As per the Latest Board Syllabus issued on 21st April, 2022

2023
EXAMINATION

OSWAL – GURUKUL

MOST LIKELY

CBSE QUESTION BANK

CHAPTERWISE & CATEGORYWISE

Scan the QR code inside for instant video solutions to all your doubts by doubtnut

MATHEMATICS (STANDARD)

CLASS X

Special Focus on New Pattern Questions	Complete Explanations to all the MCQs	Case Based Questions Included
Assertion & Reasoning Based Questions Included	Illustrated Chapter Summary	Previous Years' Board Questions Incorporated

In accordance with the latest syllabus prescribed by the Central Board of Secondary Education, New Delhi.

OSWAL – GURUKUL

MOST LIKELY

CBSE QUESTION BANK

CHAPTERWISE & CATEGORYWISE

MATHEMATICS

CLASS X

By

PANEL OF AUTHORS

As per the Latest Syllabus issued by the Board Circular No. Acad-48/2022

DISCLAIMER

With the ambition of providing standard academic resources, we have exercised extreme care in publishing the content. In case of any discrepancies in the matter, we request readers to excuse the unintentional lapse and not hold us liable for the same. Suggestions are always welcome.

EDITION : 2022

ISBN : 978-93-92563-48-5

PRINTED AT : Upkar Printing Unit, Agra

PUBLISHED BY

OSWAL PUBLISHERS

Head Office : 1/12, Sahitya Kunj, M.G. Road, Agra – 282 002

Phone : (0562) 2527771-4

Whatsapp : +91 74550 77222

E-mail : info@oswalpublishers.in

Website : www.oswalpublishers.com

The cover of this book has been designed using resources fromFreepik.com

PREFACE

Oswal – Gurukul's Most Likely CBSE Question Bank series is up-to-date with the latest syllabus given by the Central Board of Secondary Education.

This title highlights the knowledge-based and skill-based goals of the Bloom's Taxonomy by acquainting the students with relevant facts and concepts. It also teaches them ways to apply their subjective knowledge to get their best academic results.

In this book, the questions are arranged section-wise so that the students can revise the whole syllabus in less time and develop the ability of prioritising and categorising topics for effective learning. It covers all probable types of questions that can be asked in the exams. The solutions and explanations have been prepared by eminent subject experts. They follow the standard marking scheme of the CBSE Board. Additionally, questions from compartment paper, foreign paper and latest board paper have also been incorporated in the chapters.

The series is an attempt to instil confidence in students to face the board examination. The language used is simple, to the point and questions cover all the important topics as per the weightage given to them by the board.

We hope this book will be a valuable asset for the students. All suggestions towards improving the series are welcome and would be incorporated in the future editions.

—Publisher

SYLLABUS

COURSE STRUCTURE CLASS X

(Annual Examination)

Theory Paper

Time: 3 Hours Max. Marks: 80		
Unit No.	**Units**	**Marks**
I.	NUMBER SYSTEMS	06
II.	ALGEBRA	20
III.	COORDINATE GEOMETRY	06
IV.	GEOMETRY	15
V.	TRIGONOMETRY	12
VI.	MENSURATION	10
VII.	STATISTICS & PROBABILTY	11
	Total	**80**

Unit I : NUMBER SYSTEMS

1. REAL NUMBER(15) Periods

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$.

Unit II : ALGEBRA

1. POLYNOMIALS**(8) Periods**

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES**(15) Periods**

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS**(15) Periods**

Standard form of a quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS**(10) Periods**

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

Unit III : COORDINATE GEOMETRY

COORDINATE GEOMETRY **(15) Periods**

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

Unit IV : GEOMETRY

1. TRIANGLES**(15) Periods**

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to

intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.

4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES**(10) Periods**

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

Unit V : TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY**(10) Periods**

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES**(15) Periods**

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression**(10) Periods**

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation/ depression should be only 30°, 45°, and 60°.

Unit VI : MENSURATION

1. AREAS RELATED TO CIRCLES(12) Periods

Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES(12) Periods

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

Unit VII : STATISTICS AND PROBABILITY

1. STATISTICS(18) Periods

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY(10) Periods

Classical definition of probability. Simple problems on finding the probability of an event.

QUESTION PAPER DESIGN

CLASS X

(Code No. 041)

Time: 3 Hours Max. Marks: 80			
Sr, No.	**Typology of Questions**	**Total Marks**	**% Weightage**
1.	**Remembering:** Exhibit memory of	43	54

	previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas		
2.	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3..	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	Total	**80**	**100**

INTERNAL ASSESSMENT : 20 MARKS

- Pen Paper Test and Multiple Assessment (5+5):- 10 Marks
- Portfolio :- 05 Marks
- Lab Practical (Lab activities to be done from the prescribed

books):- 05 Marks

CONTENTS

केन्द्रीय माध्यमिक शिक्षा बोर्ड
(शिक्षा मंत्रालय, भारत सरकार के अधीन एक स्वायत्त संगठन)
CENTRAL BOARD OF SECONDARY EDUCATION
(An Autonomous Organisation under the Ministry of Education, Govt. of India)

CBSE/Acad/ 2022

April 21, 2022
Cir. No. Acad-48/2022

All Heads of Institutions affiliated to CBSE

Subject: Secondary and Senior School Curriculum 2022-23

1. CBSE annually provides curriculum for classes IX to XII containing academic content, syllabus for examinations with learning outcomes, pedagogical practices and assessment guidelines.

2. Considering the feedback of stakeholders and other prevailing conditions, **the Board will conduct the annual scheme of assessment at the end of the Academic Session 2022-23** and the curriculum has been designed accordingly. Details are available at the link https://cbseacademic.nic.in/curriculum_2023.html

3. It is important that schools ensure curriculum transaction as per the directions given in the initial pages of the curriculum document. The subjects should be taught as per the curriculum released by the Board with the help of suitable teaching-learning strategies such as Art-Integrated Education, Experiential Learning, and Pedagogical Plans etc. wherever possible.

4. Before making annual pedagogical plan to ensure curriculum transaction for optimal learning, it is desirable that the Head of the School may take a session with all the teachers on the important topics covered in initial pages of the curriculum document as well as the topics covered under subject-wise syllabus.

5. Sample Question Papers with detailed design of the Question Paper will be made available on CBSE's website in due course of time.

6. Schools are requested to share the curriculum available on https://cbseacademic.nic.in/curriculum_2023.html including initial pages to all the teachers and students.

With Best wishes,

Dr. Joseph Emmanuel
Director (Academics)

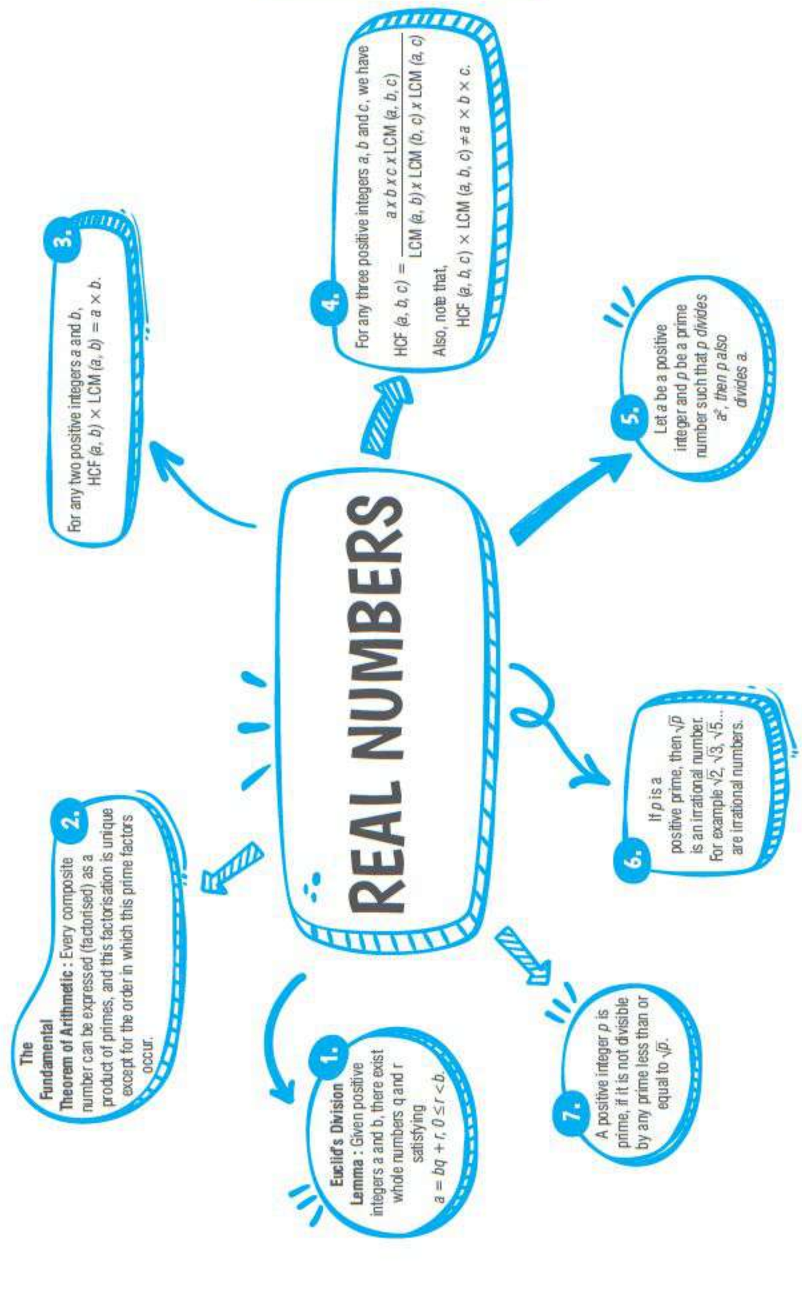
'शिक्षा सदन' ,17 राऊज़ एवेन्यू ,इंस्टीट्यूशनल एरिया, नई दिल्ली–110002
'Shiksha Sadan', 17, Rouse Avenue, Institutional Area, New Delhi - 110002
फ़ोन/Telephone: 011-23212603,23233227 वेबसाइट/Website :http://www.cbseacademic.nic.in ई-मेल/e-mail: mailto:directoracad.cbse@nic.in.

Real Numbers

Chapter

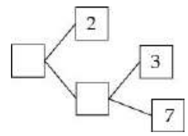
1

Basic Concepts



Multiple Choice Questions

1. Complete the missing entries in the following factor tree:



(a) 42 and 21

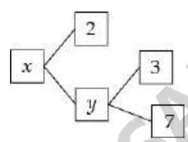
(b) 24 and 12

(c) 7 and 3

(d) 84 and 42

Ans. (a) 42 and 21

Explanation :



From the above factor tree. It is clear that

$y = 3 \times 7 = 21$

and $x = 2 \times y = 2 \times 21$

$= 42.$

2. The sum of exponents of prime factors in the prime-factorisation of 196 is:

[Board Question]

(a) 3

(b) 4

(c) 5

(d) 2

Ans. (b) 4

Explanation :

Prime factors of 196 = $2^2 \times 7^2$

So, sum of exponents = 2 + 2 = 4.

3. The total number of factors of a prime number is:

[Board Question]

(a) 1

(b) 0

(c) 2

(d) 3

Ans. (c) 2

Explanation :

As we know that prime numbers are the numbers with two factors, 1 and the number itself.

Total number of factors of a prime number will be 2.

4. The H.C.F. and the L.C.M. of 12, 21, 15 respectively are:

[Board Question]

(a) 3, 140

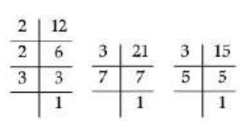
(b) 12, 420

(c) 3, 420

(d) 420, 3

Ans. (c) 3, 420

Explanation :



Prime factors of 12, 21 and 15 are

$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$

$21 = 3 \times 7 = 3^1 \times 7^1$

$15 = 3 \times 5 = 3^1 \times 5^1$

H.C.F. (12, 21, 15) = $3^1 = 3$

and L.C.M. (12, 21, 15) = $2^2 \times 3^1 \times 5^1 \times 7^1$

= 420.

5. H.C.F. of 2 numbers is 113, and their L.C.M. is 56952. If one number is 904, then the other number is:

(a) 7911

(b) 7119

(c) 7791

(d) 7971

Ans. (b) 7119

Explanation :

We know that

First number × second number

= L.C.M. × H.C.F.

⇒ 904 × second number = 113 × 56952

⇒ second number = $\frac{113 \times 56952}{904}$

= 7119.

6. For what least value of *n* natural number $(24)^n$ is divisible by 8?

(a) 1

(b) – 1

(c) 0

(d) none of these

Ans. (a) 1

Explanation :

Let n = 1 in $(24)^n$

$(24)^1$ = 24 which is divisible by 8

Hence least number = 1.

7. The H.C.F. of 306 and 1314 is:

(a) 15

(b) 16

(c) 17

(d) 18

Ans. (d) 18

Explanation :

$306 = 2^1 \times 3^2 \times 17$

$1314 = 2^1 \times 3^2 \times 73$

H.C.F. of 306 and 1314 = $2^1 \times 3^2$

= 2 × 9

= 18.

8. The H.C.F. (a, b) = 2 and L.C.M. (a, b) = 27. What is the value $a \times b$?

(a) 44

(b) 54

(c) 56

(d) 68

Ans. (b) 54

Explanation :

$a \times b$ = H.C.F. × L.C.M.

= 2 × 27

= 54.

9. The H.C.F. of $3^3 \times 5$ and $3^2 \times 5^2$ is:

(a) 135

(b) 15

(c) 225

(d) 45

Ans. (d) 45

Explanation :

H.C.F. of $3^3 \times 5$ and $3^2 \times 5^2 = 3^2 \times 5$

= 9 × 5 = 45

10. If the H.C.F. of two numbers is 1, then the two numbers are called:

(a) composite

(b) twin primes

(c) co-primes

(d) none of these

Ans. (c) co-primes

Explanation :

Two numbers are said to be co-primes if they have only one common factor, namely 1.

11. p **is:**

(a) an integer

(b) an irrational number

(c) a rational number

(d) none of the above

Ans. (b) an irrational number

Explanation :

Since, the value of p is 3.14159........

which is non-terminating and non-repating. Hence it is an irrational number.

12. Which of the following is the smallest composite number?

(a) 3

(b) 4

(c) 2

(d) 1

Ans. (b) 4

Explanation :

A composite number is a natural number which has more than two factors. Here, factors of 4 = 1, 2, 4.

4 is a composite number.

13. The H.C.F. of smallest prime number and the smallest composite number is ....

(a) 1

(b) 2

(c) 4

(d) none of these

Ans. (b) 2

Explanation :

Smallest prime no. = 2

Smallest composite no. = 4

H.C.F. (4, 2) = 2.

14. For some integer *m*, every even integer is of the form:

[NCERT Exemplar]

(a) m

(b) $m + 1$

(c) $2m$

(d) $2m + 1$

Ans. (c) $2m$

Explanation :

We know that, even integers are 2, 4, 6, …

So, it can be written in the form of $2m$.

where, m = Integer = Z

[since, integer is represented by Z]

or $m = \dots, -1, 0, 1, 2, 3, \dots$

$2m = \dots, -2, 0, 2, 4, 6, \dots$

15. $n^2 - 1$ is divisible by 8, if *n* is:

[NCERT Exemplar]

(a) an integer

(b) a natural number

(c) an odd integer

(d) an even integer

Ans. (c) an odd integer

Explanation :

Let $a = n^2 - 1$

Here n can be even or odd.

Case I: n = Even *i.e.*, $n = 2k$, where k is an integer.

$\Rightarrow a = (2k)^2 - 1$

$\Rightarrow a = 4k^2 - 1$

At $k = -1 = 4(-1)^2 - 1 = 4 - 1 = 3$, which is not divisible by 8.

At $k = 0$, $a = 4(0)^2 - 1 = 0 - 1 = -1$, which is not divisible by 8.

Case II: n = Odd *i.e.*, $n = 2k + 1$, where k is an odd integer.

$a = (2k + 1)^2 - 1$

$\Rightarrow a = 4k^2 + 4k + 1 - 1$

$\Rightarrow a = 4k^2 + 4k$

$\Rightarrow a = 4k(k + 1)$

At $k = -1$, $a = 4(-1)(-1 + 1) = 0$ which is divisible by 8.

At $k = 0$, $a = 4(0)(0 + 1) = 0$ which is divisible by 8.

At $k = 1$, $a = 4(1)(1 + 1) = 8$ which is divisible by 8.

Hence, we can conclude from above two cases, if n is odd, then $n^2 - 1$ is divisible by 8.

16. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is:

[NCERT Exemplar]

(a) 13

(b) 65

(c) 875

(d) 1750

Ans. (a) 13

Explanation :

Since, 5 and 8 are the remainders of 70 and 125, respectively. Thus, after subtracting these remainders from the numbers, we have the numbers 65 = (70 – 5) and 117 = (125 – 8), which is divisible by the required number.

Now, required number = H.C.F. of 65, 117

[for the largest number]

H.C.F. = 13

Hence, 13 is the largest number which divides 70 and 125, leaving remainders 5 and 8.

Very Short Answer Type Questions

17. Express 23150 as product of its prime factors. Is it unique?

Sol. ∴ Prime factors of 23150 is

2	23150
5	11575
5	2315
463	463
	1

$23150 = 2 \times 5^2 \times 463$

As per the fundamental theorem of Arithmetic, every number has a unique factorisation.

18. Find the largest number which divides 70 and 125 leaving remainder 5 and 8 respectively.

Sol. It is given that on dividing 70 by the required number, there is a remainder 5. This means that 70 – 5 = 65 is exactly divisible by the required number.

Similarly, 125 – 8 = 117 is also exactly divisible by required number.

Now, required number = H.C.F. (65, 117)

5	65
13	13
	1

3	117
3	39
13	13
	1

∴ 65 = 5 × 13

and 117 = 32 × 13

∴ H.C.F. (65, 117) = 13

∴ Required number = 13 **Ans.**

19. If H.C.F. (336, 54) = 6, find L.C.M. (336, 54).

[Board Question]

Sol. Given, H.C.F. (336, 54) = 6

We know,

H.C.F. × L.C.M. = one number × other number

⇒ 6 × L.C.M. = 336 × 54

⇒ L.C.M. = $\frac{336 \times 54}{6}$

= 336 × 9

= 3024 **Ans.**

20. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational number.

Sol. Given,

$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$

= $\frac{12\sqrt{5}}{2\sqrt{5}} = 6$ (a rational number)

Thus, on simplification $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ gives a rational number. **Ans.**

21. 96 defective pens are accidentally mixed with 105 good pens. What is L.C.M. of 96 and 105 ?

Sol. We have,

$96 = 2^5 \times 3$

$105 = 3 \times 7 \times 5$

$\therefore$ L.C.M. (96, 105) = $2^5 \times 3 \times 7 \times 5$

= 3360 **Ans.**

22. Atul, Ravi and Tarun go for a morning walk. They step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

Sol. We have,

$40 = 2^3 \times 5$

$42 = 2 \times 3 \times 7$

$45 = 3^2 \times 5$

$\therefore$ L.C.M. of 40, 42 and 45 = $2^3 \times 3^2 \times 5 \times 7$

= 2520

So, after the walk of 2520 cm each can cover the same distance in complete steps. **Ans.**

23. There is a circular path around a sports field, Priya takes 18 min. to drive one round of the field, while Ravish takes 12 min. for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point?

Sol. No. of minutes after which they will meet again at the starting point = L.C.M. of 18 and 12

Prime factors of 18 = 2×3^2

Prime factors of 12 = $2^2 \times 3$

$\therefore$ L.C.M. of (18, 12) = 22 × 32

= 36.

Thus, Priya and Ravish will meet again at the starting point after 36 min. **Ans.**

24. Explain why (17 × 5 × 11 × 3 × 2 + 2 × 11) is a composite number?

[Board Question]

Sol. 17 × 5 × 11 × 3 × 2 + 2 × 11

= 17 × 5 × 3 × 22 + 22

= 22 (17 × 5 × 3 + 1)

= 22 (255 + 1)

= 2 × 11 × 256 ...(i)

Equation (i) is divisible by 2, 11 and 256, which means it has more than 2 prime factors.

∴ (17 × 5 × 11 × 3 × 2 + 2 × 11) is a composite number. **Ans.**

25. Show that $3\sqrt{7}$ is an irrational number.

Sol. Let us assume, to the contrary that $3\sqrt{7}$ is rational.

That is, we can find co-prime *a* and *b* ($b \neq 0$) such that $3\sqrt{7} = \frac{a}{b}$

On rearranging, we get

$\Rightarrow \sqrt{7} = \frac{a}{3b}$

Since 3, *a* and *b* are integers, therefore, $\frac{a}{3b}$ can be written in the form of $\frac{p}{q}$, so $\frac{a}{3b}$ is rational, and so $\sqrt{7}$ is rational.

But this contradicts the fact that $\sqrt{7}$ is irrational.

So, we conclude that $3\sqrt{7}$ is irrational.

Hence Proved.

26. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

[Board Question]

Sol. As $\sqrt{2}$ = 1·414....

$\sqrt{3}$ = 1·732....

So, a rational number between $\sqrt{2}$ and $\sqrt{3}$ is 1·5 or we can take any number between 1·414 and 1·732

Ans.

27. 50 people work in a cooperative society. All of them use their own conveyance for travelling. 20 people use their scooters, 12 go by their cars, 16 go by public transport and 2 use bicycle. Find H.C.F. of 20, 16, 12 and 2.

Sol. $20 = 2^2 \times 5$

$12 = 2^2 \times 3$

$16 = 2^4$

$2 = 2^1$

∴ H.C.F. = 2 **Ans.**

28. Two positive integers *a* and *b* can be written as $a = x^3y^2$ and $b = xy^3$. *x*, *y* are prime numbers. Find L.C.M. (*a*, *b*).

[Board Question]

Sol. Given, $a = x^3y^2$ and $b = xy^3$

∴ L.C.M. (*a*, *b*) = Product of the greatestpower of each prime factors

$= x^3y^3$ **Ans.**

Short Answer Type Questions

29. Find the H.C.F. of 1260 and 7344 using Euclid's algorithm.

[Board Question]

Sol. Two numbers are 1260 and 7344

Since 7344 > 1260, we apply the Euclid division lemma to 7344 and 1260, we get

7344 = 1260 × 5 + 1044

Also, 1260 = 1044 × 1 + 216

1044 = 216 × 4 + 180

216 = 180 × 1 + 36

180 = 36 × 5 + 0

Now, remainder is 0, hence our procedure stops here.

∴ H.C.F. of 7344 and 1260 is 36. **Ans.**

30. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

[Board Question]

Sol. To find the length of the longest rod that can measure the dimensions of the room exactly, we have to find H.C.F.

Length, L = 8 m 50 cm = 850 cm = $2^1 \times 5^2 \times 17$

Breadth, B = 6 m 25 cm = 625 cm = 5^4

Height, H = 4 m 75 cm = 475 cm = $5^2 \times 19$

∴ H.C.F. of L, B and H = 5^2 = 25

∴ Length of the longest rod = 25 cm **Ans.**

31. A class of 20 boys and 15 girls is divided into *n* groups so that each group has *x* boys and *y* girls. Find *x*, *y* and *n*.

Sol. H.C.F. of 20 and 15 = 5

Therefore, the 5 students are in each group.

So, $n = \frac{20+15}{5} = \frac{35}{5} = 7$

$x = \frac{20}{5} = 4$

and $y = \frac{15}{5} = 3$

Hence, $x = 4$, $y = 3$ and $n = 7$. **Ans.**

32. The L.C.M. of two numbers is 14 times their H.C.F. The sum of L.C.M. and H.C.F. is 750. If one number is 250, then find the other number.

Sol. Let H.C.F. be 'H'

then L.C.M. = 14 H

Sum of L.C.M. and H.C.F. is 750

$\therefore$ 14H + H = 750

$\Rightarrow$ 15H = 750

$\Rightarrow H = \frac{750}{15}$

$\Rightarrow$ H = 50

$\therefore$ L.C.M. = 14H

= 14 × 50 = 700

We know,

Product of L.C.M. and H.C.F. = Product of two numbers.

Let other number be y

$700 \times 50 = 250 \times y$

$\Rightarrow y = \frac{700 \times 50}{250}$

$\Rightarrow y = 140$.

Hence, the other number is 140. **Ans.**

33. Write the smallest number which is divisible by both 306 and 657.

[Board Question]

Sol. Smallest number which is divisible by 306 and 657 is L.C.M. (657, 306) Now, 657 = 3 × 3 × 73

306 = 3 × 3 × 2 × 17

L.C.M. = 3 × 3 × 73 × 2 × 17

= 22338. **Ans.**

34. In a seminar on the topic 'liberty and equality' the numbers of participants from Hindi, Social Science and English department are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

Sol. The number of rooms will be minimum if each room accommodates maximum number of participants. Since in each room the same number of participants are to be seated and all of them must be of the same subject.

Thus, number of participants in each room

= H.C.F. of 60, 84 and 108

... $60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

and $108 = 2^2 \times 3^3$

$\therefore$ H.C.F. $= 2^2 \times 3 = 4 \times 3$

= 12

Thus, in each room 12 participants can be seated

$\therefore$ Number of rooms required

$= \frac{60+84+108}{12} = \frac{252}{12}$

= 21. **Ans.**

35. Three sets of English, Hindi and Sociology books dealing with cleanliness have to stacked in such a way that all the books are stored topic-wise and height of each stack is the same. The number of English books is 96, number of Hindi books is 240 and the number of Sociology books is 336.

Assuming that the books are of same thickness, determine the number of stacks of English, Hindi and Sociology books.

Sol. In order to arrange the books as required, we have to find the largest number that divides 96, 240 and 336 exactly.

Now, largest number = H.C.F. of 96, 240 and 336

... $96 = 2^5 \times 3$

$240 = 2^4 \times 3 \times 5$

and $336 = 2^4 \times 3 \times 7$

$\therefore$ H.C.F. $= 2^4 \times 3$

$= 16 \times 3 = 48$

So, there must be 48 books in each stack.

$\therefore$ No. of stacks of English books = $\frac{96}{48} = 2$

No. of stacks of Hindi books = $\frac{240}{48} = 5$

No. of stacks of Sociology books = $\frac{336}{48} = 7$ **Ans.**

36. Three alarm clocks ring at intervals of 4, 12 and 20 minutes respectively. If they start ringing together, after how much time will they next ring together?

Sol. To find the time when the clocks will next ring together, we have to find L.C.M. of 4, 12 and 20 minutes.

$4 = 2^2$

$12 = 2^2 \times 3$

$20 = 2^2 \times 5$

2	12	2	20
2	6	2	10
3	3	5	5
	1		1

$\therefore$ L.C.M. of 4, 12 and 20

$= 2^2 \times 3 \times 5 = 60$ minutes.

So, the clocks will ring together again after 60 minutes or 1 hour. **Ans.**

37. Find L.C.M. and H.C.F. of 3930 and 1800 by prime factorization method.

[Board Question]

Sol. By prime factorization method,

2	3930
3	1965
5	655
131	131
	1

2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$3930 = 2 \times 3 \times 5 \times 131$

$1800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$\therefore$ H.C.F. $= 2 \times 3 \times 5$

$= 30$

and L.C.M. $= 2 \times 3 \times 5 \times 131 \times 2 \times 2 \times 3 \times 5$

$= 235800$. **Ans.**

Long Answer Type Questions

38. Prove that $\sqrt{2}$ is an irrational number.

[Board Question & NCERT]

Sol. To prove that $\sqrt{2}$ is an irrational number, let us assume its opposite *i.e.*, $\sqrt{2}$ is a rational number. Also let it be $\frac{a}{b}$ in its simplest form where *a* and *b* are co-prime numbers having an H.C.F. of 1. Also let $b \neq 0$.

$\therefore \sqrt{2} = \frac{a}{b}$

$\Rightarrow (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow 2 = \frac{a^2}{b^2}$

$\Rightarrow 2b^2 = a^2$

Thus $2 \mid a^2$ [... $2 \mid 2b^2$ and $2b^2 = a^2$]

$\Rightarrow 2 \mid a$ …(i)

[From theorem, if p is a prime number and 'a' is

a positive integer then, if p divides a^2 then

p divides a]

Now let $a = 2c$ for some integer c.

Replacing the value of a in the equation $2b^2 = a^2$ we have

$2b^2 = (2c)^2$

$\Rightarrow 2b^2 = 4c^2$

$\Rightarrow b^2 = 2c^2$

$\Rightarrow 2 \mid b^2$ [... $2 \mid 2c^2$ and $2c^2 = b^2$]

$\Rightarrow 2 \mid b$ ...(ii)

[From theorem, if p is a prime number and 'a' is a positive integer then, if p divides a^2 then p divides a]

Hence, from (i) and (ii), we find that 2 is a common factor of both a and b. However, this contradicts the fact that a and b have only 1 as their common factor. Such a contradiction arises by considering $\sqrt{2}$ as a rational number. Hence, it is proved that $\sqrt{2}$ is an irrational number.

Hence Proved.

39. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

[Board Question]

Sol. Let $\frac{2+\sqrt{3}}{5}$ is a rational number.

$\therefore$ $\frac{2+\sqrt{3}}{5} = \frac{a}{b}$, where *a* and *b* are co-prime numbers.

or $2+\sqrt{3} = \frac{5a}{b}$

$\Rightarrow \sqrt{3} = \frac{5a}{b} - \frac{2}{1}$

$\Rightarrow \sqrt{3} = \frac{5a-2b}{b}$

In R.H.S., *a*, *b*, 2 and 5 are integers.

$\therefore$ R.H.S. is a rational number but given that $\sqrt{3}$ is an irrational.

So, it is a contradiction.

Hence, $\frac{2+\sqrt{3}}{5}$ is an irrational number. **Ans.**

40. Prove that $\sqrt{5}$ is an irrational number. Hence show that $3+2\sqrt{5}$ is also an irrational number.

[Board Question]

Sol. Let $\sqrt{5}$ be a rational number.

So, $\sqrt{5} = \frac{p}{q}$

On squaring both sides,

$5 = \frac{p^2}{q^2}$

$\Rightarrow q^2 = \frac{p^2}{5}$

$\Rightarrow$ 5 is a factor of p^2

$\Rightarrow$ 5 is a factor of *p*.

Now, again let $p = 5c$.

So, $\sqrt{5} = \frac{5c}{q}$

On squaring both sides,

$5 = \frac{25c^2}{q^2}$

⇒ $q^2 = 5c^2$

⇒ $c^2 = \frac{q^2}{5}$

⇒ 5 is factor of q^2

⇒ 5 is a factor of q.

Here 5 is a common factor of p and q which contradicts the fact that p and q are co-prime.

Hence our assumption is wrong, $\sqrt{5}$ is an irrational number.

Now, we have to show that 3 + $2\sqrt{5}$ is an irrational number. So let us assume that 3 + $2\sqrt{5}$ is a rational number.

⇒ $3 + 2\sqrt{5} = \frac{p}{q}$

⇒ $2\sqrt{5} = \frac{p}{q} - 3$

⇒ $2\sqrt{5} = \frac{p-3q}{q}$

⇒ $\sqrt{5} = \frac{p-3q}{2q}$

$\frac{p-3q}{2q}$ is in the rational form of $\frac{p}{q}$, so $\sqrt{5}$ is a rational number but we have already proved that $\sqrt{5}$ is an irrational number so contradiction arises because we supposed wrong that 3 + $2\sqrt{5}$ is a rational number. So we can say that 3 + $2\sqrt{5}$ is an irrational number. **Hence Proved.**

41. Prove that $2+5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Sol. Let $2+5\sqrt{3}$ = r, where, r is rational.

$(2+5\sqrt{3})^2 = r^2$

⇒ $4 + 75 + 20\sqrt{3} = r^2$

⇒ $79 + 20\sqrt{3} = r^2$

⇒ $20\sqrt{3} = r^2 - 79$

$\Rightarrow \sqrt{3} = \frac{r^2 - 79}{20}$

Now, $\frac{r^2-79}{20}$ is a rational number. So, $\sqrt{3}$ must also be a rational number. But $\sqrt{3}$ is an irrational number (Given).

So, our assumption is wrong.

$\therefore 2+5\sqrt{3}$ is an irrational number.

Hence Proved.

42. Find H.C.F. of numbers 134791, 6341 and 6339 by Euclid's division algorithm.

[Board Question]

Sol. First we find H.C.F. of 6339 and 6341 by Euclid's division method

```
6339 ) 6341 ( 1
       6339
       2 ) 6339 ( 3169
           6
           3
           2
           13
           12
            19
            18
             1 ) 2 ( 2
                 2
                 0
```

$\Rightarrow 6341 = 6339 \times 1 + 2$

Also, $6339 = 2 \times 3169 + 1$

$2 = 1 \times 2 + 0$

$\therefore$ H.C.F. of 6341 and 6339 is 1.

Now, we find the H.C.F. of 134791 and 1

$134791 = 1 \times 134791 + 0$

$\therefore$ H.C.F. of 134791 and 1 is 1.

Hence, H.C.F. of given three numbers is 1. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

43. Assertion: $\frac{123}{6250}$ is a terminating decimal.

Reason: The rational number $\frac{p}{q}$ is a terminating decimal, if $q = (2^m \times 5^n)$ for some whole numbers m and n.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

$\frac{123}{6250}$ **=** $\frac{123}{5^5 \times 2}$ is a terminating decimal

Given that, $p = 123$, $q = (2^m \times 5^n)$, where m and n are whole numbers. So, both the statements are proved to be correct and reason is the correct explanation.

44. Assertion: – 1, 0, 2, $\frac{-4}{3}$ all are example of rational numbers.

Reason: All integers and fractions are rational numbers.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

Both assertion and reason are true. It is correct explanation of that because – 1 is also a integer and rational number and – 4/3 is also fraction and rational number.

Case Based Questions

45. The department of Computer Science and Technology is conducting an International seminar. In the seminar, the number of participants in Mathematics, Science and computer Science are 60, 84 and 108 respectively. The coordinator has made the arrangement such that in each room, the same number of participants are to be seated and all of them being is of the same subject. Also they allotted the separate room for all the official other than participants.

(i) The total number of participants is:

(a) 60

(b) 84

(c) 108

(d) 252

Ans. (d) 252

Explanation :

Total number of participants

= 60 + 84 + 108

= 252

(ii) The L.C.M. of 60, 84 and 108 is:

(a) 12

(b) 504

(c) 544320

(d) 3780

Ans. (d) 3780

Explanation :

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

L.C.M. (60, 84, 108) = $2^2 \times 3^3 \times 5 \times 7$

= 3780

(iii) The H.C.F. of 60, 84 and 108 is:
(a) 12
(b) 60
(c) 84
(d) 108
Ans. (a) 12

Explanation :

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

H.C.F. (60, 84, 108) = $2^2 \times 3$

= 12

(iv) The mimimum number of rooms required, if in each room, the same number of participants are to be seated and all of them being in the same subject is:
(a) 12
(b) 20
(c) 21
(d) None of these
Ans. (c) 21

Explanation :

Minimum number of rooms required for all the participants = 252/12 = 21.

(v) Based on the above (iv) conditions, the minimum number of room required for all the participants and officials is:

(a) 12

(b) 21

(c) 22

(d) None of these

Ans. (c) 22

Explanation :

Minimum number of rooms required for all

= 21 + 1 = 22.

46. Vedika wants to organize her birthday party. She was happy on her birthday. She is very health conscious. So decided to serve fruits only to the guests. She has 36 apples, 60 bananas at home and decided to serve them. She want to distribute the fruits among guests. She does not want to discriminate among guests so she decided to distribute the fruits equally among all.

(i) How many maximum guests Vedika can invite?

(a) 12

(b) 120

(c) 6

(d) 180

Ans. (a) 12

Explanation :

H.C.F. (36, 60) = 12.

Thus, fruits will be equally distributes among 12 guests.

(ii) How many apples and bananas will each guests get ?

(a) 3 apples 5 bananas

(b) 5 apples 3 bananas

(c) 2 apples 4 bananas

(d) 4 apples 2 bananas

Ans. (a) 3 apples 5 bananas

Explanation :

Each guest will get number of apples = $\frac{36}{12}$ = 3

Each guest will get no. of bananas = $\frac{60}{12}$ = 5

Out of 36 apples and 60 bananas each guest will get 3 apples, 5 bananas.

(iii) Vedika decide to add 42 mangoes. In this case how many maximum guests Vedika invite?

(a) 12

(b) 120

(c) 6

(d) 180

Ans. (c) 6

Explanation :

H.C.F. (36, 42, 60) = 6

So, fruits will be equally distributed among 6 guests.

(iv) If Vedika decide to add 3 more mangoes and instead 6 apples, in this case how many maximum guests Vedika can invite?

(a) 12

(b) 30

(c) 15

(d) 24

Ans. (c) 15

Explanation :

Now Vedika has 30 apples, 60 bananas and 45 mangoes. H.C.F. (30, 45, 60) = 15.

(v) How many total fruits will each guest get from case (iii)?

(a) 36

(b) 60

(c) 17

(d) 23

Ans. (d) 23

Explanation :

Total apples = $\frac{36}{6}$ = 6, bananas = $\frac{60}{6}$ = 10 and mangoes = 7. So, total no. of fruits will each guest get = 6 + 10 + 7 = 23 fruits.

Passage Based Questions

47. Suresh planned a rennovation of his house. He want to renovate ceiling of his room by putting square shape tiles on it. Ceiling of the room is 8 m 25 cm long and 6m 75 cm broad.

(i) Find the dimensions of each tiles.

(ii) Find the number of tiles required for the project.

Sol. (i) Dimensions of tiles can be found by finding the H.C.F. of 8 m 25 cm and 6 m 75 cm

8 m 25 cm = 825 cm.

6 m 75 cm = 675 cm.

8 m 25 cm = 825 cm.
6 m 75 cm = 675 cm.

```
675 ) 825 ( 1
     − 675
       150 ) 675 ( 4
            − 600
               75 ) 150 ( 2
                   − 150
                     ×
```

∴ H.C.F. (825, 675) = 75.

So, side of each square tile is 75 cm.

(ii) Required number of titles

$= \frac{\text{Area of ceiling}}{\text{Area of each tile}}$

$= \frac{825 \times 675}{75 \times 75} = 99.$

48. Sandhya on the very first day of her job in a bank, noticed that there are six bells which keep on tolling at regular intervals. She noticed that toll of their intervals are 2, 4, 6, 8, 10, 12 minutes respectively. If all the six bells commence tolling together, at 10 a.m., then answer the following questions:

Based on the given information, answer the following questions:

(i) At what time will they again toll together?

(ii) How many times these bells will toll together during the working hours of Sandhya's job, if Sandhya works for 8 hours in a day?

Sol. (i) In order to calculate the time interval at which bells toll together, we need to find out the L.C.M. of 2, 4, 6, 8, 10, 12

By prime factorisation, we get

$2 = 2$, $4 = 2^2$

$6 = 2 \times 3$, $8 = 2^3$

$10 = 2 \times 5$, $12 = 2^2 \times 3$

∴ L.C.M. (2, 4, 6, 8, 10, 12) = $2^3 \times 3 \times 5$

= 120 min.

So, after every 120 min. or 2 hrs. bell will toll together.

The time, these bells will toll together again at

= 10 a.m. + 2 hours

= 12 noon

(ii) Number of times bells will toll together

$= \frac{\text{Working hours}}{\text{Time of tolling simultaneously}}$

$= \frac{8}{2} = 4$

Self-Assessment

49. State the Fundamental Theorem of Arithmetic.

Ans. The Fundamental Theorem of Arithmetic states that every positive integer is a prime number or can be expressed as a product of prime numbers, which can be arranged in different orders. (Every composite number can be expressed as a product of primes and this factorisation is unique except for the order in which the prime factors occur).

50. Find the H.C.F. of $x^2 + 3x - 10$ and $x^3 - 8x$.

Ans. $(x - 2)$.

51. Three iron rods of length 24 m, 94 m, 36 m have to be cut into poles of the same length. What is the greatest length possible?

Ans. 2 m.

52. The traffic lights at three different crossing change after 36 s, 60 s and 96 s. If at 9 pm they changed together then when will they change together again?

Ans. They will change together again after 24 minutes i.e., at 9 : 24

pm.

53. A dealer has 60 l of blue paint, 84 l of violet paint and 132 l of white paint. What would be the capacity of the cans that he would to store all the three types of paint in equal quantities ? How many such cans will there be ?

Ans. The capacity of the cans should be 12 l each.

Number of such cans = 23.

54. In a class, there are 16 boys and 18 girls. Find the number of pens required to distribute them equally among the boys and the girls.

Ans. 144.

55. Prove the irrationality of the following numbers:

(i) $\sqrt{7}+\sqrt{11}$

(ii) $\sqrt{6}+2\sqrt{3}$

(iii) $\sqrt{10}-\sqrt{5}$

56. Find the (i) H.C.F. of $2(x^3 – 4x^2 + 4x)$, $6(x^2 + x – 6)$ and $8(x^3 – 8)$ and (ii) L.C.M. of $3a^2 – 5ab – 12b^2$, $a^5 – 27a^2b^3$ and $9a^2 + 24ab + 16b^2$.

Ans. (i) $2(x – 2)$.

(ii) $a^2(a – 3b)(3a + 4b)^2(a^2 + 9b^2 + 3ab)$

57. Find the (i) H.C.F. of $8(x^2 – 4)$, $12(x^3 + 8)$ and $36(x^2 – 3x – 10)$ and (ii) L.C.M. of $(a^2 + 2a)^2$, $2a^3 – 2a + 3a^2$ and $2a^4 – 3a^3 – 14a^2$.

Ans. (i) $4(x + 2)$.

(ii) $a^2(a + 2)^2(2a – 1)(2a – 7)$.

58. Find the H.C.F. of (i) $x^4 – 2x^2 + 1$ and $x^4 – 2x^3 + 2x – 1$ and (ii) $4(x^2 – 7x + 12)$, $8(x^2 – 9)$ and $12(x^2 – 6x + 9)$.

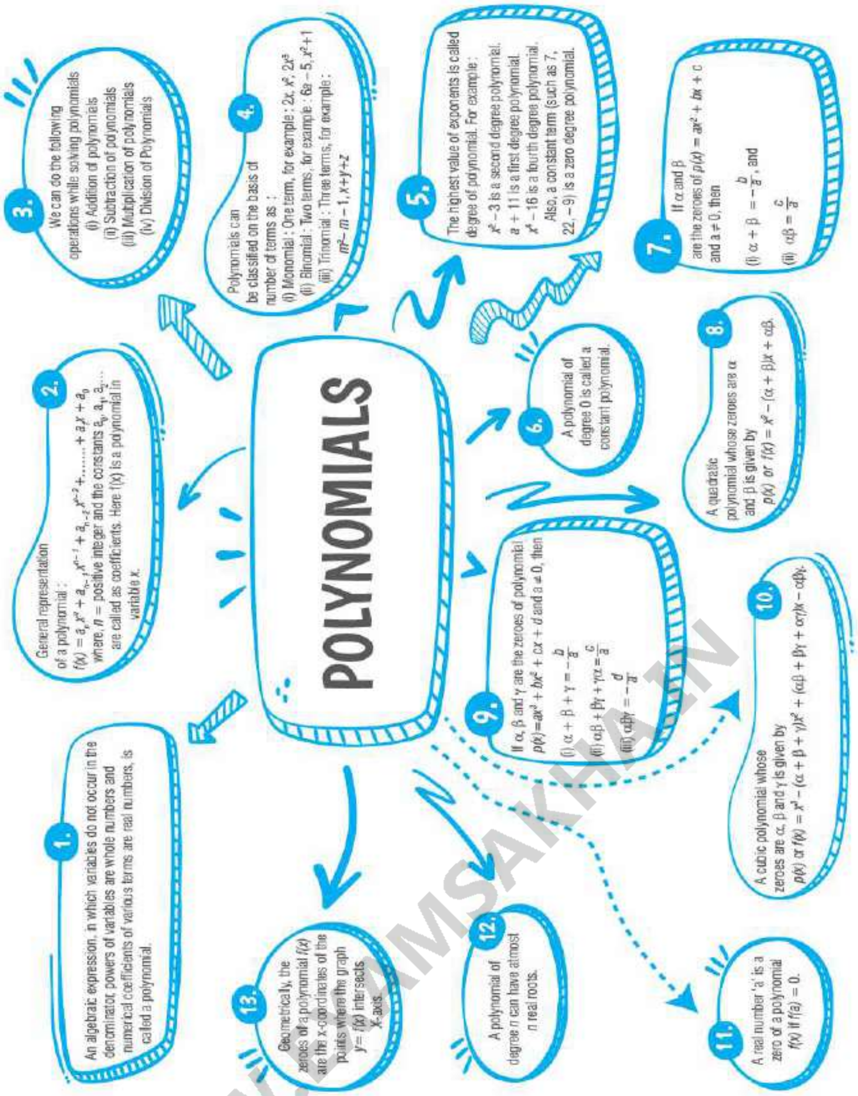
Ans. (i) $(x - 1)^2 (x + 1)$.

(ii) $4(x - 3)$.

Polynomials

Chapter
2

Basic Concepts



Multiple Choice Questions

1. If $x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 3 \times 2}}{2 \times 3}$, then the required polynomial is:

(a) $3x^2 - 8x + 2 = 0$

(b) $2x^2 - 8x - 2 = 0$

(c) $3x^2 + 8x - 2 = 0$

(d) $3x^2 + 8x + 2 = 0$

Ans. (a) $3x^2 - 8x + 2 = 0$

Explanation :

If $x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 3 \times 2}}{2 \times 3}$ then on comparing with $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we have $b = -8$, $a = 3$ and $c = 2$. So, the required polynomial is $3x^2 - 8x + 2 = 0$.

2. Find the coefficient of x^0 in $x^2 + 3x + 2 = 0$.

(a) 3

(b) – 3

(c) 2

(d) – 1

Ans. (c) 2

Explanation :

As $x^0 = 1$, so its coefficient is the constant term.

3. In which condition will the polynomial $ax^2 + bx + c = 0$ be a quadratic equation?

(a) $a \neq 0$

(b) $a = b$

(c) $a = c$

(d) $a = 0$

Ans. (a) $a \neq 0$

Explanation :

For $a \neq 0$, the given equation be a quadratic equation.

4. For which value of p will the equation

$$(p^2 - 1)x^2 + px + q = 0$$

not be a quadratic equation?

