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CBSE MCQs CHAPTERWISE FOR TERM I & II CLASS 12 MATHEMATICS

Strictly as per the Latest Term-wise Syllabus released on 22 July 2021 (CBSE Cir. No. Acad-53/2021)

ACADEMICS' CHOICE SMART BOOK AWARD WINNER

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Stand Alone MCQs

Case Based MCQs

Assertion & Reason

Strictly updated as per the CBSE Special Scheme of Assessment (SAS) released on 5th July 2021

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SYLLABUS COVERED

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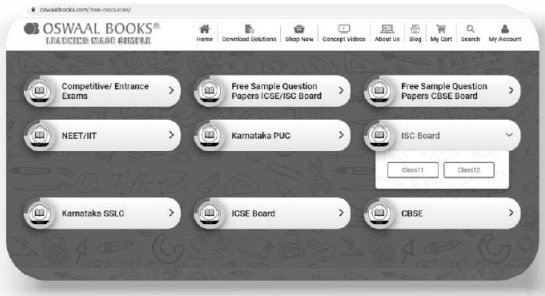

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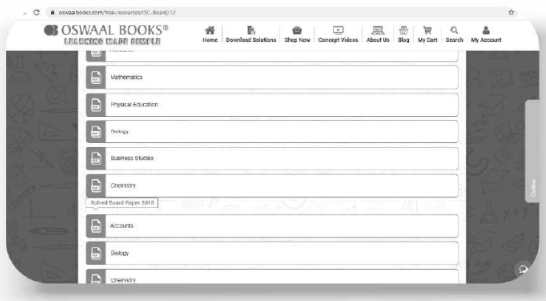

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PREPARING FOR THE CHANGE

"Progress is impossible without change, and those who cannot change their minds cannot change anything." – **George Bernard Shaw.**

A lot has changed in the last couple of years. We have had to accept new ways of not just learning but also of living. To ensure that progress doesn't stop, online classes and remote learning gained prominence. These were unprecedented changes for most schools, yet we adapted to them pretty quickly. The main reason for it was that we want to move ahead; we don't want to be tied down by limitations and problems. Finding newer and better ways is vital for human progress.

The Special Scheme of Assessment recently introduced by the Central Board of Secondary Education (CBSE) is a fresh approach to ensure that the growth of students doesn't stop, even in the testing circumstances. It has ample flexibility and various assessment criteria to make up for a just evaluation system. Unlike previous years, the academic year will be divided into two terms. The first term will have only Multiple-Choice Questions of three types; Case-based, Assertion-Reason Types & Stand-alone MCQs. Unlike the traditional 3-hour paper, this will be a 90-minute test based on the 50% of the rationalized syllabus. The Second Term will be of 2-hour duration & have questions of different formats like Case-based/ Situation based, Open-ended, Short answer/Long answer type.

This is an excellent example of positivity, ensuring that students don't get stressed and focus on learning specific areas instead of cramming loads of information. This scheme will also boost the attempts of our education system to move from 'quantity' to 'quality.'

Also, with this scheme, students will be well prepared for Multiple Choice Questions, which are the most widely applicable & helpful type of test items like in NEET, JEE, CAT, CLAT, etc. These MCQ-type questions in the test will help the educators, parents & the students measure the most critical learning outcomes like knowledge, judgment, understanding of concepts & problem-solving.

How to prepare for the MCQs introduced in the Special Assessment Scheme?

Oswaal CBSE MCQs Chapter-wise Question Banks (For Term I & II) are based on the "Special Assessment Scheme" introduced by the board on 5th July 2021 and also on the rationalised Term-wise syllabus released on 22nd July 2021.

These Question Banks include:

- Multiple Choice Questions based on the latest Typologies introduced by the board like:
  1. Stand-Alone MCQs
  2. MCQs based on Assertion-Reason
  3. Case-based MCQs
- Questions from CBSE official Question Bank released in April 2021
- Competency based Questions-Understanding, Knowledge, Application, Evaluation and Creation.
- Answers with Explanations

Oswaal CBSE MCQs Chapter-wise Question Banks (For Term I & II) are specially curated by the Oswaal Editorial Board to help the student master the Multiple-Choice Questions.

Wish you all Happy Learning and a Successful 2021-22!!

केन्द्रीय माध्यमिक शिक्षा बोर्ड
CENTRAL BOARD OF SECONDARY EDUCATION

NO.: F.1001/CBSE-Acad/Curriculum/2021 **Date: July 22, 2021**

Circular No: Acad- 53/2021

All the Heads of Schools affiliated to CBSE

Subject: Term wise syllabus for Board Examinations to be held in the academic session 2021-22 for Secondary and Senior Secondary classes and guidelines for the conduct of the Internal Assessment/Practicum/Project.

This is in continuation to Board's circular number Acad 51/2021 dated July 05, 2021 regarding Special Scheme of Assessment for Board Examination for Classes X and XII for the Session 2021- 22. The syllabus for the two terms mentioned in the scheme in all subjects for classes IX to XII are hereby notified vides this circular. In addition to syllabus for term end board examinations, guidelines for the conduct of Internal Assessment/Practicum/Project are also enclosed.

Schools are requested to share the term wise syllabus and guidelines for the conduct of board examinations and Internal Assessment / Practicum / Project available on CBSE Academic Website **http://www.cbseacademic.nic.in** at the link **http://cbseacademic.nic.in/Term-wise-curriculum_2022.html** with all their teachers and students.

(Dr. Joseph Emmanuel)
Director (Academics)

CBSE CIRCULAR 2021-22

केन्द्रीय माध्यमिक शिक्षा बोर्ड
CENTRAL BOARD OF SECONDARY EDUCATION

CBSE/DIR (ACAD)/2021 **Date: July 05, 2021**

Circular No: Acad-51/2021

All the Heads of Schools affiliated to CBSE

Subject: Special Scheme of Assessment for Board Examination Classes X and XII for the Session 2021-22

COVID 19 pandemic caused almost all CBSE schools to function in a virtual mode for most part of the academic session of 2020-21. Due to the extreme risk associated with the conduct of Board examinations during the second wave in April 2021, CBSE had to cancel both its class X and XII Board examinations of the year 2021 and results are to be declared on the basis of a credible, reliable, flexible and valid alternative assessment policy. This, in turn, also necessitated deliberations over alternative ways to look at the learning objectives as well as the conduct of the Board Examinations for the academic session 2021-22 in case the situation remains unfeasible.

CBSE has also held stakeholder consultations with Government schools as well as private independent schools from across the country, especially schools from the remote rural areas and a majority of them have requested for the rationalization of the syllabus, similar to last year in view of reduced time permitted for organizing online classes. The Board has also considered the concerns regarding differential availability of electronic gadgets, connectivity and effectiveness of online teaching and other socio-economic issues, specially with respect to students from economically weaker sections and those residing in far flung areas of the country. In a survey conducted by CBSE, it was revealed that the rationalized syllabus notified for the session 2020-21, was effective for schools in covering the syllabus and helped learners in achieving learning objectives in a less stressful manner.

In the above backdrop and in line with the Board's continued focus on assessing stipulated learning outcomes by making the examinations competencies and core concepts based, student-centric, transparent, technology-driven and having advance provision of alternatives for different future scenarios, the following schemes are introduced for the Academic Session for Class X and Class XII, 2021-22.

1. **Special Scheme for 2021-22**

   A. **Academic Session to be divided into 2 Terms with approximately 50% syllabus in each term:**

   The syllabus for the Academic Session 2021-22 will be divided into 2 terms by following a systematic approach by looking into the interconnectivity of concepts and topics by the Subject Experts and the Board will conduct examinations at the end of each term on the basis of the bifurcated syllabus. This is done to increase the probability of having the Board conducting classes X and XII examinations at the end of the academic session.

   B. **The syllabus for the Board examination 2021-22 will be rationalized,** similar to that of the last academic session to be notified in July 2021. For academic transactions, however, schools will

follow the curriculum and syllabus released by the Board vide Circular no. F. 1001/CBSE-Acad/ Curriculum/2021 dated 31 March, 2021. Schools will also use alternative academic calendar and inputs from the NCERT on transacting the curriculum.

C. **Efforts will be made to make Internal Assessment/ Practical/ Project work more credible and valid** as per the guidelines and Moderation Policy to be announced by the Board to ensure fair distribution of marks.

2. Details of Curriculum Transaction

- Schools will continue teaching in distance mode till the authorities permit in- person mode of teaching in schools.
- **Classes IX-X: Internal Assessment** (throughout the year-irrespective of Term I and II) would include the *3 periodic tests, student enrichment, portfolio and practical work/ speaking-listening activities/ project.*
- **Classes XI-XII:** Internal Assessment (throughout the year-irrespective of Term I and II) would include end of topic or unit tests/ exploratory activities/ practicals/ projects.
- Schools would create a student profile for all assessments undertaken over the year and retain the evidences in digital format.
- CBSE will facilitate schools to upload marks of Internal Assessment on the CBSE IT platform.
- Guidelines for Internal Assessment for all subjects will also be released along with the rationalized term wise divided syllabus for the session 2021-22. The Board would also provide additional resources like sample assessments, question banks, teacher training, etc., for more reliable and valid internal assessments.

3. Term I Examinations:

- At the end of the first term, the Board will organize **Term I Examination** in a flexible schedule to be conducted between November-December 2021 with a window period of 4-8 weeks for schools situated in different parts of country and abroad. Dates for conduct of examinations will be notified subsequently.
- The Question Paper will have Multiple Choice Questions (MCQs) including case-based MCQs and MCQs on assertion-reasoning type. Duration of test will be **90 minutes** and it will cover the rationalized syllabus of **Term I only** (i.e., approx. 50% of the entire syllabus).
- Question Papers will be sent by the CBSE to schools along with marking scheme.
- The exams will be conducted under the supervision of the External Center Superintendents and Observers appointed by CBSE.
- The responses of students will be captured on OMR sheets which, after scanning may be directly uploaded at CBSE portal or alternatively may be evaluated and marks obtained will be uploaded by the school on the very same day. The final direction in this regard will be conveyed to schools by the Examination Unit of the Board.
- Marks of the **Term I** Examination will contribute to the final overall score of students.

4. Term II Examination/ Year-end Examination:

- At the end of the second term, the Board would organize **Term II or Year-end Examination** based on the rationalized syllabus of Term II only (i.e., approximately 50% of the entire syllabus).
- This examination would be held around **March-April 2022** at the examination centres fixed by the Board.

- The paper will be of **2 hours duration** and have questions of different formats (case-based/ situation based, open ended- short answer/ long answer type).
- In case the situation is not conducive for normal descriptive examination, a **90 minute MCQs** based exam will be conducted at the end of the Term II also.
- Marks of the Term II Examination would contribute to the final overall score.

5. Assessment / Examination as per different situations

A. In case the situation of the pandemic improves and students are able to come to schools or centres for taking the exams.

Board would conduct Term I and Term II examinations at schools/centres and the theory marks will be distributed equally between the two exams.

B. In case the situation of the pandemic forces complete closure of schools during November-December 2021, but Term II exams are held at schools or centres.

Term I MCQ based examination would be done by students online/offline from home - in this case, the weightage of this exam for the final score would be reduced, and weightage of Term II exams will be increased for declaration of final result.

C. In case the situation of the pandemic forces complete closure of schools during March-April 2022, but Term I exams are held at schools or centres.

Results would be based on the performance of students in Term I MCQ based examination and internal assessments. The weightage of marks of Term I examination conducted by the Board will be increased to provide year end results of candidates.

D. In case the situation of the pandemic forces complete closure of schools and Board conducted Term I and II exams are taken by the candidates from home in the session 2021-22.

Results would be computed on the basis of the Internal Assessment/ Practical/ Project Work and Theory marks of Term-I and II exams taken by the candidate from home in Class X / XII, subject to the moderation or other measures to ensure validity and reliability of the assessment.

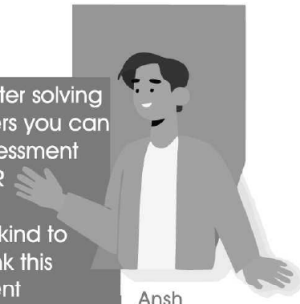
In all the above cases, data analysis of marks of students will be undertaken to ensure the integrity of internal assessments and home based exams.

(Dr. Joseph Emmanuel)
Director (Academics)

HEAR IT FROM OUR HAPPY READERS!

Good sample paper for preparation after revision. Once I finished my syllabus for boards, I used sample papers for practicing and found it really good and beneficial.

Daksh Gaba
Teacher



If you are confident after solving 5 solved sample papers you can practice the 5 self-assessment papers which have QR code so you can see answers and they are kind to provide hints and I think this worked for me. Excellent preparation tool for my Boards!!

Ansh
Teacher

Very good test papers as per new CBSE pattern. Very good mind maps and chapter wise notes. Also, toppers answer papers for reference are extremely helpful.

Rajni
Student

Rohan Mehra
Teacher

This is the best book of sample papers for 10th. Best material for the board preparation. Highly Recommended!!

Oswaal Sample Question Papers are great for practice because it contains a variety of questions synchronised with the latest syllabus. Kudos to the Oswaal Editorial Team!

Shalini Mehta
Student

This is the best till date, I completed all the sample papers and got a good practice of writing in examination, just go for it. Surely, it will help you a lot.

Prem
Student

Awesome book for preparations in board exam. 65-70% questions cracked in the board exam 2020 from this book.

Abhay Kumar
Student

I recommend Oswaal Sample Question Papers to all the students studying in the 10th grade. This book is awesome and very helpful.

Soham Roshan
Student

Awesome book! Class 10th students must buy it as soon as possible! Very helpful!

Neel
Student

SYLLABUS

Latest Syllabus for Academic Year (2021-22)

MATHEMATICS (Code No. 041)

CLASS–XII

TERM - I

One Paper

90 minutes **Max Marks : 40**

No.	Units	Marks
I.	Relations and Functions	08
II.	Algebra	10
III.	Calculus	17
V.	Linear Programming	05
	Total	40
	Internal Assessment	10
	Total	50

Unit I : Relations and Functions

1. **Relations and Functions**

   Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. **Inverse Trigonometric Functions**

   Definition, range, domain, principal value branch.

Unit II : Algebra

1. **Matrices**

   Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices, Invertible matrices; (Here all matrices will have real entries).

2. **Determinants**

   Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit III : Calculus

1. **Continuity and Differentiability**

   Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

   Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

SYLLABUS

2. Applications of Derivatives

Applications of derivatives: increasing/decreasing functions, tangents and normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

Unit-V: Linear Programming

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems. Graphical method of solution for problems in two variables, feasible and infeasible regions (bounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

INTERNAL ASSESSMENT	10 MARKS
Periodic Test	5 Marks
Mathematics Activities: Activity file record +Term end assessment of one activity & Viva	5 Marks

Note: For activities NCERT Lab Manual may be referred

TERM - II

One Paper

Max Marks : 40

No.	Units	Marks
III.	Calculus	18
IV.	Vectors and Three-Dimensional Geometry	14
VI.	Probability	8
	Total	40
	Internal Assessment	10
	Total	50

Unit-III: Calculus

1. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 \pm x^2}}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int \frac{px+q}{ax^2+bx+c}dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx, \int \sqrt{a^2 \pm x^2}\,dx, \int \sqrt{x^2-a^2}\,dx$$

Fundamental Theorem of Calculus (without proof).Basic properties of definite integrals and evaluation of definite integrals.

2. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, parabolas; area of circles /ellipses (in standard form only) (the region should be clearly identifiable).

3. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential

equations of first order and first degree of the type: $\frac{dy}{dx} = f(y/x)$. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = q$$, where p and q are functions of x or constant.

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Distance of a point from a plane.

Unit-VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.

INTERNAL ASSESSMENT	10 MARKS
Periodic Test	5 Marks
Mathematics Activities: Activity file record +Term end assessment of one activity & Viva	5 Marks

Note: For activities NCERT Lab Manual may be referred

Assessment of Activity Work:

In first term any 4 activities and in second term any 4 activities shall be performed by the student from the activities given in the NCERT Laboratory Manual for the respective class (XI or XII) which is available on the link : http://www.ncert.nic.in/exemplar/labmanuals.htmla record of the same may be kept by the student. A term end test on the activity is to be conducted.

The weightage are as under:

- The activities performed by the student in each term and record keeping: 3 marks
- Assessment of the activity performed during the term end test and Viva-voce: 2 marks

Prescribed Books:

1) Mathematics Part I - Textbook for Class XII, NCERT Publication
2) Mathematics Part II - Textbook for Class XII, NCERT Publication
3) Mathematics Exemplar Problem for Class XII, Published by NCERT
4) Mathematics Lab Manual class XII, published by NCERT

❑❑

30 DAYS OF ONLY GRATITUDE!

Take it as a challenge; practice gratitude every day.

When you'll look around yourself, you'll find umpteen number of things to be grateful for. Practicing gratitude everyday will only multiply those things in your life & will ignite positive emotions in you. Here are a few things you could be grateful for. So, get started today!

#1 About your body.	#2 What you find beautiful.	#3 A song you love.	#4 An accomplish-ment of yours.	#5 A friend.
#6 A Smell you love.	#7 Something that makes you smile.	#8 A happy memory.	#9 Something you like about where you live.	#10 A person in your family.
#11 A food you love.	#12 An ability of yours.	#13 A person.	#14 You're looking forward to.	#15 A life lesson.
#16 A person you look up to.	#17 A personality trait of yours.	#18 An item you use every day.	#19 A freedom you are grateful for.	#20 A holiday you love.
#21 A technology.	#22 Something made you laugh.	#23 Something nice.	#24 A book magazine or podcast.	#25 Another person.
#26 Something in nature.	#27 A gift you received.	#28 Something that brings hope.	#29 A compliment you have received.	#30 Something you are passionate about.

POSITIVE AFFIRMATIONS

"Affirmations are like a seed planted in soil. Poor soil, poor growth. Rich soil, abundant growth. The more you choose to think thoughts that make you feel good, the quicker the affirmations work."

- Louise Hay

- I am confident.
- I love who I am.
- I am compassionate.
- I am responsible.
- I am a leader.
- I believe in my dreams.
- I am brave.
- I choose a positive attitude.
- I am enough.
- I am great just the way I am.
- I work hard.
- I radiate joy and love.
- I am honest.
- I am patient.
- I help my family.
- I am strong.
- I can achieve my goals.
- I make good decisions.
- I am diligent.
- I am thoughtful.
- I am talented.
- I am loved.
- I am generous.
- I accept and love myself.
- I am unique.
- I am wonderfully made.
- I am creative.
- Good things happen to me.
- I am loving.
- I am kind.
- I am joyful.
- I care about others.
- I am important.
- I like myself.
- It's going to be a great day.
- I learn from my mistakes.
- I make friends easily.
- I am worthy.
- I am open to new experiences.
- I am beautiful.
- I am deserving of good things.
- I am grateful.
- I believe in me.
- I respect myself and I respect others.

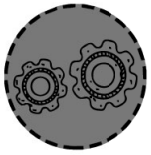
Our mind starts believing what we repeatedly think or say. We, at Oswaal Books, resonate with this belief. So, we want all our readers to create their own positive affirmations! A positive affirmation is something spoken aloud that you want to believe or want to be true. Repeating positive affirmations daily can help shift your internal dialogue from negative to positive.

So let's get started!

MIND MAPS
Learning Made Simple

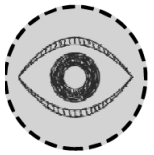
Presenting words and concepts as Pictures!!

What?



anytime, as frequency as you like till it becomes a habit!

When?



Why?

- To Unlock the imagination and come up with ideas
- To Remember facts and figures easily
- To Make clearer and better notes
- To Concentrate and save time
- To Plan with ease and ace exams

MIND MAP
AN INTERACTIVE MAGICAL TOOL

Result

Learning made simple
'a winning combination'

How?

With a blank sheet of paper coloured pens and your creative imagination!

What are Associations?

It's a technique connecting the core concept at the Centre to related concepts or ideas. Associations spreading out straight from the core concept are the First Level of Association. Then we have a Second Level of Association emitting from the first level and the chronology continues. The thickest line is the First Level of Association and the lines keep getting thinner as we move to the subsequent levels of association. This is exactly how the brain functions, therefore these Mind Maps. Associations are one powerful memory aid connecting seemingly unrelated concepts, hence strengthening memory.

CHAPTER 1

Term-I

RELATIONS AND FUNCTIONS

Syllabus

- ***Types of relations : Reflexive, Symmetric, Transitive and Equivalence relations. One-to-one and onto functions***

STAND ALONE MCQs (1 Mark each)

Q. 1. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to $b \forall a, b \in T$. Then R is

(A) reflexive but not transitive

(B) transitive but not symmetric

(C) equivalence relation

(D) None of these

Ans. Option (C) is correct.

Explanation: Consider that aRb, if a is congruent to b, $\forall\ a, b \in T$.
Then, $aRa \Rightarrow a \cong a$,
Which is true for all $a \in T$
So, R is reflexive, ...(i)
Let $aRb \Rightarrow a \cong b$
$\Rightarrow b \cong a$
$\Rightarrow bRa$
So, R is symmetric. ...(ii)
Let aRb and bRc
$\Rightarrow b \cong b$ and $b \cong a$
$\Rightarrow a \cong c \Rightarrow aRc$
So, R is transitive ...(iii)
Hence, R is equivalence relation.

Q. 2. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is

(A) symmetric but not transitive

(B) transitive but not symmetric

(C) neither symmetric nor transitive

(D) both symmetric and transitive

Ans. Option (B) is correct.

Explanation: $aRb \Rightarrow a$ is brother of b.
This does not mean b is also a brother of a as b can be a sister of a.
Hence, R is not symmetric.
$aRb \Rightarrow a$ is brother of b
and $bRc \Rightarrow b$ is a brother of c.
So, a is brother of c.
Hence, R is transitive.

Q. 3. The maximum number of equivalence relations on the set A = {1, 2, 3} are

(A) 1 **(B)** 2

(C) 3 **(D)** 5

Ans. Option (D) is correct.

Explanation: Given that, $A = \{1, 2, 3\}$
Now, number of equivalence relations are as follows:

$R_1 = \{(1, 1), (2, 2), (3, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$
$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$
$R_5 = \{(1, 2, 3) \Leftrightarrow A \times A = A^2\}$
$\therefore$ Maximum number of equivalence relations on the set $A = \{1, 2, 3\} = 5$

Q. 4. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is
(A) reflexive **(B)** transitive
(C) symmetric **(D)** None of these
Ans. Option (B) is correct.

Explanation: R on the set $\{1, 2, 3\}$ is defined by $R = \{(1, 2)\}$
It is clear that R is transitive.

Q. 5. Let us define a relation R in R as aRb if $a \geq b$. Then R is
(A) an equivalence relation
(B) reflexive, transitive but not symmetric
(C) symmetric, transitive but not reflexive
(D) neither transitive nor reflexive but symmetric.
Ans. Option (B) is correct.

Explanation: Given that, aRb if $a \geq b$
$\Rightarrow$ $aRa \Rightarrow a \geq a$ which is true
Let aRb, $a \geq b$, then $b \geq a$ which is not true as R is not symmetric.
But aRb and bRc
$\Rightarrow$ $a \geq b$ and $b \geq c$
$\Rightarrow$ $a \geq c$
Hence, R is transitive.

Q. 6. Let $A = \{1, 2, 3\}$ and consider the relation $R = (1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$.
Then R is
(A) reflexive but not symmetric
(B) reflexive but not transitive
(C) symmetric and transitive
(D) neither symmetric, nor transitive
Ans. Option (A) is correct.

Explanation: Given that $A = \{1, 2, 3\}$
and $R = (1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$.
$\because$ $(1, 1), (2, 2), (3, 3) \in R$
Hence, R is reflexive.
$(1, 2) \in R$ but $(2, 1) \notin R$
Hence, R is not symmetric.
$(1, 2) \in R$ and $(2, 3) \in R$
$\Rightarrow$ $(1, 3) \in R$
Hence, R is transitive.

Q. 7. Let R be the relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer:
(A) R is reflexive and symmetric but not transitive
(B) R is reflexive and transitive but not symmetric
(C) R is symmetric and transitive but not reflexive
(D) R is an equivalence relation
Ans. Option (B) is correct.

Explanation: Let R be the relation in the set $\{1, 2, 3, 4\}$ is given by:
$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$
(a) $(1, 1), (2, 2), (3, 3), (4, 4) \in R$
Therefore, R is reflexive.
(b) $(1, 2) \in R$ but $(2,1) \notin R$.
Therefore, R is not symmetric.
(c) If $(1, 3) \in R$ and $(3, 2) \notin R$ then $(1, 2) \in R$.
Therefore, R is transitive.

Q. 8. Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is
(A) 1 **(B)** 2
(C) 3 **(D)** 4
Ans. Option (A) is correct.

Explanation: The given set is $A = \{1, 2, 3\}$.
The smallest relation containing $(1, 2)$ and $(1, 3)$, which is reflexive and symmetric, but not transitive is given by:
$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)\}$
This is because relation R is reflexive as
$(1, 1), (2, 2), (3, 3) \in R$.
Relation R is symmetric since $(1, 2), (2, 1) \in R$ and $(1, 3), (3, 1) \in R$.
But relation R is not transitive as $(3, 1), (1, 2) \in R$, but $(3, 2) \notin R$.
Now, if we add any two pairs $(3, 2)$ and $(2, 3)$ (or both) to relation R, then relation R will become transitive.
Hence, the total number of desired relations is one.

Q. 9. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is
(A) 720 **(B)** 120
(C) 0 **(D)** None of these
Ans. Option (C) is correct.

Explanation: We know that, if A and B are two non-empty finite sets containing m and n elements, respectively, then the number of one-one and onto mapping from A to B is
$n!$ if $m = n$
0, if $m \neq n$
Given that, $m = 5$ and $n = 6$
$\therefore$ $m \neq n$
Number of one-one and onto mapping = 0

Q. 10. Let $A = \{1, 2, 3, ...n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is
(A) ${}^nP_2$ **(B)** $2^n - 2$
(C) $2^n - 1$ **(D)** None of these
Ans. Option (B) is correct.

Explanation: Total number of functions from A to $B = 2^n$
Number of into functions = 2
Number of surjections from A to $B = 2^n - 2$

Q. 11. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x}$, $\forall\, x \in R$. Then f is

(A) one-one **(B)** onto
(C) bijective **(D)** f is not defined

Ans. Option (D) is correct.

Explanation: We have, $f(x) = \frac{1}{x}$, $\forall\, x \in R$
For $x = 0$, $f(x)$ is not defined.
Hence, $f(x)$ is a not defined function.

Q. 12. Which of the following functions from Z into Z are bijections?

(A) $f(x) = x^3$ **(B)** $f(x) = x + 2$
(C) $f(x) = 2x + 1$ **(D)** $f(x) = x^2 + 1$

Ans. Option (B) is correct.

Explanation: For bijection on Z, $f(x)$ must be one-one and onto.
Function $f(x) = x^2 + 1$ is many-one as $f(1) = f(-1)$
Range of $f(x) = x^3$ is not Z for $x \in Z$.
Also $f(x) = 2x + 1$ takes only values of type
$= 2k + 1$ for $x \in k \in Z$
But $f(x) = x + 2$ takes all integral values for $x \in Z$
Hence $f(x) = x + 2$ is bijection of Z.

Q. 13. Let $f : R \to R$ be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto

Ans. Option (D) is correct.

Explanation: We know that $f : R \to R$ is defined as $f(x) = x^4$.

$$\begin{aligned} &\text{Let } x, y \in R \text{ such that } f(x) = f(y) \\ \Rightarrow\ & x^4 = y^4 \\ \Rightarrow\ & x = \pm y \\ \therefore\ & f(x) = f(y) \\ & \text{does not imply that } x = y. \\ & \text{For example, } f(1) = f(-1) = 1 \end{aligned}$$

$\therefore$ f is not one-one.
Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that $f(x) = 2$.
$\therefore$ f is not onto.
Hence, function f is neither one-one nor onto.

Q. 14. Let $f : R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto

Ans. Option (A) is correct.

Explanation: $f : R \to R$ is defined as $f(x) = 3x$.

$$\begin{aligned} &\text{Let } x, y \in R \text{ such that } f(x) = f(y) \\ \Rightarrow\ & 3x = 3y \\ \Rightarrow\ & x = y \\ \therefore\ & f \text{ is one-one.} \end{aligned}$$

Also, for any real number y in co-domain R, there exists $\frac{y}{3}$ in R such that $f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$.
$\therefore$ f is onto.
Hence, function f is one-one and onto.

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is True

Q. 1. Let W be the set of words in the English dictionary. A relation R is defined on W as
$R = \{(x, y) \in W \times W$ such that x and y have at least one letter in common$\}$.
Assertion (A): R is reflexive.
Reason (R): R is symmetric.

Ans. Option (B) is correct.

Explanation: For any word $x \in W$
x and x have atleast one (all) letter in common
$\therefore$ $(x, x) \in R, \forall\, x \in W \therefore R$ is reflexive
Symmetric: Let $(x, y) \in R, x, y \in W$
$\Rightarrow$ x and y have atleast one letter in common
$\Rightarrow$ y and x have atleast one letter in common
$\Rightarrow$ $(y, x) \in R \therefore R$ is symmetric
Hence A is true, R is true; R is not a correct explanation for A.

Q. 2. Let R be the relation in the set of integers Z given by $R = \{(a, b) : 2$ divides $a - b\}$.
Assertion (A): R is a reflexive relation.
Reason (R): A relation is said to be reflexive if xRx, $\forall x \in Z$.

Ans. Option (A) is correct.

Explanation: By definition, a relation in Z is said to be reflexive if xRx, $\forall x \in Z$. So R is true.
$a - a = 0 \Rightarrow 2$ divides $a - a \Rightarrow aRa$.
Hence R is reflexive and A is true.
R is the correct explanation for A.

Q. 3. Consider the set $A = \{1, 3, 5\}$.
Assertion (A): The number of reflexive relations on set A is 2^9.
Reason (R): A relation is said to be reflexive if xRx, $\forall x \in A$.

Ans. Option (D) is correct.

Explanation: By definition, a relation in A is said to be reflexive if xRx, $\forall x \in A$. So R is true.
The number of reflexive relations on a set containing n elements is 2^{n^2-n}.
Here $n = 3$.
The number of reflexive relations on a set $A = 2^{9-3} = 2^6$.
Hence A is false.

Q. 4. Consider the function $f : R \to R$ defined as $f(x) = x^3$
Assertion (A): $f(x)$ is a one-one function.
Reason (R): $f(x)$ is a one-one function if co-domain = range.

Ans. Option (C) is correct.

Explanation: $f(x)$ is a one-one function if
$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
Hence R is false.
Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$
$\Rightarrow (x_1)^3 = (x_2)^3$
$\Rightarrow x_1 = x_2$
Hence $f(x)$ is one-one.
Hence A is true.

Q. 5. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B.
Assertion (A): $f(x)$ is a one-one function.
Reason (R): $f(x)$ is an onto function.

Ans. Option (C) is correct.

Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f : A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$ *i.e.*, $f(1) = 4$, $f(2) = 5$ and $f(3) = 6$.
It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one.
So, A is true.
Range of $f = \{4, 5, 6\}$.
Co-domain $= \{4, 5, 6, 7\}$.
Since co-domain $\neq$ range, $f(x)$ is not an onto function. Hence R is false.

Q. 6. Consider the function $f : R \to R$ defined as
$f(x) = \dfrac{x}{x^2+1}$.
Assertion (A): $f(x)$ is not one-one.
Reason (R): $f(x)$ is not onto.

Ans. Option (B) is correct.

Explanation: Given, $f : R \to R$;
$$f(x) = \frac{x}{1+x^2}$$
Taking $x_1 = 4$, $x_2 = \dfrac{1}{4} \in R$
$$f(x_1) = f(4) = \frac{4}{17}$$
$$f(x_2) = f\left(\frac{1}{4}\right) = \frac{4}{17} \qquad (x_1 \neq x_2)$$
$\therefore$ f is not one-one.
A is true.
Let $y \in R$ (co-domain)
$f(x) = y$
$$\Rightarrow \frac{x}{1+x^2} = y$$
$$\Rightarrow y.(1+x^2) = x$$
$$\Rightarrow yx^2 + y - x = 0$$
$$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$
since, $x \in R$,
$$\therefore 1 - 4y^2 \geq 0$$
$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$
So Range $(f) \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$
Range $(f) \neq R$ (Co-domain)
$\therefore$ f is not onto.
R is true.
R is not the correct explanation for A.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:
A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever
Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

ONE – NATION
ONE – ELECTION
FESTIVAL OF DEMOCRACY
GENERAL ELECTION - 2019

$R = \{(V_1, V_2) : V_1, V_2 \in$ I and both use their voting right in general election – 2019$\}$

[CBSE QB 2021]

Q. 1. Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election - 2019. Which of the following is true?

(A) $(X, Y) \in R$ **(B)** $(Y, X) \in R$
(C) $(X, X) \notin R$ **(D)** $(X, Y) \notin R$

Ans. Option (D) is correct.

Explanation: $(X, Y) \notin$ R.
$\because$ X exercised his voting right while, Y did not cast her vote in general election-2019
And $R = \{(V_1, V_2) : V_1\ V_2 \in$ I and both use their voting right in general election-2019$\}$

Q. 2. Mr. 'X' and his wife 'W' both exercised their voting right in general election -2019, Which of the following is true?

(A) both (X, W) and $(W, X) \in R$
(B) $(X, W) \in R$ but $(W, X) \notin R$
(C) both (X, W) and $(W, X) \notin R$
(D) $(W, X) \in R$ but $(X, W) \notin R$

Ans. Option (A) is correct.

Q. 3. Three friends F_1, F_2 and F_3 exercised their voting right in general election-2019, then which of the following is true?

(A) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$
(B) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \notin R$
(C) $(F_1, F_2) \in R$, $(F_2, F_2) \in R$ but $(F_3, F_3) \notin R$
(D) $(F_1, F_2) \notin R$, $(F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

Ans. Option (A) is correct.

Q. 4. The above defined relation R is _____

(A) Symmetric and transitive but not reflexive
(B) Universal relation
(C) Equivalence relation
(D) Reflexive but not symmetric and transitive

Ans. Option (C) is correct.

Explanation: R is reflexive, since every person is friend or itself.
i.e., $(F_1, F_2) \in R$
Further, $(F_1, F_2) \in R$
$\Rightarrow F_1$ is friend of F_2
$\Rightarrow F_2$ is friend of F_1
$\Rightarrow (F_2, F_1) \in R$
$\Rightarrow R$ is symmetric
Moreover, $(F_1, F_2), (F_2, F_3) \in R$
$\Rightarrow F_1$ is friend of F_2 and F_2 is friend of F_3.
$\Rightarrow F_1$ is a friend of F_3.
$\Rightarrow (F_1, F_3) \in R$
Therefore, R is an equivalence relation.

Q. 5. Mr. Shyam exercised his voting right in General Election - 2019, then Mr. Shyam is related to which of the following?

(A) All those eligible voters who cast their votes
(B) Family members of Mr. Shyam
(C) All citizens of India
(D) Eligible voters of India

Ans. Option (A) is correct.

II. Read the following text and answer the following questions on the basis of the same:

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}$, $B = \{1, 2, 3, 4, 5, 6\}$ **[CBSE QB 2021]**

Q. 1. Let R : B $\rightarrow$ B be defined by R $= \{(x, y) : y$ is divisible by $x\}$ is

(A) Reflexive and transitive but not symmetric
(B) Reflexive and symmetric but not transitive
(C) Not reflexive but symmetric and transitive
(D) Equivalence

Ans. Option (A) is correct.

Explanation: R is reflexive, since every element of B i.e.,
$B = \{1, 2, 3, 4, 5, 6\}$ is divisible by itself.
i.e., $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \in R$
further, $(1, 2) \in R$
but $(2, 1) \notin R$
Moreover,
$(1, 2), (2, 4) \in R$
$\Rightarrow$ $(1, 4) \in R$
$\Rightarrow$ R is transitive.
Therefore, R is reflexive and transitive but not symmetric.

Q. 2. Raji wants to know the number of functions from A to B. How many number of functions are possible?

(A) 6^2 **(B)** 2^6
(C) 6! **(D)** 2^{12}

Ans. Option (A) is correct.

Q. 3. Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then R is
(A) Symmetric
(B) Reflexive
(C) Transitive
(D) None of these

Ans. Option (D) is correct.

Explanation: $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$
R is not reflexive.
Since, $(1, 1), (3, 3), (4, 4), (6, 6) \in R$
R is not symmetric.
Because, for $(1, 2) \in R$ there does not exist $(2, 1) \in R$.
R is not transitive.
Because for all element of B there does not exist, $(a, b)\ (b, c) \in R$ and $(a, c) \in R$.

Q. 4. Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?
(A) 6^2 **(B)** 2^6
(C) 6! **(D)** 2^{12}

Ans. Option (D) is correct.

Q. 5. Let $R : B \to B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$, then R is
(A) Symmetric
(B) Reflexive and Transitive
(C) Transitive and symmetric
(D) Equivalence

Ans. Option (B) is correct.

III. Read the following text and answer the following questions on the basis of the same:

An organization conducted bike race under 2 different categories–boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ $G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. **[CBSE QB 2021]**

Ravi decides to explore these sets for various types of relations and functions

Q. 1. Ravi wishes to form all the relations possible from B to G. How many such relations are possible?
(A) 2^6 **(B)** 2^5
(C) 0 **(D)** 2^3

Ans. Option (A) is correct.

Q. 2. Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of same sex$\}$, Then this relation R is_____
(A) Equivalence
(B) Reflexive only
(C) Reflexive and symmetric but not transitive
(D) Reflexive and transitive but not symmetric

Ans. Option (A) is correct.

Explanation:
$R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of same sex$\}$
R is reflexive, since, $(x, x) \in R$
R is symmetric, since, $(x, y) \in R$ and $(y, x) \in R$
R is transitive. For $a, b, c \in B$
$\exists\ (a, b)\ (b, c) \in R$
and $(a, c) \in R$.
Therefore R is equivalence relation.

Q. 3. Ravi wants to know among those relations, how many functions can be formed from B to G?
(A) 2^2 **(B)** 2^{12}
(C) 3^2 **(D)** 2^3

Ans. Option (D) is correct.

Q. 4. Let $R : B \to G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$, then R is_____
(A) Injective
(B) Surjective
(C) Neither Surjective nor Injective
(D) Surjective and Injective

Ans. Option (B) is correct.

Explanation:
$R : B \to G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$
R is surjective, since, every element of G is the image of some element of B under R, i.e., For g_1, $g_2 \in G$,
there exists an elements $b_1, b_2, b_3 \in B$,
$(b_1\ g_1)\ (b_2, g_2), (b_3, g_1) \in R$.

Q. 5. Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?
(A) 0 **(B)** 2!
(C) 3! **(D)** 0!

Ans. Option (A) is correct.

IV. Read the following text and answer the following questions on the basis of the same:

Students of Grade 9, planned to plant saplings

along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

[CBSE QB 2021]

Q. 1. Let relation R be defined by $R = \{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$ then R is_____ relation

(A) Equivalence
(B) Only reflexive
(C) Not reflexive
(D) Symmetric but not transitive

Ans. Option (A) is correct.

Explanation: Let relation R be defined by
$R = \{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$.
R is reflexive, since every line is parallel to itself.
Further, $(L_1, L_2) \in R$
$\Rightarrow L_1$ is parallel to L_2
$\Rightarrow L_2$ is parallel to L_1
$\Rightarrow (L_2, L_1) \in R$
Hence, R is symmetric.
Moreover, $(L_1, L_2), (L_2, L_3) \in R$
$\Rightarrow L_1$ is parallel to L_2 and L_2 is parallel to L_3
$\Rightarrow L_1$ is parallel to L_3
$\Rightarrow (L_1, L_3) \in R$
Therefore, R is an equivalence relation

Q. 2. Let $R = \{(L_1, L_2) : L_1 \perp L_2$ where $L_1, L_2 \in L\}$ which of the following is true?

(A) R is Symmetric but neither reflexive nor transitive
(B) R is Reflexive and transitive but not symmetric
(C) R is Reflexive but neither symmetric nor transitive
(D) R is an Equivalence relation

Ans. Option (A) is correct.

Explanation: R is not reflexive, as a line L_1 can not be perpendicular to itself, i.e., $(L_1, L_1) \notin R$.
R is symmetric as $(L_1, L_2) \in R$
As, L_1 is perpendicular to L_2
and L_2 is perpendicular to L_1
$(L_2, L_1) \in R$
R is not transitive. Indeed, it L_1 is perpendicular to L_2 and L_2 is perpendicular to L_3, then L_1 can never be perpendicular to L_3.
In fact L_1 is parallel to L_3,
i.e., $(L_1, L_2) \in R, (L_2, L_3) \in R$ but $(L_1, L_3) \notin R$
i.e., symmetric but neither reflexive nor transitive.

Q. 3. The function $f : R \to R$ defined by $f(x) = x - 4$ is_____

(A) Bijective
(B) Surjective but not injective
(C) Injective but not Surjective
(D) Neither Surjective nor Injective

Ans. Option (A) is correct.

Explanation:
The function f is one-one,
for $f(x_1) = f(x_2)$
$\Rightarrow x_1 - 4 = x_2 - 4$
$\Rightarrow x_1 = x_2$
Also, given any real number y in R, there exists $y + 4$ in R
Such that $f(y + 4) = y + 4 - 4 = y$
Hence, f is onto
Hence, function is both one-one and onto, *i.e.*, bijective.

Q. 4. Let $f : R \to R$ be defined by $f(x) = x - 4$. Then the range of $f(x)$ is _____

(A) R **(B)** Z
(C) W **(D)** Q

Ans. Option (A) is correct.

Explanation: Range of $f(x)$ is R

Q. 5. Let $R = \{(L_1, L_2) : L_1 \parallel L_2$ and $L_1 : y = x - 4\}$ then which of the following can be taken as L_2?

(A) $2x - 2y + 5 = 0$ **(B)** $2x + y = 5$
(C) $2x + 2y + 7 = 0$ **(D)** $x + y = 7$

Ans. Option (A) is correct.

Explanation: Since, $L_1 \parallel L_2$
then slope of both the lines should be same.

$$\text{Slope of } L_1 = 1$$
$$\Rightarrow \text{Slope of } L_2 = 1$$
And $2x - 2y + 5 = 0$
$$-2y = -2x - 5$$
$$y = x + \frac{5}{2}$$
Slope of $2x - 2y + 5 = 0$ is 1
So, $2x - 2y + 5 = 0$ can be taken as L_2.

V. Read the following text and answer the following questions n the basis of the same:

Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted

many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$.

[CBSE QB-2021]

Q. 1. Let $f : R \to R$ be defined by $f(x) = x^2$ is_____

(A) Neither Surjective nor Injective
(B) Surjective
(C) Injective
(D) Bijective

Ans. Option (A) is correct.

Explanation:
$f : R \to R$ be defined by $f(x) = x^2$
$\because f(-1) = f(1) = 1$, but $-1 \neq 1$
$\therefore f$ is not injective
Now, $-2 \in R$. But, there does not exist any element $x \in R$ such that $f(x) = -2$ or $x^2 = -2$
$\therefore f$ is not surjective.
Hence, function f is neither injective nor surjective.

Q. 2. Let $f : N \to N$ be defined by $f(x) = x^2$ is _____

(A) Surjective but not Injective
(B) Surjective
(C) Injective
(D) Bijective

Ans. Option (C) is correct.

Explanation: $f : N \to N$ be defined by $f(x) = x^2$
for $x, y \in N$, $f(x) = f(y)$
$\Rightarrow \quad x^2 = y^2$
$\Rightarrow \quad x = y$
$\therefore$ f is injective
Now; $2 \in N$, But, there does not exist any x in n such that $f(x) = x^2 = 2$
$\therefore$ f is not surjective
Hence, function is injective but not surjective.

Q. 3. Let $f : \{1, 2, 3, ...\} \to \{1, 4, 9, ...\}$ be defined by $f(x) = x^2$ is _____

(A) Bijective
(B) Surjective but not Injective
(C) Injective but Surjective
(D) Neither Surjective nor Injective

Ans. Option (A) is correct.

Explanation:
$f : \{1, 2, 3, ...\} \to \{1, 4, 9, ...\}$ be defined by $f(x) = x^2$
$x_1 \in \{1, 2, 3, ...\}$ and $x_2 \in \{1, 2, 9, ...\}$
$f(x_1) = f(x_2)$
$\Rightarrow \quad x_1^2 = x_2^2$
$\Rightarrow \quad x_1 = x_2$
$\therefore$ f is injective
Now, $4 \in \{1, 4, 9...\}$, there exist 2 in $\{1, 2, 3 ...\}$ such that $f(x) = 2^2 = 4$, Hence, f is surjective
Therefore f is bijective.

Q. 4. Let : $N \to R$ be defined by $f(x) = x^2$. Range of the function among the following is _____

(A) $\{1, 4, 9, 16,...\}$
(B) $\{1, 4, 8, 9, 10,...\}$
(C) $\{1, 4, 9, 15, 16,...\}$
(D) $\{1, 4, 8, 16,...\}$

Ans. Option (A) is correct.

Explanation:
Range of $f = \{1, 4, 9, 16, ...\}$
$\because \quad N = \{1, 2, 3, ...\}$

Q. 5. The function $f : Z \to Z$ defined by $f(x) = x^2$ is _____

(A) Neither Injective nor Surjective
(B) Injective
(C) Surjective
(D) Bijective

Ans. Option (A) is correct.

Explanation: $f : z \to z$ defined by $f(x) = x^2$
So, $f(-1) = f(1)$, but $1 \neq -1$
$\therefore$ f is not injective
Now, $-2 \in Z$, but, there does not exist any element $x \in z$ such that
$f(x) = -2$
or $x^2 = -2$
$\therefore$ f is not surjective
Hence, f is neither injective nor surjective.

❐❐

Term-I

CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

Syllabus

- *Definition, range, domain, principal value branch.*

STAND ALONE MCQs (1 Mark each)

Q. 1. The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is

(A) $\frac{\pi}{10}$ **(B)** $\frac{3\pi}{5}$

(C) $-\frac{\pi}{10}$ **(D)** $\frac{-3\pi}{5}$

[CBSE OD Set-I 2020]

Ans. Option (C) is correct.

Explanation:

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) \qquad \left[\because \cos\left(\frac{\pi}{2}+x\right) = -\sin x\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right) \qquad [\because \sin^{-1}(-x) = -\sin^{-1} x]$$

$$= -\frac{\pi}{10} \qquad \left[\because \sin^{-1}(\sin x) = x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$

Q. 2. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is

(A) $\frac{3+\sqrt{5}}{2}$ **(B)** $\frac{3-\sqrt{5}}{2}$

(C) $\frac{-3+\sqrt{5}}{2}$ **(D)** $\frac{-3-\sqrt{5}}{2}$

Ans. Option (B) is correct.

Explanation:

$$x = \tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$$

Let $\cos^{-1}\frac{\sqrt{5}}{3} = \theta$

$$\cos\theta = \frac{\sqrt{5}}{3}$$

$$\Rightarrow \quad x = \tan\frac{1}{2}\theta$$

$$\Rightarrow \quad x = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\because \quad \sin\frac{\theta}{2} = \frac{\sqrt{1-\frac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = \frac{\sqrt{1+\frac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$x = \frac{\sqrt{1-\frac{\sqrt{5}}{3}}}{\sqrt{1+\frac{\sqrt{5}}{3}}}$$

$$= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}}$$

$$= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}} \times \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3-\sqrt{5}}}$$

$$= \frac{3-\sqrt{5}}{\sqrt{(3)^2-(\sqrt{5})^2}}$$

$$= \frac{3-\sqrt{5}}{\sqrt{9-5}}$$

$$= \frac{3-\sqrt{5}}{2}$$

Q. 3. Which of the following is the principal value branch of $\cos^{-1}x$?

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **(B)** $\left[0, \frac{\pi}{2}\right]$

(C) $[0, \pi]$ **(D)** $(0,\pi) - \left\{\frac{\pi}{2}\right\}$

Ans. Option (C) is correct.

Explanation: As we know that the principal value of $\cos^{-1}x$ is $[0, \pi]$.

$y = \cos^{-1}x$

Q. 4. Which of the following is the principal value branch of $\text{cosec}^{-1}x$?

(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **(B)** $[0,\pi] - \left\{\frac{\pi}{2}\right\}$

(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **(D)** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Ans. Option (D) is correct.

Explanation : As we know that the principal value of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

$y = \text{cosec}^{-1}x$

Q. 5. The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is

(A) $\frac{3\pi}{5}$ **(B)** $\frac{-7\pi}{5}$

(C) $\frac{\pi}{10}$ **(D)** $\frac{-\pi}{10}$

Ans. Option (D) is correct.

Explanation: Let,

$$\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$[\because \cos(2n\pi + \theta) = \cos\theta]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right)$$

$$\left[\because \cos\left(\frac{\pi}{2} + x\right) = -\sin x\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right)$$

$$[\because \sin^{-1}(-x) = -\sin^{-1}x]$$

$$= -\frac{\pi}{10}$$

$$\left[\because \sin^{-1}(\sin x) = x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$

Q. 6. The domain of function $\cos^{-1}(2x-1)$ is

(A) $[0, 1]$ **(B)** $[-1, 1]$

(C) $(-1, 1)$ **(D)** $[0, \pi]$

Ans. Option (A) is correct.

Explanation:

We have $\cos^{-1}(2x-1)$

$\Rightarrow -1 \le 2x-1 \le 1$ $[\because x \in [-1,1]]$

$\Rightarrow 0 \le 2x \le 2$

$\Rightarrow 0 \le x \le 1$

$\Rightarrow \quad \in [0,1]$

Q. 7. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

(A) $\frac{\pi}{2}$ **(B)** $\frac{3\pi}{2}$

(C) $\frac{5\pi}{2}$ **(D)** $\frac{7\pi}{2}$

Ans. Option (A) is correct.

Explanation: We have,

$$\cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{\pi}{2}\right)\right]$$

$$\left[\because \cos\left(2\pi - \frac{\pi}{2}\right) = \cos\frac{\pi}{2}\right]$$

$$= \cos^{-1}\cos\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\left[\therefore \cos^{-1}(\cos x) = x, x \in [0,\pi]\right]$$

Q. 8. The value of expression $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{6}$ **(B)** $\frac{5\pi}{6}$

(C) $\frac{7\pi}{6}$ **(D)** 1

Ans. Option (B) is correct.

