for $\sin x = 0$

Thus, minimum value of

$$y=e^{\left(2x^2-2x+1\right)(0)}=e^0=1$$

107. (a) Let r be the radius of the given disc and A be its area. Then

$A=\pi r^2$

or $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$ (by Chain rule)

Now, approximate rate of increase of radius

$$=dr=\frac{dr}{dt}\Delta t=0.05\text{cm}/\text{s}$$

Therefore, the approximate rate of increase in area is given by

$$dA=\frac{dA}{dt}(\Delta t)=2\pi r\left(\frac{dr}{dt}\Delta t\right)$$

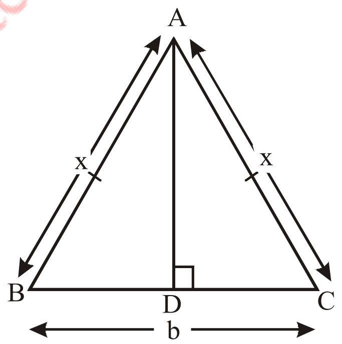
$=2\pi(3.2)(0.05)=0.320\,\pi\ \text{cm}^2/\text{s}$ (r = 3.2 cm)

108. (d) Let ΔABC be isosceles triangle where BC is the base of fixed length b.

Let the length of two equal sides of ΔABC be x.

Draw $AD\perp BC$ in figure.

Now, in ΔADC, by applying the Pythagoras theorem, we have



$$AD=\sqrt{x^2-\left(\frac{b}{2}\right)^2}=\sqrt{x^2-\frac{b^2}{4}}$$

$\therefore$ Area of triangle (A) = $\frac{1}{2}\times$ base $\times$ height

The rate of change of the area A w.r.t time t is given by

$$\frac{dA}{dt}=\frac{1}{2}b\times\frac{1}{2}\frac{2x}{\sqrt{x^2-\frac{b^2}{4}}}\times\frac{dx}{dt}=\frac{xb}{\sqrt{4x^2-b^2}}\frac{dx}{dt}$$

It is given that the two equal sides of the triangle are decreasing at the rate of 3 cm/s.

$\therefore\ \frac{dx}{dt}=-3$ cm/s (negative sign use for decreasing)

$$\therefore\ \frac{dA}{dt}=\frac{-3xb}{\sqrt{4x^2-b^2}}\text{cm}^2/\text{s}$$

When x = b, we have $\frac{dA}{dt}=\frac{-3b^2}{\sqrt{3b^2}}=-\sqrt{3}\,b\ \text{cm}^2/\text{s}$

Hence, if the two equal sides are equal to the base, then the area of the triangle is decreasing at the rate of $\sqrt{3}$ b cm^2/s.

109. (b) Let P be a point on the hypotenuse AC of right angled ΔABC.

Such that $PL \perp AB = a$ and $PM \perp BC = b$

Hence, PL = a, PM = b

Clearly, $\angle APL = \angle ACB = \theta$ (say)

AP = a sec θ, PC = b cosec θ

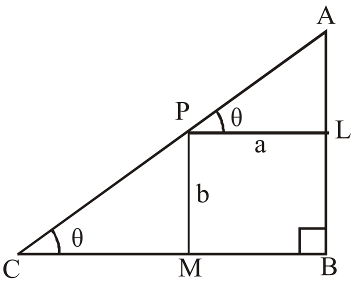
Let l be the length of the hypotenuse, then

l = AP + PC

$\Rightarrow l = a \sec\theta + b \operatorname{cosec}\theta,\ 0 < \theta < \frac{\pi}{2}$

On differentiating w.r.t. θ, we get

$$\frac{dl}{d\theta} = a\sec\theta\tan\theta - b\operatorname{cosec}\theta\cot\theta$$



For maxima or minima put $\frac{dl}{d\theta} = 0$

$\Rightarrow a\sec\theta\tan\theta = b\operatorname{cosec}\theta\cot\theta$

$\Rightarrow \frac{a\sin\theta}{\cos^2\theta} = \frac{b\cos\theta}{\sin^2\theta}$

$\Rightarrow \frac{\sin^3\theta}{\cos^3\theta} = \frac{b}{a}$

$\Rightarrow \tan^3\theta = \frac{b}{a}$

$\Rightarrow \tan\theta = \left(\frac{b}{a}\right)^{1/3}$

Now, $\frac{d^2 l}{d\theta^2} = a\left(\sec\theta \times \sec^2\theta + \tan\theta \times \sec\theta\tan\theta\right)$

$- b[\operatorname{cosec}\theta(-\operatorname{cosec}^2\theta) + \cot\theta(-\operatorname{cosec}\theta\cot\theta)$

$= a\sec\theta(\sec^2\theta + \tan^2\theta) + b\operatorname{cosec}\theta(\operatorname{cosec}^2\theta + \cot^2\theta)$

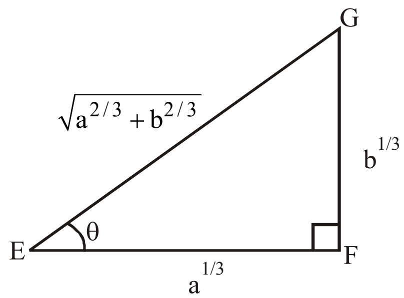
Since $0 < \theta < \frac{\pi}{2}$, so trigonometric ratios are positive.

Also, a > 0 and b > 0.

$\therefore \frac{d^2 l}{d\theta^2}$ is positive.

$\Rightarrow l$ is least when $\tan\theta = \left(\frac{b}{a}\right)^{1/3}$

Now, we have the following figure



$\therefore$ Least value of $l = a\sec\theta + b\operatorname{cosec}\theta$

$$= a\frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} + b\frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}$$

$$= \sqrt{a^{2/3}+b^{2/3}}\left(a^{2/3}+b^{2/3}\right) = \left(a^{2/3}+b^{2/3}\right)^{3/2}$$

$$\left[\begin{array}{l}\because \text{In } \Delta EFG, \sec\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} \\ \text{and } \operatorname{cosec}\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}\end{array}\right]$$

110. (d) Given: curve $y - e^{xy} + x = 0$

$\Rightarrow y = e^{xy} - x$

Differentiate w.r.t. x, we have

$\therefore \frac{dy}{dx} = e^{xy}\left[x.\frac{dy}{dx} + y.1\right] - 1$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}.x\,e^{xy} + y.e^{xy} - 1$

$\Rightarrow \frac{dy}{dx}.(1 - xe^{xy}) = ye^{xy} - 1$

$\frac{dy}{dx} = \frac{ye^{xy} - 1}{1 - xe^{xy}}$ … (i)

But for vertical tangent $\frac{dx}{dy} = 0$

$\Rightarrow \frac{ye^{xy} - 1}{1 - xe^{xy}} = \frac{1}{0} \Rightarrow 1 - x.e^{xy} = 0$

$\Rightarrow e^{xy} = \frac{1}{x}$

This equation is satisfied at point (1, 0).

111. (c) Let radius of spherical balloon = r

After increasing 0.2%, radius

$= r + r \times \frac{0.2}{100} = \frac{1002}{1000} r$

Original volume $= \frac{4}{3}\pi r^3$

and New volume $= \frac{4}{3}\pi\left(\frac{1002}{1000} r\right)^3$

$\therefore$ Increased volume$= \frac{4}{3}\pi\left(\frac{1002}{1000} r\right)^3 - \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi r^3\left[\left(\frac{1002}{1000}\right)^3 - 1\right]$

$\therefore$ % increased in volume $= \frac{\frac{4}{3}\pi r^3\left[(1.002)^3 - 1\right]}{\frac{4}{3}\pi r^3} \times 100$

$= (1.006 - 1) \times 100$

$= 0.006 \times 100$

$= 0.600 = 0.6$ %

112. (c) Given $f(x) = x^2 \log x$

Now for maximum and minimum $f'(x) = 0$

$\Rightarrow \frac{x^2}{x} + 2x \log x = 0$

$\Rightarrow x(1 + 2\log x) = 0$

$\Rightarrow$ either $x = 0$, or $\log x = \frac{-1}{2}$ or $x = e^{-1/2}$

But we have to search the minima and maxima in the interval [1, e]

$\therefore f''(x) = x \cdot \frac{2}{x} + 1 + 2\log x = 3 + 2\log x$

Now at x = 1

$f''(1) = 3 = +$ ve

$\Rightarrow f(x)$ is min. when x = 1

Therefore min. value $= 1^2 \log 1 = 0$

Now at x = e

$f''(e) = 3 + 2 = 5$ $(\because \ln e = 1)$

this is also + ve

But this is not possible that function has two min. values for two diff. values of x.

Thus, the function f(x) has no point of maximum and minimum in the interval [1, e].

113. (b) Let each side be x and area, $A = \frac{\sqrt{3}}{4}x^2$

Since, each side of an equilateral triangle expands at the rate of 2cm/s.

$\Rightarrow \left(\frac{dx}{dt}\right) = 2\text{cm/s}$ and $A = \frac{\sqrt{3}}{4}x^2$

On differentiation, we get

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} 2x \frac{dx}{dt}$, at $x = 10$

$= \frac{\sqrt{3}}{4} \times 2 \times 10 \times 2 = 10\sqrt{3}\ \text{cm}^2/\text{s}$

114. (b) If two curves intersect each other orthogonally, then the slopes of corresponding tangents at the point of intersection are perpendicular.

Let the point of intersection be (x_1, y_1).

Given curves :

$x^2 = 9A(9 - y)$ ... (i)

and $x^2 = A(y + 1)$ ... (ii)

Differentiating w.r. to x both sides equations (i) and (ii) respectively, we get

$2x = -9A\frac{dy}{dx}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{2x_1}{9A} \Rightarrow m_1 = -\frac{2x_1}{9A}$

and $2x = A\frac{dy}{dx} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{2x_1}{A}$

$\Rightarrow m_2 = \frac{2x_1}{A}$

$m_1 m_2 = -1 \Rightarrow \frac{4x_1^2}{9A^2} = 1 \Rightarrow 4x_1^2 = 9A^2$ ... (iii)

Solving equations (i) and (ii),

we find $y_1 = 8$

Substituting $y_1 = 8$ in equation (ii),

we get $x_1^2 = 9A$ ... (iv)

From equations (iii) and (iv), we get A = 4

115. (d) $4x^2 - 9y^2 = 36$ ... (i)

$\Rightarrow 8x - 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{4x}{9y}$

$\therefore$ slope of tangent $= \frac{4x}{9y}$.

Also, slope of line $5x + 2y - 10 = 0$ is $\frac{-5}{2}$

$\therefore$ Line is perpendicular to the tangent. So product of slopes = – 1

$\therefore \frac{4x}{9y} \times \left(-\frac{5}{2}\right) = -1 \Rightarrow y = \frac{10x}{9}$ ...(ii)

Using (ii) in (i), we get

$$4x^2 - \frac{100x^2}{9} = 36 \Rightarrow -64x^2 = 324$$

which gives imaginary x.

Hence, there is no point on the curve at which tangent is perpendicular to the given line.

116. (c) $y^2 = x(2-x)^2 \Rightarrow y^2 = x^3 - 4x^2 + 4x$ ...(i)

$$\Rightarrow 2y\frac{dy}{dx} = 3x^2 - 8x + 4 \Rightarrow \frac{dy}{dx} = \frac{3x^2 - 8x + 4}{2y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_P = \frac{3-8+4}{2} = -\frac{1}{2}$$

∴ Equation of tangent at P is

$$y - 1 = -\frac{1}{2}(x-1)$$

$$\Rightarrow x + 2y - 3 = 0$$

Using $y = \frac{3-x}{2}$ in (i), we get

$$\left(\frac{3-x}{2}\right)^2 = x^3 - 4x^2 + 4x$$

$$\Rightarrow 4x^3 - 17x^2 + 22x - 9 = 0 \quad \text{...(ii)}$$

which has two roots 1, 1

(Because of (ii) being tangent at (1, 1)).

$$\text{Sum of 3 roots} = \frac{17}{4}$$

$$\therefore \text{3rd root} = \frac{17}{4} - 2 = \frac{9}{4}$$

$$\text{Then, } y = \frac{3-\frac{9}{4}}{2} = \frac{3}{8}$$

$$\therefore \text{Q is } \left(\frac{9}{4}, \frac{3}{8}\right)$$

117. (d) We have,

$$y = \frac{ax-b}{(x-1)(x-4)} = \frac{ax-b}{x^2-5x+4} \quad \text{... (i)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2-5x+4)a - (ax-b)(2x-5)}{(x^2-5x+4)^2} \quad \text{... (ii)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{P(2,1)} = \frac{(4-10+4)a-(2a-b)(4-5)}{(4-10+4)^2} = -\frac{b}{4}$$

Since P is a turning point of the curve (i). Therefore,

$$\left(\frac{dy}{dx}\right)_P = 0 \Rightarrow -\frac{b}{4} = 0 \Rightarrow b = 0 \quad \text{...(iii)}$$

Since P(2, – 1) lies on $y = \frac{ax-b}{(x-1)(x-4)}$. Therefore

$$-1 = \frac{2a-b}{(2-1)(2-4)} \Rightarrow -1 = \frac{2a-b}{-2} \Rightarrow 2a - b = 2 \quad \text{...(iv)}$$

From (iii) and (iv), we get a = 1, b = 0.

118. (b) $\frac{\Delta r}{r} \times 100 = k$ (Given)

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{dr} = 4\pi r^2$$

$$\Delta V = \frac{dV}{dr} \times \Delta r$$

$$\Rightarrow \Delta V = 4\pi r^2 \Delta r$$

$$\Rightarrow \Delta V = 4\pi r^2 \frac{kr}{100}$$

$$\Rightarrow \Delta V = 4\pi r^3 \frac{k}{100}$$

$$\Rightarrow \Delta V \times 100 = \frac{4\pi r^3}{4/3\pi r^3}\frac{k}{100} \times 100 = 3k\%$$

119. (c) Let r be the radius of the sphere and Δr be the error in measuring the radius. Then,

r = 9 cm and Δr = 0.03 cm

Let V be the volume of the sphere. Then,

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$$

$$\Rightarrow \left(\frac{dV}{dr}\right)_{r=9} = 4\pi \times 9^2 = 324\pi$$

Let ΔV be the error in V due to error Δr in r. Then,

$$\Delta V = \frac{dV}{dr}\Delta r$$

$\Rightarrow \Delta V = 324\pi \times 0.03 = 9.72\,\pi\ cm^3$

Thus, the approximate error in calculating the volume is $9.72\pi\ cm^3$.

120. (b) Let the length, width and height of the open tank be x, x and y units respectively. Then, its volume is x^2y and the total surface area is $x^2 + 4xy$.

It is given that the tank can hold a given quantity of water. This means that its volume is constant. Let it be V.

$\therefore V = x^2y$ … (i)

The cost of material will be least if the total surface area is least. Let S denote the total surface area. Then,

$S = x^2 + 4xy$ … (ii)

We have to minimize S subject to the condition that the volume is constant.

Now,

$$S = x^2 + 4xy$$

$$\Rightarrow S = x^2 + \frac{4V}{x}$$

$$\Rightarrow \frac{dS}{dx} = 2x - \frac{4V}{x^2} \text{ and } \frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$$

For maximum or minimum values of S, we must have

$$\frac{dS}{dx} = 0$$

$$\Rightarrow 2x - \frac{4V}{x^2} = 0 \Rightarrow 2x^3 = 4V$$

$$\Rightarrow 2x^3 = 4x^2y$$

$$\Rightarrow x = 2y$$

Clearly, $\frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3} > 0$ for all x.

Hence, S is minimum when x = 2y i.e. the depth (height) of the tank is half of its width.

121. (d) We have,

$P(x) = 41 + 24x - 18x^2$

$$\Rightarrow \frac{dP(x)}{dx} = 24 - 36x \text{ and } \frac{d^2P(x)}{dx^2} = -36$$

For maximum or minimum, we must have

$$\Rightarrow \frac{dP(x)}{dx} = 0 \Rightarrow 24 - 36x = 0 \Rightarrow x = \frac{2}{3}$$

Also, $\left(\frac{d^2P(x)}{dx^2}\right)_{x=\frac{2}{3}} = -36 < 0$

So, profit is maximum when $x = \frac{2}{3}$.

Maximum profit = (Value of P(x) at $x = \frac{2}{3}$)

$$= 41 + 24 \times \left(\frac{2}{3}\right) - 18\left(\frac{2}{3}\right)^2 = 49$$

122. (c) A point on the parabola is at a minimum distance from the circle if and only if it is at a minimum distance from the centre of the circle. A point on the parabola $y^2 = 8x$ is of the type $P(2t^2, 4t)$. Centre C of circle $x^2 + (y + 6)^2 = 1$ is (0, –6).

$\therefore CP^2 = 4t^4 + (4t + 6)^2 = 4(t^4 + 4t^2 + 12t + 9)$

$$\Rightarrow \frac{d}{dx}(CP)^2 = 4(4t^3 + 8t + 12) = 16(t+1)(t^2 - t + 3)$$

Also $\frac{d^2}{dt^2}(CP^2) = 48t^2 + 32$

$\frac{d}{dt}(CP^2) = 0 \Rightarrow t = -1$ (real value)

and $\left.\frac{d^2}{dt^2}(CP^2)\right|_{t=1} = 80 > 0$

$\therefore$ Required point is (2, – 4).

123. (b) Let r be the radius of the base and h be the height of the cylinder ABCD which is inscribed in a sphere of radius a. It is obvious that for maximum volume the axis of the cylinder must be along the diameter of the sphere. Let O be the centre of the sphere such that OL = x. Then

$OA^2 = OL^2 + AL^2$

$$\Rightarrow AL = \sqrt{a^2 - x^2}$$

Let V be the volume of cylinder. Then,

$$V = \pi(AL)^2 \times LM$$

$$\Rightarrow V = \pi(AL)^2 \times 2(OL)$$

$$= \pi(a^2 - x^2) \times 2x$$

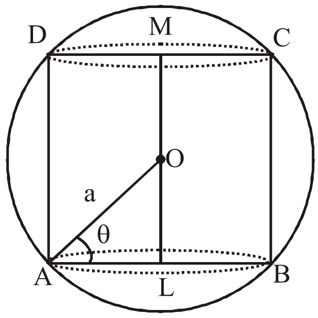
$$\Rightarrow V = 2\pi(a^2x - x^3)$$

$$\Rightarrow \frac{dV}{dx} = 2\pi(a^2 - 3x^2)$$

and $\frac{d^2V}{dx^2} = -12\pi x$

For maximum or minimum values of V, we must have

$$\frac{dV}{dx} = 0$$



$\Rightarrow 2\pi\,(a^2 - 3x^2) = 0 \Rightarrow x = \dfrac{a}{\sqrt{3}}$

$$\left[\text{neglecting } x = \frac{-a}{\sqrt{3}} \because \left.\frac{d^2V}{dx^2}\right|_{x=\frac{-a}{\sqrt{3}}} > 0\right]$$

Clearly, $\left(\dfrac{d^2V}{dx^2}\right)_{x=a/\sqrt{3}} = -12\pi \times \dfrac{a}{\sqrt{3}} < 0$

$\therefore$ V is maximum when $x = \dfrac{a}{\sqrt{3}}$

Hence, height of the cylinder LM = 2x = $\dfrac{2a}{\sqrt{3}}$

124. (c) Let $y = \left(\dfrac{1}{x}\right)^x$

$\Rightarrow \log y = x\log\left(\dfrac{1}{x}\right) \Rightarrow \log y = -\log x$

differentiating w.r.t.x

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = -(1+\log x) \Rightarrow \dfrac{dy}{dx} = -y(1+\log x)$

$\Rightarrow \dfrac{d^2y}{dx^2} = -\dfrac{dy}{dx}(1+\log x) - \dfrac{y}{x}$

$\dfrac{d^2y}{dx^2} = y(1+\log x)^2 - \dfrac{y}{x}$

$\dfrac{d^2y}{dx^2} = \left(\dfrac{1}{x}\right)^x (1+\log x)^2 - \dfrac{1}{x(x+1)}$

For maximum value $\dfrac{dy}{dx} = 0$

$\Rightarrow -y(1 + \log x) = 0$

$\Rightarrow 1 + \log x = 0 \; (\because y \neq 0) \Rightarrow \log x = -1$

$\Rightarrow x = e{-}1 \Rightarrow x = 1/e$

Also $\left[\dfrac{d^2y}{dx^2}\right]_{x=\frac{1}{e}} = e^{1/e}\left(1+\log\dfrac{1}{e}\right)^2 - e^{(1/e+1)}$

$= e^{1/e}(1-\log e)^2 - e^{1/e+1} = -e^{1/e+1} < 0$

So, x = 1/e is a point of local maxima.

Hence, local maximum value y= $(e)^{1/e}$.

CHAPTER 23

INTEGRALS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. $\int x^x (1+\log x)dx$ is equal to

(a) x^x (b) x^{2x}

(c) $x^x \log x$ (d) $1/2\ (1 + \log x)^2$

2. $\int x^{51}(\tan^{-1} x + \cot^{-1} x)\,dx$

(a) $\frac{x^{52}}{52}(\tan^{-1} x + \cot^{-1} x) + c$

(b) $\frac{x^{52}}{52}(\tan^{-1} x - \cot^{-1} x) + c$

(c) $\frac{\pi x^{52}}{104} + \frac{\pi}{2} + c$

(d) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$

3. Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}}dx = \frac{2}{3} gof(x) + C$, then

(a) $f(x) = \sqrt{x}$

(b) $f(x) = x^{3/2}$ and $g(x) = \sin^{-1}x$

(c) $f(x) = x^{2/3}$

(d) None of these

4. $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\,dx =$

(a) $-3(\tan x)^{1/3} + c$ (b) $-3(\tan x)^{-1/3} + c$

(c) $3(\tan x)^{-1/3} + c$ (d) $(\tan x)^{-1/3} + c$

5. $\int_{\log 1/2}^{\log 2} \sin\left\{\frac{e^x - 1}{e^x + 1}\right\}dx$ equals

(a) $\cos\frac{1}{3}$ (b) $\sin\frac{1}{2}$

(c) $2 \cos 2$ (d) 0

6. Evaluate: $\int 2^{2^{2^x}} 2^{2^x} 2^x\,dx$

(a) $\frac{1}{(\log 2)^3} 2^{2^{2^x}} + C$ (b) $\frac{1}{(\log 2)^3} 2^{2^x} + C$

(c) $\frac{1}{(\log 2)^2} 2^{2^x} + C$ (d) $\frac{1}{(\log 2)^4} 2^{2^{2x}} + C$

7. $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}}dx$ is equal to

(a) $10^x - x^{10} + C$ (b) $10^x + x^{10} + C$

(c) $(10^x - x^{10})^{-1} + C$ (d) $\log_e(\log^x + x^{10}) + C$

8. If $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x) + C$, then f(x) is equal to

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$

(c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

9. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ is equal to

(a) $-e^x \tan\left(\frac{x}{2}\right) + C$ (b) $-e^x \cot\left(\frac{x}{2}\right) + C$

(c) $-\frac{1}{2}e^x \tan\left(\frac{x}{2}\right) + C$ (d) $\frac{1}{2}e^x \cot\left(\frac{x}{2}\right) + C$

10. Evaluate $\int_1^2 x^2 dx$ as limit of sums.

(a) 1 (b) $\frac{7}{3}$

(c) $\frac{1}{3}$ (d) 0

11. Evaluate: $\int_0^{\pi/2} \frac{\cos x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^3}dx$

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$

(c) $3 + \sqrt{3}$ (d) $3 - \sqrt{3}$

12. $\int \frac{x^9}{\left(4x^2+1\right)^6}dx$ is equal to

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C$

(c) $\frac{1}{10x}\left(\frac{1}{x}+4\right)^{-5}+C$ (d) $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+C$

13. $\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is equal to

(a) $\frac{1}{8}(x^2-1)+k$ (b) $\frac{1}{2}x^2+k$

(c) $\frac{1}{2}x+k$ (d) None of these

14. $\int e^{3\log x}(x^4+1)^{-1}dx$ is equal to

(a) $\log(x^4+1)+C$ (b) $\frac{1}{4}\log(x^4+1)+C$

(c) $-\log(x^4+1)+C$ (d) None of these

15. The value of integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(a) 1/2 (b) 3/2 (c) 2 (d) 1

16. The value of $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ is

(a) $\frac{1}{2}\log\left|e^x+e^{-x}\right|+C$ (b) $2\log\left|e^{2x}+e^{-2x}\right|+C$

(c) $\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$ (d) None of these

17. $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$ equals

(a) $-\cot(ex^x)+C$ (b) $\tan(xe^x)+C$
(c) $\tan(e^x)+C$ (d) $\cot(e^x)+C$

18. $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ equal to

(a) $\frac{1}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$ (b) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$

(c) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x}{a}}+C$ (d) None of these

19. The value of $\int \sqrt{\frac{a-x}{a+x}}dx$ is

(a) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{x^2-a^2}+C$

(b) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$

(c) $a\sin^{-1}\left(\frac{a}{x}\right)+\sqrt{x^2-a^2}+C$

(d) None of these

20. If $\int \sin^3 x\cos^5 x\,dx = A\sin^4 x+B\sin^6 x+C\sin^8 x+D$. Then

(a) $A=\frac{1}{4},\ B=-\frac{1}{3},\ C=\frac{1}{8},\ D\in R$

(b) $A=\frac{1}{8},\ B=\frac{1}{4},\ C=\frac{1}{3},\ D\in R$

(c) $A=0,\ B=-\frac{1}{6},\ C=\frac{1}{8},\ D\in R$

(d) None of these.

21. $\int\left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$ is equal to :

(a) $\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$ (b) $\left[\frac{x^2+1}{x^2}\right]^{n+6}(n+6)+c$

(c) $\left[\frac{x}{x^2+1}\right]^{n+6}(n+6)+c$ (d) None of these

22. Value of $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}dx$ is

(a) $\log\left(\frac{1+\sqrt{3}}{5+3\sqrt{3}}\right)$ (b) $\log\left(\frac{5-3\sqrt{3}}{1-\sqrt{3}}\right)$

(c) $\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$ (d) None of these

23. $\int \sin 2x.\log\cos x\,dx$ is equal to

(a) $\cos^2 x\left(\frac{1}{2}+\log\cos x\right)+k$

(b) $\cos^2 x.\log\cos x+k$

(c) $\cos^2 x\left(\frac{1}{2}-\log\cos x\right)+k$

(d) None of these.

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

24. Which of the following is/are correct?

I. $\int \frac{dx}{x\sqrt{x^2-1}}=\text{cosec}^{-1}x+C$

II. $\int e^x dx=\log e^x+C$

III. $\int \frac{1}{x}dx=\log|x|+C$

IV. $\int a^x dx=a^x+C$

(a) I and III are correct (b) All are correct
(c) Only III is correct (d) All are incorrect

25. Consider the following statements

Statement-I: The value of $\int \frac{dx}{\sqrt{16-9x^2}}$ is $\frac{1}{3}\sin^{-1}\frac{3x}{4}+C$

Statement-II: The value of $\int \frac{dt}{\sqrt{3t-2t^2}}$ is $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{3-4t}{3}\right)+C$.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

26. Consider the following statements

Statement-I : The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is $\frac{22}{7}-\pi$.

Statement-II : The value of integral $\int_{-1}^{1} \frac{|x+2|}{x+2}dx$ is 2.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

27. Consider the following statements

Statement-I: $\int_0^{\lambda} \frac{y\,dy}{\sqrt{y+\lambda}}$ is equal to $\frac{2}{3}(2+\sqrt{2})\lambda\sqrt{\lambda}$.

Statement-II: $3a\int_0^1\left(\frac{ax-1}{a-1}\right)^2 dx$ is equal to $(a-1)+(a-1)^2$.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

28. Consider the following statements

Statement-I: $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if f is an odd function i.e., $f(-x) = -f(x)$.

Statement-II: $\int_{-a}^{a} f(x)dx = 0$, if f is an even function i.e., if $f(-x) = f(x)$.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

29. Match the following derivatives of the functions in column-I with their respective anti-derivatives in column-II.

Column - I	Column - II
A. $\frac{1}{\sqrt{1-x^2}}$	1. $\tan^{-1}x+C$
B. $\frac{-1}{\sqrt{1-x^2}}$	2. $\cot^{-1}x+C$
C. $\frac{1}{1+x^2}$	3. $\sin^{-1}x+C$
D. $\frac{-1}{1+x^2}$	4. $\cos^{-1}x+C$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	3	4	2	1
(c)	3	4	1	2
(d)	4	3	2	1

30. Match the following integrals in column-I with their corresponding values in column-II.

Column-I	Column-II
A. $\int\sqrt{ax+b}\,dx$	1. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2}+C$
B. $\int x\sqrt{x+2}\,dx$	2. $\frac{1}{6}(1+2x^2)^{3/2}+C$
C. $\int x\sqrt{1+2x^2}\,dx$	3. $\frac{4}{3}(x^2+x+1)^{3/2}+C$
D. $\int(4x+2)\sqrt{x^2+x+1}\,dx$	4. $\frac{2}{3a}(ax+b)^{3/2}+C$

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	3	4	2	1
(c)	1	3	2	4
(d)	3	2	4	1

31. Match the following integrals in column-I with their corresponding solutions in column-II.

Column - I	Column - II
A. $\int \frac{\cos x - \sin x}{1+\sin 2x} dx$	1. $\frac{1}{6}\sec^3 2x - \frac{1}{2}\sec^2 2x + C$
B. $\int \tan^3 2x \sec 2x \, dx$	2. $\tan x + C$
C. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$	3. $\frac{-1}{\sin x + \cos x} + C$
D. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$	4. $\sec x - \operatorname{cosec} x + C$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	3	1	4	2
(c)	3	4	1	2
(d)	2	1	4	3

32. Match the following definite integrals in column-I with their corresponding values in column-II.

Column - I	Column - II
A. $\int_{-1}^{1} x^{17}\cos^4 x \, dx$	1. $\frac{\pi}{2} - 1$
B. $\int_{0}^{\pi/2} \sin^3 x \, dx$	2. 0
C. $\int_{0}^{\pi/4} 2\tan^3 x \, dx$	3. $\frac{2}{3}$
D. $\int_{0}^{1} \sin^{-1} x \, dx$	4. $1 - \log 2$

Codes

	A	B	C	D
(a)	1	3	2	4
(b)	2	3	4	1
(c)	1	2	3	4
(d)	2	4	3	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

33. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x+x^2}\right) dx$ is

(a) 1 (b) 0 (c) – 1 (d) $\frac{\pi}{4}$

34. $\int_0^{2\pi} \log\left(\frac{a+b\sec x}{a-b\sec x}\right) dx =$

(a) 0 (b) $\pi/2$

(c) $\frac{\pi(a+b)}{a-b}$ (d) $\frac{\pi}{2}(a^2-b^2)$

35. Value of $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$ is

(a) 2 (b) 3 (c) 4 (d) 5

36. The value of $\int_{-1}^{1} (x-[x]) dx$ (where [.] denotes greatest integer function) is

(a) 0 (b) 1 (c) 2 (d) None of these

37. The value of definite integral $\int_0^{\frac{\pi}{2}} \log(\tan x) \, dx$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

38. If m is an integer, then $\int_0^{\pi} \frac{\sin(2mx)}{\sin x} dx$ is equal to :

(a) 1 (b) 2 (c) 0 (d) π

39. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) – 2

40. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is

(a) 1 (b) 0 (c) – 1 (d) $\frac{\pi}{4}$

41. If $\int \cos^n x \sin x \, dx = -\frac{\cos^6 x}{6} + C$, then n =

(a) 0 (b) 1 (c) 2 (d) 5

42. If $\int \frac{3x+1}{(x-3)(x-5)} dx = \int \frac{-5}{(x-3)} dx + \int \frac{B}{(x-5)} dx$,

then the value of B is

(a) 3 (b) 4 (c) 6 (d) 8

43. If $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{3} + c$, then a =

(a) 3 (b) 4 (c) 6 (d) 8

44. If $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \log(e^{2x} - 1) - Ax + C$,

then A =

(a) 0 (b) 1 (c) 2 (d) 5

45. $\int_{-a}^{4}(x^8-x^4+x^2+1)\,dx = 2\int_{0}^{4}(x^8-x^4+x^2+1)\,dx$,
then a =
(a) 3 (b) 4 (c) 6 (d) 8

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

46. **Assertion :** $I = \int_0^{\frac{\pi}{2}} \sqrt{\tan x}\,dx = \frac{\pi}{\sqrt{2}}$
Reason: $\tan x = t^2$ makes the integrand in I as a rational function.

47. **Assertion :** $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right)dx = 0$.
Reason : If f is an odd function, then $\int_{-a}^{a} f(x)dx = 0$.

48. **Assertion :** If the derivative of function x is $\frac{d}{dx}(x) = 1$, then its anti-derivatives or integral is $\int(1)dx = x + C$.
Reason : If $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$, then the corresponding integral of the function is $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$.

49. **Assertion:** It is not possible to find $\int e^{-x^2}\,dx$ by inspection method.
Reason : Function is not expressible in terms of elementary functions.

50. **Assertion :** If $\frac{d}{dx}\int f(x)dx = f(x)$, then $\int f(x)\,dx = f'(x) + C$ where C is an arbitrary constant.
Reason : Process of differentiation and integration are inverses of each other.

51. **Assertion :** Geometrically, derivative of a function is the slope of the tangent to the corresponding curve at a point.
Reason : Geometrically, indefinite integral of a function represents a family of curves parallel to each other.

52. **Assertion :** Derivative of a function at a point exists.
Reason : Integral of a function at a point where it is defined, exists.

53. **Assertion :** $\int[\sin(\log x)+\cos(\log x)]dx = x\sin(\log x) + C$
Reason : $\frac{d}{dx}[x\sin(\log x)] = \sin(\log x) + \cos(\log x)$.

54. **Assertion :** The value of $\int_a^b f(t)dt$ and $\int_a^b f(u)du$ are equal
Reason : The value of definite integral of a function over any particular interval depends on the function and the interval not on the variable of integration.

55. **Assertion :** $\int_0^{\pi} x\sin x\cos^2 x\,dx = \frac{\pi}{2}\int_0^{\pi}\sin x\cos^2 x\,dx$
Reason : $\int_a^b x\,f(x)\,dx = \frac{a+b}{2}\int_a^b f(x)\,dx$

56. **Assertion :** The value of the integral $\int e^x[\tan x + \sec^2 x]dx$ is $e^x \tan x + C$
Reason : The value of the integral $e^x\{f(x)+f'(x)\}dx$ is $e^x f(x) + C$.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

57. If $f(a+b-x) = f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$ (b) $\frac{a+b}{2}\int_a^b f(b+x)\,dx$
(c) $\frac{b-a}{2}\int_a^b f(x)\,dx$ (d) $\frac{a+b}{2}\int_a^b f(x)\,dx$

58. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, a > 0, is
(a) π (b) a π (c) π/2 (d) 2 π

59. Evaluate: $\int \sin^3 x\cos^3 x\,dx$

(a) $\frac{1}{32}\left\{\frac{3}{2}\cos 2x - \frac{1}{6}\cos 6x\right\} + C$
(b) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x + \frac{1}{6}\cos 6x\right\} + C$
(c) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x - \frac{1}{6}\cos 6x\right\} + C$
(d) None of these

60. Evaluate: $\int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$

(a) $\frac{2}{\sqrt{\tan x}} - \frac{2}{3}(\tan x)^{3/2} + C$

(b) $-\frac{2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + C$

(c) $-\frac{2}{\sqrt{\tan x}} - \frac{2}{3}(\tan x)^{3/2} + C$

(d) None of these

61. Evaluate: $\int \frac{1}{\sqrt{9+8x-x^2}} dx$

(a) $-\sin^{-1}\left(\frac{x-4}{5}\right)+C$ (b) $-\sin^{-1}\left(\frac{x+4}{5}\right)+C$

(c) $\sin^{-1}\left(\frac{x-4}{5}\right)+C$ (d) None of these

62. Evaluate: $\int \frac{1}{1+3\sin^2 x+8\cos^2 x} dx$

(a) $\frac{1}{6}\tan^{-1}(2\tan x)+C$ (b) $\tan^{-1}(2\tan x)+C$

(c) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$ (d) None of these

63. Evaluate: $\int \frac{x^3+x}{x^4-9} dx$

(a) $\frac{1}{4}\log\left|x^4-9\right|+\frac{1}{12}\log\left|\frac{x^2+3}{x^2-3}\right|+C$

(b) $\frac{1}{4}\log\left|x^4-9\right|-\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+C$

(c) $\frac{1}{4}\log\left|x^4-9\right|+\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+C$

(d) None of these

64. If $\int \frac{3x+4}{x^3-2x-4} dx = \log|x-2| + k\log f(x) + c$, then

(a) $f(x) = |x^2 + 2x + 2|$ (b) $f(x) = x^2 + 2x + 2$

(c) $k = -\frac{1}{2}$ (d) All of these

65. Evaluate: $\int \frac{1-\cos x}{\cos x(1+\cos x)} dx$

(a) $\log|\sec x+\tan x|-2\tan(x/2)+C$

(b) $\log|\sec x-\tan x|-2\tan(x/2)+C$

(c) $\log|\sec x+\tan x|+2\tan(x/2)+C$

(d) None of these

66. Evaluate: $\int_0^{\pi} \frac{1}{5+4\cos x} dx$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

67. If $\int_0^{\pi} \ln \sin x \, dx = k$, then value of $\int_0^{\pi/4} \ln(1+\tan x) dx$ is

(a) $-\frac{k}{4}$ (b) $\frac{k}{4}$ (c) $-\frac{k}{8}$ (d) $\frac{k}{8}$

68. $\int \tan^{-1}\sqrt{x}\, dx$ is equal to

(a) $(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(b) $x\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(c) $\sqrt{x}-x\tan^{-1}\sqrt{x}+C$

(d) $\sqrt{x}-(x+1)\tan^{-1}\sqrt{x}+C$

69. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} dx$ is equal to

(a) $\frac{1}{2}\sin 2x+c$ (b) $-\frac{1}{2}\sin 2x+c$

(c) $-\frac{1}{2}\sin x+c$ (d) $-\sin^2 x+c$

70. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log\sin(x-\alpha)+C$, then value of (A, B) is

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

71. If $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ \sqrt{x}, & 1 \le x \le 2 \end{cases}$ then $\int_0^2 f(x)dx =$

(a) $\frac{1}{3}$ (b) $4\sqrt{2}$

(c) $4\sqrt{2}-1$ (d) None of these

72. If $g(x) = \int_0^x \cos^4 t \, dt$, then $g(x+\pi)$ equals

(a) $g(x) + g(\pi)$ (b) $g(x) - g(\pi)$

(c) $f(x)\, g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

73. The integral $\int_0^{\pi/2} |\sin x - \cos x| dx$ is equal to :

(a) $2\sqrt{2}$ (b) $2(\sqrt{2}-1)$

(c) $\sqrt{2}+1$ (d) None of these

74. $\int_{\pi/4}^{3\pi/4} \frac{\phi d\phi}{1+\sin\phi}$ is equal to

(a) $\sqrt{2}-1$ (b) $\frac{1}{\sqrt{2}-1}$

(c) $\frac{\pi}{\sqrt{2}+1}$ (d) $\frac{\pi}{\sqrt{2}-1}$

75. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is

(a) $\frac{\pi^2}{4}$ (b) π^2 (c) zero (d) $\frac{\pi}{2}$

76. If $\int \frac{1}{(\sin x+4)(\sin x-1)}dx$

$= A\frac{1}{\left(\tan\frac{x}{2}-1\right)}+B\tan^{-1}\left[f(x)\right]+C_1$, then

(a) $A=-\frac{1}{5},\ B=\frac{-2}{5\sqrt{15}},\ f(x)=\frac{4\tan x+3}{\sqrt{15}}$

(b) $A=-\frac{1}{5},\ B=\frac{1}{\sqrt{15}},\ f(x)=\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}$

(c) $A=\frac{2}{5},\ B=-\frac{2}{5},\ f(x)=\frac{4\tan x+1}{5}$

(d) $A=\frac{2}{5},\ B=\frac{-2}{5\sqrt{15}},\ f(x)=\frac{4\tan\left(\frac{x}{2}\right)+f}{\sqrt{15}}$

77. If f and g are defined as f (x) = f (a – x) and g (x) + g (a – x) = 4, then $\int_0^a f(x)g(x)dx$ is equal to

(a) $\int_0^a f(x)dx$ (b) $2\int_0^a f(x)dx$

(c) $\int_0^a g(x)dx$ (d) $2\int_0^a g(x)dx$

78. Value of $\int \frac{dx}{\sqrt{x(a-x)}}$ is

(a) $2\sin^{-1}\sqrt{\frac{x}{a}}+c$ (b) $-2\sin^{-1}\sqrt{\frac{x}{a}}+c$

(c) $2\sin^{-1}\frac{\sqrt{x}}{a}+c$ (d) None of these

79. Value of $\int_0^{\pi}|\cos x|\,dx$ is

(a) 2 (b) –2 (c) 1 (d) None of these

80. Value of $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$

(c) $\frac{\pi}{4}$ (d) None of these

81. Value of $\int \frac{(x-x^3)^{1/3}}{x^4}dx$ is

(a) $\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}+C$ (b) $\frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{\frac{4}{3}}+C$

(c) $\frac{-3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}+C$ (d) None of these

82. Value of $\int \frac{x^2+1}{(x-1)(x-2)}dx$ is

(a) $x+\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$ (b) $x+\log\left[\frac{(x-1)^2}{(x-2)^5}\right]+C$

(c) $x-\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$ (d) None of these

83. Value of $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ if $(\beta>\alpha)$ is

(a) $2\sin^{-1}\sqrt{\frac{\beta-\alpha}{x-\alpha}}+C$ (b) $2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}}+C$

(c) $2\sin^{-1}\sqrt{\frac{x+\alpha}{\beta-\alpha}}+C$ (d) None of these

84. Value of $\int \frac{dx}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}$ is

(a) $\frac{-1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(b) $\frac{1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(c) $\frac{1}{22}\tan^{-1}\left(\frac{\tan x+2}{2}\right)+C$

(d) None of these

85. Value of $\int \frac{x^2+1}{x^4+x^2+1}dx$ is

(a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{x^2-1}\right)+C$ (b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{3}x}\right)+C$ (d) None of these

86. Value of $\int_0^1 \log\left(\frac{1}{x}-1\right)dx$ is

(a) 2I (b) –2I (c) 0 (d) None of these

87. Value of $\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\cot x}}dx$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{12}{\pi}$ (d) None of these

88. Value of $\int_{-1}^{2}\frac{|x|}{x}dx$ is

(a) 0 (b) 1 (c) –1 (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** $I=\int x^x(1+\log x)dx$

Put $x^x = t$, then $x^x(1+\log x)\,dx = dt$

$\therefore\quad I=\int dt \Rightarrow I = t + C \Rightarrow I = x^x + C.$

2. **(a)** $\int x^{51}(\tan^{-1}x+\cot^{-1}x)\,dx$

$=\int x^{51}.\frac{\pi}{2}dx \qquad \left\{\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right\}$

$=\frac{\pi x^{52}}{104}+c=\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c\,.$

3. **(b)** Put $x^{3/2}=t \Rightarrow \frac{3}{2}x^{1/2}dx=dt$

$\therefore$ integral is

$$\int\frac{\frac{2}{3}dt}{\sqrt{1-t^2}}=\frac{2}{3}\sin^{-1}t+C=\frac{2}{3}\sin^{-1}(x^{3/2})+C$$

4. **(b)** $\int \sec^{2/3}x\,\text{cosec}^{4/3}x\,dx=\int\frac{dx}{\sin^{4/3}x\cos^{2/3}x}$

Multiplying N^r and D^r by $\cos^2 x$, we get

{ Putting $\tan x = t \Rightarrow \sec^2 x\,dx = dt$}

$=\int\frac{\sec^2x\,dx}{\tan^{4/3}x}=\int\frac{dt}{t^{4/3}}=\frac{t^{-1/3}}{(-1/3)}+c=-3(\tan x)^{-1/3}+c.$

5. **(d)** $I=\int_{-\log 2}^{\log 2}\sin\left\{\frac{e^x-1}{e^x+1}\right\}dx$

If $f(x)=\sin\left\{\frac{e^x-1}{e^x+1}\right\}$

$f(-x)=\sin\left\{\frac{1-e^x}{1+e^x}\right\}=-\sin\left\{\frac{e^x-1}{e^x+1}\right\}=-f(x)$

Hence $f(x)$ is an odd function of x $\therefore I = 0$

6. **(a)** Let $I=\int 2^{2^{2^x}}2^{2^x}2^x\,dx$

Let $2^{2^{2^x}}=t \Rightarrow 2^{2^{2^x}}2^{2^x}2^x(\log 2)^3dx=dt$

$\Rightarrow I=\int\frac{1}{(\log 2)^3}dt=\frac{1}{(\log 2)^3}t+C=\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$

7. **(d)** Put $10^x + x^{10} = t$

$\therefore\quad (10^x\log_e 10+10x^9)\,dx=dt$

$\therefore\quad \int\frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx=\int\frac{dt}{t}$

$=\log_e t+c=\log_e(10^x+x^{10})+C$

8. **(c)** $\int e^x\frac{(1+\sin x)}{(1+\cos x)}dx=\int e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right]dx$

$=\frac{1}{2}\int e^x\sec^2\frac{x}{2}dx+\int e^x\tan\frac{x}{2}dx$

$=e^x\tan\frac{x}{2}+C$

But $I = e^x f(x) + C$ (given)

$\therefore\ f(x)=\tan\frac{x}{2}$

9. **(b)** $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\int e^x\left(\frac{1-\sin x}{2\sin^2\frac{x}{2}}\right)dx$

$=\int e^x\left(\frac{1}{2}\text{cosec}^2\frac{x}{2}-\cot\frac{x}{2}\right)dx$

$=\frac{1}{2}\int e^x\,\text{cosec}^2\frac{x}{2}dx-e^x\cot\frac{x}{2}-\frac{1}{2}\int e^x\,\text{cosec}^2\frac{x}{2}dx+C$

$=-e^x\cot\frac{x}{2}+C$

10. **(b)** Here, $a = 1$, $b = 2$, $f(x) = x^2$, $b - a = 1 = nh$

$\therefore\quad \int_1^2 x^2dx=\lim_{h\to\infty}\sum_{r=0}^{n-1}f(a+rh)$

$=\lim_{h\to 0}\left[h\left\{1^2+(1+h)^2+(1+2h)^2+...+(1+(n-1)h)^2\right\}\right]$

$=\lim_{h\to 0}\left[h\left\{\left(1^2+1^2+n\text{ times}\right)+h^2(1^2+2^2+......+(n-1)^2+2h(1+2+...+(n-1))\right)\right\}\right]$

$=\lim_{h\to 0}h\left\{n+h^2\frac{(n-1)n(2n-1)}{6}+2h\frac{(n-1)n}{2}\right\}$

$=\lim_{h\to 0}\left\{nh+\frac{(nh-h)nh(2nh-h)}{6}+\frac{2(hn)(nh-h)}{2}\right\}$

$=1+\frac{1}{3}+1=\frac{7}{3}$ (as $n\to\infty$, $h\to 0$)

11. **(a)** We have,

$I=\int_0^{\pi/2}\frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$

$=\int_0^{\pi/2}\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx=\int_0^{\pi/2}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}dx$

Let $\cos\frac{x}{2}+\sin\frac{x}{2}=t$. Then,

$$\frac{1}{2}\left(-\sin\frac{x}{2}+\cos\frac{x}{2}\right)dx=dt$$

$$\Rightarrow \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx=2dt$$

Also, $x=0\Rightarrow t=1$ and $x=\frac{\pi}{2}\Rightarrow t=\sqrt{2}$

$$\therefore\ I=\int_1^{\sqrt{2}}\frac{2dt}{t^2}=2\int_1^{\sqrt{2}}\frac{1}{t^2}dt$$

$$=2\left[-\frac{1}{t}\right]_1^{\sqrt{2}}=2\left[-\frac{1}{\sqrt{2}}+1\right]=\left(2-\sqrt{2}\right)$$

12. (d) We have, $I=\int\frac{x^9}{\left(4x^2+1\right)^6}dx$

$$=\int\frac{x^9dx}{x^{12}\left(4+\frac{1}{x^2}\right)^6}=\int\frac{dx}{x^3\left(4+\frac{1}{x^2}\right)^6}$$

Put $4+\frac{1}{x^2}=t\Rightarrow\frac{-2}{x^3}dx=dt$

$$I=-\frac{1}{2}\int\frac{dt}{t^6}=\frac{-1}{2}\int t^{-6}dt$$

$$=-\frac{1}{2}\frac{t^{-5}}{-5}+C=\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+C$$

13. (b) Put $x=\cos 2\theta$

$$\therefore\ I=\int\cos\{2\tan^{-1}\tan\theta\}(-2\sin 2\theta)d\theta$$

$$=-\int\sin 4\theta\, d\theta=\frac{1}{4}\cos 4\theta+c$$

$$=\frac{1}{4}(2x^2-1)+c=\frac{1}{2}x^2+k$$

14. (b) $\int e^{3\log x}(x^4+1)^{-1}dx=\int e^{\log x^3}\frac{1}{x^4+1}dx$

$$=\int\frac{x^3}{x^4+1}dx=\frac{1}{4}\log(x^4+1)+C$$

[since $e^{\log_e x^3}=x^3$]

15. (b) $I=\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

$$I=\int_3^6\frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx$$

$$2I=\int_3^6 dx=3\ \Rightarrow\ I=\frac{3}{2}$$

16. (c) $\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$

Let $e^{2x}+e^{-2x}=t$

$$\Rightarrow\ 2e^{2x}-2e^{-2x}=\frac{dt}{dx}\Rightarrow dx=\frac{dt}{2\left(e^{2x}-e^{-2x}\right)}$$

$$\therefore\ \int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx=\int\frac{e^{2x}-e^{-2x}}{t}=\frac{dt}{2\left[e^{2x}-e^{-2x}\right]}$$

$$=\frac{1}{2}\int\frac{1}{t}dt=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

17. (b) $\int\frac{e^x(1+x)}{\cos^2\left(e^x x\right)}dx$

Let $xe^x=t$

$$\Rightarrow\ \left(xe^x+e^x\right)=\frac{dt}{dx}$$

$$\Rightarrow\ dx=\frac{dt}{e^x(x+1)}$$

$$\therefore\ \int\frac{e^x(1+x)}{\cos^2\left(e^x x\right)}dx=\int\frac{e^x(1+x)}{\cos^2 t}\times\frac{dt}{e^x(1+x)}$$

$$=\int\frac{1}{\cos^2 t}dt=\int\sec^2 t\,dt$$

$$=\tan t+C=\tan\left(xe^x\right)+C$$

18. (b) We have, $I=\int\frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$

$$I=\int\frac{\sqrt{x}}{\sqrt{\left(a^{3/2}\right)^2-\left(x^{\frac{3}{2}}\right)^2}}dx$$

Put, $x^{\frac{3}{2}}=t$

$$\Rightarrow\ \frac{3}{2}x^{\frac{1}{2}}dx=dt$$

$$\Rightarrow\ \sqrt{x}\,dx=\frac{2}{3}dt$$

$$\therefore\ I=\frac{2}{3}\int\frac{dt}{\sqrt{\left(a^{\frac{3}{2}}\right)^2-t^2}}=\frac{2}{3}\sin^{-1}\left(\frac{t}{a^{\frac{3}{2}}}\right)+C$$

$$=\frac{2}{3}\sin^{-1}\left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)+C=\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$$

19. **(b)** $I=\int\sqrt{\frac{a-x}{a+x}}dx=\int\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}dx=\int\frac{a-x}{\sqrt{a^2-x^2}}dx$

$\Rightarrow\ I=\int\frac{a}{\sqrt{a^2-x^2}}dx-\int\frac{x}{\sqrt{a^2-x^2}}dx$

$\Rightarrow\ I=a\int\frac{1}{\sqrt{a^2-x^2}}dx+\frac{1}{2}\int\frac{-2x}{\sqrt{a^2-x^2}}dx$

Putting $a^2-x^2=t$, and $-2x\,dx=dt$, we get

$I=a\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\int\frac{dt}{\sqrt{t}}=a\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right)+C$

$\Rightarrow I=a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{t}+C=a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$

20. **(a)** $I=\int\sin^3 x.\cos^5 x\,dx$

Put $\sin x=t\Rightarrow\cos x\,dx=dt$

$I=\int\sin^3 x\,.\cos^4 x\,.\cos x\,dx=\int t^3\left(1-t^2\right)^2dt$

$=\int\left(t^3-2t^5+t^7\right)dt=\frac{1}{4}t^4-\frac{2}{6}t^6+\frac{1}{8}t^8+D$

$=\frac{1}{4}\sin^4 x-\frac{1}{3}\sin^6 x+\frac{1}{8}\sin^8 x+D$

21. **(a)** $I=\int\left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$

Put $x+\frac{1}{x}=t\Rightarrow\left(1-\frac{1}{x^2}\right)dx=dt$

$\Rightarrow\left(\frac{x^2-1}{x^2}\right)dx=dt$

$\therefore\ I=\int t^{n+5}\,dt=\frac{t^{n+6}}{n+6}+c=\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$

22. **(c)** We know that,

$\int_0^4\frac{1}{\sqrt{x^2+2x+3}}dx=\int_0^4\frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}dx$

$=\left[\log\left|x+1+\sqrt{(x+1)^2+(\sqrt{2})^2}\right|\right]_0^4$

$=\left[\log\left|x+1+\sqrt{x^2+2x+3}\right|\right]_0^4$

$=\log(5+\sqrt{16+8+3})-\log(1+\sqrt{3})$

$=\log(5+3\sqrt{3})-\log(1+\sqrt{3})$

$=\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$

23. **(c)** $I=\int 2\sin x.\cos x.\log\cos x\,dx$

put $\log\cos x=t$

$\therefore\ -\frac{\sin x}{\cos x}dx=dt$

$I=\int 2\sin x.\cos x.t\frac{\cos x}{-\sin x}dt$

$=-2\int\cos^2 x.t\,dt=-2\int te^{2t}dt$

$=-2\left[t.\frac{e^{2t}}{2}-\int\frac{e^{2t}}{2}.dt\right]=-t\,e^{2t}+\frac{1}{2}e^{2t}+k$

$=e^{2t}\left(\frac{1}{2}-t\right)+k=\cos^2 x\left\{\frac{1}{2}-\log\cos x\right\}+k$

STATEMENT TYPE QUESTIONS

24. **(c)** I. $\frac{d}{dx}\left(-\operatorname{cosec}^{-1}x\right)=\frac{1}{x\sqrt{x^2-1}}$

$\int\frac{dx}{x\sqrt{x^2-1}}=-\operatorname{cosec}^{-1}x+C$

II. $\frac{d}{dx}\left(e^x\right)=e^x;\int e^x dx=e^x+C$

III. $\frac{d}{dx}(\log|x|)=\frac{1}{x};\int\frac{1}{x}dx=\log|x|+C$

IV. $\frac{d}{dx}\left(\frac{a^x}{\log a}\right)=a^x;\int a^x dx=\frac{a^x}{\log x}+C$

25. **(a)** I. We have,

$I=\int\frac{dx}{\sqrt{16-9x^2}}$

$=\frac{1}{3}\int\frac{dx}{\sqrt{\left(\frac{4}{3}\right)^2-x^2}}=\frac{1}{3}\sin^{-1}\left(\frac{x}{\frac{4}{3}}\right)+C$

$\left(\because\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{x}{a}+C\right)$

$=\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$

II. We have,

$I=\int\frac{dt}{\sqrt{3t-2t^2}}$

$=\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{-\left[\left(t^2\right)-\frac{3}{2}t+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2\right]}}$

$=\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2-\left(t-\frac{3}{4}\right)^2}}$

$=\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{t-\frac{3}{4}}{\frac{3}{4}}\right)=\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4t-3}{3}\right)+C$

26. (c) I. Let, $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$

$= \int_0^1 \frac{(x^4-1)(1-x^4)+(1-x)^4}{(1+x^2)}dx$

$= \int_0^1 (x^2-1)(1-x)^4 dx + \int_0^1 \frac{(1+x^2-2x)^2}{(1+x^2)}dx$

$= \int_0^1 \left\{(x^2-1)(1-x)^4+(1+x^2)-4x+\frac{4x^2}{(1+x^2)}\right\}dx$

$= \int_0^1 \left((x^2-1)(1-x)^4+(1+x^2)-4x+4-\frac{4}{1+x^2}\right)dx$

$= \int_0^1 \left(x^6-4x^5+5x^4-4x^2+4-\frac{4}{1+x^2}\right)dx$

$= \left[\frac{x^7}{7}-4\frac{x^6}{6}+\frac{5x^5}{5}-\frac{4x^3}{3}+4x-4\tan^{-1}x\right]_0^1$

$= \frac{1}{7}-\frac{4}{6}+\frac{5}{5}-\frac{4}{3}+4-4\left(\frac{\pi}{4}-0\right) = \frac{22}{7}-\pi$

II. Let $I = \int_{-1}^{1} \frac{|x+2|}{x+2}dx$

For $-1 \le x \le 1, |x+2| = 2+x$

$\therefore \ I = \int_{-1}^{1} \frac{x+2}{x+2}dx = \int_{-1}^{1} 1\,dx$

$= [x]_{-1}^{1} = 1-(-1) = 2$

27. (d) I. Let, $I = \int_0^\lambda \frac{y\,dy}{\sqrt{y+\lambda}} = \int_0^\lambda \left[\frac{y+\lambda-\lambda}{\sqrt{y+\lambda}}\right]dy$

$= \int_0^\lambda \left[\sqrt{y+\lambda}-\frac{\lambda}{\sqrt{y+\lambda}}\right]dy$

$= \int_0^\lambda (y+\lambda)^{1/2}dy - \int_0^\lambda \frac{\lambda}{\sqrt{y+\lambda}}dy$

$= \left[\frac{(y+\lambda)^{3/2}}{3/2}\right]_0^\lambda - \left[\frac{\lambda\sqrt{y+\lambda}}{1/2}\right]_0^\lambda$

$= \frac{2}{3}\left[(2\lambda)^{3/2}-\lambda^{3/2}\right]-2\lambda\left[(2\lambda)^{1/2}-(\lambda)^{1/2}\right]$

$= 2\lambda\sqrt{\lambda}\left[\frac{2\sqrt{2}-1}{3}-(\sqrt{2}-1)\right] = \frac{2}{3}\lambda\sqrt{\lambda}\left(2-\sqrt{2}\right)$

II. Let $I = 3a\int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx = \frac{3a}{(a-1)^2}\left[\frac{(ax-1)^3}{3}\times\frac{1}{a}\right]_0^1$

$= \frac{1}{(a-1)^2}\left[(a-1)^3+1\right] = (a-1)+(a-1)^{-2}$

28. (d) We have

$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$ Then

Let t = –x in the first integral on the right hand side. dt = – dx. When, x = – a, t = a and when x = 0, t = 0. Also x = – t

Therefore, $\int_{-a}^{a} f(x)dx = -\int_a^0 f(-t)dt + \int_0^a f(x)dx$

$= \int_0^a f(-x)dx + \int_0^a f(x)dx$ by $\left[\int_0^a f(t)dt = \int_0^a f(x)dx\right]$ ...(i)

I. Now, if f is an even function, then f(– x) = f(x) and so, eq. (i) becomes

$\int_{-a}^{a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(x)dx = 2\int_0^a f(x)dx$

II. If f is an odd function, then f(– x) = – f(x) and so, eq. (i) becomes

$\int_{-a}^{a} f(x)dx = -\int_0^a f(x)dx + \int_0^a f(x)dx = 0$

MATCHING TYPE QUESTIONS

29. (c) The function in column-I are derived functions of column-II, then we say that each function of column-II is an anti-derivative of each function in column-I.

A. $\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$; $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$

B. $\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$; $\int \frac{-dx}{\sqrt{1-x^2}} = +\cos^{-1}x + C$

C. $\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$; $\int \frac{dx}{1+x^2} = \tan^{-1}x + C$

D. $\frac{d}{dx}\left(\cot^{-1}x\right) = \frac{-1}{1+x^2}$; $\int \frac{-dx}{1+x^2} = +\cot^{-1}x + C$

30. (a) A. $\int \sqrt{ax+b}\,dx = \int (ax+b)^{1/2}dx$

$= \frac{(ax+b)^{(1/2)+1}}{a\left(\frac{1}{2}+1\right)} + C = \frac{(ax+b)^{3/2}}{a\left(\frac{3}{2}\right)} + C$

$= \frac{2}{3a}(ax+b)^{3/2} + C$

B. $\int x\sqrt{x+2}\,dx = \int (x+2-2)\sqrt{x+2}\,dx$

$= \int (x+2)\sqrt{x+2}\,dx - 2\int \sqrt{x+2}\,dx$

$= \int (x+2)^{3/2}dx - 2\int (x+2)^{1/2}dx$

$= \frac{(x+2)^{(3/2)+1}}{(3/2)+1} - 2\frac{(x+2)^{(1/2)+1}}{(1/2)+1} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{2\times 2}{3}(x+2)^{3/2} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

C. Let $I = \int x\sqrt{1+2x^2}\,dx$

Let $1 + 2x^2 = t$

On differentiating w.r.t.x, we get

$4x = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{4x}$

$\therefore\ I = \int x\sqrt{t}\frac{dt}{4x} = \frac{1}{4}\int\sqrt{t}\,dt = \frac{1}{4}\int t^{1/2}dt$

$= \frac{1}{4}\frac{t^{(1/2)+1}}{(1/2)+1} + C = \frac{1}{6}(1+2x^2)^{3/2} + C$

D. Let $I = \int(4x+2)\sqrt{x^2+x+1}\,dx$

Let $x^2 + x + 1 = t$

On differentiating w.r.t. x, we get

$2x+1 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{(2x+1)}$

$\therefore\ I = \int(4x+2)\sqrt{t}\frac{dt}{(2x+1)}$

$= \int 2(2x+1)\sqrt{t}\frac{dt}{(2x+1)} = 2\int\sqrt{t}\,dt$

$= 2\frac{t^{(1/2)+1}}{(1/2)+1} + C = \frac{4}{3}(x^2+x+1)^{3/2} + C$

31. (b) A. Let $I = \int\frac{\cos x - \sin x}{1+\sin 2x}dx$

$= \int\frac{\cos x - \sin x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}dx$

$= \int\frac{\cos x - \sin x}{(\sin x + \cos x)^2}dx$

Let $\cos x + \sin x = t \Rightarrow -\sin x + \cos x = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{(\cos x - \sin x)}$

$\therefore\ I = \int\frac{\cos x - \sin x}{t^2}.\frac{dt}{(\cos x - \sin x)}$

$= \int\frac{1}{t^2}dt = \int t^{-2}dt = \frac{t^{-2+1}}{-2+1} + C$

$= \frac{-1}{\cos x + \sin x} + C$

B. $\int\tan^3 2x\sec 2x\,dx$

Let $\sec 2x = t$

$\Rightarrow 2\sec 2x\ \tan 2x = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2\sec 2x.\tan 2x}$

$\therefore\ \int\tan^3 2x\sec 2x\,dx$

$= \int\tan^3 2x\sec 2x\frac{dt}{2\sec 2x.\tan 2x}$

$= \frac{1}{2}\int\tan^2 2x\,dt = \frac{1}{2}\int\left[\sec^2 2x - 1\right]dt$

$(\because \tan^2 x = \sec^2 x - 1)$

$= \frac{1}{2}\int\left[(t^2-1)dt\right] = \frac{1}{2}\left[\frac{t^3}{3} - t\right] + C$

$= \frac{1}{2}\left[\frac{\sec^3 2x}{3} - \sec 2x\right] + C$

$= \frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + C$

C. $\int\frac{\sin^3 x + \cos^3 x}{\sin^2 x\cos^2 x}dx$

$= \int\frac{\sin^3 x}{\sin^2 x\cos^2 x}dx + \int\frac{\cos^3 x}{\sin^2 x\cos^2 x}dx$

$= \int\frac{\sin x}{\cos x\cos x}dx + \int\frac{\cos x}{\sin x\sin x}dx$

$= \int\tan x.\sec x\,dx + \int\cot x.\text{cosec}\,x\,dx$

$= \sec x - \text{cosec}\,x + C$

D. $\int\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}dx$

$= \int\frac{1-2\sin^2 x + 2\sin^2 x}{\cos^2 x}dx$

$(\because \cos 2x = 1 - 2\sin^2 x)$

$= \int\frac{1}{\cos^2 x}dx = \int\sec^2 x\,dx = \tan x + C$

32. (b) A. Let $f(x) = x^{17}\cos^4 x$,

$\Rightarrow f(-x) = (-x)^{17}\cos^4(-x) = -x^{17}\cos^4 x = -f(x)$

Therefore, f(x) is an odd function.

We know that, if f(x) is an odd function, then

$\int_{-a}^{a} f(x)dx = 0$

$\therefore\ \int_{-1}^{1} x^{17}\cos^4 x\,dx = 0$

B. Let $I = \int_0^{\pi/2}\sin^3 x\,dx = \int_0^{\pi/2}\sin^2 x.\sin x\,dx$

$= \int_0^{\pi/2}(1-\cos^2 x)\sin x\,dx$

$(\because \sin^2 x = 1 - \cos^2 x)$

Put, $\cos x = t \Rightarrow -\sin x\,dx = dt$

When, $x = 0 \Rightarrow t = \cos 0 = 1$, when $x = \frac{\pi}{2}$

$\Rightarrow\ t = \cos\frac{\pi}{2} = 0$

$\therefore\ I = \int_0^{\pi/2}(1-\cos^2 x)\sin x\,dx = \int_1^0 (1-t^2)(-dt)$

$= -\left[t - \frac{t^3}{3}\right]_1^0 = -\left\{(0-0) - \left(1-\frac{1}{3}\right)\right\} = \frac{2}{3}$

C. Let $I = \int_0^{\pi/4} 2\tan^3 x\,dx = 2\int_0^{\pi/4}\tan^2 x.\tan x\,dx$

$= 2\int_0^{\pi/4}(\sec^2 x - 1)\tan x\,dx$

$[\because 1+\tan^2 x = \sec^2 x]$

$= 2\left[\int_0^{\pi/4}\sec^2 x\tan x\,dx - \int_0^{\pi/4}\tan x\,dx\right]$

$= 2\int_0^{\pi/4}(\tan x)\sec^2 x\,dx - 2[-\log|\cos x|]_0^{\pi/4}$

$\left[\begin{array}{l}\because \text{Let } I_1 = \int(\tan x)\sec^2 x\,dx \text{ put } \tan x = t\\ \Rightarrow \sec^2 x\,dx = dt \therefore I_1 = \int t\,dt = \frac{t^2}{2} = \frac{\tan^2 x}{2}\end{array}\right]$

$= 2\left[\frac{\tan^2 x}{2}\right]_0^{\pi/4} + 2\left[\log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0|\right]$

$= \tan^2\left(\frac{\pi}{4}\right) - 0 + 2\left[\log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$

$= 1 + 2\log 2^{-1/2} - 0 \qquad (\because \log 1 = 0)$

$= 1 - 2\times\frac{1}{2}\log 2 = 1 - \log 2$

D. Let $I = \int_0^1 \sin^{-1} x\,dx = \int_0^1 \sin^{-1} x.1\,dx$

Applying rule of integration by parts taking $\sin^{-1}x$ as the first function and 1 as second function,

we get $I = \left[(\sin^{-1} x)x\right]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$

Put $1 - x^2 = t \Rightarrow -2x\,dx = dt$

When, $x = 0$

$\Rightarrow t = 1$ and when $x = 1 \Rightarrow t = 0$

$\therefore\ I = \left[x\sin^{-1} x\right]_0^1 + \frac{1}{2}\int_1^0 \frac{dt}{\sqrt{t}}$

$= \left[x\sin^{-1} x\right]_0^1 + \frac{1}{2}\left[\frac{t^{1/2}}{1/2}\right]_1^0$

$= 1\sin^{-1}(1) + \left[-\sqrt{1}\right] = \frac{\pi}{2} - 1$

INTEGER TYPE QUESTIONS

33. (b) $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx = \int_0^1 \tan^{-1}\left[\frac{x+(x-1)}{1-x(x-1)}\right]dx$

$I = \int_0^1\left[\tan^{-1} x + \tan^{-1}(x-1)\right]dx \qquad \text{... (i)}$

let $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$= \int_0^1 [\tan^{-1} x + \tan^{-1}(x-1)]\,dx$

$= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(1-x-1)]\,dx$

$= \int_0^1 [-\tan^{-1}(x-1) - \tan^{-1} x]\,dx$,

$I = -\int_0^1 [\tan^{-1} x + \tan^{-1}(x-1)]\,dx \qquad \text{...(ii)}$

Adding (i) & (ii) $2I = 0$ or $I = 0$.

34. (a) $\int_0^{2\pi}\log\left(\frac{a+b\sec x}{a-b\sec x}\right)dx = 2\int_0^{\pi}\log\left(\frac{a+b\sec x}{a-b\sec x}\right)dx$

$= 2\int_0^{\pi}\log(a+b\sec x)dx - 2\int_0^{\pi}\log(a-b\sec(\pi-x))dx$

$= 2\int_0^{\pi}\log(a+b\sec x)\,dx - 2\int_0^{\pi}\log(a+b\sec x)\,dx = 0$

35. (b) We have $I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx \qquad \text{... (i)}$

$= \int_2^8 \frac{\sqrt{10-(10-x)}}{\sqrt{10-x}+\sqrt{10-(10-x)}}dx$

$\Rightarrow \qquad I = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}dx \qquad \text{... (ii)}$

Adding (i) and (ii), we get

$2I = \int_2^8 1dx = 8 - 2 = 6$

Hence $I = 3$

36. (b) $I = \int_{-1}^1 (x - [x])dx = \int_{-1}^1 x dx - \int_{-1}^1 [x]dx$

$= \left[\frac{x^2}{2}\right]_{-1}^1 - \left[\int_{-1}^0 [x]dx + \int_0^1 [x]dx\right]$

$= \frac{1}{2}[1-1] - \left[\int_{-1}^0 (-1)dx + \int_0^1 0.dx\right]$

$\left[\begin{array}{l}\text{If } -1 \le x < 0, [x] = -1\\ \text{If } \ 0 \le x < 1,\ [x] = 0\end{array}\right]$

$= 0 - [-x]_{-1}^0 - 0 = 0 - [-0-(-1)] = 1$

37. (a) $I=\int_0^{\frac{\pi}{2}}\log(\tan x)\,dx=\int_0^{\frac{\pi}{2}}\log\left\{\tan\left(\frac{\pi}{2}-x\right)\right\}dx$

$=\int_0^{\frac{\pi}{2}}\log(\cot x)\,dx$

$\therefore\ 2I=\int_0^{\frac{\pi}{2}}\log(\tan x)\,dx+\int_0^{\frac{\pi}{2}}\log(\cot x)\,dx$

$=\int_0^{\frac{\pi}{2}}[\log\tan x+\log\cot x]\,dx$

$=\int_0^{\frac{\pi}{2}}\log(\tan x\cdot\cot x)\,dx$

$\int_0^{\frac{\pi}{2}}\log(1)\,dx=\int_0^{\frac{\pi}{2}}0\,dx=0\quad\therefore I=0$

38. (c) Use $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$\int_0^{\pi}\frac{\sin 2mx}{\sin x}dx=\int_0^{\pi}\frac{\sin(2m\pi-2mx)}{\sin(\pi-x)}dx$

$=\int_0^{\pi}\frac{-\sin 2mx}{\sin x}dx=-I\Rightarrow 2I=0\Rightarrow I=0$

39. (c) Let $I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ ... (i)

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin(\pi/2-x)}{4+3\cos(\pi/2-x)}\right)dx$

$\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow I=\int_0^{\pi/2}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$ ... (ii)

$\left[\because\sin\left(\frac{\pi}{2}-x\right)=\cos x\text{ and }\cos\left(\frac{\pi}{2}-x\right)=\sin x\right]$

On adding eqs. (i) and (ii), we get

$2I=\int_0^{\pi/2}\left[\log\left(\frac{4+3\sin x}{4+3\cos x}\right)+\log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right]dx$

$\Rightarrow 2I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\times\frac{4+3\cos x}{4+3\sin x}\right)dx$

$[\because\log m+\log n=\log mn]$

$\Rightarrow 2I=\int_0^{\pi/2}\log 1\,dx$

$\Rightarrow 2I=\int_0^{\pi/2}0\,dx\quad(\because\log 1=0)$

$\Rightarrow I=0$

40. (b) Let $I=\int_0^1\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$=\int_0^1\tan^{-1}\left(\frac{x+(x-1)}{1-x(x-1)}\right)dx$

$=\int_0^1\left\{\tan^{-1}x+\tan^{-1}(x-1)\right\}dx$

$\left[\because\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)\right]$

$\Rightarrow I=\int_0^1\left\{\tan^{-1}x-\tan^{-1}(1-x)\right\}dx$ ...(i)

Also, $I=\int_0^1\left\{\tan^{-1}(1-x)-\tan^{-1}(1-(1-x))\right\}dx$

$\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx$ ...(ii)

On adding eqs. (i) and (ii), we get

$2I=0\Rightarrow I=0$

41. (d) Let $I=\int\cos^n x\sin x\,dx$

Put cos x = t

– sin x dx = dt

$\therefore\quad I=-\int t^n dt=-\frac{t^{n+1}}{n+1}+C$

$=-\frac{\cos^{n+1}x}{n+1}+C=-\frac{\cos^6 x}{6}+C$

$\therefore$ n + 1 = 6 or n = 5

42. (d) We have,

$\frac{3x+1}{(x-3)(x-5)}=\frac{-5}{x-3}+\frac{B}{x-5}$

3x + 1 = –5(x – 5) + B(x – 3)

Put x = 5

3(5) + 1 = B(5 – 3)

16 = 2B or B = 8

43. (a) $\because\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$

But it is given that

$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{3}+c$

$\therefore$ a = 3

44. (b) $I = \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

Put $e^x + e^{-x} = t$

$(e^x + e^{-x})dx = dt$

$\therefore \quad I = \int \frac{dt}{t} = \log t + C$

$= \log(e^x - e^{-x}) + C$

$= \log\left(e^x - \frac{1}{e^x}\right) + C$

$= \log\left(\frac{e^{2x} - 1}{e^x}\right) + C$

$= \log(e^{2x} - 1) - \log e^x + C$

$= \log(e^{2x} - 1) - x + C$

$\therefore A = 1$

45. (b) If f (x) is an even function then

$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

Here $f(x) = x^8 - x^4 + x^2 + 1$ is an even function, therefore a = 4.

ASSERTION - REASON TYPE QUESTIONS

46. (a) $I = \int_0^{\frac{\pi}{2}} 2\sqrt{\tan x}\, dx$, Put $\tan x = t^2 \Rightarrow dx = \frac{2t\,dt}{1+t^4}$

If $x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \infty$

$I = \int_0^\infty \frac{2t^2 dt}{1+t^4} = \int_0^\infty \frac{t^2 + 1 + t^2 - 1}{1+t^4} dt$

$= \int_0^\infty \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} dt + \int_0^\infty \frac{1 - \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} dt$

$= \int_0^\infty \frac{d\left(t - \frac{1}{t}\right)}{\left(t - \frac{1}{t}\right)^2 + 2} + \int_0^\infty \frac{d\left(t + \frac{1}{t}\right)}{\left(t + \frac{1}{t}\right)^2 - 2} dt$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{2}}\right)\Bigg|_0^\infty + \frac{1}{2\sqrt{2}} \ln\left(\frac{t + \frac{1}{t} - \sqrt{2}}{t + \frac{1}{t} + \sqrt{2}}\right)_0^\infty$

$= \frac{\pi}{\sqrt{2}}$

47. (a) $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx = 0$

$f(x) = \log\left(\frac{1+x}{1-x}\right)$

$f(-x) = \log\left(\frac{1-x}{1+x}\right) = -\log\left(\frac{1+x}{1-x}\right) = -f(x)$

f is an odd function $\Rightarrow \int_{-a}^{a} f(x)dx = 0$

Both are true and Reason is correct explanation of Assertion.

48. (a)

Derivatives	Integrals (Anti-derivatives)
(i) $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$ Particularly, we note that $\frac{d}{dx} x = 1$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$ $\int dx = x + C$

49. (a) Sometimes, function is not expressible in terms of elementary functions viz., polynomial, logarithmic, exponential, trigonometric functions and their inverses etc. We are therefore blocked for finding $\int f(x)dx$.

Therefore, it is not possible to find $\int e^{-x^2} dx$ by inspection since, we can not find a function whose derivative is e^{-x^2}.

50. (d) The process of differentiation and integration are inverses of each other in sense of the following results.

$\frac{d}{dx}\int f(x)dx = f(x)$ and $\int f'(x)dx = f(x) + C$,

where C is any arbitrary constant

Let F be any anti-derivative of f, i.e.,

$\frac{d}{dx}F(x) = f(x)$

Then $\int f(x)dx = F(x) + C$

Therefore, $\frac{d}{dx}\int f(x)dx = \frac{d}{dx}(F(x) + C) = \frac{d}{dx}F(x) = f(x)$

Similarly, we know that

$f'(x) = \frac{d}{dx}f(x)$

and hence $\int f'(x)dx = f(x) + C$

where, C is arbitrary constant called constant of integration.

51. (b) The derivative of a function has a geometrical meaning, namely, the slope of the tangent to the corresponding curve at a point. Similarly, the indefinite integral to a function represents geometrically, a family of curves placed parallel to each other having parallel tangents at the points of intersection of the curves of the family with the lines orthogonal (perpendicular) to the axis representing the variable of integration.

52. (c) We can speak of the derivative at a point. We never speak of the integral at a point, we speak of the integral of a function over an interval on which the integral is defined.

53. (a) Here, $I=\int[\sin(\log x)+\cos(\log x)]dx$ ... (i)

By using inspection method,

$$\frac{d}{dx}\{x\sin(\log x)\}=x\frac{d}{dx}\sin(\log x)+\sin(\log x)\frac{d}{dx}(x)$$

$$=x\cos(\log x)\times\frac{1}{x}+\sin(\log x)$$

$$=\cos(\log x)+\sin(\log x) \quad \text{...(ii)}$$

From eqs. (i) and (ii), we get

$$I=\int\frac{d}{dx}\{x\sin(\log x)\}dx$$

$$=x\sin(\log x)+C$$

54. (a) The value of definite integral of a function over any particular interval depends on the function and the interval, but not on the variable of integration that we choose to represent the independent variable. If the independent variable is denoted by t or u instead of x, we simply write the integral as $\int_a^b f(t)dt$ or $\int_a^b f(u)du$ instead of $\int_a^b f(x)dx$.

Hence the variable of integration is called a dummy variable.

55. (c) $\int_a^b x\,f(x)\,dx=\int_a^b (a+b-x)\,f\,(a+b-x)\,dx$

$$=(a+b)\int_a^b f\,(a+b-x)\,dx-\int_a^b xf\,(a+b-x)\,dx$$

$\therefore$ Reason is true only when $f(a+b-x)=f(x)$ which holds in Assertion.

$\therefore$ Reason is false and Assertion is true.

56. (a) Here $f'(x)=\tan x$.

CRITICALTHINKING TYPE QUESTIONS

57. (d) Let $I=\int_a^b x\,f(x)\,dx$

Let $a+b-x=z \Rightarrow -dx=dz$

When $x=a, z=b$ and when $x=b, z=a$

$$\therefore \quad I=-\int_b^a (a+b-z)f(z)\,dz$$

$$I=(a+b)\int_b^a f(x)\,dx-\int_a^b x\,f(x)\,dx$$

$$I=(a+b)\int_b^a f(x)\,dx-I\,; \quad 2I=(a+b)\int_a^b f(x)\,dx$$

Hence, $I=\left(\frac{a+b}{2}\right)\int_a^b f(x)\,dx$

58. (c) $I=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$ ... (i)

Put $x=-y$ then $dx=-dy$

$$I=\int_{\pi}^{-\pi}\frac{\cos^2 y}{1+a^{-y}}dy=\int_{-\pi}^{\pi}\frac{a^y\cos^2 y}{1+a^y}dy$$

$$I=\int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{1+a^x}dx \quad \text{... (ii)}$$

$$\left[\because \int_a^b f(y)dy=\int_a^b f(x)dx\right]$$

Adding (i) and (ii),

$$2I=\int_{-\pi}^{\pi}\frac{(1+a^x)\cos^2 x}{(1+a^x)}dx=\int_{-\pi}^{\pi}\cos^2 x\,dx$$

$$2I=2\int_0^{\pi}\cos^2 x\,dx \quad \text{(even function)}$$

$$I=2\int_0^{\pi/2}\cos^2 x\,dx \quad \text{... (iii)}$$

$$\left[\because \int_0^{2a} f(x)dx=2\int_0^a f(x)dx \text{ if } f(2a-x)=f(x)\right]$$

$$=2\int_0^{\pi/2}\sin^2 x\,dx \quad \text{... (iv)}$$

Adding (iii) and (iv)

$$2I=2\int_0^{\pi/2}(\cos^2 x+\sin^2 x)dx=2.\pi/2=\pi$$

$\therefore I=\pi/2$

59. (b) Let $I=\int\sin^3 x\cos^3 x\,dx$. Then,

$$I=\frac{1}{8}\int(2\sin x\cos x)^3\,dx$$

$$\Rightarrow \quad I=\frac{1}{8}\int\sin^3 2x\,dx \Rightarrow I=\frac{1}{8}\int\frac{3\sin 2x-\sin 6x}{4}dx$$

$$\Rightarrow \quad I=\frac{1}{32}\int(3\sin 2x-\sin 6x)\,dx$$

$$=\frac{1}{32}\left\{-\frac{3}{2}\cos 2x+\frac{1}{6}\cos 6x\right\}+C$$

60. (b) Let $I=\int\frac{1}{\sqrt{\sin^3 x\cos^5 x}}dx$

$$\Rightarrow \quad I=\int\frac{1}{\sin^{3/2}x\cos^{5/2}x}dx \Rightarrow I=\int\frac{\sec^4 x}{\tan^{3/2}x}dx$$

[Dividing numerator and denominator by $\cos^4 x$]

$$\Rightarrow \quad I=\int\frac{(1+\tan^2 x)}{\tan^{3/2}x}\sec^2 x\,dx$$

Putting $\tan x=t \Rightarrow \sec^2 x\,dx=dt$

$$I=\int\frac{1+t^2}{t^{3/2}}dt$$

$$\Rightarrow \quad I=\int\left(t^{-3/2}+t^{1/2}\right)dt=\frac{-2}{\sqrt{t}}+\frac{t^{3/2}}{3/2}+C$$

$$=-\frac{2}{\sqrt{\tan x}}+\frac{2}{3}(\tan x)^{3/2}+C$$

61. **(c)** Let $I = \int \frac{1}{\sqrt{9+8x-x^2}}dx$. Then,

$$I = \int \frac{1}{\sqrt{-\{x^2-8x-9\}}}dx$$

$$I = \int \frac{1}{\sqrt{-\{x^2-8x+16-25\}}}dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{-\{(x-4)^2-5^2\}}}dx = \int \frac{1}{\sqrt{5^2-(x-4)^2}}dx$$

$$= \sin^{-1}\left(\frac{x-4}{5}\right)+C$$

62. **(c)** $I = \int \frac{1}{1+3\sin^2 x+8\cos^2 x}dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$I = \int \frac{\sec^2 x}{\sec^2 x+3\tan^2 x+8}dx$$

$$\Rightarrow I = \int \frac{\sec^2 x}{1+\tan^2 x+3\tan^2 x+8}dx = \int \frac{\sec^2 x}{4\tan^2 x+9}dx$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$, we get

$$I = \int \frac{dt}{4t^2+9} = \frac{1}{4}\int \frac{dt}{t^2+(3/2)^2} = \frac{1}{4}\times\frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right)+C$$

$$\Rightarrow I = \frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right)+C = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$$

63. **(c)** Let $I = \int \frac{x^3+x}{x^4-9}dx$. Then,

$$I = \int \frac{x^3}{x^4-9}dx + \int \frac{x}{x^4-9}dx = I_1 + I_2 + C(\text{say}), \text{ where}$$

$$I_1 = \int \frac{x^3}{x^4-9}dx \text{ and } I_2 = \int \frac{x}{x^4-9}dx$$

Putting $x^4 - 9 = t$ in $I_1 \Rightarrow 4x^3 dx = dt$, we get

$$I_1 = \frac{1}{4}\int \frac{1}{t}dt = \frac{1}{4}\log|t| = \frac{1}{4}\log|x^4-9|$$

$$I_2 = \int \frac{x}{x^4-9}dx = \int \frac{x}{(x^2)^2-3^2}dx$$

Putting $x^2 = t \Rightarrow 2x\,dx = dt$, we get

$$I_2 = \frac{1}{2}\int \frac{dt}{t^2-3^2} = \frac{1}{2}\cdot\frac{1}{2\times 3}\log\left|\frac{t-3}{t+3}\right| = \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|$$

Hence, $I = \frac{1}{4}\log|x^4-9| + \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+C$

64. **(d)** $\frac{3x+4}{x^3-2x-4} = \frac{3x+4}{(x-2)(x^2+2x+2)}$

$$= \frac{A}{x-2}+\frac{Bx+C}{x^2+2x+2}$$

$\Rightarrow 3x + 4 = A(x^2 + 2x + 2) + (Bx + C)(x - 2)$

$\therefore$ $A + B = 0$

$2A - 2B + C = 3$

$2A - 2C = 4$

$\Rightarrow$ $A = 1, B = C = -1$

$$\therefore \int \frac{3x+4}{x^3-2x-4}dx = \int \frac{dx}{x-2} - \frac{1}{2}\int \frac{2x+2}{x^2+2x+2}dx$$

$$= \log_e|x-2| - \frac{1}{2}\log|x^2+2x+2|+C$$

$\Rightarrow k = -\frac{1}{2}$ and $f(x) = |x^2 + 2x + 2|$

65. **(a)** Let $I = \int \frac{1-\cos x}{\cos x(1+\cos x)}dx$

Let $\cos x = y \Rightarrow \frac{1-\cos x}{\cos x(1+\cos x)} = \frac{1-y}{y(1+y)}$

Now $\frac{1-y}{y(1+y)} = \frac{A}{y}+\frac{B}{1+y}$ ...(i)

$\Rightarrow 1 - y = A(1 + y) + By$

Put $y = 0$ in (i), we get $A = 1$.

Put $y = -1$ in (i), we get $B = -2$ ...(ii)

Substituting the values of A and B in (i), we obtain

$$\frac{1-y}{y(1+y)} = \frac{1}{y} - \frac{2}{1+y}$$

$$\Rightarrow \frac{1-\cos x}{\cos x(1+\cos x)} = \frac{1}{\cos x} - \frac{2}{1+\cos x} \quad [\because y = \cos x]$$

$$\therefore I = \int \frac{1-\cos x}{\cos x(1+\cos x)}dx = \int \frac{1}{\cos x}dx - \int \frac{2}{1+\cos x}dx$$

$$\Rightarrow I = \int \sec x\, dx - \int \frac{1}{\cos^2(x/2)}dx$$

$$= \int \sec x\, dx - \int \sec^2(x/2)dx$$

$$\Rightarrow I = \log|\sec x+\tan x| - 2\tan(x/2)+C$$

66. **(a)** We have, $I = \int_0^\pi \frac{1}{5+4\cos x}dx$

$$= \int_0^\pi \frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int_0^\pi \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$$

$$= \int_0^\pi \frac{1+\tan^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx = \int_0^\pi \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$$

Let $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

Also, $x = 0 \Rightarrow t = 0$ and $x = \pi \Rightarrow t = \infty$

$$\therefore I = \int_0^\infty \frac{dt}{9+t^2}$$

$\therefore \quad I = 2\int_0^{\infty} \frac{dt}{3^2+t^2}$

$\therefore \quad I = \frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^{\infty} = \frac{2}{3}\left[\tan^{-1}\infty - \tan^{-1}0\right]$

$= \frac{2}{3}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{3}$

67. (c) Given, $\int_0^{\pi} \ell n \sin x\, dx = k$

$\therefore \quad k = 2\int_0^{\pi/2} \ell n \sin x\, dx = 2\left(-\frac{\pi}{2}\ell n 2\right)$

$\therefore \quad k = \pi\, \ell n\, 2$ ... (i)

Then, $\int_0^{\pi/4} \ell n(1+\tan x)dx = \frac{\pi}{8}\ell n 2$

$= -\frac{k}{8}$ [From eq. (i)]

68. (a) We have, $I = \int 1.\tan^{-1}\sqrt{x}\, dx$

Using by parts,

$I = \tan^{-1}\sqrt{x}.(x) - \int \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} \times x\, dx$

$= x\tan^{-1}\sqrt{x}. - \int \frac{x}{(1+x)2\sqrt{x}}dx$

$= x\tan^{-1}\sqrt{x} - \int\left(\frac{1+x}{(1+x)2\sqrt{x}} - \frac{1}{(1+x)2\sqrt{x}}\right)dx$

$= x\tan^{-1}\sqrt{x} - \int \frac{dx}{2\sqrt{x}} + \int \frac{dx}{2\sqrt{x}(1+x)}$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

69. (b) $I = \int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x}dx$

$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{1-2\sin^2 x\cos^2 x}dx$

$= \int \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x)}{1-2\sin^2 x\cos^2 x}dx$

$= \int \frac{1.(\sin^2 x - \cos^2 x)[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x]}{1-2\sin^2 x\cos^2 x}$

$= \int \frac{(\sin^2 x - \cos^2 x)(1-2\sin^2 x\cos^2 x)}{1-2\sin^2 x\cos^2 x}dx$

$= -\int \cos 2x\, dx = -\frac{1}{2}\sin 2x + c$

70. (b) $\int \frac{\sin x}{\sin(x-\alpha)}dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$= \int \frac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)}dx$

$= \int\{\cos\alpha + \sin\alpha\cot(x-\alpha)\}dx$

$= (\cos\alpha)x + (\sin\alpha)\log\sin(x-\alpha) + C$

$\therefore \quad A = \cos\alpha,\ B = \sin\alpha$

71. (d) $I = \int_0^2 f(x)dx = \int_0^1 f(x)dx - \int_1^2 f(x)dx = \int_0^1 x^2 dx + \int_1^2 \sqrt{x}dx$

$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^{3/2}}{3/2}\right]_1^2 = \left[\frac{1}{3}-0\right] + \left[2^{3/2}-1\right]\frac{2}{3}$

$= \frac{1}{3} + \frac{2}{3}.2\sqrt{2} - \frac{2}{3} = \frac{1}{3}\left(4\sqrt{2}-1\right)$

72. (a) We have $g(x) = \int_0^x \cos^4 t\, dt$

$\therefore g(x+\pi) = \int_0^{x+\pi} \cos^4 t\, dt = \int_0^{\pi} \cos^4 t\, dt + \int_{\pi}^{x+\pi} \cos^4 t\, dt$

$= g(\pi) + \int_0^x \cos^4 t\, dt$ $\left[\because \cos^4 t \text{ is periodic with period } \pi\right]$

$= g(\pi) + g(x)$

73. (b) We have $\cos x \geq \sin x$ for $0 \leq x \leq \frac{\pi}{4}$

and $\sin x \geq \cos x$ for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

$\therefore \int_0^{\frac{\pi}{2}} |\sin x - \cos x|\, dx$

$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)dx$

$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1\right] - \left[0 + 1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$

$= \sqrt{2} - 1 - 1 + \sqrt{2} = 2\sqrt{2} - 2$

74. (c) Use $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Here, $a + b = \pi$

$\therefore \quad I = \int_{\pi/4}^{3\pi/4} \frac{\phi d\phi}{1+\sin\phi} = \int_{\pi/4}^{3\pi/4} \frac{(\pi-\phi)d\phi}{1+\sin(\pi-\phi)}$

$= \int_{\pi/4}^{3\pi/4} \frac{\pi d\phi}{1+\sin\phi} - I$

$\Rightarrow \quad 2I = \pi\int_{\pi/4}^{3\pi/4} \frac{1-\sin\phi}{\cos^2\phi}d\phi$

$= \pi\int_{\pi/4}^{3\pi/4} (\sec^2\phi - \sec\phi\tan\phi)d\phi$

$= \pi[\tan\phi - \sec\phi]_{\pi/4}^{3\pi/4}$

$I = \frac{\pi}{2}(2\sqrt{2}-2) = \pi(\sqrt{2}-1) = \frac{\pi}{\sqrt{2}+1}$

75. (b) $\int_{-\pi}^{\pi}\frac{2x\,(1+\sin x)}{1+\cos^2 x}dx=\int_{-\pi}^{\pi}\frac{2x\,dx}{1+\cos^2 x}+2\int_{-\pi}^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x}$

$=0+4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x}$;

$I=4\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$

$I=4\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$

$\Rightarrow I=4\pi\int_0^{\pi}\frac{\sin x\,dx}{1+\cos^2 x}-4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x}$;

$\Rightarrow 2I=4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$

put cos x = t and solve it.