(a) $p = 1$

(b) $p = -1$

(c) Both (a) and (b)

(d) $p = 0$

Ans. (c) Both (a) and (b)

Explanation :

If coefficient of x^2 is 0, then

$p^2 - 1 = 0$

$p^2 = 1$

$p = \pm 1$.

5. Write the zero of the polynomial $f(x) = x^2 - x - 6$.

[Board Question]

(a) $-3, 2$

(b) $-3, -2$

(c) $3, 2$

(d) $3, -2$

Ans. (d) $3, -2$

Explanation :

$f(x) = x^2 - x - 6$

$= x^2 - 3x + 2x - 6$

$= x(x - 3) + 2(x - 3)$

$= (x + 2)(x - 3)$

To find zero put $f(x) = 0$

$x = -2, 3$.

6. For what value of k is – 4 a zero of the polynomial $f(x) = x^2 - x - (2k + 2)$?

(a) 6

(b) – 6

(c) 9

(d) – 9

Ans. (c) 9

Explanation :

$f(x) = x^2 - x - (2k + 2)$

Substituting x for – 4, we have

$f(-4) = (-4)^2 - (-4) - (2k + 2)$

$= 16 + 4 - 2k - 2$

$= 18 - 2k = 0$

Thus, $18 - 2k = 0$

or $2k = 18$ or $k = 9$.

7. If a and b are the zeroes of a polynomial such that a + b = – 6 and ab = – 4, then write the polynomial.

(a) $x^2 - 6x - 4 = 0$

(b) $x^2 + 6x - 4 = 0$

(c) $x^2 + 6x + 4 = 0$

(d) $x^2 - 6x + 4 = 0$

Ans. (b) $x^2 + 6x - 4 = 0$

Explanation :

Given a + b = – 6

and ab = – 4

∵ The quadratic polynomial whose zeroes are a and b is given by

$f(x) = x^2 - (a + b)x + ab$.

The polynomial is $f(x) = x^2 + 6x - 4 = 0$.

8. If a and b are the zeroes of the polynomial $f(y) = 2y^2 + 7y + 5$, write the values of a + b + ab.

(a) 1

(b) 0

(c) – 2

(d) – 1

Ans. (d) – 1

Explanation :

Given, $f(y) = 2y^2 + 7y + 5$

Here $a + b = -\frac{7}{2}$,

$ab = \frac{5}{2}$

$a + b + ab = -\frac{7}{2} + \frac{5}{2} = -1$.

9. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then *ab* equals:

(a) 3

(b) $-\frac{7}{2}$

(c) 6 (d) – 3

Ans. (a) 3

Explanation :

$f(y) = ay^2 + ay + 3 = 0$

Thus, $f(1) = a(1)^2 + a(1) + 3 = 0$

$\Rightarrow 2a + 3 = 0$

$\Rightarrow a = -\frac{3}{2}$

$f(y) = y^2 + y + b = 0$

$\Rightarrow f(1) = (1)^2 + 1 + b = 0$

$\Rightarrow b = -2$

Hence, $ab = \left(-\frac{3}{2}\right) \times (-2) = 3$.

10. The roots of the equation $x^2 + x - p(p + 1) = 0$ where p is a constant are:

(a) $p, p + 1$

(b) $-p, p + 1$

(c) $p, -(p + 1)$

(d) $-p, -(p + 1)$

Ans. (c) $p, -(p + 1)$

Explanation :

$x^2 + x - p(p + 1) = 0$

Let the roots be a and b.

Thus, a + b = – 1 and ab = $-p(p + 1)$

$\Rightarrow$ a = – (1 + b) and – (1 + b)b = $-(p + 1)p$

Hence b = p

and a = $-(p + 1)$.

11. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are:

[Board Question]

(a) $m, m + 3$

(b) $-m, m + 3$

(c) $m, -(m + 3)$

(d) $-m, -(m + 3)$

Ans. (b) $-m, m + 3$

Explanation :

$x^2 - 3x - m(m + 3)$

$= x^2 - (m + 3 - m)x - m(m + 3)$

$= x^2 - (m + 3)x + mx - m(m + 3)$

$= x\{x - (m + 3)\} + m\{x - (m + 3)\}$

$= (x + m)\{x - (m + 3)\}$

Its zeroes are $-m, (m + 3)$.

12. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value k is:

[Board Question]

(a) 10

(b) – 10

(c) – 7

(d) – 2

Ans. (b) – 10

Explanation :

Let $p(x) = x^2 + 3x + k$

$x = 2$ is a zero of $p(x)$

$p(2) = 0$

$\Rightarrow (2)^2 + 3(2) + k = 0$

$\Rightarrow 4 + 6 + k = 0 \Rightarrow 10 + k = 0$

$\Rightarrow k = -10.$

13. The quadratic polynomial, the sum of whose zeroes is – 5 and their product is 6, is:

[Board Question]

(a) $x^2 + 5x + 6$

(b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$

(d) $-x^2 + 5x + 6$

Ans. (a) $x^2 + 5x + 6$

Explanation :

Let the polynomial be

$p(x) = x^2 -$ (Sum of zeroes)x + Product of zeroes

Sum of zeroes = – 5

Product of zeroes = 6

Required polynomial is

$p(x) = x^2 - (-5)x + (6)$

$= x^2 + 5x + 6.$

14. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0,$ then the value of k is:

(a) 2

(b) – 2

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

Ans. (a) 2

Explanation :

If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then

It satisfy it *i.e.*,

⇒ $\left(\frac{1}{2}\right)^2 + \frac{1}{2}k - \frac{5}{4}$ = 0

⇒ $\frac{1}{4} + \frac{1}{2}k - \frac{5}{4}$ = 0

⇒ $\frac{-4}{4} + \frac{1}{2}k$ = 0

⇒ – 1 = $-\frac{1}{2}k$

⇒ k = 2.

15. The number of polynomials having zeroes as – 2 and 5 is:

(a) 1

(b) 2

(c) 3

(d) more than 3

Ans. (d) more than 3

Explanation :

The required quadratic polynomial is

$[x - (-2)]\ (x - 5) = (x + 2)\ (x - 5)$

$= x^2 + 2x - 5x - 10$

$= x^2 - 3x - 10$

But we can mutiply any constant to this polynomial and the resultant polynomial would have the same zeroes.

Number of polynomials having zeroes as – 2 and 5 is more than 3.

16. Which of the following is a polynomial?

(a) $x^2 - 6\sqrt{x} + 2$

(b) $\frac{5}{x^2} - 3x + 1$

(c) $\sqrt{x} + \frac{1}{\sqrt{x}}$

(d) None of these

Ans. (d) None of these

Explanation :

Since, the power of x in (a) is $\frac{1}{2}$ which is not a positive integer.

It is not a polynomial similarly the powers of x in (b) and (c) are not positive integers.

They are not polynomials.

17. A quadratic polynomial whose zeroes are – 3 and 4, is:

(a) $x^2 - x + 12$

(b) $x^2 + x + 12$

(c) $x^2 - x - 12$

(d) $2x^2 + 2x - 9$

Ans. (c) $x^2 - x - 12$

Explanation :

A quadratic polynomial

$= x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$

$= x^2 - (-3 + 4)x + (-3)(4)$

$= x^2 - x - 12.$

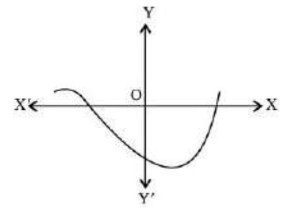
18. Number of zeroes of the polynomial $f(x)$, shown in the figure, are:

(a) 3

(b) 2

(c) 1

(d) 0



Ans. (b) 2

Explanation :

Number of zeroes of $f(x)$ = Number of times graph of $f(x)$ intersecting x-axis = 2.

19. If α and β are the zeroes of the polynomial $5x^2 - 7x + 2$, then the sum of their reciprocals is ....

(a) $\frac{7}{2}$

(b) $-\frac{7}{2}$

(c) 0

(d) $-\frac{2}{7}$

Ans. (a) $\frac{7}{2}$

Explanation :

a + b = $\frac{-(-7)}{5} = \frac{7}{5}$

ab = $\frac{2}{5}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7/5}{2/5} = \frac{7}{2}$.

20. The graph of a polynomial of degree 3 intersects X-axis at most .... points.

(a) 4

(b) 3

(c) 2

(d) none of these

Ans. (b) 3

Explanation :

The number of zeroes of polynomial with degree 3 is at most three. Graph intersects *x*-axis at most 3 points.

21. The sum and product of the zeroes of the quadratic polynomial $4x^2 - 27x + 3k^2$ are equal. Then, the value of k is ....

(a) + 3

(b) ± 3

(c) – 3

(d) 0

Ans. (b) ± 3

Explanation :

a + b = ab(Given)

$-\left(\frac{-27}{4}\right) = \frac{3k^2}{4}$

$\Rightarrow 27 = 3k^2$

$\Rightarrow k^2 = 9$

$\Rightarrow k = \pm 3.$

22. If the sum of roots of the polynomial $4x^2 - 2x + (k - 4)$ is half of their product, then the value of k is ....

(a) 8

(b) 6

(c) – 8

(d) 5

Ans. (b) 6

Explanation :

According to the question,

a + b = $\frac{\alpha\beta}{2}$ ⇒ $\frac{2}{4}$ = $\frac{(k-4)}{4}$

⇒ $2 = k - 4$

⇒ $k = 6$.

23. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is – 3, then the value of k is:

[NCERT Exemplar]

(a) $\frac{4}{3}$

(b) $\frac{-4}{3}$

(c) $\frac{2}{3}$

(d) $\frac{-2}{3}$

Ans. (a) $\frac{4}{3}$

Explanation :

Given that, one of the zeroes of the quadratic polynomial say $p(x) = (k - 1)x^2 + kx + 1$ is – 3, then

$p(-3) = 0$

⇒ $(k - 1)(-3)^2 + k(-3) + 1 = 0$

⇒ $9(k - 1) - 3k + 1 = 0$

⇒ $9k - 9 - 3k + 1 = 0$

⇒ $6k - 8 = 0$

$k = 4/3$.

24. A quadratic polynomial, whose zeroes are – 3 and 4, is:

[NCERT Exemplar]

(a) $x^2 - x + 12$

(b) $x^2 + x + 12$

(c) $\frac{x^2}{2} - \frac{x}{2} - 6$

(d) $2x^2 + 2x - 24$

Ans. (c) $\frac{x^2}{2} - \frac{x}{2} - 6$

Explanation :

Let $ax^2 + bx + c$ be a required polynomial whose zeroes are – 3 and 4.

Then, sum of zeroes = – 3 + 4 = 1

$\left[\because \text{sum of zeroes} = \frac{-b}{a}\right]$

$\Rightarrow \frac{-b}{a} = \frac{1}{1}$

$\Rightarrow \frac{-b}{a} = -\frac{(-1)}{1}$ …(i)

and product of zeroes

= – 3 × 4 = – 12

$\left[\because \text{product of zeroes} = \frac{c}{a}\right]$

$\Rightarrow \frac{c}{a} = \frac{-12}{1}$ …(ii)

From equations (i) and (ii),

$a = 1, b = -1$

and $c = -12$

Required polynomial

$= ax^2 + bx + c$

$= 1 \cdot x^2 - 1 \cdot x - 12$

$= x^2 - x - 12$

$= \frac{x^2}{2} - \frac{x}{2} - 6$

We know that, if we multiply/divide any polynomial by any constant, then the zeroes of polynomial do not change.

Alternate Method:

Let the zeroes of a quadratic polynoimal are a = – 3 and b = 4.

Then, sum of zeroes = a + b

= – 3 + 4

= 1

and product of zeroes = ab

= (– 3) (4)

= – 12

Polynomial = $x^2 - (a + b)x + ab$

= $x^2 - x - 12$

= $\frac{x^2}{2} - \frac{x}{2} - 6.$

25. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then:

[NCERT Exemplar]

(a) $a = -7$, $b = -1$

(b) $a = 5$, $b = -1$

(c) $a = 2$, $b = -6$

(d) $a = 0$, $b = -6$

Ans. (d) $a = 0$, $b = -6$

Explanation :

Let $p(x) = x^2 + (a + 1)x + b$

Given that, 2 and – 3 are the zeros of the quadratic polynomial $p(x)$.

$p(2) = 0$ and $p(-3) = 0$

$\Rightarrow 2^2 + (a + 1)(2) + b = 0$

$\Rightarrow 4 + 2a + 2 + b = 0$

$\Rightarrow 2a + b = -6$…(i)

and $(-3)^2 + (a + 1)(-3) + b = 0$

$\Rightarrow 9 - 3a - 3 + b = 0$

$\Rightarrow 3a - b = 6$…(ii)

On adding equations (i) and (ii), we get

$5a = 0$

$\Rightarrow a = 0$

Put the value of a in equation (i), we get

$2 \times 0 + b = -6$

$\Rightarrow b = -6$

required values are $a = 0$ and $b = -6$.

Very Short Answer Type Questions

26. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $-\sqrt{2}$ respectively.

Sol. Quadratic polynomial is given as,

$= x^2 -$ (Sum of zeroes) x + (Product of zeroes)

$= x^2 - (0)\,x + (-\sqrt{2})$

$= x^2 - \sqrt{2}$ **Ans.**

27. If $(x + a)$ is a factor of $f(x) = (2x^2 + 2ax + 5x + 10)$, find a.

Sol. We have,

$f(x) = 2x^2 + 2ax + 5x + 10$

Since $(x + a)$ is a factor of $f(x)$,

Then put $(x + a) = 0$

$\Rightarrow x = -a$

Put $x = -a$ in $f(x)$ and put $f(-a) = 0$

$\Rightarrow 2(-a)2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow 2a2 - 2a2 - 5a + 10 = 0$

$\Rightarrow 5a = 10$

$\Rightarrow a = 2$ **Ans.**

28. For what value of k is 3 a zero of the polynomial $f(x) = 2x^2 + x + k$?

Sol. We have, $f(x) = 2x^2 + x + k$

Substituting 3 for x, we get

$f(3) = 2(3)^2 + (3) + k = 0$

$\Rightarrow 18 + 3 + k = 0$

$\Rightarrow k = -21$ **Ans.**

29. For what value of k is -2 a zero of the polynomial $f(x) = 3x^2 + 4x + 2k$?

Sol. Given polynomial is,

$f(x) = 3x^2 + 4x + 2k$

... -2 is a zero of $f(x)$

$\because f(-2) = 0$

$\Rightarrow 3(-2)2 + 4(-2) + 2k = 0$

$\Rightarrow 12 - 8 + 2k = 0$

$\Rightarrow 2k = -4$

$\Rightarrow k = -2$ **Ans.**

30. If $x + k$ is the G.C.D. of $x^2 - 2x - 15$ and $x^3 + 27$, then find the value of k.

[Board Question]

Sol. We have

$x^3 + 27 = x^3 + 3^3$

$= (x + 3)^3 - 9x(x + 3)$

$= (x + 3)\,[(x + 3)^2 - 9x]$

$= (x + 3)\,(x^2 - 3x + 9)$

$x^2 - 2x - 15 = x^2 - 5x + 3x - 15$

$= x(x - 5) + 3(x - 5)$

$= (x + 3)\,(x - 5)$

$(x + 3)$ is the G.C.D. of both the equations.

So $x + k = x + 3$

or $k = 3$. **Ans.**

31. Show that $x = -3$ is a solution of $x2 + 6x + 9 = 0$.

Sol. Let,

$f(x) = x2 + 6x + 9 = 0$

$= x2 + 3x + 3x + 9 = 0$

$= x(x + 3) + 3(x + 3) = 0$

$\Rightarrow (x + 3)2 = 0$

$\Rightarrow x + 3 = 0$

$\Rightarrow x = -3$. **Hence Proved.**

32. If a, b **are the zeroes of a polynomial such that** a **+** b **= 6 and** ab **= 4, then write down the polynomial.**

Sol. Let the required polynomial be $f(x)$.

So $f(x) = x2 - (a + b)x + ab = 0$

or $x2 - 6x + 4 = 0$

(substituting the values of a + b and ab)

Thus, $x2 - 6x + 4 = 0$ is the required polynomial. **Ans.**

33. Sum of zeroes of the polynomial $2x2 - 4x + 5$ is 4. Navdeep at once said "it is false"

Do you agree with Navdeep ? Justify.

Sol. Yes

On comparing the given equation with $ax^2 + bx + c$, we get

Here $a = 2$, $b = -4$, $c = 5$

Sum of zeroes = $\frac{-b}{a} = \frac{-(-4)}{2} = \frac{4}{2} = 2$

Navdeep is right. **Ans.**

34. Find the value of p so that the polynomial $px(x - 3) + 9 = 0$ has two equal roots.

Sol. Given,

$px(x - 3) + 9 = 0$

or $px2 - 3px + 9 = 0$

Let a, b be the zeroes of the polynomial.

Then, a + b = 3 and ab = $\frac{9}{p}$

Also a = b

2a = 3

or a = b = $\frac{3}{2}$

$\frac{9}{4} = \frac{9}{p}$

$\Rightarrow p = 4$

Hence, the zeroes of the polynomial $px(x - 3) + 9 = 0$ will be equal when $p = 4$. **Ans.**

35. If the product of the zeroes of a polynomial ($ax2 - 6x - 6$) is 4, find the value of *a*.

Sol. Given polynomial is, $ax2 - 6x - 6$

Let the zeroes of the polynomial be a and b.

a + b = $\frac{6}{a}$

and ab = $-\frac{6}{a}$

But it is given that ab = 4.

ab = $-\frac{6}{a}$= 4

or a = $-\frac{3}{2}$. **Ans.**

36. Write the polynomial, the product and sum of whose zeroes are $-\frac{9}{2}$ and $-\frac{3}{2}$.

Sol. Let the zeroes of the polynomial be a

and b.

Then a + b = $-\frac{3}{2}$

and ab = $-\frac{9}{2}$

We know, the polynomial is given as,

$x2 - (a + b)x + ab = 0$

⇒ $x^2 + \frac{3}{2}x - \frac{9}{2}$ = 0

⇒ $2x2 + 3x - 9 = 0$. **Ans.**

37. A group consists of 12 honest people and 8 dishonest people. Write a quadratic polynomial whose roots are equal to number of honest people and number of dishonest people.

Sol. Number of honest people, (a) = 12

Number of dishonest people, (b) = 8

Sum of roots = a + b = 12 + 8

= 20

Product of roots = ab = 12 × 8

= 96

Quadratic polynomial = $x2 - 20x + 96$ **Ans.**

38. Manish engages a labour to get some repair work. Charges to be paid for this work are zeroes of the polynomial $x2 - 300x + 22500$. Find zeroes of this polynomial.

Sol. We have,

$x2 - 300x + 22500 = 0$

$\Rightarrow x2 - 150x - 150x + 22500 = 0$

$\Rightarrow x(x - 150) - 150(x - 150) = 0$

$\Rightarrow (x - 150)(x - 150) = 0$

$x = 150$ **Ans.**

39. Given $f(x + 1) = 3x + 5$, evaluate $f(-2)$ and $f(x)$.

Sol. Given,

$f(x + 1) = 3x + 5$

$f(x) = f(x + 1 - 1) = f(x - 1 + 1)$

$= 3(x - 1) + 5$

$= 3x - 3 + 5$

$= 3x + 2$

and $f(-2) = 3(-2) + 2$

$= -6 + 2$

$= -4$. **Ans.**

40. If $f(x) = \frac{x}{x^2 + 1}$, then find :

(i) $f\left(\frac{1}{x}\right)$ **and (ii)** $\boldsymbol{f(x - 1)}$**.**

Sol. Given,

$f(x) = \frac{x}{x^2+1}$

(i) Thus, $f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\left(\frac{1}{x}\right)^2+1}$

$= \frac{x^2}{x(x^2+1)}$

$= \frac{x}{x^2+1}$

(ii) $f(x - 1) = \frac{x-1}{x^2-2x+2}$

$= \frac{x-1}{x^2-2x+2}$ **Ans.**

41. Find the remainder when

$\boldsymbol{f(x) = (2x3 - 3x2 + 7x - 8)}$

is divided by $\boldsymbol{g(x) = (x - 1)}$**.**

Sol. Given, $f(x) = 2x3 - 3x2 + 7x - 8$

and $g(x) = x - 1$

Put $x - 1 = 0 \Rightarrow x = 1$

Remainder = $f(1)$

$= 2(1)3 - 3(1)2 + 7(1) - 8$

$= 2 - 3 + 7 - 8$

$= -2$

Hence, the remainder is –2. **Ans.**

42. Show that $\boldsymbol{g(x) = (x - 3)}$ **is a factor of**

$\boldsymbol{f(x) = (x3 - 7x2 + 15x - 9)}$**.**

Sol. Given, $f(x) = x3 - 7x2 + 15x - 9$

and $g(x) = x - 3$

Put $x - 3 = 0$

$\Rightarrow x = 3$

Remainder = $f(3)$

$= (3)3 - 7(3)2 + 15(3) - 9$

$= 27 - 63 + 45 - 9$

$= 72 - 72$

$= 0$

Hence, $(x - 3)$ is a factor of given polynomial of $f(x)$. **Hence Proved.**

43. Find the H.C.F. of $f(x) = 3x3 - 27x2 + 60x$ and $g(x) = x2 - 16$.

Sol. Given,

$f(x) = 3x3 - 27x2 + 60x$

and $g(x) = x2 - 16$

Now $f(x) = 3x3 - 27x2 + 60x$

$= 3x(x2 - 9x + 20)$

$= 3x(x - 5x - 4x + 20)$

$= 3x\{x(x - 5) - 4(x - 5)\}$

$= 3x(x - 5)(x - 4)$

and $g(x) = x2 - 16$

$= (x - 4)(x + 4)$

H.C.F. = $x - 4$. **Ans.**

44. Find the factors of the following polynomial

$f(x) = 2x3 + x2 - 13x + 6$.

Sol. Given,

$f(x) = 2x3 + x2 - 13x + 6$

$= 2x3 - x2 + 2x2 - x - 12x + 6$

$= x2(2x - 1) + x(2x - 1) - 6(2x - 1)$

$= (2x – 1)\ (x2 + x – 6)$

$= (2x – 1)\ (x2 + 3x – 2x – 6)$

$= (2x – 1)\ \{x(x + 3) – 2(x + 3)\}$

$= (2x – 1)\ (x + 3)\ (x – 2)$

Hence, the zeroes of $f(x)$ are – 3, 2 and $\frac{1}{2}$. **Ans.**

45. If two zeroes of the polynomial *x*3 – 4*x*2 – 3*x* + 12 are $\sqrt{3}$ and $-\sqrt{3}$, then find the third zero.

Sol. Let

$f(x) = x3 – 4x2 – 3x + 12$

$= x3 – 3x – 4x2 + 12$

$= x(x2 – 3) – 4(x2 – 3)$

$= (x2 – 3)\ (x – 4)$

As $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of $f(x)$, therefore, $(x-\sqrt{3})$ and $(x+\sqrt{3})$ are its factors. So $(x-\sqrt{3})\,(x+\sqrt{3})$ is a factor of $f(x)$.

Let $(x-\sqrt{3})(x+\sqrt{3}) = g(x)$

or $x2 – 3 = g(x)$

So, $\frac{f(x)}{g(x)} = \frac{(x^2-3)(x-4)}{(x^2-3)} = x-4$

Thus $x – 4 = 0$

or $x = 4$

Hence, the third zero of the polynomial is 4. **Ans.**

46. If *x* = $\frac{2}{3}$ and *x* = – 3 are zeroes of the polynomial *ax*2 + 7*x* + *b* = 0, find the value of *a* and *b*.

Sol. The given polynomial is,

$p(x) = ax2 + 7x + b$

... $x = \frac{2}{3}$ and $x = –3$ are the roots of the given polynomial

$p\left(\frac{2}{3}\right) = a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$

$\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0$

$\Rightarrow 4a + 42 + 9b = 0$ ...(i)

and $p(-3) = a(-3)2 + 7(-3) + b = 0$
$\Rightarrow 9a - 21 + b = 0$ ...(ii)

Multiply equation (ii) by 9 and then subtracting it from equation (i),
$4a + 42 + 9b = 0$
$81a - 189 + 9b = 0$
– + –
$-77a + 231 = 0$
$\Rightarrow a = \frac{231}{77} = 3$
Putting $a = 3$ in equation (ii), we get
$9(3) - 21 + b = 0 \Rightarrow b = -6$
$a = 3, b = -6$ **Ans.**

Short Answer Type Questions

47. Find the zeroes of given polynomial, $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0.$

[Board Question]

Sol. Given,

$f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$

$\Rightarrow x = -\frac{2}{\sqrt{3}}$ and $\frac{\sqrt{3}}{4}$

$\therefore$ The zeroes of given polynomial are $-\frac{2}{\sqrt{3}}$ and $\frac{\sqrt{3}}{4}$. **Ans.**

48. If α and β are zeroes of the quadratic polynomial $4x^2 + 4x + 1$, then find the quadratic polynomial whose zeroes are 2α and 2β.

[Board Question]

Sol. Let $p(x) = 4x^2 + 4x + 1$

... α, β are zeroes of $p(x)$

∴ $\alpha + \beta$ = Sum of zeroes = $\frac{-b}{a}$

⇒ $\alpha + \beta = \frac{-4}{4} = -1$ ...(i)

Also, $\alpha.\beta$ = Product of zeroes = $\frac{c}{a}$

⇒ $\alpha.\beta = \frac{1}{4}$ ...(ii)

Now a quadratic polynomial whose zeroes are 2α and 2β will be

x^2 – (Sum of zeroes)x + Product of zeroes

$= x^2 - (2\alpha + 2\beta)x + 2\alpha \times 2\beta$

$= x^2 - 2(\alpha + \beta)x + 4(\alpha\beta)$

$= x^2 - 2 \times (-1)x + 4 \times \frac{1}{4}$

[Using eq. (i) and (ii)]

$= x^2 + 2x + 1$. **Ans.**

49. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients of the polynomial.

[Board Question]

Sol. We have, $6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

Zeroes are :

$2x - 3 = 0$

or $3x + 1 = 0$

$\therefore$ $x = \frac{3}{2}$ and $x = \frac{-1}{3}$ **Ans.**

Verification :

Sum of zeroes (a + b) = $\frac{3}{2}+\left(\frac{-1}{3}\right)$

= $\frac{9-2}{6}$ = $\frac{7}{6}$

= $\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

= $\frac{-(-7)}{6}$ = $\frac{7}{6}$

Product of zeroes (ab)

= $\frac{3}{2}\times\frac{-1}{3}$ = $\frac{-1}{2}$

= $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

= $\frac{-3}{6}$ = $\frac{-1}{2}$.

$\therefore$ Relationship holds. **Hence Verified.**

50. Solve the quadratic polynomial $2x^2 + ax - a^2$ for x.

Sol. Let $f(x) = 2x^2 + ax - a^2 = 0$

$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x + a) - a(x + a) = 0$

$\Rightarrow (x + a)(2x - a) = 0$

$\therefore$ $x = -a$ and $\frac{a}{2}$

Thus the values of x for the polynomial $2x^2 + ax - a^2$ will be $-a$ and $\frac{a}{2}$. **Ans.**

51. If one zero of a polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, then find the value of a.

Sol. Let the zeroes of the equation be α and β.

According to the question,

$\alpha = \frac{1}{\beta}$ or $\beta = \frac{1}{\alpha}$

Thus $\alpha\beta = \alpha\frac{1}{\alpha} = \beta\frac{1}{\beta} = 1$

Now $\alpha + \beta = -\frac{13}{a^2+9}$ and $\alpha\beta = \frac{6a}{a^2+9}$

$\therefore \frac{6a}{a^2+9} = 1$

$\Rightarrow a^2 + 9 = 6a$

$\Rightarrow a^2 - 6a + 9 = 0$

$\Rightarrow a^2 - 3a - 3a + 9 = 0$

$\Rightarrow a(a - 3) - 3(a - 3) = 0$

$\Rightarrow (a - 3)^2 = 0$

$\Rightarrow a = 3$. **Ans.**

52. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:

[NCERT]

(i) $\frac{1}{4}, -1$ **(ii)** $\sqrt{2}, \frac{1}{3}$

Sol. (i) Let the zeroes of the polynomial $f(x)$ be α and β.

$\therefore \alpha + \beta = \frac{1}{4}$ and $\alpha\beta = -1$

Hence, the required polynomial is

$f(x) = x^2$ – (Sum of the zeroes) x + Product of the zeroes $\Rightarrow f(x) = x^2 - \frac{1}{4}x - 1$

$= (4x^2 - x - 4)$ **Ans.**

(ii) Let the zeroes of the polynomial $f(x)$ be α and β.

$\therefore \alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

Hence, the required polynomial is

$f(x) = x^2$ – (Sum of the zeroes) x + Product of the zeroes

$\Rightarrow f(x) = x^2 - \sqrt{2}x + \frac{1}{3}$

$= (3x^2 - 3\sqrt{2}x + 1)$. **Ans.**

53. Find a quadratic polynomial, the sum and product of whose zeros are 0 and $-\frac{3}{5}$ respectively. Hence, find the zeros.

Sol. Quadratic polynomial is given as,

$= x^2$ – (Sum of zeroes) x + Product of zeroes

$= x^2 - (0)\,x + \left(\frac{-3}{5}\right)$

$= x^2 - \frac{3}{5}$

$= (x)^2 - \left(\sqrt{\frac{3}{5}}\right)^2$

$= \left(x - \sqrt{\frac{3}{5}}\right)\left(x + \sqrt{\frac{3}{5}}\right)\left[\begin{array}{l}\text{By applying} \\ (a^2 - b^2) = (a+b)(a-b)\end{array}\right]$

Zeroes are,

$x - \sqrt{\frac{3}{5}} = 0$ or $x + \sqrt{\frac{3}{5}} = 0$

$\Rightarrow x = \sqrt{\frac{3}{5}}$ or $x = -\sqrt{\frac{3}{5}}$. **Ans.**

54. Find a cubic polynomial when the sum, sum of the products of its zeroes taken two at a time and product of its zero are 2, – 7, – 14 respectively.

[NCERT]

Sol. Let the zeroes of the cubic polynomial be α, β and γ.

$\therefore \alpha + \beta + \gamma = 2$

$\alpha\beta + \beta\gamma + \alpha\gamma = -7$

$\alpha\beta\gamma = -14$

We know, a cubic polynomial is given as,

$f(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - \alpha\beta\gamma$

$\therefore f(x) = x^3 - 2x^2 - 7x + 14$

Thus, the required cubic polynomial is

$x^3 - 2x^2 - 7x + 14$. **Ans.**

55. Solve for *x* :

$3x^2 - 2\sqrt{6}x + 2 = 0.$

Sol. Given, $3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow x = \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$ **Ans.**

56. For what value of *k* are the roots of the quadratic polynomial *kx*(*x* – 2) + 6 = 0 are equal?

[NCERT]

Sol. Given,

$kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

Let α and β be the roots of the polynomial.

Hence $\alpha + \beta = \frac{-(-2k)}{k} = 2$; $\alpha\beta = \frac{6}{k}$

But α = β [Given]

$\Rightarrow \alpha + \alpha = 2$

$\Rightarrow 2\alpha = 2$

$\Rightarrow \alpha = \beta = 1$

Hence $\alpha\beta = \frac{6}{k}$

$\Rightarrow 1 = \frac{6}{k}$ [... α = β = 1]

$\Rightarrow k = 6$

Thus, the roots of the polynomial $kx(x - 2) + 6 = 0$ will be equal if $k = 6$. **Ans.**

57. Find the value of k if $(x - 2)$ is a factor of $f(x) = x3 + 2x2 - kx + 10$. Also determine whether $(x + 5)$ is also a factor.

Sol. Given, $f(x) = x3 + 2x2 - kx + 10$

and $g(x) = x - 2$

Put $g(x) = x - 2 = 0$

or $x = 2$

Replacing x by 2 in $f(x) = x3 + 2x2 - kx + 10$, we get

$(2)3 + 2(2)2 - 2k + 10 = 0$

$\Rightarrow 8 + 8 - 2k + 10 = 0$

$\Rightarrow -2k + 26 = 0$

$\Rightarrow 2k = 26$

$\Rightarrow k = 13$

Thus $f(x) = x3 + 2x2 - 13x + 10$

Now, put $x + 5 = 0$ or $x = -5$

Replacing x with -5 in $f(x)$, we get

$f(-5) = (-5)3 + 2(-5)2 - 13(-5) + 10$

$= -125 + 50 + 65 + 10$

$= -125 + 125 = 0$

Thus, $(x + 5)$ is also a factor of

$f(x) = x3 + 2x2 - 13x + 10$. **Ans.**

Long Answer Type Questions

58. On dividing $(x^3 - 3x^2 + x + 2)$ by a polynomial $g(x)$, the quotient and remainder are $(x - 2)$ and $(-2x + 4)$ respectively. Find the value of $g(x)$.

Sol. Let

$f(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$; $r(x) = -2x + 4$

By Euclid's division lemma,

$f(x) = g(x) \times q(x) + r(x)$

$\Rightarrow g(x) \times q(x) = f(x) - r(x)$

$\Rightarrow g(x) = \frac{f(x) - r(x)}{q(x)}$ ...(i)

Now, $f(x) - r(x) = x^3 - 3x^2 + x + 2 + 2x - 4$

$= x^3 - 3x^2 + 3x - 2$

$= x^3 - 2x^2 - x^2 + 2x + x - 2$

$= x^2(x - 2) - x(x - 2) + 1(x - 2)$

$= (x - 2)(x^2 - x + 1)$

$\therefore \frac{f(x) - r(x)}{q(x)} = \frac{(x-2)(x^2 - x + 1)}{(x-2)}$

$= (x^2 - x + 1) = g(x)$

Thus, the value of $g(x)$ is $(x^2 - x + 1)$. **Ans.**

59. Verify that 3, – 1 and $-\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$ and also verify the relationship between the zeroes and their coefficients.

[NCERT]

Sol. In order to verify that 3, – 1 and $-\frac{1}{3}$ are the zeroes of the cubic polynomial

$p(x) = 3x^3 - 5x^2 - 11x - 3$,

we have to substitute $x = 3, -1, -\frac{1}{3}$ in $p(x)$ and ensure that the result is 0.

Thus $p(x) = 3x^3 - 5x^2 - 11x - 3$

$p(3) = 3(3)^3 - 5(3)^2 - 11 \times 3 - 3$

$= 3 \times 27 - 5 \times 9 - 33 - 3$

$= 81 - 45 - 36$

$= 81 - 81 = 0$

$p(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3$

$= -3 - 5 + 11 - 3$

$= -11 + 11 = 0$

$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^3 - 5\left(-\frac{1}{3}\right)^2 - 11\left(-\frac{1}{3}\right) - 3$

$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3$

$= \frac{-1-5+33-27}{9}$

$= \frac{33-33}{9} = 0$

Thus, it is verified that 3, – 1 and $-\frac{1}{3}$ are the zeroes of the cubic polynomial.

$p(x) = 3x^3 - 5x^2 - 11x - 3$

To verify the relationship between the zeroes and their coefficients, let the zeroes be α, β and γ such that,

$\alpha = 3, \beta = -1$ and $\gamma = -\frac{1}{3}$

$\therefore \alpha + \beta + \gamma = 3 - 1 - \frac{1}{3}$

$= 2 - \frac{1}{3} = \frac{5}{3}$

$= -\left\{-\frac{5}{3}\right\} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\alpha\beta + \beta\gamma + \alpha\gamma = 3(-1) + (-1)\left(-\frac{1}{3}\right) + 3\left(-\frac{1}{3}\right)$

$= -3 + \frac{1}{3} - 1$

$= -4 + \frac{1}{3}$

$= \frac{-12+1}{3} = -\frac{11}{3}$

$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\alpha\beta\gamma = 3(-1)\left(-\frac{1}{3}\right) = 1$

$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

Hence, the relationship between the zeroes and the their coefficients has also been verified.

Hence Verified.

60. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

[Board Question]

Sol. The given polynomial is

$p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$\because p(y) = 0$

$7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$\Rightarrow 21y^2 - 11y - 2 = 0$

$\Rightarrow 21y^2 - 14y + 3y - 2 = 0$

$\Rightarrow 7y(3y - 2) + 1(3y - 2) = 0$

$\Rightarrow (3y - 2)(7y + 1) = 0$

$\Rightarrow y = \frac{2}{3}, -\frac{1}{7}$

So zeroes of $p(y)$ are $\frac{2}{3}, -\frac{1}{7}$ **Ans.**

Verification : On comparing

with $ax^2 + bx + c$, we get

$a = 7$, $b = -\frac{11}{3}$, $c = -\frac{2}{3}$

Sum of zeroes = $\frac{-b}{a}$

$\Rightarrow \frac{2}{3} + \left(-\frac{1}{7}\right) = \frac{-\left(\frac{-11}{3}\right)}{7}$

$\Rightarrow \frac{14-3}{21} = \frac{11}{3\times7}$

$\Rightarrow \frac{11}{21} = \frac{11}{21}$

Product of zeroes = $\frac{c}{a}$

$\Rightarrow \frac{2}{3} \times \left(-\frac{1}{7}\right) = \frac{\left(-\frac{2}{3}\right)}{7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{3\times7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{21}$ · **Hence Verified.**

61. When divided by (x – 3), the polynomials ($x^3 - px^2 + x + 6$) and ($2x^3 - x^2 - (p + 3)x - 6$) leaves the same remainder. Find the value of p.

Sol. Let,

$f(x) = x^3 - px^2 + x + 6$

and $g(x) = 2x^3 - x^2 - (p + 3)x - 6$

Now, $x - 3 = 0$

$\Rightarrow x = 3$

Replacing x with 3 in $f(x)$, we have

$f(3) = (3)^3 - p(3)^2 + 3 + 6$

$= 27 - 9p + 9$

$= 36 - 9p = 9(4 - p)$

and in $g(x)$, we have

$g(3) = 2(3)^3 - (3)^2 - (p + 3)(3) - 6$

$= 54 - 9 - 3p - 9 - 6$

$= 30 - 3p$

$= 3(10 - p)$

As per the question,

$9(4 - p) = 3(10 - p)$

$\Rightarrow 3(4 - p) = 10 - p$

$\Rightarrow 12 - 3p = 10 - p$

$\Rightarrow 3p - p = 12 - 10$

$\Rightarrow 2p = 2$

$\Rightarrow p = 1$

Thus, the value of p is 1. **Ans.**

62. If ($x - 2$) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when divided by ($x - 3$), the remainder is 52, find the values of a and b.

Sol. Let $f(x) = 2x^3 + ax^2 + bx - 14$

Since $(x - 2)$ is a factor of $f(x)$,

... $f(2) = 0$

Replacing x by 2 in $f(x)$, we have

$\Rightarrow f(2) = 2(2)^3 + a(2)^2 + b(2) - 14$

$= 16 + 4a + 2b - 14$

$= 2(1 + 2a + b)$

$\Rightarrow 2(1 + 2a + b) = 0$

$\Rightarrow 2a + b = -1$ …(i)

Also, when $f(x)$ is divided by $(x - 3)$, the remainder is 52.

$\Rightarrow f(3) = 52$

$f(3) = 2(3)^3 + a(3)^2 + b(3) - 14$

$= 54 + 9a + 3b - 14$

$= 9a + 3b + 40$

$\Rightarrow 9a + 3b + 40 = 52$

$\Rightarrow 9a + 3b = 12$

$\Rightarrow 3a + b = 4$

$\Rightarrow a + (2a + b) = 4$

$\Rightarrow a - 1 = 4$ [From (i)]

$\Rightarrow a = 5$ and $b = -1 - 10 = -11$

$\therefore a = 5$ and $b = -11$. **Ans.**

63. Find the values of k for which the polynomial $(k + 4)x^2 + (k + 1)x + 1$ has equal roots. Also find these roots.

[Board Question]

Sol. Let $f(x) = (k + 4)x^2 + (k + 1)x + 1$

Let the roots of $f(x)$ be α and β.

Thus, $\alpha + \beta = -\frac{k+1}{k+4}$

and $\alpha\beta = \frac{1}{k+4}$

According to question,

$\alpha = \beta$

Hence, $2\alpha = -\frac{k+1}{k+4}$

$\Rightarrow \alpha = -\frac{k+1}{2(k+4)}$ ...(i) and $\alpha\beta = \frac{1}{k+4}$

$\Rightarrow -\frac{k+1}{2(k+4)}\beta = \frac{1}{k+4}$ [From (i)]

$\Rightarrow \beta = -\frac{2}{k+1}$ ...(ii)

From (i) and (ii),

$-\frac{k+1}{2(k+4)} = -\frac{2}{k+1}$

$\Rightarrow (k + 1)^2 = 4(k + 4)$

$\Rightarrow k^2 + 2k + 1 = 4k + 16$

$\Rightarrow k^2 + 2k - 4k + 1 - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k - 5) + 3(k - 5) = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

$\Rightarrow k = -3, 5$

Thus, the polynomial $(k + 4)x^2 + (k + 1)x + 1$ will have equal roots when k equals – 3 and 5. **Ans.**

64. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeroes equal to half to their product.

[Board Question]

Sol. The given quadratic polynomial is

$$x^2 - (k + 6)x + 2(2k - 1)$$

Comparing with $ax^2 + bx + c$, we get

$a = 1$, $b = -(k + 6)$ and $c = 2(2k - 1)$

Let the zeroes of the polynomial be α and β we know that

$\alpha + \beta = -\frac{b}{a}$

$= \frac{k+6}{1}$

or α + β = k + 6 ...(i)

Also, $\alpha\beta = \frac{c}{a}$

$= \frac{2(2k-1)}{1}$

or αβ = 2 (2k – 1) ...(ii)

According to question

Sum of zeroes = $\frac{1}{2}$ of their product

∴ $\alpha + \beta = \frac{1}{2}\alpha\beta$

or $k + 6 = \frac{1}{2} \times 2(2k-1)$

[using equations (i) and (ii)]

or k + 6 = 2k – 1

⇒ k = 7 **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

65. Assertion: The degree of the polynomial $(x+4)(x-2)(x-3)$ is 4.

Reason: The number of zeroes of a polynomial is the degree of that polynomial.

Ans. (d) Assertion is false but the Reason is true.

Explanation :

$p(x) = (x - 2)(x - 3)(x + 4)$

$= (x - 2)(x^2 + 4x - 3x - 12)$

$= (x - 2)(x^2 + x - 12)$

$= x^3 + x^2 - 12x - 2x^2 - 2x + 24$

$= x^3 - x^2 - 14x + 24$

So, degree of $p(x)$ is 3.

Hence, Assertion is false but the Reason is true.

66. Assertion: Degree of a zero polynomial is not defined.

Reason: Degree of a non-zero constant polynomial is 0.

Ans. (b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

We know that, the constant polynomial 0 is called a zero polynomial and the degree of a zero polynomial is not defined. So, Assertion is proved to be correct.

On the other hand, degree of a non-zero constant polynomial is always 0 which states that reason is also true, but it is not explaining the assertion statement in a detailed manner.

So, the Assertion and the Reason both are correct but Reason is not the correct explanation of Assertion.

67. Assertion: $p(x) = 4x^3 - x^2 + 5x^4 + 3x - 2$ is a polynomial of degree 3.

Reason: The highest power of x in the polynomial $p(x)$ is the degree of the polynomial.

Ans. (d) Assertion is false but the Reason is true.

Explanation :

The highest power of x in the polynomial $4x^3 - x^2 + 5x^4 + 3x - 2$ is 4.

Therefore, the degree of the polynomial $p(x)$ is 4.

Hence, Assertion is false but reason is true.

68. Assertion: If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 2$.

Reason: If $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$ *i.e.*, a is a zero of $p(x)$.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

Let a and $1/a$ be the zero of $p(x)$, then we have

Product of zeroes = $a \times \frac{1}{a} = \frac{4k}{(k^2+4)} = 1$

$\Rightarrow k^2 - 4k + 4 = 0$

$\Rightarrow (k - 2)^2 = 0$

$\Rightarrow k = 2$

Assertion is true and reason is also true.

69. Assertion: $x^2 + 7x + 12$ has no real zeroes.

Reason: A quadratic polynomial can have at the most two zeroes.

Ans. (d) Assertion is false but the Reason is true.

Explanation :

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x + 4) + 3(x + 4) = 0$

⇒ $(x + 4)(x + 3) = 0$

⇒ $x = -4$ or $x = -3$

Therefore, $x^2 + 7x + 12$ has two real zeroes.

Hence, Assertion is false but reason is true.

Case Based Questions

70. Junk food is an unhealthy food option, that is high in calories and little dietary fiber, protein, vitamins, minerals, or other important forms of nutritional value. A survay was conducted on few students consumes . If a **be the number of students who consumes junk food,** b **be the number of students who consumes healthy food such that** a **>** b **and** a **and** b **are the zeroes of the quadratic polynomial** $f(x) = x^2 - 7x + 10$**, then answer the following questions :**

(i) The type of expression of the polynomial in the above statement is:

(a) quadratic

(b) cubic

(c) linear

(d) bi-quadratic

Ans. (a) quadratic

Explanation :

Since, the degree of the polynomial is 2.

It is quadratic.

(ii) The number of students who consumes junk food are:

(a) 5

(b) 2

(c) 7

(d) none of these

Ans. (a) 5

Explanation :

Now,

$f(x) = x^2 - 7x + 10 = 0$

$\Rightarrow x^2 - 5x - 2x + 10 = 0$

$\Rightarrow x(x - 5) - 2(x - 5) = 0$

$\Rightarrow (x - 5)(x - 2) = 0$

$\Rightarrow x = 5, 2$

$\because$ a > b

a = 5 and b = 2

The number of students who take junk food = 5.

(iii) The number of students who consumes healthy food are:

(a) 5

(b) 2

(c) 7

(d) none of these

Ans. (b) 2

Explanation :

The number of students who consumes healthy food = 2.

(iv) The quadratic polynomial whose zeroes are – 3 and – 4 is:

(a) $x^2 + 4x + 2$

(b) $x^2 - x - 12$

(c) $x^2 + 7x + 12$

(d) none of these

Ans. (c) $x^2 + 7x + 12$

Explanation :

Sum of zeroes = – 3 – 4

= – 7

Product of zeroes = – 3 × – 4

= 12

Required polynomial

x^2 – (Sum of zeroes)x + (Product of zeroes)

$= x^2 - (-7)x + 12$

$= x^2 + 7x + 12.$

(v) If one zero of the polynomial $x^2 - 5x + 6$ is 2 then the other zero is:

(a) 6

(b) – 6

(c) 3

(d) none of these

Ans. (c) 3

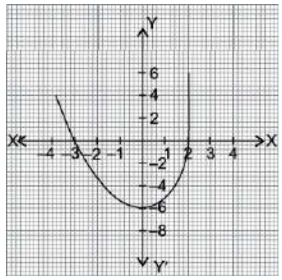
Explanation :

Sum of zeroes = a + 2 = $-\frac{(-5)}{1}$

⇒ a + 2 = 5

⇒ a = 3.