Explanation: We have,

$$2\sec^{-1}2+\sin^{-1}\left(\frac{1}{2}\right)$$
$$=2\sec^{-1}\sec\frac{\pi}{3}+\sin^{-1}\sin\frac{\pi}{6}$$
$$=2\times\frac{\pi}{3}+\frac{\pi}{6}$$
$$\left[\because \sec^{-1}(\sec x)=x \text{ and } \sin^{-1}(\sin x)=x\right]$$
$$=\frac{4\pi+\pi}{6}$$
$$=\frac{5\pi}{6}$$

Q. 9. What is the value of $\sec^2(\tan^{-1}2)$

(A) 1 **(B)** 4
(C) 5 **(D)** 3

Ans. Option (C) is correct.

Explanation :

$$\sec^2(\tan^{-1}2)=\sec^2(\sec^{-1}\sqrt{1+2^2})$$
$$=\sec^2(\sec^{-1}\sqrt{5})$$
$$=(\sqrt{5})^2$$
$$=5$$

Q. 10. The principal value of

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)+4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{ is}$$

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{6}$

(C) $\frac{4\pi}{3}$ **(D)** $\frac{3\pi}{4}$

Ans. Option (C) is correct.

Explanation :

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)+4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$=\cos^{-1}\left(\cos\frac{\pi}{3}\right)+2\sin^{-1}\left(\sin\frac{\pi}{6}\right)+4\tan^{-1}\left(\tan\frac{\pi}{6}\right)$$
$$=\frac{\pi}{3}+2\times\frac{\pi}{6}+4\times\frac{\pi}{6}$$
$$=\frac{2\pi+2\pi+4\pi}{6}$$
$$=\frac{8\pi}{6}$$
$$=\frac{4\pi}{3}$$

Q. 11. The principal value of $\cot^{-1}(-\sqrt{3})$ is

(A) $\frac{5\pi}{6}$ **(B)** $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ **(D)** $\frac{\pi}{3}$

Ans. Option (A) is correct.

Explanation :

Let $\cot^{-1}(-\sqrt{3})=\theta$

$\Rightarrow \cot\theta=-\sqrt{3}$

$\Rightarrow \cot\theta=-\cot\frac{\pi}{6}$

$=\cot\left(\pi-\frac{\pi}{6}\right)$

$\Rightarrow \cot\theta=\cot\frac{5\pi}{6}$

$\Rightarrow \theta=\frac{5\pi}{6}\in(0,\pi)$

$\therefore$ Principal value of $\cot^{-1}(-\sqrt{3})$ is $\frac{5\pi}{6}$

Q. 12. Domain of $\sin^{-1}x$ is:

(A) $[-1,\infty)$ **(B)** $[-1,1]$
(C) $(-1,1)$ **(D)** None of these.

Ans. Option (B) is correct.

Explanation : Domain of $\sin^{-1}x$ is $[-1,1]$

Q. 13. Range of $\cos^{-1}x$ is:

(A) $\left[0,\frac{\pi}{2}\right]$ **(B)** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(C) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ **(D)** $[0,\pi]$

Ans. Option (D) is correct.

Explanation : The branch with range $[0,\pi)$ is called the principal value branch of the function $\cos^{-1}x$.

Q. 14. Domain of $\sec^{-1}x$ is:

(A) $R-(-1,1)$ **(B)** R
(C) $[-1,1]$ **(D)** $R-(0,1)$

Ans. Option (A) is correct.

Q. 15. The value of $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$ is:

(A) π **(B)** $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{2\pi}{3}$

Ans. Option (B) is correct.

Explanation : $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$

$=\frac{\pi}{3}-\frac{2\pi}{3}$

$=-\frac{\pi}{3}$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin\theta)=\theta$, if $\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Ans. Option (D) is correct.

Explanation:

The principal value branch of $\sin^{-1}x$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Let $x=\sin\theta\Rightarrow\theta=\sin^{-1}x$

$\sin^{-1}(\sin\theta)=\sin^{-1}x=\theta$

$\sin^{-1}(\sin\theta)=\theta$, if $\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$.

Hence R is true.

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)\neq\frac{2\pi}{3}$, since $\frac{2\pi}{3}\notin\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Hence A is false.

Q. 2. Assertion (A): Range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

Explanation: Domain of tan x is the set $\{x : x\in R$ and $x\neq(2n+1)\frac{\pi}{2}, n\in Z\}$ and Range is R.

$\Rightarrow$ tan x is not defined for odd multiples of $\frac{\pi}{2}$.

If we restrict the domain of tangent function to $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$, then it is one-one and onto with its range as R. Actually tan x restricted to any of the intervals $\left(\frac{-3\pi}{2},\frac{-\pi}{2}\right),\left(\frac{-\pi}{2},\frac{\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ etc., is bijective and its range is R.

Thus $\tan^{-1}x$ can be defined as a function whose domain is R and range could be any of the intervals $\left(\frac{-3\pi}{2},\frac{-\pi}{2}\right),\left(\frac{-\pi}{2},\frac{\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ and soon.

$\therefore$ Both A and R are true but R is not correct explanation of A.

Q. 3. Assertion (A): Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$

Reason (R): Principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\sin^{-1}\left(\sin\frac{\pi}{4}\right)$$
$$=\frac{\pi}{4}$$
$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=y$$
$$\cot y=\frac{-1}{\sqrt{3}}$$
$$=-\cot\left(\frac{\pi}{3}\right)$$
$$=\cot\left(\pi-\frac{\pi}{3}\right)$$
$$=\cot\left(\frac{2\pi}{3}\right)$$
$$\Rightarrow\quad\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=\frac{2\pi}{3}$$

Hence Assertion is correct and Reason is incorrect.

Q. 4. Assertion (A): Range of $\cot^{-1}x$ is $(0,\pi)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

Q. 5. Assertion (A): Principal value of $\cos^{-1}(1)$ is π

Reason (R): Value of cos 0° is 1

Ans. Option (D) is correct.

Explanation: In case of Assertion:

$$\cos^{-1}(1)=y$$
$$\cos y=1$$
$$\cos y=\cos 0^\circ\qquad[\because\cos 0^\circ=1]$$
$$\therefore\quad y=0$$

$\Rightarrow$ Principal value of $\cos^{-1}(1)$ is 0

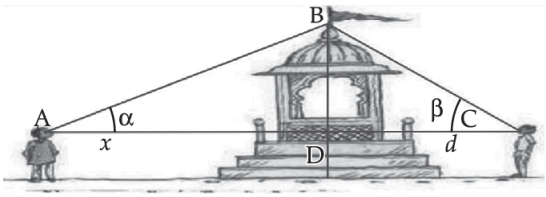
Hence Assertion is in correct.

Reason is correct.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:



Two men on either side of a temple of 30 metres high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ meters. **[CBSE QB-2021]**

Q. 1. $\angle CAB = \alpha =$

(A) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ **(B)** $\sin^{-1}\left(\frac{1}{2}\right)$

(C) $\sin^{-1}(2)$ **(D)** $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. Option (B) is correct.

Explanation: In Δ BDA

$$\sin\alpha = \frac{BD}{AB}$$

$$AB^2 = AD^2 + BD^2$$
$$= (30\sqrt{3})^2 + (30)^2$$
$$= (60)^2$$
$$AB = 60\,\text{m}$$

Now, $\sin\alpha = \frac{30}{60}$

$$\sin\alpha = \frac{1}{2}$$

i.e. $\angle \text{CAB} = \alpha = \sin^{-1}\left(\frac{1}{2}\right)$

Q. 2. $\angle CAB = \alpha =$

(A) $\cos^{-1}\left(\frac{1}{5}\right)$ **(B)** $\cos^{-1}\left(\frac{2}{5}\right)$

(C) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **(D)** $\cos^{-1}\left(\frac{4}{5}\right)$

Ans. Option (C) is correct.

Explanation: In Δ *BDA*

$$\cos\alpha = \frac{AD}{AB}$$

$$\cos\alpha = \frac{30\sqrt{3}}{60}$$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\therefore \quad \angle CAB = \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Q. 3. $\angle BCA = \beta =$

(A) $\tan^{-1}\left(\frac{1}{2}\right)$ **(B)** $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(D)** $\tan^{-1}\left(\sqrt{3}\right)$

Ans. Option (D) is correct.

Explanation:

$$DC = AC - AD$$
$$= 40\sqrt{3} - 30\sqrt{3}$$
$$= 10\sqrt{3}\,\text{m}$$

In Δ*BDC*

$$\tan\beta = \frac{BD}{DC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$$\angle BCA = \beta = \tan^{-1}(\sqrt{3})$$

Q. 4. $\angle ABC =$

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{6}$

(C) $\frac{\pi}{2}$ **(D)** $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation: Since,

$$\sin\alpha = \frac{1}{2}$$

i.e., $\sin\alpha = \sin 30°$

$$\left[\because \sin 30° = \frac{1}{2}\right]$$

$$\therefore \quad \alpha = 30°$$

we, have $\tan\beta = \sqrt{3}$

$$\tan\beta = \tan 60°$$

$$\therefore \quad \beta = 60°$$

Now, In Δ*ABC*

$$\angle ABC + \angle BCA + \angle CAB = 180°$$
$$\angle ABC + 60° + 30° = 180°$$
$$\angle ABC = 90°$$

$$\therefore \quad \angle ABC = \frac{\pi}{2}$$

Q. 5. Domain and Range of $\cos^{-1} x$ =

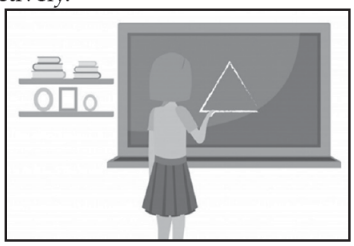
(A) $(-1, 1), (0, \pi)$ **(B)** $[-1, 1], (0, \pi)$

(C) $[-1, 1], [0, \pi]$ **(D)** $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ans. Option (C) is correct.

II. Read the following text and answer the following questions on the basis of the same:

In the school project Sheetal was asked to construct a triangle and name it as ABC. Two angles A and B were given to be equal to $\tan^{-1}\frac{1}{2}$ and $\tan^{-1}\frac{1}{3}$ respectively.



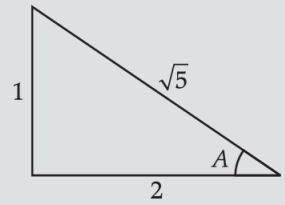
Q. 1. The value of sin A is _____.

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$

(C) $\frac{1}{\sqrt{5}}$ **(D)** $\frac{2}{\sqrt{5}}$

Ans. Option (C) is correct.

Explanation:



$$A = \tan^{-1}\frac{1}{2}$$

$$\Rightarrow \tan A = \frac{1}{2}$$

$$\therefore \sin A = \frac{1}{\sqrt{5}}$$

Q. 2. $\cos(A + B + C)$ = _____.

(A) 1 **(B)** 0

(C) –1 **(D)** $\frac{1}{2}$

Ans. Option (C) is correct.

Explanation: Since ABC is a triangle,

$$\therefore A + B + C = 180^\circ$$

$$\cos(A + B + C) = \cos 180^\circ$$

$$= -1$$

Q. 3. If $B = \cos^{-1} x$, then x = _____.

(A) $\frac{1}{\sqrt{5}}$ **(B)** $\frac{3}{\sqrt{10}}$

(C) $\frac{1}{\sqrt{10}}$ **(D)** $\frac{2}{\sqrt{5}}$

Ans. Option (B) is correct.

Explanation:

Given $B = \tan^{-1}\frac{1}{3}$

$$\Rightarrow \tan B = \frac{1}{3}$$

$$\therefore \cos B = \frac{3}{\sqrt{10}}$$

$$B = \cos^{-1}\frac{3}{\sqrt{10}}$$

$$\Rightarrow x = \frac{3}{\sqrt{10}}$$

Q. 4. If $A = \sin^{-1} x$; then the value of x is:

(A) $\frac{1}{\sqrt{5}}$ **(B)** $\frac{2}{\sqrt{5}}$

(C) $\frac{1}{\sqrt{10}}$ **(D)** $\frac{3}{\sqrt{10}}$

Ans. Option (A) is correct.

Explanation:

$$A = \tan^{-1}\frac{1}{2}$$

$$\Rightarrow \tan A = \frac{1}{2}$$

$$\therefore \sin A = \frac{1}{\sqrt{5}}$$

$$A = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$\Rightarrow x = \frac{1}{\sqrt{5}}$$

Q. 5. The third angle, $\angle C$ = _____.

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{3\pi}{4}$

Ans. Option (D) is correct.

Explanation:

$$\angle C = \pi - (A + B)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

III. Read the following text and answer the following questions on the basis of the same:

The value of an inverse trigonometric functions which lies in the range of Principal branch is called the principal value of that inverse trigonometric functions.

Q. 1. Principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ **(D)** $\frac{\pi}{2}$

Ans. Option (A) is correct.

Explanation:

$$\sin^{-1}\left(\frac{1}{2}\right) = y$$

$$\sin y = \frac{1}{2}$$

Principal value branch of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\Rightarrow$ Principal vlaue of $\sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{\pi}{6}$

Q. 2. Principal value of $\tan^{-1}(1)$

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$

(C) π **(D)** $\frac{\pi}{3}$

Ans. Option (A) is correct.

Explanation:

$$\tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}$$

Q. 1. Principal value of $\cot^{-1}(\sqrt{3})$ is :

(A) $\frac{\pi}{3}$ **(B)** π

(C) $\frac{\pi}{6}$ **(D)** $\frac{\pi}{2}$

Ans. Option (C) is correct.

Explanation:

$$\cot^{-1}(\sqrt{3}) = \cot^{-1}\left(\cot\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6}$$

Q. 4. Principal value of $\sin^{-1}(1) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is

(A) 2π **(B)** π

(C) $\frac{3\pi}{4}$ **(D)** $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\sin^{-1}(1) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2} + \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

Q. 5. Principal value of $2\cos^{-1}(1) + 5\tan^{-1}(1)$ is:

(A) $\frac{3\pi}{4}$ **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ **(D)** $\frac{5\pi}{4}$

Ans. Option (D) is correct.

Explanation:

$$2\cos^{-1}(1) + 5\tan^{-1}(1)$$

$$= 2\times 0 + 5\times\frac{\pi}{4}$$

$$= \frac{5\pi}{4}$$

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UNIT-II : ALGEBRA

CHAPTER 3

Term-I

MATRICES

Syllabus

- ***Concept, notation, order, equality, types of matrices: zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices : Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices Invertible matrices (Here, all matrices will have real entries).***

STAND ALONE MCQs (1 Mark each)

Q. 1. If $[x \ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, then x equals

(A) 0 **(B)** –2

(C) –1 **(D)** 2

[CBSE Delhi Set - II 2020]

Ans. Option (D) is correct.

Explanation:

$$[x \ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = [0 \ \ 0]$$

$$\Rightarrow \quad [x-2 \ \ 0] = [0 \ \ 0]$$

$$\Rightarrow \quad x - 2 = 0 \quad \text{[By def. of equality]}$$

$$\Rightarrow \quad x = 2$$

Q. 2. If $A = [2 \ -3 \ 4]$, $B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $X = [1 \ \ 2 \ \ 3]$ and $Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, then $AB + XY$ equals

(A) [28] **(B)** [24]

(C) 28 **(D)** 24

[CBSE OD Set - I 2020]

Ans. Option (A) is correct.

Explanation :

Given, $A = [2 \ \ -3 \ \ 4]$,

$$B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix},$$

$X = [1 \ \ 2 \ \ 3]$,

$$Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$AB + XY = [2 \ \ -3 \ \ 4]\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} + [1 \ \ 2 \ \ 3]\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= [6 - 6 + 8] + [2 + 6 + 12]$$
$$= [8] + [20]$$
$$= [28]$$

Q. 3. Suppose P and Q are two different matrices of order $3 \times n$ and $n \times p$, then the order of the matrix $P \times Q$ is ?

(A) $3 \times p$ **(B)** $p \times 3$

(C) $n \times n$ **(D)** 3×3

[CBSE SQP 2019-20]

Ans. Option (A) is correct.

Q. 4. $A = [a_{ij}]_{m\times n}$ is a square matrix, if

(A) $m < n$ **(B)** $m > n$

(C) $m = n$ **(D)** None of these

Ans. Option (C) is correct.

Explanation : It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.
Therefore,
$A = [a_{ij}]_{m\times n}$ is a square matrix, if $m = n$.

Q. 5. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}, y = 7$ **(B)** Not possible to find

(C) $y = 7, x = \frac{-2}{3}$ **(D)** $x = \frac{-1}{3}, y = \frac{-2}{3}$

Ans. Option (B) is correct.

Explanation : It is given that

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

Equating the corresponding elements, we get

$$3x+7 = 0$$
$$\Rightarrow \quad x = -\frac{7}{3}$$
$$5 = y-2$$
$$\Rightarrow \quad y = 7$$
$$y+1 = 8$$
$$\Rightarrow \quad y = 7$$
$$2-3x = 4$$
$$\Rightarrow \quad x = -\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.
Hence, it is not possible to find the values of x and y for which the given matrices are equal.

Q. 6. The number of all possible matrices of order 3×3 with each entry 0 or 1 is :

(A) 27 **(B)** 18

(C) 81 **(D)** 512

Ans. Option (D) is correct.

Explanation : The given matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.
Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$.

Q. 7. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. The restriction on n, k and p so that $PY + WY$ will be defined are :

(A) $k = 3, p = n$ **(B)** k is arbitrary, $p = 2$

(C) p is arbitrary, $k = 3$ **(D)** $k = 2, p = 3$

Ans. Option (A) is correct.

Explanation : Matrices P and Y are of the orders $p \times k$ and $3 \times k$, respectively. Therefore, matrix PY will be defined if $k = 3$. Consequently, PY will be of the order $p \times k$. Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.
Since the number of columns in W is equal to the number of rows in Y, matrix WY is well-defined and is of the order $n \times k$. Matrices PY and WY can be added only when their orders are the same.
However, PY is of the order $p \times k$ and WY is of the order $n \times k$. Therefore, we must have $p = n$.
Thus, $k = 3$ and $p = n$ are the restrictions on n, k, and p so that $PY + WY$ will be defined.

Q. 8. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is:

(A) $p \times 2$ **(B)** $2 \times n$

(C) $n \times 3$ **(D)** $p \times n$

Ans. Option (B) is correct.

Explanation: Matrix X is of the order $2 \times n$. Therefore, matrix $7X$ is also of the same order. Matrix Z is of the order $2 \times p$, *i.e.*, $2 \times n$ [Since $n = p$] Therefore, matrix $5Z$ is also of the same order. Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$. Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

Q. 9. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, then the value of α is:

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$

(C) π **(D)** $\frac{3\pi}{2}$

Ans. Option (B) is correct.

Explanation:

Given that, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Also $A + A' = I$

$$\Rightarrow \begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\cos\alpha & 0 \\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

Equating corresponding entries, we have

$$\Rightarrow \quad 2\cos\alpha = 1$$

$$\Rightarrow \quad \cos\alpha = \frac{1}{2}$$

$$\Rightarrow \quad \cos\alpha = \cos\frac{\pi}{3}$$

$$\therefore \quad \alpha = \frac{\pi}{3}$$

Q. 10. Matrices A and B will be inverse of each other only if

(A) $AB = BA$ **(B)** $AB = BA = 0$

(C) $AB = 0, BA = I$ **(D)** $AB = BA = I$

Ans. Option (D) is correct.

Explanation : We know that if A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is said to be the inverse of A.
In this case, it is clear that A is the inverse of B. Thus, matrices A and B will be the inverse of each other only if $AB = BA = I$.

Q. 11. The matrix $P=\begin{bmatrix}0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0\end{bmatrix}$ is a

(A) square matrix **(B)** diagonal matrix

(C) unit matrix **(D)** None of these

Ans. Option (A) is correct.

Explanation : We know that, in a square matrix number of rows is equal to the number of columns. So, the matrix $P=\begin{bmatrix}0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0\end{bmatrix}$ is a square matrix.

Q. 12. If A and B are symmetric matrices of same order, then $AB – BA$ is a:

(A) Skew-symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix

Ans. Option (A) is correct.

Explanation : Given that,
A and B are symmetric matrices.

$$\Rightarrow \quad A = A' \text{ and } B = B'$$

$$\text{Now, } (AB - BA)' = (AB)' - (BA)' \quad \text{...(i)}$$

$$\Rightarrow \quad (AB - BA)' = B'A' - A'B' \quad \text{[By reversal law]}$$

$$\Rightarrow \quad (AB - BA)' = BA - AB \quad \text{[From Eq. (i)]}$$

$$\Rightarrow \quad (AB - BA)' = -(AB - BA)$$

$\Rightarrow (AB – BA)$ is a skew-symmetric matrix.

Q. 13. If the matrix A is both symmetric and skew-symmetric, then

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Ans. Option (B) is correct.

Explanation : If A is both symmetric and skew-symmetric, then we have,

$$A' = A \text{ and } A' = -A$$

$$\Rightarrow \quad A = -A$$

$$\Rightarrow \quad A + A = 0$$

$$\Rightarrow \quad 2A = 0$$

$$\Rightarrow \quad A = 0$$

Therefore, A is a zero matrix.

Q. 14. The matrix $\begin{bmatrix}1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4\end{bmatrix}$ is a

(A) identity matrix
(B) symmetric matrix
(C) skew-symmetric matrix
(D) None of these

Ans. Option (B) is correct.

Explanation: $A=\begin{bmatrix}1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4\end{bmatrix}$

$$\therefore \quad A'=\begin{bmatrix}1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4\end{bmatrix}=A$$

So, the given matrix is a symmetric matrix.
[Since, in a square matrix A, if $A' = A$, then A is called symmetric matrix.]

Q. 15. The matrix $\begin{bmatrix}0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0\end{bmatrix}$ is a

(A) diagonal matrix
(B) symmetric matrix
(C) skew symmetric matrix
(D) scalar matrix

Ans. Option (C) is correct.

Explanation: We know that, in a square matrix, if $b_{ij} = 0$ when $i \neq j$ then it is said to be a diagonal matrix. Here, $b_{12}, b_{13} \ldots \neq 0$ so the given matrix is not a diagonal matrix.

Now, $$B = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$$= -B$$

So, the given matrix is a skew-symmetric matrix, since we know that in a square matrix B, if $B' = -B$, then it is called skew-symmetric matrix.

Q. 16. If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is

(A) $m \times m$ **(B)** $n \times n$

(C) $n \times m$ **(D)** $m \times n$

Ans. Option (D) is correct.

Explanation : Let, $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{p \times q}$

$\therefore \quad B' = [b_{ji}]_{q \times p}$

Now, AB′ is defined, so $n = q$ and $B'A$ is also defined, so $p = m$

$\therefore$ Order of $B' = [b_{ji}]_{n \times m}$

And order of $B = [b_{ji}]_{m \times n}$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A = I$

Reason (R): AI = IA = A

Ans. Option (A) is correct.

Explanation: AI = IA = A is true.

Hence R is true.

Given $A^2 = A$,

$$\therefore \quad (I + A)^2 - 3A = I^2 + 2IA + A^2 - 3A$$

$$= I + 2A + A - 3A$$

$$= I$$

Hence A is true.

R is the correct explanation for A.

Q. 2. Assertion (A): $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a scalar matrix.

Reason (R): If all the elements of the principal diagonal are equal, it is called a scalar matrix.

Ans. Option (C) is correct.

Explanation: In a scalar matrix the diagonal elements are equal and the non-diagonal elements are zero. Hence R is false.

A is true since the diagonal elements are equal and the non-diagonal elements are zero.

Q. 3. Assertion (A): $(A + B)^2 \neq A^2 + 2AB + B^2$.

Reason (R): Generally $AB \neq BA$

Ans. Option (A) is correct.

Explanation: For two matrices A and B, generally $AB \neq BA$.

i.e., matrix multiplication is not commutative.

$\therefore$ R is true

$$(A+B)^2 = (A+B)(A+B)$$

$$= A^2 + AB + BA + B^2$$

$$\neq A^2 + 2AB + B^2$$

$\therefore$ A is true

R is the correct explanation for A.

Q. 4. A and B are two matrices such that both AB and BA are defined.

Assertion (A): $(A + B)(A - B) = A^2 - B^2$

Reason (R): $(A + B)(A - B) = A^2 - AB + BA - B^2$

Ans. Option (D) is correct.

Explanation: For two matrices A and B, even if both AB and BA are defined, generally $AB \neq BA$.

$(A + B)(A - B) = A^2 - AB + BA - B^2$.

Since $AB \neq BA$, $(A + B)(A - B) \neq A^2 - B^2$.

Hence R is true and A is false.

Q. 5. Let A and B be two symmetric matrices of order 3.

Assertion (A): A(BA) and (AB)A are symmetric matrices.

Reason (R): AB is symmetric matrix if matrix multiplication of A with B is commutative.

Ans. Option (B) is correct.

Explanation: Generally $(AB)' = B'A'$
If $AB = BA$, then $(AB)' = (BA)' = A'B' = AB$
Since $(AB)' = AB$, AB is a symmetric matrix.
Hence R is true.

$$A(BA) = (AB)A = ABA$$
$$(ABA)' = A'B'A' = ABA.$$

$A(BA)$ and $(AB)A$ are symmetric matrices. Hence A is true.
But R is not the correct explanation for A.

Q. 6. Assertion (A): If the matrix $P = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & 3 \end{bmatrix}$ is a symmetric matrix, then $a = \frac{-2}{3}$ and $b = \frac{3}{2}$.

Reason (R): If P is a symmetric matrix, then $P' = -P$.

Ans. Option (C) is correct.

Explanation: If P is a symmetric matrix, then $P' = P$.
Hence R is false.
As P is a symmetric matrix, $P' = P$

$$\therefore \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

$\therefore$ By equality of matrices, $a = \frac{-2}{3}$ and $b = \frac{3}{2}$.
Hence A is true.

Q. 7. Assertion (A): If A is a symmetric matrix, then $B'AB$ is also symmetric.

Reason (R): $(ABC)' = C'B'A'$

Ans. Option (A) is correct.

Explanation: For three matrices A, B and C, if ABC is defined then $(ABC)' = C'B'A'$.
Hence R is true.
Given that A is symmetric $\Rightarrow A' = A$
$(B'AB)' = B'A'(B')' = B'AB$.
Hence A is true.
R is the correct explanation for A.

Q. 8. Assertion (A): If A and B are symmetric matrices, then $AB - BA$ is a skew symmetric matrix

Reason (R): $(AB)' = B'A'$

Ans. Option (A) is correct.

Explanation: $(AB)' = B'A' \Rightarrow$ R is true.
Given that A and B are symmetric matrices.
$\therefore A' = A$ and $B' = B$
$(AB - BA)' = (AB)' - (BA)'$
$= B'A' - A'B' = BA - AB$
Since $(AB - BA)' = -(AB - BA)$,
$AB - BA$ is skew symmetric.
Hence A is true.
R is the correct explanation for A.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10,000	2,000	18,000
B	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are ₹2.50, ₹1.50 and ₹1.00 respectively, and unit cost of the above three commodities are ₹2.00, ₹1.00 and ₹ 0.50 respectively, then, **[CBSE QB 2021]**

Q. 1. Total revenue of market A

(A) ₹ 64,000 **(B)** ₹ 60,400

(C) ₹ 46,000 **(D)** ₹ 40600

Ans. Option (C) is correct.

Explanation: Total revenue of

$$= [10{,}000 \quad 2{,}000 \quad 18{,}000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$
$$= [2.50 \times 10{,}000 + 1.50 \times 2{,}000 + 1.00 \times 18{,}000]$$
$$= [46{,}000]$$

Q. 2. Total revenue of market *B*

(A) ₹ 35,000 **(B)** ₹ 53,000
(C) ₹ 50,300 **(D)** ₹ 30,500

Ans. Option (B) is correct.

Explanation: Total revenue of market B

$$= [6{,}000 \quad 20{,}000 \quad 8{,}000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$
$$= [2.50 \times 6{,}000 + 1.50 \times 20{,}000 + 1.00 \times 8{,}000]$$
$$= [53{,}000]$$

Q. 3. Cost incurred in market *A*

(A) ₹ 13,000 **(B)** ₹ 30,100
(C) ₹ 10,300 **(D)** ₹ 31,000

Ans. Option (D) is correct.

Explanation: Cost incurred in market A

$$= [10{,}000 \quad 2{,}000 \quad 18{,}000]\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$= [2.00 \times 10{,}000 + 1.00 \times 2{,}000 + 0.50 \times 18{,}000]$$
$$= [31{,}000]$$

Q. 4. Profits in market A and B respectively are

(A) (₹15,000, ₹17,000) **(B)** (₹17,000, ₹15,000)
(c) (₹51,000, ₹71,000) **(D)** (₹10,000, ₹20,000)

Ans. Option (A) is correct.

Explanation: Cost incurred in market B

$$= [6{,}000 \quad 20{,}000 \quad 8{,}000]\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$= [2.00 \times 6{,}000 + 1.00 \times 20{,}000 + 0.50 \times 8{,}000]$$
$$= [36{,}000]$$

Profit of market A & B = total revenue of A and B – Cost increased in market A and B

$$\begin{bmatrix}A\\B\end{bmatrix} = \begin{bmatrix}46{,}000\\50{,}000\end{bmatrix} - \begin{bmatrix}31{,}000\\36{,}000\end{bmatrix}$$
$$= \begin{bmatrix}15{,}000\\17{,}000\end{bmatrix}$$

i.e., (₹15,000, ₹17,000)

Q. 5. Gross profit in both markets

(A) ₹23,000 **(B)** ₹20,300
(C) ₹32,000 **(D)** ₹30,200

Ans. Option (C) is correct.

Explanation:
Gross profit in both markets = Profit in A + Profit in B
= 15,000 + 17,000
= ₹32,000

II. Read the following text and answer the following questions on the basis of the same:

Amit, Biraj and Chirag were given the task of creating a square matrix of order 2.

Below are the matrices created by them. *A, B, C* are the matrices created by Amit, Biraj and Chirag respectively.

$$A = \begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix} B = \begin{bmatrix}4 & 0\\1 & 5\end{bmatrix} C = \begin{bmatrix}2 & 0\\1 & -2\end{bmatrix}$$

If $a = 4$ and $b = -2$, **[CBSE QB 2021]**

Q. 1. Sum of the matrices *A, B* and *C*, $A+(B + C)$ is

(A) $\begin{bmatrix}1 & 6\\2 & 7\end{bmatrix}$ **(B)** $\begin{bmatrix}6 & 1\\7 & 2\end{bmatrix}$

(C) $\begin{bmatrix}7 & 2\\1 & 6\end{bmatrix}$ **(D)** $\begin{bmatrix}2 & 1\\7 & 6\end{bmatrix}$

Ans. Option (C) is correct.

Explanation:

$$A+(B+C) = \begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix} + \left(\begin{bmatrix}4 & 0\\1 & 5\end{bmatrix} + \begin{bmatrix}2 & 0\\1 & -2\end{bmatrix}\right)$$
$$= \begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix} + \begin{bmatrix}6 & 0\\2 & 3\end{bmatrix}$$
$$= \begin{bmatrix}7 & 2\\1 & 6\end{bmatrix}$$

Q. 2. $(A^T)^T$ is equal to

(A) $\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$ **(B)** $\begin{bmatrix}2 & 1\\3 & -1\end{bmatrix}$

(C) $\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$ **(D)** $\begin{bmatrix}2 & 3\\-1 & 1\end{bmatrix}$

Ans. Option (A) is correct.

Explanation:

$$(A^T) = \begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$$
$$(A^T)^T = \begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$$

Q. 3. $(bA)^T$ is equal to

(A) $\begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}$ **(B)** $\begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & 2 \\ -6 & -4 \end{bmatrix}$ **(D)** $\begin{bmatrix} -6 & -2 \\ 2 & 4 \end{bmatrix}$

Ans. Option (B) is correct.

Explanation:

$$bA = -2A = \begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}$$

$$(bA)^T = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$$

Q. 4. AC − BC is equal to

(A) $\begin{bmatrix} -4 & -6 \\ -4 & 4 \end{bmatrix}$ **(B)** $\begin{bmatrix} -4 & -4 \\ 4 & -6 \end{bmatrix}$

(C) $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$ **(D)** $\begin{bmatrix} -6 & 4 \\ -4 & -4 \end{bmatrix}$

Ans. Option (C) is correct.

Explanation:

$$AC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

$$AC - BC = \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$$

Q. 5. $(a + b)B$ is equal to

(A) $\begin{bmatrix} 0 & 8 \\ 10 & 2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & 10 \\ 8 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 0 \\ 8 & 10 \end{bmatrix}$

Ans. Option (C) is correct.

Explanation:

$$(a+b)B = (4-2)\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

III. Read the following text and answer the following questions on the basis of the same:

Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹25, ₹100 and ₹50 each respectively. The numbers of articles sold are given as **[CBSE QB 2021]**



School /Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Q. 1. What is the total money (in Rupees) collected by the school DPS?

(A) 700 **(B)** 7,000

(C) 6,125 **(D)** 7,875

Ans. Option (B) is correct.

Explanation: The funds collected by the schools can be obtained by matrix multiplication :

$$\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 25 & 50 & 40 \end{bmatrix}\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

Funds collected by school DPS = 7000
Funds collected by school, CVC = 6125
Funds collected by school KVS = 7875

Q. 2. What is the total amount of money (in ₹) collected by schools CVC and KVS?

(A) 14,000 **(B)** 15,725

(C) 21,000 **(D)** 13,125

Ans. Option (A) is correct.

Explanation: Total amount of money collected by school
= 6125 + 7875
= 14000

Q. 3. What is the total amount of money collected by all three schools DPS, CVC and KVS?

(A) ₹15,775 **(B)** ₹14,000

(C) ₹21,000 **(D)** ₹17,125

Ans. Option (C) is correct.

Explanation: Total amount of money collected by all school DPS, CVC and KVS
= 7000 + 7875 + 6125
= 21000

Q. 4. If the number of handmade fans and plates are interchanged for all the schools, then what is the

total money collected by all schools?

(A) ₹18,000 **(B)** ₹6,750

(C) ₹5,000 **(D)** ₹21,250

Ans. Option (D) is correct.

Q. 5. How many articles (in total) are sold by three schools?

(A) 230 **(B)** 130

(C) 430 **(D)** 330

Ans. Option (D) is correct.

IV. Read the following text and answer the following questions on the basis of the same:

On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹).

[CBSE QB 2021]

Q. 1. The equations in terms of x and y are

(A) $5x - 4y = 40$
$5x - 8y = -80$

(B) $5x - 4y = 40$
$5x - 8y = 80$

(C) $5x - 4y = 40$
$5x + 8y = -80$

(D) $5x + 4y = 40$
$5x - 8y = -80$

Ans. Option (A) is correct.

Explanation: According to question,

$$(x-8)(y+10) = xy$$
$$\Rightarrow \quad xy + 10x - 8y - 80 = xy$$
$$\Rightarrow \quad 5x - 4y = 40 \quad \text{...(i)}$$
$$\text{and} \quad (x+16)(y-10) = xy$$
$$\Rightarrow \quad xy - 10x + 16y - 160 = xy$$
$$\Rightarrow \quad 5x - 8y = -80 \quad \text{...(ii)}$$

Q. 2. Which of the following matrix equations represent the information given above?

(A) $\begin{bmatrix} 5 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

Ans. Option (C) is correct.

Explanation: Given equations are;

$$5x - 4y = 40$$
$$5x - 8y = -80$$
$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

Q. 3. The number of children who were given some money by Seema, is

(A) 30 **(B)** 40

(C) 23 **(D)** 32

Ans. Option (D) is correct.

Explanation: Since,

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

Let $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$

$$B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

$\therefore \quad AX = B$

$$X = A^{-1}B \quad \text{...(iii)}$$
$$|A| = 5(-8) - (-4) \times 5$$
$$= -40 + 20$$
$$= -20$$
$$adj(A) = \begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}^T$$
$$= \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$
$$A^{-1} = \frac{Adj(A)}{|A|}$$
$$= \frac{1}{-20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2}{5} & \frac{-1}{5} \\ \frac{1}{4} & \frac{-1}{4} \end{bmatrix}$$
$$X = A^{-1}B$$
$$= \begin{bmatrix} \frac{2}{5} & \frac{-1}{5} \\ \frac{1}{4} & \frac{-1}{4} \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2}{5} \times 40 - \frac{1}{5} \times (-80) \\ \frac{1}{4} \times 40 - \frac{1}{4} \times (-80) \end{bmatrix}$$
$$= \begin{bmatrix} 16 + 16 \\ 10 + 20 \end{bmatrix}$$
$$= \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$\therefore \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$

$$x = 32$$
$$y = 30$$

Hence, 32 children were given some money by Seema.

Q. 4. How much amount is given to each child by Seema?
(A) ₹32 **(B)** ₹30
(C) ₹62 **(D)** ₹26

Ans. Option (B) is correct.

Explanation: ₹ 30 is given to each child by Seema [$\because y = 30$]

Q. 5. How much amount Seema spends in distributing the money to all the students of the Orphanage?
(A) ₹609 **(B)** ₹960
(C) ₹906 **(D)** ₹690

Ans. Option (B) is correct.

Explanation: Total amount Seema spends in distributing the money to all the students of the orphanage

$$= x \times y$$
$$= 32 \times 30$$
$$= ₹960$$

V. Read the following text and answer the following questions on the basis of the same:

Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in ₹) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B **[CBSE QB 2021]**


September sales (in Rupees)

$$A = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \begin{matrix} \text{Ramakrishan} \\ \text{Gurcharan} \end{matrix}$$

October sales (in Rupees)

$$B = \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \begin{matrix} \text{Ramakrishan} \\ \text{Gurcharan} \end{matrix}$$

Q. 1. The total sales in September and October for each farmer in each variety can be represented as _____.
(A) A + B **(B)** A – B
(C) A > B **(D)** A < B

Ans. Option (A) is correct.

Explanation: Combined sales in September and October for each farmer in each variety is given by

$A + B =$

Basmati	Permal	Naura	
15,000	30,000	36,000	Ramkrishan
70,000	40,000	20,000	Gurcharan singh

Q. 2. What is the value of A_{23}?
(A) 10,000 **(B)** 20,000
(C) 30,000 **(D)** 40,000

Ans. Option (A) is correct.

Explanation: $A_{23} = 10,000$

Q. 3. The decrease in sales from September to October is given by _____ .
(A) A + B **(B)** A – B
(C) A > B **(D)** A < B

Ans. Option (B) is correct.

Explanation: Change in sales from September to October is given by

$A - B =$

Basmati	Permal	Naura	
5000	10,000	24,000	Ramkishan
30,000	20,000	0	Gurcharan Singh

Q. 4. If Ramkishan receives 2% profit on gross sales, compute his profit for each variety sold in October.
(A) ₹100, ₹200 and ₹120
(B) ₹100, ₹200 and ₹130
(C) ₹100, ₹220 and ₹120
(D) ₹110, ₹200 and ₹120

Ans. Option (A) is correct.

Explanation:

$$2\% \text{ of } B = \frac{2}{100} \times B$$
$$= 0.02 \times B$$
$$= 0.02$$

Basmati	Permal	Naura	
5000	10,000	6000	Ramkishan
20,000	10,000	10,000	Gurcharn Singh

Thus, in October Ramkishan receives ₹ 100, ₹ 200 and ₹ 120 as profit in the sale of each variety of rice, respectively.

Q. 5. If Gurucharan receives 2% profit on gross sales, compute his profit for each variety sold in September.
(A) ₹100, ₹200, ₹120 **(B)** ₹1000, ₹600, ₹200
(C) ₹400, ₹200, ₹120 **(D)** ₹1200, ₹200, ₹120

Ans. Option (B) is correct.

Explanation:

$$2\% \text{ of } A = \frac{2}{100} \times A$$
$$= 0.02 \times A$$
$$= 0.02$$

$$\begin{matrix} & \text{Basmati} & \text{Permal} & \text{Naura} & \\ \end{matrix}$$
$$\begin{bmatrix} 10{,}000 & 20{,}000 & 30{,}000 \\ 50{,}000 & 30{,}000 & 10{,}000 \end{bmatrix} \begin{matrix} \text{Ramkishan} \\ \text{Gurcharn Singh} \end{matrix}$$
$$= \begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix}$$
$$\begin{bmatrix} 200 & 400 & 600 \\ 1000 & 600 & 200 \end{bmatrix} \begin{matrix} \text{Ramkishan} \\ \text{Gurcharn Singh} \end{matrix}$$

Thus, in September Gurucharan receives ₹ 1000, ₹ 600 and ₹ 200 as Profit in the sale of each variety of rice, respectively.

VI. Read the following text and answer the following questions on the basis of the same:

There are three families *A*, *B* and *C*. The number of members in these families are given in the table below.

	Men	Women	Children
Family *A*	3	2	1
Family *B*	2	4	2
Family *C*	4	3	2

The daily expenses of each man, woman and child are respectively ₹200, ₹100 and ₹50.

Q. 1. The total daily expense of family *A* is _____.

(A) 850 **(B)** 900
(C) 1,200 **(D)** 2,950

Ans. Option (A) is correct.

Explanation: Expense of family *A*

$$= [3 \quad 2 \quad 1] \begin{bmatrix} 200 \\ 100 \\ 50 \end{bmatrix} = [850]$$

Q. 2. The total daily expense of family *C* is _____.

(A) 850 **(B)** 900
(C) 1,200 **(D)** 2,950

Ans. Option (C) is correct.

Explanation: Expense of family *C*

$$= [4 \quad 3 \quad 2] \begin{bmatrix} 200 \\ 100 \\ 50 \end{bmatrix} = [1200]$$

Q. 3. The combined daily expense of all the women is _____.

(A) 850 **(B)** 900
(C) 1,200 **(D)** 2,950

Ans. Option (B) is correct.

Explanation: Combined expense of women

$$= [2 \quad 4 \quad 3] \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} = [900]$$

Q. 4. The family with highest expense is _____.

(A) *A*
(B) *B*
(C) *C*
(D) All have same expense

Ans. Option (C) is correct.

Explanation: Most expensive family is *C* with an expense of ₹1200.

Q. 5. The combined expense of men in family A and children in family *C* is _____.

(A) 600 **(B)** 700
(C) 800 **(D)** 900

Ans. Option (B) is correct.

Explanation: $[3 \quad 2] \begin{bmatrix} 200 \\ 50 \end{bmatrix} = [700]$

VII. Read the following text and answer the following questions on the basis of the same:

Three schools SNT, SNP and TKM organised a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand-made fans, mats and plates from recycled material at a cost of ₹25, ₹100 and ₹50 each. The number of articles sold are given below.

	SNT	SNP	TKM
Fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Q. 1. Funds collected by SNT is _____.

(A) ₹7000 **(B)** ₹6125
(C) ₹7875 **(D)** ₹21000

Ans. Option (A) is correct.

Explanation:

Fund raised by SNT = ₹7000

$$[40 \quad 50 \quad 20]\begin{bmatrix}25\\100\\50\end{bmatrix} = [7{,}000]$$

Q. 2. Funds collected by SNP is _____.

(A) ₹7000 **(B)** ₹6125

(C) ₹7875 **(D)** ₹21000

Ans. Option (B) is correct.

Explanation:

$$[25 \quad 40 \quad 30]\begin{bmatrix}25\\100\\50\end{bmatrix} = [6{,}125]$$

Fund raised by SNP = ₹6,125

Q. 3. The total fund raised by all the three schools together is _____.

(A) ₹7000 **(B)** ₹6125

(C) ₹7875 **(D)** ₹21000

Ans. Option (D) is correct.

Explanation:

$$[35 \quad 50 \quad 40]\begin{bmatrix}25\\100\\50\end{bmatrix} = [7{,}875]$$

Fund raised by TKM = ₹7,875

Total amount raised = 7,000 + 6,125 + 7,875

= ₹21,000

Q. 4. The total fund raised by selling fans is _____

(A) ₹4,000 **(B)** ₹2,000

(C) ₹2,500 **(D)** ₹35,000

Ans. Option (C) is correct.

Explanation:

[40 + 25 + 35] [25] = [2,500]

Fund raised by selling fans = ₹2,500

Q. 5. TKM collected ₹ _____ by selling plates.

(A) 4000 **(B)** 2000

(C) 2500 **(D)** 3500

Ans. Option (B) is correct.

Explanation:

[40] [50] = [2000]

TKM collected ₹2000 by selling plates.

❐❐

CHAPTER

4

Term-I

DETERMINANTS

Syllabus

➢ ***Determinant of a square matrix (up to 3 × 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.***

STAND ALONE MCQs (1 Mark each)

Q. 1. If A is a square matrix of order 3, such that $A(adj\ A) = 10I$, then $|adj\ A|$ is equal to

(A) 1 **(B)** 10

(C) 100 **(D)** 101

[CBSE Delhi Set-I 2020]

Ans. Option (C) is correct.

Explanation: Consider the equation

$$A\ (adj\ A) = |A|\ I$$

Here, $A\ (adj\ A) = 10\ I$

Then, $|A| = 10$

Since, $|adj\ A| = |A|^{n-1}$

Where n is order of matrix

Here, $= |A|^{3-1}$

$= 10^2$

$= 100$

Q. 2. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals

(A) 8 **(B)** 24

(C) 72 **(D)** 216

[CBSE Delhi Set-I 2020]

Ans. Option (D) is correct.

Explanation:

Here $|A| = 8$

Then $|3A| = 3^3|A| = 27 \times 8 = 216$

Q. 3. If A is skew symmetric matrix of order 3, then the value of $|A|$ is

(A) 3 **(B)** 0

(C) 9 **(D)** 27

[CBSE Delhi Set-III 2020]

Ans. Option (B) is correct.

Explanation: Determinant value of skew symmetric matrix is always '0'.

Q. 4. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is

(A) 3 **(B)** 0

(C) – 1 **(D)** 1

[CBSE OD Set-I 2020]

Ans. Option (C) is correct.

Explanation:

$$\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$

On expanding along R_1

$2(x - 9x) - 3(x - 4x) + 2(9x - 4x) + 3 = 0$

$2(-8x) - 3(-3x) + 2(5x) + 3 = 0$

$-16x + 9x + 10x + 3 = 0$

$3x + 3 = 0$

$$3x = -3$$
$$x = -\frac{3}{3}$$
$$x = -1$$

Q. 5. Let $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$, then $|AB|$ is equal to

(A) 460 **(B)** 2000
(C) 3000 **(D)** – 7000

[CBSE OD Set-I 2020]

Ans. Option (D) is correct.

Explanation:

$$A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$$
$$B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$$
$$AB = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}\begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 10000+100 & 8000+150 \\ 500+4 & 400+6 \end{bmatrix}$$
$$AB = \begin{bmatrix} 10100 & 8150 \\ 504 & 406 \end{bmatrix}$$
$$|AB| = (10100)(406) - (504)(8150)$$
$$= 4100600 - 4107600$$
$$= -7000$$

Q. 6. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det $(adj\ A)$ equals

(A) a^{27} **(B)** a^9
(C) a^6 **(D)** a^2

[CBSE OD Set-III 2020]

Ans. Option (C) is correct.

Explanation:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$
$$\text{Det}(A) = a(a \times a - 0 \times 0) - 0 + 0$$
$$= a^3$$
$$\text{Det}(adj\ A) = (a^3)^2$$
$$= a^6$$

Q. 7. If A is any square matrix of order 3 × 3 such that $|A| = 3$, then the value of $|adj\ A|$ is?

(A) 3 **(B)** $\frac{1}{3}$
(C) 9 **(D)** 27

[CBSE SQP 2019-20]

Ans. Option (C) is correct.

Explanation:

$$|A| = 3, n = 3$$
$$|adj\ A| = |A|^2 = 3^2 = 9$$

Q. 8. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of x is

(A) 3 **(B)** ±3
(C) ±6 **(D)** 6

Ans. Option (C) is correct.

Explanation: Given that

$$\therefore \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix},$$
$$\Rightarrow 2x^2 - 40 = 18 + 14$$
$$\Rightarrow 2x^2 = 32 + 40$$
$$\Rightarrow x^2 = \frac{72}{2}$$
$$x^2 = 36$$
$$\therefore x = \pm 6$$

Q. 9. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is

(A) $a^3 + b^3 + c^3$ **(B)** $3bc$
(C) $a^3 + b^3 + c^3 - 3abc$ **(D)** None of these

Ans. Option (D) is correct.

Explanation: We have

$$\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix} = \begin{vmatrix} a+c & b+c+a & a \\ b+c & c+a+b & b \\ c+b & a+b+c & c \end{vmatrix}$$
$$[\because C_1 \to C_1 + C_2 \text{ and } C_2 \to C_2 + C_3]$$
$$= (a+b+c)\begin{vmatrix} a+c & 1 & a \\ b+c & 1 & b \\ c+b & 1 & c \end{vmatrix}$$
[Taking $(a+b+c)$ common from C_2]
$$[\because R_2 \to R_2 - R_3 \text{ and } R_1 \to R_1 - R_3]$$
$$= (a+b+c)\begin{vmatrix} a-b & 0 & a-c \\ 0 & 0 & b-c \\ c+b & 1 & c \end{vmatrix}$$
[Expanding along R_2]
$$= (a+b+c)[(b-c)(a-b)]$$
$$= (a+b+c)(b-c)(a-b)$$

Q. 10. The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. Then, the value of k will be

(A) 9 **(B)** 3
(C) –9 **(D)** 6

Ans. Option (B) is correct.

Explanation: We know that, area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \quad \Delta = \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$$

[Expanding along R_1]

$$9 = \frac{1}{2}\left[-3(-k) - 0 + 1(3k)\right]$$

$$\Rightarrow 18 = 3k + 3k$$

$$18 = 6k$$

$$\therefore \quad k = \frac{18}{6}$$

$$= 3$$

Q. 11. The determinant $\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$ is equal to

(A) $abc(b-c)(c-a)(a-b)$

(B) $(b-c)(c-a)(a-b)$

(C) $(a+b+c)(b-c)(c-a)(a-b)$

(D) None of these

Ans. Option (D) is correct.

Explanation: We have

$$\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix} = \begin{vmatrix} b(b-a) & b - c & c(b-a) \\ a(b-a) & a - b & b(b-a) \\ c(b-a) & c - a & a(b-a) \end{vmatrix}$$

$$= (b-a)^2 \begin{vmatrix} b & b - c & c \\ a & a - b & b \\ c & c - a & a \end{vmatrix}$$

[On taking $(b-a)$ common from C_1 and C_3 each]

$$= (b-a)^2 \begin{vmatrix} b - c & b - c & c \\ a - b & a - b & b \\ c - a & c - a & a \end{vmatrix}$$

$[\because C_1 \to C_1 - C_3]$

$$= 0$$

[Since, two columns C_1 and C_2 are identical, so the value of determinant is zero.]