76. (d) We have, $I=\int\frac{1}{(\sin x+4)(\sin x-1)}dx$

$=\frac{1}{5}\int\frac{(\sin x+4)-(\sin x-1)}{(\sin x+4)(\sin x-1)}dx$

$=\frac{1}{5}\int\frac{1}{\sin x-1}dx-\frac{1}{5}\int\frac{1}{\sin x+4}dx$

$=\frac{1}{5}\int\frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}-1-\tan^2\frac{x}{2}}dx$

$-\frac{1}{5}\int\frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}+4+4\tan^2\frac{x}{2}}dx$

Put, $\tan\frac{x}{2}=t$

$\Rightarrow \sec^2\frac{x}{2}dx=2\,dt$

$\therefore I=\frac{1}{5}\int\frac{2dt}{2t-1-t^2}-\frac{1}{5}\int\frac{2dt}{\left[2t+4\left(1+t^2\right)\right]}$

$\therefore I=-\frac{2}{5}\int\frac{dt}{t^2-2t+1}-\frac{1}{10}\int\frac{dt}{t^2+\frac{1}{2}t+1}$

$=-\frac{2}{5}\int\frac{1}{(t-1)^2}dt-\frac{1}{10}\int\frac{dt}{\left(t+\frac{1}{4}\right)^2+\left(\frac{\sqrt{15}}{4}\right)^2}$

$=\frac{2}{5}\cdot\frac{1}{t-1}-\frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right)+C$

$=\frac{2}{5}\frac{1}{\tan\frac{x}{2}-1}-\frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}\right)+C$ ...(i)

But, given that

$I=A\frac{1}{\left(\tan\frac{x}{2}-1\right)}+B\tan^{-1}\left[f(x)\right]+C$ ...(ii)

From eqs. (i) and (ii), we get

$A=\frac{2}{5},\ B=\frac{-2}{5\sqrt{15}},\ f(x)=\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$

77. (b) We have, f(x) = f(a – x) and g(x) + g(a – x) = 4

Let $I=\int_0^a f(x)g(x)dx$ ... (i)

$\Rightarrow I=\int_0^a f(a-x)g(a-x)dx$

$\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow I=\int_0^a f(x)\{4-g(x)\}dx$ ... (ii)

[∵ f(x) = f(a – x) and g(x) + g(a – x) = 4 (given)]

On adding eqs. (i) and (ii), we get

$2I=\int_0^a 4f(x)dx \Rightarrow I=2\int_0^a f(x)dx$

78. (a) Let $x=a\sin^2\theta$

then $dx=2a\sin\theta\cos\theta\,d\theta$

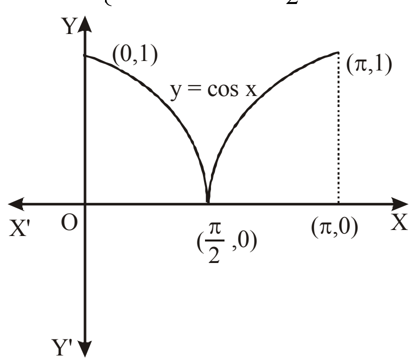
$\therefore \quad I=\int\frac{2a\sin\theta.\cos\theta}{\sqrt{a\sin^2\theta.a\cos^2\theta}}d\theta$

$=2\int d\theta=2\theta+c$

$=2\sin^{-1}\left(\sqrt{x/a}\right)+c$

79. (a) We have

$|\cos x|=\begin{cases}\cos x & \text{when } 0\le x\le\frac{\pi}{2}\\ -\cos x & \text{when } \frac{\pi}{2}\le x\le\pi\end{cases}$



$\therefore \int_0^{\pi}|\cos x|\,dx=\int_0^{\pi/2}|\cos x|\,dx+\int_{\pi/2}^{\pi}|\cos x|\,dx$

$=\int_0^{\pi/2}\cos x\,dx+\int_{\pi/2}^{\pi}(-\cos x)dx$

$=[\sin x]_0^{\pi/2}-[\sin x]_{\pi/2}^{\pi}=1+1=2$

80. **(c)** Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ … (i)

Then, $I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ … (ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_0^{\pi/2} 1.dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$\Rightarrow I = \frac{\pi}{4} \Rightarrow \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$$

81. **(b)** $\int \frac{(x - x^3)^{1/3}}{x^4} dx = \int \frac{1}{x^3}\left(\frac{1}{x^2} - 1\right)^{1/3} dx$

$= \frac{-1}{2} \int t^{1/3} \, dt \quad \left[\text{Putting } \frac{1}{x^2} - 1 = t \Rightarrow \frac{-2}{x^3} dx = dt\right]$

$= \frac{-1}{2} . \frac{t^{4/3}}{4/3} + C = \frac{-3}{8}\left(\frac{1}{x^2} - 1\right)^{4/3} + C$

82. **(a)** Here since the highest powers of x in numerator and denominator are equal and coefficients of x^2 are also equal, therefore

$$\int \frac{x^2 + 1}{(x-1)(x-2)} \equiv 1 + \frac{A}{x-1} + \frac{B}{x-2}$$

On solving we get A = – 2, B = 5

Thus $\int \frac{x^2 + 1}{(x-1)(x-2)} \equiv 1 - \frac{2}{x-1} + \frac{5}{x-2}$

The above method is used to obtain the value of constant corresponding to non-repeated linear factor in the denominator.

Now, $I = \int \left(1 - \frac{2}{x-1} + \frac{5}{x-2}\right) dx$

$= x - 2 \log (x - 1) + 5 \log (x - 2) + C$

$= x + \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + C$

83. **(b)** Put $x - \alpha = t^2 \Rightarrow dx = 2t \, dt$

$\therefore \quad I = 2\int \frac{t \, dt}{\sqrt{t^2(\beta - \alpha - t^2)}}$

$= 2\int \frac{dt}{\sqrt{(\beta - \alpha) - t^2}} = 2\sin^{-1}\frac{t}{\sqrt{\beta - \alpha}} + C$

$= 2\sin^{-1}\sqrt{\frac{x - \alpha}{\beta - \alpha}} + C$

84. **(b)** After dividing by $\cos^2 x$ to numerator and denominator of integration

$$I = \int \frac{\sec^2 x \, dx}{4\tan^2 x + 4\tan x + 5}$$

$$= \int \frac{\sec^2 x \, dx}{(2\tan x + 1)^2 + 4}$$

$$= \frac{1}{22} \tan^{-1}\left(\frac{2\tan x + 1}{2}\right) + C$$

85. **(b)** $I = \int \frac{1 + 1/x^2}{x^2 + 1 + 1/x^2} dx = \int \frac{d\,(x - 1/x)}{(x - 1/x)^2 + 3}$

$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x - 1/x}{\sqrt{3}} + C$

$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{(x^2 - 1)}{\sqrt{3}x}\right) + C$

86. **(c)** $I = \int_0^1 \log\left(\frac{1-x}{x}\right) dx$ ...(i)

$\Rightarrow I = \int_0^1 \log\left[\frac{1 - (1-x)}{1-x}\right] dx$

$= \int_0^1 \log\left(\frac{x}{1-x}\right) dx = -\int_0^1 \log\left(\frac{1-x}{x}\right) dx = I$

$\Rightarrow 2I = 0 \Rightarrow I = 0$

87. **(b)** $I = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \sqrt{\cot x}} dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ … (i)

Then, $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$

$\Rightarrow \quad I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ … (ii)

Adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$\Rightarrow \quad 2I = \int_{\pi/6}^{\pi/3} 1.dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow I = \pi/12$$

88. **(b)** $\because \quad \frac{|x|}{x} = \begin{cases} -1 \text{ when } -1 < x < 0 \\ 1 \text{ when } 0 < x < 2 \end{cases}$

$\therefore I = \int_{-1}^{0} \frac{|x|}{x} dx + \int_0^2 \frac{|x|}{x} dx = \int_{-1}^{0} (-1) dx + \int_0^2 1 dx$

$= -[x]_{-1}^{0} + [x]_0^2 = -1 + 2 = 1$

CHAPTER

24

APPLICATION OF INTEGRALS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The area of the region bounded by the ellipse

$\frac{x^2}{16} + \frac{y^2}{9} = 1$ is

(a) 12π (b) 3π

(c) 24π (d) π

2. The area of the region bounded by the parabola $y = x^2$ and $y = |x|$ is

(a) 3 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) 2

3. The area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$ is

(a) $\frac{2}{21}$ (b) 21

(c) $\frac{21}{2}$ (d) $\frac{9}{2}$

4. The area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x-axis, is

(a) $\frac{2}{9}$ (b) $\frac{9}{2}$

(c) 9 (d) 2

5. AOB is a positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where OA = a, OB = b. The area between the arc AB and chord AB of the ellipse is

(a) πab sq. units (b) $(\pi - 2)ab$ sq. units

(c) $\frac{ab(\pi+2)}{2}$ sq. units (d) $\frac{ab(\pi-2)}{4}$ sq. units

6. The area bounded by the line y = x, x-axis and lines x = – 1 to x = 2, is

(a) 0 sq. unit (b) 1/2 sq. units

(c) 3/2 sq. units (d) 5/2 sq. units

7. The area bounded by the line $y = 2x - 2$, $y = -x$ and x-axis is given by

(a) $\frac{9}{2}$ sq. units (b) $\frac{43}{6}$ sq. units

(c) $\frac{35}{6}$ sq. units (d) None of these

8. The area bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is

(a) 1 sq. unit (b) $\frac{1}{3}$ sq. units

(c) $\frac{2}{3}$ sq. units (d) $\frac{4}{3}$ sq. units

9. The area bounded by the curves $y = \sin x$, $y = \cos x$ and $x = 0$ is

(a) $(\sqrt{2} - 1)$ sq. units (b) 1 sq. unit

(c) $\sqrt{2}$ sq. units (d) $(1 + \sqrt{2})$ sq. units

10. The area enclosed between the graph of $y = x^3$ and the lines $x = 0$, $y = 1$, $y = 8$ is

(a) $\frac{45}{4}$ (b) 14

(c) 7 (d) None of these

11. The area bounded by $f(x) = x^2$, 0 £ x £ 1, $g(x) = -x + 2, 1 \le x \le 2$ and x - axis is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{8}{3}$ (d) None of these

12. The area bounded by $y - 1 = |x|$, $y = 0$ and $|x| = \frac{1}{2}$ will be :

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$

(c) $\frac{5}{4}$ (d) None of these

13. Area bounded by the curve $y = \log x$ and the coordinate axes is

(a) 2 (b) 1 (c) 5 (d) $2\sqrt{2}$

14. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is
(a) 2 (b) 1 (c) 1/2 (d) 1/4

15. Area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is
(a) $\frac{2}{3}a^2 - 5$ (b) $\frac{15}{4}a^2 + 5$
(c) $\frac{16}{3}a^2 + 2$ (d) $\frac{16}{3}a^2$

16. Area between the curves $y = x$ and $y = x^3$ is
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) $\frac{1}{4}$

17. Area between the parabola $x^2 = 4y$ and line $x = 4y - 2$ is
(a) $\frac{8}{9}$ (b) $\frac{9}{7}$ (c) $\frac{7}{9}$ (d) $\frac{9}{8}$

18. Area between the curve $y = \cos^2 x$, x-axis and ordinates $x = 0$ and $x = p$ in the interval $(0, p)$ is
(a) $\frac{2\pi}{3}$ (b) 2π (c) π (d) $\frac{\pi}{2}$

19. The area (sq. units) bounded by the parabola $y^2 = 4ax$ and the line $x = a$ and $x = 4a$ is :
(a) $\frac{35a^2}{3}$ (b) $\frac{4a^2}{3}$
(c) $\frac{7a^2}{3}$ (d) $\frac{56a^2}{3}$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

20. Consider the following statements
Statement I : The area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2p$ is 2 sq. units.
Statement II : The area bounded by the curve $y = 2 \cos x$ and the x-axis from $x = 0$ to $x = 2p$ is 8 sq. units.
(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

21. Using the method of integration the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3) is
(a) 2 (b) 4
(c) 7 (d) 8

22. Area bounded by the lines $y = |x| - 2$ and $y = 1 - |x - 1|$ is equal to
(a) 4 sq. units (b) 6 sq. units
(c) 2 sq. units (d) 8 sq. units

23. Area of the region bounded by $y = |x - 1|$ and $y = 1$ is
(a) 2 sq. units (b) 1 sq. unit
(c) $\frac{1}{2}$ sq. units (d) None of these

24. The area bounded by the curve $y^2 = 16x$ and line $y = mx$ is $\frac{2}{3}$, then m is equal to
(a) 3 (b) 4
(c) 1 (d) 2

25. Area bounded by the curve $y = \cos x$ between $x = 0$ and $x = \frac{3\pi}{2}$ is
(a) 1 sq. unit (b) 2 sq. units
(c) 3 sq. units (d) 4 sq. units

26. The area of the region bounded by the curve $x = 2y + 3$ and lines $y = 1$ and $y = -1$ is
(a) 4 sq. units (b) $\frac{3}{2}$ sq. units
(c) 6 sq. units (d) 8 sq. units

27. What is the area of the triangle bounded by the lines $y = 0$, $x + y = 0$ and $x = 4$?
(a) 4 units (b) 8 units
(c) 12 units (d) 16 units

28. The area of the region bounded by the curves $y = |x - 2|, x = 1, x = 3$ and the x-axis is
(a) 4 (b) 2
(c) 3 (d) 1

29. The area bounded by the parabola $y^2 = 36x$, the line $x = 1$ and x-axis is ______ sq. units.
(a) 2 (b) 4
(c) 6 (d) 8

30. The area of the ellipse

$\frac{x^2}{9} + \frac{y^2}{4} = 1$ in first quadrant is 6π sq. units.

The ellipse is rotated about its centre in anti-clockwise direction till its major axis coincides with y-axis. Now the area of the ellipse in first quadrant is______ π sq. units.
(a) 2 (b) 4
(c) 6 (d) 8

31. The area bounded by the curve $y = \sin x$ from 0 to π and x-axis is ______ sq. units.
(a) 2 (b) 4
(c) 6 (d) 8

32. The area under the curve $y = x^2$ and the line $x = 3$ and x axis is ______ sq. units.
(a) 0 (b) 1
(c) 3 (d) 9

33. For the area bounded by the curve $y = ax$, the line $x = 2$ and x-axis to be 2 sq. units, the value of a must be equal to
(a) 2 (b) 4
(c) 6 (d) 8

34. The area bounded by the curve $y=\frac{3}{2}\sqrt{x}$, the line $x = 1$ and x-axis is _______ sq. units.

(a) 2 (b) 4
(c) 6 (d) 8

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

35. **Assertion :** The area bounded by the curves $y^2 = 4a^2(x – 1)$ and lines $x = 1$ and $y = 4a$ is $\frac{16a}{3}$ sq. units.

Reason : The area enclosed between the parabola $y^2 = x^2 – x + 2$ and the line $y = x + 2$ is $\frac{8}{3}$ sq. units.

36. **Assertion :** The area bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant is given by

$$\int_0^a \sqrt{a^2 - x^2}\,dx$$

Reason : The same area can also be found by

$$\int_0^a \sqrt{a^2 - y^2}\,dy.$$

37. **Assertion :** The area bounded by the circle $y = \sin x$ and $y = –\sin x$ from 0 to π is 3 sq. unit.

Reason : The area bounded by the curves is symmetric about x-axis.

38. **Assertion :** The area bounded by the curve $y = \cos x$ in I quadrant with the coordinate axes is 1 sq. unit.

Reason : $\int_0^{\pi/2} \cos x\,dx = 1$

CRITICAL THINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

39. The area bounded by curves $(x – 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ is

(a) $\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ (b) $\frac{2\pi}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$

40. The area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ is

(a) $3(\pi – 2)$ (b) $\frac{3}{2}\pi$

(c) $\frac{3}{2}(\pi-2)$ (d) $\frac{2}{3}(\pi-2)$

41. The area of the region $\{(x, y) : y^2 \le 4x,\ 4x^2 + 4y^2 \le 9\}$ is

(a) $\frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$

(b) $\frac{\sqrt{2}}{6}-\frac{9\pi}{8}$

(c) $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$

(d) None of these

42. The area bounded by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \le x \le \frac{\pi}{2}$ is

(a) $2\left(\sqrt{2}-1\right)$ (b) $\sqrt{2}-1$

(c) $\sqrt{2}+1$ (d) $\sqrt{2}$

43. The area of the region enclosed by the lines $y = x$, $x = e$ and curve $y=\frac{1}{x}$ and the positive x-axis is

(a) 1 sq. unit (b) $\frac{3}{2}$ sq. units

(c) $\frac{5}{2}$ sq. units (d) $\frac{1}{2}$ sq. units

44. Area of triangle whose two vertices formed from the x-axis and line $y = 3 – |x|$ is

(a) 9 sq. units (b) 9/4 sq. units
(c) 3 sq. units (d) None of these

45. The area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of parabola $y^2 = 4x$ is

(a) $\frac{1}{3}(2+3\pi)$ sq. units (b) $\frac{2}{3}(4+3\pi)$ sq. units

(c) $6 + 3\pi$ sq. units (d) $\frac{4}{3}(8+3\pi)$ sq. units

46. Area of the region between the curves $x^2 + y^2 = \pi^2$, $y = \sin x$ and y-axis in first quadrant is

(a) $\frac{\pi^3-8}{4}$ sq. units (b) $\frac{\pi^3-4}{8}$ sq. units

(c) $\frac{\pi^2-8}{4}$ sq. units (d) $\frac{\pi^2-4}{8}$ sq. units

47. Area enclosed between the curves $y = \sin^2 x$, $y = \cos^2 x$ and $y = 0$ in the interval $[0, \pi/2]$ is

(a) $\frac{1}{3}(2\pi - 1)$ sq. units (b) $\frac{1}{2}(\pi - 3)$ sq. units

(c) $\frac{1}{4}(\pi - 2)$ sq. units (d) $(2\pi + 3)$ sq. units

48. The area included between the parabolas $y^2 = 4a(x + a)$ and $y^2 = 4b(x - a)$, $b > a > 0$, is

(a) $\frac{4\sqrt{2}}{3} b^2 \sqrt{\frac{a}{b-a}}$ sq. units

(b) $\frac{8\sqrt{8}}{3} a^2 \sqrt{\frac{b}{b-a}}$ sq. units

(c) $\frac{4\sqrt{2}}{3} a^2 \sqrt{\frac{b}{b-a}}$ sq. units

(d) $\frac{8\sqrt{8}}{3} b^2 \sqrt{\frac{a}{b-a}}$ sq. units

49. The area common to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $0 < b < a$ is

(a) $(a+b)^2 \tan^{-1}\frac{b}{a}$ (b) $(a+b)^2 \tan^{-1}\frac{a}{b}$

(c) $4ab \tan^{-1}\frac{b}{a}$ (d) $4ab \tan^{-1}\frac{a}{b}$

50. The area of the triangle formed by the tangent and normal at the point $(1, \sqrt{3})$ on the circle $x^2 + y^2 = 4$ and the x-axis is

(a) 3 sq. units (b) $2\sqrt{3}$ sq. units

(c) $3\sqrt{2}$ sq. units (d) 4 sq. units

51. The ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$ is

(a) 15 : 49 (b) 13 : 37

(c) 15 : 23 (d) 17 : 50

52. The line $y = mx$ bisects the area enclosed by lines $x = 0$, $y = 0$ and $x = 3/2$ and the curve $y = 1 + 4x - x^2$. Then the value of m is

(a) $\frac{13}{6}$ (b) $\frac{13}{2}$

(c) $\frac{13}{5}$ (d) $\frac{13}{7}$

53. Area bounded by the circle $x^2 + y^2 = 1$ and the curve $|x| + |y| = 1$ is

(a) 2π (b) $\pi - 2$ (c) π (d) $\pi + 3$

54. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(a) $\frac{5}{3}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{3}$

55. Area bounded by the parabola $y = x^2 - 2x + 3$ and tangents drawn to it from the point P(1, 0) is equal to

(a) $4\sqrt{2}$ sq. units (b) $\frac{4\sqrt{2}}{3}$ sq. units

(c) $\frac{8\sqrt{2}}{3}$ sq. units (d) $\frac{16}{3}\sqrt{2}$ sq. units

56. The area (in sq. units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$ and x-axis lying in the first quadrant is

(a) 9 (b) 36

(c) 18 (d) $\frac{27}{4}$

57. Area of the region bounded by the curve $y = |x + 1| + 1$, $x = -3$, $x = 3$ and $y = 0$ is

(a) 8 sq units (b) 16 sq units

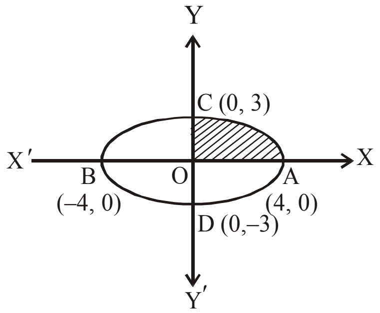
(c) 32 sq units (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) The equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$

The given ellipse is symmetrical about both axis as it contains only even powers of y and x.



Now, $\frac{x^2}{16}+\frac{y^2}{9}=1 \Rightarrow \frac{y^2}{9}=1-\frac{x^2}{16} \Rightarrow y^2=\frac{9}{16}(16-x^2)$

$\Rightarrow \quad y=\pm\frac{3}{4}\left(\sqrt{16-x^2}\right)$

Now, area bounded by the ellipse = 4 (area of ellipse in first quardant)

= 4 (area OAC)

$=4\int_0^4 y\,dx=\int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$ [$\because$ $y \geq 0$ in first quadrant]

Put $x = 4 \sin\theta$ so that $dx = 4\cos\theta\, d\theta$,

Now when $x = 0, \theta = 0$ and when $x = 4, \theta = \frac{\pi}{2}$

$\therefore$ Required area $=\frac{4\times 3}{4}$

$$\int_0^{\frac{\pi}{2}}\sqrt{16-16\sin^2\theta}\cdot 4\cos\theta\, d\theta$$

$$=3\int_0^{\frac{\pi}{2}}4\sqrt{1-\sin^2\theta}\cdot 4\cos\theta\, d\theta$$

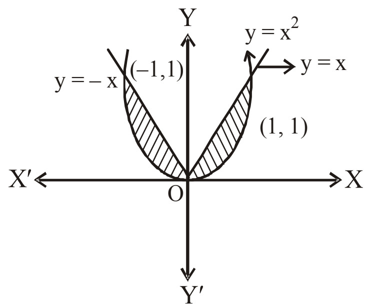
$$=48\int_0^{\frac{\pi}{2}}\cos^2\theta d\theta=48\int_0^{\frac{\pi}{2}}\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$=24\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)\,d\theta=24\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$=24\left[\left(\frac{\pi}{2}-0\right)+\frac{1}{2}(0-0)\right]=12\pi \text{ sq. units.}$$

2. (c) Clearly $x^2 = y$ represents a parabola with vertex at (0, 0) positive direction of y-axis as its axis opens upwards.

$y=|x|$ i.e., $y = x$ and $y = -x$ represent two lines passing through the origin and making an angle of 45° and 135° with the positive direction of the x-axis.



The required region is the shaded region as shown in the figure. Since both the curve are symmetrical about y-axis. So, required area = 2 (shaded area in the first quardant)

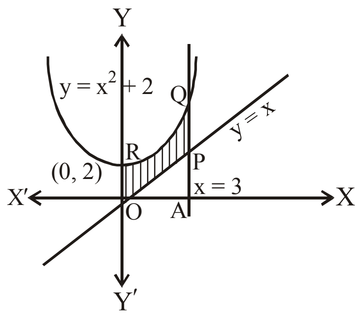
$$=2\int_0^1(x-x^2)\,dx=2\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$$

$$=2\left(\frac{1}{2}-\frac{1}{3}\right)=2\times\frac{1}{6}=\frac{1}{3}\text{ sq. units.}$$

3. (c) Equation of the parabola is $y = x^2 + 2$

or $x^2 = (y - 2)$

Its vertex is (0, 2) and axis is y-axis.



Boundary lines are $y = x$, $x = 0$, $x = 3$. Graphs of the curve and lines have been shown in the figure.

Area of the region PQRO

= Area of the region OAQR – Area of region OAP

$$=\int_0^3(x^2+2)\,dx-\int_0^3 x\,dx=\left[\frac{x^3}{3}+2x\right]_0^3-\left[\frac{x^2}{2}\right]_0^3$$

$$=\left[\left(\frac{27}{3}+6\right)-0\right]-\left(\frac{9}{2}-0\right)=\frac{21}{2}\text{ sq. units.}$$

4. (b) $y = x^2$ is a parabola with vertex (0, 0) and axis of parabola is y-axis.

	A	O	B	C
x	– 1	0	1	2
y	1	0	1	4

For intersection of $y = x^2$ and $y = x + 2$

$x^2 = x + 2$

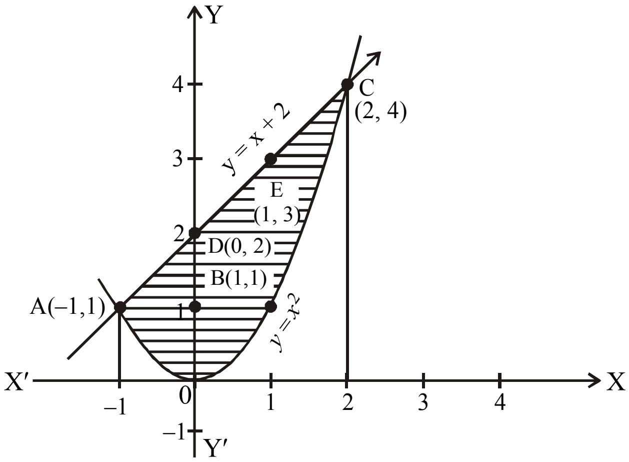
$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = -1, 2$

For $y = x + 2$

	A	D	E	C
x	– 1	0	1	2
y	1	2	3	4



Shaded required area

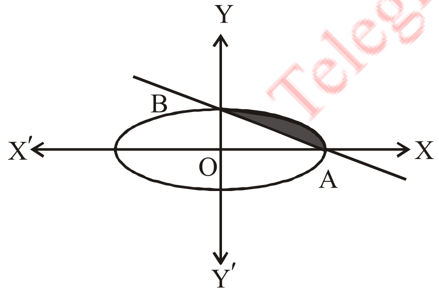
$$= \int_{-1}^{2} (x+2)\,dx - \int_{-1}^{2} x^2\,dx$$

$$= \left[\frac{(x+2)^2}{2}\right]_{-1}^{2} + \left[-\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{2}[16-1] - \left(\frac{8}{3}+\frac{1}{3}\right)$$

$$= \frac{15}{2} - 3 = \frac{9}{2} \text{ sq. units.}$$

5. **(d)** The required area is the shaded region in the figure.



$\therefore$ The required area

$$= \int_0^a \left(\frac{b}{a}\sqrt{a^2-x^2} - \frac{b}{a}(a-x)\right)dx$$

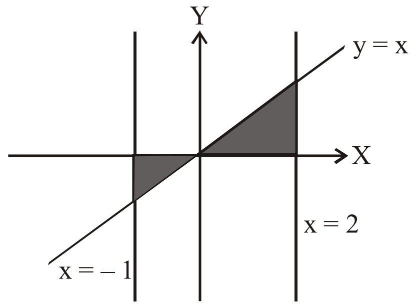
$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx - \frac{b}{a}\int_0^a (a-x)\,dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\frac{\sin^{-1}x}{a}\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1}1\right] - \frac{b}{a}\left[a^2 - \frac{a^2}{2}\right]$$

$$= \frac{ab}{2}\frac{\pi}{2} - ba\left(\frac{1}{2}\right) = \frac{ab}{4}(\pi - 2) \text{ sq. units.}$$

6. **(d)** We have, y = x, a line



Required area = area of shaded region

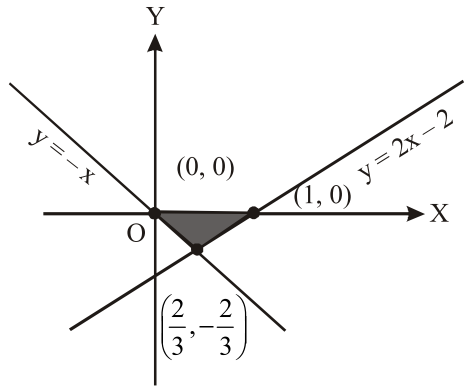
$$A = \left|\int_{-1}^{0} x\,dx\right| + \left|\int_0^2 x\,dx\right|$$

$$= \left|\frac{x^2}{2}\right|_{-1}^{0} + \left|\frac{x^2}{2}\right|_0^2 = \left|-\frac{1}{2}\right| + |2| = 2 + \frac{1}{2} = \frac{5}{2} \text{ sq. units.}$$

7. **(d)** We have, $y = 2x - 2$ ...(i)

$y = -x$ ...(ii)

Solving (i) and (ii), we get, $x = \frac{2}{3},\ y = \frac{-2}{3}$



Required area = area of shaded region

$$A = \int_0^{2/3} y\,dx + \int_{2/3}^{1} y\,dx$$

$$= \left|\int_0^{2/3} (-x)\,dx\right| + \left|\int_{2/3}^{1} (2x-2)\,dx\right|$$

$$= \left|\left[\frac{-x^2}{2}\right]_0^{2/3}\right| + \left|\left[x^2 - 2x\right]_{2/3}^{1}\right|$$

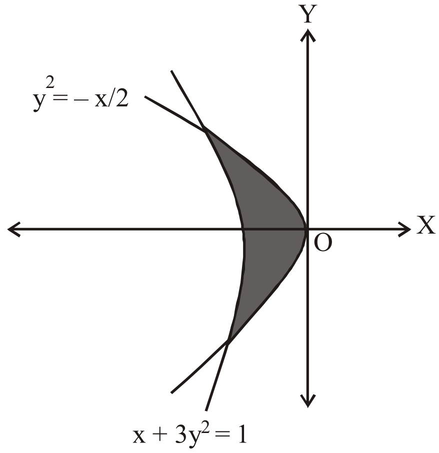
$$= \left|\frac{-1}{2}\left(\frac{4}{9} - 0\right)\right| + \left|\left[1 - 2 - \frac{4}{9} + \frac{4}{3}\right]\right|$$

$$= \frac{4}{18} + \frac{1}{9} = \frac{1}{3} \text{ sq. units}$$

8. (d) We have, $x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$ ...(i), a parabola with vertex (0, 0) and $x + 3y^2 = 1$

$\Rightarrow y^2 = \frac{1-x}{3} = -\left(\frac{x-1}{3}\right)$ ...(ii), a parabola with vertex (1, 0)

Solving (i) and (ii), we get $y = \pm 1$



$$A = \int_{-1}^{1}\left(\left(1-3y^2\right)-\left(-2y^2\right)\right)dy$$

$$= 2\int_0^1\left(1-y^2\right)dy = 2\left[y-\frac{y^3}{3}\right]_0^1 = \frac{4}{3}\text{sq.units}$$

9. (a) The given equation of curves are

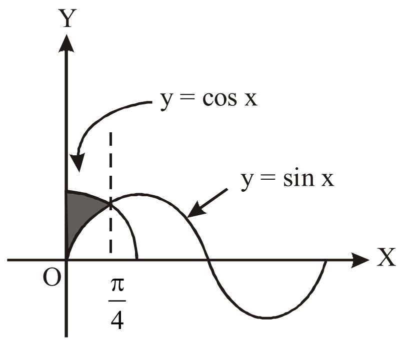
$y = \sin x$ ...(i)

and $y = \cos x$ ...(ii)

From equations (i) and (ii), we get

$\sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

$\therefore$ Required area $= \int_0^{\pi/4}(\cos x - \sin x)\,dx$

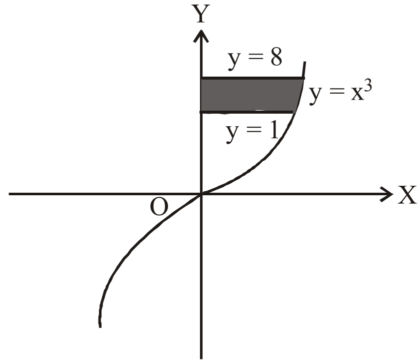


$$= [\sin x + \cos x]_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}-\sin 0-\cos 0\right) = \left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right]$$

$$= \frac{2}{\sqrt{2}}-1 = \left(\sqrt{2}-1\right)\text{sq. units}$$

10. (a) Given curve is $y = x^3$ or $x = y^{1/3}$

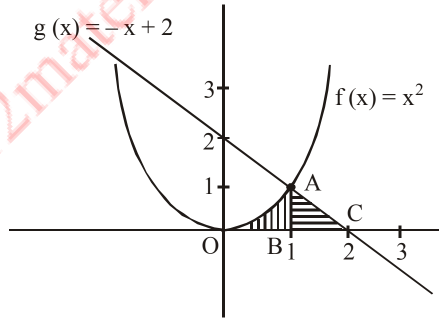


Considering the areas with y-axis, we find that required area

$$= \int_1^8 y^{1/3}dy = \left[\frac{y^{4/3}}{4/3}\right]_1^8 = \frac{3}{4}\left[8^{4/3}-1^{4/3}\right]$$

$$= \frac{3}{4}\times(16-1) = \frac{3}{4}\times 15 = \frac{45}{4}\text{sq.units}$$

11. (d) Required area = Area of OAB + Area of ABC



Now, Area of OAB $= \int_0^1 f(x)\,dx + \int_1^2 g(x)dx$

$$= \int_0^1 x^2dx + \int_1^2(-x+2)\,dx$$

$$= \left.\frac{x^3}{3}\right|_0^1 + \left[\frac{-x^2}{2}+2x\right]_1^2$$

$$= \frac{1}{3}+\left[\left(\frac{-4}{2}+4\right)-\left(\frac{-1}{2}+2\right)\right]$$

$$= \frac{1}{3}+\left[(-2+4)-\left(\frac{3}{2}\right)\right]$$

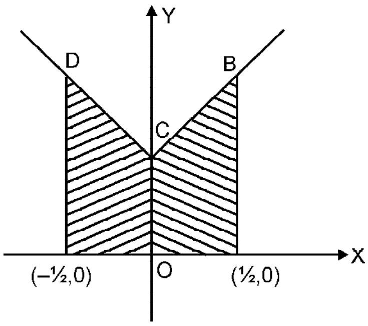
$$= \frac{1}{3}+\frac{1}{2} = \frac{5}{6}\text{sq unit}$$

12. (c) The given lines are,

$y - 1 = x, x \geq 0;\quad y - 1 = -x, x < 0$

$y = 0;\quad x = -\frac{1}{2}, x < 0;\quad x = \frac{1}{2}, x \geq 0$

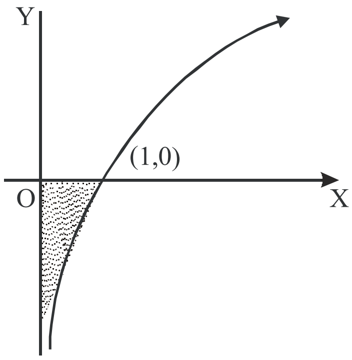
so that the area bounded is as shown in the figure.



Required area $= 2\int_0^{1/2}(1+x)\,dx$

$$= 2\left(x+\frac{x^2}{2}\right)_0^{1/2} = 2\left(\frac{1}{2}+\frac{1}{8}\right) = \frac{5}{4}$$

13. (b) Observing the graph of log x, we find that the required area lies below x-axis between x = 0 and x = 1.



So required area $= \left|\int_0^1 \log x\,dx\right|$

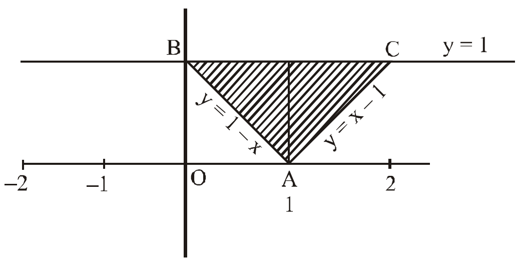
$$= |(x\log x - x)|_0^1 = |-1| = 1$$

14. (b) The given region is represented by the equations

$y = 1 - x,\ x \le 1$

$\ \ = x - 1,\ x \ge 1$

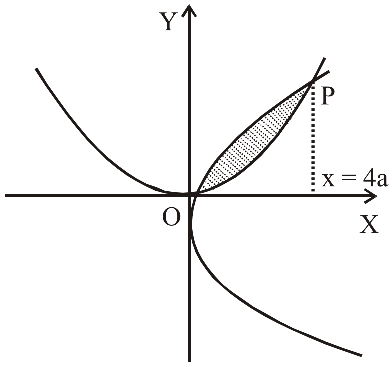
and y = 1; C = (2, 1) and B = (0, 1)



∴ the shaded area in the figure

$$= \frac{1}{2}\,BC\,.\,AC = \frac{1}{2}\,2\,.\,1 = 1.$$

15. (d) Solving the equation of the given curves for x, we get x = 0 and x = 4a.

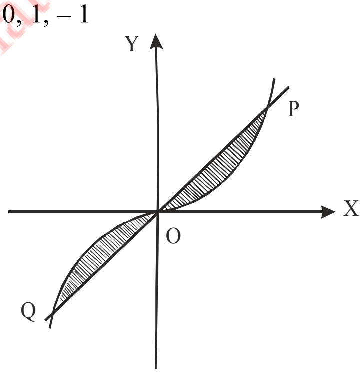


∴ Reqd area $= \int_0^{4a}\left(\sqrt{4ax}-\frac{x^2}{4a}\right)dx$

$$= \left[2\frac{2}{3}\sqrt{a}\,x^{3/2} - \frac{x^3}{12a}\right]_0^{4a}$$

$$= \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{16}{3}a^2$$

16. (b) Solving the equation of the given curves for x, we get x = 0, 1, – 1

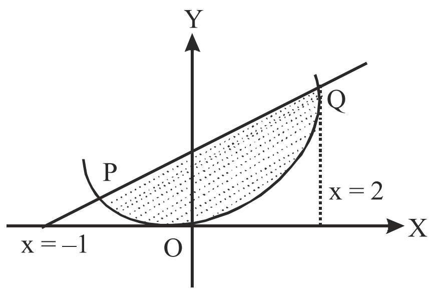


The required area is symmetrical about the origin as shown in the diagram. So

Reqd. area $= 2\int_0^1 (x - x^3)\,dx$

$$= 2\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{2}-\frac{1}{4}\right] = \frac{1}{2}$$

17. (d) Solving the equation of the given curves for x, we get



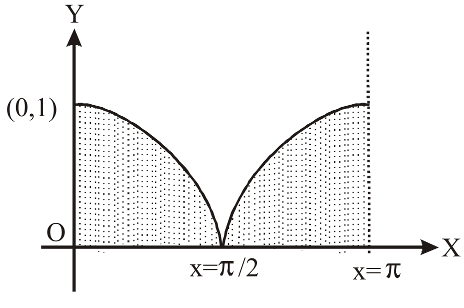
$x^2 = x + 2 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$

So, reqd. area $= \int_{-1}^{2}\left[\frac{x+2}{4}-\frac{x^2}{4}\right]dx$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\,[(2 + 4 - 8/3) - (1/2 - 2 + 1/3)] = 9/8$$

18. (d)



Required area $= \int_0^{\pi} \cos^2 x\, dx$

$$= \int_0^{\pi/2} \cos^2 x\, dx + \int_{\pi/2}^{\pi} \cos^2 x\, dx$$

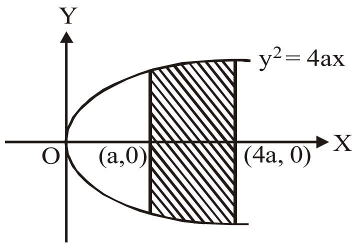
$$= \frac{1}{2} \times \frac{\pi}{2} + \frac{1}{2}\int_{\pi/2}^{\pi} (1 + \cos 2x)\, dx$$

$$= \frac{\pi}{4} + \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_{\pi/2}^{\pi} = \frac{\pi}{4} + \frac{1}{2}\left[\left(\pi - \frac{\pi}{2}\right)\right]$$

$$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

19. (d) Required area

= area of shaded portion of given curve



= 2 area of curve above x-axis

$$= 2\int_a^{4a} y\, dx = 2\int_a^{4a} \sqrt{4ax}\, dx$$

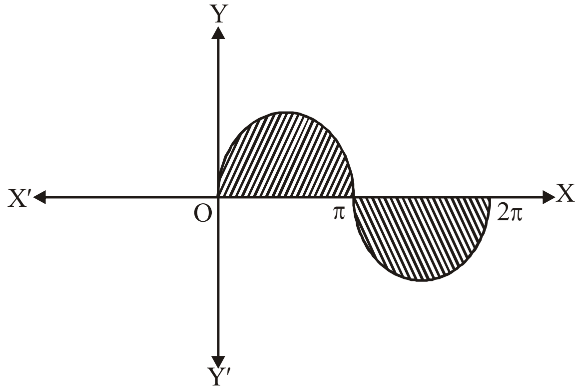
$$= 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a} = \frac{8}{3}\sqrt{a}\,[(4a)^{3/2} - a^{3/2}]$$

$$= \frac{8}{3}\sqrt{a}\,(8a^{3/2} - a^{3/2}) = \frac{56}{3}a^2 \text{ sq unit}$$

STATEMENT TYPE QUESTIONS

20. (b) I. We have, y = sin x

Let us draw a graph of sin x between 0 to 2π.



∴ Area of shaded region

$$= \int_0^{\pi} \sin x\, dx + \left|\int_{\pi}^{2\pi} \sin x\, dx\right|$$

$$= [-\cos x]_0^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$$

= [–cosπ + cos0] + | – cos 2π + cos π|

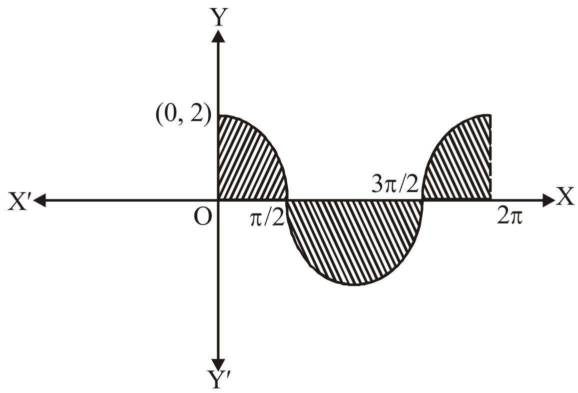
= |1 + 1| + |–1 –1|

= 2 + 2

= 4 sq units

II. We have y = 2 cos x

Let us draw the graph of 2 cos x between 0 to 2π.



∴ Area of shaded region

$$= \int_0^{\pi/2} 2\cos x\,dx + \left|\int_{\pi/2}^{3\pi/2} 2\cos x\,dx\right| + \int_{3\pi/2}^{2\pi} 2\cos x\, dx$$

$$= 2[\sin x]_0^{\pi/2} + \left|[2\sin x]_{\pi/2}^{3\pi/2}\right| + [2\sin x]_{3\pi/2}^{2\pi}$$

$$= 2\left[\sin\frac{\pi}{2} - 0\right]\left[\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right] + 2\left[\sin 2\pi - \frac{3\pi}{2}\right]$$

= 2 + 2 × 2 + 2

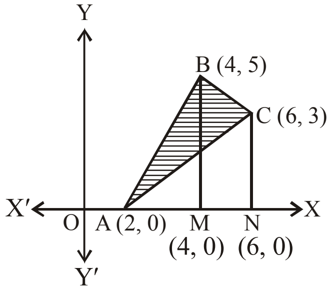
= 2 + 4 + 2

= 8 sq units

INTEGER TYPE QUESTIONS

21. (c) Equation of the line AB is $y = \frac{5}{2}(x - 2)$

Equation of the line BC is $y = -x + 9$



Equation of the line CA is $y = \frac{3}{4}(x - 2)$

Required area = area of the region bounded by ΔABC

= area of the region AMB + Area of region BMNC – area of the region ANC

$$= \frac{5}{2}\int_2^4 (x-2)\,dx + \int_4^6 -(x-9)\,dx - \frac{3}{4}\int_2^6 (x-2)\,dx$$

$$= \frac{5}{2}\left[\frac{(x-2)^2}{2}\right]_2^4 - \left[\frac{(x-9)^2}{2}\right]_4^6 - \frac{3}{4}\left[\frac{(x-2)}{2}\right]_2^6$$

$$= \frac{5}{4}[2^2 - 0] - \frac{1}{2}[(-3)^2 - (-5)^2] - \frac{3}{8}[(4)^2 - 0]$$

= 7 sq. units.

22. (a) We have, $y = -x - 2$ ...(i)

$y = x - 2$ ...(ii)

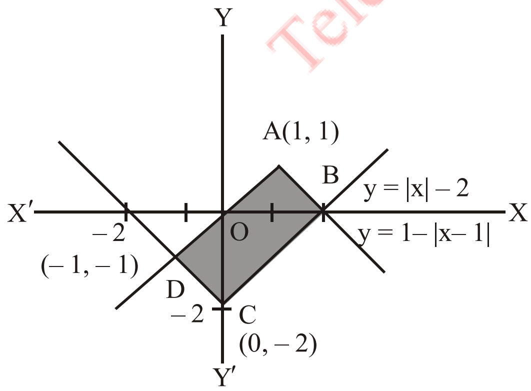
$y = 2 - x$ ...(iii)

$y = x$ ...(iv)

Solving (iii) and (iv), we get A(1, 1)

Solving (i) and (iv), we get D(– 1, – 1)

Required area = area of ΔAOB + area of ΔOCB + area of ΔOCD



$$\text{Area of } \Delta AOB = \int_0^1 x\,dx + \int_1^2 (2-x)\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 = \frac{1}{2} + \left[4 - \frac{4}{4}\right] - \left[2 - \frac{1}{2}\right]$$

$$= \frac{1}{2} + \frac{4}{2} - \frac{3}{2} = 1 \text{ sq. unit}$$

$$\text{Area of } \Delta OCB = \left|\int_0^2 (x-2)\,dx\right|$$

$$= \left|\left[\frac{x^2}{2} - 2x\right]_0^2\right| = 2 \text{ sq. units}$$

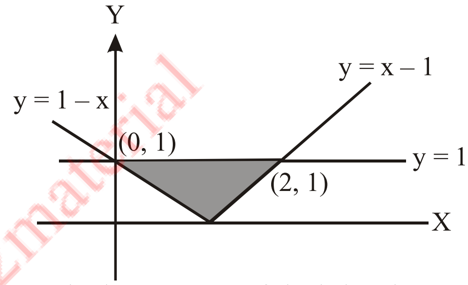
$$\text{Area of } \Delta OCD = \left|\int_{-1}^{0} (-x-2)\,dx - \int_{-1}^{0} x\,dx\right|$$

$$= \left|-\left[\frac{x^2}{2} + 2x\right]_{-1}^{0} - \left[\frac{x^2}{2}\right]_{-1}^{0}\right| = 1 \text{ sq. unit}$$

Required area = 1 + 2 + 1 = 4 sq. units

23. (b) We have, $y = x - 1$, if $x - 1 \geq 0$

$y = -x + 1$, if $x - 1 < 0$



Required area = area of shaded region

$$A = \int_0^2 1\,dx - \left[\int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx\right]$$

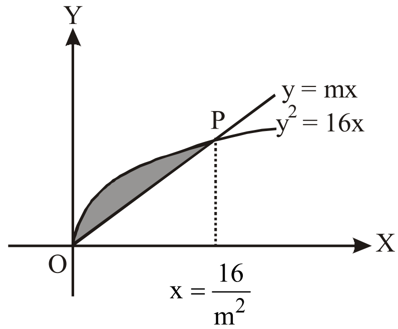
$$= [x]_0^2 - \left[x - \frac{x^2}{2}\right]_0^1 - \left[\frac{x^2}{2} - x\right]_1^2$$

$$= 2 - \frac{1}{2} - \frac{1}{2} = 1 \text{ sq. unit}$$

24. (b) We have, $y^2 = 16x$, a parabola with vertex (0, 0) and line $y = mx$.

Required area = area of shaded region

$$\Rightarrow \int_0^{16/m^2} \left(\sqrt{16x} - mx\right)dx = \frac{2}{3}$$



$$\Rightarrow \left[4 \times \frac{2}{3}x^{3/2} - m\left(\frac{x^2}{2}\right)\right]_0^{16/m^2} = \frac{2}{3}$$

$$\Rightarrow \frac{8}{3}\times\frac{64}{m^3}-\frac{m}{2}\frac{256}{m^4}=\frac{2}{3} \Rightarrow \frac{1}{m^3}\left[\frac{512}{3}-128\right]=\frac{2}{3}$$

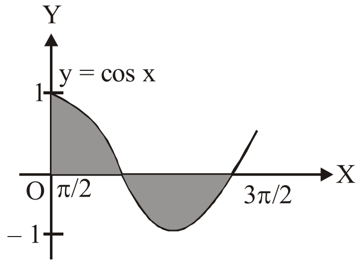
$\Rightarrow m = 4$

25. (c) We have, $y = \cos x$

between $x = 0$ to $x = \frac{3\pi}{2}$

Required area = area of shaded region

$$A = \int_0^{\pi/2} \cos x\, dx + \left|\int_{\pi/2}^{3\pi/2} \cos x\, dx\right|$$

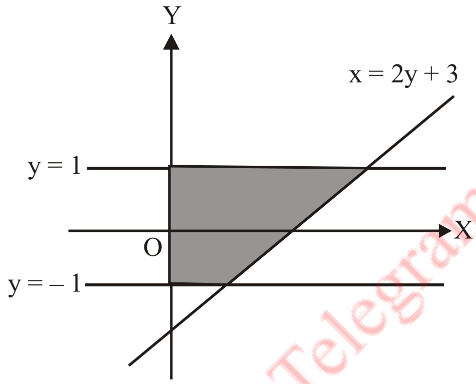


$= [\sin x]_0^{\pi/2} + [\sin x]_{\pi/2}^{3\pi/2}$

$= 1 + |(-1-1)|$

$= 1 + 2 = 3$ sq. units

26. (c) We have $x = 2y + 3$, a straight line



Required area = area of shaded region

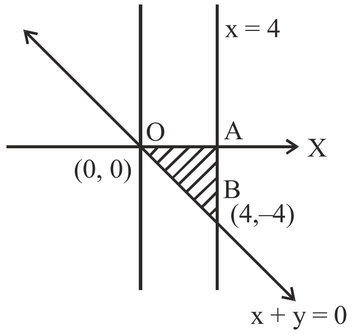
$$= \int_{-1}^{1} (2y+3)\, dy = \left[y^2+3y\right]_{-1}^{1}$$

$= (1 + 3) - (1 - 3) = 4 + 2 = 6$ sq. units

27. (b) Graph of these three lines, shows the area.

$x = 4$, cuts $x + y = 0$ at point B $(4, -4)$ as shown below.

The area OAB has been shown by shaded region.

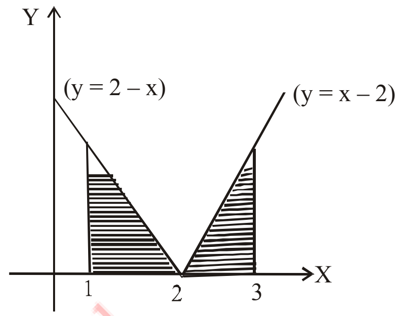


Required Area = Area of ΔOAB

$$= \frac{1}{2}\times AB\times OA = \frac{1}{2}\times 4\times 4$$

$$= \frac{16}{2} = 8 \text{ sq. units}$$

28. (d) The required area is shown by shaded region



Required Area

$$A = \int_1^3 |x-2|\, dx = 2\int_2^3 (x-2)\, dx$$

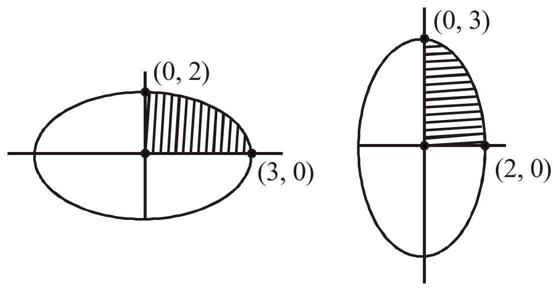
$$= 2\left[\frac{x^2}{2}-2x\right]_2^3 = 1$$

29. (b) $\text{Area} = \int_0^1 y\, dx = \int_0^1 \sqrt{36x}\, dx$

$$= 6\int_0^1 \sqrt{x}\, dx = 6\left[\frac{2}{3}x^{3/2}\right]_0^1$$

$$= 4\left[x^{3/2}\right]_0^1 = 4\left(1^{3/2}-0\right) = 4 \text{ sq. units.}$$

30. (b)



When the ellipse is rotated then its area in first quadrant still remains 6π sq. units.

31. (a) $\text{Area} = \int_0^{\pi} y\, dx = \int_0^{\pi} \sin x\, dx$

$$= -[\cos x]_0^{\pi} = -[\cos\pi - \cos 0]$$

$$= -[-1-1] = -(-2)$$

$= 2$ sq. units.

32. (d) Area $= \int_0^3 y\,dx = \int_0^3 x^2\,dx$

$$= \left[\frac{x^3}{3}\right]_0^3 = \left[\frac{27}{3} - 0\right] = 9 \text{ sq. units.}$$

33. (b) Area $= 2 = \int_0^2 ax\,dx$

$$2 = a\left[\frac{x^2}{2}\right]_0^2$$

$$2 = a\left[\frac{4}{2} - 0\right] = 2a$$

which gives a = 1

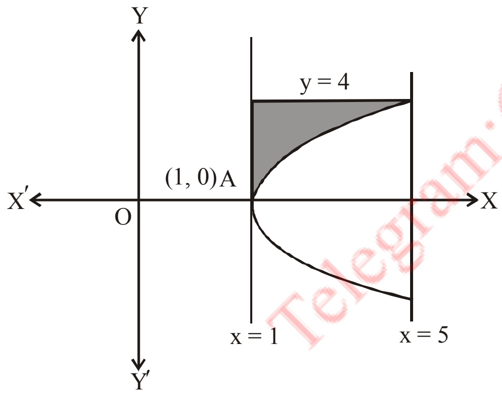
34. (b) Area $= \int_0^1 y\,dx = \int_0^1 \frac{3}{2}\sqrt{x}\,dx$

$$= \frac{3}{2}\left[\frac{2}{3}x^{3/2}\right]_0^1 = \left[x^{3/2}\right]_0^1$$

$$= 1^{3/2} - 0 = 1 \text{ sq. unit}$$

ASSERTION - REASON TYPE QUESTIONS

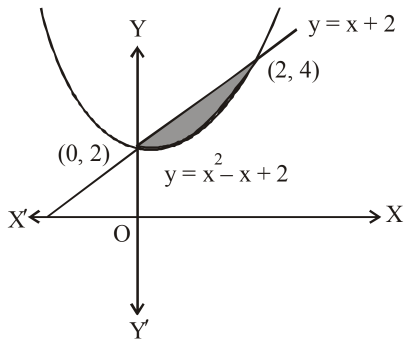
35. (c) **Assertion :** On solving $y^2 = 4a^2(x-1)$ and $y = 4a$, we get x = 5



$$\therefore \text{ Required area } = \int_1^5 \left(4a - 2a\sqrt{x-1}\right)dx$$

$$= \left[4ax - 2a.\frac{(x-1)^{3/2}}{3/2}\right]_1^5 = \frac{16a}{3} \text{sq.units}$$

Reason : Given equation of parabola can be rewritten as



$$\left(x - \frac{1}{2}\right)^2 = y - \frac{7}{4}$$

Parabola $y^2 = x^2 - x + 2$ and the line $y = x + 2$ intersects at the point (0, 0) and (2, 4)

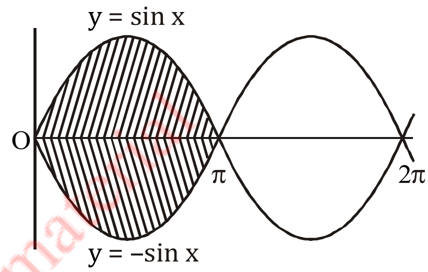
$$\therefore \text{ Required area } = \int_0^2 \left[(x+2) - \left(x^2 - x + 2\right)\right]dx$$

$$= \int_0^2 \left(-x^2 + 2x\right)dx$$

$$= \left[\frac{-x^3}{3} + x^2\right]_0^2 = -\frac{8}{3} + 4 = \frac{4}{3} \text{sq.units}$$

36. (b)

37. (d)



Shaded regions is the required area which is symmetric about x-axis.

$$\text{Required area} = 2\int_0^{\pi} \sin x\,dx$$

$$= 2\left[-\cos y\right]_0^{\pi} = -2\left[(-1) - (1)\right]$$

$$= -2(-2) = 4 \text{ sq. units.}$$

38. (a) $\int_0^{\pi/2} \cos x\,dx = \left[\sin x\right]_0^{\pi/2} = \left[\sin\frac{\pi}{2} - \sin x\right]$

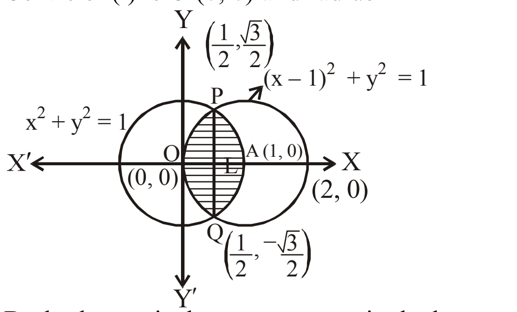
$$= 1 - 0 = 1 \text{ sq. unit.}$$

CRITICAL THINKING TYPE QUESTIONS

39. (a) Given circles are $x^2 + y^2 = 1$ ...(i)

and $(x-1)^2 + y^2 = 1$ ...(ii)

Centre of (i) is O (0, 0) and radius =1



Both these circle are symmetrical about x-axis

solving (i) and (ii), we get, $-2x + 1 = 0 \Rightarrow x = \frac{1}{2}$

then $y^2 = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow \quad y = \frac{\sqrt{3}}{2}$

$\therefore$ The points of intersection are

$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

It is clear from the figure that the shaded portion in region whose area is required.

$\therefore$ Required area = area OQAPO

= 2 × area of the region OLAP

= 2 × (area of the region OLPO + area of LAPL)

$$= 2\left[\int_0^{1/2}\sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1}\sqrt{1-x^2}\,dx\right]$$

$$= 2\left[\frac{(x-1)\sqrt{1-(x-1)^2}}{2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{1/2} + 2\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_{1/2}^{1}$$

$$= -\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(-1) + 0 + \sin^{-1}(1) - \left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{1}{2}\right)\right)$$

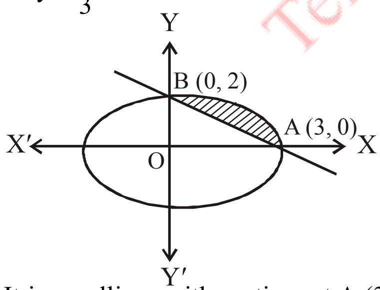
$$= \frac{\sqrt{3}}{4} - \frac{\pi}{6} - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \frac{\sqrt{3}}{4} - \frac{\pi}{6}$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ sq. units.}$$

40. (c) The given ellipse is

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \Rightarrow y^2 = \frac{4}{9}(9 - x^2)$$

$$\Rightarrow y = \frac{2}{3}\sqrt{9 - x^2}$$



It is an ellipse with vertices at A (3, 0) and B (0, 2) and length of the major axis = 2 (3) = 6 and length of the minor axis = 2 (2) = 4

Line $\frac{x}{3} + \frac{y}{2} = 1 \Rightarrow y = \left(\frac{6-2x}{3}\right)$

It is a straight line passing thorugh A (3, 0) and B (0, 2).
Smaller area common to both is shaded.
Shaded Area

$$= \frac{2}{3}\int_0^3\sqrt{9-x^2}\,dx - \int_0^3\left(\frac{6-2x}{3}\right)dx = \frac{2}{3}I_1 - \frac{1}{3}I_2$$

where, $I_1 = \int_0^3\sqrt{9-x^2}\,dx$ and $I_2 = \int_0^3(6-2x)\,dx$

For I_1, put x = 3 sin θ so that dx = 3 cos θ dθ

When, x = 0, θ = 0 and when x = 3, $\theta = \frac{\pi}{2}$

$$\therefore I_1 = \int_0^{\frac{\pi}{2}}\sqrt{9 - 9\sin^2\theta}\cdot 3\cos\theta\,d\theta$$

$$= \frac{9}{2}\int_0^{\frac{\pi}{2}}(1 + \cos 2\theta)\,d\theta$$

$$= \frac{9}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} = \frac{9}{2}\left(\frac{\pi}{2} - 0\right) = \frac{9\pi}{4} \text{ and}$$

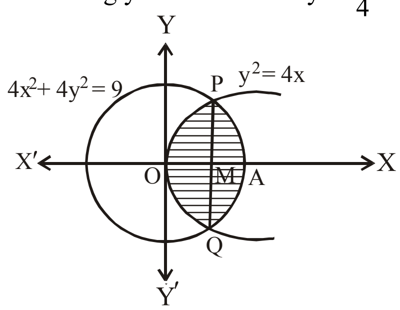
$$I_2 = \int_0^3(6-2x)\,dx = \left[6x - x^2\right]_0^3 = 18 - 9 = 9$$

$$\text{Required area} = \frac{2}{3}\times\frac{9\pi}{4} - \frac{1}{3}\times 9 = \frac{3\pi}{2} - 3$$

$$= \frac{3}{2}(\pi - 2) \text{ sq. units.}$$

41. (a) $y^2 = 4x$ is a parabola where vertex is the origin and $4x^2 + 4y^2 = 9$ represents circle whose centre is (0, 0) and radius $= \frac{3}{2}$

on solving $y^2 = 4x$ and $x^2 + y^2 = \frac{9}{4}$.



The points of intersection are $P\left(\frac{1}{2}, \sqrt{2}\right)$ and $Q\left(\frac{1}{2}, -\sqrt{2}\right)$. Both the curves are symmetrical about x-axis.

Required Area = area of the shaded region

= 2 (area of the region OAPO)

= [(area of the region OMPO) + (area of the region MAPM)]

$$= 2\left(\int_0^{1/2}2\sqrt{x}\,dx + \int_{1/2}^{3/2}\sqrt{\frac{9}{4} - x^2}\,dx\right)$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^{1/2} + 2\left[\frac{x\sqrt{\frac{9}{4} - x^2}}{2} + \frac{1}{2}\cdot\frac{9}{4}\sin^{-1}\left(\frac{x}{3/2}\right)\right]_{1/2}^{3/2}$$

$$= \frac{8}{3}\left(\frac{1}{2\sqrt{2}} - 0\right) + \left[x\sqrt{\frac{9}{4} - x^2} + \frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)\right]_{1/2}^{3/2}$$

$$= \frac{2\sqrt{2}}{3} + 0 + \frac{9}{4}\sin^{-1}(1) - \left(\frac{1}{2}\sqrt{2} + \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right)$$

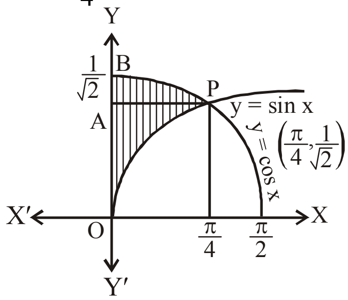
$$= \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{2} + \frac{9}{4}\cdot\frac{\pi}{2} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$$

$$= \left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right] \text{ sq. units.}$$

42. (b) The curves are $y = \cos x$, $y = \sin x$, $0 \le x \le \frac{\pi}{2}$

The curves meet where $\sin x = \cos x$ or $\tan x = 1$

$\Rightarrow x = \frac{\pi}{4}$



$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, Graphs of three curves is as shown in the figure. They intersect at $P\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right)$.

The area bounded by y-axis,

$y = \cos x$ and $y = \sin x$ $\left(0 \le x \le \frac{\pi}{2}\right)$

= Shaded region = Area of region OPBO

= Area of region PAO + Area of region APBA

$$= \int_0^{1/\sqrt{2}} 2\sin^{-1} y\,dy + \int_{1/\sqrt{2}}^{1} \cos^{-1} y\,dy$$

Integrate each by parts taking 1 as first function.

$$A = \left[y\sin^{-1} y - \int \frac{1}{\sqrt{1-y^2}}\,y\,dx\right]_0^{1/\sqrt{2}}$$

$$+ \left[y\cos^{-1} y + \int \frac{1}{\sqrt{1-y^2}}\,y\,dy\right]_{1/\sqrt{2}}^{1}$$

$$= \left[y\sin^{-1} y + \sqrt{1-y^2}\right]_0^{\frac{1}{\sqrt{2}}} + \left[y\cos^{-1} y - \sqrt{1-y^2}\right]_{1/\sqrt{2}}^{1}$$

$$= \left[\frac{1}{\sqrt{2}}\sin^{-1}\frac{1}{\sqrt{2}} + \sqrt{\frac{1}{\sqrt{2}}} - 1\right]$$

$$+ \left[(\cos^{-1} 1 - 0) - \left(\frac{1}{\sqrt{2}}\cos^{-1}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)\right]$$

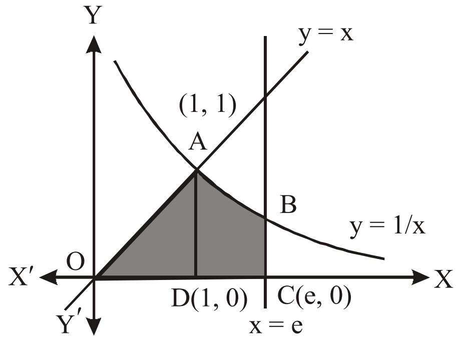
$$= \left(\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4} + \frac{1}{\sqrt{2}} - 1\right) + \left(0 - \frac{1}{\sqrt{2}}\cdot\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2} - 1.$$

43. (b) We have, $y = x$ ...(i)

$x = e$ ...(ii)

and $y = \frac{1}{x}$, $x \ge 0$ ...(iii)



Since $y = x$ and $x \ge 0 \Rightarrow y \ge 0$

$\therefore$ Area to be calculated in first quadrant shown in figure.

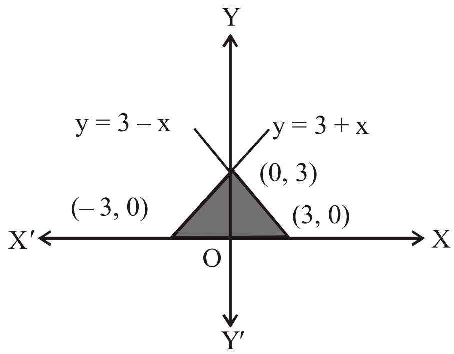
$\therefore$ Area = Area of ΔODA + Area of ABCD

$$= \frac{1}{2}(1\times 1) + \int_1^e \frac{1}{x}dx = \frac{1}{2} + \left[\log|x|\right]_1^e = \frac{1}{2} + 1 = \frac{3}{2}\text{sq.units}$$

44. (d) We have, $y = 3 - |x|$

$\Rightarrow y = 3 + x, \ \forall \ x < 0$ ...(i)

$y = 3 - x, \ \forall \ x > 0$ ...(ii)



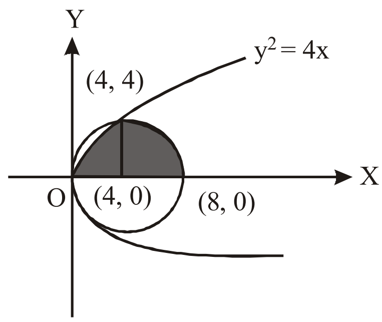
$\therefore$ Required area = area of shaded region

$$= 2\int_{-3}^{0}(3-x)\,dx = 2\left[3x - \frac{x^2}{2}\right]_{-3}^{0}$$

$$= -2\left[3(-3) - \frac{9}{2}\right] = -2\left[-9 - \frac{9}{2}\right]$$

$$= -2\left[-\frac{27}{2}\right] = 27 \text{ sq. units}$$

45. (d)



We have $x^2 + y^2 = 8x$ ...(i),
a circle with centre (4, 0) and radius 4.
and $y^2 = 4x$ ...(ii)
a parabola with vertex (0, 0).
Points of intersection of (i) and (ii) are (0, 0) and (4, 4).

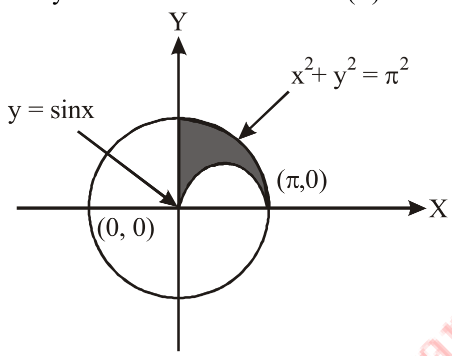
$$\text{Area} = \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2-(x-4)^2}\,dx$$

$$= 2\left[\frac{2x^{3/2}}{3}\right]_0^4 + \left[\frac{(x-4)}{2}\sqrt{4^2-(x-4)^2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$$

$$= \frac{4}{3}(4)^{3/2} + \left(8\sin^{-1}1\right) = 4\times\frac{8}{3} + 8\times\frac{\pi}{2}$$

$$= \frac{4}{3}(8+3\pi) \text{ sq. units.}$$

46. (a) We have $x^2 + y^2 = \pi^2$ ...(i) is a circle with vertex (0, 0) and radius π.
and $y = \sin x$ ...(ii)



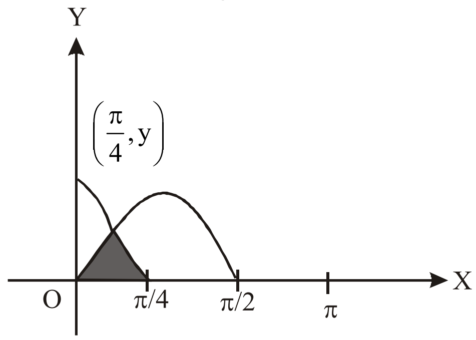
Required area = area of shaded region

$$= \text{Area of the circle in 1st quadrant} - \int_0^\pi \sin x\,dx$$

$$= \frac{\pi\times\pi^2}{4} + [\cos x]_0^\pi = \frac{\pi^3}{4} + \{-1-1\} = \frac{\pi^3-8}{4} \text{ sq. units}$$

47. (c) $y = \sin^2 x$ ...(i) and $y = \cos^2 x$ ...(ii)
Solving (i) and (ii), we get
$\sin^2 x = \cos^2 x$
$\tan^2 x = 1$

$$\tan x = \pm 1 \Rightarrow x = \frac{\pi}{4}$$



Required area = area of shaded region

$$= \int_0^{\pi/4} \sin^2 x\,dx + \int_{\pi/4}^{\pi/2} \cos^2 x\,dx$$

$$= \int_0^{\pi/4} \frac{1-\cos 2x}{2}dx + \int_{\pi/4}^{\pi/2} \frac{1+\cos 2x}{2}dx$$

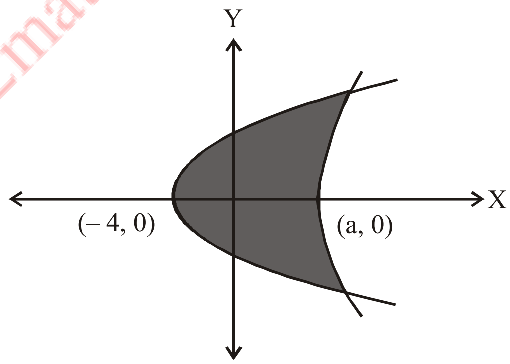
$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/4} + \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_{\pi/4}^{\pi/2}$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}\right] + \frac{1}{2}\left[\frac{\pi}{2} - 0 - \frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{4} - \frac{1}{2}\right] = \frac{\pi}{4} - \frac{1}{2} = \frac{1}{4}(\pi - 2) \text{ sq. units.}$$

48. (b) We have $y^2 = 4a(x + a)$ ...(i), a parabola with vertex (– a, 0) and $y^2 = 4b\,(x – a)$ ...(ii), a parabol a with vertex (a, 0)

Solving (i) and (ii), we get $y = \pm a\sqrt{\dfrac{8b}{b-a}}$



$$A = 2\int_0^{\frac{a\sqrt{8b}}{\sqrt{b-a}}} \left(\left(\frac{y^2}{4b} + a\right) - \left(\frac{y^2}{4b} - a\right)\right)dy$$

$$= 2\int_0^{\frac{a\sqrt{8b}}{\sqrt{b-a}}} \left(2a - \frac{(b-a)y^2}{4ab}\right)dy$$

$$= 2\left[2ay - \frac{b-a}{12ab}y^3\right]_0^{\frac{a\sqrt{8b}}{\sqrt{b-a}}}$$

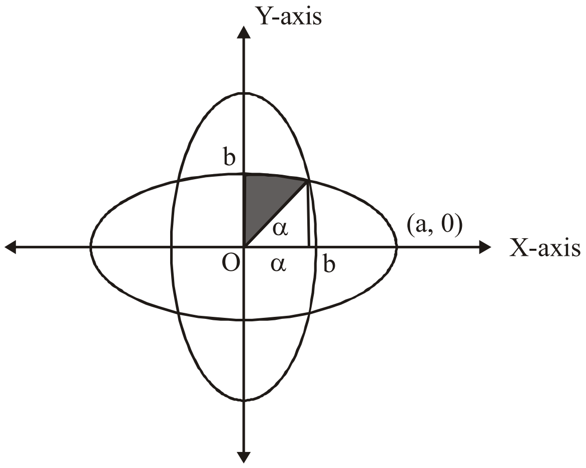
$$= 2\left[2a\times\frac{a\sqrt{8b}}{\sqrt{b-a}} - \frac{b-a}{12ab}\left(\frac{a\sqrt{8b}}{b-a}\right)^3\right]$$

$$= 4a^2\sqrt{\frac{8b}{b-a}} - \frac{4}{3}a^2\sqrt{\frac{8b}{b-a}} = \frac{8a^2}{3}\sqrt{\frac{8b}{b-a}} \text{ sq. units}$$

49. (c) We have, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ both are ellipse with centre (0, 0), vertex (a, 0), (– a, 0) and (0, b), (0, – b)

The curves intersect at $(\pm\alpha, \pm\alpha)$

where, $\alpha = \dfrac{ab}{\sqrt{a^2+b^2}}$



The area of shaded region

$$I = \int_0^{\alpha} \frac{b}{a}\sqrt{a^2-x^2}\,dx - \frac{\alpha^2}{2}$$

$$= \frac{b}{2a}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right]_0^{\alpha} - \frac{\alpha^2}{2}$$

$$= \frac{b}{2a}\left[\alpha\sqrt{a^2-\alpha^2} + a^2\sin^{-1}\frac{\alpha}{a}\right] - \frac{\alpha^2}{2}$$

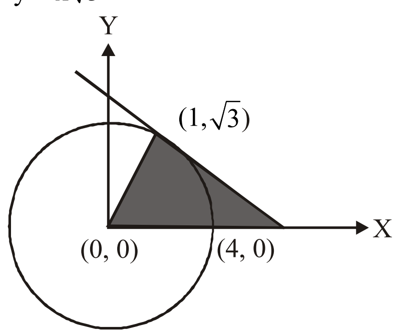
$$= \frac{b}{2a}\cdot\frac{ab}{\sqrt{a^2+b^2}}\cdot\frac{a^2}{\sqrt{a^2+b^2}} +$$

$$\frac{ab}{2}\sin^{-1}\frac{b}{\sqrt{a^2+b^2}} - \frac{a^2b^2}{2\left(a^2+b^2\right)}$$

$$= \frac{ab}{2}\tan^{-1}\frac{b}{a}$$

Required area $= 8I = 4ab\tan^{-1}\dfrac{b}{a}$ sq. units

50. (b) The tangent on $x^2 + y^2 = 4$ at $\left(1, \sqrt{3}\right)$ is $\left[x + \sqrt{3}y = 4\right]$ and equation of normal at $\left(1, \sqrt{3}\right)$ is $y = x\sqrt{3}$



Required area $= \displaystyle\int_0^1 x\sqrt{3}\,dx + \int_1^4 \frac{4-x}{\sqrt{3}}\,dx$

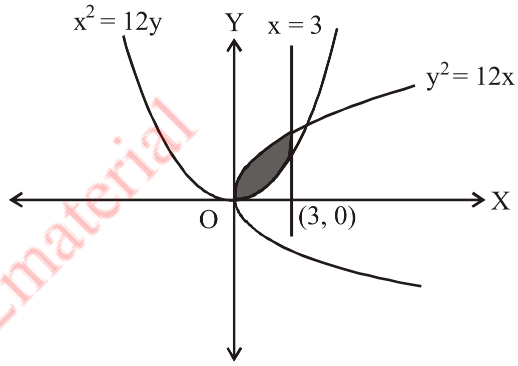
$$= \sqrt{3}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{\sqrt{3}}\left[4x - \frac{x^2}{2}\right]_1^4$$

$$= \sqrt{3}\times\frac{1}{2} + \frac{1}{\sqrt{3}}\left[4(4-1) - \frac{1}{2}(16-1)\right]$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[12 - \frac{15}{2}\right] = \frac{\sqrt{3}}{2} + \frac{9}{2\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 2\sqrt{3} \text{ sq. units}$$

51. (b) We have $y^2 = 12x$ and $x^2 = 12y$, parabolas with vertex (0, 0).

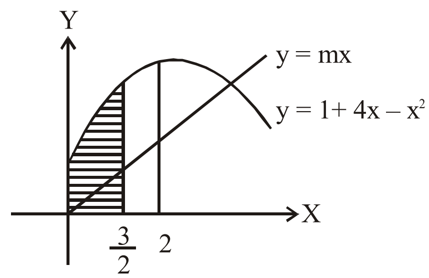


The curves intersect at x = 0, 12

$$\text{Required ratio} = \frac{\displaystyle\int_0^3\left(\sqrt{12x} - \frac{x^2}{12}\right)dx}{\displaystyle\int_3^{12}\left(\sqrt{12x} - \frac{x^2}{12}\right)dx} = \frac{\left|\sqrt{12}.\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{36}\right|_0^3}{\left|\sqrt{12}.\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{36}\right|_3^{12}}$$

$$= \frac{15}{49} \text{ sq. units}$$

52. (a) $y = 1 + 4x - x^2 = 5 - (x-2)^2$



We have $\displaystyle\int_0^{3/2}(1+4x-x^2)dx = 2\int_0^{3/2} mx\,dx$

$$\Rightarrow \frac{3}{2} + 2\left(\frac{9}{4}\right) - \frac{1}{3}\left(\frac{27}{8}\right) = m\cdot\frac{9}{4}$$

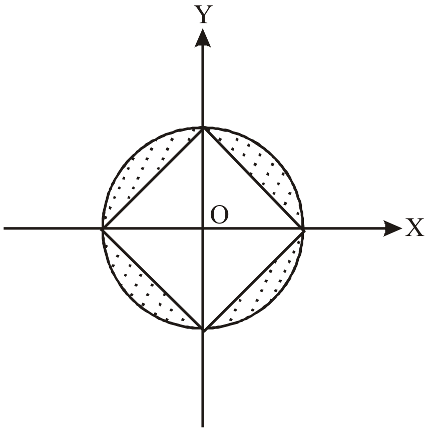
On solving, we get $m = \dfrac{13}{6}$

53. (b) By changing x as –x and y as–y, both the given equation remains unchanged so required area will be symmetric with respect to both the axis, which is shown in the figure. So,

Required area $= 4\int_0^1\left[\sqrt{1-x^2}-(1-x)\right]dx$

$= 4\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x - x+\frac{x^2}{2}\right]_0^1$

$= 4\left[0+\frac{1}{2}.\frac{\pi}{2}-1+\frac{1}{2}\right] = \pi - 2$



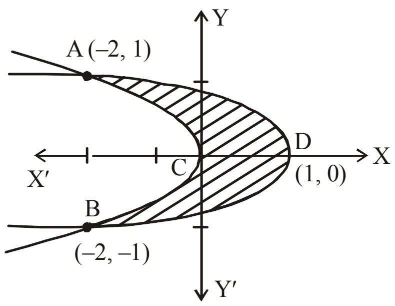
54. (d) $x+2y^2=0 \Rightarrow y^2=-\frac{x}{2}$

[Left handed parabola with vertex at (0, 0)]

$x+3y^2=1 \Rightarrow y^2=-\frac{1}{3}(x-1)$

[Left handed parabola with vertex at (1, 0)]

Solving the two equations we get the points of intersection as (–2, 1), (–2, –1)

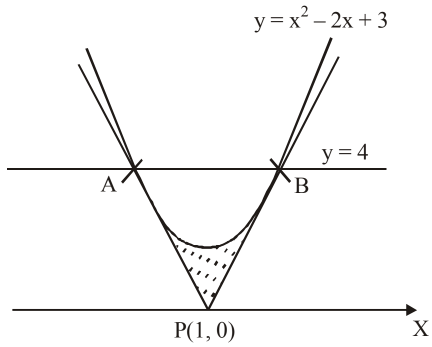


The required area is ACBDA, given by

$= \left|\int_{-1}^{1}(1-3y^2-2y^2)dy\right| = \left|\left[y-\frac{5y^3}{3}\right]_{-1}^{1}\right|$

$= \left|\left(1-\frac{5}{3}\right)-\left(-1+\frac{5}{3}\right)\right| = 2\times\frac{2}{3}=\frac{4}{3}$ sq. units.

55. (c) Let the drawn tangents be PA and PB. AB is clearly the chord of contact of point P.



Thus equation of AB is

$\frac{1}{2}.(y+0) = x.1-(2+1)+3$ i.e., y = 4

x coordinates of points A and B will be given by,

$x^2-2x+3=4$ i.e., $x^2-2x-1=0 \Rightarrow x = 1 \pm \sqrt{2}$

Thus AB = $2\sqrt{2}$ units.

Hence $\Delta_{PAB} = \frac{1}{2}(2\sqrt{2}).4 = 4\sqrt{2}$ sq. units

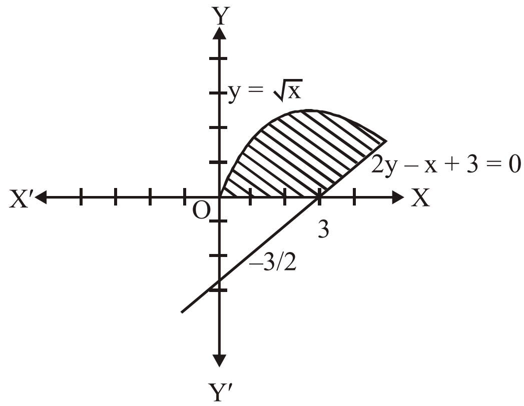
Now area bounded by line AB and parabola is equal to

$\int_{1-\sqrt{2}}^{1+\sqrt{2}}(4\sqrt{2}-(x^2-2x+3))\,dx = \frac{4\sqrt{2}}{3}$ sq. units.

Thus required area $= 4\sqrt{2}-\frac{4\sqrt{2}}{3}=\frac{8\sqrt{2}}{3}$ sq. units.