71. Venika saw a creeper on the wall of his grandmother's house, which was in the shape as shown in the figure.



(i) How many zeroes does the polynomial have?

(a) 0

(b) 1

(c) 2

(d) 3

Ans. (c) 2

Explanation :

As the graph cuts the X-axis at two points.

It has 2 zeroes.

(ii) The zeroes of the polynomial are:

(a) 2, – 3

(b) – 2, 3

(c) 2, 1

(d) – 3, 1

Ans. (a) 2, – 3

Explanation :

Since, the curve cuts X-axis at 2 and – 3.

Zeroes of given polynomial are 2, – 3.

(iii) The expression of the polynomial is:

(a) $x^2 - x - 6$

(b) $x^2 + x - 6$

(c) $x^3 - x + 6$

(d) $x^3 - x^2 + x + 6$

Ans. (b) $x^2 + x - 6$

Explanation :

Zeroes of the polynomial are 2 and – 3

So, α + b = (2 – 3)

= – 1

ab = 2 × (– 3)

= – 6

So, $x^2 - (a + b)x + ab$

$= x^2 - (-1)x + (-6) = x^2 + x - 6.$

(iv) The type of expression of the polynomial is:

(a) linear

(b) cubic

(c) quadratic

(d) biquadratic

Ans. (c) quadratic

Explanation :

Since, the degree of polynomial is 2.

It is a quadratic polynomial.

(v) For what value of x, the value of polynomial is 6?

(a) $x = 3$

(b) $x = -4$

(c) Both (a) and (b)

(d) $x = 2$

Ans. (c) Both (a) and (b)

Explanation :

$f(x) = x^2 + x - 6$

At $x = 3$, $f(3) = 3^2 + 3 - 6 = 9 + 3 - 6 = 6$

At $x = -4$, $f(-4) = (-4)^2 + (-4) - 6$

$= 16 - 4 - 6 = 6$

At $x = 2$, $f(2) = 2^2 + 2 - 6 = 0$

Thus, for $x = 3$ and $x = -4$, the value of polynomial is 6.

72. For the box to satisfy certain requirements, its length must be three metre greater than the width, and its height must be two metre less than the width.

(i) It width is taken as x, which of the following polynomial represent volume of box?

(a) $x^2 - 5x - 6$

(b) $x^3 + x^2 - 6x$

(c) $x^3 - 6x^2 - 6x$

(d) $x^2 + x + 6$

Ans. (b) $x^3 + x^2 - 6x$

Explanation :

$V(x) = x(x + 3)(x - 2)$

$= x(x^2 + x - 6)$

$= x^3 + x^2 - 6x$

(ii) Which of the following polynomial represent the area of paper sheet used to make box?

(a) $x^2 - 5x - 6$

(b) $6x^2 + 4x - 12$

(c) $x^2 - 6x^2 - 6x$

(d) $6x^2 + 3x - 4$

Ans. (b) $6x^2 + 4x - 12$

Explanation :

$S(x) = 2[LB + BH + HL]$

$= 2[x(x + 3) + (x + 3)(x - 2) + x(x - 2)]$

$= 2[x^2 + 3x + x^2 + x - 6 + x^2 - 2x]$

$= 2[3x^2 + 2x - 6] = [6x^2 + 4x - 12]$

(iii) If the width of box is 3 units, what must be its height?

(a) 1 unit

(b) 3 units

(c) 2 units

(d) 4 units

Ans. (a) 1 unit

Explanation :

Height $= x - 2 = 3 - 2 = 1$ unit.

(iv) At the volume of 18 units, what must be its length?

(a) 6 units

(b) 3 units

(c) 4 units

(d) 2 units

Ans. (a) 6 units

Explanation :

$V(x) = x^3 + x^2 - 6$

$\Rightarrow 18 = x^3 + x^2 - 6x$

$\Rightarrow x^3 + x^2 - 6x - 18 = 0$

$\Rightarrow x^3 - 3x^2 + 4x^2 - 12x + 6x - 18 = 0$

$\Rightarrow x^2(x - 3) + 4x(x - 3) + 6(x - 3)$

$\Rightarrow (x - 3)(x^2 + 4x + 6) = 0$

$\Rightarrow (x - 3)(x^2 + 4x + 6) = 0$

$x = 3$

Thus, width is 3 units.

So, length = $x + 3 = 3 + 3$

= 6 units.

(v) If box is made of a paper sheet, and paper sheet costs ` 100 per square unit, then what is the cost of paper used in making box?

(a) ` 5400

(b) ` 10800

(c) ` 2700

(d) ` 3400

Ans. (a) ` 5400

Explanation :

$S(x) = 6x^2 + 4x - 12$

= 6 × 3 × 3 + 4 × 3 – 12

= 54 sq. unit

Cost of paper = 100 × 54

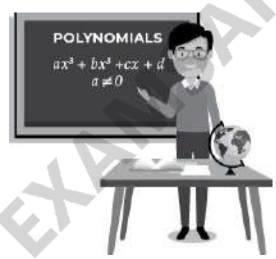
= ` 5400.

73. The tutor in a coaching centre was explaining the concept of cubic polynomial as a cubic polynomial is of the form $ax^3 + bx^2 + cx + d$, $a \neq 0$ and it has maximum three zeroes. The zeroes of a cubic polynomial are namely the *x*- coordinates of the points where the graph of the polynomial intersects the *x*-axis. If a, b **and** g **are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then the relations between their zeroes and their coefficients are**

α **+** b **+** g **= –** ***b*/*a***

α b **+** bg **+** αg **=** ***c*/*a***

αbg **= –*d*/*a***



(i) Which of the following are the zeroes of the polynomial $x^3 - 4x^2 - 7x + 10$?

(a) – 3, 1 and 3

(b) –1, 2 and – 3

(c) 2, – 1 and 5

(d) – 2, 1 and 5

Ans. (d) – 2, 1 and 5

Explanation :

For finding zeroes, check whether $p(x) = x^3 - 4x^2 - 7x + 10$ is 0 for given values.

Let $p(x) = x^3 - 4x^2 - 7x + 10$. Then,

Clearly $p(-2) = p(1) = p(5) = 0$

So, the zeroes are – 2, 1 and 5.

(ii) If $-\frac{1}{2}$, – 2 and 5 are zeroes of a cubic polynomial, then the sum of product of zeroes taken two at a time is:

(a) $\frac{23}{2}$

(b) $-\frac{1}{2}$

(c) – 23

(d) $-\frac{23}{2}$

Ans. (d) $-\frac{23}{2}$

Explanation :

Here α = $\frac{-1}{2}$, b = – 2 and g = 5

∴ Sum of product of zeroes taken two at a time

= ab + bg + ga

$= \left(\frac{-1}{2}\right)(-2) + (-2)(5) + \left(\frac{-1}{2}\right) \times 5$

$= 1 - 10 - \frac{5}{2} = \frac{-23}{2}$

(iii) In which of the following polynomials the sum and product of zeroes are equal?

(a) $x^3 - x^2 + 5x - 1$

(b) $x^3 - 4x$

(c) $3x^3 - 5x^2 - 11x - 3$

(d) Both (a) and (b)

Ans. (d) Both (a) and (b)

Explanation :

Consider $x^3 - x^2 + 5x - 1$

Sum of zeroes = 1 = Product of zeroes

Now, consider $x^3 - 4x$

Sum of zeroes = 0

= Product of zeroes.

(iv) The polynomial whose all the zeroes are same is :

(a) $x^3 + x^2 + x - 1$

(b) $x^3 - 3x^2 + 3x - 1$

(c) $x^3 - 5x^2 + 6x - 1$

(d) $3x^3 + x^2 + 2x - 1$

Ans. (b) $x^3 - 3x^2 + 3x - 1$

Explanation :

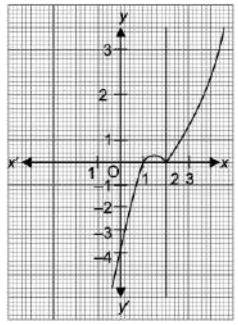
Let α, α, α, be the zeroes of the cubic polynomial.[∵ All zeroes are same]

Then, $\alpha^3 = 1$[Using given options]

$\Rightarrow \alpha = 1$

So, the required polynomial is $(x - 1)^3 = x^3 - 3x^2 + 3x - 1$

(v) The cubic polynomial, whose graph is as shown below, is:



(a) $x^3 - 5x^2 + 8x - 4$

(b) $x^3 - 7x^2 + 11x + 9$

(c) $3x^3 - 4x^2 + x - 5$

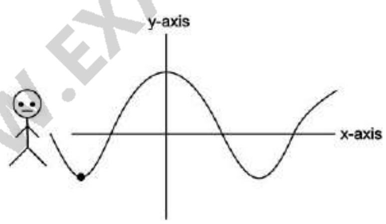
(d) $x^3 - 9$

Ans. (a) $x^3 - 5x^2 + 8x - 4$

Explanation :

Clearly $x = 1$ and $x = 2$ are the zeroes of given polynomial, both of which satisfies $x^3 - 5x^2 + 8x - 4$.

74. A boy is playing with rope which moves as shown in the figure below:



Now give the answers of following questions:

(i) The number of zeroes of the above figure:

(a) 2

(b) – 2

(c) 3

(d) 1

Ans. (c) 3

Explanation :

Figure cuts x-axis at 3 points, then number of zeroes are 3.

(ii) If above rope cuts x-axis at – 2, $\frac{1}{2}$, 1, the polynomial $p(x)$ will be:

(a) $2x^3 + x^2 - 5x + 2$

(b) $2x^2 - x^2 - 5x + 2$

(c) $x^3 - 2x^2 - 5x + 2$

(d) $2x^3 - x^2 - 5x - 2$

Ans. (a) $2x^3 + x^2 - 5x + 2$

Explanation :

$P(x) = k[x^3 -$ (sum of zeroes)x^2 + (sum of product of two zeroes)x – product of zeroes]

Zeroes are – 2, $\frac{1}{2}$, 1

$\alpha + \beta + \gamma = -2+\frac{1}{2}+1=-1+\frac{1}{2}=\frac{-1}{2}$

$\alpha\beta + \beta\gamma + \gamma\alpha = -2\times\frac{1}{2}+\frac{1}{2}\times 1+(-2)\times 1$

$= -1+\frac{1}{2}-2=\frac{1}{2}-3=\frac{-5}{2}$

$\alpha\beta\gamma = -2\times\frac{1}{2}\times 1=-1$

$p(x) = x^3+\frac{1}{2}x^2-\frac{5}{2}x+1$

$= \frac{1}{2}(2x^3+x^2-5x+2)$

$k = \frac{1}{2}$.

(iii) If the boy moves the above rope with more force and it moves along with polynomial $P(x)$ **=** $x^3 - 4x^2 + 5x - 2$, then what are the points on which it cuts x-axis:

(a) 2, 1, 1

(b) 2, – 1, 1

(c) 2, 1, 1

(d) – 2, – 1, + 1

Ans. (a) 2, 1, 1

Explanation :

$$
\begin{array}{r|l}
 & x^2 - 2x + 1 \\
\hline
x - 2 & x^3 - 4x^2 + 5x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -2x^2 + 5x \\
 & -2x^2 + 4x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

$x = 2$ is a root of P(x).
Now P(x) = $(x^2 – 2x + 1)(x – 2)$
$= (x – 1)^2 (x – 2)$
$= (x – 1)(x – 1)(x – 2)$
Roots are 1, 1, 2.
(iv) What are the sum and product of zeros?

(a) 4, 2

(b) – 4, 2

(c) 5, 4

(d) 4, – 2

Ans. (a) 4, 2

Explanation :

Sum of roots = 2 + 1 + 1 = 4

Product of roots = 2 × 1 × 1 = 2.

Passage Based Questions

75. Read the following passage and answer the questions that follows:

A teacher told 10 students to write a polynomial on the black board.

Students wrote

1. $x^2 + 2$ 6. $x - 3$

2. $2x + 3$ 7. $x^4 + x^2 + 1$

3. $x^3 + x^2 + 1$ 8. $x^2 + 2x + 1$

4. $x^3 + 2x^2 + 1$ 9. $2x^3 - x^2$

5. $x^2 - 2x - 1$ 10. $x^4 - 1$

Based on the information, answer the following questions :

(i) How many students wrote cubic polynomial?

(ii) Divide the polynomial ($x^2 + 2x + 1$) by ($x + 1$).

Sol. (i) 3 students wrote cubic polynomial

(ii) $x+1)\ x^2+2x+1\ (x+1$

x^2+x

$-\quad -$

$x+1$

$x+1$

$-\quad -$

x

$\frac{x^2+2x+1}{x+1}$ = $x + 1$ **Ans.**

76. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students:

[Board Question]

$2x + 3$, $3x^2 + 7x + 2$, $4x^3 + 3x^2 + 2$, $x^2+\sqrt{3x}+7$, $7x+\sqrt{7}$, **$5x^3 - 7x + 2$,** $2x^2+3-\frac{5}{x}$, $5x-\frac{1}{2}$,

$ax^3 + bx^2 + cx + d$, $x+\frac{1}{x}$

Based on the information, answer the following questions:

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

Sol. (i) $3\left(x^2+\sqrt{3x}+7, 2x^2+3-\frac{5}{x} \text{ and } x+\frac{1}{x}\right)$

(ii) There is only one quadratic polynomial which is $3x^2 + 7x + 2$.

Self-Assessment

77. If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, then evaluate:

(i) $\alpha - \beta$

(ii) $\alpha^2\beta + \alpha\beta^2$

(iii) $\alpha^4 + \beta^4$

(iv) $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

(v) $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$

(vi) $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

Ans. (i) $\frac{\sqrt{b^2-4ac}}{a}$ (ii) $\frac{-bc}{a^2}$

(iii) $\frac{(b^2-2ac)^2-2a^2c^2}{a^4}$ (iv) $\frac{b}{ac}$

(v) $-\left(\frac{b}{c}+\frac{2c}{a}\right)$ (vi) b

78. Show that

$$f(x) = (x^4 - 16x^3 + 86x^2 - 176x + 105)$$

is exactly divisible by $g(x) = (x - 3)$.

79. Show that $f(x) = (x^4 + 2x^3 - 13x^2 - 14x + 24)$ is exactly divisible by $g(x) = (x + 4)$.

80. Find the relation between a and b, if

$$f(x) = (4x^3 - 3x^2 + 2ax + b)$$

be exactly divisible by (x + 2).

Ans. – 4a + b = 44.

81. Find the relation between a and b, if

$$f(x) = (ax^5 + 3bx^3 + 8)$$

be exactly divisible by (x – 2).

Ans. 4a + 3b = – 1.

82. Express:

(i) $f(x) = 3x^3 - 4x^2 + 5x + 6$ as a polynomial of x + 1

(ii) $f(x) = x^4 - x^3 + 2x^2 - 3x + 1$ as a polynomial of x – 3

(iii) $f(x) = 3x^4 + 4x^3 + 7x^2 + 8x - 8$ as a polynomial of x + 1

(iv) $f(x) = 4x^5 - 6x^4 + 3x^3 - 5x + 2$ as a polynomial of x + 2.

Ans. (i) $3x^2(x + 1) - 7x(x + 1) + 12(x + 1) - 6$.

(ii) $x^3(x - 3) + 2x^2(x - 3) + 4x(x - 3) + 9(x - 3) + 28$.

(iii) $3x^3(x - 1) + 7x^2(x - 1) + 14x(x - 1) + 22(x - 1) + 14$.

(iv) $4x^5(x + 2) - 14x^4(x + 2) + 31x^3(x + 2) - 62x(x + 2) + 119(x + 2) - 236$.

83. (i) If $f(x) = x^4 - 3x^3 + 4x^2 - 5x - 9$, then show that

$$f(x + 2) = x^4 + 5x^3 + 10x^2 + 7x - 11.$$

(ii) If $f(x) = 2x^4 - x^3 - 2x^2 + 5x - 1$, then show that $f(x + 3) = 2x^4 + 23x^3 + 97x^2 + 182x + 131$.

84. For $f(x) = (2x^3 + x^2 - 5x + 2)$, find the values for f(x) = – 1, 1, – 2 and prove that

$$f(1) = f(-2) \text{ and } 2f(-1) = f(2).$$

Ans. $f(-1) = 6$, $f(1) = 0$, $f(-2) = 0$

85. Obtain all zeroes of the polynomial

$$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$$

if two of its zeroes are – 2 and – 1.

Ans. $-\frac{1}{2}$, 3, – 2, – 1.

86. Find the polynomial whose zeroes are 1, – 2, 3 and – 4.

Ans. $f(x) = (x^4 + 2x^3 - 13x^2 - 14x + 24)$.

87. Find all the zeroes of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

Ans. $-\sqrt{2}, \sqrt{2}, -3$

88. Find the polynomial whose zeroes are 2, – 3, 4 and – 1.

Ans. $f(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$

89. In the equation $f(x) = (x^4 + 2x^3 - 13x^2 - 14x + 24)$, two zeros are 1 and – 2. Find the other zeros.

Ans. – 4 and 3.

90. Find the zeroes of the following polynomials:

(i) $(x^4 - 9x^2 + 4x + 12) = 0$

(ii) $(x^4 - 6x^3 + 12x^2 - 10x + 3) = 0$.

Ans. (i) – 1, – 3, 2, 2. (ii) 1, 1, 1, 3.

91. Find the zeroes of the polynomial such that the sum of two of the zeroes in each polynomial is 0:

(i) $f(x) = x^3 - 5x^2 - 16x + 80$;

(ii) $f(x) = 4x^3 + 16x^2 - 9x - 36$.

Ans. (i) – 4, 4 and 5 (ii) $-\frac{3}{2}, \frac{3}{2}$ and – 4.

92. Find the zeroes of x in $\frac{x+3}{x-3}+6\left(\frac{x-3}{x+3}\right)$ = 5.

Ans. 6 and 9.

93. Find the zeroes of x in

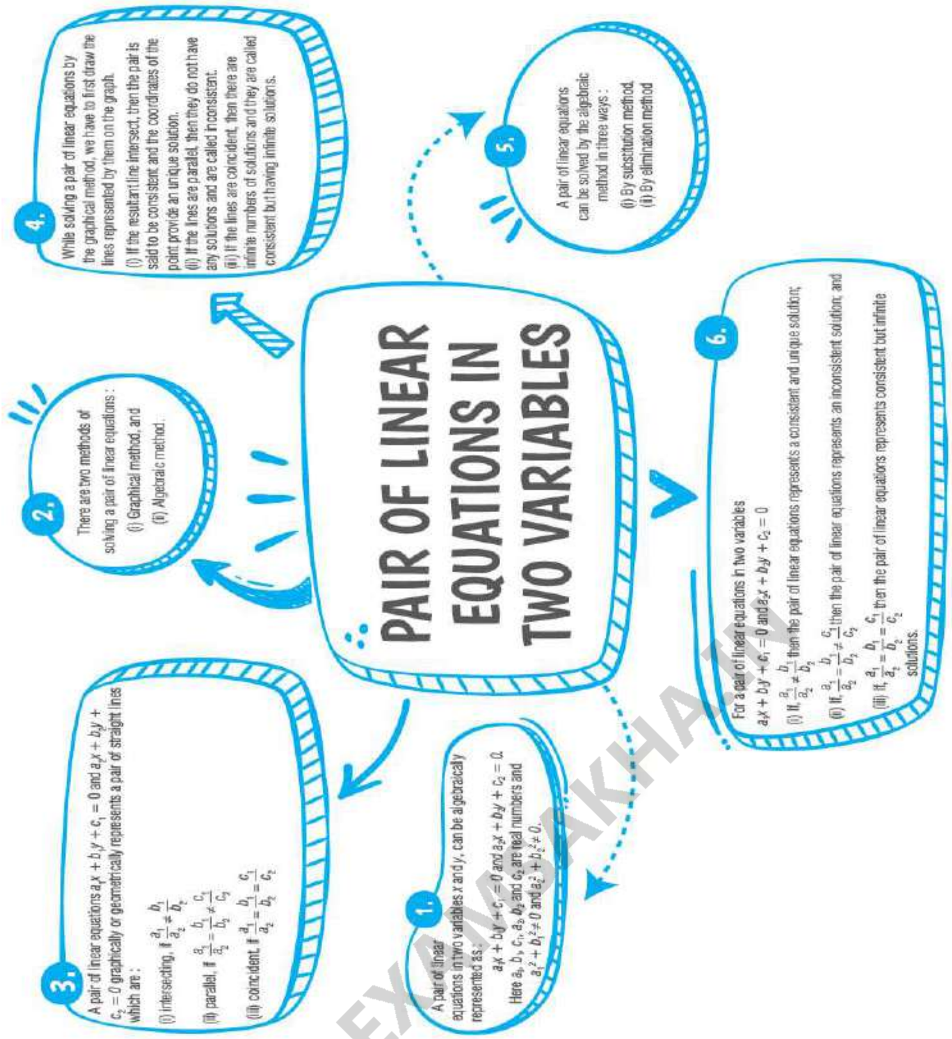
$$\frac{x+1}{2}+\frac{2}{x+1}=\frac{x+1}{3}+\frac{3}{x+1}-\frac{5}{6}$$

Ans. 0, – 7.

Pair of Linear Equations in Two Variables

Chapter 3

Basic Concepts



Multiple Choice Questions

1. **The values of *x* and *y* in 2*x* + 3*y* = 2 and *x* – 2*y* = 8 are:**

(a) – 4, 2

(b) – 4, – 2

(c) 4, – 2

(d) 4, 2

Ans. (c) 4, – 2

Explanation :

As per the question,

$2x + 3y = 2$ …(i)

$x - 2y = 8$ …(ii)

Multiplying equation (i) with 2 and equation (ii) with 3, we get

$4x + 6y = 4$ …(iii)

and $3x - 6y = 24$ …(iv)

Thus, adding the equation (iii) & (iv), we get

$7x = 28$

$\Rightarrow x = 4$

Putting $x = 4$ in equation (ii), we get

$4 - 2y = 8$

$\Rightarrow -2y = 4$

$\Rightarrow y = -2$

Hence, the values of x and y are 4, – 2 respectively.

2. **The values of *x* and *y* in $2x + y + 1 = 0$ and $2x - 3y + 8 = 0$ are:**

(a) 1, 2

(b) $\frac{-11}{8}, \frac{7}{4}$

(c) $\frac{11}{8}, \frac{7}{4}$

(d) 2, 3

Ans. (b) $\frac{-11}{8}, \frac{7}{4}$

Explanation :

As per the question,

$2x + y + 1 = 0$

$\Rightarrow 2x + y = -1$ …(i)

and $2x - 3y + 8 = 0$

$\Rightarrow 2x - 3y = -8$ …(ii)

Thus, subtracting equation (ii) from (i), we get

$4y = 7$

$\Rightarrow y = \frac{7}{4}$

Putting $y = \frac{7}{4}$ in equation (ii), we have

$2x - 3\left(\frac{7}{4}\right) = -8$

$\Rightarrow 2x = \frac{21}{4} - 8$

$\Rightarrow 2x = \frac{21-32}{4}$

$= \frac{-11}{4}$

$\Rightarrow x = \frac{-11}{8}$

Hence, the values of x and y are $\frac{-11}{8}, \frac{7}{4}$ respectively.

3. **The perimeter of an isosceles triangle is 65 cm and the unequal side is thrice as large as each of the equal sides. The lengths of the sides are:**

(a) 13, 13, 39

(b) 39, 39, 26

(c) 13, 26, 26

(d) 13, 13, 26

Ans. (a) 13, 13, 39

Explanation :

Given, perimeter of an isosceles triangle = 65 cm

Let the sides of the triangle be a cm, a cm and b cm.

Now, $b = 3a$…(i)

and $a + b + a = 65$

$\Rightarrow a + 3a + a = 65$[from (i)]

$\Rightarrow 5a = 65$

$\Rightarrow a = 13$ cm

$b = 39$ cm.

4. **The value of k for which the system of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent is:**

[Board Question]

(a) $-\frac{14}{3}$ (c) 5

(b) $\frac{2}{5}$ (d) 10

Ans. (d) 10

Explanation :

For the system of linear equations to be inconsistent, we have
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Here $a_1 = 1$, $b_1 = 2$, $c_1 = -3$ and $a_2 = 5$, $b_2 = k$, $c_2 = 7$
So, $\frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$
$\Rightarrow k = 10$.

5. **The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is:**

[Board Question]

(a) – 2

(b) ≠ 2

(c) 3

(d) 2

Ans. (d) 2

Explanation :

Given,

$x + y - 4 = 0$

$2x + ky - 3 = 0$

$a_1 = 1, b_1 = 1, c_1 = -4$

$a_2 = 2, b_2 = k, c_2 = -3$

For no solution, the condition is

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$

$\Rightarrow \frac{1}{2} = \frac{1}{k}$ and $\frac{1}{k} \neq \frac{4}{3}$

$\Rightarrow k = 2$ and $k \neq \frac{3}{4}$

$\Rightarrow k = 2.$

6. The pair of linear equations $2x + 3y = 4$ and $3x + 4y = 9$ has:

(a) infinitely many solutions

(b) no solution

(c) one unique solution

(d) two solutions

Ans. (c) one unique solution

Explanation :

$2x + 3y = 4$

$3x + 4y = 9$

Here, $a_1 = 2, a_2 = 3, b_1 = 3, b_2 = 4, c_1 = -4, c_2 = -9$

$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{2}{3} \neq \frac{3}{4}$

$\Rightarrow$ one unique solution.

7. A system of simultaneous linear equations is said to be inconsistent, if it has:

(a) one solution

(b) two solutions

(c) no solution

(d) infinite solutions

Ans. (c) no solution

Explanation :

If the lines are parallel then they do not have any solution and are called inconsistent.

8. If a pair of linear equations is inconsistent, then the lines will be:

(a) coincident

(b) intersecting

(c) parallel

(d) can't say

Ans. (c) parallel

Explanation :

If a pair of linear equations is inconsistent, then the system has no solution or we can say that lines do not intersect. Therefore, lines are parallel.

9. **The system of equations $3x + y - 4 = 0$ and $6x + 2y - 8 = 0$ has :**

(a) a unique solution x = 1, y = 1

(b) a unique solution x = 0, y = 4

(c) no solution

(d) infinite solution

Ans. (d) infinite solution

Explanation :

Given equation of system

$3x + y = 4$

$6x + 2y = 8$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, the system of equation has infinite solutions because it is a coincident line.

10. **The value of k, for which the system of equations $3x - ky - 20 = 0$ and $6x - 10y + 40 = 0$ has no solution, is:**

(a) 10

(b) 6

(c) 5

(d) 3

Ans. (c) 5

Explanation :

Given equations are

$3x - ky - 20 = 0$ and $6x - 10y + 40 = 0$

Here, $a_1 = 3$, $b_1 = -k$, $c_1 = -20$, $a_2 = 6$

$b_2 = -10$ and $c_2 = 40$

For no solution,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{6} = \frac{-k}{-10}$

$\Rightarrow k = 5.$

11. If the pair of linear equations $2x + 3y = 11$ and $(m + n)x + (2m - n)y - 33 = 0$ has infinitely many solutions, then the values of m and n, are .... and .... respectively.

(a) 5, 1

(b) 1, 2

(c) – 1, 5

(d) 1, – 5

Ans. (a) 5, 1

Explanation :

It has infinite many solution

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{m+n} = \frac{3}{2m-n} = \frac{11}{33}$

$\Rightarrow \frac{2}{m+n} = \frac{3}{2m-n} = \frac{1}{3}$

$\Rightarrow \frac{2}{m+n} = \frac{1}{3}$

$\Rightarrow m + n = 6$ ...(i)

and $2m - n = 9$ ...(ii)

on adding $3m = 15$

$\Rightarrow m = 5$

Putting m in equation (i),

$5 + n = 6$

$\Rightarrow n = 1$.

12. The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is ....

(a) 36

(b) 63

(c) 27

(d) 72

Ans. (a) 36

Explanation :

Let two digit be x and y, so number = $10x + y$

According to question

$x + y = 9$ ...(i)

and $10x + y + 27 = 10y + x$

$\Rightarrow 9x - 9y = -27$

$\Rightarrow x - y = -3$ ...(ii)

On adding equations (i) and (ii),

$2x = 6$

$\Rightarrow x = 3$

Put $x = 3$ in equation (i),

$3 + y = 9$

$\Rightarrow y = 9 - 3$

$= 6$

Required number is 36.

13. **The pair of equation $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have:**

[NCERT Exemplar]

(a) a unique solution

(b) exactly two solutions

(c) infinitely many solutions

(d) no solution

Ans. (d) no solution

Explanation :

Given, equations are $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$

and $a_2 = -3$, $b_2 = -6$, $c_2 = 1$

$$\frac{a_1}{a_2} = -\frac{1}{3}, \quad \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{5}{1}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.

14. **If a pair of linear equations is consistent, then the lines will be:**

[NCERT Exemplar]

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting

Ans. (c) intersecting or coincident

Explanation :

If a pair of linear equations in two variables is consistent then the lines represented by two equations are intersecting or coincident *i.e.*, either it has a unique solution or infinitely many solutions.

Very Short Answer Type Questions

15. Find the values of x and y in $3x + 2y = 4$ and $2x - 3y = 7$.

Sol. Given equations are,

$3x + 2y = 4$ …(i)

$2x - 3y = 7$ …(ii)

Multiplying equation (i) by 3 and equation (ii) by 2, we get

by $9x + 6y = 12$ …(iii)

and $4x - 6y = 14$ …(iv)

Thus, adding equations (iii) and (iv), we get

$13x = 26$

$\Rightarrow x = 2$

Replacing $x = 2$ in equation (i), we get

$3(2) + 2y = 4$

$\Rightarrow 6 + 2y = 4$

$\Rightarrow 2y = -2$

$\Rightarrow y = -1$

Hence, the values of x and y are 2, – 1 respectively.

Ans.

16. Find the values of x and y in $2x - 5y + 4 = 0$ and $2x + y - 8 = 0$.

Sol. Given equations are,

$2x - 5y + 4 = 0$

$\Rightarrow 2x - 5y = -4$ …(i)

and $2x + y - 8 = 0$

$\Rightarrow 2x + y = 8$ …(ii)

Thus, subtracting equation (ii) from (i), we get

$-6y = -12$

$\Rightarrow y = 2$

Replacing $y = 2$ in equation (ii), we get

$2x + 2 = 8$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

Hence, the values of x and y are 3, 2 respectively.

Ans.

17. Given the linear equation $x - 2y - 6 = 0$, write another linear equation in these two variables, such that the geometrical representation of the pair so formed is:

(i) coincident lines

(ii) intersecting lines

Sol. (i) Given, $x - 2y - 6 = 0$

For lines to be coincident, the condition is:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Thus, one possible option can be:

$2x - 4y - 12 = 0$

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -6$.

$a_2 = 2$, $b_2 = -4$, $c_2 = -12$.

$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{-6}{-12} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, it shows coincident lines. **Ans.**

(ii) Given, $x - 2y - 6 = 0$

For intersecting lines

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, one possible option can be:

$2x - 7y - 13 = 0$

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -6$

$a_2 = 2$, $b_2 = -7$, $c_2 = -13$

$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-2}{-7} = \frac{2}{7}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, they represents intersecting lines. **Ans.**

18. A number consists of 2 digits. The sum of the digits is 12 and the units digital when divided by the tens digit gives the

result as 3. Find the number.

Sol. Let the number be $10x + y$

Given, $x + y = 12$ ...(i)

and $\frac{y}{x} = 3$

$\Rightarrow y = 3x$ ...(ii)

Hence, $x + 3x = 12$ [From (i)]

$\Rightarrow 4x = 12$

$\Rightarrow x = 3$

and $y = 9$ [From (ii)]

So, the number is $3(10) + 9 = 39$ **Ans.**

19. The cost of 2 kg apples and 1 kg grapes was ` 160. After a month, the cost of 4 kg apples and 2 kg grapes was ` 300. Represent the situation algebraically.

[NCERT]

Sol. Let the cost of 1 kg of apples be ` a

and the cost of 1 kg of grapes be ` g.

Thus, $2a + g = 160$ …(i)

and $4a + 2g = 300$

or $2a + g = 150$ …(ii)

Thus, equations are $2a + g = 160$

and $2a + g = 150$ **Ans.**

20. Cost of 2 pens and 3 pencils together is ` 40 and cost of 6 pens and 9 pencils together is ` 130. Express above statement in the form of linear equations.

Sol. Let the cost of 1 pen be ` $x$ and cost of 1 pencil be ` y

Then,

$2x + 3y = 40$

and $6x + 9y = 130$ **Ans.**

21. Cost of a burger is ` 20 more than the cost of juice of one glass of orange. If cost of one burger and one glass of orange juice is ` 60. Find the cost of each.

Sol. Let the cost of burger be ` $x$ and the cost of orange juice be ` y

Then, $x = 20 + y$

$\Rightarrow x - y = 20$ ...(i)

and $x + y = 60$ ...(ii)

On adding equations (i) and (ii), we get

$2x = 80$

$\Rightarrow x = 40$

Replacing the value of x in eq. (ii), we get

$40 + y = 60$

$\Rightarrow y = 20$

Thus, the cost of a burger is ` 40 and orange juice is ` 20. **Ans.**

22. Solve for *x* and *y* :

$$\boldsymbol{mx - ny = m2 + n2 \text{ and } x + y = 2m.}$$

Sol. Given, $mx - ny = m^2 + n^2$ …(i)

and $x + y = 2m$

$\Rightarrow nx + ny = 2mn$ …(ii)

[multiplying both sides by n]

Adding equations (i) and (ii), we get

$mx + nx = m^2 + n^2 + 2mn$

$\Rightarrow (m + n)x = (m + n)^2$

$\Rightarrow x = m + n$

Substituting $x = m + n$ in $x + y = 2m$, we get

$m + n + y = 2m$

$\Rightarrow y = m - n$

Thus, $x = m + n$ and $y = m - n$. **Ans.**

23. Solve for *x* and *y*:

$$\sqrt{2}x - \sqrt{3}y = 0 \text{ and } \sqrt{3}x - \sqrt{8}y = 0.$$

Sol. Given,

$\sqrt{2}x - \sqrt{3}y = 0$ …(i)

and $\sqrt{3}x - \sqrt{8}y = 0$

$\Rightarrow \sqrt{3}x - 2\sqrt{2}y = 0$ …(ii)

Multiplying (i) with $2\sqrt{2}$ and (ii) with $\sqrt{3}$ we get

$4x - 2\sqrt{6}y = 0$ …(iii)

and $3x - 2\sqrt{6}y = 0$ …(iv)

Subtracting (iv) from (iii), we get

$x = 0$

Substituting $x = 0$ in (i), we get

$0 - \sqrt{3}y = 0$

or $y = 0$

Thus, $x = y = 0$. **Ans.**

24. Solve for *x* and *y* :

$152x - 378y = -74$ and $-378x + 152y = -604$.

[NCERT]

Sol. Given,

$152x - 378y = -74$ …(i)

and $-378x + 152y = -604$ …(ii)

Adding (i) and (ii), we get

$-226x - 226y = -678$

$\Rightarrow x + y = 3$ …(iii)

Subtracting (ii) from (i), we get

$530x - 530y = 530$

$\Rightarrow x - y = 1$ …(iv)

Adding (iii) and (iv), we get

$2x = 4$

$\Rightarrow x = 2$

Substituting $x = 2$ in (iii), we get

$2 + y = 3$

$\Rightarrow y = 1$

Thus, $x = 2$ and $y = 1$. **Ans.**

25. Determine the value of *k*, for which the given system of equations has infinitely many solutions:

$$kx + 3y = k - 3 \text{ and } 12x + ky = k.$$

[NCERT]

Sol. Given,

$kx + 3y - (k - 3) = 0$ …(i)

and $12x + ky - k = 0$ …(ii)

For this pair of equations to have infinite number of solutions, it must satisfy the condition of

$\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$

$\Rightarrow k^2 = 12(k - 3)$

$\Rightarrow k^2 - 12k + 36 = 0$

$\Rightarrow k^2 - 6k - 6k + 36 = 0$

$\Rightarrow k(k - 6) - 6(k - 6) = 0$

$\Rightarrow (k - 6)^2 = 0$

$\Rightarrow k = 6$

Also $k^2 = 36$

$k = \pm 6$

Common value $k = 6$

Thus, the pair of equations will have infinite number of solutions for $k = 6$. **Ans.**

26. Determine the value of *k*, for which the given system of equations has no solution:

$$3x + y = 1 \text{ and } (2k - 1)x + (k - 1)y = 2k + 1.$$

[NCERT]

Sol. Given, $3x + y = 1$ …(i)

and $(2k - 1)x + (k - 1)y = 2k + 1$ …(ii)

For this pair of equations not to have any solution, it must satisfy the condition of

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$\Rightarrow 3(k - 1) = 2k - 1$

$\Rightarrow 3k - 3 - 2k + 1 = 0$

$\Rightarrow k - 2 = 0$

$\Rightarrow k = 2$

Thus, the pair of equations will have no solution for $k = 2$. **Ans.**

27. For what value of *k* will the system of equations $x + 2y = 5$ and $3x + ky - 15 = 0$ has (i) a unique solution and (ii) infinite solutions?

Sol. Given, $x + 2y - 5 = 0$ ...(i)

and $3x + ky - 15 = 0$ ...(ii)

(i) The system of equations will have a unique solution, if

$\frac{1}{3} \neq \frac{2}{k}$

or $k \neq 6$ **Ans.**

(ii) The system of equations will have infinite solutions, if

$\frac{1}{3} = \frac{2}{k} = \frac{-5}{-15}$

or $\frac{1}{3} = \frac{2}{k} = \frac{1}{3}$

Thus, $k = 6$. **Ans.**

28. Determine the value of a and b for which the given system of equations has infinitely many solutions : $2x + 3y = 7$ and $2ax + (a + b)y = 28$.

Sol. Given, $2x + 3y - 7 = 0$ …(i)

and $2ax + (a + b)y - 28 = 0$ …(ii)

For this pair of equations to have infinite number of solution, it must satisfy the condition of

$\frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{7}{28}$

$\Rightarrow \frac{1}{\alpha} = \frac{3}{\alpha+\beta} = \frac{1}{4}$

$\Rightarrow a + b = 12$ …(iii)

and $a = 4$

From equation (iii),

$4 + b = 12$

$\Rightarrow b = 8$

Thus, the pair of equations will have infinite number of solutions for $a = 4$ and $b = 8$. **Ans.**

29. For what value of k will the system of equations $(3k + 1)x + 3y - 2 = 0$ and $(k2 + 1)x + (k - 2)y - 5 = 0$ has no solution?

Sol. Given, $(3k + 1)x + 3y - 2 = 0$ ...(i)

and $(k^2 + 1)x + (k - 2)y - 5 = 0$ ...(ii)

The system of equations will have no solution, if

$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{2}{5}$

$\Rightarrow (3k + 1)(k - 2) = 3(k^2 + 1)$

$\Rightarrow 3k^2 + k - 6k - 2 = 3k^2 + 3$

$\Rightarrow -5k = 5$

$\Rightarrow k = -1$

Clearly, for $k = -1$, $\frac{3}{k-2} \neq \frac{2}{5}$

Thus, for $k = -1$ the pair of equations will have no solution. **Ans.**

30. For what value of *k* will the system of equations $4x + ky + 8 = 0$ and $2x + 2y + 2 = 0$ has a unique solution?

[NCERT]

Sol. The system of equations will have a unique solution, if

$\frac{4}{2} \neq \frac{k}{2}$

or $k \neq 4$. **Ans.**

31. For what value of a will the system of equations $ax + 3y = a - 3$ and $12x + ay = a$ has no solution?

[Board Question]

Sol. The system of equations will have no solution, if

$\frac{\alpha}{12} = \frac{3}{\alpha} \neq \frac{\alpha-3}{\alpha}$

$\Rightarrow a^2 = 36$

$a = \pm 6$

Also $3a \neq a(a - 3)$

$\Rightarrow 3a \neq a^2 - 3a$

$\Rightarrow a^2 \neq 6a$

$\Rightarrow a \neq 6$

Since a = 6 and $a \neq 6$ is not possible, so for a = – 6, the pair of equations will have no solution.

Ans.

Short Answer Type Questions

32. Solve for x and y: $ax + by - a + b = 0$ and $bx - ay - a - b = 0$.

Sol. The equations $ax + by - a + b = 0$ and $bx - ay - a - b = 0$ can be written as

$ax + by = a - b$ …(i)

and $bx - ay = a + b$ …(ii)

Multiplying equation (i) with a and equation (ii) with b, we have

$a^2x + aby = a^2 - ab$ …(iii)

$b^2x - aby = ab + b^2$ …(iv)

On adding equation (iii) and (iv), we get

$(a^2 + b^2)x = a^2 + b^2$

$\Rightarrow x = 1$

Substituting $x = 1$ in equation (i), we get

$a + by = a - b$

$\Rightarrow by = -b$

$\Rightarrow y = -1$

Thus, $x = 1$ and $y = -1$. **Ans.**

33. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

[Board Question]

Sol. Given, $x + 2y = 5$

$3x + ky + 15 = 0$

Comparing above equations with

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

We get, $a_1 = 1$, $b_1 = 2$, $c_1 = -5$

$a_2 = 3$, $b_2 = k$, $c_3 = 15$

Condition for the pair of equations to have unique solution is

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{1}{3} \neq \frac{2}{k}$

$\Rightarrow k \neq 6$

k can have any value except 6. **Ans.**

34. Solve for x and y : $0.4x - 1.5y = 6.5$ and $0.3x + 0.2y = 0.9$.

Sol. Given,

$0.4x - 1.5y = 6.5$

and $0.3x + 0.2y = 0.9$

Multiplying given equations by 10, we get

$4x - 15y = 65$ ...(i)

and $3x + 2y = 9$ ...(ii)

Now, multiplying equation (i) with 2 and equation (ii) with 15, we get

$8x - 30y = 130$ …(iii)

and $45x + 30y = 135$ …(iv)

Adding equations (iii) and (iv), we get

$53x = 265$

$\Rightarrow x = 5$

Substituting $x = 5$ in equation (ii), we get

$3(5) + 2y = 9$

$\Rightarrow 15 + 2y = 9$

$\Rightarrow 2y = -15 + 9$

$= -6$

$\Rightarrow y = -3$

Thus, $x = 5$ and $y = -3$. **Ans.**

35. Solve for *x* and *y*:

$$\sqrt{2}x - \sqrt{3}y = 0 \text{ and } \sqrt{5}x + \sqrt{2}y = 0.$$

Sol. Given, $\sqrt{2}x - \sqrt{3}y = 0$ …(i)

and $\sqrt{5}x + \sqrt{2}y = 0$ …(ii)

Multiplying equation (i) with $\sqrt{2}$ and equation (ii) with $\sqrt{3}$, we get

$2x - \sqrt{6}y = 0$ …(iii)

and $\sqrt{15}x + \sqrt{6}y = 0$ …(iv)

Adding equations (iii) and (iv), we get

$(2 + \sqrt{15})x = 0$

$\Rightarrow x = 0$

Substituting $x = 0$ in equation (i), we get

$\Rightarrow 0 \times \sqrt{2} - \sqrt{3}y = 0$

$\Rightarrow y = 0$

Thus, $x = y = 0$. **Ans.**

36. Solve for *x* and *y*:

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$

and $(a + b)(x + y) = a^2 + b^2$.

[Board Question]

Sol. Given,

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$ …(i) and $(a + b)(x + y) = a^2 + b^2$

or $(a + b)x + (a + b)y = a^2 + b^2$ …(ii)

Subtracting equation (ii) from equation (i), we get

$(a - b)x - (a + b)x = a^2 - 2ab - b^2 - a^2 - b^2$

$\Rightarrow (a - b - a - b)x = -2ab - 2b^2$

$\Rightarrow -2bx = -2b(a + b)$

$\Rightarrow x = a + b$

Substituting $x = a + b$ in equation (i), we get

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$\Rightarrow a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$\Rightarrow (a + b)y = -2ab$

$\Rightarrow y = -\frac{2ab}{a+b}$

Thus, $x = a + b$ and $y = -\frac{2ab}{a+b}$. **Ans.**

37. Determine the value of *a* and *b* for which the given system of equations has infinitely many solutions:

$$(2a - 1)x + 3y - 5 = 0 \text{ and } 3x + (b - 1)y - 2 = 0.$$

Sol. Given,

$(2a - 1)x + 3y - 5 = 0$ …(i)

and $3x + (b - 1)y - 2 = 0$ …(ii)

For this pair of equations to have infinite number of solutions, it must satisfy the condition of

$\frac{2a-1}{3} = \frac{3}{b-1} = \frac{5}{2}$

$\Rightarrow 2(2a - 1) = 15$

$\Rightarrow 4a - 2 = 15$

$\Rightarrow 4a = 17$

$\Rightarrow a = 4\frac{1}{4}$

and $5(b - 1) = 6$

$\Rightarrow 5b - 5 = 6$

$\Rightarrow 5b = 11$

$\Rightarrow b = 2\frac{1}{5}$

Thus, the pair of equations will have infinite number of solutions for $a = 4\frac{1}{4}$ and $b = 2\frac{1}{5}$.

Ans.

38. Find the value of *k* for which the following pair of linear equations have infinitely many solutions.

$$\mathbf{2x + 3y = 7, (k + 1)x + (2k - 1)y = 4k + 1.}$$

[Board Question]

Sol. Given,

$$2x + 3y = 7 \text{ and } (k + 1)x + (2k - 1)y = 4k + 1$$

On comparing above equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 2, b_1 = 3, c_1 = -7,$

$a_2 = k + 1, b_2 = 2k - 1, c_2 = -(4k + 1)$

For infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \frac{2}{k+1} = \frac{3}{2k-1} = \frac{-7}{-(4k+1)}$

$\Rightarrow 2(2k - 1) = 3(k + 1)$

$\Rightarrow 4k - 2 = 3k + 3$

$\Rightarrow k = 5$

or $3(4k + 1) = 7(2k - 1)$

$\Rightarrow k = 5$

Hence, $k = 5$. **Ans.**

39. Determine the values of *a* and *b* for which the given system of equations has infinitely many solutions: $2x + 3y = 7$ and $(a - b)x + (a + b)y = 3a + b - 1$.