Q. 12. If $A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$. Then A^{-1} exist if

(A) $\lambda = 2$ **(B)** $\lambda \neq 2$

(C) $\lambda \neq -2$ **(D)** None of these

Ans. Option (D) is correct.

Explanation: Given that,

$$A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$$

Expanding along R_1,

$$|A| = 2(6-5) - \lambda(-5) - 3(-2)$$

$$= 2 + 5\lambda + 6$$

We know that A^{-1} exists, if A is non-singular matrix, *i.e.*, $|A| \neq 0$

$$\therefore \quad 2 + 5\lambda + 6 \neq 0$$

$$\Rightarrow \quad 5\lambda \neq -8$$

$$\therefore \quad \lambda \neq \frac{-8}{5}$$

So, A^{-1} exists if and only if $\lambda \neq \frac{-8}{5}$.

Q. 13. If A and B are invertible matrices, then which of the following is not correct?

(A) $adj\ A = |A|.A^{-1}$ **(B)** $\det(A^{-1}) = [\det(A)]^{-1}$

(C) $(AB)^{-1} = B^{-1}A^{-1}$ **(D)** $(A+B)^{-1} = B^{-1} + A^{-1}$

Ans. Option (D) is correct.

Explanation: Since, A and B are invertible matrices, so, we can say that

$$(AB)^{-1} = B^{-1}A^{-1} \quad \text{...(i)}$$

Also, $$A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\Rightarrow \quad adj\ A = A^{-1}.|A| \quad \text{...(ii)}$$

Also, $$\det(A)^{-1} = [\det(A)]^{-1}$$

$$\Rightarrow \quad \det(A)^{-1} = \frac{1}{[\det(A)]}$$

$$\Rightarrow \quad \det(A).\det(A)^{-1} = 1 \quad \text{...(iii)}$$

From equation (iii), we conclude that it is true.

Again, $$(A+B)^{-1} = \frac{1}{|(A+B)|}adj\ (A+B)$$

$$\Rightarrow \quad (A+B)^{-1} = B^{-1} + A^{-1} \quad \text{...(iv)}$$

Q. 14. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to

(A) 6 **(B)** ± 6

(C) -6 **(D)** 0

Ans. Option (B) is correct.

Explanation: Given that,

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$\Rightarrow \quad x^2 - 36 = 36 - 36$$

$$\Rightarrow \quad x^2 - 36 = 0$$

$$\Rightarrow \quad x = \pm 6$$

Q. 15. Let A be a non-singular square matrix of order 3×3. Then $|adj\, A|$ is equal to

(A) $|A|$ **(B)** $|A|^2$

(C) $|A|^3$ **(D)** $3|A|$

Ans. Option (B) is correct.

Explanation: We know that,

$$(adj\, A)A = |A|I = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$\Rightarrow \quad |(adj\, A)A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\Rightarrow |adj\, A|\,|A| = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |A|^3 \ (I)$$

$$\therefore \quad |adj\, A| = |A|^2$$

Q. 16. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

(A) det (A) **(B)** $\dfrac{1}{\det(A)}$

(C) 1 **(D)** 0

Ans. Option (B) is correct.

Explanation: Given that A is an invertible matrix, A^{-1} exists and $A^{-1} = \dfrac{1}{|A|}\, adj.\, A.$

As matrix A is of order 2, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then, $|A| = ad - bc$ and $adj\, A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now

$$A^{-1} = \frac{1}{|A|} adj.A = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\therefore \quad |A^{-1}| = \begin{vmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{vmatrix}$$

$$= \frac{1}{|A|^2} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$= \frac{1}{|A|^2} . |A|$$

$$= \frac{1}{|A|}$$

$$\therefore \quad \det(A^{-1}) = \frac{1}{\det(A)}$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Let A be a 2×2 matrix.

Assertion (A): $adj\,(adj\, A) = A$

Reason (R): $|adj\, A| = |A|$

Ans. Option (B) is correct.

Explanation:

$$adj\,(adj\, A) = |A|^{n-2} A$$

Here $n = 2 \Rightarrow adj\,(adj\, A) = A$

Hence A is true.

$$|adj\, A| = |A|^{n-1}$$

$n = 2 \Rightarrow |adj\, A| = |A|$

Hence R is true.

R is not the correct explanation for A.

Q. 2. Assertion (A): If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}.$$

Reason (R): The inverse of an invertible diagonal matrix is a diagonal matrix.

Ans. Option (B) is correct.

Explanation:

$$|A| = 24$$

$$Adj\, A = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\,A) = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

Hence A is true.

A is a diagonal matrix and its inverse is also a diagonal matrix. Hence R is true.

But R is not the correct explanation of A.

Q. 3. Assertion (A): If every element of a third order determinant of value Δ is multiplied by 5, then the value of the new determinant is 125Δ.

Reason (R): If k is a scalar and A is an $n \times n$ matrix, then $|kA| = k^n |A|$

Ans. Option (A) is correct.

Explanation: If k is a scalar and A is an $n \times n$ matrix, then $|kA| = k^n |A|$.

This is a property of the determinant. Hence R is true.

Using this property, $|5\Delta| = 5^3 \Delta = 125\Delta$

Hence A is true.

R is the correct explanation of A.

Q. 4. Assertion (A): If the matrix $A = \begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then $\lambda = 4$.

Reason (R): If A is a singular matrix, then $|A| = 0$.

Ans. Option (A) is correct.

Explanation: A matrix is said to be singular if $|A| = 0$.

Hence R is true.

$$\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix} = 0$$

$$\Rightarrow \quad 1(40-40) - 3(20-24) + (\lambda+2)(10-12) = 0$$

$$0 + 12 - 2\lambda - 4 = 0$$

$$\Rightarrow \quad \lambda = 4.$$

Hence A is true.

R is the correct explanation for A.

Q. 5. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$.

Assertion (A): $2A^{-1} = 9I - A$

Reason (R): $A^{-1} = \frac{1}{|A|}(adj\,A)$

Ans. Option (A) is correct.

Explanation: $A^{-1} = \frac{1}{|A|}(adj\,A)$ is true.

Hence R is true

$|A| = 2$,

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix},$$

$$\text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$\therefore \quad 2A^{-1} = 9I - A$ is true.

R is the correct explanation for A.

Q. 6. Assertion (A): If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ and $A^{-1} = kA$, then $k = \frac{1}{9}$

Reason (R): $|A^{-1}| = \frac{1}{|A|}$

Ans. Option (D) is correct.

Explanation:

$$|A| = -4 - 15 = -19$$

$$A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$\Rightarrow \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2k & 3k \\ 5k & -2k \end{bmatrix}$$

$$\Rightarrow \quad k = \frac{1}{19}$$

A is false

$$|A^{-1}| = \frac{1}{|A|} \text{ is true.}$$

R is true.

CASE-BASED MCQs

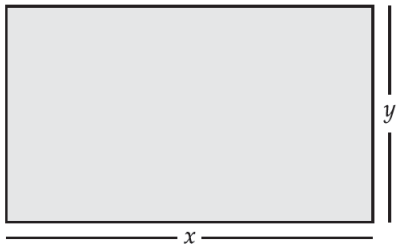
Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by

50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2

[CBSE QB 2021]



Q. 1. The equations in terms of X and Y are

(A) $x - y = 50, 2x - y = 550$

(B) $x - y = 50, 2x + y = 550$

(C) $x + y = 50, 2x + y = 550$

(D) $x + y = 50, 2x + y = 550$

Ans. Option (B) is correct.

Q. 2. Which of the following matrix equation is represented by the given information

(A) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -50 \\ -550 \end{bmatrix}$

Ans. Option (A) is correct.

Q. 3. The value of x (length of rectangular field) is

(A) 150 m **(B)** 400 m

(C) 200 m **(D)** 320 m

Ans. Option (C) is correct.

Explanation: We have,

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$,

$X = \begin{bmatrix} x \\ y \end{bmatrix}$

Now $AX = B$

$X = A^{-1}B$

$Adj(A) = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$

$|A| = 1 - [2 \times (-1)]$

$= 1 + 2$

$= 3$

$A^{-1} = \frac{Adj(A)}{|A|}$

$= \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$

$= \begin{bmatrix} \frac{50}{3} + \frac{550}{3} \\ \frac{-100}{3} + \frac{550}{3} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$

$\Rightarrow \quad x = 200$

$y = 150$

Q. 4. The value of y (breadth of rectangular field) is

(A) 150 m **(B)** 200 m

(C) 430 m. **(D)** 350 m

Ans. Option (A) is correct.

Q. 5. How much is the area of rectangular field?

(A) 60000 sq.m. **(B)** 30000 sq.m.

(C) 30000m **(D)** 3000m

Ans. Option (B) is correct.

Explanation: Area of rectangular field

$= xy$

$= 200 \times 150$

$= 30000$ sqm.

II. Read the following text and answer the following questions on the basis of the same:

The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to kept the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. The sum of the number of awardees for honesty and supervision is twice the number of awardees for helping.

Q. 1. $x + y + z =$ _____.

(A) 3 **(B)** 5

(C) 7 **(D)** 12

Ans. Option (D) is correct.

Explanation:

$$x + y + z = 12 \quad ...(i)$$

$$2x + 3y + 3z = 33 \quad ...(ii)$$

$$x - 2y + z = 0 \quad ...(iii)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$$
$$= 9 + 1 - 7$$
$$= 3$$

$$A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$= \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$\Rightarrow x = 3, y = 4, z = 5$

$x + y + z = 12$ [from (i)]

Q. 2. $x - 2y =$ _____.

(A) z **(B)** $-z$

(C) $2z$ **(D)** $-2z$

Ans. Option (B) is correct.

Explanation: $x - 2y = -z$ [from (iii)]

Q. 3. The value of z is _____.

(A) 3 **(B)** 4

(C) 5 **(D)** 6

Ans. Option (C) is correct.

Explanation: $z = 5$

Q. 4. The value of $x + 2y =$ _____.

(A) 9 **(B)** 10

(C) 11 **(D)** 12

Ans. Option (C) is correct.

Explanation: $x + 2y = 3 + 8 = 11$

Q. 5. The value of $2x + 3y + 5z =$ _____.

(A) 40 **(B)** 43

(C) 50 **(D)** 53

Ans. Option (B) is correct.

Explanation:

$$2x + 3y + 5z = 6 + 12 + 25$$
$$= 43$$

III. Read the following text and answer the following questions. On the basis of the same:

Two schools Oxford and Navdeep want to award their selected students on the values of sincerity, truthfulness and helpfulness. Oxford wants to award ₹x each, ₹y each and ₹z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹1600. Navdeep wants to spend ₹2300 to award its 4, 1 and 3 students on the respective values (by giving the same amount to the three values as before). The total amount of the award for one prize on each is ₹900.

Q. 1. $x + y + z =$ _____.

(A) 800 **(B)** 900

(C) 1000 **(D)** 1200

Ans. Option (B) is correct.

Explanation:
From the above information, we have

$3x + 2y + z = 1600$ ...(i)

$4x + y + 3z = 2300$ ...(ii)

$x + y + z = 900$ ...(iii)

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = 3(1-3) - 2(4-3) + 1(4-1)$$
$$= -6 - 2 + 3$$
$$= -5$$

$$A^{-1} = \frac{1}{|A|}(adj\, A)$$

$$= \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{-1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$= \frac{-1}{5}\begin{bmatrix} -1000 \\ -1500 \\ -2000 \end{bmatrix}$$

$$= \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

$\therefore x = 200, y = 300, z = 400$

$x + y + z = 900$ [from (iii)]

Q. 2. $4x + y + 3z =$ _____.

(A) 1600 **(B)** 2300

(C) 900 **(D)** 1200

Ans. Option (B) is correct.

Explanation: $4x + y + 3z = 2300$

Q. 3. The value of y is _____.

(A) 200 **(B)** 250

(C) 300 **(D)** 350

Ans. Option (C) is correct.

Explanation: $4x + y + 3z = 2300$ [from (ii)]
$y = 300$

Q. 4. The value of $2x + 3y$ is _____.

(A) 1000 **(B)** 1100

(C) 1200 **(D)** 1300

Ans. Option (D) is correct.

Explanation: $2x + 3y = 400 + 900$
$= 1300$

Q. 5. $y - x =$ _____.

(A) 100 **(B)** 200

(C) 300 **(D)** 400

Ans. Option (A) is correct.

Explanation:
$y - x = 300 - 200$
$= 100$

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UNIT–III : CALCULUS

CHAPTER 5

Term-I

CONTINUITY & DIFFERENTIABILITY

Syllabus

- ***Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.***
- ***Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.***

STAND ALONE MCQs (1 Mark each)

Q. 1. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$ then which of the following can be a discontinuous function?

(A) $f(x) + g(x)$ **(B)** $f(x) - g(x)$

(C) $f(x).g(x)$ **(D)** $\frac{g(x)}{f(x)}$

Ans. Option (D) is correct.

Explanation: Since $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$ are continuous functions, then by using the algebra of continuous functions , the functions $f(x) + g(x)$, $f(x) - g(x)$, $f(x).g(x)$ are also continuous functions but $\frac{g(x)}{f(x)}$ is discontinuous function at $x = 0$.

Q. 2. The function $f(x) = \frac{4 - x^2}{4x - x^3}$

(A) discontinuous at only one point

(B) discontinuous at exactly two points

(C) discontinuous at exactly three points

(D) none of these

Ans. Option (C) is correct.

Explanation: Given that,

$$f(x) = \frac{4 - x^2}{4x - x^3},$$

then it is discontinuous if

$$\Rightarrow \quad 4x - x^3 = 0$$

$$\Rightarrow \quad x(4 - x^2) = 0$$

$$\Rightarrow x(2 + x)(2 - x) = 0$$

$$\Rightarrow \quad x = 0, -2, 2$$

Thus, the given function is discontinuous at exactly three points.

Q. 3. The function $f(x) = \cot x$ is discontinuous on the set

(A) $\{x = n\pi; n \in Z\}$

(B) $\{x = 2n\pi; n \in Z\}$

(C) $\left\{x = (2n + 1)\frac{\pi}{2}; n \in Z\right\}$

(D) $\left\{x = \frac{n\pi}{2}; n \in Z\right\}$

Ans. Option (A) is correct.

Explanation: Given that,

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$

It is discontinuous at

$$\sin x = 0$$

$$\Rightarrow \quad x = n\pi,\ n \in Z$$

Thus, the given function is discontinuous at $\{x = n\pi : n \in Z\}$.

Q. 4. If $f(x) = \begin{cases} mx + 1 & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$, is continuous at $x = \frac{\pi}{2}$

then

(A) $m = 1, n = 0$ **(B)** $m = \frac{n\pi}{2} + 1$

(C) $n = \frac{m\pi}{2}$ **(D)** $m = n = \frac{\pi}{2}$

Ans. Option (C) is correct.

Explanation: Given that,

$$f(x) = \begin{cases} mx + 1 & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$$

is continuous function at $x = \frac{\pi}{2}$, then

$$\text{LHL} = \text{RHL}$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x)$$

$$\Rightarrow \quad \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right)$$

$$\Rightarrow \lim_{h \to 0} m\left(\frac{\pi}{2} - h\right) + 1 = \lim_{h \to 0} \sin\left(\frac{\pi}{2} + h\right) + n$$

$$\Rightarrow \lim_{h \to 0} m\left(\frac{\pi}{2} - h\right) + 1 = \lim_{h \to 0} \cos h + n$$

$$\Rightarrow \quad m\left(\frac{\pi}{2}\right) + 1 = 1 + n$$

$$\Rightarrow \quad n = \frac{m\pi}{2}$$

Q. 5. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to

(A) $25y$ **(B)** $5y$

(C) $-25y$ **(D)** $15y$

[CBSE Delhi Set-I 2020]

Ans. Option (A) is correct.

Explanation:

$$y = Ae^{5x} + Be^{-5x}$$

$$\Rightarrow \quad \frac{dy}{dx} = 5Ae^{5x} - 5Be^{-5x}$$

$$\Rightarrow \quad \frac{d^2y}{d^2x} = 25Ae^{5x} + 25Be^{-5x}$$

$$= 25y$$

Q. 6. If $y = \log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ equals

(A) $-\frac{1}{x}$ **(B)** $-\frac{1}{x^2}$

(C) $\frac{2}{x^2}$ **(D)** $-\frac{2}{x^2}$

[CBSE Delhi Set-III 2020]

Ans. Option (D) is correct.

Explanation:

Given, $y = \log_e\left(\frac{x^2}{e^2}\right)$

$$\Rightarrow \quad y = 2\log_e x - \log_e e^2$$

$$\Rightarrow \quad y = 2\log_e x - 2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2}{x}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-2}{x^2}$$

Q. 7. The set of points where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable is

(A) R **(B)** $R - \left\{\frac{1}{2}\right\}$

(C) $(0, \infty)$ **(D)** none of these

Ans. Option (B) is correct.

Explanation: Given that,

$$f(x) = |2x - 1| \sin x$$

The function $\sin x$ is differentiable.

The function $|2x - 1|$ is differentiable, except

$$2x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1}{2}$$

Thus, the given function is differentiable $R - \left\{\frac{1}{2}\right\}$.

Q. 8. The function $f(x) = e^{|x|}$ is

(A) continuous everywhere but not differentiable at $x = 0$

(B) continuous and differentiable everywhere

(C) not continuous at $x = 0$

(D) none of these

Ans. Option (A) is correct.

Explanation: Given that,

$$f(x) = e^{|x|}$$

The functions e^x and $|x|$ are continuous functions for all real value of x.
Since e^x is differentiable everywhere but $|x|$ is non-differentiable at $x = 0$.
Thus, the given functions $f(x) = e^{|x|}$ is continuous everywhere but not differentiable at $x = 0$.

Q. 9. Let $f(x) = |\sin x|$, then
(A) f is everywhere differentiable
(B) f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$.
(C) f is everywhere continuous but not differentiable at $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$.
(D) none of these

Ans. Option (B) is correct.

Explanation: Given that,
$$f(x) = |\sin x|$$
The functions $|x|$ and $\sin x$ are continuous function for all real value of x.
Thus, the function $f(x) = |\sin x|$ is continuous function everywhere.
Now, $|x|$ is non-differentiable function at $x = 0$.
Since $f(x) = |\sin x|$ is non-differentiable function at $\sin x = 0$
Thus, f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$.

Q. 10. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{4x^3}{1-x^4}$ **(B)** $\frac{-4x}{1-x^4}$
(C) $\frac{1}{4-x^4}$ **(D)** $\frac{-4x^3}{1-x^4}$

Ans. Option (B) is correct.

Explanation: Given that,
$$y = \log\left(\frac{1-x^2}{1+x^2}\right)$$
$$\Rightarrow \quad y = \log(1-x^2) - \log(1+x^2).$$
Differentiate with respect to x, we have
$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(1-x^2)\right] - \frac{d}{dx}\left[\log(1+x^2)\right]$$
$$= \frac{-2x}{1-x^2} - \frac{2x}{1+x^2}$$
$$= -2x\left(\frac{2}{(1-x^2)(1+x^2)}\right)$$
$$= -\frac{4x}{1-x^4}$$

Q. 11. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\cos x}{2y-1}$ **(B)** $\frac{\cos x}{1-2y}$
(C) $\frac{\sin x}{1-2y}$ **(D)** $\frac{\sin x}{2y-1}$

Ans. Option (A) is correct.

Explanation: Given that,
$$y = \sqrt{\sin x + y}$$
$$\Rightarrow y^2 = \sin x + y \quad .$$
Differentiate with respect to x, we have
$$\Rightarrow \quad 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$
$$\Rightarrow (2y-1)\frac{dy}{dx} = \cos x$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

Q. 12. The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1} x$ is

(A) 2 **(B)** $\frac{-1}{2\sqrt{1-x^2}}$
(C) $\frac{2}{x}$ **(D)** $1 - x^2$

Ans. Option (A) is correct.

Explanation: Let
$$u = \cos^{-1}\left(2x^2 - 1\right)$$
$$\Rightarrow \quad \frac{du}{dx} = -\frac{4x}{\sqrt{1-\left(2x^2-1\right)^2}}$$
$$\Rightarrow \quad \frac{du}{dx} = -\frac{4x}{\sqrt{1-4x^4+4x^2-1}}$$
$$\Rightarrow \quad \frac{du}{dx} = -\frac{4x}{\sqrt{-4x^4+4x^2}}$$
$$\Rightarrow \quad \frac{du}{dx} = -\frac{2}{\sqrt{1-x^2}}$$
And, $v = \cos^{-1} x$
$$\frac{dv}{dx} = -\frac{1}{\sqrt{1-x^2}}$$
Thus, $\frac{du}{dv} = 2$

Q. 13. If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$ is

(A) $\frac{3}{2}$ **(B)** $\frac{3}{4t}$
(C) $\frac{3}{2t}$ **(D)** $\frac{3}{4}$

Ans. Option (A) is correct.

Explanation: Given that,

$x = t^2$ and $y = t^3$

Then, $\frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = 3t^2$

Thus,

$$\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{3}{2}$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $|\sin x|$ is continuous for all $x \in R$.

Reason (R): $\sin x$ and $|x|$ are continuous in R.

Ans. Option (A) is correct.

Explanation: $\sin x$ and $|x|$ are continuous in R. hence R is true.

Consider the functions $f(x) = \sin x$ and $g(x) = |x|$ both of which are continuous in R.

$$gof(x) = g(f(x)) = g(\sin x) = |\sin x|.$$

Since $f(x)$ and $g(x)$ are continuous in R, $gof(x)$ is also continuous in R.

Hence A is true.

R is the correct explanation of A.

Q. 2. Assertion (A): $f(x) = \tan^2 x$ is continuous at $x = \frac{\pi}{2}$.

Reason (R): $g(x) = x^2$ is continuous at $x = \frac{\pi}{2}$.

Ans. Option (D) is correct.

Explanation: $g(x) = x^2$ is a polynomial function. It is continuous for all $x \in R$.

Hence R is true.

$f(x) = \tan^2 x$ is not defined when $x = \frac{\pi}{2}$.

Therefore $f\left(\frac{\pi}{2}\right)$ does not exist and hence $f(x)$ is not continuous at $x = \frac{\pi}{2}$.

A is false.

Q. 3. Consider the function $f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$

which is continuous at $x = 0$.

Assertion (A): The value of k is – 3.

Reason (R): $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

Ans. Option (A) is correct.

Explanation:

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}.$$

This is the definition for modulus function and hence true.

Hence R is true.

Since f is continuous at $x = 0$,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

Here $f(0) = 3$,

$$\text{LHL} = \lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0^-} \frac{kx}{|x|} = \lim_{x \to 0^-} \frac{kx}{-x} = -k$$

$\therefore \quad -k = 3$ or $k = -3$.

Hence A is true.

R is the correct explanation of A.

Q. 4. Consider the function

$$f(x) = \begin{cases} \frac{x^2 + 3x - 10}{x - 2}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$$

which is continuous at $x = 2$.

Assertion (A): The value of k is 0.

Reason (R): $f(x)$ is continuous at $x = a$, if

$$\lim_{x \to a} f(x) = f(a).$$

Ans. Option (D) is correct.

Explanation:

$f(x)$ is continuous at $x = a$, if $\lim_{x \to a} f(x) = f(a)$.

$\therefore$ R is true.

$$\lim_{x \to 2} f(x) = f(2) = k$$

$$\lim_{x \to 2} \frac{(x+5)(x-2)}{x-2} = k$$

$$\therefore \quad k = 7$$

Hence A is false.

Q. 5. Assertion (A): $|\sin x|$ is continuous at $x = 0$.

Reason (R): $|\sin x|$ is differentiable at $x = 0$.

Ans. Option (C) is correct.

Explanation: Since $\sin x$ and $|x|$ are continuous functions in R, $|\sin x|$ is continuous at $x = 0$.
Hence A is true.

$$|\sin x| = \begin{cases} -\sin x, & \text{if } x < 0 \\ \sin x, & \text{if } x \geq 0 \end{cases}$$

$$f(0) = |\sin 0| = 0$$

$$\text{LHD} = f'(0^-) = \lim_{x \to 0} \frac{-\sin x - 0}{x} = -1$$

$$\text{RHD} = f'(0^+) = \lim_{x \to 0} \frac{\sin x - 0}{x} = 1$$

At $x = 0$, LHD $\neq$ RHD.
So $f(x)$ is not differentiable at $x = 0$.
Hence R is false.

Q. 6. Assertion (A): $f(x) = [x]$ is not differentiable at $x = 2$.
Reason (R): $f(x) = [x]$ is not continuous at $x = 2$.

Ans. Option (A) is correct.

Explanation: $f(x) = [x]$ is not continuous when x is an integer.
So $f(x)$ is not continuous at $x = 2$. Hence R is true.
A differentiable function is always continuous.
Since $f(x) = [x]$ is not continuous at $x = 2$, it is also not differentiable at $x = 2$.
Hence A is true.
R is the correct explanation of A.

Q. 7. Assertion (A): A continuous function is always differentiable.
Reason (R): A differentiable function is always continuous.

Ans. Option (D) is correct.

Explanation: The function $f(x)$ is differentiable at $x = a$, if it is continuous at $x = a$ and
LHD = RHD at $x = a$.
A differentiable function is always continuous.
Hence R is true.
A continuous function need not be always differentiable.
For example, $|x|$ is continuous at $x = 0$, but not differentiable at $x = 0$.
Hence A is false.

Q. 8. Assertion (A): If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, then $\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$
Reason (R): $\sin^{-1}(6x\sqrt{1-9x^2}) = 3\sin^{-1}(2x)$

Ans. Option (C) is correct.

Explanation:
put $3x = \sin\theta$ or $\theta = \sin^{-1} 3x$

$$y = \sin^{-1}(6x\sqrt{1-9x^2}) = \sin^{-1}(\sin 2\theta)$$
$$= 2\theta$$
$$= 2\sin^{-1} 3x$$

$$\therefore \quad \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

A is true. R is false.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

Ms. Remka of city school is teaching chain rule to her students with the help of a flow-chart

The chain rule says that if h and g are functions and $f(x) = g(h(x))$, then

$$f'(x) = (g(h(x)))' = g'(h(x))\, h'(x)$$

- keep the inside
- take derivative of outside

by derivative of the inside

Let $f(x) = \sin x$ and $g(x) = x^3$

Q. 1. $fog(x) =$ _____.

(A) $\sin x^3$ **(B)** $\sin^3 x$
(C) $\sin 3x$ **(D)** $3\sin x$

Ans. Option (A) is correct.

Explanation:

$$fog(x) = f(g(x)) = f(x^3) = \sin(x^3)$$

Q. 2. $gof(x) =$ _____.

(A) $\sin x^3$ **(B)** $\sin^3 x$
(C) $\sin 3x$ **(D)** $3\sin x$

Ans. Option (B) is correct.

Explanation:

$$gof(x) = g(f(x)) = g(\sin x) = \sin^3 x$$

Q. 3. $\frac{d}{dx}(\sin^3 x) =$ _____.

(A) $\cos^3 x$ **(B)** $3\sin x\cos x$

(C) $3\sin^2 x \cos x$ **(D)** $-\cos^3 x$

Ans. Option (C) is correct.

Explanation:

$$\frac{d}{dx}(\sin^3 x) = 3\sin^2 x \frac{d}{dx}(\sin x)$$
$$= 3\sin^2 x \cos x$$

Q. 4. $\frac{d}{dx}\sin x^3$ _____.

(A) $\cos(x^3)$ **(B)** $-\cos(x^3)$

(C) $3x^2 \sin(x^3)$ **(D)** $3x^2 \cos(x^3)$

Ans. Option (D) is correct.

Explanation:

$$\frac{d}{dx}(\sin x^3) = \cos x^3 \frac{d}{dx}(x^3)$$
$$= 3x^2 \cos x^3$$

Q. 5. $\frac{d}{dx}(\sin 2x)$ at $x = \frac{\pi}{2}$ is _____.

(A) 0 **(B)** 1

(C) 2 **(D)** –2

Ans. Option (D) is correct.

Explanation:

$$\frac{d}{dx}(\sin 2x) = \cos 2x \frac{d}{dx}(2x)$$
$$= 2\cos 2x$$
$$\left.\frac{d}{dx}(\sin 2x)\right|_{x=\frac{\pi}{4}} = 2\cos 2 \times \frac{\pi}{2} = 2\cos\pi$$
$$= 2(-1)$$
$$= -2$$

II. Read the following text and answer the following questions on the basis of the same:

A potter made a mud vessel, where the shape of the pot is based on $f(x) = |x-3| + |x-2|$, where $f(x)$ represents the height of the pot.

[CBSE QB 2021]

Q. 1. When $x > 4$ what will be the height in terms of x?

(A) $x - 2$ **(B)** $x - 3$

(C) $2x - 5$ **(D)** $5 - 2x$

Ans. Option (C) is correct.

Explanation: The given function can be written as

$$f(x) = \begin{cases} 5-2x, & \text{if } x < 2 \\ 1, & \text{if } 2 \le x < 3 \\ 2x-5, & \text{if } x \ge 3 \end{cases}$$

When $x > 4$, $f(x) = 2x - 5$

Q. 2. Will the slope vary with x value?

(A) Yes **(B)** No

(C) Can't say **(D)** In complete data

Ans. Option (A) is correct.

Explanation:

$$f'(x) = \begin{cases} -2, & \text{if } x < 2 \\ 0, & \text{if } 2 \le x < 3 \\ 2, & \text{if } x \ge 3 \end{cases}$$

Q. 3. What is $\frac{dy}{dx}$ at $x = 3$

(A) 2

(B) –2

(C) Function is not differentiable

(D) 1

Ans. Option (C) is correct.

Explanation: $f(x)$ is not differentiable at $x = 2$ and $x = 3$.

Q. 4. When the value of x lies between (2, 3) then the function is

(A) $2x - 5$ **(B)** $5 - 2x$

(C) 1 **(D)** 5

Ans. Option (C) is correct.

Explanation: In (2, 3), $f(x) = 1$

Q. 5. If the potter is trying to make a pot using the function $f(x) = [x]$, will he get a pot or not? Why?

(A) Yes, because it is a continuous function

(B) Yes, because it is not continuous

(C) No, because it is a continuous function

(D) No, because it is not continuous

Ans. Option (D) is correct.

Explanation: $[x]$ is not continuous at integral values of x.

❐❐

CHAPTER

6

Term-I

APPLICATIONS OF DERIVATIVES

Syllabus

- ***Applications of derivatives: increasing/decreasing functions, tangents & normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).***

STAND ALONE MCQs (1 Mark each)

Q. 1. A ladder, 5 metre long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10 cm/sec, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 metre from the wall is :

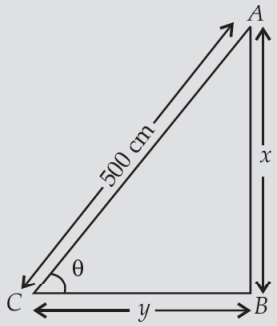
(A) $\frac{1}{10}$ radian/sec **(B)** $\frac{1}{20}$ radian/sec

(C) 20 radian/sec **(D)** 10 radian/sec

Ans. Option (B) is correct.

Explanation : Let the angle between floor and the ladder be θ.

Let $AB = x$ cm and $BC = y$ cm



$\therefore \quad \sin\theta = \frac{x}{500}$ and $\cos\theta = \frac{y}{500}$

$\Rightarrow \quad x = 500\sin\theta$ and $y = 500\cos\theta$

Also, $\frac{dx}{dt} = 10$ cm/s

$\Rightarrow \quad 500.\cos\theta.\frac{d\theta}{dt} = 10$ cm/s

$\Rightarrow \quad \frac{d\theta}{dt} = \frac{10}{500\cos\theta} = \frac{1}{50\cos\theta}$

For $y = 2$ m $= 200$ cm,

$$\frac{d\theta}{dt} = \frac{1}{50.\frac{y}{500}} = \frac{10}{y} = \frac{10}{200} = \frac{1}{20} \text{ rad/s}$$

Q. 2. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then at $x = 3$ the slope of curve is changing at _____ units/sec.

(A) –72 **(B)** –36

(C) 24 **(D)** 48

Ans. Option (A) is correct.

Explanation: Given

curve is $y = 5x - 2x^3$

or $\frac{dy}{dx} = 5 - 6x^2$

or $m = 5 - 6x^2 \quad \left[\text{slope } m = \frac{dy}{dx}\right]$

$$\frac{dm}{dt} = -12x\frac{dx}{dt}$$
$$= -24x$$
$$\left.\frac{dm}{dt}\right|_{x=3} = -72$$

Q. 3. The contentment obtained after eating x units of a new dish at a trial function is given by the function $f(x) = x^3 + 6x^2 + 5x + 3$. The marginal contentment when 3 units of dish are consumed is _____.

(A) 60 **(B)** 68

(C) 24 **(D)** 48

Ans. Option (B) is correct.

Explanation:

$$f(x) = x^3 + 6x^2 + 5x + 3$$
$$\frac{df(x)}{dx} = 3x^2 + 12x + 5$$

At $x = 3$,

Marginal contentment

$$= 3\times(3)^2 + 12\times 3 + 5$$
$$= 27 + 36 + 5$$
$$= 68 \text{ units.}$$

Q. 4. A particle moves along the curve $x^2 = 2y$. The point at which, ordinate increases at the same rate as the abscissa is _____

(A) (1, 2) **(B)** $\left(\frac{1}{2}, 1\right)$

(C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ **(D)** $\left(1, \frac{1}{2}\right)$

Ans. Option (D) is correct.

Explanation:

$$x^2 = 2y \quad \ldots(1)$$
$$\Rightarrow \quad 2x\frac{dx}{dt} = 2\frac{dy}{dt} \quad \left(\text{given } \frac{dy}{dt} = \frac{dx}{dt}\right)$$
$$\Rightarrow \quad 2x\frac{dx}{dt} = 2\frac{dx}{dt}$$
$$\Rightarrow \quad x = 1$$

from (1), $y = \frac{1}{2}$

so point is $\left(1, \frac{1}{2}\right)$

Q. 5. The curve $y = x^{1/5}$ has at (0, 0)

(A) a vertical tangent (parallel to y-axis)

(B) a horizontal tangent (parallel to x-axis)

(C) an oblique tangent

(D) no tangent

Ans. Option (A) is correct.

Explanation : Given that, $y = x^{1/5}$

On differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{5}x^{\frac{1}{5}-1} = \frac{1}{5}x^{-4/5}$$
$$\therefore \left(\frac{dy}{dx}\right)_{(0,0)} = \frac{1}{5}\times(0)^{-4/5} = \infty$$

So, the curve $y = x^{1/5}$ has a vertical tangent at (0, 0), which is parallel to y-axis.

Q. 6. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$ is

(A) $3x - y = 8$ **(B)** $3x + y + 8 = 0$

(C) $x + 3y \pm 8 = 0$ **(D)** $x + 3y = 0$

Ans. Option (C) is correct.

Explanation : We have, the equation of the curve is $3x^2 - y^2 = 8$ ....(i)

Also, the given equation of the line is $x + 3y = 8$.

$$\Rightarrow 3y = 8 - x$$
$$\Rightarrow \quad y = -\frac{x}{3} + \frac{8}{3}$$

Thus, slope of the line is $-\frac{1}{3}$ which should be equal to slope of the equation of normal to the curve.

On differentiating equation (i) with respect to x, we get

$$6x - 2y\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{dy}{dx} = \frac{6x}{2y} = \frac{3x}{y} = \text{Slope of the curve}$$

Now, slope of normal to the curve

$$= -\frac{1}{\left(\frac{dy}{dx}\right)}$$
$$= -\frac{1}{\left(\frac{3x}{y}\right)}$$
$$= -\frac{y}{3x}$$
$$\therefore \quad -\left(\frac{y}{3x}\right) = -\frac{1}{3}$$
$$\Rightarrow \quad -3y = -3x$$
$$\Rightarrow \quad y = x$$

On substituting the value of the given equation of the curve, we get

$$3x^2 - x^2 = 8$$
$$\Rightarrow \quad 2x^2 = 8$$
$$\Rightarrow \quad x^2 = 4$$

$\Rightarrow \quad x = \pm 2$

For $x = 2$

$3(2)^2 - y^2 = 8$

$\Rightarrow \quad y^2 = 4$

$\Rightarrow \quad y = \pm 2$

and for $x = -2$,

$3(-2)^2 - y^2 = 8$

$\Rightarrow \quad y^2 = 4$

$\Rightarrow \quad y = \pm 2$

So, the points at which normal is parallel to the given line are $(\pm 2, \pm 2)$.

Hence, the equation of normal at $(\pm 2, \pm 2)$ is

$$\Rightarrow \quad y - (\pm 2) = -\frac{1}{3}[x - (\pm 2)]$$

$$\Rightarrow 3[y - (\pm 2)] = -[x - (\pm 2)]$$

$$\therefore \quad x + 3y \pm 8 = 0$$

Q. 7. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1, 1), then the value of a is :

(A) 1 **(B)** 0

(C) –6 **(D)** 6

Ans. Option (D) is correct.

Explanation : Given that, $ay + x^2 = 7$ and $x^3 = y$

On differentiating both equations with respect to x, we get

$$a.\frac{dy}{dx} + 2x = 0 \quad \text{and} \quad 3x^2 = \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2x}{a} \quad \text{and} \quad \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{-2}{a} = m_1$$

and $\left(\frac{dy}{dx}\right)_{(1,1)} = 3.1$

$= 3 = m_2$

Since, the curve cuts orthogonally at (1, 1).

$$\therefore \quad m_1 m_2 = -1$$

$$\Rightarrow \quad \left(\frac{-2}{a}\right).3 = -1$$

$$\therefore \quad a = 6$$

Q. 8. The equation of tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses x-axis is :

(A) $x + 5y = 2$ **(B)** $x - 5y = 2$

(C) $5x - y = 2$ **(D)** $5x + y = 2$

Ans. Option (A) is correct.

Explanation : Given that the equation of curve is

$$y(1 + x^2) = 2 - x \qquad ...(i)$$

On differentiating with respect to x, we get

$$\therefore \quad y.(0 + 2x) + (1 + x^2).\frac{dy}{dx} = 0 - 1$$

$$\Rightarrow \quad 2xy + (1 + x^2)\frac{dy}{dx} = -1$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-1 - 2xy}{1 + x^2} \qquad ...(ii)$$

Since, the given curve passes through x-axis,

i.e., $y = 0$

$$\therefore \quad 0(1 + x^2) = 2 - x$$

[By using Eq. (i)]

$$\Rightarrow \quad x = 2$$

So the curve passes through the point (2, 0).

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(2,\,0)} = \frac{-1 - 2 \times 0}{1 + 2^2} = -\frac{1}{5}$$

= Slope of the curve

$\therefore$ Slope of tangent to the curve $= -\frac{1}{5}$

$\therefore$ Equation of tangent to the curve passsing through (2, 0) is

$$y - 0 = -\frac{1}{5}(x - 2)$$

$$\Rightarrow \quad y + \frac{x}{5} = +\frac{2}{5}$$

$$\Rightarrow \quad 5y + x = 2$$

Q. 9. The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to x-axis are :

(A) (2, –2), (–2, –34) **(C)** (2, 34), (–2, 0)

(B) (0, 34), (–2, 0) **(D)** (2, 2), (–2, 34)

Ans. Option (D) is correct.

Explanation : The equation of the curve is given by

$$y = x^3 - 12x + 18$$

On differentiating with respect to x, we get

$$\therefore \quad \frac{dy}{dx} = 3x^2 - 12$$

So, the slope of line parallel to the x-axis,

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 3x^2 - 12 = 0$$

$$\Rightarrow \quad x^2 = \frac{12}{3}$$

$$= 4$$

$$\therefore \quad x = \pm 2$$

For $x = 2$,

$$y = 2^3 - 12 \times 2 + 18$$

$$= 2$$

and for $x = -2$,

$$y = (-2)^3 - 12 \times (-2) + 18$$

$$= 34$$

So, the points are (2, 2) and (−2, 34).

Q. 10. The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets x-axis at :

(A) (0, 1) **(B)** $\left(-\frac{1}{2}, 0\right)$

(C) (2, 0) **(D)** (0, 2)

Ans. Option (B) is correct.

Explanation : The equation of the curve is given by $y = e^{2x}$

Since, it passes through the point (0, 1).

$$\therefore \quad \frac{dy}{dx} = e^{2x}.2$$

$$= 2.e^{2x}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(0,1)} = 2.e^{2.0}$$

$$= 2$$

= Slope of tangent to the curve.

$\therefore$ Equation of tangent is

$$y - 1 = 2(x - 0)$$

$$\Rightarrow \quad y = 2x + 1$$

Since, tangent to the curve $y = e^{2x}$ at the point (0, 1) meets x-axis, *i.e.*, $y = 0$.

$$\therefore \quad 0 = 2x + 1$$

$$\Rightarrow \quad x = -\frac{1}{2}$$

So, the required point is $\left(-\frac{1}{2}, 0\right)$.

Q. 11. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is :

(A) $[-1, \infty)$ **(B)** $[-2, -1]$

(C) $(-\infty, -2]$ **(D)** $[-1, 1]$

Ans. Option (B) is correct.

Explanation : Given that,

$$f(x) = 2x^3 + 9x^2 + 12x - 1$$

$$f'(x) = 6x^2 + 18x + 12$$

$$= 6(x^2 + 3x + 2)$$

$$= 6(x + 2)(x + 1)$$

So, $f'(x) \le 0$, for decreasing.

On drawing number lines as below :

We see that $f'(x)$ is decreasing in $[-2, -1]$.

Q. 12. $y = x(x-3)^2$ decreases for the values of x given by :

(A) $1 < x < 3$ **(B)** $x < 0$

(C) $x > 0$ **(D)** $0 < x < \frac{3}{2}$

Ans. Option (A) is correct.

Explanation : Given that,

$$y = x(x-3)^2$$

$$\therefore \quad \frac{dy}{dx} = x.2(x-3).1 + (x-3)^2.1$$

$$= 2x^2 - 6x + x^2 + 9 - 6x$$

$$= 3x^2 - 12x + 9$$

$$= 3(x^2 - 3x - x + 3)$$

$$= 3(x-3)(x-1)$$

So, $y = x(x-3)^2$ decreases for (1, 3).

[Since, $y' < 0$ for all $x \in (1, 3)$, hence y is decreasing on (1, 3)].

Q. 13. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

(A) increasing in $\left(\pi, \frac{3\pi}{2}\right)$

(B) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(C) decreasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(D) decreasing in $\left(0, \frac{\pi}{2}\right)$

Ans. Option (B) is correct.

Explanation : Given that,

$$f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$$

On differentiating with respect to x, we get

$$f'(x) = 12\sin^2 x.\cos x - 12\sin x.\cos x + 12\cos x$$

$$= 12[\sin^2 x.\cos x - \sin x.\cos x + \cos x]$$

$$= 12\cos x[\sin^2 x - \sin x + 1]$$

$$\Rightarrow f'(x) = 12\cos x[\sin^2 x + 1(1 - \sin x)]$$

$\Rightarrow 1 - \sin x \ge 0$ and $\sin^2 x \ge 0$

$\Rightarrow \sin^2 x + 1 - \sin x \ge 0$

Hence, $f'(x) > 0$, when $\cos x > 0$, *i.e.*, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

So, $f(x)$ is increasing when $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

and $f'(x) < 0$, when $\cos x < 0$, *i.e.*, $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

Hence, $f'(x)$ is decreasing when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Since $\left(\frac{\pi}{2}, \pi\right) \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, $f(x)$ is decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Q. 14. Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$.

(A) $\sin 2x$ **(B)** $\tan x$
(C) $\cos x$ **(D)** $\cos 3x$

Ans. Option (C) is correct.

Explanation : In the given interval $\left(0,\frac{\pi}{2}\right)$
$$f(x) = \cos x$$
On differentiating with respect to x, we get
$$f'(x) = -\sin x$$
which gives $f'(x) < 0$ in $\left(0,\frac{\pi}{2}\right)$
Hence, $f(x) = \cos x$ is decreasing in $\left(0,\frac{\pi}{2}\right)$.

Q. 15. The function $f(x) = \tan x - x$
(A) always increases
(B) always decreases
(C) never increases
(D) sometimes increases and sometimes decreases

Ans. Option (A) is correct.

Explanation : We have,
$$f(x) = \tan x - x$$
On differentiating with respect to x, we get
$$f'(x) = \sec x - 1$$
$$\Rightarrow \quad f'(x) > 0, \forall x \in R$$
So, $f(x)$ always increases.

Q. 16. Let the $f : R \to R$ be defined by $f(x) = 2x + \cos x$, then f :
(A) has a minimum at $x = \pi$
(B) has a maximum, at $x = 0$
(C) is a decreasing function
(D) is an increasing function

Ans. Option (D) is correct.

Explanation : Given that,
$$f(x) = 2x + \cos x$$
Differentiating with respect to x, we get
$$f'(x) = 2 + (-\sin x)$$
$$= 2 - \sin x$$
Since, $f'(x) > 0$, $\forall\, x \in R$
Hence, $f(x)$ is an increasing function.

Q. 17. If x is real, the minimum value of $x^2 - 8x + 17$ is
(A) –1 **(B)** 0
(C) 1 **(D)** 2

Ans. Option (C) is correct.

Explanation : Let,
$$f(x) = x^2 - 8x + 17$$
On differentiating with respect to x, we get
$$f'(x) = 2x - 8$$
So, $f'(x) = 0$
$\Rightarrow \quad 2x - 8 = 0$
So, $f'(x) = 0$
$\Rightarrow \quad 2x - 8 = 0$
$\Rightarrow \quad 2x = 8$
$\therefore \quad x = 4$

Now, Again on differentiating with respect to x, we get
$$f''(x) = 2 > 0, \forall x$$
So, $x = 4$ is the point of local minima.
Minimum value of $f(x)$ at $x = 4$
$$f(4) = 4 \cdot 4 - 8 \cdot 4 + 17 = 1$$

Q. 18. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0, 9] is
(A) 126 **(B)** 0
(C) 135 **(D)** 160

Ans. Option (B) is correct.

Explanation : Given that, the smallest value of polynomial is $f(x) = x^3 - 18x^2 + 96x$
On differentiating with respect to x, we get
$$f'(x) = 3x^2 - 36x + 96$$
So,
$$f'(x) = 0$$
$$\Rightarrow \quad 3x^2 - 36x + 96 = 0$$
$$\Rightarrow 3(x^2 - 12x + 32) = 0$$
$$\Rightarrow \quad (x - 8)(x - 4) = 0$$
$$\Rightarrow \quad x = 8, 4 \in [0, 9]$$
We shall now calculate the value of $f(x)$ at these points and at the end points of the interval [0, 9], *i.e.*, at $x = 4$ and $x = 8$ and at $x = 0$ and at $x = 9$.
$$f(4) = 4^3 - 18 \times 4^2 + 96 \times 4$$
$$= 64 - 288 + 384$$
$$= 160$$
$$f(8) = 8^3 - 18 \times 8^2 + 96 \times 8$$
$$= 128$$
$$f(9) = 9^3 - 18 \times 9^2 + 96 \times 9$$
$$= 729 - 1458 + 864$$
$$= 135$$
$$\text{and } f(0) = 0^3 - 18 \times 0^2 + 96 \times 0$$
$$= 0$$
Thus, we conclude that absolute minimum value of $f(x)$ in [0, 9] is 0 occurring at $x = 0$.

Q. 19. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has
(A) two points of local maximum
(B) two points of local minimum
(C) one maxima and one minima
(D) no maxima or minima

Ans. Option (C) is correct.

Explanation : We have,

$$f(x) = 2x^3 - 3x^2 - 12x + 4$$
$$f'(x) = 6x^2 - 6x - 12$$

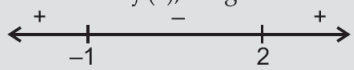
Now, $f'(x) = 0$

$\Rightarrow \quad 6(x^2 - x - 2) = 0$

$\Rightarrow 6(x+1)(x-2) = 0$

$\Rightarrow \quad x = -1$ and $x = +2$

On number line for $f'(x)$, we get



Hence, $x = -1$ is point of local maxima and $x = 2$ is point of local minima.

So, $f(x)$ has one maxima and one minima.

Q. 20. The maximum value of $\sin x . \cos x$ is

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$

(C) $\sqrt{2}$ **(D)** $2\sqrt{2}$

Ans. Option (B) is correct.

Explanation : Let us assume that,

$$f(x) = \sin x . \cos x$$

Now, we know that

$$\sin x . \cos x = \frac{1}{2}\sin 2x$$

$$\therefore \quad f'(x) = \frac{1}{2}.\cos 2x.2 = \cos 2x$$

Now, $f'(x) = 0$

$\Rightarrow \quad \cos 2x = 0$

$$\Rightarrow \quad \cos 2x = \cos\frac{\pi}{2}$$

$$\Rightarrow \quad x = \frac{\pi}{4}$$

Also, $f''(x) = \frac{d}{dx}.\cos 2x = -2.\sin 2x$

$$\therefore \left[f''(x)\right]_{\text{at } x=\frac{\pi}{4}} = -2\sin 2.\frac{\pi}{4}$$

$$= -2\sin\frac{\pi}{2}$$
$$= -2 < 0$$

$\therefore x \ \frac{\pi}{4}$ is point of maxima.

$$f\left(\frac{\pi}{4}\right) = \frac{1}{2}.\sin.2.\frac{\pi}{4}$$
$$= \frac{1}{2}$$

Q. 21. Maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is :

(A) 0 **(B)** 12

(C) 16 **(D)** 32

Ans. Option (B) is correct.

Explanation : Given that,

$$y = -x^3 + 3x^2 + 9x - 27$$

$$\therefore \quad \frac{dy}{dx} = -3x^2 + 6x + 9$$

$$= \text{Slope of the curve}$$

and $\quad \frac{d^2y}{dx^2} = -6x + 6 = -6(x-1)$

$$\therefore \quad \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \quad -6(x-1) = 0$$

$$\Rightarrow \quad x = 1 > 0$$

Now, $\quad \frac{d^3y}{dx^3} = -6 < 0$

So, the maximum slope of given curve is at $x = 1$.

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(x=1)} = -3\times 1^2 + 6\times 1 + 9$$

$$= 12$$

Q. 22. The maximum value of $\left(\frac{1}{x}\right)^x$ is :

(A) e **(B)** e^e

(C) $e^{1/e}$ **(D)** $\left(\frac{1}{e}\right)^{1/e}$

Ans. Option (C) is correct.