56. (a) Given curves are $y=\sqrt{x}$ .... (i)

and 2y – x + 3 = 0 .... (ii)



On solving eqs. (i) and (ii), we get

$2\sqrt{x}-(\sqrt{x})^2+3=0$

$\Rightarrow (\sqrt{x})^2-2\sqrt{x}-3=0$

$\Rightarrow (\sqrt{x}-3)(\sqrt{x}+1)=0$

$\Rightarrow \sqrt{x}=3$ ($\because \sqrt{x}=-1$ is not possible)

$\therefore$ y = 3

$\therefore$ Required area

$$= \int_0^3 (x_2 - x_1)\,dy = \int_0^3 \{(2y+3) - y^2\}\,dy$$

$$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9 + 9 - 9 = 9$$

57. (b) Given equation of the curves are

$$y = |x+1| + 1 = \begin{cases} (x+1)+1, \text{ if } x+1 \geq 0 \\ -(x+1)+1, \text{ if } x+1 < 0 \end{cases}$$

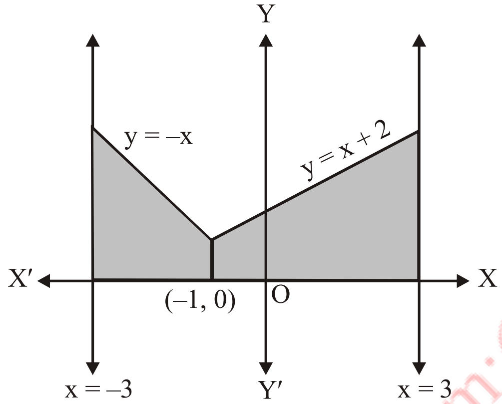
$$= \begin{cases} x+2, & \text{if } x \geq -1 \\ -x. & \text{if } x < -1 \end{cases} \quad \text{....(i)}$$

$x = -3$ ....(ii)

$x = 3$ ....(iii)

and $y = 0$ ....(iv)

eq. (ii) represents the line parallel to y-axis and passes through the point (–3, 0).



eq. (iii) represents the line parallel to y-axis and passes through the point (3,0)

$\therefore$ Required area $= \int_{-3}^{-1} y\,dx + \int_{-1}^{3} y\,dx$

$$= \int_{-3}^{-1} -x\,dx + \int_{-1}^{3} (x+2)\,dx$$

$$= \left[\frac{-x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\frac{1}{2}(1-9) + \left[\left(\frac{9}{2}+6\right) - \left(\frac{1}{2}-2\right)\right]$$

$$= 4 + \frac{21}{2} + \frac{3}{2} = 16 \text{ sq. units}$$

CHAPTER 25

DIFFERENTIAL EQUATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis is

(a) $y^2y'' - 2xy' = 0$ (b) $y^2 - 2xyy'' = 0$
(c) $y^2 - 2xyy' = 0$ (d) None of these

2. The differential equation of all non-horizontal lines in a plane is

(a) $\frac{d^2y}{dx^2}$ (b) $\frac{d^2x}{dy^2} = 0$ (c) $\frac{dy}{dx} = 0$ (d) $\frac{dx}{dy} = 0$

3. The differential equation which represent the family of curves $y = ae^{bx}$, where a and b are arbitrary constants.

(a) $y' = y^2$ (b) $y'' = y\,y'$
(c) $y\,y'' = y'$ (d) $y\,y'' = (y')^2$

4. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circle with

(a) variable radii and fixed centre (0, 1)
(b) variable radii and fixed centre (0, –1)
(c) fixed radius 1 and variable centre on x-axis
(d) fixed radius 1 and variable centre on y-axis

5. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(a) $e^x = \frac{y^3}{3} + e^y + c$ (b) $e^y = \frac{x^2}{3} + e^x + c$
(c) $e^y = \frac{x^3}{3} + e^x + c$ (d) None of these

6. Solution of differential equation $(x^2 - 2x + 2y^2)\,dx + 2xy\,dy = 0$ is

(a) $y^2 = 2x - \frac{1}{4}x^2 + \frac{c}{x^2}$ (b) $y^2 = \frac{2}{3}x - x^2 + \frac{c}{x^2}$
(c) $y^2 = \frac{2}{3}x - \frac{x^2}{4} + \frac{c}{x^2}$ (d) None of these

7. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$, respectively, are

(a) 2 and not defined (b) 2 and 2
(c) 2 and 3 (d) 3 and 3

8. $\tan^{-1}x + \tan^{-1}y = c$ is the general solution of the differential equation

(a) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$
(b) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$
(c) $(1 + x^2)\,dy + (1 + y^2)\,dx = 0$
(d) $(1 + x^2)\,dx + (1 + y^2)\,dy = 0$

9. Integrating factor of the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is

(a) cos x (b) sec x
(c) $e^{\cos x}$ (d) $e^{\sec x}$

10. General solution of $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is

(a) $y(1 + x^2) = c + \tan^{-1}x$
(b) $\frac{y}{1+x^2} = c + \tan^{-1}x$
(c) $y\log(1 + x^2) = c + \tan^{-1}x$
(d) $y(1 + x^2) = c + \sin^{-1}x$

11. A homogeneous differential equation of the $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution

(a) y = vx (b) v = yx
(c) x = vy (d) x = v

12. Which of the following equation has $y = c_1e^x + c_2e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$
(c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$

13. The particular solution of $\log\frac{dy}{dx} = 3x + 4y$, y(0) = 0 is

(a) $e^{3x} + 3e^{-4y} = 4$ (b) $4e^{3x} - 3^{-4y} = 3$
(c) $3e^{3x} + 4e^{4y} = 7$ (d) $4e^{3x} + 3e^{-4y} = 7$

14. The solution of the equation $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$ is
(a) $(x - y^2) + c = \log(3x - 4y + 1)$
(b) $x - y + c = \log(3x - 4y + 4)$
(c) $(x - y + c) = \log(3x - 4y - 3)$
(d) $x - y + c = \log(3x - 4y + 1)$

15. The order of the differential equation of a family of curves represented by an equation containing four arbitrary constants, will be
(a) 2 (b) 4 (c) 6 (d) None of these

16. The order and degree of the differential equation
$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ is
(a) order = 3, degree = 2 (b) order = 2, degree = 3
(c) order = 2, degree = 2 (d) order = 3, degree = 3

17. The differential equation representing the family of curves $y = A\cos(x + B)$, where A, B are parameters, is
(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$
(c) $\frac{d^2y}{dx^2} = \frac{dy}{dx} + y$ (d) $\frac{dy}{dx} + y = 0$

18. The order and degree of the differential equation
$y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$ is
(a) order = 1, degree = 2 (b) order = 2, degree = 1
(c) order = 2, degree = 2 (d) None of these

19. The order and degree of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter, is
(a) order = 4, degree = 4 (b) order = 4, degree = 1
(c) order = 1, degree = 4 (d) None of these

20. An equation which involves variables as well as derivative of the dependent variable with respect to independent variable, is known as
(a) differential equation (b) integral equation
(c) linear equation (d) quadratic equation

21. For the differential equation $\frac{d^2y}{dx^2} + y = 0$, if there is a function $y = \phi(x)$ that will satisfy it, then the function $y = \phi(x)$ is called
(a) solution curve only
(b) integral curve only
(c) solution curve or integral curve
(d) None of the above

22. The differential equation obtained by eliminating the arbitrary constants a and b from $xy = ae^x + be^{-x}$ is
(a) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$ (b) $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} - xy = 0$
(c) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$ (d) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

23. A first order-first degree differential equation is of the form
(a) $\frac{d^2y}{dx^2} = F(x, y)$ (b) $\frac{d^3y}{dx^2} = F(x, y)$
(c) $\left(\frac{dy}{dx}\right)^2 = F(x, y)$ (d) $\frac{dy}{dx} = F(x, y)$

24. The equation of a curve whose tangent at any point on it different from origin has slope $y + \frac{y}{x}$, is
(a) $y = e^x$ (b) $y = kx.\ e^x$
(c) $y = kx$ (d) $y = k.\ e^{x^2}$

25. The solution of the differential equation
$(1 + e^{\frac{x}{y}})\,dx + e^{\frac{x}{y}}\left(1 + \frac{x}{y}\right)dy = 0$ is
(a) $ye^x + x = C$ (b) $xe^y + y = C$
(c) $ye^{\frac{y}{x}} + y = C$ (d) $ye^{\frac{x}{y}} + y = C$

26. The differential equation of the form
$\frac{dy}{dx} + Py = Q$
where, P and Q are constants or functions of x only, is known as a
(a) first order differential equation
(b) linear differential equation
(c) first order linear differential equation
(d) None of the above

27. Which of the following is/are first order linear differential equation?
(a) $\frac{dy}{dx} + y = \sin x$ (b) $\frac{dy}{dx} + \left(\frac{1}{x}\right)y = e^x$
(c) $\frac{dy}{dx} + \left(\frac{y}{x\log x}\right) = \frac{1}{x}$ (d) All the above

28. If p and q are the order and degree of the differential equation $y\frac{dy}{dx} + x^3\frac{d^2y}{dx^2} + xy = \cos x$, then
(a) $p < q$ (b) $p = q$
(c) $p > q$ (d) None of these

29. The differential equation obtained by eliminating arbitrary constants from $y = ae^{bx}$ is
(a) $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
(c) $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$ (d) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

30. The elimination of constants A, B and C from $y = A + Bx - Ce^{-x}$ leads the differential equation:
(a) $y'' + y''' = 0$ (b) $y'' - y''' = 0$
(c) $y' + e^x = 0$ (d) $y'' + e^x = 0$

31. The integrating factor of the differential equation
$x\frac{dy}{dx} - y = 2x^2$ is
(a) e^{-x} (b) e^{-y} (c) $\frac{1}{x}$ (d) x

32. The degree of the differential equation

$y_3^{2/3}+2+3y_2+y_1=0$ is :

(a) 1 (b) 2 (c) 3 (d) None of these

33. In order to solve the differential equation

$$x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1$$

the integrating factor is:

(a) x cos x (b) x sec x
(c) x sin x (d) x cosec x

34. The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$, where c > 0, is a parameter, is of order and degree as follows :

(a) order 1, degree 2 (b) order 1, degree 1
(c) order 1, degree 3 (d) order 2, degree 2

35. Consider the differential equation

$y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If y (1) = 1, then x is given by :

(a) $4-\frac{2}{y}-\frac{e^{\frac{1}{y}}}{e}$ (b) $3-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$

(c) $1+\frac{1}{y}-\frac{e^{\frac{1}{y}}}{e}$ (d) $1-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$

36. The equation of the curve through the point (1, 2) and whose slope is $\frac{y-1}{x^2+x}$, is

(a) $(y-1)(x+1)-2x=0$ (b) $2x(y-1)+x+1=0$

(c) $x(y-1)(x+1)+2=0$ (d) None of these

37. Differential equation of all straight lines which are at a constant distance from the origin is

(a) $(y+xy_1)^2=p^2(1+y_1^2)$ (b) $(y-xy_1)^2=p^2(1-y_1^2)$

(c) $(y-xy_1)^2=p^2(1+y_1^2)$ (d) None of these

38. General solution of the differential equation

$\frac{dy}{dx}+y\,g'(x)=g(x).\,g'(x)$, where g(x) is a function of x is

(a) $g(x)-\log[1-y-g(x)]=C$

(b) $g(x)-\log[1+y-g(x)]=C$

(c) $g(x)+[1+y-\log g(x)]=C$

(d) $g(x)+\log[1+y-g(x)]=C$

39. If $x\frac{dy}{dx}$ = y (log y – log x + 1), then the solution of the equation is

(a) $y\log\left(\frac{x}{y}\right)=cx$ (b) $x\log\left(\frac{y}{x}\right)=cy$

(c) $\log\left(\frac{y}{x}\right)=cx$ (d) $\log\left(\frac{x}{y}\right)=cy$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

40. Consider the following statements

I. The order of the differential equation $\frac{dy}{dx}=e^x$ is 1.

II. The order of the differential equation $\frac{d^2y}{dx^2}+y=0$ is 2.

III. The order of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)+x^2\left(\frac{d^2y}{dx^2}\right)^3=0 \text{ is 3.}$$

Choose correct option.

(a) I and II are true (b) II and III are true
(c) I and III are true (d) All are true

41. To solve first order linear differential equation, we use following steps

I. Write the solution of the given differential equation as

$y\,(IF)=\int(Q\times IF)\,dx+C$

II. Write the given differential equation in the from $\frac{dy}{dx}+Py=Q$, where P and Q are constants or functions of x only.

III. Find the integrating factor (IF) $e^{\int P\,dx}$

The correct order of the above steps is

(a) II, III, I (b) II, I, III
(c) III, I, II (d) I, III, II

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

42.

	Column-I Differential equations	Column-II Degree
A.	$\frac{dy}{dx}=e^x$	1. 1
B.	$\frac{d^2y}{dx^2}+y=0$	2. 2
C.	$\frac{d^3y}{dx^3}+x^2\left(\frac{d^2y}{dx^2}\right)^3=0$	3. not defined
D.	$\frac{d^3y}{dx^3}+2\left(\frac{d^2y}{dx^2}\right)^2-\frac{dy}{dx}+y=0$	4. 3
E.	$\left(\frac{dy}{dx}\right)^2+\left(\frac{dy}{dx}\right)-\sin^2 y=0$	
F.	$\frac{dy}{dx}+\sin\left(\frac{dy}{dx}\right)=0$	

Codes

	A	B	C	D	E	F
(a)	2	2	4	3	1	1
(b)	1	1	1	1	2	3
(c)	3	4	1	1	2	3
(d)	1	1	1	3	4	2

43.

Column-I (Differential equations)	Column-II (Order and degree respectively)
A. $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$	1. 2,1
B. $y''' + 2y'' + y' = 0$	2. 1, 1
C. $y' + y = e^x$	3. 3, 1
D. $y'' + (y')^2 + 2y = 0$	4. 3, 2
	5. 1, not defined

Codes

	A	B	C	D
(a)	5	4	1	3
(b)	2	4	1	3
(c)	4	2	3	1
(d)	4	3	2	1

44.

Column-I (Solutions)	Column-II Differential equations
A. $y = e^x + 1$	1. $y' + \sin x = 0$
B. $y = x^2 + 2x + C$	2. $xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0 \text{ and } x > y \text{ or } x < -y)$
C. $y = \cos x + C$	3. $y'' - y' = 0$
D. $y = \sqrt{1+x^2}$	4. $xy' = y (x \neq 0)$
E. $y = Ax$	5. $y' - 2x - 2 = 0$
F. $y = x \sin x$	6. $y' = \dfrac{xy}{1+x^2}$

Codes

	A	B	C	D	E	F
(a)	2	1	4	3	6	5
(b)	2	1	4	6	5	3
(c)	5	6	1	4	3	2
(d)	3	5	1	6	4	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

45. The order of the differential equation of all tangent lines to the parabola $y = x^2$, is
(a) 1 (b) 2 (c) 3 (d) 4

46. The order of the differential equation whose general solution is given by

$y = (C_1 + C_2) \cos (x + C_3) - C_4 e^{x + C_5}$

where C_1, C_2, C_3, C_4, C_5 are arbitrary constant, is
(a) 5 (b) 4 (c) 3 (d) 2

47. Family $y = Ax + A^3$ of curves will correspond to a differential equation of order
(a) 3 (b) 2 (c) 1 (d) not infinite

48. The degree of the differential equation satisfied by the curve

$\sqrt{1+x} - a\sqrt{1+y} = 1$, is
(a) 0 (b) 1 (c) 2 (d) 3

49. If $y = e^x(\sin x + \cos x)$, then the value of $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y$, is
(a) 0 (b) 1 (c) 2 (d) 3

50. A differential equation of the form $\dfrac{dy}{dx} = F(x, y)$ is said to be homogeneous, if $F(x, y)$ is a homogeneous function of degree,
(a) 0 (b) 1 (c) 2 (d) 3

51. The order of the differential equation whose solution is : $y = a \cos x + b \sin x + ce^{-x}$ is :
(a) 3 (b) 2 (c) 1 (d) None of these

52. The degree of the differential equation

$\left(\dfrac{d^3y}{dx^3}\right)^4 + \left(\dfrac{d^2y}{dx^2}\right)^5 + \dfrac{dy}{dx} + y = 0$ is
(a) 2 (b) 4 (c) 6 (d) 8

53. If the I.F. of the differential equation $\dfrac{dy}{dx} + 5y = \cos x$ is $\int e^{Adx}$, then A =
(a) 0 (b) 1 (c) 3 (d) 5

54. For $y = \cos kx$ to be a solution of differential equation $\dfrac{d^2y}{dx^2} + 4y = 0$, the value of k is
(a) 2 (b) 4 (c) 6 (d) 8

55. For the function $y = Bx^2$ to be the solution of differential equation $\left(\dfrac{dy}{dx}\right)^3 - 15x^2\dfrac{dy}{dx} - 2xy = 0$, the value of B is __________, given that $B \neq 0$.
(a) 2 (b) 4
(c) 6 (d) 8

56. In the particular solution of differential equation

$\dfrac{dy}{dx} = \dfrac{1}{x(3y^2 - 1)}$, the value of constant term is __________, given that $y = 2$ when $x = 1$.
(a) 2 (b) 4 (c) 6 (d) 8

57. A family of curves is given by the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.
The differential equation representing this family of curves is given by $xy\dfrac{d^2y}{dx^2} + Ax\left(\dfrac{dy}{dx}\right)^2 - y\dfrac{dy}{dx} = 0$. The value of A is
(a) 0 (b) 1 (c) 3 (d) 5

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

58. **Assertion:** Order of the differential equation whose solution is $y = c_1 e^{x+c_2} + c_3 e^{x+c_4}$ is 4.
Reason: Order of the differential equation is equal to the number of independent arbitrary constant mentioned in the solution of differential equation.

59. **Assertion:** $x \sin x \dfrac{dy}{dx} + (x + x\cos x + \sin x)\, y = \sin x,\ y\left(\dfrac{\pi}{2}\right) = 1 - \dfrac{2}{\pi} \Rightarrow \lim_{x\to 0} y(x) = \dfrac{1}{3}$
Reason: The differential equation is linear with integrating factor $x(1 - \cos x)$.

60. **Assertion:** The differential equation of all circles in a plane must be of order 3.
Reason: If three points are non-collinear, then only one circle always passing through these points.

61. **Assertion:** The degree of the differential equation $\dfrac{d^2y}{dx^2} + 3\left(\dfrac{dy}{dx}\right)^2 = x^2 \log\left(\dfrac{d^2y}{dx^2}\right)$ is not defined.
Reason : If the differential equation is a polynomial in terms of its derivatives, then its degree is defined.

62. **Assertion:** The differential equation $x^2 = y^2 + xy\dfrac{dy}{dx}$ is an ordinary differential equation.
Reason: An ordinary differential equation involves derivatives of the dependent variable with respect to only one dependent variable.

63. For the differential equation $\dfrac{d^2y}{dx^2} + y = 0$, let its solution be $y = \phi_1(x) = 2\sin\left(x + \dfrac{\pi}{4}\right)$.
Assertion: The function $y = \phi_1(x)$ is called the particular solution.
Reason: The solution which is free from arbitrary constant, is called a particular solution.

64. **Assertion :** The differential equation
$$\frac{dx}{dy} + x = \cos y \text{ and } \frac{dx}{dy} + \frac{-2x}{y} = y^2 e^{-y}$$
are first order linear differential equations.
Reason : The differential equation of the form
$$\frac{dx}{dy} + P_1 x = Q_1$$
where, P_1 and Q_1 are constants or functions of y only, is called first order linear differential equation.

65. **Assertion:** The differential equation $y^3\,dy + (x + y^2)\,dx = 0$ becomes homogeneous if we put $y^2 = t$.
Reason: All differential equation of first order first degree becomes homogeneous if we put $y = tx$.

66. Let a solution $y = y(x)$ of the differential equation
$$x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0 \text{ satisfy } y(2) = \frac{2}{\sqrt{3}}.$$
Assertion: $y(x) = \sec\left(\sec^{-1} x - \dfrac{\pi}{6}\right)$
Reason: y(x) is given by $\dfrac{1}{y} = \dfrac{2\sqrt{3}}{x} - \sqrt{1 - \dfrac{1}{x^2}}$

67. **Assertion :** The number of arbitrary constants in the solution of differential equation $\dfrac{d^2y}{dx^2} = 0$ are 2.
Reason: The solution of a differential equation contains as many arbitrary constants as is the order of differential equation.

68. **Assertion :** The degree of the differential equation
$$\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} + 2y = 0 \text{ is zero}$$
Reason: The degree of a differential equation is not defined if it is not a polynomial eq in its derivatives.

69. **Assertion :** $\dfrac{dy}{dx} = \dfrac{x^3 - xy^2 + y^3}{x^2y - x^3}$ is a homogeneous differential equation.
Reason: The function $F(x,y) = \dfrac{x^3 - xy^2 + y^3}{x^2y - x^3}$ is homogeneous.

70. **Assertion :** $\dfrac{dy}{dx} + x^2y = 5$ is a first order linear differential equation.
Reason: If P and Q are functions of x only or constant then differential equation of the form $\dfrac{dy}{dx} + Py = Q$ is a first order linear differential equation.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

71. Which of the following differential equation has y = x as one of its particular solution ?
(a) $\dfrac{d^2y}{dx^2} - x^2\dfrac{dy}{dx} + xy = x$ (b) $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + xy = x$
(c) $\dfrac{d^2y}{dx^2} - x^2\dfrac{dy}{dx} + xy = 0$ (d) $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + xy = 0$

72. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000 if the rate of growth of bacteria is proportional to the number present.

(a) $\frac{2}{\log\frac{11}{10}}$ (b) $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$

(c) $\frac{\log 2}{\log 11}$ (d) $\frac{\log 2}{\log\left(\frac{11}{10}\right)}$

73. The equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is
sin x cos y dx + cos x sin y dy = 0, is

(a) sec x sec y = $\sqrt{2}$ (b) cos x cos y = $\sqrt{2}$

(c) sec x = $\sqrt{2}$ cos y (d) cos y = $\sqrt{2}$ sec y

74. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20, 000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?

(a) 3125 (b) 31250

(c) 21350 (d) 12350

75. Solution of the differential equation

$$\frac{dx}{dy} - \frac{x\log x}{1+\log x} = \frac{e^y}{1+\log x}, \text{ if } y(1) = 0, \text{ is}$$

(a) $x^x = e^{ye^y}$ (b) $e^y = x^{e^y}$

(c) $x^x = ye^y$ (d) None of these

76. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of 'a' is

(a) 2 (b) –2

(c) 3 (d) –4

77. If $\left(1+e^{x/y}\right)dx + \left(1-\frac{x}{y}\right)e^{x/y}dy = 0$, then

(a) $x - ye^{x/y} = c$ (b) $y - xe^{x/y} = c$

(c) $x + ye^{x/y} = c$ (d) $y + xe^{x/y} = c$

78. The solution of the differential equation

$$\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2} \text{ is}$$

(a) $\frac{y}{4} + \frac{1}{x^2+y^2} = c$ (b) $\frac{y}{x} - \frac{1}{x^2+y^2} = c$

(c) $\frac{x}{y} - \frac{1}{x^2+y^2} = c$ (d) None of these

79. The equation of the curve satisfying the differential equation $y_2(x^2+1) = 2xy$, passing through the point (0, 1) and having slope of tangent at x = 0 as 3, is

(a) $y = x^3 + 3x + 1$ (b) $y = x^2 + 3x + 1$

(c) $y = x^3 + 3x$ (d) $y = x^3 + 1$

80. If y(t) is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and y(0) = –1, then the value of y (1) is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) 1

81. The female-male ratio of a village decreases continuously at the rate proportional to their ratio at any time. If the ratio of female : male of the villages was 980 : 1000 in 2001 and 920 : 1000 in 2011. What will be the ratio in 2021 ?

(a) 864 : 1000 (b) 864 : 100

(c) 1000 : 864 (d) 100 : 864

82. Solution of the differential equation

$$xdy - ydx = \sqrt{x^2+y^2}\,dx \text{ is}$$

(a) $y = cx^2$ (b) $y = cx^2 + \sqrt{x^2+y^2}$

(c) $y + \sqrt{x^2+y^2} = cx^2$ (d) $y - \sqrt{x^2-y^2} = c$

83. The order and degree of the differential equation $\frac{d^4y}{dx^4} + \sin(y''') = 0$ are respectively

(a) 4 and 1 (b) 1 and 2

(c) 4 and 4 (d) 4 and not defined

84. The degree of the equation $e^x\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 3$ is

(a) 2 (b) 0

(c) not defined (d) 1

85. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$ (c) $0 - y$ (d) $2x^2y$

86. The equation of the curve passing through the point (1, 1) whose differential equation is $x\,dy = (2x^2 + 1)\,dx$ $(x \neq 0)$ is

(a) $x^2 = y + \log|x|$ (b) $y = x^2 + \log|x|$

(c) $y^2 = x + \log|x|$ (d) $y = x + \log|x|$

87. At any point (x, y) of a curve, the slope of tangent is twice the slope of the line segment joining the point of contact to the point (–4, –3). The equation of the curve given that it passes through (–2, 1) is

(a) $x + 4 = (y + 3)^2$ (b) $(x + 4)^2 = (y - 3)$

(c) $x - 4 = (y - 3)^2$ (d) $(x + 4)^2 = |y + 3|$

88. In a bank, principal increases continuously at the rate of 5% per year. In how many years ` 1000 double itself?

(a) 2 (b) 20

(c) $20\log_e 2$ (d) $2\log_e 20$

89. The equation of curve through the point (1, 0), if the slope of the tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$, is

(a) (y + 1) (x – 1) + 2x = 0

(b) (y + 1) (x – 1) – 2x = 0

(c) (y – 1) (x – 1) + 2x = 0

(d) (y – 1) (x + 1) + 2x = 0

90. The general solution of the homogeneous differential equation of the type.

$\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, when $y = v : x$ is

(a) $\int \frac{dv}{g(v)+v} = \int \frac{1}{x} dx + C$

(b) $\int \frac{dv}{g(v)-v} = \int \frac{1}{x} dx + C$

(c) $\int \frac{dv}{g(v)} = \int \frac{1}{x} dx + C$

(d) $\int \frac{dv}{vg(v)} = \int \frac{1}{x} dx + C$

91. If $dx + dy = (x + y)(dx - dy)$, then $\log(x + y)$ is equal to

(a) $x + y + C$ (b) $x + 2y + C$
(c) $x - y + C$ (d) $2x + y + C$

92. If the slope of the tangent to the curve at any point P(x, y) is $\frac{y}{x} - \cos^2 \frac{y}{x}$, then the equation of a curve passing through $\left(1, \frac{\pi}{4}\right)$ is

(a) $\tan\left(\frac{y}{x}\right) + \log x = 1$ (b) $\tan\left(\frac{y}{x}\right) + \log y = 1$

(c) $\tan\left(\frac{x}{y}\right) + \log x = 1$ (d) $\tan\left(\frac{x}{y}\right) + \log y = 1$

93. The general solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$ is

(a) $x = (\tan^{-1} y + 1) + Ce^{-\tan^{-1} y}$

(b) $x = (\tan^{-1} y - 1) + Ce^{-\tan^{-1} y}$

(c) $x = (\tan^{-1} x - 1) + Ce^{-\tan^{-1} x}$

(d) $x = (\tan^{-1} x + 1) + Ce^{-\tan^{-1} x}$

94. The solution of differential equation

$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1; x \neq 0$ is

(a) $ye^{2\sqrt{x}} = 2\sqrt{x} + C$ (b) $ye^{\sqrt{x}} = \sqrt{x} + C$

(c) $ye^{2\sqrt{x}} = \sqrt{y} + C$ (d) $ye^{2\sqrt{x}} = 2\sqrt{x} + C$

95. Solution of differential equation $x\,dy - y\,dx = 0$ represents:

(a) rectangular hyperbola.
(b) parabola whose vertex is at origin.
(c) circle whose centre is at origin.
(d) straight line passing through origin.

96. The order and degree of the differential equation

$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ are respectively

(a) 2, 2 (b) 2, 3
(c) 2, 1 (d) None of these

97. The solution of the differential equation $y' = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is

(a) $x\,\phi(y/x) = k$ (b) $\phi(y/x) = kx$
(c) $y\,\phi(y/x) = k$ (d) $\phi(y/x) = ky$

98. The differential equation

$(1 + y^2)x\,dx - (1 + x^2)y\,dy = 0$

represents a family of :

(a) ellipses of constant eccentricity
(b) ellipses of variable eccentricity
(c) hyperbolas of constant eccentricity
(d) hyperbolas of variable eccentricity

99. The solution of the differential equation

$y\,dx - x\,dy + 3x^2y^2e^{x^3}\,dx = 0$ is

(a) $\frac{x}{y} + e^{x^3} = c$ (b) $\frac{x}{y} - e^{x^3} = c$

(c) $\frac{y}{x} + e^{x^3} = c$ (d) $\frac{y}{x} - e^{x^3} = c$

100. The solution of $x^3 \frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying $y(1) = 0$ is :

(a) $\tan y = (x-2)e^x \log x$ (b) $\sin y = e^x (x-1)x^{-4}$

(c) $\tan y = (x-1)e^x x^{-3}$ (d) $\sin y = e^x (x-1)x^{-3}$

101. The differential equations of all conics whose axes coincide with the co-ordinate axis is

(a) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

(b) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = 0$

(c) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(d) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

102. The expression satisfying the differential equation

$(x^2 - 1)\frac{dy}{dx} + 2xy = 1$ is

(a) $x^2y - xy^2 = c$ (b) $(y^2 - 1)x = y + c$

(c) $(x^2 - 1)y = x + c$ (d) None of these

103. The differential equation $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3 \cos^2 y$ represents a family of curves given by the equation

(a) $x^6 + 6x^2 = C \tan y$ (b) $6x^2 \tan y = x^6 + C$

(c) $\sin 2y = x^3 \cos^2 y + C$ (d) none of these

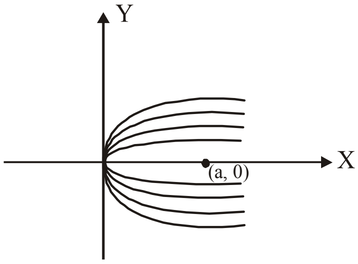
104. A steam boat is moving at velocity V when steam is shut off. Given that the retardation at any subsequent time is equal to the magnitude of the velocity at that time. The velocity v in time t after steam is shut off is

(a) $v = Vt$ (b) $v = Vt - V$
(c) $v = Ve^t$ (d) $v = Ve^{-t}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** Family of parabola satisfying given conditions can be represented graphically as shown below:



Equation is $y^2 = 4ax$ … (i)

Differentiating w.r.t. x, we get $2yy' = 4a$

Substituting 4a from equation (i)

$$2yy' = \frac{y^2}{x}$$

$\Rightarrow y^2 - 2xyy' = 0$

2. **(b)** The general equation of all non-horizontal lines in a plane is $ax + by = 1$, where $a \neq 0$.

Now, $ax + by = 1$

$\Rightarrow a\frac{dx}{dy} + b = 0$ [Differentiating w.r.t. y]

$\Rightarrow a\frac{d^2x}{dy^2} = 0$ [Differentiating w.r.t. y]

$\Rightarrow \frac{d^2x}{dy^2} = 0$ [$\because a \neq 0$]

Hence, the required differential equation is $\frac{d^2x}{dy^2} = 0$

3. **(d)** $\ln y = \ln a + bx$

Differentiating w.r.t. x, we get

$$\frac{1}{y}y' = b$$

Again differentiating w.r.t. x, we get

$$\frac{y''}{y} - \frac{1}{y^2}(y')^2 = 0$$

$\Rightarrow yy'' = (y')^2$

4. **(c)** $\frac{ydy}{\sqrt{1-y^2}} = dx$

On integration, we get $-\sqrt{1-y^2} = x + c$

$1 - y^2 = (x + c)^2 \Rightarrow (x + c)^2 + y^2 = 1$, radius 1 and centre on the x-axis.

5. **(c)** From given differential equation

$$\frac{dy}{dx} = \frac{e^x + x^2}{e^y}$$

Using variable separable form, we have

$e^y dy = (e^x + x^2)\,dx$

Integrating, we get

$$e^y = e^x + \frac{x^3}{3} + c$$

6. **(c)** As $(x^2 - 2x + 2y^2)\,dx = -2xy\,dy$

$\Rightarrow 2xy\frac{dy}{dx} + 2y^2 + x^2 - 2x = 0$

$\Rightarrow 2xy\frac{dy}{dx} + 2y^2 = 2x - x^2$

$\Rightarrow x\left(2y\frac{dy}{dx}\right) + 2y^2 = 2x - x^2$

By putting $y^2 = v$

$\because 2y.\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow x\frac{dv}{dx} + 2v = 2x - x^2.$

$\Rightarrow \frac{dv}{dx} + v\left(\frac{2}{x}\right) = \frac{2x - x^2}{x}$

I.F.$= e^{\int \frac{2}{x}dx} = x^2$

Now, required solution is

$$v.x^2 = \int \frac{(2x - x^2)x^2 dx}{x} = \int x^2(2 - x)\,dx$$

$\Rightarrow v.x^2 = \frac{2x^3}{3} - \frac{x^4}{4} + c$

$\Rightarrow v = \frac{2x}{3} - \frac{1}{4}x^2 + \frac{c}{x^2}$

$\therefore y^2 = \frac{2x}{3} - \frac{1}{4}x^2 + \frac{c}{x^2}$ which is required solution.

7. **(a)** $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$

Clearly, order of given differential equation is 2 and degree is not defined.

8. **(c)** $\tan^{-1}x + \tan^{-1}y = c$

differentiating w.r.t. x

$$\frac{1}{1+x^2} + \frac{1}{1+y^2}\frac{dy}{dx} = 0$$

$\Rightarrow \frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$

$\Rightarrow (1 + y^2)dx + (1 + x^2)\,dy = 0$

9. **(b)** $\frac{dy}{dx} + y\tan x - \sec x = 0$

$\frac{dy}{dx} + (\tan x)\,y = \sec x$

I.F. $= e^{\int \tan x\,dx} = e^{\log \sec x} = \sec x$

10. (a) $\dfrac{dy}{dx}+\dfrac{2xy}{1+x^2}=\dfrac{1}{\left(1+x^2\right)^2}$

It is linear differential equation with

$$\text{I.F.}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log\left(1+x^2\right)}=1+x^2$$

Now, solution is

$$y(1+x^2)=\int 1+x^2.\frac{1}{\left(1+x^2\right)^2}dx+c=\int\frac{dx}{1+x^2}+c$$

$y(1 + x^2) = \tan^{-1}x + c$

11. (c) For solving the homogeneous equation of the form

$\dfrac{dx}{dy}=h\left(\dfrac{x}{y}\right)$, to make the substitution $x = vy$

12. (b) Family of curves is $y = c_1 e^x + c_2 e^{-x}$ ... (i)

Differentiating w.r.t. x

$y' = c_1 e^x - c_2 e^{-x}$, $y'' = c_1 e^x + c_2 e^{-x} = y$

$\therefore\ y'' - y = 0$

Solution is $\dfrac{d^2y}{dx^2}-y=0$

13. (d) $\dfrac{dy}{dx}=e^{3x+4y}=e^{3x}\cdot e^{4y}$

$$\Rightarrow\ e^{-4y}dy=e^{3x}dx\Rightarrow\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+c$$

put $x = 0$

We have $-\dfrac{1}{4}-\dfrac{1}{3}=c\ \Rightarrow c=-\dfrac{7}{12}$,

$$\therefore\ \frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}-\frac{7}{12}\Rightarrow 7=3e^{-4y}+4e^{3x}$$

14. (d) Hint : Put $3x - 4y = X$

$$\Rightarrow 3-4\frac{dy}{dx}=\frac{dX}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{4}\left(3-\frac{dX}{dx}\right)$$

$$\Rightarrow\ \frac{3}{4}-\frac{1}{4}\frac{dX}{dx}=\frac{X-2}{X-3}$$

$$\Rightarrow\ -\frac{1}{4}\frac{dX}{dx}=\frac{4X-8-3(X-3)}{4(X-3)}$$

$$=-\frac{1}{4}\frac{dX}{dx}=\frac{X+1}{4(X-3)}$$

15. (b) It is obvious.

16. (b) Clearly order of the differential equation is 2.

Again $\dfrac{d^2y}{dx^2}+x^{1/4}=-\left(\dfrac{dy}{dx}\right)^{1/3}$

$$\Rightarrow\ \left(\frac{d^2y}{dx^2}+x^{1/4}\right)^3=-\frac{dy}{dx}$$

which shows that degree of the differential equation is 3.

17. (a) Since $y = A\cos(x + B)$

$$\therefore\ \frac{dy}{dx}=-A\sin(x+B)$$

$$\Rightarrow\ \frac{d^2y}{dx^2}=-A\cos(x+B)=-y$$

$$\Rightarrow\ \frac{d^2y}{dx^2}+y=0$$

18. (a) Given differential equation can be written as

$$y^2+x^2\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}=a^2\left(\frac{dy}{dx}\right)^2+b^2$$

Clearly, it is a 1st order and 2nd degree differential equation.

19. (c) $y=cx+c^2-3c^{3/2}+2$ ... (i)

Differentiating above with respect to x, we get $\dfrac{dy}{dx}=c$.

Putting this value of c in (i), we get

$$y=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2-3\left(\frac{dy}{dx}\right)^{3/2}+2$$

Clearly its order is ONE and after removing the fractional power we get the degree FOUR.

20. (a) In general, an equation involving derivative of the dependent variable with respect to independent variable (variables) is called a differential equation.

21. (c) Consider the differential equation $\dfrac{d^2y}{dx^2}+y=0$

The solution of this differential equation is a function ϕ that will satisfy it i.e., when the function ϕ is substituted for the unknown y (dependent variable) in the given differential equation, LHS becomes equal to RHS.

The curve $y = \phi(x)$ is called the solution curve (integral curve) of the given differential equation.

22. (a) The given function is

$xy = ae^x + be^{-x}$ …(i)

On differentiating equation (i) w.r.t. x, we get

$$x\frac{dy}{dx}+y=ae^x-be^{-x}$$

Again, differentiating w.r.t. x, we get

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=ae^x-be^{-x}$$

$$\Rightarrow\ x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0\quad\text{[using eq. (i)]}$$

23. (d) A first order-first degree differential equation is of the form

$$\frac{dy}{dx}=F(x,y)$$

24. (b) According to the equation,

$$\frac{dy}{dx}=y+\frac{y}{x}=y\left(1+\frac{1}{x}\right)$$

$$\Rightarrow\ \frac{dy}{y}=\left(1+\frac{1}{x}\right)dx$$

On integrating both sides, we get

$$\int \frac{dy}{y} = \int \left(1+\frac{1}{x}\right) dx$$

$\Rightarrow$ $\log y = x + \log x + C$

$\Rightarrow$ $\log\left(\frac{y}{x}\right) = x + C$

$\Rightarrow$ $\frac{y}{x} = e^{x+C} = e^x . e^C$

$\Rightarrow$ $\frac{y}{x} = ke^x$

$\Rightarrow$ $y = kx. e^x$

25. **(d)** The given differential equation is

$$(1+e^{\frac{x}{y}})\,dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$$

$$\frac{dx}{dy} = \frac{e^{\frac{x}{y}}\left(\frac{x}{y}-1\right)}{(e^{x/y}+1)} \qquad \text{...(i)}$$

$$= g\left(\frac{x}{y}\right)$$

$\because$ $\frac{dx}{dy} = g\left(\frac{x}{y}\right)$

$\therefore$ eq. (i) is the homogeneous differential equation

so, put $\frac{x}{y} = v$

i.e., $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

Then, eq. (i) becomes

$$v + y\frac{dv}{dy} = \frac{e^v(v-1)}{e^v+1}$$

$\Rightarrow$ $y\frac{dv}{dy} = \frac{e^v(v-1)}{e^v+1} - v$

$\Rightarrow$ $y\frac{dv}{dy} = \frac{ve^v - e^v - ve^v - v}{e^v+1} - v$

$\Rightarrow$ $\frac{e^v+1}{e^v+v}dv = -\frac{1}{y}dy$

On integrating both sides, we get

$$\int \frac{e^v+1}{e^v+v}dv = -\int \frac{1}{y}dy$$

Put $e^v + v = t$

$\Rightarrow$ $e^v + 1 = \frac{dt}{dv}$

$\Rightarrow$ $dv = \frac{dt}{e^v+1}$

$\therefore$ $\int \frac{e^v+1}{t}\frac{dt}{e^v+1} - \log|y| + \log C$

$\Rightarrow$ $\log|t| + \log|y| = \log C$

$\Rightarrow$ $\log|e^v + v| + \log|y| = \log C$ $\quad (\because t = e^v + v)$

$\Rightarrow$ $\log|(e^v + v)y| = C \Rightarrow |(e^v + v)\,y| = C$

$\Rightarrow$ $(e^v + v)y = C$

So, put $v = \frac{x}{y}$, we get

$$\left(e^{x/y} + \frac{x}{y}\right)y = C \;\Rightarrow\; ye^{x/y} + x = C$$

This is the required solution of the given differential equation.

26. **(c)** A differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where, P and Q are constants or functions of x only, is known as a first order linear differential equation.

27. **(d)** The differential equation

$$\frac{dy}{dx} + y = \sin x$$

$\Rightarrow$ $\frac{dy}{dx} + \left(\frac{1}{x}\right)y = e^x$

$\Rightarrow$ $\frac{dy}{dx} + \left(\frac{y}{x\log x}\right) = \frac{1}{x}$

are the first order linear different equation since, all the equation are of the type $\frac{dy}{dx} + Py = Q,$ where P and Q, are constants or functions of x only.

28. **(c)** The given differential equation is

$$y\frac{dy}{dx} + x^3\frac{d^2y}{dx^2} + xy = \cos x$$

Its order is 2 and its degree is 1.

$\therefore$ $p = 2$ and $q = 1$

Hence, $p > q$

29. **(c)** The given equation is

$y = ae^{bx}$

$\Rightarrow$ $\frac{dy}{dx} = abe^{bx}$ ...(i)

$\Rightarrow$ $\frac{d^2y}{dx^2} = ab^2e^{bx}$ ...(ii)

$\Rightarrow$ $ae^{bx}\frac{d^2y}{dx^2} = a^2b^2e^{bx}$

$\Rightarrow$ $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ [from eq. (ii)]

$\Rightarrow$ $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$

30. **(a)** Given, $y = A + Bx - Ce^{-x}$

$\Rightarrow$ $y' = B + C.e^{-x}$ ..... (i)

and $y'' = -Ce^{-x}$ .... (ii)

and $y''' = Ce^{-x}$ ... (iii)

From eqs. (ii) and (iii),

$y''' = -y'' \Rightarrow y''' + y'' = 0$

31. **(c)** $x\frac{dy}{dx} - y = 2x^2$ or $\frac{dy}{dx} - \frac{y}{x} = 2x$

$$\text{I.F.} = e^{\int \frac{-1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$$

32. **(b)** Given : Differential equation is

$y_3^{\frac{2}{3}} + 2 + 3y_2 + y_1 = 0$

We know that the degree of a differential equation is the degree of highest order derivative.

$\therefore$ degree = 2.

33. **(b)** Given differential equation is :

x cos x dy/dx + y (x sin x + cos x) = 1

Dividing both the sides by x cos x,

$$\Rightarrow \frac{dy}{dx} + \frac{xy\sin x}{x\cos x} + \frac{y\cos x}{x\cos x} = \frac{1}{x\cos x}$$

$$\Rightarrow \frac{dy}{dx} + y\tan x + \frac{y}{x} = \frac{1}{x\cos x}$$

$$\Rightarrow \frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{\sec x}{x}$$

which is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \tan x + \frac{1}{x}$ and $Q = \frac{\sec x}{x}$

Integrating factor $= e^{\int P dx}$

$= e^{\int \tan x + \frac{1}{x} dx}$

$= e^{(\log \sec x + \log x)} = e^{\log (\sec x \,.\, x)}$

= x sec x

34. **(c)** $y^2 = 2c(x + \sqrt{c})$ ... (i)

$2yy' = 2c.1$ or $yy' = c$ ... (ii)

$\Rightarrow y^2 = 2yy'(x + \sqrt{yy'})$

[On putting value of c from (ii) in (i)]

On simplifying, we get

$(y - 2xy')^2 = 4yy'^3$ ... (iii)

Hence equation (iii) is of order 1 and degree 3.

35. **(c)** $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

I.F. $= e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$

So, $x.e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$

$\Rightarrow x.e^{-\frac{1}{y}} = I$

where $I = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$

Let $\frac{-1}{y} = t \Rightarrow \frac{1}{y^2} dy = dt$

$\Rightarrow I = -\int te^t dt = e^t - te^t = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$

$\Rightarrow xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$

$\Rightarrow x = 1 + \frac{1}{y} + c.e^{1/y}$

Since $y(1) = 1$

$\therefore c = -\frac{1}{e}$

$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$

36. **(a)** Given $\frac{dy}{dx} = \frac{y-1}{x^2 + x} \Rightarrow \frac{dy}{y-1} = \frac{dx}{x(x+1)}$

Integrating we get,

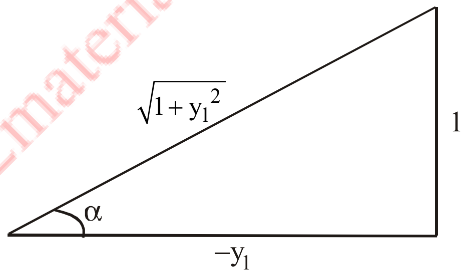
$\ln(y-1) = 2\ln\left(\frac{x}{x+1}\right) + c$

It passes through (1, 2), so, c = log 2

Required equation is $\ln(y-1) = \ln\left(\frac{2x}{x+1}\right)$

$\Rightarrow (y-1)(x+1) - 2x = 0$

37. **(c)** Any straight lines which is at a constant distance p from the origin is



x cos α + y sin α = p ... (i)

Diff. both sides w.r.t.'x', we get

$\cos\alpha + \sin\alpha \frac{dy}{dx} = 0$

$\Rightarrow \tan\alpha = -\frac{1}{y_1}$ $\left(\text{where } y_1 = \frac{dy}{dx}\right)$

$\therefore \sin\alpha = \frac{1}{\sqrt{1+y_1^2}}$; $\cos\alpha = -\frac{y_1}{\sqrt{1+y_1^2}}$

Putting the value of sin α and cos α in (i), we get

$x.\frac{-y_1}{\sqrt{1+y_1^2}} + y\frac{1}{\sqrt{1+y_1^2}} = p$

$\Rightarrow (y - xy_1)^2 = p^2(1 + y_1^2)$

38. **(d)** We have, $\frac{dy}{dx} = [g(x) - y]\, g'(x)$

Put $g(x) - y = V \Rightarrow g'(x) - \frac{dy}{dx} = \frac{dV}{dx}$

Hence, $g'(x) - \frac{dV}{dx} = V.g'(x)$

$\Rightarrow \frac{dV}{dx} = (1-V).g'(x) \Rightarrow \frac{dV}{1-V} = g'(x)dx$

$\Rightarrow \int \frac{dV}{1-V} = \int g'(x)\, dx \Rightarrow -\log(1-V) = g(x) - C$

$\Rightarrow g(x) + \log(1-V) = C$

$\therefore g(x) + \log[1 + y - g(x)] = C$

39. (c) $\frac{xdy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$

Put y = vx

$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow v + \frac{xdv}{dx} = v(\log v + 1)$

$\frac{xdv}{dx} = v \log v \Rightarrow \frac{dv}{v \log v} = \frac{dx}{x}$

Put log v = z

$\frac{1}{v} dv = dz \Rightarrow \frac{dz}{z} = \frac{dx}{x}$

ln z = ln x + ln c

x = cx or log v = cx or $\log\left(\frac{y}{x}\right) = cx.$

STATEMENT TYPE QUESTIONS

40. (d) I. The differential equation $\frac{dy}{dx} = e^x$ involves the highest derivative of first order.

∴ Its order is 1.

II. The order of the differential equation $\frac{d^2y}{dx^2} + y = 0$ is 2 since it involves highest derivative of second order.

III. The differential equation $\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$ involves highest derivative of third order

∴ Its order is 3.

41. (a) Steps involved to solve first order linear differential equation.

I. Write the given differential equation in the form

$\frac{dy}{dx} + Py = Q$

where P, Q are constants or functions of x only.

II. Find the Integrating Factor (IF) = $e^{\int P\,dx}$.

III. Write the solution of the given differential equation as

$y(IF) = \int (Q \times IF)\,dx + C$

MATCHING TYPE QUESTIONS

42. (b) The degree of the differential equation

$\frac{dy}{dx} = e^x$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$

$\Rightarrow \frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$

is 1 since, the highest power of highest order derivative is 1.

The degree of the differential equation

$\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) - \sin^2 y = 0$

is 2 since, the highest power of highest order derivative is 2.

The degree of the differential equation

$\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$

cannot defined since, this differential equation is not a polynomial in y′, y″, y‴,...etc.

43. (d) A. The highest order derivative which occurs in the given differential equation is y‴. Therefore, its order is three. The given differential equation is a polynomial equation in y‴, y″ and y′.
The highest power raised to y‴ is 2. Hence, its degree is 2.

B. The highest order derivative which occurs in the given differential equation is y‴. Therefore, its order is three. It is a polynomial equation in y‴, y″ and y′. The highest power raised to y‴ is 1. Hence, its degree is 1.

C. The highest order derivative present in the given differential equation is y′. Therefore, its order is one. The given differential equation is a polynomial equation in y′. The highest power raised to y′ is 1. Hence, its degree is 1.

D. The highest order derivative present in the differential equation is y″. Therefore, its order is two. The given differential equation is a polynomial equation in y″ and y′ and the highest power raised to y″is 1. Hence, its degree is 1.

44. (d) A. Given, $y = e^x + 1$ … (i)

On differentiating both sides of this equation w.r.t. x, we get

$y' = \frac{d}{dx}(e^x + 1) = e^x$

Again, differentiating both sides w.r.t. x, we get

$y'' = \frac{d}{dx}(e^x) = e^x$

$\Rightarrow y'' = e^x \Rightarrow y'' - y' = e^x - e^x = 0$

Hence $y = e^x + 1$ is a solution of the differential equation

$y'' - y' = 0$.

B. Given $y = x^2 + 2x + C$ … (i)

On differentiating both sides w.r.t. x, we get

$\Rightarrow y' - 2x - 2 = 0$

Hence, $y = x^2 + 2x + C$ is a solution of the differential equation $y' - 2x - 2 = 0$

C. Given, $y = \cos x + C$ … (i)
On differnetiating both sides w.r.t. x, we get
$y' = -\sin x$
$\Rightarrow y' + \sin x = 0$
Hence, $y = \cos x + C$ is a solution of the differential equation $y' + \sin x = 0$

D. Given $y = \sqrt{1+x^2}$ … (i)
On differentiating both sides of eq. (i) w.r.t. x, we get

$$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

$$\Rightarrow y' = \frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}(2x)$$

$$\Rightarrow y' = \frac{2x}{2\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{x}{\sqrt{1+x^2}} = \frac{xy}{\sqrt{1+x^2}.\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{xy}{\sqrt{1+x^2}}$$

Hence, $y = \sqrt{1+x^2}$ is the solution of the differential equation $y' = \frac{xy}{\left(1+x^2\right)}$.

E. The given function is $y = Ax$ … (i)
On differentiating both sides, we get
$y' = A$
$\Rightarrow y' = \frac{y}{x}$ $(\because y = Ax, x \neq o)$
$\Rightarrow xy' = y$
Hence, $y = Ax$ is the solution of the differential equation
$xy' = y'$ $(x \neq 0)$

F. Given $y = x \sin x$ ....(i)
On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = y' = \frac{d}{dx}(x \sin x)$$

$$\Rightarrow y' = \sin x.\frac{d}{dx}\ x + x\frac{d}{dx}\ (\sin x)$$

(using product rule of differentiation)
$\Rightarrow y' = x \cos x + \sin x$
On substituting the value of y and y′ in the equation
$xy' = y + x\sqrt{x^2 - y^2}$, we get
LHS $= xy' = x\ (x\cos x + \sin x)$
$= x^2 \cos x + x \sin x$

$$\Rightarrow xy' = x^3\sqrt{1-\sin^2 x} + y$$

$(\because \cos^2 x = 1 - \sin^2 x$ and $x \sin x = y)$

$$\Rightarrow xy' = x^2\sqrt{1-\left(\frac{y}{x}\right)^2} + y$$

$$\Rightarrow xy' = x\sqrt{x^2 - y^2} + y = \text{RHS}$$

Hence, $y = x \sin x$ is a solution of the differential equation
$(x \neq 0, x > y$ or $x < -y)$
$xy' = y + x\sqrt{x^2 - y^2}$

INTEGER TYPE QUESTIONS

45. **(a)** The parametric form of the given equation is $x = t$, $y = t^2$.
The equation of any tangent at t is $2xt = y + t^2$.
On differentiating, we get $2t = y_1$.
On putting this value in the above equation, we get

$$xy_1 = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1^2$$

The order of this equation is 1.

46. **(c)** The given equation can be written as
$y = A\cos(x + C_3) - Be^x$.
where $A = C_1 + C_2$ and $B = C_4e^{C_5}$
Here, there are three independent variables, (A, B, C_3).
Hence, the differential equation will be of order 3.

47. **(b)** $y = Ax + A^3$
differentiating w.r.t. x

$$\frac{dy}{dx} = A$$

Again differentiating w.r.t.x

$$\frac{d^2y}{dx^2} = 0$$

which is differential equation of order 2.

48. **(b)** Differentiate the given equation

$$\frac{1}{2}(1+x)^{-1/2} - \frac{a}{2}(1+y)^{-1/2}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{1}{\sqrt{1+x}} = \frac{a}{\sqrt{1+y}}\frac{dy}{dx}$$

$$\Rightarrow a = \frac{\sqrt{1+y}}{\sqrt{1+x}}\frac{1}{dy/dx}$$

Putting this value in the given equation

$$\frac{dy}{dx}\sqrt{1+x} - \frac{1+y}{\sqrt{1+x}} = \frac{dy}{dx}$$

$$\Rightarrow (1+x)\frac{dy}{dx} = 1 + y\ \sqrt{1+x}\ \frac{dy}{dx}$$

The degree of this equation is one.

49. **(a)** Hint : $y = e^x(\sin x + \cos x)$

$$\Rightarrow \frac{dy}{dx} = 2e^x \cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2e^x\ (\cos x - \sin x)$$

$$\text{L.H.S.}\ \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$$

$= 2e^x\ [\cos x - \sin x - 2\cos x + \sin x + \cos x]$
$= 2e^x \times 0 = 0 = \text{R.H.S.}$

50. (a) A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero.

51. (a) Given : $y = a \cos x + b \sin x + ce^{-x}$
This equation has three parameters.
$\therefore$ The order of differential equation is 3.

52. (b) The degree of a differential equations is the exponent of the highest order in the differential equation. Therefore the degree of the given differential equation is 4.

53. (d) The I.F. of the differential equation $\frac{dy}{dx} + Py = Q$ is

$e^{\int P dx}$. Here P = 5 therefore I.F. = $e^{\int 5 dx}$. Hence A = 5.

54. (a) Given that $y = \cos kx$, therefore $\frac{dy}{dx} = -k \sin kx$ and

$\frac{d^2y}{dx^2} = -k^2 \cos kx$ Putting this value of $\frac{d^2y}{dx^2}$ and

$y = \cos kx$ in $\frac{d^2y}{dx^2} + 4y = 0$, we get

$-k^2 \cos kx + 4 \cos kx = 0$
or $k^2 = 4$
or $k = \pm 2$, or $k = 2$.

55. (a) $\frac{dy}{dx} = B.2x$, Putting this value of $\frac{dy}{dx}$ in equation

$\left(\frac{dy}{dx}\right)^3 - 15x^2 \frac{dy}{dx} - 2xy = 0$, we get

$(B.2x)^3 - 15x^2 (B.2x) - 2x (Bx^2) = 0$
or $B^3.8x^3 - B.30x^3 - B.2x^3 = 0$
or $B^3 . 8x^3 - B.32x^3 = 0$
or $B (B^2.8x^3 - 32x^3) = 0$
$B \neq 0$
$\therefore B^2 8x^3 - 32x^3 = 0$
or $B^2 8x^3 = 32x^3$
or $B^2 = 4$
or $B = \pm 2$ or $B = 2$

56. (c) $\because \frac{dy}{dx} = \frac{1}{x(3y^2 - 1)}$

$\therefore \int (3y^2 - 1) dy = \int \frac{dx}{x}$

$y^3 - y = \ln x + c$
when x = 1, y = 2
$\therefore 2^3 - 2 = \ln 1 + c$
$8 - 2 = 0 + c$ or $c = 6$

57. (b) Differentiating the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ w.r.t. x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

or $\frac{x^2}{a^2} + \frac{xy}{b^2}\frac{dy}{dx} = 0$

or $1 - \frac{y^2}{b^2} + \frac{xy}{b^2}\frac{dy}{dx} = 0$ $\left[\because \frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}\right]$

Differentiating again w.r.t. x, we get

$$\frac{-2y}{b^2}\frac{dy}{dx} + \frac{y}{b^2}\frac{dy}{dx} + \frac{x}{b^2}\frac{dy}{dx}.\frac{dy}{dx} + \frac{xy}{b^2}\frac{d^2y}{dx^2} = 0$$

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

comparing with the given differential equation, we get A = 1.

ASSERTION - REASON TYPE QUESTIONS

58. (d) $\because y = \left(c_1 e^{c_2} + c_3 e^{c_4}\right) e^x = ce^x$ (say)

$\therefore \frac{dy}{dx} = ce^x = y$

$\therefore$ Order is 1.

59. (a) $\frac{dy}{dx} + \left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right) y = \frac{1}{x}$

$$\text{I.F.} = \exp \int \left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right) dx$$

$$= \exp \ell n \left(x \tan \frac{x}{2} \sin x\right)$$

$$= x \tan \frac{x}{2} \times 2 \sin \frac{x}{2} \cos \frac{x}{2} = x(1 - \cos x)$$

Solution, $yx(1 - \cos x)$

$$= \int \frac{1}{x}.x (1 - \cos x)\, dx = x - \sin x + c$$

$y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi} \Rightarrow c = 0$

$\therefore y(x) = \frac{x - \sin x}{x(1 - \cos x)}$

$$y = \frac{x - \left(x - \frac{x^3}{6} ...\right)}{x\left(1 - \left(1 - \frac{x^2}{2} ...\right)\right)} = \frac{x^2}{6} \frac{1}{\frac{x^2}{2}} \text{ as } x \to 0, y \to \frac{1}{3}$$

60. **(b)** Let $x^2 + y^2 + 2gx + 2fy + c = 0$
Here in this equation, there are three constants.
$\therefore$ Order = 3
Reason is also true.

61. **(a)** The given differential equation is

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

This is not a polynomial equation in terms of its derivatives.
$\therefore$ Its degree is not defined.

62. **(a)** The given differential equation is

$$x^2 = y^2 + xy\frac{dy}{dx}$$

Since, this equation involves the derivative of the dependent variable y with respect to only one independent variable x.
$\therefore$ It is an ordinary differential equation.

63. **(a)** The solution free from arbitrary constants, i.e., the solution obtained from the general solution by giving particular values to the arbitrary constant is called a particular solution of the differential equation.
Here, function ϕ_1 contains no arbitrary constants but only the particular values of the parameters a and b and hence is called a particular solution of the given differential equation.

64. **(a)** Another form of first order linear differential equation is

$$\frac{dx}{dy} + P_1 y = Q_1$$

where P_1 and Q_1 are constants or function of y only.
This type of differential equation are

$$\frac{dx}{dy} + x = \cos y$$

$$\frac{dx}{dy} + \frac{-2x}{y} = y^2 e^{-y}$$

65. **(c)** **Assertion:** $y^3\frac{dy}{dx} + (x + y^2) = 0$
$y^2 = t$

$$2y\frac{dy}{dx} = \frac{dt}{dx}$$

$\frac{1}{2}\frac{dt}{dx}.t + x + t = 0$ is homogeneous equation.
Reason is obviously false.

66. **(c)** **67.** **(a)**

68. **(d)** The given differential equaiton is not a polynomial equation in its derivatives so its degree is not defined.

69. **(a)**

70. **(a)** Here $P = x^2$ and $Q = 5$. P is a function of x only and Q is a constant.

CRITICALTHINKING TYPE QUESTIONS

71. **(c)** $y = x \Rightarrow \frac{dy}{dx} = 1, \frac{d^2y}{dx^2} = 0$

Now $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

72. **(b)** Let y denote the number of bacteria at any instant t · then according to the question

$\frac{dy}{dt} \alpha y \Rightarrow \frac{dy}{y} = k\,dt$ ... (i)

k is the constant of proportionality, taken to be + ve on integrating (i), we get
$\log y = kt + c$ ... (ii)
c is a parameter. let y_0 be the initial number of bacteria i.e., at t = 0 using this in (ii), $c = \log y_0$
$\Rightarrow \log y = kt + \log y_0$

$\Rightarrow \log\frac{y}{y_0} = kt$ ...(iii)

$y = \left(y_0 + \frac{10}{100}y_0\right) = \frac{11y_0}{10}$, when t = 2

So, from (iii), we get $\log\frac{\frac{11y_0}{10}}{y_0} = k\,(2)$

$\Rightarrow k = \frac{1}{2}\log\frac{11}{10}$ ... (iv)

Using (iv) in (iii) $\log\frac{y}{y_0} = \frac{1}{2}\left(\log\frac{11}{10}\right)t$ ... (v)

let the number of bacteria become 1, 00, 000 to 2,00,000 in t_1 hours. i.e., $y = 2y_0$ when $t = t_1$ hours. from (v)

$\log\frac{2y_0}{y_0} = \frac{1}{2}\left(\log\frac{11}{10}\right)t_1 \Rightarrow t_1 = \frac{2\log 2}{\log\frac{11}{10}}$

Hence, the reqd. no. of hours $= \frac{2\log 2}{\log\frac{11}{10}}$

73. **(a)** The given differential equation is
$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$

dividing by $\cos x \cos y \Rightarrow \frac{\sin x}{\cos x}dx + \frac{\sin y}{\cos y}dy = 0$

Integrating, $\int \tan x\, dx + \int \tan y\, dy = \log c$
or $\log \sec x \sec y = \log c$ or $\sec x \sec y = c$

curve passes through the point $\left(0, \frac{\pi}{4}\right)$

$\sec 0 \sec\frac{\pi}{4} = c = \sqrt{2}$

Hence, the reqd. equ. of the curve is $\sec x \sec y = \sqrt{2}$

74. **(b)** Let y be the population at an instant t. Now population increase at a rate α no. of inhabitants

$\therefore \frac{dy}{dt} \alpha y$ or $\frac{dy}{dt} = ky$

$\therefore \frac{dy}{y} = kdt$ Integrating $\int\frac{dy}{y} = \int kdt + c$

or $\log y = kt + c$ ...(i)
In 1999, t = 0, population = 20,000
$\therefore$ $\log 20{,}000 = c$ Put the value of c in (i)

log y = kt + log 20,000 or log y – log 20000 = kt

or $\log \frac{y}{20000} = kt$ ...(ii)

In 2004, t = 5, y = 25000

$\log \frac{25000}{20000} = k \times 5 \Rightarrow k = \frac{1}{5} \log \frac{5}{4}$

Equ (ii) as $\log \frac{y}{20000} = \left(\frac{1}{5} \log \frac{5}{4}\right) t$

In 2009, t = 10

$\Rightarrow \log \frac{y}{20000} = \left(\frac{1}{5} \log \frac{5}{4}\right) \times 10 = 2 \log \frac{5}{4}$

$\Rightarrow \log \left(\frac{5}{4}\right)^2 = \log \frac{25}{16} \Rightarrow \frac{y}{20000} = \frac{25}{16}$

$\Rightarrow y = \frac{25}{16} \times 20000 = 25 \times 1250 = 31250$

75. (a) $(1 + \log x)\frac{dx}{dy} - x \log x = e^y$

putting x log x = t ⇒ (1 + log x) dx = dt

$\therefore \frac{dt}{dy} - t = e^y$

Now, I.F. $= e^{\int -1dy} = e^{-y}$

$\Rightarrow te^{-y} = \int e^{-y} e^y dy + C$

$\Rightarrow t = Ce^y + ye^y$

$\Rightarrow x \log x = (C + y) e^y$,

Since, y(1) = 0, then

0 = (C + 0) 1 ⇒ C = 0

$\therefore ye^y = x \log x$

$\Rightarrow x^x = e^{ye^y}$

76. (b) We have,

$\frac{dy}{dx} = \frac{ax+3}{2y+f} \Rightarrow (ax + 3)\, dx = (2y + f)\, dy$

$\Rightarrow a\frac{x^2}{2} + 3x = y^2 + fy + C$ (Integrating)

$\Rightarrow -\frac{a}{2}x^2 + y^2 - 3x + fy + C = 0$

This will represent a circle, if

$-\frac{a}{2} = 1$ [∵ Coeff. of x^2 = Coeff. of y^2]

and, $\frac{9}{4} + \frac{f^2}{4} - C > 0$ [Using : $g^2 + f^2 - c > 0$]

$\Rightarrow a = -2$ and $9 + f^2 - 4C > 0$

77. (c) $\frac{dx}{dy} = \frac{\left(\frac{x}{y} - 1\right) e^{\frac{x}{y}}}{1 + e^{\frac{x}{y}}}$

Substitute $x = vy \Rightarrow \frac{dx}{dy} = \frac{ydv}{dy} + v$

Now given equation becomes

$\frac{ydv}{dy} + v = \frac{(v-1)e^v}{1+e^v}$

$\Rightarrow \frac{ydv}{dy} = \frac{(v-1)e^v}{1+e^v} - v = \frac{-(v+e^v)}{1+e^v}$

$\Rightarrow \frac{(1+e^v)dv}{v+e^v} + \frac{dy}{y} = 0$

$\Rightarrow \ln (v + e^v)\, y = \ln c \Rightarrow (v + e^v)y = c$

$\Rightarrow x + ye^{x/y} = c$

78. (b) Given $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$

$\Rightarrow \frac{d(x^2+y^2)}{(x^2+y^2)^2} = 2\frac{d\left(\frac{x}{y}\right)}{\left(\frac{x}{y}\right)^2}$

Integrating, we get

$-\frac{1}{x^2+y^2} = \frac{-1}{x/y} + c \Rightarrow c = \frac{y}{x} - \frac{1}{x^2+y^2}$

79. (a) The given differential equation can be written as $y_2/y_1 = 2x/(x^2 + 1)$.

Integrating both the sides we have

$\log y_1 = \log (x^2 + 1) + c$

which implies $\log y_1 (0) = \log 1 + c$, i.e., c = log 3.

Therefore, $\log y_1 = \log (x^2 + 1) + \log 3$ which implies $y_1 = 3 (x^2 + 1)$ or $y = x^3 + 3x + A$,

so, 1 = y (0) = 0 + 0 + A, i.e., A = 1.

Hence the required equation of curve is

$y = x^3 + 3x + 1$.

80. (b) By multiplying e^{-t} and rearranging the terms, we get

$e^{-t}(1+t)dy + y(e^{-t} - (1+t)e^{-t})dt = e^{-t}dt$

$\Rightarrow d(e^{-t}(1+t)y) = d(-e^{-t}) \Rightarrow ye^{-t}(1+t) = -e^{-t} + c.$

Also $y_0 = -1 \Rightarrow c = 0 \Rightarrow y(1) = -1/2$

81. (a) Let female-male ratio at any time be r

$\frac{dr}{dt} \propto r \Rightarrow \frac{dr}{dt} = -k\, r$

where k is the constant of proportionality and k > 0

We have $\frac{dr}{r} = -k\, dt$

Integrating both sides, we have

$\int \frac{dr}{r} = -k\int dt$

log r = –kt + log C

$\log r - \log C = -kt \Rightarrow \log\left(\frac{r}{C}\right) = -kt$

where log C is the constant of integration

$\Rightarrow r = Ce^{-kt}$ ...(i)

Let us start time from the year 2001,

So in 2001, t = 0, $r = \frac{980}{1000} = \frac{49}{50}$

Putting t = 0 in (i), we have

$\frac{49}{50} = C \Rightarrow r = \frac{49}{50}e^{-kt}$ ...(ii)

Also in the year 2011, t = 10 and

$r = \frac{920}{1000} = \frac{23}{25}$

Putting in (ii), we have

$\frac{23}{25} = \frac{49}{50}e^{-10k} \Rightarrow e^{10k} = \frac{49}{50} \times \frac{25}{23} = \frac{49}{46}$

or $e^{-10k} = \frac{46}{49}$

Hence, $r = \frac{49}{50}e^{-10k \times \frac{t}{10}} \Rightarrow r = \frac{49}{50}\left(\frac{46}{49}\right)^{\frac{t}{10}}$

In the year 2021, $t = 20 \therefore r = \frac{49}{50}\left(\frac{46}{49}\right)^{\frac{20}{10}}$

$= \frac{49}{50} \times \frac{46}{49} \times \frac{46}{49} = 0.864$

Thus, at this trend female : male ≃ 864 : 1000

82. (c) Given equation can be written as

$xdy = \left(\sqrt{x^2 + y^2} + y\right)dx$, i.e.,

$\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x}$ ...(i)

Substituting y = vx, we get from (i)

$v + x\frac{dv}{dx} = \frac{\sqrt{x^2 + v^2 + x^2} + vx}{x}$

$v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$

$x\frac{dv}{dx} = \sqrt{1 + v^2} \Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ ....(ii)

Intergrating both sides of (ii), we get

$\log\ (v + \sqrt{1 + v^2}) = \log x + \log c$

$\Rightarrow v + \sqrt{1 + v^2} = cx \Rightarrow \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx$

$\Rightarrow y + \sqrt{x^2 + y^2} = cx^2$

83. (d) $\frac{d^4y}{dx^4} + \sin(y''') = 0$

$\Rightarrow y'''' + \sin(y''') = 0$

The highest order derivative which occurs in the given differential equation is y'''', therefore its order is 4. As the given differential equation is not a polynomial equation in derivatives of y w.r.t. x (i.e., y'''), therefore its degree is not defined.

84. (c) The given differential equation is

$e^x \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 3$

Since, this differential equation is not a polynomial in terms of its derivatives.

∴ Its degree is not defined

85. (a) Given, $y = \left(x + \sqrt{1 + x^2}\right)^n$ ...(i)

$\Rightarrow \frac{dy}{dx} = n\left[x + \sqrt{1 + x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2 + 1}}\right)$

$= \frac{n\left[x + \sqrt{1 + x^2}\right]^n}{\sqrt{1 + x^2}}$

$\Rightarrow \left(\frac{dy}{dx}\right)^2(1 + x^2) = n^2y^2$ [using eq. (i) and squaring]

Again, differentiating, we get

$2\frac{dy}{dx}.\frac{d^2y}{dx^2}(1 + x^2) + 2x\left(\frac{dy}{dx}\right)^2 = 2n^2y\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2}(1 + x^2) + x\frac{dy}{dx} = n^2y$ $\left(\text{divide by } 2\frac{dy}{dx}\right)$

86. (b) The given differential equation can be expressed as

$dy = \frac{2x^2 + 1}{x}dx$

or $dy = \left(2x + \frac{1}{x}\right)dx$ ... (i)

On integrating both sides of eq. (i), we get

$\int dy = \left(2x + \frac{1}{x}\right)dx$

$\Rightarrow y = x^2 + \log|x| + C$ ... (ii)

Eq. (ii) represents the family of solution curves of the given differential equation but we are interested in finding the equation of a particular member of the family which passes through the point (1, 1). Therefore, substituting x = 1, y = 1 in eq. (ii), we get C = 0

87. (d) It is given that (x, y) is the point of contact of the curve and its tangent.

The slope of the line segment joining the points.

$(x_2, y_2) \to (x, y)$ and $(x_1, y_1) \to (-4, -3)$

$= \frac{y - (-3)}{x - (-4)} = \frac{y + 3}{x + 4}$ $\left(\because \text{slope of a tangent} = \frac{y_2 - y_1}{x_2 - x_1}\right)$

According to the question, (slope of tangent is twice the slope of the line), we must have

$\frac{dy}{dx} = 2\left(\frac{y + 3}{x + 4}\right)$

Now, separating the variable, we get

$\frac{dy}{y + 3} = \left(\frac{2}{x + 4}\right)dx$

On integrating both sides, we get

$\int\frac{dy}{y + 3} = \int\left(\frac{2}{x + 4}\right)dx$

$\Rightarrow \log|y + 3| = 2\log|x + 4| + \log|C|$

$\Rightarrow \log|y + 3| = \log|x + 4|^2 + \log|C|$

$\Rightarrow \log\frac{|y + 3|}{|x + 4|^2} = \log|C|$ $\left(\because \log m - \log n = \log\frac{m}{n}\right)$

$\Rightarrow \frac{|y + 3|}{|x + 4|^2} = C$ ...(i)

The curve passes through the point (–2, 1) therefore

$$\frac{|1+3|}{|-2+4|^2} = C \Rightarrow C = 1$$

On substituting C = 1 in eq. (i), we get

$$\frac{|y+3|}{|x+4|^2} = 1$$

$\Rightarrow |y + 3| = (x + 4)^2$

Which is the required equation of curve

88. (c) Let P be the principal at any time T.

According to the given problem,

$$\frac{dP}{dt} = \left(\frac{5}{100}\right) \times P$$

$$\Rightarrow \frac{dP}{dt} = \frac{P}{20} \quad \ldots (i)$$

On separating the variables in eq. (i), we get

$$\frac{dP}{P} = \frac{dt}{20} \quad \ldots (ii)$$

On integrating both sides of eq.(ii), we get

$$\log P = \frac{t}{20} + C_1$$

$$\Rightarrow P = e^{\frac{t}{20}}.e^{C_1}$$

$$\Rightarrow P = Ce^{\frac{t}{20}} \left(\text{where, } e^{C_1} = C\right) \quad \ldots (iii)$$

Now, P = 1000, when t = 0

On substituting the values of P and t in eq. (iii), we get C = 1000. Therefore, eq. (iii), gives

$$P = 1000e^{\frac{t}{20}}$$

Let t years be the time required to double the principal. Then,

$$2000 = 1000e^{\frac{t}{20}}$$

$\Rightarrow t = 20 \log_e 2$

89. (d) Here, the slope of the tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$.

$$\therefore \frac{dy}{dx} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

On integrating both sides, we get

$$\int \frac{dy}{y-1} = \int \frac{dx}{x(x-1)}$$

$\Rightarrow \log (y – 1) = \log x – \log (x + 1) + \log C$

$$\log (y-1) = \log\left(\frac{xC}{x+1}\right)$$

$\Rightarrow (y – 1) (x + 1) = xC$

Since, the above curve passes through (1, 0)

$\Rightarrow (–1) (2) = 1.C$

$\Rightarrow C = –2$

$\therefore$ Required equation of the curve is

$(y – 1) (x + 1) + 2x = 0$

90. (b) To solve a homogenous differential equation of the type

$$\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right) \quad \ldots (i)$$

We make the substitution

$$y = v . x \quad \ldots (ii)$$

On differntiating eq. (ii) w.r.t. x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \ldots (iii)$$

On substituting the value of $\frac{dy}{dx}$ from eq. (iii) in eq. (i), we get

$$v + x\frac{dv}{dx} = g(v)$$

or $$x\frac{dv}{dx} = g(v) - v \quad \ldots (iv)$$

On separating the variables in eq. (iv) we get

$$\frac{dv}{g(v)-v} = \frac{dx}{x} \quad \ldots (v)$$

On integrating both sides of Eq. (v), we get

$$\int \frac{dv}{g(v)-v} = \int \frac{1}{x}dx + C \quad \ldots (vi)$$

eq (vi) gives general solution (primitive) of the differential eq. (i) when we replace v by $\frac{v}{x}$.

91. (c) The given differential equation is

dx + dy = (x + y) (dx – dy)

$$\Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x+y+1} \quad \ldots (i)$$

Put x + y = t

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$$

So, from equation (i), we have

$$\frac{dt}{dx} - 1 = \frac{t-1}{t+1} \Rightarrow \frac{dt}{dx} = \frac{t-1}{t+1} + 1$$

$$\Rightarrow \frac{dt}{dx} = \frac{t-1+t+1}{t+1} \Rightarrow \frac{1}{2}\left(1+\frac{1}{t}\right)dt = dx$$

On integrating both sides, we get

$$\int \frac{1}{2}\left(1+\frac{1}{t}\right)dt = \int dx \Rightarrow \frac{1}{2}.(t + \log t) = x + \frac{C}{2}$$

$\Rightarrow t + \log t = 2x + C$

$\Rightarrow \log (x + y) = x – y + C$

92. (a) According to the condition,

$$\frac{dy}{dx}=\frac{y}{x}-\cos^2\frac{y}{x} \quad \ldots (i)$$

This is a homogeneous differential equation
Substituting y = vx, we get

$$v+x\frac{dv}{dx}=v-\cos^2 v$$

$$\Rightarrow \; x\frac{dv}{dx}=-\cos^2 v$$

$$\Rightarrow \int \sec^2 v \, dv = -\int\frac{dx}{x}$$

$$\Rightarrow \tan v = -\log x + C$$

$$\Rightarrow \tan\frac{y}{x}+\log x = C \quad \ldots(ii)$$

Substituting $x = 1, y=\frac{\pi}{4}$, we get C = 1. Thus, we get

$$\tan\left(\frac{y}{x}\right)+\log x = 1$$

which is the required solution

93. (b) The given differential equation can be written as

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2} \quad \ldots(i)$$

Now, eq. (i) is a linear differential equation of the form $\frac{dx}{dy}+P_1 x = Q_1$

where $P_1=\frac{1}{1+y^2}$ and $Q_1=\frac{\tan^{-1}y}{1+y^2}$

Therefore, $I.F = e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

Thus, the solution of the given differential equation is given by

$$x\,e^{\tan^{-1}y}=\int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy+C \quad \ldots (ii)$$

Let $I=\int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy$

On substituting $\tan^{-1}y = t$, so that $\left(\frac{1}{1+y^2}\right)dy = dt$,

we get

$$I=\int te^t dt = te^t - \int 1.e^t dt$$

$$= te^t - e^t = e^t(t-1)$$

or $\quad I = e^{\tan^{-1}y}\left(\tan^{-1}y-1\right)$

On substituting the value of I in equation (ii), we get

$$x.e^{\tan^{-1}y} = .e^{\tan^{-1}y}\left(\tan^{-1}y-1\right)+C$$

or $\quad x = (\tan^{-1}y - 1) + Ce^{\tan^{-1}y}$

which is the general solution of the given differentail equation.

94. (a) The given differential equation is

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1 \quad \ldots (i)$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}=\frac{y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{x}}y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}} \quad \ldots (ii)$$

On comparing with the form $\frac{dy}{dx}+Py=Q,$ we get

$$P=\frac{1}{\sqrt{x}} \text{ and } Q=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$\therefore \; I.F = e^{\int\frac{1}{\sqrt{x}}dx} \Rightarrow I.F = e^{2\sqrt{x}}$

The general solution of the given differential equation is given by

$$y.I.F=\int(Q\times I.F)\,dx + C$$

$$\Rightarrow \; ye^{2\sqrt{x}}=\int e^{2\sqrt{x}}\times\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx + C$$

$$\Rightarrow \; ye^{2\sqrt{x}}=\int\frac{1}{\sqrt{x}}dx+C$$

$$\Rightarrow \; ye^{2\sqrt{x}}=2\sqrt{x}+C$$

95. (d) Given: xdy – ydx = 0
Dividing by xy on both sides,
we get:

$$\frac{dy}{y}-\frac{dx}{x}=0$$

$$\Rightarrow \frac{dy}{y}=\frac{dx}{x}$$

By integrating on both sides, we get,
log y = log x + log c

$$\Rightarrow \log\frac{y}{x}=\log c \Rightarrow y = cx \text{ or } y - cx = 0$$

which represents a straight line passing through origin.

96. (a) Differential equation is given by

$$\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

order of a differential equation is the order of the highest derivative appearing in the equation. Hence the order is 2.

To find the degree of the differential equation, it has to be expressed as a polynomial in derivatives. For this we square both the sides of differential eq^n.

$$\rho\left(\frac{d^2y}{dx^2}\right)^2 = \left[1+\left(\frac{dy}{dx}\right)^2\right]^3$$

Here power of highest derivative is 2, hence order is 2 and degree is also 2.

97. (b) Put $\frac{y}{x} = u$ we have $\frac{dy}{dx} = u + x\frac{du}{dx}$

$$u + x\frac{du}{dx} = u + \frac{\phi(u)}{\phi'(u)} \Rightarrow x\frac{dy}{dx} = \frac{\phi(u)}{\phi'(u)}$$

$\Rightarrow \frac{\phi'(u)}{\phi(u)}du = \frac{dx}{x}$ and Integrate it

Required solution is $\phi\left(\frac{y}{x}\right) = kx$

98. (d) Given $\frac{x\,dx}{1+x^2} = \frac{y\,dy}{1+y^2}$

Integrating we get,

$$\frac{1}{2}\log(1+x^2) = \frac{1}{2}\log(1+y^2)+a$$

$\Rightarrow 1+x^2 = c(1+y^2)$,

Where $c = e^{2a}$

$$x^2 - cy^2 = c-1 \Rightarrow \frac{x^2}{c-1} - \frac{y^2}{\left(\frac{c-1}{c}\right)} = 1 \quad \ldots(i)$$

Clearly $c > 0$ as $c = e^{2a}$

Hence, the equation (i) gives a family of hyperbolas with eccentricity

$$= \sqrt{\frac{c-1+\frac{c-1}{c}}{c-1}} = \sqrt{\frac{c^2-1}{c-1}} = \sqrt{c+1} \text{ if } c \neq 1$$

Thus ecentricity varies from member to member of the family as it depends on c.

99. (a) Divide the equation by y^2, we get

$$\frac{ydx - xdy}{y^2} = -3x^2e^{x^3}dx \Rightarrow \frac{d}{dx}\left(\frac{x}{y}\right) = -\frac{d}{dx}\left(e^{x^3}\right)$$

On integrating we get,

$$\frac{x}{y} = -e^{x^3} + c \Rightarrow \frac{x}{y} + e^{x^3} = c$$

100. (b) Rewriting the given equation in the form

$$x^4\cos y\frac{dy}{dx} + 4x^3\sin y = xe^x \Rightarrow \frac{d}{dx}(x^4\sin y) = xe^x$$

$$\Rightarrow x^4\sin y = \int xe^x dx + c = (x-1)e^x + c$$

Since, $y(1) = 0$ so, $c = 0$.

Hence, $\sin y = x^{-4}(x-1)e^x$

101. (c) Any conic whose axes coincide with co-ordinate axis is $ax^2 + by^2 = 1$ ..(i)

Diff. both sides w.r.t. 'x', we get

$$2ax + 2by\frac{dy}{dx} = 0 \text{ i.e. } ax + by\frac{dy}{dx} = 0 \quad \ldots(ii)$$

$$\text{Diff. again, } a + b\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = 0 \quad \ldots(iii)$$

From (ii), $\frac{a}{b} = -\frac{ydy/dx}{x}$

From (iii), $\frac{a}{b} = -\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right)$

$$\therefore \frac{y\frac{dy}{dx}}{x} = y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

102. (c) Rewrite the given differential equation as follows :

$\frac{dy}{dx} + \frac{2x}{x^2-1}y = \frac{1}{x^2-1}$, which is a linear form

The integrating factor

$$\text{I.F.} = e^{\int\frac{2x}{x^2-1}dx} = e^{\ell n(x^2-1)} = x^2-1$$

Thus multiplying the given equation by $(x^2 - 1)$, we get

$$(x^2-1)\frac{dy}{dx} + 2xy = 1 \Rightarrow \frac{d}{dx}[y(x^2-1)] = 1$$

On integrating we get $y(x^2-1) = x + c$

103. (b) The given equation can be converted to linear form by dividing both the sides by $\cos^2 y$. We get

$$\sec^2 y\frac{dy}{dx} + \frac{1}{x}2\tan y = x^3 \quad ;$$

Put $\tan y = z \Rightarrow \sec^2 y\frac{dy}{dx} = \frac{dz}{dx}$

The equation becomes $\frac{dz}{dx} + \frac{2}{x}z = x^3$, which is linear in z

The integrating factor is

$$\text{I.F.} = e^{\int\frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$$

Hence, the solution is

$$z(x^2) = \int x^3(x^2)dx + a \Rightarrow zx^2 = \frac{x^6}{6} + a,$$

a is constant of integration.

$$\therefore (6\tan y)x^2 = x^6 + 6a \Rightarrow 6x^2\tan y = x^6 + c, \quad [c = 6a]$$

104. (d) The retardation at time $t = -\frac{dv}{dt}$. Hence, the differential equation is $-\frac{dv}{dt} = v \Rightarrow \frac{dv}{v} = -dt$ ...(i)

Integrating, we get $\log v = -t + c$ ...(ii)

When $t = 0$, $v = V \Rightarrow C = \log V$

The equation (ii) becomes $\log v = -t + \log V$

$$\Rightarrow \log\frac{v}{V} = -t \Rightarrow \frac{v}{V} = e^{-t} \Rightarrow v = Ve^{-t}$$

CHAPTER 26
VECTOR ALGEBRA

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then the vectors $\vec{a}$ and $\vec{b}$ are adjacent sides of
(a) a rectangle (b) a square
(c) a rhombus (d) None of these

2. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of magnitudes 3, 4 and 5 respectively. If each one is perpendicular to the sum of the other two vectors, then $|\vec{a}+\vec{b}+\vec{c}|=$
(a) 5 (b) $3\sqrt{2}$
(c) $5\sqrt{2}$ (d) 12

3. A unit vector perpendicular to the plane ABC, where A, B and C are respectively the points (3, –1, 2), (1, –1, –3) and (4, –3, 1), is
(a) $-\frac{1}{\sqrt{29}}(2\hat{i}+5\hat{k})$ (b) $\frac{1}{\sqrt{6}}(\hat{i}-2\hat{j}-\hat{k})$
(c) $\frac{1}{\sqrt{26}}(4\hat{i}-3\hat{j}+\hat{k})$ (d) $-\frac{1}{\sqrt{165}}(10\hat{i}+7\hat{j}-4\hat{k})$

4. The perpendicular distance of A(1, 4, –2) from BC, where coordinates of B and C are respectively (2, 1, –2) and (0, –5, 1) is
(a) $\frac{3}{7}$ (b) $\frac{\sqrt{26}}{7}$
(c) $\frac{3\sqrt{26}}{7}$ (d) $\sqrt{26}$

5. ABC is a triangle and P is any point on BC such that $\overrightarrow{PQ}$ is the resultant of the vectors $\overrightarrow{AP}$, $\overrightarrow{PB}$ and $\overrightarrow{PC}$, then
(a) the position of Q depends on position of P
(b) Q is a fixed point
(c) Q lies on AB or AC
(d) None of these

6. ABCDEF is a regular hexagon where centre O is the origin. If the position vectors of A and B are $\hat{i}-\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ respectively, then $\overrightarrow{BC}$ is equal to
(a) $\hat{i}+\hat{j}-2\hat{k}$ (b) $-\hat{i}+\hat{j}-2\hat{k}$
(c) $3\hat{i}+3\hat{j}-4\hat{k}$ (d) None of these

7. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$ and $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=-3(\hat{i}-\hat{k})$, then the ordered triplet (α, β, γ) is
(a) (2, –1, –1) (b) (–2, 1, 1)
(c) (–2, –1, 1) (d) (2, 1, –1)

8. A unit vector perpendicular to the plane formed by the points (1, 0, 1), (0, 2, 2) and (3, 3, 0) is
(a) $\frac{1}{5\sqrt{3}}(5\hat{i}-\hat{j}-7\hat{k})$ (b) $\frac{1}{5\sqrt{3}}(5\hat{i}-\hat{j}+7\hat{k})$
(c) $\frac{1}{5\sqrt{3}}(5\hat{i}+\hat{j}+7\hat{k})$ (d) None of these

9. If $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$, then the value of $|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}|$, is equal to (given that $\vec{a}+\vec{b}+\vec{c}=0$)
(a) 25 (b) 50
(c) –25 (d) –50

10. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|$
(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$
(c) 34 : 39 : 45 (d) 39 : 35 : 34

11. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$, $|\vec{b}|=2$, $|\vec{c}|=3$, , then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to
(a) 1 (b) 0
(c) – 7 (d) 7

12. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a
(a) parallelogram but not a rhombus
(b) square
(c) rhombus
(d) None of these

13. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$, then
(a) $\alpha = 1, \beta = -1$ (b) $\alpha = 1, \beta = \pm 1$
(c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

14. Three points (2, –1, 3), (3, –5, 1) and (– 1, 11, 9) are
(a) Non-collinear (b) Non-coplanar
(c) Collinear (d) None of these

15. If three points A, B and C have position vectors (1, x, 3), (3, 4, 7) and (y, – 2, – 5) respectively and, if they are collinear, then (x – y) is equal to
(a) (2, – 3) (b) (– 2, 3)
(c) (2, 3) (d) (– 2, – 3)

16. The vectors $\vec{a}=x\hat{i}+2\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}+y\hat{j}-z\hat{k}$ are collinear, if
(a) x = 1, y = – 2, z = – 5 (b) x = 1/2, y = – 4, z = – 10
(c) x = – 1/2, y = 4, z = 10 (d) All of these

17. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=4$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $[\vec{a}\ \vec{b}\ \vec{c}]$ is equal to
(a) $4\sqrt{3}$ (b) $6\sqrt{3}$
(c) $12\sqrt{3}$ (d) $18\sqrt{3}$

18. The angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}$ = (1, 1, 4) and $\vec{b}$ = (1, –1, 4) is
(a) 90° (b) 45°
(c) 30° (d) 15°

19. If $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2=676$ and $|\vec{b}|=2$, then $|\vec{a}|$ is equal to
(a) 13 (b) 26
(c) 39 (d) None of these

20. Let $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}-\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$.
Then, $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on
(a) neither x nor y (b) both x and y
(c) only x (d) only y

21. If $\overrightarrow{AB}\times\overrightarrow{AC}=2\hat{i}-4\hat{j}+4\hat{k}$, then the area of ΔABC is
(a) 3 sq. units (b) 4 sq. units
(c) 16 sq. units (d) 9 sq. units

22. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to
(a) $\vec{a}^2$ (b) $3\vec{a}^2$
(c) $4\vec{a}^2$ (d) $2\vec{a}^2$

23. If $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a}.\vec{b}=12$, then the value of $|\vec{a}\times\vec{b}|$ is
(a) 5 (b) 10
(c) 14 (d) 16

24. The vector in the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$ that has magnitude 9 is
(a) $\hat{i}-2\hat{j}+2\hat{k}$ (b) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$
(c) $3(\hat{i}-2\hat{j}+2\hat{k})$ (d) $9(\hat{i}-2\hat{j}+2\hat{k})$

25. In triangle ABC, which of the following is not true?
(a) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$
(b) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$
(c) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\vec{0}$
(d) $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

26. If $\vec{a}$ is a non-zero vector of magnitude a and l a non-zero scalar, then l $\vec{a}$ is a unit vector if.
(a) l = 1 (b) l = – 1
(c) a = |l| (d) $a=\frac{1}{|\lambda|}$

27. A zero vector has
(a) any direction (b) no direction
(c) many directions (d) None of these

28. If two vertices of a triangle are i – j and j + k, then the third vertex can be
(a) i + k (b) i –2j – k and –2i – j
(c) i – k (d) All the above

29. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct ?
(a) $\vec{a}\times\vec{b}=b\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$
(b) $\vec{a}\times\vec{b}=b\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$
(c) $\vec{a}\times\vec{b}=b\times\vec{c}=\vec{a}\times\vec{c}\neq\vec{0}$
(d) $\vec{a}\times\vec{b}, b\times\vec{c}, \vec{c}\times\vec{a}$ are mutually perpendicular

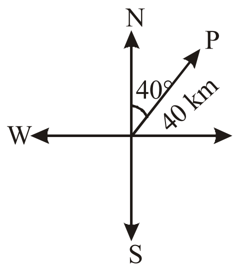
30. Which among the following is correct statement?
(a) A quantity that has only magnitude is called a vector
(b) A directed line segment is a vector, denoted as |AB| or |a|
(c) The distance between initial and terminal points of a vector is called the magnitude of the vector
(d) None of the above

31. Two or more vectors having the same initial point are called
(a) unit vectors (b) zero vectors
(c) coinitial vectors (d) collinear vectors

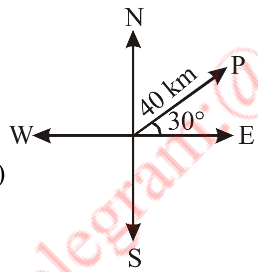
32. If two vectors a and b are such that a = b, then
(a) they have same magnitude and direction regardless of the positions of their initial points
(b) they have same magnitude and different directions
(c) Both (a) and (b) are true
(d) Both (a) and (b) are false

33. A vector whose magnitude is the same as that of a given vector, but direction is opposite to that of it, is called
(a) negative of the given vector
(b) equal vector
(c) null vector
(d) collinear vector

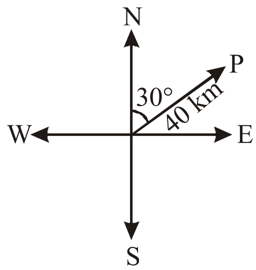
34. Which of the following represents graphically the displacement of 40 km, 30° East of North?