[NCERT]

Sol. Given, $2x + 3y = 7$ …(i)

and $(a - b)x + (a + b)y = 3a + b - 1$ …(ii)

For this pair of equations to have infinite number of solutions, it must satisfy the condition of

$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-1}$

$\Rightarrow 2(a + b) = 3(a - b)$

$\Rightarrow 2a + 2b = 3a - 3b$

$\Rightarrow a = 5b$ …(iii)

and $2(3a + b - 1) = 7(a - b)$

$\Rightarrow 6a + 2b - 2 = 7a - 7b$

$\Rightarrow a - 9b = -2$ …(iv)

Substituting $a = 5b$ in equation (iv), we get

$5b - 9b = -2$

$\Rightarrow -4b = -2$

$\Rightarrow 2b = 1$

$\Rightarrow b = \frac{1}{2}$

Thus, $a = 5\left(\frac{1}{2}\right) = \frac{5}{2}$

Thus, the pair of equations will have infinite number of solutions for $a = \frac{5}{2}$ and $b = \frac{1}{2}$. **Ans.**

40. Determine the value of *a* for which the given system of equations has infinitely many solutions:

$$2x + 3y = 7 \text{ and } (a - 1)x + (a + 2)y = 3a.$$

[Board Question]

Sol. Given, $2x + 3y = 7$ …(i)

and $(a - 1)x + (a + 2)y = 3a$ …(ii)

For this pair of equations to have infinite number of solutions, it must satisfy the condition of

$\frac{2}{a-1} = \frac{3}{a+2} = \frac{7}{3a}$

$\Rightarrow 2(a + 2) = 3(a - 1)$

$\Rightarrow 2a + 4 = 3a - 3$

$\Rightarrow a = 7$

Also $\frac{3}{a+2} = \frac{7}{3a}$

$\Rightarrow 9a = 7a + 14$

$\Rightarrow 2a = 14$

$\Rightarrow a = 7$

Thus, the pair of equations will have infinite number of solutions for $a = 7$.**Ans.**

41. $7x - 5y - 4 = 0$ is given. Write another linear equation, so that the lines represented by the pair are:

[Board Question]

(i) Intersecting (ii) Coincident (iii) Parallel.

Sol. Given equation is,

$7x - 5y - 4 = 0$

(i) $7x + 3y + 2 = 0$ $\left[\because \text{ Here, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$

(ii) $14x - 10y - 8 = 0$ $\left[\because \text{ Here, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right]$

(iii) $7x - 5y + 3 = 0$ $\left[\because \text{ Here, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right]$

42. Find *c* if the system of equations *cx* + 3*y* + (3 – *c*) = 0, 12*x* + *cy* – *c* = 0 has infinitely many solutions ?

[Board Question]

Sol. The given equations are

$cx + 3y + (3 - c) = 0$

and $12x + cy - c = 0$

On comparing with equation $a_1x + b_1y + c_1 = 0$ and equation $a_2x + b_2y + c_2 = 0$, we get

$a_1 = c, b_1 = 3, c_1 = 3 - c$

and $a_2 = 12, b_2 = c, c_2 = -c$

For infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

or $\frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$

$\frac{c}{12} = \frac{3}{c}$ or $\frac{3}{c} = \frac{3-c}{-c}$

$\frac{c}{12} = \frac{3}{c}$	or $\frac{3}{c} = \frac{3-c}{-c}$
$\Rightarrow c^2 = 36$	$\Rightarrow -3c = 3c - c^2$
$\Rightarrow c = \pm 6$	$\Rightarrow -6c = -c^2$
	$\Rightarrow c^2 - 6c = 0$
	$\Rightarrow c(c - 6) = 0$
	$\Rightarrow c = 0$ or $c = 6$

Since $c = 6$ satisfies both the cases

So, $c = 6$. **Ans.**

43. Determine the value of *k* for which the given system of equations has infinitely many solutions:

$$x + (k + 1)y = 5 \text{ and } (k + 1)x + 9y = (8k - 1).$$

Sol. Given, $x + (k + 1)y = 5$ …(i)

and $(k + 1)x + 9y = (8k - 1)$ …(ii)

For this pair of equations to have infinite number of solutions, it must satisfy the condition of

$\frac{1}{k+1} = \frac{k+1}{9} = \frac{5}{8k-1}$

$\Rightarrow \frac{1}{k+1} = \frac{5}{8k-1}$

$\Rightarrow 5(k + 1) = 8k - 1$

$\Rightarrow 5k + 5 = 8k - 1$

$\Rightarrow 3k = 6$

$\Rightarrow k = 2$

and $\frac{1}{k+1} = \frac{k+1}{9}$

$(k + 1)^2 = 9$

$k + 1 = \pm 3$

$\Rightarrow k = 2$

or $k = -4$

(Rejected as it doesn't satisfy the condition)

Thus, the pair of equations will have infinite number of solutions for $k = 2$.**Ans.**

44. Solve for *x* and *y*:

$$\frac{bx}{a} - \frac{ay}{b} + a + b = 0 \text{ and } bx - ay + 2ab = 0.$$

Sol. Given,

$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$

$\Rightarrow b^2x - a^2y = -ab(a + b)$ …(i)

and $bx - ay + 2ab = 0$

$\Rightarrow abx - a^2y = -2a^2b$ …(ii)

[Multiplying by 'a' on both sides]

Subtracting (ii) from (i), we get

$b^2x - abx = -ab(a + b) - (-2a^2b)$

$\Rightarrow (b - a)bx = -a^2b - ab^2 + 2a^2b$

$\Rightarrow (b - a)bx = a^2b - ab^2$

$\Rightarrow (b - a)bx = ab(a - b)$

$\Rightarrow (b - a)x = -a(b - a)$

$\Rightarrow x = -a$

Substituting with $x = -a$ in equation (ii), we get

$ab(-a) - a^2y = -2a^2b$

$\Rightarrow -a^2b - a^2y = -2a^2b$

$\Rightarrow -a^2y = -a^2b$

$\Rightarrow y = b$

Thus, $x = -a$ and $y = b$. **Ans.**

45. Solve for *x* and *y* :

$$\frac{x}{a} + \frac{y}{b} = 2 \text{ and } ax - by = a2 - b2.$$

Sol. Given,

$\frac{x}{a} + \frac{y}{b} = 2$

or $bx + ay = 2ab$ …(i)

and $ax - by = a^2 - b^2$ …(ii)

Multiplying (i) with b and (ii) with a, we have

$b^2x + aby = 2ab^2$ …(iii)

$a^2x - aby = a(a^2 - b^2)$ …(iv)

Adding (iii) and (iv), we get

$(a^2 + b^2)x = 2ab^2 + a(a^2 - b^2)$

$\Rightarrow (a^2 + b^2)x = a(2b^2 + a^2 - b^2)$

$\Rightarrow (a^2 + b^2)x = a(a^2 + b^2)$

$\Rightarrow x = a$

Substituting $x = a$ in (ii), we get

$a^2 - by = a^2 - b^2$

$\Rightarrow -by = -b^2$

$\Rightarrow y = b$

Thus, $x = a$ and $y = b$. **Ans.**

46. For what value of k will the system of equations $kx + 3y = (2k + 1)$ and $2(k + 1)x + 9y = (7k + 1)$ has infinite solutions?

Sol. Given, $kx + 3y - (2k + 1) = 0$ ...(i)

and $2(k +1)x + 9y - (7k + 1) = 0$ ...(ii)

For pair of equations to have infinite number of solutions, it must satisfy the condition of

$\frac{k}{2(k+1)} = \frac{3}{9} = \frac{2k+1}{7k+1}$

$\Rightarrow \frac{3}{9} = \frac{2k+1}{7k+1}$

$21k + 3 = 18k + 9$

$3k = 6$

$k = 2$

Also, $9k = 6(k + 1)$

$\Rightarrow 9k - 6k = 6$

$\Rightarrow 3k = 6$

$\Rightarrow k = 2$

Thus, the pair of equations will have infinite number of solutions for $k = 2$. **Ans.**

Long Answer Type Questions

47. Solve the following system of equations graphically : $2x + 3y = 2$ and $x - 2y = 8$.

Sol. Let XOX′ and YOY′ be the X-axis and Y-axis respectively.

Now, $2x + 3y = 2 \Rightarrow 2x = 2 - 3y$

$\Rightarrow x = \frac{2-3y}{2}$

If $y = 0$, $x = 1$

If $y = 2$, $x = -2$

If $y = -2$, $x = 4$

Therefore,

x	-2	1	4
y	2	0	-2

Thus, plotting the points P(– 2, 2), Q(1, 0) and R(4, – 2) on the graph paper, we get the graph of $2x + 3y = 2$, which is represented by PR.

Now, $x - 2y = 8 \Rightarrow 2y = x - 8$

$\Rightarrow y = \frac{x-8}{2}$

If $x = 4$, $y = -2$

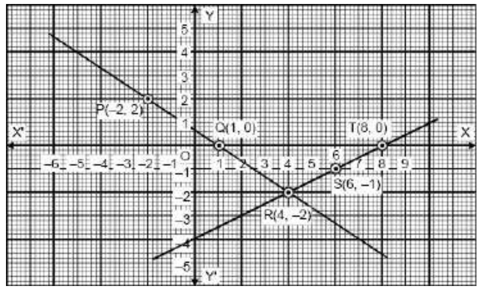
If $x = 6$, $y = -1$

If $x = 8$, $y = 0$

Therefore,

x	4	6	8
y	-2	-1	0

Thus, plotting the points R(4, – 2), S(6, – 1) and T(8, 0) on the graph paper, we get the graph of $x – 2y = 8$, which is represented by RT.



Hence, the solution is (4, – 2) as this is the point of intersection of both the lines. **Ans.**

48. Check graphically whether the pair of equations $3x – 2y + 2 = 0$ and $\frac{3}{2}x – y + 3 = 0$ is consistent. Also find the coordinates of the points where the graphs of the equations meet the *Y*-axis.

[Board Question]

Sol. Let XOX′ and YOY′ be the *X*-axis and Y-axis respectively.

Now, $3x – 2y + 2 = 0 \Rightarrow 3x = 2y – 2$

$\Rightarrow x = \frac{2(y-1)}{3}$

If $y = – 2, x = – 2$

If $y = 1, x = 0$

If $y = 4, x = 2$

Therefore,

x	– 2	0	2
y	– 2	1	4

Thus, plotting the points P(– 2, – 2), Q(0, 1) and R(2, 4) on the graph paper, we get the graph of $3x – 2y + 2 = 0$, which is represented by PR.

Now, $\frac{3}{2}x – y + 3 = 0 \Rightarrow 3x – 2y + 6 = 0$

$\Rightarrow y = \frac{3(x+2)}{2}$

If $x = 0$, $y = 3$

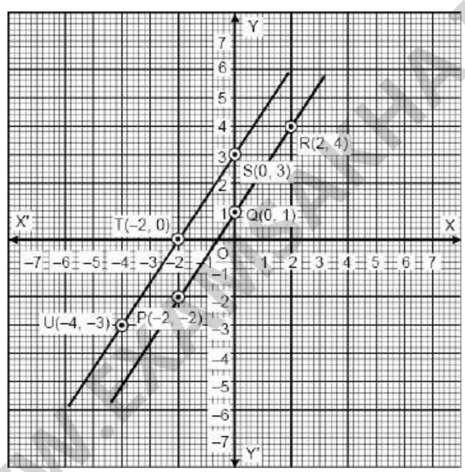
If $x = -2$, $y = 0$

If $x = -4$, $y = -3$

Therefore,

x	0	– 2	– 4
y	3	0	– 3

Thus, plotting the points S(0, 3), T(– 2, 0) and U(– 4, – 3) on the graph paper, we get the graph of $\frac{3}{2}x - y + 3 = 0$, which is represented by SU.



Since, the lines do not intersect each other at any point, so the pair of equations is inconsistent.

Also the lines representing the graphs of the equations, meet the *Y*-axis at points (0, 1) and (0, 3).**Ans.**

49. Solve the following system of linear equations graphically : $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Calculate the area of the region bounded by these lines and the *X*-axis.

[Board Question]

Sol. Let XOX′ and YOY′ be the X-axis and Y-axis respectively.
Now, $x - y + 1 = 0 \Rightarrow x = y - 1$

If $y = 1$, $x = 0$
If $y = 0$, $x = -1$
If $y = 3$, $x = 2$

Therefore,

x	0	– 1	2
y	1	0	3

Thus, plotting the points P(0, 1), Q(– 1, 0) and R(2, 3) on the graph paper, we get the graph of $x - y + 1 = 0$, which is represented by PR.

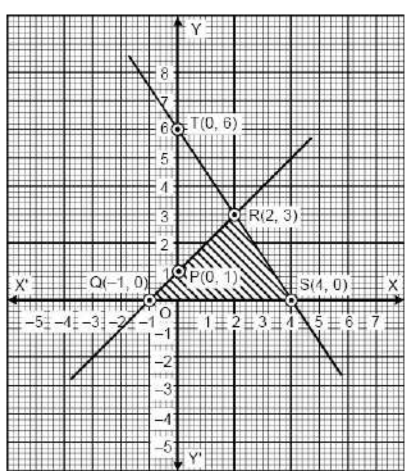
Now, $3x + 2y - 12 = 0 \Rightarrow 2y = 12 - 3x$

$\Rightarrow y = \frac{3(4-x)}{2}$
If $x = 4$, $y = 0$
If $x = 2$, $y = 3$
If $x = 0$, $y = 6$

Therefore,

x	4	2	0
y	0	3	6

Thus, plotting the points S(4, 0), R(2, 3) and T(0, 6) on the graph paper, we get the graph of $3x + 2y - 12 = 0$, which is represented by ST.



The solution of the pair of equations is given by (2, 3).

Hence, the coordinates of the triangle formed by these two lines and the X-axis are Q(– 1, 0), R(2, 3) and S(4, 0).

Area = $\frac{1}{2}$ × Base × Height

Now, Base = 4 – (– 1)

= 5

Height = 3

Thus, Area = $\frac{1}{2}$ × 5 × 3

= 7.5 sq. units. **Ans.**

50. The path of a train A is given by the equation $x + 2y - 4 = 0$ and of B by the equation $2x + 4y - 12 = 0$. Represent this situation graphically.

[NCERT]

Sol. Let XOX′ and YOY′ be the X-axis and Y-axis respectively.

Now, $x + 2y - 4 = 0$

$\Rightarrow x = 4 - 2y$

If $y = 2$, $x = 0$

If $y = 1$, $x = 2$

If $y = 0$, $x = 4$

Therefore,

x	0	2	4
y	2	1	0

Thus, plotting the points P(0, 2), Q(2, 1) and R(4, 0) on the graph paper, we get the graph of $x + 2y - 4 = 0$, which is represented by PR.

Now, $2x + 4y - 12 = 0$

$\Rightarrow 4y = 12 - 2x$

$\Rightarrow 2y = 6 - x$

$\Rightarrow y = \frac{6-x}{2}$

If $x = 6$, $y = 0$

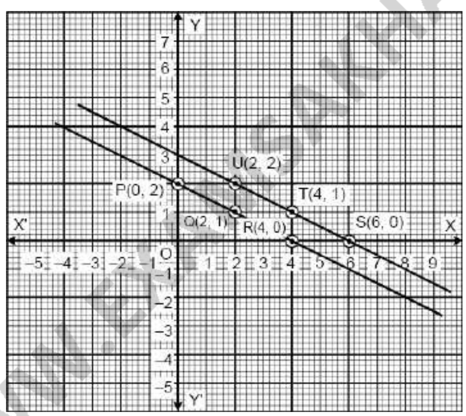
If $x = 4$, $y = 1$

If $x = 2$, $y = 2$

Therefore,

x	6	4	2
y	0	1	2

Thus, plotting the points S(6, 0), T(4, 1) and U(2, 2) on the graph paper, we get the graph of $2x + 4y = 12$, which is represented by SU.



Since the lines do not meet, it is evident that they are parallel and so do not have any solution.

Ans.

51. Draw the graph of the following pair of linear equations:

$$x + 3y = 6 \text{ and } 2x - 3y = 12$$

Find the ratio of the areas of the two triangles formed by first line, $x = 0$, $y = 0$ and second line, $x = 0$, $y = 0$.

[Board Question]

Sol. First line Second line

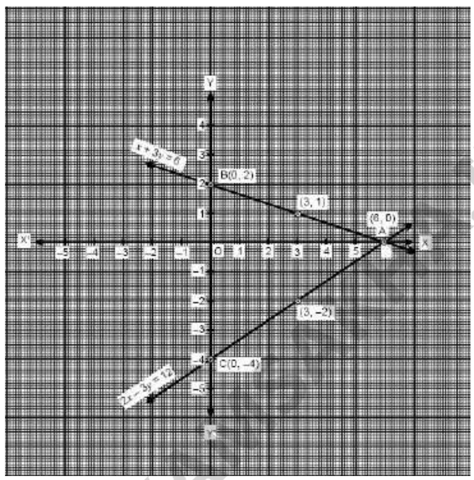
$x + 3y = 6$ $2x - 3y = 12$

$\Rightarrow x = 6 - 3y$ $\Rightarrow 2x = 12 + 3y$

$\Rightarrow x = \frac{12+3y}{2}$

x	6	3	0		x	6	3	0
y	0	1	2		y	0	– 2	– 4

(6, 0), (3, 1), (0, 2) (6, 0), (3, – 2), (0, – 4),



Area of triangle = $\frac{1}{2}$ × Base × Corresponding altitude

$\therefore \frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta AOC} = \frac{1/2 \times OA \times OB}{1/2 \times OA \times OC}$

$\Rightarrow \frac{OB}{OC} = \frac{2}{4}$

$= \frac{1}{2}$

$\therefore$ Required ratio = 1 : 2 **Ans.**

52. Draw graph of the following pair of linear equations:

$\mathbf{y = 2(x - 1)}$

$\mathbf{4x + y = 4}$

Also write the coordinate of the points where these lines meets *X*-axis and *Y*-axis.

[Board Question]

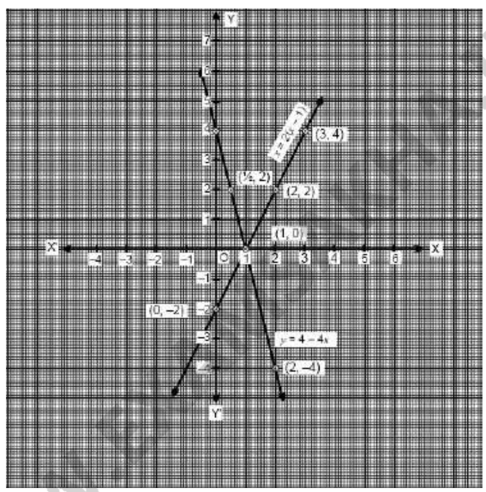
Sol. $y = 2(x - 1)$

So,	x	1	2	3	4
	y	0	2	4	6

and $4x + y = 4$

or $y = 4 - 4x$

x	1	2	1/2
y	0	– 4	2



Coordinates of point where lines meets

Line 1: X-axis = (1, 0)

and Y-axis = (0, – 2)

Line 2 : X-axis = (1, 0)

Y-axis = (0, 4) **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

53. Assertion: $x + y - 4 = 0$ and $2x + ky - 3 = 0$ has no solution if $k = 2$.

Reason: $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are consistent if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Ans. (b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

For the lines to be parallel,

$$\frac{a_1}{b_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The equations are

$x + y - 4 = 0$, and $2x + ky - 3 = 0$

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -4$,

$a_2 = 2$, $b_2 = k$, $c_2 = -3$,

$\therefore \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{-4}{-3} = \frac{4}{3}$

$\therefore \frac{1}{k} = \frac{1}{2}$

$\therefore k = 2$

So, the assertion is correct.

But, when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the equation has a unique solution.

So, the system is consistent and thus the reason is also correct.

54. Assertion: If the system of equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ has infinitely many solutions, then $2a – b = 0$.

Reason: The system of equations $3x – 5y = 9$ and $6x – 10y = 8$ has a unique solution.

Ans. (c) Assertion is true but the Reason is false.

Explanation :

Suppose the given system of equations has infinitely many solutions if

$\frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$

$\therefore \frac{1}{a} = \frac{3}{a+b} = \frac{1}{4}$

$\Rightarrow 3a = a + b$

$\Rightarrow 2a – b = 0$

Also clearly $a = 4$ and $a + b = 12$

$\Rightarrow b = 8$

$\therefore 2a – b = 8 – 8 = 0$

So, the assertion is true.

On the other hand, A pair of equations have a unique solution, if

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$,

$\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

So, reason is false. Hence correct option is (c).

Self-Assessment

55. Given the linear equation $2x + 3y – 8 = 0$, write another equation in two variables such that the graphical representation of the pair so formed is (i) intersecting, (ii) parallel and (iii) coincident.

[NCERT]

Ans. (i) $2x + 4y = 6$

(ii) $4x + 6y = 8$

(iii) $6x + 9y = 24$

56. Solve for x and y : $\frac{x}{6} - \frac{y}{3} = \frac{x}{12} - \frac{2y}{3} = 4.$

Ans. x = 16 and y = – 4.

57. Solve for x and y :

$$\frac{x}{a} + \frac{y}{b} = 2 \text{ and } ax - by = a^2 - b^2.$$

Ans. x = a and y = b.

58. Find the values of x and y in $2x + 3y = 8$ and $x - 2y + 3 = 0$ are:

Ans. x = 1, y = 2.

59. The values of x and y in $2x - 5y + 4 = 0$ and $2x + y - 8 = 0$ are:

Ans. x = 3, y = 2.

60. Find the values of x and y in $2x + 3y = 2$ and $x - 2y = 8$.

Ans. x = 4, y = – 2.

61. Determine the value of k for which the given system of equations has infinitely many solutions:

$$x + (k + 1)y = 5 \text{ and } (k + 1)x + 9y = 8k - 1.$$

Ans. The pair of equations will have infinite number of solutions for k = 2.

62. Determine the value of k for which the given system of equations have no solution:

$$3x - y - 5 = 0 \text{ and } 6x - 2y + k = 0 \text{ where } k \neq 0.$$

Ans. The pair of equations will have no solution for $k \neq -10$.

63. Determine the values of a and b for which the given system of equations have infinitely many solutions : 2x – 3y = 7 and (a + b)x – (a + b – 3)y = 4a + b.

Ans. The pair of equations will have infinite number of solutions for a = – 5 and b = – 1.

64. For what value of k, will the following pair of equations have infinitely many solutions :

$$2x + 3y = 7 \text{ and } (k + 2)x - 3(1 - k)\, y = 5k + 1$$

Ans. The given system of equations has infinitely many solutions when k = 4.

65. Solve the following linear equations algebraically : (a – b)x + (a + b)y = a2 – 2ab – b2 and (a + b) (x + y) = a2 + b2.

Ans. x = a + b and y = $-\frac{-2ab}{a+b}$

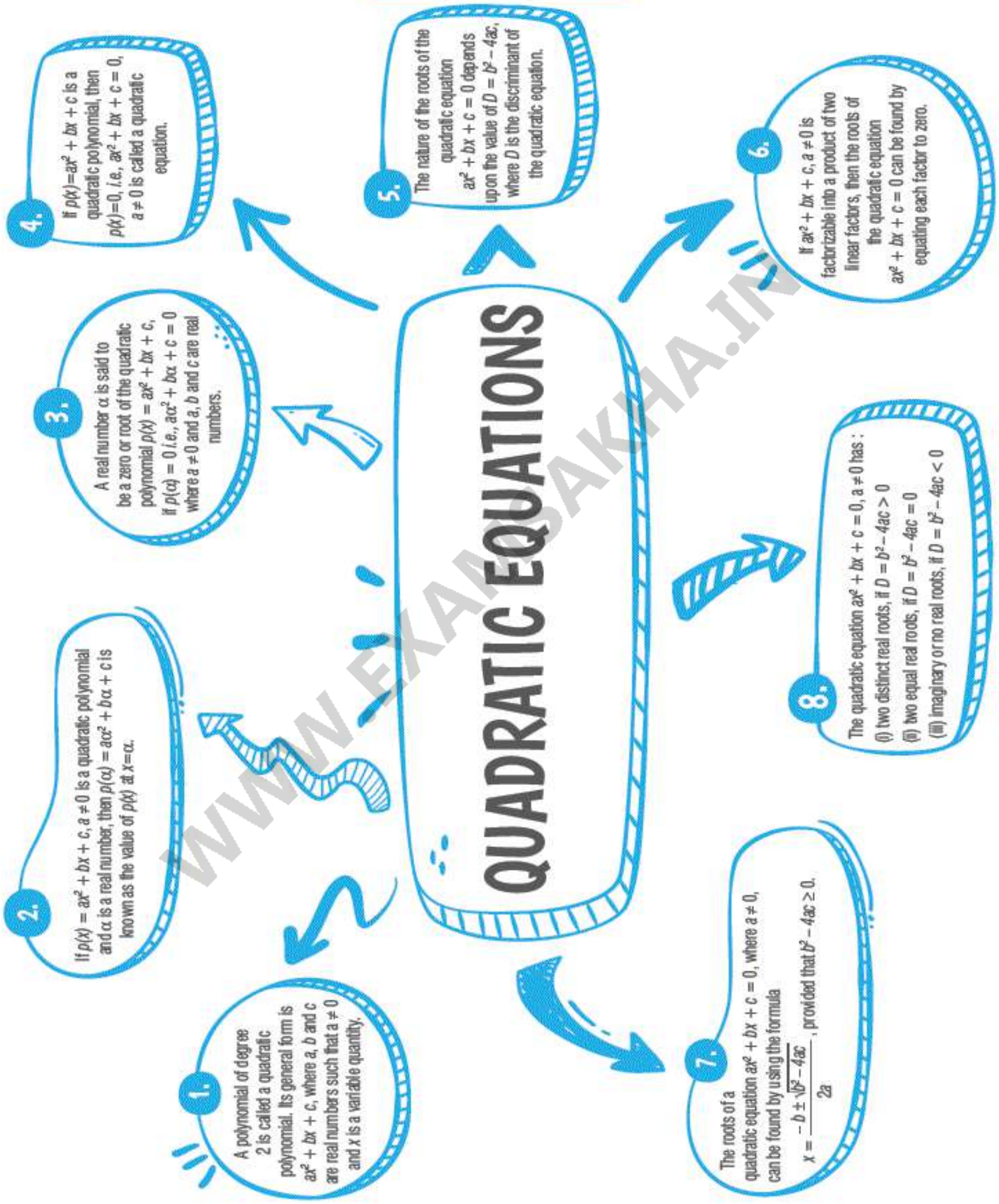
66. For what value of k will the system of equations 2x + (k – 2)y = k and 6x + (2k – 1)y = 2k + 5 has infinite solutions ?

Ans. The pair of equations will have infinite number of solutions for k = 5.

Quadratic Equations

Chapter 4

Basic Concepts



Multiple Choice Questions

1. If $y = 1$ is a common root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals:

[Board Question]

(a) 3

(b) $-\frac{7}{2}$

(c) 6

(d) – 3

Ans. (a) 3

Explanation :

The given equations are

$$ay^2 + ay + 3 = 0 \text{ and } y^2 + y + b = 0$$

Substituting $y = 1$ in them we have

$a(1)^2 + a(1) + 3 = 0$ and $(1)^2 + (1) + b = 0$

$\Rightarrow 2a + 3 = 0$ and $2 + b = 0$

$\Rightarrow a = -\frac{3}{2}$ and $b = -2$

Thus, $ab = \left(-\frac{3}{2}\right)(-2) = 3$

2. The roots of the equation $x^2 - 3x - 9 = 0$ are:

[Board Question]

(a) real and unequal
(b) real and equal
(c) roots are not equal
(d) imaginary roots

Ans. (a) real and unequal

Explanation :

$D = b^2 - 4ac$

$= (-3)^2 - 4(1)(-9)$

$= 9 + 36 = 45 > 0$

Thus, the roots are real and distinct.

3. The value of x in $x - \frac{18}{x} = 6$ is:

[Board Question]

(a) real and unequal

(b) real and equal

(c) roots are imaginary

(d) roots are not equal

Ans. (a) real and unequal

Explanation :

Given,

$x - \frac{18}{x} = 6$

or $x^2 - 6x - 18 = 0$

Thus, $D = b^2 - 4ac$

$= (-6)^2 - 4(-18)$

$= 36 + 72 = 108 > 0$

Thus, the roots are real and distinct.

4. The roots of the quadratic equation $x^2 - 0.04 = 0$ are:

[Board Question]

(a) real and unequal

(a) $\pm$ 0.2

(b) $\pm$ 0.02

(c) 0.4

(d) 2

Ans. (a) $\pm$ 0.2

Explanation :

Given : $x^2 - 0.04 = 0$

$\Rightarrow x^2 - (0.2)^2 = 0$

$\Rightarrow (x + 0.2)(x - 0.2) = 0$

$\Rightarrow x = -0.2, 0.2$

5. Value of *k* for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is:

(a) 0 only

(b) 4

(c) 8 only

(d) 0, 8

Ans. (d) 0, 8

Explanation :

For equal roots, D = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow k^2 - 4 \times 2\, k = 0$

$\Rightarrow k^2 - 8k = 0$

$\Rightarrow k(k - 8) = 0$

$\Rightarrow k = 0$ or 8.

6. If α and β are the zeroes of $x^2 + 5x + 8$, then the value of α + β is:

(a) 5

(b) 8

(c) – 5

(d) – 8

Ans. (c) – 5

Explanation :

Sum of roots = a + b = $\left(\frac{-5}{1}\right)$ = – 5

7. The discriminant of the quadratic equation $3x^2 – 4x – 2 = 0$ is equal to:

(a) 40

(b) 20

(c) 24

(d) 48

Ans. (a) 40

Explanation :

D = $b^2 – 4ac$
= $(– 4)^2 – 4 × 3 × (– 2)$
= 16 + 24
= 40.

8. If one root of the equation $2x^2 + 3x + c = 0$ is 0.5, then what is the value of c?

(a) – 1

(b) – 2

(c) – 3

(d) – 4

Ans. (b) – 2

Explanation :

$2x^2 + 3x + c = 0$

Put, $x = 0.5$
Þ $2(0.5)^2 + 3(0.5) + c = 0$

Þ 0.5 + 1.5 + c = 0
Þ c = – 2

9. The equation whose roots are twice the roots of the equation $x^2 - 2x + 4 = 0$ is:

(a) $x^2 - 2x + 4 = 0$

(b) $x^2 - 2x + 16 = 0$

(c) $x^2 - 4x + 8 = 0$

(d) $x^2 - 4x + 16 = 0$

Ans. (d) $x^2 - 4x + 16 = 0$

Explanation :

$\alpha + \beta = 2$ and $\alpha\beta = 4$

On taking $\alpha \rightarrow 2\alpha$, $\beta \rightarrow 2\beta$

$2\alpha + 2\beta = 4$ and $2\alpha . 2\beta = 4 \times 4 = 16$

$x^2 - 4x + 16 = 0$

10. The difference in the roots of the equation $2x^2 - 11x + 5 = 0$ is:

(a) 4.5

(b) 4

(c) 3.5

(d) 3

Ans. (a) 4.5

Explanation :

Let α and β be the root of this quadratic equation

Then, $\alpha + \beta = (11/2)$, $\alpha\beta = (5/2)$

We know that,

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= \left(\frac{11}{2}\right)^2 - 4\left(\frac{5}{2}\right)$

$= \frac{121}{4} - \frac{20}{2} = \left(\frac{9}{2}\right)^2$

Difference of roots = $(\alpha - \beta) = 4.5$

11. In the quadratic equation $x^2 + ax + b = 0$, *a* and *b* can take any value from the set {1, 2, 3, 4}. How many pairs of values of *a* and *b* are possible in order that the quadratic equation has real roots?

(a) 6

(b) 7

(c) 8

(d) 16

Ans. (b) 7

Explanation :

For real roots.

$B^2 - 4AC \geq 0$
So, by equation,

$a^2 - 4b \geq 0$

$a^2 \geq 4b$

When $b = 1$, then $a^2 \geq 4$

$a^2 - 4 \geq 0$
$b = 1$ and $a = 2, 3, 4$

When $b = 2$, then $a^2 - 8 \geq 0$
$b = 2$ and $a = 3, 4$

When $b = 3$, then $a^2 - 12 \geq 0$
$b = 3$ and $a = 4$

When $b = 4$, then $a^2 - 16 \geq 0$
$a = 4$ and $b = 4$

Hence, 7 pairs of values of *a* and *b* are possible.

12. For which value of k does the pair of equations $x^2 - y^2 = 0$ and $(x - k)^2 + y^2 = 1$ yield a unique positive solution of x?

(a) 2

(b) 0

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

Ans. (c) $\sqrt{2}$

Explanation :

$x^2 - y^2 = 0$ ...(i)

and $(x - k)^2 + y^2 = 1$

$x^2 + k^2 - 2kx + y^2 - 1 = 0$ ...(ii)

From equations (i) and (ii),

$2x^2 - 2kx + k^2 - 1 = 0$

For unique solution $b^2 - 4ac = 0$ must satisfy

$(-2k)^2 - 4 \times 2 \times (k^2 - 1) = 0$

Þ $4k^2 = 8$
Þ $k = \sqrt{2}$

13. If the root of the equation $Ax^2 - Bx + C = 0$ are -1 and 1, then which one of the following is correct?
(a) A and C are both zero
(b) A and B are both positive
(c) A and C are both negative
(d) A and C are of opposite sign
Ans. (d) A and C are of opposite sign

Explanation :

$Ax^2 – Bx + C = 0$
Since, the given roots are – 1 and 1
Sum of roots = – 1 + 1 = 0
Product of roots = 1 × (–1) = – 1
x^2 – (sum of root)x + product of roots = 0
$x^2 - \left(\frac{B}{A}\right)x + \left(\frac{C}{A}\right) = 0$

Þ $\frac{C}{A}$ = Product of roots = – 1

Þ C = –A

14. If the roots of the equation $(a^2 – bc)x^2 + 2(b^2 – ac)x + (c^2 – ab) = 0$ are equal, where $b \neq 0$, then which one of the following is correct?

(a) $a + b + c = abc$

(b) $a^2 + b^2 + c^2 = 0$

(c) $a^3 + b^3 + c^3 = 3a$

(d) $a^3 + b^3 + c^3 = 3abc$

Ans. (d) $a^3 + b^3 + c^3 = 3abc$

Explanation :

$(a^2 – bc)x^2 + 2(b^2 – ac)x + (c^2 – ab) = 0$
The given roots are equal, and then D must be zero
$[2(b^2 – ac)]^2 – 4(a^2 – bc)(c^2 – ab) = 0$

Þ $4b^4 – 12ab^2c + 4bc^3 + 4a^3b = 0$

Þ $a^3 + b^3 + c^3 = 3abc$

15. If one root of $(a^2 – 5a + 3)x^2 + (3a – 1)x + 2 = 0$ is twice the other, then what is the value of a?

(a) $\frac{2}{3}$

(b) $-\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{3}$

Ans. (a) $\frac{2}{3}$

Explanation :

Let α and 2α be the roots of the given equation

α + 2α = $\frac{-(3a-1)}{a^2-5a+3}$

Þ 3α = $\frac{-(3a-1)}{a^2-5a+3}$

Þ α = $\frac{-(3a-1)}{3(a^2-5a+3)}$ ...(i)

Also, (α)(2α) = $\frac{2}{a^2-5a+3}$

Þ α^2 = $\frac{1}{a^2-5a+3}$...(ii)

From equations (i) and (ii), we get

$\left[\frac{-(3a-1)}{3(a^2-5a+3)}\right]^2 = \frac{1}{a^2-5a+3}$

Þ $\frac{(3a-1)^2}{9(a^2-5a+3)^2} = \frac{1}{a^2-5a+3}$

Þ $(3a - 1)^2 = 9(a^2 - 5a + 3)$

Þ $9a^2 - 6a + 1 = 9a^2 - 45a + 27$

Þ $39a = 26$

Þ $a = \frac{26}{39} = \frac{2}{3}$.

16. If 3 is a root of the equation $kx^2 - kx - 3 = 0$, then the value of *k* is ....

(a) 3

(b) 1/2

(c) 4

(d) 2

Ans. (b) $\frac{1}{2}$

Explanation :

If 3 is the root of $kx^2 - kx - 3 = 0$, then it satisfy it *i.e.*,

Þ $9k - 3k - 3 = 0$

Þ $6k - 3 = 0$

Þ $6k = 3$

Þ $k = \frac{3}{6}$

Þ $k = \frac{1}{2}$.

17. If the zeroes of the quadratic equation $x^2 + (a + 1)x + b + 1 = 0$ are 2 and – 3, then the values of *a* and *b* are .... and .... respectively.

(a) 0, 7

(b) 0, – 7

(c) 2, 3

(d) none of these

Ans. (b) 0, – 7

Explanation :

If 2 and – 3 are the roots of $x^2 + (a + 1)x + b + 1 = 0$ then it satisfy it.

Put $x = 2$

$4 + (a + 1)2 + b + 1 = 0$

Þ $4 + 2a + 2 + b + 1 = 0$

Þ $2a + b + 7 = 0$

Þ $2a + b = -7$ ...(i)

and put $x = -3$

$9 - 3(a + 1) + b + 1 = 0$

Þ $9 - 3a - 3 + b + 1 = 0$

Þ $-3a + b + 7 = 0$

Þ $-3a + b = -7$ ...(ii)

On subtracting (i) from (ii), we get

Þ $-5a = 0$

Þ $a = 0$

Putting $a = 0$ in equation (i), we get $b = -7$

Hence a and b are 0 and – 7.

18. The roots of the equation $\sqrt{2x+9} + x$ = 13 are .... and ....

(a) – 8, – 20

(b) – 8, 20

(c) 8, 20

(d) 8, – 20

Ans. (c) 8, 20

Explanation :

$\sqrt{2x+9} + x = 13$

Þ $\sqrt{2x+9} = 13 - x$

On squaring both sides,

$2x + 9 = (13 - x)^2$

$2x + 9 = 169 + x^2 - 26x$

Þ $x^2 - 26x - 2x - 9 + 169 = 0$

Þ $x^2 - 28x + 160 = 0$

Þ $x^2 - 20x - 8x + 160 = 0$

Þ $x(x - 20) - 8(x - 20) = 0$

Þ $(x - 8)(x - 20) = 0$

$x = 8, x = 20.$

19. Which of the following is not a quadratic equation?

(a) $2(x + 1)^2 = 4x^2 - 2x + 1$

(b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x + \sqrt{3}x)^2 + x^2 = 3x^2 - 5x$

(d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

Ans. (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

Explanation :

Given that,

$(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

$\Rightarrow x^4 + 4x^2 + 4x^3 = x^4 + 3 + 4x^2$

$\Rightarrow 4x^3 - 3 = 0$

which is not of the form $ax^2 + bx + c$, $a \neq 0$.

Thus, the equation is not quadratic. This is a cubic equation.

20. Which of the following equation has 2 as a root?

(a) $x^2 - 4x + 5 = 0$

(b) $x^2 + 3x - 12 = 0$

(c) $2x^2 - 7x + 6 = 0$

(d) $3x^2 - 6x - 2 = 0$

Ans. (c) $2x^2 - 7x + 6 = 0$

Explanation :

Substituting $x = 2$ in $2x^2 - 7x + 6$, we get

$2(2)^2 - 7(2) + 6 = 2(4) - 14 + 6$

$= 8 - 14 + 6 = 14 - 14 = 0$

So, $x = 2$ is root of the equation $2x^2 - 7x + 6 = 0$.

21. Which one of the following is a quadratic equation?

(a) $(a+1)x^2 - \frac{3}{5}x = 11$, where $a\ ^{1} - 1$

(b) $(3 - x)^2 - 5 = x^2 + 2x + 1$

(c) $8x^3 - x^2 = (x - 1)^3$

(d) $-3x^2 = (2-x)\left(3x - \frac{1}{2}\right)$

Ans. (a) $(a+1)x^2 - \frac{3}{5}x = 11$, where *a* ¹ – 1

Explanation :

We will check for all the options

For (a), $(a+1)x^2 - \frac{3}{5}x = 11$

So, if *a* ¹ – 1, then coefficient of x^2 will not be zero.

So, it will be a quadratic equation and in rest of 3 options, on expanding coefficient of x^2 becomes zero.

22. Which of the following is a solution of quadratic equation $x^2 - b^2 = a(2x - a)$?

(a) $a + b$

(b) $2b - a$

(c) ab

(d) $\frac{a}{b}$

Ans. (a) $a + b$

Explanation :

$x^2 - b^2 = a(2x - a)$

Þ $x^2 - b^2 = 2ax - a^2$

Þ $x^2 - 2ax + a^2 - b^2 = 0$

Þ $x^2 - \{(a + b) + (a - b)\}\, x + (a^2 - b^2) = 0$

Þ $x^2 - \{(a + b) + (a - b)\}\, x + (a - b)(a + b) = 0$

Þ $x^2 - (a + b)x - (a - b)x + (a - b)(a + b) = 0$

Þ $x\{x – (a + b)\} – (a – b)\{x – (a + b)\} = 0$

Þ $\{x – (a + b)\}\{x – (a – b)\} = 0$

$x = (a + b)$ or $x = (a – b)$

23. The roots of the quadratic equation $x^2 – 3x – m(m + 3) = 0$, where *m* is a constant are:

(a) $m, m + 3$

(b) $– m, m + 3$

(c) $m, – (m + 3)$

(d) $– m, – (m + 3)$

Ans. (b) $– m, m + 3$

Explanation :

Given quadratic equation is

$x^2 – 3x – m(m + 3) = 0$

$\Rightarrow x^2 – \{(m + 3) – m\} x – m(m + 3) = 0$

$\Rightarrow x^2 – (m + 3)x + mx – m(m + 3) = 0$

$\Rightarrow x\{x – (m + 3)\} + m\{x – (m + 3)\} = 0$

$\Rightarrow (x + m)\{x – (m + 3)\} = 0$

So, $x = – m, m + 3$.

24. The general form of a quadratic equation is:

(a) $ax^2 + bx + c$

(b) $ax^2 + bx + c = 0$

(c) $a^2x + b$

(d) $ax^2 + bx + c = 0$, a ¹ 0

Ans. (d) $ax^2 + bx + c = 0$, a ¹ 0

Explanation :

By the definition of quadratic equation it is the form $ax^2 + bx + c$, *a* ¹ 0.

25. The number of possible solutions of a quadratic equation are:

(a) exactly two

(b) at most two

(c) at least two

(d) none of these

Ans. (b) at most two

Explanation :

A quadratic equation cannot have more than 2 solution *i.e.*, either 2, 1 or 0.

26. If a, b **are the roots of $x^2 + px + q = 0$, then the value of** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ **is:**

(a) $\frac{p^2 - 2q}{q}$

(b) $\frac{2q - p^2}{q}$

(c) $\frac{p^2 + 2q}{q}$

(d) none of these

Ans. (a) $\frac{p^2 - 2q}{q}$

Explanation :

Here, a + b = – *p*

and ab = *q*

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{p^2 - 2q}{q}$$

27. If the sum of the roots of an equation is 6 and one root is $3-\sqrt{5}$, then the equation is:

(a) $x^2 - 6x + 4 = 0$

(b) $x^2 - 4x + 6 = 0$

(c) $x^2 - 6x + 5 = 0$

(d) none of the above

Ans. (a) $x^2 - 6x + 4 = 0$

Explanation :

Here a + b = 6 and a = $3-\sqrt{5}$

So, b = 6 – a

= $6-3+\sqrt{5}$

= $3+\sqrt{5}$

Now, ab = $(3-\sqrt{5})(3+\sqrt{5})$ = 4

Required quadratic equation is $x^2 - 6x + 4 = 0$

28. The quadratic equation whose roots are $a, \frac{1}{a}$ is:

(a) $ax^2 - (a^2 + 1)x + a = 0$

(b) $ax^2 - (a^2 - 1)x + a = 0$

(c) $ax^2 - (a^2 - 1)x - a = 0$

(d) none of these

Ans. (a) $ax^2 - (a^2 + 1)x + a = 0$

Explanation :

Quadratic equation whose roots are $a, \frac{1}{a}$ be

$x^2 - \left(a+\frac{1}{a}\right)x+1$ = 0

Þ $x^2 - \left(\frac{a^2+1}{a}\right)x + 1$ = 0

Þ $ax^2 - (a^2 + 1)x + a = 0$.

29. Find the values of *k* for which the quadratic equation *kx*(*x* – 3) + 9 = 0 has real equal roots:

(a) $k = 0$ or $k = 4$

(b) $k = 1$ or $k = 4$

(c) $k = -3$ or $k = 3$

(d) $k = -4$ or $k = 4$

Ans. (a) $k = 0$ or $k = 4$

Explanation :

For real and equal roots D = $b^2 - 4ac = 0$

Given quadratic equation $kx^2 - 3kx + 9 = 0$

D = $(3k)^2 - 4(k)(9) = 0$

$\Rightarrow 9k^2 - 36k = 0$

$\Rightarrow 9k(k - 4) = 0$

$\Rightarrow k = 0$ or 4.

30. The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number:

(a) 3

(b) $\frac{1}{3}$

(c) both (a) and (c)

(d) none of these

Ans. (c) both (a) and (c)

Explanation :

Solution: Let number be x

$x + \frac{1}{x} = \frac{10}{3}$

$\Rightarrow \frac{x^2 + 1}{x} = \frac{10}{3}$

$\Rightarrow 3x^2 + 3 = 10x$

$\Rightarrow 3x^2 - 10x + 3 = 0$

$\Rightarrow 3x^2 - 9x - x + 3 = 0$

$\Rightarrow 3x(x - 3) - 1(x - 3) = 0$

$\Rightarrow (3x - 1)(x - 3) = 0$

$x = 3$ or $x = \frac{1}{3}$

31. The condition for the sum and the product of the roots of the quadratic equation $ax^2 - bx + c = 0$ to be equal, is:

(a) $b + c = 0$

(b) $b - c = 0$

(c) $a + c = 0$

(d) $a + b + c = 0$

Ans. (b) $b - c = 0$

Explanation :

∵ Sum of roots = Product of roots

$-\left(\frac{-b}{a}\right) = \frac{c}{a}$

$\Rightarrow b = c$

$\Rightarrow b - c = 0$

32. Find the value of $\sqrt{30 + \sqrt{30 + \sqrt{30 + \ldots\ldots \infty}}}$

(a) 6

(b) – 5

(c) Either (a) or (b)

(d) Neither (a) nor (b)

Ans. (a) 6

Explanation :

Let $y = \sqrt{30+\sqrt{30+\sqrt{30+\ldots\ldots\ldots\infty}}}$

$y = \sqrt{30+y}$

On squaring both sides,

$\Rightarrow y^2 = 30 + y$

$\Rightarrow y^2 - y - 30 = 0$

$\Rightarrow y^2 - 6y + 5y - 30 = 0$

$\Rightarrow y(y - 6) + 5(y - 6) = 0$

$\Rightarrow (y + 5)(y - 6) = 0$

$\Rightarrow y = -5$ or $y = 6$

But y must be positive

$y = 6$

33. The roots of $x^2 - (a + 1)x + b^2 = 0$ are equal. Then choose the correct value of *a* and *b* from the following options:

(a) 5, 2

(b) 3, 4

(c) 5, – 3

(d) 5, 4

Ans. (c) 5, – 3

Explanation :

The roots of $x^2 - (a + 1)x + b^2 = 0$ are equal

$\Rightarrow (a + 1)^2 - 4b^2 = 0$

⇒ $a + 1 = \pm 2b$

From the options

$a = 5$, $b = -3$ satifes the above relation.