Explanation:

Let $\quad y = \left(\frac{1}{x}\right)^x$

$$\Rightarrow \quad \log y = x.\log\frac{1}{x}$$

$$\therefore \quad \frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{\frac{1}{x}}.\left(-\frac{1}{x^2}\right) + \log\frac{1}{x}.1$$

$$= -1 + \log\frac{1}{x}$$

$$\therefore \quad \frac{dy}{dx} = \left(\log\frac{1}{x} - 1\right).\left(\frac{1}{x}\right)^x$$

Now, $\quad \frac{dy}{dx} = 0$

$$\Rightarrow \quad \log\frac{1}{x} = 1 = \log e$$

$$\Rightarrow \quad \frac{1}{x} = e$$

$$\Rightarrow \quad x = \frac{1}{e}$$

Hence, the maximum value of $f\left(\frac{1}{e}\right) = (e)^{1/e}$.

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees.

Assertion (A): The marginal revenue when $x = 5$ is 66.

Reason (R): Marginal revenue is the rate of change of total revenue with respect to the number of items sold at an instance.

Ans. Option (A) is correct.

Marginal revenue is the rate of change of total revenue with respect to the number of items sold at an instance. Therefore R is true.

$$R'(x) = 6x + 36$$

$$R'(5) = 66$$

$\therefore$ A is true.

R is the correct explanation of A.

Q. 2. The radius r of a right circular cylinder is increasing at the rate of 5 cm/min and its height h, is decreasing at the rate of 4 cm/min.

Assertion (A): When $r = 8$ cm and $h = 6$ cm, the rate of change of volume of the cylinder is 224π cm^3/min

Reason (R): The volume of a cylinder is $V = \frac{1}{3}\pi r^2 h$

Ans. Option (C) is correct.

Explanation: The volume of a cylinder is $V = \pi r^2 h$. So R is false.

$$\frac{dr}{dt} = 5 \text{ cm/min}, \quad \frac{dh}{dt} = -4 \text{ cm/min}$$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right)$$

$$\frac{dV}{dt} = \pi[64 \times (-4) + 2 \times 6 \times 8 \times 5]$$

$$\left.\frac{dV}{dt}\right)_{r=8,\ h=6} = 224\pi \text{ cm}^3/\text{min}$$

$\therefore$ Volume is increasing at the rate of 224π cm^3/min.

$\therefore$ A is true.

Q. 3. Assertion (A): For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then at $x = 3$ the slope of curve is decreasing at 36 units/sec.

Reason (R): The slope of the curve is $\frac{dy}{dx}$.

Ans. Option (D) is correct.

Explanation: The slope of the curve $y = f(x)$ is $\frac{dy}{dx}$. R is true.

Given curve is $y = 5x - 2x^3$

or $\frac{dy}{dx} = 5 - 6x^2$

or $m = 5 - 6x^2 \quad \left[\text{slope } m = \frac{dy}{dx}\right]$

$$\frac{dm}{dt} = -12x\frac{dx}{dt} = -24x \quad \left[\because \frac{dx}{dt} = 2 \text{ units/sec}\right]$$

$$\left.\frac{dm}{dt}\right|_{x=3} = -72$$

Rate of Change of the slope is decreasing by 72 units/s.

A is false.

Q. 4. A particle moves along the curve $6y = x^3 + 2$.

Assertion (A): The curve meets the Y axis at three points.

Reason (R): At the points $\left(2, \frac{5}{3}\right)$ and $(-2, -1)$ the ordinate changes two times as fast as the abscissa.

Ans. Option (D) is correct.

Explanation:

On Y axis, $x = 0$. The curve meets the Y axis at only one point, *i.e.*, $\left(0, \frac{1}{3}\right)$.

Hence A is false.

$$6y = x^3 + 2$$

or $$6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

Given, $$\frac{dy}{dt} = 2\frac{dx}{dt}$$

or $$12 = 3x^2$$

or $$x = \pm 2$$

Put $x = 2$ and -2 in the given equation to get y

$\therefore$ The points are $\left(2, \frac{5}{3}\right), (-2, -1)$

R is true.

Q. 5. Assertion (A): At $x = \frac{\pi}{6}$, the curve $y = 2\cos^2(3x)$ has a vertical tangent.

Reason (R): The slope of tangent to the curve $y = 2\cos^2(3x)$ at $x = \frac{\pi}{6}$ is zero.

Ans. Option (D) is correct.

Explanation:

Given $y = 2\cos^2(3x)$

$$\frac{dy}{dx} = 2 \times 2 \times \cos(3x) \times (-\sin 3x) \times 3$$

$$\frac{dy}{dx} = -6\sin 6x$$

$$\left[\frac{dy}{dx}\right]_{x=\frac{\pi}{6}} = -6\sin\pi$$

$$= -6 \times 0$$

$$= 0$$

$\therefore$ R is true.

Since the slope of tangent is zero, the tangent is parallel to the X-axis. That is the curve has a horizontal tangent at $x = \frac{\pi}{6}$. Hence A is false.

Q. 6. Assertion (A): The equation of tangent to the curve $y = \sin x$ at the point $(0, 0)$ is $y = x$.

Reason (R): If $y = \sin x$, then $\frac{dy}{dx}$ at $x = 0$ is 1.

Ans. Option (A) is correct.

Explanation: Given $y = \sin x$

$$\frac{dy}{dx} = \cos x$$

Slope of tangent at $(0, 0) = \left[\frac{dy}{dx}\right]_{(0,0)}$

$$= \cos 0°$$

$$= 1$$

$\therefore$ R is true.

Equation of tangent at $(0, 0)$ is

$$y - 0 = 1(x - 0)$$

$\Rightarrow$ $y = x$.

Hence A is true.

R is the correct explanation of A.

Q. 7. Assertion (A): The slope of normal to the curve $x^2 + 2y + y^2 = 0$ at $(-1, 2)$ is -3.

Reason (R): The slope of tangent to the curve $x^2 + 2y + y^2 = 0$ at $(-1, 2)$ is $\frac{1}{3}$.

Ans. Option (A) is correct.

Explanation:

Given $x^2 + 2y + y^2 = 0$

$$2x + 2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2 + 2y) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2(1+y)}$$

$$= -\frac{x}{1+y}$$

Slope of tangent at $(-1, 2)$

$$\left[\frac{dy}{dx}\right]_{(-1,2)} = \frac{-(-1)}{1+2}$$

$$= \frac{1}{3}$$

Hence R is true.

Slope of normal at $(-1, 2)$

$$= \frac{-1}{\text{Slope of tangent}}$$

$$= -3.$$

Hence A is true.

R is the correct explanation for A.

Q. 8. The equation of tangent at $(2, 3)$ on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$.

Assertion (A): The value of a is ± 2

Reason (R): The value of b is ± 7

Ans. Option (C) is correct.

Explanation:

$$y^2 = ax^3 + b$$

Differentiate with respect to x,

$$2y\frac{dy}{dx} = 3ax^2$$

or $$\frac{dy}{dx} = \frac{3ax^2}{2y}$$

or $$\frac{dy}{dx} = \frac{3ax^2}{\pm 2\sqrt{ax^3 + b}} \quad [\because y^2 = ax^3 + b]$$

or $$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{3a(2)^2}{\pm 2\sqrt{a(2)^3 + b}}$$

$$= \frac{12a}{\pm 2\sqrt{8a + b}}$$

$$= \frac{6a}{\pm\sqrt{8a + b}}$$

Since $(2, 3)$ lies on the curve

$$y^2 = ax^3 + b$$

or $$9 = 8a + b \quad \ldots(i)$$

Also from equation of tangent

$$y = 4x - 5$$

slope of the tangent $= 4$

$\therefore$ $\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{6a}{\pm\sqrt{8a + b}}$ becomes

$$4 = \frac{6a}{\pm\sqrt{9}} \quad \{\text{from (i)}\}$$

$$\therefore \quad 4 = \frac{6a}{\pm 3}$$

$$\therefore \quad 4 = \frac{6a}{3} \text{ or } 4 = \frac{6a}{-3}$$

either, $a = 2$ or $a = -2$

For $a = 2$,

$9 = 8(2) + b$

or $b = -7$

$\therefore$ $a = 2$ and $b = -7$

and for $a = -2$,

$9 = 8(-2) + b$

or $b = 25$

or $a = -2$ and $b = 25$

Hence A is true and R is false.

Q. 9. Assertion (A): The function $f(x) = x^3 - 3x^2 + 6x - 100$ is strictly increasing on the set of real numbers.
Reason (R): A strictly increasing function is an injective function.

Ans. Option (B) is correct.

Explanation:

$$f(x) = x^3 - 3x^2 + 6x - 100$$

$$f'(x) = 3x^2 - 6x + 6$$

$$= 3[x^2 - 2x + 2]$$

$$= 3[(x-1)^2 + 1]$$

since $f'(x) > 0$; $x \in R$

$f(x)$ is strictly increasing on R.

Hence A is true.

For a strictly increasing function,

$x_1 > x_2$

$\Rightarrow \quad f(x_1) > f(x_2)$

i.e.; $x_1 = x_2$

$\Rightarrow \quad f(x_1) = f(x_2)$

Hence, a strictly increasing function is always an injective function.

So R is true.

But R is not the correct explanation of A.

Q. 10. Consider the function $f(x) = \sin^4 x + \cos^4 x$.

Assertion (A): $f(x)$ is increasing in $\left[0, \frac{\pi}{4}\right]$

Reason (R): $f(x)$ is decreasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Ans. Option (B) is correct.

Explanation:

$$f(x) = \sin^4 x + \cos^4 x$$

or $$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$$

$$= -4\sin x\cos x\,[-\sin^2 x + \cos^2 x]$$

$$= -2\sin 2x \cos 2x$$

$$= -\sin 4x$$

On equating,

$f'(x) = 0$

or $-\sin 4x = 0$

or $4x = 0, \pi, 2\pi, \ldots\ldots$

or $x = 0, \frac{\pi}{4}, \frac{\pi}{2}$.

Sub-intervals are $\left[0, \frac{\pi}{4}\right], \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

or $f'(x) < 0$ in $\left[0, \frac{\pi}{4}\right]$

or $f(x)$ is decreasing in $\left[0, \frac{\pi}{4}\right]$

and, $f'(x) > 0$ in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$\therefore f'(x)$ is increasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

Both A and R are true. But R is not the correct explanation of A.

Q. 11. Assertion (A): The function $y = [x(x-2)]^2$ is increasing in $(0, 1) \cup (2, \infty)$

Reason (R): $\frac{dy}{dx} = 0$, when $x = 0, 1, 2$.

Ans. Option (B) is correct.

Explanation:

$$y = [x(x-2)]^2$$

$$= [x^2 - 2x]^2$$

$$\therefore \quad \frac{dy}{dx} = 2(x^2 - 2x)(2x - 2)$$

or $$\frac{dy}{dx} = 4x(x-1)(x-2)$$

On equating $\frac{dy}{dx} = 0$,

$4x(x-1)(x-2) = 0 \Rightarrow x = 0, x = 1, x = 2$

$\therefore$ Intervals are $(-\infty, 0), (0,1), (1,2), (2, \infty)$

Since, $\frac{dy}{dx} > 0$ in $(0,1)$ or $(2, \infty)$

$\therefore f(x)$ is increasing in $(0,1) \cup (2, \infty)$

Both A and R are true. But R is not the correct explanation of A.

Q. 12. Assertion (A): The function $y = \log(1 + x) - \frac{2x}{2+x}$ is a decreasing function of x throughout its domain.
Reason (R): The domain of the function

$f(x) = \log(1 + x) - \frac{2x}{2+x}$ is $(-1, \infty)$

Ans. Option (D) is correct.

Explanation:

$\log (1 + x)$ is defined only when $x + 1 > 0$ or $x > -1$.

Hence R is true.

$y = \log(1+x) - \frac{2x}{2+x}$

Diff. w.r.t. 'x',

$$\frac{dy}{dx} = \frac{1}{1+x} - \frac{[(2+x)(2)-2x]}{(2+x)^2}$$
$$= \frac{1}{1+x} - \frac{[4-2x-2x]}{(2+x)^2}$$
$$= \frac{1}{1+x} - \frac{4}{(2+x)^2}$$
$$= \frac{(2+x)^2 - 4(1+x)}{(2+x)^2(1+x)}$$
$$= \frac{4+x^2+4x-4-4x}{(2+x)^2(1+x)}$$
$$= \frac{x^2}{(2+x)^2(1+x)}$$

For increasing function,

$$\frac{dy}{dx} \geq 0$$

or $\frac{x^2}{(2+x)^2(x+1)} \geq 0$

or $\frac{(2+x)^2(x+1)x^2}{(2+x)^4(x+1)^2} \geq 0$

or $(2+x)^2(x+1)x^2 \geq 0$

When $x > -1$,

$\frac{dy}{dx}$ is always greater than zero.

$\therefore \quad y = \log(1+x) - \frac{2x}{2+x}$

is always increasing throughout its domain.

Hence A is false.

Q. 13. The sum of surface areas (S) of a sphere of radius 'r' and a cuboid with sides $\frac{x}{3}$, x and $2x$ is a constant.

Assertion (A): The sum of their volumes (V) is minimum when x equals three times the radius of the sphere.

Reason (R): V is minimum when $r = \sqrt{\frac{S}{54+4\pi}}$

Ans. Option (A) is correct.

Explanation:

Given $S = 4\pi r^2 + 2\left[\frac{x^2}{3} + 2x^2 + \frac{2x^2}{3}\right]$

$S = 4\pi r^2 + 6x^2$

or $x^2 = \frac{S - 4\pi r^2}{6}$

and $V = \frac{4}{3}\pi r^3 + \frac{2x^3}{3}$

$\therefore \quad V = \frac{4}{3}\pi r^3 + \frac{2}{3}\left(\frac{S-4\pi r^2}{6}\right)^{3/2}$

$\frac{dV}{dr} = 4\pi r^2 + \left(\frac{S-4\pi r^2}{6}\right)^{1/2}\left(\frac{-8\pi r}{6}\right)$

$\frac{dV}{dr} = 0$

or $r = \sqrt{\frac{S}{54+4\pi}}$

Now $\frac{d^2V}{dr^2} = 8\pi r + \left(\frac{-8\pi}{6}\right)\left(\frac{S-4\pi r^2}{6}\right)^{1/2} + \frac{1}{2}\left(\frac{S-4\pi r^2}{6}\right)^{-1/2}\cdot\left(\frac{-8\pi r}{6}\right)$

at $r = \sqrt{\frac{S}{54+4\pi}}$; $\frac{d^2V}{dr^2} > 0$

$\therefore$ for $r = \sqrt{\frac{S}{54+4\pi}}$ volume is minimum

i.e., $r^2(54 + 4\pi) = S$

or $r^2(54 + 4\pi) = 4\pi r^2 + 6x^2$

or $6x^2 = 54r^2$

or $x^2 = 9r^2$

or $x = 3r$

Hence both A and R are true.

R is the correct explanation of A.

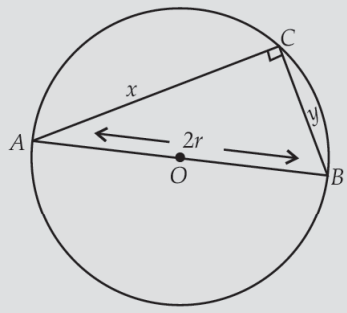
Q. 14. AB is the diameter of a circle and C is any point on the circle.

Assertion (A): The area of ΔABC is maximum when it is isosceles.

Reason (R): ΔABC is a right-angled triangle.

Ans. Option (A) is correct.

Explanation:



Let the sides of rt. ΔABC be x and y.

$\therefore \quad x^2 + y^2 = 4r^2$

and $A = \text{Area of } \Delta = \frac{1}{2}xy$

Let, $S = A^2$

$= \frac{1}{4}x^2y^2$

$$= \frac{1}{4}x^2(4r^2 - x^2)$$

$$= \frac{1}{4}(4r^2x^2 - x^4)$$

$\therefore \quad \frac{dS}{dx} = \frac{1}{4}[8r^2x - 4x^3]$

or $\quad \frac{dS}{dx} = 0$

or $\quad x^2 = 2r^2$ or $x = \sqrt{2}r$

and $\quad y^2 = 4r^2 - 2r^2 = 2r^2$

or $\quad y = \sqrt{2}r$

i.e., $\quad x = y$ and $\frac{d^2S}{dx^2} = (2r^2 - 3x^2)$

$= 2r^2 - 6r^2 < 0$

or Area is maximum, when Δ is isosceles.

Hence A is true.

Angle in a semicircle is a right angle.

$\therefore \angle C = 90°$

$\Rightarrow \Delta ABC$ is a right-angled triangle.

$\therefore$ R is true.

R is the correct explanation of A.

Q. 15. A cylinder is inscribed in a sphere of radius R.

Assertion (A): Height of the cylinder of maximum volume is $\frac{2R}{\sqrt{3}}$ units.

Reason (R): The maximum volume of the cylinder is $\frac{4\pi R^3}{\sqrt{3}}$ cubic units.

Ans. Option (C) is correct.

Explanation: Let the radius and height of cylinder be r and h respectively

$\therefore \quad V = \pi r^2 h$ ...(i)

But $\quad r^2 = R^2 - \frac{h^2}{4}$

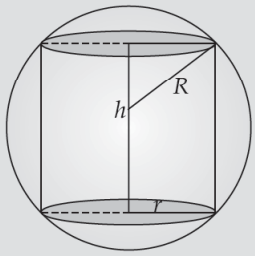
$\therefore \quad \pi h\left(R^2 - \frac{h^2}{4}\right) = \pi\left(R^2h - \frac{h^3}{4}\right)$

or $\quad \frac{dV}{dh} = \pi\left(R^2 - \frac{3h^2}{4}\right)$

For maximum or minimum

$\therefore \quad \frac{dV}{dh} = 0$ or $h^2 = \frac{4R^2}{3}$

or $\quad h = \frac{2R}{\sqrt{3}}$



and $\quad \frac{d^2V}{dh^2} = \pi\left(-\frac{6h}{4}\right) < 0$

Maximum volume $= \pi \cdot \left[R^2 \cdot \frac{2R}{\sqrt{3}} - \frac{1}{4}\left(\frac{2R}{\sqrt{3}}\right)^3\right]$

$= \frac{4\pi R^3}{3\sqrt{3}}$ cubic units

Hence A is true and R is false.

Q. 16. Assertion (A): The altitude of the cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Reason (R): The maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

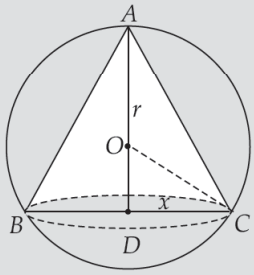
Ans. Option (B) is correct.

Explanation: Let radius of cone be x and its height be h.

$\therefore \quad OD = (h - r)$

Volume of cone

$(V) = \frac{1}{3}\pi x^2 h$ ...(i)



In ΔOCD, $x^2 + (h - r)^2 = r^2$ or $x^2 = r^2 - (h - r)^2$

$\therefore \quad V = \frac{1}{3}\pi h\{r^2 - (h - r)^2\}$

$= \frac{1}{3}\pi(-h^3 + 2h^2r)$

or $\quad \frac{dV}{dh} = \frac{\pi}{3}(-3h^2 + 4hr)$

$\therefore \quad \frac{dV}{dh} = 0$ or $h = \frac{4r}{3}$

$$\frac{d^2V}{dh^2} = \frac{\pi}{3}(-6h+4r)$$
$$= \frac{\pi}{3}\left(-6\left(\frac{4r}{3}\right)+4r\right)$$
$$= -\frac{4\pi r}{3} < 0$$

$\therefore$ at $h = \frac{4r}{3}$, Volume is maximum

Maximum volume
$$= \frac{1}{3}\pi\cdot\left\{-\left(\frac{4r}{3}\right)^3 + 2\left(\frac{4r}{3}\right)^2 r\right\}$$
$$= \frac{8}{27}\cdot\left(\frac{4}{3}\pi r^3\right)$$
$$= \frac{8}{27} \text{ (volume of sphere)}$$

Hence both A and R are true.
R is not the correct explanation of A.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions, on the basis of the same:

The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

[CBSE QB 2021]

Q. 1. The rate of growth of the plant with respect to sunlight is _____ .

(A) $4x - \left(\frac{1}{2}\right)x^2$ **(B)** $4 - x$

(C) $x - 4$ **(D)** $x - \frac{1}{2}x^2$

Ans. Option (B) is correct.

Explanation:

$$y = 4x - \frac{1}{2}x^2$$

$\therefore$ rate of growth of the plant with respect to sunlight
$$= \frac{dy}{dx}$$
$$= \frac{d}{dx}\left[4x - \frac{1}{2}x^2\right]$$
$$= (4-x) \text{ cm / day}$$

Q. 2. What is the number of days it will take for the plant to grow to the maximum height?

(A) 4 **(B)** 6

(C) 7 **(D)** 10

Ans. Option (A) is correct.

Explanation:

$$\frac{dy}{dx} = 4 - x$$

The number of days it will take for the plant to grow to the maximum height,
$$\frac{dy}{dx} = 0$$
$$4 - x = 0$$
$$x = 4 \text{ Days.}$$

Q. 3. What is the maximum height of the plant?

(A) 12 cm **(B)** 10 cm

(C) 8 cm **(D)** 6 cm

Ans. Option (C) is correct.

Explanation: We have, number of days for maximum height of plant
$$= 4 \text{ Days}$$
$\therefore$ Maximum height of plant

$\Rightarrow$ $y_{(x=4)} = 4\times4 - \frac{1}{2}\times4\times4$
$$= 16 - 8$$
$$= 8 \text{ cm}$$

Q. 4. What will be the height of the plant after 2 days?

(A) 4 cm **(B)** 6 cm

(C) 8 cm **(D)** 10 cm

Ans. Option (B) is correct.

Explanation: Height of plant after 2 days

$= y_{(x=2)} = 4\times2 - \frac{1}{2}\times2\times2$
$$= 8 - 2$$
$$= 6 \text{ cm}$$

Q. 5. If the height of the plant is 7/2 cm, the number of days it has been exposed to the sunlight is _____ .

(A) 2 **(B)** 3
(C) 4 **(D)** 1

Ans. Option (D) is correct.

Explanation:

Given, $y = \frac{7}{2}$

i.e., $4x - \frac{1}{2}x^2 = \frac{7}{2}$

$8x - x^2 = 7$

$x^2 - 8x + 7 = 0$

$x^2 - 7x - x + 7 = 0$

$x(x-7) - (x-7) = 0$

$x = 1, 7$

We will take $x = 1$, because it will take 4 days for the plant to grow to the maximum height i.e. 8 cm and $\frac{7}{2}$ cm is not maximum height so, it will take less than 4 days. *i.e.*, 1 Day.

II. Read the following text and answer the following questions on the basis of the same:

$P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company. **[CBSE QB 2021]**

Q. 1. What will be the production when the profit is maximum?

(A) 37,500 **(B)** 12.5
(C) – 12.5 **(D)** – 37,500

Ans. Option (B) is correct.

Explanation: We, have

$P(x) = -5x^2 + 125x + 37500$

$P'(x) = -10x + 125$

For maximum profit

$P'(x) = 0$

$-10x + 125 = 0$

$-10x = -125$

$x = \frac{125}{10}$

$= 12.5$

Q. 2. What will be the maximum profit?

(A) ₹ 38,28,125 **(B)** ₹ 38,281.25
(C) ₹ 39,000 **(D)** None of these

Ans. Option (B) is correct.

Explanation: Maximum profit

$= P(12.5)$

$= -5(12.5)^2 + 125 \times 12.5 + 37500$

$= -781.25 + 1562.5 + 37500$

$= 38,281.25$

Q. 3. Check in which interval the profit is strictly increasing .

(A) (12.5, ∞)
(B) for all real numbers
(C) for all positive real numbers
(D) (0, 12.5)

Ans. Option (D) is correct.

Q. 4. When the production is 2 units what will be the profit of the company?

(A) 37,500 **(B)** 37,730
(C) 37,770 **(D)** None of these

Ans. Option (B) is correct.

Explanation: When production is 2 units, then profit of company $= P(2)$

$= -5 \times 2^2 + 125 \times 2 + 37500$

$= -20 + 250 + 37500$

$= 37,730$

Q. 5. What will be production of the company when the profit is ₹ 38,250?

(A) 15 **(B)** 30
(C) 10
(D) data is not sufficient to find

Ans. Option (C) is correct.

Explanation:

Profit $= 38,250$

i.e., $-5x^2 + 125x + 37,500 = 38,250$

$5x^2 - 125x + 750 = 0$

$x^2 - 25x + 150 = 0$

$x(x-15) - 10(x-15) = 0$

$(x-10)(x-15) = 0$

$x = 10, 15$

$P(x) = -5x^2 + 125x + 37500$

$P(10) = -5 \times 10^2 + 125 \times 10 + 37500$

$= -500 + 1250 + 37500$

$= ₹ 38,250$

Hence, production of company is 10 units when the profit is ₹38250.

III. Read the following text and answer the following questions on the basis of the same:

The shape of a toy is given as $f(x) = 6(2x^4 - x^2)$. To make the toy beautiful 2 sticks which are perpendicular to each other were placed at a point (2, 3), above the toy. **[CBSE QB-2021]**

Q. 1. Which value from the following may be abscissa of critical point?

(A) $\pm 1/4$

(B) ± 12

(C) ± 1

(D) None of these

Ans. Option (B) is correct.

Q. 2. Find the slope of the normal based on the position of the stick.

(A) 360

(B) –360

(C) $\frac{1}{360}$

(D) $\frac{-1}{360}$

Ans. Option (D) is correct.

Explanation: Slope of the normal based on the position of the stick

$$= \frac{-1}{f'(x)}$$

$$f'(x) = 6[8x^3 - 2x]$$

$$f'(2) = 6[8 \times 8 - 2 \times 2]$$

$$= 6[64 - 4]$$

$$= 360$$

$$\therefore \quad \text{Slope} = \frac{-1}{360}$$

Q. 3. What will be the equation of the tangent at the critical point if it passes through (2, 3)?

(A) $x + 360y = 1082$

(B) $y = 360x - 717$

(C) $x = 717y + 360$

(D) None of these

Ans. Option (B) is correct.

Explanation: We have

$$\left.\frac{dy}{dx}\right]_{(2,\,3)} = 360$$

$$\therefore \quad (y - y') = \frac{dy}{dx}(x - x')$$

$$(y - 3) = 360\,(x - 2)$$

$$y - 3 = 360x - 720$$

$$y = 360x - 717$$

Q. 4. Find the second order derivative of the function at $x = 5$.

(A) 598 **(B)** 1,176

(C) 3,588 **(D)** 3,312

Ans. Option (C) is correct.

Explanation:

$$f(x) = 6(2x^4 - x^2)$$

$$f'(x) = 6[8x^3 - 2x]$$

$$f''(x) = 6[24x^2 - 2]$$

$$f''(5) = 6[24 \times 25 - 2]$$

$$= 6[600 - 2]$$

$$= 3588$$

Q. 5. At which of the following intervals will $f(x)$ be increasing?

(A) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

(B) $\left(\frac{-1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

(C) $\left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

(D) $\left(-\infty, \frac{-1}{2}\right) \cup \left(0, \frac{1}{2}\right)$

Ans. Option (B) is correct.

Explanation: For increasing

$$f'(x) > 0$$

$$6(8x^3 - 2x) > 0$$

i.e., $x(4x^2 - 1) > 0$

$\Rightarrow \quad 4x^2 - 1 > 0$

and $x > 0$

$$4x^2 > 1$$

$\Rightarrow \quad x^2 > \frac{1}{4}$

$\Rightarrow \quad x > \frac{1}{2}$

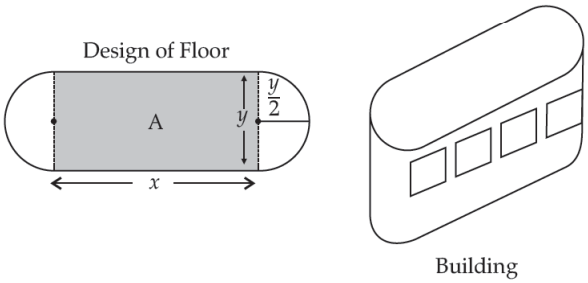
and $x > -\frac{1}{2}$

i.e., $x \in \left(\frac{-1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

VI. Read the following text and answer the following questions, on the basis of the same:

An architect designs a building for a multi-national company. The floor consists of a rectangular region

with semicircular ends having a perimeter of 200 m as shown below:



Q. 1. If x and y represents the length and breadth of the rectangular region, then the relation between the variables is :

(A) $x + \pi y = 100$ **(B)** $2x + \pi y = 200$

(C) $\pi x + y = 50$ **(D)** $x + y = 100$

Ans. Option (B) is correct.

Explanation:

$$\text{Perimeter} = x + x + \frac{\pi y}{2} + \frac{\pi y}{2}$$

$$200 = 2x + \frac{2\pi y}{2}$$

$$200 = 2x + \pi y \quad \text{...(i)}$$

Q. 2. The area of the rectangular region A expressed as a function of x is :

(A) $\frac{2}{\pi}(100x - x^2)$ **(B)** $\frac{1}{\pi}(100x - x^2)$

(C) $\frac{x}{\pi}(100 - x)$ **(D)** $\pi y^2 + \frac{2}{\pi}(100x - x^2)$

Ans. Option (A) is correct.

Explanation:

$$\text{Area (A)} = x \times y$$

$$= x \times \left(\frac{200 - 2x}{\pi}\right) \quad \text{[from (i)]}$$

$$= \frac{2}{\pi}[100x - x^2] \quad \text{...(ii)}$$

Q. 3. The maximum value of area A is :

(A) $\frac{\pi}{3200}\text{m}^2$ **(B)** $\frac{3200}{\pi}\text{m}^2$

(C) $\frac{5000}{\pi}\text{m}^2$ **(D)** $\frac{1000}{\pi}\text{m}^2$

Ans. Option (C) is correct.

Explanation:

$$\frac{dA}{dx} = \frac{2}{\pi}[100 - 2x]$$

$$\frac{dA}{dx} = \frac{4}{\pi}[500 - x]$$

For maxima,

$$\frac{dA}{dx} = 0$$

$$x = 50 \quad \text{...(i)}$$

$$A = \frac{2}{\pi}[100 \times 50 - 50 \times 50] \quad \text{[from (ii)]}$$

$$= \frac{2}{\pi}[5000 - 2500]$$

$$= \frac{2}{\pi} \times 2500$$

$$= \frac{5000}{\pi}\ \text{m}^2$$

Q. 4. The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the value of x should be

(A) 0 m **(B)** 30 m

(C) 50 m **(D)** 80 m

Ans. Option (A) is correct.

Q. 5. The extra area generated if the area of the whole floor is maximized is :

(A) $\frac{3000}{\pi}\text{m}^2$

(B) $\frac{5000}{\pi}\text{m}^2$

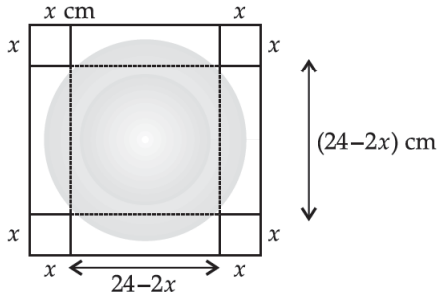
(C) $\frac{7000}{\pi}\text{m}^2$

(D) No change. Both areas are equal.

Ans. Option (D) is correct.

V. Read the following text and answer the following questions. On the basis of the same:

An open box is to be made out of a piece of cardboard measuring (24 cm × 24 cm) by cutting of equal squares from the corners and turning up the sides. **[CBSE SQP 2020-21]**



Q. 1. Find the volume of that open box ?

(A) $4x^3 - 96x^2 + 576x$ **(B)** $4x^3 + 96x^2 - 576x$

(C) $2x^3 - 48x^2 + 288x$ **(D)** $2x^3 + 48x^2 + 288x$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned}\text{Volume of open box} &= \text{length} \times \text{breadth} \times \text{height}\\ &= (24-2x)\times(24-2x)\times x\\ &= (4x^3-96x^2+576x)\ \text{cm}^3\end{aligned}$$

Q. 2. Find the value of $\frac{dV}{dx}$?

(A) $12(x^2+16x-48)$ **(B)** $12(x^2-16x+48)$
(C) $6(x^2+8x-24)$ **(D)** $6(x^2-8x+24)$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned}\frac{dV}{dx} &= \frac{d}{dx}[4x^3-96x^2+576x]\\ &= 12x^2-2\times96x+576\\ &= 12[x^2-16x+48]\end{aligned}$$

Q. 3. Find the value of $\frac{d^2V}{dx^2}$?

(A) $24(x+8)$ **(B)** $12(x-4)$
(C) $24(x-8)$ **(D)** $12(x+4)$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}\frac{d^2V}{dx} &= \frac{d}{dx}\left[\frac{dV}{dx}\right]\\ &= \frac{d}{dx}[12(x^2-16x+48)]\\ &= [12(2x-16)]\\ &= 24(x-8)\end{aligned}$$

Q. 4. Find the value of x other than 12 ?

(A) 3 **(B)** 9
(C) 1 **(D)** 4

Ans. Option (D) is correct.

Q. 5. Volume is maximum at what height of that open box ?

(A) 3 cm **(B)** 9 cm
(C) 1 cm **(D)** 4 cm

Ans. Option (D) is correct.

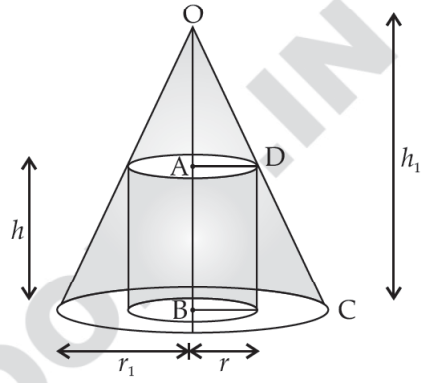
Explanation: For maximum value,

$$\begin{aligned}\frac{dV}{dx} &= 0\\ i.e.,\ 12(x^2-16x+48) &= 0\\ x^2-16x+48 &= 0\\ x^2-4x-12x+48 &= 0\\ x(x-4)-12(x-4) &= 0\\ (x-4(x-12) &= 0\\ x &= 4, 12\\ V(x=4) &= (24-2\times4)(24-2\times4)\times4\\ &= 16\times16\times4\\ &= 1024\ \text{cm}^3\\ V(x=12) &= (24-2\times12)(24-2\times12)\times12\\ &= 0\end{aligned}$$

Hence, volume is maximum at height 4 cm of the open box.

VI. Read the following text and answer the following questions on the basis of the same:

A right circular cylinder is inscribed in a cone.



S = Curved Surface Area of Cylinder.

Q. 1. $\frac{r}{r_1}=?$

(A) $\frac{h-h_1}{h_1}$ **(B)** $\frac{h_1-h}{h_1}$

(C) $\frac{h-h_1}{h}$ **(D)** $\frac{h+h_1}{h_1}$

Ans. Option (B) is correct.

Explanation: In Δ DEC and Δ OBC

$$\begin{aligned}\frac{DE}{OB} &= \frac{EC}{BC} \quad [\text{Since } \Delta DEC \sim \Delta OBC]\\ \frac{h}{h_1} &= \frac{r_1-r}{r_1}\\ r_1h &= r_1h_1-rh_1\\ r_1(h-h_1) &= -rh_1\end{aligned}$$

or $$\frac{r}{r_1} = \frac{h_1-h}{h_1}$$

Q. 2. Find the value of 'S' ?

(A) $\frac{2\pi r}{h}(h_1-h)h$ **(B)** $\frac{2\pi r}{h_1}(h_1-h)h$

(C) $\frac{2\pi r_1}{h_1}(h_1-h)h$ **(D)** $\frac{2\pi r_1}{h_1}(h_1+h)h$

Ans. Option (C) is correct.

Explanation: Curved surface area of cylinder,

$$S = \frac{2\pi r h_1(r_1-r)}{r_1}$$

$$= \frac{2\pi r}{r_1}(r_1 - r)h_1$$

$$= 2\pi r h_1 \times \frac{h}{h_1} \qquad \left[\because \frac{h}{h_1} = \frac{r_1 - r}{r_1}\right]$$

$$\frac{2\pi r_1 (h_1 - h).h}{h_1} \qquad \left[\because r = r_1\frac{(h_1 - h)}{h_1}\right]$$

$$\therefore \quad S = \frac{2\pi r_1}{h_1}(h_1 - h).h$$

Q. 3. What is the value of $\frac{dS}{dh}$?

(A) $\frac{2\pi r_1}{h}(h_1 - 2h)$ **(B)** $\frac{2\pi r_1}{h_1}(h - 2h_1)$

(C) $\frac{2\pi r}{h}(h_1 - 2h)$ **(D)** $\frac{2\pi r_1}{h_1}(h_1 - 2h)$

Ans. Option (D) is correct.

Explanation:

$$\frac{dS}{dh} = \frac{2\pi r_1}{h_1}(h_1 - 2h)$$

Q. 4. Find the value of $\frac{d^2S}{dh^2}$?

(A) $-\frac{4\pi r_1}{h_1}$ **(B)** $-\frac{4\pi r}{h}$

(C) $-\frac{4\pi r_1}{h}$ **(D)** $\frac{4\pi r_1}{h}$

Ans. Option (A) is correct.

Explanation:

$$\frac{d^2S}{dh^2} = \frac{2\pi r_1}{h_1}(0 - 2)$$

$$= \frac{-4\pi r_1}{h_1}$$

Q. 5. What is the relation between r_1 and r ?

(A) $r_1 = \frac{r}{2}$ **(B)** $2r_1 = 3r$

(C) $r_1 = 2r$ **(D)** $\frac{r_1}{2} = \frac{r}{3}$

Ans. Option (C) is correct.

Explanation:

$$S = \frac{2\pi r}{r_1}(r_1 - r)h_1$$

$$S = \frac{2\pi h_1 (rr_1 - r^2)}{r_1}$$

$$\frac{dS}{dr} = \frac{2\pi h_1 (r_1 - 2r)}{r_1}$$

$$\frac{dS}{dr} = 0$$

$$\frac{2\pi h_1 (r_1 - 2r)}{r_1} = 0$$

$$\Rightarrow \quad r_1 - 2r = 0$$

$$r_1 = 2r$$

❐❐

CHAPTER

7

Term-II

INTEGRALS

Syllabus

- ***Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts. Evaluation of simple integrals of the following types and problems based on them.***

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{px + q}{ax^2 + bx + c} dx,$$

$$\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

STAND ALONE MCQs (1 Mark each)

Q. 1. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to

(A) $2(\sin x + x\cos\theta) + C$
(B) $2(\sin x - x\cos\theta) + C$
(C) $2(\sin x + 2x\cos\theta) + C$
(D) $2(\sin x - 2x\cos\theta) + C$

Ans. Option (A) is correct.

Explanation : Let,

$$I = \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$$

$$= \int \frac{(2\cos^2 x - 1 - 2\cos^2 \theta + 1)}{\cos x - \cos \theta} dx$$

$$= 2\int \frac{(\cos x + \cos\theta)(\cos x - \cos\theta)}{(\cos x - \cos\theta)} dx$$

$$= 2\int (\cos x + \cos\theta) dx$$

$$= 2\sin x + 2x\cos\theta + C$$

Q. 2. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\, dx$ is

(A) 0 **(B)** 2
(C) π **(D)** 1

Ans. Option (C) is correct.

Explanation : Let,

$$I = \int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1) dx$$

$$= \int_{-\pi/2}^{\pi/2} x^3 dx + \int_{-\pi/2}^{\pi/2} x\cos x + \int_{-\pi/2}^{\pi/2} \tan^5 x dx + \int_{-\pi/2}^{\pi/2} 1 \cdot dx$$

It is known that if $f(x)$ is an even function, then

$$\int_{-a}^{a} f(x) dx = 2\int_{0}^{a} f(x) dx$$

and if $f(x)$ is an odd function, then

$$\int_{-a}^{a} f(x) dx = 0$$

$$\therefore\ I = 0 + 0 + 0 + 2\int_{0}^{\pi/2} 1 \cdot dx$$

$$= 2[x]_0^{\pi/2} = \frac{2\pi}{2} = \pi$$

Q. 3. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(A) $\tan^{-1}(e^x) + C$
(B) $\tan^{-1}(e^{-x}) + C$
(C) $\log(e^x - e^{-x}) + C$
(D) $\log(e^x + e^{-x}) + C$

Ans. Option (A) is correct.

Explanation:

Let $I = \int \frac{dx}{e^x + e^{-x}}\,dx$

$= \int \frac{e^x}{e^{2x}+1}\,dx$

Also, let $e^x = t$

$\Rightarrow e^x dx = dt$

$\Rightarrow I = \int \frac{dt}{1+t^2}$

$= \tan^{-1} t + C$

$= \tan^{-1}(e^x) + C$

Q. 4. $\int \frac{\cos 2x}{(\sin x + \cos x)^2}\,dx$ is equal to

(A) $\frac{-1}{\sin x + \cos x} + C$
(B) $\log|\sin x + \cos x| + C$
(C) $\log|\sin x - \cos x| + C$
(D) $\frac{1}{(\sin x + \cos x)^2}$

Ans. Option (B) is correct.

Explanation:

Let $I = \frac{\cos 2x}{(\cos x + \sin x)^2}$

$I = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2}\,dx$

$= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}\,dx$

$= \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$

Let $\cos x + \sin x = t$

$\Rightarrow (\cos x - \sin x)\,dx = dt$

$\Rightarrow I = \int \frac{dt}{t}$

$= \log|t| + C$

$= \log|\cos x + \sin x| + C$

Q. 5. If $f(a + b - x) = f(x)$, then $\int_a^b xf(x)\,dx$ is equal to

(A) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$ **(B)** $\frac{a+b}{2}\int_a^b f(b+x)\,dx$

(C) $\frac{b-a}{2}\int_a^b f(x)\,dx$ **(D)** $\frac{a+b}{2}\int_a^b f(x)\,dx$

Ans. Option (D) is correct.

Explanation:

Let $I = \int_a^b xf(x)\,dx$ ...(i)

$I = \int_a^b (a+b-x)f(a+b-x)\,dx$

$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

$\Rightarrow I = \int_a^b (a+b-x)f(x)\,dx$

$\Rightarrow I = (a+b)\int_a^b f(x)\,dx - I$ [Using Eq. (i)]

$\Rightarrow I + I = (a+b)\int_a^b f(x)\,dx$

$\Rightarrow 2I = (a+b)\int_a^b f(x)\,dx$

$\Rightarrow I = \left(\frac{a+b}{2}\right)\int_a^b f(x)\,dx$

Q. 6. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

(A) 1 **(B)** 0
(C) –1 **(D)** $\frac{\pi}{4}$

Ans. Option (B) is correct.

Explanation :

Let $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$\Rightarrow I = \int_0^1 \tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$

$\Rightarrow I = \int_0^1 [\tan^{-1} x - \tan^{-1}(1-x)]\,dx$ ...(i)

$\Rightarrow I = \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(1-1+x)]\,dx$

$\Rightarrow I = \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)]\,dx$

$\Rightarrow I = \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)]\,dx$ ...(ii)

Adding equations (i) and (ii), we obtain

$2I = \int_0^1 (\tan^{-1} x + \tan^{-1}(1-x) - \tan^{-1}(1-x) - \tan^{-1} x)\,dx$

$\Rightarrow 2I = 0$

$\Rightarrow I = 0$

Q. 7. $\frac{dx}{\sin(x-a)\,\sin(x-b)}$ is equal to

(A) $\sin(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$

(B) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

(C) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$

(D) $\sin(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

Ans. Option (C) is correct.

Explanation : Let,

$$I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a-x+b)}{\sin(x-a)\sin(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cos(x-b) - \cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int [\cot(x-b) - \cot(x-a)] dx$$

$$= \frac{1}{\sin(b-a)} [\log|\sin(x-b)| - \log|\sin(x-a)|] + C$$

$$= \operatorname{cosec}(b-a) \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| + C$$

Q. 8. $\int \sqrt{1+x^2}\, dx$ is equal to

(A) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right| + C$

(B) $\frac{2}{3}(1+x^2)^{3/2} + C$

(C) $\frac{2}{3}x(1+x^2)^{3/2} + C$

(D) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right) + C$

Ans. Option (A) is correct.

Explanation : It is known that,

$$\int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\therefore \int \sqrt{1+x^2}\, dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x+\sqrt{1+x^2}\right| + C$$

Q. 9. $\int \frac{x\,dx}{(x-1)(x-2)}$ equals

(A) $\log\left|\frac{(x-1)^2}{x-2}\right| + C$

(B) $\log\left|\frac{(x-2)^2}{x-1}\right| + C$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$

(D) $\log|(x-1)(x-2)| + C$

Ans. Option (B) is correct.

Explanation :

Let $\frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$

$x = A(x-2) + B(x-1)$ ...(i)

Substituting $x = 1$ and 2 in Eq. (i), we obtain

A = –1 and B = 2

$$\therefore \frac{x}{(x-1)(x-2)} = -\frac{1}{(x-1)} + \frac{2}{(x-2)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)} dx = \int \left\{ \frac{-1}{(x-1)} + \frac{2}{(x-2)} \right\} dx$$

$$= -\log|x-1| + 2\log|x-2| + C$$

$$= \log\left|\frac{(x-2)^2}{x-1}\right| + C$$

Q. 10. If $f(x) = \int_0^x t\sin t\, dt$, then $f'(x)$ is

(A) $\cos x + x\sin x$

(B) $x\sin x$

(C) $x\cos x$

(D) $\sin x + x\cos x$

Ans. Option (B) is correct.

Explanation :

$$f(x) = \int_0^x t\sin t\, dt$$

Integrating by parts, we obtain

$$f(x) = t\int_0^x \sin t\, dt - \int_0^x \left\{ \left(\frac{d}{dt}t\right) \int \sin t\, dt \right\} dt$$

$$= [t(-\cos t)]_0^x - \int_0^x (-\cos t)\, dt$$

$$= [-t\cos t + \sin t]_0^x$$

$$= -x\cos x + \sin x$$

$$\Rightarrow f'(x) = -[\{x(-\sin x)\} + \cos x] + \cos x$$

$$= x\sin x - \cos x + \cos x$$

$$= x\sin x$$

Q. 11. $\int \tan^{-1}\sqrt{x}\, dx$ is equal to

(A) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(B) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(C) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$

(D) $\sqrt{x} - (x+\ )\tan\ \sqrt{x} + C$

Ans. Option (A) is correct.

Explanation : Let,

$$I = \int 1\cdot\tan^{-1}\sqrt{x}\, dx$$

$$= \tan^{-1}\sqrt{x}\cdot x - \frac{1}{2}\int \frac{1}{(1+x)}\cdot\frac{2}{\sqrt{x}} dx$$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{2}{\sqrt{x}(1+x)} dx$$

Put $x = t^2$

$\Rightarrow dx = 2t\,dt$

$\therefore \quad I = x\tan^{-1}\sqrt{x} - \int \frac{t}{t(1+t^2)}dt$

$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}dt$

$= x\tan^{-1}\sqrt{x} - \int\left(1 - \frac{1}{1+t^2}\right)dt$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}t + C$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

Q. 12. $\int x^2 e^{x^3}dx$ is equal to

(A) $\frac{1}{3}e^{x^3} + C$ **(B)** $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$ **(D)** $\frac{1}{2}e^{x^2} + C$

Ans. Option (A) is correct.

Explanation:

Let $I = \int x^2 e^{x^3}dx$

Also, let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$\Rightarrow I = \frac{1}{3}\int e^t dt$

$= \frac{1}{3}(e^t) + C$

$= \frac{1}{3}e^{x^3} + C$

Q. 13. $\int e^x \sec x(1+\tan x)dx$ is equal to

(A) $e^x \cos x + C$ **(B)** $e^x \sec x + C$

(C) $e^x \sin x + C$ **(D)** $e^x \tan x + C$

Ans. Option (B) is correct.

Explanation : $\int e^x \sec x(1+\tan x)dx$

Let

$I = \int e^x \sec x(1+\tan x)dx = \int e^x(\sec x + \sec x\tan x)dx$

Also, let $\sec x = f(x) \Rightarrow \sec x\tan x = f'(x)$

It is known that, $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Q. 14. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x\ \cos^2 x}dx$ is equal to

(A) $\tan x + \cot x + C$

(B) $\tan x + \text{cosec}\ x + C$

(C) $-\tan x + \cot x + C$

(D) $\tan x + \sec x + C$

Ans. Option (A) is correct.

Explanation:

$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x\cos^2 x}dx = \int\left(\frac{\sin^2 x}{\sin^2 x\cos^2 x} - \frac{\cos^2 x}{\sin^2 x\cos^2 x}\right)dx$

$= \int(\sec^2 x - \text{cosec}^2 x)dx$

$= \tan x + \cot x + C$

Q. 15. $\int \frac{dx}{\sin^2 x\cos^2 x}$ is equal to

(A) $\tan x + \cot x + C$ **(B)** $\tan x - \cot x + C$

(C) $\tan x\cot x + C$ **(D)** $\tan x - \cot 2x + C$

Ans. Option (B) is correct.

Explanation:

Let $I = \int \frac{dx}{\sin^2 x\cos^2 x}$

$= \int \frac{1}{\sin^2 x\cos^2 x}dx$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x\cos^2 x}dx$

$= \int \frac{\sin^2 x}{\sin^2 x\cos^2 x}dx + \int \frac{\cos^2 x}{\sin^2 x\cos^2 x}dx$

$= \int \sec^2 x dx + \int \text{cosec}^2\ x dx$

$= \tan x - \cot x + C$

Q. 16. $\int \frac{x^9}{(4x^2+1)^6}dx$ is equal to

(A) $\frac{1}{5x}\left(4 + \frac{1}{x^2}\right)^{-5} + C$ **(B)** $\frac{1}{5}\left(4 + \frac{1}{x^2}\right)^{-5} + C$

(C) $\frac{1}{10x}(1+4)^{-5} + C$ **(D)** $\frac{1}{10}\left(\frac{1}{x^2} + 4\right)^{-5} + C$

Ans. Option (D) is correct.