(a)

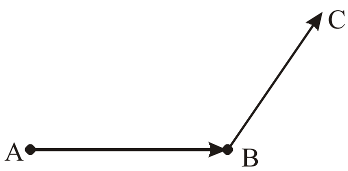


(b)



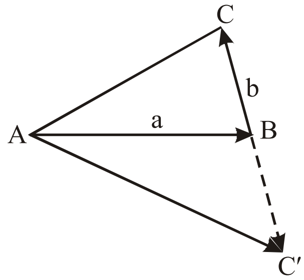
(c) (d) None of these

35. If a girl moves from A to B and then from B to C (as shown). Then, net displacement made by the girl from A to C, is



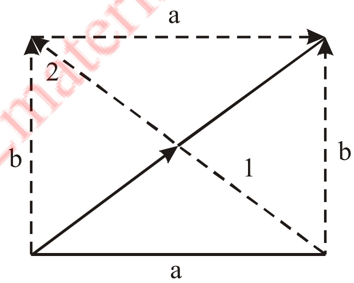
(a) AB – BC = AC (b) AC = AB – BC
(c) AC = AB + BC (d) None of these

36. Consider the figure given below



Here, it is shown that a vector BC′ is having same magnitude as the vector BC, but its direction is opposite to that of it.
Based on above information which of the following is true?
(a) AC′ = a + b
(b) AC′ = a – b
(c) Difference of a and b is AC
(d) None of these

37. If two vectors a and b represented by two adjacent sides of a parallelogram in magnitude and direction then a + b is represented as



(a) diagonal 1 (as shown)
(b) diagonal 2 (as shown)
(c) sides opposite to either of the side
(d) None of the above

38. If l, m and n are the direction cosines of a vector, and α, β and γ are the angles which the vector makes with X, Y and Z-axes respectively, then the unit vector in the direction of that vector is

(a) $l\hat{i}+m\hat{j}+n\hat{k}=\dfrac{\hat{i}}{\cos\alpha}+\dfrac{\hat{j}}{\cos\beta}+\dfrac{\hat{k}}{\cos\gamma}$

(b) $l\hat{i}+m\hat{j}+n\hat{k}=(\cos\alpha)\hat{i}+(\cos\beta)\hat{j}+(\cos\gamma)\hat{k}$

(c) $l\hat{i}+m\hat{j}+n\hat{k}=(l\cos\alpha)\hat{i}+(m\cos\beta)\hat{j}+(n\cos\gamma)\hat{k}$

(d) None of these

39. Which of the following is an example of two different vectors with same magnitude?

(a) $(2\hat{i}+3\hat{j}+\hat{k})$ and $(2\hat{i}+3\hat{j}-\hat{k})$

(b) $(3\hat{i}+5\hat{j}+\hat{k})$ and $(3\hat{i}+4\hat{j}+\hat{k})$

(c) $(\hat{j}+\hat{k})$ and $(2\hat{j}+3\hat{k})$

(d) None of the above

40. ABCD be a parallelogram and M be the point of intersection of the diagonals, if O is any point, then OA + OB + OC + OD is equal to
(a) 3 OM (b) 2 OM
(c) 4 OM (d) OM

41. If a is a vector of magnitude 50, collinear with the vector $b = 6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}$ and makes an acute angle with the positive direction of Z-axis, then a is equal to

(a) $-24\hat{i} + 32\hat{j} + 30\hat{k}$ (b) $24\hat{i} - 32\hat{j} - 30\hat{k}$

(c) $-12\hat{i} + 16\hat{j} - 15\hat{k}$ (d) $12\hat{i} - 16\hat{j} - 15\hat{k}$

42. If ABCDE is a pentagon, then resultant of AB, AE, BC, DC, ED and AC is

(a) 2AC (b) 3AC

(c) AB (d) None of these

43. The non-zero vectors a, b and c are related by a = 8b and c = – 7b, then the angle between a and c is

(a) π (b) 0

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

44. Which of the following is true?

(a) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 0$

(b) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(c) Both (a) and (b) are true

(d) Both (a) and (b) are not true

45. Multiplication of two vectors is defined in two ways, namely

(a) scalar product and dot product

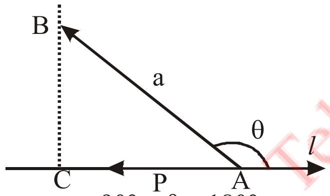
(b) vector product and cross product

(c) scalar product and vector product

(d) None of the above

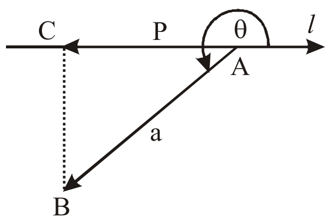
46. Which among the following figure correctly represents projection of AB on a line *l* ?

(a)



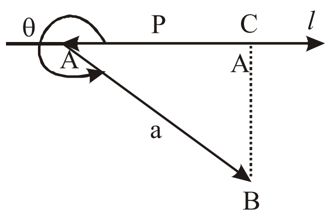
$90° < \theta < 180°$

(b)



$180° < \theta < 270°$

(c)



$270° < \theta < 360°$

(d) All of these

47. A unit vector in xy- plane makes an angle of 45° with the vector $\hat{i} + \hat{j}$ and an angle of 60° with the vector $3\hat{i} - 4\hat{j}$ is

(a) $\frac{13}{7}\hat{i} + \frac{1}{7}\hat{j}$ (b) $\frac{7}{13}\hat{i} + \frac{7}{15}\hat{j}$

(c) $\frac{13}{14}\hat{i} + \frac{1}{14}\hat{j}$ (d) None of the above

48. The vector product of two non zero vector a and b, is denoted by a × b and is equal to

(a) $|a|\,|b| \cos\theta$ (b) $|a||b|\sin\theta\hat{n}$

(c) $|a||b|\cos\theta\hat{n}$ (d) None of these

49. Which of the following is true?

(a) $\hat{j} \times \hat{i} = \hat{k}$ (b) $\hat{k} \times \hat{j} = \hat{i}$

(c) $\hat{i} \times \hat{k} = -\hat{j}$ (d) All of these

50. Which of the following statement is correct?

(a) [a b c] is a scalar quantity

(b) The magnitude of the scalar triple product is the volume of a parallelopiped formed by adjacent sides given by three vectors a, b and c

(c) The volume of a parallelopiped form by three vectors a, b and c is equal to |a. (b × c)|

(d) All are correct

51. If $a = 2\hat{i} + \hat{j} + 3\hat{k}$, $b = -\hat{i} + 2\hat{j} + \hat{k}$ and $c = 3\hat{i} + \hat{j} + 2\hat{k}$ then a.(b × c) is equal to

(a) – 15 (b) 15

(c) – 10 (d) – 5

52. Which of the following statement is correct?

(a) $a.(b \times c) = [b \quad c \quad a]$

(b) $[a \quad b \quad c] = [c \quad a \quad b]$

(c) $[c \quad a \quad b] = c.(a \times b) = (a \times b).c$

(d) All are correct

53. Which of the following is correct?

(a) $[a \quad b \quad c] = [a \quad c \quad b]$

(b) $[a \quad c \quad b] = 0$

(c) Both (a) and (b) are correct

(d) Both (a) and (b) are incorrect

54. Magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

(a) $(x_2 - x_1) + (y_2 - y_1) + (z_2 - z_1)$

(b) $(x_2 - y_2 + z_2) + (x_1 + y_1 + z_1)$

(c) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(d) None of the above

55. If θ is the angle between any two vectors a and b, then $|a.b| = |a \times b|$, where θ is equal to

(a) zero (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

56. The vector $\vec{a} \times (\vec{b} \times \vec{c})$ is:

(a) parallel to $\vec{a}$.

(b) perpendicular to $\vec{a}$.

(c) parallel to $\vec{b}$.

(d) perpendicular to $\vec{b}$.

57. $\vec{a}.(\vec{a} \times \vec{b})$ is equal to:

(a) 0 (b) $a^2 + ab$

(c) a^2b (d) $\vec{a}.\vec{b}$

58. If the vectors $a\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + 5\hat{j} + a\hat{k}$ are perpendicular to each other then a is equal to:

(a) 5 (b) – 6

(c) – 5 (d) 6

59. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit-vector, then $\left|\vec{a}+\vec{b}-\vec{c}\right|$ equals :

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

60. If $|\vec{a}| = 3, |\vec{b}| = 4,$ then a value of λ for which $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b}$ is :

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$

(c) $\frac{3}{2}$ (d) $\frac{4}{3}$

61. Which one of the following statement is not correct ?

(a) Vector product is commutative

(b) Vector product is not associative

(c) Vector product is distributive over addition

(d) Scalar product is commutative

62. If $\vec{a}$ and $\vec{b}$ are the two vectors such that $\vec{a}\cdot\vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, then

(a) $\vec{a}$ is parallel to $\vec{b}$.

(b) $\vec{a}$ is perpendicular to $\vec{b}$.

(c) either $\vec{a}$ or $\vec{b}$ is a null vector .

(d) None of these.

63. If $\vec{a} = (\hat{i} + \hat{j} + \hat{k}), \vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is

(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $2\hat{j} - \hat{k}$

(c) $\hat{i}$ (d) $2\hat{i}$

64. If p, q, r be three non-zero vectors, then equation p .q = p . r implies:

(a) q = r

(b) p is orthogonal to both q and r.

(c) p is orthogonal to q – r.

(d) either q = r or p is perpendicular to q – r.

65. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}] = 4$, then $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] =$

(a) 16 (b) 64

(c) 4 (d) 8

66. Two vectors $\vec{a}$ and $\vec{b}$ are non-zero and non-collinear. What is the value of x for which the vectors $\vec{p} = (x-2)\vec{a} + \vec{b}$ and $\vec{q} = (x+1)\vec{a} - \vec{b}$ are collinear?

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) 2

67. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle of 30° to each other, then which one of the following is correct ?

(a) $|\vec{a}+\vec{b}| > 1$ (b) $1 < |\vec{a}+\vec{b}| < 2$

(c) $|\vec{a}+\vec{b}| = 2$ (d) $|\vec{a}+\vec{b}| > 2$

68. If C is the middle point of AB and P is any point outside AB, then

(a) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$ (b) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$

(c) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$ (d) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$

69. ABCD is a parallelogram whose diagonals meet at P. If O is a fixed point, then $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$ equals

(a) $\overrightarrow{OP}$ (b) $2\overrightarrow{OP}$

(c) $3\overrightarrow{OP}$ (d) $4\overrightarrow{OP}$

70. If the vertices of any tetrahedron be $\vec{A} = \vec{J} + 2\vec{K}, \vec{B} = 3\vec{I} + \vec{K},$ $\vec{C} = 4\vec{I} + 3\vec{J} + 6\vec{K}$ and $\vec{D} = 2\vec{I} + 3\vec{J} + 2\vec{K},$ then its volume is

(a) $\frac{1}{6}$ units (b) 6 units

(c) 36 units (d) None of these

71. For any two vectors a and b, $(a \times b)^2$ equals

(a) $a^2b^2 - (a.b)^2$ (b) $a^2 + b^2$

(c) $a^2 - b^2$ (d) None of these

72. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals

(a) – 4 (b) – 2 (c) 0 (d) 1.

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

73. Which of the following is/are true?

I. In a zero vector, initial and terminal points coincide.

II. Zero vector is denoted as O.

III. Zero vector has zero magnitude.

(a) Only II is true (b) I and III are true

(c) II and III are true (d) All are true

74. Which of the following is/are true?

I. To add two vectors a and b, they are positioned such that the initial point of one does not coincide with the terminal point of the other.

II. The resultant of the vectors AB and BC is represented by the third side AC of a triangle.

III. If sides of a triangle are taken in order, then it leads to zero resultant.

(a) Only I is true (b) Only II is true
(c) II and III are true (d) All are true

75. **Statement - I:** Scalar components of the vector with initial point (2, 1) and terminal point (–5, 7) are – 6 and 7.

Statement - II: Vector components of the vector with initial point (2, 1) and terminal point (–5, 7) are $-7\hat{i}$ and $6\hat{j}$.

(a) Only statement I is true
(b) Only statement II is true
(c) Both statements are true
(d) Both statements are false

76. **Statement I :** The position vector of point R which divides the line joining two points P(2a + b) and Q(a – 3b) externally in the ratio 1 : 2, is 3a + 5b.

Statement II : P is the mid-point of the line segment RQ.

(a) Only statement I is true
(b) Only statement II is true
(c) Both statements are true
(d) Both statements are false

77. **Statement I :** The point A(1, –2, – 8), B(5, 0, – 2) and C(11, 3, 7) are collinear.

Statement II : The ratio in which B divides AC, is 2 : 3

(a) Only statement I is true
(b) Only statement II is true
(c) Both statements are true
(d) Both statements are false

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

78. For the following table $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$

	Column-I		**Column-II**
A.	$\lvert \vec{a}+\vec{b} \rvert = \lvert \vec{a}-\vec{b} \rvert$	1.	$\vec{a} \perp \vec{b}$
B.	$\lvert \vec{a}+\vec{b} \rvert = \lvert \vec{a} \rvert + \lvert \vec{b} \rvert$	2.	$\vec{a}$ is parallel to $\vec{b}$.
C.	$\lvert \vec{a}+\vec{b} \rvert^2 = \lvert \vec{a} \rvert^2 + \lvert \vec{b} \rvert^2$		
D.	$\vec{a}.\vec{b} = 0$		
E.	$\lvert \vec{a}\times\vec{b} \rvert = \lvert \vec{a} \rvert + \lvert \vec{b} \rvert$		

Codes

	A	B	C	D	E
(a)	2	2	2	1	1
(b)	1	2	1	2	1
(c)	1	2	1	1	1
(d)	2	1	2	1	2

79. In the following table $\vec{a} \neq \vec{0}, \vec{b} = \vec{0}$ and l_1, m_1, n_1 and l_2, m_2, n_2 are their d.c. and a_1, b_1, c_1 and a_2, b_2, c_2 are their d. r.s respectively.

	Column-I		**Column-II**
A.	$\vec{a}\times\vec{b} = \vec{0}$	1.	$\vec{a}$ and $\vec{b}$ are collinear
B.	$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$ where l_1, m_1, n_1 and l_2, m_2, n_2 are d.c.s. of two vectors	2.	$\vec{a}$ and $\vec{b}$ are perpendiculars
C.	$l_1 = l_2, m_1 = m_2, n_1 = n_2$	3.	The angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$
D.	$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$		
E.	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$		
F.	$\vec{a}.\vec{b} = \frac{\lvert \vec{a} \rvert \lvert \vec{b} \rvert}{2}$		

Codes

	A	B	C	D	E	F
(a)	2	1	2	2	1	3
(b)	3	1	2	1	2	1
(c)	1	3	2	1	1	2
(d)	1	2	1	2	1	3

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

80. If ABCDEF is a regular hexagon and

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF} = n\overrightarrow{AD}$. Then n is

(a) 1 (b) 2 (c) 3 (d) $\frac{5}{2}$

81. Two forces whose magnitudes are 2 gm wt, and 3 gm wt act on a particle in the directions of the vectors $2\hat{i}+4\hat{j}+4\hat{k}$ and $4\hat{i}+4\hat{j}+2\hat{k}$ resepectively. If the particle is displaced from the origin to the point (1, 2, 2), the work done is (the unit of length being 1 cm) :

(a) 6 gm-cm (b) 4 gm-cm
(c) 5 gm-cm (d) None of these

82. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b},\ 2\vec{b}-\vec{c},\ 2\vec{c}-\vec{a}\right]=$

(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$

83. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then l + m is equal to

(a) 0 (b) 1
(c) 2 (d) 3

84. Area of rectangle having vertices A, B, C and D with position vector $\left(-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right)$, $\left(\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right)$, $\left(\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}\right)$ and $\left(-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}\right)$ is

(a) $\frac{1}{2}$ sq. units (b) 1 sq. units
(c) 2 sq units (d) 4 sq. units.

85. ABCDEF is a regular hexagon with centre at origin such that AD + EB + FC = λED, then λ is equal to

(a) 2 (b) 4
(c) 6 (d) 3

86. If the scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one then the value of λ is

(a) 0 (b) – 1
(c) $\frac{1}{2}$ (d) 1

87. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$.

If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$, then $|\vec{w}.\hat{n}|$ is equal to

(a) 3 (b) 0
(c) 1 (d) 2

88. For what value of m, are the points with position vector $10\hat{i}+3\hat{j}, 12\hat{i}-5\hat{j}$ and $m\hat{i}+11\hat{j}$ collinear ?

(a) – 8 (b) 8
(c) 4 (d) – 4

89. If G is the centroid of triangle ABC, then the value of $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$ is

(a) $\frac{1}{2}\left(\overrightarrow{GB}+\overrightarrow{GC}\right)$ (b) 0
(c) $\frac{1}{2}\left(\overrightarrow{GB}-\overrightarrow{GC}\right)$ (d) None of these

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

90. Assertion : In ΔABC, $\overline{AB}+\overline{BC}+\overline{CA}=0$.

Reason : If $\overline{OA}=\bar{a}$, $\overline{OB}=\bar{b}$, then $\overline{AB}=\bar{a}+\bar{b}$ (triangle law of addition)

91. Assertion : If I is the incentre of ΔABC, then $|\overline{BC}|\,\overline{IA}+|\overline{CA}|\,\overline{IB}+|\overline{AB}|\,\overline{IC}=0$.

Reason : The position vector of centroid of ΔABC is $\frac{\overline{OA}+\overline{OB}+\overline{OC}}{3}$.

92. Assertion : $\bar{a}=i+pj+2k$ and $\bar{b}=2i+3j+qk$ are parallel vectors if $p=\frac{3}{2}, q=4$

Reason: If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are parallel $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

93. Assertion : If the point $\vec{P}=(\vec{a}+\vec{b}-\vec{c})$, $\vec{Q}=(2\vec{a}+\vec{b})$ and $\vec{R}=(\vec{b}+t\vec{c})$ are collinear, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vectors, then the value of t is –2.

Reason : If P, Q, R are collinear, then $\overline{PQ}\,||\,\overline{PR}$ or $\overline{PQ}=\lambda\overline{PR}$, $\lambda\in R$

94. Assertion : The adjacent sides of a parallelogram are along $\vec{a}=\hat{i}+2\hat{j}$ and $\vec{b}=2\hat{i}+\hat{j}$. The angle between the diagonal is 150°.

Reason : Two vectors are perpendicular to each other if their dot product is zero.

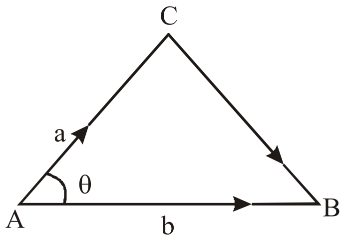
95. Assertion : If $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|=9$.

Reason : If $\vec{a}$ and $\vec{b}$ are any two vectors, then $(\vec{a}\times\vec{b})^2$ is equal to $(\vec{a})^2(\vec{b})^2-(\vec{a}.\vec{b})^2$

96. Assertion : The projection of the vector $a=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ is $\frac{5}{3}\sqrt{6}$.

Reason The projection of vector a on vector b is $\frac{1}{|a|}(a.b)$.

97. Consider the shown figure.



Assertion : If a and b represent the adjacent sides of a triangle as shown, then its area is $\frac{1}{2}|a \times b|$

Reason : Area of $\Delta ABC = \frac{1}{2}|b||a|\sin\theta$ where, θ is the angle between the adjacent sides a and b (as shown).

98. **Assertion :** For any three vectors a, b and c,

$[a \ \ b \ \ c] = [b \ \ c \ \ a] = [c \ \ a \ \ b]$

Reason : Cyclic permutation of three vectors does not change the value of the scalar triple product.

99. **Assertion :** Let $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$ then OABC is a tetrahedron.

Reason : Let $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar then OABC is a tetrahedron, where O is the origin.

CRITICAL THINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

100. The resultant moment of three forces $\hat{i} + 2\hat{j} - 3\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $-\hat{i} - \hat{j} + \hat{k}$ acting on a particle at a point P (0, 1, 2) about the point A (1, –2, 0) is

(a) $6\sqrt{2}$ (b) $\sqrt{140}$ (c) $\sqrt{21}$ (d) None

101. The two vectors $(x^2 - 1)\hat{i} + (x + 2)\hat{j} + x^2\hat{k}$ and $2\hat{i} - x\hat{j} + 3\hat{k}$ are orthogonal

(a) for no real value of x (b) for x = –1

(c) for $x = \frac{1}{2}$ (d) for $x = -\frac{1}{2}$ and x = 1

102. Let there be two points A, B on the curve $y = x^2$ in the plane OXY satisfying $\overrightarrow{OA} \cdot \hat{i} = 1$ and $\overrightarrow{OB} \cdot \hat{i} = -2$, then the length of the vector $2\overrightarrow{OA} - 3\overrightarrow{OB}$ is

(a) $\sqrt{14}$ (b) $2\sqrt{51}$ (c) $3\sqrt{41}$ (d) $2\sqrt{41}$

103. The acute angle between the medians drawn through the acute angle of an isosceles right angled triangle is

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{3}{4}\right)$

(c) $\cos^{-1}\left(\frac{4}{5}\right)$ (d) $\cos^{-1}\left(\frac{5}{6}\right)$

104. The acute angle that the vector $2\hat{i} - 2\hat{j} + \hat{k}$ makes with the plane contained by the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is given by

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

(c) $\tan^{-1}\left(\sqrt{2}\right)$ (d) $\cot^{-1}(\sqrt{2})$

105. If $\vec{b}$ and $\vec{c}$ are any two non-collinear mutually perpendicular unit vectors and $\vec{a}$ is any vector, then

$(\vec{a} \cdot \vec{b})\vec{b} + (\vec{a} \cdot \vec{c})\vec{c} + \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|^2}(\vec{b} \times \vec{c})$ is equal to :

(a) $\vec{a}$ (b) $2\vec{a}$ (c) $3\vec{a}$ (d) None

106. The three vectors $\hat{i} + \hat{j}$, $\hat{j} + \hat{k}$, $\hat{k} + \hat{i}$ taken two at a time form three planes. The three unit vectors drawn perpendicular to these three planes form a parallelopiped of volume :

(a) $\frac{1}{3}$ (b) 4 (c) $\frac{3\sqrt{3}}{4}$ (d) $\frac{4}{3\sqrt{3}}$

107. If $\vec{\alpha} = x(\vec{a} \times \vec{b}) + y(\vec{b} \times \vec{c}) + z(\vec{c} \times \vec{a})$ and $[\vec{a}\,\vec{b}\,\vec{c}] = \frac{1}{8}$, then $x + y + z =$

(a) $8\vec{\alpha} \cdot (\vec{a} + \vec{b} + \vec{c})$ (b) $\vec{\alpha} \cdot (\vec{a} + \vec{b} + \vec{c})$

(c) $8(\vec{a} + \vec{b} + \vec{c})$ (d) None of these.

108. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are the position vectors of points A, B, C and D respectively such that $(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) = (\vec{b} - \vec{d}) \cdot (\vec{c} - \vec{a}) = 0$, then D is the

(a) centroid of Δ ABC (b) circumcentre of Δ ABC

(c) orthocentre of Δ ABC (d) None of these

109. The angle between any two diagonal of a cube is

(a) 45° (b) 60°

(c) 30° (d) $\tan^{-1}(2\sqrt{2})$

110. A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$, which is displaced from the point $\hat{i} + 2\hat{j} + \hat{k}$ to the point $5\hat{i} + 4\hat{j} - \hat{k}$. The total work done by the forces is

(a) 50 units (b) 20 units

(c) 30 units (d) 40 units

111. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ & $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(a) $\sqrt{288}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

112. If position vector of a point A is $\vec{a} + 2\vec{b}$ and any point $P(\vec{a})$ divides $\overline{AB}$ in the ratio of 2 : 3, then position vector of B is

(a) $2\vec{a} - \vec{b}$ (b) $\vec{b} - 2\vec{a}$
(c) $\vec{a} - 3\vec{b}$ (d) $\vec{b}$

113. If $\vec{a}$ is any vector, then $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k})$ is equal to

(a) $\vec{a}$ (b) $2\vec{a}$
(c) $3\vec{a}$ (d) 0

114. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are perpendicular to $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$ and $\vec{a} + \vec{b}$ respectively and if $|\vec{a} + \vec{b}| = 6$, $|\vec{b} + \vec{c}| = 8$ and $|\vec{c} + \vec{a}| = 10$, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

(a) $5\sqrt{2}$ (b) 50
(c) $10\sqrt{2}$ (d) 10

115. If unit vector $\vec{c}$ makes an angle $\frac{\pi}{3}$ with $\hat{i} + \hat{j}$, then minimum and maximum values of $(\hat{i} \times \hat{j}).\vec{c}$ respectively are

(a) $0, \frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$
(c) $-1, \frac{\sqrt{3}}{2}$ (d) None of these

116. The dot product of a vector with the vectors $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.

(a) $\hat{i} + 2\hat{j} + \hat{k}$ (b) $-\hat{i} + 3\hat{j} - 2\hat{k}$
(c) $\hat{i} + 2\hat{j} + 3\hat{k}$ (d) $\hat{i} - 3\hat{j} - 3\hat{k}$

117. The moment about the point $\hat{i} + 2\hat{j} + 3\hat{k}$ of a force represented by $\hat{i} + \hat{j} + \hat{k}$ acting through the point 2i + 3j + k is

(a) $3\hat{i} + 3\hat{j}$ (b) $3\hat{i} + \hat{j}$
(c) $-\hat{i} + \hat{j}$ (d) $3\hat{i} - 3\hat{j}$

118. A girls walks 4 km towards West. Then, she walks 3 km in a direction 30° East to North and stops. The girls displacement from her initial point of departure is

(a) $-\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$ (b) $-\frac{5}{2}\hat{i} + \frac{3}{2}\hat{j}$
(c) $-\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$ (d) None of these

119. In a parallelogram ABCD, $|\overrightarrow{AB}| = a, |\overrightarrow{AD}| = b$ and $|\overrightarrow{AC}| = c$, the value of $\overline{DB}.\overline{AB}$ is

(a) $\frac{3a^2 + b^2 - c^2}{2}$ (b) $\frac{a^2 + 3b^2 - c^2}{2}$
(c) $\frac{a^2 - b^2 + 3c^2}{2}$ (d) $\frac{a^2 + 3b^2 + c^2}{2}$

120. $|(a \times b).c| = |a|\,|b||c|$, if

(a) a.b = b. c = 0 (b) b.c = c. a = 0
(c) c.a = a.b = 0 (d) a.b = b.c = c.a = 0

121. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{b}$ is not parallel to $\vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, then the angle between $\vec{a}$ and $\vec{c}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

122. If $\vec{a} = \hat{i} + \hat{j}, \hat{b} = 2\hat{j} - \hat{k}$ and $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$, $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$, then what is the value of $\frac{\vec{r}}{|\vec{r}|}$?

(a) $\frac{(\hat{i} + 3\hat{j} - \hat{k})}{\sqrt{11}}$ (b) $\frac{(\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{11}}$
(c) $\frac{(\hat{i} + 3\hat{j} + \hat{k})}{\sqrt{11}}$ (d) $\frac{(\hat{i} - 3\hat{j} - \hat{k})}{\sqrt{11}}$

123. A vector perpendicular to the plane containing the vectors $\hat{i} - 2\hat{j} - \hat{k}$ and $3\hat{i} - 2\hat{j} - \hat{k}$ is inclined to the vector $\hat{i} + \hat{j} + \hat{k}$ at an angle

(a) $\tan^{-1}\sqrt{14}$ (b) $\sec^{-1}\sqrt{14}$
(c) $\tan^{-1}\sqrt{15}$ (d) None of these

124. The altitude through vertex C of a triangle ABC, with position vectors of vertices $\vec{a}, \vec{b}, \vec{c}$ respectively is :

(a) $\frac{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|}{|\vec{b} - \vec{a}|}$ (b) $\frac{|\vec{a} + \vec{b} + \vec{c}|}{|\vec{b} - \vec{a}|}$
(c) $\frac{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|}{|\vec{b} \times \vec{a}|}$ (d) None of these

125. If $\vec{p} = \lambda(\vec{u} \times \vec{v}) + \mu(\vec{v} \times \vec{w}) + \nu(\vec{w} \times \vec{u})$ and $[\vec{u}\ \vec{v}\ \vec{w}] = \frac{1}{5}$, then $\lambda + \mu + \nu$ is equal to

(a) 5 (b) 10
(c) 15 (d) None of these

126. Let $\vec{A} = 2\hat{i} + \hat{k}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$. The vector $\vec{R}$ which satisfies the equations $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} . \vec{A} = 0$ is given by

(a) $-2\hat{i} + \hat{k}$ (b) $-\hat{i} - 8\hat{j} + 2\hat{k}$
(c) $\frac{1}{\sqrt{6}}(\hat{i} - \hat{j} + 2\hat{k})$ (d) None of these

127. Force $\vec{i} + 2\vec{j} - 3k, 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $-\vec{i} - \vec{j} + \vec{k}$ are acting at the point P (0, 1, 2). The moment of these forces about the point A (1, – 2, 0) is

(a) $2\vec{i} - 6\vec{j} + 10\vec{k}$ (b) $-2\vec{i} + 6\vec{j} - 10\vec{k}$
(c) $2\vec{i} + 6\vec{j} - 10\vec{k}$ (d) None of these

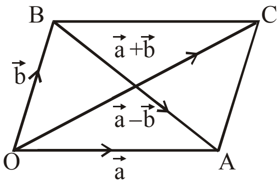
128. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a)



Let $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$. Complete the parallelogram OACB.

$\vec{a} + \vec{b} = \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC} \Rightarrow |\vec{a} + \vec{b}| = OC$

Again $\vec{a} - \vec{b} = \overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{BA} \Rightarrow |\vec{a} - \vec{b}| = BA$

Given $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \Rightarrow OC = BA$

$\therefore$ Diagonals of the parallelogram OACB are equal.
$\therefore$ OACB is a rectangle.

$\therefore$ $\vec{a}$ and $\vec{b}$ are adjacent sides of a rectangle.

2. (c) We have $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$. It is given that

$\vec{a} \perp (\vec{b} + \vec{c}), \ \vec{b} \perp (\vec{c} + \vec{a})$ and $\vec{c} \perp (\vec{a} + \vec{b})$

$\Rightarrow \vec{a}.(\vec{b} + \vec{c}) = 0, \ \vec{b}.(\vec{c} + \vec{a}) = 0, \ \vec{c}.(\vec{a} + \vec{b}) = 0$

$\Rightarrow \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0, \ \vec{b}.\vec{c} + \vec{b}.\vec{a} = 0, \ \vec{c}.\vec{a} + \vec{c}.\vec{b} = 0$

Adding all these, we get, $2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$

Now, $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$= 3^2 + 4^2 + 5^2 + 0 = 50$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{50} = 5\sqrt{2}$.

3. (d) Let O be the origin. Then $\overrightarrow{OA} = 3\hat{i} - \hat{j} + 2\hat{k}$,

$\overrightarrow{OB} = \hat{i} - \hat{j} - 3\hat{k}$ and $\overrightarrow{OC} = 4\hat{i} - 3\hat{j} + \hat{k}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (\hat{i} - \hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 0\hat{j} - 5\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (4\hat{i} - 3\hat{j} + \hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = \hat{i} - 2\hat{j} - \hat{k}$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$

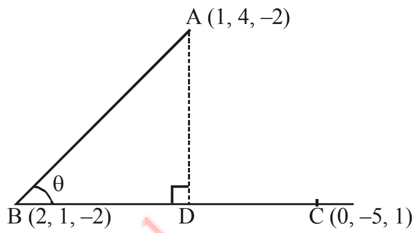
$= (0 - 10)\hat{i} - (2 + 5)\hat{j} + (4 - 0)\hat{k} = -10\hat{i} - 7\hat{j} + 4\hat{k}$

$\Rightarrow |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{100 + 49 + 16} = \sqrt{165}$

A unit vector perpendicular to the plane of ΔABC is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Hence, a unit vector perpendicular, to the plane of

$\Delta ABC = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|} = \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}} = -\frac{1}{\sqrt{165}}(10\hat{i} + 7\hat{j} - 4\hat{k})$.

4. (c)



$AD = AB \sin\theta = AB.\frac{|\overrightarrow{BC} \times \overrightarrow{BA}|}{|\overrightarrow{BC}|.|\overrightarrow{BA}|} = \frac{|\overrightarrow{BC} \times \overrightarrow{BA}|}{|\overrightarrow{BC}|}$

$[\because |\overrightarrow{BA}| = BA = AB]$

Now $\overrightarrow{BC} = -2\hat{i} - 6\hat{j} + 3\hat{k}$ and $\overrightarrow{BA} = -\hat{i} + 3\hat{j}$

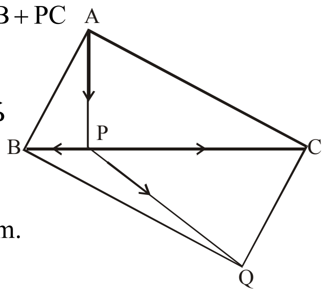
$\therefore \overrightarrow{BC} \times \overrightarrow{BA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -6 & 3 \\ -1 & 3 & 0 \end{vmatrix} = -9\hat{i} - 3\hat{j} - 12\hat{k}$

$|\overrightarrow{BC} \times \overrightarrow{BA}| = \sqrt{9^2 + 3^2 + (12)^2} = 3\sqrt{26}$ and

$|\overrightarrow{BC}| = \sqrt{4 + 36 + 9} = 7$

$\therefore AD = \frac{3\sqrt{26}}{7}$

5. (b) We have $\overrightarrow{PQ} = \overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PC}$



$\Rightarrow \overrightarrow{PQ} = \overrightarrow{AB} + \overrightarrow{PC}$

$\Rightarrow \overrightarrow{AB} = \overrightarrow{PQ} - \overrightarrow{PC} = \overrightarrow{PQ} + \overrightarrow{CP}$

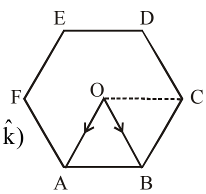
$= \overrightarrow{CP} + \overrightarrow{PQ} = \overrightarrow{CQ}$.

$\therefore AB = CQ$ and $AB \| CQ$

$\therefore$ ABQC is a parallelogram.

$\therefore$ Q is a fixed point.

6. (b) $\overrightarrow{OA} = \hat{i} - \hat{j} + 2\hat{k}$, $\overrightarrow{OB} = 2\hat{i} + \hat{j} - \hat{k}$



$\therefore \overrightarrow{OC} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\therefore \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (2\hat{i} + \hat{j} - \hat{k})$

$= -\hat{i} + \hat{j} - 2\hat{k}$

7. (a) Equating the components in

$\alpha(\hat{i} + 2\hat{j} + 3\hat{k}) + \beta(2\hat{i} + 3\hat{j} + \hat{k}) + \gamma(3\hat{i} + \hat{j} + 2\hat{k})$

$= -3(\hat{i}-\hat{k})$, we have

$\alpha+2\beta+3\gamma=-3$ ....(i) $\qquad 2\alpha+3\beta+\gamma=0$ ....(ii)

$3\alpha+\beta+2\gamma=3$ ....(iii)

Solving the equations (i), (ii), & (iii), we get

$\alpha=2,\ \beta=-1,\ \gamma=-1$.

8. (b) Let A (1, 0, 1), B(0, 2, 2) and C (3, 3, 0) be the given points,

then $\overrightarrow{AB}=-\hat{i}+2\hat{j}+\hat{k}$, $\overrightarrow{BC}=3\hat{i}+\hat{j}-2\hat{k}$

$$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 1\\ 3 & 1 & -2\end{vmatrix}=-5\hat{i}+\hat{j}-7\hat{k}$$

$\therefore$ unit vector $\perp$ to the plane

$$ABC=\pm\frac{1}{5\sqrt{3}}(-5\hat{i}+\hat{j}-7\hat{k})$$

9. (a) We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}\Rightarrow(\vec{a}+\vec{b}+\vec{c})^2=0$

$\Rightarrow|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow 25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-25$

$\therefore|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|=25$

10. (b) We have $\vec{a}\times\vec{b}=39\vec{k}=\vec{c}$

Also $|\vec{a}|=\sqrt{34}, |\vec{b}|=\sqrt{45}, |\vec{c}|=39$;

$\therefore|\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39$

11. (c) $\vec{a}+\vec{b}+\vec{c}=0\Rightarrow(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\frac{-1-4-9}{2}=-7$$

12. (d) $A=(7,-4,7), B=(1,-6,10), C=(-1,-3,4)$ and $D=(5,-1,5)$

$$AB=\sqrt{(7-1)^2+(-4+6)^2+(7-10)^2}=\sqrt{36+4+9}=7$$

Similarly, $BC=7,\ CD=\sqrt{41},\ DA=\sqrt{17}$

13. (d) If $\vec{a},\vec{b},\vec{c}$ are linearly dependent vectors, then $\vec{c}$ should be a linear combination of $\vec{a}$ and $\vec{b}$.

Let $\vec{c}=p\vec{a}+q\vec{b}$

i.e. $\hat{i}+\alpha\hat{j}+\beta\hat{k}=p(\hat{i}+\hat{j}+\hat{k})+q(4\hat{i}+3\hat{j}+4\hat{k})$

Equating coefficients of $\hat{i},\hat{j},\hat{k}$ we get

$1=p+4q,\ \alpha=p+3q,\ \beta=p+4q$

from first and third, $\beta=1$

Now, $|\vec{c}|=\sqrt{3}$

$\therefore 1+\alpha^2+\beta^2=3$

$\Rightarrow 1+\alpha^2+1=3$

$\Rightarrow \alpha=\pm1$ $\qquad$ [Using $\beta=1$]

Hence, $\alpha=\pm1$, $\beta=1$

14. (c) Let A,B and C be three points whose coordinates are (2, – 1, 3),(3, –5, 1) and (–1, 11, 9) respectively, then

$\overrightarrow{OA}=2\hat{i}-\hat{j}+3\hat{k}$, $\overrightarrow{OB}=3\hat{i}-5\hat{j}+\hat{k}$ and

$\overrightarrow{OC}=-\hat{i}+11\hat{j}+9\hat{k}$

$\therefore\ \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(3\hat{i}-5\hat{j}+\hat{k})-(2\hat{i}-\hat{j}+3\hat{k})$

$=\hat{i}-4\hat{j}-2\hat{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(-\hat{i}+11\hat{j}+9\hat{k})-(2\hat{i}-\hat{j}+3\hat{k})$

$=-3\hat{i}+12\hat{j}+6\hat{k}$

$\Rightarrow\ \overrightarrow{AC}=-3\overrightarrow{AB}$

Thus, the vector $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel having the same initial point A.

Hence, the point, A, B C are collinear.

15. (a) Given that $\overrightarrow{OA}=\hat{i}+x\hat{j}+3\hat{k}$

$\overrightarrow{OB}=3\hat{i}+4\hat{j}+7\hat{k}$ and $\overrightarrow{OC}=y\hat{i}-2\hat{j}-5\hat{k}$

Since, A, B, C are collinear, Then, $\overrightarrow{AB}=\lambda\overrightarrow{BC}$

$\Rightarrow\ 2\hat{i}+(4-x)\hat{j}+4\hat{k}=\lambda\left[(y-3)\hat{i}-6\hat{j}-12\hat{k}\right]$

On comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$2=(y-3)\lambda$ ....(i)

$4-x=-6\lambda$ ....(ii)

and $4=-12\lambda\Rightarrow\lambda=-\dfrac{1}{3}$ ....(iii)

On putting the value of λ in eqs. (i) and (ii), we get

$y=-3$ and $x=2$

16. (d) Since, $\vec{a}$ and $\vec{b}$ are collinear.

$\therefore\ \vec{a}=\lambda\vec{b}$

$\Rightarrow(x\hat{i}-2\hat{j}+5\hat{k})=\lambda(\hat{i}+y\hat{j}-z\hat{k})$

On comparing

$x=\lambda,\ -2=\lambda y$ and $5=-\lambda z$

For $\lambda=1$

$x=1,\ y=-2$ and $z=-5$

For $\lambda=1/2$

$x=1/2,\ y=-4$ and $z=-10$

For $\lambda=-1/2$

$x=-1/2,\ y=4$ and $z=-10$

all options are correct

17. (c) Given that, $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=4$

$$\therefore\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=\vec{a}.\left(|\vec{b}||\vec{c}|\sin\frac{2\pi}{3}\hat{n}\right)$$

$$=|\vec{a}||\vec{b}||\vec{c}|\left(\sin\frac{2\pi}{3}\right)\qquad\left[\because\vec{a}.\hat{n}=|\vec{a}||\hat{n}|\cos0^\circ=|\vec{a}|\right]$$

$$=2\times3\times4\times\frac{\sqrt{3}}{2}=12\sqrt{3}$$

18. (a) Given that $\vec{a} = (1,1,4) = \hat{i} + \hat{j} + 4\hat{k}$

and $\vec{b} = (1,-1,4) = \hat{i} - \hat{j} + 4\hat{k}$

$\therefore\ \vec{a} + \vec{b} = 2\hat{i} + 8\hat{k} \Rightarrow \vec{a} - \vec{b} = 2\hat{j}$

Let θ be the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, then

$$\cos\theta = \frac{(\vec{a}+\vec{b})(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|} = \frac{(2\hat{i}+0\hat{j}+8\hat{k}).(0\hat{i}+2\hat{j}+0\hat{k})}{\sqrt{2^2+0^2+8^2}\sqrt{0^2+2^2+0^2}}$$

$$= \frac{0+0+0}{\sqrt{4+64}\sqrt{4}} = 0$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} = 90°$$

19. (a) Since, $(\vec{a}\times\vec{b})^2 + (\vec{a}.\vec{b})^2 = 676$

$$\Rightarrow (|\vec{a}|.|\vec{b}|\sin\theta\hat{n})^2 + (|\vec{a}|.|\vec{b}|\cos\theta)^2 = 676$$

$$\Rightarrow |\vec{a}|^2.|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 676$$

$$\Rightarrow |\vec{a}|^2(2)^2 = 676 \Rightarrow |\vec{a}|^2 = 169 \Rightarrow |\vec{a}| = 13$$

20. (a) $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Now, $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix} = 1[(1 + x) - x] = 1$$

21. (a) Area of ΔABC = $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \frac{1}{2}|2\hat{i} - 4\hat{j} + 4\hat{k}|$

$$= \frac{1}{2}\left[\sqrt{(2)^2 + (-4)^2 + (4)^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{4+16+16}\right] = \frac{1}{2}\left[\sqrt{36}\right]$$

$$= \frac{1}{2}(6) = 3 \text{ sq. units}$$

22. (d) Since, $(\vec{a}\times\hat{i}).(\vec{a}\times\hat{i}) = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\hat{i} \\ \hat{i}.\vec{a} & 1 \end{vmatrix} = |\vec{a}|^2 - a_1^2$

Similarly, $(\vec{a}\times\hat{j})^2 = |\vec{a}|^2 - a_2^2$

$(\vec{a}\times\hat{k})^2 = |\vec{a}|^2 - a_3^2$

$$\therefore\ (\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$$

$$= 3|\vec{a}|^2 - (a_1^2 + a_2^2 + a_3^2)$$

$$= 3|\vec{a}|^2 - |\vec{a}|^2 = 2\vec{a}^2$$

23. (d) $|\vec{a}| = 10$, $|\vec{b}| = 2$, $\vec{a}.\vec{b} = 12$

We know, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$12 = 10 \times 2\cos\theta \Rightarrow \cos\theta = \frac{3}{5}$

$\therefore\ \sin\theta = \frac{4}{5}$

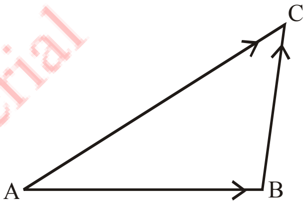
Now, $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 10\times2\times\frac{4}{5} = 16$

24. (c) Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{a}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$\therefore$ Required vector = $\frac{9(\hat{i}-2\hat{j}+2\hat{k})}{3} = 3(\hat{i}-2\hat{j}+2\hat{k})$

25. (c) By triangle law of vector addition $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



or $\overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$ or $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.

26. (d) $\vec{a}$ is a non-zero, vector of magnitude a

$\Rightarrow\ |\vec{a}| = a$

Now, $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$ or $|\lambda||\vec{a}| = 1$

or $|\lambda|\, a = 1 \Rightarrow a = \frac{1}{|\lambda|}$

27. (b) Direction is not determined.

28. (d) Since, no vector given in options in collinear with the given vectors. Therefore all vectors can be third vertex of the triangle.

29. (b) Since, $\vec{a}+\vec{b}+\vec{c} = 0$ and $\vec{a},\vec{b},\vec{c}$ are unit vectors, therefore $\vec{a},\vec{b},\vec{c}$ form an equilateral triangle.

$\Rightarrow\ \vec{a}\times(\vec{a}+\vec{b}+\vec{c}) = \vec{0}$

$\Rightarrow\ \vec{a}\times\vec{a} + \vec{a}\times\vec{b} + \vec{a}\times\vec{c} = \vec{0}$

$\Rightarrow\ \vec{a}\times\vec{b} = \vec{c}\times\vec{a}$

Similarly, $\vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

$\therefore\ \vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

Also since $\vec{a},\vec{b},\vec{c}$ are non-parallel (these form an equilateral Δ).

$\therefore\ \vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} \neq \vec{0}$

30. (c) A quantity that has magnitude as well as direction is called a vector.

A directed line segment is a vector denoted as AB or simply as a and read as 'vector AB' or vector a.

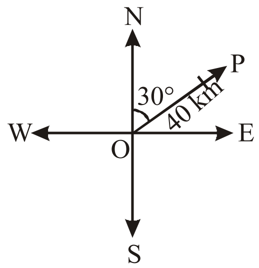
The point A from where the vector AB starts is called its initial point and the point B where it ends is called its terminal point. The distance between initial point and terminal points of a vector is called the magnitude (or length) of the vector, denoted as |AB|, or |a|, or a.

31. (c) Two or more vectors having the same initial point are called coinitial vectors.

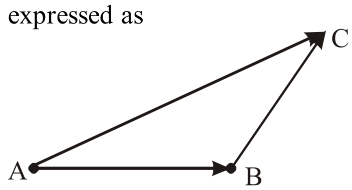
32. (a) Two vectors a and b are said to be equal vectors if they have the same magnitude and direction regardless of the positions of their initial points and written as a = b.

33. (a) A vector whose magnitude is the same as that of a given vector (say, AB), but direction is opposite to that of it,is called negative of the given vector.
e.g.,vector BA is negative of the vector AB, and written as BA= – AB

34. (c) The displacement is 30° east of North so, we have to draw a straight line making 30° with north.
Here, vector OP represents the displacement of 40 km, 30° East of North.

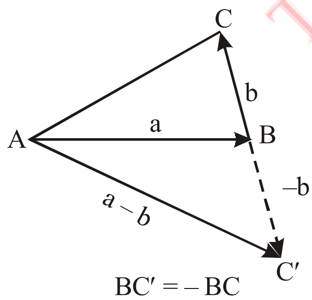


35. (c) If a girl moves from A to B and then from B to C(see the fig.). The net displacement made by the girl from point A to the point C, is given by vector AC and expressed as



this is known as the triangle law of vector addition.

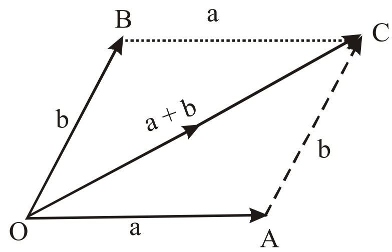
36. (b) A vector BC′ is given such that its magnitude is same as the vector BC, but its direction is opposite to that of it.



BC′ = – BC

Then, on applying triangle law from the figure, we have
AC′ = AB + BC′ = AB + (–BC) = a – b
The vector AC′ is said to represent the difference of a and b.

37. (a) If we have two vectors a and b represented by the two adjacent sides of a parallelogram in magnitude and direction then their sum a + b is represented in magnitude and direction by the diagonal of the parallelogram through their common point. This is known as the parallelogram law of vector addition.



38. (b) In case if it is given that l, m, n are direction cosines of a vector, then
$l\hat{i}+m\hat{j}+n\hat{k}=(\cos\alpha)\hat{i}+(\cos\beta)\hat{j}+(\cos\gamma)\hat{k}$ is the unit vector in the direction of that vector where α, β and γ are the angles which the vector makes with X, Y and Z-axes, respectively.

39. (a) Two vectors can have same magnitude, if the sum of the squares of coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$ is same. The vectors $a=(2\hat{i}+3\hat{j}+\hat{k})$ and $b=(2\hat{i}+3\hat{j}-\hat{k})$ are different vectors having the same magnitude.
Magnitude of Ist vector
$=\sqrt{(2)^2+(3)^2+(1)^2}=\sqrt{4+9+1}+\sqrt{14}$
and magnitude of IInd vector = $\sqrt{(2)^2+(1)^2+(-3)^2}$
$=\sqrt{4+1+9}=\sqrt{14}$
i.e., they have same magnitude.

40. (c) We know that the diagonal of a parallelogram bisect each other. Therefore, M is the mid-point of AC and BD both.
∴ OA + OC = 2 OM
and OB + OD = 2 OM
⇒ OA +OB + OC + OD = 4 OM

41. (a) Since a = mb for some scalar m i.e.,
$a=m\left(6\hat{i}-8\hat{j}-\frac{15}{2}\hat{k}\right)\Rightarrow |a|=|m|\sqrt{36+64+\frac{225}{4}}$
$\Rightarrow 50=\frac{25}{2}|m|$
$\Rightarrow |m|=4 \Rightarrow m=\pm 4$
Since, a makes an acute angle with the positive direction of Z-axis, so its z component must be positive and hence, m must be – 4.
$\therefore\ a=-4\left(6\hat{i}+8\hat{j}-\frac{15}{2}\hat{k}\right)=-24\hat{i}+32\hat{j}+30\hat{k}$

42. (b) R = AB + AE + BC + DC + ED + AC
= (AB + BC) + (AE + ED + DC) + AC
= AC + AC + AC
= 3AC

43. (a) Since, a = 8b and c = – 7 b
a is parallel to b and c is anti – parallel to b.
⇒ a and c are anti-parallel.
⇒ Angle between a and c is π.

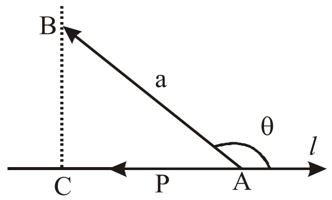
44. (d) Since $\hat{i}, \hat{j}$ and $\hat{k}$ are mutually perpendicular unit vectors, so we have

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

and $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

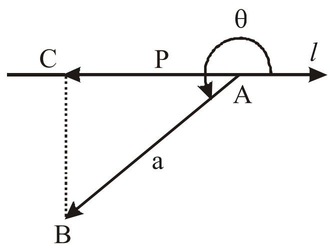
45. (c) Multiplication of two vectors is defined in two ways, namely, scalar (or dot) product where the result is a scalar, and vector (or cross) product where the result is a vector. Based upon these two types of products for vectors, they have found various applications in geometry, mechanics and engineering.

46. (d)



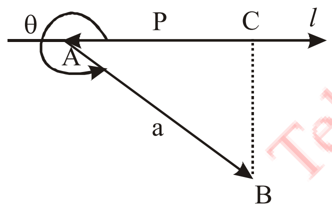
(90° < θ < 180°)

(i)



(180° < θ < 270°)

(ii)



270° < θ < 360°

(iii)

For example, in each of the above figures (figure) (i) to (iii), projection vector of AB along the line l is vector AC.

47. (c) Let the required unit vector be $r = a\hat{i} + b\hat{j}$.

Then, $|r| = 1 \Rightarrow a^2 + b^2 = 1$

Since, r makes an angle of 45° with $\hat{i} + \hat{j}$ and an angle of 60° with $3\hat{i} - 4\hat{j}$, therefore

$$\cos\frac{\pi}{4} = \frac{r.(\hat{i}+\hat{j})}{|r|(\hat{i}+\hat{j})}$$

and $$\cos\frac{\pi}{3} = \frac{r.(3\hat{i}-4\hat{j})}{|r|(3\hat{i}-4\hat{j})}$$

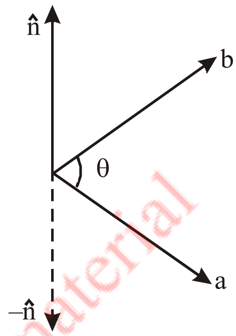
$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{a+b}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{3a-4b}{5}$$

$$\Rightarrow a + b = 1 \text{ and } a - 4b = \frac{5}{2}$$

$$\Rightarrow a = \frac{13}{14},\ b = \frac{1}{14} \qquad \therefore\ r = \frac{13}{14}\hat{i} + \frac{1}{14}\hat{j}$$

48. (b) The vector product of two non-zero vectors a and b, is denoted by a,b and defined as

$a \times b = |a||b| \sin\theta\hat{n}$



where θ is angle between a and b, $0 \le \theta \le \pi$ and $\hat{n}$ is a unit vector perpendicular to both a and b,such that a,b and $\hat{n}$ form a right handed system i.e., the right handed system rotated from a to b moves in the direction of $\hat{n}$.

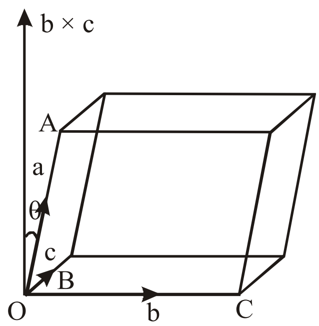
49. (c) We know that, A × B = – B × A

$\hat{i} \times \hat{j} = \hat{k}$ then $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{i} = \hat{j}$ then $\hat{i} \times \hat{k} = -\hat{j}$

$\hat{j} \times \hat{k} = \hat{i}$ then $\hat{k} \times \hat{j} = -\hat{i}$

50. (d) 1. Since dot product is a scalar quantity, a. (b × c) is a scalar quantity, i.e., $[a \quad b \quad c]$ is a scalar quantity



2. Geometrically, the magnitude of the scalar triple product is the volume of the parallelopiped, formed by adjacent sides given by the three vectors a, b and c Fig. Indeed, the area of the parallelogram forming the base of the parallelopiped is |b × c|. The height is the projection of a along the normal to the plane containing b and c which is the magnitude of the component of a in the direction

of b × c i.e., $\frac{|a.(b\times c)|}{|(b\times c)|}$. So the required volume of the parallelopiped is $\frac{|a.(b\times c)|}{|(b\times c)|}|b\times c| = |a.(b\times c)|$

51. (c) We have $a.(b\times c) = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

$= 2(4-1) + 1(3+2) + 3(-1-6)$
$= 6 + 5 - 21$
$= 11 - 21$
$= -10$

52. (d) In scalar triple product a.(b × c), the dot and cross can be interchanged. Indeed.

$a.(b\times c) = [b \ \ c \ \ a] = [c \ \ a \ \ b] = c.(a\times b) = (a\times b).c$

53. (a) (i) $[a \ \ b \ \ c] = -[a \ \ c \ \ b]$

Indeed $[a \ \ b \ \ c] = a.(b\times c)$

$= (a.(-c\times b)) = -[a \ \ c \ \ b]$

(ii) $-[a \ \ c \ \ b] = 0$

Indeed $[a \ \ a \ \ b] = [a \ \ b \ \ a]$

$= [b \ \ a \ \ a]$

$= b.(a\times a)$

$= b.0 = 0$ (as a × a = 0)

Note: The above result is true irrespective of the position of two equal vectors.

54. (d) The vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ can be obtained by

PQ = PV of Q – PV of P

$PQ = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$

$PQ = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

∴ Scalar components of PQ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

∴ Magnitude of

$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

55. (b) We have $|a.b| = |a\times b|$

$\Rightarrow |a||b|\cos\theta = |a||b|\sin\theta \Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

56. (a) Consider $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} + (\vec{a}.\vec{b})\vec{c}$

$\Rightarrow$ $\vec{a}\times(\vec{b}\times\vec{c})$ lie in the plane of $\vec{b}$ and $\vec{c}$ which is parallel to $\vec{a}$.

57. (a) Let $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

and $\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Thus $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

$= \hat{i}(y_1z_2 - y_2z_1) - \hat{j}(x_1z_2 - x_2z_1) + \hat{k}(x_1y_2 - x_2y_1)$

Thus $\vec{a}.(\vec{a}\times\vec{b})$

$= x_1y_1z_2 - x_1y_2z_1 - x_1y_1z_2 + x_2y_1z_1 + x_1y_2z_1 - x_2y_1z_1 = 0$

58. (c) Let $\vec{X} = a\hat{i} + 2\hat{j} + 3\hat{k}$ and

$\vec{Y} = -\hat{i} + 5\hat{j} + a\hat{k}$

Note: If $\vec{X}$ and $\vec{Y}$ are perpendicular to each other, then $\vec{X}.\vec{Y} = 0$

$\Rightarrow -a + 10 + 3a = 0 \Rightarrow 2a + 10 = 0$

Thus, a = – 5.

59. (c) Let $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vector

$\therefore \vec{a}.\vec{b} = 0, \vec{a}.\vec{c} = 0, \vec{b}.\vec{c} = 0$

and | a | = 1, | b | = 1, | c | = 1

Consider

$|\vec{a}+\vec{b}-\vec{c}|^2 = |a|^2 + |b|^2 + |c|^2 + 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{c}.\vec{a}$

$= 1 + 1 + 1 - 0 = 3$

$\Rightarrow |\vec{a}+\vec{b}-\vec{c}| = \sqrt{3}$

60. (b) If $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$, then

$(\vec{a}+\lambda\vec{b}).(\vec{a}-\lambda\vec{b}) = |\vec{a}+\lambda\vec{b}||\vec{a}-\lambda\vec{b}|.\cos 90°$

$\Rightarrow (\vec{a}+\lambda\vec{b}).(\vec{a}-\lambda\vec{b}) = 0$

$\Rightarrow \vec{a}.\vec{a} - \lambda.\vec{a}.\vec{b} + \lambda.\vec{b}.\vec{a} - \lambda^2.\vec{b}.\vec{b} = 0$

$\Rightarrow a^2 - \lambda^2 b^2 = 0$

$\Rightarrow \lambda^2 = \frac{a^2}{b^2} \Rightarrow \lambda^2 = \frac{3^2}{4^2}$ $(\because |\vec{a}| = 3, |\vec{b}| = 4)$

$\Rightarrow \lambda = \frac{3}{4}$.

61. (a) Since vector product is not commutative.

62. (c) Either $\vec{a}$ or $\vec{b}$ is a null vector.

63. (c) $\because (\vec{a}\times\vec{b})\times\vec{a} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\therefore (\hat{j}-\hat{k})\times(\hat{i}+\hat{j}+k) = (\sqrt{3})^2(\vec{b}) - (\hat{i}+\hat{j}+k)$

$\Rightarrow 3\vec{b} = 3\hat{i} \Rightarrow \vec{b} = \hat{i}$

64. (d) Given: $\vec{p}, \vec{q}, \vec{r}$ be three non-zero vectors, then

$\vec{p}.\vec{q} = \vec{p}.\vec{r}$

$\Rightarrow$ p. (q – r) = 0

$\Rightarrow p \perp (q - r)$

$\Rightarrow$ p = 0 or q – r = 0

$\Rightarrow$ q = r

65. (a) We have, $[\vec{a}\times\vec{b} \ \ \vec{b}\times\vec{c} \ \ \vec{c}\times\vec{a}]$

$= (\vec{a}\times\vec{b}).\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\}$

$= (\vec{a}\times\vec{b}).\{(\vec{m}\cdot\vec{a})\vec{c} - (\vec{m}\cdot\vec{c})\vec{a}\}$

(where $\vec{m} = \vec{b}\times\vec{c}$)

$= \{(\vec{a} \times \vec{b}).\vec{c}\}.\{(\vec{a} \cdot (\vec{b} \times \vec{c})\}$

$= [\vec{a}\ \vec{b}\ \vec{c}]^2 = 4^2 = 16$ ·

66. (b) Since, $\vec{p}$ and $\vec{q}$ are collinear, then

$\vec{p} = k\vec{q}$ [where k is a scalar]

$\Rightarrow$ $(x-2)\ \vec{a} + \vec{b} = k(x+1)\vec{a} - k\vec{b}$

On equating the coefficients

$x - 2 = k(x + 1)$ and $-k = 1$,

putting value of k

we get, $x - 2 = -(x + 1) \Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

67. (b) $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}\cos\theta$

$\Rightarrow |\vec{a}+\vec{b}|^2 = 1+1+2|\vec{a}||\vec{b}|\cos 30°$

$= 1 + 1 + 2 \times \frac{\sqrt{3}}{2}$

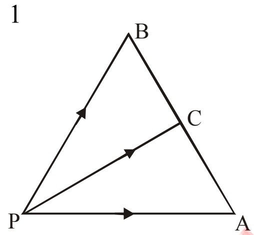
$\Rightarrow |\vec{a}+\vec{b}|^2 = 2+\sqrt{3}$

$\Rightarrow |\vec{a}+\vec{b}| = \sqrt{2+\sqrt{3}}$

$1 < \sqrt{2+\sqrt{3}} < 2$

$\Rightarrow 1 < |\vec{a}+\vec{b}| < 2$

68. (b) $\because \frac{AC}{CB} = \frac{1}{1}$



$\Rightarrow \overrightarrow{AC} = \overrightarrow{CB}$

$\Rightarrow \overrightarrow{AP} + \overrightarrow{PC} = \overrightarrow{CP} + \overrightarrow{PB}$

$\Rightarrow \overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$

69. (d) Since, P bisects both the diagonals AC and BD, so

$\therefore \overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OP}$ and $\overrightarrow{OB} + \overrightarrow{OD} = 2\overrightarrow{OP}$

$\Rightarrow \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$

70. (b) Let the position vector of the vertices A,B,C,D with respect to O are a,b,c and d respectively then

$\overrightarrow{AB} = b - a = 3\vec{i} - \vec{j} - \vec{k}$, $\overrightarrow{AC} = 4\vec{I} + 2\vec{J} + 4\vec{K}$ and

$\overrightarrow{AD} = 2\vec{I} + 2\vec{J}$

Now, volume of tetrahedron $= \frac{1}{6}\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right]$

$= \frac{1}{6}\begin{vmatrix} 3 & -1 & -1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = -6$

$\therefore$ Required volume = 6 units

71. (a) $(a \times b)^2 = |a \times b|^2 = (ab\sin\theta)^2$

$= a^2b^2\sin^2\theta = a^2b^2(1 - \cos^2\theta)$

$= a^2b^2 - (ab\cos\theta)^2 = a^2b^2 - (a.b)^2$

72. (b) Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and

$\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

If $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

i.e. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & (x-2) & -1 \end{vmatrix} = 0$

$\Rightarrow 1[1 - 2(x-2)] - 1[-1 - 2x] + 1[x - 2 + x] = 0$

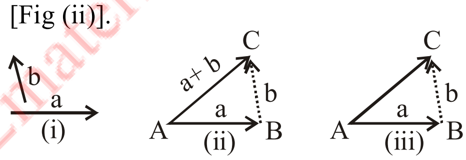
$\Rightarrow 1 - 2x + 4 + 1 + 2x + 2x - 2 = 0$

$\Rightarrow 2x = -4 \Rightarrow x = -2$

STATEMENT TYPE QUESTIONS

73. (d) A vector whose initial and terminal points coincide, is called a zero vector (or null vector) and denoted as O. Zero vector cannot be assigned a definite direction as it has zero magnitude.

74. (c) If two vectors a and b are given [Fig. (i)], then to add them, they are positioned so that the initial point of one coincides with the terminal point of the other [Fig (ii)].



For example, in Fig (ii), we have shifted vector b without changing its magnitude and direction, so that it's initial point coincides with the terminal point of a. Then, the vector a + b, represented by the third side AC of the ΔABC, gives us the sum (or resultant) of the vectors a and b i.e., in ΔABC [Fig (ii)], we have

AC = AB + BC

Now again, since **AC = – CA,** from the above equation, we have

AB + BC + CA = AA = 0

This means that when the sides of a triangle are taken in order, it leads to zero resultant as the initial and terminal points get coincides [Fig. (iii)].

75. (b) A vector with initial point (x_1, y_1) and final point (x_2, y_2) is given $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

Vector with initial point A(2, 1) and final (terminal) point B(–5, 7) can be given by

$AB = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$= (-5-2)\hat{i} + (7-1)\hat{j} = (-7)\hat{i} + 6\hat{j}$

Hence, the required scalar components (coefficients of $\hat{i}$ and $\hat{j}$) are 7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$.

76. (c) It is given that OP = 2a+ b, OQ = a – 3b

If a point divides the line joining point P and Q externally in the ratio m : n, then position vector of the point is $\frac{m(\text{PV of Q}) - n(\text{PV of P})}{m - n}$

It is given that point R divides a line segment joining two points P and Q externally in the ratio 1 : 2. Then, on using the section formula, we get

Position vector of point $R = \frac{(a-3b)\times 1-(2a+b)\times 2}{1-2}$

$= \frac{a-3b-4a-2b}{-1} = \frac{-3a-5b}{-1} = 3a + 5b$

Now, position vector of mid-point of RQ $= \frac{OQ+OR}{2}$

$= \frac{(3a+5b)+(a-3b)}{2} = \frac{4a+2b}{2} = 2a+b$

Also, the position vector of point P = 2a + b

Which shows that p is mid-point of line segment RQ.

77. (c) Three points A, B and C are collinear if $|AC| = |AB| + |BC|$

The given points are A(1, – 2, – 8), B (5, 0, – 2) and (11, 3, 7).

$AB = \text{PV of B} - \text{PV of A} = (5\hat{i}+0\hat{j}-2\hat{k}) - (\hat{i}-2\hat{j}-8\hat{k})$

$= 4\hat{i}+2\hat{j}+6\hat{k}$

$|AB| = \sqrt{4^2+2^2+6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$

$BC = \text{PV of C} - \text{PV of B} = (11\hat{i}+3\hat{j}+7\hat{k})-(5\hat{i}+0\hat{j}-2\hat{k})$

$= 6\hat{i}+3\hat{j}+9\hat{k}$

$|BC| = \sqrt{6^2+3^2+9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$

$AC = \text{PV of C} - \text{PV of A} = (11\hat{i}+3\hat{j}+7\hat{k})-(\hat{i}-2\hat{j}-8\hat{k})$

$= 10\hat{i}+5\hat{j}+15\hat{k}$

$|AC| = \sqrt{10^2+5^2+15^2} = 5\sqrt{14}$

$\therefore\ |AC| = |AB| + |BC|$

Thus, the given points A, B and C are collinear.

Let P be the point (on the line AC) which divides AC in the ratio λ : 1, then PV of the point

$P = \frac{\lambda \times \text{PV of C} + 1\times \text{PV of A}}{\lambda+1}$

$= \frac{1}{\lambda+1}\left\{\lambda(11\hat{i}+3\hat{j}+7\hat{k})+1(\hat{i}-2\hat{j}-8\hat{k})\right\}$

$= \left(\frac{11\lambda+1}{\lambda+1}\right)\hat{i}+\left(\frac{3\lambda-2}{\lambda+1}\right)\hat{j}+\left(\frac{7\lambda-8}{\lambda+1}\right)\hat{k}$

B lies on line AC i.e., B is collinear with A and C, if P = B for a unique λ.

$\Rightarrow \left(\frac{11\lambda+1}{\lambda+1}\right)\hat{i}+\left(\frac{3\lambda-2}{\lambda+1}\right)\hat{j}+\left(\frac{7\lambda-8}{\lambda+1}\right)\hat{k} = 5\hat{i}+0\hat{j}-2\hat{k}$

$\Rightarrow \frac{11\lambda+1}{\lambda+1} = 5,\ \frac{3\lambda-2}{\lambda+1} = 0$ and $\frac{7\lambda-8}{\lambda+1} = -2$

$\Rightarrow 11\lambda+1 = 5\lambda+5,\ 3\lambda = 2,\ 7\lambda-8 = -2\lambda-2$

$\Rightarrow 6\lambda = 4,\ \lambda = \frac{2}{3},\ 9\lambda = 6\ \Rightarrow \lambda = \frac{2}{3}$

Hence, A, B, C are collinear and B divides AC in the ratio $\frac{2}{3}$: 1 i.e. 2 : 3.

So, both the statements are true.

MATCHING TYPE QUESTIONS

78. (c)

A. $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| \Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 4|\vec{a}||\vec{b}|\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

$\Rightarrow \vec{a}\perp\vec{b}$

B. $|\vec{a}+\vec{b}| = |\vec{a}|+|\vec{b}| \Rightarrow |\vec{a}+\vec{b}|^2 = [|\vec{a}|+|\vec{b}|]^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|$

$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta = 2|\vec{a}||\vec{b}|$

$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0 \Rightarrow \vec{a}\parallel\vec{b}$

C. $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|+2|\vec{a}||\vec{b}|\cos\theta = |\vec{a}|^2+|\vec{b}|^2$

$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta = 0$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90° \Rightarrow \vec{a}\perp\vec{b}$

D. $\vec{a}.\vec{b} = 0$

$\Rightarrow ab\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

$\Rightarrow \vec{a}\perp\vec{b}$

E. $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|$

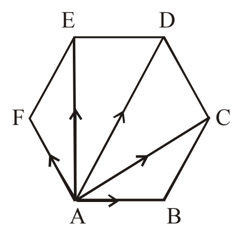
$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = |\vec{a}||\vec{b}|$

$\Rightarrow \sin\theta = 1 \Rightarrow \theta = 90°$ or $\vec{a}\perp\vec{b}$

79. (d)

INTEGER TYPE QUESTIONS

80. (c)



$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$

$= \overrightarrow{ED}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{CD}$

$= (\overrightarrow{AC}+\overrightarrow{CD})+\overrightarrow{AD}+(\overrightarrow{AE}+\overrightarrow{ED})$

$= \overrightarrow{AD}+\overrightarrow{AD}+\overrightarrow{AD} = 3\overrightarrow{AD}$

$\therefore n = 3$.

81. (a) The unit vectors in the given direction being $\frac{1}{6}(2\hat{i}+4\hat{j}+4\hat{k})$ and $\frac{1}{6}(4\hat{i}+4\hat{j}+2\hat{k})$, the vectors representing the forces are $\frac{1}{3}(2\hat{i}+4\hat{j}+4\hat{k})$ and $\frac{1}{2}(4\hat{i}-4\hat{j}+2\hat{k})$ respectively, of which the resultant is

$$\left(\frac{2}{3}+2\right)\hat{i}+\left(\frac{4}{3}-2\right)\hat{j}+\left(\frac{4}{3}+1\right)\hat{k} \text{ i.e., } \frac{1}{3}(8\hat{i}-2\hat{j}+7\hat{k}).$$

The displacement is represented by the vector $\hat{i}+2\hat{j}+2\hat{k}$.

Hence the work done

$$= \frac{1}{3}(8\hat{i}-2\hat{j}+7\hat{k}).(\hat{i}+2\hat{j}+2\hat{k})$$

$$= \frac{1}{3}(8-4+14) = 6 \text{ gm-cm}$$

82. (a) $\vec{a},\vec{b},\vec{c}$ are coplanar vectors, $2\vec{a}-\vec{b},\ 2\vec{b}-\vec{c}$, and $2\vec{c}-\vec{a}$ are also coplanar vectors.

Thus $\left[2\vec{a}-\vec{b}\quad 2\vec{b}-\vec{c}\quad 2\vec{c}-\vec{a}\right]=0$

83. (a) $\vec{a}.\vec{c}=(\hat{i}+\hat{j}+\hat{k}).\hat{i}=1$ and $\vec{b}.\vec{c}=(\hat{i}+\hat{j}).\hat{i}=1$

Now, $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{c}.\vec{a})\vec{b}-(\vec{c}.\vec{b})\vec{a}=\mu\vec{b}+\lambda\vec{a}$

$\Rightarrow \mu=\vec{c}\cdot\vec{a}$ and $\lambda=-\vec{c}\cdot\vec{b}$

$\therefore \mu+\lambda = 1-1 = 0$

84. (c) The position vectors of A and B are $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$ and $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$

$\therefore \overrightarrow{AB} = [1-(-1)]\,\hat{i}+0\cdot\hat{j}+0\cdot\hat{k}$

$\therefore |\overrightarrow{AB}| = 2$

The position vectors of A and D are

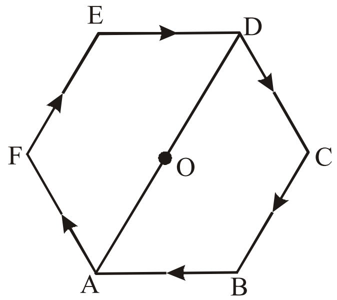
$-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$

$\therefore \overrightarrow{AD}=(-\hat{i}+\hat{i})+\left(-\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4\hat{k}-4\hat{k})=-\hat{j}$

$|\overrightarrow{AD}| = 1$

Area of rectangle ABCD $= |\overrightarrow{AB}|\,|\overrightarrow{AD}| = 2\times 1 = 2$

85. (b) Given, AD + ED + FC = λED



$\Rightarrow$ (AE + ED) + (ED + DB) + 2ED = λED

$\Rightarrow$ 4ED + (AE + DB) = λED

$\Rightarrow$ 4ED = λED $\Rightarrow$ λ = 4($\because$ AE = – DB)

86. (d) Let $a=\hat{i}+\hat{j}+\hat{k}$, $b=2\hat{i}+4\hat{j}-5\hat{k}$ and $c=\lambda\hat{i}+2\hat{j}+3\hat{k}$

Now, $b+c=2\hat{i}+4\hat{j}-5\hat{k}+\lambda\hat{i}+2\hat{j}+3\hat{k}$

$=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\therefore |b+c|=\sqrt{(2+\lambda)^2+(6)^2+(-2)^2}$

$=\sqrt{4+\lambda^2+4\lambda+36+4}=\sqrt{\lambda^2+4\lambda+44}$

The unit vector along (b + c), i.e.,

$$\frac{b+c}{|b+c|}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

Scalar product $(\hat{i}+\hat{j}+\hat{k})$ with this unit vector is 1.

$$\therefore (\hat{i}+\hat{j}+\hat{k}).\frac{b+c}{|b+c|}=1$$

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k}).\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow \frac{1(2+\lambda)+1(6)+1(-2)}{\sqrt{\lambda^2+4\lambda+44}}=1\Rightarrow\frac{(2+\lambda)+6+-2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$\Rightarrow \lambda+6=\sqrt{\lambda^2+4\lambda+44}$

$\Rightarrow (\lambda+6)^2=\lambda^2+4\lambda+44$

$\Rightarrow \lambda^2+12\lambda+36=\lambda^2+4\lambda+44$

$\Rightarrow 8\lambda = 8 \Rightarrow \lambda = 1$

Hence, the value of λ is 1.

87. (a) Since $\vec{n}$ is perpendicular to $\vec{u}$ and $\vec{v}$, $\vec{n}=\frac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|}$

$$\hat{n}=\frac{\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-\hat{k}$$

$|\vec{w}.\hat{n}|=|(i+2j+3k).(-k)|=|-3|=3$

88. (b) The vectors $10\hat{i}+3\hat{j}, 12\hat{i}-5\hat{j}$ and $m\hat{i}+11\hat{j}$ are collinear, if area of triangle formed by their position vectors is zero.

$$\Rightarrow \begin{vmatrix} 10 & 3 & 1 \\ 12 & -5 & 1 \\ m & 11 & 1 \end{vmatrix}=0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 10 & 3 & 1 \\ 2 & -8 & 0 \\ m-10 & 8 & 0 \end{vmatrix}=0$$

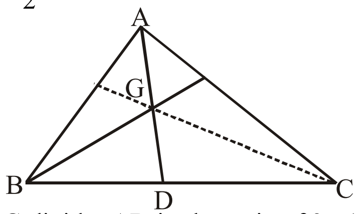
$\Rightarrow$ 16 + 8m – 80 = 0

$\Rightarrow$ 8m – 64 = 0

$\Rightarrow$ m = 8

89. (b) If D is middle point of side BC, then

$\overrightarrow{GD} = \frac{1}{2}(\overrightarrow{GB} + \overrightarrow{GC})$



$\because$ G divides AD in the ratio of 2 : 1

$\therefore$ $\overrightarrow{AG} = 2\,\overrightarrow{GD}$

$\Rightarrow$ $-\overrightarrow{GA} = \overrightarrow{GB} + \overrightarrow{GC}$ $\Rightarrow$ $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = 0$

ASSERTION - REASON TYPE QUESTIONS

90. (d) In ΔABC, $\overline{AB} + \overline{BC} = \overline{AC} = -\overline{CA}$

$\Rightarrow \overline{AB} + \overline{BC} + \overline{CA} = \overline{0}$

$\overline{OA} + \overline{AB} = \overline{OB}$ is triangle law of addition, A is true

R is false.

91. (c) I is incentre

$\Rightarrow$ OI $= \dfrac{|\overline{BC}|\overline{OA} + |\overline{CA}|\overline{OB} + |\overline{AB}|\overline{OC}}{|\overline{AB}| + |\overline{BC}| + |\overline{CA}|}$

$\Rightarrow$ $|\overline{BC}|\overline{IA} + |\overline{CA}|\overline{IB} + |\overline{AB}|\overline{IC} = 0$

Position vector of centroid, $\overline{OG} = \dfrac{\overline{OA} + \overline{OB} + \overline{OC}}{3}$

92. (b) $\overline{a} = a_1 i + a_2 j + a_3 k$, $\overline{b} = b_1 i + b_2 j + b_3 k$ are parallel

$\Rightarrow \dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

$\overline{a} = i + pj + 2k$, $\overline{b} = 2i + 3j + qk$ are parallel

$\Rightarrow \dfrac{1}{2} = \dfrac{p}{3} = \dfrac{2}{q}$, $p = \dfrac{3}{2}$, $q = 4$

93. (a) P, Q, R are collinear

$\Rightarrow$ $\overline{PQ} \parallel \overline{PR} \Rightarrow \overline{PQ} \parallel \lambda\overline{PR}$, $\lambda \in R$

$(2\vec{a} + \vec{b}) - (\vec{a} + \vec{b} - \vec{c}) = \lambda\left[(\vec{b} + t\vec{c}) - (\vec{a} + \vec{b} - \vec{c})\right]$

$\left[(\vec{a} + \vec{c}) = \lambda\vec{a} + (t+1)\vec{c}\right]$

$\Rightarrow \vec{a} + \vec{c} = -\lambda\vec{a} + \lambda(t+1)\vec{c}$

On Comparing,

$-\lambda = 1 \Rightarrow \lambda = -1$

and $\lambda(t + 1) = 1 \Rightarrow -(t + 1) = 1 \Rightarrow t = -2$

Hence, both are true and Reason is the correct explanation of Assertion.

94. (d) $\vec{a} = \hat{i} + 2\hat{j}$, $\vec{b} = 2\hat{i} + \hat{j}$

Diagonals of the parallelogram are along

$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

Now, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j}) + (2\hat{i} + \hat{j}) = 3\hat{i} + 3\hat{j}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j}) - (2\hat{i} + \hat{j}) = -\hat{i} + \hat{j}$

Let θ be angle between these vectors, then

$\cos\theta = \dfrac{(3\hat{i} + 3\hat{j}).(-\hat{i} + \hat{j})}{\sqrt{9+9}\sqrt{1+1}} = \dfrac{-3+3}{\sqrt{18}\sqrt{2}} = 0$

$\Rightarrow \theta = 90°$

Hence, Assertion is false

95. (d) $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = 144$ $|\vec{a}| = 4$

We know that

$(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

$\Rightarrow 144 = (4)^2 |\vec{b}|^2 \Rightarrow 16|\vec{b}|^2 = 144$

$|\vec{b}|^2 = 9 \Rightarrow |\vec{b}|^2 = 3$

Hence, Assertion is false.

$(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = |\vec{a} \times \vec{b}|^2 + (\vec{a} \times \vec{b})^2$

$= (ab\sin\theta)^2 + (ab\cos\theta)^2 = \vec{a}^2.\vec{b}^2$

$\Rightarrow (\vec{a} \times \vec{b})^2 = \vec{a}^2.\vec{b}^2 - (\vec{a}.\vec{b})^2$

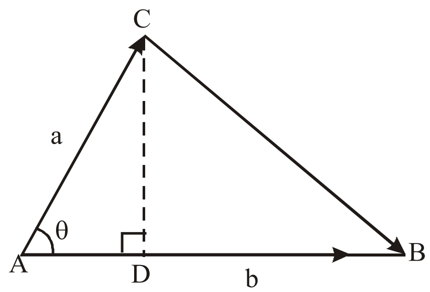
Hence Reason is true

96. (c) The projection of vector a on the vector b is given by

$\dfrac{(a.b)}{|b|} = \dfrac{(2\times1 + 3\times2 + 2\times1)}{\sqrt{(1)^2 + (2)^2 + (1)^2}} = \dfrac{10}{\sqrt{6}} = \dfrac{5}{3}\sqrt{6}$

97. (a) If a and b represent the adjacent sides of a triangle, then this area is given as $\frac{1}{2}|a \times b|$

By definition of the area of a triangle, we have



Area of $\Delta ABC = \dfrac{1}{2}AB.CD$

But $AB = |b|$ and $CD = |a|\sin\theta$

Thus, area of $\Delta ABC = \dfrac{1}{2}|b||a|\sin\theta = \dfrac{1}{2}|a \times b|$

so both the Assertion and Reason are true and Reason is the correct explanation of Assertion.

98. (a) If a,b and c be any three vectors, then

$[a \ b \ c]=[b \ c \ a]=[c \ a \ b]$

(cyclic permutation of three vectors does not change the value of the scalar triple product).

Let $a=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, b=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

and $c=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

$$[a \ b \ c]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= a_1(b_2c_3-b_3c_2)+a_2(b_3c_1-b_1c_3)+a_3(b_1c_2-b_2c_1)$$

$$= b_1(a_3c_2-a_2c_3)+b_2(a_1c_3-a_3c_1)+b_3(a_2c_1-a_1c_2)$$

$$=\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

$=[b \ c \ a]$

Similarly, $[a \ b \ c]=[c \ a \ b]$

Hence $[a \ b \ c]=[b \ c \ a]=[c \ a \ b]$

99. (d) Assertion is false and Reason is true.

Since $\vec{a}.(\vec{b}\times\vec{c})=0$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar.

CRITICAL THINKING TYPE QUESTIONS

100. (b) If $\vec{F}$ be the resultant of the three given forces then

$\vec{F}=(\hat{i}+2\hat{j}-3\hat{k})+(2\hat{i}+3\hat{j}+4\hat{k})+(-\hat{i}-\hat{j}+\hat{k})=2\hat{i}+4\hat{j}+2\hat{k}$

If O be the origin, then $\overrightarrow{OP}$ = p.v. of P = $\hat{j}+2\hat{k}$

$\overrightarrow{OA}$ = p.v. of A = $\hat{i}-2\hat{j}$

$\therefore \overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=(\hat{j}+2\hat{k})-(\hat{i}-2\hat{j})=-\hat{i}+3\hat{j}+2\hat{k}$

$\therefore$ Vector moment of the given forces about

A = vector moment of $\vec{F}$ about A = $\overrightarrow{AP}\times\vec{F}$

$=(-\hat{i}+3\hat{j}+2\hat{k})\times(2\hat{i}+4\hat{j}+2\hat{k})=-2\hat{i}+6\hat{j}-10\hat{k}$

The magnitude of the moment = $\sqrt{4+36+100}=\sqrt{140}$

101. (d) For orthogonality, the scalar product = 0

$\Rightarrow 2(x^2-1)+(-x)(x+2)+3x^2=0$

$\Rightarrow 2(2x+1)(x-1)=0 \Rightarrow x=-\frac{1}{2}, 1$

102. (d) Let $\overrightarrow{OA}=x_1\hat{i}+y_1\hat{j}$ and $\overrightarrow{OB}=x_2\hat{i}+y_2\hat{j}$

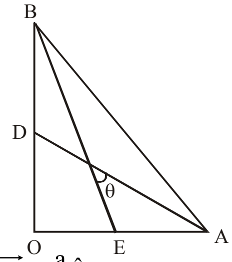
Since, $1=\overrightarrow{OA}.\hat{i}=x_1$ and $-2=\overrightarrow{OB}.\hat{i}=x_2$

Moreover, $y_1=x_1^2=1$ and $y_2=x_2^2=4$

So, $\overrightarrow{OA}=\hat{i}+\hat{j}$ and $\overrightarrow{OB}=-2\hat{i}+4\hat{j}$

Hence, $|2\overrightarrow{OA}-3\overrightarrow{OB}|=|8\hat{i}-10\hat{j}|=\sqrt{164}=2\sqrt{41}$

103. (c) Without loss of generality, let the right angled ΔOAB be such that OA = OB = a units.



Along OA take unit vector as $\hat{i}$ and along OB taken unit vector as $\hat{j}$, so that

$\overrightarrow{OA}=a\hat{i};\ \overrightarrow{OE}=\frac{a}{2}\hat{i};\ \overrightarrow{OB}=a\hat{j};\ \overrightarrow{OD}=\frac{a}{2}\hat{j};$

$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=\frac{a}{2}\hat{j}-a\hat{i}\ ;\ \overrightarrow{BE}=\overrightarrow{OE}-\overrightarrow{OB}=\frac{a}{2}\hat{i}-a\hat{j}$

$\overrightarrow{AD}.\overrightarrow{BE}=\left|\left(\frac{a}{2}\hat{j}-a\hat{i}\right).\left(\frac{a}{2}\hat{i}-a\hat{j}\right)\right| \Rightarrow \sqrt{\frac{5a^2}{4}}\times\sqrt{\frac{5a^2}{4}}\cos\theta=a^2$

$\cos\theta=\frac{4}{5};\ \therefore \theta=\cos^{-1}\left(\frac{4}{5}\right)$

104. (d) Let $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$, then

$$\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix}=5\hat{i}-5\hat{j}-5\hat{k}$$

unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$. If θ is the required angle, then

$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{2\hat{i}-2\hat{j}+\hat{k}}{3}.\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})=\frac{1}{\sqrt{3}}$

$\Rightarrow \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)=\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)=\cot^{-1}(\sqrt{2})$

105. (a) Since, $\vec{b}, \vec{c}$ and $\vec{b}\times\vec{c}$ are mutually perpendicular vectors, therefore any vector $\vec{a}$ can be expressed in terms of $\vec{b}, \vec{c}$ and $\vec{b}\times\vec{c}$.

Let $\vec{a}=x\vec{b}+y\vec{c}+z(\vec{b}\times\vec{c})$ ....(i)

Taking dot product with $\vec{b}\times\vec{c}$ in eq. (i),

we get, $\vec{a}.(\vec{b}\times\vec{c})=0+0+z|\vec{b}\times\vec{c}|^2$

$\Rightarrow z=\frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}$

Taking dot product with $\vec{b}$ in eq. (i), we get

$\vec{a}.\vec{b}=x\vec{b}.\vec{b}+y\vec{c}.\vec{b}+z.0=x$

Taking dot product with $\vec{c}$ in eq. (i), we get

$\vec{a}.\vec{c}=0+y+0 \Rightarrow y=\vec{a}.\vec{c}$

$\therefore \vec{a}=(\vec{a}.\vec{b})\vec{b}+(\vec{a}.\vec{c})\vec{c}+\frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}(\vec{b}\times\vec{c})$

106. (d) $(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})=\hat{i}-\hat{j}+\hat{k}$; so the unit vector $\perp$ to the plane of $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$. Similarly, the other two unit vectors are $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ and $\frac{1}{\sqrt{3}}(-\hat{i}+\hat{j}+\hat{k})$.

Hence, the required volume $=\frac{1}{3\sqrt{3}}\begin{vmatrix}1 & -1 & 1\\ 1 & 1 & -1\\ -1 & 1 & 1\end{vmatrix}=\frac{4}{3\sqrt{3}}$

107. (a) We have $\vec{\alpha}=x(\vec{a}\times\vec{b})+y(\vec{b}\times\vec{c})+z(\vec{c}\times\vec{a})$

Taking dot products with $\vec{a},\vec{b},\vec{c}$, we get

$\vec{\alpha}.\vec{a}=y[\vec{a}\,\vec{b}\,\vec{c}]\Rightarrow y=8(\vec{\alpha}.\vec{a})$

$\vec{\alpha}.\vec{b}=z[\vec{a}\,\vec{b}\,\vec{c}]\Rightarrow z=8(\vec{\alpha}.\vec{b})$

$\vec{\alpha}.\vec{c}=x[\vec{a}\,\vec{b}\,\vec{c}]\Rightarrow x=8(\vec{\alpha}.\vec{c})$

$\therefore x+y+z=8\vec{\alpha}.(\vec{a}+\vec{b}+\vec{c})$

108. (c) $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=0$

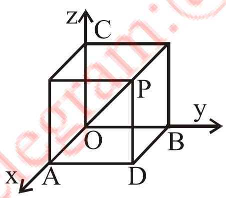
$\Rightarrow \overrightarrow{DA}$ and $\overrightarrow{CB}$ are perpendicular

$(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a})=0$

$\Rightarrow \overrightarrow{DB}$ and $\overrightarrow{AC}$ are perpendicular

$\therefore$ D is orthocentre of ΔABC

109. (d) For a unit cube, unit vector along the diagonal



$OP=\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

unit vector along the diagonal

$CD=\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

$\therefore \cos\theta=\frac{1}{3}(1+1-1)=\frac{1}{3}$

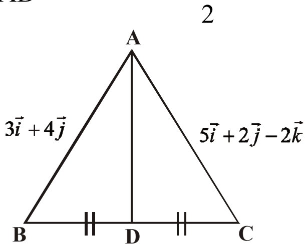
$\tan\theta=2\sqrt{2}$

110. (d) $\vec{F}=\vec{F}_1+\vec{F}_2=7i+2j-4k$

$\vec{d}=P.V\ of\ \vec{B}-P.V\ of\ \vec{A}=4i+2j-2k$

$W=\vec{F}.\vec{d}=28+4+8=40$ unit

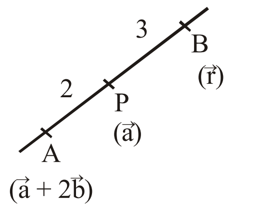
111. (d) P.V of $\overrightarrow{AD}=\frac{(3+5)i+(0-2)j+(4+4)k}{2}$



$=4i-j+4k$ or $|\overrightarrow{AD}|=\sqrt{16+16+1}=\sqrt{33}$

112. (c) Let the position vector of B is $\vec{r}$.