34. The solution of the equation $x^2 + x + 1 = 1$ is:

(a) $x = 0, -1$

(b) $x = -1, 2$

(c) $x = 0, 1$

(d) cannot be determined

Ans. (a) $x = 0, -1$

Explanation :

Given equation is

$x^2 + x + 1 = 1$

⇒ $x^2 + x = 0$

⇒ $x(x + 1) = 0$

$x = 0$ or $x = -1$.

35. The roots of the equation $x^2 + 5x + 1 = 0$ are:

(a) $\frac{5+\sqrt{21}}{2}, \frac{5-\sqrt{21}}{2}$

(b) $\frac{-5-\sqrt{21}}{2}, \frac{5+\sqrt{21}}{2}$

(c) $\frac{-5+\sqrt{21}}{2}, \frac{-5-\sqrt{21}}{2}$

(d) $\frac{-5+\sqrt{29}}{2}, \frac{-5-\sqrt{29}}{2}$

Ans. (c) $\frac{-5+\sqrt{21}}{2}, \frac{-5-\sqrt{21}}{2}$

Explanation :

Given equation

$x^2 + 5x + 1 = 0$

By quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-5 \pm \sqrt{25-4}}{2}$

$\Rightarrow x = \frac{-5 \pm \sqrt{21}}{2}$

36. The age of a father is 25 years more than his son's age. The product of their ages is 84 in years. What will be son's age after 10 years?

(a) 3

(b) 28

(c) 13

(d) 18

Ans. (c) 13

Explanation :

Let age of son be x years

So, age of father be $(x + 25)$years

According to the question

$\Rightarrow x(x + 25) = 84$

$\Rightarrow x^2 + 25x – 84 = 0$

$\Rightarrow x^2 + 28x – 3x + 84 = 0$

$\Rightarrow (x + 28)(x – 3) = 0$

$\Rightarrow x = – 28$ or $x = 3$

$x = 3$

($\because$ Age can not be negative)

Son age after 10 years (3 + 10) = 13 years.

Very Short Answer Type Questions

37. Find the marks obtained by Neha in two subjects, if their average is 75 and their product is 5600.

Sol. Let the marks obtained by Neha in the two subjects be x and y.

Hence, $xy = 5600$ and $\frac{x+y}{2} = 75$

Thus, $x + y = 150$

or $x = 150 - y$

and $xy = 5600$

$\Rightarrow (150 - y)y = 5600$

$\Rightarrow 150y - y^2 = 5600$

$\Rightarrow y^2 - 150y + 5600 = 0$

$\Rightarrow y^2 - 80y - 70y + 5600 = 0$

$\Rightarrow y(y - 80) - 70(y - 80) = 0$

$\Rightarrow (y - 80)(y - 70) = 0$

$\Rightarrow y = 70$ or 80

and $x = 80$ or 70 **Ans.**

38. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

[Board Question]

Sol. Given, $2x^2 - 4x + 3 = 0$

Comparing it with quadratic equation

$ax^2 + bx + c = 0$

Here, $a = 2$, $b = -4$ and $c = 3$

□ $D = b^2 - 4ac = (-4)^2 - 4 \times (2)(3) = 16 - 24 = -8 < 0$

Hence, D < 0 this shows that roots are imaginary. **Ans.**

39. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

[Board Question]

Sol. The given equation is $x^2 + 4x + k = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 4$ and $c = k$

For real roots, D ≥ 0

or $b^2 - 4ac \geq 0$

$16 - 4k \geq 0$

or $k \leq 4$

∴ For $k \leq 4$, equation $x^2 + 4x + k$ will have real roots. **Ans.**

40. The solution of a quadratic equation is as follows:

***x* =** $\frac{8 \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$.

Then, find the quadratic equation.

Sol. Given, $x = \frac{8 \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$

Thus $a = 3$, $b = -8$ and $c = 2$

The equation is given by $ax^2 + bx + c = 0$

Hence, equation is $3x^2 - 8x + 2 = 0$.**Ans.**

41. Solve: $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0.$

Sol. Given, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow x = \frac{2\sqrt{2} \pm \sqrt{(-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})}}{2(\sqrt{3})}$

$\left[\text{applying the formula : } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$

$\Rightarrow x = \frac{2\sqrt{2} \pm \sqrt{8+24}}{2\sqrt{3}} = \frac{2\sqrt{2} \pm \sqrt{32}}{2\sqrt{3}} = \frac{\sqrt{2} \pm 2\sqrt{2}}{\sqrt{3}} = \sqrt{6}$ and $-\sqrt{\frac{2}{3}}$. **Ans.**

42. Solve: $abx^2 + (b^2 - ac)x - bc = 0$.

Sol. Given,

$abx^2 + (b^2 - ac)x - bc = 0$

$\Rightarrow abx^2 + b^2x - acx - bc = 0$

$\Rightarrow bx(ax + b) - c(ax + b) = 0$

$\Rightarrow (ax + b)(bx - c) = 0$

$\Rightarrow x = -\frac{b}{a}$ and $\frac{c}{b}$.

Ans.

43. Solve for *x*: $\sqrt{6x+7} - (2x - 7) = 0$

Sol. We have,

$\sqrt{6x+7} - (2x - 7) = 0$

$\sqrt{6x+7} = (2x - 7)$

On squaring both sides, we get

$(\sqrt{6x+7})^2 = (2x - 7)^2$

$\Rightarrow 6x + 7 = 4x^2 + 49 - 28x$

$\Rightarrow 4x^2 + 42 - 34x = 0$

$\Rightarrow 2x^2 - 17x + 21 = 0$

$\Rightarrow 2x^2 - 14x - 3x + 21 = 0$

$\Rightarrow 2x(x - 7) - 3(x - 7) = 0$

$\Rightarrow (2x - 3)(x - 7) = 0$

$\Rightarrow x = \frac{3}{2}$ or 7

$\therefore x = 7$ (as $x = 3/2$ doesn't satisfy the given equation)**Ans.**

44. Solve: $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$.

Sol. Given,

$12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$\Rightarrow 12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$\Rightarrow 3ax(4bx - 3a) + 2b(4bx - 3a) = 0$

$\Rightarrow (3ax + 2b)(4bx - 3a) = 0$

$\Rightarrow x = -\frac{2b}{3a}$ and $\frac{3a}{4b}$. **Ans.**

45. Find the value of *k* for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.

Sol. Given equation is,

$x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow x^2 + 2kx + k(k - 1) + 2 = 0$

Here $a = 1$, $b = 2k$ and $c = k(k - 1) + 2$

For real and equal roots

$b^2 - 4ac = 0$

$\Rightarrow (2k)^2 - 4.1.[(k(k - 1) + 2)] = 0$

$\Rightarrow 4k^2 - 4(k^2 - k + 2) = 0$

$\Rightarrow 4k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow 4k = 8$

$\Rightarrow k = \frac{8}{4} = 2$. **Ans.**

46. A teacher on attempting to arrange the students for mass drill in the form of a solid square found that 24 students were left. When he increased the square by one row and one column, he was short of 25 students. Find the number of students.

Sol. Let the number of rows be *x* and the number of students in each row also be *x*.

Thus $x^2 + 24 = (x + 1)^2 - 25$

$\Rightarrow (x + 1)^2 - x^2 = 24 + 25$

$\Rightarrow (x + 1 – x)(x + 1 + x) = 49$

$\Rightarrow 2x + 1 = 49$
$\Rightarrow 2x = 48$
$\Rightarrow x = 24$

Thus, the total number of students = $x^2 + 24$
= $(24)^2 + 24 = 24(24 + 1) = 24 \times 25 = 600$. **Ans.**

47. Solve : $\sqrt{2}x^2 + 7x + 5\sqrt{2}$ = 0.

[NCERT]

Sol. Given,
$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$
$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$
$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$
$\Rightarrow x = -\frac{5}{\sqrt{2}}$ and $-\sqrt{2}.$

Ans.

48. Find the value of k for which the roots of the equation $3x^2 – 10x + k = 0$ are reciprocal of each other.

[Board Question]

Ans. The given equation is $3x^2 – 10x + k = 0$

On comparing it with $ax^2 + bx + c = 0$, we get
$a = 3$, $b = – 10$, $c = k$

Let the roots of the equation are α and $\frac{1}{\alpha}$

□ Product of the roots = $\frac{c}{a}$

$\therefore \alpha . \frac{1}{\alpha} = \frac{k}{3}$ or $k = 3$ **Ans.**

49. If the quadratic equation $px^2 – 2\sqrt{5}\, px + 15 = 0$, has two equal roots, then find the value of p.

Sol. The given quadratic equation is,

$px^2 – 2\sqrt{5}\, px + 15 = 0$

This equation is of the form

$ax^2 + bx + c = 0$

where, $a = p$, $b = -2\sqrt{5}\, p$, $c = 15$

We know , $D = b^2 - 4ac$

$= (-2\sqrt{5}p)^2 - 4 \times p \times 15$

$= 20p^2 - 60p$

$= 20p\,(p - 3)$

For real and equal roots, we must have

$D = 0 \Rightarrow 20p\,(p - 3) = 0$

$\Rightarrow p = 0, p = 3$

$p = 0$, is not possible as whole equation will be zero.

Hence, 3 is the required value of p. **Ans.**

50. Find two consecutive numbers whose squares have the sum 85.

Sol. Let the two numbers be x and $(x + 1)$.

Thus, $x^2 + (x + 1)^2 = 85$

$\Rightarrow x^2 + x^2 + 2x + 1 = 85$

$\Rightarrow 2x^2 + 2x - 84 = 0$

$\Rightarrow x^2 + x - 42 = 0$

$\Rightarrow x^2 + 7x - 6x - 42 = 0$

$\Rightarrow x(x + 7) - 6(x + 7) = 0$

$\Rightarrow (x + 7)(x - 6) = 0$

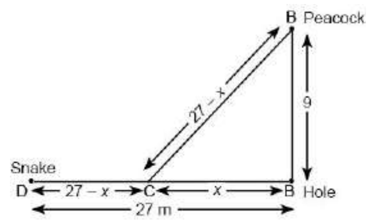
$\Rightarrow x = 6, -7$

Thus, the two numbers are either 6 and 7 or – 7 and – 6.**Ans.**

51. A peacock is sitting on the top of a pillar which is 9 m high. From a point, 27 m away from the bottom of a pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake, the

peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

Sol. Let the distance, where the snake is caught from the base of the pillar be x m. So the distance travelled by the snake before being caught is $(27 – x)$ m.



As the speed of the peacock and the snake is the same, so the distance travelled by the two is also same. Thus, the peacock travels $(27 – x)$ m from the top of the tower to catch the snake. Hence, $(27 – x)$ is the hypotenuse. Applying Pythagoras theorem,

$(9)^2 + x^2 = (27 – x)^2$

$\Rightarrow x^2 + 81 = 729 + x^2 – 54x$

$\Rightarrow 54x = 729 – 81$

$\Rightarrow 54x = 648$

$\Rightarrow x = 12$ m

Hence, the snake is caught 12 m from the pillar. **Ans.**

52. Show that $x = – 2$ is a solution of $3x^2 + 13x + 14 = 0$.

Sol. The given equation is,

$$3x^2 + 13x + 14 = 0$$

Substituting $x = – 2$, we have

L.H.S. $= 3(– 2)^2 + 13(– 2) + 14$

$= 12 – 26 + 14$

$= 0 =$ R.H.S.

Thus, $x = – 2$ is a solution of $3x^2 + 13x + 14 = 0$.

Hence Proved.

53. Solve the following quadratic equation for *x* :

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

Sol. $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$\Rightarrow x = \frac{6b^2 \pm \sqrt{36b^4 + 4\times9\times(a^4 - b^4)}}{2\times9}$

$\left[\because x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$

$\Rightarrow x = \frac{6b^2 \pm \sqrt{36b^4 + 36a^4 - 36b^4}}{2\times9}$

$\Rightarrow x = \frac{6b^2 \pm \sqrt{36a^4}}{2\times9}$

$\Rightarrow x = \frac{6b^2 \pm 6a^2}{2\times3\times3}$

$\Rightarrow x = \frac{b^2 \pm a^2}{3}$

$\Rightarrow x = \frac{b^2 + a^2}{3}, \frac{b^2 - a^2}{3}$.

54. Write the sum of the real roots of the equation $x^2 + |x| - 6 = 0$.

Sol. $x^2 + |x| - 6 = 0$

$\Rightarrow x^2 + x - 6 = 0 (\because |x| = x)$

$\Rightarrow x^2 + 3x - 2x - 6 = 0$

$\Rightarrow x(x + 3) - 2(x + 3) = 0$

$\Rightarrow (x - 2)(x + 3) = 0$

$\Rightarrow x = -3, 2$

Sum of roots = – 3 + 2 = – 1.

55. In there any real value of *a* for which the equation $x^2 + 2x + (a^2 + 1) = 0$ has real roots?

Sol. $x^2 + 2x + (a^2 + 1) = 0$

D = $(-b)^2 - 4ac$

= $(2)^2 - 4 \times 1\ (a^2 + 1)$

= $4 - 4a^2 - 4 = -4a^2$

For real value of x,

D ³ 0

But – $4a^2$ £ 0

So, it is not possible.

There is no real value of a.

56. Write the condition to be satisfied for which equation $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ have equal roots.

Sol. In $ax^2 + 2bx + c = 0$

$D_1 = (2b)^2 - 4 \times a \times c$

$= 4b^2 - 4ac$

∵ Roots are equal,

$D_1 = 0$

$4b^2 - 4ac = 0$

$\Rightarrow 4b^2 = 4ac$

$\Rightarrow b^2 = ac$

and in $bx^2 - 2\sqrt{ac}x + b = 0$

$D_2 = (-2\sqrt{ac})^2 - 4 \times b \times b$

$= 4ac - 4b^2$

∵ Roots are equal,

$D_2 = 0$

$4ac = 4b^2$

$\Rightarrow b^2 = ac$

The required condition is $b^2 = ac$.

Short Answer Type Questions

57. Solve: $\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}$, where $x \neq -1, \frac{1}{3}$.

[Board Question]

Sol. Given, $\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}$

$\Rightarrow \frac{6-(x+1)}{2(x+1)} = \frac{2}{3x-1}$

$\Rightarrow [6 - (x + 1)] (3x - 1) = 4(x + 1)$

$\Rightarrow 6(3x - 1) - (3x - 1) (x + 1) = 4(x + 1)$

$\Rightarrow 18x - 6 - (3x^2 - x + 3x - 1) = 4x + 4$

$\Rightarrow 14x - 10 - 3x^2 - 2x + 1 = 0$

$\Rightarrow -3x^2 + 12x - 9 = 0$

$\Rightarrow 3x^2 - 12x + 9 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow x(x - 3) - 1(x - 3) = 0$

$\Rightarrow (x - 3) (x - 1) = 0$

$\Rightarrow x = 1, 3$. **Ans.**

58. Solve: $\frac{4}{x} - 3 = \frac{5}{2x+3}$, where $x \neq 0, -\frac{3}{2}$.

[Board Question]

Sol. Given, $\frac{4}{x} - 3 = \frac{5}{2x+3}$

$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$

$\Rightarrow 4(2x + 3) - 3x(2x + 3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow -6x^2 - 6x + 12 = 0$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x + 2)(x - 1) = 0$

$\Rightarrow x = 1, -2$. **Ans.**

59. Solve for *x* :

[Board Question]

$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)}$ **= 0, *x* ≠ 3,** $-\frac{3}{2}$

Sol. We have,

$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow 2x(2x + 3) + (x - 3) + (3x + 9) = 0$

$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$

$\Rightarrow 4x^2 + 10x + 6 = 0$

$\Rightarrow 2x^2 + 5x + 3 = 0$

$\Rightarrow 2x^2 + 2x + 3x + 3 = 0$

$\Rightarrow 2x(x + 1) + 3(x + 1) = 0$

$\Rightarrow (2x + 3)(x + 1) = 0$

$\Rightarrow x = -1, \frac{-3}{2}$

$\Rightarrow x = -1$

[Q Given $x \neq -3/2$]

Ans.

60. Solve : $x^2 - 4ax - b^2 + 4a^2 = 0$.

[Board Question]

Ans. Given, $x^2 - 4ax - b^2 + 4a^2 = 0$

$\Rightarrow x^2 - 4ax - (b^2 - 4a^2) = 0$

$\Rightarrow x^2 - (b + 2a)x + (b - 2a)x - (b - 2a)(b + 2a) = 0$

$\Rightarrow x[x - (b + 2a)] + (b - 2a)[x - (b + 2a)] = 0$

$\Rightarrow x[x - b - 2a] + (b - 2a)[x - b - 2a] = 0$

$\Rightarrow [x - b - 2a][x + b - 2a] = 0$

$\Rightarrow x = 2a + b$ and $2a - b$.**Ans.**

61. The sum of two natural numbers is 8 and their product is 15. Find the numbers.

[Board Question]

Ans. Let one of the numbers be x.

So, the other number is $(8 - x)$.

Now, $x(8 - x) = 15$

$\Rightarrow 8x - x^2 = 15$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow x(x - 5) - 3(x - 5) = 0$

$\Rightarrow (x - 5)(x - 3) = 0$

$\Rightarrow x = 3$ or 5

∴ $8 - x = 5$ or 3

Thus, the two numbers are 3 and 5. **Ans.**

62. Solve: $4x^2 - 4ax + (a^2 - b^2) = 0$.

[Board Question]

Sol. Given,

$4x^2 - 4ax + (a^2 - b^2) = 0$

$\Rightarrow 4x^2 - 4ax + (a - b)(a + b) = 0$

$\Rightarrow 4x^2 - 2(a - b)x - 2(a + b)x + (a - b)(a + b) = 0$

$\Rightarrow 2x[2x - (a - b)] - (a + b)[2x - (a - b)] = 0$

$\Rightarrow [2x - (a - b)][2x - (a + b)] = 0$

$\Rightarrow 2x = a - b$ and $a + b$

$\Rightarrow x = \frac{a-b}{2}$ and $\frac{a+b}{2}$ · **Ans.**

63. Solve: $3x^2 - 2\sqrt{6}x + 2 = 0$.

[NCERT]
[Board Question]

Sol. Given, $3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2})^2 = 0$

$\Rightarrow \sqrt{3}x - \sqrt{2} = 0$

$\Rightarrow \sqrt{3}x = \sqrt{2}$

$\Rightarrow x = \sqrt{\frac{2}{3}}$

∴ $x = \sqrt{\frac{2}{3}}$ and $\sqrt{\frac{2}{3}}$ **Ans.**

64. Divide 12 into two parts such that the sum of their squares is 74.

Sol. Let the two parts be x and $(12 - x)$.

So, $x^2 + (12 - x)^2 = 74$

$\Rightarrow x^2 + 144 + x^2 - 24x - 74 = 0$

$\Rightarrow 2x^2 - 24x + 70 = 0$

$\Rightarrow x^2 - 12x + 35 = 0$

$\Rightarrow x^2 - 7x - 5x + 35 = 0$

$\Rightarrow x(x - 7) - 5(x - 7) = 0$

$\Rightarrow (x - 7)(x - 5) = 0$

$\Rightarrow x = 5$ and 7. **Ans.**

65. The sum of two numbers is 15 and their reciprocals is $\frac{3}{10}$. Find the numbers.

Sol. Let the numbers be x and $(15 - x)$. So, their reciprocals are $\frac{1}{x}$ and $\frac{1}{15-x}$ respectivley.

Now, $\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$

$\Rightarrow \frac{15 - x + x}{x(15 - x)} = \frac{3}{10}$

$\Rightarrow \frac{5}{x(15 - x)} = \frac{1}{10}$

$\Rightarrow x(15 - x) = 50$

$\Rightarrow 15x - x^2 = 50$

$\Rightarrow x^2 - 15x + 50 = 0$

$\Rightarrow x^2 - 10x - 5x + 50 = 0$

$\Rightarrow x(x - 10) - 5(x - 10) = 0$

$\Rightarrow (x - 5)(x - 10) = 0$

$\Rightarrow x = 5$ and 10. **Ans.**

66. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Sol. Let the two consecutive odd numbers be x and $(x + 2)$.

Thus $x^2 + (x + 2)^2 = 394$

$\Rightarrow x^2 + x^2 + 4x + 4 = 394$

$\Rightarrow 2x^2 + 4x - 390 = 0$

$\Rightarrow x^2 + 2x - 195 = 0$

$\Rightarrow x^2 + 15x - 13x - 195 = 0$

$\Rightarrow x(x + 15) - 13(x + 15) = 0$

$\Rightarrow (x + 15)(x - 13) = 0$

$\Rightarrow x = 13$ and -15

So, the numbers are either 13 and 15 or – 15 and – 13. **Ans.**

67. A dealer sells a toy for ` 24 and gains as much percent as the cost price of the toy. Find the cost price of the toy.

Sol. Suppose the cost price of the toy be ` x then gain is x%

We know Gain = $\left(x \times \frac{x}{100}\right)$

= $\left(\frac{x^2}{100}\right)$

Now, S.P. = C.P. + Gain

= $x + \frac{x^2}{100}$

As, S.P. = ` 24 (Given)

$x + \frac{x^2}{100} = 24$

$\Rightarrow x^2 + 100x - 2400 = 0$

$\Rightarrow x^2 + 120x - 20x - 2400 = 0$

$\Rightarrow x(x + 120) - 20(x + 120) = 0$

$\Rightarrow (x - 20)(x + 120) = 0$

$x = 20$ ($\because x > 0$)

Hence, the cost price of the toy will be ` 20.

Ans.

68. The difference of the squares of two natural numbers is 45. The square of the smaller number is 4 times the larger number. Find the numbers.

Sol. Let the smaller number be x and the larger be y.

Now, $x^2 = 4y$

Also, $y^2 - x^2 = 45$

$\Rightarrow y^2 - 4y = 45$

$\Rightarrow y^2 - 4y - 45 = 0$

$\Rightarrow y^2 - 9y + 5y - 45 = 0$

$\Rightarrow y(y - 9) + 5(y - 9) = 0$
$\Rightarrow (y - 9)(y + 5) = 0$
$\Rightarrow y = 9$ or -5
As natural numbers are not negative, so the larger number is 9.
Thus, $x^2 = 4(9) = 36$
$\Rightarrow x = 6$ or -6
Again as natural numbers are not negative, so the smaller number is 6.
So, the two natural numbers are 9 and 6. **Ans.**
69. Three-eighth of the students of a class opted for visiting an old age home. Sixteen students opted for having a nature walk. Square root of total number of students in the class opted for tree plantation in the school. The number of students who visited the old age home is same as the number of students who went for a nature walk and did tree plantation. Find the total number of students.

Sol. Suppose the total number of students is x

Number of students who opted for visiting old age home = $\frac{3}{8}x$

Number of students who opted for having a nature walk = 16

Number of students who opted for tree plantation = $\sqrt{x}$

So, $\frac{3}{8}x$ = 16 + $\sqrt{x}$

∴ $3x - 8\sqrt{x} - 128 = 0$

Suppose, $\sqrt{x} = y$

∴ $3y^2 - 8y - 128 = 0$

⇒ $(y - 8)(3y + 16) = 0$

∴ $y = 8$ [∵ $y > 0$]

Hence, $x = y^2 = 64$

Number of students = 64. **Ans.**

70. A charity trust decides to build the player hall having a carpet area of 300 sq m with its length 1 m more than twice its breadth. Find the length and breadth of the hall.

Sol. Let breadth of the hall be x m
∴ Length of the hall = $(2x + 1)$ m
Since, Area = $l \times b$
⇒ $(2x + 1) \times x = 300$
⇒ $2x^2 + x - 300 = 0$
⇒ $2x^2 + 25x - 24x - 300 = 0$
⇒ $x(2x + 25) - 12(2x + 25) = 0$
⇒ $(x - 12)(2x + 25) = 0$
⇒ $x = 12$, $x = -\frac{25}{2}$ (Rejected)
∴ $x = 12$ m ⇒ breadth = 12 m,
Length = $(2 \times 12 + 1) = 25$ m **Ans.**

71. Students of class X collected ` 18000. They wanted to divide it equally among a certain number of students residing in slums area. When they started distributing the amount, 20 more students from nearby slums also joined. Now each student got ` 240 less. Find the number of students living in the slum.

Sol. Let number of students living in slums area be x

∴ Share per student = ` $\frac{18000}{x}$

When number of students = $x + 20$

Then, share per student = ` $\frac{18000}{x+20}$

According to question,

$\frac{18000}{x} - \frac{18000}{x+20} = 240$

$\Rightarrow \frac{18000x + 360000 - 18000x}{(x+20)(x)} = 240$

$\Rightarrow 360000 = 240(x^2 + 20x)$

$\Rightarrow 1500 = x^2 + 20x$

$\Rightarrow x^2 + 20x - 1500 = 0$

$\Rightarrow x^2 + 50x - 30x - 1500 = 0$

$\Rightarrow x(x + 50) - 30(x + 50) = 0$

$\Rightarrow (x + 50)(x - 30) = 0$

$\Rightarrow x = 30$ or $x = -50$ (Rejected)

$\Rightarrow x = 30$

∴ Number of students living in the slum area = 30.**Ans.**

72. The distance between Mumbai and Pune is 192 km. Travelling by the Deccan Queen, it takes 48 minutes less than another train. Calculate the speed of the Deccan Queen if the speeds of the two trains differ by 20 km/hr.

[Board Question]

Sol. Let the speed of the other train be x km/hr.

Thus, the speed of the Deccan Queen is $(x + 20)$ km/hr.

Time taken by the other train = $\frac{192}{x}$ hr.

Time taken by the Deccan Queen = $\frac{192}{x+20}$ hr.

According to question,

$\Rightarrow \frac{192}{x} - \frac{192}{x+20} = \frac{48}{60}$

$\Rightarrow 192\left(\frac{x+20-x}{x(x+20)}\right) = \frac{48}{60}$

$\Rightarrow 3840\left(\frac{60}{48}\right) = x(x + 20)$

$\Rightarrow 4800 = x^2 + 20x$

$\Rightarrow x^2 + 20x – 4800 = 0$

$\Rightarrow x^2 + 80x – 60x – 4800 = 0$
$\Rightarrow x(x + 80) – 60(x + 80) = 0$
$\Rightarrow (x + 80) (x – 60) = 0$
$\Rightarrow x = 60$ or $– 80$
As speed cannot be negative, so the correct answer is 60 km/hr. **Ans.**

73. The difference of the squares of two numbers is 88. If the larger number is 5 more than twice the smaller number then find the two numbers.

[Board Question]

Sol. Let the larger number be x and the smaller number be y.
Given, $x = 2y + 5$…(i)
and $x^2 – y^2 = 88$…(ii)
From (i) and (ii)
$\Rightarrow (2y + 5)^2 – y^2 = 88$
$\Rightarrow 4y^2 + 25 + 20y – y^2 = 88$
$\Rightarrow 3y^2 + 20y – 63 = 0$
$\Rightarrow 3y^2 + 27y – 7y – 63 = 0$
$\Rightarrow 3y(y + 9) – 7(y + 9) = 0$
$\Rightarrow (y + 9) (3y – 7) = 0$
$\Rightarrow y = – 9$ or $\frac{7}{3}$

Thus, $x = – 18 + 5 = – 13$

or $x = 2\left(\frac{7}{3}\right)+5 = \frac{29}{3}$

Hence, the two numbers are – 13 and – 9 or $\frac{7}{3}$ and $\frac{29}{3}$.**Ans.**

74. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of

the other two by 60. Find the numbers.

[Board Question]

Sol. Let the three consecutive natural numbers be x, $x + 1$ and $x + 2$.

According to the question,

$\Rightarrow (x + 1)^2 - [(x + 2)^2 - x^2] = 60$

$\Rightarrow x^2 + 1 + 2x - [(x + 2 - x)(x + 2 + x)] = 60$

$\Rightarrow x^2 + 2x + 1 - [2(2 + 2x)] = 60$

$\Rightarrow x^2 + 2x + 1 - 4 - 4x = 60$

$\Rightarrow x^2 - 2x - 63 = 0$

$\Rightarrow x^2 - 9x + 7x - 63 = 0$

$\Rightarrow x(x - 9) + 7(x - 9) = 0$

$\Rightarrow (x + 7)(x - 9) = 0$

$\therefore x = 9$ or -7

$\Rightarrow x = 9$

($\because$ numbers are natural)

$\therefore$ Numbers are 9, 10, 11 **Ans.**

75. If the roots of the quadratic equation

$$(a - b)x^2 + (b - c)x + (c - a) = 0$$

are equal, prove that $2a = b + c$.

[Board Question]

Sol. Given equation is,

$$(a - b)x^2 + (b - c)x + (c - a) = 0$$

By comparing the given equation with $ax^2 + bx + c = 0$, we get

$$A = a - b, B = b - c, C = c - a$$

Since the roots of the given quadratic equation are equal,

$(b - c)^2 - 4(c - a)(a - b) = 0$

[$\because B^2 - 4AC = 0$]

$\Rightarrow b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$

$\Rightarrow (b^2 + c^2 + 2bc) - 4a(b + c) + 4a^2 = 0$

$\Rightarrow (b + c)^2 - 4a(b + c) + (2a)^2 = 0$

$\Rightarrow [(b + c) - 2a]^2 = 0$

$\Rightarrow b + c - 2a = 0$

i.e., $2a = b + c$

Hence Proved.

76. A girl is twice as old as her sister. Four years hence the product of their ages (in years) will be 160. Find their present ages.

[Board Question]

Sol. Let the sister's age be x. Thus, the girl's age is $2x$.

Four years hence,

Sister's age = $(x + 4)$

and Girl's age = $(2x + 4)$

Now $(x + 4)(2x + 4) = 160$

$\Rightarrow 2x^2 + 8x + 4x + 16 = 160$

$\Rightarrow 2x^2 + 12x + 16 = 160$

$\Rightarrow 2x^2 + 12x - 144 = 0$

$\Rightarrow x^2 + 6x - 72 = 0$

$\Rightarrow x^2 + 12x - 6x - 72 = 0$

$\Rightarrow x(x + 12) - 6(x + 12) = 0$

$\Rightarrow (x + 12)(x - 6) = 0$

$\Rightarrow x = 6$ or -12

$\Rightarrow x = 6$

(as age is always positive)

Thus, the sister's age is 6 years and the girl's age is $2 \times 6 = 12$ year. **Ans.**

77. If $ad \neq bc$, then prove that the equation

$$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$$

has no real roots.

[Board Question]

Sol. Given,

$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$, $ad \neq bc$

$D = b^2 - 4ac$

$= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= 4[a^2c^2 + b^2d^2 + 2abcd] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$

$= 4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$

$= 4[-a^2d^2 - b^2c^2 + 2abcd]$

$= -4(a^2d^2 + b^2c^2 - 2abcd)$

$= -4(ad - bc)^2$

... D is negative

Hence, given equation has no real roots.

Hence Proved.

Long Answer Type Questions

78. Solve for x:

[Board Question]

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}, \; x \neq -1, -2, -4$$

Sol. We have,

$$\frac{1}{x+1}+\frac{2}{x+2} = \frac{4}{x+4}, x \neq -1, -2, -4$$

$\Rightarrow (x + 2)(x + 4) + 2(x + 1)(x + 4) = 4(x + 1)(x + 2)$

$\Rightarrow x^2 + 2x + 4x + 8 + 2(x^2 + x + 4x + 4)$

$= 4(x^2 + x + 2x + 2)$

$\Rightarrow x^2 + 6x + 8 + 2x^2 + 10x + 8 = 4x^2 + 12x + 8$

$\Rightarrow 3x^2 + 16x + 16 = 4x^2 + 12x + 8$

$\Rightarrow x^2 - 4x - 8 = 0$

$\therefore x = \frac{4 \pm \sqrt{16+32}}{2}$

$\Rightarrow x = \frac{4 \pm \sqrt{48}}{2}$

$\therefore x = 2 \pm 2\sqrt{3}$. **Ans.**

79. Solve for *x*: $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)$ **= 5, where *x* ≠** $\frac{1}{2}$, **– 3.**

[Board Question]

Sol. Given,

$2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right) = 5$

$\Rightarrow \frac{2(2x-1)^2-3(x+3)^2}{(x+3)(2x-1)} = 5$

$\Rightarrow 2(2x - 1)^2 - 3(x + 3)^2 = 5(x + 3)(2x - 1)$

$\Rightarrow 2[4x^2 + 1 - 4x] - 3[x^2 + 9 + 6x]$

$= 5[2x^2 + 6x - x - 3]$

$\Rightarrow 8x^2 + 2 - 8x - 3x^2 - 27 - 18x$

$= 10x^2 + 25x - 15$

$\Rightarrow 5x^2 - 25 - 26x = 10x^2 + 25x - 15$

$\Rightarrow 5x^2 + 51x + 10 = 0$

$\Rightarrow 5x^2 + 50x + x + 10 = 0$

$\Rightarrow 5x(x + 10) + 1(x + 10) = 0$

$\Rightarrow (5x + 1)(x + 10) = 0$

$\Rightarrow x = -10, -\frac{1}{5}$ · **Ans.**

80. Solve for *x*:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}.$$

Sol. Given,

$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} = \frac{bx+2ax+ab}{2abx}$

$2abx = (2a + b + 2x)(bx + 2ax + ab)$

$\Rightarrow 2abx = 2abx + b^2x + 2bx^2 + 4a^2x$

$+ 2bax + 4ax^2 + 2a^2b + ab^2 + 2abx$

$\Rightarrow 0 = 2abx + b^2x + 2bx^2 + 4a^2x$

$+ 2abx + 4ax^2 + 2a^2b + ab^2$

$\Rightarrow (4ax^2 + 2bx^2) + (4a^2x + 4abx + b^2x)$

$+ (2a^2b + ab^2) = 0$

$\Rightarrow 2x^2(2a + b) + x[4a^2 + 4ab + b^2] + ab(2a + b) = 0$

$\Rightarrow 2x^2(2a + b) + x[4a^2 + 2ab + 2ab + b^2]$

$+ ab(2a + b) = 0$

$\Rightarrow 2x^2(2a + b) + x[2a(2a + b) + b(2a + b)]$

$+ ab(2a + b) = 0$

$\Rightarrow 2x^2(2a + b) + x(2a + b)^2 + ab(2a + b) = 0$

$\Rightarrow 2x^2 + x(2a + b) + ab = 0$

$\Rightarrow x = \frac{-(2a+b) \pm \sqrt{(2a+b)^2 - 4(2ab)}}{4}$

[Applying $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$]

$\Rightarrow x = \frac{-(2a+b) \pm \sqrt{4a^2 + 4ab + b^2 - 8ab}}{4}$

$\Rightarrow x = \frac{-(2a+b) \pm \sqrt{4a^2+b^2-4ab}}{4}$

$\Rightarrow x = \frac{-(2a+b) \pm (2a-b)}{4}$

$x = \frac{-2a-b+2a-b}{4}$ and $\frac{-2a-b-2a+b}{4}$

$\Rightarrow x = \frac{-2b}{4}$ and $\frac{-4a}{4}$

$\Rightarrow x = -\frac{b}{2}$ and $-a$. **Ans.**

81. A shopkeeper buys some books for ` 80. If he had bought 4 more books for the same amount then the price of each book would have been reduced by ` 1. Find the number of books he bought.

Sol. Let the number of books be x.

Thus, the price of 1 book = ` $\frac{80}{x}$ ·

If the number of books were $x + 4$, then the price per book = ` $\frac{80}{x+4}$

According to the question,

$\frac{80}{x+4} = \frac{80}{x} - 1$

$\Rightarrow \frac{80}{x} - \frac{80}{x+4} = 1$

$\Rightarrow 80\left(\frac{1}{x} - \frac{1}{x+4}\right) = 1$

$\Rightarrow 80\left(\frac{x+4-x}{x(x+4)}\right) = 1$

$\Rightarrow x(x + 4) = 320$

$\Rightarrow x^2 + 4x - 320 = 0$

$\Rightarrow x^2 + 20x - 16x - 320 = 0$

$\Rightarrow x(x + 20) - 16(x + 20) = 0$

$\Rightarrow (x + 20)(x - 16) = 0$

$\Rightarrow x = 16, -20$

As the number of books bought cannot be negative, so the correct answer is 16.**Ans.**

82. *A* takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it?

Sol. Let B can finish a work in x days

So, A can finish work in $(x - 6)$ days.

Together they finish the work in 4 days

According to the question,

$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$

$\Rightarrow \frac{x-6+x}{(x)(x-6)} = \frac{1}{4}$

$\Rightarrow 4(2x - 6) = x^2 - 6x$

$\Rightarrow 8x - 24 = x^2 - 6x$

$\Rightarrow x^2 - 14x + 24 = 0$

$\Rightarrow x^2 - 12x - 2x + 24 = 0$

$\Rightarrow x(x - 12) - 2(x - 12) = 0$

$\Rightarrow (x - 12)(x - 2) = 0$

Either $x - 12 = 0$ or $x - 2 = 0$

$x = 12$ or $x = 2$; Rejected

Hence, B can finish work in 12 days.

A can finish work in 6 days. **Ans.**

83. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

[Board Question]

Sol. Given, equation is $x^2 + px + 16 = 0$

This is of the form $ax^2 + bx + c = 0$

where, $a = 1$, $b = p$ and $c = 16$

 D = $b^2 - 4ac$

$= p^2 - 4 \times 1 \times 16$

$= p^2 - 64$
for equal roots, we have
$D = 0 \Rightarrow p^2 - 64 = 0$
$\Rightarrow p^2 = 64 \Rightarrow p = \pm 8$
Putting $p = 8$ in given equation we have, $x^2 + 8x + 16 = 0$
$\Rightarrow (x + 4)^2 = 0 \Rightarrow x + 4 = 0 \Rightarrow x = -4$
Now, putting $p = -8$ in the given equation, we get
$x^2 - 8x + 16 = 0$
$\Rightarrow (x - 4)^2 = 0 \Rightarrow x = 4$
 Required roots are – 4 and – 4 or 4 and 4.

Ans.

84. Two water taps together take 6 hours to fill a tank. If the tap with the larger diameter takes 9 hours lesser than the tap with the smaller diameter, then find the time in which each tap can separately fill the tap.

[Board Question]

Sol. Let the smaller tap fill the tank in x hr and the total capacity of the tank be 1 unit.

Then in 1 hr, it can fill $\frac{1}{x}$ unit.

So the larger tank takes $(x - 9)$ hours to fill the tank.

Then in 1 hr, it can fill $\frac{1}{x-9}$ unit.
Together they can fill the tank in 6 hours.

Then in 1 hour, they can fill $\frac{1}{6}$ unit of the tank.

Thus, $\frac{1}{x} + \frac{1}{x-9} = \frac{1}{6}$

$\Rightarrow \frac{x - 9 + x}{x(x-9)} = \frac{1}{6}$

$\Rightarrow \frac{2x-9}{x(x-9)} = \frac{1}{6}$

$\Rightarrow 6(2x - 9) = x(x - 9)$

$\Rightarrow 12x - 54 = x^2 - 9x$

$\Rightarrow x^2 – 9x – 12x + 54 = 0$

$\Rightarrow x^2 – 21x + 54 = 0$

$\Rightarrow x^2 – 18x – 3x + 54 = 0$

$\Rightarrow x(x – 18) – 3(x – 18) = 0$
$\Rightarrow (x – 18)(x – 3) = 0$
$\Rightarrow x = 3, 18$
As 9 hours less than 3 hours is – 6 which is not possible, so the smaller tap takes 18 hours to fill the tank and the larger tap takes 9 hours to do so.**Ans.**
85. Solve for x:

[Board Question]

$$\frac{1}{x+1}+\frac{3}{5x+1} = \frac{5}{x+4}, x^{1} – 1, -\frac{1}{5}, – 4$$

Sol. Given,

$\frac{1}{x+1}+\frac{3}{5x+1} = \frac{5}{x+4}$

$\Rightarrow \frac{1}{x+1}-\frac{5}{x+4} = \frac{-3}{5x+1}$

$\Rightarrow \frac{(x+4)-5(x+1)}{(x+1)(x+4)} = \frac{-3}{5x+1}$

$\Rightarrow \frac{x+4-5x-5}{x^2+5x+4} = \frac{-3}{5x+1}$

$\Rightarrow \frac{-(4x+1)}{x^2+5x+4} = \frac{-3}{5x+1}$

$\Rightarrow (4x + 1) (5x + 1) = 3(x^2 + 5x + 4)$

$\Rightarrow 20x^2 + 4x + 5x + 1 = 3x^2 + 15x + 12$

$\Rightarrow 17x^2 – 6x – 11 = 0$

$\Rightarrow 17x^2 – 17x + 11x – 11 = 0$
$\Rightarrow 17x(x – 1) + 11 (x – 1) = 0$
$\Rightarrow (x – 1) (17x + 11) = 0$

Either $x = 1$ or $x = \frac{-11}{17}$

Given $x \neq 1$, so $x = \frac{-11}{17}$ **Ans.**

86. Solve $\frac{x-1}{x-2}+\frac{x-3}{x-4}=\frac{10}{3}$, where $x \neq 2, 4$.

[Board Question]

Sol. Given, $\frac{x-1}{x-2}+\frac{x-3}{x-4}=\frac{10}{3}$

$\Rightarrow \frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$

$\Rightarrow 3[(x-1)(x-4)+(x-3)(x-2)]$

$= 10[(x-2)(x-4)]$

$\Rightarrow 3[x^2 - x - 4x + 4 + x^2 - 3x - 2x + 6]$

$= 10[x^2 - 2x - 4x + 8]$

$\Rightarrow 3[2x^2 - 10x + 10] = 10[x^2 - 6x + 8]$

$\Rightarrow 6x^2 - 30x + 30 = 10x^2 - 60x + 80$

$\Rightarrow 4x^2 - 30x + 50 = 0$

$\Rightarrow 2x^2 - 15x + 25 = 0$

$\Rightarrow 2x^2 - 10x - 5x + 25 = 0$

$\Rightarrow 2x(x-5) - 5(x-5) = 0$

$\Rightarrow (x-5)(2x-5) = 0$

$\Rightarrow x = 5, \frac{5}{2}$. **Ans.**

87. Solve: $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$.

[Board Question]

Sol. Given,

$9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$, we get

$A = 9$, $B = -9(a+b)$ and $C = (2a^2 + 5ab + 2b^2)$

$\therefore D = B^2 - 4AC$

$\Rightarrow D = [-9(a+b)]^2 - 4(9)(2a^2 + 5ab + 2b^2)$

$\Rightarrow D = 81(a+b)^2 - 36(2a^2 + 5ab + 2b^2)$

$\Rightarrow D = 81[a^2 + b^2 + 2ab] - 72a^2 - 180ab - 72b^2$

$\Rightarrow D = 81a^2 + 81b^2 + 162ab - 72a^2 - 180ab - 72b^2$

$\Rightarrow D = 9a^2 + 9b^2 - 18ab$

$\Rightarrow D = 9(a^2 + b^2 - 2ab)$

$\Rightarrow D = 9(a - b)^2 \geq 0$

Thus, $x = \frac{-B+\sqrt{D}}{2A}$ and $x = \frac{-B-\sqrt{D}}{2A}$

$x = \frac{9(a+b)+3(a-b)}{18}$ and $\frac{9(a+b)-3(a-b)}{18}$

$\Rightarrow x = \frac{6(2a+b)}{18}$ and $\frac{6(a+2b)}{18}$

$\Rightarrow x = \frac{(2a+b)}{3}$ and $\frac{(a+2b)}{3}$ · **Ans.**

88. Solve for x: $x^2 + 5x - (a^2 + a - 6) = 0$.

[Board Question]

Sol. Taking $(a^2 + a - 6) = a^2 + 3a - 2a - 6$

$= a(a + 3) - 2(a + 3)$

$= (a + 3)(a - 2)$

$\therefore x^2 + 5x - (a + 3)(a - 2) = 0$

$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$

$\Rightarrow x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$\Rightarrow (x - a + 2)(x + a + 3) = 0$

Hence, $x - a + 2 = 0$ and $x + a + 3 = 0$

$\Rightarrow x = a - 2$ and $x = -(a + 3)$

Required values of x are $(a - 2)$, $-(a + 3)$ **Ans.**

89. In a class test, the sum of Arun's marks in Hindi and English is 30. He had got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

[Board Question]

Sol. Let Arun marks in hindi be x and marks in english be y.

Then, according to question, we have $x + y = 30$ ...(i)

$(x + 2)(y - 3) = 210$ ...(ii)

From equation (i) put $x = 30 - y$ in equation (ii),

$\Rightarrow (30 - y + 2)(y - 3) = 210$

$\Rightarrow (32 - y)(y - 3) = 210$

$\Rightarrow 32y - 96 - y^2 + 3y = 210$

$\Rightarrow y^2 - 35y + 306 = 0$

$\Rightarrow y^2 - 18y - 17y + 306 = 0$

$\Rightarrow y(y - 18) - 17(y - 18) = 0$

$\Rightarrow (y - 18)(y - 17) = 0$

$y = 18, 17$

Put $y = 18$ and 17 in equation (i), we get

$x = 12, 13$

Hence, his marks in hindi can be 12 and 13 and in english his marks can be 18 and 17. **Ans.**

90. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Sol. Let the numerator be x. So the denominator is $2x + 1$.