Explanation : Let

$I = \int \frac{x^9}{(4x^2+1)^6}dx$

$= \int \frac{x^9}{x^{12}\left(4 + \frac{1}{x^2}\right)^6}dx$

$= \int \frac{dx}{x^3\left(4 + \frac{1}{x^2}\right)^6}$

Put $4 + \frac{1}{x^2} = t$

$\Rightarrow \frac{-2}{x^3}dx = dt$

$\Rightarrow \frac{1}{x^3}dx = -\frac{1}{2}dt$

$\therefore I = -\frac{1}{2}\int \frac{dt}{t^6}$

$= -\frac{1}{2}\left[\frac{t^{-6+1}}{-6+1}\right] + C$

$$= \frac{1}{10}\left[\frac{1}{t^5}\right] + C$$
$$= \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5} + C$$

Q. 17. $\int \frac{x^3}{x+1}$ is equal to

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$

(B) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$

(C) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$

(D) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

Ans. Option (D) is correct.

Explanation : Let,

$$I = \int \frac{x^3}{x+1} dx$$
$$= \int \left((x^2 - x + 1) - \frac{1}{(x+1)}\right) dx$$
$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + C$$

Q. 18. If $\int \frac{x^3 dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$, then

(A) $a = \frac{1}{3}, \ b = 1$ **(B)** $a = \frac{-1}{3}, \ b = 1$

(C) $a = \frac{-1}{3}, \ b = -1$ **(D)** $a = \frac{1}{3}, \ b = -1$

Ans. Option (D) is correct.

Explanation: Let,

$$I = \int \frac{x^3}{\sqrt{1+x^2}} dx$$
$$= a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$$
$$\therefore \quad I = \int \frac{x^3}{\sqrt{1+x^2}} dx$$
$$= \int \frac{x^2 \cdot x}{\sqrt{1+x^2}} dx$$

Put $1 + x^2 = t^2$

$\Rightarrow \quad 2xdx = 2tdt$

$$\therefore \quad I = \int \frac{t(t^2-1)}{t} dt$$
$$= \frac{t^3}{3} - t + C$$
$$= \frac{1}{3}(1+x^2)^{3/2} - \sqrt{1+x^2} + C$$
$$\therefore \quad a = \frac{1}{3} \text{ and } b = -1$$

Q. 19. $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ is equal to

(A) 1 **(B)** 2

(C) 3 **(D)** 4

Ans. Option (A) is correct.

Explanation : Let

$$I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$$
$$= \int_{-\pi/4}^{\pi/4} \frac{dx}{2\cos^2 x}$$
$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4} \sec^2 x dx$$
$$= \int_0^{\pi/4} \sec^2 x dx$$
$$= [\tan x]_0^{\pi/4}$$
$$= 1$$

Q. 20. $\int \frac{dx}{x^2+2x+2}$ equals

(A) $x\tan^{-1}(x+1) + C$ **(B)** $\tan^{-1}(x+1) + C$

(C) $(x+1)x\tan^{-1} + C$ **(D)** $\tan^{-1} + C$

Ans. Option (B) is correct.

Explanation :

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x^2+2x+1)+1}$$
$$= \int \frac{1}{(x+1)^2 + (1)^2} dx$$
$$= \left[\tan^{-1}(x+1)\right] + C$$

Q. 21. $\int_0^{\pi/2} \sqrt{1 - \sin 2x} dx$ is equal to

(A) $2\sqrt{2}$ **(B)** $2(\sqrt{2}+1)$

(C) 2 **(D)** $2(\sqrt{2}-1)$

Ans. Option (D) is correct.

Explanation: Let

$$I = \int_0^{\pi/2} \sqrt{1 - \sin 2x} dx$$
$$= \int_0^{\pi/4} \sqrt{(\cos x - \sin x)^2} \, dx + \int_{\pi/4}^{\pi/2} \sqrt{(\sin x - \cos x)^2} \, dx$$
$$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$
$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 + \left(-0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$
$$= 2\sqrt{2} - 2$$
$$= 2(\sqrt{2} - 1)$$

Q. 22. The anti-derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{1/3} + 2x^{1/2} + C$ **(B)** $\frac{2}{3}x^{2/3} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{2/3} + 2x^{1/2} + C$ **(D)** $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + C$

Ans. Option (C) is correct.

Explanation:

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx = \int x^{1/2}dx + \int x^{-1/2}dx$$

$$= \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}} + C$$

$$= \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

Q. 23. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(A) $\frac{\pi}{3}$ **(B)** $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$ **(D)** $\frac{\pi}{12}$

Ans. Option (D) is correct.

Explanation:

$$\int \frac{dx}{1+x^2} = \tan^{-1}x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = F(\sqrt{3}) - F(1)$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Q. 24. $\int_0^{2/3} \frac{dx}{4+9x^2}$ equals

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{12}$

(C) $\frac{\pi}{24}$ **(D)** $\frac{\pi}{4}$

Ans. Option (C) is correct.

Explanation :

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2+(3x)^2}$$

Put $3x = t$

$\Rightarrow 3dx = dt$

$$\therefore \int \frac{dx}{(2)^2+(3x)^2} = \frac{1}{3}\int \frac{dt}{(2)^2+(t)^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{2/3} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$

$$= \frac{1}{6}\tan^{-1}\left[\frac{3}{2}\times\frac{2}{3}\right] - \frac{1}{6}\tan^{-1}(0)$$

$$= \frac{1}{6}\tan^{-1}(1)$$

$$= \frac{1}{6}\tan^{-1}\left[\tan\frac{\pi}{4}\right]$$

$$= \frac{\pi}{24}$$

Q. 25. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ **(B)** $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ **(D)** $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Ans. Option (A) is correct.

Explanation : It is given that,

$$\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$$

$\therefore$ Anti-derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore \quad f(x) = \int 4x^3 - \frac{3}{x^4}dx$$

$$f(x) = 4\int x^3dx - 3\int\left(x^{-4}\right)dx$$

$$\therefore \quad f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also,

$$f(2) = 0$$

$$\therefore \quad f(2) = (2)^4 + \frac{1}{(2)^3} + C$$

$$= 0$$

$$\Rightarrow \quad 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow \quad C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow \quad C = \frac{-129}{8}$$

$$\therefore \quad f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $\int \frac{dx}{x^2+2x+3} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c.$$

This is a standard integral and hence R is true.

$$\int \frac{dx}{x^2+2x+3} = \int \frac{dx}{(x+1)^2+(\sqrt{2})^2}$$
$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$$

Hence A is true and R is the correct explanation for A.

Q. 2. Assertion(A): $\int e^x[\sin x - \cos x]\,dx = e^x \sin x + C$

Reason (R): $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+c$

Ans. Option (D) is correct.

Explanation:

$$\int e^x[f(x)+f'(x)]dx = \int e^x f(x)dx + \int e^x f'(x)dx$$
$$= f(x)e^x - \int f'(x)e^x dx + \int f'(x)e^x dx$$
$$= e^x f(x)+c$$

Hence R is true.

$$\int e^x(\sin x - \cos x)dx = e^x(-\cos x)+c$$
$$= -e^x \cos x + c$$
$$\left[\because \frac{d}{dx}(-\cos x) = \sin x\right]$$

Hence A is false.

Q. 3. Assertion (A): $\int x^x(1+\log x)dx = x^x + c$

Reason (R): $\frac{d}{dx}(x^x) = x^x(1+\log x)$

Ans. Option (A) is correct.

Explanation : Let $y = x^x$

$\Rightarrow \log y = x\log x$

Differentiating w.r.t. x

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right)+\log x(1)$$
$$\frac{dy}{dx} = y(1+\log x)$$
$$= x^x(1+\log x)$$

Hence R is true.

Since $\frac{d}{dx}(x^x) = x^x(1+\log x)$

$\int x^x(1+\log x)dx = x^x + c$

Using the concept of anti-derivative, A is true.
R is the correct explanation for A.

Q. 4. Assertion (A): $\int x^2 dx = \frac{x^3}{3}+c$

Reason (R): $\int e^{x^2}dx = e^{x^3/3}+c$

Ans. Option (C) is correct.

Explanation:

Since $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c,$

$$\int x^2 dx = \frac{x^{2+1}}{2+1}+c$$
$$= \frac{x^3}{3}+c$$

$\therefore A$ is true.

$\int e^{x^2}\,dx$ is a function which can not be integrated.

$\therefore$ R is false.

Q. 5. Assertion (A): $\int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x}\,dx = \frac{\pi}{4}$

Reason (R): $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx = \frac{\pi}{4}$

Ans. Option (A) is correct.

Explanation:

Let $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$ ...(i)

$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\therefore I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right)dx}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$

$I = \int \frac{\cos x}{\cos x + \sin x}\,dx$ ...(ii)

Adding equations (i) + (ii),

$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$

$= \int_0^{\pi/2} 1\,dx$

$= [x]_0^{\pi/2}$

$= \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$

Hence R is true.
From (ii), A is also true.
R is the correct explanation for A.

Q. 6. Assertion (A): $\int_{-3}^{3} (x^3+5)\,dx = 30$

Reason (R): $f(x) = x^3 + 5$ is an odd function.

Ans. Option (C) is correct.

Explanation:

Let $f(x) = x^3 + 5$

$f(-x) = (-x)^3 + 5$

$= -x^3 + 5$

$f(x)$ is neither even nor odd. Hence R is false.

$\int_{-3}^{3} x^3\,dx = 0$ [$\because x^3$ is odd]

$\int_{-3}^{3} 5\,dx = 5[x]_{-3}^{3} = 30$

$\therefore \int_{-3}^{3} (x^3+5)\,dx = 0 + 30 = 30$

Hence A is true.

Q. 7. Assertion (A): $\frac{d}{dx}\left[\int_0^{x^2} \frac{dt}{t^2+4}\right] = \frac{2x}{x^4+4}$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

Ans. Option (A) is correct.

Explanation:

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c.$

This is a standard integral and hence true.
So R is true.

$\int_0^{x^2} \frac{dt}{t^2+4} = \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{x^2}$

$= \frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)$

$\frac{d}{dx}\left[\int_0^{x^2} \frac{dt}{t^2+4}\right] = \frac{d}{dx}\left[\frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)\right]$

$= \frac{1}{2} \times \frac{1}{1+\frac{x^4}{4}} \times \frac{2x}{2}$

$= \frac{x}{2} \times \frac{4}{4+x^4}$

$= \frac{2x}{4+x^4}$

Hence A is true and R is the correct explanation for A.

Q. 8. Assertion (A): $\int_{-1}^{1} (x^3 + \sin x + 2)\,dx = 0$

Reason (R):

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function } i.e., (-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function } i.e., f(-x) = -f(x) \end{cases}$$

Ans. Option (D) is correct.

Explanation:

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function } i.e., (-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function } i.e., f(-x) = -f(x) \end{cases}$$

This is a property of the definite integrals and hence R is true.

$\int_{-1}^{1} (x^3 + \sin x + 2)\,dx$

$= \underset{\text{Odd function}}{\int_{-1}^{1} (x^3 + \sin x)\,dx} + \underset{\text{Even function}}{\int_{-1}^{1} 2\,dx}$

$= 0 + 2[x]_{-1}^{1}$

$= 2 \times 2$

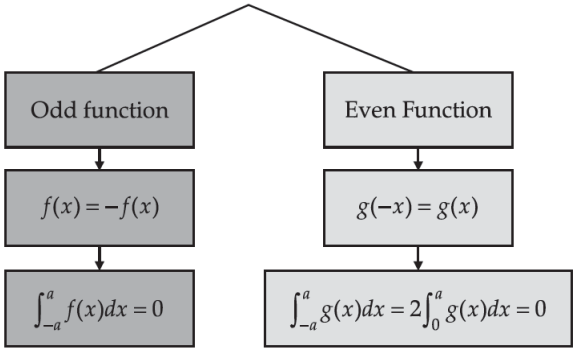
$= 4$

Hence A is false.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:



Q. 1. $\int_{-1}^{1} x^{99}dx =$ _____.

(A) 0 **(B)** 1
(C) –1 **(D)** 2

Ans. Option (A) is correct.

Explanation:

$\int_{-1}^{1} x^{99}dx = 0$, since x^{99} is an odd function.

Q. 2. $\int_{-\pi}^{\pi} x\cos x\,dx =$ _____.

(A) 1 **(B)** 0
(C) –1 **(D)** $\frac{\pi}{2}$

Ans. Option (B) is correct.

Explanation:

$\int_{-\pi}^{\pi} x\cos x\,dx = 0$, since $x\cos x$ is an odd function.

Q. 3. $\int_{-\pi/2}^{\pi/2} \sin^3 x\,dx =$ _____.

(A) 1 **(B)** 0
(C) –1 **(D)** π

Ans. Option (B) is correct.

Explanation:

$\int_{-\pi/2}^{\pi/2} \sin^3 x\,dx = 0$, since $\sin^3 x$ is an odd function.

Q. 4. $\int_{-\pi}^{\pi} x\sin x\,dx =$ _____.

(A) π **(B)** 0
(C) 2π **(D)** $\frac{\pi}{2}$

Ans. Option (C) is correct.

Explanation: Since, $x\sin x$ is an even function

$$\begin{aligned}\int_{-\pi}^{\pi} x\sin x\,dx &= 2\int_{0}^{\pi} x\sin x\,dx \\ &= 2\left[-x\cos x + \int(1\times\cos x\,dx)\right]_0^{\pi} \\ &= 2\left[-x\cos x + \sin x\right]_0^{\pi} \\ &= 2[(\pi+0)-(0+0)] \\ &= 2\pi\end{aligned}$$

Q. 5. $\int_{-\pi}^{\pi} \tan x\sec^2 x\,dx =$ _____.

(A) 1 **(B)** –1
(C) 0 **(D)** 2

Ans. Option (C) is correct.

Explanation:

$\int_{-\pi}^{\pi} \tan x\sec^2 x\,dx = 0$, Since it is an odd function

II. Read the following text and answer the following questions on the basis of the same:

$$\begin{aligned}\int e^x[f(x)+f'(x)]dx &= \int e^x f(x)dx + \int e^x f'(x)dx \\ &= f(x)e^x - \int f'(x)e^x dx \\ &\quad + \int f'(x)e^x dx \\ &= e^x f(x) + c\end{aligned}$$

Q. 1. $\int e^x(\sin x + \cos x)dx =$ _____.

(A) $e^x\cos x + c$ **(B)** $e^x\sin x + c$
(C) $e^x + c$ **(D)** $e^x(-\cos x + \sin x) + c$

Ans. Option (B) is correct.

Explanation:

$$\int e^x(\underbrace{\sin x}_{f(x)} + \underbrace{\cos x}_{f'(x)})dx = e^x\sin x + c$$

Q. 2. $\int e^x\left(\frac{x-1}{x^2}\right)dx =$ _____.

(A) $e^x + c$ **(B)** $\frac{e^x}{x} + c$
(C) $\frac{e^x}{x^2} + c$ **(D)** $\frac{-e^x}{x^2} + c$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned}\int e^x\left(\frac{x-1}{x^2}\right)dx &= \int e^x\left(\underbrace{\frac{1}{x}}_{f(x)} - \underbrace{\frac{1}{x^2}}_{f'(x)}\right)dx \\ &= \frac{e^x}{x} + c\end{aligned}$$

Q. 3. $\int e^x(x+1)dx$ = _____.

(A) $xe^x + c$ **(B)** $e^x + c$

(C) $e^{-x} + c$ **(D)** None of these

Ans. Option (A) is correct.

Explanation:

$$\int e^x(\underbrace{x}_{f(x)} + \underbrace{1}_{f'(x)})dx = xe^x + c$$

Q. 4. $\int_0^{\pi} e^x(\tan x + \sec^2 x)dx$ = _____.

(A) 0 **(B)** 1

(C) –1 **(D)** $-e^{\pi}$

Ans. Option (A) is correct.

Explanation:

$$\int_0^{\pi} e^x(\tan x + \sec^2 x)dx = \left[e^x \tan x\right]_0^{\pi} = 0$$

Q. 5. $\int \frac{xe^x}{(1+x)^2}dx$ = _____.

(A) $x\,e^x + c$ **(B)** $\frac{e^x}{(x+1)^2} + c$

(C) $\frac{x\,e^x}{x+1} + c$ **(D)** $\frac{e^x}{x+1} + c$

Ans. Option (D) is correct.

Explanation:

$$\int e^x\left[\frac{(x+1)-1}{(x+1)^2}\right]dx = \int e^x\left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right]dx$$

$$= \frac{e^x}{x+1} + c$$

III. Read the following text and answer the following questions on the basis of the same:

Let's say that we want to evaluate $\int [P(x)/Q(x)]\,dx$, where $P(x)/Q(x)$ is a proper rational fraction. In such cases, it is possible to write the integrand as a sum of simpler rational functions by using partial fraction decomposition. Post this, integration can be carried out easily. The following image indicates some simple partial fractions which can be associated with various rational functions:

S. No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$,
	where, x^2+bx+c cannot be factorised further	

In the above table, A, B and C are real numbers to be determined suitably.

Q. 1. $\int \frac{dx}{(x+1)(x+2)}$

(A) $\log\left|\frac{x+1}{x+2}\right| + C$ **(B)** $\log\left|\frac{x-1}{x+2}\right| + C$

(C) $\log\left|\frac{x+1}{x-2}\right| + C$ **(D)** $\log\left|\frac{x+2}{x+1}\right| + C$

Ans. Option (A) is correct.

Explanation: We write,

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \quad \ldots(i)$$

where, real number A and B are to be determined suitably. This gives

$$1 = A(x+2) + B(x+1)$$

Equating the coefficients of x and the constant term, we get

$$A + B = 0$$

and $$2A + B = 1$$

Solving these equations, we get $A = 1$ and $B = -1$. Thus, the integrand is given by

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$$

Therefore,

$$\int \frac{dx}{(x+1)(x+2)} = \int \frac{dx}{x+1} - \int \frac{dx}{x+2}$$

$$= \log|x+1| - \log|x+2| + C$$

$$= \log\left|\frac{x+1}{x+2}\right| + C$$

Q. 2. Integration of $\frac{x}{(x+1)(x+2)}$

(A) $\log\frac{(x+1)^2}{(x+2)} + C$ **(B)** $\log\frac{(x+2)^2}{(x+1)} + C$

(C) $\log\frac{(x)^2}{(x+1)} + C$ **(D)** $\log\frac{(x-2)^2}{(x+1)} + C$

Ans. Option (B) is correct.

Explanation:

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow$ $x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term, we obtain

$$A + B = 1$$
$$2A + B = 0$$

On solving, we obtain

$$A = -1 \text{ and } B = 2$$

$$\therefore \quad \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)}dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)}dx$$
$$= -\log|x+1| + 2\log|x+2| + C$$
$$= \log(x+2)^2 - \log|x+1| + C$$
$$= \log\frac{(x+2)^2}{(x+1)} + C$$

Q. 3. $\int \frac{1}{x^2-9}dx$

(A) $\frac{1}{6}\log\left|\frac{x-3}{x+3}\right| + C$ **(B)** $\frac{1}{6}\log\left|\frac{x-2}{x+3}\right| + C$

(C) $\frac{1}{6}\log\left|\frac{x+3}{x-3}\right| + C$ **(D)** $\frac{1}{3}\log\left|\frac{x-3}{x+3}\right| + C$

Ans. Option (A) is correct.

Explanation:

Let $\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

$$1 = A(x-3) + B(x+3)$$

Equating the coefficients of x and constant term, we obtain

$$A + B = 0$$
$$-3A + 3B = 1$$

On solving, we obtain

$$A = -\frac{1}{6} \text{ and } B = \frac{1}{6}$$

$$\therefore \quad \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x^2-9)}dx = \int\left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)}\right)dx$$
$$= -\frac{1}{6}\log|x+3| + \frac{1}{6}\log|x-3| + C$$
$$= \frac{1}{6}\log\left|\frac{(x-3)}{(x+3)}\right| + C$$

Q. 4. $\int \frac{1}{e^x-1}dx =$

(A) $\log\left|\frac{e^x-1}{2}\right| + C$ **(B)** $\log\left|\frac{e^x-1}{2e^x}\right| + C$

(C) $\log\left|\frac{e^x-1}{2x}\right| + C$ **(D)** $\log\left|\frac{e^x-1}{e^x}\right| + C$

Ans. Option (D) is correct.

Explanation:

Let $e^x = t$

$$\Rightarrow \quad e^x dx = dt$$

$$\Rightarrow \quad \int \frac{1}{e^x-1}dx = \int \frac{1}{t-1} \times \frac{dt}{t}$$
$$= \int \frac{1}{t(t-1)}dt$$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

$$1 = A(t-1) + Bt \quad \ldots(i)$$

Substituting $t = 1$ and $t = 0$ in equation (i), we obtain

$A = -1$ and $B = 1$

$$\therefore \quad \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \quad \int \frac{1}{t(t-1)}dt = \log\left|\frac{t-1}{t}\right| + C$$
$$= \log\left|\frac{e^x-1}{e^x}\right| + C$$

Q. 5. $\int \frac{dx}{x(x^2+1)} =$

(A) $\log|x| + \frac{1}{2}\log|x^2+1| + C$

(B) $\log|x| - \frac{1}{4}\log|x^2+1| + C$

(C) $\log|x| - \frac{1}{2}\log|x^2+1| + C$

(D) $\log|x| - \frac{1}{3}\log|x^2-1| + C$

Ans. Option (C) is correct.

Explanation:

Let $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$1 = A(x^2+1) + (Bx+C)x$$

Equating the coefficients of x^2, x and constant term, we obtain

$$A + B = 0$$
$$C = 0$$
$$A = 1$$

On solving these equations, we obtain

$A = 1$, $B = -1$, and $C = 0$

$$\therefore \quad \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$

$$\Rightarrow \quad \int \frac{1}{x(x^2+1)}dx = \int\left\{\frac{1}{x} - \frac{x}{x^2+1}\right\}dx$$
$$= \log|x| - \frac{1}{2}\log|x^2+1| + C$$

❑❑

CHAPTER

8

Term-II

APPLICATIONS OF THE INTEGRALS

Syllabus

➢ *Applications in finding the area under simple curves, especially lines, parabolas; area of circles/ parabolas/ellipses (in standard form only). (the region should be clearly identifiable).*

STAND ALONE MCQs (1 Mark each)

Q. 1. The area of the region bounded by the y-axis, $y = \cos x$ and $y = \sin x$, $0 \le x \le \pi/2$ is

(A) $\sqrt{2}$ sq. units **(B)** $(\sqrt{2}+1)$ sq. units

(C) $(\sqrt{2}-1)$ sq. units **(D)** $(2\sqrt{2}-1)$ sq. units

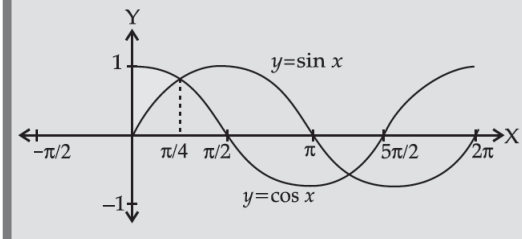
Ans. Option (C) is correct.

Explanation : We have $y = \cos x$ and $y = \sin x$, where $0 \le x \le \frac{\pi}{2}$.

We get $\cos x = \sin x$

$\Rightarrow \quad x = \frac{\pi}{4}$

From the figure, area of the shaded region,



$$A = \int_0^{\pi/4} (\cos x + \sin x)\,dx$$

$$= \left[\sin x + \cos x\right]_0^{\pi/4}$$

$$= \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0\right]$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= (\sqrt{2}-1) \text{ sq. units}$$

Q. 2. The area of the region bounded by the curve $x^2 = 4y$ and the straight-line $x = 4y - 2$ is

(A) $\frac{3}{8}$ sq. units **(B)** $\frac{5}{8}$ sq. units

(C) $\frac{7}{8}$ sq. units **(D)** $\frac{9}{8}$ sq. units

Ans. Option (D) is correct.

Explanation:

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

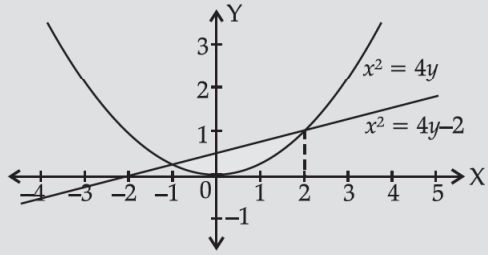
$$(x-2)(x+1) = 0$$

$$x = -1,\ 2$$

For $x = -1$, $y = \frac{1}{4}$ and for $x = 2$, $y = 1$

Points of intersection are $(-1, \frac{1}{4})$ and $(2, 1)$.

Graphs of parabola $x^2 = 4y$ and $x = 4y - 2$ are shown in the following figure :



$$A = \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right]dx$$
$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$
$$= \frac{1}{4}\left[8 - \frac{1}{2} - 3\right]$$
$$= \frac{1}{4}\left[8 - \frac{1}{2} - 3\right]$$
$$= \frac{9}{8} \text{ sq. units}$$

Q. 3. Area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is

(A) 16π sq. units **(B)** 4π sq. units
(C) 32π sq. units **(D)** 24π sq. units

Ans. Option (B) is correct.

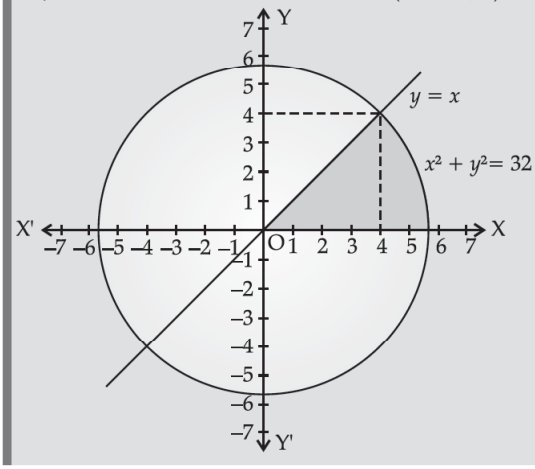
Explanation: We have $y = 0$, $y = x$ and the circle $x^2 + y^2 = 32$ in the first quadrant.
Solving $y = x$ with the circle

$$x^2 + x^2 = 32$$
$$x^2 = 16$$
$$x = 4 \quad \text{(In the first quadrant)}$$

When $x = 4$, $y = 4$ for the point of intersection of the circle with the x-axis.
Put $y = 0$

$$x^2 + 0 = 32$$
$$x = \pm 4\sqrt{2}$$

So, the circle intersects the x-axis at $(\pm 4\sqrt{2}, 0)$.



From the above figure, area of the shaded region,

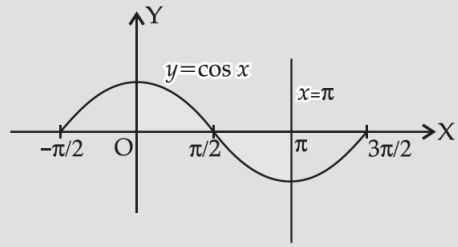
$$A = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$$
$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$
$$= \left[\frac{16}{2}\right] + \left[\begin{matrix} 0 + 16\sin^{-1}1 - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16^2} \\ -16\sin^{-1}\frac{4}{4\sqrt{2}} \end{matrix}\right]$$
$$= 8 + \left[\frac{16\pi}{2} - 2\sqrt{16} - 16\frac{\pi}{4}\right]$$
$$= 8 + [8\pi - 8 - 4\pi]$$
$$= 4\pi \text{ sq. units}$$

Q. 4. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

(A) 2 sq. units **(B)** 4 sq. units
(C) 3 sq. units **(D)** 1 sq. unit

Ans. Option (A) is correct.

Explanation : We have $y = \cos x$, $x = 0$, $x = \pi$



From the figure, area of the shaded region,

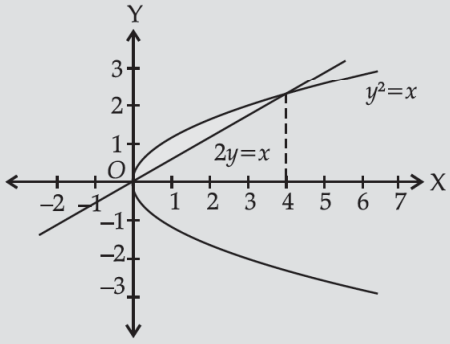
$$A = \int_0^{\pi} |\cos x|\,dx + \int_0^{\pi/2} \cos x\,dx$$
$$= 2[\sin x]_0^{\pi/2}$$
$$= 2 \text{ sq. units}$$

Q. 5. The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is

(A) $\frac{4}{3}$ sq. units **(B)** 1 sq. unit
(C) $\frac{2}{3}$ sq. unit **(D)** $\frac{1}{3}$ sq. unit

Ans. Option (A) is correct.

Explanation : When $y^2 = x$ and $2y = x$
Solving we get $y^2 = 2y$
$\Rightarrow y = 0, 2$ and when $y = 2$, $x = 4$
So, points of intersection are (0, 0) and (4, 2).
Graphs of parabola $y^2 = x$ and $2y = x$ are as shown in the following figure :



From the figure, area of the shaded region,

$$A = \int_0^4 \left[\sqrt{x} - \frac{x}{2}\right] dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{2}\cdot\frac{x^2}{2}\right]_0^4$$

$$= \frac{2}{3}\cdot(4)^{3/2} - \frac{16}{4} - 0$$

$$= \frac{16}{3} - 4$$

$$= \frac{4}{3} \text{ sq. unit}$$

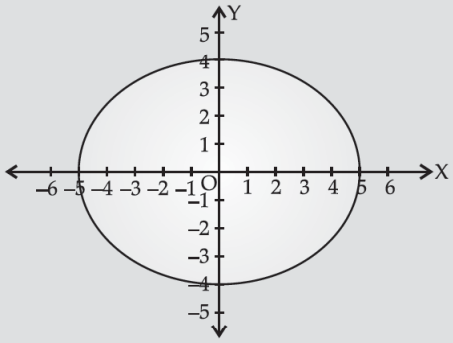
Q. 6. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

(A) 20π sq. units **(B)** $20\pi^2$ sq. units

(C) $16\pi^2$ sq. units **(D)** 25π sq. units

Ans. Option (A) is correct.

Explanation: We have $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$, which is ellipse with its axes as coordinate axes.



$$\frac{y^2}{4^2} = 1 - \frac{x^2}{5^2}$$

$$y^2 = 16\left(1 - \frac{x^2}{25}\right)$$

$$y = \frac{4}{5}\sqrt{5^2 - x^2}$$

From the figure, area of the shaded region,

$$A = 4\int_0^5 \frac{4}{5}\sqrt{5^2 - x^2}\,dx$$

$$= \frac{16}{5}\left[\frac{x}{2}\sqrt{5^2 - x^2} - \frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$$

$$= \frac{16}{5}\left[0 + \frac{5^2}{2}\sin^{-1}1 - 0 - 0\right]$$

$$= \frac{16}{5}\cdot\frac{25}{2}\cdot\frac{\pi}{2}$$

$$= 20\pi \text{ sq. units}$$

Q. 7. The area of the region bounded by the circle $x^2 + y^2 = 1$ is

(A) 2π sq. units **(B)** π sq. units

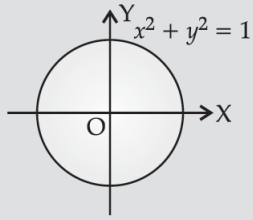
(C) 3π sq. units **(D)** 4π sq. units

Ans. Option (B) is correct.

Explanation : We have, $x^2 + y^2 = 1$, which is a circle having centre at (0, 0) and radius '1' unit.

$$\Rightarrow \quad y^2 = 1 - x^2$$

$$y = \sqrt{1 - x^2}$$



From the figure, area of the shaded region,

$$A = 4\int_0^1 \sqrt{1^2 - x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{1^2 - x^2} - \frac{1^2}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$= 4\left[0 + \frac{1^2}{2}\times\frac{\pi}{2} - 0 - 0\right]$$

$$= \pi \text{ sq. units}$$

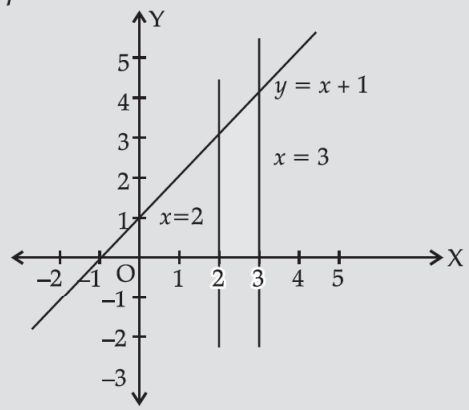
Q. 8. The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is

(A) $\frac{7}{2}$ sq. units **(B)** $\frac{9}{2}$ sq. units

(C) $\frac{11}{2}$ sq. units **(D)** $\frac{13}{2}$ sq. units

Ans. Option (A) is correct.

Explanation:



From the figure, area of the shaded region,

$$A = \int_2^3 (x+1)\,dx$$
$$= \left[\frac{x^2}{2} + x\right]_2^3$$
$$= \left[\frac{9}{2} + 3 - \frac{4}{2} - 2\right]$$
$$= \frac{7}{2} \text{ sq. units}$$

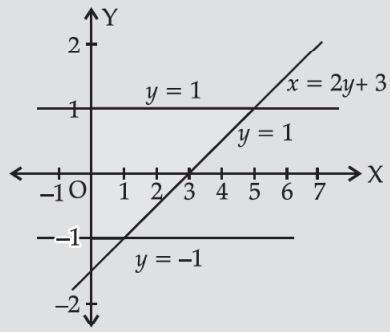
Q. 9. The area of the region bounded by the curve $x = 2y + 3$ and the y lines $y = 1$ and $y = -1$ is,

(A) 4 sq. units **(B)** $\frac{3}{2}$ sq. units

(C) 6 sq. units **(D)** 8 sq. units

Ans. Option (C) is correct.

Explanation:



From the figure, area of the shaded region,

$$A = \int_{-1}^{1} (2y+3)\,dy$$
$$= \left[y^2 + 3y\right]_{-1}^{1}$$
$$= [1+3-1+3]$$
$$= 6 \text{ sq. units}$$

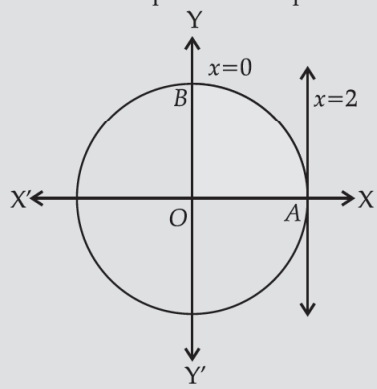
Q. 10. Area lying in the first quadrant and bounded by circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(A) π **(B)** $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{4}$

Ans. Option (A) is correct.

Explanation : The area bounded by the circle and the lines in the first quadrant is represented as :



$$A = \int_0^2 y\,dx$$
$$= \int_0^2 \sqrt{4-x^2}\,dx$$
$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$
$$= \pi \text{ sq. units}$$

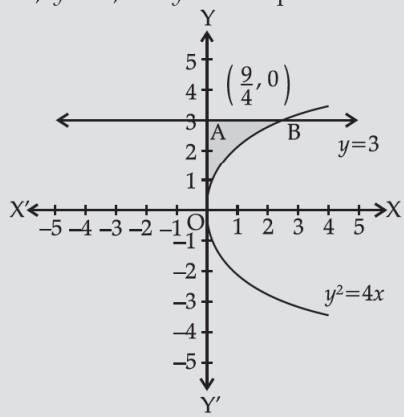
Q. 11. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2 **(B)** $\frac{9}{4}$

(C) $\frac{9}{3}$ **(D)** $\frac{9}{2}$

Ans. Option (B) is correct.

Explanation: The area bounded by the curve, $y^2 = 4x$, y-axis, and $y = 3$ is represented as :



Area of $OAB = \int_0^3 x\,dy$

$$= \int_0^3 \frac{y^2}{4} dy$$
$$= \frac{1}{4}\left[\frac{y^3}{4}\right]_0^3$$
$$= \frac{1}{12} \times 27$$
$$= \frac{9}{4} \text{ sq. units}$$

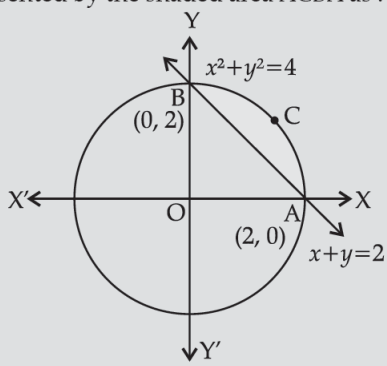
Q. 12. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$

(A) $2(\pi - 2)$ **(B)** $\pi - 2$

(C) $2\pi - 1$ **(D)** $2(\pi + 2)$

Ans. Option (B) is correct.

Explanation: The smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line, $x + y = 2$ is represented by the shaded area ACBA as :



It can be observed that

Area of $ACBA$ = Area of $OACBO$ − Area of ΔAOB

$$A = \int_0^2 \sqrt{4 - x^2}\, dx - \int_0^2 (2 - x) dx$$
$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x}{2}\right]_0^2$$
$$= \left[2 \times \frac{\pi}{2}\right] - [4 - 2]$$
$$= \pi - 2 \text{ sq. units}$$

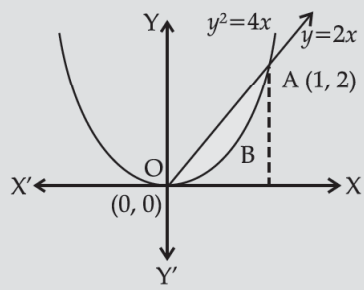
Q. 13. Area lying between the curve $y^2 = 4x$ and $y = 2x$

(a) $\frac{2}{3}$ **(B)** $\frac{1}{3}$

(C) $\frac{1}{4}$ **(D)** $\frac{3}{4}$

Ans. Option (B) is correct.

Explanation: The area lying between the curve $y^2 = 4x$ and $y = 2x$ is represented by the shaded area $OBAO$ as



The points of intersection of the curves are $O(0, 0)$ and $A(1, 2)$.

We draw AC perpendicular to x-axis such that coordinate of C is $(1, 0)$.

Area of $OBAO$=Area of ΔOCA − Area of $OCABO$

$$A = \int_0^1 2x dx - \int_0^1 2\sqrt{x} dx$$
$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^1$$
$$= \left|\left[1 - \frac{4}{3}\right]\right|$$
$$= \left|-\frac{1}{3}\right|$$
$$= \frac{1}{3} \text{ sq. unit}$$

Q. 14. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

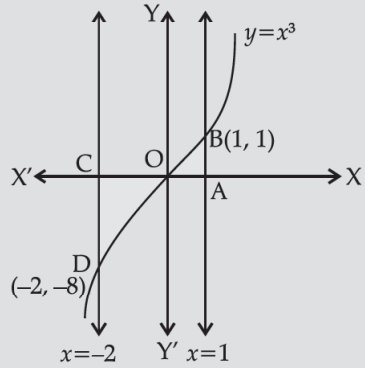
(A) -9 **(B)** $-\frac{15}{4}$

(C) $\frac{15}{4}$ **(D)** $\frac{17}{4}$

Ans. Option (C) is correct.

Explanation: Required area,

$$A = \int_{-2}^1 y dx$$
$$= \int_{-2}^1 x^3 dx$$



$$= \left[\frac{x^4}{4}\right]_{-2}^{1}$$

$$= \left(\frac{1}{4} - 4\right)$$

$$= -\frac{15}{4}$$

$\therefore$ Area $= \left|-\frac{15}{4}\right|$

$$= \frac{15}{4} \text{ sq. units}$$

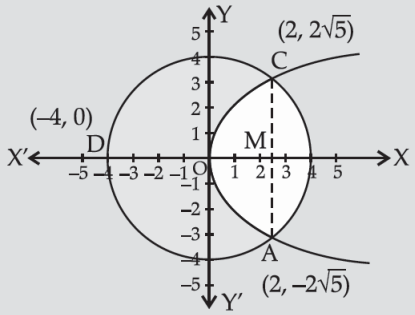
Q. 15. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

(A) $\frac{4}{3}\left(4\pi - \sqrt{3}\right)$ **(B)** $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$

(C) $\frac{4}{3}\left(8\pi - \sqrt{3}\right)$ **(D)** $\frac{4}{3}\left(8\pi + \sqrt{3}\right)$

Ans. Option (C) is correct.

Explanation:



Area bounded by the circle and parabola

$= 2[\text{area } (OAMO) + \text{area } (AMBA)]$

$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$

$= 2\int_0^2 \sqrt{6x}\,dx + 2\int_2^4 \sqrt{16 - x^2}\,dx$

$= 2\sqrt{6}\int_0^2 \sqrt{x}\,dx + 2\int_2^4 \sqrt{16 - x^2}\,dx$

$= 2\sqrt{6} \times \frac{2}{3}\left[x^{3/2}\right]_0^2 + 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4$

$= \frac{4\sqrt{6}}{2}\left(2\sqrt{2} - 0\right) +$

$2\left[\left\{0 + 8\sin^{-1}(1)\right\} - \left\{2\sqrt{3} + 8\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$

$= \frac{16\sqrt{3}}{3} + 2\left[8 \times \frac{\pi}{2} - 2\sqrt{3} - 8 \times \frac{\pi}{6}\right]$

$= \frac{16\sqrt{3}}{3} + 2\left(4\pi - 2\sqrt{3} - \frac{4\pi}{3}\right)$

$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8\pi}{3}$

$= \frac{16\sqrt{3} + 24\pi - 4\sqrt{3} - 8\pi}{3}$

$= \frac{16\pi + 12\sqrt{3}}{3}$

$= \frac{4}{3}\left[4\pi + \sqrt{3}\right]$ sq. units

Area of circle $= \pi(r)^2$

$= \pi(4)^2$

$= 16\pi$ sq. units

$\therefore$ Required area $= 16\pi - \frac{4}{3}\left(4\pi + \sqrt{3}\right)$

$= 16\pi - \frac{16\pi}{3} - \frac{4\sqrt{3}}{3}$

$= \frac{32\pi}{3} - \frac{4\sqrt{3}}{3}$

$= \frac{4}{3}\left[8\pi - \sqrt{3}\right]$ sq. units

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

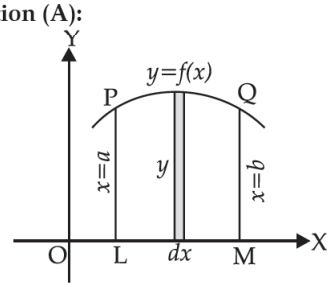
(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

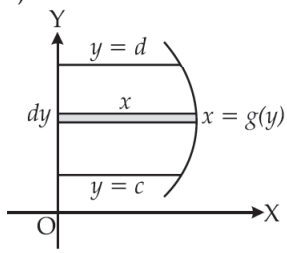
(D) A is false and R is True

Q. 1. Assertion (A):



The area of region $PQML = \int_a^b y\,dx = \int_a^b f(x)\,dx$

Reason (R):



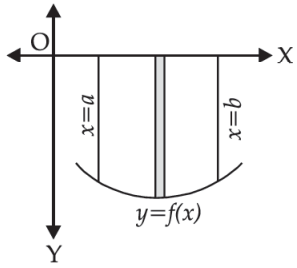
The area A of the region bounded by curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$ is given by

$$A = \int_c^d x\,dy$$

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.

Q. 2. Assertion (A):



$$\text{Area} = \left|\int_a^b f(x)\,dx\right|$$

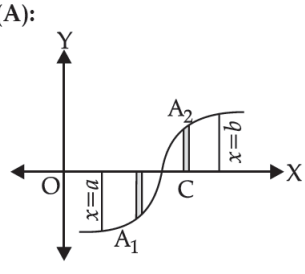
Reason (R): If the curve under consideration lies below x-axis, then $f(x) < 0$ from $x = a$ to $x = b$, the area bounded by the curve $y = f(x)$ and the ordinates $x = a$, $x = b$ and x-axis is negative. But, if the numerical value of the area is to be taken into consideration, then

$$\text{Area} = \left|\int_a^b f(x)\,dx\right|$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 3. Assertion (A):



$$\text{Area} = |A_1| + |A_2|$$

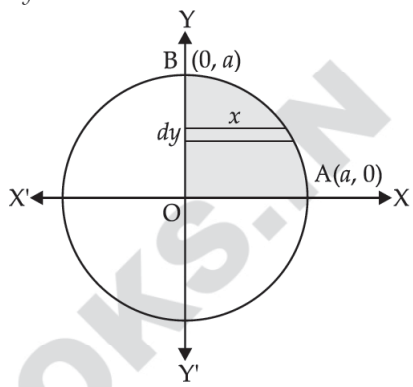
Reason (R): It may happen that some portion of the curve is above x-axis and some portion is below x-axis as shown in the figure. Let A_1 be the area below x-axis and A_2 be the area above the x-axis. Therefore, area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$ is given by

$$\text{Area} = |A_1| + |A_2|$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 4. Assertion (A): The area enclosed by the circle $x^2 + y^2 = a^2$ is πa^2.



Reason (R): The area enclosed by the circle

$$= 4\int_0^a x\,dy$$

$$= 4\int_0^a \sqrt{a^2 - y^2}\,dy$$

$$= 4\left[\frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}\right]_0^a$$

$$= 4\left[\left(\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}1\right) - 0\right]$$

$$= 4\frac{a^2}{2}\frac{\pi}{2}$$

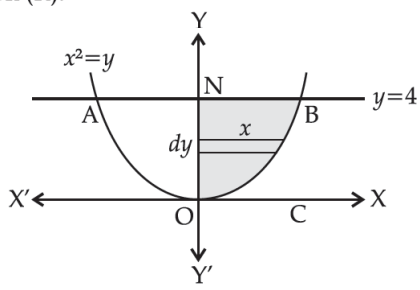
$$= \pi a^2$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 5. Assertion (A): The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is $\frac{3}{32}$.

Reason (R):



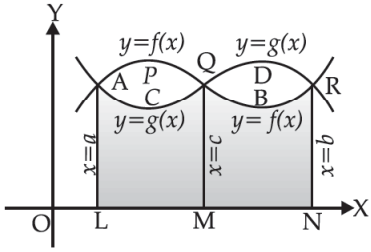
Since the given curve represented by the equation $y = x^2$ is a parabola symmetrical about y-axis only, therefore, from figure, the required area of the region $AOBA$ is given by

$$A = 2\int_0^4 x\,dy$$
$$= 2\int_0^4 \sqrt{y}\,dy$$
$$= 2\times\frac{2}{3}\left[y^{3/2}\right]_0^4$$
$$= \frac{4}{3}\times 8$$
$$= \frac{32}{3}$$

Ans. Option (D) is correct.

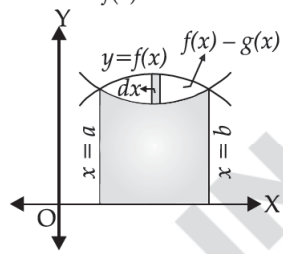
> ***Explanation:*** Assertion (A) is wrong. Reason (R) is the correct solution of Assertion (A).

Q. 6. Assertion (A): If the two curves $y = f(x)$ and $y = g(x)$ intersect at $x = a$, $x = c$ and $x = b$, such that $a < c < b$.



If $f(x) > g(x)$ in $[a, c]$ and $g(x) \le f(x)$ in $[c, b]$, then Area of the regions bounded by the curve
= Area of region $PACQP$ + Area of region $QDRBQ$.
$= \int_a^c |f(x) - g(x)|\,dx + \int_c^b |g(x) - f(x)|\,dx.$

Reason (R): Let the two curves by $y = f(x)$ and $y = g(x)$, as shown in the figure. Suppose these curves intersect at $f(x)$ with width dx.



$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$$
$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$
= Area bounded by the curve $\{y = f(x)\}$
–Area bounded by the curve $\{y = g(x)\}$,
where $f(x) > g(x)$.

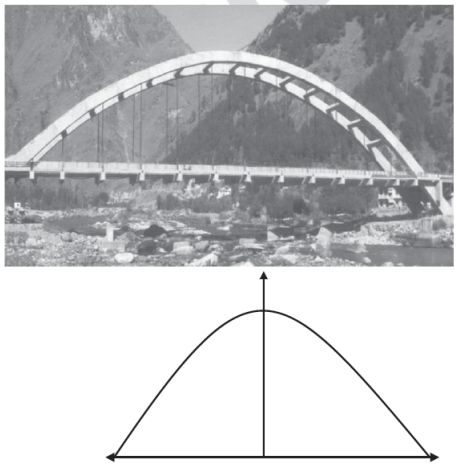
Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:



The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure.

[CBSE QB-2021]

Q. 1. The equation of the parabola designed on the bridge is

(A) $x^2 = 250y$ **(B)** $x^2 = -250y$
(C) $y^2 = 250x$ **(D)** $y^2 = 250y$

Ans. Option (C) is correct.

Q. 2. The value of the integral $\int_{-50}^{50} \frac{x^2}{250}$ is

(A) $\frac{1000}{3}$ **(B)** $\frac{250}{3}$
(C) 1200 **(D)** 0

Ans. Option (A) is correct.

> ***Explanation:***
> $$\int_{-50}^{50} \frac{x^2}{250} = \frac{1}{250}\left[\frac{x^3}{3}\right]_{-50}^{50}$$
> $$= \frac{1}{250}\times\frac{1}{3}\left[(50)^3 - (-50)^3\right]$$
> $$= \frac{1}{750}[125000 + 125000]$$
> $$= \frac{1000}{3}$$

Q. 3. The integrand of the integral $\int_{-50}^{50} x^2\,dx$ is _____ function.

(A) Even **(B)** Odd
(C) Neither odd nor even **(D)** None of these

Ans. Option (A) is correct.

Explanation:

$$f(x) = x^2$$
$$f(-x) = x^2$$
$\therefore f(x)$ is even function.

Q. 4. The area formed by the curve $x^2 = 250y$, x-axis, $y = 0$ and $y = 10$ is

(A) $\frac{1000\sqrt{2}}{3}$ **(B)** $\frac{4}{3}$

(C) $\frac{1000}{3}$ **(D)** 0

Ans. Option (C) is correct.

Explanation:

$$x^2 = 250y$$
$$y = \frac{1}{250}x^2$$
at $y = 0$ $x = 0$
at $y = 10$ $x = 50, -50$
$\therefore$ Area formed by curve
$$= \int_{-50}^{50} \frac{1}{250}x^2 dx$$
$$= \frac{1}{250} \times \frac{1}{3}\left[x^3\right]_0^{50}$$
$$= \frac{1}{750}[250,000]$$
$$= \frac{1000}{3} \text{ sq. units}$$

Q. 5. The area formed between $x^2 = 250y$, y-axis, $y = 2$ and $y = 4$ is

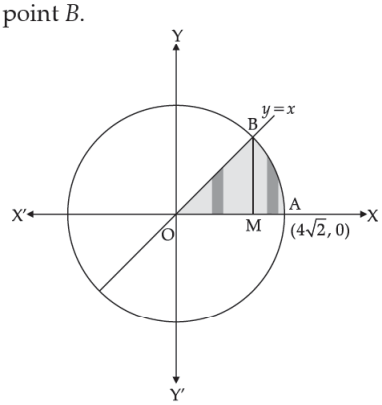
(A) $\frac{1000}{3}$ **(B)** 0

(C) $\frac{1000\sqrt{2}}{3}$ **(D)** None of these

Ans. Option (D) is correct.