$\therefore\ \vec{a}=\frac{2\vec{r}+3(\vec{a}+2\vec{b})}{2+3}$



$\Rightarrow 5\vec{a}=2\vec{r}+3\vec{a}+6\vec{b}$

$\Rightarrow 2\vec{r}=2\vec{a}-6\vec{b}\ \therefore\ \vec{r}=\vec{a}-3\vec{b}$

113. (b) Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

Now, $\hat{i}\times(\vec{a}\times\hat{i})=(\hat{i}.\hat{i})\vec{a}-(\hat{i}.\vec{a})\hat{i}=\vec{a}-a_1\hat{i}$

Similarly, $\hat{j}\times(\vec{a}\times\hat{j})=\vec{a}-a_2\hat{j}$

and $\hat{k}\times(\vec{a}\times\hat{k})=\vec{a}-a_3\hat{k}$

$\therefore\ \hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})$

$=3\vec{a}-(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})=3\vec{a}-\vec{a}=2\vec{a}$

114. (d) $|\vec{a}+\vec{b}|=6$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=36$ ...(i)

Similarly, $|\vec{b}|^2+|\vec{c}|^2+2\vec{b}.\vec{c}=64$ ...(ii)

and $|\vec{c}|^2+|\vec{a}|^2+2\vec{c}.\vec{a}=100$ ...(iii)

On adding eqs.(i), (ii) and (iii), we get

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=100$ ...(iv)

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2=100\quad(\because \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0)$

Now, $|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2=100$ [using (iv)]

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|=10$

115. (b) Angle between $\hat{i}+\hat{j}$ and $(\hat{i}\times\hat{j})$ is $\frac{\pi}{2}$

$\Rightarrow$ Angle between $\vec{c}$ and $(\hat{i}\times\hat{j})\geq\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$

$\Rightarrow (\hat{i}\times\hat{j}).\vec{c} \le |\hat{i}\times\hat{j}||\vec{c}|\cos\frac{\pi}{6}$

$\Rightarrow -\frac{\sqrt{3}}{2} \le (\hat{i}\times\hat{j}).\vec{c} \le \frac{\sqrt{3}}{2}$

116. (a) Let the required vector be

$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$, then

$\vec{v}.(\hat{i}+\hat{j}-3\hat{k}) = 0 \Rightarrow x + y - 3z = 0$ ...(i)

$\vec{v}.(\hat{i}+3\hat{j}-2\hat{k}) = 5 \Rightarrow x + 3y - 2z = 5$ ...(ii)

and $\vec{v}.(2\hat{i}+\hat{j}+4\hat{k}) = 8 \Rightarrow 2x + y + 4z = 8$ ...(iii)

Subtracting (ii) from (i),we have

$-2y - z = -5 \Rightarrow 2y + z = 5$ ...(iv)

Multiply (ii) by 2 and subtracting (iii) from it, we obtain

$5y - 8z = 2$ ...(v)

Multiply (iv) by 8 and adding (v) to it, we have

$21y = 42 \Rightarrow y = 2$ ...(v)

Substituting y = 2 in(iv), we get

$2 \times 2 + z = 5 \Rightarrow z = 5 - 4 = 1$

Substituting these values in (i),we get

$x + 2 - 3 = 0 \Rightarrow x = 3 - 2 = 1$

Hence, the required vector is

$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k} = \hat{i} + 2\hat{j} + \hat{k}$

117. (d) Here, $r = (2\hat{i}+3\hat{j}+\hat{k}) - (\hat{i}+2\hat{j}+3\hat{k})$

$\Rightarrow r = \hat{i}+\hat{j}-2\hat{k}$ and $F = \hat{i}+\hat{j}+\hat{k}$

Then, the required moment is given by

$r \times F = (\hat{i}+\hat{j}-2\hat{k}) \times (\hat{i}+\hat{j}+\hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j}$

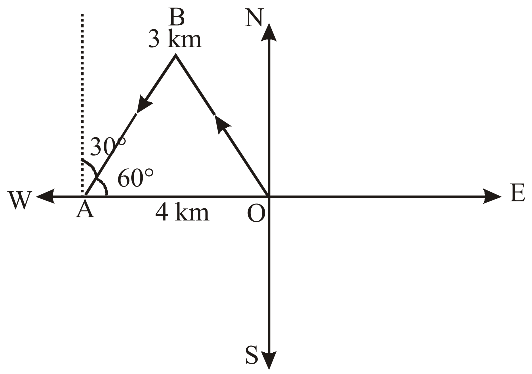
$\therefore$ Moment about given point $= 3\hat{i} - 3\hat{j}$

118. (c) Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as in the figure.

Now, we have $OA = -4\hat{i}$

$AB = \hat{i}|AB|\cos 60° + \hat{j}|AB|\sin 60°$



(AB cos 60° is component of AB along X-axis and AB sin 60° is component of AB along Y-axis)

$= \hat{i}\left(3\times\frac{1}{2}\right) + \hat{j}\left(3\times\frac{\sqrt{3}}{2}\right) = \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

By the triangle law of vector addition, we have

$OB = OA + AB = (-4\hat{i}) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$

$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j} = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Hence the girl's displacement from her initial point of departure is $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

119. (a) Let $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{AD} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$ when $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-collinear coplanar vectors.

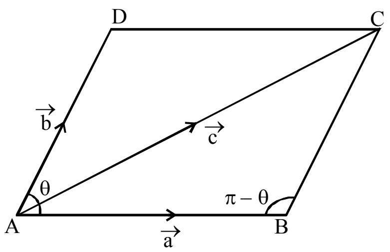
$\overrightarrow{DB} = \overrightarrow{AB} - \overrightarrow{AC} = \vec{a} - \vec{b}$

Now, $\overrightarrow{DB}.\overrightarrow{AB} = (\vec{a} - \vec{b}).(\vec{a}) = \vec{a}.\vec{a} - \vec{b}.\vec{a}$

$a^2 - ab\cos\theta = a^2 - \frac{c^2 - a^2 - b^2}{2}$

$= \frac{3a^2 + b^2 - c^2}{2}$

$\left[\because \text{In } \Delta ABC, \cos(\pi - \theta) = \frac{a^2 + b^2 - c^2}{2ab}\right]$



120. (d) We have | (a × b).c | = | a || b || c |

$\Rightarrow$ | a || b || sin θ n.c.| = | a || b || c |

$\Rightarrow$ | a || b | | c || sin θ cos α | = | a || b || c |

$\Rightarrow$ |sin θ | |cos α | =1 $\Rightarrow \theta = \frac{\pi}{2}$ and α = 0

$\Rightarrow$ a ⊥ b and c | | n

$\Rightarrow$ a ⊥ b and c is perpendicular to both a and b

$\therefore$ a, b, c are mutually perpendicular

Hence, a.b. = b.c = c.a. = 0

121. (a) It is given that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}\vec{b}$

Note: $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$\Rightarrow \frac{1}{2}\vec{b} = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$\Rightarrow \quad (\vec{a}.\vec{c}-\frac{1}{2})\vec{b}-(\vec{a}.\vec{b})\vec{c}=0 \quad ...(i)$

Since $\vec{b}$ and $\vec{c}$ are non-parallel, therefore for the existence of relation (i), the coeff. of $\vec{b}$ and $\vec{c}$ should vanish separately. Therefore, we get

$\vec{a}.\vec{c}-\frac{1}{2}=0$ i.e., $\vec{a}.\vec{c}=\frac{1}{2}$ and $\vec{a}.\vec{b}=0$

Since $\vec{a},\vec{b},\vec{c}$ are the unit vectors therefore

$\vec{a}.\vec{b}=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}.\ (\because \cos^{-1}\left(\frac{1}{2}\right)=\theta)$

122. (a) As given, $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}\Rightarrow(\vec{r}-\vec{b})\times\vec{a}=0$

$\Rightarrow \vec{r}-\vec{b}$ is parallel to $\vec{a}\Rightarrow\vec{r}-\vec{b}=n\vec{a},\ n\in R$

$\Rightarrow \vec{r}=\vec{b}+n\vec{a} \quad ...(i)$

Similarly, $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ can be written as $\vec{r}=\vec{a}+m\vec{b}$ where $m\in R \quad ...(ii)$

$\therefore$ From equations (i) and (ii), we get

$m=1=n$ and $\vec{r}=\vec{a}+\vec{b}$

$\Rightarrow \vec{r}=\hat{i}+3\hat{j}-\hat{k}$ and $|\vec{r}|=\sqrt{9+1+1}=\sqrt{11}$

$\therefore \frac{\vec{r}}{|\vec{r}|}=\frac{\hat{i}+3\hat{j}-\hat{k}}{\sqrt{11}}$

123. (a) A vector perpendicular to the plane is

$(\hat{i}-2\hat{j}-\hat{k})\times(3\hat{i}-2\hat{j}-\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & -1\\ 3 & -2 & -1\end{vmatrix}=-2\hat{j}+4\hat{k}$$

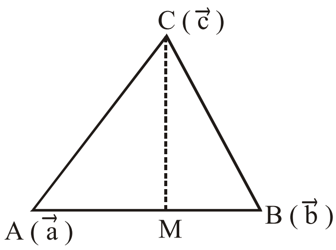
$\Rightarrow$ unit vector $\hat{a}=\frac{-2\hat{j}+4\hat{k}}{\sqrt{4+16}}=\frac{-2\hat{j}+4\hat{k}}{2\sqrt{5}}$

Angle between the unit vector and $\vec{r}=\hat{i}+\hat{j}+\hat{k}$

$=\cos^{-1}\frac{\vec{r}.\hat{a}}{|\vec{r}|.|\hat{a}|}$

$=\cos^{-1}\frac{1}{\sqrt{15}}=\sec^{-1}\sqrt{15}=\tan^{-1}\sqrt{14}$

124. (a) Let CM be the altitude through C.



Then, area of triangle ABC

$=\frac{1}{2}(AB)(CM)=\frac{1}{2}|\vec{b}-\vec{a}|CM \quad ....(i)$

Again, area of triangle ABC

$=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{1}{2}|(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})|$

$=\frac{1}{2}|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}| \quad ...(ii)$

From (i) and (ii),

$CM=\frac{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}{|\vec{b}-\vec{a}|}$

125. (d) $\vec{p}=\lambda(\vec{u}\times\vec{v})+\mu(\vec{v}\times\vec{w})+\nu(\vec{w}\times\vec{u})$

$\Rightarrow \vec{p}.\vec{w}=\lambda(\vec{u}\times\vec{v}).\vec{w}+\mu(\vec{v}\times\vec{w}).\vec{w}+\nu(\vec{w}\times\vec{u}).\vec{w}$

$=\lambda[\vec{u}\ \vec{v}\ \vec{w}]+0+0=\frac{\lambda}{5} \Rightarrow \lambda=5(\vec{p}.\vec{w})$

Similarly, $\mu=5(\vec{p}.\vec{u})$ and $\nu=5(\vec{p}.\vec{v})$

$\therefore \lambda+\mu+\nu=5(\vec{p}.\vec{w})+5(\vec{p}.\vec{u})+5(\vec{p}.\vec{v})$

$5\vec{p}.(\vec{u}+\vec{v}+\vec{w})$

Hence, $\lambda+\mu+\nu$ depends on the vectors

126. (b) Let $\vec{R}=x\hat{i}+y\hat{j}+z\hat{k}$. Then

$\vec{R}\times\vec{B}=\vec{C}\times\vec{B}\Rightarrow(\vec{R}-\vec{C})\times\vec{B}=\vec{0}$

$$\Rightarrow\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x-4 & y+3 & z-7\\ 1 & 1 & 1\end{vmatrix}=\vec{0}$$

$\Rightarrow (y-z+10)\hat{i}+(z-x-3)\hat{j}+(x-y-7)\hat{k}=\vec{0}$

$\Rightarrow y-z=-10,\ z-x=3,\ x-y=7$

Also

$\vec{R}.\vec{A}=0\Rightarrow 2x+0.y+z=0\Rightarrow z=-2x$.

Solving, we obtain

$x=-1,\ y=-8,\ z=2.$

Hence $\vec{R}=-\hat{i}-8\hat{j}+2\hat{k}$.

127. (b) If F be the resultant force, then

$\vec{F}+2\vec{i}+4\vec{j}+\vec{k}$

and, $r=\overrightarrow{AP}=-2\vec{i}+6\vec{j}-10\vec{k}$

$$\therefore \text{Required moment}=r\times F=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -1 & 3 & 2\\ 2 & 4 & 2\end{vmatrix}$$

$=-2\vec{i}+6\vec{j}-10\vec{k}$

128. (d) Clearly $\vec{a}=-\frac{8}{7}\vec{c}$

$\Rightarrow \vec{a}\,||\,\vec{c}$ and are opposite in direction

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

CHAPTER 27

THREE DIMENSIONAL GEOMETRY

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(a) -5 (b) 5 (c) 2 (d) -2

2. The line which passes through the origin and intersect the two lines

$\frac{x-1}{2}=\frac{y+3}{4}=\frac{z-5}{3}, \frac{x-4}{2}=\frac{y+3}{3}=\frac{z-14}{4}$, is

(a) $\frac{x}{1}=\frac{y}{-3}=\frac{z}{5}$ (b) $\frac{x}{-1}=\frac{y}{3}=\frac{z}{5}$

(c) $\frac{x}{1}=\frac{y}{3}=\frac{z}{-5}$ (d) $\frac{x}{1}=\frac{y}{4}=\frac{z}{-5}$

3. The points A(1, 2, 3), B (–1, –2, –3) and C(2, 3, 2) are three vertices of a parallelogram ABCD. The equation of CD is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{2}$ (b) $\frac{x+2}{1}=\frac{y+3}{2}=\frac{z-2}{2}$

(c) $\frac{x}{2}=\frac{y}{3}=\frac{z}{2}$ (d) $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-2}{2}$

4. The vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ is

(a) $\vec{r}=(3\hat{i}+7\hat{j}+2\hat{k})+\mu(5\hat{i}+4\hat{j}-6\hat{k})$

(b) $\vec{r}=(5\hat{i}+4\hat{j}-6\hat{k})+\mu(3\hat{i}+7\hat{j}+2\hat{k})$

(c) $\vec{r}=(5\hat{i}-4\hat{j}-6\hat{k})+\mu(3\hat{i}-7\hat{j}-2\hat{k})$

(d) $\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\mu(3\hat{i}+7\hat{j}+2\hat{k})$

5. The angle between a line whose direction ratios are in the ratio 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12) is

(a) $\cos^{-1}(2/3)$ (b) $\cos^{-1}(-2/3)$

(c) $\tan^{-1}(2/3)$ (d) None of these

6. The equation of the plane passing through three non-collinear points with position vectors $\vec{a}, \vec{b}, \vec{c}$ is

(a) $\vec{r}.(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})=0$

(b) $\vec{r}.(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})=[\vec{a}\ \vec{b}\ \vec{c}]$

(c) $\vec{r}.(\vec{a}\times(\vec{b}+\vec{c}))=[\vec{a}\ \vec{b}\ \vec{c}]$

(d) $\vec{r}.(\vec{a}+\vec{b}+\vec{c})=0$

7. The vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i}-3\hat{j}+4\hat{k}$, is

(a) $\vec{r}.(2\hat{i}-3\hat{j}+4\hat{k})=\frac{6}{\sqrt{29}}$

(b) $\vec{r}.\left(\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$

(c) Both (a) and (b)

(d) None of the above

8. Two lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are said to be coplanar, if

(a) $(a_2 - a_1) . (b_1 \times b_2) = 0$

(b) $\begin{vmatrix} x_1 & y_1 & z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0,$

where (x_1, y_1, z_1) are the coordinates of a point on any of the line and a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratio of b_1 and b_2

(c) Both (a) and (b)

(d) None of the above

9. If the directions cosines of a line are k, k, k, then

(a) $k > 0$ (b) $0 < k < 1$

(c) $k = 1$ (d) $k=\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

10. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) $\frac{10}{6\sqrt{5}}$ (b) $\frac{4}{5\sqrt{2}}$ (c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{\sqrt{2}}{10}$

11. If a line makes an angle of $\pi/4$ with the positive directions of each of x- axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

12. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$. Then (α, β) equals

(a) $(-6, 7)$ (b) $(5, -15)$ (c) $(-5, 5)$ (d) $(6, -17)$

13. If the equations of two lines l_1 and l_2 are given by $r=\vec{a}_1+\lambda\vec{b}_1$ and $r=\vec{a}_2+\lambda\vec{b}_2$, where λ, μ are parameter then angle θ between them is given by

(a) $\cos\theta=\frac{\vec{a}_1\cdot\vec{a}_2}{|\vec{a}_1||\vec{a}_2|}$ (b) $\cos\theta=\frac{\vec{b}_2\cdot\vec{b}_1}{|\vec{b}_1||\vec{b}_2|}$

(c) $\cos\theta=\frac{\vec{a}_1\cdot\vec{b}_2}{|\vec{a}_1||\vec{b}_2|}$ (d) $\cos\theta=\frac{\vec{a}_2\cdot\vec{b}_1}{|\vec{a}_2||\vec{b}_1|}$

14. If a plane passes through the point (1, 1, 1) and is perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$, then its perpendicular distance from the origin is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{7}{5}$ (d) 1

15. Distance between the parallel planes $2x-y+3z+4=0$ and $6x-3y+9z-3=0$ is:

(a) $\frac{5}{\sqrt{3}}$ (b) $\frac{4}{\sqrt{6}}$ (c) $\frac{5}{\sqrt{14}}$ (d) $\frac{3}{2\sqrt{3}}$

16. The angle between two planes is equal to

(a) The angle between the tangents to them from any point.
(b) The angle between the normals to them from any point.
(c) The angle between the lines parallel to the planes from any point.
(d) None of these.

17. Direction ratios of two lines are a, b, c and $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$, The lines are

(a) Mutually perpendicular (b) Parallel
(c) Coincident (d) None of these

18. Under what condition do $\left\langle\frac{1}{\sqrt{2}}, \frac{1}{2}, k\right\rangle$ represent direction cosines of a line?

(a) $k=\frac{1}{2}$ (b) $k=-\frac{1}{2}$
(c) $k=\pm\frac{1}{2}$ (d) k can take any value

19. What is the condition for the plane $ax+by+cz+d=0$ to be perpendicular to xy-plane ?

(a) $a=0$ (b) $b=0$
(c) $c=0$ (d) $a+b+c=0$

20. The projection of the line segment joining the points $(-1, 0, 3)$ and $(2, 5, 1)$ on the line whose direction ratios are $(6, 2, 3)$ is

(a) 6 (b) 7 (c) $\frac{22}{7}$ (d) 3

21. A line makes angles of 45° and 60° with the positive axes of X and Y respectively. The angle made by the same line with the positive axis of Z, is.

(a) 30° or 60° (b) 60° or 90°
(c) 90° or 120° (d) 60° or 120°

22. The angle between the line $\frac{x-2}{a}=\frac{y-2}{b}=\frac{z-2}{c}$ and the plane $ax+by+cz+6=0$ is

(a) $\sin^{-1}\left(\frac{1}{\sqrt{a^2+b^2+c^2}}\right)$ (b) 45°
(c) 60° (d) 90°

23. The projections of the segment PQ on the co-ordinate axes are –9, 12, –8 respectively. The direction cosines of the line PQ are

(a) $<\frac{-9}{\sqrt{17}}, \frac{12}{\sqrt{17}}, \frac{-8}{\sqrt{17}}>$ (b) $<-9, 12, -8>$

(c) $<-\frac{9}{289}, \frac{12}{289}, \frac{-8}{289}>$ (d) $<-\frac{9}{17}, \frac{12}{17}, \frac{-8}{17}>$

24. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\frac{\pi}{4}$ with plane $x+y=3$ are

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$
(c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$

25. A plane meets the coordinate axes in points A, B, C and the centroid of the triangle ABC is (α, β, γ). The equation of the plane is

(a) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$ (b) $\alpha x+\beta y+\gamma z=3\alpha\beta\gamma$

(c) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=\frac{1}{2}$ (d) None of these

26. The direction ratios of the line OP are equal and the length $OP=\sqrt{3}$. Then the coordinates of the point P are :

(a) $(-1, -1, -1)$ (b) $(\sqrt{3}, \sqrt{3}, \sqrt{3})$
(c) $(\sqrt{2}, \sqrt{2}, \sqrt{2})$ (d) (2, 2, 2)

27. Which one of the following planes contains the z-axis?

(a) $x-z=0$ (b) $z+y=0$
(c) $3x+2y=0$ (d) $3x+2z=0$

28. The coordinates of the point where the line through the points A (3, 4, 1) and B (5, 1, 6) crosses the XY - plane are

(a) $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$ (b) $\left(-\frac{13}{5}, \frac{23}{5}, 0\right)$

(c) $\left(\frac{13}{5}, -\frac{23}{5}, 0\right)$ (d) $\left(-\frac{13}{5}, \frac{-23}{5}, 0\right)$

29. The ordered pair (λ, μ) such that the points $(\lambda, \mu, -6)$, $(3, 2, -4)$ and $(9, 8, -10)$ become collinear is

(a) (3, 4) (b) (5, 4) (c) (4, 5) (d) (4, 3)

30. Any three numbers which are proportional to the direction cosines of a line, are called.

(a) direction angles
(b) direction ratios
(c) another set of direction cosines
(d) None of the above

31. The coordinates of a point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of $\frac{6}{\sqrt{2}}$ from the point (1, 2, 3) is

(a) (56,,43, 111) (b) $\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$
(c) (2, 1, 3) (d) (–2, –1, –3)

32. If $r = a_1 + \lambda b_1$ and $r = a_2 + \mu b_2$ are the equations of two lines, then cos θ =

(a) $\left|\frac{a_1 \cdot a_2}{|a_1||a_2|}\right|$ (b) $\left|\frac{b_1 \cdot b_2}{|b_1||b_2|}\right|$
(c) $\left|\frac{a_1 \cdot a_2}{|b_1||b_2|}\right|$ (d) $\left|\frac{b_1 \cdot b_2}{|a_1||a_2|}\right|$

33. The lines in a space which are neither intersecting nor parallel, are called

(a) concurrent lines (b) intersecting lines
(c) skew lines (d) parallel lines

34. If the plane x – 3y + 5z = d passes through the point (1, 2, 4), then the length of intercepts cut by it on the axes of X, Y, Z are respectively, is

(a) 15, –5, 3 (b) 1, –5, 3
(c) –15, 5, –3 (d) 1, –6, 20

35. Two planes $r \cdot n_1 = d_1$ and $r \cdot n_2 = d_2$ are perpendicular to each other, if

(a) $n_1 = n_2$ (b) n_1 is parallel to n_2
(c) $n_1 \cdot n_2 = 0$ (d) None of the above

36. The angle θ between two planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by cos θ equal to

(a) $A_1A_2 + B_1B_2 + C_1C_2$

(b) $\left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}}\right|$

(c) $\left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

(d) $\left|\frac{A_1A_2+B_1B_2+C_1C_2}{\left(A_1^2+B_1^2+C_1^2\right)\left(A_2^2+B_2^2+C_2^2\right)}\right|$

37. The distance of a point (2, 5, –3) from the plane $r\cdot\left(6\hat{i}-3\hat{j}+2\hat{k}\right)=4$ is

(a) 13 (b) $\frac{13}{7}$ (c) $\frac{13}{5}$ (d) $\frac{37}{7}$

38. The distance of the point (–5, –5, –10) from the point of intersection of the line $r. = 2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $r\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$ is

(a) 13 (b) 12 (c) $4\sqrt{15}$ (d) $10\sqrt{2}$

39. If O be the origin and the coordinates of P be (1, 2, –3), then the equation of the plane passing through P and perpendicular to OP is

(a) x + 2y + 3z = –5 (b) x + 2y + 3z = – 14
(c) x + 2y – 3z =14 (d) x + 2y – 3z =5

40. The shortest distance between the Z-axis and the line $x+y+2z-3=0,\ 2x+3y+4z-4=0$ is

(a) 2 (b) $\frac{1}{2}$
(c) 0 (d) 1

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

41. Consider the following statements:

I. Equations ax + by + cz + d = 0, a'x + b'y + c'z + d' = 0 represent a straight line.

II. Equation of the form

$$\frac{x-\alpha}{\ell}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

represent a straight line passing through the point (α, β, γ) and having direction ratio proportional to *l*, m, n.

Which of the statements given above is/are correct ?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

42. Consider the following statements

Statement I : If a line in space does not pass through the origin, the direction cosines of the line does not exist.

Statement II : Two parallel lines have same set of direction cosines.

Choose the correct option.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

43. Which of the following are true?

I. If a, b and c are the direction ratios of a line, then ka, kb and kc is also a set of direction ratios.

II. The two sets of direction ratios of a line are in proportion.

III. There exists two sets of direction ratios of a line.

(a) I and II are true
(b) II and III are true
(c) I and III are true
(d) All are true

44. Consider the following statements

Statement I : The points (1, 2, 3), (–2, 3, 4) and (7, 0, 1) are collinear.

Statement II : If a line makes angles $\frac{\pi}{2},\frac{3\pi}{4}$ and $\frac{\pi}{4}$ with X, Y and Z-axes respectively, then its direction cosines are 0, $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

Choose the correct option.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

45. Consider the following statements

Statement I : The vector equation of a line passing through two points whose position vectors are a and b, is $r = a + \lambda (b - a)\ \forall\ \lambda \in R$.

Statement II : The cartesian equation of a line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Choose the correct option.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

46. Consider the following statements

Statement I : The angle between two planes is twice the angle between their normals.

Statement II : If θ is the angle between two planes, then $180 - \theta$ is also the angle between same planes.

Choose the correct option.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

47. Consider the following statements

Statement I: The angle between two planes $x + 2y + 2z = 3$ and $-5x + 3y + 4z = 9$ is $\cos^{-1}\left(\frac{19\sqrt{2}}{30}\right)$.

Statement II: The angle between the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$ and the plane $x + y + 4 = 0$ is 45°.

Choose the correct option.

(a) Statement I is true (b) Statement II is true
(c) Both statements are true (d) Both statements are false

48. Which of the following is/are true?

I. The vector equation of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ is $r = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

II. If a plane has intercepts a, b, c and is at a distance of p units from the origin, then $p^2 = a^2 + b^2 + c^2$.

(a) Only I is true (b) Only II are true
(c) Both I and III are true (d) Neither I nor II is true

49. Which of the following is/are true?

I. The reflection of the point (α, β, γ) in the xy-plane is $(\alpha, \beta, -\gamma)$.

II. The locus represented by $xy + yz = 0$ is a pair of perpendicular lines.

III. The line $r = 2i - 3\hat{j} - \hat{k} + \lambda(\hat{i} - j + 2\hat{k})$ lies in the plane $r = (3\hat{i} + j - \hat{k}) + 2 = 0$.

(a) Only I is true (b) I and II are true
(c) I and III are true (d) II and III are true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

50.

	Column-I (Lines)		Column-II (Direction cosines)
A.	A line makes angles 90°, 135°, 45° with X, Y and Z-axis, respectively.	1.	$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
B.	A line which makes equal angles with coordinates axis.	2.	$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$
C.	A line has the direction ratio –18, 12 and –4.	3.	$0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Codes

	A	B	C
(a)	1	3	2
(b)	2	3	1
(c)	3	1	2
(d)	2	1	3

51.

	Column-I (Planes)		Column-II (Their equations)
A.	The plane which cut equal intercepts of unit length on the axes	1.	$by + cz + d = 0$
B.	The plane through (2, 3, 4) and parallel to the plane $x + 2y + 4z = 5$.	2.	$x + y + z = 1$
C.	The plane parallel to X-axis	3.	$x + 2y + 4z = 24$

Codes

	A	B	C
(a)	1	2	3
(b)	3	1	2
(c)	2	3	1
(d)	2	1	3

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

52. If vector equation of the line $\frac{x-2}{2} = \frac{2y-5}{-3} = z+1$, is $\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - \frac{3}{2}\hat{j} + p\hat{k}\right)$ then p is equal to

(a) 0 (b) 1 (c) 2 (d) 3

53. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{2}$

54. If two points are P (7, –5, 11) and Q (–2, 8, 13), then the projection of PQ on a straight line with direction cosines $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ is

(a) $\frac{1}{2}$ (b) $\frac{26}{3}$ (c) $\frac{4}{3}$ (d) 7

55. The lines whose vector equations are

$r = 2\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(2\hat{i} + p\hat{j} + 5\hat{k})$

and $r = \hat{i} - 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + p\hat{j} + p\hat{k})$

are perpendicular for all values of λ and μ if p =

(a) 1 (b) –1 (c) – 6 (d) 6

56. What is the length of the projection of $3\hat{i} + 4\hat{j} + 5\hat{k}$ on the xy-plane ?

(a) 3 (b) 5 (c) 7 (d) 9

57. What is the angle between the line 6x = 4y = 3z and the plane 3x + 2y – 3z = 4 ?

(a) 0 (b) $\pi/6$ (c) $\pi/3$ (d) $\pi/2$

58. A rectangular parallelopiped is formed by drawing planes through the points (–1, 2, 5) and (1, –1, –1) and parallel to the coordinate planes. The length of the diagonal of the parallelopiped is

(a) 2 (b) 3 (c) 6 (d) 7

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion

(c) Assertion is correct, Reason is incorrect

(d) Assertion is incorrect, Reason is correct.

59. **Assertion:** If a variable line in two adjacent positions has direction cosines l, m, n, and $l + \delta l$, m + δm, n + δn, then the small angle $\delta\theta$ between the two positions is given by

$\delta\theta = \delta l^2 + \delta m^2 + \delta n^2$

Reason: If O is the origin and A is (a, b, c), then the equation of plane through at right angle to OA is given by

$ax + by + cz = a^2 + b^2 + c^2$.

60. **Assertion:** The pair of lines given by $\vec{r} = \hat{i} - \hat{j} + \lambda(2i + k)$ and $\vec{r} = 2\hat{i} - \hat{k} + \mu(i + \hat{j} - k)$ intersect.

Reason: Two lines intersect each other, if they are not parallel and shortest distance = 0.

61. Consider the lines

$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$, $L_2 : \frac{x-2}{2} = \frac{y-2}{2} = \frac{z-3}{3}$,

Assertion: The distance of point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is $\frac{13}{5\sqrt{3}}$.

Reason: The unit vector perpendicular to both the lines L_1 and L_2 is $\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$.

62. **Assertion :** Distance of a point with position vector a from a plane r. N = d is given by |a . N – d|.

Reason : The length of perpendicular from origin O to the plane r . N = d is $\frac{|d|}{|N|}$.

63. Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5.

Assertion : The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.

Reason : The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes.

64. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = 1$

$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

Assertion : At least two of the lines L_1, L_2 and L_3 are non-parallel

Reason : The three planes doe not have a common point.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

65. The distance between the lines given by

$\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is

(a) $\sqrt{\frac{59}{14}}$ (b) $\sqrt{\frac{59}{7}}$ (c) $\sqrt{\frac{118}{7}}$ (d) $\frac{\sqrt{59}}{7}$

66. Four points (0, –1, –1) (–4, 4, 4) (4, 5, 1) and (3, 9, 4) are coplanar. Find the equation of the plane containing them.

(a) 5x + 7y + 11z – 4 = 0 (b) 5x – 7y + 11z + 4 = 0

(c) 5x – 7y –11z – 4 = 0 (d) 5x + 7y – 11z + 4 = 0

67. A variable plane remains at constant distance p from the origin.If it meets coordinate axes at points A, B, C then the locus of the centroid of Δ ABC is

(a) $x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$

(b) $x^{-3} + y^{-3} + z^{-3} = 9p^{-3}$

(c) $x^2 + y^2 + z^2 = 9p^2$

(d) $x^3 + y^3 + z^3 = 9p^3$

68. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then cos α equals

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

69. The line $\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-1}{3}$ and the plane $x+2y+z=6$ meet at

(a) no point.
(b) only one point.
(c) infinitely many points.
(d) none of these.

70. The plane passing through the point (–2, –2, 2) and containing the line joining the points (1, 1, 1) and (1, –1, 2) makes intercepts on the coordiantes axes, the sum of whose lengths is

(a) 3 (b) – 4
(c) 6 (d) 12

71. The three planes $x + y = 0$, $y + z = 0$ and $x + z = 0$

(a) meet in a unique point
(b) meet in a line
(c) meet taken two at a time in parallel lines
(d) None of these

72. The projections of a line segment on the coordinate axes are 12, 4, 3. The direction cosine of the line are:

(a) $-\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$ (b) $\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$

(c) $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$ (d) None of these

73. If the points (1, 1, p) and (–3, 0, 1) be equidistant from the plane $r.(3\hat{i}+4j-12\hat{k})+13=0$, then the value of p is

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

74. The equation of the line passing through (1, 2, 3) and parallel to the planes $r.(\hat{i}-\hat{j}+2\hat{k})=5$ and $r.(3\hat{i}+\hat{j}+\hat{k})=6$ is

(a) $r=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$

(b) $r=(-3\hat{i}+5\hat{j}+4\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$

(c) $r=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(-3\hat{i}+5\hat{j}+4\hat{k})$

(d) $r=\lambda(-3\hat{i}+5\hat{j}+4\hat{k})$

75. The equation of two lines through the origin, which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$ each, are

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{1};\frac{x}{1}=\frac{y}{1}=\frac{z}{2}$

(b) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1};\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$

(c) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1};\frac{x}{1}=\frac{y}{-1}=\frac{z}{-2}$

(d) None of the above

76. The length intercepted by a line with direction ratios 2, 7, –5 between the lines

$\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}$ and $\frac{x+3}{-3}=\frac{y-3}{2}=\frac{z-6}{4}$ is

(a) $\sqrt{75}$ (b) $\sqrt{78}$

(c) $\sqrt{83}$ (d) None of these

77. The lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular if

(a) $aa' + bb' + cc' + 1 = 0$ (b) $aa' + bb' + 1 = 0$
(c) $bb' + cc' + 1 = 0$ (d) $aa' + cc' + 1 = 0$

78. The equation of the right bisector plane of the segment joining (2, 3, 4) and (6, 7, 8) is

(a) $x + y + z + 15 = 0$ (b) $x + y + z – 15 = 0$
(c) $x – y + z – 15 = 0$ (d) None of these

79. The locus of a point, such that the sum of the squares of its distances from the planes $x + y + z = 0$, $x – z = 0$ and $x – 2y + z = 0$ is 9, is

(a) $x^2+y^2+z^2=3$ (b) $x^2+y^2+z^2=6$

(c) $x^2+y^2+z^2=9$ (d) $x^2+y^2+z^2=12$

80. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}\,z+4=0$ is such that $\sin\theta=\frac{1}{3}$ then the value of λ is

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) The lines intersect if

$$\begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$\Rightarrow 2k^2 + 5k - 25 = 0$

$\Rightarrow k = -5, \frac{5}{2}.$

2. (a) Let the line be $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ ... (i)

If line (i) intersects with the line $\frac{x-1}{2} = \frac{y+3}{4} = \frac{z-5}{3}$, then

$$\begin{vmatrix} a & b & c \\ 2 & 4 & 3 \\ 4 & -3 & 14 \end{vmatrix} = 0$$

$\Rightarrow 9a - 7b - 10c = 0$ ...(ii)

from (i) and (ii), we have

$$\frac{a}{1} = \frac{b}{-3} = \frac{c}{5}$$

$\therefore$ The line is $\frac{x}{1} = \frac{y}{-3} = \frac{z}{5}$

3. (d) Given A(1, 2, 3), B(–1, –2, –1) and C(2, 3, 2), Let D be (α, β, γ). Since ABCD is a parallelogram, diagonals AC and BD bisect each other i.e., mid-point of segment AC is same as mid-point of segment BD.

$$\Rightarrow \left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = \left(\frac{\alpha-1}{2}, \frac{\beta-2}{2}, \frac{\gamma-1}{2}\right)$$

$\Rightarrow \alpha - 1 = 3, \beta - 2 = 5, \gamma - 1 = 5$

$\Rightarrow \alpha = 4, \beta = 7, \gamma = 6$

Hence, the point D is (4, 7, 6).

We have C(2, 3, 2) and D(4, 7, 6).

$\therefore$ Equation of line CD is

$$\frac{x-2}{4-2} = \frac{y-3}{7-3} = \frac{z-2}{6-2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-3}{4} = \frac{z-2}{4}$$

i.e., $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-2}{2}$

is the required equation of line.

4. (d) $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ have vector form

$= (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

Required equation in vector form is

$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$

5. (a) Let $a_1 = 2x, b_1 = 2x, c_1 = x$

and $a_2 = 7 - 3 = 4, b_2 = 2 - 1 = 1, c_2 = 12 - 4 = 8$

$$\therefore \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2x \times 4 + 2x \times 1 + x \times 8}{\sqrt{4x^2 + 4x^2 + x^2}\sqrt{16 + 1 + 64}}$$

$$= \frac{18x}{3x \times 9} = \frac{2}{3}$$

$\Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$

6. (b) Let $P(\vec{r})$ be any point on the plane.

Clearly $\vec{r} - \vec{a}$ will be in linear combination of $\vec{b} - \vec{a}$ and $\vec{c} - \vec{a}$

$\Rightarrow \vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}$ will be coplanar

$\Rightarrow (\vec{r} - \vec{a}).\{(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\} = 0$

$\Rightarrow (\vec{r} - \vec{a}) \cdot \{\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\} = 0$

$\Rightarrow \vec{r}.\{\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}\} = [\vec{a}\ \vec{b}\ \vec{c}]$

7. (b) Let $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then,

$$\hat{n} = \frac{n}{|\hat{n}|} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4 + 9 + 16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

Hence, the required equation of the plane is

$$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} + \frac{-3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

8. (a)

9. **(d)** (k, k, k) is direction cosines of a line. Then,
$k^2 + k^2 + k^2 = 1$
$\Rightarrow 3k^2 = 1$
$\Rightarrow k = \pm\frac{1}{\sqrt{3}}$

10. **(d)** D.r.'s of the line is (3, 4, 5) ans d.r's of the normal of the plane is (2, –2, 1).
Let θ be the angle between line and plane, then (90 – θ) be the angle between the line and normal of the plane.

$$\therefore \cos(90-\theta) = \frac{3\times 2+4(-2)+5\times 1}{\sqrt{3^2+4^2+5^2}\sqrt{2^2+(-2)^2+1^2}}$$

$$\Rightarrow \sin\theta = \frac{6-8+5}{\sqrt{50}\sqrt{9}}$$

$$\Rightarrow \sin\theta = \frac{3}{5\sqrt{2}\times 3}$$

$$\Rightarrow \sin\theta = \frac{1}{5\sqrt{2}} \Rightarrow \sin\theta = \frac{\sqrt{2}}{10}$$

11. **(b)** Let the angle of line makes with the positive direction of *z*-axis is α. Direction cosines of line with the +ve directions of *x*-axis, *y*-axis, and *z*-axis is ℓ, *m*, *n* respectively.

$$\therefore \ell = \cos\frac{\pi}{4}, m = \cos\frac{\pi}{4}, n = \cos\alpha$$

as we know that, $\ell^2 + m^2 + n^2 = 1$

$$\therefore \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\alpha = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\alpha = 1$$

$$\Rightarrow \cos^2\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$$

Hence, angle with positive direction of the *z*-axis is $\frac{\pi}{2}$

12. **(a)** $\because$ The line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane
$x + 3y - \alpha z + \beta = 0$
$\therefore$ Point (2, 1, – 2) lies on the plane
i.e. $2 + 3 + 2\alpha + \beta = 0$
or $2\alpha + \beta + 5 = 0$ ....(i)
Also normal to plane will be perpendicular to line,
$\therefore 3 \times 1 - 5 \times 3 + 2 \times (-\alpha) = 0$
$\Rightarrow \alpha = -6$
From equation (i) we have, $\beta = 7$
$\therefore (\alpha, \beta) = (-6, 7)$

13. **(b)** $\cos\theta = \frac{b_1 \cdot b_2}{|b_1||b_2|}$

14. (c) Equation of plane passing through (1, 1, 1) and perpendicular to line

$\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ is $3x + 4z - 7 = 0$

Distance of plane from the origin $= \left|\frac{0+0-7}{\sqrt{9+16}}\right| = \frac{7}{5}$

15. **(c)** **Note:** The perpendicular distance from the origin to the plane ax + by + cz + d = 0 is given by

$$\frac{d}{\sqrt{a^2+b^2+c^2}}$$

Thus the perpendicular distance from origin to the plane $2x - y + 3z + 4 = 0$ is

$$P_1 = \frac{4}{\sqrt{14}}$$

Similarly, $P_2 = \frac{-3}{3\sqrt{14}} = \frac{-1}{\sqrt{14}}$

Thus the distance between the two parallel planes

$$= |P_2 - P_1| = \frac{5}{\sqrt{14}}.$$

16. **(b)** It is a fundamental concept.

17. **(b)** As $\frac{a}{(1/bc)} = \frac{b}{(1/ca)} = \frac{c}{(1/ab)}$,

Hence lines are parallel.

18. **(c)** For $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, k\right)$ to represent direction cosines, we should have

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + k^2 = 1$$

or, $\frac{1}{2} + \frac{1}{4} + k^2 = 1$

$$\Rightarrow k = \pm\frac{1}{2}$$

19. **(c)** If the plane ax + by + cz + d = 0 to be perpendicular to xy- plane, the coefficient of z is equal to zero.
$\Rightarrow c = 0$

20. (c) Direction cosines of the line are

$$\frac{6}{\sqrt{\{(6)^2+(2)^2+(3)^2\}}}, \frac{2}{\sqrt{\{(6)^2+(2)^2+(3)^2\}}},$$

$$\frac{3}{\sqrt{\{(6)^2+(2)^2+(3)^2\}}} \text{ i.e., } \frac{6}{7}, \frac{2}{7}, \frac{3}{7}$$

$\therefore$ Projection of the line segment joining the points on the given line

$$= \frac{6}{7}(2+1)+\frac{2}{7}(5-0)+\frac{3}{7}(1-3)=\frac{22}{7}.$$

21. (d) Given $\alpha = 45°, \beta = 60°, \gamma = ?$

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\therefore \cos^2\gamma = 1-\frac{1}{2}-\frac{1}{4}=\frac{1}{4} \Rightarrow \gamma = 60° \text{ or } 120°$$

22. (d) Obviously the line perpendicular to the plane because

$\frac{a}{a}=\frac{b}{b}=\frac{c}{c}$ i.e., their direction ratios are proportional.

23. (d) The projection of the segment on the coordinates axes are –9, 12, –8. Thus the direction ratios of the segment PQ are –9, 12, –8. Hence the direction cosines are

$$-\frac{9}{17}, \frac{12}{17}, \frac{-8}{17}$$

24. (b) Equation of plane through (1, 0, 0) is

$a(x-1)+by+cz=0$ ...(i)

(i) passes through (0, 1, 0).

$-a+b=0 \Rightarrow b=a.;$

Also, $\cos 45° = \frac{a+a}{\sqrt{2(2a^2+c^2)}}$

$\Rightarrow 2a = \sqrt{2a^2+c^2} \Rightarrow 2a^2 = c^2$

$\Rightarrow c = \sqrt{2}a.$

So, d.r of normal are a, a $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

25. (a) Let the equation of the required plane be

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ...(i)

It meets co-ordinate axes in points

A (a, 0, 0), B(0, b, 0), C(0, 0, c).

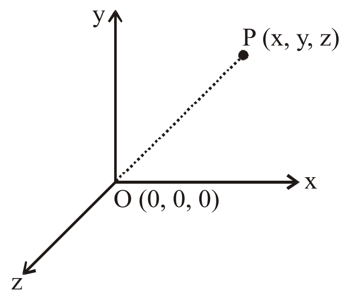
The centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$$\Rightarrow \frac{a}{3}=\alpha, \frac{b}{3}=\beta, \frac{c}{3}=\gamma \Rightarrow a=3\alpha, b=3\beta, c=3\gamma$$

Hence the required plane is

$$\frac{x}{3\alpha}+\frac{y}{3\beta}+\frac{z}{3\gamma}=1 \text{ i.e., } \frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3.$$

26. (a) Let coordinates of P be (x, y, z), O is origin.



Direction ratios of OP are,

$a = 0 - x$

$b = 0 - y$

$c = 0 - z$

As given: a, b, c are equal

$\Rightarrow x = y = z$

$\Rightarrow OP = \sqrt{(0-x)^2+(0-y)^2+(0-z)^2} = \sqrt{3}$

[$\because OP = \sqrt{3}$]

$\Rightarrow \sqrt{3x^2} = \sqrt{3} \Rightarrow 3x^2 = 3$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

$\Rightarrow x = -1, y = -1, z = -1$ or $x = 1, y = 1, z = 1$

$\therefore$ Coordinates of P = (– 1, – 1, – 1) is given in the choice.

27. (c) The equation of plane which contains z-axis is $3x + 2y = 0$ as z is absent in this equation.

28. (a) Equation of the line through the given points is

$$\frac{x-3}{5-3}=\frac{y-4}{1-4}=\frac{z-1}{6-1}$$

$$\Rightarrow \frac{x-3}{2}=\frac{y-4}{-3}=\frac{z-1}{5}$$

Any point on this line can be taken as

$(3 + 2\lambda, 4 - 3\lambda, 1 + 5\lambda)$

If this point lies on XY-plane then the z-coordinate is zero

$$\Rightarrow 1 + 5\lambda = 0 \Rightarrow \lambda = -\frac{1}{5}$$

Thus the required coordinates of the point are

$$\left(3-\frac{2}{5}, 4-3\left(-\frac{1}{5}\right), 0\right) \equiv \left(\frac{13}{5}, \frac{23}{5}, 0\right)$$

29. (b) If the given points $(\lambda, \mu, -6)$, $(3, 2, -4)$ and $(9, 8, -10)$ are collinear then

$$\frac{\lambda-3}{9-3}=\frac{\mu-2}{8-2}=\frac{-6+4}{-10+4} \Rightarrow \lambda=5, \mu=4$$

30. (b) Any three numbers which are proportional to the direction cosines of a line, are called the direction ratios of the line. If l, m and n are direction cosines and a, b and c are direction ratios of a line, then $a = \lambda l$, $b = \lambda m$ and $c = \lambda n$ for any non-zero $\lambda \in R$.

31. (b) Given equation of line is

$$\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}=\lambda \text{ (let)}$$

$\Rightarrow x = 3\lambda - 2 y = 2\lambda - 1, z = 2\lambda + 3$ ...(i)

$\therefore$ Coordinates of any point on the line are $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$

The distance between this point and (1, 2, 3) is $\frac{6}{\sqrt{2}}$

$$\therefore \sqrt{(3\lambda-2-1)^2+(2\lambda-1-2)^2+(2\lambda+3-3)^2}=\frac{6}{\sqrt{2}}$$

$$\Rightarrow (3\lambda-3)^2+(2\lambda-3)^2+(2\lambda)^2=\frac{36}{2}$$

Squaring on both sides

$\Rightarrow 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 9 - 12\lambda + 4\lambda^2 = 18$

$\Rightarrow \lambda(17\lambda - 30) = 0$

$\Rightarrow \lambda = 0, \frac{30}{17}$

Substituting the values of λ in eq. (i) we get the required point $(-2, -1, 3)$ and $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$

32. (b) We find the angle between two lines when their equation are given if θ is the acute angle between the lines $r = a_1 + \lambda b_1$ and $r = a_2 + \mu b_2$,

then $\cos\theta = \left|\frac{b_1 \cdot b_2}{|b_1||b_2|}\right|$

33. (c) In a space, these are lines which are neither intersecting nor parallel. Infact, such pair of lines are non-coplanar and are called skew-lines.

34. (a) Given equation of plane is

$x - 3y + 5z = d$

Since, it passes through (1, 2, 4), then

$1 - 3(2) + 5(4) = d$

$\Rightarrow 1 - 6 + 20 = d$

$d = 15$

Therefore, the equation of the plane will be

$x - 3y + 5z = 15$

$$\Rightarrow \frac{x}{15}-\frac{3y}{15}+\frac{5z}{15}=1$$

$$\Rightarrow \frac{x}{15}+\frac{y}{-5}+\frac{z}{3}=1$$

Hence, the intercept cut by the plane on axes X, Y, Z are 15, –5 and 3, respectively

35. (c) The planes are perpendicular to each other, if $n_1 . n_2 = 0$ and parallel, if n_1 is parallel to n_2.

36. (c) Let θ be the angle between the planes.

$A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$

The direction ratios of the normal to the planes are A_1, B_1, C_1 and A_2, B_2, C_2, respectively

$$\text{Therefore, } \cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$$

37. (b) Here, $a = 2\hat{i}+5\hat{j}-3\hat{k}$, $N = 6\hat{i}-3\hat{j}+2\hat{k}$ and $d = 4$.

Therefore, the distance of the point (2, 5, –3) from the given plane is $\frac{\left|(2\hat{i}+5\hat{j}-3\hat{k}).(6\hat{i}-3\hat{j}+2\hat{k})-4\right|}{\left|6\hat{i}-3\hat{j}+2\hat{k}\right|}$

$$=\frac{|12-15-6-4|}{\sqrt{36+9+4}}=\frac{13}{7} \quad \left(\because \text{distance} = \left|\frac{a.N-d}{N}\right|\right)$$

38. (a) Given equation of line is

$r = 2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$

$\left(x\hat{i}+y\hat{j}+z\hat{k}\right) = (2+3\lambda)\hat{i}+(-1+4\lambda)\hat{j}+(2+2\lambda)\hat{k}$

Any point on the line is

$(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$

Since it also lie on the plane $r \cdot \left(\hat{i}-\hat{j}+\hat{k}\right)$

So, $\left[(2+3\lambda)\hat{i}+(-1+4\lambda)\hat{j}+(2+2\lambda)\hat{k}\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$\Rightarrow 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5$

$\Rightarrow \lambda = 0$

Therefore, coordinate of the point of intersection of line and plane is (2,–1, 2).

$$\therefore \text{Distance } d=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$$
$$=13$$

39. (c) Since, OP is perpendicular to the plane

$\therefore N = OP = \hat{i}+2\hat{j}-3\hat{k}$

Plane is passing through P

$\therefore a = \hat{i}+2\hat{j}-3\hat{k}$

Therefore, equation of the plane is

$r.N = a.N$

$\Rightarrow r.(\hat{i}+2\hat{j}-3\hat{k}) = (\hat{i}+2\hat{j}-3\hat{k}).(\hat{i}+2\hat{j}-3\hat{k})$

$x + 2y - 3z = 14$

40. (a) Solving the equations of the planes we get y = –2. Put z = 0 and y = –2 in any of the plane we get x = 5 so (5, –2, 0) is a point on the line of intersection. If the line has direction cosines proportional to l, m, n, then $l + m + 2n = 0$ and $2l + 3m + 4n = 0$

On solving we get, $\frac{l}{-2} = \frac{m}{0} = \frac{n}{1}$ and so the equation of line is $\frac{x-5}{-2} = \frac{y+2}{0} = \frac{z}{1}$

Equation of z-axis is $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$

$$\therefore \text{ shortest distance} = \frac{\begin{vmatrix} 5 & -2 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}}{\sqrt{(0-0)^2+(0+2)^2+(0-0)}}$$

$$= \frac{4}{2} = 2$$

STATEMENT TYPE QUESTIONS

41. (c) Equations

$ax + by + cz + d = 0, \ a'x + b'y + c'z + d' = 0$

represent a straight line and equation of the form

$$\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

represent a straight line passing through the point (α, β, γ) and having direction ratios proportional to ℓ, m, n. Thus, both statements are correct.

42. (b) If the given line in space does not pass through the origin, then in order to find its direction cosines, we draw a line through the origin and parallel to the given line. Now, take one of the direction lines from the origin and find its direction cosines as two parallel line have same set of direction cosines.

43. (a) For any line, if a, b and c are direction ratios of a line then ka, kb, kc; $k \neq 0$ is also a set of direction ratios. So, any two sets of direction ratios of a line are also proportional. Also, for any line there are infinitely many sets of direction ratios.

44. (c) I. We have $x_1 = 1, y_1 = 2, z_1 = 3$

$x_2 = -2, y_2 = 3, z_2 = 4$

$x_3 = 7, y_3 = 0, z_3 = 1$

$$\Rightarrow \frac{x_2 - x_1}{x_3 - x_2} = \frac{y_2 - y_1}{y_3 - y_2} = \frac{z_2 - z_1}{z_3 - z_2}$$

$$\Rightarrow \frac{-2-1}{7-(-2)} = \frac{3-2}{0-3} = \frac{4-3}{1-4}$$

$$\Rightarrow \frac{-1}{3} = \frac{-1}{3} = \frac{-1}{3}$$

$\therefore$ Given points are collinear.

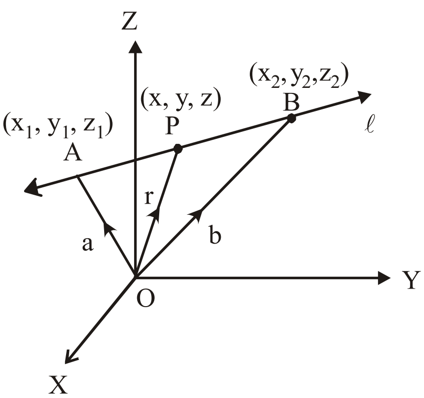
II. $\ell = \cos\frac{\pi}{2} = 0$

$$m = \cos\frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$$

$$n = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$\therefore$ Direction cosines are $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

45. (c) I. Let a and b be the position vectors of two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, respectively that are lying on a line



Let r be the position vector of an arbitrary point P(x, y, z) then P is a point on the line if and only if AP = r – a and AB = b – a are collinear vectors. Therefore, P is on the line if and only if

$r - a = \lambda(b - a)$

or $r = a + \lambda(b - a), \lambda \in R$ ... (i)

This is the vector equation of the line.

II. We have,

$r = x\hat{i} + y\hat{j} + z\hat{k}$

$a = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

and $b = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

On substituting these values in eq. (i), we get

$$x\hat{i}+y\hat{j}+z\hat{k}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}+\lambda(x_2-x_1)\hat{i}$$

Equating the like coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$, we get

$$x = x_1+\lambda(x_2-x_1)$$

$$y = y_1+\lambda(y_2-y_1)$$

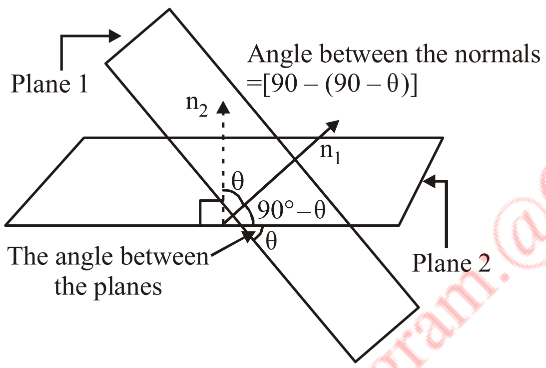
$$z = z_1+\lambda(z_2-z_1)$$

On eliminating λ, we obtain

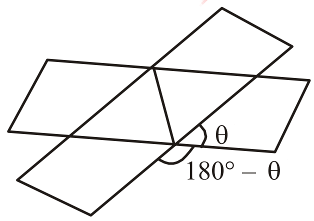
$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

which is the equation of the line in cartesian form.

46. (b) The angle between two planes is defined as the angle between their normals [Fig (a)]. Observe that, if θ is an angle between the two planes, then so is $180 - \theta$ [Fig (b)].



(a)



(b)

We shall take the acute angle as the angles between two planes.

47. (b) I. Given equation of planes are

$x + 2y + 2z = 3$

and $-5x + 3y + 4z = 9$

Here, $a_1 = 1, b_1 = 2, c_1 = 2$

and $a_2 = -5, b_2 = 3, c_2 = 4$

$$\because \quad \cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$=\frac{1(-5)+2(3)+2(4)}{\sqrt{1^2+2^2+2^2}\sqrt{(-5)^2+3^2+4^2}}$$

$$=\frac{-5+6+8}{\sqrt{9}\sqrt{50}}=\frac{9}{15\sqrt{2}}=\frac{3\sqrt{2}}{10}$$

II. Given, Line

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$$

and plane $x + y + 4 = 0$

Here, $b = 2\hat{i}+\hat{j}-2\hat{k}$ and $n = \hat{i}+\hat{j}$

$$\sin\theta=\frac{b\,.\,n}{|b||n|}$$

$$=\frac{(2\hat{i}+\hat{j}-2\hat{k}).(\hat{i}+\hat{j})}{\sqrt{2^2+1^2+(-2)^2}\sqrt{1^2+1^2}}$$

$$=\frac{2+1}{\sqrt{9}\sqrt{2}}=\frac{3}{3\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\theta=\sin^{-1}\frac{1}{\sqrt{2}}=\frac{\pi}{4}\text{or}45°$$

48. (a) I. Any line passing through (1, 2, –4) can be written as

$$\frac{x-1}{a}=\frac{y-2}{b}=\frac{z+4}{c} \quad ...(i)$$

where, a, b, c are the direction ratios of line (i).
Now, the line (i) is perpendicular to the lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$$

and $\frac{x-15}{3}=\frac{y+20}{8}=\frac{z-5}{-5}$

In above lines direciton ratios are (3, –16, 7) and (3, 8, –5) respectively, which is perpendicular with the eq. (i).

$3a - 16b + 7c = 0$ ...(ii)

and $3a + 8b - 5c = 0$ ...(iii)

By cross-multiplication, we have

$$\frac{a}{80-56}=\frac{b}{21+15}=\frac{c}{24+48}$$

$$\Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{6}=\lambda$$

$a = 2\lambda, b = 3\lambda$, and $c = 6\lambda$

The equation of required line which passes through the point (1, 2, –4) and parallel to vector $2\hat{i}+3\hat{j}+6\hat{k}$ is $r=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$

49. (a) I. It is obvious the reflection of the point (α, β, γ) in xy-plane is (α, β, –γ).

II. It is false, since xy + yz = 0 represent a pair of perpendicular planes.

III. If the line $r=2i-3\hat{j}-\hat{k}+\lambda(\hat{i}-j+2\hat{k})$ lies in the plane $r=(3\hat{i}+j-\hat{k})+2=0$, then $(2\hat{i}-3j-\hat{k})$ must satisfy the plane.

$\Rightarrow\ (2\hat{i}-3j-\hat{k}).(3\hat{i}+\hat{j}-\hat{k})+2=0$

$\Rightarrow\ 6=0$

which is not true

∴ the given line does not lie in the given plane.

MATCHING TYPE QUESTIONS

50. (c) A. Let direction cosines of the line be l, m and n with the X, Y, and Z-axis respectively and given that $\alpha=90°;\ \beta=135°$ and $\gamma=45°$

Then, $l=\cos\alpha=\cos 90°=0$

$m=\cos\beta=\cos 135°=\frac{-1}{\sqrt{2}}$

and $n=\cos\gamma=\cos 45°=\frac{1}{\sqrt{2}}$

Therefore, the direction cosines of the line are

$0,-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

B. Let line make an angle α with each of the three coordinate axes. Then its direction cosines are

$l=\cos\alpha,\ m=\cos\alpha,\ n=\cos\alpha.$

We know that, $l^2+m^2+n^2=1$

$\Rightarrow\ \cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$ $(\because\ l=m=n=\cos\alpha)$

$\Rightarrow\ \cos\alpha=\pm\frac{1}{\sqrt{3}}$

∴ Direction cosines of the line are either

$\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

or $-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}$.

C. Given direction ratios are – 18, 12, and –4.

Here, a = –18, b = 12 and c = – 4. Then direction cosines of a line are

$$\left(\frac{a}{\sqrt{a^2+b^2+c^2}},\frac{b}{\sqrt{a^2+b^2+c^2}},\frac{c}{\sqrt{a^2+b^2+c^2}}\right)$$

$$=\left(\frac{18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}.\frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}},\frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}\right)$$

$$=\left(\frac{-9}{11},\frac{6}{11},\frac{-2}{11}\right)$$

Thus, the direction cosines are $-\frac{9}{11},\frac{6}{11}$ and $\frac{-2}{11}$.

51. (c) A. The equation of the plane cut off equal intercept is given by $\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$

$\Rightarrow\ \frac{x}{1}+\frac{y}{1}+\frac{z}{1}=1$ $(\because\ a=1,\text{ given})$

$\Rightarrow\ x+y+z=1$

B. Since, the plane is parallel to x + 2y + 4z = 5

∴ Normal vector N = $\hat{i}+2\hat{j}+4\hat{k}$

∴ Equation of the plane passing through (2, 3, 4) and parallel to x + 2y + 4z = 5 is

(x – 2) + 2(y – 3) + 4(z – 4) = 0

$\Rightarrow$ x + 2y + 4z – 24 = 0

C. Let the equation of plane is given by

ax + by + cz + d = 0

Since, it is parallel to X-axis, then a = 0

So, required equation of plane is

by + cz + d = 0

INTEGER TYPE QUESTIONS

52. (a) The given line is

$$\frac{x-2}{2}=\frac{2y-5}{-3}=z+1,$$

$$\Rightarrow\ \frac{x-2}{2}=\frac{y-\frac{5}{2}}{-\frac{3}{2}}=\frac{z+1}{0}$$

This shows that the given line passes through the point $\left(2, \frac{5}{2}, -1\right)$ and has direction ratios $\left(2, \frac{-3}{2}, 0\right)$. Thus, given line passes through the point having position vector $\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}$ and is parallel to the vector $\vec{b} = \left(2\hat{i} - \frac{3}{2}\hat{j} - 0\hat{k}\right)$. So, its vector equation is

$$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - \frac{3}{2}\hat{j} - 0\hat{k}\right).$$

Hence, p = 0

53. (b) The lines are $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$

and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

Here,

$\vec{a}_1 = -2\hat{j} + \hat{k}, b_1 + 6\hat{i} + 6\hat{j} + \hat{k}, \vec{a}_2 = -\hat{i},$

$\vec{b}_2 = 12\hat{i} + 6\hat{j} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

$$\text{Shortest distance} = \frac{\left|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 - \vec{b}_2)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$= \frac{\left|(-\hat{i} + 2\hat{j} - \hat{k}).(-12i + 18\hat{j} - 36\hat{k})\right|}{\sqrt{(-12)^2 + (18)^2 + (-36)^2}}$$

$$= \frac{|+12 + 36 + 36|}{\sqrt{1764}} = \frac{84}{42} = 2$$

54. (d) The projection of line joining the points P (7, – 5, 11) and Q (–2, 8, 13) on a line with direction cosines $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ is equal to $(-2-7)\frac{1}{3} + (8+5)\frac{2}{3} + (13-11)\frac{2}{3}$

= 7

55. (d) $2\hat{i} - p\hat{j} + 5\hat{k}$ and $3\hat{i} + p\hat{j} + p\hat{k}$ are perpendicular

$\Rightarrow$ 2 × 3 + p (–p) + 5 (p) = 0

$\Rightarrow$ p = –1 or p = 6

Hence for p = 6, the lines are perpendicular.

56. (b) xy-plane is perpendicular to z - axis. Let the vector $\vec{a}$ = 3i + 4j + 5k make angle θ with z - axis, then it makes 90 – θ with xy-plane.

unit vector along z-axis is k.

$$\text{So, } \cos\theta = \frac{\vec{a}.\hat{k}}{|\vec{a}|.|\hat{k}|} = \frac{(3i + 4j + 5k).k}{|3i + 4j + 5k|}$$

$$= \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}.$$

Hence angle with xy- plane $\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

projection of $\vec{a}$ on xy plane $= |\vec{a}|.\cos\frac{\pi}{4}$

$$= 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5.$$

57. (a) The equation of the given line is 6x = 4y = 3z which is written in symmetric form as

$$\frac{x-0}{1/6} = \frac{y-0}{1/4} = \frac{z-0}{1/3}$$

Direction ratios of this line are $\left(\frac{1}{6}, \frac{1}{4}, \frac{1}{3}\right)$ and equation of the plane is 3x + 2y –3z – 4 = 0

If θ be the angle between line and plane, then direction ratios of the normal to this plane is (3, 2, – 3)

$$\sin\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$= \left|\frac{\frac{1}{6} \times 3 + \frac{1}{4} \times 2 + \frac{1}{3}(-3)}{\sqrt{\frac{1}{36} + \frac{1}{16} + \frac{1}{9}}\sqrt{9 + 4 + 9}}\right| = 0$$

$\Rightarrow \theta = 0°$

58. (d) The planes forming the parallelopiped are

x = –1, x = 1; y = 2, y = –1 and z = 5, z = –1

Hence, the lengths of the edges of the parallelopiped are 1– (–1) = 2, | – 1–2| = 3 and |–1–5| = 6

(Length of an edge of a rectangular parallelopiped is the distance between the parallel planes perpendicular to the edge)

∴ Length of diagonal of the parallelopiped

$$= \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7.$$

ASSERTION - REASON TYPE QUESTIONS

59. (b) Let θ be the angle between the two adjacent positions.

$\therefore \cos\theta = l(l+\delta l) + m(m+\delta m) + n(n+\delta n)$

$\Rightarrow \cos\theta = (l^2+m^2+n^2) + l\delta l + m\delta m + n\delta n$

$\Rightarrow \cos\theta = 1 + l\delta l + m\delta m + n\delta n$

Differentiating both sides, we get

$\sin\theta\delta\theta = 0 + \delta l\delta l + \delta m\delta m + \delta n\delta n$

Neglecting the higher order derivatives for the very small angle, we have

$\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$.

$\vec{N} = \overrightarrow{OA} = a\hat{i} + b\hat{j} + c\hat{k}$

The required equation of plane is given by

$[\vec{r} - (a\hat{i} + b\hat{j} + c\hat{k})]\cdot\vec{N} = 0$

$\Rightarrow \vec{r}.(a\hat{i}+bj+c\hat{k}) - (a\hat{i}+bj+c\hat{k}).(a\hat{i}+bj+c\hat{k}) = 0$

$\Rightarrow (x\hat{i}+yj+z\hat{k}).(a\hat{i}+bj+c\hat{k}) - (a^2+b^2+c^2) = 0$

$\Rightarrow ax + by + cz = a^2 + b^2 + c^2$.

60. (a) Here, $\vec{a}_1 = \hat{i} - \hat{j}, \vec{b}_1 = 2\hat{i} + \hat{k}$

$\vec{a}_2 = 2\hat{i} - \hat{k}, \vec{b}_2 = \hat{i} + \hat{j} - \hat{k}$

$\because \vec{b}_1 \neq \lambda\vec{b}_2$, for any scalar λ

$\therefore$ Given lines are not parallel.

$\vec{a}_2 - \vec{a}_1 = (2\hat{i} - \hat{k}) - (\hat{i} - j) = \hat{i} + j - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$= \hat{i}(0-1) - \hat{j}(-2-1) + \hat{k}(2-0)$

$= -\hat{i} + 3\hat{j} + 2\hat{k}$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + (3)^2 + (2)^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

$$SD = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_2 - \vec{b}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$= \left|\frac{(\hat{i} + j - \hat{k}).(-\hat{i} + 3j + 2\hat{k})}{\sqrt{14}}\right|$$

$$= \left|\frac{-1+3-2}{\sqrt{14}}\right| = 0$$

Hence, two lines intersect each other.

Two lines intersect each other, if they are not parallel and shortest distance = 0.

61. (a) Line L_1 and L_2 are parallel to the vectors $\vec{a} = 3i + \hat{j} + 2\hat{k}$ and $\vec{b} = i + 2\hat{j} + 3\hat{k}$ respectively. The unit vector perpendicular to both L_1 and L_2 is

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{1+49+25}} = \frac{-i-7j+5k}{5\sqrt{3}}$$

Now, eq. of plane through (–1, –2, –1) is –(x + 1) – 7(y + 2) + 5 (z + 1) = 0 whose distance from (1, 1, 1) is $\frac{13}{5\sqrt{3}}$.

62. (d) If the equation of the plane is in the form r N = d. where N is normal to the plane, then the perpendicular distance is

$$\frac{|a\,.\,N - d|}{|N|}$$

$\therefore$ Assertion is incorrect.

The length of the perpendicular from origin O to the plane r . N = d is $\frac{|d|}{|N|}$ (since a = 0)

$\therefore$ Reason is correct.

63. (d) **64. (d)**

CRITICALTHINKING TYPE QUESTIONS

65. (b) The given lines are parallel and

$\vec{a}_1 = \hat{i} + \hat{j}, \vec{a}_2 = 2\hat{i} - 3\hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = (2\hat{i} - 3\hat{k}) - (\hat{i} + \hat{j}) = \hat{i} - \hat{j} - 3\hat{k}$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 1 & -1 & -3 \end{vmatrix}$$

$= \hat{i}(6+3) - j(-3-3) + \hat{k}(-1+2)$

$= 9\hat{i} + 6j + \hat{k}$

$|\vec{b}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

Distance, $$d = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right| = \left|\frac{9\hat{i} + 6\hat{j} + \hat{k}}{\sqrt{14}}\right|$$

$$= \frac{1}{\sqrt{14}}\sqrt{(9)^2 + (6)^2 + (1)^2}$$

$$= \sqrt{\frac{59}{7}}$$

66. (b) We shall find the equation of a plane passing through any three out of the given four points and show that the fourth point satisfies the equation.

Now, any plane passing through (0, –1, –1) is:

$a(x - 0) + b(y + 1) + c(z + 1) = 0$ ...(i)

If it passes through (–4, 4, 4); we have

$a(-4) + b(5) + c(5) = 0$ ...(ii)

Also, if plane passes through (4, 5, 1) we have

$a(4) + b(6) + c(2) = 0$

$\Rightarrow 2a + 3b + c = 0$ ...(iii)

Solving (ii) and (iii) by cross multiplication method, we obtain

$$\frac{a}{-5} = \frac{b}{7} = \frac{c}{-11} = k$$

Putting in (i), we get

$-5kx + 7k(y + 1) - 11k(z + 1) = 0$

(Required equation of plane)

Clearly the fourth point namely (3, 9, 4) satisfies this equation hence the given points are coplanar and the equation of plane containing those points is

$5x - 7y + 11z + 4 = 0$

67. (a) Let $A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$, $C \equiv (0, 0, c)$, then equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Its distance from the origin, $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$ ... (i)

If (x, y, z) be centroid of Δ ABC, then

$x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3}$ ... (ii)

Eliminating a,b,c from (i) and (ii) required locus is

$x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$

68. (c) Let the direction cosines of line *L* be *l*, *m*, *n*, then

$2l + 3m + n = 0$ … (i)

and $l + 3m + 2n = 0$ … (ii)

on solving equations (i) and (ii), we get

$$\frac{l}{6-3} = \frac{m}{1-4} = \frac{n}{6-3} \Rightarrow \frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$$

Now $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{3^2 + (-3)^2 + 3^2}}$

$\because l^2 + m^2 + n^2 = 1$

$\therefore \frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{1}{\sqrt{27}}$

$\Rightarrow l = \frac{3}{\sqrt{27}} = \frac{1}{\sqrt{3}}, m = -\frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$

Line *L*, makes an angle α with +ve *x*-axis

$\therefore l = \cos\alpha \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

69. (c) Direction ratios. of given line are 1, –2, 3 and the d.r. of normal to the given plane are 1, 2, 1.

Since $1 \times 1 + (-2) \times 2 + 3 \times 1 = 0$, therefore, the line is parallel to the plane.

Also, the base point of the line (1, 2, 1) lies in the given plane.

$(1 + 2 \times 2 + 1 = 6$ is true)

Hence, the given line lies in the given plane.

70. (b) Equation of any plane passing through (–2, –2, 2) is $a(x + 2) + b(y + 2) + c(z - 2) = 0$

since it contains the line joining the points (1, 1, 1) and (1, –1, 2), it contains these points as well so that $3a + 3b - c = 0$ and $3a + b + 0 = 0$

on solving we get $\frac{a}{1} = \frac{b}{-3} = \frac{c}{-6}$ and thus the equation of the plane is

$x + 2 - 3(y + 2) - 6(z - 2) = 0$

or $\frac{x}{-8} + \frac{y}{8/3} + \frac{z}{8/6} = 1$

The requd. sum $= -8 + \frac{8}{3} + \frac{8}{6} = -4.$

71. (a) The planes $x + y = 0$ i.e. $x = -y$ and $y + z = 0$ i.e. $z = -y$ meet in the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$.

Any point on this line is $(t, -t, t)$. This point lies in the plane $x + z = 0$ if $t + t = 0 \Rightarrow t = 0$.

So the three planes meet in a unique point (0, 0, 0).

72. (c) Let direction cosine of the line be *l*, m, n where Given,

$\cos\theta = \frac{DC's}{r}$

$\Rightarrow DC's = r\cos\theta = rl$

$\Rightarrow rl = 12$ … (i)

similarly $rm = 4$ … (ii)

and $rn = 3$ … (iii)

Squaring and adding equations (i), (ii) and (iii), we get

$r^2(l^2 + m^2 + n^2) = 12^2 + 4^2 + 3^2$

$\Rightarrow r^2 = 169$ $(\because l^2 + m^2 + n^2 = 1)$

$\Rightarrow r = 13$

Now, $l = \frac{\text{projection on } x-\text{axis}}{\text{length of line segment}} = \frac{12}{13}$

Similarly, $m = \frac{4}{13}, n = \frac{3}{13}$

Hence, Direction cosine are $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$.

73. (b) The distance of point (1, 1, p) from the plane $r.(3\hat{i}+4\hat{j}-12\hat{k})+13=0$ or (in cartesian form) $3x + 4y - 12z + 13 = 0$, is

$$d_1 = \left|\frac{3\times1+4\times1-12\times p+13}{\sqrt{3^2+4^2+(-12)^2}}\right|$$

$$= \left|\frac{3+4-12p+13}{\sqrt{169}}\right| = \left|\frac{20-12p}{13}\right| \quad \ldots(i)$$

The distance of the point (–3, 0, 1) from the plane $3x + 4y - 12z + 13 = 0$ is

$$d_2 = \left|\frac{3\times(-3)+4\times0-12\times1+13}{\sqrt{3^2+4^2+(-12)^2}}\right| = \frac{8}{13}$$

According to the given condition,

$d_1 = d_2$

$$\Rightarrow \left|\frac{20-12p}{13}\right| = \frac{8}{13}$$

$$\Rightarrow \frac{20-12p}{13} = \pm\frac{8}{13}$$

Taking +ve sign, we get

$$\frac{20-12p}{13} = \frac{8}{13} \quad \Rightarrow p = 1$$

Taking –ve sign, we get

$$\frac{20-12p}{13} = \frac{8}{13} \quad \Rightarrow p = \frac{28}{12} = \frac{7}{3}$$

74. (c) The equation of line passing through (1, 2, 3) and parallel to b is given by

$$r = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}) \quad \ldots(i)$$

The equations of the given planes are

$$r.(\hat{i}-\hat{j}+2\hat{k}) = 5 \quad \ldots(ii)$$

$$\text{and } r.(3\hat{i}+\hat{j}+\hat{k}) = 6 \quad \ldots(iii)$$

The line in eq. (i) and plane in eq. (ii) are parallel. Therefore, the normal to the plane of eq. (ii) and the given line are perpendicular.

$$\therefore \quad (\hat{i}-\hat{j}+2\hat{k}).\lambda(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}) = 0$$

$$\Rightarrow \lambda(b_1-b_2+2b_3) = 0$$

$$\Rightarrow (b_1-b_2+2b_3) = 0 \quad \ldots(iv)$$

Similarly, $(3\hat{i}+\hat{j}+\hat{k}).\lambda(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}) = 0$

$$\Rightarrow \lambda(3b_1+b_2+b_3) = 0 \quad \ldots(v)$$

From eqs. (iv) and (v), we obtain

$$\frac{b_1}{(-1)\times1-1\times2} = \frac{b_2}{2\times3-1\times1} = \frac{b_3}{1\times1-3(-1)}$$

$$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$$

Therefore, the direction ratios of b are –3, 5 and 4.

$$\therefore \quad b = b_1\hat{i}+b_2\hat{j}+b_3\hat{k} = -3\hat{i}+5\hat{j}+4\hat{k}$$

Substituting the value of b in eq. (i), we obtain

$$r = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(-3\hat{i}+5\hat{j}+4\hat{k})$$

This is the equation of the required line.

75. (b) Given equation of line is

$$\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$$

$\Rightarrow$ DR's of the given line are 2, 1, 1

$\Rightarrow$ DC's of the given line are $\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

Since, required lines make an angle $\frac{\pi}{3}$ with the given line

The DC's of the required lines are

$\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$ and $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ respectively.

Also, both the required lines pass through the origin.

$\therefore$ Equation of required lines are

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} \text{ and } \frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$$

76. (b) The general points on the given lines are respectively $P(5+3t, 7-t, -2+t)$ and $Q(-3-3s, 3+2s, 6+4s)$.

Direction ratios of PQ are

$$< -3-3s-5-3t,\ 3+2s-7+t,\ 6+4s+2-t >$$

i.e., $< -8-3s-3t,\ -4+2s+t,\ 8+4s-t >$

If PQ is the desired line then direction ratios of PQ should be proportional to < 2, 7, –5>, therefore,

$$\frac{-8-3s-3t}{2} = \frac{-4+2s+t}{7} = \frac{8+4s-t}{-5}$$

Taking first and second numbers, we get

$$-56-21s-21t=-8+4s+2t$$

$$\Rightarrow 25s+23t=-48 \quad \ldots (i)$$

Taking second and third members, we get

$$20-10s-5t=56+28s-7t$$

$$\Rightarrow 38s-2t=-36 \quad \ldots (ii)$$

Solving (i) and (ii) for t and s, we get

$s=-1$ and $t=-1$.

The coordinates of P and Q are respectively

$$(5+3(-1),\ 7-(-1),\ -2-1)=(2,\ 8,\ -3)$$

and $(-3-3(-1),\ 3+2(-1),\ 6+4(-1))=(0,\ 1,\ 2)$

$\therefore$ The required line intersects the given lines in the points (2, 8, –3) and (0, 1, 2) respectively.

Length of the line intercepted between the given lines

$$=|PQ|=\sqrt{(0-2)^2+(1-8)^2+(2+3)^2}=\sqrt{78}.$$

77. (d) The equations of the given lines are not in symmetrical form. We first put them in symmetrical form. Equations of the first line are $x=ay+b, z=cy+d$.

These equation can be written as

$$\frac{x-b}{a}=y=\frac{z-b}{c}$$

or $\dfrac{x-b}{a}=\dfrac{y-0}{1}=\dfrac{z-b}{c}$ ...(i)

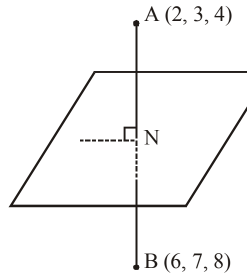
Similarly, equation of second line

$x=a'y+b', z=c'y+d'$ can be written as

$$\frac{x-b'}{a'}=\frac{y-0}{1}=\frac{z-b'}{c'} \quad \text{...(ii)}$$

Clearly, d.r'.s of the first line are a, 1, c and those of the second line are a′, 1, c′. Now, lines (i) and (ii) are perpendicular if $aa'+cc'+1=0$

78. (b) If the given points be A (2, 3, 4) and B (6, 7, 8), then their mid-point N(4, 5, 6) must lie on the plane. The direction ratios of AB are 4, 4, 4, i.e. 1, 1, 1.



$\therefore$ The required plane passes through N (4, 5, 6) and is normal to AB. Thus its equation is

$$1(x-4)+1(y-5)+1(z-6)=0 \Rightarrow x+y+z=15$$

79. (c) Let the variable point be (α, β, γ) then according to question

$$\left(\frac{|\alpha+\beta+\gamma|}{\sqrt{3}}\right)^2+\left(\frac{|\alpha-\gamma|}{\sqrt{2}}\right)^2+\left(\frac{|\alpha-2\beta+\gamma|}{\sqrt{6}}\right)^2=9$$

$$\Rightarrow \alpha^2+\beta^2+\gamma^2=9.$$

So, the locus of the point is $x^2+y^2+z^2=9$

80. (a) Angle between line and normal to plane is

$$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

Where θ is angle between line and plane

$$\Rightarrow \sin\theta=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}=\frac{1}{3} \Rightarrow \lambda=\frac{5}{3}.$$

CHAPTER 28

LINEAR PROGRAMMING

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. L.P.P is a process of finding
(a) Maximum value of objective function
(b) Minimum value of objective function
(c) Optimum value of objective function
(d) None of these

2. L.P.P. has constraints of
(a) one variables
(b) two variables
(c) one or two variables
(d) two or more variables

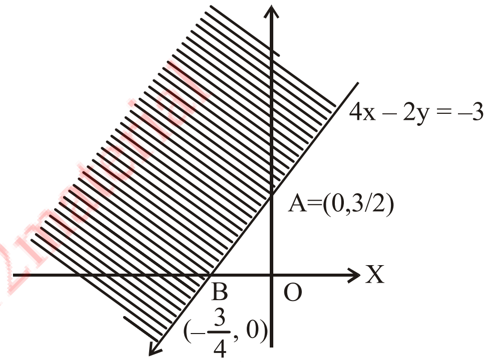
3. Corner points of feasible region of inequalities gives
(a) optional solution of L.P.P.
(b) objective function
(c) constraints.
(d) linear assumption

4. The optimal value of the objective function is attained at the points
(a) Given by intersection of inequations with axes only
(b) Given by intersection of inequations with x- axis only
(c) Given by corner points of the feasible region
(d) None of these.

5. Which of the following statement is correct?
(a) Every L.P.P. admits an optimal solution
(b) A L.P.P. admits a unique optimal solution
(c) If a L.P.P. admits two optimal solutions, it has an infinite number of optimal solutions
(d) The set of all feasible solutions of a L.P.P. is not a convex set.

6. If a point (h, k) satisfies an inequation $ax + by \geq 4$, then the half plane represented by the inequation is
(a) The half plane containing the point (h, k) but excluding the points on $ax + by = 4$
(b) The half plane containing the point (h, k) and the points on $ax + by = 4$
(c) Whole xy-plane
(d) None of these

7. Shaded region is represented by

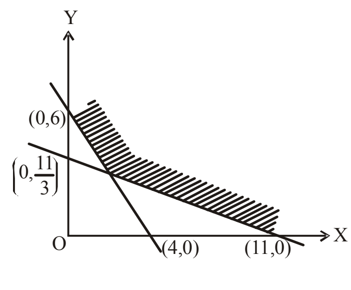


(a) $4x - 2y \leq 3$ (b) $4x - 2y \leq -3$
(c) $4x - 2y \geq 3$ (d) $4x - 2y \geq -3$

8. The maximum value of $z = 5x + 2y$, subject to the constraints $x + y \leq 7$, $x + 2y \leq 10$, $x, y \geq 0$ is
(a) 10 (b) 26
(c) 35 (d) 70

9. The maximum value of $P = x + 3y$ such that $2x + y \leq 20$, $x + 2y \leq 20$, $x \geq 0$, $y \geq 0$ is
(a) 10 (b) 60
(c) 30 (d) None of these

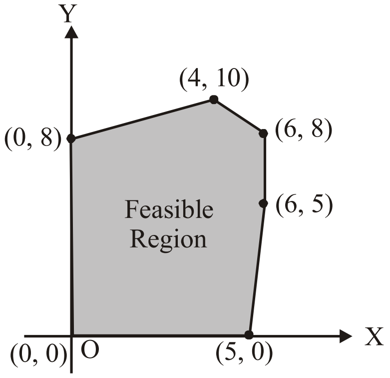
10. For the following feasible region, the linear constraints are



(a) $x \geq 0,\ y \geq 0,\ 3x + 2y \geq 12,\ x + 3y \geq 11$
(b) $x \geq 0,\ y \geq 0,\ 3x + 2y \leq 12,\ x + 3y \geq 11$
(c) $x \geq 0,\ y \geq 0,\ 3x + 2y \leq 12,\ x + 3y \leq 11$
(d) None of these

11. Objective function of a L.P.P. is
(a) a constant
(b) a function to be optimised
(c) a relation between the variables
(d) None of these

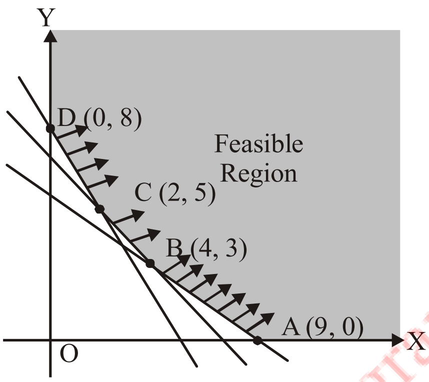
12. The feasible region for an LPP is shown shaded in the figure. Let Z = 3 x – 4 y be the objective function. Minimum of Z occurs at



(a) (0, 0) (b) (0, 8)
(c) (5, 0) (d) (4, 10)

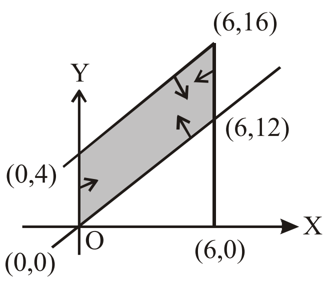
13. Region represented by $x \geq 0$, $y \geq 0$ is
(a) first quadrant (b) second quadrant
(c) third quadrant (d) fourth quadrant

14. Feasible region for an LPP is shown shaded in the following figure. Minimum of Z = 4 x + 3 y occurs at the point.



(a) (0, 8) (b) (2, 5)
(c) (4, 3) (d) (9, 0)

15. The feasible region for LPP is shown shaded in the figure. Let f = 3 x – 4 y be the objective function, then maximum value of f is



(a) 12 (b) 8 (c) 0 (d) –18

16. Maximize Z = 3x + 5y, subject to $x + 4y \leq 24$, $3x + y \leq 21$, $x + y, \leq 9$, $x \geq 0$, $y \geq 0$, is
(a) 20 at (1, 0) (b) 30 at (0, 6)
(c) 37 at (4, 5) (d) 33 at (6, 3)

17. Z = 6x + 21y, subject to $x + 2y \geq 3$, $x + 4y \geq 4$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at
(a) (4, 0) (b) (28, 8)
(c) $\left(2, \frac{1}{2}\right)$ (d) (0, 3)

18. Maximize Z = 4x + 6y, subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$, is
(a) 16 at (4,0) (b) 24 at (0, 4)
(c) 24 at (6, 0) (d) 36 at (0, 6)

19. Shamli wants to invest ₹50,000 in saving certificates and PPE. She wants to invest atleast ₹15,000 in saving certificates and at least ₹20,000 in PPF. The rate of interest on saving certificates is 8% p.a. and that on PPF is 9% p.a. Formulation of the above problem as LPP to determine maximum yearly income, is

(a) Maximize Z = 0.08x + 0.09y
Subject to, $x + y \leq 50{,}000$, $x \geq 15000$, $y \geq 20{,}000$

(b) Maximize Z = 0.08x + 0.09y
Subject to, $x + y \leq 50{,}000$, $x \geq 15000$, $y \leq 20{,}000$

(c) Maximize Z = 0.08x + 0.09y
Subject to, $x + y \leq 50{,}000$, $x \leq 15000$, $y \geq 20{,}000$

(d) Maximize Z = 0.08x + 0.09y
Subject to, $x + y \leq 50{,}000$, $x \leq 15000$, $y \leq 20{,}000$

20. A furniture manufacturer produces tables and bookshelves made up of wood and steel. The weekly requirement of wood and steel is given as below.

Material / Product ↓	Wood	Steel
Table (x)	8	2
Book shelf (y)	11	3

The weekly variability of wood and steel is 450 and 100 units respectively. Profit on a table ₹1000 and that on a bookshelf is ₹1200. To determine the number of tables and bookshelves to be produced every week in order to maximize the total profit, formulation of the problem as L.P.P. is

(a) Maximize Z = 1000x + 1200 y
Subject to
$8x + 11y \geq 450$, $2x + 3y \leq 100$, $x \geq 0$, $y \geq 0$

(b) Maximize Z = 1000x + 1200 y
Subject to
$8x + 11y \leq 450$, $2x + 3y \leq 100$, $x \geq 0$, $y \geq 0$

(c) Maximize Z = 1000x + 1200 y
Subject to
$8x + 11y \leq 450$, $2x + 3y \geq 100$, $x \geq 0$, $y \geq 0$

(d) Maximize Z = 1000x + 1200 y
Subject to
$8x + 11y \geq 450$, $2x + 3y \geq 100$, $x \geq 0$, $y \geq 0$

21. Corner points of the feasible region for an LPP are (0, 2) (3, 0) (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function.
The minimum value of F occurs at
(a) (0, 2) only
(b) (3, 0) only
(c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only
(d) any point on the line segment joining the points (0, 2) and (3, 0)

22. The point at which the maximum value of $(3x + 2y)$ subject to the constraints $x + y \le 2,\ x \ge 0, y \ge 0$ is obtained, is
(a) (0, 0) (b) (1.5, 1.5) (c) (2, 0) (d) (0, 2)

23. For the constraint of a linear optimizing function $z = x_1 + x_2$, given by $x_1 + x_2 \le 1, 3x_1 + x_2 \ge 3$ and $x_1, x_2 \ge 0$,
(a) There are two feasible regions
(b) There are infinite feasible regions
(c) There is no feasible region
(d) None of these.