Now $\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21}$

$\Rightarrow \frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}$

$\Rightarrow \frac{x^2 + (2x+1)^2}{x(2x+1)} = \frac{58}{21}$

$\Rightarrow \frac{x^2 + 4x^2 + 4x + 1}{x(2x+1)} = \frac{58}{21}$

$\Rightarrow \frac{5x^2 + 4x + 1}{x(2x+1)} = \frac{58}{21}$

$\Rightarrow 21(5x^2 + 4x + 1) = 58x(2x + 1)$

$\Rightarrow 105x^2 + 84x + 21 = 116x^2 + 58x$

$\Rightarrow 11x^2 - 26x - 21 = 0$

$\Rightarrow 11x^2 - 33x + 7x - 21 = 0$

$\Rightarrow 11x(x - 3) + 7(x - 3) = 0$

$\Rightarrow (x - 3)(11x + 7) = 0$

$\Rightarrow x = 3$ and $-\frac{7}{11}$

So, the numerator is 3. [As a fraction cannot be negative]

Hence, the fraction is $\frac{3}{7}$. **Ans.**

91. 7 years ago, Varun's age was five times the square of Swati's age. 3 years hence, Swati's age will be two-fifths of Varun's age. Find their present ages.

Sol. Let Swati's present age be x years and Varun's be y years.

7 years ago, Swati's age was $(x - 7)$ years and Varun's was $(y - 7)$.

According to the question,

$y - 7 = 5(x - 7)^2$ ...(i)

3 years later, Swati's age will be $(x + 3)$ and Varun's will be $(y + 3)$.
According to the question,

$x + 3 = \frac{2}{5}(y + 3)$

$\Rightarrow 5x + 15 = 2y + 6$

$\Rightarrow 2y = 5x + 9$

$\Rightarrow y = \frac{5x+9}{2}$ ...(ii)

Substituting the value of y in equation (i),

Hence, $5(x - 7)^2 = \frac{5x+9}{2} - 7$

$\Rightarrow 10(x - 7)^2 = 5x + 9 - 14$

$\Rightarrow 10[x^2 - 14x + 49] = 5x - 5$

$\Rightarrow 10x^2 - 140x + 490 = 5x - 5$

$\Rightarrow 10x^2 - 145x + 495 = 0$

$\Rightarrow 2x^2 - 29x + 99 = 0$

$\Rightarrow 2x^2 - 18x - 11x + 99 = 0$

$\Rightarrow 2x(x - 9) - 11(x - 9) = 0$
$\Rightarrow (x - 9)(2x - 11) = 0$
$\Rightarrow x = 9$ and $\frac{11}{2}$

If Swati's present age is $5\frac{1}{2}$ years, then 7 years ago it would be – 1, $\frac{1}{2}$ years, which is not possible. Thus, her present age is 9 years.

Thus, Varun's present age is:

$y = \frac{5(9) + 9}{2} = \frac{45 + 9}{2}$

$= \frac{54}{2} = 27$ years. **Ans.**

92. The sum of the areas of two squares is 260 m^2. If the difference of their perimeters is 24 m, then find the sides of the two squares.

Sol. Let the sides of the two squares be x m and y m respectively.

Thus area of first square = x^2 m^2

and perimeter of first square = $4x$ m

Also, area of second square = y^2 m^2

and perimeter of second square = $4y$ m

Now, $x^2 + y^2 = 260$ ...(i)

and $4(x - y) = 24$

$\Rightarrow y = x - 6$ ...(ii)

Now, $x^2 + y^2 = 260$ [From (i)]

$\Rightarrow x^2 + (x - 6)^2 = 260$ [Using (ii)]

$\Rightarrow x^2 + x^2 - 12x + 36 = 260$

$\Rightarrow 2x^2 - 12x - 224 = 0$

$\Rightarrow x^2 - 6x - 112 = 0$

$\Rightarrow x^2 - 14x + 8x - 112 = 0$

$\Rightarrow x(x - 14) + 8(x - 14) = 0$

$\Rightarrow (x + 8)(x - 14) = 0$

$\Rightarrow x = -8$ and 14

As length cannot be negative, so we take $x = 14$ m

and $y = 14 - 6 = 8$ m

Thus, the sides of the two squares are 14 m and 8 m long. **Ans.**

93. The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 30 m more than the shorter side, find the sides of the field.

Sol. As the field is rectangular, so all the angles are of 90°.

Hence the triangle formed by the diagonal and the two sides is a right-angled triangle.

Let the length of the shorter side be x m.

Thus the length of the longer side is $(x + 30)$ m and the diagonal is $(x + 60)$ m.

Applying Pythagoras' theorem,

$(x + 60)^2 = (x + 30)^2 + x^2$
$\Rightarrow x^2 = (x + 60)^2 - (x + 30)^2$
$\Rightarrow x^2 = (x + 60 - x - 30)(x + 60 + x + 30)$
$\Rightarrow x^2 = 30(2x + 90)$
$\Rightarrow x^2 = 60x + 2700$
$\Rightarrow x^2 - 60x - 2700 = 0$
$\Rightarrow x^2 - 90x + 30x - 2700 = 0$
$\Rightarrow x(x - 90) + 30(x - 90) = 0$
$\Rightarrow (x - 90)(x + 30) = 0$
$\Rightarrow x = 90$ and -30

As length cannot be negative, so the length of the smaller side is 90 m and the length of the larger side is 120 m. **Ans.**

94. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

[Board Question]

Sol. Let the length of the rectangular park be x m. Then, breadth = $(x - 3)$ m

Area of rectangular park = $x(x - 3)$ m^2

and area of isosceles triangular park

= $\frac{1}{2}(x - 3) \times 12$ m^2

= $6(x - 3)$ m^2

Acoording to the question,

$x(x - 3) - 6(x - 3) = 4$

$\Rightarrow x^2 - 3x - 6x + 18 = 4$

$\Rightarrow x^2 - 9x + 14 = 0$

$\Rightarrow x^2 - 7x - 2x + 14 = 0$

$\Rightarrow x(x - 7) - 2(x - 7) = 0$

$\Rightarrow (x - 2)(x - 7) = 0$

$\Rightarrow x = 2$ or 7

$\therefore x = 7$ m

(as breadth can't be negative)

and $x - 3 = (7 - 3)$ m = 4 m

Hence, length and breadth of the rectangular park is 7 m and 4 m respectively. **Ans.**

95. 300 apples were distributed equally among a certain number of students. There had been 10 more students, each would have received one apple less. Find the number of students.

Sol. Let the number of students be x.

So, the number of apples per student is $\frac{300}{x}$.

If the number of students were $x + 10$ then the number of apples per student would have been $\frac{300}{x+10}$.

According to the question,

$\frac{300}{x+10} + 1 = \frac{300}{x}$

$\Rightarrow \frac{300}{x} - \frac{300}{x+10} = 1$

$\Rightarrow 300\left(\frac{x+10-x}{x(x+10)}\right) = 1$

$\Rightarrow 300\left(\frac{10}{x(x+10)}\right) = 1$

$\Rightarrow x(x + 10) = 3000$

$\Rightarrow x^2 + 10x = 3000$

$\Rightarrow x^2 + 10x - 3000 = 0$

$\Rightarrow x^2 + 60x - 50x - 3000 = 0$

$\Rightarrow x(x + 60) - 50(x + 60) = 0$

$\Rightarrow (x + 60)(x - 50) = 0$

$\Rightarrow x = 50$ and -60

Since number of students cannot be negative, so the correct answer is 50. **Ans.**

96. The total cost of a certain length of a piece of cloth is ` 200. If the piece was 5 m longer and each metre of cloth costs ` 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

[Board Question]

Sol. Let the original length of piece of cloth is x m and rate of cloth is ` y per metre.

Then according to question, we have $x \times y = 200$ ...(i)

and if length be 5 m longer and each meter of cloth be ` 2 less then

$(x + 5)(y - 2) = 200 \Rightarrow xy - 2x + 5y - 10 = 200$ ...(ii)

On equating equation (i) and (ii), we have

$xy = xy - 2x + 5y - 10$

$\Rightarrow 2x - 5y = -10$ ...(iii)

$\left(y = \frac{200}{x}\right)$ from equation (i),

$\Rightarrow 2x - 5 \times \frac{200}{x} = -10$

$\Rightarrow 2x - \frac{1000}{x} = -10$

$\Rightarrow 2x^2 - 1000 = -10x$

$\Rightarrow 2x^2 + 10x - 1000 = 0$

$\Rightarrow x^2 + 5x - 500 = 0$

$\Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x(x + 25) - 20(x + 25) = 0$

$\Rightarrow (x + 25)(x - 20) = 0$ $x = 20$

($x \neq -25$ length of cloth can never be negative)

$\therefore x \times y = 200 \Rightarrow 20 \times y = 200$

$\Rightarrow y = 10$

Thus, length of the piece of cloth is 20 m and original price per metre is ` 10. **Ans.**

97. A passenger while boarding a plane hurt herself and the captain called for immediate medical attention. Thus, the plane left 30 minutes behind schedule. In order to reach its

destination 1500 km away on time, the speed was increased by 100 km/hr from its usual speed. Find the usual speed.

[Board Question]

Sol. Let the usual speed be x km/hr.

We know, Time = $\frac{\text{Distance}}{\text{Speed}}$

So, Usual time taken = $\frac{1500}{x}$ hr.

Increased speed = (x + 100) km/hr

Thus, new time taken = $\frac{1500}{x+100}$

Now, according to the question

$$\frac{1500}{x+100} + \frac{1}{2} = \frac{1500}{x}$$

$$\Rightarrow \frac{1500}{x} - \frac{1500}{x+100} = \frac{1}{2}$$

$$\Rightarrow 1500\left(\frac{1}{x} - \frac{1}{x+100}\right) = \frac{1}{2}$$

$$\Rightarrow 15000\left(\frac{x+100-x}{x(x+100)}\right) = \frac{1}{2}$$

$$\Rightarrow 1500\left(\frac{100}{x(x+100)}\right) = \frac{1}{2}$$

$$\Rightarrow x(x + 100) = 300000$$

$$\Rightarrow x^2 + 100x - 300000 = 0$$

$$\Rightarrow x^2 + 600x - 500x - 300000 = 0$$

$$\Rightarrow x(x + 600) - 500(x + 600) = 0$$

$$\Rightarrow (x + 600)(x - 500) = 0$$

$$\Rightarrow x = 500 \text{ and } -600$$

As speed cannot be negative, so the original speed of the plane is 500 km/hr. **Ans.**

98. Two pipes together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill tank separately, then find the time in which each pipe can fill the tank separately.

[Board Question]

Sol. Let the smaller tap fill the tank in x minutes and the total capacity of the tank be 1 sq. unit.

Then in 1 minute, smaller tap can fill $\frac{1}{x}$ sq. unit.

So the other pipe takes (x – 5) minutes to fill the tank.

Then in 1 minute, it can fill $\frac{1}{x-5}$ sq. unit.

Together they can fill the tank is $11\frac{1}{9}$ minutes

$\left(= \frac{100}{9} \text{ minutes}\right)$

Then in 1 minute, they can fill $\frac{9}{100}$ of the tank.

Thus, $\frac{1}{x} + \frac{1}{x-5} = \frac{9}{100}$

$\Rightarrow \frac{x-5+x}{x(x-5)} = \frac{9}{100}$

$\Rightarrow \frac{2x-5}{x(x-5)} = \frac{9}{100}$

$\Rightarrow 200x - 500 = 9x^2 - 45x$

$\Rightarrow 9x^2 - 245x + 500 = 0$

$\Rightarrow 9x^2 - 225x - 20x + 500 = 0$

$\Rightarrow 9x(x - 25) - 20(x - 25) = 0$

$\Rightarrow (x - 25)(9x - 20) = 0$

$\Rightarrow x = 25$ or $\frac{20}{9}$.

As 5 minutes less than $\frac{20}{9}$ minutes is less than 0 which is not possible, so the first pipe takes 25 minutes to fill the tank and the second takes 20 minutes to so do. **Ans.**

99. The difference between two natural numbers is 5 and the difference between their reciprocals is $\frac{5}{14}$. Find the numbers.

[Board Question]

Sol. Let the natural numbers be x and y respectively.

Now, $x - y = 5$ …(i)

Also, $\frac{1}{y} - \frac{1}{x} = \frac{5}{14}$

⇒ $14x - 14y = 5xy$

⇒ $14(x - y) = 5xy$

⇒ $14 \times 5 = 5xy$ [From (i)]

⇒ $xy = 14$

⇒ $x = \frac{14}{y}$

Substituting $x = \frac{14}{y}$ in equation (i), we get

$\frac{14}{y} - y = 5$

⇒ $y - \frac{14}{y} + 5 = 0$

⇒ $y^2 + 5y - 14 = 0$

⇒ $y^2 + 7y - 2y - 14 = 0$

⇒ $y(y + 7) - 2(y + 7) = 0$

⇒ $(y + 7)(y - 2) = 0$

⇒ $y = 2, -7$

Since natural numbers cannot be –ve, so $y = 2$.

Thus, from equation (i),

$x - y = 5$

⇒ $x - 2 = 5$

⇒ $x = 7$

So, the two numbers are 7 and 2. **Ans.**

100. A two-digit number is such that the product of its digits is 20. If 9 is added to the number, the digit interchange their places. Find the number.

[Board Question]

Sol. Let the ten's digit be x and the unit's digit be y.

Thus, the given number is $(10x + y)$ and its reverse is $(10y + x)$.

So, $xy = 20$

$\Rightarrow x = \frac{20}{y}$ ...(i)

and $10x + y + 9 = 10y + x$

$\Rightarrow 9x - 9y = -9$

$\Rightarrow x - y = -1$

$\Rightarrow \frac{20}{y} - y = -1$ [From (i)]

$\Rightarrow 20 - y^2 = -y$

$\Rightarrow y^2 - y - 20 = 0$

$\Rightarrow y^2 - 5y + 4y - 20 = 0$

$\Rightarrow y(y - 5) + 4(y - 5) = 0$

$\Rightarrow (y - 5)(y + 4) = 0$

$\Rightarrow y = 5, -4$

So, $x = \frac{20}{5}, -\frac{20}{4}$

$\Rightarrow x = 4, -5$

Thus, the number is 45 or – 54. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

101. Assertion: If a and b are integers and the roots of $x^2 + ax + b = 0$ are rational, then they must be integers.

Reason: If the coefficient of x^2 in a quadratic equation is unity then its roots must be integers.

Ans. (c) Assertion is true but the Reason is false.

Explanation :

As given in assertion, a and b are integers.
$x^2 + ax + b = 0$,
Given that roots are rational
$x = -a \pm \sqrt{a^2 - 4b}$

As roots are rational $a^2 - 4b$ must be perfect square

Let, $a^2 - 4b = m^2$

M is integer as a and b are integers

$\therefore x = -a \pm m$

So, roots are also integers ($\because$ a and m are integers)

So, assertion is true.

As given in reason,

In a quadratic equation if coefficient of x^2 is unity. Then it is not necessary that roots must be integer.

For example; $x^2 + 2x - 1$ have irrational roots.

Therefore, reason is false.

102. Assertion: If $f(x)$ is a quadratic expression such that $f(1) + f(2) = 0$. If -1 is a root of $f(x) = 0$, then the other root is 8/5.

Reason: If $f(x) = ax^2 + bx + c$, then $\alpha + \beta$

$= -ba$ and $\alpha\beta = \frac{c}{a}$

Ans. (b) Both the Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

Given, $f(1) + f(2) = 0$

For $f(x) = ax^2 + bx + c$, we have

$a + b + c + 4a + 2b + c = 0$

$\Rightarrow 5a + 3b + 2c = 0$ ...(1)

Since – 1 is a root of the equation

$f(-1) = a - b + c = 0$ ...(2)

By eliminating c and b in turn by solving equations (1) and (2), we have

$$\frac{-b}{a} = \frac{3}{5}, \frac{c}{a} = \frac{-8}{5}$$

Since -1 is one root, the other root is $\frac{8}{5}$.

Case Based Questions

103. Chandana and Sohana are very close friends. Chandana's parents own a Maruti Alto. Sohana's parents own a Toyota Livo. Both the families decided to go Somanath tample in Gujrat by their own cars.

Chandana's car travels x km/hr while Sohana's car travels 5 km/h more than Chandana's car. Chandana car took 4 hrs more than Sohana's car in covering 400 km.

(i) What will be the distance covered by Sohana's car in two hour?

(a) $2(x + 5)$ km

(b) $(x + 5)$ km

(c) $2(x + 10)$ km

(d) $(2x + 5)$ km

Ans. (a) $2(x + 5)$ km

Explanation :

Chandana's car travels x km/h while Sohana's car travels 5 km/h more than Chandana's car. Thus Sohana's car speed is $(x + 5)$ km/hour Distance covered in two hour is $2(x + 5)$ km.

(ii) Which of the following quadratic equation describe the speed of Chandana's car?

(a) $x^2 - 5x - 500 = 0$

(b) $x^2 + 4x - 400 = 0$

(c) $x^2 + 5x - 500 = 0$

(d) $x^2 - 4x + 400 = 0$

Ans. (c) $x^2 + 5x - 500 = 0$

Explanation :

As per question

$\frac{400}{x} = \frac{400}{x+5} + 4 \Rightarrow 400(x + 5) = 400x + 4x(x + 5)$

$\Rightarrow 2000 = 4x^2 + 20x$

$\Rightarrow 500 = x^2 + 5x$

$\Rightarrow x^2 + 5x - 500 = 0$

(iii) What is the speed of Chandana's car?

(a) 20 km/hour

(b) 15 km/hour

(c) 25 km/hour

(d) 10 km/hour

Ans. (a) 20 km/hour

Explanation :

We have

$x^2 + 5x - 500 = 0$

$\Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x(x + 25) - 20(x + 25) = 0$

$\Rightarrow (x + 25)(x - 20) = 0$

$x = 20, x = -25$

Since $x = -25$ is not possible, we get $x = 20$ km/hr.

(iv) How much time did Sohana's car took to travel 400 km?

(a) 20 hour

(b) 40 hour

(c) 25 hour

(d) 16 hour

Ans. (d) 16 hour

Explanation :

Sohana's car speed = 20 + 5 = 25 km/hour

$$\text{Time taken} = \frac{400}{25} = 16 \text{ hour}$$

(v) How much time did Chandana's car took to travel 400 km?

(a) 15 hour

(b) 18 hour

(c) 20 hour

(d) 16 hour

Ans. (c) 20 hour

Explanation :

Chanda's car speed = 20

$$\text{Time taken} = \frac{400}{20} = 20 \text{ hour.}$$

104. In an auditorium, seats are arranged in rows and columns. Initially, the number of rows were equal to the number of seats in each row. When the number of rows become twice and the number of seats in each row was reduced by 10, the total number of seats increased by 300.

(i) If x is taken as number of row in original arrangement which of the following quadratic equation describe the situation?

(a) $x^2 - 20x - 300 = 0$

(b) $x^2 + 20x - 300 = 0$

(c) $x^2 - 20x + 300 = 0$

(d) $x^2 + 20x + 300 = 0$

Ans. (a) $x^2 - 20x - 300 = 0$

Explanation :

Since number of rows were equal to the number of seats in each row in original arrangement so total seats are x^2. In new arrangement row are $2x$ and seats in each row $x - 10$. Total seats are 300 more than previous seats. So total number of seats are $x^2 + 300$

Thus $2x(x - 10) = x^2 + 300$

$\Rightarrow 2x^2 - 20x = x^2 + 300$

$\Rightarrow x^2 - 20x - 300 = 0$

(ii) How many number of rows are there in the original arrangement?

(a) 20

(b) 40

(c) 10

(d) 30

Ans. (d) 30

Explanation :

We have

$x^2 - 20x - 300 = 0$

$\Rightarrow x^2 - 30x + 10x - 300 = 0$

$\Rightarrow x(x - 30) + 10(x - 30) = 0$

$\Rightarrow (x - 30)(x + 10) = 0$

$\Rightarrow x = 30, -10$

$x = 30$ (since rows can't be negative)

(iii) How many number of seats are there in the auditorium in original arrangement?

(a) 725

(b) 400

(c) 900

(d) 680

Ans. (c) 900

Explanation :

Number of seats in original arrangement

$x^2 = 30^2 = 900$

(iv) How many number of seats are there in the auditorium after re-arrangement.

(a) 860

(b) 990

(c) 1200

(d) 680

Ans. (c) 1200

Explanation :

Total seats in rearrangement = $30^2 + 300$
= 900 + 300 = 1200.
(v) How many number of columns are there in the auditorium after re-arrangement?
(a) 42
(b) 20

(c) 25

(d) 32

Ans. (b) 20

Explanation :

Number of column after rerrangement = $\frac{\text{Total seats}}{\text{row}} = \frac{1200}{60}$ = 20 column.

105. Amit is preparing for his upcoming semester exam. For this, he has to practice the chapter of Quadratic equations. So he started with factorization method. Let two linear factors of $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$.
How he factorized each of the following quadratic equations and found the roots.

(i) $6x^2 + x - 2 = 0$.
(a) 1, 6
(b) $\frac{1}{2}, \frac{-2}{3}$
(c) $\frac{1}{3}, \frac{-1}{2}$
(d) $\frac{3}{2}, -2$

Ans. (b) $\frac{1}{2}, \frac{-2}{3}$

Explanation :

We have, $6x^2 + x - 2 = 0$
$\Rightarrow 6x^2 + 4x - 3x - 2 = 0$
$\Rightarrow (3x + 2)(2x - 1) = 0$

⇒ $x = \frac{-2}{3}, \frac{1}{2}$

(ii) $2x^2 + x - 300 = 0$

(a) $30, \frac{2}{15}$

(b) $60, \frac{-2}{5}$

(c) $12, \frac{-25}{2}$

(d) None of these

Ans. (c) $12, \frac{-25}{2}$

Explanation :

$2x^2 + x - 300 = 0$

$\Rightarrow 2x^2 + 25x - 24x - 300 = 0$

$\Rightarrow (x - 12)(2x + 25) = 0$

$\Rightarrow x = 12, \frac{-25}{2}$

(iii) $x^2 - 8x + 16 = 0$

(a) 3, 3

(b) 3, – 3

(c) 4, – 4

(d) 4, 4

Ans. (d) 4, 4

Explanation :

$x^2 - 8x + 16 = 0$

$\Rightarrow (x - 4)^2 = 0$

$\Rightarrow (x - 4)(x - 4) = 0$

$\Rightarrow x = 4, 4$

(iv) $6x^2 - 13x + 5 = 0$

(a) $2, \frac{3}{5}$

(b) $-2, \frac{-5}{3}$

(c) $\frac{1}{2}, \frac{-3}{5}$

(d) $\frac{1}{2}, \frac{5}{3}$

Ans. (d) $\frac{1}{2}, \frac{5}{3}$

Explanation :

$6x^2 - 13x + 5 = 0$
$\Rightarrow 6x^2 - 3x - 10x + 5 = 0$
$\Rightarrow (2x - 1)(3x - 5) = 0$
$\Rightarrow x = \frac{1}{2}, \frac{5}{3}$
(v) $100x^2 - 20x + 1 = 0$
(a) $\frac{1}{10}, \frac{1}{10}$
(b) – 10, – 10
(c) $-10, \frac{1}{10}$
(d) $\frac{1}{10}, \frac{1}{10}$

Ans. (a) $\frac{1}{10}, \frac{1}{10}$

Explanation :

$100x^2 - 20x + 1 = 0$
$\Rightarrow (10x - 1)^2 = 0$
$\Rightarrow x = \frac{1}{10}, \frac{1}{10}$

106. If $p(x)$ is a quadratic polynomial *i.e.,* $p(x) = ax^2 + bx + c$, $a \neq 0$, then $p(x) = 0$ is called a quadratic equation. Now, answer the following questions.

(i) Which of the following is correct about the quadratic equation $ax^2 + bx + c = 0$?
(a) *a*, *b* and *c* are real numbers, $c \neq 0$
(b) *a*, *b* and *c* are rational numbers, $a \neq 0$
(c) *a*, *b* and *c* are integers, *a*, *b* and $c \neq 0$
(d) *a*, *b* and *c* real numbers, $a \neq 0$

Ans. (d) *a*, *b* and *c* real numbers, $a \neq 0$.

Explanation :

This is a necessary condition for equation to be quadratic.

(ii) The degree of a quadratic equation is:

(a) 1

(b) 2

(c) 3

(d) other than 1

Ans. (b) 2

Explanation :

The standard quadratic equation is of the form $ax^2 + bx + c = 0$.

(iii) Which of the following is a quadratic equation?

(a) $x(x + 3) + 7 = 5x – 11$

(b) $(x – 1)^2 – 9 = (x – 4)(x + 3)$

(c) $x^2(2x + 1) – 4 = 5x^2 – 10$

(d) $x(x – 1)(x + 7) = x(6x – 9)$

Ans. (a) $x(x + 3) + 7 = 5x – 11$

Explanation :

(a) $x^2 + 3x + 7 = 5x – 11$

$\Rightarrow x^2 – 2x + 18 = 0$, is a quadratic equation.

(b) $x^2 + 1 – 2x – 9 = x^2 – 4x + 3x – 12$

$\Rightarrow – x + 4 = 0$

$\Rightarrow x – 4 = 0$, is not a quadratic equation.

(c) $x^2(2x + 1) – 4 = 5x^2 – 10$

$\Rightarrow 2x^3 + x^2 – 4 = 5x^2 – 10$

$\Rightarrow 2x^3 – 4x^2 + 6 = 0$

(d) is not a quadratic equation.

$x(x - 1)(x + 7) = x(6x - 9)$

$\Rightarrow x^3 + 6x^2 - 7x = 6x^2 - 9x$

$\Rightarrow x^3 + 2x = 0$

is not a quadratic equation.

(iv) Which of the following is incorrect about the quadratic equation $ax^2 + bx + c = 0$?

(a) If $a\alpha^2 + b\alpha + c = 0$, then $x = -\alpha$ is the solution of the given quadratic equation.

(b) The additive inverse of zeroes of the polynomial $ax^2 + bx + c$ is the roots of the given equation.

(c) If α is a root of the given quadratic equation, then its other root is – α.

(d) All of the above

Ans. (d) All of the above.

Explanation :

All of the above sitautions are incorrect.

(v) Which of the following is a method of finding solutions of the given quadratic equation?

(a) Factorisation method

(b) Completing the square method

(c) Formula method

(d) All of the above

Ans. (d) All of the above

Explanation :

There are three methods for finding solution of the given quadratic.

(i) Factorisation method

(ii) Completing the square method

(iii) Formula method

107. Point A and B representing Chandigarh and Kurukshetra respectively are almost 90 km apart from each other on the highway. A car starts from Chandigarh and another from

Kurukshetra at the same time. If these cars go in the same direction they meet in 9 hours and if these cars go in opposite direction, they meet in 9/7 hours. Let X and Y be two cars starting from points A and B respectively and their speed be x km/hr and y km/hr.

Then, answer the following questions,

(i) When both cars move in the same direction, then the situation can be represented algebraically as:

(a) $x - y = 10$

(b) $x + y = 10$

(c) $x + y = 9$

(d) $x - y = 9$

Ans. (a) $x - y = 10$

Explanation :

Suppose two cars meet at point Q.

Then, distance travelled by car X = AQ

and distance travelled by car Y = BQ.

It is given that two cars meet in 9 hours.

Distance travelled by car X in 9 hours

= $9x$ km

$\Rightarrow AQ = 9x$

Distance travelled by car Y in 9 hours

= $9y$ km

$\Rightarrow BQ = 9y$

Clearly, AQ – BQ = AB

$\Rightarrow 9x - 9y = 90$

$\Rightarrow x - y = 10$

(ii) When both cars move in opposite direction, then the situation can be represented algebraically as:

(a) $x - y = 70$

(b) $x + y = 90$

(c) $x + y = 70$

(d) $x + y = 10$

Ans. (c) $x + y = 70$

Explanation :

Suppose two cars meet at point P.

Then, Distance travelled by car X = AP

and distance travelled by car Y = BP

In this case, two cars meet in $\frac{9}{7}$ hours.

Distance travelled by car X in 9/7 hours = $\frac{9}{7}x$ km

AP = $\frac{9}{7}x$ km

Distance travelled by car Y in $\frac{9}{7}$ hours is $\frac{9}{7}y$ km

BP = $\frac{9}{7}y$

Clearly, AP + BP = AB

$\Rightarrow \frac{9}{7}x + \frac{9}{7}y = 90$

$\Rightarrow \frac{9}{7}(x + y) = 90$

$\Rightarrow x + y = 70$

(iii) Speed of car X is:

(a) 30 km/hr

(b) 40 km/hr

(c) 50 km/hr

(d) 60 km/hr

Ans. (b) 40 km/hr

Explanation :

We have $x - y = 10$

and $x + y = 70$

On solving both the equations, we get

$x = 40$ km/hr

Hence, speed of car X is 40 km/hr.

(iv) Speed of car Y is:

(a) 50 km/hr

(b) 40 km/hr

(c) 30 km/hr

(d) 60 km/hr

Ans. (c) 30 km/hr

Explanation :

We have $x - y = 10$
$\Rightarrow 40 - y = 10$
$\Rightarrow y = 30$
Hence, speed of car y is 30 km/hr.
(v) If the speed of car X and car Y, each is increased by 10 km/hr, and cars are moving in opposite direction, then after how much time they will meet?

(a) 5 hrs

(b) 4 hrs

(c) 2 hrs

(d) 1 hr

Ans. (d) 1 hr

Explanation :

New speed of car X = 50 km/hr

New speed of car Y = 40 km/hr

Then, $t = \frac{90}{50+40} = 1 \text{ hour}$

108. A quadratic equation can be defined as an equation of degree 2. This means that the highest exponent of the polynomial in it is 2. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers are a ^{1} 0.

Every quadratic equation has two roots depending of the nature of its discriminant, D = $b^2 - 4ac$. Based on the above information, answer the following questions.

(i) Which of the following quadratic equation have no real roots?

(a) $-4x^2 + 7x - 4 = 0$

(b) $-4x^2 + 7x - 2 = 0$

(c) $-2x^2 + 5x - 2 = 0$

(d) $3x^2 + 6x + 2 = 0$

Ans. (a) $-4x^2 + 7x - 4 = 0$

Explanation :

To have no real roots,

Discriminant ($D = b^2 - 4ac$) should be < 0.

(a) $D = 7^2 - 4(-4)(-4)$

$= 49 - 64 = -15 < 0$

(b) $D = 7^2 - 4(-4)(-2)$

$= 49 - 32 = 17 > 0$

(c) $D = 5^2 - 4(-2)(-2)$

$= 25 - 16 = 9 > 0$

(d) $D = 6^2 - 4(3)(2)$

$= 36 - 24 = 12 > 0$

(ii) Which of the following quadratic equation have rational roots?

(a) $x^2 + x - 1 = 0$

(b) $x^2 - 5x + 6 = 0$

(c) $4x^2 - 3x - 2 = 0$

(d) $6x^2 - x + 11 = 0$

Ans. (b) $x^2 - 5x + 6 = 0$

Explanation :

To have rational roots

Discriminant (D = $b^2 - 4ac$) should be > 0 and also a perfect square.

(a) D = $1^2 - 4(1)(-1)$

= 1 + 4 = 5 > 0

but is not a perfect square.

(b) D = $(-5)^2 - 4(1)(6)$

= 25 – 24 = 1 > 0

and it is a perfect square.

(c) D = $(-3)^2 - 4(4)(-2)$

= 9 + 32 = 41 > 0

but is not a perfect square.

(d) D = $(-1)^2 - 4(6)(11)$

= 1 – 264 = – 263 < 0

which is not a perfect square.

(iii) Which of the following quadratic equation have irrational roots?

(a) $3x^2 + 2x + 2 = 0$

(b) $4x^2 - 7x + 3 = 0$

(c) $6x^2 - 3x - 5 = 0$

(d) $2x^2 + 3x - 2 = 0$

Ans. (c) $6x^2 - 3x - 5 = 0$

Explanation :

To have irrational roots,

Discriminant (D = $b^2 - 4ac$) should be > 0 but not a perfect square.

(a) D = $2^2 – 4(3)(2)$

= 4 – 24

= – 20 < 0

(b) D = $(– 7)^2 – 4(4)(3)$

= 49 – 48

= 1 > 0 and a perfect square.

(c) D = $(– 3)^2 – 4(6)(– 5)$

= 9 + 120

= 129 > 0

and not a perfect square.

(d) D = $3^2 – 4(2)(– 2)$

= 9 + 16

= 25 > 0

and a perfect square.

(iv) Which of the following quadratic equations have equal roots?

(a) $x^2 – 3x + 4 = 0$

(b) $2x^2 – 2x + 1 = 0$

(c) $5x^2 – 10x + 1 = 0$

(d) $9x^2 + 6x + 1 = 0$

Ans. (d) $9x^2 + 6x + 1 = 0$

Explanation :

To have equal roots,

Discriminant (D = $b^2 – 4ac$) should be = 0.

(a) D = $(– 3)^2 – 4(1)(4)$

= 9 – 16

= – 7 < 0

(b) D = $(-2)^2 - 4(2)(1)$

= 4 – 8

= – 4 < 0

(c) D = $(-10)^2 - 4(5)(1)$

= 100 – 20

= 80 > 0

(d) D = $6^2 - 4(9)(1)$

= 36 – 36 = 0

(v) Which of the following quadratic equations has two distinct equal roots?

(a) $x^2 + 3x + 1 = 0$

(b) $-x^2 + 3x - 3 = 0$

(c) $4x^2 + 8x + 4 = 0$

(d) $3x^2 + 6x + 4 = 0$

Ans. (a) $x^2 + 3x + 1 = 0$

Explanation :

To have two distinct real roots,

Discriminant (D = $b^2 - 4ac$) should be > 0.

(a) D = $3^2 - 4(1)(1) = 9 - 4$
= 5 > 0
(b) D = $3^2 - 4(-1)(-3)$
= 9 – 12
= – 3 < 0

(c) D = $8^2 - 4(4)(4)$
$= 64 - 64 = 0$
(d) D = $6^2 - 4(3)(4) = 36 - 48$
$= -12 < 0$

109. In our daily life we use quadratic formula as for calculating areas, determining a product's profit or formulating the speed of an object and many more.

Based on the above information, answer the following questions :

(i) If the roots of the quadratic equation are 2, – 3, then its equation is :

(a) $x^2 - 2x + 3 = 0$

(b) $x^2 + x - 6 = 0$

(c) $2x^2 - 3x + 1 = 0$

(d) $x^2 - 6x - 1 = 0$

Ans. (b) $x^2 + x - 6 = 0$

Explanation :

Roots of the quadratic equation are 2 and – 3.

The required quadratic equation is

$\Rightarrow (x - 2)(x + 3) = 0$

$\Rightarrow x^2 + x - 6 = 0$

(ii) If one root of the quadratic equation $2x^2 + kx + 1 = 0$ is $\frac{-1}{2}$, then k =

(a) 3

(b) – 5

(c) – 3

(d) 5

Ans. (a) 3

Explanation :

We have, $2x^2 + kx + 1 = 0$

Since, $\frac{-1}{2}$ is the root of the equation, so it will satisfy the given equation.

$2\left(\frac{-1}{2}\right)^2 + k\left(\frac{-1}{2}\right) + 1 = 0$

$\Rightarrow 1 - k + 2 = 0$

$\Rightarrow k = 3$

(iii) Which of the following quadratic equation has equal and opposite roots?

(a) $x^2 - 4 = 0$

(b) $16x^2 - 9 = 0$

(c) $3x^2 + 5x - 5 = 0$

(d) Both (a) and (b)

Ans. (d) Both (a) and (b)

Explanation :

If the roots of the quadratic equations are opposites to each other then coefficient of x (sum of roots) is 0.

So, both (a) and (b) have the coefficient of $x = 0$.

(iv) Which of the following quadratic equation can be represented as $(x - 2)^2 + 19 = 0$?

(a) $x^2 + 4x - 15 = 0$

(b) $x^2 - 4x + 15 = 0$

(c) $x^2 - 4x + 23 = 0$

(d) $x^2 + 4x + 23 = 0$

Ans. (c) $x^2 - 4x + 23 = 0$

Explanation :

The given equation is

$(x - 2)^2 + 19 = 0$

$\Rightarrow x^2 - 4x + 4 + 19 = 0$

$\Rightarrow x^2 - 4x + 23 = 0$

(v) If one root of a quadratic equation is $1+5\sqrt{7}$, then the other is:

(a) $1+5\sqrt{7}$

(b) $1-5\sqrt{7}$

(c) $-1+5\sqrt{7}$

(d) $-1-5\sqrt{7}$

Ans. (b) $1-5\sqrt{7}$

Explanation :

If one root of a quadratic equation is irrational, then its other root is also irrational and also its conjugate *i.e.*, if one root is $p+(\sqrt{q})$ then its other root is $p-(\sqrt{q})$

Passage Based Questions

110. During the battle of Mahabharta, Arjun carried some arrows for fighting with Bheeshma. With half of the arrows, he cut down the arrows thrown by Bheeshma on him and with six other arrows he killed the rath driver of Bheeshma. With one arrow each, he knocked down respectively the rath, flag and the bow of Bheeshma. Finally, with one more than four times the square root of total arrows, he laid Bheeshma unconscious on an arrow bed. Based on above information answer the following questions :

(i) Find the total number of arrows Arjun had.

(ii) Find the number of arrows that Arjun used to lay unconsicious Bheeshma.

Sol. (i) Suppose Arjun had x arrows.

Number of arrows used to cut arrows of Bheeshma = $\frac{x}{2}$

Number of arrows used to kill the rath driver = 6

Number of arrows used to knock down the rath, flag and bow of Bheeshma = 3

Number of arrows used to lay Bheeshma unconscious = $4\sqrt{x}+1$

According to question we have

$\therefore \frac{x}{2}+6+3+4\sqrt{x}+1 = x$

$\Rightarrow x+20+8\sqrt{x} = 2x$

$\Rightarrow x = 20+8\sqrt{x}$

Putting $x = y^2$, the above equation becomes

$y^2 = 20 + 8y$

$\Rightarrow y^2 - 8y - 20 = 0$

$\Rightarrow y^2 - 10y + 2y - 20 = 0$

$\Rightarrow (y - 10)(y + 2) = 0$

$\Rightarrow y = 10$ or $y = -2$

$\Rightarrow y = 10$

[$\because$ y cannot be negative]

$x = y^2 \Rightarrow x = 100$

Hence, Arjun had 100 arrows.

(ii) Required no. of arrows = $4\sqrt{x}+1$

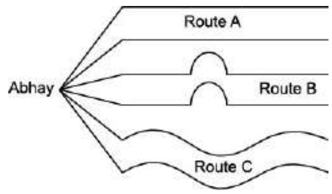
= $4\sqrt{100}+1$

= 40 + 1 = 41. **Ans.**

111. Abhay is planning a campaign. He investigates three possible routes.

• If he takes route A, which is 600 km long, he expects to cover *x* km per day.

• Route B, whose distance is equal to the route A, has more difficult conditions and he would only expect to cover (x – 5) km per day.
• Route C, which is 200 km longer than route A, has easier conditions and he would expect to cover (x + 5) km per day.



Based on the following figure and given information, answer the following questions :
(i) Abhay takes 20 days less, if he takes route C instead of route B. According to this statement, form an equation in x and reduce it to standard form.
(ii) Find the number of days taken by Abhay, if he choose route A.

Sol. (i) Number of days taken by Abhay, if he chooses route C = $\frac{800}{x+5}$
Number of days taken by Abhay, if he chooses route B

$= \frac{600}{x-5}$

$\Rightarrow \frac{800}{x+5} + 20 = \frac{600}{x-5}$

$\Rightarrow \frac{800}{x+5} - \frac{600}{x-5} = -20$

$\Rightarrow \frac{40(x-5) - 30(x+5)}{(x+5)(x-5)} = -1$

$\Rightarrow \frac{40x - 200 - 30x - 150}{x^2 - 25} = -1$

$\Rightarrow 10x - 350 = -x^2 + 25$

$\Rightarrow x^2 + 10x - 375 = 0$

(ii) On solving the equation $x^2 + 10x - 375 = 0$

We have,

$x = \frac{-10 \pm \sqrt{100 + 1500}}{2}$

$= \frac{-5 \pm 40}{2}$

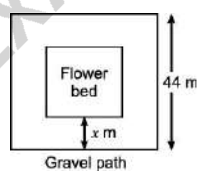
= 17·5 or – 22·5 ($x \neq$ – 22·5)

So, the number of days, Abhay takes on route A

$= \frac{600}{x}$

$= \frac{600}{17.5} \approx$ 34 days.**Ans.**

112. In the courtyard at the back of the house. Sunita prepared a small beautiful garden. The courtyard is of square shaped whose each side is 44 m. At the centre, she prepared a square flower bed leaving a gravel path all around it. One day a friend of her visited and praised the garden a lot. She also asked for the cost of laying the flower bed and gravelling the path, then sunita told her that the total cost of laying the flower bed and gravelling the path at ` 2.75 and ` 1.50 per square metre, respectively, is ` 4904. Using the above information find :
(i) The width of the path for gravelling.
(ii) The cost of laying the flower bed.
(iii) The cost of gravelling the path.
Sol. Let the width of the gravel path be x metres. Then, each side of the square flower bed is (44 – 2x) metres.



Now,

Area of the square field = 44 × 44 = 1936 m^2

Area of the flower bed = $(44 - 2x)^2$ m^2

Area of gravel path

= Area of square field – Area of flower bed

= 1936 – $(44 - 2x)^2$

= 1936 – (1936 – 176x + 4x^2)

$= (176x - 4x^2)\ m^2$

Cost of laying the flower bed

$= (44 - 2x)^2 \times \frac{275}{100}$

$= \frac{11}{4}(44 - 2x)^2$

$= 11(22 - x)^2$

Cost of gravelling the path = $(176x - 4x^2)\frac{150}{100}$

$= 6(44x - x^2)$

It is given that the total cost of laying the flower bed and gravelling the path is ` 4904

$\therefore 11(22 - x)^2 + 6(44x - x^2) = 4904$

$\Rightarrow 11[484 - 44x + x^2] + 264x - 6x^2 = 4904$

$\Rightarrow 5x^2 - 220x + 5324 = 4904$

$\Rightarrow 5x^2 - 220x + 420 = 0$

$\Rightarrow x^2 - 44x + 84 = 0$

$\Rightarrow x^2 - 42x - 2x + 84 = 0$

$\Rightarrow x(x - 42) - 2(x - 42) = 0$

$\Rightarrow (x - 2)(x - 42) = 0$

$\Rightarrow x = 2$ or $x = 42$

By $x \neq 42$, as the side of the square is 44 m.

Therefore, $x = 2$

Hence, the width of the gravel path is 2.

(ii) Cost of laying flower bed = Area of flower bed × rate per m^2.

Area of flower bed = $[44 - 2 \times 2]^2$

$= (44 - 4)^2$

$= (40)^2 = 1600 \text{ m}^2$.

Hence, cost of laying flower bed = 1600 × 2.75

= ` 4400.

Ans.

(iii) Cost of gravelling the path

= Area of path × Cost per m^2

Area of path = Area of square field

– Area of flower bed

$= [(44)^2 - (40)^2]\text{m}^2$

$= [1936 - 1600]\text{m}^2$

$= 336 \text{ m}^2$

Total cost of gravelling the path = 336 × 1.50

= ` 504

Hence, the cost of gravelling the path is ` 504.

Ans.

113. An industry produces a certain number of toys in a day. On a particular day, the cost of production of each toy was 9 less than twice the number of toys produced on that day. The total cost of production on that day was ` 143.
Based on the given information, answer the following questions :
(i) Find the number of toys produced in the industry on that day.
(ii) What is the cost of each toy?
Sol. (i) Let the number of toys be n
According to the question,
Cost of each toy = $2n - 9$
Total cost of toys = $n(2n - 9)$
But, total cost = ` 143
$\therefore n(2n - 9) = 143$

$\Rightarrow 2n^2 - 9n - 143 = 0$

$\Rightarrow n = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times (-143)}}{2 \times 2}$

$= \frac{9 \pm \sqrt{81 + 1144}}{4}$

$= \frac{9 \pm \sqrt{1225}}{4} = \frac{9 \pm 35}{4} = \frac{44}{4}, -\frac{26}{4}$

$\Rightarrow n = 11, -\frac{26}{4}$

Since, the number of toys cannot be negative or fraction,

$\therefore$ 11 toys were produced on that day.

(ii) Cost of each toy = ` (2 × 11) – 9) = ` 13

114. In a society, there is a big swimming pool. It has three pipes with uniform flow to fill the swimming pool. If first two pipes operate simultaneously then they fill the pool in same time during which the pool is filled by the third pipe alone. If second pipe is operated alonge it fills the pool five hour faster than the first pipe and four hours slower than the third pipe. Pool is closed for monthly maintenance. Based on the above information, answer the following questions:

(i) Find the time required by each pipe to fill the pool separately.

(ii) If all the three pipes are opened simultaneously then in how much time the pool be filled?

Sol. (i) Let x be the number of hours required by the second pipe along to fill the pool. Then, the first pipe takes (x + 5) hours, while the third pipe takes (x – 4) hours to fill the pool. So, the parts of the pool filled by the first, second and third pipes in one hour are $\frac{1}{x+5}, \frac{1}{x}, \frac{1}{x-4}$ respectively.

Let the time taken by the first and second pipes to fill the pool simultaneously by t hours then, the third pipe also takes the same time to fill the pool.

$\therefore \left(\frac{1}{x+5} + \frac{1}{x}\right) t = \left(\frac{1}{x-4}\right) t$

$\Rightarrow \frac{1}{x+5} + \frac{1}{x} = \frac{1}{x-4}$

$\Rightarrow (2x + 5)(x - 4) = x^2 + 5x$

$\Rightarrow x^2 - 8x - 20 = 0$

$\Rightarrow x^2 - 10x + 2x - 20 = 0$

$\Rightarrow (x - 10)(x + 2) = 0$

$\Rightarrow x = 10$ or $x = -2$

But time cannot be negative. So, $x = 10$.

Hence, the time required to fill the pool by first pipe is 15 hrs. second pipe is 10 hrs and third pipe is 6 hrs.**Ans.**

(ii) Tank filled by first pipe in 1 hour

$= \frac{1}{x+5} = \frac{1}{15}$

Tank filled by second pipe in 1 hour

$= \frac{1}{x} = \frac{1}{10}$

Tank filled by third pipe in 1 hour

$= \frac{1}{x-4} = \frac{1}{6}$

Tank filled by all the three pipes in one hour

$= \frac{1}{15} + \frac{1}{10} + \frac{1}{6}$

$= \frac{2+3+5}{30} = \frac{10}{30} = \frac{1}{3}$

i.e., $\frac{1}{3}$rd part of the full tank.