II. Read the following text and answer the following questions on the basis of the same:

In the figure $O(0, 0)$ is the centre of the circle. The line $y = x$ meets the circle in the first quadrant at the point B.



Q. 1. The equation of the circle is _____.

(A) $x^2 + y^2 = 4\sqrt{2}$ **(B)** $x^2 + y^2 = 16$

(C) $x^2 + y^2 = 32$ **(D)** $(x - 4\sqrt{2})^2 + 0$

Ans. Option (C) is correct.

Explanation:

Centre = (0, 0),
$r = 4\sqrt{2}$
Equation of circle is
$$x^2 + y^2 = (4\sqrt{2})^2$$
$$\Rightarrow \quad x^2 + y^2 = 32$$

Q. 2. The co-ordinates of B are _____.

(A) (1, 1) **(B)** (2, 2)

(C) $(4\sqrt{2}, 4\sqrt{2})$ **(D)** (4, 4)

Ans. Option (D) is correct.

Explanation:

$$x^2 + y^2 = 32 \quad ...(i)$$
$$y = x \quad ...(ii)$$
Solving (i) and (ii),
$$\Rightarrow \quad x^2 + y^2 = 32$$
$$\Rightarrow \quad x^2 = 16$$
$$\Rightarrow \quad x = 4,$$
$$\Rightarrow \quad y = x = 4$$
$$\therefore \quad B = (4, 4)$$

Q. 3. Area of ΔOBM is _____ sq. units

(A) 8 **(B)** 16

(C) 32 **(D)** 32π

Ans. Option (A) is correct.

Explanation:

$$\text{Ar } (\Delta OBM) = \int_0^4 x dx$$
$$= \left[\frac{x^2}{2}\right]_0^4$$
$$= 8 \text{ sq. units}$$

Q. 4. *Ar (BAMB)* = _____ sq. units

(A) 32π **(B)** 4π

(C) 8 **(D)** $4\pi - 8$

Ans. Option (D) is correct.

Explanation:

$$Ar\ (BAMB) = \int_4^{4\sqrt{2}} \sqrt{32 - x^2} dx$$
$$= \left[\frac{x}{2}\sqrt{32 - x^2} + 16 \sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$
$$= (4\pi - 8) \text{ sq. units.}$$

Q. 5. Area of the shaded region is _____ sq. units.

(A) 32π **(B)** 4π

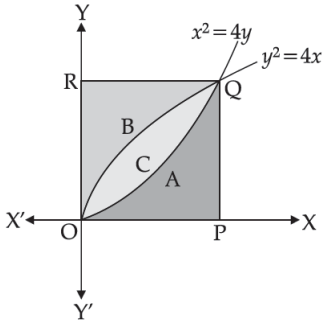
(C) 8 **(D)** $4\pi - 8$

Ans. Option (B) is correct.

Explanation:
Area of shaded region
$= Ar\,(\Delta OBM) + Ar\,(BAMB)$
$= 8 + 4\pi - 8$
$= 4\pi$ sq. units

III. Read the following text and answer the following questions on the basis of the same:

A farmer has a square plot of land. Three of its boundaries are $x = 0$, $y = 0$ and $y = 4$. He wants to divide this land among his three sons A, B and C as shown in figure.



Q. 1. Equation of PQ is _____.
(A) $x = 0$ **(B)** $x = 2$
(C) $x = 4$ **(d)** $y = 4$

Ans. Option (C) is correct.

Explanation: Equation of PQ is $x = 4$.

Q. 2. The co-ordinates of Q are _____.
(A) (2, 2) **(B)** (4, 4)
(C) (1, 1) **(D)** (5, 5)

Ans. Option (B) is correct.

Explanation: Q = (4, 4)

Q. 3. Area received by son B is _____ sq. units.
(A) 4 **(B)** 16
(C) $\frac{16}{3}$ **(D)** $\frac{8}{3}$

Ans. Option (C) is correct.

Explanation:

$$\text{Ar (son B)} = \int_0^4 x\,dy$$
$$= \int_0^4 \frac{y^2}{4}\,dy$$
$$= \left[\frac{y^3}{12}\right]_0^4$$
$$= \frac{1}{12}[4^3 - 0]$$
$$= \frac{64}{12}$$
$$= \frac{16}{3} \text{ sq. units}$$

Q. 4. Area received by son A is _____ sq. units.
(A) 4 **(B)** 16
(C) $\frac{16}{3}$ **(D)** $\frac{8}{3}$

Ans. Option (C) is correct.

Explanation:

$$\text{Ar(son A)} = \int_0^4 y\,dx$$
$$= \int_0^4 \frac{x^2}{4}dx$$
$$= \frac{1}{12}[x^3]_0^4$$
$$= \frac{16}{3} \text{ sq. units}$$

Q. 5. Total area of the square field is _____ sq. units.
(A) 4 **(B)** 16
(C) $\frac{16}{3}$ **(D)** $\frac{8}{3}$

Ans. Option (B) is correct.

Explanation:

$$\text{Total area} = 4 \times 4$$
$$= 16 \text{ sq. units}$$

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CHAPTER

9

Term-II

DIFFERENTIAL EQUATIONS

Syllabus

- ***Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree type:*** $\frac{dy}{dx} + f(y/x)$ ***. Solutions of linear differential equations of the type:***

$$\frac{dy}{dx} + py = q, \text{ where } p \text{ and } q \text{ are the functions of } x \text{ or constants}$$

STAND ALONE MCQs (1 Mark each)

Q. 1. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is

(A) 1 **(B)** 2

(C) 3 **(D)** not defined

Ans. Option (D) is correct.

Explanation: The degree of above differential equation is not defined because when we expand $\sin\left(\frac{dy}{dx}\right)$ we get an infinite series in the increasing powers of $\frac{dy}{dx}$. Therefore its degree is not defined.

Q. 2. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$ respectively, are

(A) 2 and 4 **(B)** 2 and 2

(C) 2 and 3 **(D)** 3 and 3

Ans. Option (A) is correct.

Explanation :

Given that, $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} = -x^{1/5}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} = -x^{1/5}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^{1/4} = -\left(x^{1/5} + \frac{d^2y}{dx^2}\right)$$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^{1/2} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^2$$

Again, on squaring both sides, we have

$$\frac{dy}{dx} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^4$$

Order = 2, degree = 4

Q. 3. If $y = e^{-x}(A\cos x + B\sin x)$, then y is a solution of

(A) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ **(B)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ **(D)** $\frac{d^2y}{dx^2} + 2y = 0$

Ans. Option (C) is correct.

Explanation:

Given that, $y = e^{-x}(A\cos x + B\sin x)$

On differentiating both sides w.r.t., x we get

$$\frac{dy}{dx} = -e^{-x}(A\cos x + B\sin x) + e^{-x}(-A\sin x + B\cos x)$$

$$\frac{dy}{dx} = -y + e^{-x}(-A\sin x + B\cos x)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-dy}{dx} + e^{-x}(-A\cos x - B\sin x) - e^{-x}(-A\sin x + B\cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y\left[\frac{dy}{dx} + y\right]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y - \frac{dy}{dx} - y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 2y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

Q. 4. The solution of differential equation $xdy - ydx = 0$ represents

(A) a rectangular hyperbola

(B) parabola whose vertex is at origin

(C) straight line passing through origin

(D) a circle whose centre is at origin

Ans. Option (C) is correct.

Explanation: Given that,

$$xdy - ydx = 0$$

$$\Rightarrow \quad xdy = ydx$$

$$\Rightarrow \quad \frac{dy}{y} = \frac{dx}{x}$$

On integrating both sides, we get

$$\log y = \log x + \log C$$

$$\Rightarrow \quad \log y = \log Cx$$

$$\Rightarrow \quad y = Cx$$

which is a straight line passing through the origin.

Q. 5. The integrating factor of differential equation $\cos x\frac{dy}{dx} + y\sin x = 1$ is

(A) $\cos x$ **(B)** $\tan x$

(C) $\sec x$ **(D)** $\sin x$

Ans. Option (C) is correct.

Explanation: Given that,

$$\cos x\frac{dy}{dx} + y\sin x = 1$$

$$\Rightarrow \quad \frac{dy}{dx} + y\tan x = \sec x$$

Here, $P = \tan x$ and $Q = \sec x$

$$\text{IF} = e^{\int P dx} = e^{\int \tan x dx} = e^{\ln \sec x}$$

$$\therefore \quad \text{IF} = \sec x$$

Q. 6. Family $y = Ax + A^3$ of curves is represented by the differential equation of degree :

(A) 1 **(B)** 2

(C) 3 **(D)** 4

Ans. Option (A) is correct.

Explanation:

Given that, $y = Ax + A^3$

$$\Rightarrow \quad \frac{dy}{dx} = A$$

[We can differentiate above equation only once because it has only one arbitrary constant.]

$\therefore$ Degree = 1

Q. 7. Which of the following is a second-order differential equation?

(A) $(y')^2 + x = y^2$ **(B)** $y'y'' + y = \sin x$

(C) $y''' + (y'')^2 + y = 0$ **(D)** $y' = y^2$

Ans. Option (B) is correct.

Explanation: The second-order differential equation is $y'y'' + y = \sin x$.

Q. 8. The integrating factor of differential equation $(1-x^2)\frac{dy}{dx} - xy = 1$ is

(A) $-x$ **(B)** $\frac{x}{1+x^2}$

(C) $\sqrt{1-x^2}$ **(D)** $\frac{1}{2}\log(1-x^2)$

Ans. Option (C) is correct.

Explanation: Given that,

$$(1-x^2)\frac{dy}{dx} - xy = 1$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{1}{1-x^2}$$

which is a linear differential equation.

$$\text{IF} = e^{-\int \frac{x}{1-x^2}dx}$$

Put $1 - x^2 = t$

$$\Rightarrow \quad -2xdx = dt$$

$$\Rightarrow \quad xdx = -\frac{dt}{2}$$

Now, $\text{IF} = e^{\frac{1}{2}\int \frac{dt}{t}}$

$$= e^{\frac{1}{2}\log t}$$
$$= e^{\frac{1}{2}\log(1-x^2)}$$
$$= \sqrt{1-x^2}$$

Q. 9. The degree of differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+6y^5=0$ is

(A) 1 **(B)** 2
(C) 3 **(D)** 5

Ans. Option (A) is correct.

Explanation:

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+6y^5=0$$

We know that, the degree of a differential equation is exponent of highest order derivative.

$\therefore$ Degree = 1

Q. 10. The integrating factor of differential equation $\frac{dy}{dx}+y\tan x-\sec x=0$ is

(A) $\cos x$ **(B)** $\sec x$
(C) $e^{\cos x}$ **(D)** $e^{\sec x}$

Ans. Option (B) is correct.

Explanation: Given that,

$$\frac{dy}{dx}+y\tan x-\sec x=0$$

Here, $P = \tan x$, $Q = \sec x$

$$\text{IF} = e^{\int P dx} = e^{\int \tan x dx}$$
$$= e^{(\log \sec x)}$$
$$= \sec x$$

Q. 11. $y=ae^{mx}+be^{-mx}$ satisfies which of the following differential equation

(A) $\frac{dy}{dx}+my=0$ **(B)** $\frac{dy}{dx}-my=0$
(C) $\frac{d^2y}{dx^2}-m^2y=0$ **(D)** $\frac{d^2y}{dx^2}+m^2y=0$

Ans. Option (C) is correct.

Explanation: Given that,

$$y = ae^{mx}+be^{-mx}$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = mae^{mx}-bme^{-mx}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = m^2ae^{mx}-bm^2e^{-mx}$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} = m^2(ae^{mn}+be^{-mn})$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} = m^2y$$
$$\Rightarrow \quad \frac{d^2y}{dx^2}-m^2y = 0$$

Q. 12. The solution of $x\frac{dy}{dx}+y=e^x$ is

(A) $y=\frac{e^x}{x}+\frac{k}{x}$ **(B)** $y = xe^x + Cx$
(C) $y = xe^x + k$ **(D)** $x=\frac{e^y}{y}+\frac{k}{y}$

Ans. Option (A) is correct.

Explanation: Given that,

$$x\frac{dy}{dx}+y = e^x$$
$$\Rightarrow \quad \frac{dy}{dx}+\frac{y}{x} = \frac{e^x}{x}$$

which is a linear differential equation.

$$\therefore \quad \text{IF} = e^{\int \frac{1}{x}dx}$$
$$= e^{(\log x)}$$
$$= x$$

The general solution is

$$y\cdot x = \int\left(\frac{e^x}{x}\cdot x\right)dx$$
$$\Rightarrow \quad y\cdot x = \int e^x dx$$
$$\Rightarrow \quad y\cdot x = e^x + k$$
$$\Rightarrow \quad y = \frac{e^x}{x}+\frac{k}{x}$$

Q. 13. The general solution of $\frac{dy}{dx}=2xe^{x^2-y}$ is

(A) $e^{x^2-y}=C$ **(B)** $e^{-y}+e^{x^2}=C$
(C) $e^y=e^{x^2}+C$ **(D)** $e^{x^2+y}=C$

Ans. Option (C) is correct.

Explanation : Given that,

$$\frac{dy}{dx} = 2xe^{x^2-y}$$
$$= 2xe^{x^2}\cdot e^{-y}$$
$$\Rightarrow \quad e^y\frac{dy}{dx} = 2xe^{x^2}$$
$$\Rightarrow \quad e^y dy = 2xe^{x^2}dx$$

On integrating both sides, we get

$$\int e^y dy = 2\int xe^{x^2}dx$$

Put $x^2 = t$ in RHS integral, we get

$$2xdx = dt$$

$$\int e^y dy = \int e^t dt$$

$$\Rightarrow \quad e^y = e^t + C$$

$$\Rightarrow \quad e^y = e^{x^2} + C$$

Q. 14. The solution of equation $(2y-1)dx - (2x+3)dy = 0$ is

(A) $\frac{2x-1}{2y+3} = k$ **(B)** $\frac{2y+1}{2x-3} = k$

(C) $\frac{2x+3}{2y-1} = k$ **(D)** $\frac{2x-1}{2y-1} = k$

Ans. Option (C) is correct.

Explanation: Given that,

$$(2y-1)dx - (2x+3)dy = 0$$

$$\Rightarrow \quad (2y-1)dx = (2x+3)dy$$

$$\Rightarrow \quad \frac{dx}{2x+3} = \frac{dy}{2y-1}$$

On integrating both sides, we get

$$\frac{1}{2}\log(2x+3) = \frac{1}{2}\log(2y-1) + \log C$$

$$\Rightarrow \quad \frac{1}{2}[\log\cdot(2x+3) - \log(2y-1)] = \log C$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{2x+3}{2y-1}\right) = \log C$$

$$\Rightarrow \quad \left(\frac{2x+3}{2y-1}\right)^{1/2} = C$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = C^2$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = k, \quad \text{where, } k = C^2$$

Q. 15. The solution of $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$ is

(A) $y = e^{-x}(x-1)$ **(B)** $y = xe^x$

(C) $y = xe^{-x} + 1$ **(D)** $y = xe^{-x}$

Ans. Option (D) is correct.

Explanation: Given that,

$$\frac{dy}{dx} + y = e^{-x}$$

which is a linear differential equation.

Here, $P = 1$ and $Q = e^{-x}$

$$\text{IF} = e^{\int dx} = e^x$$

The general solution is

$$y \cdot e^x = \int e^{-x} \cdot e^x dx + C$$

$$\Rightarrow \quad ye^x = \int dx + C$$

$$\Rightarrow \quad ye^x = x + C \qquad \ldots(i)$$

When $x = 0$ and $y = 0$ then, $0 = 0 + C \Rightarrow C = 0$

eqn. (i) becomes $y \cdot e^x = x \Rightarrow y = xe^{-x}$

Q. 16. The general solution of $\frac{dy}{dx} + y\tan x = \sec x$ is

(A) $y\sec x = \tan x + C$

(B) $y\tan x = \sec x + C$

(C) $\tan x = y\tan x + C$

(D) $x\sec x = \tan y + C$

Ans. Option (A) is correct.

Explanation: Given differential equation is

$$\frac{dy}{dx} + y\tan x = \sec x$$

which is a linear differential equation

Here, $P = \tan x$, $Q = \sec x$,

$$\therefore \quad \text{IF} = e^{\int \tan x dx} = e^{\log|\sec x|} = \sec x$$

The general solution is

$$y \cdot \sec x = \int \sec x \cdot \sec x + C$$

$$\Rightarrow \quad y \cdot \sec x = \int \sec^2 x \, dx + C$$

$$\Rightarrow \quad y \cdot \sec x = \tan x + C$$

Q. 17. The general solution of differential equation $(e^x+1)ydy = (y+1)e^x dx$ is

(A) $(y+1) = k(e^x+1)$

(B) $y + 1 = e^x + 1 + k$

(C) $y = \log\{k(y+1)(e^x+1)\}$

(D) $y = \log\frac{x+1}{y+1} + k$

Ans. Option (C) is correct.

Explanation: Given differential equation

$$(e^x+1)ydy = (y+1)e^x dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x(1+y)}{(e^x+1)y}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{(e^x+1)y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{e^x y}{e^x(1+y)} + \frac{y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y} + \frac{y}{(1+y)e^x}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(1 + \frac{1}{e^x}\right)$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(\frac{e^x+1}{e^x}\right)$$

$$\Rightarrow \quad \left(\frac{y}{1+y}\right)dy = \left(\frac{e^x}{e^x+1}\right)dx$$

On integrating both sides, we get

$$\int \frac{y}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \quad \int \frac{1+y-1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \int 1dy - \int \frac{1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \quad y - \log|(1+y) = \log|(1+e^x) + \log k$$

$$\Rightarrow \quad y = \log(1+y) + \log(1+e^x) + \log(k)$$

$$\Rightarrow \quad y = \log\left\{k(1+y)(1+e^x)\right\}$$

Q. 18. The solution of differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is

(A) $y(1+x^2) = C + \tan^{-1}x$

(B) $\frac{y}{1+x^2} = C + \tan^{-1}x$

(C) $y\log(1+x^2) = C + \tan^{-1}x$

(D) $y(1+x^2) = C + \sin^{-1}x$

Ans. Option (A) is correct.

Explanation: Given that,

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$$

Here, $P = \frac{2x}{1+x^2}$

and $Q = \frac{1}{(1+x^2)^2}$

which is a linear differential equation.

$$\therefore \quad \text{IF} = e^{\int \frac{2x}{1+x^2}dx}$$

Put $1+x^2 = t$

$$\Rightarrow \quad 2xdx = dt$$

$$\therefore \quad \text{IF} = e^{\int \frac{dt}{t}} = e^{\log t} = e^{\log(1+x^2)} = 1+x^2$$

The general solution is

$$y\cdot(1+x^2) = \int (1+x^2)\frac{1}{(1+x^2)^2} + C$$

$$\Rightarrow \quad y(1+x^2) = \int \frac{1}{1+x^2}dx + C$$

$$\Rightarrow \quad y(1+x^2) = \tan^{-1}x + C$$

Q. 19. The order of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is

(A) 2 **(B)** 1

(C) 0 **(D)** not defined

Ans. Option (A) is correct.

Explanation :

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

Q. 20. The numbers of arbitrary constants in the general solution of a differential equation of fourth order are :

(A) 0 **(B)** 2

(C) 3 **(D)** 4

Ans. Option (D) is correct.

Explanation: We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth-order differential equation is four.

Q. 21. The numbers of arbitrary constants in the particular solution of a differential equation of third order are :

(A) 3 **(B)** 2

(C) 1 **(D)** 0

Ans. Option (D) is correct.

Explanation : In the particular solution of a differential equation, there are no arbitrary constants.

Q. 22. Which of the following differential equations has $y = x$ as one of its particular solution?

(A) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$ **(B)** $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

(C) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ **(D)** $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

Ans. Option (C) is correct.

Explanation: The given equation of curve is $y=x$

Differentiating with respect to x, we get :

$$\frac{dy}{dx} = 1 \qquad \text{...(i)}$$

Again, differentiating with respect to x, we get :

$$\frac{d^2y}{dx^2} = 0 \qquad \text{...(ii)}$$

Now, on substituting the values of y, $\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (i) and (ii) in each of the given alternatives, we find that only the differential equation given in alternative C is correct.

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 - x^2 \cdot 1 + x \cdot x$$
$$= -x^2 + x^2$$
$$= 0$$

Q. 23. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(A) $e^x + e^{-y} = C$ **(B)** $e^x + e^y = C$

(C) $e^{-x} + e^y = C$ **(D)** $e^{-x} + e^{-y} = C$

Ans. Option (A) is correct.

Explanation:

$$\frac{dy}{dx} = e^{x+y}$$
$$= e^x \cdot e^y$$
$$\Rightarrow \quad \frac{dy}{e^y} = e^x dx$$
$$\Rightarrow \quad e^{-y}dy = e^x dx$$

Integrating both sides, we get:

$$\int e^{-y}dy = \int e^x dx$$
$$\Rightarrow \quad -e^{-y} = e^x + k$$
$$\Rightarrow \quad e^x + e^{-y} = -k$$
$$\Rightarrow \quad e^x + e^{-y} = C \qquad (\text{where}, C = -k)$$

Q. 24. The Integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(A) e^{-x} **(B)** e^{-y}

(C) $\frac{1}{x}$ **(D)** x

Ans. Option (C) is correct.

Explanation: The given differential equation is:

$$x\frac{dy}{dx} - y = 2x^2$$
$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = 2x$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q$$

$$(\text{where } p = -\frac{1}{x} \text{ and } Q = 2x)$$

The integrating factor (IF) is given by the relation,

$$\text{IF} = \int p dx$$
$$\therefore \quad \text{IF} = e^{\int -\frac{1}{x}dx}$$
$$= e^{-\log x}$$
$$= e^{\log(x^{-1})}$$
$$= x^{-1}$$
$$= \frac{1}{x}$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): The order of the differential equation given by $\frac{dy}{dx} + 4y = \sin x$ is 1.

Reason (R): Since the order of a differential equation is defined as the order of the highest derivative occurring in the differential equation, *i.e.*, for nth derivative $\frac{d^n y}{d^x n}$ if $n = 1$. then it's order = 1.

Given differential equation contains only $\frac{dy}{dx}$ derivative with variables and constants.

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A): The degree of the differential equation given by $\frac{dy}{dx} = \frac{x^4 - y^4}{(x^2 + y^2)xy}$ is 1.

Reason (R): The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radicals and fraction.

The given differential equation has first order derivative which is free from radical and fraction with power = 1, thus it has a degree of 1.

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 3. Assertion (A): Solution of the differential equation $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$ is $\frac{e^{2y}}{3} = \frac{e^{3x}}{3} + \frac{x^2}{2} + C$

Reason (R):

$$\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$$

$$\frac{dy}{dx} = e^{-2y}(e^{3x} + x^2)$$

separating the variables

$$e^{2y}dy = (e^{3x} + x^2)dx \qquad \text{[integrating]}$$

$$\int e^{2y}dy = \int (e^{3x} + x^2)dx$$

$$\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + C.$$

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong. The correct solution is given in Reason (R).

Q. 4. Assertion (A): The solution of differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ is $\cos\left(\frac{y}{x}\right) = xc$

Reason (R): $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ we can clearly see that it is an homogeneous equation substituting

$$y = vx$$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v + \tan v$$

separating the variables and integrating we get

$$\int \frac{1}{\tan v}dv = \int \frac{1}{x}dx$$

$$\log(\sin v) = \log x + \log C$$

$$\sin(v) = xC$$

$$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = xC$$

is the solution where, C is constant.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong. The correct solution is given in Reason (R).

Q. 5. Assertion (A): The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx} + 5}$ are 2 and 1 respectively

Reason (R): The differential equation

$$\left(\frac{dx}{dy}\right)^3 + 2y^{1/2} = x$$

is of order 1 and degree 3.

Ans. Option (B) is correct.

Explanation: Squaring both sides of the given differential equation,

$$\left(\sqrt{\frac{d^2y}{dx^2}}\right)^2 = \left(\sqrt{\frac{dy}{dx} + 5}\right)^2$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} + 5$$

The highest order is 2 and its power is 1

$\therefore$ Order is 2, degree is 1

Hence, Assertion (A) is true.

The equation given in reason (R) is,

$$\left(\frac{1}{\frac{dy}{dx}}\right)^3 + 2\sqrt{y} = x$$

$$\Rightarrow \quad \frac{1 + 2\sqrt{y}\left(\frac{dy}{dx}\right)^3}{\left(\frac{dy}{dx}\right)^3} = x$$

$$\Rightarrow \quad 1 + 2\sqrt{y}\left(\frac{dy}{dx}\right)^3 = x\left(\frac{dy}{dx}\right)^3$$

Highest order is 1 and its power is 3

$\therefore$ Order is 1 and degree is 3.

Hence, reason (R) is also true.

Q. 6. Assertion (A): The differential equation formed by eliminating a and b from $y = ae^x + be^{-x}$ is $\frac{d^2y}{dx^2} - y = 0$

Reason (R):

$$y = ae^x + be^{-x} \qquad \text{...(i)}$$

Differentiating w.r.t. 'x'

$$\frac{dy}{dx} = ae^x - be^{-x}$$

Differentiating again w.r.t. 'x'

$$\frac{d^2y}{dx^2} = ae^x + be^{-x} \qquad \text{...(ii)}$$

Subtracting eqn. (i) from eqn. (ii)

$$\frac{d^2y}{dx^2} - y = ae^x + be^{-x} - ae^x - be^{-x}$$

$$= 0$$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation: $\frac{dT}{dt} \propto (T-70)$, where 70°F is the room temperature and T is the temperature of the object at time t.

Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k(T-70)$ where k is a constant of proportion, time of death is calculated. **[CBSE QB-2021]**

Q. 1. What will be the degree of the above given differential equation.

(A) 2 **(B)** 1

(C) 0 **(D)** 3

Ans. Option (B) is correct.

Q. 2. Which method of solving a differential equation helped in calculation of the time of death?

(A) Variable separable method

(B) Solving Homogeneous differential equation

(C) Solving Linear differential equation

(D) all of the above

Ans. Option (A) is correct.

Q. 3. If the temperature was measured 2 hours after 11.30 pm, what will be the change in time of death?

(A) No change

(B) Death time increased

(C) Death time decreased

(D) Death time always constant

Ans. Option (A) is correct.

Q. 4. The solution of the differential equation $\frac{dT}{dt} = k(T-70)$ is given by,

(A) $\log|T-70| = kt + C$

(B) $\log|T-70| = \log|kt| + C$

(C) $T - 70 = kt + C$

(D) $T - 70 = kt\,C$

Ans. Option (A) is correct.

Q. 5. If $t = 0$ when T is 72, then the value of C is

(A) –2 **(B)** 0

(C) 2 **(D)** log 2

Ans. Option (D) is correct.

II. Read the following text and answer the following questions on the basis of the same:

Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$ where x denotes the number of weeks and y the number of children who have been given the drops. **[CBSE QB-2021]**

Q. 1. State the order of the above given differential equation.

(A) 2 **(B)** 1

(C) 0 **(D)** Can't define

Ans. Option (B) is correct.

Q. 2. Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(50 - y)$?

(A) Variable separable method

(B) Solving Homogeneous differential equation

(C) Solving Linear differential equation

(D) all of the above

Ans. Option (A) is correct.

Q. 3. The solution of the differential equation $\frac{dy}{dx} = k(50 - y)$ is given by,

(A) $\log|50-y| = kx + C$

(B) $-\log|50-y| = kx + C$

(C) $\log|50-y| = \log|kx| + C$

(D) $50 - y = kx + C$

Ans. Option (B) is correct.

Explanation:

$$\frac{dy}{dx} = k(50 - y)$$
$$\int \frac{dy}{50 - y} = \int K dx$$
$$-\log|50 - y| = Kx + C$$

Q. 4. The value of C in the particular solution given that $y(0) = 0$ and $k = 0.049$ is

(A) $\log 50$ **(B)** $\log \frac{1}{50}$

(C) 50 **(D)** -50

Ans. Option (B) is correct.

Explanation:

Given, $y(0) = 0$ and $k = 0.049$

We have, $-\log|50 - y| = Kx + C$

$\log|50 - y| = -Kx - C$

$\log|50 - 0| = 0 - C$

$[\because x = 0, K = 0.049, y(0) = 0]$

$\log 50 = -C$

$C = \log \frac{1}{50}$

Q. 5. Which of the following solutions may be used to find the number of children who have been given the polio drops?

(A) $y = 50 - e^{kx}$ **(B)** $y = 50 - e^{-kx}$

(C) $y = 50(1 - e^{-kx})$ **(D)** $y = 50(e^{-kx} - 1)$

Ans. Option (C) is correct.

Explanation: We have

$$-\log|50 - y| = Kx + C$$
$$-\log|50 - y| = Kx + \log \frac{1}{50}$$
$$\log \frac{50 - y}{50} = -Kx$$
$$\frac{50 - y}{50} = e^{-Kx}$$
$$50 - y = 50e^{-Kx}$$
$$y = 50 - 50e^{-Kx}$$
$$y = 50(1 - e^{-Kx})$$

III. Read the following text and answer the following questions on the basis of the same:

The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours.

Q. 1. $\int \frac{1}{Kx} dx =$ _____.

(A) $\log|x| + C$ **(B)** $\log|Kx| + C$

(C) $\frac{1}{K}\log|x| + C$ **(D)** $\frac{-1}{Kx^2} + C$

Ans. Option (C) is correct.

Explanation:

$$\int \frac{1}{Kx} dx = \frac{1}{K}\int \frac{1}{x} dx$$
$$= \frac{1}{K}\log|x| + C$$

Q. 2. If 'N' is the number of bacteria, the corresponding differential equation is _____.

(A) $\frac{dN}{dt} = Kt$ **(B)** $\frac{dN}{dt} = KN$

(C) $\frac{dK}{dt} = N$ **(D)** $\frac{dK}{dN} = t$

Ans. Option (B) is correct.

Explanation: Given that N is the number of bacteria.

$$\frac{dN}{dt} \propto N$$
$$\Rightarrow \quad \frac{dN}{dt} = KN$$

Q. 3. The general solution is _____.

(A) $\log|N| = Kt + C$ **(B)** $\log|Nt| = K + C$

(C) $\log|N| = t$ **(D)** $\log|Kt| = N + C$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dN}{N} = K\int dt$$
$$\log|N| = Kt + C \qquad ...(i)$$

Q. 4. If N_0 is the initial count of bacteria, after 10 hours the count is _____.

(A) $\frac{1}{5}\log 3$ **(B)** $3\log N_0$

(C) $9N_0$ **(D)** $2N_0$

Ans. Option (C) is correct.

Explanation: Given when $t = 0, N = N_0$.

From (i), $\log|N_0| = C$

$\therefore$ (i) $\Rightarrow$ $\log|N| = Kt + \log|N_0|$

$\Rightarrow$ $\log\left|\frac{N}{N_0}\right| = Kt$ ...(ii)

Given when $t = 5, N = 3N_0$.

From (ii), $\log|3| = 5K$

$\Rightarrow$ $K = \frac{1}{5}\log 3$

$\therefore$ The particular solution is

$$\log\left|\frac{N}{N_0}\right| = \frac{t}{5}\log 3 \quad ...(3)$$

When $t = 10$,

$$\log\left|\frac{N}{N_0}\right| = 2\log 3 = \log 9$$

$$\frac{N}{N_0} = 9$$

$\Rightarrow$ $N = 9N_0$

Q. 5. The bacteria becomes 10 times in _____ hours.

(A) $5\log 7$ **(B)** $\frac{5\log 10}{\log 3}$

(C) $\frac{5}{\log 3}$ **(D)** $\log\left(\frac{10^5}{3}\right)$

Ans. Option (B) is correct.

Explanation:

Given $N = 10N_0$,

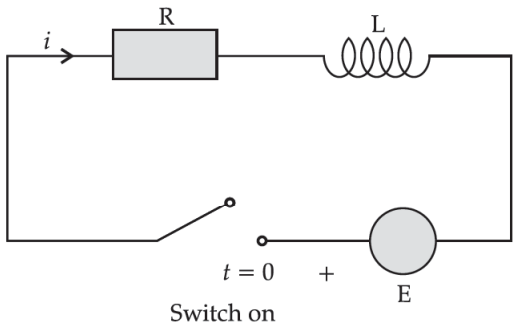
$$\log\left|\frac{N}{N_0}\right| = \frac{t}{5}\log 3$$

(iii) $\Rightarrow \log 10 = \frac{t}{5}\log 3$

$$t = \frac{5\log 10}{\log 3}$$

IV. Read the following text and answer the following questions on the basis of the same:

A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone, is called a first order linear differential equation. It has many applications in physics including RL circuits.



Switch on

Consider the linear differential equation

$$x\frac{dy}{dx} + 2y = x^2$$

Q. 1. $\int Q\,dx =$ _____.

(A) $\log|x| + C$ **(B)** $2x + C$

(C) $\log x^2 + C$ **(D)** $\frac{x^2}{2} + C$

Ans. Option (D) is correct.

Explanation: The given differential equation can be expressed as

$$\frac{dy}{dx} + \frac{2y}{x} = x$$

$$P = \frac{2}{x},\ Q = x$$

$$\int Q\,dx = \int x\,dx$$

$$= \frac{x^2}{2} + C$$

Q. 2. The value of $\int P\,dx =$ _____.

(A) $\frac{x^3}{3} + C$ **(B)** $\log x^2 + C$

(C) $\log|x| + C$ **(D)** $\frac{x^2}{2} + C$

Ans. Option (B) is correct.

Explanation:

$$\int P\,dx = \int \frac{2}{x}dx$$

$$= 2\log|x| + C$$

$$= \log x^2 + C$$

Q. 3. The integrating factor is _____.

(A) $\log|x|$ **(B)** $2x$

(C) x^2 **(D)** $e^{\log x}$

Ans. Option (C) is correct.

Explanation:

$$\text{IF} = e^{\int P\,dx}$$

$$= e^{\log x^2}$$

$$= x^2$$

Q. 4. The general solution is _____.

(A) $y = \frac{x^2}{4} + C$ **(B)** $yx^2 = \frac{x^4}{4} + C$

(C) $xy^2 = \frac{x^3}{3} + C$ **(D)** $y^2 = \frac{x^2}{3} + C$

Ans. Option (B) is correct.

Explanation: The general solution is

$$y(x^2) = \int (x^2 \times x)\,dx$$

$$x^2 y = \frac{x^4}{4} + C$$

Q. 5. If $y(1) = 0$, then $y(2) =$ _____.

(A) 0 **(B)** 1

(C) $\frac{15}{4}$ **(D)** $\frac{15}{16}$

Ans. Option (D) is correct.

Explanation:

Given $y(1) = 0$

$$\Rightarrow \quad 0 = \frac{1}{4} + C$$

$$\Rightarrow \quad C = \frac{-1}{4}$$

The particular solution is

$$yx^2 = \frac{x^4 - 1}{4}$$

When $x = 2$,

$$4y = \frac{15}{4}$$

$$\Rightarrow \quad y = \frac{15}{16}$$

❐❐

CHAPTER 10

Term-II

VECTORS

Syllabus

- ***Vectors and Scalars, Magnitude and direction of a vector. Direction cosines and direction ratios of a vector , Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.***

STAND ALONE MCQs (1 Mark each)

Q. 1. If θ be the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when

(A) $0 < \theta < \frac{\pi}{2}$ **(B)** $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$ **(D)** $0 \leq \theta \leq \pi$

Ans. Option (B) is correct.

Explanation: Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

It is known that, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$\therefore \quad \vec{a}.\vec{b} \geq 0$

$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \quad \cos\theta \geq 0$

$\left[\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}\right]$

$\Rightarrow \quad 0 \leq \theta \leq \frac{\pi}{2}.$

Q. 2. Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

(A) $\theta = \frac{\pi}{4}$ **(B)** $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$ **(D)** $\theta = \frac{2\pi}{3}$

Ans. Option (D) is correct.

Explanation: Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ be the angle between them.

Then, $|\vec{a} + \vec{b}| = |\vec{b}| = 1.$

Now, $\vec{a} + \vec{b}$ is a unit vector if

$$|\vec{a} + \vec{b}| = 1$$

$$\Rightarrow (\vec{a} + \vec{b})^2 = 1$$

$$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = 1$$

$$\Rightarrow \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}\vec{b} = 1$$

$$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 1$$

$$\Rightarrow 1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 = 1$$

$$\Rightarrow \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

So that, $|\vec{a} + \vec{b}|$ is a unit vector if $\theta = \frac{2\pi}{3}$.

Q. 3. The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$ is

(A) 0 **(B)** –1

(C) 1 **(D)** 3

Ans. Option (C) is correct.

Explanation :

$$\begin{aligned}\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j}) &= \hat{i}.\hat{i}+\hat{j}.(-\hat{j})+\hat{k}.\hat{k}\\ &= 1-\hat{j}.\hat{j}+1\\ &= 1-1+1\\ &= 1\end{aligned}$$

Q. 4. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ when θ is equal to

(A) 0 **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ **(D)** π

Ans. Option (B) is correct.

Explanation: Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$\begin{aligned}&|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|\\ \Rightarrow\quad &|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta\\ \Rightarrow\quad &\cos\theta = \sin\theta\\ &\left[\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}\right]\\ \Rightarrow\quad &\tan\theta = 1\\ \Rightarrow\quad &\theta = \frac{\pi}{4}\end{aligned}$$

So that, $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ when θ is equal to $\frac{\pi}{4}$.

Q. 5. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

Ans. Option (B) is correct.

Explanation :

It is given that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$.

We know that $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a}\times\vec{b}$ is a unit vector if

$$\begin{aligned}&|\vec{a}\times\vec{b}| = 1\\ \Rightarrow\quad &|\vec{a}||\vec{b}|\sin\theta\hat{n}| = 1\\ \Rightarrow\quad &|\vec{a}||\vec{b}||\sin\theta| = 1\\ \Rightarrow\quad &3\times\frac{\sqrt{2}}{3}\times\sin\theta = 1\\ \Rightarrow\quad &\sin\theta = \frac{1}{\sqrt{2}}\\ \Rightarrow\quad &\theta = \frac{\pi}{4}\end{aligned}$$

So that, $\vec{a}\times\vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

Q. 6. Area of a rectangle having vertices *A*, *B*, *C* and *D* with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively is

(A) $\frac{1}{2}$ **(B)** 1

(C) 2 **(D)** 4

Ans. Option (C) is correct.

Explanation : The position vectors of vertices *A*, *B*, *C* and *D* of rectangle *ABCD* are given as :

$$\begin{aligned}\overrightarrow{OA} &= -\hat{i}+-\hat{j}+\hat{k}\\ \overrightarrow{OB} &= \hat{i}+\frac{1}{2}\hat{j}+4\hat{k},\\ \overrightarrow{OC} &= \hat{i}-\frac{1}{2}\hat{j}+4\hat{k},\\ \overrightarrow{OD} &= -\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}\end{aligned}$$

The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of the given rectangle are given as :

$$\begin{aligned}\overrightarrow{AB} &= (1+1)\hat{i}+\left(\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k}\\ &= 2\hat{i}\\ \overrightarrow{BC} &= (1-1)\hat{i}+\left(-\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k}\\ &= -\hat{j}\\ \therefore\quad \overrightarrow{AB}\times\overrightarrow{BC} &= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 0\\ 0 & -1 & 0\end{vmatrix}\\ &= \hat{k}(-2)\\ &= -2\hat{k}\\ \Rightarrow\quad \left|\overrightarrow{AB}\times\overrightarrow{BC}\right| &= 2\end{aligned}$$

Now, it is known that the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a}\times\vec{b}|$.

Therefore, the area of the given rectangle is $\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = 2$sq. units.

Q. 7. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector if

(A) $\lambda = 1$ **(B)** $\lambda = -1$

(C) $a = |\lambda|$ **(D)** $a = \frac{1}{|\lambda|}$

Ans. Option (D) is correct.

Explanation :

Vector $\lambda\vec{a}$ is a unit vector if

$$|\lambda\vec{a}| = 1$$

$$\Rightarrow |\lambda||\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \quad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \quad [|\vec{a}| = a]$$

Therefore, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$.

Q. 8. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect :

(A) $\vec{b} = \lambda\vec{a}$, for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

Ans. Option (D) is correct.

Explanation :

If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then they are parallel.

Therefore, we have

$\vec{b} = \lambda\vec{a}$ (For some scalar λ)

If $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then,

$$\vec{b} = \lambda\vec{a}$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

So that, the respective components of $\vec{a}$ and $\vec{b}$ are proportional. However, vectors $\vec{a}$ and $\vec{b}$ can have different directions.

Q. 9. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

(A) $\hat{i} - 2\hat{j} + 2\hat{k}$ **(B)** $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

(C) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ **(D)** $9(\hat{i} - 2\hat{j} + 2\hat{k})$

Ans. Option (C) is correct.

Explanation :

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}}$$

$$= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude 9

$$= 9\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

$$= 3(\hat{i} - 2\hat{j} + 2\hat{k})$$

Q. 10. The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 :1 is

(A) $\frac{3\vec{a} - 2\vec{b}}{2}$ **(B)** $\frac{7\vec{a} - 8\vec{b}}{4}$

(C) $\frac{3\vec{a}}{4}$ **(D)** $\frac{5\vec{a}}{4}$

Ans. Option (D) is correct.

Explanation :

Let the position vector of the R divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$.

$\therefore$ Position vector, $R = \frac{3(\vec{a} + \vec{b}) + 1(2\vec{a} - 3\vec{b})}{3 + 1}$

Since, the position vector of a point R dividing the line segments joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ration $m : n$ internally, is given by $\frac{m\vec{q} + n\vec{p}}{m + n}$.

$$\therefore R = \frac{5\vec{a}}{4}$$

Q. 11. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is :

(A) $-\hat{i} + 12\hat{j} + 4\hat{k}$ **(B)** $5\hat{i} + 2\hat{j} - 4\hat{k}$

(C) $-5\hat{i} + 2\hat{j} + 4\hat{k}$ **(D)** $\hat{i} + \hat{j} + \hat{k}$

Ans. Option (C) is correct.

Explanation:

Required vector $= (-3 - 2)\hat{i} + (7 - 5)\hat{j} + (4 - 0)\hat{k}$

$$= -5\hat{i} + 2\hat{j} + 4\hat{k}$$

Q. 12. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b} = 2\sqrt{3}$ is :

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{2}$

Ans. Option (B) is correct.

Explanation :

Here, $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 4$ and $\vec{a}.\vec{b} = 2\sqrt{3}$ [Given]

We know that,

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\Rightarrow \quad 2\sqrt{3} = \sqrt{3}.4.\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{2\sqrt{3}}{4\sqrt{3}}$$

$$= \frac{1}{2}$$

$$\therefore \quad \theta = \frac{\pi}{3}$$

Q. 13. Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal.

(A) 0 **(B)** 1

(D) $\frac{3}{2}$ (D) $\frac{-5}{2}$

Ans. Option (D) is correct.

Explanation :

Since, two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal, *i.e.*, $\vec{a}.\vec{b} = 0$

$$\therefore (2\hat{i} + \lambda\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow \quad 2 + 2\lambda + 3 = 0$$

$$\therefore \quad \lambda = \frac{-5}{2}$$

Q. 14. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) $\frac{5}{2}$ (D) $\frac{2}{5}$

Ans. Option (A) is correct.

Explanation:

Let $\vec{a} = 3\hat{i} - 6\hat{j} + \hat{k}$

and $\vec{b} = 2\vec{i} - 4\vec{j} + \lambda\hat{k}$

Since, $\vec{a} \parallel \vec{b}$

$$\Rightarrow \quad \frac{3}{2} = \frac{-6}{-4} = \frac{1}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{2}{3}$$

Q. 15. The vectors from origin to the points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then the area of triangle OAB is

(A) 340 (B) $\sqrt{25}$

(C) $\sqrt{229}$ (D) $\frac{1}{2}\sqrt{229}$

Ans. Option (D) is correct.

Explanation :

$$\text{Area of } \Delta OAB = \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$$

$$= \frac{1}{2}\left|(2\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})\right|$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|\left[\hat{i}(-3-6) - \hat{j}(2-4) + \hat{k}(6+6)\right]\right|$$

$$= \frac{1}{2}\left|-9i + 2\hat{j} + 12\hat{k}\right|$$

$$\text{Area of } \Delta OAB = \frac{1}{2}\sqrt{(81+4+144)}$$

$$= \frac{1}{2}\sqrt{229}$$

Q. 16. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to

(A) $\vec{a}^2$ **(B)** $3\vec{a}^2$

(C) $4\vec{a}^2$ **(D)** $2\vec{a}^2$

Ans. Option (D) is correct.

Explanation :

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\therefore \quad \vec{a}^2 = x^2 + y^2 + z^2$$

$$\therefore \quad \vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}[0] - \hat{j}[-z] + \hat{k}[-y]$$

$$= z\hat{j} - y\hat{k}$$

$$\therefore \quad (\vec{a} \times \hat{i})^2 = (z\hat{j} - y\hat{k})(z\hat{j} - y\hat{k})$$

$$= y^2 + z^2$$

Similarly, $(\vec{a} \times \hat{j})^2 = x^2 + z^2$ and $(\vec{a} \times \hat{k})^2 = x^2 + z^2$

$$(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = y^2 + z^2 + x^2 + z^2 + x^2 + y$$

$$= 2(x^2 + y^2 + z^2)$$

$$= 2\vec{a}^2$$

Q. 17. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is

(A) 5 **(B)** 10

(C) 14 **(D)** 16

Ans. Option (D) is correct.

Explanation :

Here, $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$ [Given]

$$\therefore \quad \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$12 = 10 \times 2\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{12}{20}$$

$$= \frac{3}{5}$$

$$\Rightarrow \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{25}}$$

$$\sin\theta = \pm\frac{4}{5}$$

$$\therefore \quad |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}||\sin\theta|$$

$$= 10 \times 2 \times \frac{4}{5}$$

$$= 16$$

Q. 18. The vectors $\lambda\hat{i}+\hat{j}+2\hat{k}$, $\hat{i}+\lambda\hat{j}-\hat{k}$ and $2\hat{i}-\hat{j}+\lambda\hat{k}$ are coplanar, if

(A) $\lambda = -2$ **(B)** $\lambda = 0$

(C) $\lambda = 1$ **(D)** $\lambda = -1$

Ans. Option (A) is correct.

Explanation : Let $\vec{a} = \lambda\hat{i}+\hat{j}+2\hat{k}$, $\vec{b} = \hat{i}+\lambda\hat{j}-\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+\lambda\hat{k}$

For $\vec{a}, \vec{b}$ and $\vec{c}$ to be coplanar,

$$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda(\lambda^2-1)-1(\lambda+2)+2(-1-2\lambda) = 0$$

$$\Rightarrow \quad \lambda^3-\lambda-\lambda-2-2-4\lambda = 0$$

$$\Rightarrow \quad \lambda^3-6\lambda-4 = 0$$

$$\Rightarrow \quad (\lambda+2)(\lambda^2-2\lambda-2) = 0$$

$$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = \frac{2\pm\sqrt{12}}{2}$$

$$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = \frac{2\pm 2\sqrt{3}}{2}$$

$$= 1\pm\sqrt{3}$$

Q. 19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(A) 1 **(B)** 3

(C) $\frac{-3}{2}$ **(D)** None of these

Ans. Option (C) is correct.

Explanation :

We have, $\vec{a}+\vec{b}+\vec{c} = 0$ and , $\vec{b}^2 = 1, \vec{c}^2 = 1$

$$\therefore \quad (\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c}) = 0$$

$$\Rightarrow \quad \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+$$

$$\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}^2 = 0$$

$$\Rightarrow \quad \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}, \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{b} \text{ and } \vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{c}]$

$$\Rightarrow \quad 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -\frac{3}{2}$$

Q. 20. The projection vector of $\vec{a}$ and $\vec{b}$ is

(A) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\vec{b}$ **(B)** $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ **(D)** $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\hat{b}$

Ans. Option (A) is correct.

Explanation: Projection vector of $\vec{a}$ on $\vec{b}$ is given by,

$$= \vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\vec{b}$$

$$= \left(\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\right)\cdot\vec{b}$$

Q. 21. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $|\vec{a}| = 2, |\vec{b}| = 3$ and $|\vec{c}| = 5$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(A) 0 **(B)** 1

(C) –19 **(D)** 38

Ans. Option (C) is correct.

Explanation :

Here, $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $\vec{a}^2 = 4, \vec{b}^2 = 9, \vec{c}^2 = 25$

$$\therefore \quad (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = \vec{0}$$

$$\Rightarrow \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+$$

$$\vec{c}\cdot\vec{b}+\vec{c}^2 = \vec{0}$$

$$\Rightarrow \quad \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$$

$$\Rightarrow \quad 4+9+25+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = \frac{-38}{2}$$

$$= -19$$

Q. 22. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then the range of $|\lambda\vec{a}|$ is

(A) [0, 8] **(B)** [–12, 8]

(C) [0, 12] **(D)** [8, 12]

Ans. Option (C) is correct.

Explanation:

We have, $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$

$$\therefore \quad |\lambda\vec{a}| = |\lambda||\vec{a}|$$

$$= \lambda|4|$$

$$\Rightarrow \quad |\lambda\vec{a}| = |-3|4$$

$$= 12,$$

at $\lambda = -3$

$|\lambda\vec{a}| = |0|4 = 0,$

at $\lambda = 0$

and $|\lambda\vec{a}| = |2|$

$4 = 8,$

at $\lambda = 2$

So, the range of $|\lambda\vec{a}|$ is [0,12].

Q. 23. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(A) one **(B)** two

(C) three **(D)** infinite

Ans. Option (B) is correct.

Explanation : The number of vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is c (say)

i.e., $\vec{c} = \pm(\vec{a} \times \vec{b})$.

So, there will be two vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

Q. 24. Which of the following statement is true.

(A) $\vec{a}$ and $\overrightarrow{-a}$ are collinear

(B) Two collinear vectors are always equal in magnitude

(C) Two vectors having same magnitude are collinear

(D) Two collinear vectors having the same magnitude are equal

Ans. Option (A) is correct.

Explanation :

(A) True

Vectors $\vec{a}$ and $-\vec{a}$ are parallel to the same line.