24. Which of the following is not a vertex of the positive region bounded by the inequalities $2x + 3y \le 6$, $5x + 3y \le 15$ and $x, y \ge 0$?
(a) (0, 2) (b) (0, 0) (c) (3, 0) (d) None

25. The area of the feasible region for the following constraints $3y + x \ge 3, x \ge 0, y \ge 0$ will be
(a) Bounded (b) Unbounded
(c) Convex (d) Concave

26. The maximum value of $z = 4x + 2y$ subject to constraints $2x + 3y \le 18,\ x + y \ge 10$ and $x, y \ge 0$, is
(a) 36 (b) 40 (c) 20 (d) None

27. The maximum value of $P = x + 3y$ such that $2x + y \le 20$, $x + 2y \le 20, x \ge 0, y \ge 0$ is
(a) 10 (b) 60 (c) 30 (d) None

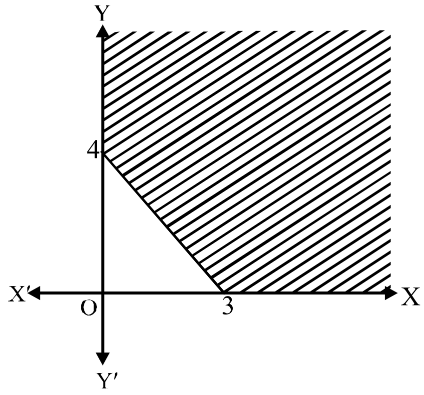
28. The maximum value of $z = 6x + 8y$ subject to constraints $2x + y \le 30, x + 2y \le 24$ and $x \ge 0, y \ge 0$ is
(a) 90 (b) 120 (c) 96 (d) 240

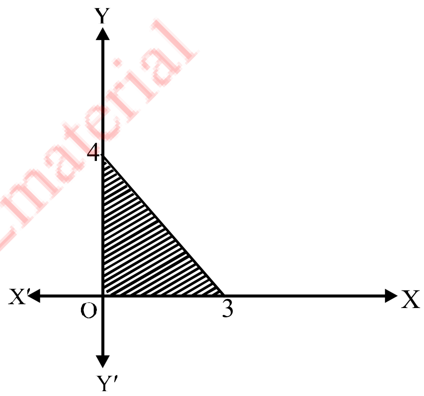
29. A wholesale merchant wants to start the business of cereal with ` 24000. Wheat is ` 400 per quintal and rice is ` 600 per quintal. He has capacity to store 200 quintal cereal. He earns the profit ` 25 per quintal on wheat and ` 40 per quintal on rice. If he store x quintal rice and y quintal wheat, then for maximum profit, the objective function is
(a) $25x + 40y$ (b) $40x + 25y$
(c) $400x + 600y$ (d) $\frac{400}{40}x + \frac{600}{25}y$

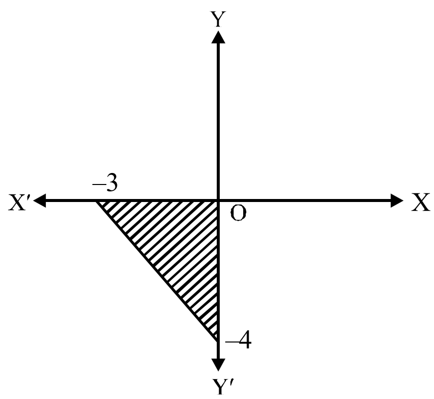
30. The value of objective function is maximum under linear constraints, is
(a) At the centre of feasible region
(b) At (0, 0)
(c) At any vertex of feasible region
(d) The vertex which is at maximum distance from (0, 0)

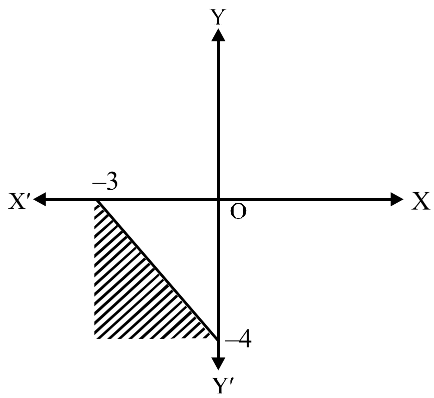
31. The feasible solution of a L.P.P. belongs to
(a) Only first quadrant (b) First and third quadrant
(c) Second quadrant (d) Any quadrant

32. Graph of the constraints $\frac{x}{3} + \frac{y}{4} \le 1, x \ge 0, y \ge 0$ is

(a)


(b)


(c)


(d)


33. The lines $5x + 4y \geq 20$, $x \leq 6$, $y \leq 4$ form

(a) A square (b) A rhombus

(c) A triangle (d) A quadrilateral

34. The graph of inequations $x \leq y$ and $y \leq x + 3$ is located in

(a) II quadrant (b) I, II quadrants

(c) I, II, III quadrants (d) II, III, IV quadrants

35. A linear programming of linear functions deals with

(a) Minimizing (b) Optimizing

(c) Maximizing (d) None of these

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

36. The number of corner points of the L.P.P. Max $Z = 20x + 3y$ subject to the constraints $x + y \leq 5$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$ are

(a) 4 (b) 3 (c) 2 (d) 1

37. Consider the objective function $Z = 40x + 50y$. The minimum number of constraints that are required to maximize Z are

(a) 4 (b) 2 (c) 3 (d) 1

38. The no. of convex polygon formed bounding the feasible region of the L.P.P. Max. $Z = 30x + 60y$ subject to the constraints $5x + 2y \leq 10$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$ are

(a) 2 (b) 3 (c) 4 (d) 1

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion

(c) Assertion is correct, Reason is incorrect

(d) Assertion is incorrect, Reason is correct.

39. **Assertion :** The region represented by the set $\{(x, y) : 4 \leq x^2 + y^2 \leq 9\}$ is a convex set.

Reason : The set $\{(x, y) : 4 \leq x^2 + y^2 \leq 9\}$ represents the region between two concentric circles of radii 2 and 3.

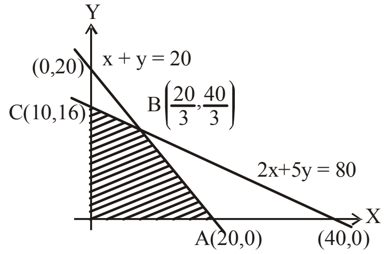
40. **Assertion :** If a L.P.P. admits two optimal solutions then it has infinitely many optimal solutions.

Reason : If the value of the objective function of a LPP is same at two corners then it is same at every point on the line joining two corner points.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

41. Shaded region in the following figure is represented by



(a) $2x + 5y \geq 80$, $x + y \leq 20$, $x \geq 0$, $y \leq 0$

(b) $2x + 5y \geq 80$, $x + y \geq 20$, $x \geq 0$, $y \geq 0$

(c) $2x + 5y \leq 80$, $x + y \leq 20$, $x \geq 0$, $y \geq 0$

(d) $2x + 5y \leq 80$, $x + y \leq 20$, $x \leq 0$, $y \leq 0$

42. The maximum value of $z = 2x + 5y$ subject to the constraints $2x + 5y \leq 10$, $x + 2y \geq 1$, $x - y \leq 4$, $x \geq y \geq 0$, occurs at

(a) exactly one point

(b) exactly two points

(c) infinitely many points

(d) None of these

43. Consider Max. $z = -2x - 3y$ subject to

$\frac{x}{2} + \frac{y}{3} \leq 1$, $\frac{x}{3} + \frac{y}{2} \leq 1$, $x, y \geq 0$

The max value of z is :

(a) 0 (b) 4 (c) 9 (d) 6

44. The solution region satisfied by the inequalities $x + y \leq 5$, $x \leq 4$, $y \leq 4$,

$x \geq 0$, $y \geq 0$, $5x + y \geq 5$, $x + 6y \geq 6$,

is bounded by

(a) 4 straight lines (b) 5 straight lines

(c) 6 straight lines (d) unbounded

45. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(a) $p = 2q$ (b) $p = \frac{q}{2}$

(c) $p = 3q$ (d) $p = q$

46. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5) (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(a) $p = q$ (b) $p = 2q$ (c) $q = 2p$ (d) $q = 3p$

47. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function.

The minimum value of F occurs at

(a) (0, 2) only

(b) (3, 0) only

(c) the mid point of the line segment joining the points (0, 2) and (3, 0).

(d) any point on the line segment joining the points (0, 2) and (3, 0).

48. The region represented by the inequalities $x \geq 6$, $y \geq 2$, $2x + y \leq 10$, $x \geq 0$, $y \geq 0$ is

(a) unbounded (b) a polygon

(c) exterior of a triangle (d) None of these

49. $Z = 7x + y$, subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at

(a) (3, 0) (b) $\left(\frac{1}{2}, \frac{5}{2}\right)$

(c) (7, 0) (d) (0, 5)

50. A brick manufacture has two depots A and B, with stocks of 30000 and 20000 bricks respectively. He receive orders from three builders P, Q and R for 15000, 20,000 and 15000 bricks respectively. The cost (in `) of transporting 1000 bricks to the builders from the depots as given in the table.

To / From	Transportation cost per 1000 bricks (in `)		
	P	Q	R
A	40	20	20
B	20	60	40

The manufacturer wishes to find how to fulfill the order so that transportation cost is minimum. Formulation of the L.P.P., is given as

(a) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \geq 15$, $y \leq 20$, $x \geq 0$, $y \geq 0$

(b) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \leq 15$, $y \geq 20$, $x \geq 0$, $y \geq 0$

(c) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \leq 15$, $y \leq 20$, $x \geq 0$, $y \geq 0$

(d) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \geq 15$, $y \geq 20$, $x \geq 0$, $y \geq 0$

51. The solution set of the following system of inequations: $x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$, is

(a) bounded region (b) unbounded region

(c) only one point (d) empty set

52. A company manufactures two types of products A and B. The storage capacity of its godown is 100 units. Total investment amount is ` 30,000. The cost price of A and B are ` 400 and ` 900 respectively. Suppose all the products have sold and per unit profit is ` 100 and ` 120 through A and B respectively. If x units of A and y units of B be produced, then two linear constraints and iso-profit line are respectively

(a) $x + y = 100$; $4x + 9y = 300$, $100x + 120y = c$

(b) $x + y \leq 100$; $4x + 9y \leq 300$, $x + 2y = c$

(c) $x + y \leq 100$; $4x + 9y \leq 300, 100x + 120y = c$

(d) $x + y \leq 100$; $9x + 4y \leq 300$, $x + 2y = c$

53. Which of the following cannot be considered as the objective function of a linear programming problem?

(a) Maximize $z = 3x + 2y$

(b) Minimize $z = 6x + 7y + 9z$

(c) Maximize $z = 2x$

(d) Minimize $z = x^2 + 2xy + y^2$

54. Inequation $y - x \leq 0$ represents

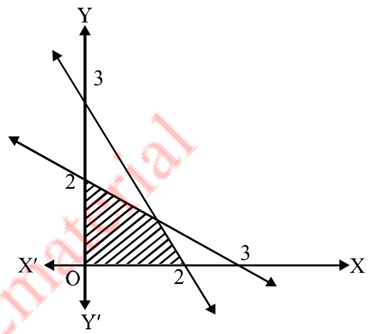
(a) The half plane that contains the positive X-axis

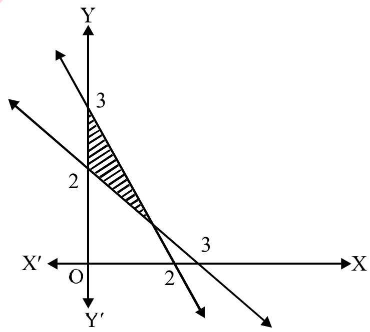
(b) Closed half plane above the line $y = x$, which contains positive Y-axis

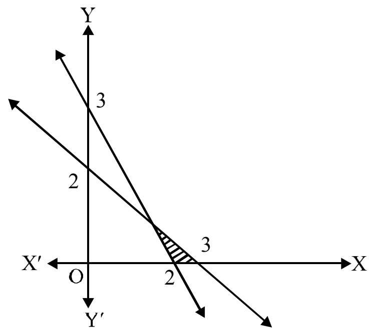
(c) Half plane that contains the negative X-axis

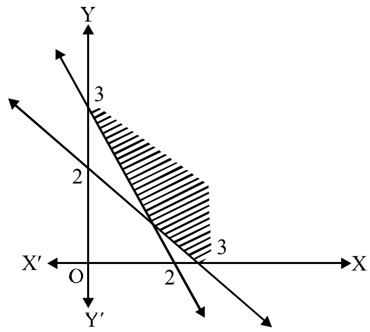
(d) None of these

55. Graph of the inequalities $x \geq 0$, $y \geq 0$, $2x + 3y \geq 6$, $3x + 2y \geq 6$ is

(a)


(b)


(c)


(d)


56. A printing company prints two types of magazines A and B. The company earns `10 and `15 on each magazine A and B respectively. These are processed on three machines I, II & III and total time in hours available per week on each machine is as follows:

Magzine → ↓ Machine	A(x)	B(y)	Time available
I	2	3	36
II	5	2	50
III	2	6	60

The number of constraints is

(a) 3 (b) 4

(c) 5 (d) 6

57. Children have been invited to a birthday party. It is necessary to give them return gifts. For the purpose, it was decided that they would be given pens and pencils in a bag. It was also decided that the number of items in a bag would be atleast 5. If the cost of a pen is `10 and cost of a pencil is `5, minimize the cost of a bag containing pens and pencils. Formulation of LPP for this problem is

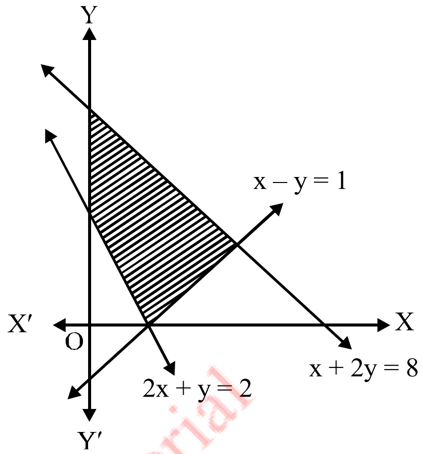
(a) Minimize $C = 5x + 10y$ subject to $x + y \le 10, x \ge 0, y \ge 0$

(b) Minimize $C = 5x + 10y$ subject to $x + y \ge 10, x \ge 0, y \ge 0$

(c) Minimize $C = 5x + 10y$ subject to $x + y \ge 5, x \ge 0, y \ge 0$

(d) Minimize $C = 5x + 10y$ subject to $x + y \le 5, x \ge 0, y \ge 0$

58. The linear inequations for which the shaded area in the following figure is the solution set, are



(a) $x + y \le 1, 2x + y \ge 2, x - 2y \ge 8, x \le 0, y \ge 0$

(b) $x - y \ge 1, 2x + y \ge 2, x + 2y \ge 8, x \ge 0, y \ge 0$

(c) $x - y \le 1, 2x + y \ge 2, x + 2y \le 8, x \ge 0, y \ge 0$

(d) $x + y \ge 1, 2x + y \le 2, x + 2y \ge 8, x \ge 0, y \ge 0$

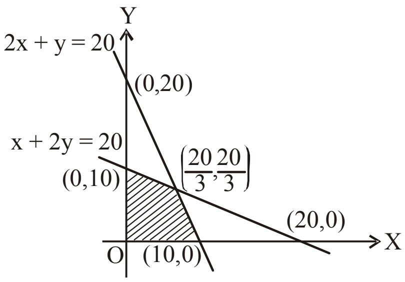
HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (c) 2. (d) 3. (a) 4. (c)
5. (c) 6. (b) 7. (d)

8. (c) Change the inequalities into equations and draw the graph of lines, thus we get the required feasible region. The region bounded by the vertices A(0, 5), B(4, 3), C (7, 0). The objective function is maximum at C(7,0) and Max $z = 5\times7+2\times0=35$.

9. (c) Obviously, $P=x+3y$ will be maximum at (0, 10). $\therefore P=0+3\times10=30$.



10. (a)

11. (b) Objective function is a linear function (of the variable involved) whose maximum or minimum value is to be found.

12. (b) Construct the following table of the values of the objective function :

Corner Point	(0, 0)	(5, 0)	(6, 5)	(6, 8)	(4, 10)	(0, 8)
Value of $Z=3x-4y$	0	**15**	–2	–14	–28	**–32**

(Maximum) **(Minimum)**

13. (a) Solution set of the given inequalities is $\{(x, y) : x \geq 0\} \cap \{(x, y) : y \geq 0\} = \{(x, y) : x \geq 0, y \geq 0\}$, i.e., the set of all these points whose both coordinates are non-negative. All these points lie in the first quadrants (including points on +ve X-axis, +ve Y-axis and the origin).

14. (b) Construct the following table of functional values :

Corner Point	(0, 8)	(2, 5)	(4, 3)	(9, 0)
Value of $Z=4x+3y$	24	**23**	25	36

(Minimum)

15. (c) Construct the following table of values of objective function f.

Corner Point	(0,0)	(6,12)	(6,16)	(0,4)
Value of $f=3x-4y$	**0**	–30	**–46**	–16

Maximum **Minimum**

16. (c) We have, maximize $Z=3x+5y$

Subject to constraints :

$x+4y\leq 24,\ 3x+y\leq 21,\ x+y\leq 9,\ x\geq 0,\ y\geq 0$

Let $\ell_1 : x+4y=24$

$\ell_2 : 3x+y=21$

$\ell_3 : x+y=9$

$\ell_4 : x=0$ and $\ell_5 : y=0$

On solving these equations we will get points as O(0, 0), A(7, 0), B(6, 3), C(4, 5), D(0, 6)

Now maximize $Z=3x+5y$

Z at O(0, 0) = 3(0) + 5(0) = 0

Z at A(7, 0) = 3(7) + 5(0) = 21

Z at B(6, 3) = 3(6) + 5(3) = 33

Z at C(4, 5) = 3(4) + 5(5) = 37

Z at D(0, 6) = 3(0) + 5(6) = 30

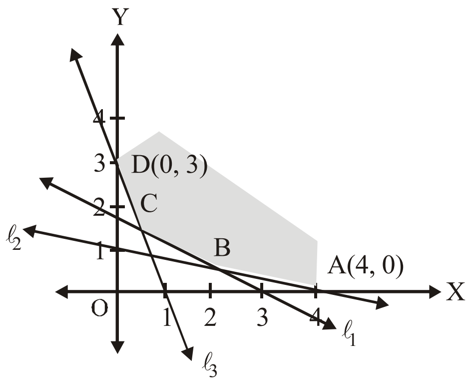
Thus, Z is maximized at C(4, 5) and its maximum value is 37.

17. (c) We have, minimize $Z=6x+21y$

Subject to $x+2y\geq 3,\ x+4y\geq 4,\ 3x+y\geq 3,\ x\geq 0,\ y\geq 0$

Let $\ell_1 : x+2y=3$: $\ell_2 = x+4y=4$, $\ell_3 : 3x+y=3$

Shaded portion is the feasible region, where A (4, 0), $B\left(2,\frac{1}{2}\right)$, C(0.6, 1.2), D (0, 3).



For B: Solving ℓ_1 and ℓ_2, we get B(2, 1/2)

For C: Solving ℓ_1 and ℓ_3, we get C(0.6, 1.2)

Now, minimize $Z=6x+21y$

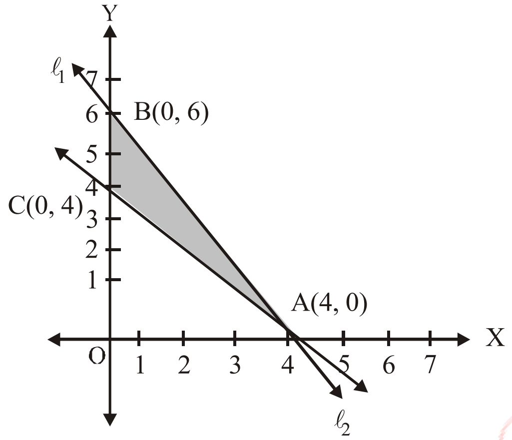
Z at A(4, 0) = 6(4) + 21(0) = 24

Z at B $\left(2, \frac{1}{2}\right) = 6(2) + 21\left(\frac{1}{2}\right) = 22.5$

Z at C $(0.6, 1.2) = 6(0.6) + 21(1.2) = 3.6 + 25.2 = 28.8$
Z at D $(0, 3) = 6(0) + 21(3) = 63$

Thus, Z is minimized at B $\left(2, \frac{1}{2}\right)$ and its minimum value is 22.5.

18. (d) We have, minimized $Z = 4x + 6y$
Subject to $3x + 2y \le 12$, $x + y \ge 4$, $x, y \ge 0$
Let $\ell_1 : 3x + 2y = 12$
$\ell_2 : x + y = 4$
$\ell_3 : x = 0$ and $\ell_4 : y = 0$
Shaded portion ABC is the feasible region, where A(4, 0), C(0, 4), B(0, 6).



Now maximize $Z = 4x + 6y$
Z at A$(4, 0) = 4(4) + 6(0) = 16$
Z at B $(0, 6) = 4(0) + 6(6) = 36$
Z at C $(0, 4) = 4(0) + 6(4) = 24$
Thus, Z is maximized at B(0, 6) and its maximum value is 36.

19. (a) Let Shamali invest ` x in saving certificate and ` y in PPF.
$\therefore x + y \le 50000$, $x \ge 15000$ and $y \ge 20000$

Total income $= \frac{8}{100}x + \frac{9}{100}y$

$\therefore$ Given problem can be formulated as
Maximize $Z = 0.08x + 0.09y$
Subject to, $x + y \le 50000$, $x \ge 15000$, $y \ge 20000$.

20. (b) Given x and y units of tables and bookshelves are produced
Profit on one table is ` 1000
$\therefore$ Profit on x table is ` 1000x
Profit on one bookshelf is ` 1200
$\therefore$ Profit on y bookshelves is ` 1200y
$\therefore$ Profit $Z = 1000x + 1200y$

Material \ Product	Table (x)	Bookshelf (y)	Availability
Wood	8	11	450
Steel	2	3	100

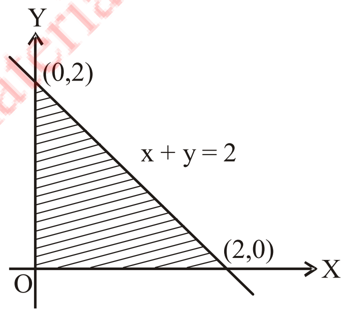
$\therefore$ Constraints are $8x + 11y \le 450$, $2x + 3y \le 100$, $x \ge 0$, $y \ge 0$
$\therefore$ Given problem can be formulated as
Maximize $Z = 1000x + 1200y$
Subject to, $8x + 11y \le 450$, $2x + 3y \le 100$, $x \ge 0$, $y \ge 0$

21. (d) Construct the following table of objective function

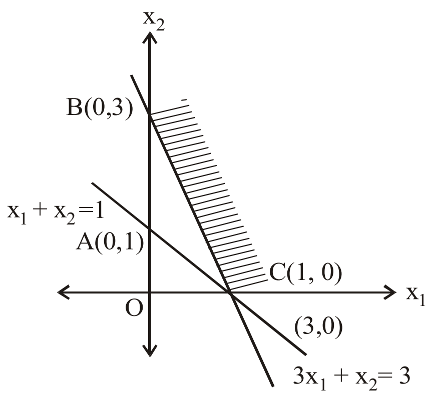
Corner Point	Value of F = 4x + 6y	
(0, 2)	$4 \times 0 + 6 \times 2 = 12$	} ← minimum
(3, 0)	$4 \times 3 + 6 \times 0 = 12$	
(6, 0)	$4 \times 6 + 6 \times 0 = 24$	
(6, 8)	$4 \times 6 + 6 \times 8 = 72$	← maximum
(0, 5)	$4 \times 0 + 6 \times 5 = 30$	

Since the minimum value (F) = 12 occurs at two distinct corner points, it occurs at every points of the segment joining these two points.

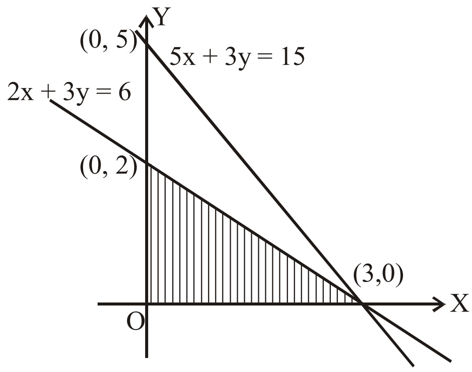
22. (c) Hence maximum z is at (2, 0).



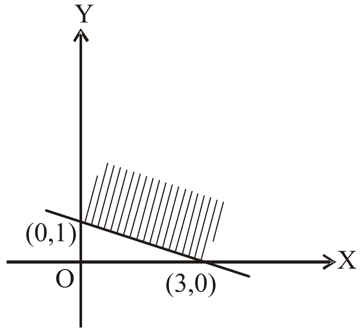
23. (c) Clearly from graph there is no feasible region.



24. (d) Here (0, 2), (0, 0) and (3, 0) all are vertices of feasible region. Hence option (d) is correct.

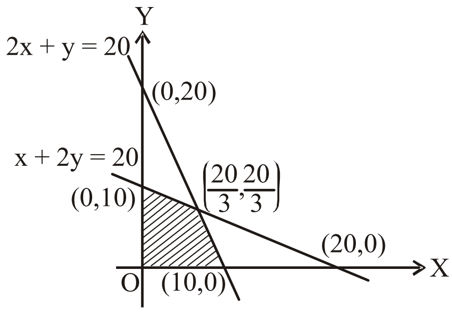


25. **(b)**

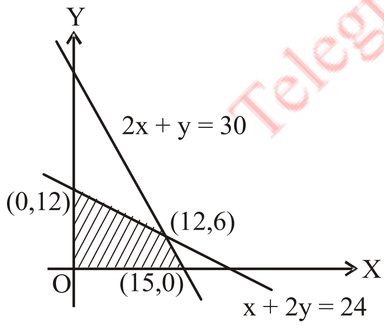


26. **(d)** After drawing the graph, we get the points on the region are (9,0), (0,6), (10, 0), (0, 10) and (12, –2) But there is no feasible point as no point satisfy all the inequations simultaneously.

27. **(c)** Obviously, $P = x + 3y$ will be maximum at (0, 10).
$\therefore P = 0 + 3 \times 10 = 30.$



28. **(b)** Here, $2x + y \leq 30$, $x + 2y \leq 24$, $x, y \geq 0$
The shaded region represents the feasible region, hence
$z = 6x + 8y$. Obviously it is maximum at (12, 6).
Hence $z = 12 \times 6 + 8 \times 6 = 120$



29. **(b)** For maximum profit, $z = 40x + 25y$.

30. **(c)** **31.** **(d)**

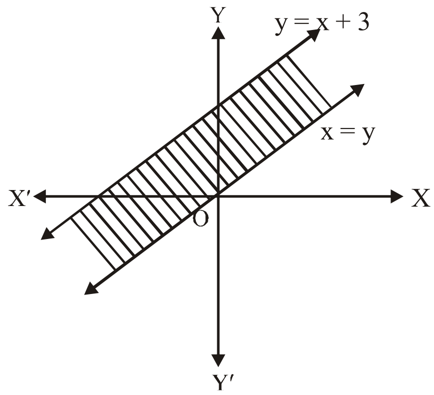
32. **(b)** Take a test point O (0,0).

Equation of the constraint is $\frac{x}{3} + \frac{y}{4} \leq 1$

$\Rightarrow 4x + 3y \leq 12$
Since $4(0) + 3(0) \leq 12$, the feasible region lies below the line $4x + 3y = 12$
Since $x \geq 0$, $y \geq 0$
the feasible region lies in the first quadrant.

33. **(d)** Common region is quadrilateral.

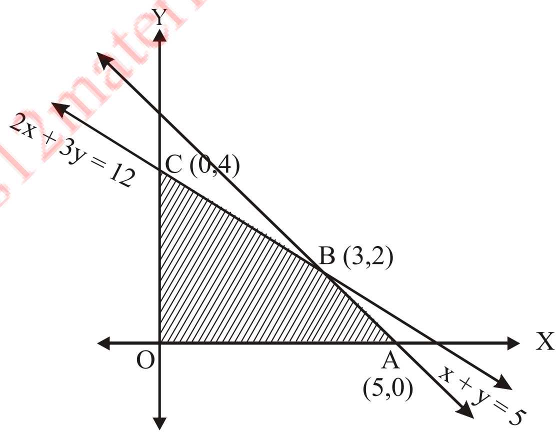
34. **(c)** The shaded area is the required area given in graph as below.



35. **(b)**

INTEGER TYPE QUESTIONS

36. **(a)** As is obvious from the figure.

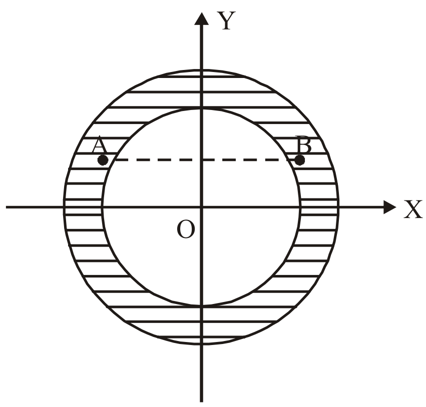


37. **(c)** Two constraints are $x \geq 0$, $y \geq 0$ and the third one will be of the type $ax + by \leq c$.

38. **(d)**

ASSERTION - REASON TYPE QUESTIONS

39. **(d)** From the figure it is clear that the region is not a convex set.

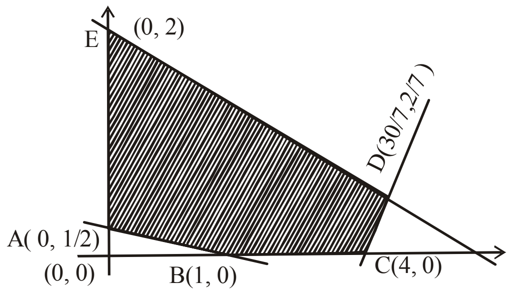


40. **(a)** It is a standard result.

CRITICALTHINKING TYPE QUESTIONS

41. (c) In given all equations, the origin is present in shaded area, answer (c) satisfy this condition.

42. (c) We find that the feasible region is on the same side of the line $2x + 5y = 10$ as the origin, on the same side of the line $x - y = 4$ as the origin and on the opposite side of the line $x + 2y = 1$ from the origin. Moreover, the lines meet the coordinate axes at (5, 0), (0, 2); (1, 0), (0, 1/2) and (4, 0). The lines $x - y = 4$ and $2x + 5y = 10$ intersect at $\left(\frac{30}{7}, \frac{2}{7}\right)$.



The values of the objective function at the vertices of the pentagon are:

(i) $Z = 0 + \frac{5}{2} = \frac{5}{2}$ (ii) $Z = 2 + 0 = 2$

(iii) $Z = 8 + 0 = 8$ (iv) $Z = \frac{60}{7} + \frac{10}{7} = 10$

(v) $Z = 0 + 10 = 10$

The maximum value 10 occurs at the points $D\left(\frac{30}{7}, \frac{2}{7}\right)$ and E(0, 2). Since D and E are adjacent vertices, the objective function has the same maximum value 10 at all the points on the line DE.

43. (a) Given problem is max $z = -2x - 3y$

Subject to $\frac{x}{2} + \frac{y}{3} \le 1$, $\frac{x}{3} + \frac{y}{2} \le 1$, x, y $\ge$ 0

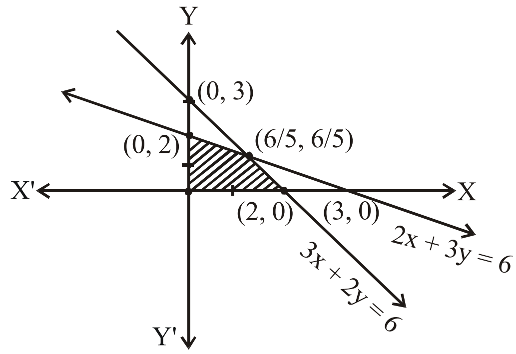
First convert these inequations into equations we get

$3x + 2y = 6$ ...(i)

$2x + 3y = 6$ ...(ii)

on solving these two equation, we get point of intersection is $\left(\frac{6}{5}, \frac{6}{5}\right)$.

Now, we draw the graph of these lines.



Shaded portion shows the feasible region.

Now, the corner points are

(0, 2), (2,0), $\left(\frac{6}{5}, \frac{6}{5}\right)$, (0, 0).

At (0, 2),
value of z $= -2(0) - 3(2) = -6$

At (2, 0),
value of z $= -2(2) - 3(0) = -4$

At $\left(\frac{6}{5}, \frac{6}{5}\right)$, Value of z $= -2\left(\frac{6}{5}\right) - 3\left(\frac{6}{5}\right)$

$= \frac{-30}{5} = -6$

At (0, 0), value of z $= -2(0) - 3(0) = 0$

$\therefore$ The max value of z is 0.

44. (b) We find that the solution set satisfies $x \ge 0$, $y \ge 0$, $x \le 4$, $y \le 4$ so that the solution region lies within the square enclosed by the lines x = 0, y = 0, x = 4, y = 4. Moreover, the solution region is bounded by the lines

$x + y = 5$, ...(i)

$5x + y = 5$ ...(ii)

$x + 6y = 6$ ...(iii)

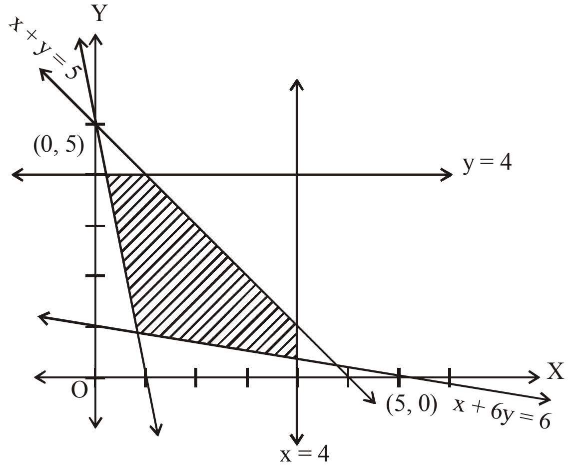
Line (i) meets the coordinate axes in (5, 0) and (0, 5) and the lines x = 4 and y = 4 in (4, 1) and (1, 4), and $0 < 5$ is true.

Hence (0, 0) belongs to the half plane $x + y \le 5$.

But (0, 0) does not belong to the half planes $5x + y \ge 5$ and $x + 6y \ge 6$. The line $5x + y = 5$, meets the coordinate axes in (1, 0) and (0, 5), and meets the line x = 4 in (4, 1), where as it meets the line y = 4 in (1/5, 4).

Similarly $x + 6y = 6$ meets x = 4 in (4, 1/3) and y = 4 in (–18, 4).

The solution is marked as the shaded region.



45. **(b)** We must have value of Z at (3, 0) = value of Z at (1, 1) and this value must be less than the value (0, 3)

$\Rightarrow 3p + 0q = 1p + 1q$ and $3p < 3q$

$\Rightarrow 3p = p + q$ and $p < q$

$\Rightarrow p = \frac{1}{2}q.$

46. **(d)** We must have the value of Z at (0, 20) equal to the value of Z at (15, 15) and this common value must be greater than the values at (0, 10) and (5, 5), i.e.,

$15p + 15q = 20q > 10q$ and $20q > 5p + 5q$

$\Rightarrow q = 3p.$

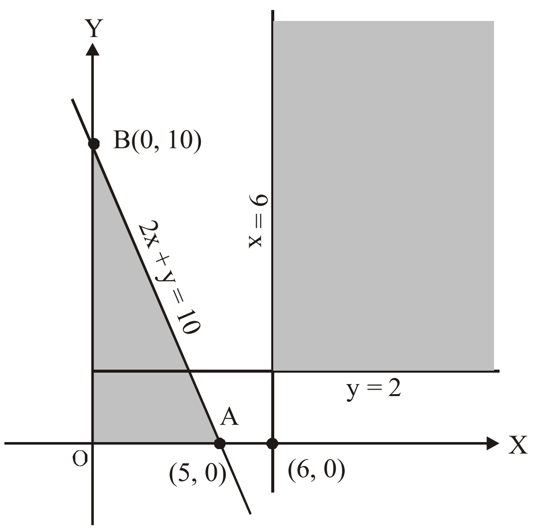
47. **(d)** Construct the following table of functional values :

Corner Point	(0, 2)	(3, 0)	(6, 0)	(6, 8)	(0, 5)
Value of F = 4 x + 6 y	**12**	**12**	24	**72**	30

↑minimum ↑ maximum

Since the minimum value (F) = 12 occurs at two distinct corner points, it occurs at every point of the segment joining these two points.

48. **(d)** The graph of the inequalities $2x + y \le 10, x \ge 0, y \ge 0$ is the region bounded by ΔAOB. This region has no point common with the region $\{(x, y) : x \ge 6, y \ge 2\}$ as is clear from the figure . Hence, the region of the given inequalities is the empty set.



49. **(d)** We have, maximize $Z = 7x + y$,

Subject to :

$5x + y \ge 5, x + y \ge 3, x, y \ge 0.$

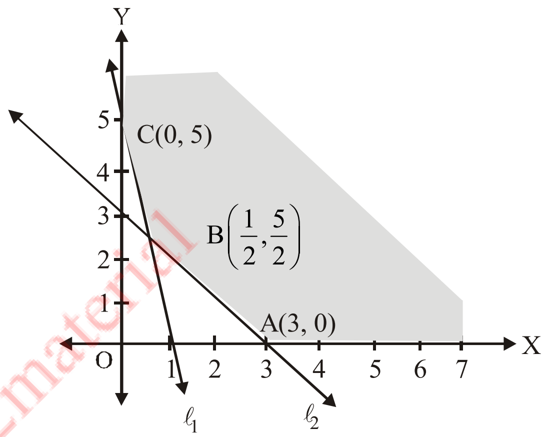
Let $\ell_1 : 5x + y = 5$

$\ell_2 : x + y = 3$

$\ell_3 : x = 0$ and $\ell_4 :\ y = 0$

Shaded portion is the feasible region,

Where A(3, 0), $B\left(\frac{1}{2}, \frac{5}{2}\right)$, C(0, 5)



For B : Solving ℓ_1 and ℓ_2, we get $B\left(\frac{1}{2}, \frac{5}{2}\right)$

Now maximize $Z = 7x + y$

Z at $A(3, 0) = 7(3) + 0 = 21$

Z at $B\left(\frac{1}{2}, \frac{5}{2}\right) = 7\left(\frac{1}{2}\right) + \frac{5}{2} = 6$

Z at $C(0, 5) = 7(0) + 5 = 5$

Thus Z, is minimized at C(0, 5) and its minimum value is 5

50. **(c)** The given information can be expressed as given in the diagram:

In order to simply, we assume that 1 unit = 1000 bricks

Suppose that depot A supplies x units to P and y units to Q, so that depot A supplies (30 – x – y) bricks to builder R.

Now, as P requires a total of 15000 bricks, it requires (15 – x) units from depot B.

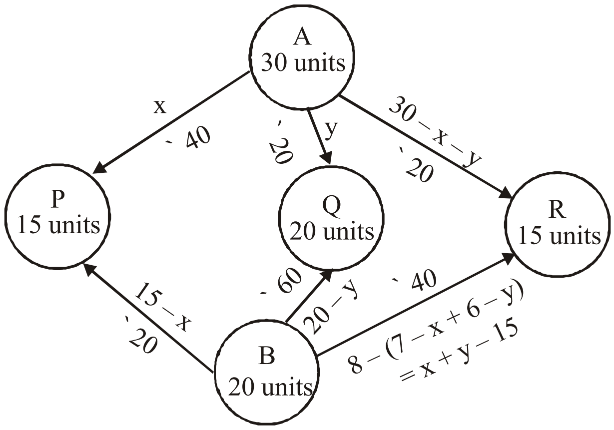
Similarly, Q requires (20 – y) units from B and R requires 15 – (30 – x – y) = x + y – 15 units from B.

Using the transportation cost given in table, total transportation cost.

$Z = 40x + 20y + 20(30 - x - y) + 20(15 - x) + 60(20 - y) + 40(x + y - 15)$

$= 40x - 20y + 1500$

Obviously the constraints are that all quantities of bricks supplied from A and B to P, Q, R are non-negative.



$\therefore$ $x \ge 0, y \ge 0, 30 - x - y \ge 0, 15 - x \ge 0, 20 - y \ge 0,$
$x + y - 15 \ge 0.$

Since, 1500 is a constant, hence instead of minimizing $Z = 40x - 20y + 1500$, we can minimize $Z = 40x - 20y$.

Hence, mathematical formulation of the given LPP is

Minimize $Z = 40x - 20y$,

subject to the constraints:

$x + y \ge 15$,
$x + y \le 30$,
$x \le 15, y \le 20$,
$x \ge 0, y \ge 0$

51. (d) The solution region is bounded by the straight lines

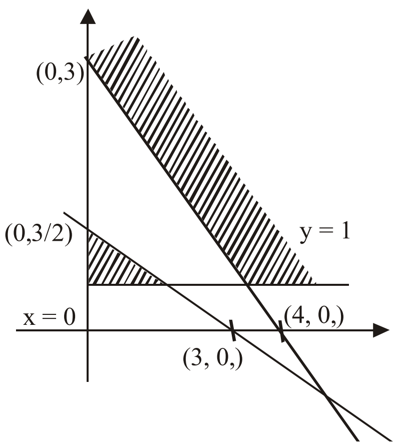
$x + 2y = 3$ …(i)
$3x + 4y = 12$ …(ii)
$x = 0$ …(iii)
$y = 1$ …(iv)

The straight lines (i) and (ii) meet the x-axis in (3, 0) and (4,0) and for (0, 0), $x + 2y \le 3 \Rightarrow 0 \le 3$ which is true.

Hence (0, 0) lies in the half plane $x + 2y \le 3$. Also the lines (1) and (2) meet the y-axis in (0, 3/2) and (0, 3) and for (0,0) $3x + 4y \ge 12 \Rightarrow 0 \ge 12$ which is not true. Hence (0, 0) doesn't belong to the half plane $3x + 4y \ge 0$.

Also $x \ge 0$, $y \ge 1 \Rightarrow$ the solution set belongs to the first quadrant. Moreover all the boundary lines are part of the solution.

From the shaded region, We find that there is no solution of the given system. Hence the solution set is an empty set.

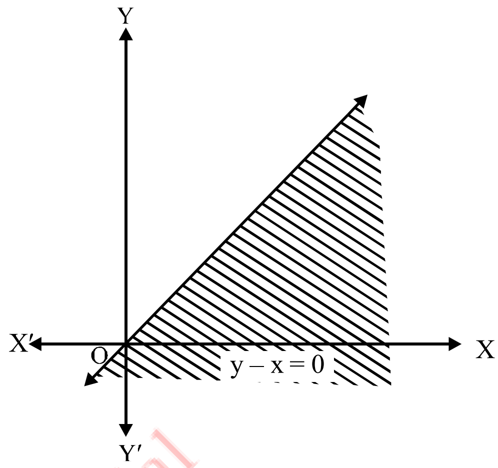


52. **(c)** $x + y \le 100;\ 400x + 900y \le 30000$ or

$4x + 9y \le 300$ and $100x + 120y = c$.

53. **(d)** d is the only option which is not linear.

54. **(a)**



55. **(c)** Take a test point O(0, 0)

Since, $2(0) + 3(0) \le 6$, the feasible region lies below the line $2x + 3y = 6$

Since, $3(0) + 2(0) \ge 6$ is incorrect, the feasible region lies above the line $3x + 2y = 6$

$\therefore$ the feasible region lies in the common region between the lines $2x + 3y = 6$ and $3x + 2y = 6$

Since $x \ge 0$, $y \ge 0$, the feasible region lies in the first quadrant.

56. **(c)** Constraints are $2x + 3y \le 36$; $5x + 2y \le 50$; $2x + 6y \le 60$, $x \le 0, y \le 0$

$\therefore$ The number of constraints are 5.

57. **(a)** Let the no. of pencils in a bag be x

Let the no. of pens in a bag be y. There should be at least 5 items in a bag

$\therefore$ we have $x + y \ge 5$

cost of pencils in a bag = \`5x

cost of pens in bag \`10y

$\therefore$ Total cost of a bag = 5x + 10y,

The total cost has to minimized

$\therefore$ Objective function is minimize $C = 5x + 10y$ subject to $x + y \ge 5$, $x \ge 0$, $y \ge 0$

58. **(c)** Let $L_1 : x + 2y = 8$;
L_2 $2x + y = 2$;
$L_3 : x - y = 1$

Since the shaded area is below the line L_1, we have $x + 2y \le 8$. Since the shaded area is above the line L_2, we have $2x + y \ge 2$. Since the common region is to the left of the line L_3, we have $x - y \le 1$

CHAPTER 29

PROBABILITY-II

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If P (A) = $\frac{1}{2}$, P (B) = 0, then P (A/B) is

(a) 0 (b) $\frac{1}{2}$
(c) not defined (d) 1

2. A and B are events such that P (A/B) = P (B/A) then

(a) $A \subset B$ (b) B = A
(c) $A \cap B = \phi$ (d) P(A) = P (B)

3. If two events A and B are such that
P(A') = 0.3, P(B) = 0.4 and $P(A \cap B') = 0.5$, then

$P\left(\frac{B}{A \cup B'}\right) =$

(a) 1/4 (b) 1/5 (c) 3/5 (d) 2/5

4. It is given that the events *A* and *B* are such that $P(A) = \frac{1}{4}, P(A \mid B) = \frac{1}{2}$ and $P(B \mid A) = \frac{2}{3}$. Then *P*(*B*) is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

5. If A and B are 2 events such that P(A) > 0 and P(B) ≠ 1, then $P(\overline{A} \mid \overline{B}) =$

(a) $1 - P(A \mid B)$ (b) $1 - P(A \mid \overline{B})$

(c) $\frac{1 - P(A \cup B)}{P(B)}$ (d) $\frac{P(\overline{A})}{P(B)}$

6. $P(E \cap F)$ is equal to

(a) P(E). P(F|E) (b) P(F). P(E|F)
(c) Both (a) and (b) (d) None of these

7. Let three fair coins be tossed. Let A = {all heads or all tails}, B = {atleast two heads}, and C = {atmost two tails}. Which of the following events are independent?

(a) A and C (b) B and C
(c) A and B (d) None of these

8. Which one is not a requirement of a binomial distribution?

(a) There are 2 outcomes for each trial
(b) There is a fixed number of trials
(c) The outcomes must be dependent on each other
(d) The probability of success must be the same for all the trial

9. Examples of some random variables are given below :

1. Number of sons among the children of parents with five children.
2. Number of sundays in some randomly selected months with 30 days.
3. Number of apples in some 3 kg packets, purchased from a retail shop.

Which of the above is expected to follow binomial distribution?

(a) Variable 1 (b) Variable 2
(c) Variable 3 (d) None of these

10. If A and B are two events such that P(A) ≠ 0 and P(B) ≠ 1, then $P\left(\frac{\overline{A}}{\overline{B}}\right) =$

(a) $1 - P\left(\frac{A}{B}\right)$ (b) $1 - P\left(\frac{\overline{A}}{B}\right)$

(c) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (d) $\frac{P(\overline{A})}{P(B)}$

11. A coin is tossed three times is succession. If E is the event that there are at least two heads and F is the event in which first throw is a head, then P(E/F) equal to:

(a) $\frac{3}{4}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

12. The probability of safe arrival of one ship out of five is $\left(\frac{1}{5}\right)$. The probability of safe arrival of atleast 3 ship is:

(a) $\frac{3}{52}$ (b) $\frac{1}{31}$ (c) $\frac{184}{3125}$ (d) $\frac{181}{3125}$

13. The mean and variance of a random variable *X* having binomial distribution are 4 and 2 respectively, then *P* (*X* = 1) is

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

14. Consider the following statement:
"The mean of a binomial distribution is 3 and variance is 4."
Which of the following is correct regarding this statement?
(a) It is always true
(b) It is sometimes true
(c) It is never true
(d) No conclusion can be drawn

15. The probability that a man hits a target is p = 0.1. He fires n = 100 times. The expected number n of times he will hit the target is :
(a) 33 (b) 30 (c) 20 (d) 10

16. The probability of the simultaneous occurrence of two events A and B is p. If the probability that exactly one of the events occurs is q, then which of the following is not correct?
(a) $P(A')+P(B')=2+2q-p$
(b) $P(A')+P(B')=2-2p-q$
(c) $P(A\cap B\,|\,A\cup B)=\dfrac{p}{p+q}$
(d) $P(A'\cap B')=1-p-q$

17. Girl students constitute 10% of I year and 5% of II year at Roorkee University. During summer holidays 70% of the I year and 30% of II year students are given a project. The girls take turns on duty in canteen. The chance that I year girl student is on duty in a randomly selected day is
(a) $\frac{3}{17}$ (b) $\frac{14}{17}$ (c) $\frac{3}{10}$ (d) $\frac{7}{10}$

18. One ticket is selected at random from 50 tickets numbered 00,01,02,...,49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals
(a) $\frac{1}{7}$ (b) $\frac{5}{14}$ (c) $\frac{1}{50}$ (d) $\frac{1}{14}$

19. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is
(a) 0.2 (b) 0.7 (c) 0.06 (d) 0.14

20. In a college, 30% students fail in physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics, if she has failed in mathematics, is
(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{9}{20}$ (d) $\frac{1}{3}$

21. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0, if he opposed and X = 1, if he is in favour. Then, E(X) and Var (X) respectively are
(a) $\frac{3}{7},\frac{5}{17}$ (b) $\frac{13}{15},\frac{2}{15}$
(c) $\frac{7}{10},\frac{21}{100}$ (d) $\frac{7}{10},\frac{23}{100}$

22. If A and B be two events such that P(A) = 0.6, P(B) = 0.2 and P(A/B) = 0.5, then $P(A'/B')$ is equal to
(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

23. If A and B are independent events, then which of the following is not true ?
(a) P(A/B) = P(A) (b) P(B/A) = P(B)
(c) P(A/B) = P(B/A) (d) None of these

24. If $P(A\cap B)=0.15$, $P(B')=0.10$, then P(A/B) =
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

25. Two dice are thrown. If it is known that the sum of the numbers on the dice is less than 6, the probability of getting a sum 3 is
(a) $\frac{1}{8}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{5}{18}$

26. Five defective mangoes are accidently mixed with 15 good ones. Four mangoes are drawn at random from this lot. Then the probability distribution of the number of defective mangoes is:

(a)

X:	0	1	2	3	4
P(X):	$\frac{85}{323}$	$\frac{5}{323}$	$\frac{1}{969}$	$\frac{2}{969}$	$\frac{3}{969}$

(b)

X:	0	1	2	3	4
P(X):	$\frac{91}{323}$	$\frac{85}{969}$	$\frac{3}{323}$	$\frac{1}{969}$	$\frac{3}{969}$

(c)

X:	0	1	2	3	4
P(X):	$\frac{91}{323}$	$\frac{455}{969}$	$\frac{70}{323}$	$\frac{10}{323}$	$\frac{1}{969}$

(d)

X:	0	1	2	3	4
P(X):	$\frac{455}{969}$	$\frac{85}{323}$	$\frac{263}{323}$	$\frac{25}{969}$	$\frac{2}{969}$

27. Two dice are thrown, simultaneously. If X denotes the number of sixes, then the expected value of X is
(a) $E(X)=\frac{1}{3}$ (b) $E(X)=\frac{2}{3}$
(c) $E(X)=\frac{1}{6}$ (d) $E(X)=\frac{5}{6}$

28. The random variable X has the following probability distribution:

x :	−3	−1	0	1	3
$P(X=x)$:	0.05	0.45	0.20	0.25	0.05

Then, its mean is
(a) – 0.2 (b) 0.2 (c) – 0.4 (d) 0.4

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

29. Consider the following statements

Statement I: An experiment succeeds twice as often as it fails. Then, the probability that in the next six trials, there will be atleast 4 successes is $\frac{31}{9}\left(\frac{2}{3}\right)^4$.

Statement II: The number of times must a man toss a fair coin so that the probability of having atleast one head is more than 90% is 4 or more than 4.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

30. I. Bernoulli's trials of a random experiment can be infinite in number.

II The appearance of 50 students in a test that whether they pass or fail can be considered as 50 Bernoulli trials

III The 5 trials of drawing balls from a bag containing 8 white and 12 black balls with replacement can be considered as Bernoulli's trials.

(a) Only I and II are correct
(b) Only II and III are correct
(c) Only III is correct
(d) All are correct

31. I. Partition of a sample space is unique.

II If *n* events represent position of a sample space then it is not necessary for them to be pairwise disjoint.

(a) Only I is correct
(b) Only II is correct
(c) Both I and II are correct
(d) Both I and II are incorrect

32. I. Independent events and mutually exclusive events have one and the same meaning.

II If $E_1, E_2, ...E_n$ represent partition of a sample space then more than one of them can occur simultaneously.

(a) Only I is correct
(b) Only II is correct
(c) Both I and II are correct
(d) Both I and II are incorrect

33. A fair coin is tossed two times

I The first and second tosses are independent of each other.

II The sample space for the experiment is S = {HH, HT, TH, TT}

III Getting head in both the tosses is a sure event.

(a) Only I is correct
(b) Only I and II are correct
(c) All are correct
(d) Only III is correct

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

34. Given, two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Then, match the terms of column I with their respectively values in column II .

Column-I		**Column-II**	
A.	P(A and B)	1.	0.28
B.	P(A and not B)	2.	0.72
C.	P(A or B)	3.	0.12
D.	P(neither A nor B)	4.	0.18

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	2	1	3
(c)	1	2	3	4
(d)	4	3	2	1

35. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X=x)=\begin{cases}0.1, \text{if } x=0\\ kx, \text{if } x=1 \text{ or } 2\\ k(5-x), \text{if } x=3 \text{ or } 4\\ 0, \text{otherwise}\end{cases}$$

Then, match the terms of column I with their respective values in column II.

Column-I		**Column-II**	
A.	The value of k	1.	0.55
B.	P(you study atleast 2 hours)	2.	0.3
C.	P(you study exactly 2 hours)	3.	0.75
D.	P(you study atmost 2 hours)	4.	0.15

Codes

	A	B	C	D
(a)	1	4	3	2
(b)	4	3	1	2
(c)	4	2	3	1
(d)	4	3	2	1

36. A fair die is rolled. Consider events E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}.

	Column-I		**Column-II**
A.	P(E/F) and P(F/E)	1.	$\frac{3}{4}$ and $\frac{1}{4}$
B.	P(E/G) and P(G/E)	2.	$\frac{1}{2}$ and $\frac{1}{3}$
C.	$P[(E \cup F)/G]$ and $P[(E \cap F)/G]$	3.	$\frac{1}{2}$ and $\frac{2}{3}$

Codes

	A	B	C
(a)	2	3	1
(b)	1	2	3
(c)	2	1	3
(d)	3	1	2

37. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both hindi and English news papers. A student is selected at random. Then, match the terms of column I with their respective values in column II.

	Column-I		Column-II
A.	The probability that she reads neither Hindi nor English newspapers.	1.	$\frac{1}{3}$
B.	If she reads Hindi newspaper, then the probability that she reads English newspaper	2.	$\frac{1}{2}$
C.	If she reads English newspaper, then the probability that she reads Hindi newspaper	3.	$\frac{1}{5}$

Codes

	A	B	C
(a)	3	1	2
(b)	1	2	3
(c)	2	3	1
(d)	3	2	1

38. For a loaded die, the probabilities of outcomes are given as under.

$P(1) = P(2) = 0.2$

$P(3) = P(5) = P(6) = 0.1$ and $P(4) = 0.3$

The dice is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more" respectively. Then, match the terms of column I with their respective values in column II.

	Column-I		Column-II
A.	P(A)	1.	Independent
B.	P(B)	2.	0.02
C.	$P(A \cap B)$	3.	0.10
D.	Events A and B are	4.	0.20
		5.	Dependent

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	4	3	1	5
(c)	4	1	3	5
(d)	2	4	1	3

39. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Then, match the probability that out of 5 such bulbs will fuse after 150 days of use in column I with their respective values in column II.

	Column-I		Column-II
A.	P(none)	1.	$1-(0.95)^5$
B.	P(not more than one)	2.	$1-(0.95)^4 \times 1.2$
C.	P(more than one)	3.	$(0.95)^4 \times 1.2$
D.	P(atleast one)	4.	$(0.95)^5$

Codes

	A	B	C	D
(a)	2	4	1	3
(b)	4	3	2	1
(c)	4	3	1	2
(d)	4	1	3	2

40.

	Column-I		Column-II
A.	Conditional probability of event A given that event B has already occurred	1.	$P(A \cap B) = P(A).P(B)$
B.	A and B are disjoint events	2.	$P\{(A \cup B)/F\} = P(A/F)+P(B/F)$
C.	The probability of event A and B	3.	$\frac{P(A \cap B)}{P(B)}$
D.	Events A and B are independent	4.	P(A/B)
		5.	$P(A \cap B)$

Codes

	A	B	C	D
(a)	1	2,3	4	5
(b)	1,2	3	5	4
(c)	2	3,4	1	5
(d)	3,4	2	5	1

41.

	Column - I		Column - II
A.	$P(A \cap B)$	1.	P (neither A nor B)
B.	$P(A \cap \bar{B})$	2.	P (A or B)
C.	$P(A \cup B)$	3.	P (A and not B)
D.	$P(\bar{A} \cap \bar{B})$	4.	P (A and B)

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	4	3	2	1
(c)	4	3	1	2
(d)	1	3	4	2

42.

	Column - I		Column - II
A.	The event having probability 1	1.	pairwise disjoint
B.	The event having probability 0	2.	mutually exclusive
C.	$E_i \cap E_j = \phi$, $i \neq j$ $i,j = 1,2,3,...n$	3.	sure event
D.	$E_1 \cup E_2 \cup ... \cup E_n = S$	4.	impossible event

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	3	1	2	4
(c)	1	3	4	2
(d)	4	3	1	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

43. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(a) 1 (b) 2 (c) 5 (d) $\frac{8}{3}$

44. If E and F are events such that 0 < P(F) < 1, then

(a) $P(E \mid F) + P(\bar{E} \mid F) = 1$

(b) $P(E \mid F) + P(E \mid \bar{F}) = 1$

(c) $P(\bar{E} \mid F) + P(E \mid \bar{F}) = 1$

(d) $P(E \mid \bar{F}) + P(\bar{E} \mid \bar{F}) = 0$

45. If $P(B)=\frac{3}{5}$, $P(A \mid B)=\frac{1}{2}$ and $P(A \cup B)=\frac{4}{5}$, then $P(A \cup B)' + P(A' \cup B) =$

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2}$ (d) 1

46. In a binomial distribution, the mean is 4 and variance is 3. Then its mode is :

(a) 4 (b) 5 (c) 6 (d) 7

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion

(c) Assertion is correct, Reason is incorrect

(d) Assertion is incorrect, Reason is correct.

47. Let A and B be two events associated with an experiment such that

$P(A \cap B) = P(A)P(B)$

Assertion : P(A|B) = P(A) and P(B|A) = P(B)

Reason : $P(A \cup B) = P(A) + P(B)$

48. **Assertion :** The mean of a random variable X is also called the expectation of X, denoted by E(X).

Reason : The mean or expectation of a random variable X is not sum of the products of all possible values of X by their respective probabilities.

49. **Assertion :** Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely.

If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively.

then $P(E|F) = \frac{1}{4}$ and $P(F|E) = \frac{1}{13}$

Reason : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

50. **Assertion :** For a binomial distribution B(n, p), Mean > Variance

Reason : Probability is less than or equal to 1

51. Consider the two events E and F which are associated with the sample space of a random experiment.

Assertion : $P(E/F) = \frac{n(E \cap F)}{n(F)}$.

Reason : $P(E/F) = \frac{P(E \cap F)}{P(F)}$.

52. Consider the following statements

Assertion : Let A and B be two independent events. Then $P(A \cap B) = P(A) + P(B)$

Reason : Three events A, B and C are said to be independent, if $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$.

CRITICAL THINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

53. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is :

(a) 10^{-1} (b) $\left(\frac{1}{2}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\frac{9}{10}$

54. If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$, then choose the incorrect statement

(a) E_1 and E_2 are independent

(b) E_1 and E_2 are exhaustive

(c) E_2 is twice as likely to occur as E_1

(d) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

55. A bag contains 12 white pearls and 18 black pearls. Two pearls are drawn in succession without replacement. The probability that the first pearl is white and the second is black, is

(a) $\frac{32}{145}$ (b) $\frac{28}{143}$ (c) $\frac{36}{145}$ (d) $\frac{36}{143}$

56. Bag P contains 6 red and 4 blue balls and bag Q contains 5 red and 6 blue balls. A ball is transferred from bag P to bag Q and then a ball is drawn from bag Q. What is the probability that the ball drawn is blue?

(a) $\frac{7}{15}$ (b) $\frac{8}{15}$ (c) $\frac{4}{19}$ (d) $\frac{8}{19}$

57. A die is thrown again and again until three sixes are obtained. The probability of obtaining third six in the sixth throw of the die, is

(a) $\frac{625}{23329}$ (b) $\frac{621}{25329}$ (c) $\frac{625}{23328}$ (d) $\frac{620}{23328}$

58. If the chance that a ship arrives safely at a port is $\frac{9}{10}$; the chance that out of 5 expected ships, at least 4 will arrive safely at the port 2, is

(a) $\frac{91854}{100000}$ (b) $\frac{32805}{100000}$

(c) $\frac{59049}{100000}$ (d) $\frac{26244}{100000}$

59. Five bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement, then the probability distribution of the number of good eggs drawn, is

(a)

X	0	1	2	3
P(X)	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{7}{9}$	$\frac{1}{9}$

(b)

X	0	1	2	3
P(X)	$\frac{5}{54}$	$\frac{7}{54}$	$\frac{2}{27}$	$\frac{7}{27}$

(c)

X	0	1	2	3
P(X)	$\frac{1}{13}$	$\frac{2}{13}$	$\frac{9}{26}$	$\frac{3}{26}$

(d)

X	0	1	2	3
P(X)	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{27}$

60. A drunken man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps, he is one step away from the starting point.

(a) $24 \times (0.36)^6$ (b) $462 \times (0.24)^6$

(c) $24 \times (0.36)^5$ (d) $462 \times (0.24)^5$

61. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A'|B') \cdot P(B'|A')$ is equal to

(a) $\frac{5}{6}$ (b) $\frac{5}{7}$ (c) $\frac{25}{42}$ (d) 1

62. Two events E and F are independent. If P(E) = 0.3, $P(E \cup F) = 0.5$, then P(E | F) – P(F | E) equals

(a) $\frac{2}{7}$ (b) $\frac{3}{35}$ (c) $\frac{1}{70}$ (d) $\frac{1}{7}$

63. Three persons, A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

(a) 0.024 (b) 0.188 (c) 0.336 (d) 0.452

64. Suppose X follows a binomial distribution with parameters n and p, where $0 < p < 1$, if $P(X = r)/P(X = n - r)$ is independent of n and r, then

(a) $p = \frac{1}{2}$ (b) $p = \frac{1}{3}$

(c) $p = \frac{1}{4}$ (d) None of these

65. One hundred identical coins, each with probability p of showing up heads, are tossed. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins. The value of p is

(a) $\frac{1}{2}$ (b) $\frac{49}{101}$ (c) $\frac{50}{101}$ (d) $\frac{51}{101}$

66. The probability of a man hitting a target is 1/4. The number of times he must shoot so that the probability he hits the target, at least once is more than 0.9, is

[use log 4 = 0.602 and log 3 = 0.477]

(a) 7 (b) 8 (c) 6 (d) 5

67. Two dice are tossed 6 times. Then the probability that 7 will show an exactly four of the tosses is:

(a) $\frac{225}{18442}$ (b) $\frac{116}{20003}$ (c) $\frac{125}{15552}$ (d) $\frac{117}{17442}$

68. A box contains 20 identical balls of which 10 are blue and 10 are green. The balls are drawn at random from the box one at a time with replacement. The probability that a blue ball is drawn 4th time on the 7th draw is

(a) $\frac{27}{32}$ (b) $\frac{5}{64}$ (c) $\frac{5}{32}$ (d) $\frac{1}{2}$

69. If X follows Binomial distribution with mean 3 and variance 2, then $P(X \geq 8)$ is equal to :

(a) $\frac{17}{3^9}$ (b) $\frac{18}{3^9}$ (c) $\frac{19}{3^9}$ (d) $\frac{20}{3^9}$

70. If the mean and variance of a binomial variate x are respectively $\frac{35}{6}$ and $\frac{35}{36}$, then the probability of $x > 6$ is :

(a) $\frac{1}{6^2}$ (b) $\frac{5^7}{6^7}$

(c) $\frac{1}{7^6}$ (d) $\frac{1}{6^7} + \frac{1}{6^7}$

71. There is 30% chance that it rains on any particular day. Given that there is at least one rainy day, then the probability that there are at least two rainy days is

(a) $\dfrac{\frac{14}{5} \times \left(\frac{7}{10}\right)^6}{1 + \left(\frac{7}{10}\right)^7}$ (b) $\left(\frac{7}{10}\right)^6 - \frac{14}{17}$

(c) $\frac{13}{5} \times \left(\frac{7}{10}\right)^6$ (d) $\dfrac{1 - \frac{14}{15} \times \left(\frac{7}{10}\right)^6}{1 - \left(\frac{7}{10}\right)^7}$

72. In a binomial distribution, mean is 3 and standard deviation is $\frac{3}{2}$, then the probability function is

(a) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$ (b) $\left(\frac{1}{4}+\frac{3}{4}\right)^{12}$

(c) $\left(\frac{1}{4}+\frac{3}{4}\right)^{9}$ (d) $\left(\frac{3}{4}+\frac{1}{4}\right)^{9}$

73. An urn contains five balls. Two balls are drawn and found to be white. The probability that all the balls are white is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$

74. A signal which can be green or red with probability 4/5 and 1/5 respectively, is received by station A and then trasmitted to station B. The probability of each station receiving the signal correctly is 3/4. If the signal received at station B is given, then the probability that the original signal is green, is

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{20}{23}$ (d) $\frac{9}{20}$

75. By examining the chest X-ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB, A person is selected at random and is diagnosed to have TB. Then, the probability that the person actually has TB is

(a) $\frac{110}{221}$ (b) $\frac{2}{223}$ (c) $\frac{110}{223}$ (d) $\frac{1}{221}$

76. Coloured balls are distributed in four boxes as shown in the following table

Box	Colour			
	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black. Probability that the ball drawn from Box III, is

(a) 0.161 (b) 0.162 (c) 0.165 (d) 0.104

77. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. Then, the probability that he will knock down fewer than 2 hurdles is

(a) $\frac{5^9}{2\times6^9}$ (b) $\frac{5^{10}}{2\times6^{10}}$

(c) $\frac{5^9}{2\times6^{10}}$ (d) $\frac{5^{10}}{2\times6^9}$

78. For a biased dice, the probability for the different faces to turn up are

Face	1	2	3	4	5	6
P	0.10	0.32	0.21	0.15	0.05	0.17

The dice is tossed and it is told that either the face 1 or face 2 has shown up, then the probability that it is face 1, is

(a) $\frac{16}{21}$ (b) $\frac{1}{10}$ (c) $\frac{5}{16}$ (d) $\frac{5}{21}$

79. The random variable X has the following probability distribution

x	0	1	2	3	4
P(X = x)	k	3k	5k	2k	k

Then the value of $P(X \geq 2)$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** $P(A) = \frac{1}{2}, P(B) = 0$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$

= Not defined

2. **(d)** P (A/B) = P (B/A)

$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A) = P(B)$

3. **(a)** $P(B / A \cup B') = \frac{P(B \cap (A \cup B'))}{P(A \cup B')}$

$= -\frac{P(A \cap B)}{P(A) + P(B') - P(A \cap B')}$

$= \frac{P(A) - P(A \cap B')}{0.7 + 0.6 - 0.5}$

$= \frac{0.7 - 0.5}{0.8} = \frac{1}{4}$

4. **(b)** $P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A)\, P(B/A) = P(B) P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

5. **(b)** $P(A | \overline{B}) + P(\overline{A} | \overline{B}) = 1 \Rightarrow P(\overline{A} | \overline{B}) = 1 - P(A | \overline{B})$

6. **(c)** We know that, conditional probability of event E given that F has occurred is denoted by P(E|F) and is given by

$P(E | F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$

From this result, we can write

$P(E \cap F) = P(F).P(E | F)$

Also, we know that

$P(F | E) = \frac{P(E \cap F)}{P(E)}, P(E) \neq 0$

Thus, $P(E \cap F) = P(E).P(F / E)$

7. **(c)** The events can be written explicitly

A = {HHH, TTT}, B = {HHH, HHT, HTH, THH}

C = {HHH, HHT, HTH, THH, HTT, THT, TTH}

$P(A \cap B) = 1/8$

Also, P(A).P(B) = (2/8)(4/8) = 1/8 = $P(A \cap B)$

So, A and B are independent.

$P(A \cap C) = 1/8$

Also, P(A).P(C) = (2/8) (7/8) = 7/32 ≠ $P(A \cap C)$

So, A and C are dependent.

$P(B \cap C) = \frac{4}{8}$

Also, $P(B) \cdot P(C) = \frac{7}{16} \neq P(B \cap C) \Rightarrow$ B and C are dependent.

8. **(c)** For a Binomial distribution, outcomes at different trials must be independent.

9. **(b)** Number of Sundays in some randomly selected months with 30 days follow binomial distribution.

10. **(c)** **Remember:** The following relationships:

$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

consider $P(\overline{A} / \overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}$

(by defn of conditional prob.)

$= \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})}$.

11. **(a)** **Note:** $P(A/B) = \frac{P(A \cap B)}{P(B)}$

The sample space of tossing a coin three time is:

H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

Now, P(E) = probability of having at least 2 heads

$= \frac{4}{8} = \frac{1}{2}$ and

P(F) = Prob. of the first throw to be head

$= \frac{4}{8} = \frac{1}{2}$

$P(E \cap F) = \frac{3}{8}$

Now, $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$

$\Rightarrow P\left(\frac{E}{F}\right) = \frac{3/8}{4/8} = \frac{3}{4}$.

12. (d) Given: Let A be the event of arrival of ship safely

and $P(A) = \frac{1}{5}$ (given)

$\therefore \quad P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore \quad P(A \geq 3) = P(3) + P(4) + P(5)$

$= {}^5C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3 + {}^5C_4\left(\frac{1}{5}\right)^4\frac{4}{5} + {}^5C_5\left(\frac{1}{5}\right)^5$

$= \frac{5!}{3!\,2!}.\frac{4^2}{5^3.5^2} + \frac{5!}{4!}.\frac{1}{5^4}.\frac{4}{5} + \frac{1}{5^5}$

$= \frac{5.4}{2}.\frac{4^2}{5^5} + \frac{5.4}{5^5} + \frac{1}{5^5}$

$= \frac{1}{5^5}[10 \times 16 + 20 + 1] = \frac{181}{3125}$.

13. (b) $\left.\begin{array}{l} np = 4 \\ npq = 2 \end{array}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$P(X = 1) = {}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8.\frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$

14. (c) Given, that np = 3 and npq = 4
where p is the probability of success in one trial and q is the probability of failure and n is the number of trials

$\Rightarrow \quad q = \frac{4}{3}$

and this is not possible.
Thus , the given statement is never true.

15. (d) Given :
Probability of hitting the target = 0.1
i.e. p = 0.1
$\therefore$ q = 1 – p = 0.9
Also given n = 100
$\therefore$ By Binomial distribution, we have
Mean = μ = np, variance = npq
$\therefore$ Expected number = μ = 100 × 0.1 = 10

16. (a) It is given that

$P(A \cap B) = p$ and $P(A' \cap B) + P(A \cap B') = q$.

since $P(A' \cap B) = P(B) - P(A \cap B)$, we get

$= P(B) - P(A \cap B) + P(A) - P(A \cap B)$

$q = P(A) + P(B) = q + 2p$

$P(A') + P(B') = 1 - P(A) + 1 - P(B)$

$= 2 - q - 2p,$

showing that (b) is correct. The answer (c) is also correct because

$P(A \cap B \mid A \cup B) = \frac{P[(A \cap B) \cap (A \cup B)]}{P(A \cup B)}$

$= \frac{P(A \cap B)}{P(A \cup B)}$

$= \frac{P(A \cap B)}{P(A) + P(B) - P(A \cap B)} = \frac{p}{q + 2p - p} = \frac{p}{p + q}$

Finally, (d) is correct because

$P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A' - P(A \cap B)]$

$= 1 - (q + 2p - p) = 1 - p - q.$

17. (b) The desired probability

$= \frac{\frac{10}{100} \times \frac{70}{100}}{\frac{10}{100} \times \frac{70}{100} + \frac{5}{100} \times \frac{30}{100}} = \frac{14}{17}$

18. (d) Let $A \equiv$ Sum of the digits is 8
$B \equiv$ Product of the digits is 0
Then A = {08, 17, 26, 35, 44}
B = {00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40,}

$A \cap B$ = { 08 }

$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{14}{50}} = \frac{1}{14}$

19. (d) Given : P(I) = 0.3 and P(II) = 0.2

$\therefore$ $P(\overline{I}) = 1 - 0.3 = 0.7$ and $P(\overline{II}) = 1 - 0.2 = 0.8$

$\therefore$ The required probability

$= P(\overline{I} \cap II) = P(\overline{I}).P(II) = 0.7 \times 0.2 = 0.14$

20. (b) Let A : the student fails in Physics
and B : The student fails in Mathematics.

$\therefore \quad P(A) = \frac{30}{100}, \quad P(B) = \frac{25}{100}$

and $P(A \cap B) = \frac{10}{100}$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{\frac{10}{100}}{\frac{25}{100}} = \frac{10}{100} \times \frac{100}{25} = \frac{10}{25} = \frac{2}{5}$$

21. (c) It is given that, $P(X = 0) = 30\% = \frac{30}{100} = \frac{3}{10}$

$P(X = 1) = 70\% = \frac{70}{100} = \frac{7}{10}$

Therefore, the probability distribution is as follows

X	0	1
P(X)	$\frac{3}{10}$	$\frac{7}{10}$

$\therefore$ Mean of X = E(X) = ΣXP(X)

$= 0 \times \frac{3}{10} + 1 \times \frac{7}{10} = \frac{7}{10}$

Variance of X = $\Sigma X^2 P(X) - (\text{Mean})^2$

$= (0)^2 \times \frac{3}{10} + (1)^2 \times \frac{7}{10} - \left(\frac{7}{10}\right)^2$

$= \frac{7}{10} - \frac{49}{100} = \frac{21}{100}$

22. (c) Given $P(A/B) = 0.5 \Rightarrow \frac{P(A \cap B)}{P(B)} = 0.5$

$\Rightarrow P(A \cap B) = (0.5) \times P(B) = 0.5 \times 0.2 = 0.1$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.2 - 0.1 = 0.7$

Hence $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P((A \cup B)')}{1 - P(B)}$

$= \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - 0.7}{1 - 0.2} = \frac{3}{8}$

23. (c) When A and B are independent events then

$P(A/B) = \frac{P(A \cap B)}{P(B)} = P(A)$

and $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B)$

But, in general $P(A) \neq P(B)$ i.e., $P(A/B) \neq P(B/A)$.

24. (c) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{1 - P(B')} = \frac{0.15}{1 - 0.1} = \frac{15}{90} = \frac{1}{6}$

25. (c) Let E_1 : 'sum is less than 6' and 'E_2 : 'sum is 3', then E_1 = {(1, 1), (1, 2), (2, 1), (2, 2), (1, 3), (3, 1), (2, 3), (3, 2), (1, 4), (4, 1)} and E_2 = {(1, 2), (2, 1)}

Hence $E_1 \cap E_2 = \{(1,2)(2,1)\}$

and required probability

$= P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{\frac{2}{36}}{\frac{10}{36}} = \frac{2}{10} = \frac{1}{5}$

26. (c) Let X denote the number of defective mangoes from the bag. X can take values 0, 1, 2, 3 and 4.

P(X = 0) = Probability of getting no defective mango

$= \frac{^{15}C_4}{^{20}C_4} = \frac{91}{323}$

P(X = 1) = Probability of getting one defective mango

$= \frac{^5C_1 \times {}^{15}C_3}{^{20}C_4} = \frac{455}{969}$

P(X = 2) = Probability of getting two defective mango

$= \frac{^5C_2 \times {}^{15}C_2}{^{20}C_4} = \frac{70}{323}$

P(X = 3) = Probability of getting three defective mangoes $= \frac{^5C_3 \times {}^{15}C_1}{^{20}C_4} = \frac{10}{323}$

P(X = 4) = Probability of getting four defective mangoes $= \frac{^5C_4}{^{20}C_4} = \frac{1}{969}$

27. (a) X can take values 0, 1, 2.

P(X = 0) = Probability of not getting six on any dice $= \frac{25}{36}$

P(X = 1) = Probability of getting one six $= \frac{10}{36}$

P(X = 2) = Probability of getting two six $= \frac{1}{36}$

Thus, the probability distribution is

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

$\therefore E(X) = 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$

$= \frac{10}{36} + \frac{2}{36} = \frac{1}{3}$

28. (a) Mean = E(X) = $\sum x_i.P(x_i)$

= (0.05) (– 3) + (0.45) (– 1) + (0.20)0 + (0.25)1 + (0.05)3

= – 0.15 – 0.45 + 0 + 0.25 + 0.15 = – 0.2

STATEMENT TYPE QUESTIONS

29. (c) I. If p is the probability of a success and q, that of a failure, then p + q = 1 and p = 2q

$\Rightarrow 2q + q = 1$

$\Rightarrow q = \frac{1}{3}$ and hence, $p = \frac{2}{3}$

Let X be the random variable that represents the number of successes in six trials.

Also, n = 6

By binomial distribution, we get

$P(X = r) = {}^nC_r p^r q^{n-r}$.

$= {}^6C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r}$

P(atleast 4 success in 6 trials)

$= P(X \geq 4) = P(4) + P(5) + P(6)$

$= {}^6C_4\, p^4q^2 + {}^6C_5\, p^5q^1 + {}^6C_6\, p^6q^0$

$= p^4\, \{{}^6C_2 q^2 + {}^6C_1\, pq + {}^6C_0\, p^2\}$

$(\because {}^nC_r = {}^nC_{n-r})$

$$= \left(\frac{2}{3}\right)^4 \left[\frac{6\times5}{1\times2}\left(\frac{1}{3}\right)^2 + \frac{6}{1}.\frac{2}{3}.\frac{1}{3} + \left(\frac{2}{3}\right)^2\right]$$

$$= \left(\frac{2}{3}\right)^4 \left[\frac{15}{9}+\frac{12}{9}+\frac{4}{9}\right] = \frac{31}{9}\left(\frac{2}{3}\right)^4$$

II. let the man tosses the coin n times.

Probability (p) of getting a head at the toss of a coin

$= \frac{1}{2}$

$p = \frac{1}{2}$ and $q = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore P(X = r) = {}^nC_r\, p^r q^{n-r}$

$$= {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n$$

It is given that

P(atleast one head) > 90%

$$\Rightarrow 1 - P(0) > \frac{90}{100} \Rightarrow 1 - {}^nC_0 p^0 q^n > \frac{9}{10}$$

$$\Rightarrow 1 - {}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n > \frac{9}{10}$$

$$\Rightarrow 1 - \frac{9}{10} > \frac{1}{2^n} \Rightarrow 2^n > 10$$

The minimum value of n that satisfies the given inequality is 4. Thus, the man should toss the coin 4 or more than 4 times.

30. (c) I - Bernoulli's trials of a random experiment are finite in number.

II - The probability of pass or fail of each student is not same.

31. (d) I - Partition of a sample space is not unique.

II - It is necessary for them to be pairwise disjoint.

32. (d) I - Independent events and mutually exclusive events do not have one and the same meaning.

II - Only one of them can occur at a time.

33. (b) III- It is not sure to get head in both the tosses.

MATCHING TYPE QUESTIONS

34. (d) If A and B are two independent events, then A′ and B, A and B′, A′ and B′ are all independent events. It is given that P(A) = 0.3 and P(B) = 0.6

Also, A and B are independent events.

A. P(A and B) = $P(A \cap B) = P(A) \times P(B)$

$= 0.3 \times 0.6 = 0.18$

B. P(A and not B) = $P(A \cap B') = P(A) \times P(B')$

$= 0.3 \times (1 - 0.6) = 0.3 \times 0.4 = 0.12$

C. P(A or B) = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) \times P(B)$

$= 0.3 + 0.6 - 0.3 \times 0.6 = 0.9 - 0.18 = 0.72$

D. P(neither A nor B) = P(A′ and B′)

$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - 0.72 = 0.28$

35. (d) The probability distribution of X is

X	0	1	2	3	4
P(X)	0.1	k	2k	2k	k

A. We know that $\sum_{i=1}^{n} p_i = 1$

Therefore, 0.1 + k + 2k + 2k + k = 1

i.e., k = 0.15

B. P(you study for atleast two hours) = $P(X \geq 2)$

$= P(X = 2) + P(X = 3) + P(X = 4)$

$= 2k + 2k + k = 5k = 5 \times 0.15 = 0.75$

C. P(you study exactly two hours) = P(X = 2)

$= 2k = 2 \times 0.15 = 0.3$

D. P(you study atmost two hours) = $P(X \leq 2)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.1 + k + 2k = 0.1 + 3k = 0.1 + 3 \times 0.15 = 0.55$

36. (a) Here, the sample space is S = {1, 2, 3, 4, 5, 6}

Given, E = {1, 3, 5}, F = {2, 3}

and G = {2, 3, 4, 5}

$\Rightarrow E \cap F = \{3\}, E \cap G = \{3, 5\}$

$E \cup F = \{1, 2, 3, 5\}, (E \cup F) \cap G = \{2, 3, 5\}$

and $(E \cap F) \cap G = \{3\}$

$\Rightarrow$ n (S) = 6, n(E) = 3, n (F) = 2, n (G) = 4

$n(E \cap F) = 1, n(E \cap G) = 2, n(E \cup F) = 4$

$n[(E \cup F) \cap G] = 3, n[(E \cap F) \cap G] = 1$

$\therefore$ By the formula,

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

We have, $P(E) = \frac{3}{6} = \frac{1}{2}$, $P(F) = \frac{2}{6} = \frac{1}{3}$,

$P(G) = \frac{4}{6} = \frac{2}{3}$

$P(E \cap F) = \frac{1}{6}, P(E \cap G) = \frac{2}{6} = \frac{1}{3}$

$P[(E \cup F) \cap G] = \frac{3}{6} = \frac{1}{2}$ and $P[(E \cap F) \cap G] = \frac{1}{6}$

A. $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{1/3} = \frac{1}{2}$

and $P\left(\frac{F}{E}\right) = \frac{P(E \cap F)}{P(E)} = \frac{1/6}{1/2} = \frac{1}{3}$

B. $P\left(\frac{E}{G}\right) = \frac{P(E \cap G)}{P(G)} = \frac{1/3}{2/3} = \frac{1}{2}$

and $P\left(\frac{G}{E}\right) = \frac{P(E \cap G)}{P(E)} = \frac{1/3}{1/2} = \frac{2}{3}$

C. $P\left(\frac{E \cup F}{G}\right) = \frac{P[(E \cup F) \cap G]}{P(G)} = \frac{1/2}{2/3} = \frac{3}{4}$

and $P\left(\frac{E \cap F}{G}\right) = \frac{P[(E \cap F) \cap G]}{P(G)} = \frac{1/6}{2/3} = \frac{1}{4}$

37. (a) Let H : set of students reading Hindi newspaper and E : set of students reading English newspaper.
Let n(S) = 100, then n (H) = 60
n(E) = 40 and $n(H \cap E) = 20$

$\therefore P(H) = \frac{n(H)}{n(S)} = \frac{60}{100} = \frac{3}{5}$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{40}{100} = \frac{2}{5}$

and $P(H \cap E) = \frac{n(H \cap E)}{n(S)} = \frac{20}{100} = \frac{1}{5}$

A. Required probability = P (student reads neither Hindi nor English newspaper)
$= P(H' \cap E') = P(H \cup E)' = 1 - P(H \cup E)$

$= 1 - [P(H) + P(E) - P(H \cap E)] = 1 - \left[\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right] = \frac{1}{5}$

B. Required probability = P(a randomly choose student reads English newspaper, if she reads Hindi newspaper)

$P\left(\frac{E}{H}\right) = \frac{P(E \cap H)}{P(H)} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$

C. Required probability = P (student reads Hindi newspaper when it is given that she reads English newspaper)

$\therefore P\left(\frac{H}{E}\right) = \frac{P(H \cap E)}{P(E)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$

38. (a) Here, P(1) = P(2) = 0. 2,
P(3) = P(5) = P(6) = 0.1 and P(4) = 0.3
Let A be the event that same number each time and B the event that a total score is 10 or more.

A. P(A) = P(1, 1) + P(2, 2) + P(3, 3) + P(4, 4) + (P(5, 5) + P(6, 6)
= P(1) . P(1) + P(2) . P(2) + P(3) . P (3) + P(4) . P(4) + P(5) . P(5) + P(6). P(6)
$= (0.2)^2 + (0.2)^2 + (0.1)^2 + (0.3)^2 + (0.1)^2 + (0.1)^2$
= 0.20

B. P(B) = P(4, 6) + P(5, 5) + P(6, 4) + P(5, 6) + P(6, 5) + P(6, 6)
= P(4) . P(6) + P(5) . P(5) + P(6) . P(4) + P(5) P(6) + P(6) . P(5) + P(6) . P(6)
= 0.03 + 0.01 + 0.03 + 0.01 + 0.01 + 0.01
= 0.10

C. $P(A \cap B)$ = P(5, 5) + P(6, 6)
= P(5) . P(5) + P(6) . P(6)
= 0.01 + 0.01
= 0.02

D. Since, P(A) × P(B)
= 0.20 × 0.10
$= 0.020 = 0.02 = P(A \cap B)$
Therefore, the events are independent.

39. (b) Let X represents the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.
p = P (success) = 0.05 and q = 1 – p = 1 – 0.05 = 0.95
X has a binomial distribution with n = 5,
p = 0.05 and q = 0.95
$P(X = r) = {}^5C_r. (0.05)^r (0.95)^{5-r}$.

A. Required probability = P(X = 0) ${}^5C_0 p^0 q^5$
$= q^5 = (0.95)^5$.

B. Required probability
$= P(X \leq 1) = P(0) + P(1) = {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4$
$= q^5 + 5pq^4 = q^4 (q + 5p)$
$= (0.95)^4 [0.95 + 5 \times (0.05)]$
$= (0.95)^4 [0.95 + 0.25] = (0.95)^4 \times 1.2$

C. Required probability = P(X > 1)
= 1 – {P(0) + P(1)}
$= 1 - (0.95)^4 \times 1.2$

D. Required probability = P (atleast one)
$= P(X \geq 1) = 1 - P(0) = 1 - (0.95)^5$.

40. (d) 41. (b) 42. (a)

INTEGER TYPE QUESTIONS

43. (b) The variables are 1, 2 and 5,
1 is written on 3 faces

$\therefore$ Probability of getting 1 $= \frac{3}{6} = \frac{1}{2}$

2 is written on two faces

$\therefore$ Probability of getting 2 is $\frac{2}{6} = \frac{1}{3}$

5 is written on 1 face

$\therefore$ Probability of getting 5 $= \frac{1}{6}$

Probability distribution is

X	1	2	5
$P(X)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Mean = E (X) = $\sum P_i X_i = \frac{1}{2} \times 1 + \frac{1}{3} \times 2 + \frac{1}{6} \times 5$

$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3+4+5}{6} = \frac{12}{6} = 2.$

44. (a) $P(E|F) + P(\bar{E}|F)$

$$= \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)} = \frac{P((E \cup \bar{E}) \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

45. (d) $P(B) = \frac{3}{5}$, $P(A|B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$

$P(A \cap B) = P(A|B)P(B) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

$P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

We know, $P(A \cap B) + P(A' \cap B) = P(B)$

[as $A \cap B$ and $A' \cap B$ are mutually exclusive events]

$\Rightarrow \frac{3}{10} + P(A' \cap B) = \frac{3}{5}$

$\Rightarrow P(A' \cap B) = \frac{3}{5} - \frac{3}{10} = \frac{3}{10}$

Now, $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$

$= \frac{1}{2} + \frac{3}{5} - \frac{3}{10} = \frac{5+6-3}{10} = \frac{4}{5}$

$P((A \cup B)') = 1 - P(A \cup B) = 1 - \frac{4}{5} = \frac{1}{5}$

$\therefore P((A \cup B)') + P(A' \cup B) = \frac{1}{5} + \frac{4}{5} = 1$

46. (a) Given, mean = 4 i.e., np = 4
and variance = 3, i.e., npq = 3
so, q = 3/4 and p = 1 – q = 1/4
and n = 16
Hence the Binomial distribution is given by

$${}^{16}C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{16-r} ; \; r = 0, 1, ... 16$$

$\therefore$ Mode of the distribution is a number that repeats for the maximum time.
Thus, Mode = 4.

ASSERTION - REASON TYPE QUESTIONS

47. (c) Since, $P(A \cap B) = P(A)P(B)$, therefore A and B are independent events.