Hence, time required to fill the pool by all three pipes is 3 hrs.

Self-Assessment

115. In a flight of 6000 km, an aircraft was slowed down due to bad weather. The average speed for the trip was reduced by 400 km/hr and the time of the flight was increased by 30 minutes. Find the original duration of the flight.

Ans. $2\frac{1}{2}$ hrs.

116. A train travels a distance of 480 km at uniform speed. If the speed had been 8 km/hr less, it would have taken 3 hours more to cover the same distance. Formulate the quadratic equation in terms of the speed of the train.

[NCERT]

Ans. $x^2 - 8x - 1280 = 0$.

117. The product of two consecutive positive integers is 306. Form the quadratic equation to find the integers, if x denotes the smaller integer.

[NCERT]

Ans. $x^2 + x - 306 = 0$.

118. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be the product of the numbers of toys produced per day and 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ` 750. If x denotes the number of toys produced that day, form the quadratic equation to find x.

[NCERT]

Ans. $x^2 - 55x + 750 = 0$.

119. The height of a right-triangle is 7 cm less than the base. If the hypotenuse is 13 cm form the quadratic equation to find the base of the triangle.

[NCERT]

Ans. $x^2 - 7x - 60 = 0$.

120. Solve : $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$, x ¹ 0, 1, 2

[NCERT]

Ans. $x = 3$ and $\frac{4}{3}$.

121. Solve : $x + \frac{1}{x} = 3$, where $x \neq 0$.

[NCERT]

Ans. $x = \frac{3 \pm \sqrt{5}}{2}$.

122. Determine the roots of the equation

$2x^2 - 6x + 3 = 0$.

[NCERT]

Ans. $x = \frac{3 \pm \sqrt{3}}{2}$.

123. Find the value of k for which the equation $kx(x - 2) + 6 = 0$ has real and equal roots.

[NCERT]

Ans. $k = 6$.

124. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/hr more than the passenger train, find the speeds of the two trains.

[NCERT]

Ans. The speed of the passenger train is 33 km/hr and the speed of the express train is 44 km / hr.

125. The sum of the reciprocals of Rehman's ages 3 years ago and 5 years hence is $\frac{1}{3}$. Find his present age.

[NCERT]

Ans. 7 years.

126. A pole has to be erected at a point on the boundary of a circular park of diameter 13 m in such a way that the difference of its distances from two diametrically opposite fixed gates *A* and *B* on the boundary is 7 . If it is possible to do so, at what distances from the gates should the pole be erected?

[NCERT]

Ans. 5 metres and 12 metres.

127. Is it possible to design a rectangular mango grove whose length is twice its breadth and area is 800 m^2 ? If so, find the length and the breadth.

[NCERT]

Ans. Breadth = 20 m and length = 40 m.

128. Two water taps together take $9\frac{3}{8}$ hours to fill a tank. If the tap with the larger diameter takes 10 hours lesser than the tap with the smaller diameter, then find the time in which each tap can separately fill the tap.

[NCERT]

Ans. The smaller tap takes 25 hours to fill the tank and the larger one takes 15 hours to do so.

129. In a class test, the sum of the marks obtained by Shefali in Mathematics and English was 30. Had she secured 2 more marks in Mathematics and 3 less in English then the product of the marks in both the tests would have been 210. Find the marks obtained by her in the two subjects separately.

[NCERT]

Ans. The marks obtained in Mathematics are either 13 or 12 and in English are 17 or 18.

130. The nature of roots of the equation $2x^2 + \sqrt{5x} - 1 = 0$ is ....

(a) real and equal

(b) imaginary and equal

(c) imaginary and unequal

(d) real and unequal

Ans. (d) real and unequal

131. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then:

(a) k £ 4

(b) $k < 4$

(c) $k > 4$

(d) k ³ 4

Ans. (b) $k < 4$

132. The quadratic equation $49x^2 + 21x + \frac{9}{4} = 0$ has:

(a) real and equal roots

(b) four real roots

(c) real and unequal roots

(d) no real roots

Ans. (a) real and equal roots

133. The roots of the equation $\sqrt{2x+9}+x$ = 13 are:

(a) 8, – 20

(b) 20, – 8

(c) – 20, – 8

(d) 20, 8

Ans. (d) 8, 20

134. For what value of *k* is one root of the quadratic equation $9x^2 - 18x + k = 0$ double the other?

(a) 36

(b) 9

(c) 12

(d) 8

Ans. (d) 8

135. Find the value of *p* for which the quadratic equation $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$, $p \neq 1$ has equal roots. Hence, find the roots of the equation.

Sol. $p = 3$, roots are 3 and 3.

136. Solve : $\sqrt{7}y^2 - 6y - 13\sqrt{7} = 0.$

Sol. $y = \frac{13}{\sqrt{7}}, -\sqrt{7}.$

Chapter 5

Arithmetic Progressions

Basic Concepts

ARITHMETIC PROGRESSIONS

1. **Sequence** : A sequence is an arrangement of numbers or objects in a definite order.

2. A sequence $a_1, a_2, a_3, a_4, \ldots, a_n$ is called an Arithmetic Progression, if there exists a constant d such that $a_2 - a_1 = d$, $a_3 - a_2 = d$, $a_4 - a_3 = d, \ldots, a_{n+1} - a_n = d$ and so on. The constant d is called the common difference.

3. A sequence $a_1, a_2, a_3, a_4, \ldots, a_n$ is called an Arithmetic Progression, if $a_{n+1} - a_n$ is independent of n.

4. A sequence $a_1, a_2, a_3, \ldots, a_n$ is an A.P., if and only if its n^{th} term a_n is a linear expression in n and is such that the coefficient of n is the common difference.

5. The n^{th} term a_n of an A.P. with the first term a and common difference d is given by $a_n = a + (n-1)d$.

6. If there are m terms of an A.P. with the first term as a and the common difference as d then, n^{th} term from the end $= (m - n + 1)^{th}$ term from the beginning $= a + (m - n)\,d$. Also, n^{th} term from the end $= l - (n-1)d$, where l denotes the last term.

7. The sum of n terms of an A.P. with the first term a and common difference d is given by

$$S_n = \frac{n}{2}\{2a + (n-1)d\},$$

or, $S_n = \frac{n}{2}\{a + l\}$, where l = last term $= a + (n-1)d$

Multiple Choice Questions

1. The first three terms of A.P. are ($3y-1$), ($3y+5$) and ($5y+1$). Then y equals:

[Board Question]

(a) – 3

(b) 4

(c) 5

(d) 2

Ans. (c) 5

Explanation :

The terms of an A.P. are

$$(3y-1),\ (3y+5) \text{ and } (5y+1)$$

Thus, $d = (3y+5)-(3y-1) = (5y+1)-(3y+5)$

$\Rightarrow 6 = 2y - 4$

$\Rightarrow 2y = 10$

$\Rightarrow y = 5$

2. If k, $2k-1$ and $2k+1$ are three consecutive terms of an A.P., then the value of k is:

[Board Question]

(a) 2

(b) 3

(c) – 3

(d) 5

Ans. (b) 3

Explanation :

The terms of an A.P. are

$$k, (2k - 1) \text{ and } (2k + 1)$$

Thus, $d = (2k - 1) - k = 2k + 1 - (2k - 1)$

$\Rightarrow 2k - 1 - k = 2k + 1 - 2k + 1$

$\Rightarrow k - 1 = 2$

$\Rightarrow k = 3$

3. The next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}$, ... will be:

[Board Question]

(a) $\sqrt{70}$

(b) $\sqrt{84}$

(c) $\sqrt{97}$

(d) $\sqrt{112}$

Ans. (d) $\sqrt{112}$

Explanation :

The given series is $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$

Thus, a = $\sqrt{7}$ and d = $\sqrt{28} - \sqrt{7} = \sqrt{63} - \sqrt{28}$

$\Rightarrow d$ = $2\sqrt{7} - \sqrt{7}$ = $3\sqrt{7} - 2\sqrt{7}$ = $\sqrt{7}$

Thus, $t_4 = a + (4 - 1)d$

$\Rightarrow t_4$ = $\sqrt{7} + (4 - 1)\sqrt{7}$

$\Rightarrow t_4$ = $\sqrt{7} + 3\sqrt{7}$ = $4\sqrt{7}$

= $\sqrt{(16)7}$ = $\sqrt{112}$

4. The sum of first 20 odd natural numbers is:

[Board Question]

(a) 100

(b) 210

(c) 400

(d) 420

Ans. (c) 400

Explanation :

The given A.P. is

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39

Thus $a = 1$, $d = 2$ and $n = 20$

We know that,

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$= \frac{20}{2}\{2(1) + (20 - 1)2\}$

$= 10\{2 + 38\}$

$= 400$

5. If the n^{th} term of an A.P. is $(2n + 1)$, then the sum of its first three terms is:

[Board Question]

(a) 6

(b) 15

(c) 12

(d) 21

Ans. (b) 15

Explanation :

We know that,

$t_n = a + (n - 1)d = 2n + 1$

$\Rightarrow a + nd - d = 2n + 1$

$\Rightarrow (a - d) + nd = 1 + 2n$

On comparing we get

Thus, $a - d = 1$ and $d = 2$

$a = 2 + 1$

$= 3$

Then, $S_3 = \frac{3}{2}\{2(3) + (3 - 1)2\}$

$= \frac{3}{2}\{6 + 4\}$

$= 15$

6. The value of $t_{30} - t_{20}$ for the A.P. 2, 7, 12, 17, … is:

[Board Question]

(a) 100

(b) 10

(c) 50

(d) 20

Ans. (c) 50

Explanation :

$a = 2, d = 5$

So, $t_{30} = 2 + (30 - 1)5$

$= 2 + 145 = 147$

and $t_{20} = 2 + (20 - 1)5$

$= 2 + 95 = 97$

Thus, $t_{30} - t_{20} = 147 - 97$

$= 50.$

7. If the common difference of an A.P. is 3, then $t_{20} - t_{15}$ is:

[Board Question]

(a) 5

(b) 3

(c) 15

(d) 20

Ans. (c) 15

Explanation :

Given, $d = 3$

$t_{20} = a + (20 - 1)3 = a + 57$

$t_{15} = a + (15 - 1)3 = a + 42$

Thus, $t_{20} - t_{15} = a + 57 - (a + 42) = 15$

8. The first term of an A.P. is *p* and its common difference is *q*. Find the 10th term.

[Board Question]

(a) $p + 9q$

(b) $p + 11q$

(c) $p + 10q$

(d) $p + q$

Ans. (a) $p + 9q$

Explanation :

Given, $a = p$ and $d = q$

$\therefore t_{10} = p + (10 - 1)(q)$

$= p + 9q$

9. If the sum of first p terms of A.P. is $ap^2 + bp$, find its common difference.

[Board Question]

(a) $\frac{2b}{p-1} + 2a$

(b) $\frac{2a}{p-1} + 2b$

(c) $\frac{2b}{a-1} + 2p$

(d) $\frac{2a}{b-1} + 2p$

Ans. (a) $\frac{2b}{p-1} + 2a$

Explanation :

Let the first term be a and the common difference be d.

We know that,

$S_p = \frac{p}{2}\{2a + (p - 1)d\}$

$= ap^2 + bp$

$\Rightarrow p\{2a + (p - 1)d\} = 2p(ap + b)$

$\Rightarrow 2a + (p - 1)d = 2ap + 2b$

$\Rightarrow (p - 1)d = 2a(p - 1) + 2b$

$\Rightarrow (p - 1)(d - 2a) = 2b$

$\Rightarrow d - 2a = \frac{2b}{p-1}$

$\Rightarrow d = \frac{2b}{p-1} + 2a$

10. The common difference of the A.P.

[Board Question]

$\frac{1}{2q}, \frac{1-2q}{2q}, \frac{1-4q}{2q} \dots$ **is :**

(a) – 1
(b) 1

(c) q

(d) $2q$

Ans. (a) – 1

Explanation :

The given A.P. is $\frac{1}{2q}, \frac{1-2q}{2q}, \frac{1-4q}{2q} \dots$

Thus, common difference

$= \frac{1-2q}{2q} - \frac{1}{2q}$

$= -\frac{2q}{2q} = -1$

11. The n^{th} term of the A.P. a, $3a$, $5a$, ... is:

[Board Question]

(a) na
(b) $(2n - 1)a$
(c) $(2n + 1)a$
(d) $2na$

Ans. (b) $(2n - 1)a$

Explanation :

In the A.P., first terms, A = a and common difference, D = $3a - a = 2a$

We know, n^{th} term of an A.P. is given by,

$a_n = A + (n - 1) D$

$\Rightarrow a_n = a + (n - 1)2a$

$\Rightarrow a_n = a + 2an - 2a$

$\Rightarrow a_n = 2an - a$

$\Rightarrow a_n = a(2n - 1)$.

12. The sum of first five multiples of 3 is:

(a) 45
(b) 55
(c) 65
(d) 75

Ans. (a) 45

Explanation :

Here $a = 3$, $d = 3$, $n = 5$

$S_5 = \frac{n}{2}[2a+(n-1)d]$

$= \frac{5}{2}[6+4\times3]$

$= \frac{5}{2}[6+12]$

$= \frac{5}{2}[18]$

$= 5 \times 9$

$= 45$.

13. 11th term of the A.P.: – 3, $-\frac{1}{2}$, 2, ... is:

(a) 28
(b) 22
(c) – 38
(d) $-48\frac{1}{2}$.

Ans. (b) 22

Explanation :

Here, $a = -3$, $d = -\frac{1}{2}-(-3)$

$= -\frac{1}{2}+3$

$a = -3$, $d = \frac{5}{2}$, $n = 11$

11^{th} term is, $a_{11} = a + 10d$

$= -3+\frac{5}{2}\times 10$

$= -3 + 25 = 22$.

14. In an A.P., if $d = -4$, $n = 7$, $a_n = 4$, then a is:

(a) 6
(b) 7
(c) 20
(d) 28

Ans. (d) 28

Explanation :

Here $d = -4$, $n = 7$, $a_n = 4$

Since, $a_n = a + (n-1)d$

$\therefore a_7 = a + 6d$

$\Rightarrow 4 = a + 6 \times (-4)$

$\Rightarrow 4 = a - 24$

$a = 28$.

15. If $S_n = 2n^2 - 7n$, then the n^{th} term is ....

(a) $4n + 9$
(b) $2n + 3$
(c) $4n - 9$
(d) $9 - 4n$

Ans. (c) $4n - 9$

Explanation :

$S_n = 2n^2 - 7n$

$\Rightarrow \frac{n}{2}[2a+(n-1)d] = 2n^2 - 7n$

where, a is the first term of AP, d is common difference and n is the number of terms

$\Rightarrow an+\frac{n}{2}(n-1)d = 2n^2 - 7n$

$\Rightarrow na+\frac{n^2}{2}d-\frac{n}{2}d = 2n^2 - 7n$

$\frac{d}{2}n^2+n\left(a-\frac{d}{2}\right) = 2n^2 - 7n$

On comparing, we get

$\frac{1}{2}d = 2$

$\Rightarrow d = 4$

and $a-\frac{1}{2}d = -7$

$a-\frac{1}{2}\times 4 = -7$

$\Rightarrow a - 2 = -7$

$\Rightarrow a = -7 + 2$

$\Rightarrow a = -5$

Now, $a_n = a + (n - 1)d$

$= -5 + (n - 1)4$

$= -5 + 4n - 4$

$= 4n - 9.$

16. If $S_{n+1} = n^2 + 9n$, then the second term of the A.P. is ....

(a) 12

(b) 8

(c) – 10

(d) 10

Ans. (d) 10

Explanation :

$a_2 = S_2 - S_1$

$= [(1)^2 + 9(1)] - [0^2 + 9 \times 0]$

$= 10 - 0 = 10.$

17. If the sum of three consecutive terms of an increasing A.P. is 51 and the product of the first and third of these terms is 273, then the third term is:

(a) 13

(b) 9

(c) 21

(d) 17

Ans. (c) 21

Explanation :

Let three consecutive terms of an increasing A.P. be $a - d$, a, $a + d$ where a is the first term and d be the common difference

According to question,

$a - d + a + a + d = 51$

$\Rightarrow 3a = 51$

$\Rightarrow a = \frac{51}{3} = 17$

Now, product of the first and third terms

$(a - d)(a + d) = 273$

$\Rightarrow a^2 - d^2 = 273$

$\Rightarrow (17)^2 - d^2 = 273$

$\Rightarrow 289 - d^2 = 273$

$\Rightarrow d^2 = 289 - 273$

$= 16 = (\pm 4)^2$

$d = \pm 4$

... A.P. is increasing,

therefore, $d = 4$

Now, third term = $a + d$

$= 17 + 4 = 21$

18. If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are:

(a) 5, 10, 15, 20
(b) 4, 10, 16, 22
(c) 3, 7, 11, 15
(d) none of these

Ans. (a) 5, 10, 15, 20

Explanation :

Consider, the four numbers are in A.P. as

$$a - 3d,\ a - d,\ a + d,\ a + 3d$$

Where a is the first term and $2d$ is the common difference.

Now their sum = 50

$a - 3d + a - d + a + d + a + 3d = 50$

$\Rightarrow 4a = 50$

$\Rightarrow a = \frac{25}{2}$ ...(i)

Given, $a + 3d = 4(a - 3d)$

$\Rightarrow a + 3d = 4a - 12d$

$\Rightarrow 4a - a = 3d + 12d$

$\Rightarrow 3a = 15d$

$\Rightarrow a = 5d$

$\Rightarrow \frac{25}{2} = 5d$ [From (i)]

$\Rightarrow d = \frac{25}{2\times 5} = \frac{5}{2}$

Numbers are

$= \frac{25}{2} - 3\times\frac{5}{2}, \frac{25}{2} - \frac{5}{2}, \frac{25}{2} + \frac{5}{2}, \frac{25}{2} + 3\times\frac{5}{2}$

$= \frac{10}{2}, \frac{20}{2}, \frac{30}{2}, \frac{40}{2}$

= 5, 10, 15, 20

19. The 9th term of an A.P. is 449 and 449th term is 9. The term which is equal to zero is:

(a) 50th

(b) 502nd

(c) 508th

(d) none of these

Ans. (d) none of these

Explanation :

Since, $a_n = a + (n - 1)d$

Here, $a_9 = a + (9 - 1)d$

$449 = a + 8d$...(i)

and, $a_{449} = a + (449 - 1)d$

$9 = a + 448d$...(ii)

Subtracting equation (i) from equation (ii),

$440d = -440$

$\Rightarrow d = \frac{-440}{440} = -1$

Put value of d in (i), we get

$a + 8 \times (-1) = 449$

$\Rightarrow a = 449 + 8 = 457$

For $a_n = 0$

$0 = a + (n - 1)d$

$\Rightarrow 0 = 457 + (n - 1)(-1)$

$\Rightarrow 0 = 457 - n + 1$

$\Rightarrow n = 458$

458th term = 0

20. If the first term of an A.P. is *a* and *n*th term is *b*, then its common difference is:

(a) $\frac{b-a}{n+1}$

(b) $\frac{b-a}{n-1}$

(c) $\frac{b-a}{n}$

(d) $\frac{b+a}{n-1}$

Ans. (b) $\frac{b-a}{n-1}$

Explanation :

In the given A.P., first term = a

and nth term = b

$a + (n - 1)d = b$

⇒ $(n - 1)d = b - a$

⇒ $d = \frac{b-a}{n-1}$

21. Two A.P.'s have the same common difference. The first term of one of these is 8 and that of the other is 3. The difference between their 30th term is:

(a) 11

(b) 3

(c) 8

(d) 5

Ans. (d) 5

Explanation :

In two A.P.'s common-difference is same

Let A and *a* be two A.P.'s

First term of A is 8 and first term of *a* is 3

$A_{30} - a_{30} = 8 + (30 - 1)d - 3 - (30 - 1)d$

$= 5 + 29d - 29d$

= 5.

22. If 18th and 11th term of an A.P. are in the ratio 3 : 2, then its 21st and 5th terms are in the ratio:

(a) 3 : 2

(b) 3 : 1

(c) 1 : 3

(d) 2 : 3

Ans. (b) 3 : 1

Explanation :

Given 18th term : 11th term = 3 : 2

⇒ $\frac{a_{18}}{a_{11}} = \frac{3}{2}$

⇒ $\frac{a+17d}{a+10d} = \frac{3}{2}$

where '*a*' is the first term and '*d*' is the common difference.

$\Rightarrow 2a + 34d = 3a + 30d$

$\Rightarrow 34d - 30d = 3a - 2a$

$\Rightarrow a = 4d$

Now $\frac{a_{21}}{a_5} = \frac{a+20d}{a+4d} = \frac{4d+20d}{4d+4d}$

$= \frac{24d}{8d} = \frac{3}{1}$

$a_{21} : a_5 = 3 : 1.$

23. The first three terms of an A.P. respectively are 3*y* – 1, 3*y* + 5 and 5*y* + 1. Then, *y* equals:
(a) – 3
(b) 4

(c) 5

(d) 2

Ans. (c) 5

Explanation :

Since, 3 terms are in A.P.

$2(3y + 5) = 3y - 1 + 5y + 1$

(If a, b, c are in A.P., $b - a = c - b \Rightarrow 2b = a + c$)

$\Rightarrow 6y + 10 = 8y$

$\Rightarrow 10 = 2y$

$\Rightarrow y = 5$

24. The list of numbers – 10, – 6, – 2, 2, ... is:
(a) an A.P. with $d = -16$
(b) an A.P. with $d = 4$
(c) an A.P. with $d = -4$
(d) not an A.P.

Ans. (b) an A.P. with $d = 4$

Explanation :

The given list of numbers

– 10, – 6, – 2, 2, ... is an A.P. with

$d = -6 - (-10)$

$\Rightarrow d = -6 + 10 = 4.$

25. The 15th term from the last of the A.P. 7, 10, 13, ... 130 is:

(a) 49
(b) 85
(c) 88
(d) 110

Ans. (c) 88

Explanation :

15th term from the end of A.P. 7, 10, 13, ..., 130

Here, $a = 7$, $d = 10 - 7 = 3$, $l = 130$

15th term from the end

$= l - (n - 1)d$

$= 130 - (15 - 1) \times 3$

$= 130 - 42 = 88$

26. In an A.P., if $a_{18} - a_{14} = 32$, then the common difference is:

(a) 8

(b) – 8

(c) – 4

(d) 4

Ans. (a) 8

Explanation :

Given, $a_{18} - a_{14} = 32$

$\Rightarrow (a + 17d) - a - 13d = 32$

$\Rightarrow a + 17d - a - 13d = 32$

$\Rightarrow 4d = 32$

$\Rightarrow d = \frac{32}{4} = 8$

27. In an A.P., if *a* = 3.5, *d* = 0, *n* = 101, then a_n will be:

(a) 0
(b) 3.5
(c) 103.5
(d) 104.5

Ans. (b) 3.5

Explanation :

In an A.P.

$a = 3.5$, $d = 0$, $n = 101$, then $a_n = ?$

Since, $a_n = a_{101} = a + (101 - 1)d$

$= 3.5 + 100d = 3.5 + 100 \times 0$

$= 3.5 + 0 = 3.5$

28. Which term of the A.P. 21, 42, 63, 84, ... is 210?

(a) 9th

(b) 10th

(c) 11th

(d) 12th

Ans. (b) 10th

Explanation :

Here, $a = 21$

$d = 42 - 21 = 21$

Let 210 be *n*th term,

$210 = a + (n - 1)d$

$\Rightarrow 210 = 21 + (n - 1) \times 21$

$\Rightarrow 210 - 21 = 21(n - 1)$

$\Rightarrow n - 1 = \frac{189}{21}$

$\Rightarrow 9 = n - 1$

$\Rightarrow n = 9 + 1 = 10$

It is 10th term.

29. If the last term of the A.P. 5, 3, 1, – 1, ... is – 41, then the A.P. consists of:

(a) 46 terms

(b) 25 terms

(c) 24 terms

(d) 23 terms

Ans. (c) 24 terms

Explanation :

Last term of an A.P. 5, 3, 1, – 1, ... is – 41

Let, A.P. has '*n*' terms

Here, $a = 5$, $d = 3 - 5 = -2$ and $n = ?$

Then, – 41 will be the n^{th} term.

$l = a + (n - 1)d$

$\Rightarrow -41 = 5 + (n - 1)(-2)$

$\Rightarrow -41 - 5 = (n - 1)(-2)$

$\Rightarrow n - 1 = \frac{-46}{-2}$

$\Rightarrow n - 1 = 23$

$\Rightarrow n = 23 + 1 = 24$

A.P. consists of 24 terms.

30. The 21st term of an A.P., whose first two terms are – 3 and 4, is:
(a) 17
(b) 137
(c) 143
(d) – 143

Ans. (b) 137

Explanation :

First two terms of an A.P. are – 3 and 4

$a = -3$, $d = 4 - (-3) = 4 + 3 = 7$

Then, $a_{21} = a + 20d$

$= -3 + 20(7)$

$= -3 + 140$

$= 137$

31. If the 2nd term of an A.P. is 13 and the 5th term is 25, then its 7th term is:
(a) 30
(b) 33
(c) 37
(d) 38

Ans. (b) 33

Explanation :

In an A.P., consider 'a' as first term and 'd' as common difference

Now, $a_2 = 13$

$\Rightarrow a + d = 13$...(i)

$a_5 = 25$

$\Rightarrow a + 4d = 25$...(ii)

On subtracting equation (i) from (ii), we get

$3d = 12$

$\Rightarrow d = 4$

Substitute the value of d in eq. (i), we get

$a = 13 – 4 = 9$

$a_7 = a + 6d$

$= 9 + 6 \times 4 = 9 + 24 = 33$

32. If the first term of an A.P. is – 5 and the common difference is 2, then the sum of its first 6 terms is:

(a) 0

(b) 5

(c) 6

(d) 15

Ans. (a) 0

Explanation :

First term (a) of an A.P. is – 5 and common difference (d) is 2

$\because$ Sum of n terms = $\frac{n}{2}[2a+(n-1)d]$

$S_6 = \frac{6}{2}[2\times(-5)+(6-1)\times 2]$

$= 3[– 10 + 5 \times 2]$

$= 3 \times [– 10 + 10]$

$= 3 \times 0 = 0$

33. The number of two-digit numbers which are divisible by 3, is:

(a) 33

(b) 31

(c) 30

(d) 29

Ans. (c) 30

Explanation :

Two digit numbers which are divisible by 3 are 12, 15, 18, 21, ... 99

Here, $a = 12$, $d = 3$, $l = 99$

$l = a_n = a + (n - 1)d$

$\Rightarrow 12 + (n - 1) \times 3 = 99$

$\Rightarrow (n - 1)3 = 99 - 12 = 87$

$\Rightarrow n - 1 = \frac{87}{3} = 29$

$\Rightarrow n = 29 + 1 = 30$.

34. The sum of first *n* natural number is:

(a) $0.5n(n + 1)$

(b) $\frac{n^2}{2}$

(c) $n + 2$

(d) $0.5 + (n + 1)$

Ans. (a) $0.5n(n + 1)$

Explanation :

Series of natural numbers 1 + 2 + 3 + 4 + ... + n

Here $a = 1$, $d = 1$ and $l = n$

So, S = $\frac{n}{2}(a+l)$

= $\frac{n}{2}(1+n)$ = $0.5n(n + 1)$

35. What is the sum of first *n* odd natural numbers?

(a) $n^2 - 1$

(b) n^2

(c) $n^2 - 2$

(d) None of these

Ans. (b) n^2

Explanation :

Series of odd natural numbers

= 1 + 3 + 5 + 7 + ... + n.

Here, a = 1, d = 2

So, S = $\frac{n}{2}\{2a+(n-1)d\}$

= $\frac{n}{2}\{2+(n-1)2\}$

= $\frac{n}{2}\{2+2n-2\}$ = n^2

36. If t_n = $n(n + 3)$, then difference of its 5^{th} term and 2^{nd} term is:

(a) 20

(b) 30

(c) – 30

(d) 10

Ans. (b) 30

Explanation :

$t_n = n(n + 3)$

$t_5 = 5(5 + 3) = 40$

$t_2 = 2(2 + 3) = 10$

$t_5 - t_2 = 40 - 10 = 30$

37. The n^{th} term of an A.P.

$\frac{1}{m}, \frac{m+1}{m}, \frac{2m+1}{m},$ **............. is:**

(a) $\frac{m+1-mn}{m}$

(b) $\frac{mn-m+1}{m}$

(c) $\frac{mn-m-n}{m}$

(d) $\frac{mn+m-n}{m}$

Ans. (b) $\frac{mn-m+1}{m}$

Explanation :

$a = \frac{1}{m}$

$d = \frac{m+1}{m} - \frac{1}{m} = \frac{m+1-1}{m} = \frac{m}{m} = 1$

$t_n = a + (n-1)d$

$= \frac{1}{m} + (n-1) \times 1 = \frac{1+m(n-1)}{m}$

$= \frac{mn-m+1}{m}$

38. If the k^{th} term of the arithmetic progression 25, 50, 75, 100, ... is 1000, then k is ....

(a) 20
(b) 30
(c) 40
(d) 50

Ans. (c) 40

Explanation :

Here, $t_k = a + (k-1)d$

$\Rightarrow 1000 = 25 + (k-1)25$

$\Rightarrow 1000 = 25 + 25k - 25$

$\Rightarrow 1000 = 25k$

$\Rightarrow k = \frac{1000}{25} = 40$

39. A man saves ` 320 during the first month, ` 360 in the second month, ` 400 in the third month. If he continues his savings in this sequence, in how many months will he save ` 20,000?

(a) 28
(b) 25
(c) 22
(d) 20

Ans. (b) 25

Explanation :

a = 320, d = 40
S_n = 20,000
It is required to find n.
S_n = 20,000

$\Rightarrow 20{,}000 = \frac{n}{2}[2 \times 320 + (n - 1) \times 40]$
$\Rightarrow 1000 = n[16 + (n - 1)]$

$\Rightarrow 1000 = 16n + n^2 - n$

$\Rightarrow n^2 + 15n - 1000 = 0$
$\Rightarrow n = 25.$

40. If t_n is the n^{th} term of an A.P., then $t_{2n} - t_n$ is:
(a) $(n + 1)d$
(b) nd
(c) $(n - 1)d$
(d) none of these

Ans. (b) nd

Explanation :

$t_{2n} - t_n = \{a + (2n - 1)d\} - \{a + (n - 1)d\}$

$\Rightarrow t_{2n} - t_n = a + (2n - 1)d - a - (n - 1)d$

$\Rightarrow t_{2n} - t_n = (2n - 1)d - (n - 1)d$

$\Rightarrow t_{2n} - t_n = d\{(2n - 1) - (n - 1)\}$

$\Rightarrow t_{2n} - t_n = d\{2n - 1 - n + 1\}$

$\Rightarrow t_{2n} - t_n = nd$

41. The common difference of a constant A.P. is:

(a) 1

(b) 2

(c) 0

(d) none of these

Ans. (c) 0

Explanation :

Constant AP means no increasing and no decreasing.
So common difference is 0.

42. If *a* and *l*, are first and last terms of an A.P., then number of terms:

(a) $\left(\frac{l-a}{d}\right)+1$

(b) $\left(\frac{l-a}{d}\right)-1$

(c) $\left(\frac{l+a}{d}\right)+1$

(d) none of these

Ans. (a) $\left(\frac{l-a}{d}\right)+1$

Explanation :

We know,

$l = a + (n - 1)d$

$\Rightarrow l - a = (n - 1)d$

$\Rightarrow n - 1 = \frac{l-a}{d}$

$\Rightarrow n = \frac{l-a}{d}+1$

43. A mother divides ` 207 into three parts such that the amounts are in A.P. and gives it to her three children. The product of the two least amounts that the children had ` 4623. Find the amount received by each child.

(a) ` 66, ` 68 and ` 70

(b) ` 67, ` 69 and ` 71

(c) ` 60, ` 64 and ` 68

(d) ` 57, ` 59 and ` 61

Ans. (b) ` 67, ` 69 and ` 71

Explanation :

Let the amount received by the three children be in the form of A.P. is given by $a - d$, a, $a + d$. Since, sum of the amount is ` 207, we have

$(a - d) + a + (a + d) = 207$

$\Rightarrow 3a = 207$

$\Rightarrow a = 69$

It is given that product of the two least amounts is 4623.

$(a - d)a = 4623$

$\Rightarrow (69 - d)69 = 4623$

$\Rightarrow d = 2$

Therefore, amount given by the mother to her three children are

` (69 – 2), ` 69, ` (69 + 2) *i.e.*, is ` 67, ` 69 and ` 71.

44. If every term of an A.P. is multiplied by 3, then the common difference of the new A.P., is:

(a) same

(b) increase by three

(c) 3 times of the previous A.P.

(d) none of these

Ans. (c) 3 times of the previous A.P.

Explanation :

Let the A.P. be a, b, c then

common difference = $b - a$ …(i)

Now multiply each term by 3, we get

$$3a, 3b, 3c$$

then, common difference = $3b - 3a$

$= 3(b - a)$ …(ii)

Thus, by equations (i) and (ii), we can clearly see that common difference of the new AP. is 3 times of the previous A.P.

45. If x, 10, y, 24, z are in A.P. then value of x, y and z are:

(a) 3, 17, 31,
(b) 31, 17, 3
(c) 3, 17, 30
(d) none of these
Ans. (a) 3, 17, 31

Explanation :

10 = $x + d$ ...(i)

24 = $x + 3d$ ...(ii)

$2d = 14$ [from (i) and (ii)]

$d = 7$ and $x = 3$ so $y = 17$ and $z = 31$

46. The sum of the series 0.40 + 0.43 + 0.46 + ... + 1 is:
(a) 14.7
(b) 1.47
(c) 7.41
(d) none of these
Ans. (a) 14.7

Explanation :

Here the value of n is not given.

But the last term is given.

From this, we can find the value of n.

Given, $a = 0.40$ and $l = 1$,

we find $d = 0.43 - 0.40 = 0.03$

Therefore, $n = \left(\frac{l-a}{d}\right)+1$

$= \left(\frac{1-0.40}{0.03}\right)+1 = 21$

Sum of first n terms of an A.P.

$S_n = \frac{n}{2}[a+l]$

$\because n = 21$

Therefore, $S_n = \frac{21}{2}[0.40 + 1] = 14.7$

So, the sum of 21 terms of the given series is 14.7.

Very Short Answer Type Questions

47. Find the 9^{th} term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185.

Sol. Given, A.P. is 5, 9, 13, ..., 185

Here, $l = 185$ and $d = 9 - 5 = 13 - 9 = 4$

Then, $a_9 = l + (n - 1)d$

$= 185 - (9 - 1)(4)$

$= 185 - 8(4)$

... $a_9 = 153$ **Ans.**

48. For what value of p are $2p + 1$, 13, $5p - 3$ the three consecutive terms of an A.P.?

Sol. For $2p + 1$, 13, $5p - 3$ to be consecutive, the common difference should be the same.

Thus $13 - (2p + 1) = 5p - 3 - 13$

$\Rightarrow 12 - 2p = 5p - 16$

$\Rightarrow 7p = 28$

$\Rightarrow p = 4$. **Ans.**

49. Find the common difference of the arithmetic progression (A.P.)

[Board Question]

$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}$, ... $(a \neq 0)$.

Sol. Given, A.P. is $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}$, ...

$d = \frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a}$

$= \frac{-a}{3a} = \frac{-1}{3}$ **Ans.**

50. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?

Sol. Given, $a_{21} - a_7 = 84$

$\Rightarrow (a + 20d) - (a + 6d) = 84$

$\Rightarrow a + 20d - a - 6d = 84$

$\Rightarrow 20d - 6d = 84$

$\Rightarrow 14d = 84$

$\Rightarrow d = \frac{84}{14} = 6$

Hence, common difference = 6. **Ans.**

51. The first and the last terms of an A.P. are 5 and 45 respectively. If the sum of all the terms is 400, find the common difference.

Sol. Given,

$S_n = 400$, $a = 5$ and $l = 45$

We know $S_n = \frac{n}{2}[a + l]$

$\Rightarrow 400 = \frac{n}{2}[5 + 45]$

$\Rightarrow n = \frac{400 \times 2}{50} = 16$

Thus, applying $t_n = a + (n - 1)d$, we have

$45 = 5 + (16 - 1)d$

$\Rightarrow 15d = 40 \Rightarrow 3d = 8$

$\Rightarrow d = \frac{8}{3}$

Thus, the common difference is $\frac{8}{3}$. **Ans.**

52. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Sol. As the numbers that are divisible by both 2 and 5, have their last digits as always 0. Hence, the series is 110, 120, 130, …, 990.

Here $a = 110$, $d = 10$ and $t_n = 990$

We know that,

$t_n = a + (n - 1)d$

$\Rightarrow 990 = 110 + (n - 1)10$

$\Rightarrow (n - 1)10 = 880$

$\Rightarrow n - 1 = 88$

$\Rightarrow n = 89$

Thus, the number of natural numbers between 101 and 999 which are divisible by both 2 and 5 is 89. **Ans.**

53. The sum of the first *n* terms of an A.P. is $3n^2 + 6n$. Find the n^{th} term of this A.P.

Sol. We have,

$S_n = 3n^2 + 6n$

$\therefore S_{n-1} = 3(n - 1)^2 + 6(n - 1)$

Now, $t_n = S_n - S_{n-1}$

$= 3n^2 + 6n - 3(n - 1)^2 - 6(n - 1)$

$= 3n^2 + 6n - 3n^2 + 6n - 3 - 6n + 6$

$t_n = 6n + 3$ **Ans.**

OR

We know that

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$

Also, $S_n = 3n^2 + 6n$(Given)

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 3n^2 + 6n$

$\Rightarrow [2a + (n - 1)d] = 6(n + 2)$

$\Rightarrow 2a + dn - d = 6n + 12$

$\Rightarrow dn + (2a - d) = 6n + 12$

On comparing we get

$\therefore d = 6$

and $2a - d = 12$

$\Rightarrow 2a - 6 = 12$

$\Rightarrow 2a = 18$

$\Rightarrow a = 9$

Thus, $t_n = 9 + (n - 1)6 = 9 + 6n - 6 = 6n + 3$. **Ans.**

54. Find the sum of all three-digit natural numbers which are multiples of 11.

Sol. The given A.P. series is

$$110, 121, 132, 143, \ldots, 990$$

Here, $a = 110$, $d = 11$ and $t_n = 990$

We know that

$t_n = a + (n - 1)d$

$\Rightarrow 990 = 110 + (n - 1)11$

$\Rightarrow 90 = 10 + n - 1$

$\Rightarrow n = 81$

Now $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{81} = \frac{81}{2}(110 + 990)$

$= \frac{81}{2}(1100) = 81 \times 550$

= 44550. **Ans.**

55. Find an A.P. whose fourth term is 9 and the sum of its sixth term and thirteenth term is 40.

Sol. Let the first term of an A.P. be *a* and the common difference be *d*.

Thus, $t_4 = \{a + (4 – 1)d\} = 9$(Given)

$\Rightarrow a + 3d = 9$…(i)

Also $t_6 + t_{13} = 40$(Given)

$\Rightarrow a + 5d + a + 12d = 40$

$\Rightarrow 2a + 17d = 40$…(ii)

Multiplying (i) with 2 and then subtracting from (ii), we have

$11d = 22$

$\Rightarrow d = 2$

Thus, $a = 9 – 3(2) = 3$

Thus, the A.P. is 3, 5, 7, 9, 11, … **Ans.**

56. Find whether – 150 is a term of the A.P. 17, 12, 7, 2, …?

Sol. Given A.P. is 17, 12, 7, 2, ...

Here, $a = 17$, $d = 12 – 17 = – 5$

Let – 150 be the n^{th} term.

Thus $t_n = 17 + (n – 1)(– 5)$

$\Rightarrow 17 – 5n + 5 = – 150$

$\Rightarrow – 5n = – 150 – 22$

$\Rightarrow 5n = 172$

$\Rightarrow n = \frac{172}{5} = 34.4$

Thus, – 150 is not a term of this A.P. as n is not a natural number. **Ans.**

57. Which term of the A.P. 5, 9, 13, 17, …is 81 ? Also, find the sum.

Sol. From the given A.P, we have

$a = 5$ and $d = 9 - 5 = 4$

Let 81 be the n^{th} term of the A.P.

Thus, $t_n = 5 + (n - 1)(4) = 81$

$\Rightarrow 5 + 4n - 4 = 81$

$\Rightarrow 4n = 80$

$\Rightarrow n = 20$

Thus, 81 is the 20^{th} term of the A.P.

Hence $S_{20} = \frac{20}{2}\{2(5) + (20 - 1)4\}$

$= 20(5 + 38)$

$= 20 \times 43$

$= 860$. **Ans.**

58. Which term of the A.P. 3, 15, 27, 39,… will be 120 more than its 21^{st} term?

Sol. From the given A.P., we have

$a = 3$, $d = 15 - 3 = 12$

Thus, 21^{st} term, $t_{21} = 3 + (21 - 1)(12)$

$= 3 + 240 = 243$

Thus, the number 120 more than its 21^{st} term is $243 + 120 = 363$

Let 363 be the m^{th} term.

Hence, $t_m = 3 + (m - 1)(12) = 363$

$\Rightarrow 3 + 12m - 12 = 363$

$\Rightarrow 12m - 9 = 363$

$\Rightarrow 12m = 372$

$\Rightarrow m = 31$

Thus, the 31^{st} term will be 120 more than the 21^{st} term. **Ans.**

59. Find the value of the middle term of the following A.P., 10, 7, 4, …, (– 62).

Sol. From the given A.P., we have

$a = 10$, $d = -3$ and $t_n = -62$

Thus $t_n = 10 + (n - 1)(-3) = -62$

$\Rightarrow 10 - 3n + 3 = -62$

$\Rightarrow -3n = -75$

$\Rightarrow n = 25$

Thus, the middle term would be 13.

Hence $t_{13} = 10 + (13 - 1)(-3)$

$= 10 - 36 = -26$. **Ans.**

60. The 8^{th} term of an A.P. is zero. Prove that the 38^{th} term is triple its 18^{th} term.

Sol. Let the first term of the A.P. be *a* and the common difference be *d*.

Thus, $t_8 = a + (8 - 1)d = 0$[Given]

or $a + 7d = 0$...(i)

Also $t_{18} = a + (18 - 1)d$

$= a + 17d$

$= (a + 7d) + 10d = 10d$[from (i)]

and $t_{38} = a + (38 - 1)d$

$= a + 37d = (a + 7d) + 30d$

$= 30d$ [from (i)]

$= 3(10d)$

$= 3t_{18}$. **Hence Proved.**

61. If *m* times the m^{th} term of an A.P. is equal to *n* times the n^{th} term and $m \neq n$, then show that its $(m + n)^{th}$ term is zero.

Sol. Let the first term be *a* and the common difference be *d*.

Hence $t_m = a + (m - 1)d$ and $t_n = a + (n - 1)d$

Now $m\{a + (m - 1)d\} = n\{a + (n - 1)d\}$

$\Rightarrow ma + dm^2 - dm = na + dn^2 - dn$

$\Rightarrow (m - n)a + (m - n)(m + n)d - (m - n)d = 0$

$\Rightarrow a + (m + n)d - d = 0$

$\Rightarrow a + \{(m + n) - 1\}d = 0$

$\Rightarrow t_{m+n} = 0.$

Hence Proved.

62. For what value of *n*, are the *n*th terms of two A.P's. 63, 65, 67,....... and 3, 10, 17,...... equal?

Sol. 1^{st} A.P. is 63, 65, 67, ...

Here, $a = 63$, $d = 65 - 63 = 2$

$\therefore a_n = a + (n - 1)d$

$= 63 + (n - 1)(2)$

$= 63 + 2n - 2 = 61 + 2n$

2^{nd} A.P. is 3, 10, 17....

Here, $a = 3$, $d = 10 - 3 = 7$

$\therefore a_n = a + (n - 1)d$

$= 3 + (n - 1)(7)$

$= 3 + 7n - 7 = 7n - 4$

According to the question,

$61 + 2n = 7n - 4$

$\Rightarrow 61 + 4 = 7n - 2n \Rightarrow 65 = 5n$

$\Rightarrow n = \frac{65}{5}$

$\Rightarrow n = 13$

Hence, 13^{th} term of both A.P. is equal. **Ans.**

63. Which term of the A.P. 3, 8, 13, 18, … will be 55 more than its 20^{th} term?

Sol. Given, $a = 3$, $d = 5$

Thus, $t_{20} = 3 + (20 - 1)5$

$= 3 + 95 = 98$

So $98 + 55 = 153$

Let the required term be n.

So $t_n = 3 + (n - 1)5 = 153$

$\Rightarrow (n - 1)5 = 150 \Rightarrow n - 1 = 30$

$\Rightarrow n = 31$. **Ans.**

64. Find the 6^{th} term from the end of the A.P. 17, 14, 11, …, – 40.

Sol. Given, $a = 17$, $d = -3$ and $t_n = -40$

Now taking from the end

$a = -40$, $d = -3$ and $t_n = 17$

Hence, the 6^{th} term from the end is

$t_6 = \{-40 + (6-1)3\}$

$= -40 + 15 = -25$

Thus, the 6^{th} term from the end is -25. **Ans.**

65. Find the 21^{st} term of the A.P.

[Board Question]

$-4\frac{1}{2}, 3, -1\frac{1}{2}, \ldots$

Sol. Given, $-4\frac{1}{2}, -3, -1\frac{1}{2}, \ldots$

or $-\frac{9}{2}, -3, -\frac{3}{2}, \ldots$

Here $a = -\frac{9}{2}$, $d = -3-\left(-\frac{9}{2}\right) = -3+\frac{9}{2}$

$= \frac{-6+9}{2} = \frac{3}{2}$

... $T_n = a + (n-1)d$

$\Rightarrow T_{21} = -\frac{9}{2}+(21-1)\frac{3}{2}$

$\Rightarrow T_{21} = -\frac{9}{2}+20\times\frac{3}{2}$

$\Rightarrow T_{21} = -\frac{9}{2}+30$

$\Rightarrow T_{21} = \frac{-9+60}{2} = \frac{51}{2} = 25\frac{1}{2}$ **Ans.**

66. Write the value of x for which $(x + 2)$, $2x$ and $(2x + 3)$ are three consecutive terms of an A.P.

Sol. The terms of an A.P. are $(x + 2)$, $2x$ and $(2x + 3)$.