(B) False

Collinear vectors are those vectors that are parallel to the same line.

(C) False

It is not necessary for two vectors having the same magnitude to be parallel to the same line.

(D) False

Two vectors are said to be equal if they have the same magnitude and direction, regardless of the positions of their initial points.

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): The position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector be $2\hat{i} + 5\hat{j} + 3\hat{k}$.

Reason (R): The displacement vector of the particle that moves from point $P(2, 3, 5)$ to point $Q(3, 4, 5)$ is $\hat{i} + \hat{j}$.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong.

The position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector be $3\hat{i} + 2\hat{j} + 5\hat{k}$.

Reason (R) is correct.

The displacement vector of the particle that moves from point $P(2, 3, 5)$ to point $Q(3, 4, 5)$

$= (3-2)\hat{i} + (4-3)\hat{j} + (5-5)\hat{k}$

$= \hat{i} + \hat{j}$

Q. 2. Assertion (A): The direction of cosines of vector $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ are $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, -\frac{5}{\sqrt{45}}$.

Reason (R): A vector having zero magnitude and arbitrary direction is called 'zero vector' or 'null vector'.

Ans. Option (B) is correct.

Explanation: Assertion (A) is correct.

Direction cosines of $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ are :

$$\frac{2}{\sqrt{2^2+4^2+(-5)^2}}, \frac{4}{\sqrt{2^2+4^2+(-5)^2}}, \frac{-5}{\sqrt{2^2+4^2+(-5)^2}}$$

Or, $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}$

Q. 3. Assertion (A): The vectors which can undergo parallel displacement without changing its magnitude and direction are called free vectors.

Reason (R): $\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are individually correct.
Reason (R) is the distributive property of dot product.

Q. 4. Assertion (A): The area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a}\times\vec{b}|$.

Reason (R): If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating $|\vec{a}\times\vec{b}|$.

Ans. Option (C) is correct.

Explanation: If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating $\frac{1}{2}|\vec{a}\times\vec{b}|$.

Q. 5. Assertion (A): For any two vectors $\vec{a}$ and $\vec{b}$, we always have $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$.

Reason (R): The given inequality holds trivially when either $\vec{a} = 0$ or $\vec{b} = 0$ *i.e.*, in such a case

$$|\vec{a}+\vec{b}| = 0 = |\vec{a}| + |\vec{b}|.$$

Then consider

So, let us check it for $|\vec{a}| \neq 0 \neq |\vec{b}|$.

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$$

or $$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|\,|\vec{b}|\cos\theta$$

For $\cos\theta \le 1$, we have :

$$2|\vec{a}|\,|\vec{b}|\cos\theta \le 2|\vec{a}|\,|\vec{b}|$$

or $$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|\,|\vec{b}|\cos\theta \le |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|\,|\vec{b}|$$

or $$|\vec{a}+\vec{b}|^2 \le \left(|\vec{a}| + |\vec{b}|\right)^2$$

or $$|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 6. Assertion (A): The position vector of a point say $P(x, y, z)$ is $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude is $|\vec{r}| = \sqrt{x^2+y^2+z^2}$.

Reason (R): If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then coefficient of $\hat{i}, \hat{j}, \hat{k}$ in $\vec{r}$ *i.e.*, x, y, z are called the direction ratios of vector $\vec{r}$.

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are individually correct.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters $P_1(6, 8, 4)$, $P_2(21, 8, 4)$, $P_3(21, 16, 10)$ and $P_4(6, 16, 10)$ **[CBSE QB-2021]**

Q. 1. What are the components to the two edge vectors defined by $\vec{A}$ = *PV* of P_2 – *PV* of P_1 and $\vec{B}$ = *PV* of P_4 – *PV* of P_1? (where PV stands for position vector)

(A) 0, 0, 15 : 0, 8, 6 **(B)** 15, 0, 0 : 0, 8, 6

(C) 0, 8, 6 : 0, 0, 15 **(D)** 15, 0, 0 : 6, 8, 8

Ans. Option (B) is correct.

Explanation:

$$\vec{A} = PV \text{ of } P_2 - PV \text{ of } P_1$$
$$= 21\hat{i} + 8\hat{j} + 4\hat{k} - (6\hat{i} + 8\hat{k} + 4\hat{k})$$
$$= 15\hat{i} + 0\hat{j} + 0\hat{k}$$
$$\vec{B} = PV \text{ of } P_4 - PV \text{ of } P_1$$
$$= 6\hat{i} + 16\hat{j} + 10\hat{k} - (6\hat{i} + 8\hat{j} + 4\hat{k})$$
$$= (0\hat{i} + 8\hat{j} + 6\hat{k})$$

$\therefore A(15,0,0)$ and $B(0,8,6)$

Q. 2. Write the vector in standard notation with $\hat{i}, \hat{j}$ and $\hat{k}$ (where $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along the three axes).

(A) $15\hat{i}+0\hat{j}+0\hat{k}, 0\hat{i}+8\hat{j}+6\hat{k}$

(B) $0\hat{i}+6\hat{j}+8\hat{k}, 15\hat{i}+0\hat{j}+0\hat{k}$

(C) $0\hat{i}+0\hat{j}+0\hat{k}, 0\hat{i}+8\hat{j}+6\hat{k}$

(D) $15\hat{i}+0\hat{j}+0\hat{k}, 6\hat{i}+8\hat{i}+0\hat{k}$

Ans. Option (A) is correct.

Q. 3. What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and in what units?

(A) 9, 10 **(B)** 15, 5

(C) 15, 10 **(D)** 10, 20

Ans. Option (C) is correct.

Explanation:

$$|\vec{A}| = \sqrt{(15)^2+0^2+0^2}$$
$$= 15 \text{ units}$$
$$|\vec{B}| = \sqrt{0^2+8^2+6^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

Q. 4. What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

(A) –90, 90 **(B)** 120, 18

(C) –90, 100 **(D)** –90, 120

Ans. Option (D) is correct.

Explanation:

$$\vec{N} = \vec{A}\times\vec{B}$$
$$N = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix}$$
$$= -15(6\hat{j}-8\hat{k})$$
$$= -90\hat{j}+120\hat{k};$$

Q. 5. What is the magnitude of $\vec{N}$ and in what units?

(A) 100 **(B)** 150

(C) 50 **(D)** 90

Ans. Option (B) is correct.

Explanation:

$$\vec{N} = -90\hat{j}+120\hat{k}$$
$$|\vec{N}| = \sqrt{(90)^2-(120)^2}$$
$$= \sqrt{8100+14400}$$
$$= \sqrt{22500}$$
$$= 150 \text{ units}$$

II. Read the following text and answer the following questions on the basis of the same:

A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non zero vectors.

[CBSE QB 2021]

Q. 1. If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then

(A) $\vec{a} \perp \vec{b}$ **(B)** $\vec{a} \parallel \vec{b}$

(C) $\vec{a} = \vec{b}$ **(D)** None of these

Ans. Option (A) is correct.

Q. 2. If $\vec{a} = \hat{i}-2\hat{j}$, $\vec{b} = 2\hat{i}+\hat{j}+3\hat{k}$ then evaluate $(2\vec{a}+\hat{b})\cdot[(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b})]$

(A) 0 **(B)** 4

(C) 3 **(D)** 2

Ans. Option (A) is correct.

Explanation:

$$\vec{a}+\vec{b} = 3\hat{i}-\hat{j}+3\hat{k}$$
$$\vec{a}-2\vec{b} = -3\hat{i}-4\hat{j}-6\hat{k}$$
$$(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 3 \\ -3 & -4 & -6 \end{vmatrix}$$
$$= (6+12)\hat{i}-(-18+9)\hat{j}+(-12-3)\hat{k}$$
$$= 18\hat{i}+9\hat{j}-15\hat{k}$$
$$(2\vec{a}+\vec{b}) = 2(\hat{i}-2\hat{j})+(2\hat{i}+\hat{j}+3\hat{k})$$
$$= 4\hat{i}-3\hat{j}+3\hat{k}$$
$$(2\vec{a}+\vec{b}).[(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b})]$$
$$= (4\hat{i}+3\hat{j}+3\hat{k})(18\hat{i}+9\hat{j}-15\hat{j})$$
$$= 72-27-45$$
$$= 0$$

Q. 3. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between them then $|\vec{a}-\vec{b}|$ is

(A) $\sin\frac{\theta}{2}$ **(B)** $2\sin\frac{\theta}{2}$

(C) $2\cos\frac{\theta}{2}$ **(D)** $\cos\frac{\theta}{2}$

Ans. Option (B) is correct.

Explanation:

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$$
$$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}-\vec{b}\cos\theta$$
$$= 1+1-2(1)(1)\cos\theta$$
$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2-2\cos\theta$$
$$= 2(1-\cos\theta)$$
$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2\left(2\sin^2\frac{\theta}{2}\right)$$
$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 4\sin^2\frac{\theta}{2}$$
$$|\vec{a}-\vec{b}|^2 = 2\sin\frac{\theta}{2}$$

Q. 4. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ then $\vec{a}$ =

(A) $2(\vec{b}\times\vec{c})$ **(B)** $-2(\vec{b}\times\vec{c})$

(C) $\pm 2(\vec{b}\times\vec{c})$ **(D)** $2(\vec{b}\pm\vec{c})$

Ans. Option (C) is correct.

Q. 5. The area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ as diagonals is

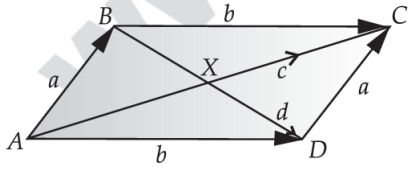
(A) 70 **(B)** 35

(C) $\frac{\sqrt{70}}{2}$ **(D)** $\sqrt{70}$

Ans. Option (C) is correct.

III. Read the following text and answer the following questions on the basis of the same:

ABCD is a parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$. Three of its vertices are $A(1, 2, 3)$, $B(2, 0, 5)$ and $D(-1, 4, 1)$.



Q. 1. The vector $\vec{a}$ = _____.

(A) $2\hat{i}-\hat{j}+\hat{k}$ **(B)** $\hat{i}-2\hat{j}-2\hat{k}$

(C) $\hat{i}-2\hat{j}+2\hat{k}$ **(D)** $2\hat{i}-\hat{j}+2\hat{k}$

Ans. Option (C) is correct.

Explanation:

$$\vec{a} = \overrightarrow{AB}$$
$$= \overrightarrow{OB}-\overrightarrow{OA}$$
$$= (2\hat{i}+0\hat{j}+5\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$$
$$= \hat{i}-2\hat{j}+2\hat{k}$$

Q. 2. The length of side *CD* is _____ units.

(A) 3 **(B)** 4

(C) 5 **(D)** 6

Ans. Option (A) is correct.

Explanation:

$$|\overrightarrow{CD}| = |\vec{a}|$$
$$= \sqrt{1+4+4}$$
$$= 3 \text{ units}$$

Q. 3. The diagonal $\vec{c}$ = _____.

(A) $\hat{i}+3\hat{j}+5\hat{k}$ **(B)** $\hat{i}+\hat{j}-\hat{k}$

(C) $-\hat{i}$ **(D)** $-\hat{j}$

Ans. Option (C) is correct.

Explanation:

$$\vec{a} = \hat{i}-2\hat{j}+2\hat{k}$$
$$\vec{b} = -2\hat{i}+2\hat{j}-2\hat{k}$$
$$\vec{c} = \vec{a}+\vec{b}$$
$$= -\hat{i}$$

Q. 4. The diagonal $\vec{d}$ = _____.

(A) $-\hat{i}$ **(B)** $-3\hat{i}+4\hat{j}-4\hat{k}$

(C) $\hat{i}+2\hat{j}-\hat{k}$ **(D)** $\hat{i}$

Ans. Option (B) is correct.

Explanation:

$$\vec{d} = \vec{b}-\vec{a}$$
$$= -3\hat{i}+4\hat{j}-4\hat{k}$$

Q. 5. Area of ABCD = _____ sq. units.

(A) 8 **(B)** 4

(C) $2\sqrt{2}$ **(D)** 16

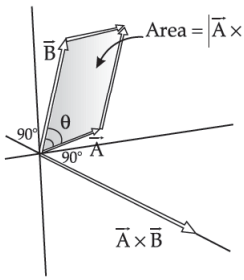
Ans. Option (C) is correct.

Explanation:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ -2 & 2 & -2 \end{vmatrix}$$
$$= -2\hat{j}-2\hat{k}$$
$$\text{Area} = |\vec{a}\times\vec{b}|$$
$$= \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ sq. units}$$

IV. Read the following text and answer the following questions on the basis of the same:

The cross product of two vectors gives a vector perpendicular to the plane containing the vectors.



$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{a}_n$

$\hat{a}_n$ is a unit vector perpendicular to the plane defined by $\vec{A}$ and $\vec{B}$.

Q. 1. $\hat{i}\times\hat{j}$ = _____.

(A) $\hat{k}$ **(B)** $-\hat{k}$

(C) 0 **(D)** $\vec{0}$

Ans. Option (A) is correct.

Explanation: $\hat{i}\times\hat{j} = \hat{k}$

Q. 2. $\hat{i}\cdot(\hat{j}\times\hat{k})$ = _____.

(A) $\hat{k}$ **(B)** 0

(C) 1 **(D)** –1

Ans. Option (C) is correct.

Explanation:
$$\hat{i}\cdot(\hat{j}\times\hat{k}) = \hat{i}\cdot\hat{i} = 1$$

Q. 3. The angle between $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ is _____.

(A) 0 **(B)** π

(C) $\frac{\pi}{2}$ **(D)** $\frac{\pi}{4}$

Ans. Option (B) is correct.

Explanation: $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
The angle between $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ is π.

Q. 4. If $\vec{a}$ and $\vec{b}$ are non-zero vectors, then $\vec{a}\cdot(\vec{a}\times\vec{b})$ = _____.

(A) 0 **(B)** $\vec{0}$

(C) $\vec{a}$ **(D)** $\vec{b}$

Ans. Option (A) is correct.

Explanation:
$$\vec{a}\cdot(\vec{a}\times\vec{b}) = [\vec{a},\vec{a},\vec{b}] = 0$$

Q. 5. Area of parallelogram whose adjacent sides are represented by the vectors $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}$ is _____ sq. units.

(A) 3 **(B)** $\sqrt{10}$

(C) $\sqrt{11}$ **(D)** $2\sqrt{3}$

Ans. Option (C) is correct.

Explanation:
$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -1 & 0\end{vmatrix} = -\hat{i}+3\hat{j}+\hat{k}$$

$$\text{Area} = |\vec{a}\times\vec{b}| = \sqrt{1+9+1} = \sqrt{11} \text{ sq. units}$$

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CHAPTER

11

Term-II

THREE DIMENSIONAL GEOMETRY

Syllabus

- ***Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Distance of a point from a plane.***

STAND ALONE MCQs (1 Mark each)

Q. 1. Distance of the point (α, β, γ) from y-axis is

(A) β **(B)** $|\beta|$

(C) $|\beta| + |\gamma|$ **(D)** $\sqrt{\alpha^2 + \gamma^2}$

Ans. Option (D) is correct.

Explanation :

The foot of perpendicular from point $P(\alpha, \beta, \gamma)$ on y-axis is $Q(0, \beta, 0)$.

$\therefore$ Required distance,

$PQ = \sqrt{(\alpha - 0)^2 + (\beta - \beta)^2 + (\gamma - 0)^2} = \sqrt{\alpha^2 + \gamma^2}$

Q. 2. If the direction cosines of a line are k, k, k, then

(A) $k > 0$ **(B)** $0 < k < 1$

(C) $k = 1$ **(D)** $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

Ans. Option (D) is correct.

Explanation :

Since, direction cosines of a line are k, k and k.

$\therefore l = k, m = k$ and $n = k$

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow k^2 + k^2 + k^2 = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\therefore k = \pm\frac{1}{\sqrt{3}}$

Q. 3. The distance of the plane $\vec{r} \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 1$ from the origin is

(A) 1 **(B)** 7

(C) $\frac{1}{7}$ **(D)** None of these

Ans. Option (A) is correct.

Explanation:

The distance of the plane $\vec{r} \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 1$ from the origin is 1.

[Since $\vec{r} \cdot \hat{n} = d$ is the form of above equation, where d represents the distance of plane from the origin, *i.e.*, $d = 1$]

Q. 4. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) $\frac{10}{6\sqrt{5}}$ **(b)** $\frac{4}{5\sqrt{2}}$

(c) $\frac{2\sqrt{3}}{5}$ **(d)** $\frac{\sqrt{2}}{10}$

Ans. Option (D) is correct.

Explanation : We have, the equation of line as

$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

This line is parallel to the vector, $\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Equation of plane is $2x - 2y + z = 5$.

Normal to the plane is $\vec{n} = 2\hat{i} - 2\hat{j} + \hat{k}$.

The angle between line and plane is 'θ'.

Then,

$$\sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}$$

$$= \frac{|(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})|}{\sqrt{3^2 + 4^2 + 5^2}\sqrt{4 + 4 + 1}}$$

$$= \frac{|6 - 8 + 5|}{\sqrt{50}\sqrt{9}}$$

$$= \frac{3}{15\sqrt{2}}$$

$$= \frac{1}{5\sqrt{2}}$$

$$\sin\theta = \frac{\sqrt{2}}{10}$$

Q. 5. The reflection of the point (α, β, γ) in the ***xy***-plane is

(A) $(\alpha, \beta, 0)$ **(B)** $(0, 0, \gamma)$

(C) $(-\alpha, -\beta, \gamma)$ **(D)** $(\alpha, \beta, -\gamma)$

Ans. Option (D) is correct.

Explanation : In *xy*-plane, the reflection of the point (α, β, γ) is $(\alpha, \beta, -\gamma)$.

Q. 6. The area of the quadrilateral *ABCD*, where *A*(0, 4, 1), *B*(2, 3, –1), *C*(4, 5, 0) and *D*(2, 6, 2), is equal to

(A) 9 sq. units **(B)** 18 sq. units

(C) 27 sq. units **(D)** 81 sq. units

Ans. Option (A) is correct.

Explanation :

We have, *A*(0, 4, 1), *B*(2, 3, –1), *C*(4, 5, 0) and *D*(2, 6, 2)

$$\overline{AB} = (2-0)\hat{i} + (3-4)\hat{j} + (-1-1)\hat{k}$$
$$= 2\hat{i} - \hat{j} - 2\hat{k}$$
$$\overline{BC} = (4-2)\hat{i} + (5-3)\hat{j} + (0+1)\hat{k}$$
$$= 2\hat{i} + 2\hat{j} + \hat{k}$$
$$\overline{CD} = (2-4)\hat{i} + (6-5)\hat{j} + (2-0)\hat{k}$$
$$= -2\hat{i} + \hat{j} + 2\hat{k}$$
$$\overline{DA} = (0-2)\hat{i} + (4-6)\hat{j} + (1-2)\hat{k}$$
$$= -2\hat{i} - 2\hat{j} - \hat{k}$$

Thus quadrilateral formed is parallelogram.

∴Area of quadrilateral *ABCD*

$$= |\overline{AB} \times \overline{BC}|$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{vmatrix}$$
$$= |3\hat{i} - 6\hat{j} + 6\hat{k}|$$
$$= \sqrt{9 + 36 + 36}$$
$$= 9 \text{ sq. units}$$

Q. 7. The locus represented by $xy + yz = 0$ is

(A) A pair of perpendicular lines

(B) A pair of parallel lines

(C) A pair of parallel planes

(D) A pair of perpendicular planes

Ans. Option (D) is correct.

Explanation :

We have,

$$xy + yz = 0$$
$$\Rightarrow \quad x(y + z) = 0$$
$$\Rightarrow \quad x = 0 \text{ and } y + z = 0$$

Above are equations of planes.

Normal to the plane $x = 0$ is $\hat{i}$.

And normal to the plane $y + z = 0$ is $\hat{j} + \hat{k}$.

Now, $\hat{i} \cdot (\hat{j} + \hat{k}) = 0$

So, planes are perpendicular.

Q. 8. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with *x*-axis. The value of α is equal to

(A) $\frac{\sqrt{3}}{2}$ **(B)** $\frac{\sqrt{2}}{3}$

(C) $\frac{2}{7}$ **(D)** $\frac{3}{7}$

Ans. Option (C) is correct.

Explanation :

We have equation of plane as $2x - 3y + 6z - 11 = 0$.

Normal to the plane is $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$.

Also x-axis is along the vector $\vec{a} = \hat{i} + 0\hat{j} + 0\hat{k}$.

According to the question,

$$\sin\alpha = \frac{|\vec{a} \cdot \vec{n}|}{|\vec{a}||\vec{n}|}$$
$$= \frac{|\hat{i} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k})|}{\sqrt{1}\sqrt{4 + 9 + 36}}$$
$$= \frac{2}{7}$$

Q. 9. Distance between the two planes : $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

(A) 2 units **(B)** 4 units

(C) 8 units **(D)** $\frac{2}{\sqrt{29}}$ unit

Ans. Option (D) is correct.

Explanation :

Distance between two parallel planes,

$Ax + By + Cz = d_1$ and $Ax + By + Cz = d_2$ is

$$\left|\frac{d_1 - d_2}{\sqrt{A^2 + B^2 + C^2}}\right|$$

$2x + 3y + 4z = 4$

Comparing with $Ax + By + Cz = d_1$

$A = 2, B = 3, C = 4, d_1 = 4$

And now, $4x + 6y + 8z = 12$

$2(2x + 3y + 4z) = 12$

Dividing by 2

$2x + 3y + 4z = 6$

Comparing with $Ax + By + Cz = d_2$

$A = 2, B = 3, C = 4, d_2 = 6$

So,

Distance between the two planes

$$= \left| \frac{4-6}{\sqrt{2^2+3^2+4^2}} \right|$$

$$= \left| \frac{-2}{\sqrt{4+9+16}} \right|$$

$$= \frac{2}{\sqrt{29}}$$

Q. 10. The planes : $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

(A) Perpendicular

(B) Parallel

(C) Intersect y-axis

(D) Passes through $\left(0, 0, \frac{5}{4}\right)$

Ans. Option (B) is correct.

Explanation :

Angle between two planes $A_1x + B_1y + C_1z = d_1$ and $A_2x + B_2y + C_2z = d_2$ is given by

$$\cos\theta = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

Given that plane,

$2x - 1y + 4z = 5$

Comparing with $A_1x + B_1y + C_1z = d_1$

$A_1 = 2, B_1 = -1, C_1 = 4, d_1 = 5$

$5x - 2.5y + 10z = 16$

Multiplying by 2 on both sides,

$10x - 5y + 20z = 12$

Comparing with $A_2x + B_2y + C_2z = d_2$

$A_2 = 10, B_2 = -5, C_2 = 20, d_2 = 12$

So, $\cos\theta = \left| \frac{(2\times10) + (-1\times-5) + (4\times20)}{\sqrt{2^2 + (-1)^2 + 4^2}\sqrt{10^2 + (-5)^2 + 20^2}} \right|$

$$= \left| \frac{20 + 5 + 80}{\sqrt{4+1+16}\sqrt{100+25+400}} \right|$$

$$= \left| \frac{105}{\sqrt{21}\sqrt{525}} \right|$$

$$= \left| \frac{105}{\sqrt{21} \times \sqrt{25 \times 21}} \right|$$

$$= \left| \frac{105}{\sqrt{21} \times 5\sqrt{21}} \right|$$

$$= \left| \frac{105}{21 \times 5} \right|$$

$$= 1$$

So, $\cos\theta = 1$

$\therefore \quad \theta = 0°$

Since angle between the planes is 0°.

Therefore, the planes are parallel.

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$ the equation to the sphere whose centre is at (–2, 3, 4) and radius is 6 units.

Reason (R): Given:

Centre is at (–2, 3, 4) and $r = 6$

$\Rightarrow (x_0, y_0, z_0) = (-2, 3, 4)$ and $r = 6$

We know that general equation of sphere is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

$\Rightarrow (x - (-2))^2 + (y - 3)^2 + (z - 4)^2 = 6^2$

$\Rightarrow (x + 2)^2 + (y - 3)^2 + (z - 4)^2 = 6^2$

$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 + z^2 - 8z + 16 = 36$

$\Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z + 29 = 36$

$\Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A): If two lines are in the same plane *i.e.*, they are coplanar, they will intersect each other if they are non-parallel. Hence the shortest distance between them is zero.

If the lines are parallel then the shortest distance between them will be the perpendicular distance between the lines *i.e.*, the length of the perpendicular drawn from a point on one line onto the other line.

Reason (R): The angle between the lines with direction ratio $\langle a_1, b_1, c_1\rangle$ and $\langle a_2, b_2, c_2\rangle$ is given by:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are individually correct.

Q. 3. Assertion (A): Direction cosines of a line are the sines of the angles made by the line with the negative directions of the coordinate axes.

Reason (R): The acute angle between the lines $x - 2 = 0$ and $\sqrt{3x - y - 2}$ is 30°.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong.
Since, direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.
Reason (R) is correct.
Since, the slope of the line $x - 2 = 0$ is ∞.
The slope of line $\sqrt{3}x - y - 2 = 0$ is $\sqrt{3}$.
Let $m_1 = \infty$, $m_2 = \sqrt{3}$ and the angle between the given lines is θ.

$$\Rightarrow \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{m_2}{m_1} - 1}{\frac{1}{m_1} + m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30°$$

Q. 4. Assertion (A): P is a point on the line segment joining the points (3, 2, –1) and (6, –4, –2). If x coordinate of P is 5, then its y coordinate is –2.

Reason (R): The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ will be perpendicular, iff $aa' + bb' + cc' = 0$.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.
Since $P = (5, y, z)$
Equation of line joining (3, 2, –1) and (6, –4, –2) is

$$\frac{x-3}{6-3} = \frac{y-2}{-4-2} = \frac{z+1}{-2+1} = \frac{x-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

so if point P lies on the line then it must satisfy the above equation

$$\frac{5-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

$$\frac{5-3}{3} = \frac{y-2}{-6}$$

Hence y co-ordinate of P is –2.
Reason (R) is false.
Since, the two lines $x = ay + b, z = cy + d$ and $= a'y + b', z = c'y + d'$ will be perpendicular, iff $aa' + cc' + 1 = 0$.

Q. 5. Assertion (A): The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$ is 90°

Reason (R): Skew lines are lines in different planes which are parallel and intersecting.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.
Given : $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$

and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$

Direction ratios of lines are $a_1 = 2, b_1 = 5, c_1 = 4$ and $a_2 = 1, b_2 = 2, c_2 = -3$
As we know, The angle between the lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right).\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

$$\Rightarrow \quad \cos\theta = \frac{2\times1 + 5\times2 + 4\times-3}{\left(\sqrt{2^2 + 5^2 + 4^2}\right).\left(\sqrt{1^2 + 2^2 + (-3)^3}\right)}$$

$$= 0$$

$$\therefore \quad \theta = 90°$$

Reason (R) is wrong.
In the space, there are lines neither intersecting nor parallel, such pairs of lines are non-coplanar and are called skew lines.

Q. 6. Assertion (A): The length of the intercepts on the co-ordinate axes made by the plane $5x + 2y + z - 13 = 0$ are $\frac{13}{5}, \frac{13}{2}, 13$ unit

Reason (R): Given:
Equation of plane

$$5x + 2y + z - 13 = 0$$

$$\Rightarrow \quad 5x + 2y + z = 13$$

$$\Rightarrow \quad \frac{5x + 2y + z}{13} = 1$$

$$\Rightarrow \quad \frac{x}{\frac{13}{5}} + \frac{y}{\frac{13}{2}} + \frac{z}{\frac{13}{1}} = 1$$

$\therefore$ Length of intercepts are $\frac{13}{5}, \frac{13}{2}, 13$ units

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A cricket match is organized between two Clubs *A* and *B* for which a team from each club is chosen. Remaining players of Club *A* and Club *B* are respectively sitting on the plane represented by the equation $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})=3$ and $\vec{r}.(\hat{i}+3\hat{j}+2\hat{k})=8$, to cheer the team of their own clubs.

[CBSE QB-2021]

Q. 1. The Cartesian equation of the plane on which players of Club *A* are seated is

(A) $2x-y+z=3$ **(B)** $2x-y+2z=3$

(C) $2x-y+z=-3$ **(D)** $x-t+z=3$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned}\vec{r} &= x\hat{i}+y\hat{j}+z\hat{k}\\ \vec{r}.(2\hat{i}-\hat{j}+\hat{k}) &= 3\\ (x\hat{i}+y\hat{i}+z\hat{k})(2\hat{i}-\hat{j}+\hat{k}) &= 3\\ 2x-y+z &= 3\end{aligned}$$

Q. 2. The magnitude of the normal to the plane on which players of club *B* are seated, is

(A) $\sqrt{15}$ **(B)** $\sqrt{14}$

(C) $\sqrt{17}$ **(D)** $\sqrt{20}$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned}\vec{n} &= 2\hat{i}+3\hat{j}+2\hat{k}\\ |\vec{n}| &= \sqrt{1+9+4}\\ &= \sqrt{14}\text{ units}\end{aligned}$$

Q. 3. The intercept form of the equation of the plane on which players of Club *B* are seated is

(A) $\frac{x}{8}+\frac{y}{8/3}+\frac{z}{8/3}=1$ **(B)** $\frac{x}{5}+\frac{y}{8/3}+\frac{z}{8/3}=1$

(C) $\frac{x}{8}+\frac{y}{8/3}+\frac{z}{4}=1$ **(d)** $\frac{x}{8}+\frac{y}{7}+\frac{z}{2}=1$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}\vec{r}.(\hat{i}+3\hat{j}+2\hat{k}) &= 8\\ (x\hat{i}+y\hat{j}+2\hat{k})(\hat{i}+3\hat{j}+2\hat{k}) &= 8\\ x+3y+2z &= 8\\ \frac{x}{8}+\frac{3y}{8}+\frac{2z}{8} &= 1\\ \frac{x}{8}+\frac{y}{8/3}+\frac{z}{4} &= 1\end{aligned}$$

Q. 4. Which of the following is a player of Club *B*?

(A) Player sitting at (1, 2, 1)

(B) Player sitting at (0, 1, 2)

(C) Player sitting at (1, 4, 1)

(D) Player sitting at (1, 1, 2)

Ans. Option (D) is correct.

Q. 5. The distance of the plane, on which players of Club *B* are seated, from the origin is

(A) $\frac{8}{\sqrt{14}}$ units **(B)** $\frac{6}{\sqrt{14}}$ units

(C) $\frac{7}{\sqrt{14}}$ units **(D)** $\frac{9}{\sqrt{14}}$ units

Ans. Option (A) is correct.

Explanation: We know

$$\begin{aligned}\vec{r}.\hat{n} &= d\\ \therefore\quad \vec{r}.\frac{(\hat{i}+3\hat{j}+2\hat{k})}{\sqrt{14}} &= \frac{8}{\sqrt{14}}\\ \therefore\quad d &= \frac{8}{\sqrt{14}}\text{ units}\end{aligned}$$

II. Read the following text and answer the following questions on the basis of the same:

The Indian coast guard, while patrolling, saw a suspicious boat with people. They were nowhere looking like fishermen. The coast guard were closely observing the movement of the boat for an opportunity to seize the boat. They observed that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of the coast guard helicopter and the boat is (1, 3, 5) and (2, 5, 3) respectively. **[CBSE QB 2021]**

Q. 1. If the line joining the positions of the helicopter and the boat is perpendicular to the plane in which the boat moves, then the equation of the plane is

(A) $-x + 2y - 2z = 6$ **(B)** $x + 2y + 2z = 6$

(C) $x + 2y - 2z = 6$ **(D)** $x - 2y - 2z = 6$

Ans. Option (C) is correct.

Q. 2. If the coast guard decide to shoot the boat at that given instant of time, then what is the distance (in meters) that the bullet has to travel?

(A) 5m **(B)** 3m

(C) 6m **(D)** 4m

Ans. Option (B) is correct.

Explanation:

$$\vec{n} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Distance that the bullet has to travel

$$= \sqrt{1+4+4}$$

$$= 3\text{m}$$

Q. 3. If the coast guard decides to shoot the boat at that given instant of time, when the speed of bullet is 36 m/sec, then what is the time taken for the bullet to travel and hit the boat?

(A) $\frac{1}{8}$ seconds **(B)** $\frac{1}{14}$ seconds

(C) $\frac{1}{10}$ seconds **(D)** $\frac{1}{12}$ seconds

Ans. Option (D) is correct.

Explanation: Time taken for the bullet to travel and hit the boat

$$= \frac{3\text{m}}{36\text{m/sec}}$$

$$= \frac{1}{12} \text{ seconds}$$

Q. 4. At that given instant of time, the equation of line passing through the positions of the helicopter and boat is

(A) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-5}{-2}$

(B) $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-2}$

(C) $\frac{x+1}{-2} = \frac{y-3}{-1} = \frac{z-5}{-2}$

(D) $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+5}{2}$

Ans. Option (A) is correct.

Explanation: Here, direction cosines are 1, 2, –2. Equation of line passing through the positions of the helicopter and boat is

$$\Rightarrow \frac{x-x_1}{l} + \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\frac{x-1}{1} = \frac{y-3}{2}$$

$$= \frac{z-5}{-2}$$

Q. 5. At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat if the coast guard shoots the bullet along the line whose equation is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{1}$ for the bullet to hit the boat?

(A) $\left(\frac{-8}{3}, \frac{19}{3}, \frac{-14}{3}\right)$ **(B)** $\left(\frac{8}{3}, \frac{-19}{3}, \frac{-14}{3}\right)$

(C) $\left(\frac{8}{3}, \frac{-19}{3}, \frac{14}{3}\right)$ **(D)** None of these

Ans. Option (D) is correct.

III. Read the following text and answer the following questions on the basis of the same:

The equation of motion of a missile are $x = 3t$, $y = -4t$, $z = t$, where the time 't' is given in seconds, and the distance is measured in kilometres.

[CBSE QB 2021]

Q. 1. What is the path of the missile?

(A) Straight line **(B)** Parabola

(C) Circle **(D)** Ellipse

Ans. Option (A) is correct.

Q. 2. Which of the following points lie on the path of the missile?

(A) (6, 8, 2) **(B)** (6, –8, –2)

(C) (6, –8, 2) **(D)** (–6, –8, 2)

Ans. Option (C) is correct.

Explanation: (6, –8, 2) point lie on the path of the missile.

$\because$ $x = 3t, y = -4t, z = t$

at $t = 2$

$x = 6, y = -8, z = 2$

i.e., (6, –8, 2)

Q. 3. At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?

(A) $\sqrt{550}$ kms **(B)** $\sqrt{650}$ kms

(C) $\sqrt{450}$ kms **(D)** $\sqrt{750}$ kms

Ans. Option (B) is correct.

Explanation: Here,

$$t = 5 \text{ seconds}$$
$$x = 3t = 3 \times 5 = 15$$
$$y = -4t = -4 \times 5 = -20$$
$$z = t = 5$$
$$(x, y, z) \equiv (15, -20, 5)$$

Distance from starting point (0, 0, 0)

$$= \sqrt{(15-0)^2 + (-20-0)^2 + (5-0)^2}$$
$$= \sqrt{225 + 400 + 25}$$
$$= \sqrt{650} \text{ kms}$$

Q. 4. If the position of rocket at a certain instant of time is (5, –8, 10), then what will be the height of the rocket from the ground? (The ground is considered as the xy-plane).

(A) 12 km **(B)** 11 km

(C) 20 km **(D)** 10 km

Ans. Option (D) is correct.

Explanation: Height of the rocket from the ground (*i.e.*, xy-plane)

= 10 km

Q. 5. At a certain instant of time, if the missile is above the sea level, where the equation of the surface of sea is given by $2x + y + 3z = 1$ and the position of the missile at that instant of time is (1, 1, 2), then the image of the position of the rocket in the sea is

(A) $\left(\frac{-9}{7}, \frac{-1}{7}, \frac{-10}{7}\right)$ **(B)** $\left(\frac{9}{7}, \frac{-1}{7}, \frac{-10}{7}\right)$

(C) $\left(\frac{-9}{7}, \frac{-1}{7}, \frac{10}{7}\right)$ **(D)** $\left(\frac{-9}{7}, \frac{-1}{7}, \frac{10}{7}\right)$

Ans. Option (A) is correct.

IV. Read the following text and answer the following questions on the basis of the same:

Suppose the floor of a hotel is made up of mirror polished Salvatore stone. There is a large crystal chandelier attached to the ceiling of the hotel room. Consider the floor of the hotel room as a plane having the equation $x - y + z = 4$ and the crystal chandelier is suspended at the point (1, 0, 1). **[CBSE QB 2021]**

Q. 1. Find the direction ratios of the perpendicular from the point (1, 0, 1) to the plane $x - y + z = 4$.

(A) (–1, –1, 1) **(B)** (1, –1, –1)

(C) (–1, –1, –1) **(D)** (1, –1, 1)

Ans. Option (D) is correct.

Q. 2. Find the length of the perpendicular from the point (1, 0, 1) to the plane $x - y + z = 4$.

(A) $\frac{2}{\sqrt{3}}$ units **(B)** $\frac{4}{\sqrt{3}}$ units

(C) $\frac{6}{\sqrt{3}}$ units **(D)** $\frac{8}{\sqrt{3}}$ units

Ans. Option (A) is correct.

Q. 3. The equation of the perpendicular from the point (1, 0, 1) to the plane $x - y + z = 4$ is

(A) $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+5}{2}$

(B) $\frac{x-1}{-2} = \frac{y+3}{-1} = \frac{z-5}{2}$

(C) $\frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{1}$

(D) $\frac{x-1}{2} = \frac{y}{-2} = \frac{z-1}{1}$

Ans. Option (C) is correct.

Explanation: Here,

$$l = 1, m = -1, n = 1$$
$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$
$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{1}$$
$$\Rightarrow \quad \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{1}$$

Q. 4. The equation of the plane parallel to the plane $x - y + z = 4$, which is at a unit distance from the point (1, 0, 1) is

(A) $x - y + z + (2 - \sqrt{3})$

(B) $x - y + z - (2 + \sqrt{3})$

(C) $x - y + z + (2 + \sqrt{3})$

(D) Both (A) and (C)

Ans. Option (D) is correct.

Q. 5. The direction cosine of the normal to the plane $x - y + z = 4$ is

(A) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ **(B)** $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ **(D)** $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

Ans. Option (B) is correct.

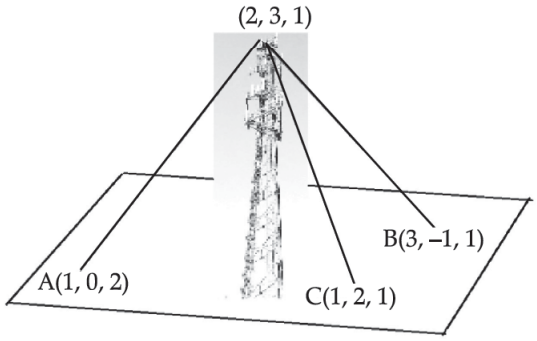
Explanation: Direction cosine of normal to the plane,

$$\frac{1}{\sqrt{1^2+(-1)^2+1^2}}, \frac{-1}{\sqrt{1^2+(-1)^2+1^2}}, \frac{1}{\sqrt{1^2+(-1)^2+1^2}}$$

$$\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

V. Read the following text and answer the following questions on the basis of the same:

A mobile tower stands at the top of a hill. Consider the surface on which the tower stands as a plane having points $A(1, 0, 2)$, $B(3, -1, 1)$ and $C(1, 2, 1)$ on it. The mobile tower is tied with 3 cables from the points A, B and C such that it stands vertically on the ground. The top of the tower is at the point $(2, 3, 1)$ as shown in the figure. **[CBSE QB 2021]**



Q. 1. The equation of the plane passing through the points A, B and C is

(A) $3x - 2y + 4z = -11$

(B) $3x + 2y + 4z = 11$

(C) $3x - 2y - 4z = 11$

(d) $-3x + 2y + 4z = -11$

Ans. Option (B) is correct.

Explanation:

$A(1, 0, 2)$, $B(3, -1, 1)$ and $C(1, 2, 1)$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-0 & z-2 \\ 3-1 & -1-0 & 1-2 \\ 1-1 & 2-0 & 1-2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y & z-2 \\ 2 & -1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = 0$$

$$(x-1)(1+2) - y(-2-0) + (z-2)(4-0) = 0$$

$$3x - 3 + 2y + 4z - 8 = 0$$

$$3x + 2y + 4z = 11$$

Q. 2. The height of the tower from the ground is

(A) $\frac{5}{\sqrt{29}}$ units **(B)** $\frac{7}{\sqrt{29}}$ units

(C) $\frac{6}{\sqrt{29}}$ units (D) $\frac{8}{\sqrt{29}}$ units

Ans. Option (A) is correct.

Q. 3. The equation of the perpendicular line drawn from the top of the tower to the ground is

(A) $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-2}$

(B) $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z-1}{4}$

(C) $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$

(D) $\frac{x+1}{-2} = \frac{y+3}{-1} = \frac{z-5}{2}$

Ans. Option (C) is correct.

Explanation:

$a = 3, b = 2, c = 4$ and

$(x_1, y_1, z_1) \equiv (2, 3, 1)$

$$\therefore \quad \frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$$

Q. 4. The co-ordinates of the foot of the perpendicular drawn from the top of the tower to the ground are

(A) $\left(\frac{43}{29}, \frac{-77}{29}, \frac{-9}{29}\right)$ **(B)** $\left(\frac{9}{7}, \frac{-1}{7}, \frac{-10}{7}\right)$

(C) $\left(\frac{-43}{29}, \frac{77}{29}, \frac{-9}{29}\right)$ **(D)** $\left(\frac{43}{29}, \frac{77}{29}, \frac{9}{29}\right)$

Ans. Option (D) is correct.

Q. 5. The area of ΔABC is

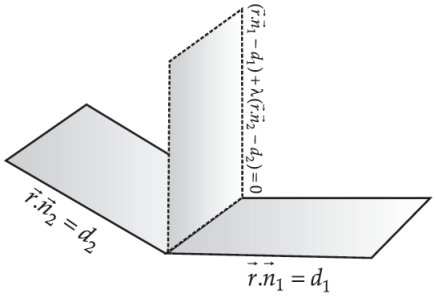
(A) $\frac{\sqrt{29}}{4}$ sq. units **(B)** $\frac{\sqrt{29}}{2}$ sq. units

(C) $\frac{\sqrt{39}}{2}$ sq. units **(D)** $\frac{\sqrt{39}}{4}$ sq. units

Ans. Option (B) is correct.

VI. Read the following text and answer the following questions on the basis of the same:

$P_1 : x + 3y - z = 0$ and $P_2 : y + 2z = 0$ are two intersecting planes. P_3 is a plane passing through the point $(2, 1, -1)$ and through the line of intersection of P_1 and P_2.



Q. 1. The angle between P_1 and P_2 is _____.

(A) $\cos^{-1}\left(\frac{1}{5}\right)$ **(B)** $\cos^{-1}\left(\frac{1}{\sqrt{55}}\right)$

(C) $\cos^{-1}\left(\frac{2}{\sqrt{11}}\right)$ **(D)** $\cos^{-1}\left(\frac{3}{\sqrt{11}}\right)$

Ans. Option (B) is correct.

Explanation: M

$$\vec{n_1} = \hat{i}+3\hat{j}-\hat{k},\ \vec{n_2} = \hat{j}+2\hat{k}$$

$$\cos\theta = \left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right| = \left|\frac{0+3-2}{\sqrt{11}\times\sqrt{5}}\right| = \frac{1}{\sqrt{55}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{55}}\right)$$

Q. 2. Equation of P_3 is _____.

(A) $4x + y - 2z = 10$ **(B)** $x + y - 2z = 3$

(C) $x + 9y + 11z = 0$ **(D)** $4x - y + z = 0$

Ans. Option (C) is correct.

Explanation:

Let the equation of P_3 be $P_1 + \lambda P_2 = 0$

i.e., $(x + 3y - z) + \lambda(y + 2z) = 0$

P_3 passes through the point $(2, 1, -1)$

$\therefore\ (2 + 3 + 1) + \lambda(1 - 2) = 0$

$\Rightarrow\ \lambda = 6$

$\therefore$ The equation of P_3 is $x + 9y + 11z = 0$

Q. 3. Equation of plane parallel to P_3 and passing through $(1, 2, 3)$ is _____.

(A) $x + 9y + 11z - 52 = 0$

(B) $x + 9y + 11z - 20 = 0$

(C) $4x + y - 2z + 10 = 0$

(D) $4x + y - 2z + 1 = 0$

Ans. Option (A) is correct.

Explanation: The required equation is

$(x - 1) + 9(y - 2) + 11(z - 3) = 0$

i.e., $x + 9y + 11z - 52 = 0$

Q. 4. _____ is a point on P_3.

(A) $(1, 2, 3)$ **(B)** $(-1, 4, 3)$

(C) $(-6, -3, 3)$ **(D)** $(6, 3, -3)$

Ans. Option (C) is correct.

Explanation: The equation of P_3 is $x + 9y + 11z = 0$.

$\therefore$ $(-6, -3, 3)$ is a point on P_3.

Q. 5. Distance of P_3 from origin is _____ units.

(A) 0 **(B)** 1

(C) $\frac{1}{\sqrt{5}}$ **(D)** $\frac{11}{10}$

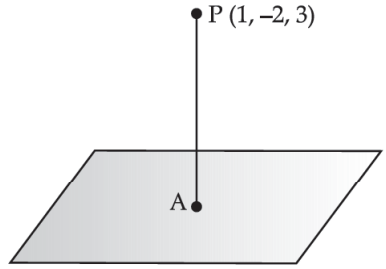
Ans. Option (A) is correct.

Explanation: $(0, 0, 0)$ is a point on P_3.

$\therefore$ Distance of P_3 from origin = 0

VII. Read the following text and answer the following questions on the basis of the same:

Consider the plane $\pi_1 : 2x - 3y + 4z + 9 = 0$ and the point $P(1, -2, 3)$. π_1 is a plane parallel to π_1 and containing the point P.



Q. 1. Equation of π_2 is _____.

(A) $2x - 3y + 4z + 9 = 0$

(B) $2x - 3y + 4z + 20 = 0$

(C) $2x - 3y + 4z - 20 = 0$

(D) $2x - 3y + 4z - 9 = 0$

Ans. Option (C) is correct.

Explanation: Equation of π_2 is

$2(x - 1) - 3(y + 2) + 4(z - 3) = 0$

i.e., $2x - 3y + 4z - 20 = 0$

Q. 2. Distance between π_1 and π_2 is _____ units.

(A) 5 **(B)** $\sqrt{29}$

(C) $\sqrt{13}$ **(D)** $2\sqrt{3}$

Ans. Option (B) is correct.

Explanation: Distance between π_1 and π_2

$$= \frac{|-20-9|}{\sqrt{4+9+16}}$$

$$= \sqrt{29} \text{ units}$$

Q. 3. A is the foot of perpendicular from P to π_1. Equation of PA is _____.

(A) $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4}$

(B) $\frac{x+1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$

(C) $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k})$

(D) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

Ans. Option (A) is correct.

Explanation:

Here $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b} = \vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

Equation of PA is

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k})$$

or $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4} = \lambda$

Q. 4. The co-ordinates of A are _____.

(A) (0, 0, 0) **(B)** (–1, 2, 3)

(C) (–1, 1, –1) **(D)** (1, 4, –2)

Ans. Option (C) is correct.

Explanation:

Generally $A = (2\lambda + 1, -3\lambda - 2, 4\lambda + 3)$.

A is a point on π_1.

$\Rightarrow 2(2\lambda + 1) - 3(-3\lambda - 2) + 4(4\lambda + 3) + 9 = 0$

$\Rightarrow 29\lambda + 29 = 0$

$\Rightarrow \lambda = -1$

$\therefore A = (-1, 1, -1)$

Q. 5. The image of P on π_1 is _____.

(A) (–1, 1, –1) **(B)** (–1, 2, –3)

(C) (–3, 4, –5) **(D)** (0, 0, 0)

Ans. Option (C) is correct.

Explanation:

Let $P'(\alpha, \beta, \gamma)$ be the image A is mid-point of PP'.

i.e., $(-1, 1, -1) = \left(\frac{\alpha+1}{2}, \frac{\beta-2}{2}, \frac{\gamma+3}{2}\right)$

$\Rightarrow \alpha = -3, \ \beta = 4, \ \gamma = -5$

$\therefore P' = (-3, 4, -5)$

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CHAPTER 12

Term-I

LINEAR PROGRAMMING

Syllabus

- ***Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, graphical method of solution for problems in two variables, feasible and infeasible region (bounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).***

STAND ALONE MCQs (1 Mark each)

Q. 1. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is $Z = 4x + 3y$.

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

(A) The quantity in column A is greater.
(B) The quantity in column B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined on the basis of the information supplied.

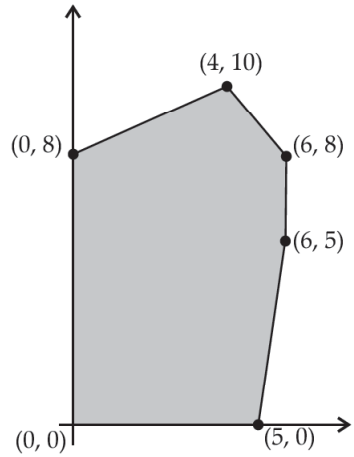
Ans. Option (B) is correct.

Explanation :

Corner points	Corresponding value of $Z = 4x + 3y$
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 ← Maximum
(60, 0)	240

Hence, maximum value of $Z = 300 < 325$
So, the quantity in column B is greater.

Q. 2. The feasible solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function Minimum of Z occurs at



(A) (0, 0) **(B)** (0, 8)
(C) (5, 0) **(D)** (4, 10)

Ans. Option (B) is correct.

Explanation:

Corner points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0

(5, 0)	15 ← Maximum
(6, 5)	–2
(6, 8)	–14
(4, 10)	–28
(0, 8)	–32 ← Minimum

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (–32).

Q. 3. Refer to Q.2 of multiple choice questions, maximum of Z occurs at

(A) (5, 0) **(B)** (6, 5)
(C) (6, 8) **(D)** (4, 10)

Ans. Option (A) is correct.

Explanation: Maximum of Z occurs at (5, 0).

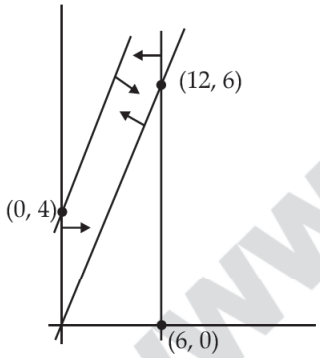
Q. 4. Refer to Q.2 of multiple choice questions, (Maximum value of Z + Minimum value of Z) is equal to

(A) 13 **(B)** 1
(C) –13 **(D)** –17

Ans. Option (D) is correct.