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$

Similarly, P(B|A) = P(B).
Thus, Assertion is true.
However, Reason is not true for independent events. For example, when a dice is rolled once, then the events.
A : 'an even number' shows up
and B : 'a multiple of 3' show up
are independent as

$P(A)P(B) = \frac{3}{6} \times \frac{2}{6} = \frac{1}{6} = P(A \cap B)$

$(\because A = \{2, 4, 6\}, B = \{3, 6\})$

$A \cup B = \{2, 3, 4, 6\}$

But $P(A \cup B) = \frac{4}{6} \neq P(A) + P(B)$

$\left(\because P(A) + P(B) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6} \neq \frac{4}{6}\right)$

48. (c)

49. (a) We have, $P(E) = \frac{13}{52} = \frac{1}{4}$ and $P(F) = \frac{4}{52} = \frac{1}{13}$

Also, E and F is the event 'the card drawn is the ace of spades, so that

$P(E \cap F) = \frac{1}{52}$

Hence, $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{52}}{\frac{1}{13}} = \frac{1}{4}$

Since, $P(E) = \frac{1}{4} = P(E|F)$, we can say that the occurrence of event F has not affected the probability of occurrence of the event E. We also have,

$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{52}}{\frac{1}{4}} = \frac{1}{13} = P(F)$

Again, $P(F) = \frac{1}{13} = P(F|E)$ shows that occurrence of event E has not affected the probability of occurrence of the event F. Thus, E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

50. (a) Mean = np and Variance = npq < np ($\because$ q < 1)

51. (a) **Assertion :** The conditional probability of E given that F has occurred is

$$P\left(\frac{E}{F}\right)=\frac{\text{Number of elementary event favourable to } E\cap F}{\text{Number of elementary events which are favourable to F}}$$

$$=\frac{n(E\cap F)}{n(F)}$$

Dividing the numerator and the denominator by total number of elementary events of the sample space, we see that $P\left(\frac{E}{F}\right)$ can also be written as

$$P\left(\frac{E}{F}\right)=\frac{\frac{n(E\cap F)}{n(S)}}{\frac{n(F)}{n(S)}}=\frac{P(E\cap F)}{P(F)} \quad ....(i)$$

Note that I is valid only when P(F) ≠ 0 i.e., F ≠ ϕ.

Reason : If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e.,

$$P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)} \text{ provided } P(F)\neq 0$$

52. (d) Assertion is false. If A and B are independent events then, P(A ∩ B) = P(A) . P(B)

Reason is false. Reason is that A, B, C will be independent, if they are pairwise independent and P(A ∩ B ∩ C) = P(A) P (B) P(C).

CRITICAL THINKING TYPE QUESTIONS

53. (c) Number of bulbs in the box = 100

Number of defective bulbs = 10

Probability of occuring a defective bulb $=\frac{10}{100}=\frac{1}{10}$

⇒ Probability of occurrence of a good bulb

$=1-\frac{1}{10}=\frac{9}{10}$

In a sample of 5 bulbs, the distribution of defective bulbs $=\left(\frac{9}{10}+\frac{1}{10}\right)^5$

Probability that no bulb is defective in this sample

$=\left(\frac{9}{10}\right)^5$

54. (b) $P(E_2/E_1)=\frac{P(E_1\cap E_2)}{P(E_1)}$

$\Rightarrow \frac{1}{2}=\frac{P(E_1\cap E_2)}{1/4}$

$\Rightarrow P(E_1\cap E_2)=\frac{1}{8}=P(E_2).P(E_1/E_2)$

$=P(E_2).\frac{1}{4}$

$\Rightarrow P(E_2)=\frac{1}{2}$

Since $P(E_1\cap E_2)=\frac{1}{8}=P(E_1).P(E_2)$

⇒ events are independent

Also $P(E_1\cup E_2)=\frac{1}{2}+\frac{1}{4}-\frac{1}{8}=\frac{5}{8}$

⇒ E_1 & E_2 are non exhaustive

55. (c) Let A and B be the events of getting a white pearl in the first draw and a black pearl in the second draw. Now

P(A) = P(getting a white pearl in the first draw)

$=\frac{12}{30}=\frac{2}{5}$

When second pearl is drawn without replacement, the probability that the second pearl is black is the conditional probability of the event B occurring when A has already occurred.

$\therefore P(B|A)=\frac{18}{29}$

By multiplication rule of probability, we have

$$P(A\cap B)=P(A).P(B|A)=\frac{2}{5}\times\frac{18}{29}=\frac{36}{145}$$

56. (b) Let E_1, E_2 and A be the events defined as follows:

E_1 = red ball is transferred from bag P to bag Q

E_2 = blue ball is transferred from bag P to bag Q

A = the ball drawn from bag Q is blue

As the bag P contains 6 red and 4 blue balls,

$P(E_1)=\frac{6}{10}=\frac{3}{5}$ and $P(E_2)=\frac{4}{10}=\frac{2}{5}$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.

When E_1 has occurred i.e., a red ball has already been transferred from bag P to Q, then bag Q will contain 6 red and 6 blue balls,

So, $P(A|E_1)=\frac{6}{12}=\frac{1}{2}$

When E_2 has occurred i.e., a blue ball has already been transferred from bag P to Q, then bag Q will contain 5 red and 7 blue balls,

So, $P(A|E_2)=\frac{7}{12}$

By using law of total probability, we get

$P(A)=P(E_1)\,P(A|E_1)+P(E_2)\,P(A|E_2)$

$=\frac{3}{5}\times\frac{1}{2}+\frac{2}{5}\times\frac{7}{12}=\frac{8}{15}$

57. (c) Here we do not have a strictly binomial distribution. However, each trial is independent and the probability of obtaining a six in a throw.

$p=\frac{1}{6}$ and $q=\frac{5}{6}$

Now, as the third six is obtained in the six throw, the required probability

= Prob(exactly two sixes in 5 throws). Prob (a six in sixth throw).

$$= {}^5C_2p^2q^3.p = {}^5C_2p^3q^3 = \frac{5.4}{1.2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3 = \frac{625}{23328}$$

58. (a) Let E = the event that a ship will arrive safely at the port then

p = probability of occurrence of event E in one trial

$$= \frac{9}{10}$$

$$\therefore\ q = 1 - p = \frac{1}{10}$$

Here number of trials = number of ship = 5

∴ The probability that event E will occur exactly 4 times in 5 trials is given by

$$P(4) = {}^5C_4p^4q^1 = 5\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^1 = \frac{32805}{100000}$$

Probability that event E will occur exactly 5 times in 5 trials is given by

$$P(5) = {}^5C_5p^5q^0 = 1\times\left(\frac{9}{10}\right)^5 \times 1 = \frac{59049}{100000}$$

$$\therefore \text{Required probability} = \frac{32805}{100000} + \frac{59049}{100000} = \frac{91854}{100000}$$

59. (d) Since, the eggs are drawn one by one with replacement, the events are independent, therefore, it is a problem of binomial distribution.

Total number of eggs = 5 + 10 = 15, out of which 10 are good.

If p = probability of drawing a good egg, then

$$p = \frac{10}{15} = \frac{2}{3},\ \therefore\ q = 1 - \frac{2}{3} = \frac{1}{3}$$

Thus, we have a binomial distribution with $p = \frac{2}{3}$, $q = \frac{1}{3}$ and n = 3.

If X denotes the number of good eggs drawn, then X can take values 0, 1, 2, 3.

$$P(0) = {}^3C_0q^3 = 1\times\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$P(1) = {}^3C_1pq^2 = 3\times\frac{2}{3}\times\left(\frac{1}{3}\right)^2 = \frac{2}{9}$$

$$P(2) = {}^3C_2p^2q = 3\times\left(\frac{2}{3}\right)^2\times\frac{1}{3} = \frac{4}{9} \text{ and}$$

$$P(3) = {}^3C_3p^3 = 1\times\left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

∴ The required probability distribution is

X	0	1	2	3
P(X)	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{27}$

60. (d) The man is one step away from starting point after 11 steps. This can happen in following two ways:

(i) he takes 5 steps forward and 6 steps backward

(ii) he takes 6 steps forward and 5 steps backward

Probability in first case = ${}^{11}C_5(0.4)^5(0.6)^6$

Probability in second case = ${}^{11}C_6(0.4)^6 = (0.6)^5$

Hence, required probability

= ${}^{11}C_5(0.4)^5(0.6)^6 + {}^{11}C_6(0.4)^6(0.6)^5$

= ${}^{11}C_5(0.4)^5(0.6)^5$ (0.6 + 0.4) = ${}^{11}C_5(0.24)^5 1$

= 462 × $(0.24)^5$

61. (c) $P(A) = \frac{2}{5},\ P(B) = \frac{3}{10},\ P(A\cap B) = \frac{1}{5}$

$$P(A') = 1 - P(A) = 1 - \frac{2}{5} = \frac{3}{5}$$

$$P(B') = 1 - P(B) = 1 - \frac{3}{10} = \frac{7}{10}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{2}{5} + \frac{3}{10} - \frac{1}{5} = \frac{1}{2}$$

$$P(A'\cap B') = P(A\cup B)' = 1 - P(A\cup B) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore\ P(A'|B')\cdot P(B'|A') = \frac{P(A'\cap B')}{P(B')}.\frac{P(A'\cap B')}{P(A')}$$

$$= \frac{1/2}{7/10}.\frac{1/2}{3/5} = \frac{1}{4}\times\frac{50}{21} = \frac{25}{42}$$

62. (c) Since, E and F are independent events

$\therefore\ P(E\cap F) = P(E)P(F)$

⇒ P(E|F) P(F) = P(E) P(F) and P(F|E)P(E) = P(E)P(F)

⇒ P(E|F) = P(E) and P(F|E) = P(F)

Now, $P(E\cup F) = P(E) + P(F) - P(E\cap F)$

= P(E) + P(F) – P(E)P(F)

⇒ 0.5 = 0.3 + P(F) – 0.3P(F)

P(F) (1 – 0.3) = 0.5 – 0.3

$$P(F) = \frac{0.2}{0.7} = \frac{2}{7}$$

∴ P(E|F) – P(F|E)= P(E) – P(F) = $0.3 - \frac{2}{7}$

$$= \frac{3}{10} - \frac{2}{7} = \frac{1}{70}$$

63. (b) Required probability

$$P(A'\cap B\cap C)+P(A\cap B'\cap C)+P(A\cap B\cap C')$$

$= [(1 - P(A)]P(B)P(C) + P(A)\ [(1 - P(B)]P(C) + P(A)P(B)[(1 - P(C)]$

$= (1 - 0.4)\ (0.3)\ (0.2) + (0.4)\ (1 - 0.3)\ (0.2) + (0.4)\ (0.3)\ (1 - 0.2)$

$= 0.036 + 0.056 + 0.096 = 0.188$

64. (a) $$\frac{P(X=r)}{P(X=n-r)}=\frac{^nC_r p^r(1-p)^{n-r}}{^nC_{n-r}p^{n-r}(1-p)^r}$$

$$=\frac{(1-p)^{n-2r}}{p^{n-2r}}=\left(\frac{1-p}{p}\right)^{n-2r}=\left(\frac{1}{p}-1\right)^{n-2r}$$

and $\left(\frac{1}{p}\right)-1>0$

$\therefore$ ratio will be independent of n and r if $(1/p)-1 = 1 \Rightarrow p = 1/2$

65. (d) Let $X \sim B(100, p)$ be the number of coins showing heads, and let $q = 1 - p$. Then, since $P(X = 51) = P(X=50)$, we have

$$^{100}C_{51}(p^{51})(q^{49})={}^{100}C_{50}(p^{50})(q^{50})$$

$$\Rightarrow \frac{p}{q}=\left(\frac{100!}{50!50!}\right)\left(\frac{51!\ 49!}{100!}\right)$$

$$\Rightarrow \frac{p}{1-p}=\frac{51}{50}\Rightarrow 50p=51-51p \Rightarrow p=\frac{51}{101}$$

66. (b) Let n denote the required number of shots and X the number of shots that hit the target. Then $X \sim B(n, p)$, with $p = 1/4$. Now,

$$P(X\ge 1)\ge 0.9 \Rightarrow 1-P(X=0)\ge 0.9$$

$$\Rightarrow 1-{}^nC_0\left(\frac{3}{4}\right)^n\ge 0.9 \Rightarrow \left(\frac{3}{4}\right)^n\le\frac{1}{10}$$

$$\Rightarrow \left(\frac{4}{3}\right)^n\ge 10 \Rightarrow n(\log 4-\log 3)\ge 1$$

$$\Rightarrow n(0.602-0.477)\ge 1 \Rightarrow n\ge\frac{1}{0.125}=8$$

Therefore the least number of trials required is 8.

67. (c) This is the question of binomial distribution because dice tossed 6 times

we know, $P(x) = {}^nC_r\, p^r\, q^{n-r}$ where

p = prob of success

q = prob of failure

n = no. of toss

Now, Here n = 6,

p = prob of getting 7 = $\frac{1}{6}$

and $q = 1 - p = 1-\frac{1}{6}=\frac{5}{6}$, r = 4

$$\therefore \text{Required prob} = {}^6C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2=\frac{125}{15552}$$

68. (c) Probability of getting a blue ball at any draw

$$= p=\frac{10}{20}=\frac{1}{2}$$

P [getting a blue ball 4th time in 7th draw] = P [getting 3 blue balls in 6 draw] × P [a blue ball in the 7th draw].

$$={}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3.\frac{1}{2}$$

$$=\frac{6\times 5\times 4}{1\times 2\times 3}\left(\frac{1}{2}\right)^7=20\times\frac{1}{32\times 4}=\frac{5}{32}$$

69. (c) As given,

Mean = np = 3 and Variance = npq = 2

$$\therefore \frac{npq}{np}=\frac{2}{3}\Rightarrow q=\frac{2}{3}$$

$$\text{and } p=(1-q)=1-\frac{2}{3}=\frac{1}{3}$$

$$\therefore n.\frac{1}{3}=3 \Rightarrow n=9$$

$$\therefore P(X\ge 8)=P(X=8)+P(X=9)$$

$$={}^9C_8\left(\frac{1}{3}\right)^8\times\frac{2}{3}+{}^9C_9\left(\frac{1}{3}\right)^9$$

$$=9\left(\frac{1}{3}\right)^8\frac{2}{3}+1\left(\frac{1}{3}\right)^9=\frac{18+1}{3^9}=\frac{19}{3^9}$$

70. (b) Given mean = np = $\frac{35}{6}$ ...(i)

and variance = npq = $\frac{35}{36}$ ...(ii)

Solving (i) and (ii), we get

$$q=\frac{1}{6},\ p=1-\frac{1}{6}=\frac{5}{6}$$

$\therefore$ we get n = 7

$\therefore$ $P(x = r) = {}^nC_r\, p^r\, q^{n-r}$ for Binomial distribution where r = 7

$$\therefore P(x>6)={}^7C_7\left(\frac{5}{6}\right)^7.\left(\frac{1}{6}\right)^{7-7}=\left(\frac{5}{6}\right)^7$$

71. (d) $P(r)=\frac{3}{10}$, so $P(\overline{r})=\frac{7}{10}$

The probability of at least one rainy day in 7 days

$$P(A)=1-\left(\frac{7}{10}\right)^7$$

Now the probability that at least two rainy days in 7 days

$$P(B) = 1-\left(\frac{7}{10}\right)^7 - {}^7C_1\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^6$$

Hence,

$$P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{1-\left(\frac{7}{10}\right)^7 - {}^7C_1\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^6}{1-\left(\frac{7}{10}\right)^7}$$

$$= \frac{1-\frac{14}{15}\times\left(\frac{7}{10}\right)^6}{1-\left(\frac{7}{10}\right)^7}$$

72. (a) In a binomial distribution,

Mean = np, Var = npq

and standard deviation = $\sqrt{npq}$

$\therefore$ Mean = 3, S.D = $\frac{3}{2}$

$\Rightarrow q = \frac{npq}{np}$ (multiply and divide by np)

$= \frac{9}{4\times 3} = \frac{3}{4}$ $\left(\because \sqrt{npq} = \frac{3}{2}\right)$

$\Rightarrow p = 1-\frac{3}{4} = \frac{1}{4}$ $(\because p = 1-q)$

Now, np = 3 $\Rightarrow n.\frac{1}{4} = 3 \Rightarrow n = 12$

Therefore, binomial function is given as

$$(q+p)^n = \left(\frac{3}{4}+\frac{1}{4}\right)^{12}$$

73. (d) Let A_i (i = 2, 3, 4, 5) be the event that urn contains 2, 3, 4, 5 white balls and let B be the event that two white balls have been drawn then we have to find $P(A_5/B)$.

Since the four events A_2, A_3, A_4 and A_5 are equally likely we have $P(A_2) = P(A_3) = P(A_4) = P(A_5) = \frac{1}{4}$.

$P(B/A_2)$ is probability of event that the urn contains 2 white balls and both have been drawn.

$$\therefore P(B/A_2) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$$

Similarly, $P(B/A_3) = \frac{{}^3C_2}{{}^5C_2} = \frac{3}{10}$,

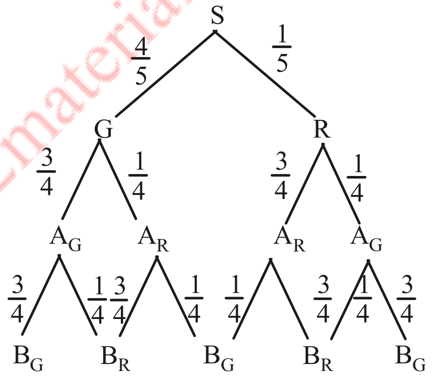
$P(B/A_4) = \frac{{}^4C_2}{{}^5C_2} = \frac{3}{5}$, $P(B/A_5) = \frac{{}^5C_2}{{}^5C_2} = 1$.

By Baye's theorem,

$$P(A_5/B) = \frac{P(A_5)P(B/A_5)}{P(A_2)P(B/A_2)+P(A_3)P(B/A_3)+P(A_4)(B/A_4)+P(A_5)P(B/A_5)}$$

$$= \frac{\frac{1}{4}.1}{\frac{1}{4}\left[\frac{1}{10}+\frac{3}{10}+\frac{3}{5}+1\right]} = \frac{10}{20} = \frac{1}{2}.$$

74. (c) From the tree diagram, it follows that



$P(B_G) = \frac{46}{80}, P(G) = \frac{4}{5}$

$P(B_G/G) = \frac{10}{16} = \frac{5}{8}$

$\therefore P(B_G\cap G) = \frac{5}{8}\times\frac{4}{5} = \frac{1}{2}$ $\left[\because P(B_G\cap G) = P\left(\frac{B_G}{G}\right)\times P(G)\right]$

Now, $P(G/B_G) = \frac{P(B_G\cap G)}{P(B_G)} = \frac{1}{2}\times\frac{80}{40} = \frac{20}{23}$

75. (a) Let A denote the event that the person has TB

Let B denote the event that the person has not TB.

Let E denote the event that the person is diagnosed to have TB.

$\therefore P(A) = \frac{1}{1000}, P(B) = \frac{999}{1000}$

$P\left(\frac{E}{A}\right) = 0.99, P\left(\frac{E}{B}\right) = 0.001$

The required probability is given by

$$P\left(\frac{A}{E}\right)=\frac{P(A)\times\left(\frac{E}{A}\right)}{P(A)\times P\left(\frac{E}{A}\right)+P(B)\times P\left(\frac{E}{B}\right)}$$

$$=\frac{\frac{1}{1000}\times 0.99}{\frac{1}{1000}\times 0.99+\frac{999}{1000}\times 0.001}$$

$$=\frac{0.99}{0.99+0.001\times 0.999}=\frac{0.99}{0.99+0.999}$$

$$=\frac{990}{990+999}=\frac{990}{1989}=\frac{110}{221}$$

76. (c) Let A, E_1, E_2, E_3 and E_4 be the events as defined below

A : a black ball is selected

E_1 : box I is selected

E_2 : box II is selected

E_3 : box III is selected

E_4 : box IV is selected

Since, the boxes are chosen at random,

Therefore, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

Also, $P(A/E_1) = \frac{3}{18}$, $P(A/E_2) = \frac{2}{8}$, $P(A/E_3) = \frac{1}{7}$

and $P(A/E_4) = \frac{4}{13}$

P(box III is selected, given that the drawn ball is black) = $P(E_3/A)$ By Bayes' theorem,

$$P(E_3/A)=\frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)+P(E_4)P(A/E_4)}$$

$$=\frac{\frac{1}{4}\times\frac{1}{7}}{\frac{1}{4}\times\frac{3}{18}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{4}{13}}=0.165$$

77. (d) It is a case of Bernoulli trials, where success is not crossing a hurdle successfully. Here, n = 10.

$$p = P\text{ (success)} = 1-\frac{5}{6}=\frac{1}{6}\Rightarrow q=\frac{5}{6}$$

let X be the random variable that represents the number of times the player will knock down the hurdle.

Clearly, X has a binomial distribution with n = 10 and $p = \frac{1}{6}$

$$\therefore P(X=r) = {}^nC_r q^{n-r}.\, p^r = {}^{10}C_r\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{10-r}$$

P (player knocking down less than 2 hurdles)

$= P(X < 2) = P(X = 0) + P(X = 1)$

$$= {}^{10}C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{10-0}+{}^{10}C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^9$$

$$=\left(\frac{5}{6}\right)^9\left(\frac{5}{6}+\frac{10}{6}\right)=\frac{5^{10}}{2\times 6^9}$$

78. (d) Let E : 'face 1 comes up' and F: 'face 1 or 2 comes up'

$\Rightarrow E\cap F=E \qquad (\because E\subset F)$

$\therefore$ P(E) = 0.10 and P(F) = P(1) + P(2)

= 0.10 + 0.32 = 0.42

Hence, required probability

$$=P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{P(E)}{P(F)}=\frac{0.10}{0.42}=\frac{5}{21}$$

79. (b) Since, $\sum P_i(X=x)=1$

$\therefore$ k + 3k + 5k + 2k + k = 1

$\therefore$ 12k = 1 $\qquad \therefore k=\frac{1}{12}$

Now, $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$

= 5k + 2k + k

$$= 8k = 8\left(\frac{1}{12}\right)=\frac{2}{3}$$

Mock Test-1

Time : 1 hr **Max. Marks -180**

1. All possible two factors products are formed from the numbers 1, 2, 3, 4, ..., 200. The number of factors out of total obtained which are multiples of 5 is

(a) 5040 (b) 7180
(c) 8150 (d) None

2. If z be a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$ then |z| is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$
(c) 1 (d) No unique value

3. The number of real solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$ is

(a) 0 (b) 1
(c) 2 (d) None of these

4. The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + \frac{1}{x^2}$, $0 \le x \le \frac{\pi}{2}$ has

(a) one real solution
(b) no real solution
(c) more than one real solution
(d) none of these

5. If log a, log b, and log c are in A.P. and also log a – log 2b, log 2b – log 3c, log 3c – log a are in A.P., then :

(a) a, b, c, are in H.P.
(b) a, 2b, 3c are in A.P.
(c) a, b, c are the sides of a triangle
(d) none of the above

6. Which of the following is correct ?

(a) If $y + \frac{y^3}{3} + \frac{y^5}{5} + ...\infty = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + ...\infty\right)$ then $y = 2x^2 - x$

(b) $\left(\frac{a-b}{a}\right) + \frac{1}{2}\left(\frac{a-b}{a}\right)^2 + \frac{1}{3}\left(\frac{a-b}{a}\right)^3 + ...\infty = \log ab$

(c) $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + ..\infty = \frac{x}{1-x} + \log(1-x)$

(d) $\log_4 2 - \log_8 2 + \log_{16} 2 - .....\infty = -\log 2$

7. Incorrect sum from the following is :

(a) $1 + \frac{1+a}{2\,!} + \frac{1+a+a^2}{3!} + ...\infty = \frac{e^a - e}{a-1}$

(b) $9 + \frac{19}{2!} + \frac{35}{3!} + \frac{57}{4!} + \frac{85}{5!} + ...\infty = 12e - 5$

(c) $\frac{1}{1.2} + \frac{1.3}{1.2.3.4} + \frac{1.3.5}{1.2.3.4.5.6} + ...\infty = \sqrt{e}$

(d) $1.3 + \frac{2.4}{1.2} + \frac{3.5}{1.2.3} + \frac{4.6}{1.2.3.4} + ...\infty = 4e$

8. The value of the determinant

$$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix} \text{ is}$$

(a) independent of α
(b) independent of β
(c) independent of α & β
(d) None of these

9. The number of solution of the equation

$\tan 3x = \sin 6x$ in $\left[0, \frac{\pi}{6}\right] \cup \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ is :

(a) 5 (b) 4
(c) 3 (d) 1

10. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ then the value of $x^{100} + y^{100} + z^{100} - \frac{3}{x^{101} + y^{101} + z^{101}}$ is

(a) 0 (b) 1
(c) 2 (d) 3

11. If A and B are positive acute angles satisfying $3\cos^2 A + 2\cos^2 B = 4$ and $\frac{3\sin A}{\sin B} = \frac{2\cos B}{\cos A}$,

Then the value of A + 2B is equal to :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

12. If $\alpha = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3}$

and $\beta = \cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{3}$, then :

(a) $\alpha < \beta$ (b) $\alpha = \beta$
(c) $\alpha > \beta$ (d) $\alpha + \beta = 2\pi$

13. The principal value of $\cos^{-1}(-\sin 7\pi/6)$ is

(a) $\frac{5\pi}{3}$ (b) $\frac{7\pi}{6}$

(c) $\frac{\pi}{3}$ (d) None

14. If α, β, γ are the altitudes of a triangle, then $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ =

(a) Δ / s (b) s / Δ

(c) sD (d) s / Δ^2

15. A harbour lies in a direction 60° south of west from a fort and at a distance 30 km from it, a ship sets out from the harbour at noon and sails due east at 10 km an hour. The ship will be 70 km from the fort at :

(a) 7 p.m. (b) 8 p.m.

(c) 5 p.m. (d) 10 p.m.

16. The points with co-ordinates (2a, 3a), (3b, 2b) and (c, c) are collinear

(a) for no value of a, b, c

(b) for all values of a, b, c

(c) iff a, $\frac{c}{5}$, b are in H.P.

(d) iff a, $\frac{2c}{5}$, b are in H.P.

17. The equation of the pair of straight lines parallel to the y-axis and which are tangents to the circle $x^2 + y^2 - 6x - 4y - 12 = 0$ is

(a) $x^2 - 4y - 21 = 0$ (b) $x^2 - 5x + 6 = 0$

(c) $x^2 - 6x - 16 = 0$ (d) None of these

18. The length of the chord x + y = 3 intercepted by the circle $x^2 + y^2 - 2x - 2y - 2 = 0$ is

(a) $\frac{7}{2}$ (b) $\frac{3\sqrt{3}}{2}$

(c) $\sqrt{14}$ (d) $\frac{\sqrt{7}}{2}$

19. If the eccentricity of the hyperbola $x^2 - y^2$ coces$^2 \alpha = 25$ is $\sqrt{5}$ times the eccentricity of the ellipse $x^2 \operatorname{cosec}^2 \alpha + y^2 = 5$, then α is equal to :

(a) $\tan^{-1}\sqrt{2}$ (b) $\sin^{-1}\sqrt{\frac{3}{4}}$

(c) $\tan^{-1}\sqrt{\frac{2}{5}}$ (d) $\sin^{-1}\sqrt{\frac{2}{5}}$

20. The point of intersection of the tangents to the parabola $y^2 = 4ax$ at the points 't_1' and 't_2' is

(a) $(at_1t_2, a(t_1 + t_2))$

(b) $(at_1t_2, at_1t_2(t_1 + t_2))$

(c) $(at_1t_2(t_1 + t_2), a(t_1 + t_2))$

(d) None of these

21. Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on circle for

(a) all integral values of k (b) 0 < k < 1

(c) k < 0 (d) For one value of k

22. The condition that the straight line $cx - by + b^2 = 0$ may touch the circle $x^2 + y^2 = ax + by$, is

(a) abc = 1 (b) a = c

(c) b = ac (d) None of these

23. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where [x] stands for the greatest integer function, then

(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$

(c) $f(-\pi) = 1$ (d) $f\left(\frac{\pi}{4}\right) = 2$

24. Let f"(x) be continuous at x = 0 and f" (0) = 4.

Then value of $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is

(a) 12 (b) 10

(c) 6 (d) 4

25. If $x^p y^q = (x + y)^{p+q}$, then dy/dx is equal to

(a) $\frac{y}{x}$ (b) $\frac{py}{qx}$

(c) $\frac{x}{y}$ (d) $\frac{qy}{px}$

26. Let $f(x) = x^n$, n being a non-negative integer. The value of n for which equality

f' (a + b) = f' (a) + f' (b) is valid for all a, b > 0 is :

(a) 5 (b) 1

(c) 2 (d) 4

27. The angle at which the curve $y = ke^{kx}$ intersects the y-axis is :

(a) $\tan^{-1}(k^2)$ (b) $\cot^{-1}(k^2)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$ (d) $\sec^{-1}\sqrt{1+k^4}$

28. The maximum value of the function

$y = \frac{a^2}{x} + \frac{b^2}{a - x}$, a > 0, b > 0 in (0, a) is :

(a) a + b (b) $\frac{1}{a+b}$

(c) $\frac{1}{a}(a+b)^2$ (d) $\frac{1}{a^2}(a+b)$

29. The set of all points, where the function

$f(x) = \frac{x}{(1 + |x|)}$ is differentiable, is

(a) (–¥, ¥) (b) (0,¥)

(c) (–¥, 0) È (0,¥) (d) none of these

30. If $y = \sin^{-1} x + \cos^{-1}\sqrt{(1 - x^2)}$, then dy/dx at $x = -1/\sqrt{2}$, is

(a) 0 (b) 2

(c) does not exist (d) none of these.

31. If $\int \frac{dx}{\sqrt{x^2+2x+1}} = A \log|x+1| + C$ for $x < -1$

then A is

(a) 0 (b) 1

(c) –1 (d) None of these

32. $\int \left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$ is equal to :

(a) $\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$

(b) $\left[\frac{x^2+1}{x^2}\right]^{n+6}(n+6)+c$

(c) $\left[\frac{x}{x^2+1}\right]^{n+6}(n+6)+c$

(d) none of these

33. $\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is equal to

(a) $\frac{1}{8}(x^2-1)+k$ (b) $\frac{1}{2}x^2+k$

(c) $\frac{1}{2}x+k$ (d) None of these.

34. The value of the integral $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi^2}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi^2}{2}$

35. $\int_{\pi/3}^{\pi/2} x\sin(\pi[x]-x)\,dx$ is equal to :

(a) $\frac{1}{2}+\frac{\pi}{6}$ (b) $1-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$

(c) $-\frac{1}{2}-\frac{\pi}{6}$ (d) $\frac{\sqrt{3}}{2}-1-\frac{\pi}{6}$

36. The solution to the differential equation $\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$ is

(a) $e^{2y}=e^{3x}+x^3+C$ (b) $3e^{2y}=2(e^{3x}+x^3)+c$

(c) $e^{3x+2y}=x^3+c$ (d) none of these.

37. The solution of the differential equation $(xy^2+x)dx+(yx^2+y)dy=0$ is

(a) $(x^2+1)(y^2+1)=c$

(b) $e^{x^2}+e^{y^2}=c$

(c) $(y^2+1)=c(x^2+1)$

(d) none of these.

38. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$ and $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=-3(\hat{i}-\hat{k})$, then the number triple (α, β, γ) is

(a) (2, –1, –1) (b) (–2, 1, 1)

(c) (–2, –1, 1) (d) (2, 1, –1)

39. For any vector $\vec{p}$, the value of $\frac{3}{2}\left\{|\vec{p}\times\hat{i}|^2+|\vec{p}\times\hat{j}|^2+|\vec{p}\times\hat{k}|^2\right\}$ is

(a) $\vec{p}^{\,2}$ (b) $2\vec{p}^{\,2}$

(c) $3\vec{p}^{\,2}$ (d) $4\vec{p}^{\,2}$

40. The position vector of A and B are $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$

The length of the internal bisector of $\angle BOA$ of triangle AOB is

(a) $\sqrt{\frac{136}{9}}$ (b) $\frac{\sqrt{136}}{9}$

(c) $\frac{20}{3}$ (d) $\sqrt{\frac{217}{9}}$

41. If α, β, γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ is equal to

(a) 1 (b) 2

(c) 0 (d) –1.

42. The shortest distance from the point (1, 2, –1) to the surface of the sphere $x^2+y^2+z^2=24$ is

(a) $3\sqrt{6}$ (b) $2\sqrt{6}$

(c) $\sqrt{6}$ (d) 12

43. If the probability that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is

(a) $p+q$ (b) $p+q-2pq$

(c) $p+q-pq$ (d) $p+q+pq$.

44. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$.

If $|\vec{u}|=3$, $|\vec{v}|=4$ and $|\vec{w}|=5$, then $\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}$ is

(a) 47 (b) –25 (c) 0 (d) 25

45. A rectangular parallelopiped is formed by drawing planes through the points (–1, 2, 5) and (1, –1, –1) and parallel to the coordinate planes. The length of the diagonal of the parallelopiped is

(a) 2 (b) 3 (c) 6 (d) 7

ANSWER KEY

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (c)
11. (b)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (c)	**19.** (a)	**20.** (a)
21. (d)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (a)	**26.** (c)	**27.** (b)	**28.** (c)	**29.** (a)	**30.** (a)
31. (c)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (a)	**38.** (a)	**39.** (c)	**40.** (b)
41. (b)	**42.** (c)	**43.** (b)	**44.** (b)	**45.** (d)					

HINTS & SOLUTIONS

1. **(b)** The total number of two factor products = ${}^{200}C_2$. The number of numbers from 1 to 200 which are not multiples of 5 is 160. Therefore the total number of two factor products which are not multiple of 5 is ${}^{160}C_2$. Hence, the required number of factors which are multiples of 5 = ${}^{200}C_2 - {}^{160}C_2 = 7180$.

2. **(c)** $z^4 + z^3 + 2z^2 + z + 1 = 0$

$\Rightarrow z^4 + z^3 + z^2 + z^2 + z + 1 = 0$

or $z^2(z^2 + z + 1) + (z^2 + z + 1) = 0$

or $(z^2 + z + 1)(z^2 + 1) = 0$

$\therefore z = i, -i, \omega, \omega^2$, For each, $|z| = 1$.

3. **(a)** $\because \sin\theta \le 1$ for all $\theta \Rightarrow \sin(e^x) \le 1$ for all x

Also $5^x + 5^{-x} = \left(5^{\frac{x}{2}}\right)^2 + \left(5^{-\frac{x}{2}}\right)^2 - 2 + 2$

$= \left(5^{\frac{x}{2}} - 5^{-\frac{x}{2}}\right)^2 + 2 \ge 2$ for all x

$\therefore$ LHS ≤ 1 and RHS ≥ 2

Hence LHS $\ne$ RHS for any real x.

Therefore the equation $\sin(e^x) = 5^x + 5^{-x}$ has no real solution.

4. **(b)** We have for any real x, $\cos^2\frac{x}{2} \le 1$ and $\sin^2 x \le 1$

$\therefore\ 2\cos^2\frac{x}{2}\sin^2 x \le 2 \Rightarrow \text{LHS} \le 2$

Again for any real x,

$x^2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} - 2 + 2 = \left(x - \frac{1}{x}\right)^2 + 2 \ge 2$

$\therefore$ RHS ≥ 2 Thus, the given equation can have solution only if LHS = RHS = 2

$\Rightarrow 2\cos^2\frac{x}{2}\sin^2 x = x^2 + \frac{1}{x^2} = 2$

Now, $x^2 + \frac{1}{x^2} = 2 \Rightarrow x = \pm 1$

but for $x = \pm 1$, $\cos^2\frac{x}{2}\sin^2 x \ne 1$.

That is, LHS and RHS cannot be simultaneously equal to 2 for any value of x.

$\therefore$ The equation has no real solution.

5. **(c)** log a, log b, log c are in A.P.

$\Rightarrow 2\log b = \log a + \log c$

$\Rightarrow \log b^2 = \log(ac)$

$\Rightarrow b^2 = ac$

$\Rightarrow$ a, b, c, are in G.P.

log a – log 2b, log 2b – log 3c, log 3c – log a are in A.P.

$\Rightarrow 2(\log 2b - \log 3c) = (\log a - \log 2b) + (\log 3c - \log a)$

$\Rightarrow 3\log 2b = 3\log 3c$

$\Rightarrow 2b = 3c$

Now, $b^2 = ac \Rightarrow b^2 = a \cdot \frac{2b}{3}$

$\Rightarrow b = \frac{2a}{3},\ c = \frac{4a}{9}$

i.e. $a = a, b = \frac{2a}{3},\ c = \frac{4a}{9}$

$\Rightarrow a : b : c = 1 : \frac{2}{3} : \frac{4}{9} = 9 : 6 : 4$

Since, sum of any two is greater than the 3rd, a, b, c, form a triangle.

6. **(c)** (a) We have

$y + \frac{y^3}{3} + \frac{y^5}{5} + \ldots\infty = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)$

$\Rightarrow \frac{1}{2}\log\frac{1+y}{1-y} = 2 \cdot \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$\Rightarrow \frac{1+y}{1-y} = \left(\frac{1+x}{1-x}\right)^2$

Applying componendo and dividendo

$$\frac{2y}{2}=\frac{(1+x)^2-(1-x)^2}{(1+x)^2+(1-x)^2}=\frac{2x}{1+x^2}$$

$\therefore x^2y=2x-y$

(b) $\left(\frac{a-b}{a}\right)+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+....\infty$

Let $\frac{a-b}{a}=x$, then the series is

$$x+\frac{x^2}{2}+\frac{x^3}{3}+....=-\log(1-x)$$

$$=-\log\left[1-\frac{a-b}{a}\right]=-\log\left[\frac{b}{a}\right]=\log\left(\frac{a}{b}\right)$$

(c) $T_n=\frac{n}{n+1}x^{n+1}=\left[1-\frac{1}{n+1}\right]x^{n+1}$

$$T_n=x^{n+1}-\frac{x^{n+1}}{n+1}$$

$$S=\sum_{n=1}^{\infty}T_n=(x^2+x^3+x^4+....)-\left\{\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+....\right\}$$

$$=\frac{x^2}{1-x}+\{\log(1-x)+x\}$$

$$=\frac{x}{1-x}+\log(1-x)$$

(d) General Term $T_n=\log_{2^n}2$

But $\log_{2^n}2=\frac{1}{n}\log_2 2=\frac{1}{n}$

$\therefore S=\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-.....$

Using

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.....$$

$$\log 2=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...=1-S$$

$\therefore S=1-\log_e 2$

7. **(c)**

(a) The given series is

$$1+\frac{1+a}{2!}+\frac{1+a+a^2}{3!}+\frac{1+a+a^2+a^3}{4!}+....$$

Here, $T_n=\frac{1+a+a^2+a^3+....\text{to n terms}}{n!}$

$$=\frac{1(1-a^n)}{(1-a)(n!)}=\frac{1}{1-a}\left(\frac{1-a^n}{n!}\right)$$

$\therefore T_1+T_2+T_3+.....\text{to }\infty$

$$=\frac{1}{1-a}\left[\frac{1-a}{1!}+\frac{1-a^2}{2!}+\frac{1-a^3}{3!}+....\text{to }\infty\right]$$

$$=\frac{1}{1-a}\left[\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....\text{to}\infty\right)-\left(\frac{a}{1!}+\frac{a^2}{2!}+\frac{a^3}{3!}+....\text{ to }\infty\right)\right]$$

$$=\frac{1}{1-a}[(e-1)-(e^a-1)]$$

$$=\frac{e-e^a}{1-a}=\frac{e^a-e}{a-1}$$

(b) First of all, we find the nth term of the series

$9+19+35+57+85+.......$

Let $S_n=9+19+35+57+85+....+T'_{n-1}+T'_n$ .....(1)

Also, $S_n=9+19+35+27+....+T'_{n-1}+T'_n$ .....(2)

On subtraction, we get

$0=9+10+16+22+28+....\text{to n terms}-T'_n$

$\Rightarrow T'_n=9+[10+16+22+28+.....\text{ to (n–1) terms}]$

$$=9+\frac{n-1}{2}[2\times10+(n-2)6]$$

$$=9+\frac{n-1}{2}[6n+8]\ =9+(n-1)(3n+4)$$

$$=3n^2+n+5$$

$\therefore T_n=\frac{3n^2+n+5}{n!}$ (i.e., nth term of the given series)

$$\Rightarrow S_n=\sum_{n=1}^{\infty}T_n=\sum_{n=1}^{\infty}\frac{3n^2+n+5}{n!}$$

$$=3\sum_{n=1}^{\infty}\frac{n^2}{n!}+\sum_{n=1}^{\infty}\frac{n}{n!}+5\sum_{n=1}^{\infty}\frac{1}{n!}$$

$$=3(2e)+e+5(e-1)=12e-5$$

(c) $T_n=\frac{1.3.5......(2n-1)}{(2n)!}$

$$=\frac{1.2.3.4.....(2n-1).2n}{(2n)!\,2.4.6.....2n}$$

$$=\frac{(2n)!}{(2n)!2^n\,n!}=\frac{1}{2^n\,n!}$$

Now putting n = 1, 2, 3, ... we see that the sum of series

$$S=\frac{1}{2}+\frac{(1/2)^2}{2!}+\frac{(1/2)^3}{3!}+....$$

$$=e^{\frac{1}{2}}-1=\sqrt{e}-1$$

(d) $T_n = \frac{n(n+2)}{n!} = \frac{n-1+3}{(n-1)!}$

$= \frac{1}{(n-2)!} + \frac{3}{(n-1)!}$

$S = \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty}\left[\frac{1}{(n-2)!} + \frac{3}{(n-1)!}\right]$

$= e + 3e = 4e$

8. (a) *Let* $\Delta = \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$

Apply $R_3 \to R_3 - R_1\cos\beta + R_2\sin\beta$

$\Delta = \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1+\sin\beta-\cos\beta \end{vmatrix}$

$= (1+\sin\beta-\cos\beta)(\cos^2\alpha+\sin^2\alpha)$

$= 1+\sin\beta-\cos\beta$, which is independent of α.

9. (a) Given, $\tan 3x = \sin 6x$

$\Rightarrow \frac{\sin 3x}{\cos 3x} = 2\sin 3x\cos 3x$

$\Rightarrow \sin 3x = 2\sin 3x\cos^2 3x,\ x \neq \frac{\pi}{6}$

$\Rightarrow \sin 3x = 0$ or $\cos^2 3x = \frac{1}{2}$

$\Rightarrow x = 0, \frac{\pi}{3}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$

Hence, the number of solution is 5.

10. (c) We have $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ it is possible only when

$\sin^{-1}x = \frac{\pi}{2} \Rightarrow x = 1 \quad \because \sin^{-1}x \le \frac{\pi}{2}$

$\sin^{-1}y = \frac{\pi}{2} \Rightarrow y = 1$

$\sin^{-1}z = \frac{\pi}{2} \Rightarrow z = 1$

$\therefore x^{100} + y^{100} + z^{100} - \frac{3}{x^{101}+y^{101}+z^{101}}$

$= 1+1+1-\frac{3}{3} = 3-1 = 2.$

11. (b) Given, $3\cos^2 A + 2\cos^2 B = 4$

$\Rightarrow 2\cos^2 B - 1 = 4 - 3\cos^2 A - 1$

$\Rightarrow \cos 2B = 3(1-\cos^2 A) = 3\sin^2 A$ ...(1)

and $2\cos B\sin B = 3\sin A\cos A$

$\sin 2B = 3\sin A\cos A$ ......(2)

Now, $\cos(A+2B)$

$= \cos A\cos 2B - \sin A\sin 2B$

$= \cos A(3\sin^2 A) - \sin A(3\sin A\cos A) = 0$

[using eqs. (1) and (2)]

$\Rightarrow A + 2B = \frac{\pi}{2}$

12. (a) We have

$\alpha+\beta = \left(\sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2}\right) + \left(\sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}\right)$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

Since $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for all x

Also, $\alpha = \frac{\pi}{3} + \sin^{-1}\frac{1}{3} < \frac{\pi}{3} + \sin^{-1}\frac{1}{2}$

as $\sin\theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$

$\therefore \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

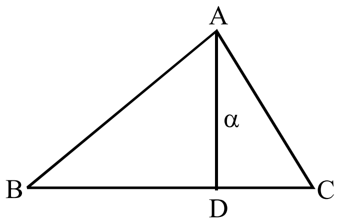
$\Rightarrow \beta > \frac{\pi}{2} > \alpha \Rightarrow \alpha < \beta$

13. (c) $\cos^{-1}(-\sin 7\pi/6) = \cos^{-1}\left\{\cos\left(\frac{\pi}{2}+\frac{7\pi}{6}\right)\right\}$

$= \cos^{-1}\left(\cos\frac{5\pi}{3}\right) = \cos^{-1}\left\{\cos\left(2\pi - \frac{5\pi}{3}\right)\right\}$

$= \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}.$

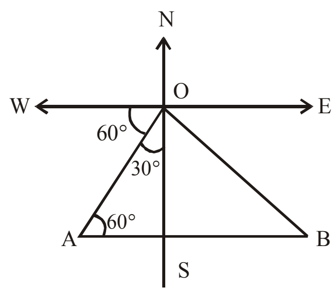
14. (b) $\Delta = \left(\frac{1}{2}\right)a\alpha \Rightarrow \alpha^{-1} = \frac{a}{2\Delta}$



$\Sigma\alpha^{-1} = \frac{a+b+c}{2\Delta} = \frac{2s}{2\Delta} = \frac{s}{\Delta}$

$\therefore \alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{s}{\Delta}$

15. (b) Let A be the position of the harbour and O be the fort. OA = 30, OB = 70. Let AB = x.



Applying cosine formula in $\angle OAB$,

$$\cos 60° = \frac{OA^2 + AB^2 - OB^2}{2OA \cdot AB}$$

$$\Rightarrow \frac{1}{2} = \frac{900 + x^2 - 4900}{2 \times 30 \times x}$$

$\Rightarrow x^2 - 30x - 4000 = 0 \Rightarrow x = 80$

AB = 80 km

Speed of ship = 10 km/hr

$\therefore$ Time = 8 hr

Hence, the ship will reach at 8 p.m.

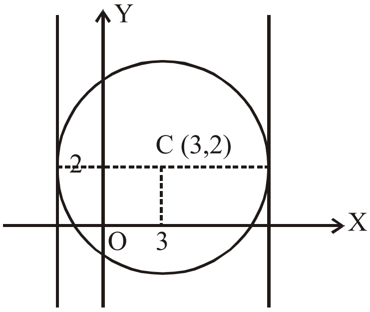
16. (d) For collinearity $\begin{vmatrix} 1 & 2a & 3a \\ 1 & 3b & 2b \\ 1 & c & c \end{vmatrix} = 0$

$\Rightarrow ac + bc - 5ab = 0$; $\Rightarrow a^{-1} + b^{-1} = 2.\frac{5}{2c}$

so that $\frac{1}{a}, \frac{5}{2c}, \frac{1}{b}$ are in A.P.

$\Rightarrow a, \frac{2c}{5}, b$ are in H.P.

17. (c)

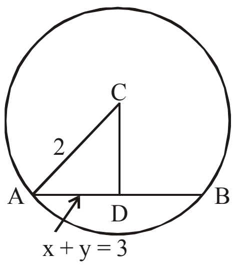


Circle has its centre at (3, 2) and radius $\sqrt{9+4+12} = 5$

One tangent parallel to the y-axis is

x = 3 + 5 and the other x = –(5 – 3). Their combined equation is $(x-8)(x+2) = 0$ i.e. $x^2 - 6x - 16 = 0$.

18. (c) The centre of the circle is C(1, 1) and radius of the circle is 2, perpendicular distance from C on AB, the length of the chord x + y = 3



$$CD = \left|\frac{1+1-3}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

$$\therefore AD = \sqrt{4 - \frac{1}{2}} = \sqrt{\frac{7}{2}} \quad [\because AD = \sqrt{AC^2 - CD^2}]$$

Hence, the length of the chord

$$AB = 2AD = 2\sqrt{\frac{7}{2}} = \sqrt{14}$$

19. (a) Eccentrcity of $\frac{x^2}{25} - \frac{y^2}{25\sin^2\alpha} = 1$

is $\sqrt{1+\sin^2\alpha}$.

Eccentricity of $\frac{x^2}{5\sin^2\alpha} + \frac{y^2}{5} = 1$

is $\sqrt{1-\sin^2\alpha}$

Given, $\sqrt{1+\sin^2\alpha} = \sqrt{5}\sqrt{1-\sin^2\alpha}$

$\Rightarrow \sin^2\alpha = \frac{2}{3} \Rightarrow \alpha = \sin^{-1}\sqrt{\frac{2}{3}} = \tan^{-1}\sqrt{2}$

20. (a) The point 't_1' is $(at_1^2, 2at_1)$ and the point 't_2' is $(at_2^2, 2at_2)$.

The equation to the tangent at

't_1' is $t_1 y = x + at_1^2$ ...(i)

similary, the equation to the tangent at

't_2' is $t_2 y = x + at_2^2$ .....(ii)

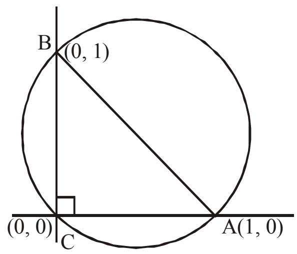
The point of intersection of (i) and (ii) is given by

$$y = \frac{at_1^2 - at_2^2}{t_1 - t_2} = a(t_1 + t_2)$$

and $x = t_1 y - at_1^2 = t_1 a(t_1 + t_2) - at_1^2 = at_1 t_2$.

Hence, the point of intersection of the tangents to the parabola $y^2 = 4ax$ at the points 't_1' and 't_2' is $(at_1 t_2, a(t_1 + t_2))$.

21. (d) The equation of the circle passing through (1, 0), (0, 1) and (0, 0) is



$(x-1)(x-0) + (y-0)(y-1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$ ($\because$ AB is diameter)

which passes through (2k, 3k) then

$4k^2 + 9k^2 - 2k - 3k = 0 \Rightarrow 13k^2 - 5k = 0$

$k \neq 0, \therefore k = \frac{5}{13}$ ($k \neq 0$ as the four points are distinct)

22. (b) Circle is given as $x^2 + y^2 = ax + by$

or $x^2 + y^2 - ax - by = 0$ ....(1)

Then line is given as $cx - by + b^2 = 0$

or $y = \left(\frac{c}{b}x + b\right)$ ....(2)

Substituting the value of y from (2) in (1), we get

$$x^2+\left(\frac{c}{b}x+b\right)^2-ax-b\left(\frac{c}{b}x+b\right)=0$$

or $x^2+\frac{c^2}{b^2}x^2+2cx+b^2-ax-cx-b^2=0$

$$x^2\left(1+\frac{c^2}{b^2}\right)+x(c-a)=0 \quad ....(3)$$

If it is a perfect square, then c – a = 0 or c = a

23. (a) We have $\pi^2=9.86\Rightarrow[\pi^2]=9$

Also $-\pi^2=-9.86\Rightarrow[-\pi^2]=-10$

$\therefore f(x)=\cos 9x+\cos(-10)x$

$=\cos 9x+\cos 10x$

Now,

$f\left(\frac{\pi}{2}\right)=\cos\frac{9\pi}{2}+\cos 5\pi=0-1=-1$

$f(\pi)=\cos\pi+\cos 10\pi=-1+1=0$

$f(-\pi)=\cos 9\pi+\cos 10\pi=-1+1=0$

$f\left(\frac{\pi}{4}\right)=\cos\frac{9\pi}{4}+\cos\frac{5\pi}{2}=\frac{1}{\sqrt2}+0=\frac{1}{\sqrt2}$

24. (a) Given f'' (x) is continuous at x = 0

$=\lim_{x\to0} f''(x)=f''(0)=4$

Now, $\lim_{x\to0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}\left[\frac{0}{0}form\right]$

$=\lim_{x\to0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}\left[\frac{0}{0}\text{form}\right]$

$=\lim_{x\to0}\frac{2f''(x)-12f''(x)+16f''(4x)}{2}$

[Using L' Hospital Rule successively]

$=\frac{2f''(0)-12f''(0)+16f''(0)}{2}=12$

25. (a) We have, $x^p y^q=(x+y)^{p+q}$

$\Rightarrow p\log x+q\log y=(p+q)\log(x+y)$

Diff. w.r.t. x, we get $\frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow\frac{dy}{dx}\left(\frac{q}{y}-\frac{p+q}{x+y}\right)=\frac{p+q}{x+y}-\frac{p}{x}\Rightarrow\frac{dy}{dx}=\frac{y}{x}.$

26. (c) $f(x)=x^n$

$\Rightarrow f'(x)=nx^{n-1}$

$f'(a+b)=f'(a)+f'(b)$

$\Rightarrow n(a+b)^{n-1}=na^{n-1}+nb^{n-1}$

$\Rightarrow(a+b)^{n-1}=a^{n-1}+b^{n-1}$

Which is true for n = 2 and false for n = 1 and n = 4.

Also for n = 0, f(x) = 1, so, f'(x) = 0

f'(a + b) = 0

So, f' (a + b) = f' (a) + f'(b)

Hence, there are two values of n.

27. (b) Given $y=ke^{kx}$. The curve intersects the y-axis at (0, k) So,

$\left(\frac{dy}{dx}\right)_{(0,k)}=k^2$

If θ is the angle at which the given curve intersects the y-axis, then

$\tan\left(\frac{\pi}{2}-\theta\right)=\frac{k^2-0}{1+0.k^2}=k^2\Rightarrow\theta=\cot^{-1}(k^2)$

28. (c) Given, $y=\frac{a^2}{x}+\frac{b^2}{a-x}$

$\Rightarrow\frac{dy}{dx}=-\frac{a^2}{x^2}+\frac{b^2}{(a-x)^2}=0$

$\Rightarrow x=\frac{a^2}{a\pm b},$

out of which only one $x=\frac{a^2}{a+b}$ is in (0, a).

Also $\frac{d^2y}{dx^2}=\frac{2a^2}{x^3}+\frac{2b^2}{(a-x)^3}>0$ in (0, a)

∴ Maximum value attained is

$a+b+\frac{a+b}{ab}b^2=\frac{1}{a}(a+b)^2$

29. (a) |x| = x, x > 0, |x| = –x, x < 0, |x| = 0, x = 0

$\therefore f(x)=\frac{x}{1-x},\ x<0$

$=0\quad, x=0$

$=\frac{x}{1+x},\ x>0$

LHD at x = 0

$=\lim_{h\to0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to0}\frac{\frac{-h}{1+h}-0}{-h}=1$

RHD at x = 0

$=\lim_{h\to0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to0}\frac{\frac{h}{1+h}-0}{h}=1$

∴ f is differentiable at x = 0.

Also, for all other values of x ≠ 0, the function is differentiable. Hence, the function is differentiable in (–∞, ∞).

30. (a) $y=\sin^{-1}\sin\theta+\cos^{-1}\cos\theta=\theta+(-\theta)=0$

Where $x=\sin\theta,\ \because x=-1/\sqrt2\Rightarrow\theta=-\pi/4$

$\Rightarrow -\pi/2\le\theta\le\pi/2$

$\therefore\ \sin^{-1}\sin\theta=\theta,\quad\text{when } -\pi/2\le\theta\le\pi/2$

$\cos^{-1}\cos\theta=-\theta,\quad\text{when } -\pi\le\theta\le0$

$\therefore\frac{dy}{dx}=0$

31. (c) $\int\frac{dx}{\sqrt{x^2+2x+1}}=\int\frac{dx}{\sqrt{(x+1)^2}}=\int\frac{dx}{|x+1|}$

$= \int \frac{dx}{-(x+1)}$ [for $x<-1$] $= -\log|x+1| + c$

$\therefore A = -1$

32. (a) $I = \int \left(x + \frac{1}{x}\right)^{n+5} \left(\frac{x^2-1}{x^2}\right) dx$

Put $x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt$

$\Rightarrow \left(\frac{x^2-1}{x^2}\right) dx = dt$

$\therefore I = \int t^{n+5}\, dt = \frac{t^{n+6}}{n+6} + c$

$= \frac{\left(x + \frac{1}{x}\right)^{n+6}}{n+6} + c$

33. (b) Put $x = \cos 2\theta$

$\therefore I = \int \cos\{2\tan^{-1} \tan\theta\}(-2\sin 2\theta)d\theta$

$= -\int \sin 4\theta d\theta = \frac{1}{4}\cos 4\theta + c$

$= \frac{1}{4}(2x^2 - 1) + c = \frac{1}{2}x^2 + k$

34. (b) $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1 + \cos^2(\pi - x)}$

$= \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx - I$

$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$

Put $\cos x = t$, then $-\sin x\, dx = dt$

$\therefore 2I = \pi \int_1^{-1} \frac{-dt}{1+t^2} = \pi \int_{-1}^{1} \frac{dt}{1+t^2}$

$= \pi\left[\tan^{-1} t\right]_{-1}^{1} = \pi\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \pi . \frac{\pi}{2} = \frac{\pi^2}{2}$

$\therefore I = \frac{\pi^2}{4}$

35. (b) In the interval $\frac{\pi}{3}$ to $\frac{\pi}{2}$, $[x] = 1$

$\therefore I = \int_{\pi/3}^{\pi/2} x \sin(\pi - x)\, dx = \int_{\pi/3}^{\pi/2} x \sin x\, dx$

$= [-x \cos x + \sin x]_{\pi/3}^{\pi/2} = 1 - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$

36. (b) We have $\frac{dy}{dx} = (e^{3x} + x^2)e^{-2y}$

$\Rightarrow e^{2y}dy = (e^{3x} + x^2)dx$

$\Rightarrow \int e^{2y} dy = \int (e^{3x} + x^2)dx + a$

$\Rightarrow \frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + a$

$\Rightarrow 3e^{2y} = 2(e^{3x} + x^3) + c,\ c = 6a$

37. (a) $(xy^2 + x)dx + (yx^2 + y)dy = 0$

$\Rightarrow x(y^2+1)dx + y(x^2+1)dy = 0$

$\Rightarrow \frac{x}{x^2+1}dx + \frac{y}{y^2+1}dy$

$\Rightarrow \int \frac{x}{x^2+1}dx + \int \frac{y}{y^2+1}dy = a$

$\Rightarrow \frac{1}{2}\log(x^2+1) + \frac{1}{2}\log(y^2+1) = a$

$\Rightarrow \log(x^2+1)(y^2+1) = 2a$

$\Rightarrow (x^2+1)(y^2+1) = e^{2a} = c$

38. (a) Equating the components in

$\alpha(\hat{i} + 2\hat{j} + 3\hat{k}) + \beta(2\hat{i} + 3\hat{j} + \hat{k}) + \gamma(3\hat{i} + \hat{j} + 2\hat{k})$

$= -3(\hat{i} - \hat{k})$, we have

$\alpha + 2\beta + 3\gamma = -3$ ....(i)

$2\alpha + 3\beta + \gamma = 0$ ....(ii)

$3\alpha + \beta + 2\gamma = 3$ ....(iii)

Solving the equations (i), (ii), & (iii) we get

$\alpha = 2,\ \beta = -1,\ \gamma = -1.$

39. (c) Suppose $\vec{p} = p_1\hat{i} + p_2\hat{j} + p_3\hat{k}$

$\vec{p} \times \hat{i} = p_2\hat{j} \times \hat{i} + p_3\hat{k} \times \hat{i} = -p_2\hat{k} + p_3\hat{j}$

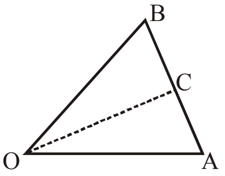
$|\vec{p} \times \hat{i}|^2 = p_2^2 + p_3^2$

Similarly, $|\vec{p} \times \hat{j}|^2 = p_3^2 + p_1^2$, $|\vec{p} \times \hat{k}|^2 = p_1^2 + p_2^2$

$\therefore \frac{3}{2}\left\{|\vec{p} \times \hat{i}|^2 + |\vec{p} \times \hat{j}|^2 + |\vec{p} \times \hat{k}|^2\right\}$

$= 3(p_1^2 + p_2^2 + p_3^2) = 3\vec{p}^{\,2}$

40. (b)



We have

$OA = |2\hat{i} + 2\hat{j} + \hat{k}| = 3$; $OB = |2\hat{i} + 4\hat{j} + 4\hat{k}| = 6$

Since the internal bisector divides opposite side in the ratio of adjacent sides

$\therefore \frac{AC}{BC} = \frac{3}{6} = \frac{1}{2}$

where OC is the bisector of $\angle BOA$.

$\therefore$ Position vector of C is

$$\frac{2(2\hat{i}+2\hat{j}+\hat{k})+(2\hat{i}+4\hat{j}+4\hat{k})}{2+1}$$

$$= 2\hat{i} + \frac{8}{3}\hat{j} + 2\hat{k}$$

$$\therefore OC = \left|2\hat{i} + \frac{8}{3}\hat{j} + 2\hat{k}\right| = \sqrt{\frac{136}{9}}$$

41. **(b)** $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

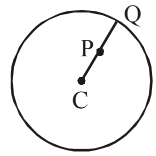
$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 3 - 1 = 2.$

42. **(c)** Radius of the sphere $= 2\sqrt{6}$ and distance of the given point from the centre of the sphere is

$\sqrt{(1-0)^2 + (2-0)^2 + (-1-0)^2} = \sqrt{6}.$

So the point lies within the sphere and its distance from the surface (along the radius which is Shortest) is $2\sqrt{6} - \sqrt{6} = \sqrt{6}$.



$PQ = CQ - CP$

43. **(b)** Only one of A and B can be alive in the following, mutually exclusive ways.

E_1 : A will die and B will live

E_2 : B will die and A will live

So, required probability $= P(E_1) + P(E_2)$

$= p(1-q) + q(1-p) = p + q - 2pq.$

44. **(b)** We have $\vec{u} + \vec{v} + \vec{w} = \vec{0}$

$\therefore |\vec{u} + \vec{v} + \vec{w}| = 0 \Rightarrow |\vec{u} + \vec{v} + \vec{w}|^2 = 0$

$\Rightarrow |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}) = 0$

$\Rightarrow \vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u} = \frac{-1}{2}[9+16+25] = -25$

45. **(d)** The planes forming the parallelopiped are

$x = -1, x = 1;\ y = 2,\ y = -1$ and $z = 5, z = -1$

Hence, the lengths of the edges of the parallelopiped are $1-(-1) = 2,\ |-1-2| = 3$ and $|-1-5| = 6$

(Length of an edge of a rectangular parallelopiped is the distance between the parallel planes perpendicular to the edge)

$\therefore$ Length of diagonal of the parallelopiped

$= \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7.$

Mock Test-2

Time : 1 hr **Max. Marks -180**

1. The number of all possible selections of one or more questions from 10 given questions, each question having one alternative is

(a) 3^{10} (b) $2^{10} - 1$
(c) $3^{10} - 1$ (d) 2^{10}

2. The polynomial $x^{3m} + x^{3n+1} + x^{3k+2}$, is exactly divisible by $x^2 + x + 1$ if

(a) m n, k are rational
(b) m, n, k are integers
(c) m, n, k are positive integers
(d) none of these.

3. If $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in A.P., then a_1, b_1, c_1 are in

(a) A.P. (b) GP.
(c) H.P. (d) none

4. If α an β be the roots of $x^2 + px + q = 0$

then $\dfrac{(\omega\alpha + \omega^2\beta)(\omega^2\alpha + \omega\beta)}{\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}}$

(a) $-\frac{q}{p}$ (b) $\alpha\beta$
(c) $\frac{p}{q}$ (d) ω

[ω, ω^2 are complex cube roots of unity]

5. If n arithmetic means are inserted between 1 and 31 such that the ratio of the 1^{st} mean and n^{th} mean is 3 : 29, then the value of n is

(a) 14 (b) 15
(c) 30 (d) 13

6. The number of dissimilar terms in the expansion of $(a + b)^n$ is n + 1, therefore number of dissimilar terms in the expansion of $(a + b + c)^{12}$ is

(a) 13 (b) 39
(c) 78 (d) 91

7. $a\,C_0 + (a + b)\,C_1 + (a + 2b)\,C_2 + ... + (a + nb)\,C_n$ is equal to

(a) $(2a + nb)\,2^n$ (b) $(2a + nb)\,2^{n-1}$
(c) $(na + 2b)\,2^n$ (d) $(na + 2b)\,2^{n-1}$

8. Which of the following is INCORRECT?

(a) If A is a skew-symmetric matrix and n is a positive integer, then A^n is always symmetric
(b) If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2 = A + B$
(c) If A and B are two matrices such that AB = O, then |A| = 0 or |B| = 0.
(d) All three are correct

9. The general solution of the equation $\cos x \cos 6x = -1$ is :

(a) $x = (2n + 1)\pi,\ n \in \mathbf{I}$
(b) $x = 2n\pi,\ n \in \mathbf{I}$
(c) $x = \left(2n - \frac{1}{2}\right)\pi,\ n \in \mathbf{I}$
(d) none of these.

10. The value of $\cos\left(2\cos^{-1} x + \sin^{-1} x\right)$ at $x = \frac{1}{5}$ is

(a) $-\frac{2\sqrt{6}}{5}$ (b) $-2\sqrt{6}$
(c) $-\frac{\sqrt{6}}{5}$ (d) None

11. The set of values of x for which

$\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is

(a) ϕ (b) $\left\{\frac{\pi}{4}\right\}$
(c) $\left\{n\pi + \frac{\pi}{4} : n = 1, 2, 3, ...\right\}$ (d) $\left\{2n\pi + \frac{\pi}{4} : n = 1, 2, 3.....\right\}$

12. The value of $\sin(\cot^{-1}(\cos(\tan^{-1}x)))$ is

(a) $\sqrt{\frac{x^2 + 2}{x^2 + 1}}$ (b) $\sqrt{\frac{x^2 + 1}{x^2 + 2}}$
(c) $\frac{x}{\sqrt{x^2 + 2}}$ (d) $\frac{1}{\sqrt{x^2 + 2}}$

13. If $A = \sin^8 \theta + \cos^{14} \theta$, then for all values of θ :

(a) A ³ 1 (b) 0 < A £ 1
(c) 1< 2A £ 3 (d) none of these

14. Two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$. The angle between the sides is $\frac{\pi}{3}$. The perimeter of the triangle is

(a) $2\sqrt{3}$ (b) $\sqrt{6}$
(c) $2\sqrt{3} + \sqrt{6}$ (d) $2(\sqrt{3} + \sqrt{6})$

15. The value of

$\cos\frac{\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$

is equal to

(a) $\frac{1}{64}$ (b) $\frac{1}{65}$

(c) $\frac{1}{2}$ (d) None of these

16. The equation of the straight line whose intercepts on the axes are twice the intercepts of the straight line $3x + 4y = 6$ on the axes is

(a) $3x+44y=3$ (b) $3x+4y=12$

(c) $6x+8y=9$ (d) None of these

17. The equation of pair of lines through origin and perpendicular to the pair of lines $ax^2+2hxy+by^2=0$ is

(a) $ax^2-2hxy+by^2=0$ (b) $bx^2+2hxy-ay^2=0$

(c) $bx^2-2hxy-ay^2=0$ (d) $bx^2-2hxy+ay^2=0$

18. The four points of intersection of the lines $(2x-y+1)(x-2y+3)=0$ with the axes lie on a circle whose centre is the point

(a) $\left(\frac{3}{4},\frac{5}{4}\right)$ (b) $\left(-\frac{7}{4},\frac{5}{4}\right)$

(c) $(2,3)$ (d) None of these

19. If the line $y=mx+\sqrt{a^2m^2-b^2}$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point φ. Then $\varphi=$

(a) $\sin^{-1}(m)$ (b) $\sin^{-1}\left(\frac{a}{bm}\right)$

(c) $\sin^{-1}\left(\frac{b}{am}\right)$ (d) $\sin^{-1}\left(\frac{bm}{a}\right)$

20. The equation of the ellipse whose axes are along the coordinate axes, the length of the latus rectum is 5 and the eccentricity is $\frac{2}{3}$ is

(a) $\frac{x^2}{81}+\frac{y^2}{45}=4$ (b) $\frac{x^2}{81}+\frac{y^2}{45}=\frac{1}{4}$

(c) $\frac{x^2}{81}+\frac{y^2}{45}=3$ (d) $\frac{x^2}{81}+\frac{y^2}{45}=\frac{3}{2}$

21. The line joining (5, 0) to $(10\cos\theta, 10\sin\theta)$ is divided internally in the ratio 2 : 3 at P. If θ varies, then the locus of P is

(a) a pair of straight lines (b) a circle

(c) a straight line (d) None of these

22. If the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ cut the co-ordinate axes in concylic points, then

(a) $a_1b_1=a_2b_2$ (b) $\frac{a_1}{a_2}=\frac{b_1}{b_2}$

(c) $a_1+a_2=b_1+b_2$ (d) $a_1a_2=b_1b_2$

23. The function $f(x)=\begin{cases}6.5^x & \text{for } x\le 0\\ 2a+x & \text{for } x>0\end{cases}$

will be continuous at $x = 0$ if $a =$

(a) 3 (b) 2 (c) 1 (d) 0

24. The number of points at which the function $f(x)=|x-0.5|+|x-1|+\tan x$ does not have a derivative in the interval (0, 2) is

(a) 0 (b) 1 (c) 2 (d) 3

25. If $y=\tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right)$, then $\frac{dy}{dx}$ at $x=0$ is :

(a) 1 (b) 2

(c) log 2 (d) None of these

26. The abscissa of the point on the curve $ay^2 = x^3$, the normal at which cuts off equal intercepts from the coordinate axes is

(a) $\frac{2a}{9}$ (b) $\frac{4a}{9}$

(c) $-\frac{4a}{9}$ (d) $-\frac{2a}{9}$

27. If $A > 0$, $B > 0$ and $A + B = \pi/3$, then the maximum value of $\tan A \tan B$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$

(c) 3 (d) $\sqrt{3}$

28. The derivative of $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$ with respect to $\sin^{-1}x$ is :

(a) 1 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) None of these

29. Let f(x) = x |x|. The set of points where f(x) is twice differentiable is

(a) **R** (b) **R** – {0}

(c) **R** – {1} (d) None of these.

30. If $y^{1/m}=\left[x+\sqrt{1+x^2}\right]$,

then $(1+x^2)y_2+xy_1$ is equal to

(a) m^2y (b) my^2

(c) m^2y^2 (d) None

31. If $I=\int\frac{1}{2p}\sqrt{\frac{p-1}{p+1}}\,dp=f(p)+c$, then f(p) is equal to :

(a) $\frac{1}{2}\ell n\left[p-\sqrt{p^2-1}\right]$

(b) $\frac{1}{2}\cos^{-1}p+\frac{1}{2}\sec^{-1}p$

(c) $\ell n\sqrt{p+\sqrt{p^2-1}}-\frac{1}{2}\sec^{-1}p$

(d) none of the above.

32. If $\int \frac{x+\left(\cos^{-1}3x\right)^2}{\sqrt{1-9x^2}}\,dx = A\sqrt{1-9x^2}+B\left(\cos^{-1}3x\right)^3+c$, where c is integration constant, then the values of A and B are :

(a) $A=-\frac{1}{9},\ B=-\frac{1}{9}$ (b) $A=-\frac{1}{9},\ B=\frac{1}{9}$

(c) $A=\frac{1}{9},\ B=-\frac{1}{9}$ (d) none of the above.

33. If $\phi(x)=\int \cot^4 x\,dx+\frac{1}{3}\cot^3 x-\cot x$ and $\phi\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$ then $\phi(x)$ is

(a) $\pi-x$ (b) $x-\pi$

(c) $\pi/2-x$ (d) None

34. If $f(x)=\int_{1/x}^{\sqrt{x}}\cos t^2\,dt\,(x>0)$, then $\frac{df(x)}{dx}$ is

(a) $\frac{\sqrt{3}\cos x+2\cos(x^{-2})}{2x\sqrt{x}}$ (b) $\frac{x\sqrt{x}\cos x+2\cos(x^{-2})}{2x^2}$

(c) $2\sqrt{x}\cos x-\frac{2}{x}\cos\left(\frac{1}{x}\right)$ (d) none of the above

35. The value of $\int_{1/n}^{(an-1)/n}\frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}\,dx$ is

(a) a/2 (b) $\frac{1}{2n}(na+2)$

(c) $\frac{na-2}{2n}$ (d) none of these.

36. The function $f(\theta)=\frac{d}{d\theta}\int_0^{\theta}\frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation ;

(a) $\frac{df}{d\theta}+2f(\theta)\cot\theta=0$ (b) $\frac{df}{d\theta}-2f(\theta)\cot\theta=0$

(c) $\frac{df}{d\theta}+2f(\theta)=0$ (d) $\frac{df}{d\theta}-2f(\theta)=0$

37. The solution to the differential equation $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$ is

(a) $y=(x+1)e^{3x}+c$ (b) $3y=(x+1)+e^{3x}+c$

(c) $\frac{3y}{x+1}=e^{3x}+c$ (d) $ye^{-3x}=3(x+1)+c$

38. A vector of magnitude 3, bisecting the angle between the vectors $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$ and making an obtuse angle with $\vec{b}$ is

(a) $\frac{3\hat{i}-\hat{j}}{\sqrt{6}}$ (b) $\frac{\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{14}}$

(c) $\frac{3(\hat{i}+3\hat{j}-2\hat{k})}{\sqrt{14}}$ (d) $\frac{3\hat{i}-\hat{j}}{\sqrt{10}}$

39. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar vectors, and let $\vec{p}$, $\vec{q}$ and $\vec{r}$ be the vectors defined by the relations $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ and $\vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$.

Then the value of the expression $(\vec{a}+\vec{b}).\vec{p}+(\vec{b}+\vec{c}).\vec{q}+(\vec{c}+\vec{a}).\vec{r}$ is equal to

(a) 0 (b) 1

(c) 2 (d) 3

40. Consider the parallelopiped with side $\vec{a}=3\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{c}=\hat{i}+3\hat{j}+3\hat{k}$ then the angle between $\vec{a}$ and the plane containing the face determined by $\vec{b}$ and $\vec{c}$ is

(a) $\sin^{-1}\frac{1}{3}$ (b) $\cos^{-1}\frac{9}{14}$

(c) $\sin^{-1}\frac{9}{14}$ d) $\sin^{-1}\frac{2}{3}$

41. The projection of a line PQ, where P is (0, –1, 3) and Q is (4, 5, 6) on a line whose direction ratios are 1, 2, 3 is

(a) $\frac{12}{\sqrt{14}}$ (b) $\frac{9}{\sqrt{14}}$

(c) $\frac{25}{\sqrt{14}}$ (d) None of these

42. Let d_1, d_2, be the intercepts of the sphere $x^2+y^2+z^2-5x-13y-14=0$ on x-axis and y-axis, then

(a) $d_1=9, d_2=13$ (b) $d_1=5, d_2=13$

(c) $d_1=9, d_2=15$ (d) none of these.

43. A speaks the truth in 70 percent cases and B in 80 percent. The probability that they will contradict each other when describing a single event is

(a) 0.36 (b) 0.38 (c) 0.4 (d) 0.42

44. The two vectors $(x^2-1)\hat{i}+(x+2)\hat{j}+x^2\hat{k}$ and $2\hat{i}-x\hat{j}+3\hat{k}$ are orthogonal

(a) for no real value of x

(b) for $x=-1$

(c) for $x=\frac{1}{2}$

(d) for $x=-\frac{1}{2}$ and $x=1$

45. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$. If $|\vec{u}|=3$, $|\vec{v}|=4$ and $|\vec{w}|=5$, then $\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}$ is

(a) 47 (b) –25 (c) 0 (d) 25

ANSWER KEY

1. (c)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (d)	**9.** (a)	**10.** (a)
11. (a)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (b)
21. (b)	**22.** (d)	**23.** (a)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (a)
31. (c)	**32.** (a)	**33.** (d)	**34.** (b)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (c)	**39.** (d)	**40.** (c)
41. (c)	**42.** (c)	**43.** (b)	**44.** (d)	**45.** (b)					

HINTS & SOLUTIONS

1. **(c)** Since each question can be dealt with in 3 ways, by selecting it or by selecting its alternative or by rejecting it. Thus, the total number of ways of dealing with 10 given questions is 3^{10} including a way in which we reject all the questions.
Hence, the number of all possible selections is $3^{10}-1$.

2. **(b)** We know that ω and ω² are roots of $x^2+x+1=0$.

Therefore, $x^{3m}+x^{3n+1}+x^{3k+2}$ will be exactly divisible by x^2+x+1, if ω and ω² are its roots.
For $x=\omega$, we have

$x^{3m}+x^{3n+1}+x^{3k+2}$

$=\omega^{3m}+\omega^{3n+1}+\omega^{3k+2}$

$=1+\omega+\omega^2=0$,

provided m, n, k are integers.

Similarly, $x=\omega^2$ will be a root of

$x^{3m}+x^{3n+1}+x^{3k+2}$ if m, n, k are integers.

3. **(b)** Let α be the common root :

$\therefore\ a\alpha^2+2b\alpha+c=0$ *and* $a_1\alpha^2+2b_1\alpha+c_1=0$

$$\therefore\ \frac{\alpha^2}{2(bc_1-b_1c)}=\frac{\alpha}{a_1c-ac_1}=\frac{1}{2(b_1a-ba_1)}$$

$\Rightarrow (a_1c-ac_1)^2=4(ab_1-ba_1)(bc_1-b_1c)$ ...(1)

$\because \frac{a}{a_1},\frac{b}{b_1},\frac{c}{c_1}$ are in A.P.

$$\therefore\ \frac{b}{b_1}-\frac{a}{a_1}=\frac{c}{c_1}-\frac{b}{b_1}=d \quad \text{(say)}$$

$$\Rightarrow \frac{ba_1-ab_1}{b_1a_1}=\frac{cb_1-c_1b}{c_1b_1}=d,$$

$\Rightarrow ba_1-ab_1=db_1a_1$ and $cb_1-c_1b=c_1b_1d$,

also $\frac{c}{c_1}-\frac{a}{a_1}=2d$;

$\therefore\ ca_1-c_1a=2dc_1a_1$

$\therefore$ form (1) $4d^2c_1^2a_1^2=4(-db_1a_1)(-dc_1b_1)$

$\Rightarrow c_1a_1=b_1^2$

4. **(a)** Consider $(\omega\alpha+\omega^2\beta)(\omega^2\alpha+\omega\beta)$

$=\alpha^2+\beta^2+(\omega^4+\omega^2)\alpha\beta \quad (\because \omega^3=1)$

$=\alpha^2+\beta^2-\alpha\beta=(\alpha+\beta)^2-3\alpha\beta$

$=p^2-3q$

Also,

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(\alpha+\beta)-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{p(3q-p^2)}{q}$$

∴ The given expression

$$=\frac{(p^2-3q)}{\frac{p(3q-p^2)}{q}}=-\frac{q}{p}$$

5. **(a)** We have

1 . A_1, A_2, ..., A_n, 31 are in A.P.

$\therefore\ 31=x_{n+2}=1+(n+1)d$

$$\therefore\ d=\frac{30}{n+1}$$

Now $A_1=1+d=1+\frac{30}{n+1}=\frac{31+n}{n+1}$

Also $A_n=1+nd=1+\frac{30n}{n+1}=\frac{31n+1}{n+1}$

Given $\frac{A_1}{A_n}=\frac{3}{29}$

$$\Rightarrow \frac{31+n}{31n+1}=\frac{3}{29}\Rightarrow n=14$$

6. **(d)** $(a+b+c)^{12}=[(a+b)+c]^{12}$

$={}^{12}C_0(a+b)^{12}+{}^{12}C_1(a+b)^{11}c+...+{}^{12}C_{12}c^{12}$

The R.H.S. contains, $13+12+11+....+1$ terms

$$=\frac{13(13+1)}{2}=91 \text{ terms}$$

Also no. of term in the expansion of $(a+b+c)^n$ is given by ${}^{n+2}C_2$.
Thus for n = 12

$${}^{n+2}C_2 = {}^{14}C_2 = \frac{14\times 13}{2} = 91.$$

7. **(b)** Let $S = aC_0 + (a+b)C_1 + (a+2b)C_2 + ... + (a+nb)C_n$

$$S = (a+nb)C_n + \{a+(n-1)b\}C_{n-1} + \{a+(n-2)b\}C_{n-2} + ... + aC_0$$

Add, $2S = (2a+nb)(C_0 + C_1 + C_2 + C_3 + ... + C_n)$ $[\because C_r = C_{n-r}]$

$$\therefore 2S = (2a+nb).2^n \Rightarrow S = (2a+nb)2^{n-1}$$

8. **(d)**
(a) Since A is a skew symmetric matrix

$$\Rightarrow A^T = -A \Rightarrow (A^T)^n = (-A)^n$$
$$\Rightarrow (A^n)^T = (-1)^n A^n$$
$$\Rightarrow (A^n)^T = \begin{cases} A^n & \text{if } n \text{ is even} \\ -A^n & \text{if } n \text{ is odd} \end{cases}$$

(b) We have to make use of given relation AB = B and BA = A

$$A^2 + B^2 = AA + BB = A(BA) + B(AB)$$
$$= (AB)A + (BA)B = BA + AB$$
$$= A + B$$

(c) $AB = 0 \Rightarrow \det(AB) = 0$
$\Rightarrow |A||B| = 0$
$\Rightarrow$ either $|A| = 0$ or $|B| = 0$

9. **(a)** We have
$\cos x \cos 6x = -1 \Rightarrow 2\cos x \cos 6x = -2$
$\Rightarrow \cos 7x + \cos 5x = -2$
which is possible only when
$\cos 7x = -1$ & $\cos 5x = -1$
The values of x satisfying these two equations simultaneously and lying between 0 and 2π is π. Therefore the general solution is
$x = (2n+1)\pi, n \in \mathbf{I}$.

10. **(a)** $\cos\left[2\cos^{-1}x + \sin^{-1}x\right]$

$$= \cos\left[\cos^{-1}x + \cos^{-1}x + \sin^{-1}x\right]$$
$$= \cos\left[\cos^{-1}x + \pi/2\right] = -\sin\cos^{-1}x$$
$$= -\sin\sin^{-1}\sqrt{1-x^2} = -\sqrt{1-x^2}$$
$$= -\sqrt{1-\left(\frac{1}{5}\right)^2} = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$$

11. **(a)** The given equation can be written as
$\tan(3x-2x) = 1$
$\Rightarrow \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}$

But for these values of x, tan 2x is not defined so the given equation has no solutions.

12. **(b)** We have, $\sin[\cot^{-1}(\cos(\tan^{-1}x))]$

$$= \sin\left[\cot^{-1}\left(\frac{1}{\sqrt{1+\tan^2(\tan^{-1}x)}}\right)\right]$$
$$= \sin\left(\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$
$$= \frac{1}{\sqrt{1+\cot^2\left\{\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right\}}}$$
$$= \frac{1}{\sqrt{1+\frac{1}{1+x^2}}} = \sqrt{\frac{1+x^2}{2+x^2}}$$

13. **(b)** We have, $\sin^8\theta \geq 0$ and $\cos^{14}\theta \geq 0$
$\therefore$ $A = \sin^8\theta + \cos^{14}\theta \geq 0$
But $\sin^8\theta + \cos^{14}\theta = 0$ is possible only if $\sin\theta = 0$ and $\cos\theta = 0$ simultaneously which is not true for any value of θ
$\therefore$ $A \neq 0$ or $A > 0$
Also, since $0 \leq \sin^2\theta \leq 1$

$$\therefore \sin^8\theta \leq \sin^2\theta \text{ and } \cos^{14}\theta \leq \cos^2\theta$$
$$\Rightarrow \sin^8\theta + \cos^{14}\theta \leq \sin^2\theta + \cos^2\theta = 1$$
$$\therefore A \leq 1$$

Hence, we get $0 < A \leq 1$

14. **(c)** Let the two sides a and b are the roots of the equation
$x^2 - 2\sqrt{3}x + 2 = 0$.

$\therefore a + b = 2\sqrt{3}$ and ab = 2. Also $\angle C = \frac{\pi}{3}$

Using cosine rule :

$$\cos C = \frac{a^2+b^2-c^2}{2ab} \Rightarrow \frac{1}{2} = \frac{a^2+b^2-c^2}{2ab}$$
$$\Rightarrow a^2+b^2-c^2 = ab \Rightarrow (a+b)^2 - c^2 = 3ab$$
$$\Rightarrow (2\sqrt{3})^2 - c^2 = 3\times 2 \Rightarrow c = \sqrt{6}$$
$$\therefore \text{ Perimeter} = a+b+c = 2\sqrt{3}+\sqrt{6}.$$

15. **(a)** The given expression can be written as

$$\frac{1}{2\sin\frac{\pi}{65}}\left[2\sin\frac{\pi}{65}\cos\frac{\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}\right]$$
$$= \frac{1}{2\sin\frac{\pi}{65}}\left[\sin\frac{2\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}\right]$$

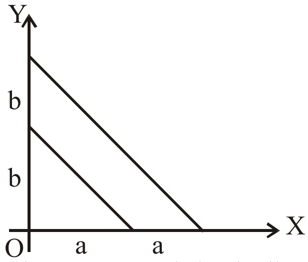
$$= \frac{1}{2^2 \sin\frac{\pi}{65}}\left[\sin\frac{4\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}\right]$$

=..................

$$= \frac{1}{2^6 \sin\frac{\pi}{65}}\left[\sin\frac{64\pi}{65}\right] = \frac{1}{64}\frac{\sin\left(\pi-\frac{\pi}{65}\right)}{\sin\frac{\pi}{65}} = \frac{1}{64}$$

16. **(b)** The equation of the given line is $3x+4y=6$;

$$\Rightarrow \frac{3x}{6}+\frac{4y}{6}=1 \Rightarrow \frac{x}{2}+\frac{y}{\frac{3}{2}}=1.$$



$\therefore$ The intercepts made by the line on the axes are $a=2, b=\frac{3}{2}$.

$\therefore$ The intercepts made by the required line on the axes are 2a, 2b i.e. 4, 3.

$\therefore$ The equation of the required line is

$$\frac{x}{4}+\frac{y}{3}=1; \Rightarrow 3x+4y=12.$$

17. **(d)** If the lines represented by the given equation be $y=m_1x$ and $y=m_2x$

then $m_1+m_2=-\frac{2h}{b}$ and $m_1m_2=\frac{a}{b}$

The lines perpendicular to these lines and passing through origin are $m_1y+x=0$ and $m_2y+x=0$.

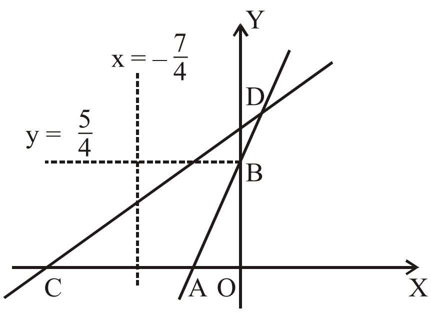
Their combined equation is $(m_1y+x)(m_2y+x)=0$

$$\Rightarrow m_1m_2y^2+(m_1+m_2)xy+x^2=0$$

$$\Rightarrow \frac{a}{b}y^2+\left(-\frac{2h}{b}\right)xy+x^2=0$$

$$\Rightarrow bx^2-2hxy+ay^2=0$$

18. **(b)**



The separate equations of the lines are $2x-y+1=0$ and $x-2y+3=0$.

They intersect the axes respectively at the points $\left(-\frac{1}{2}, 0\right)$; (0, 1) and (–3, 0); $\left(0, \frac{3}{2}\right)$, say A, B, C and D.

Since, A, B, C, D are concyclic. That is AC and BD are chords of a circle. The centre of circle must lie on the perpendicular bisectors of the chords, which are $x=-\frac{7}{4}$ and $y=\frac{5}{4}$.

$\therefore$ The centre is $\left(-\frac{7}{4}, \frac{5}{4}\right)$.

19. **(c)** Equation of tangent at point 'φ' on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{x}{a}\sec\varphi-\frac{y}{b}\tan\varphi=1$

or $y=\frac{b}{a}x\,\text{cosec}\,\varphi-b\cot\varphi$ ...(1)

If $y=mx+\sqrt{a^2m^2-b^2}$ ....(2)

also touches the hyperbola then on comparing (1) & (2)

$$1=\frac{\frac{b}{a}\text{cosec}\,\varphi}{m}=\frac{-b\cot\varphi}{\sqrt{a^2m^2-b^2}}$$

Hence, $m=\frac{b}{a}\text{cosec}\,\varphi$; or $\text{cosec}\,\varphi=\frac{am}{b}$

or $\sin\varphi=\frac{b}{am}$, or $\varphi=\sin^{-1}\frac{b}{am}$

20. **(b)** Given that $e=\frac{2}{3}$,

length of latus rectum = 5

in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

$$\therefore \frac{2b^2}{a}=5 \text{ or } \frac{2a^2(1-e^2)}{a}=5,$$

$$\therefore 2a\left(1-\frac{4}{9}\right)=5, \text{ or } a=\frac{9}{2}$$

and $b^2=\frac{5a}{2}=\frac{5}{2}\cdot\frac{9}{2}=\frac{45}{4}$

Now substituting the values of a and b in the equation of an ellipse (1).

$$\frac{x^2}{\frac{81}{4}}=\frac{y^2}{\frac{45}{4}}=1, \text{ or } \frac{x^2}{81}+\frac{y^2}{45}=\frac{1}{4}$$

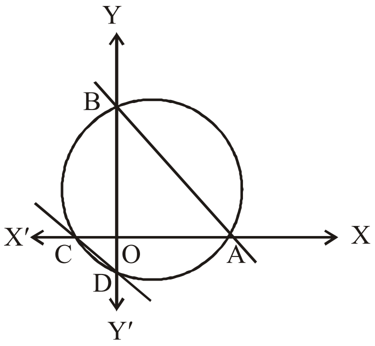
21. **(b)** Let P(x, y) be the point dividing the join of A and B in the ratio 2 : 3 internally, then

$$x = \frac{20\cos\theta + 15}{5} = 4\cos\theta + 3 \Rightarrow \cos\theta = \frac{x-3}{4} \quad ....(i)$$

$$y = \frac{20\sin\theta + 0}{5} = 4\sin\theta \Rightarrow \sin\theta = \frac{y}{4} \quad ...(ii)$$

Squaring and adding (i) and (ii), we get the required locus $(x-3)^2 + y^2 = 16$, which is a circle.

22. **(d)**



Let the given lines be $L_1 = a_1x + b_1y + c_1 = 0$ and $L_2 = a_2x + b_2y + c_2 = 0$, suppose L_1 meets the co-ordinates axes at A and B and L_2 meets at C & D. Then coordinates of A, B, C, D are

$$A\left(-\frac{c_1}{a_1}, 0\right), B\left(0, -\frac{c_1}{b_1}\right), C\left(-\frac{c_2}{a_2}, 0\right)$$

and $D\left(0, -\frac{c_2}{b_2}\right)$. Since A, B, C, D are concyclic, therefore OA . OC = OD . OB

$$\Rightarrow \left(-\frac{c_1}{a_1}\right)\left(-\frac{c_2}{a_2}\right) = \left(-\frac{c_2}{b_2}\right)\left(-\frac{c_1}{b_1}\right)$$

$$\Rightarrow a_1a_2 = b_1b_2$$

23. **(a)** f(x) will be continuous at x = 0 if

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0).$$

$$\Rightarrow \lim_{x\to 0} 6.5^x = \lim_{x\to 0} 2a + x \Rightarrow 6.5° = 2a + 0$$

$$\Rightarrow a = 3$$

24. **(d)** |x – a| is not differentiable at x = a.
Also tan x is not differentiable if

$$x = (2k+1)\frac{\pi}{2}, \ k \in \mathbf{I}$$

∴ In the interval (0, 2), f(x) is not derivable at x = 0.5,

$$x = 1 \text{ and } x = \frac{\pi}{2}$$

25. **(d)** $y = \tan^{-1}\left[\frac{2^x(2-1)}{1 + 2^x.2^{x+1}}\right] = \tan^{-1}\left[\frac{2^{x+1} - 2^x}{1 + 2^x.2^{x+1}}\right]$

$$= \tan^{-1}(2^{x+1}) - \tan^{-1}(2^x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^{x+1}\log 2}{1 + 2^{2(x+1)}} - \frac{2^x \log 2}{1 + 2^{2x}}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = (\log 2)\left(\frac{2}{5} - 1\right) = \log 2\left(-\frac{3}{5}\right)$$

26. **(b)** We have, $ay^2 = x^3$.

$$2ay.\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

Let (x_1, y_1) be a point on $ay^2 = x^3$.
Then $ay_1^2 = x_1^3$ ....(i)
The equation of the normal at (x_1, y_1) is
$y - y_1 = -(2ay_1/3x_1^2)(x - x_1)$
This meets the coordinate axes at

$$A\left(x_1 + \frac{3x_1^2}{2a}, 0\right) \text{ and } B\left(0, y_1 + \frac{2ay_1}{3x_1}\right)$$

Since the normal cuts off equal intercepts with the coordinate axes, therefore

$$x_1 + \frac{3x_1^2}{2a} = y_1 + \frac{2ay_1}{3x_1}$$

$$\Rightarrow x_1\frac{(2a + 3x_1)}{2a} = y_1\frac{(3x_1 + 2a)}{3x_1}$$

$$\Rightarrow 3x_1^2 = 2ay_1 \Rightarrow 9x_1^4 = 4a^2y_1^2 \quad ...(ii)$$

From (i) & (ii),

$$9x_1^4 = 4a^2\left(\frac{x_1^3}{a}\right) \Rightarrow x_1 = \frac{4a}{9}$$

27. **(b)** We have, $A + B = \frac{\pi}{3}$

$$\therefore B = \frac{\pi}{3} - A \Rightarrow \tan B = \frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$$

Let Z = tan A. tan B. Then,

$$Z = \tan A.\frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A} = \frac{\sqrt{3}\tan A - \tan^2 A}{1 + \sqrt{3}\tan A}$$

$$\Rightarrow Z = \frac{\sqrt{3}x - x^2}{1 + \sqrt{3}x}, \text{ where } x = \tan A$$

$$\Rightarrow \frac{dZ}{dx} = -\frac{(x + \sqrt{3})(\sqrt{3}x - 1)}{(1 + \sqrt{3}x)^2}$$

For max Z, $\frac{dZ}{dx} = 0 \Rightarrow x = \frac{1}{\sqrt{3}}, -\sqrt{3}.$

$x \neq -\sqrt{3}$ because $A + B = \pi/3$ which implies that x = tan A > 0. It can be easily checked that

$\frac{d^2Z}{dx^2} < 0$ for $x = \frac{1}{\sqrt{3}}$. Hence, Z is maximum for $x = \frac{1}{\sqrt{3}}$ i.e. $\tan A = \frac{1}{\sqrt{3}}$ or $A = \pi/6$.

For this value of x, we have $Z = \frac{1}{3}$.

28. (b) $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$, put $x = \cos\theta$

$= \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \tan^{-1}\sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}} = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$

let $z = \sin^{-1}x \Rightarrow \frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{-\frac{1}{2\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}} = -\frac{1}{2}$

29. (b) $f(x) = x|x|$

$\therefore f(x) = \begin{cases} x(-x) = -x^2, x < 0 \\ x(x) = x^2, x \ge 0 \end{cases}$

Clearly $f(x)$ is twice differentiable in $(-\infty, 0)$ and $(0, \infty)$. Hence we have to discuss the differentiability at $x = 0$.

$Rf'(0) = \lim_{h\to 0}\frac{(0+h)^2 - 0}{h} = \lim_{h\to 0} h = 0$

$Lf'(0) = \lim_{h\to 0}\frac{-(0-h)^2 - 0}{-h} = \lim_{h\to 0} h = 0$

Since $Rf'(0) = Lf'(0)$, so the function is differentiable once at $x = 0$ and $f'(0) = 0$. Further,
Let $F(x) = f'(x) = -2x, x < 0$ and
$f'(x) = 2x, x > 0$. Also $f'(0) = 0$

$RF'(0) = \lim_{h\to 0}\frac{2(0-h)-0}{h} = 2$

$LF'(0) = \lim_{h\to 0}\frac{-2(0-h)-0}{-h} = -2$

Since $RF'(0) \ne LF'(0)$ so the function F(x) is not differetiable at $x = 0$. In other words, $f(x)$ is not twice differentiable at $x = 0$. But it is twice differentiable in $R - \{0\}$. i.e. the set of all real numbers except 0.

30. (a) We have,

$y^{1/m} = \left[x + \sqrt{1+x^2}\right] \Rightarrow y = \left[x + \sqrt{1+x^2}\right]^m$

$\Rightarrow \frac{dy}{dx} = m\left[x+\sqrt{1+x^2}\right]^{m-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$

$= m\frac{[x+\sqrt{1+x^2}]^m}{\sqrt{x^2+1}}$

$\Rightarrow \frac{dy}{dx} = \frac{my}{\sqrt{1+x^2}} \Rightarrow y_1^2(1+x^2) = m^2y^2$

$\Rightarrow 2y_1y_2(1+x^2) + 2xy_1^2 = 2m^2yy_1$

$\Rightarrow y_2(1+x^2) + xy_1 = m^2y.$

31. (c) $I = \int\frac{1}{2p}\sqrt{\frac{p-1}{p+1}}dp$

$= \frac{1}{2}\int\frac{p-1}{p\sqrt{(p+1)(p-1)}}dp$

$= \frac{1}{2}\int\frac{pdp}{p\sqrt{p^2-1}} - \frac{1}{2}\int\frac{dp}{p\sqrt{p^2-1}}$

$= \frac{1}{2}\int\frac{dp}{\sqrt{p^2-1}} - \frac{1}{2}\int\frac{dp}{p\sqrt{p^2-1}}$

$= \frac{1}{2}\log_e\left(p + \sqrt{p^2-1}\right) - \frac{1}{2}\sec^{-1}p$

$\Rightarrow f(p) = \log\sqrt{p+\sqrt{p^2-1}} - \frac{1}{2}\sec^{-1}p$

32. (a) $I = \int\frac{x + (\cos^{-1}3x)^2}{\sqrt{1-9x^2}}dx$

Put $3x = \cos\theta \Rightarrow 3dx = -\sin\theta d\theta$

$I = -\frac{1}{3}\int\frac{\frac{\cos\theta}{3} + \theta^2}{\sin\theta}\sin\theta\, d\theta$

$= -\frac{1}{3}\left[\frac{1}{3}\cos\theta + \theta^2\right]d\theta = -\frac{1}{9}\sin\theta - \frac{\theta^3}{9} + c$

$= -\frac{1}{9}\sqrt{1-9x^2} - \frac{1}{9}(\cos^{-1}3x)^3 + c$

$\therefore A = B = -\frac{1}{9}$

33. (d) $\int\cot^4 x dx = \int\cot^2 x.(\operatorname{cosec}^2 x - 1)dx$

$= \int\cot^2 x \operatorname{cosec}^2 x\, dx - \int(\operatorname{cosec}^2 x - 1)dx$

$$= -\frac{1}{3}\cot^3 x + \cot x + x + c$$

$$\therefore \phi(x) = -\frac{1}{3}\cot^3 x + \cot x + x + c + \frac{1}{3}\cot^3 x - \cot x$$

$$= x + c$$

$$\therefore \phi\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + c, \quad \therefore c = 0, \quad \therefore \phi(x) = x$$

34. **(b)** $\frac{df(x)}{dx} = \cos(\sqrt{x})^2 \frac{d}{dx}(\sqrt{x}) - \cos\left(\frac{1}{x}\right)^2 \frac{d}{dx}\left(\frac{1}{x}\right)$

[Using Leibnitz rule]

$$= \frac{1}{2\sqrt{x}}\cos x + \frac{\cos x^{-2}}{x^2}$$

$$= \frac{x\sqrt{x}\cos x + 2\cos(x^{-2})}{2x^2}$$

35. **(c)** Let $I = \int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$ ....(i)

Then, $I = \int_{1/n}^{(an-1)/n} \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$ .....(ii)

$$\left[\text{Using} \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

Adding (i) and (ii), we get

$$2I = \int_{1/n}^{(an-1)/n} 1.dx = \frac{an-2}{n} \Rightarrow 1 = \left(\frac{an-2}{2n}\right)$$

36. **(a)** $f(\theta) = \frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos\theta.\cos x}$

$$= \frac{1}{1-\cos\theta.\cos\theta} = \text{cosec}^2\theta$$

$$\therefore \frac{df}{d\theta} = -2\,\text{cosec}\theta.\cot\theta\,\text{cosec}\theta$$

$$= -2f(\theta)\cot\theta$$

$$\Rightarrow \frac{df}{d\theta} + 2f(\theta)\cot\theta = 0$$

37. **(c)** The given equation can be rewritten as

$$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$$

$$\text{I.F.} = e^{\int -\frac{1}{x+1}dx} = e^{-\log(x+1)} = \frac{1}{x+1}$$

The solution is

$$y\left(\frac{1}{x+1}\right) = \int e^{3x}(x+1).\frac{1}{x+1}dx + a$$

$$\Rightarrow \frac{y}{x+1} = \int e^{3x}dx + a = \frac{e^{3x}}{3} + a$$

$$\Rightarrow \frac{3y}{x+1} = e^{3x} + c, \quad c = 3a$$

38. **(c)** A vector bisecting the angle between

$\vec{a}$ and $\vec{b}$ is $\frac{\vec{a}}{|\vec{a}|} \pm \frac{\vec{b}}{|\vec{b}|}$; in this case

$$\frac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}} \pm \frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}} \text{ i.e.,}$$

$$\frac{3\hat{i}-\hat{j}}{\sqrt{6}} \text{ or } \frac{\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{6}}$$

A vector of magnitude 3 along these vectors is

$$\frac{3(3\hat{i}-\hat{j})}{\sqrt{10}} \text{ or } \frac{3(\hat{i}+3\hat{j}-2\hat{k})}{\sqrt{14}}$$

Now, $\frac{3}{\sqrt{14}}(\hat{i}+3\hat{j}-2\hat{k}).(\hat{i}-2\hat{j}+\hat{k})$ is negative and hence $\frac{3}{\sqrt{14}}(\hat{i}+3\hat{j}-2\hat{k})$ makes an obtuse angle with $\vec{b}$.

39. **(d)** $\vec{a}.\vec{p} = \frac{\vec{a}.(\vec{b}\times\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} = \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} = 1 = \vec{b}.\vec{q} = \vec{c}.\vec{r}$

$$\vec{b}.\vec{p} = \frac{\vec{b}.(\vec{b}\times\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} = \frac{0}{[\vec{a}\ \vec{b}\ \vec{c}]} = 0 = \vec{c}.\vec{p} = \vec{a}.\vec{r}$$

Therefore, the given expression is equal to $1+0+1+0+1+0=3$.

[Also see the system of reciprocal vectors]

40. **(c)** $\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{vmatrix} = -3\hat{i} - \hat{j} + 2\hat{k}$

If θ is the angle between $\vec{a}$ and the plane containing $\vec{b}$ and $\vec{c}$, then

$$\cos(90° - \theta) = \left|\frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{a}||\vec{b}\times\vec{c}|}\right|$$

$$= \frac{1}{\sqrt{14}}.\frac{1}{\sqrt{14}}|(-9-2+2)| = \frac{9}{14}$$

$$\Rightarrow \sin\theta = \frac{9}{14} \Rightarrow \theta = \sin^{-1}\left(\frac{9}{14}\right)$$

41. **(c)** The direction cosines of the given line are

$$\frac{1}{\sqrt{1^2+2^2+3^2}}, \frac{2}{\sqrt{1^2+2^2+3^2}}, \frac{3}{\sqrt{1^2+2^2+3^2}}$$

i.e., $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

Therefore the projection of PQ on the line

$$= (4-0)\times\frac{1}{\sqrt{14}} + (5+1)\times\frac{2}{\sqrt{14}} + (16-3)\times\frac{3}{\sqrt{14}}$$

$$= \frac{25}{\sqrt{14}}$$

42. **(c)** The sphere meets x-axis where y = 0 = z ie. where $x^2 - 5x - 14 = 0 \Rightarrow x = 7, -2$, So the sphere meets x-axis in (–2, 0, 0) and (7, 0, 0) which are at a distance of 9 units. Similarly, the sphere meets y-axis where x = 0 = z i.e. (0, 14, 0) and (0, –1, 0), which are at a distance of 15 units.

$\therefore\ d_1 = 9$ and $d_2 = 15$.

43. **(b)** A and B will contradict each other if one of the events $A \cap B'$ or $A' \cap B$ occurs. The probability of this happening is

$$P[(A\cap B')\cup(A'\cap B)] = P(A\cap B') + P(A'\cap B)$$

$$= P(A)P(B') + P(A')P(B),$$

because A and B are independent. Therefore, putting P(A) = 0.7 and P(B) = 0.8 the required probability is $(0.7)(0.2) + (0.3)(0.8) = 0.38$.

44. **(d)** For orthogonality, the scalar product = 0

$$\Rightarrow 2(x^2 - 1) + (-x)(x+2) + 3x^2 = 0$$

$$\Rightarrow 2(2x+1)(x-1) = 0$$

$$\Rightarrow x = -\frac{1}{2}, 1$$

45. **(b)** We have $\vec{u} + \vec{v} + \vec{w} = \vec{0}$

$$\therefore |\vec{u} + \vec{v} + \vec{w}| = 0 \Rightarrow |\vec{u} + \vec{v} + \vec{w}|^2 = 0$$

$$\Rightarrow |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}) = 0$$

$$\Rightarrow \vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u} = \frac{-1}{2}[9 + 16 + 25] = -25$$

Mock Test-3

Time : 1 hr **Max. Marks -120**

1. Area of triangle formed by the vertices (0, 0), (6, 0), (4, 3) is
(a) 6 (b) 9
(c) 18 (d) 24

2. In a box containing 100 bulbs, 10 are faulty. The probability that from a sample of 5 bulbs none are defective.
(a) $\left(\frac{1}{10}\right)^5$ (b) $\left(\frac{9}{10}\right)^5$
(c) $\frac{9}{10^5}$ (d) $\frac{1}{5}$

3. If $2\sec 2\alpha = \tan\beta + \cot\beta$ then one of the values of $(\alpha+\beta) =$
(a) p (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) None of these

4. The probability of getting the sum more than 7 when a pair of dice is tossed is
(a) $\frac{1}{9}$ (b) $\frac{1}{4}$
(c) $\frac{7}{12}$ (d) $\frac{5}{12}$

5. The value of $\sum_{r=1}^{5} r\frac{{}^nC_r}{{}^nC_{r-1}} =$
(a) $5(n-3)$ (b) $5(n-2)$
(c) $5n$ (d) $5(2n-9)$

6. ${}^{14}C_7 + \sum_{i=1}^{3} {}^{17-i}C_6 =$
(a) ${}^{16}C_7$ (b) ${}^{17}C_7$
(c) ${}^{17}C_8$ (d) ${}^{16}C_8$

7. If $y = A\sin\omega t$ then $\frac{d^5y}{dt^5} =$
(a) $A\omega^5\cos\left(\omega t-\frac{\pi}{2}\right)$ (b) $A\omega^5\sin\left(\omega t-\frac{\pi}{2}\right)$
(c) $A\omega^5\cos\left(\omega t+\frac{\pi}{2}\right)$ (d) $A\omega^5\sin\left(\omega t+\frac{\pi}{2}\right)$

8. λ for which $\frac{x-2}{-3}=\frac{y-4}{7}=\frac{z-8}{\lambda}$ and $\frac{x-1}{\lambda}=\frac{y-2}{-3}=\frac{z-3}{6}$ are perpendicular equals
(a) 5 (b) 6 (c) 7 (d) 8

9. The angle between the two lines $\frac{x+1}{2}=\frac{y+3}{2}=\frac{z-4}{-1}$ & $\frac{x-4}{1}=\frac{y+4}{2}=\frac{z+1}{2}$ is
(a) $\cos^{-1}\left(\frac{4}{9}\right)$ (b) $\cos^{-1}\left(\frac{3}{9}\right)$
(c) $\cos^{-1}\left(\frac{2}{9}\right)$ (d) $\cos^{-1}\left(\frac{1}{9}\right)$

10. The contrapositive of $(p\vee q)\Rightarrow r$ is
(a) $r\Rightarrow(p\vee q)$ (b) $\sim r\Rightarrow(p\vee q)$
(c) $\sim r\Rightarrow \sim p\wedge\sim q$ (d) $p\Rightarrow(q\vee r)$

11. If (2, 3, 5) are ends of the diameter of a sphere $x^2+y^2+z^2-6x-12y-2z+20=0$. Then coordinates of the other end are
(a) (4, 9, –3) (b) (4, 3, 5)
(c) (4, 3, –3) (d) (4, –3, 9)

12. Three persons A, B, C throw a die in succession. The one getting 'six' wins. If A starts then the probability of B winning is
(a) $\frac{36}{91}$ (b) $\frac{25}{91}$
(c) $\frac{41}{91}$ (d) $\frac{30}{91}$

13. The eccentricity of the ellipse represented by $25x^2+16y^2-150x-175=0$ is
(a) $\frac{2}{5}$ (b) $\frac{3}{5}$
(c) $\frac{4}{5}$ (d) $\frac{1}{5}$

14. If $f(x)=|x-2|$ and $g(x)=f(f(x))$ then for $x>10$, $g'(x)$ equal
(a) –1 (b) 0
(c) 1 (d) $2x-4$

15. If a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. then a, c, e are in

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

16. The coefficient of x^{10} in the expansion of $\left(3x^2 - \frac{1}{x^2}\right)^{15}$ is

(a) $\frac{15!}{10!\,5!}3^{10}$ (b) $-\frac{15!}{10!\,5!}3^{10}$
(c) $-\frac{15!\,3^5}{10!\,5!}$ (d) $\frac{15!}{7!\,8!}3^8$

17. The mean of discrete observations $y_1, y_2, y_3, ..., y_n$ is given by

(a) $\frac{\sum_{i=1}^{n} y_i}{n}$ (b) $\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$
(c) $\frac{\sum_{i=1}^{n} y_i f_i}{n}$ (d) $\frac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

18. $\int \frac{dx}{(x-\beta)\sqrt{(x-\alpha)(\beta-x)}}$ is

(a) $\frac{2}{\alpha-\beta}\sqrt{\frac{x-\alpha}{\beta-x}}+c$ (b) $\frac{2}{\alpha-\beta}\sqrt{(x-\alpha)(\beta-x)}+c$
(c) $\frac{\alpha-\beta}{2}(x-\alpha)\sqrt{\beta-x}$ (d) none of these.

19. The spheres $x^2+y^2+z^2+x+y+z-1=0$ and $x^2+y^2+z^2+x+y+z-5=0$

(a) intersect in a plane
(b) intersect in five points
(c) do not intersect
(d) None of these

20. Let $I_n = \int_1^e (\ell nx)^n \, dx, n \in N$

Statement-1 : $I_1, I_2, I_3 \ldots$ is an increasing sequence.
Statement-2 : ln x is an increasing function.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

21. **Statement–1 :** Range of $f(x) = \sqrt{4-x^2}$ is [0, 2]

Statement–2 : f(x) is increasing for $0 \le x \le 2$ and decreasing for $-2 \le x \le 0$.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

22. Let x, y, z are three integers lying between 1 and 9 such that x 51, y41 and z31 are three digit numbers.

Statement-1 : The value of the determinant

$$\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$$ is zero.

Statement-2 : The value of a determinant is zero if the entries in any two rows (or columns) of the determinant are correspondingly proportional.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

23. **Statement-1 :** Slope of tangents drawn from (4, 10) to parabola $y^2 = 9x$ are $\frac{1}{4}, \frac{9}{4}$

Statement-2 : Every parabola is symmetric about its directrix

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

24. **Statement-1 :** If $x^2 + x + 1 = 0$ then the value of $\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+....+\left(x^{27}+\frac{1}{x^{27}}\right)^2$ is 54.

Statement-2 : ω, ω^2 are the roots of given equation and $x+\frac{1}{x}=-1,\ x^2+\frac{1}{x^2}=-1,\ x^3+\frac{1}{x^3}=2$

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

25. If AB = 0, then for the matrices

$$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$ and

$$B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}, \theta - \phi \text{ is}$$

(a) an odd multiple of $\frac{\pi}{2}$ (b) an odd multiple of p
(c) an even multiple of $\frac{\pi}{2}$ (d) 0

26. If $f'(x) = (x - 1)(x - 2)(x - 3)$ then f(x) in monotonically increasing in

(a) $x < 1$ (b) $x > 3, x < 1$
(c) $x > 3, 1 < x < 2$ (d) $x < 1, 2 < x < 3$

27. The area enclosed by the curve $y = x^5$, the x-axis and the ordinates $x = -1, x = 1$ is

(a) 0 (b) $\frac{1}{3}$
(c) $\frac{1}{6}$ (d) None

28. An inverted cone is 10 cm in diameter and 10 cm deep. Water is poured into it at the rate of $4cm^3$/min. When the depth of water level is 6 cm, then it is rising at the rate

(a) $\frac{9}{4\pi} cm^3 / min.$ (b) $\frac{1}{4\pi} cm^3 / min.$
(c) $\frac{1}{9\pi} cm^3 / min.$ (d) $\frac{4}{9\pi} cm^3 / min.$

29. The equation of tangent to $4x^2 - 9y^2 = 36$ which are perpendicular to straight line $5x + 2y - 10 = 0$ are

(a) $5(y-3) = 2\left(x - \frac{\sqrt{117}}{2}\right)$
(b) $2y - 5x + 10 - 2\sqrt{18} = 0$
(c) $2y - 5x - 10 - 2\sqrt{18} = 0$
(d) None of these

30. $\int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x} \sec^2\left(\frac{1}{3}e^{2x}\right) dx$ is equal to :

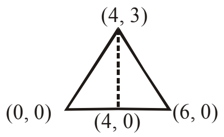
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{3\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$

ANSWER KEY

1. (b)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (c)
11. (a)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (a)
21. (d)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (a)	**26.** (c)	**27.** (b)	**28.** (d)	**29.** (d)	**30.** (a)

HINTS & SOLUTIONS

1. (b)



Area $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 6 \times 3 = 9$ square unit

2. (b) $p = \frac{1}{10}$ and $q = \frac{9}{10}$

$\therefore$ Probability that none are defective $= \left(\frac{9}{10}\right)^5$

3. (c) $\frac{2}{\cos 2\alpha} = \frac{\tan^2\beta + 1}{\tan\beta} = \frac{1}{\sin\beta\,\cos\beta}$

$\Rightarrow \sin 2\beta = \cos 2\alpha = \sin(90\text{-}2\alpha) \Rightarrow \alpha + \beta = \frac{\pi}{4}$

4. (d) Sum of 7 can be obtained when S = {(2,6), (3,5) (3, 6), (4, 4), (4,5), (4, 6)(5, 3)(5, 4) (5, 5)(5, 6)(6, 2)(6, 3) (6,4)(6,5)(6, 6)}

$\therefore$ Probability of sum greater than 7 $= \frac{15}{36} = \frac{5}{12}$

5. (b) $r\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{r.|\underline{n}}{|\underline{r}.|\underline{n-r}} . \frac{|\underline{r-1}.|\underline{n-r+1}}{|\underline{n}} = \frac{|\underline{n-r+1}}{|\underline{n-r}}$

$= \frac{|\underline{n-r+1}}{|\underline{n-r}} = n - r + 1$

$\therefore \sum_{r=1}^{5} = n + (n-1) + (n-2) + (n-3) + (n-4)$

$= 5n - 10 = 5(n-2)$

6. (b) ${}^{14}C_7 + \sum_{i=1}^{3} {}^{17-i}C_6 = {}^{14}C_7 + {}^{14}C_6 + {}^{15}C_6 + {}^{16}C_6$

$= {}^{15}C_7 + {}^{15}C_6 + {}^{16}C_6 = {}^{16}C_7 + {}^{16}C_6 = {}^{17}C_7$

7. (d) $y = A\sin\omega t.\quad \therefore \frac{dy}{dx} = A\omega\cos\omega t$

$\frac{d^2y}{dx^2} = -A\omega^2\sin\omega t$

$\frac{d^3y}{dx^3} = -A\omega^3\cos\omega t$

$\frac{d^4y}{dx^4} = +A\omega^4\sin\omega t$

$\therefore \frac{d^5y}{dx^5} = A\omega^5\cos\omega t = A\omega^5\sin\left(\omega t + \frac{\pi}{2}\right)$

8. (c) For perpendicularity $-3\lambda - 21 + 6\lambda = 0 \Rightarrow \lambda = 7.$

9. (a) $a_1 = 2, b_1 = 2, c_1 = -1$ and $a_2 = 1, b_2 = 2, c_2 = 2$

$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \frac{2 + 4 - 2}{\sqrt{4+4+1}\sqrt{1+4+4}} = \pm\frac{4}{9}.$

10. (c) Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

$\therefore$ contrapositive of $(p \vee q) \Rightarrow r$ is

$\sim r \Rightarrow \sim (p \vee q)$ i.e. $\sim r \Rightarrow (\sim p \wedge \sim q)$

11. (a) Let the co-ordiante of other ends are (x, y, z).

The centre of sphere is C(3, 6, 1)

Therefore, $\frac{x+2}{2} = 3 \Rightarrow x = 4$

$\frac{y+3}{2} = 6 \Rightarrow y = 9$ and $\frac{z+5}{2} = 1 \Rightarrow z = -3$

12. (d) $P(\bar{E}\,E) + P(\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E)$

$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \left(\frac{5}{6}\right)^8 \frac{1}{6} \ldots\infty$

$= \frac{5}{36}\left[1 + \left(\frac{5}{6}\right)^3 \ldots\ldots\right] = \frac{30}{91}$

13. (b) The equation of ellipse can be rewritten as

$\frac{(x-3)^2}{16} + \frac{y^2}{25} = 1 \quad \therefore \frac{16}{25} = 1 - e^2 \Rightarrow e = \frac{3}{5}.$

14. (c) For $x > 10$, $f(x) = x - 2$.

Therefore, $g(x) = x - 2 - 2 = x - 4$

$\therefore g'(x) = 1.$

15. **(b)** a, b, c in A.P. $\Rightarrow$ a + c = 2b;

b, c, d in G.P.

$\Rightarrow$ bd = c^2;

c, d, e in H.P. $\Rightarrow d = \frac{2ce}{c+e}$

$\therefore \frac{a+c}{2} \times \frac{2ce}{c+e} = c^2 \Rightarrow (a+c)e = (c+e)c \Rightarrow c^2 = ae.$

Therefore, a, c and e are in G.P.

16. **(b)** $\left(3x^2 - \frac{1}{x^2}\right)^{15}$

$T_{r+1} = {}^{15}C_r (3x^2)^{15-r}\left(-\frac{1}{x^2}\right)^r$

$= {}^{15}C_r 3^{15-r}(-1)^r x^{30-2r-2r}$

Therefore, 30 – 4r = 10 $\Rightarrow$ r = 5.

Therefore, $T_6 = -{}^{15}C_5 3^{10} = \frac{-15!}{10!\,5!}3^{10}$.

17. **(a)** Requred mean = $\frac{\sum_{i=1}^{n} y_i}{n}$

18. **(a)** $I = \int \frac{dx}{(x-\beta)\sqrt{(x-\alpha)(\beta-x)}}$

Put $x = \alpha\sin^2\theta + \beta\cos^2\theta$

[see the standard substitutions]

$dx = 2(\alpha-\beta)\sin\theta\cos\theta\, d\theta$

Also, $(x-\alpha) = (\beta-\alpha)\cos^2\theta$

$(x-\beta) = (\alpha-\beta)\sin^2\theta$

$\therefore I = \int \frac{2(\alpha-\beta)\sin\theta\cos\theta\, d\theta}{(\alpha-\beta)\sin^2\theta(\beta-\alpha)\sin\theta\cos\theta}$

$= \frac{2}{\beta-\alpha}\int \frac{d\theta}{\sin^2\theta} = \frac{2}{\beta-\alpha}\int \text{cosec}^2\theta\, d\theta$

$= \frac{2}{\beta-\alpha}(-\cot\theta) + c = \frac{2}{\alpha-\beta}\cot\theta + c$

Now, $x = \alpha\sin^2\theta + \beta\cos^2\theta$

$\Rightarrow x\,\text{cosec}^2\theta = \alpha + \beta\cot^2\theta$

$\Rightarrow x(1+\cot^2\theta) = \alpha + \beta\cot^2\theta$

$\therefore \cot\theta = \sqrt{\frac{x-\alpha}{\beta-x}}$; $\therefore I = \frac{2}{\alpha-\beta}\sqrt{\frac{x-\alpha}{\beta-x}} + c$

19. **(c)** As the given spheres both have same centre and different radii therefore they are concentric and they do not have any point in common. Hence they do not intersect.

20. **(a)** Statement – II is true, as if $f(x) = \ln x$, then

$f'(x) = \frac{1}{x} > 0$ (as x > 0, so that f (x) is defined)

Statement – I is not true as $0 < \ln x < 1,\ \forall\ x \in (1, e)$ and hence $(\ln x)^n$ decreases as n is increasing. So that I_n is a decreasing sequence.

21. **(d)** $f'(x) = \frac{-x}{\sqrt{4-x^2}}$

$\therefore$ f (x) is increasing for $-2 \le x \le 0$ and decreasing for $0 \le x \le 2$.

22. **(d)** $\Delta = \begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$

$= \begin{vmatrix} 5 & 4 & 3 \\ 100x+51 & 100y+41 & 100z+31 \\ x & y & z \end{vmatrix}$

$= \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$ $[R_2 \to R_2 - 100R_3 - 10R_1]$

which is zero provided x, y, z are in A.P.

23. **(d)** $y = mx + \frac{a}{m}$

$10 = 4m - 1\frac{9/4}{m} \Rightarrow 16m^2 - 40m + 9 = 0$

$m_1 + m_2 = \frac{40}{16} = \frac{5}{2};\ m_1 m_2 = \frac{9}{16}$

$\Rightarrow m_1 = \frac{1}{4},\ m_2 = \frac{9}{4}$

every parabola is symmetric about its axis only Statement 1 is true.

24. **(b)** $x + \frac{1}{x} = -1,\ x^2 + \frac{1}{x^2} = -1,$

$x^3 + \frac{1}{x^3} = 2,\ x^4 + \frac{1}{x^4} = -1$

$x^5 + \frac{1}{x^5} = -1,\ x^6 + \frac{1}{x^6} = 2$, etc.

$\Rightarrow \left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)^2$

$+ \left(x^4 + \frac{1}{x^4}\right)^2 + \left(x^5 + \frac{1}{x^5}\right)^2$

$+ \left(x^6 + \frac{1}{x^6}\right)^2 + \left(x^7 + \frac{1}{x^7}\right)^2 + \ldots\ldots + \left((x^3)^9 + \frac{1}{(x^3)^9}\right)^2$

$= (1+1+4) + (1+1+4) + (1+1+4) + \ldots\ldots$ 9 times

$= 6 \times 9 = 54.$

25. (a) We have,

$$AB = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\cos\phi\sin\theta\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\varphi\sin\phi & \cos\theta\cos\phi\sin\theta\sin\phi + \sin^2\theta\sin^2 v \end{bmatrix}$$

$$= \cos(\theta - \phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\theta\sin\phi \end{bmatrix}$$

Since, $AB = 0$, $\therefore \cos(\theta - \phi) = 0$

$\therefore \theta - \phi$ is an odd multiple of $\frac{\pi}{2}$

26. (c) When $x > 3$, $f'(x) > 0$;
when $2 < x < 3$, $f'(x) < 0$;
when $1 < x < 2$, $f'(x) > 0$;
when $x < 1$, $f'(x) < 0$.

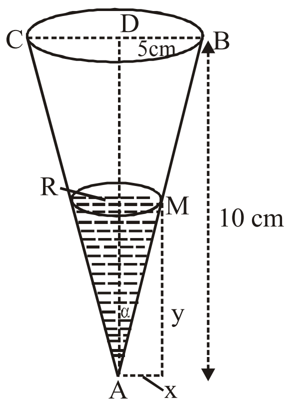
27. (b) Required area

$$= \int_{-1}^{1} |y|\,dx = \int_{-1}^{1} |x^5|\,dx = 2\int_{0}^{1} |x^5|\,dx$$

$$= 2\int_{0}^{1} x^5\,dx = 2\left[\frac{x^6}{6}\right]_0^1 = \frac{2}{6} = \frac{1}{3}$$

28. (d) Let y be the level of water at time t and x the radius of the surface and V, the volume of water.
We know that the volume of cone

$= \frac{1}{3}\pi$ (radius)$^2 \times$ height



$\therefore V = \frac{1}{3}\pi x^2 y$. Let $\angle BAD = \alpha$

$\Rightarrow \tan\alpha = \frac{BD}{AD} = \frac{5}{10} = \frac{1}{2}$.

Again, from right angled ΔAMR, we have

$\tan\alpha = \frac{MR}{AR} = \frac{x}{y}; \Rightarrow \frac{1}{2} = \frac{x}{y}; \therefore x = \frac{y}{2}$.

$\therefore V = \frac{1}{3}\pi . x^2 y = \frac{1}{3}\pi . \left(\frac{y}{2}\right)^2 . y = \frac{\pi}{12} y^2$ ......(1).

By question, the rate of change of volume

$= \frac{dV}{dt} . = 4$ cub.cm./min.

We have to find out the rate of increase of water-level i.e. $\frac{dy}{dt}$.

Differentiating (1) with respect to t, we get

$\frac{dV}{dt} = \frac{\pi}{12}.3y^2.\frac{dy}{dt}; \therefore 4 = \frac{\pi}{4}y^2.\frac{dy}{dt}; \therefore \frac{dy}{dt} = \frac{16}{\pi y^2}$.

When $y = 6$ cm, $\frac{dy}{dt} = \frac{16}{\pi 6^2} = \frac{4}{9\pi}$ cub.cm./min.

29. (d) Slope of the equations $4x^2 - 9y^2 = 36$

$8x - 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{4x}{9y}$ or $m_1 = \frac{4x}{9y}$

Slope of the straight line, $5x + 2y - 10 = 0$ is $m_2 = -\frac{5}{2}$

Therefore, for the perpendicularity, $m_1 m_2 = -1$

Now, $\frac{4x}{9y} \times \frac{-5}{2} = -1 \Rightarrow y = \frac{10x}{9}$.

Putting $y = \frac{10x}{9}$ in $4x^2 - 9y^2 = 36$ gives imaginary roots resulting in no tangents.

30. (a) $I = \int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$

Put $e^{2x} = t \Rightarrow 2e^{2x}dx = dt$

When $x = \log\sqrt{\pi/2}$, $t = e^{2\log\sqrt{\pi/2}} = e^{\log\pi/2} = \frac{\pi}{2}$

When $x = \log\sqrt{\pi}$, $t = e^{2\log\sqrt{\pi}} = \pi$

$\therefore I = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\sec^2\left(\frac{1}{3}t\right)dt = \frac{1}{2}.\frac{1}{\frac{1}{3}}\left[\tan\frac{t}{3}.\right]_{\pi/2}^{\pi}$

$= \frac{3}{2}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right] = \frac{3}{2}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = \sqrt{3}$

Mock Test-4

Time : 1 hr **Max. Marks -120**

1. Equation of straight line $ax + by + c = 0$ where $3a+4b+ c = 0$, which is at maximum distance from (1, –2), is

(a) $3x + y - 17 = 0$ (b) $4x + 3y - 24 = 0$
(c) $3x + 4y - 25 = 0$ (d) $x + 3y - 15 = 0$

2. Given $f(x) = \begin{cases} \sqrt{10 - x^2} & \text{if } -3 < x < 3 \\ 2 - e^{x-3} & \text{if } x \ge 3 \end{cases}$

The graph of $f(x)$ is

(a) continuous and differentiable at x = 3
(b) continuous but not differentiable at x = 3
(c) differentiable but not continuous at x = 3
(d) neither differentiable nor continuous at x = 3

3. The solution of the equation $2z = |z| + 2i$, where z is a complex number, is

(a) $z = \frac{\sqrt{3}}{3} - i$ (b) $z = \frac{\sqrt{3}}{3} + i$
(c) $z = \frac{\sqrt{3}}{3} \pm i$ (d) None of these

4. If $x \ne 2, y \ne 2, z \ne 2$ and $\begin{vmatrix} 2 & y & z \\ x & 2 & z \\ x & y & 2 \end{vmatrix} = 0$, then the value of

$\frac{2}{2-x} + \frac{y}{2-y} + \frac{z}{2-z} =$

(a) 1 (b) 0 (c) 3 (d) 4

5. Box contains 2 one rupee, 2 five rupee, 2 ten rupee and 2 twenty rupee coin. Two coins are drawn at random simultaneously. The probability that their sum is ` 20 or more, is

(a) 1/4 (b) 1/2 (c) 3/4 (d) 1/8

6. The equation $(5x - 1)^2 + (5y - 2)^2 = (\lambda^2 - 4\lambda + 4)(3x + 4y - 1)^2$ represents an ellipse if $\lambda \in$

(a) (0, 1] (b) (–1, 2) (c) (2, 3) (d) (–1, 0)

7. The value of the definite integral, $\int_{\theta_1}^{\theta_2} \frac{d\theta}{1 + \tan\theta} = \frac{501\pi}{K}$

where $\theta_2 = \frac{1003\pi}{2008}$ and $\theta_1 = \frac{\pi}{2008}$. The value of K equals

(a) 2007 (b) 2006 (c) 2009 (d) 2008

8. The straight line $y = m(x - a)$ meets the parabola $y^2 = 4ax$ in two distinct points for

(a) all m Î R (b) all m Î [–1, 1]
(c) all m Î R – {0} (d) None of these

9. The expansion of $(1 + x)^n$ has 3 consecutive terms with coefficients in the ratio 1 : 2 : 3 and can be written in the form ${}^nC_k : {}^nC_{k+1} : {}^nC_{k+2}$. The sum of all possible values of (n + k) is

(a) 18 (b) 21 (c) 28 (d) 32

10. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new standard deviation of the resulting observations.

(a) 12 (b) 18 (c) 24 (d) 144

11. $p \vee (p \wedge q)$ is equivalent to

(a) q (b) p (c) ~p (d) ~q

12. Value of $\int e^{\sin x} \left(\frac{x\cos^3 x - \sin x}{\cos^2 x} \right) dx$ is

(a) $x\, e^{\sin x} - e^{\sin x} \sec x + C$
(b) $x\, e^{\cos x} - e^{\sin x} \sec x + C$
(c) $x^2 e^{\sin x} + e^{\sin x} \sec x + C$
(d) $2x\, e^{\sin x} - e^{\sin x} \tan x + C$

13. The function $f : [2, \infty) \to (0, \infty)$ defined by

$f(x) = x^2 - 4x + a$, then the set of values of 'a' for which f(x) becomes onto is

(a) (4,¥) (b) [4,¥) (c) {4} (d) f

14. If α and β are the real roots of the equation

$x^2 - (k - 2)x + (k^2 + 3k + 5) = 0 \ (k \in R)$.

Find the maximum and minimum values of $(\alpha^2 + \beta^2)$.

(a) 18, 50/9 (b) 18, 25/9
(c) 27, 50/9 (d) None of these

15. The sum of the coefficient of all the terms in the expansion of $(2x - y + z)^{20}$ in which y do not appear at all while x appears in even powers and z appears in odd powers is

(a) 0 (b) $\frac{2^{20} - 1}{2}$ (c) 2^{19} (d) $\frac{3^{20} - 1}{2}$

16. All the five digit numbers in which each successive digit exceeds is predecessor are arranged in the increasing order. The $(105)^{th}$ number does not contain the digit

(a) 1 (b) 2
(c) 6 (d) All of these

17. $\lim_{x \to 0} \frac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3 . \sqrt{x}}$ equals

(a) 1/4 (b) 3/4 (c) 1/2 (d) 1

18. Three people each flip two fair coins. The probability that exactly two of the people flipped one head and one tail, is
(a) 1/2 (b) 3/8 (c) 5/8 (d) 3/4

19. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vector such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{\sqrt{2}}(\vec{b} + \vec{c})$ then the angle between the vectors $\vec{a}, \vec{b}$ is
(a) 3p/4 (b) p/4 (c) p/8 (d) p/2

20. **Statement-1 :** If $|z_1| = 30, |z_2 - (12 + 5i)| = 6$, then maximum value of $|z_1 - z_2|$ is 49.

Statement-2 : If z_1, z_2 are two complex numbers, then $|z_1 - z_2| \le |z_1| + |z_2|$ and equality holds when origin, z_1 and z_2 are collinear and z_1, z_2 are on the opposite side of the origin.
(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

21. Consider the family of straight lines
$2x \sin^2\theta + y \cos^2\theta = 2 \cos 2\theta$

Statement-1 : All the lines of the given family pass through the point (3, –2).

Statement-2 : All the lines of the given family pass through a fixed point.
(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

22. Consider $I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 - \sin x}$

Statement-1 : $I = 0$

Statement-2 : $\int_{-a}^{a} f(x)\,dx = 0$, wherever $f(x)$ is an odd function.
(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

23. **Statement-1 :** Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is one-one.

Statement-2 : $f(x)$ neither increasing nor decreasing function.
(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

24. **Statement-1 :** If the lengths of subtangent and subnormal at point (x, y) on y = f (x) are respectively 9 and 4.
Then $x = \pm 6$

Statement-2 : Product of sub tangent and sub normal is square of the ordinate of the point.
(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

25. Consider the following statements :
S_1 : Number of integrals values of 'a' for which the roots of the equation $x^2 + ax + 7 = 0$ are imaginary with positive real parts is 5.
S_2 : Let α, β are roots $x^2 - (a + 3)x + 5 = 0$ and α, a, β are in A.P. then roots are 2 and 5/2
S_3 : Solution set of $\log_x (2 + x) \le \log_x (6 - x)$ is (1, 2]
State, in order, whether S_1, S_2, S_3 are true or false.
(a) FFT (b) TFT (c) TFF (d) TTT

26. If the substitution $x = \tan^{-1}(t)$ transforms the differential equation $\frac{d^2y}{dx^2} + xy\frac{dy}{dx} + \sec^2 x = 0$ into a differential equation $(1 + t^2)\frac{d^2y}{dt^2} + (2t + y\tan^{-1}(t))\frac{dy}{dt} = k$ then k is equal to
(a) –2 (b) 2 (c) –1 (d) 0

27. Let $f : R \to R$ and $f_n(x) = f(f_{n-1}(x))\ \forall\ n \ge 2, n \in N$, the roots of equation $f_3(x)\, f_2(x)\, f(x) - 25 f_2(x)\, f(x) + 175\, f(x) = 375$ which also satisfy equation $f(x) = x$ will be
(a) 5 (b) 15
(c) 10 (d) Both (a) and (b)

28. A triangle ABC satisfies the relation
$2 \sec 4C + \sin^2 2A + \sqrt{\sin B} = 0$ and a point P is taken on the longest side of the triangle such that it divides the side in the ratio 1 : 3. Let Q and R be the circumcentre and orthocentre of Δ ABC. If PQ : QR : RP = 1 : α : β, then the value of $\alpha^2 + \beta^2$, is
(a) 9 (b) 8 (c) 6 (d) 7

29. The value of $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$ is
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) 1

30. If a is real and $\sqrt{2}ax + \sin By + \cos Bz = 0$,
$x + \cos By + \sin Bz = 0$, $-x + \sin By - \cos Bz = 0$, then the set of all values of a for which the system of linear equations has a non-trivial solution, is
(a) [1, 2] (b) [–1, 1]
(c) $[1, \infty)$ (d) $[2^{-1/2}, 2^{1/2}]$

ANSWER KEY

1. (d)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (a)
11. (b)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (c)
21. (a)	**22.** (a)	**23.** (d)	**24.** (a)	**25.** (b)	**26.** (c)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (b)

HINTS & SOLUTIONS

1. **(d)** It passes through a fixed point (3, 4)
Slope of line joining (3, 4) and (1, –2) is –6/–2 = 3
$\therefore$ Slope of required line = –1/3

Equation is $y - 4 = -\frac{1}{3}(x-3)$

$x + 3y - 15 = 0$

2. **(b)** $f'(3^+) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$

$= \lim_{h\to 0}\frac{(2-e^h)-1}{h} = -\lim_{h\to 0}\left(\frac{e^h-1}{h}\right) = -1$

$f'(3^-) = \lim_{h\to 0}\frac{f(3-h)-f(3)}{-h}$

$= \lim_{h\to 0}\frac{\sqrt{10-(3-h)^2}-1}{-h} = -\lim_{h\to 0}\frac{\sqrt{1+(6h-h^2)}-1}{-h}$

$= \lim_{h\to 0}\frac{6h-h^2}{-h(\sqrt{1+6h-h^2}+1)}$

$= \lim_{h\to 0}\frac{h(h-6)}{h(\sqrt{1+6h-h^2}+1)} = \frac{-6}{2} = -3$

Hence, $f'(3^+) \neq f(3^-)$

3. **(b)** $2(x+iy) = \sqrt{x^2+y^2} + 2i$

$2x = \sqrt{x^2+y^2}$ and $2y = 2$ i.e., $y = 1$

$4x^2 = x^2 + 1$ i.e., $3x^2 = 1$ i.e., $x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{1}{\sqrt{3}}$ $(\because x \geq 0)$ $\therefore$ $z = \frac{1}{\sqrt{3}} + i = \frac{\sqrt{3}}{3} + i$

4. **(b)** $0 = \begin{vmatrix} 2 & y & z \\ x & 2 & z \\ x & y & 2 \end{vmatrix} = \begin{vmatrix} 2 & y & z \\ x-2 & 2-y & 0 \\ x-2 & 0 & 2-z \end{vmatrix}$

$= (x-2)(2-y)(2-z)\begin{vmatrix} \frac{2}{x-2} & \frac{y}{2-y} & \frac{z}{2-z} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

$\Rightarrow 0 = \frac{2}{x-2} - \frac{y}{2-y} - \frac{z}{2-z} \Rightarrow \frac{2}{2-x} + \frac{y}{2-y} + \frac{z}{2-z} = 0$

5. **(b)** Let A be the event such that sum is ` 20 or more
$\therefore$ P (1) = 1 – P (Total value is < 20)

$= 1 - \frac{{}^6C_2 - {}^2C_2}{{}^8C_2} = 1 - \frac{14}{28} = 1 - \frac{1}{2} = \frac{1}{2}$

8 → 1,1; 5,5; 10,10; 20,20

6. **(c)** $\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2 = (\lambda^2 - 4\lambda + 4)\left(\frac{3x+4y-1}{5}\right)^2$

i.e., $\sqrt{\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2} = |\lambda - 2|\left|\frac{3x+4y-1}{\sqrt{5}}\right|$

is an ellipse.
If $0 < |\lambda - 2| < 1$ i.e., $\lambda \in (1, 2) \cup (2, 3)$

7. **(d)** $\theta_1 + \theta_2 = \frac{\pi}{2}$

$\therefore I = \int_{\theta_1}^{\theta_2}\frac{d\theta}{1+\tan\left(\frac{\pi}{2}-\theta\right)} = \int_{\theta_1}^{\theta_2}\frac{\tan\theta\, d\theta}{1+\tan\theta}$

and also $I = \int_{\theta_1}^{\theta_2}\frac{d\theta}{1+\tan\theta}$

$\therefore 2I = \int_{\theta_1}^{\theta_2} d\theta = \theta_2 - \theta_1 = \frac{1002\pi}{2008} \Rightarrow I = \frac{501\pi}{2008}$

Hence, K = 2008.

8. **(c)** $y^2 = 4a\left(\frac{y+am}{m}\right)$ i.e., $my^2 - 4ay - 4a^2m = 0$

$m \neq 0$; $16a^2 + 16a^2m^2 > 0$ which is true $\forall$ m.
$\therefore$ $m \in R - \{0\}$

9. **(a)** $\frac{{}^nC_k}{{}^nC_{k+1}} = \frac{1}{2} \Rightarrow \frac{n!}{k!(n-k)!}\frac{(k+1)!(n-k-1)!}{n!} = \frac{1}{2}$

or $\frac{k+1}{n-k} = \frac{1}{2}$

$2k + 2 = n - k$

$n - 3k = 2$ .... (1)

Similarly, $\frac{{}^nC_{k+1}}{{}^nC_{k+2}} = \frac{2}{3}$

$$\frac{n!}{(k+1)!(n-k-1)!}\cdot\frac{(k+2)!(n-k-2)!}{n!}=\frac{2}{3}$$

$$\frac{k+2}{n-k-1}=\frac{2}{3}$$

$3k+6=2n-2k-2$

$2n-5k=8$ .... (2)

From (1) and (2)

$n = 14$ and $k = 4$

$\therefore n+k=18$

10. **(a)** Let the observations be x_1, x_2, x_3, x_4, x_5 and x_6, so

their mean $\bar{x}=\frac{\sum_{i=1}^{6}x_i}{6}=8$

$$\Rightarrow \sum_{i=1}^{6}x_i=8\times 6\Rightarrow \sum_{i=1}^{6}x_i=48$$

On multiplying each observation by 3, we get the new observations as $3x_1, 3x_2, 3x_3, 3x_4, 3x_5$ and $3x_6$.

Now, their mean $=\bar{x}=\frac{\sum_{i=1}^{6}3x_i}{6}=\frac{3\sum_{i=1}^{6}x_i}{6}$

$$\Rightarrow \bar{x}=\frac{3\times 48}{6}=24$$

Variance of new observations

$$=\frac{\sum_{i=1}^{6}(3x_i-24)^2}{6}=\frac{3^2\sum_{i=1}^{6}(x_i-8)^2}{6}$$

$=\frac{9}{1}\times$ Variance of old observations $=9\times 4^2=144$

Thus, standard deviation of new observations

$=\sqrt{\text{Variance}}=\sqrt{144}=12$

11. **(b)** $p\vee(p\wedge q)$ is equivalent to p.

12. **(a)** $\int e^{\sin x}\left(\frac{x\cos^3 x-\sin x}{\cos^2 x}\right)dx$

$=\int e^{\sin x}\ x\cos x\,dx-\int e^{\sin x}\ \tan x\sec x\,dx$

$=\int x\,d\left(e^{\sin x}\right)-\int e^{\sin x}\ d(\sec x)$

$=\left\{x\,e^{\sin x}-\int e^{\sin x}\,dx\right\}$

$-\left\{e^{\sin x}\sec x-\int e^{\sin x}\sec x\cos x\,dx\right\}$

$=x\,e^{\sin x}-e^{\sin x}\sec x+C$

13. **(d)** $f(x)=x^2-4x+a$ always attains its minimum value.

So its range must be closed.

So, $a=\{\phi\}$

14. **(a)** For real roots, $D\geq 0$

$(k-2)^2-4(k^2+3k+5)\geq 0$

$\Rightarrow (k^2+4-4k)-4k^2-12k-20\geq 0$

$\Rightarrow -3k^2-16k-16\geq 0\Rightarrow 3k^2+16k+16\leq 0$

$$\Rightarrow\left(k+\frac{4}{3}\right)(k+4)\leq 0$$

Now $E=\alpha^2+\beta^2$; $E=(\alpha+\beta)^2-2\alpha\beta$

$E=(k-2)^2-2(k^2+3k+5)=-k^2-10k-6$

$E=-(k^2+10k+6)=-[(k+5)^2-19]=19-(k+5)^2$

$\therefore E_{min}$ occurs when $k=-4/3$

$$\therefore E_{min}=19-\frac{121}{9}=\frac{171-121}{9}=\frac{50}{9}$$

E_{max} occurs when $k=-4$

$E_{max}=19-1=18$

15. **(a)** $\frac{20!}{p!q!r!}(2x)^p(-y)^q(z)^r=\frac{20!}{p!q!r!}2^p(-1)^q\ x^py^qz^r$

$p+q+r=20,\ q=0$

$p+r=20$ (p is even and r is odd).

even + odd = even (never possible)

Coefficient of such power never occur

$\therefore$ coefficient is zero

16. **(a)**

Starting with 1	1					2 3 4 5 6 7 8 9

$={}^8C_4=70$

Starting with 2	2					3 4 5 6 7 8 9

$={}^7C_4=35$

Total = 105

$(105)^{th}$ number 26789

17. **(b)** $\lim_{x\to 0}\frac{(\tan x)^{3/2}[1-(\cos x)^{3/2}]}{x^{3/2}.x^2}$.

$$=1\times\lim_{x\to 0}\frac{1-\cos^3 x}{x^2}.\frac{1}{1+(\cos x)^{3/2}}$$

$$=\frac{1}{2}.\frac{1}{2}(1+\cos x+\cos^2 x)=\frac{3}{4}$$

18. **(b)** $n=3$, P (success) = P (HT or TH) = 1/2

$$\Rightarrow p=q=\frac{1}{2}\text{ and } r=2$$

$$P(r=2)={}^3C_2\left(\frac{1}{2}\right)^2.\ \frac{1}{2}=\frac{3}{8}$$

19. **(a)** $(\hat{a}.\hat{c})\hat{b}-(\hat{a}.\hat{b})\hat{c}=\frac{1}{\sqrt{2}}\hat{b}+\frac{1}{\sqrt{2}}\hat{c}$

$$\therefore\ \hat{a}.\hat{c}=\frac{1}{\sqrt{2}}\text{ and }\hat{a}.\hat{b}=-\frac{1}{\sqrt{2}}$$

$\Rightarrow$ angle between $\hat{a}$ and $\hat{c} = \frac{\pi}{4}$ and angle between $\hat{a}$ and $\hat{b} = \frac{3\pi}{4}$

20. **(c)** $C_1C_2 = 13, r_1 = 30, r_2 = 6$

$C_1C_2 < r_1 - r_2$

$\therefore$ The circle $|z_2 - (12+5i)| = 6$

lies with in the circle $|z_1| = 30$

$\therefore$ max $|z_1 - z_2| = 30 + 13 + 6 = 49$

$\therefore$ Statement-1 is true.

Statement-2. $|z_1 - z_2| \le |z_1| + |z_2|$ is always true.

Equality sign holds if z_1, z_2 origin are collinear and z_1 and z_2 lies on opposite sides of the origin.

$\therefore$ Statement-2 is true.

21. **(a)** $2\sin^2\theta\, x + \cos^2\theta\, y = 2\cos 2\theta$

Statement-1: The line passes through the point (3, –2)

If $6\sin^2\theta - 2\cos^2\theta = 2\cos 2\theta$

i.e. $6(1-\cos^2\theta) - 2\cos^2\theta = 4\cos^2\theta - 2$

i.e. $12\cos^2\theta = 8$

$\therefore$ Statement-1 is false.

Statement : 2 $(1-\cos^2\theta)\,x + \cos^2\theta\, y = 4\cos^2\theta - 2$

$\therefore \cos^2\theta(-2x + y - 4) + 2x + 2 = 0$

Family of lines passes through the point of intersection of line $2x - y + 4 = 0$ and $x = -1$

$\therefore$ The point is (–1, 2)

$\therefore$ Statement-2 is true.

22. **(a)** $f(x) = \frac{1}{1-\sin x}$ and $f(-x) = \frac{1}{1+\sin x}$

$\therefore I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1-\sin x}$

Now, $f(x) + f(-x) = 2I = \int_{-\pi/4}^{\pi/4} \frac{2\,dx}{1-\sin^2 x}$

$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \frac{dx}{\cos^2 x}$. This is an even function

$\therefore I = 2\int_0^{\pi/4} \sec^2 x\, dx \ne 0 \Rightarrow$ Statement-1 is false.

23. **(d)** Every increasing or decreasing function is one-one

$$f'(x) = 3x^2 + 2x + 3 + \cos x = 3\left(x + \frac{1}{3}\right)^2 + \frac{8}{3} + \cos x > 0$$

$[\because |\cos x| < 1$ and $3\left(x+\frac{1}{3}\right)^2 + \frac{8}{3} \ge \frac{8}{3}]$

$\therefore f(x)$ is strictly increasing

24. **(a)** $\left|\frac{y_1}{m}\right| = 9$ and $|y_1 m| = 4$

$\Rightarrow |y_1|^2 = 36 \quad \Rightarrow y_1 = \pm 6$

Product of subtangent and sub normal is y_1^2.

Statement 1 is false. Statement 2 is true.

25. **(b)** S_1 : if $x^2 + ax + 7 = 0$ has imaginary roots with positive real parts then $D < 0$ and sum of roots > 0

$\Rightarrow a^2 - 28 < 0$ and $-a > 0$

$\Rightarrow -\sqrt{28} < a < \sqrt{28}$ and $a < 0$

$\Rightarrow a = -1, -2, -3, -4, -5$

S_2 : $x^2 - (a+3)x + 5 = 0$ has roots α, a, β

If α, a, β are in AP. then

$2a = \alpha + \beta \Rightarrow 2a = a + 3 \Rightarrow a = 3$

The equation becomes $x^2 - 6x + 5 = 0$ which has roots 1 and 5.

S_3 : Case- I :

If $0 < x < 1$, then $2 + x \ge 6 - x > 0 \Rightarrow 2x \ge 4$ and $x < 6$

$\Rightarrow x \ge 2$ and $x < 6 \Rightarrow x \in [2, 6)$

$\therefore x \in (0,1) \cap [2,6) = \phi \quad \therefore x \in \phi$

Case II : If $x > 1$, then $0 < 2 + x \le 6 - x$

$\Rightarrow x > -2$ and $x \le 2 \qquad \therefore x \in (1, 2]$

26. **(c)** $x = \tan^{-1} t \Rightarrow \frac{dx}{dt} = \frac{1}{1+t^2}$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{dy}{dt}(1+t^2) \qquad \text{..........(1)}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{dy}{dt}(1+t^2)\right]\cdot\frac{dt}{dx}$$

$$= \left[\frac{dy}{dt}2t + (1+t^2)\frac{d^2y}{dt^2}\right](1+t^2) \qquad \text{..........(2)}$$

Hence the given differential equation

$\frac{d^2y}{dx^2} + xy\frac{dy}{dx} + \sec^2 x > 0$, becomes

$$(1+t^2)\left[2t\frac{dy}{dt} + (1+t^2)\frac{d^2y}{dt^2}\right] + y\tan^{-1}t\left[\frac{dy}{dt}(1+t^2)\right] + (1+t^2) = 0$$

Cancelling $(1+t^2)$ throughout we get

$$(1+t^2)\frac{d^2y}{dt^2} + (2t + y\tan^{-1}t)\frac{dy}{dt} = -1 \Rightarrow k = -1$$

27. **(d)** $f_2(x) = f(f(x)) = f(x) = x$

$f_3(x) = f(f_2(x)) = f(x) = x$

$\Rightarrow x^3 - 25x^2 + 175x - 375 = 0$

$(x-5)(x^2 - 20x + 75) = 0$

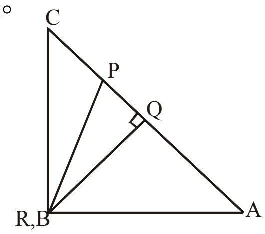
$(x-5)^2(x-15) = 0 \Rightarrow x = 5, 15$

28. **(a)** $2\sec 4C + \sin^2 2A + \sqrt{\sin B} = 0$

$A = 45°, B = 90°$ and $C = 45°$

Let AQ = a, then $BP = \frac{a}{2}$,

$PQ = \frac{a}{2}$ and QR = a



$\therefore\ PR = \sqrt{a^2 + \frac{a^2}{4}} = \frac{\sqrt{5}a}{2}$

$\therefore\ 1 : \alpha : \beta = \frac{a}{2} : a : \frac{\sqrt{5}a}{2} = 1 : 2 : \sqrt{5}$

$\therefore\ \alpha = 2$ and $\beta = \sqrt{5}$ $\quad\therefore\ \alpha^2 + \beta^2 = 9$

29. **(c)** Let $I_1 = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\ dt$

Put $t = \sin^2 u \Rightarrow dt = 2\sin u \cos u\, du$

$\Rightarrow dt = \sin 2u du$

$\therefore I_1 = \int_0^x u \sin 2u\, du$

Let $I_2 = \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\ dt$

Put $t = \cos^2 v \Rightarrow dt = -2\cos v \sin v dv$

$\Rightarrow dt = -\sin 2v\, dv$

$\therefore I_2 = \int_{\frac{\pi}{2}}^{x} v(-\sin 2v)dv = -\int_{\frac{\pi}{2}}^{x} v \sin 2v dv$

$= -\int_{\frac{\pi}{2}}^{x} u \sin 2u du$ [change of variable]

$\therefore\ I = I_1 + I_2 = \int_0^x u \sin 2u du - \int_{\frac{\pi}{2}}^{x} u \sin 2u du$

$= \int_0^{\frac{\pi}{2}} u \sin 2u du + \int_{\frac{\pi}{2}}^{x} u \sin 2u du - \int_{\frac{\pi}{2}}^{x} u \sin 2u du$

$= \int_0^{\frac{\pi}{2}} u \sin 2u du = \frac{\pi}{4}$ [Integrate by parts]

30. **(b)**

For non-trivial solution, $\Delta = \begin{vmatrix} \sqrt{2}a & \sin B & \cos B \\ 1 & \cos B & \sin B \\ -1 & \sin B & -\cos B \end{vmatrix} = 0$

$\Rightarrow\ a\sqrt{2}\,[-\cos^2 B - \sin^2 B] - \sin B\,[-\cos B + \sin B]$
$+ \cos B\,[\sin B + \cos B] = 0$

$\Rightarrow\ -a\sqrt{2} + \sin 2B + \cos 2B = 0 \Rightarrow a \in [-1, 1]$

Mock Test-5

Time : 1 hr **Max. Marks -120**

1. Let L_1 be a straight line passing through the origin and L_2 be the straight line x + y = 1. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equation can represent L_1 ?
(a) $x + 7y = 0$ (b) $x - y = 0$
(c) $x - 7y = 0$ (d) Both (a) and (b)

2. Let $P = \begin{bmatrix} 3 & -5 \\ 7 & -12 \end{bmatrix}$ and $Q = \begin{bmatrix} 12 & -5 \\ 7 & -3 \end{bmatrix}$ then incorrect about the matrix $(PQ)^{-1}$ is
(a) nilpotent (b) idempotent
(c) involutory (d) symmetric

3. The equation sin x + x cos x = 0 has at least one root in
(a) $\left(-\frac{\pi}{2}, 0\right)$ (b) (0, p)
(c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right)$

4. The area enclosed by the parabola $y^2 = 12x$ and its latus rectum is
(a) 36 (b) 24 (c) 18 (d) 12

5. Number of permutations 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time are such that the digit. 1 appearing somewhere to the left of 2, 3 appearing to the left of 4 and 5 somewhere to the left of 6, is
(e.g., 815723946 would be one such permutation)
(a) 9 . 7! (b) 8! (c) 5! . 4! (d) 8! . 4!

6. If the function f : [0, 16] → R is differentiable. If $0 < \alpha < 1$ and $1 < \beta < 2$, then $\int_0^{16} f(t)\, dt$ is equal to
(a) $4[a^3 f(a^4) - b^3 f(b^4)]$ (b) $4[a^3 f(a^4) + b^3 f(b^4)]$
(c) $4[a^4 f(a^3) + b^4 f(b^3)]$ (d) $4[a^2 f(a^2) + b^2 f(b^2)]$

7. Three distinct points $P(3u^2, 2u^3)$, $Q(3v^2, 2v^3)$ and $R(3w^2, 2w^3)$ are collinear then
(a) uv + vw + wu = 0 (b) uv + vw + wu = 3
(c) uv + vw + wu = 2 (d) uv + vw + wu = 1

8. Let 'a' denote the root of equation

$$\cos(\cos^{-1} x) + \sin^{-1} \sin\left(\frac{1+x^2}{2}\right) = 2\sec^{-1}(\sec x)$$

then possible values of [| 10a |] where [.] denotes the greatest integer function will be
(a) 1 (b) 5
(c) 10 (d) Both (a) and (c)

9. The two of the straight lines represented by the equation $ax^3 + bx^2y + cxy^2 + dy^3 = 0$ will be at right angle if
(a) $a^2 + c^2 = 0$ (b) $a^2 + ac + bd + d^2 = 0$
(c) $a^2c^2 + bd + d^2 = 0$ (d) None of these

10. If $x^2 - 2x\cos\theta + 1 = 0$, then the value of $x^{2n} - 2x^n \cos n\theta + 1$, $n \in N$ is equal to
(a) cos 2nq
(b) sin 2nq
(c) 0
(d) some real number greater than 0

11. Evaluate $\int \frac{8}{(x+2)(x^2+4)} dx$
(a) $\log|x+2| - \frac{1}{2}\log(x^2+4) + \tan^{-1}\frac{x}{2} + C$
(b) $\log|x+2| - \frac{1}{2}\log(x^2+4) + \sin^{-1}\frac{x}{2} + C$
(c) $\log|x+2| - \frac{1}{2}\log(x^2+4) + \cos^{-1}\frac{x}{2} + C$
(d) $\log|x+2| - \frac{1}{2}\log(x^3+4) + \tan^{-1}\frac{x}{2} + C$

12. Vertices of a parallelogram taken in order are A (2, –1, 4), B (1, 0, –1), C (1, 2, 3) and D. Distance of the point P (8, 2, –12) from the plane of the parallelogram is
(a) $\frac{4\sqrt{6}}{9}$ (b) $\frac{32\sqrt{6}}{9}$
(c) $\frac{16\sqrt{6}}{9}$ (d) None of these

13. Given $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$.
Compute the value of $|\vec{A} \times [\vec{A} \times (\vec{A} \times \vec{B})].\vec{C}|$.
(a) 343 (b) 512 (c) 221 (d) 243

14. The value of the definite integral

$$\int_0^{3\pi/4} [(1+x)\sin x + (1-x)\cos x]\, dx \text{ is}$$

(a) $2(\sqrt{2}+1)$ (b) $2(\sqrt{2}-1)$
(c) $\sqrt{2}+1$ (d) $\sqrt{2}-1$

15. Area of triangle formed by common tangents to the circle $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 2x = 0$ is
(a) $3\sqrt{3}$ (b) $2\sqrt{3}$ (c) $9\sqrt{3}$ (d) $6\sqrt{3}$

16. The locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is
(a) $9x + 10y - 7 = 0$ (b) $x - y + 2 = 0$
(c) $9x - 10y + 11 = 0$ (d) $9x + 10y + 7 = 0$

17. The sum to infinity of the series

$$\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \ldots\ldots, \text{ is equal to}$$

(a) 3 (b) 1 (c) 2 (d) 3/2

18. The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is

(a) $x^2 + 2y^2 - ax = 0$ (b) $2x^2 + y^2 - 2ax = 0$
(c) $2x^2 + 2y^2 - ay = 0$ (d) $2x^2 + y^2 - 2ay = 0$

19. A box contains 6 red, 5 blue and 4 white marbles. Four marbles are chosen at random without replacement. The probability that there is atleast one marble of each colour among the four chosen, is

(a) $\frac{48}{91}$ (b) $\frac{44}{91}$ (c) $\frac{88}{91}$ (d) $\frac{24}{91}$

20. **Statement-1 :** If a, b, c are non real complex and α, β are the roots of the equation $ax^2 + bx + c = 0$ then $\text{Im}(\alpha\beta) \neq 0$.
because
Statement-2 : A quadratic equation with non real complex coefficient do not have root which are conjugate of each other.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

21. **Statement-1 :** The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at some point $x = x_0$. because
Statement-2 : $\frac{dy}{dx}$ exists at $x = x_0$.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

22. Let C be a circle with centre O and HK is the chord of contact of pair of the tangents from points A. OA intersects the circle C at P and Q and B is the midpoint of HK, then
Statement-1 : AB is the harmonic mean of AP and A
because
Statement-2 : AK is the Geometric mean of AB and AO, OA is the arithmetic mean of AP and A

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

23. **Statement-1 :** The statement $(p \vee q) \wedge \sim p$ and $\sim p \wedge q$ are logically equivalent.
Statement-2 : The end columns of the truth table of both statements are identical.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

24. **Statement-1 :** Period of $f(x) = \sin 4\pi\{x\} + \tan \pi[x]$, where, [x] & {x} denote the G.I.F. & fractional part respectively is 1.
Statement-2: A function f(x) is said to be periodic if there exist a positive number T independent of x such that $f(T + x) = f(x)$. The smallest such positive value of T is called the period or fundamental period.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is false.

25. The value of the expression

$$\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)+\ldots+\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$$

where ω is an imaginary cube root of unity, is

(a) $\frac{n(n^2-2)}{3}$ (b) $\frac{n(n^2+2)}{3}$
(c) $\frac{n(n^2-1)}{3}$ (d) None of these

26. If s, s' are the length of the perpendicular on a tangent from the foci, a, a' are those from the vertices is that from the centre and e is the eccentricity of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $\frac{ss' - c^2}{aa' - c^2} =$

(a) e (b) 1/e (c) $1/e^2$ (d) e^2

27. One percent of the population suffers from a certain disease. There is blood test for this disease, and it is 99% accurate, in other words, the probability that it gives the correct answer is 0.99, regardless of whether the person is sick or healthy. A person takes the blood test, and the result says that he has the disease. The probability that he actually has the disease, is

(a) 0.99% (b) 25% (c) 50% (d) 75%

28. Set of values of m for which two points P and Q lie on the line $y = mx + 8$ so that $\angle APB = \angle AQB = \frac{\pi}{2}$ where $A \equiv (-4, 0)$, $B \equiv (4, 0)$ is

(a) $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty) - \{-2, 2\}$
(b) $[-\sqrt{3}, -\sqrt{3}] - \{-2, 2\}$
(c) $(-\infty, -1) \cup (1, \infty) - \{-2, 2\}$
(d) $\{-\sqrt{3}, \sqrt{3}\}$

29. The trace $T_r(A)$ of a 3 × 3 matrix $A = (a_{ij})$ is defined by the relation $T_r(A) = a_{11} + a_{22} + a_{33}$ (i.e., $T_r(A)$ is sum of the main diagonal elements). Which of the following statements cannot hold ?

(a) $T_r(kA) = kT_r(A)$ (k is a scalar)
(b) $T_r(A + B) = T_r(A) + T_r(B)$
(c) $T_r(I_3) = 3$
(d) $T_r(A^2) = T_r(A)^2$

30. Let $a_n = \int_0^{\pi/2} (1 - \sin t)^n \sin 2t \, dt$ then $\lim_{n\to\infty} \sum_1^n \frac{a_n}{n}$ is equal to

(a) 1/2 (b) 1 (c) 4/3 (d) 3/2

ANSWER KEY

1. (d)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (c)
11. (a)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (a)
21. (c)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (a)

HINTS & SOLUTIONS

1. (d) Centre of the circle is $\left(\frac{1}{2},-\frac{3}{2}\right)$.

Its distance from the line $x + y - 1 = 0$ is $\sqrt{2}$

Let the required line be $mx - y = 0$

$$\therefore \left|\frac{\frac{m}{2}+\frac{3}{2}}{\sqrt{m^2+1}}\right| = \sqrt{2} \Rightarrow m = 1, -1/7$$

$\therefore$ The lines are $x - y = 0$, $x + 7y = 0$

2. (b) $PQ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. (b) Let $f(x) = \sin x + x\cos x$

Consider

$$g(x) = \int_0^x (\sin t + t\cos t)\,dt = t\sin t]_0^x = x\sin x$$

$g(x) = x\sin x$ which is differentiable

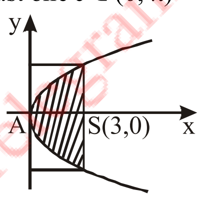
Now, $g(0) = 0$ and $g(\pi) = 0$, using Rolles Theorem

$\exists$ atleast one $c \in (0, \pi)$ such that $g'(c) = 0$

i.e. $c\cos c + \sin c = 0$ for atleast one $c \in (0, \pi)$

4. (b) Required area $= 2\int y\,dx$

$$= 2\int_0^3 \sqrt{12}\,\sqrt{x}\,dx = 24$$



5. (a) Number of digits are 9

Select 2 places for the digit 1 and 2 in ${}^9C_2$ ways from the remaining 7 places select any two places for 3 and 4 in ${}^7C_2$ ways and from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways

Now, the remaining 3 digits can be filled in 3! ways

$\therefore$ Total ways $= {}^9C_2 \cdot {}^7C_2 \cdot {}^5C_2 \cdot 3!$

$$= \frac{9!}{2!.7!}\cdot\frac{7!}{2!.5!}\cdot\frac{5!}{2!.3!}.3! = 9.7!$$

6. (b) Let $I = \int_0^{16} f(t)\,dt$

Consider $g(x) = \int_0^{x^4} f(t)\,dt \Rightarrow g(0) = 0$

LMVT for g in [0, 1] gives, some $\alpha \in (0, 1)$ such that

$$\frac{g(1)-g(0)}{1-0} = g'(\alpha) \quad \text{..........(1)}$$

Similarly, LMVT in [1, 2] gives, some $\beta \in (1, 2)$ such that $\frac{g(2)-g(1)}{2-1} = g'(\beta)$ ....(2)

Eq. (1) + Eq. (2)

$g'(\alpha) + g'(\beta) = g(2) - \underbrace{g(0)}_{\text{zero}}$; but $g'(x) = f(x^4).4x^3$

$$\therefore 4\left[\alpha^3 f(\alpha^4) + \beta^3 f(\beta^4)\right] = \int_0^{16} f(t)\,dt$$

7. (a)

$$\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\begin{vmatrix} u^2 - v^2 & u^3 - v^3 & 0 \\ v^2 - w^2 & v^3 - w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} u+v & u^2+v^2+vu & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\begin{vmatrix} u-w & (u^2-w^2)+v(u-w) & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & u+w+v & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$\Rightarrow (v^2 + w^2 + vw) - (v + w)\,[(v + w) + u] = 0$

$\Rightarrow v^2 + w^2 + vw = (v + w)^2 + u\,(v + w)$

$\Rightarrow uv + vw + wu = 0$

8. (d) Case I : $x \in [-1, 0]$

$$x + \frac{1+x^2}{2} = -2x$$

$\Rightarrow x^2 + 6x + 1 = 0$

$\Rightarrow x = 2\sqrt{2} - 3 \Rightarrow |10a| = [|20\sqrt{2} - 30|] = 30 - 20\sqrt{2}$

Case II : $x \in [0, 1]$

$$x + \frac{1 + x^2}{2} = 2x$$

$\Rightarrow 1 + x^2 = 2x \Rightarrow x = 1 \Rightarrow |10a| = 10$

$|10a| = 10, \; |20\sqrt{2} - 30|$

$\Rightarrow [|10a|] = 1, 10$

9. (b) Let $y = mx$ be any line represented by the equation $ax^3 + bx^2y + cxy^2 + dy^3 = 0$

$\Rightarrow ax^3 + bx^2(mx) + cx(m^2x^2) + dm^3x^3 = 0$

$\Rightarrow a + bm + cm^2 + dm^3 = 0$ which is a cubic equation.

It represents three lines out of which two are perpendicular. Hence

$$m_1m_2 = -1 \text{ and } m_1m_2m_3 = -\frac{a}{d} \Rightarrow m_3 = \frac{a}{d}$$

and m_3 is the root of the given equation

Hence, $a + b\left(\frac{a}{d}\right) + c\left(\frac{a}{d}\right)^2 + d\left(\frac{a}{d}\right)^3 = 0$

$\Rightarrow d^2 + bd + ca + a^2 = 0$

10. (c) $x^2 - 2x\cos\theta + 1 = 0$,

$$\therefore\; x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \cos\theta \pm i\sin\theta$$

Let $x = \cos\theta + i\sin\theta$

$\therefore\; x^{2n} - 2x^n\cos n\theta + 1$

$= \cos 2n\theta + i\sin 2n\theta - 2(\cos n\theta + i\sin n\theta)\cos n\theta + 1$

$= \cos 2n\theta + 1 - 2\cos^2 n\theta + i(\sin 2n\theta - 2\sin n\theta\cos n\theta)$

$= 0 + i\,0 = 0$

11. (a) Let $\dfrac{8}{(x+2)(x^2+4)} = \dfrac{A}{x+2} + \dfrac{Bx + C}{x^2+4}$ .... (i)

Then, $8 = A(x^2+4) + (Bx + C)(x+2)$ ....(ii)

Putting $x + 2 = 0$ i.e. $x = -2$ in (ii), we get $8 = 8A \Rightarrow A = 1$

Putting $x = 0$ and 1 respectively in (ii), we get

$8 = 4A + 2C$ and $8 = 5A + 3B + 3C$

Solving these equation, we obtain $A = 1, C = 2, B = -1$

Substituting the values of A, B and C in (i) , we obtain

$$\frac{8}{(x+2)(x^2+4)} = \frac{1}{x+2} + \frac{-x+2}{x^2+4}$$

$$\therefore \int \frac{8}{(x+2)(x^2+4)}dx = \int \frac{1}{(x+2)}dx + \int \frac{-x+2}{x^2+4}dx$$

$$= \int \frac{1}{(x+2)}dx - \int \frac{x}{x^2+4}dx + 2\int \frac{1}{x^2+4}dx$$

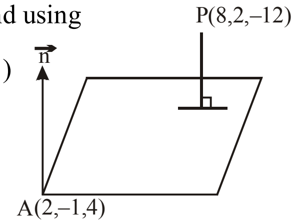
$$= \log|x+2| - \frac{1}{2}\int \frac{1}{t}dt + 2.\frac{1}{2}\tan^{-1}\frac{x}{2} + C, \; (\text{where } t = x^2+4)$$

$$= \log|x+2| - \frac{1}{2}\log t + \tan^{-1}\frac{x}{2} + C$$

$$= \log|x+2| - \frac{1}{2}\log(x^2+4) + \tan^{-1}\frac{x}{2} + C$$

12. (b) $\vec{n} = 7\hat{i} + 2\hat{j} - \hat{k}$ is normal to plane

(Assuming $n = a\hat{i} + b\hat{j} + c\hat{k}$ and using $\vec{n}.\overrightarrow{AB} = 0, \vec{n}.\overrightarrow{BC} = 0, \vec{n}.\overrightarrow{AC} = 0$)



$P = (8, 2, -12)$

$\overrightarrow{AP} = 6\hat{i} + 3\hat{j} - 16\hat{k}$

$\therefore$ Distance

$$d = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \left|\frac{42 + 6 + 16}{\sqrt{49+4+1}}\right| = \frac{64}{\sqrt{54}} = \frac{64}{3\sqrt{6}} = \frac{64\sqrt{6}}{18} = \frac{32\sqrt{6}}{9}$$

13. (a)

$$\vec{V} = \vec{A} \times \left[(\vec{A}.\vec{B})\vec{A} - (\vec{A}.\vec{A})\vec{B}\right].\vec{C}$$

$$= \Big(\underbrace{\vec{A} \times (\vec{A}.\vec{B})\vec{A}}_{\text{zero}} - (\vec{A}.\vec{A})\vec{A} \times \vec{B}\Big).\vec{C} = -|\vec{A}|^2 [\vec{A}\;\vec{B}\;\vec{C}]$$

Now, $|\vec{A}|^2 = 4 + 9 + 36 = 49$

$$[\vec{A}\;\vec{B}\;\vec{C}] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix}$$

$= 2(1+4) - 1(3-12) + 1(-6-6)$

$= 10 + 9 - 12 = 7$

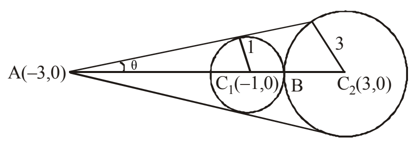
$\therefore\; \left|-|\vec{A}|^2 [\vec{A}\;\vec{B}\;\vec{C}]\right| = 49 \times 7 = 343$

14. (a) $I = \displaystyle\int_0^{3\pi/4} (\sin x + \cos x)\,dx + \int_0^{3\pi/4} \underset{I}{x}\,\underbrace{(\sin x - \cos x)}_{II}\,dx$

$$= \int_0^{3\pi/4} (\sin x + \cos x)\,dx + \underbrace{x(-\cos x - \sin x)|_0^{3\pi/4}}_{\text{zero}} + \int_0^{3\pi/4} (\sin x + \cos x)\,dx$$

$$= 2\int_0^{3\pi/4} (\sin x + \cos x)\,dx = 2(\sqrt{2} + 1)$$

15. (a) A divides C_1C_2 externally in the ratio 1 : 3.



$\therefore$ coordinate of A are (–3, 0)

We have $\sin\theta = 1/2 \quad \therefore\; \theta = 30°$

Area $= 3 \times 3\tan 30° = 3\sqrt{3}$

16. **(c)** Given circles are $x^2+y^2+4x-6y+9=0$
and $x^2+y^2-5x+4y-2=0$
$\therefore$ locus of centres is
$9x-10y+11=0$

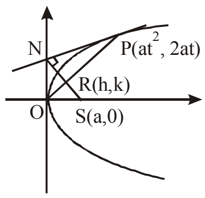
17. **(c)** Given series is

$$\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+.....,$$

$$t_n=\frac{1}{1+2+3+.....n}=\frac{2}{n(n+1)}=2\left[\frac{1}{n}-\frac{1}{n+1}\right]$$

$$\therefore \text{Sum}=S=2\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+.......\right]=2$$

18. **(b)**



$T: ty=x+at^2$ .... (1)
Line perpendicular to (1) through (a, 0) is
$tx+y=ta$ .... (2)

Equation of OP : $y-\frac{2}{t}x=0$ .... (3)

From equations (2) and (3) eliminating t we get

$$\frac{2x}{y}(x)+y=\frac{2x}{y}(a)$$

$\Rightarrow 2x^2+y^2=2ax \Rightarrow 2x^2+y^2-2ax=0$

19. **(a)** Box ⟨ 6R, 5B, 4W

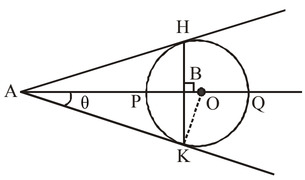
$P(E)=P(RRBW \text{ or } BBRW \text{ or } WWRB)$
$n(E)={}^6C_2\cdot{}^5C_1\cdot{}^4C_1+{}^5C_2\cdot{}^6C_1\cdot{}^4C_1+{}^4C_2\cdot{}^6C_1\cdot{}^5C_1$
$n(S)={}^{15}C_4$

$$\therefore\ P(E)=\frac{720\cdot 4!}{15\cdot 14\cdot 13\cdot 12}=\frac{48}{91}$$

20. **(a)** $ix^2+(1+i)x+i=0$
$\Rightarrow \alpha\beta=1$
$\Rightarrow \text{Im}(\alpha\beta)=0$

21. **(c)** Line touches the curve at (0, b) and $\left.\frac{dy}{dx}\right]_{x=0}$ also exists but even if $\frac{dy}{dx}$ fails to exist tangents line can be drawn.

22. **(b)**



$$\frac{(AK)}{(OA)}=\cos\theta=\frac{AB}{AK}$$

$\Rightarrow (AK)^2=(AB)(OA)=(AP)(AQ)$ ...(1)
$[AK^2=AP\,.\,AQ$ using power of point A]

Also, $OA=\frac{AP+AQ}{2}$

$[AQ-AO=r=AO-AP \Rightarrow 2AO=AQ+AP]$

$$\Rightarrow (AP)(AQ)=AB\left(\frac{AP+AQ}{2}\right)\ \text{(from (1))}$$

$$\Rightarrow\ AB=\frac{2(AP)(AQ)}{(AP+AQ)}$$

23. **(b)** Truth table has been given below :

p	q	~ p	p ∨ q	(p ∨ q) ∧ ~ p	~ p ∧ q
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	F

24. **(b)** Clearly, $\tan\pi[x]=0$ for all $x\in R$ and period of $\sin 4\pi\{x\}=1$.

25. **(b)** $\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)$

$$+\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)+....+\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$$

Consider $\left(r+\frac{1}{\omega}\right)\left(r+\frac{1}{\omega^2}\right)$

$=r^2+(\omega+\omega^2)r+1=(r^2-r+1)$

$$=\sum_{r=1}^{n}(r^2-r+1)=\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}+n$$

$$=\frac{n}{6}[2n^2+3n+1-3n-3+6]=\frac{n}{6}(2n^2+4)=\frac{n(n^2+2)}{3}$$

26. **(d)** Let the equation of tangent is $y=mx+\sqrt{a^2m^2+b^2}$
Foci $\equiv(\pm ae, 0)$, vertices $\equiv(\pm a, 0)$, $C\equiv(0,0)$

$$\therefore\ s=\left|\frac{mae+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|,\ s'=\left|\frac{-mae+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|$$

$$a=\left|\frac{ma+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|,\ a'=\left|\frac{-ma+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|,$$

$$c=\left|\frac{\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|$$

$$\therefore\ \frac{ss'-c^2}{aa'-c^2}=\frac{-\frac{m^2a^2e^2}{1+m^2}}{-\frac{m^2a^2}{1+m^2}}=e^2$$

27. (c) A: blood result says positive about the disease

B_1: Person suffers from the disease $\therefore\ P(B_1)=\dfrac{1}{100}$

B_2 : person does not suffer $\therefore\ P(B_2)=\dfrac{99}{100}$

$P(A/B_1)=\dfrac{99}{100}$, $P(A/B_2)=\dfrac{1}{100}$

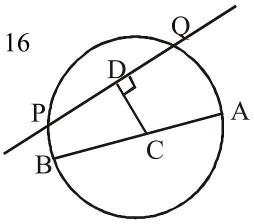
$$P(B_1/A)=\frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1)+P(B_2).P(A/B_2)}$$

$$=\frac{\frac{1}{100}\cdot\frac{99}{100}}{\frac{1}{100}\cdot\frac{99}{100}+\frac{99}{100}\cdot\frac{1}{100}}=\frac{99}{2(99)}=\frac{1}{2}=50\%$$

28. (a) Since, $\angle APB=\angle AQB=\dfrac{\pi}{2}$ so $y=mx+8$ intersect the circle whose diameter is AB.

Equation of circle is $x^2+y^2=16$

$CD<4$



$$\Rightarrow \frac{8}{\sqrt{1+m^2}}<4\Rightarrow 1+m^2>4$$

$\Rightarrow m\in(-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)$

If the line passing throw the point A (–4, 0), B (4, 0) then $\angle APB=\angle AQB=\dfrac{\pi}{2}$ does not formed.

$\therefore\ m\neq\pm 2$

29. (d)

(a) $T_r(kA)=k(a_{11}+a_{22}+a_{33})=kT_r(A)$

(b) $T_r(A+B)=a_{11}+b_{11}+a_{22}+b_{22}+a_{33}+b_{33}$
$=T_r(A)+T_r(B)$

(c) $T_r(I_3)=1+1+1=3$

(d) $T_r(A^2)=\Sigma a_{11}^2+\Sigma a_{12}^2\neq(a_{11}+a_{22}+a_{33})^2$

30. (a) $a_n=\displaystyle\int_0^{\pi/2}(1-\sin t)^n\sin 2t\,dt$

Let $1-\sin t=u\Rightarrow -\cos t\,dt=du$

$$=2\int_0^1 u^n(1-u)\,du=2\left(\int_0^1 u^n du-\int_0^1 u^{n+1}du\right)$$

$$=2\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

Hence, $\dfrac{a_n}{n}=2\left(\dfrac{1}{n(n+1)}-\dfrac{1}{n(n+2)}\right)$

$$\lim_{n\to\infty}\sum_1^n\frac{a_n}{n}=2\left(\sum\left(\frac{1}{n}-\frac{1}{n+1}\right)\right)-\frac{1}{2}\sum\left(\frac{1}{n}-\frac{1}{n+2}\right)$$

$$=2\left(\sum_1^n\left(\frac{1}{n}-\frac{1}{n+1}\right)\right)-\sum_1^n\left(\frac{1}{n}-\frac{1}{n+2}\right)$$

$$=2(1)-\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+.....\right]$$

$$=2-\frac{3}{2}=\frac{1}{2}$$