Explanation: Maximum value of Z + Minimum value of $Z = 15 - 32 = -17$

Q. 5. The feasible region for an LPP is shown in the given Figure. Let $F = 3x - 4y$ be the objective function. Maximum value of F is



(A) 0 **(B)** 8
(C) 12 **(D)** –18

Ans. Option (C) is correct.

Explanation: The feasible region as shown in the figure, has objective function $F = 3x - 4y$.

Corner points	Corresponding value of $F = 3x - 4y$
(0, 0)	0
(12, 6)	12 ← Maximum
(0, 4)	–16 ← Minimum

Hence, the maximum value of F is 12.

Q. 6. Refer to Q.5 of multiple choice questions, minimum value of F is

(A) 0 **(B)** –16
(C) 12 **(D)** does not exist

Ans. Option (B) is correct.

Explanation: Minimum value of F is -16 at (0, 4).

Q. 7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function.

The minimum value of F occurs at

(A) (0, 2) only
(B) (3, 0) only
(C) the mid-point of the line segment joining the points (0, 2) and (3, 0) only
(D) any point on the line segment joining the points (0, 2) and (3, 0)

Ans. Option (D) is correct.

Explanation :

Corner points	Corresponding value of $F = 4x+6y$
(0, 2)	12 ← Minimum
(3, 0)	12 ← Minimum
(6, 0)	24
(6, 8)	72 ← Maximum
(0, 5)	30

Hence, minimum value of F occurs at any points on the line segment joining the points (0, 2) and (3, 0).

Q. 8. Refer to Q. 7 above, Maximum of F – Minimum of F =

(A) 60 **(B)** 48
(C) 42 **(D)** 18

Ans. Option (A) is correct.

Explanation: Maximum of F – Minimum of $F = 72 - 12 = 60$

Q. 9. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(A) $p = 2q$ **(B)** $p = q/2$
(C) $p = 3q$ **(D)** $p = q$

Ans. Option (B) is correct.

Explanation :

Corner points	Corresponding value of $Z = px + qy$; $p, q > 0$
(0, 3)	$3q$
(1, 1)	$p + q$
(3, 0)	$3p$

So, condition of p and q, so that the minimum of Z occurs at (3, 0) and (1, 1) is

$$p + q = 3p$$
$$\Rightarrow \quad 2p = q$$
$$\therefore \quad p = \frac{q}{2}$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): Feasible region is the set of points which satisfy all of the given constraints and objective function too.

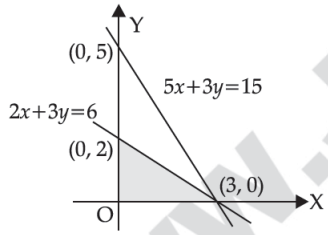
Reason (R): The optimal value of the objective function is attained at the points on *X*-axis only.

Ans. Option (C) is correct.

Explanation: The optimal value of the objective function is attained at the corner points of feasible region.

Q. 2. Assertion (A): The intermediate solutions of constraints must be checked by substituting them back into objective function.

Reason (R):



Here (0, 2); (0, 0) and (3, 0) all are vertices of feasible region.

Ans. Option (D) is correct.

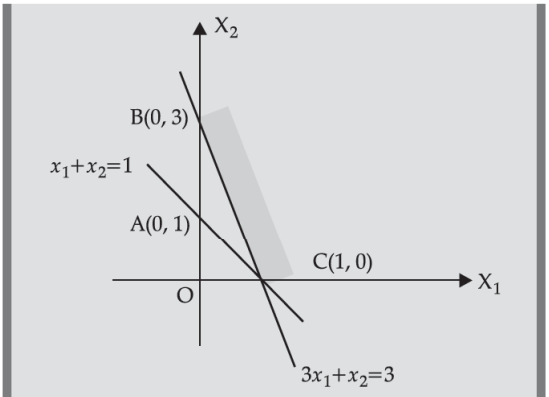
Explanation: The intermediate solutions of constraints must be checked by substituting them back into constraint equations.

Q. 3. Assertion (A) : For the constraints of linear optimizing function $Z = x_1 + x_2$ given by $x_1 + x_2 \le 1$, $3x_1 + x_2 \ge 1$, there is no feasible region.

Reason (R): $Z = 7x + y$, subject to $5x + y \le 5$, $x + y \ge 3$, $x \ge 0$, $y \ge 0$. Out of the corner points of feasible region (3, 0), $\left(\frac{1}{2}, \frac{5}{2}\right)$, (7, 0) and (0,5), the maximum value of Z occurs at (7, 0).

Ans. Option (B) is correct.

Explanation: Assertion (A) is correct.
Clearly from the graph below that there is no feasible region.



Reason (R) is also correct.

Corner Points	$Z = 7x + y$
(3, 0)	21
$\left(\frac{1}{2}, \frac{5}{2}\right)$	6
(7, 0)	49 maximum
(0, 5)	5

Q. 4. Assertion (A): $Z = 20x_1 + 20x_2$, subject to $x_1 \ge 0$, $x_2 \ge 2$, $x_1 + 2x_2 \ge 8$, $3x_1 + 2x_2 \ge 15$, $5x_1 + 2x_2 \ge 20$.
Out of the corner points of feasible region (8, 0), $\left(\frac{5}{2}, \frac{15}{2}\right), \left(\frac{7}{2}, \frac{9}{4}\right)$ and (0,10), the minimum value of Z occurs at $\left(\frac{7}{2}, \frac{9}{4}\right)$.

Reason (R) :

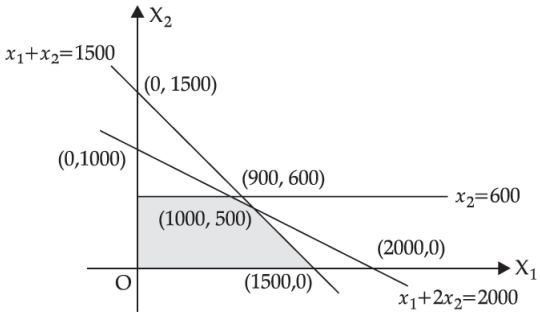
Corner Points	$Z = 20x_1 + 20x_2$
(8, 0)	160
$\left(\frac{5}{2}, \frac{15}{4}\right)$	125
$\left(\frac{7}{2}, \frac{9}{4}\right)$	115 minimum
(0, 10)	200

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 5. Assertion (A): For the constraints of a LPP problem given by
$x_1 + 2x_2 \le 2000$, $x_1 + x_2 \le 1500$, $x_2 \le 600$ and $x_1, x_2 \ge 0$, the points (1000, 0), (0, 500), (2, 0) lie in the positive bounded region, but point (2000, 0) does not lie in the positive bounded region.

Reason (R):



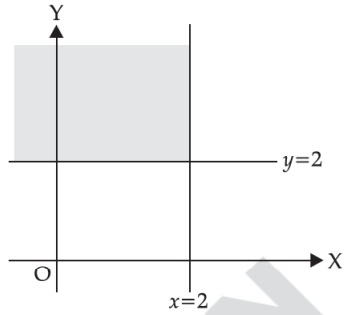
From the graph, it is clear that the point (2000, 0) is outside.

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 6. Assertion (A) : The graph of $x \le 2$ and $y \ge 2$ will be situated in the first and second quadrants.

Reason (R):



Ans. Option (A) is correct.

Explanation: It is clear from the graph given in the Reason (R) that Assertion (A) is true.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class tickets is x and that of economy class tickets is y.

Q. 1. The maximum value of $x + y$ is_____.

(A) 100 **(B)** 200

(C) 20 **(D)** 80

Ans. Option (B) is correct.

Q. 2. The relation between x and y is _____.

(A) $x < y$ **(B)** $y > 80$

(C) $x \ge 4y$ **(D)** $y \ge 4x$

Ans. Option (D) is correct.

Q. 3. Which among these is not a constraint for this LPP?

(A) $x \ge 0$ **(B)** $x + y \le 200$

(C) $x \ge 80$ **(D)** $4x - y \le 0$

Ans. Option (C) is correct.

Q. 4. The profit when $x = 20$ and $y = 80$ is _____.

(A) ₹60,000 **(B)** ₹68,000

(C) ₹64,000 **(D)** ₹1,36,000

Ans. Option (B) is correct.

Q. 5. The maximum profit is ₹ _____.

(A) 1,36,000 **(B)** 1,28,000

(C) 68,000 **(D)** 1,20,000

Ans. Option (A) is correct.

Explanation:

Objective function :

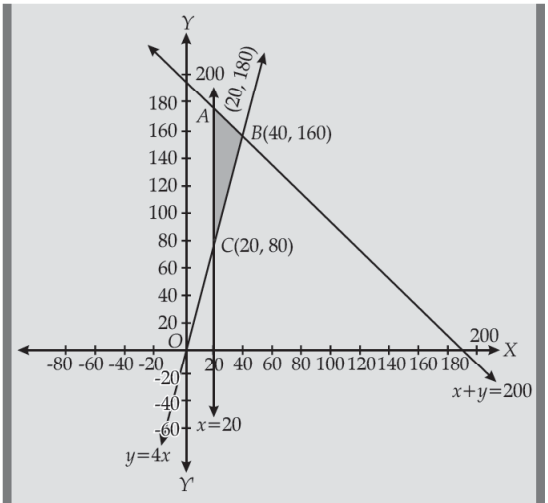
Maximise $Z = 1000x + 600y$

Constraints:

$x + y \ge 200$

$y \ge 20, x \ge 0$

$y \ge 4x$



The corner points are $A(20, 180)$, $B(40, 160)$, $C(20, 80)$

Evaluating the objective function

$Z = 1{,}000x + 600y$ at A, B and C

At $A(20, 180)$, $Z = 1{,}000 \times 20 + 600 \times 180$
$= 20{,}000 + 1{,}08{,}000$
$= ₹\ 1{,}28{,}000$

At $B(40, 160)$, $Z = 1{,}000 \times 40 + 600 \times 160$
$= 40{,}000 + 96{,}000$
$= ₹\ 1{,}36{,}000$ (max.)

At $C(20, 80)$, $Z = 1000 \times 20 + 600 \times 80$
$= 20{,}000 + 48{,}000$
$= ₹\ 68{,}000$

or Z is maximum, when $x = 40$, $y = 160$.

or 40 tickets of executive class and 160 tickets of economy class should be sold to get the maximum profit of ₹ 1,36,000.

II. Read the following text and answer the following questions on the basis of the same:

A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹5,760 to invest and has space for at most 20 items for storage. An electronic sewing machine cost him ₹360 and a manually operated sewing machine ₹240. He can sell an electronic sewing machine at a profit of ₹22 and a manually operated machine at a profit of ₹18. Assume that the electronic sewing machines he can sell is x and that of manually operated machines is y.

Q. 1. The objective function is _____.

(A) Maximise $Z = 360x + 240y$

(B) Maximise $Z = 22x + 18y$

(C) Minimise $Z = 360x + 240y$

(D) Minimise $Z = 22x + 18y$

Ans. Option (B) is correct.

Q. 2. The maximum value of $x + y$ is _____.

(A) 5760 **(B)** 18

(C) 22 **(D)** 20

Ans. Option (D) is correct.

Q. 3. Which of the following is not a constraint?

(A) $x + y \geq 20$

(B) $360x + 240y \leq 5{,}760$

(C) $x \geq 0$

(D) $y \geq 0$

Ans. Option (A) is correct.

Q. 4. The profit is maximum when (x, y) = _____.

(A) (5, 15) **(B)** (8, 12)

(C) (12, 8) **(D)** (15, 5)

Ans. Option (B) is correct.

Q. 5. The maximum profit is ₹ _____.

(A) 5,760 **(B)** 392

(C) 362 **(D)** 290

Ans. Option (B) is correct.

Explanation:

Objective function :

Maximise $Z = 22x + 18y$

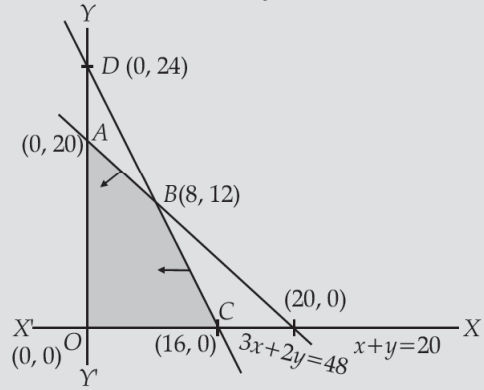
Constraints :

$x + y \leq 20$

$360x + 240y \leq 5{,}760$

Or $3x + 2y \leq 48$

$x \geq 0,\ y \geq 0$



Vertices of feasible region are :

$A(0, 20)$, $B(8, 12)$, $C(16, 0)$ & $O(0, 0)$

$P(A) = 360$, $P(B) = 392$, $P(C) = 352$

$\therefore$ For Maximum P, Electronic machines = 8, and Manual machines = 12. Max. profit ₹392

III. Read the following text and answer the following questions on the basis of the same:

A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

kg per bag		
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Q. 1. The Objective function to minimise the amount of nitogen added to garden?

(A) Maximise $Z = 3x + 4y$

(B) Minimise $Z = 3x + 3.5y$

(C) Maximise $Z = 4x + 3.5y$

(D) Minimise $Z = 3x + 4y$

Ans. Option (B) is correct.

Q. 2. If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of brand P should be used?

(A) 40 **(B)** 50

(C) 100 **(D)** 60

Ans. Option(A) is correct.

Q. 3. If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of brand Q should be used?

(A) 40 **(B)** 50

(C) 100 **(D)** 60

Ans. Option (C) is correct.

Q. 4. What is the minimum amount of nitrogen added in the garden?

(A) 595 kg **(B)** 550 kg

(C) 400 kg **(D)** 470 kg

Ans. Option (D) is correct.

Q. 5. What is the total number of bags used by fruit grower to minimise the amount of nitrogen ?

(A) 160 **(B)** 190

(C) 140 **(D)** 130

Ans. Option (C) is correct.

Explanation: Let the fruit grower use x bags of brand P and y bags of brand Q.

The problem can be formulated as follows:

Minimise $Z = 3x + 3.5y$ ...(i)

Subject to the constraings,

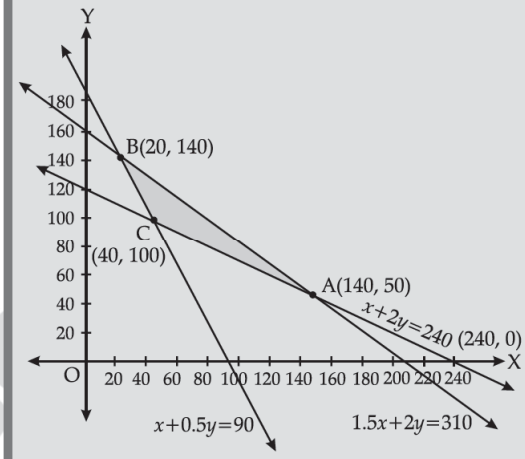
$x + 2y \geq 240$ ...(ii)

$x + 0.5y \geq 90$ ...(iii)

$1.5x + 2y \leq 310$ ...(iv)

$x, y \geq 0$ ...(v)

The feasible region determined by the system of constraints is as follows:



The corner points are $A(240, 50)$, $B(20, 140)$, and $C(40, 100)$

Corner points	$Z = 3x + 3.5y$	
$A(140, 50)$	595	
$B(20, 140)$	550	
$C(40, 100)$	470	← Minimum

The minimum value of Z is 470 at (40, 100).

Thus, 40 bags of brand P and 100 bags of brand Q should be added to the garden to minimise the amount of nitrogen.

The minimum amount of nitrogen added to the garden is 470 kg.

❐❐

UNIT–VI : PROBABILITY

CHAPTER 13

Term-II

PROBABILITY

Syllabus

- ***Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.***

STAND ALONE MCQs (1 Mark each)

Q. 1. If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then

(A) $A \subset B$ **(B)** $B \subset A$

(C) $B = \varphi$ **(D)** $A = \varphi$

Ans. Option (A) is correct.

Explanation:

$$P(A) \neq 0$$

and $$P(B|A) = 1$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$1 = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = P(B \cap A)$$

$$\therefore \quad A \subset B$$

Q. 2. If $P(A|B) > P(A)$, then which of the following is correct :

(A) $P(B|A) < P(B)$

(B) $P(A \cap B) < P(A).P(B)$

(C) $P(B|A) > P(B)$

(D) $P(B|A) = P(B)$

Ans. Option (C) is correct.

Explanation:

$$P(A|B) > P(A)$$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} > P(A)$$

$$\Rightarrow \quad P(A \cap B) > P(A).P(B)$$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(A)} > P(B)$$

$$\Rightarrow \quad P(B|A) > P(B)$$

Q. 3. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(A) $P(B|A) = 1$ **(B)** $P(A|B) = 1$

(C) $P(B|A) = 0$ **(D)** $P(B|A) = 0$

Ans. Option (B) is correct.

Explanation :

$$P(A) + P(B) - P(A \text{ and } B) = P(A)$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$$

$$\Rightarrow \quad P(B) - P(A \cap B) = 0$$

$$\Rightarrow \quad P(A \cap B) = P(B)$$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$= 1$$

Q. 4. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(A) 10^{-1} **(B)** $\left(\frac{1}{2}\right)^5$

(C) $\left(\frac{9}{10}\right)^5$ **(D)** $\frac{9}{10}$

Ans. Option (C) is correct.

Explanation : The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denotes the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb,

$$p = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{10}$.

$$\therefore \quad P(X = x) = {}^nC_x q^{n-x} p^x = {}^5C_x \left(\frac{9}{10}\right)^{5-x} \left(\frac{1}{10}\right)^x$$

P (none of the bulbs is defective) $= P(X=0)$

$$= {}^5C_0 \cdot \left(\frac{9}{10}\right)^5 = 1.\left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$$

Q. 5. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 **(B)** 2

(C) 5 **(D)** $\frac{8}{3}$

Ans. Option (B) is correct.

Explanation :

Let X be the random variable representing a number on the die.

The total number of observations is 6. Therefore,

$$P(X = 1) = \frac{3}{6} = \frac{1}{2}$$

$$P(X = 2) = \frac{2}{6} = \frac{1}{3}$$

$$P(X = 5) = \frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
$P(X)$	1/2	1/3	1/6

$$\text{Mean} = E(X) = \sum p_i x_i = \frac{1}{2}\times 1 + \frac{1}{3}\times 2 + \frac{1}{6}\times 5 = \frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3+4+5}{6} = \frac{12}{6} = 2$$

Q. 6. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 **(B)** $\frac{1}{3}$

(C) $\frac{1}{12}$ **(D)** $\frac{1}{36}$

Ans. Option (D) is correct.

Explanation : When two dices are rolled, the number of outcomes is 36. The only even prime number is 2.

Let E be the event of getting an even prime number on each die.

$\therefore \quad E = \{(2, 2)\}$

$\Rightarrow P(E) = \frac{1}{36}$

Q. 7. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

(A) 0.24 **(B)** 0.3

(C) 0.48 **(D)** 0.96

Ans. Option (D) is correct.

Explanation :

Here,

$P(A) = 0.4,\ P(B) = 0.8$ and $P(A|B) = 0.6$

$$\because \quad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow \quad P(B \cap A) = P(B|A).P(A) = 0.6 \times 0.4 = 0.24$$

$$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.8 - 0.24 = 1.2 - 0.24 = 0.96$$

Q. 8. A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

(A) $\left(\frac{9}{10}\right)^5$ **(B)** $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(C) $\frac{1}{2}\left(\frac{9}{10}\right)^5$ **(D)** $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$

Ans. Option (D) is correct.

Explanation : Here,

$n = 5,\ p = \frac{10}{100} = \frac{1}{10}$ and $q = \frac{9}{10}$

$r \le 1$

$\Rightarrow \quad r = 0, 1$

Also,

$P(X = r) = {}^{n}C_r p^r q^{n-r}$

$P(X = r) = P(r = 0) + P(r = 1)$

$= {}^{5}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^{5}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4$

$= \left(\frac{9}{10}\right)^5 + 5.\frac{1}{10}.\left(\frac{9}{10}\right)^4$

$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$

Q. 9. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is, $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

(A) $\frac{1}{12}$ **(B)** $\frac{1}{40}$

(C) $\frac{13}{120}$ **(D)** $\frac{10}{13}$

Ans. Option (D) is correct.

Explanation : Let E_1 = Event that both A and B solve the problem

$\therefore \quad P(E_1) = \frac{1}{3} \times \frac{1}{4}$

$= \frac{1}{12}$

Let E_2 = Event that both A and B got incorrect solution of the problem

$\therefore \quad P(E_2) = \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{2}$

Let E = Event that they got same answer

Here,

$P(E / E_1) = 1,$

$P(E / E_2) = \frac{1}{20}$

$P(E_1 / E) = \frac{P(E_1 \cap E)}{P(E)}$

$= \frac{P(E_1).P(E / E_1)}{P(E_1).P(E / E_1) + P(E_2)P(E / E_1)}$

$= \frac{\frac{1}{12} \times 1}{\frac{1}{12} \times 1 + \frac{1}{2} \times \frac{1}{20}}$

$= \frac{\frac{1}{12}}{\frac{10+3}{120}}$

$= \frac{120}{12 \times 13}$

$= \frac{10}{13}$

Q. 10. In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in physics if she has failed in mathematics is

(A) $\frac{1}{10}$ **(B)** $\frac{2}{5}$

(C) $\frac{9}{20}$ **(D)** $\frac{1}{3}$

Ans. Option (B) is correct.

Explanation : Here,

$P_{(Ph)} = \frac{30}{100}$

$= \frac{3}{10},$

$P_{(M)} = \frac{25}{100}$

$= \frac{1}{4}$

and $P_{(M \cap Ph)} = \frac{10}{100}$

$= \frac{1}{10}$

$\therefore \quad P\left(\frac{Ph}{M}\right) = \frac{P(Ph \cap M)}{P(M)}$

$= \frac{\frac{1}{10}}{\frac{1}{4}}$

$= \frac{2}{5}$

Q. 11. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability, that both cards are queens, is

(A) $\frac{1}{13} \times \frac{1}{13}$ **(B)** $\frac{1}{13} + \frac{1}{13}$

(C) $\frac{1}{13} \times \frac{1}{17}$ **(D)** $\frac{1}{13} \times \frac{4}{51}$

Ans. Option (A) is correct.

Explanation :

Required probability $= \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$

Q. 12. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Ans. Option (C) is correct.

Explanation : Let,

E_1 = Event that the sum of numbers on the dice was less than 6 and

E_2 = Event that the sum of numbers on the dice is 3.

$\therefore$ $E_1 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2,3), (3, 1), (3, 2), (4,1)\}$

$\Rightarrow$ $n(E_1) = 10$

and $E_2 = \{(1, 2), (2, 1)\}$

$\Rightarrow$ $n(E_2) = 2$

$\therefore$ Required probability $= \frac{2}{10}$

$= \frac{1}{5}$

Q. 13. Eight coins are tossed together. The probability of getting exactly 3 heads is

(A) $\frac{1}{256}$ (B) $\frac{7}{32}$

(C) $\frac{5}{32}$ (D) $\frac{3}{32}$

Ans. Option (B) is correct.

Explanation :

We know that, probability distribution

$$P(X = r) = {}^nC_r (p)^r \, q^{n-r}$$

Here, $n = 8$, $r = 3$, $p = \frac{1}{2}$ and $q = \frac{1}{2}$

$\therefore$ Required probability $= {}^8C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{8-3}$

$= \frac{8!}{5!3!}\left(\frac{1}{2}\right)^8$

$= \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} \cdot \frac{1}{2^8}$

$= \frac{7}{32}$

Q. 14. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(A) $\frac{3}{28}$ (B) $\frac{2}{21}$

(C) $\frac{1}{28}$ (D) $\frac{167}{168}$

Ans. Option (A) is correct.

Explanation :

Probability of drawing 2 green balls and one blue ball

$= P(G) \cdot P(G) \cdot P(B) + P(B) \cdot P(G) \cdot P(G) + P(G) \cdot P(B) \cdot P(G)$

$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6} + \frac{2}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6}$

$= \frac{3}{28}$

Q. 15. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{3}{4}$

Ans. Option (C) is correct.

Explanation : Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore$ $P(E_1) = \frac{3}{6}$

$= \frac{1}{2}$

and $P(E_2) = \frac{13}{52}$

$= \frac{1}{4}$

Then, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$= \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

Q. 16. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4}{7}$

Ans. Option (D) is correct.

Explanation : We have,

$S = \{B, B, B), (G, G, G), (B, G, G), (G, B, G), (G, G, B), (G, B, B), (B, G, B), (B, B, G)\}$

E_1 = Event that a family has at least one girl, then

$E_1 = \{(G, B, B), (B, G, B), (B, B, G), (G, G, B), (B, G, G), (G, B, G), (G, G, G)\}$

E_2 = Event that the eldest child is a girl, then

$E_2 = \{(G, B, B), (G, G, B), (G, B, G), (G, G, G)\}$

$\therefore \quad E_1 \cap E_2 = \{(G,B,B),(G,G,B),(G,B,G),$
$(G,G,G)\}$

$\therefore \quad P(E_2 \mid E_1) = \dfrac{P(E_1 \cap E_2)}{P(E_1)}$

$= \dfrac{\frac{4}{8}}{\frac{7}{8}}$

$= \dfrac{4}{7}$

Q. 17. Two events E and F are independent. If $P(E) = 0.3$, $P(E \cup F) = 0.5$, then $P(E|F) - P(F|E)$ equals

(A) $\frac{2}{7}$ **(B)** $\frac{3}{35}$

(C) $\frac{1}{70}$ **(D)** $\frac{1}{7}$

Ans. Option (C) is correct.

Explanation : We have,

$P(E) = 0.3$

and $P(E \cup F) = 0.5$

Also, E and F are independent.

Now,

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow \quad 0.5 = 0.3 + P(F) - 0.3P(F)$

$\Rightarrow \quad P(F) = \dfrac{0.5 - 0.3}{0.7}$

$= \dfrac{2}{7}$

$\therefore \quad P(E/F) - P(F/E) = P(E) - P(F)$

(as E and F are independent)

$= \dfrac{3}{10} - \dfrac{2}{7}$

$= \dfrac{1}{70}$

Q. 18. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{5}$, then $P(A \cap B)$ equals

(A) $\frac{4}{15}$ **(B)** $\frac{8}{45}$

(C) $\frac{1}{3}$ **(D)** $\frac{2}{9}$

Ans. Option (D) is correct.

Explanation : Since A and B are independent events, A' and B' are also independent. Therefore,

$P(A' \cap B') = P(A') \cdot (B')$

$= (1 - P(A)(1 - P(B))$

$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right)$

$= \frac{2}{5} \cdot \frac{5}{9}$

$= \frac{2}{9}$

Q. 19. If A and B are such events that $P(A) > 0$ and $P(B) \neq 1$, then $P(A|B)$ equals

(A) $1 - P(A|B)$ **(B)** $1 - P(A|B)$

(C) $\dfrac{1 - P(A \cup B)}{P(B')}$ **(D)** $P(A)|P(B)$

Ans. Option (C) is correct.

Explanation : We have,

$P(A) > 0$ and $P(B) \neq 1$

$P(A'/B') = \dfrac{P(A' \cap B')}{P(B')}$

$= \dfrac{1 - P(A \cup B)}{P(B')}$

Q. 20. Let $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$. Then $P(A|B)$ is equal to

(A) $\frac{6}{13}$ **(B)** $\frac{4}{13}$

(C) $\frac{4}{9}$ **(D)** $\frac{5}{9}$

Ans. Option (D) is correct.

Explanation : Here,

$P(A) = \dfrac{7}{13}$,

$P(B) = \dfrac{9}{13}$

and $P(A \cap B) = \dfrac{4}{13}$

$P(A'|B) = \dfrac{P(A' \cap B)}{P(B)}$

$= \dfrac{P(B) - P(A \cap B)}{P(B)}$

$= \dfrac{\frac{9}{13} - \frac{4}{13}}{\frac{9}{13}}$

$= \dfrac{5}{9}$

Q. 21. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$. Then $P(B \cap A)$ equals

(A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$

(C) $\frac{3}{10}$ **(D)** $\frac{1}{5}$

Ans. Option (D) is correct.

Explanation : We have,

$P(A) = 0.4,$

$P(B) = 0.3$

and $P(A \cup B) = 0.5$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.4 + 0.3 - 0.5$

$= 0.2$

$$
\begin{aligned}
P(A) &= 0.4,\\
P(B) &= 0.3\\
\text{and}\quad P(A\cup B) &= 0.5\\
\text{Now,}\quad P(A\cup B) &= P(A)+P(B)-P(A\cap B)\\
\Rightarrow\quad P(A\cap B) &= 0.4+0.3-0.5\\
&= 0.2\\
\therefore\quad P(B'\cap A) &= P(A)-P(A\cap B)\\
&= 0.4-0.2\\
&= 0.2\\
&= \frac{1}{5}
\end{aligned}
$$

Q. 22. If $P(A)=\frac{2}{5}$ $P(B)=\frac{3}{10}$ and $P(A\cap B)=\frac{1}{5}$, then $P(A'|B').P(B'|A')$ is equal to

(A) $\frac{5}{6}$ **(B)** $\frac{5}{7}$

(C) $\frac{25}{42}$ **(D)** 1

Ans. Option (C) is correct.

Explanation : We have,

$$
\begin{aligned}
P(A) &= \frac{2}{5},\\
P(B) &= \frac{3}{10}\\
\text{and}\quad P(A\cap B) &= \frac{1}{5}\\
P(A'|B')\cdot P(B'|A') &= \frac{P(A'\cap B')}{P(B')}\cdot\frac{P(A'\cap B')}{P(A')}\\
&= \frac{(P((A\cup B)'))^2}{P(A')P(B')}\\
&= \frac{(1-P(A\cup B))^2}{(1-P(A))(1-P(B))}\\
&= \frac{(1-P(A)+P(B)-P(A\cap B))^2}{(1-P(A))(1-P(B))}\\
&= \frac{\left[1-\left(\frac{2}{5}+\frac{3}{10}-\frac{1}{5}\right)\right]^2}{\left(1-\frac{1}{2}\right)\left(1-\frac{3}{10}\right)}\\
&= \frac{\left(1-\frac{1}{2}\right)^2}{\frac{3}{5}\cdot\frac{7}{10}}\\
&= \frac{25}{42}
\end{aligned}
$$

Q. 23. If $P(A)=\frac{4}{5}$ and $P(A\cap B)=\frac{7}{10}$, then $(P(B|A)$ is equal to

(A) $\frac{1}{10}$ **(B)** $\frac{1}{8}$

(C) $\frac{7}{8}$ **(D)** $\frac{17}{20}$

Ans. Option (C) is correct.

Explanation :

$$
\begin{aligned}
\because\quad P(A) &= \frac{4}{5},\\
P(A\cap B) &= \frac{7}{10}\\
\therefore\quad P(B|A) &= \frac{P(A\cap B)}{P(A)}\\
&= \frac{\frac{7}{10}}{\frac{4}{5}}\\
&= \frac{7}{8}
\end{aligned}
$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): Let A and B be two events such that $P(A)=\frac{1}{5}$, while $P(A \text{ or } B)=\frac{1}{2}$. Let $P(B)=P$, then for $P=\frac{3}{8}$, A and B independent.

Reason (R) : For independent events,

$$
\begin{aligned}
P(A\cap B) &= P(A)\,P(B)\\
P(A\cup B) &= P(A)+P(B)-P(A\cap B)\\
&= P(A)+P(B)-P(A)\,P(B)\\
&= \frac{1}{5}+P-\left(\frac{1}{5}\right)P\\
\Rightarrow\quad \frac{1}{2} &= \frac{1}{5}+\frac{4}{5}P\\
\Rightarrow\quad P &= \frac{3}{8}.
\end{aligned}
$$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A) : If A and B are two mutually exclusive events with $P(\bar{A})=\frac{5}{6}$ and $P(B)=\frac{1}{3}$. Then $P(A/\bar{B})$ is equal to $\frac{1}{4}$.

Reason (R) : If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.6$ and $P(A|B) = 0.2$ then the value of $P(A|\overline{B})$ is 0.2.

Ans. Option (B) is correct.

Explanation: Assertion (A) is correct.

$$P(A|\overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})}$$

$$P(A|\overline{B}) = \frac{P(A)}{P(\overline{B})}$$

[since, given A and B are two mutually exclusive events]

$$P\left(\frac{A}{\overline{B}}\right) = \frac{\left(1-\frac{5}{6}\right).}{\left(1-\frac{1}{3}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{2}{3}}$$

$$= \frac{1}{4}$$

Reason (R) is also correct.

For independent events,

$$P(A|\overline{B}) = P(A)$$

$$= 0.2.$$

Q. 3. Assertion (A) : Let A and B be two events such that the occurrence of A implies occurrence of B, but not vice-versa, then the correct relation between $P(A)$ and $P(B)$ is $P(B) \geq P(A)$.

Reason (R) : Here, according to the given statement

$$A \subseteq B$$

$$P(B) = P(A \cup (A \cap B))$$

$$(\because A \cap B = A)$$

$$= P(A) + P(A \cap B)$$

Therefore, $P(B) \geq P(A)$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 4. Assertion (A) : If $A \subset B$ and $B \subset A$ then, $P(A) = P(B)$.

Reason (R) : If $A \subset B$ then $P(\overline{A}) \leq P(\overline{B})$.

Ans. Option (C) is correct.

Explanation : Assertion (A) is correct.

$A \subset B$ and $B \subset A \Rightarrow A = B$

Hence, $P(A) = P(B)$.

But (R) is wrong.

$$A \subset B \Rightarrow \overline{B} \subset \overline{A}$$

Therefore, $P(\overline{A}) \geq P(\overline{B})$

Q. 5. Assertion (A) : The probability of an impossible event is 1.

Reason (R) : If A is a perfect subset of B and $P(A) < P(B)$, then $P(B - A)$ is equal to $P(B) - P(A)$.

Ans. Option (D) is correct.

Explanation : Assertion (A) is wrong.

If the probability of an event is 0, then it is called as an impossible event.

But Reason (R) is correct.

From Basic Theorem of Probability,

$P(B - A) = P(B) - P(A)$, this is true only if the condition given in the question is true.

Q. 6. Assertion (A) : If $A = A_1 \cup A_2 \ldots \cup A_n$, where $A_1 \ldots A_n$ are mutually exclusive events then

$\sum_{i=1}^{n} P(A_i) = P(A)$

Reason (R) :

Given, $A = A_1 \cup A_2 \ldots \cup A_n$

Since $A_1 \ldots A_n$ are mutually exclusive

$$P(A) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

Therefore $P(A) = \sum_{i=1}^{n} P(A_i)$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. **[CBSE QB 2021]**

Q. 1. Let the target is hit by *A, B*: the target is hit by *B* and, *C*: the target is hit by A and C. Then, the probability that *A, B* and, *C* all will hit, is

(A) $\frac{4}{5}$ **(B)** $\frac{3}{5}$

(C) $\frac{2}{5}$ **(D)** $\frac{1}{5}$

Ans. Option (C) is correct.

Explanation:

$P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$, $P(C) = \frac{2}{3}$

Probability that *A, B* and *C* all will hit the target

$= P(A \cap B \cap C)$

$= P(A)P(B)P(C)$

$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{2}{5}$

Q. 2. What is the probability that B, C will hit and A will lose?

(A) $\frac{1}{10}$ **(B)** $\frac{3}{10}$

(C) $\frac{7}{10}$ **(D)** $\frac{4}{10}$

Ans. Option (A) is correct.

Explanation:

$P(\bar{A}) = 1 - \frac{4}{5}$

$= \frac{1}{5}$

Probability that *B, C* will hit and *A* will lose

$= P(\bar{A} \cap B \cap C)$

$= P(\bar{A}).P(B).P(C)$

$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{1}{10}$

Q. 3. What is the probability that 'any two of *A, B* and *C* will hit?

(A) $\frac{1}{30}$ **(B)** $\frac{11}{30}$

(C) $\frac{17}{30}$ **(D)** $\frac{13}{30}$

Ans. Option (D) is correct.

Explanation:

$P(\bar{B}) = 1 - \frac{3}{4}$

$= \frac{1}{4}$,

$P(\bar{C}) = 1 - \frac{2}{3}$

$= \frac{1}{3}$

Probability that any two of *A, B* and *C* will hit

$= P(\bar{A})P(B)P(C) + P(A)P(\bar{B})P(C)$

$+ P(A)P(B)P(\bar{C})$

$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$

$= \frac{1}{10} + \frac{2}{15} + \frac{1}{5}$

$= \frac{3+4+6}{30}$

$= \frac{13}{30}$

Q. 4. What is the probability that 'none of them will hit the target'?

(A) $\frac{1}{30}$ **(B)** $\frac{1}{60}$

(C) $\frac{1}{15}$ **(D)** $\frac{2}{15}$

Ans. Option (B) is correct.

Explanation: Probability that none of them will hit the target

$= P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= P(\bar{A}).P(\bar{B}).P(\bar{C})$

$= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{60}$

Q. 5. What is the probability that at least one of *A, B* or *C* will hit the target?

(A) $\frac{59}{60}$ **(B)** $\frac{2}{5}$

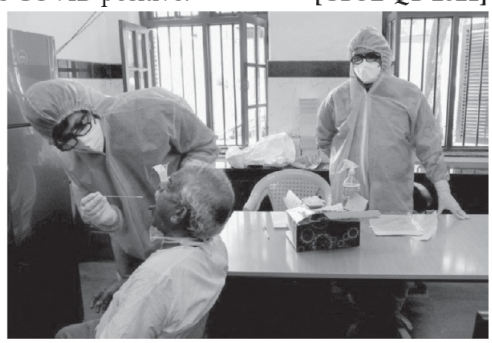
(C) $\frac{3}{5}$ **(D)** $\frac{1}{60}$

Ans. Option (A) is correct.

II. Read the following text and answer the following questions on the basis of the same:

The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10% goes undetecte. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive. **[CBSE QB 2021]**



Q. 1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?

(A) 0.001 **(B)** 0.1

(C) 0.8 **(D)** 0.9

Ans. Option (D) is correct.

Explanation:

E = Person selected has Covid

F = Does not have Covid

G = Test judge Covid positive

Probability of the person to be tested as Covid positive given that he is actually having Covid

$$= P(G/E) = 90\% = \frac{90}{100} = 0.9$$

Q. 2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

(A) 0.01 **(B)** 0.99

(C) 0.1 **(D)** 0.001

Ans. Option (A) is correct.

Explanation: Probability of person to be tested as Covid positive given that he is actually not having Covid

$$= P(G/E) = 1\% = \frac{1}{100} = 0.01$$

Q. 3. What is the probability that the 'person is actually not having COVID'?

(A) 0.998 **(B)** 0.999

(C) 0.001 **(D)** 0.111

Ans. Option (B) is correct.

Explanation:

$$P(E) = 1 - P(E)$$

$$= 1 - 0.001 \quad \left[\because P(F) = 0.1\% = \frac{0.1}{100} = 0.001\right]$$

$$= 0.999$$

Q. 4. What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?

(A) 0.83 **(B)** 0.0803

(C) 0.083 **(D)** 0.089

Ans. Option (C) is correct.

Explanation:

$$P(E/G) = \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01}$$

$$= \frac{9 \times 10^{-4}}{9 \times 10^{-4} + 99.9 \times 10^{-4}}$$

$$= \frac{9 \times 10^{-4}}{10^{-4}(9 + 99.9)}$$

$$= \frac{9}{108.9}$$

$$= 0.083 \text{ (approx)}$$

Q. 5. What is the probability that the 'person selected will be diagnosed as COVID positive'?

(A) 0.1089 **(B)** 0.01089

(C) 0.0189 **(D)** 0.189

Ans. Option (B) is correct.

III. Read the following text and answer the following questions on the basis of the same:

In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

[CBSE QB 2021]

Q. 1. What is the value of $P(E_1)$?

(A) $\frac{2}{5}$ **(B)** $\frac{1}{3}$

(C) 1 **(D)** $\frac{3}{5}$

Ans. Option (D) is correct.

Q. 2. Value of $P(E|E_1)$ is

(A) $\frac{1}{3}$ **(B)** 1

(C) $\frac{2}{3}$ **(D)** $\frac{4}{5}$

Ans. Option (B) is correct.

Explanation:

$$P(E_1/E) = 1$$

Q. 3. $\sum_{k=1}^{k=2} P(E|E_k)\, P(E_k)$ Equals

(A) $\frac{11}{15}$ **(B)** $\frac{4}{15}$

(C) $\frac{1}{5}$ **(D)** 1

Ans. Option (A) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E \mid E_k)P(E_K) = P(E \mid E_1)P(E) + P(E \mid E_2) + P(E_2)$$
$$= 1 \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5}$$
$$= \frac{11}{15}$$

Q. 4. Value of $\sum_{k=1}^{k=2} P(E_k)$

(A) $\frac{1}{3}$ **(B)** $\frac{1}{5}$

(C) 1 **(D)** $\frac{3}{5}$

Ans. Option (C) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E_k) = P(E_1) + P(E_2)$$
$$= \frac{3}{5} + \frac{2}{5}$$
$$= \frac{5}{5}$$
$$= 1$$

Q. 5. What is the probability that the student knows the answer given that he answered it correctly?

(A) $\frac{2}{11}$ **(B)** $\frac{5}{3}$

(C) $\frac{9}{11}$ **(D)** $\frac{13}{3}$

Ans. Option (C) is correct.

Explanation:

$$P(E \mid E_1) = \frac{P(E_1).P(E_1 \mid E)}{P(E_1).P(E_1 \mid E) + P(E_2)P(E \mid E_2)}$$
$$= \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}}$$
$$= \frac{\frac{3}{5}}{\frac{11}{15}}$$
$$= \frac{9}{11}$$

IV. Read the following text and answer the following questions on the basis of the same:

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

[CBSE QB 2021]

Q. 1. The conditional probability that an error is committed in processing given that Sonia processed the form is :

(A) 0.0210 **(B)** 0.04

(C) 0.47 **(D)** 0.06

Ans. Option (B) is correct.

Q. 2. The probability that Sonia processed the form and committed an error is :

(A) 0.005 **(B)** 0.006

(C) 0.008 **(D)** 0.68

Ans. Option (C) is correct.

Explanation:

P (sonia processed the form and committed an error) = 20% × 0.4

$$= \frac{20}{100} \times 0.04$$
$$= \frac{1}{5} \times 0.04$$
$$= 0.008$$

Q. 3. The total probability of committing an error in processing the form is :

(A) 0 **(B)** 0.047

(C) 0.234 **(D)** 1

Ans. Option (B) is correct.

Q. 4. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is :

(A) 1 **(B)** $\frac{30}{47}$

(C) $\frac{20}{47}$ **(D)** $\frac{17}{47}$

Ans. Option (D) is correct.

Q. 5. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A) = 1$ is :

(A) 0 **(B)** 0.03

(C) 0.06 **(D)** 1

Ans. Option (D) is correct.

Explanation:

$$\sum_{i=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = 1$$

[$\because$ sum of all occurrence of an event is equal to 1]

V. Read the following text and answer the following questions on the basis of the same:

A group of people start playing cards. And as we know a well shuffled pack of cards contains a total of 52 cards. Then 2 cards are drawn simultaneously (or successively without replacement).

Q. 1. If x = no. of kings = 0, 1, 2. Then $P(x = 0)$ = ?

(A) $\frac{188}{221}$ **(B)** $\frac{198}{223}$

(C) $\frac{197}{290}$ **(D)** $\frac{187}{221}$

Ans. Option (A) is correct.

Explanation:

$$P(x=0) = \frac{48}{52}\times\frac{47}{51} = \frac{188}{221}$$

Q. 2. If x = no. of kings = 0, 1, 2. Then $P(x = 1)$ = ?

(A) $\frac{32}{229}$ **(B)** $\frac{32}{227}$

(C) $\frac{32}{221}$ **(D)** $\frac{32}{219}$

Ans. Option (C) is correct.

Explanation:

$$P(x=1) = \frac{48}{52}\times\frac{47}{51} = \frac{188}{221}$$

Q. 3. If x = no. of kings = 0, 1, 2. Then $P(x = 2)$ = ?

(a) $\frac{2}{219}$ **(B)** $\frac{1}{221}$

(C) $\frac{3}{209}$ **(D)** $\frac{1}{209}$

Ans. Option (B) is correct.

Explanation:

$$P(x=2) = \frac{4}{52}\times\frac{3}{51} = \frac{1}{221}$$

Q. 4. Find the mean of the number of kings ?

(A) $\frac{2}{13}$ **(B)** $\frac{1}{13}$

(C) $\frac{1}{17}$ **(D)** $\frac{2}{17}$

Ans. Option (A) is correct.

Explanation:

$$\text{Mean} = \Sigma x_i p_i = \left(0\times\frac{188}{221}\right)+\left(1\times\frac{32}{221}\right)+\left(2\times\frac{1}{221}\right) = \frac{34}{221} = \frac{2}{13}$$

Q. 5. Find the variance of the number of kings ?

(A) $\frac{400}{2873}$ **(B)** $\frac{400}{2877}$

(C) $\frac{400}{2879}$ **(D)** $\frac{400}{2871}$

Ans. Option (A) is correct.

Explanation:

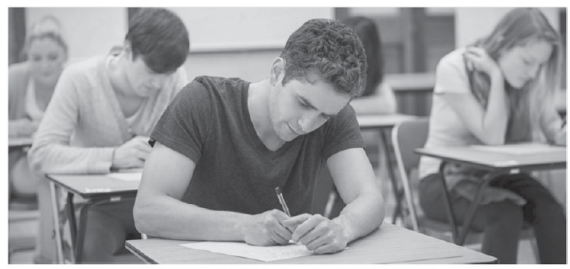
$$\text{Variance} = \Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$$

$$\Sigma x_i^2 p_i = \left(0\times\frac{188}{221}\right)+\left(1\times\frac{32}{221}\right)+\left(4\times\frac{1}{221}\right) = \frac{36}{221}$$

$$\text{Variance} = \frac{36}{221} - \left(\frac{2}{13}\right)^2 = \frac{400}{2873}$$

VI. Read the following text and answer the following questions on the basis of the same:

Anand, Samanyu and Shah of SHORTCUTS classes were given a problem in Mathematics whose respective probabilities of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. They were asked to solve it independently.



Based on the above data, answer any four of the following questions.

Q. 1. The probability that Anand alone solves it is _____.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{11}{24}$ **(D)** $\frac{17}{24}$

Ans. Option (A) is correct.

Explanation:

Let $A \rightarrow$ event that Anand solves

$B \rightarrow$ event that Samanyu solves

$C \rightarrow$ event that Shah solves

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4}$$

$$\therefore P(A') = \frac{1}{2},\ P(B') = \frac{2}{3},\ P(C') = \frac{3}{4}$$

$$P(A \cap B' \cap C') = P(A)\,P(B')\,P(C')$$
$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$$
$$= \frac{1}{4}$$

Q. 2. The probability that the problem is not solved is _____.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) 0 **(D)** $\frac{11}{24}$

Ans. Option (A) is correct.

Explanation:

$$P(A' \cap B' \cap C') = P(A')\,P(B')\,P(C')$$
$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$$
$$= \frac{1}{4}$$

Q. 3. The probability that the problem is solved is _____.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{17}{24}$ **(D)** $\frac{11}{24}$

Ans. Option (B) is correct.

Explanation:

$$P(A \cup B \cup C) = 1 - P(A')\,P(B')\,P(C')$$
$$= 1 - \frac{1}{4}$$
$$= \frac{3}{4}$$

Q. 4. The probability that exactly one of them solves it is _____.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{17}{24}$ **(D)** $\frac{11}{24}$

Ans. Option (D) is correct.

Explanation:

$$P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$$
$$= P(A)\,P(B')\,P(C') + P(A')\,P(B)\,P(C') + P(A')\,P(B')\,P(C)$$
$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$$
$$= \frac{11}{24}$$

Q. 5. The probability that exactly two of them solves it is _____.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{17}{24}$ **(D)** $\frac{11}{24}$

Ans. Option (A) is correct.

Explanation:

$$P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)$$
$$= P(A)\,P(B)\,P(C') + P(A)\,P(B')\,P(C) + P(A')\,P(B)\,P(C)$$
$$= \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$$
$$= \frac{6}{24}$$
$$= \frac{1}{4}$$

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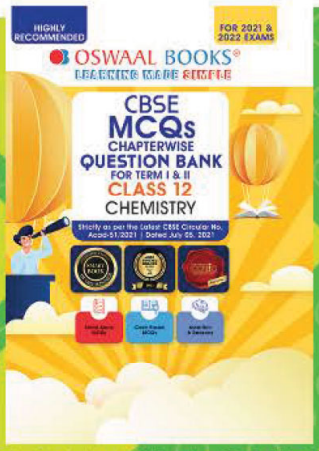

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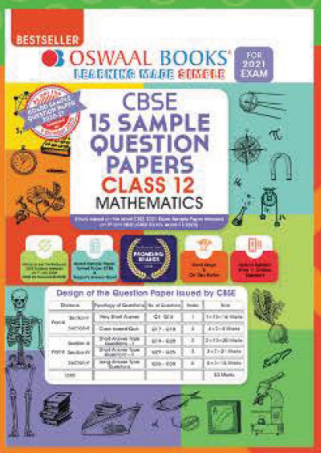

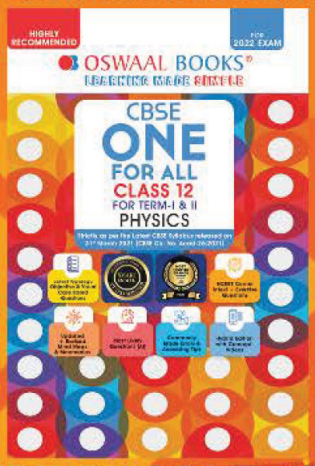

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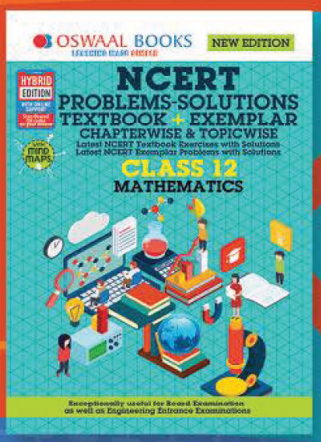

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