Thus, $d = 2x - (x + 2) = (2x + 3) - 2x$

$\Rightarrow x - 2 = 3$

$\Rightarrow x = 5$. **Ans.**

67. Find the sum of 25 + 28 + 31 + … + 100.

Sol. Here, $a = 25$, $d = 3$ and $t_n = 100$

Now, $t_n = 25 + (n - 1)3 = 100$

$\Rightarrow 3(n - 1) = 75$

$\Rightarrow n - 1 = 25$

$\Rightarrow n = 26$

$\therefore S_n = \frac{26}{2}\{2(25) + (26 - 1)3\}$

$= 13[50 + 75]$

$= 13 \times 125$

$= 1625$. **Ans.**

68. If the n^{th} term of an A.P. is $2n + 1$, find the sum of the first n terms of an A.P.

Sol. Given, $t_n = 2n + 1$

$t_1 = 2 \times 1 + 1 = 3$

and $t_2 = 2 \times 2 + 1 = 5$

Thus, $a = 3$ and $d = t_2 - t_1 = 2$

Hence, $S_n = \frac{n}{2}\{2(3) + (n - 1)(2)\}$

$= n\{3 + n - 1\}$

$= n(2 + n)$. **Ans.**

69. How many terms of the A.P. 65, 60, 55,.... be taken so that their sum is zero?

Sol. Given, A.P. is 65, 60, 55......

There, $a = 65$, $d = 60 - 65 = 55 - 60 = -5$

Now, $S_n = 0$

... $S_n = \frac{n}{2}[2a + (n-1)d] = 0$

$\Rightarrow \frac{n}{2}[2(65) + (n-1)(-5)] = 0$

$\Rightarrow 130 - 5n + 5 = 0$

$\Rightarrow 135 - 5n = 0$

$\Rightarrow 5n = 135$

$\therefore n = 27$

Hence, the number of terms are 27. **Ans.**

70. Find the sum of the first hundred even natural numbers, which are divisible by 5.

Sol. First hundred even natural numbers divisible by 5 are 10, 20, 30, 40, …

This is an A.P.

Here, $a = 10$, $d = 10$ and $n = 100$

$S_{100} = \frac{100}{2}\{2(10) + (100 - 1)(10)\}$

$= 50\{20 + 990\}$

$= 50\{1010\}$

$= 50500$. **Ans.**

71. The first and last terms of an A.P. are 4 and 81 respectively. If its common difference is 7, how many terms are there and what is their sum?

Sol. Given, $a = 4$, $d = 7$ and $t_n = 81$

We know $t_n = a + (n - 1)d$

$\Rightarrow 4 + (n - 1)7 = 81$

$\Rightarrow 7n - 3 = 81$

$\Rightarrow 7n = 84$

$\Rightarrow n = 12$

Thus, $S_{12} = \frac{12}{2}\{2(4) + (12 - 1)7\}$

$= 6\{8 + 77\}$

$= 6\{85\} = 510$. **Ans.**

72. Which term of the progression 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$..... is the first negative term?

Sol. Given, A.P. is 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$

$= 20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, \ldots$

Here, $a = 20$, $d = \frac{77}{4} - 20 = \frac{77-80}{4} = \frac{-3}{4}$

Let a_n be its first negative term

$a_n < 0$

Then $a + (n - 1)d < 0$

$\Rightarrow 20 + (n - 1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow 20 - \frac{3}{4}n + \frac{3}{4} < 0$

$\Rightarrow 20 + \frac{3}{4} < \frac{3}{4}n$

$\Rightarrow \frac{83}{4} < \frac{3}{4}n$

$\Rightarrow n > \frac{83}{4} \times \frac{4}{3}$

$\Rightarrow n > \frac{83}{3} = 27.66...$

28th term will be first negative term of given A.P.

Ans.

73. The n^{th} term of an A.P. is (7 – 4*n*). Find its common difference.

Sol. Given, $t_n = 7 - 4n$

$\therefore t_{n-1} = 7 - 4(n - 1)$

$= 7 - 4n + 4$

$= 11 - 4n$

Now, $d = t_n - t_{n-1}$

$\Rightarrow d = 7 - 4n - (11 - 4n)$

$\Rightarrow d = -4$. **Ans.**

OR

Given, $t_n = (7 - 4n)$.

Let the first term be a and the common difference be d.

Then, $t_n = a + (n - 1)d = (7 - 4n)$

$\Rightarrow \{a + (n - 1)d\} = 7 - 4n$

$\Rightarrow a + dn - d = 7 - 4n$

$\Rightarrow (a - d) + dn = 7 - 4n$

On comparing we get

$\Rightarrow d = -4$. **Ans.**

74. Write the next term of the A.P.

$\sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

Sol. The given A.P. is $\sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

Thus, $a = \sqrt{8}$

and $d = \sqrt{18} - \sqrt{8}$

$= 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Hence, $t_4 = \sqrt{8} + (4 - 1)\sqrt{2}$

(*i.e.,* next term)

= $2\sqrt{2}+3\sqrt{2}$

= $5\sqrt{2}$

= $\sqrt{50}$

Thus, the next term is $\sqrt{50}$. **Ans.**

75. Which term of the A.P. 21, 18, 15, … is zero?

Sol. The given A.P. is 21, 18, 15, …

Thus, a = 21 and d = 18 – 21 = – 3

Let the n^{th} term be zero.

Hence, $t_n = 21 - (n - 1)3 = 0$

$\Rightarrow 21 - 3n + 3 = 0$

$\Rightarrow 24 - 3n = 0$

$\Rightarrow 3n = 24$

$\Rightarrow n = 8$. **Ans.**

76. Find the number of all three-digit natural number disivible by 9.

Sol. Three digit natural numbers divisible by 9 are 108, 117, 126, 135, …, 999.

The above series is an A.P.

with, a = 108, d = 9 and t_n = 999

Thus, $t_n = 108 + (n - 1)9 = 999$

$\Rightarrow 108 + 9n - 9 = 999$

$\Rightarrow 9n = 999 - 99$

$\Rightarrow 9n = 900$

$\Rightarrow n = 100$

Thus, the number of three-digit natural numbers divisible by 9 are 100. **Ans.**

77. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of the A.P. is three times its 11^{th} term.

Sol. We know that

$T_n = a + (n - 1)d$

Given, $T_4 = a + (4 - 1)d = 0$

$\Rightarrow a + 3d = 0$

$\Rightarrow a = -3d$...(i)

Now, $T_{25} = a + (25 - 1)d$

$= a + 24d = (-3d) + 24d$

$= 21d$

and $T_{11} = a + (11 - 1)d$

$= a + 10d$

then, $3T_{11} = 3(a + 10d)$

$= 3a + 30d$

$= 3(-3d) + 30d$[From (i)]

$= 30d - 9d = 21d = T_{25}$

$\therefore 3T_{11} = T_{25}$ **Hence Proved.**

78. Find the middle term of the A.P. 6, 13, 20, ..., 216.

Sol. The given A.P. is 6, 13, 20, ..., 216

Let n be the number of terms,

$d = 13 - 6 = 7$

$a_n = 216$

Since, $a_n = a + (n - 1)d$

$216 = 6 + (n - 1)7$

$\Rightarrow 216 - 6 = (n - 1)7$

$\Rightarrow \frac{210}{7} = n - 1$

$\Rightarrow 30 + 1 = n$

$\Rightarrow n = 31$

Middle term = $\left(\frac{n+1}{2}\right)^{th}$ term

= $\left(\frac{32}{2}\right)^{th}$ term

= 16^{th} term

Now, $a_{16} = a + 15d$

= 6 + 15 × 7 = 111 **Ans.**

79. Find the A.P. whose n^{th} term is 7 – 3*k*. Also find the 20^{th} term.

Sol. From the question it is given that, n^{th} term is 7 – 3*k*

So, $T_n = 7 - 3n$

Now, we start giving values, 1, 2, 3, ... in the place of *n*, we get

$T_1 = 7 - (3 \times 1) = 7 - 3 = 4$

$T_2 = 7 - (3 \times 2) = 7 - 6 = 1$

$T_3 = 7 - (3 \times 3) = 7 - 9 = -2$

$T_4 = 7 - (3 \times 4) = 7 - 12 = -5$

$T_{20} = 7 - (3 \times 20) = 7 - 60 = -53$

Therefore, A.P. is 4, 1, – 2, – 5, ...

And 20^{th} term is – 53 **Ans.**

80. The angles of a triangle are in A.P., the least being half the greatest. Find the angles.

Sol. Let the angles be $a - d$, a, $a + d$; $a > 0$, $d > 0$

... Sum of angles = 180°

$a - d + a + a + d = 180°$

$\Rightarrow 3a = 180°$

$\Rightarrow a = 60°$...(i)

By the given condition

$a - d = \frac{a+d}{2}$

$\Rightarrow 2a - 2d = a + d$

$\Rightarrow 2a - a = d + 2d$

$\Rightarrow a = 3d$

$\Rightarrow d = \frac{a}{3} = \frac{60}{3} = 20°$[From (i)]

Angles are 60° – 20°, 60°, 60° + 20° *i.e.,* 40°, 60°, 80°. **Ans.**

81. Which term of the progression 4, 9, 14, 19, ... is 109?

Sol. Here, $d = 9 - 4 = 14 - 9 = 19 - 14 = 5$

Difference between consecutive terms is constant.

Hence it is an A.P.

Given : First term, $a = 4$, $d = 5$, $a_n = 109$

Since, $a_n = a + (n - 1)d$

$109 = 4 + (n - 1)5$

$\Rightarrow 105 = 5(n - 1)$

$\Rightarrow n - 1 = \frac{105}{5} = 21$

$\Rightarrow n = 21 + 1 = 22$

109 is the 22^{nd} term **Ans.**

82. How many natural numbers are there between 200 and 500, which are divisible by 7?

Sol. Natural numbers, divisible by 7 between 200 and 500 is,

$$203, 210, 217, ..., 497$$

Here, $a = 203$, $d = 210 - 203 = 7$, $a_n = 497$

$a + (n - 1)d = a_n$

$\Rightarrow 203 + (n - 1)7 = 497$

$\Rightarrow (n - 1)7 = 497 - 203 = 294$

$\Rightarrow n - 1 = \frac{294}{7} = 42$

$n = 42 + 1 = 43$

There are 43 natural numbers between 200 and 500 which are divisible by 7. **Ans.**

83. The sum of the 5th and the 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, find the A.P.

Sol. $a_5 + a_9 = 30$[Given]

$a + 4d + a + 8d = 30$[Q $a_n = a + (n - 1)d$]

$\Rightarrow 2a + 12d = 30$

$\Rightarrow a + 6d = 15$

$\Rightarrow a = 15 - 6d$...(i)

Now, $a_{25} = 3(a_8)$

$\Rightarrow a + 24d = 3(a + 7d)$

$\Rightarrow 15 - 6d + 24d = 3(15 - 6d + 7d)$

[From (i)]

$\Rightarrow 15 + 18d = 3(15 + d)$

$\Rightarrow 15 + 18d = 45 + 3d$

$\Rightarrow 18d - 3d = 45 - 15$

$\Rightarrow 15d = 30$

$d = \frac{30}{15} = 2$

From (i),

$a = 15 - 6(2) = 15 - 12 = 3$

A.P. is $a, a + d, a + 2d, a + 3d, ...$

$= 3, 5, 7, 9, ...$ **Ans.**

84. If p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, then show that $(a - b)r + (b - c)p + (c - a)q = 0$.

Sol. Let A be the first term and D be the common difference of the given A.P.

p^{th} term $= A + (p - 1)D = a$...(i)

q^{th} term $= A + (q - 1)D = b$...(ii)

r^{th} term $= A + (r - 1)D = c$...(iii)

L.H.S. $= (a - b)r + (b - c)p + (c - a)q$

$= [A + (p - 1)D - (A + (q - 1)D)]r$

$+ [A + (q - 1)D - (A + (r - 1)D)]p$

$+ [A + (r - 1)D - (A + (p - 1)D)]q$

$= [(p - 1 - q + 1)D]r + [(q - 1 - r + 1)D]p$

$+ [(r - 1 - p + 1)D]q$

$= D[(p - q)r + (q - r)p + (r - p)q]$

$= D[pr - qr + qp - rp + rq - pq]$

$= D[0] = 0 =$ R.H.S. **Ans.**

Short Answer Type Questions

85. If the 8^{th} term of an A.P. is 31 and the 15^{th} term is 16 more than the 11^{th} term, find the A.P.

[Board Question]

Sol. Let the first term be a and common difference be d.
Thus, $t_8 = a + (8 - 1)d = 31$
$\Rightarrow a + 7d = 31$…(i)
and $t_{15} = 16 + t_{11}$
$\Rightarrow a + (15 - 1)d = 16 + a + (11 - 1)d$
$\Rightarrow a + 14d = 16 + a + 10d$
$\Rightarrow 4d = 16$
$\Rightarrow d = 4$
Substituting $d = 4$ in (i), we get
$a + 7(4) = 31$
$\Rightarrow a + 28 = 31$
$\Rightarrow a = 3$
Thus, the A.P. is 3, 7, 11, 15 … **Ans.**

86. Which term of the A.P. 8, 14, 20, … will be 72 more than the 41^{st} term?

[Board Question]

Sol. Given, $a = 8$, $d = 6$
Thus, $t_{41} = 8 + (41 - 1)6$
$\Rightarrow t_{41} = 8 + 240$
$\Rightarrow t_{41} = 248$
Now, 72 + 248 = 320
Let the n^{th} term be 320.
So, $t_n = 8 + (n - 1)6 = 320$
$\Rightarrow 2 + 6n = 320$
$\Rightarrow 6n = 318$
$\Rightarrow n = 53$
Thus, the required term is 53^{rd} term. **Ans.**

87. The sum of the 5^{th} and the 9^{th} term of an A.P. is 30. If its

25^{th} term is three times the 8^{th} term, find the A.P.

[Board Question]

Sol. Let a be the first term and d be the common difference.

$\therefore T_5 = a + (5 - 1)d$

$\Rightarrow T_5 = a + 4d$

and $T_9 = a + (9 - 1)d$

$\Rightarrow T_9 = a + 8d$

Now, $T_5 + T_9 = 30$ [Given]

$\Rightarrow a + 4d + a + 8d = 30$

$\Rightarrow 2a + 12d = 30$

$\Rightarrow a + 6d = 15$…(i)

Also, $T_{25} = a + (25 - 1)d$

$\Rightarrow T_{25} = a + 24d$

and $T_8 = a + (8 - 1)d$

$\Rightarrow T_8 = a + 7d$

Now, $T_{25} = 3T_8$[Given]

$\Rightarrow a + 24d = 3(a + 7d)$

$\Rightarrow a + 24d = 3a + 21d$

$\Rightarrow 2a - 3d = 0$

On multiplying the above equation by 2, we get

$4a - 6d = 0$…(ii)

Adding equations (i) and (ii), we get

$5a = 15$

$\Rightarrow a = 3$

and $d = \frac{2(3)}{3} = 2$

So the required A.P. is 3, 5, 7, 9, … **Ans.**

88. The sum of the first three terms of an A.P. is 48. If the product of the first and the second terms exceeds four times

the third term by 12, find the A.P.

[Board Question]

Sol. Given, $S_3 = 48$

$S_3 = \frac{3}{2}\{2a + (3 - 1)d\}$

$\Rightarrow 48 = \frac{3}{2}\{2a + 2d\}$

$\Rightarrow 48 = 3(a + d)$

$\Rightarrow a + d = 16$

$\Rightarrow d = 16 - a$

Now, $T_2 = a + (2 - 1)\, d = a + d$

$T_3 = a + (3 - 1)\, d = a + 2d$

Now, $a(a + d) = 4(a + 2d) + 12$[Given]

$\Rightarrow 16a = 4a + 8d + 12$[$\because a + d = 16$]

$\Rightarrow 12a = 8d + 12$

$\Rightarrow 3a = 2d + 3$

$\Rightarrow 3a = 2(16 - a) + 3$[$\because d = 16 - a$]

$\Rightarrow 3a = 32 - 2a + 3$

$\Rightarrow 5a = 35$

$\Rightarrow a = 7$

Hence, $d = 16 - 7 = 9$

Hence, the required A.P. is 7, 16, 25,..... **Ans.**

89. In a given A.P., if the p^{th} term is q and the q^{th} term is p, then show that the n^{th} term is $(p + q - n)$.

[Board Question]

Sol. Let the first term be a and the common difference be d.

Thus, $T_p = a + (p - 1)d = q$

⇒ $a - d = q - pd$...(i)

and $T_q = a + (q - 1)d = p$

⇒ $a - d = p - qd$...(ii)

From equations (i) and (ii),

$q - pd = p - qd$

⇒ $d(q - p) = p - q$

⇒ $d = -1$

and $a = p + q - 1 = q + p - 1$

So, $T_n = \{a + (n - 1)d\}$

$= p + q - 1 + (n - 1)(-1)$

$= p + q - 1 - n + 1$

$= p + q - n$. **Hence Proved.**

90. The sum of the three numbers in A.P. is 21 and their product is 231. Find the numbers.

Sol. Let the numbers be $(a - d)$, a, $(a + d)$.

So, the first term = $a - d$

and the common difference = d

We know, $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

⇒ $S_3 = \frac{3}{2}\{2(a - d) + (3 - 1)d\}$

⇒ $21 = \frac{3}{2}\{2a - 2d + 2d\}$ [Given]

⇒ $3a = 21$

⇒ $a = 7$

Now, $(a - d)\,a(a + d) = 231$ [Given]

⇒ $(7 - d)\,7(7 + d) = 231$

$\Rightarrow 49 - d^2 = 33$

$\Rightarrow d^2 = 16$

$\Rightarrow d = \pm 4$

Thus, the numbers are either 3, 7, 11 or 11, 7, 3. **Ans.**

91. Find the sum of all two-digit odd positive numbers.

Sol. Two-digit odd positive integers are, 11, 13, 15, ... 99.

The above series is an A.P. with

$a = 11$, $d = 2$ and $T_n = 99$

Now, $T_n = 11 + (n - 1)2 = 99$

$\Rightarrow (n - 1)2 = 88$

$\Rightarrow n - 1 = 44$

$\Rightarrow n = 45$

$\therefore S_{45} = \frac{45}{2}\{2(11) + (45 - 1)(2)\}$

$= \frac{45}{2}\{2(11) + (44)(2)\}$

$= 45\{11 + 44\}$

$= 45 \times 55$

= 2475. **Ans.**

92. How many terms of the A.P. 3, 5, 7, 9, … must be added to get the sum of 120 ?

Sol. Given A.P. is, 3, 5, 7, 9, ...

Thus, $a = 3$ and $d = 2$

Now, $S_n = \frac{n}{2}\{2(3) + (n - 1)(2)\}$

$\Rightarrow n\{3 + (n - 1)\} = 120$

$\Rightarrow n(n + 2) = 120$

$\Rightarrow n^2 + 2n - 120 = 0$

$\Rightarrow n^2 + 12n - 10n - 120 = 0$

$\Rightarrow n(n + 12) - 10(n + 12) = 0$

$\Rightarrow (n + 12)(n - 10) = 0$

$\Rightarrow n = 10$ or -12

Here, the given A.P. series is increasing, so n cannot be negative.

So, the required answer is 10. **Ans.**

93. If the sum of the first *n* terms of an A.P. is given by $S_n = \frac{n}{2}(3n + 5)$, then find its 25^{th} term.

Sol. Let the first term be a and the common difference be d.

It is given that,

$S_n = \frac{n}{2}\{2a + (n - 1)d\} = \frac{n}{2}(3n + 5)$

$\Rightarrow 2a + (n - 1)d = 3n + 5$

$\Rightarrow (2a - d) + dn = 5 + 3n$

On comparing, we get

$d = 3$ and $2a - d = 5$

$\Rightarrow 2a = 8$

$\Rightarrow a = 4$

$\therefore T_{25} = 4 + (25 - 1)3$

$= 4 + 24 \times 3$

$= 76$. **Ans.**

94. The sum of the first 7 terms of an A.P. is 49 and the sum of the first 17 terms is 289. Find the sum of first *n* terms.

[NCERT & Board Question]

Sol. Let the first terms be a and the common difference be d.

Now, it is given that,

$S_7 = \frac{7}{2}\{2a + (7 - 1)d\} = 49$

$\Rightarrow 49 = \frac{7}{2}\{2a + 6d\}$

$\Rightarrow a + 3d = 7$…(i)

and $S_{17} = \frac{17}{2}\{2a + (17 - 1)d\} = 289$

$\Rightarrow 289 = \frac{17}{2}\{2a + (16)d\}$

$\Rightarrow a + 8d = 17$…(ii)

Subtracting equation (i) from equation (ii), we have

$5d = 10$

$\Rightarrow d = 2$

From equation (i),

$a = 7 - 3d$

$= 7 - 3(2)$

$= 7 - 6 = 1$

$\therefore S_n = \frac{n}{2}\{2(1) + (n - 1)2\}$

$\Rightarrow S_n = \frac{n}{2}\{2 + 2n - 2\} = n^2$

Hence, the sum of n terms will be n^2. **Ans.**

95. If the sum of first *n* terms of an A.P. is n^2, then find its 10^{th} term.

[Board Question]

Sol. Given, $S_n = n^2$, $S_{n-1} = (n - 1)^2$ $a_n = S_n - S_{n-1} = n^2 - (n - 1)^2$

$= n^2 - [n^2 - 2n + 1]$

$= n^2 - n^2 + 2n - 1$ $a_n = 2n - 1$

Put $n = 10$,

$\therefore a_{10} = 2 \times 10 - 1 = 19$

Hence 10th term = 19. **Ans.**

96. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

[Board Question]

Sol. Let the three digits of a positive number be $a - d$, a, $a + d$

$\therefore a - d + a + a + d = 15$[Given]

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

Original number = $100 (a - d) + 10 (a) + a + d$

$= 100a - 100d + 10a + a + d$

$= 111a - 99d$

and number obtained by reversing the digits

$= 100 (a + d) + 10 (a) + a - d$

$= 100a + 100d + 10a + a - d$

$= 111a + 99 d$

According to the question,

$(111a - 99d) - (111a + 99d) = 594$

$\Rightarrow -198d = 594$

$\Rightarrow d = -3$

$\therefore$ Original number is $111 (5) - 99 (-3)$

i.e, 852 **Ans.**

97. If S_n, denotes the sum of first *n* terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

[Board Question]

Sol. Let be the first term be *a* and the common difference be *d,*

We know, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{12} = \frac{12}{2}[2a + (12 - 1)d\,]$

$= 6\,(2a + 11d\,)$

$\Rightarrow S_{12} = 12a + 66d$...(i)

$S_8 = \frac{8}{2}[2a + (8 - 1)d\,]$

$= 4(2a + 7d)$

$\Rightarrow S_8 = 8a + 28d$...(ii)

and $S_4 = \frac{4}{2}[2a + (4 - 1)d]$

$\Rightarrow S_4 = 2(2a + 3d) = 4a + 6d$...(iii)

Given, $3(S_8 - S_4) = 3(8a + 28d - 4a - 6d)$

[from (ii) and (iii)]

$= 3(4a + 22d) = 12a + 66d$

$= S_{12}$[from (i)]

Hence Proved.

98. Pradeep repays the total loan of ` 1,18,000 by paying every month starting with the first instalment of ` 1000. He increases the instalment by ` 100 every month. What amount will he pay as the last instalment of loan?

Sol. Ist instalment = ` 1000

IInd instalment = ` (1000 + 100) = ` 1100

IIIrd instalment = ` (1100 + 100) = ` 1200

Let the number of instalments be n.

∴ Sum = 1000 + 1100 + 1200 + ...+ n^{th} term

These terms are in AP with a = 1000, d = 100.

∴ Sum = $\frac{n}{2}$ [2 × 1000 + (n – 1)100]

⇒ $\frac{n}{2}(2000 + 100n - 100) = 118000$

⇒ $n(1900 + 100n) = 236000$

⇒ $100n^2 + 1900n - 236000 = 0$

⇒ $n^2 + 19n - 2360 = 0$

⇒ $n^2 + 59n - 40n - 2360 = 0$

⇒ $(n + 59)(n - 40) = 0$

⇒ $n = 40$ or $n = -59$ (Rejected)

∴ Last instalment, $a_{40} = a + 39d$

= 1000 + 39 × 100

= ` 4900 **Ans.**

99. Saurav gets pocket money from his father every day. Out of the pocket money, he saves ` 2.75 on first day and on each succeeding day he increases his saving by 25 paise. Find:

(i) the amount saved by Saurav on 14th day,

(ii) the amount saved by Saurav on 25th day,

(iii) the total amount saved by Saurav in 30 days.

Sol. Money saved on Ist day = ` 2.75

Money saved on IInd day = ` (2.75 + 0.25)

= ` 3.00

Money saved on IIIrd day = ` (3.00 + 0.25)

= ` 3.25.

Money saved by Saurav forms an AP with a = 2.75 and d = 0.25

(i) Money saved on 14th day

= $a + 13d$

= 2.75 + 13 × 0.25

= 2.75 + 3.25

= ` 6.00 **Ans.**

(ii) Money saved on 25th day

= $a + 24d$

= 2.75 + 24 × 0.25

= ` 8.75 **Ans.**

(iii) Total amount of money saved in 30 days

= $\frac{30}{2}$(2 × 2.75 + 29 × 0.25)

= ` 191.25 **Ans.**

100. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, *e.g.,* a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Sol. Number of trees planted by students of class I
= 3 × 1 = 3
Number of trees planted by students of class II
= 3 × 2 = 6
Number of trees planted by students of class III
= 3 × 3 = 9

Number of trees planted by students of class XII
= 3 × 12 = 36
Total number of trees planted
= 3 + 6 + 9 + ... + 36

There are 12 terms of an A.P. with a = 3 and last term = 36

$\Rightarrow l = 36$

$\therefore$ Sum = $\frac{n}{2}(a + l) = \frac{12}{2}(3 + 36)$

= 234 **Ans.**

101. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.

Each competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run ?

Sol. We have,

d_1 = Distance run by the competitor to pick up first potato = 2 × 5 m

d_2 = Distance run by the competitor to pick up second potato = 2(5 + 3) m

d_3 = Distance run by the competitor to pick up third potato = 2(5 + 2 × 3) m

d_4 = Distance run by the competitor to pick up fourth potato = 2(5 + 3 × 3) m

d_n = Distance run by the competitor to pick up nth potato = 2{5 + (n – 1) × 3} m

Total distance run by the competitor to pick up potatoes

$= d_1 + d_2 + d_3 + \ldots\ldots\ldots\ldots + d_n$

$= 2 \times 5 + 2(5 + 3) + 2(5 + 2 \times 3) + 2\ (5 + 3 \times 3) + \ldots\ldots + 2\ \{5 + (n - 1) \times 3\}$ metres

$= 2[5 + \{5 + 3\} + \{5 + (2 \times 3)\} + \{5 + (3 \times 3)\} + \ldots\ldots + \{5 + (n - 1) \times 3\}]$
$= 2[(5 + 5 + \ldots\ldots + 5) + \{3 + (2 \times 3) + (3 \times 3) + \ldots\ldots + (n - 1) \times 3]$
$= 2\ [(5 + 5 + \ldots\ldots + 5)\} + 3\ \{1 + 2 + 3 + \ldots\ldots\ (n - 1)\}]$
$= 2\left[5n + 3\left(\frac{n-1}{2}\right)\{2 + (n-2)\}\right]$
$= 2\left[5n + \frac{3n(n-1)}{2}\right]$
$= [10n + 3n\ (n - 1)]$
$= 3n^2 + 7n$
$= n\ (3n + 7)$
Since, $n = 10$
∴ Total distance run by the competitor
$= 10(3(10) + 7)$
= 370 m. **Ans.**

102. Shyam was late by 5 minutes in joining his duty on the first working day. On the second day, he was late by 10 minutes, on third day by 15 minutes and so on. After 25 working days he was shunted out of the job. Find the total working time avoided by Shyam.

Sol. On first day, Shyam was late by 5 minutes
On second day, he was late by 10 minutes
On third day, he was late by 15 minutes and so on
The time by which Shyam was late for his office forms an A.P.
∴ $a = 5$, $d = 10 - 5 = 5$, $n = 25$
So, total working time avoided by Shyam, in 25 days was

$S_{25} = \frac{25}{2}\{2a + (25 - 1)\ d\}$

$= \frac{25}{2}\{2 \times 5 + (25 - 1)\ 5\}$

$= \frac{25}{2}\{2 \times 5 + 24 \times 5\}$

$= 25\ \{5 + 12 \times 5\}$

= 25 {5 + 60}

= 25 × 65

= 1625 minutes **Ans.**

103. If in an A.P., $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of its first n terms.

Sol. Given, $S_5 + S_7 = 167$

$\Rightarrow \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$

$\Rightarrow \frac{5}{2} \times 2(a + 2d) + \frac{7}{2} \times 2(a + 3d) = 167$

$\Rightarrow 5a + 10d + 7a + 21d = 167$

$\Rightarrow 12a + 31d = 167$...(i)

Also, $S_{10} = 235$

$\Rightarrow \frac{10}{2}(2a + 9d) = 235$

$\Rightarrow 10a + 45d = 235$

$\Rightarrow 2a + 9d = 47$...(ii)

On multiplying equation (ii) by 6, we get

$12a + 54d = 282$...(iii)

On subtracting equation (i) from (iii), we get

$12a + 54d = 282$

$12a + 31d = 167$

– – –

$23d = 115$

$\Rightarrow d = 5$

Substituting value of d in equation (i), we get

$12a + 31 \times 5 = 167$

$\Rightarrow 12a + 155 = 167$

$\Rightarrow 12a = 12$

$\Rightarrow a = 1$

Hence, A.P. is 1, 6, 11....

Long Answer Type Questions

104. In an A.P. of 50 terms, the sum of the first 10 terms is 210 and the sum of the last 15 terms is 2565. Find the A.P.

[Board Question]

Sol. Let a be the first term and d be the common difference.

Now, $S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

$\Rightarrow 210 = 5(2a + 9d)$

$\Rightarrow 2a + 9d = 42$…(i)

and 15^{th} term from the end is $(50 - 15 + 1) = 36^{th}$ term from the beginning.

So, $t_{36} = a + (36 - 1)d$

$t_{36} = a + 35d$

Hence, the sum of the last 15 terms

$= \frac{15}{2}\{2t_{36} + (15 - 1)d\}$

$\Rightarrow \frac{15}{2}\{2(a + 35d) + 14d\} = 2565$

$\Rightarrow 15\{a + 35d + 7d\} = 2565$

$\Rightarrow a + 42d = 171$

$\Rightarrow 2a + 84d = 342$…(ii)

Subtracting (i) from (ii), we have

$75d = 300$

$\Rightarrow d = 4$

From (i), $2a + 9d = 42$

$\Rightarrow 2a = 42 - 9d$

$\Rightarrow 2a = 42 - 9 \times 4$

$\Rightarrow 2a = 42 - 36 = 6$

$\Rightarrow a = 3$

$t_{50} = a + (n - 1)d$

$= 3 + 49 \times 4 = 3 + 196 = 199$

Thus, the A.P. is 3, 7, 11, … 199. **Ans.**

105. If S_n denotes the sum of the first n terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$.

[Board Question]

Sol. Let a be the first term of the series and d be the common difference.

$\therefore S_n = \frac{n}{2}\{2a + (n - 1)d\}$

So, $S_{30} = \frac{30}{2}\{2a + (30 - 1)d\} \Rightarrow S_{30} = 15\{2a + 29d]$ …(i)

$S_{20} = \frac{20}{2}\{2a + (20 - 1)d\}$

$\Rightarrow S_{20} = 10\{2a + 19d\}$…(ii)

and $S_{10} = \frac{10}{2}\{2a + (10 - 1)d\}$

$\Rightarrow S_{10} = 5\{2a + 9d\}$…(iii)

Now, $S_{20} - S_{10} = 10\{2a + 19d\} - 5\{2a + 9d\}$

$= 20a + 190d - 10a - 45d$

$= 10a + 145d = 5(2a + 29d)$

$\Rightarrow 3(S_{20} - S_{10}) = 3[5(2a + 29d]$

$S_{30} = 15(2a + 29b)$[From (i)] Thus, $3(S_{20} - S_{10}) = S_{30}$ **Hence Proved.**

106. Find the sum of the middlemost terms of the A.P.

[Board Question]

$-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$.

Sol. The given A.P. is $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$

Thus, $a = -\frac{4}{3}$, $d = -1-\left(-\frac{4}{3}\right)$

$= -1 + \frac{4}{3} = \frac{1}{3}$

and $t_n = 4\frac{1}{3}$

$t_n = a + (n - 1)d$

$\Rightarrow 4\frac{1}{3} = -\frac{4}{3}+(n-1)\frac{1}{3}$

$\Rightarrow \frac{13}{3} = -\frac{4}{3}+\frac{1}{3}n-\frac{1}{3}$

$\Rightarrow \frac{13}{3} = -\frac{5}{3}+\frac{1}{3}n$

$\Rightarrow \frac{13}{3}+\frac{5}{3} = \frac{1}{3}n$

$\Rightarrow n = 18$

Thus, the middlemost terms of the A.P. are the 9^{th} and 10^{th} terms.

So $t_9 = a + (9 - 1)d = a + 8d$

$= -\frac{4}{3}+\frac{8}{3}=\frac{4}{3}$

and $t_{10} = a + (10 - 1)d = a + 9d$

$= -\frac{4}{3}+\frac{9}{3}=\frac{5}{3}$

Thus, $t_9 + t_{10} = \frac{4}{3}+\frac{5}{3}=\frac{9}{3}=3$ **Ans.**

107. Find the common difference of an A.P. whose first term is 5 and the sum of the first 4 terms is half of the sum of the next four terms.

[Board Question]

Sol. Given, $a = 5$

Thus, $S_4 = \frac{4}{2}\{2a + (4 - 1)d\}$

$= 2(2a + 3d)$

$t_5 = \{a + (5 - 1)d\}$

$= (a + 4d)$

For the next set of 4 numbers,

$a = t_5$ and d remains the same

Thus, $S_4 = \frac{4}{2}\{2(a + 4d) + (4 - 1)d\}$

$= 2(2a + 8d + 3d)$

$= 2(2a + 11d)$

According to the question,

$\frac{1}{2} \times 2(2a + 11d) = 2(2a + 3d)$

$\Rightarrow 2a + 11d = 4a + 6d$

$\Rightarrow 5d = 2a = 2(5)$

$\Rightarrow d = 2$

Thus, the common difference is 2. **Ans.**

108. The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

[Board Question]

Sol. Let the first term be a, the common difference be d and the number of terms be n.

Thus, $t_4 = a + (4 - 1)d = a + 3d$

$t_8 = a + (8 - 1)d = a + 7d$

$t_6 = a + (6 – 1)d = a + 5d$

$t_{10} = a + (10 – 1)d = a + 9d$

Now, $t_4 + t_8 = 24$

$\Rightarrow a + 3d + a + 7d = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12$…(i)

and $t_6 + t_{10} = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22$…(ii)

Subtracting (i) from (ii), we have

$2d = 10$

$\Rightarrow d = 5$

From (i) $a = 12 – 5(5)$

$= 12 – 25$

$= – 13$

Hence, first three terms of the A.P. are – 13, – 8, – 3. **Ans.**

109. The sum of the first 6 terms of an A.P. is 42. If its 10^{th} and 30^{th} terms are in the ratio 1 : 3, find the 1^{st} and 13^{th} terms.

[Board Question]

Sol. Let a be the first term and d be the common difference of the A.P.

So, $S_6 = \frac{6}{2}\{2a + (6 – 1)d\}$

$\Rightarrow 42 = \frac{6}{2}\{2a + 5d\}$

$\Rightarrow 42 = 3(2a + 5d)$

$\Rightarrow 2a + 5d = 14$…(i)

$t_{10} = a + (10 - 1)d$

$\Rightarrow t_{10} = a + 9d$

$t_{30} = a + (30 - 1)d$

$\Rightarrow t_{30} = a + 29d$

Now $t_{10} : t_{30} = 1 : 3$

$\Rightarrow a + 9d : a + 29d = 1 : 3$

$\Rightarrow \frac{a+9d}{a+29d} = \frac{1}{3}$

$\Rightarrow 3a + 27d = a + 29d$

$\Rightarrow 2a = 2d$

$\Rightarrow a = d$…(ii)

Substituting (ii) in (i),

$2a + 5a = 14$

$\Rightarrow 7a = 14$

$\Rightarrow a = 2$

Thus, $d = 2$

Now $t_{13} = 2 + (13 - 1)2$

$\Rightarrow t_{13} = 2 + 24 = 26$

So the first term is 2 and the 13^{th} term is 26. **Ans.**

110. If the ratio of the sum of the first *n* terms of two A.Ps. is (7*n* + 1) : (4*n* + 27), then find the ratio of their 9th terms.

[Board Question]

Sol. Let the first terms be *a* and *a*’ and *d* and *d* ‘ be their respective common differences.

Then, $\frac{S_n}{S'_n} = \frac{\frac{n}{2}\{2a+(n-1)d\}}{\frac{n}{2}\{2a'+(n-1)d'\}}$

$= \frac{7n+1}{4n+27}$

$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right)d}{a'+\left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27}$

To get ratio of 9th terms, replacing $\frac{n-1}{2}$ = 8

⇒ *n* = 17

Hence $\frac{t_9}{t'_9} = \frac{a+8d}{a'+8d'}$

$= \frac{7\times17+1}{4\times17+27}$

$= \frac{120}{95} = \frac{24}{19}$ **Ans.**

111. Find the number of terms of the A.P. 64, 60, 56, … so that sum is 544. Explain the double answer.

[Board Question]

Sol. Given A.P. is 64, 60, 56, ...

So, *a* = 64, *d* = – 4 and S_n = 544

Let the number of terms be *n*.

Thus, $S_n = \frac{n}{2}\{2(64) + (n-1)(-4)\} = 544$

$\Rightarrow \frac{n}{2}\{128 - 4n + 4\} = 544$

$\Rightarrow \frac{n}{2}(132 - 4n) = 544$

$\Rightarrow \frac{4n}{2}\{33 - n\} = 544$

$\Rightarrow \frac{n}{2}\{33 - n\} = 136$

$\Rightarrow 33n - n^2 = 272$

$\Rightarrow n^2 - 33n + 272 = 0$

$\Rightarrow n^2 - 17n - 16n + 272 = 0$

$\Rightarrow n(n - 17) - 16(n - 17) = 0$

$\Rightarrow (n - 17)(n - 16) = 0$

$\Rightarrow n = 16$ or 17

The double answer is because of the zero. It does not add to the total sum but is definitely a number in the series (17^{th} term).

$t_n = 64 + (n - 1)(-4) = 0$

$\Rightarrow 64 - (4n - 4) = 0$

$\Rightarrow 68 - 4n = 0$

$\Rightarrow n = 17.$

i.e., 17^{th} term is zero. So sum of all 16 and 17 terms is zero as 0 will not contribute to the sum. **Ans.**

112. Find the sum of the following:

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right) + \ldots \text{ upto } n \text{ terms.}$$

[Board Question]

Sol. Given,

$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots\left(1-\frac{n}{n}\right)$

$= (1 + 1 + 1 + \ldots + 1) - \left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\frac{4}{n}+\ldots+\frac{n}{n}\right)$

$= n-\left[\frac{n}{2}\left\{2\left(\frac{1}{n}\right)+(n-1)\frac{1}{n}\right\}\right]$

$= n-\left[\frac{n}{2}\left\{\frac{2}{n}-\frac{1}{n}+1\right\}\right]$

$= n-\left[\frac{n}{2}\left\{\frac{1}{n}+1\right\}\right]$

$= n-\frac{1+n}{2}$

$= \frac{2n-n-1}{2n}$

$= \frac{n-1}{2}$

Thus, the sum of

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots\left(1-\frac{n}{n}\right) \text{ is } \frac{n-1}{2}.$$

Ans.

113. If *m*th term of an A.P. is $\frac{1}{n}$ and *n*th term is $\frac{1}{m}$, then find the sum of its first *mn* terms.

[Board Question]

Sol. Let a and d be the first term and common difference respectively of the given A.P.

Then, $\frac{1}{n}$ = m^{th} term ⇒ $\frac{1}{n} = a + (m-1)\,d$ ...(i)

$\frac{1}{m}$ = n^{th} term ⇒ $\frac{1}{m} = a + (n-1)\,d$...(ii)

Subtracting equation (ii) from equation (i),

$\frac{1}{n}-\frac{1}{m} = (m-n)\,d$

⇒ $\frac{m-n}{nm} = (m-n)\,d$

⇒ $d = \frac{1}{mn}$

Putting $d = \frac{1}{mn}$ in equation (i), we get

$\frac{1}{n} = a + (m-1)\frac{1}{mn}$

⇒ $\frac{1}{n} = a + \frac{1}{n} - \frac{1}{mn}$

⇒ $a = \frac{1}{mn}$

∴ Sum of first *mn* terms

$= \frac{mn}{2}[2a + (mn-1)d]$

$= \frac{mn}{2}\left[\frac{2}{mn}+(mn-1)\frac{1}{mn}\right]$

$= \frac{mn}{2}\left[\frac{2}{mn}+1-\frac{1}{mn}\right]$

$= \frac{mn}{2}\left(\frac{1}{mn}+1\right)$

$= \frac{1+mn}{2}$ **Ans.**

114. Which term of the Arithmetic Progression – 7, – 12, – 17, – 22, ... will be – 82 ? Is – 100 any term of the A.P. ? Give reason for your answer.

[Board Question]

Sol. Given A.P. is– 7, – 12, – 17, – 22, ...

Here $a = -7$, $d = -12 - (-7)$

$= -12 + 7$

$= -5$

Let $T_n = -82$

$\therefore T_n = a + (n-1)d$

$\Rightarrow -82 = -7 + (n-1)(-5)$

$\Rightarrow -82 = -7 - 5n + 5$

$\Rightarrow -82 = -2 - 5n \Rightarrow -82 + 2 = -5n$

$\Rightarrow -80 = -5n$

$\Rightarrow n = 16$

Therefore, 16^{th} term will be – 82.

Let $T_n = -100$

Again, $T_n = a + (n-1)d$

$\Rightarrow -100 = -7 + (n-1)(-5)$

$\Rightarrow -100 = -7 - 5n + 5$

$\Rightarrow -100 = -2 - 5n$

$\Rightarrow -100 + 2 = -5n$

$\Rightarrow -98 = -5n$

$\Rightarrow n = \frac{98}{5}$

But the number of terms can not be in fraction.

So, – 100 can not be the term of this A.P. **Ans.**

115. In an A.P., the sum of the first 10 terms is – 150 and the sum of the next ten terms is – 550. Find the A.P.

[Board Question]

Sol. Let the first term be *a* and the common difference be *d*.

Thus, $S_{10} = \frac{10}{2}\{2a + (10 - 1)d\} = -150$

$\Rightarrow 2a + 9d = -30$…(i)

and $S_{20} = \frac{20}{2}\{2a + (20 - 1)d\}$

$= 10\{2a + 19d\}$

Sum of next ten terms

$S_{10}' = S_{20} - S_{10}$

$\Rightarrow -550 = S_{20} + 150$

$\Rightarrow S_{20} = -550 - 150 = -700$

$\Rightarrow 10\{2a + 19d\} = -700$

$\Rightarrow 2a + 19d = -70$…(ii)

Subtracting (i) from (ii), we have

$10d = -40$

$\Rightarrow d = -4$

Substituting $d = -4$ in (i), we have

$2a + 9(-4) = -30$

$\Rightarrow 2a - 36 = -30$

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$

Thus, the A.P. is 3, – 1, – 5, – 9, … **Ans.**

116. The sum of first *n* terms of an A.P. is $5n^2 + 3n$. If m^{th} term is 168, find the value of *m* and also the 20^{th} term.

[Board Question]

Sol. Given, $S_n = 5n^2 + 3n$ and $t_m = 168$

Thus, $S_n = \frac{n}{2}\{2a + (n - 1)d\} = 5n^2 + 3n$

$\Rightarrow n\{2a + (n - 1)d\} = 2n(5n + 3)$

$\Rightarrow 2a + dn - d = 10n + 6$

$\Rightarrow (2a - d) + dn = 6 + 10n$

On comparing both sides, we get

$d = 10$

and $2a - d = 6$

$\Rightarrow 2a = d + 6$

$\Rightarrow 2a = 10 + 6$

$\Rightarrow a = 8$

Now, $t_m = a + (m - 1)d = 168$

$\Rightarrow 8 + (m - 1)10 = 168$

$\Rightarrow 10(m - 1) = 160$

$\Rightarrow m - 1 = 16$

$\Rightarrow m = 17$

Also, $t_{20} = 8 + (20 - 1)10 = 198$. **Ans.**

117. If the sum of first *m* terms of an A.P. is as the same as the

sum of its first *n* terms, show that the sum of its first ($m + n$) terms is zero.

[Board Question]

Sol. Let a be first term and d be common difference of given A.P.

Then, $S_m = S_n$(Given)

$\frac{m}{2}\{2a + (m - 1)d\} = \frac{n}{2}\{2a + (n - 1) d\}$

$\Rightarrow \frac{2am}{2} + \frac{m}{2}(m - 1)d - \frac{2an}{2} - \frac{n}{2}(n - 1) d = 0$

$\Rightarrow 2am - 2an + \{m(m - 1) - n (n - 1)\} d = 0$

$\Rightarrow 2a (m - n) + (m^2 - m - n^2 + n) d = 0$

$\Rightarrow 2a (m - n) + \{m^2 - n^2 - (m - n)\} d = 0$

$\Rightarrow 2a (m - n) + (m - n) (m + n - 1) d = 0$

$\Rightarrow (m - n) \{2a + (m + n - 1) d\} = 0$

$\Rightarrow 2a + (m + n - 1) d = 0$

Now, $S_{m+n} = \frac{m+n}{2} \{2a + (m + n - 1) d \}$

$= \frac{m+n}{2} \times 0 = 0$. **Hence Proved.**

118. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following *X*.

[Board Question]

Sol. Given, the houses in a row numbered consecutively from 1 to 49.

Now, sum of numbers preceding the number X

$= \frac{X(X-1)}{2}$

and sum of numbers following the number X

$= \frac{49(50)}{2} - \frac{X(X-1)}{2} - X$

$= \frac{2450 - X^2 + X - 2X}{2}$

$= \frac{2450 - X^2 - X}{2}$

According to the question,

Sum of no's preceding *X* = Sum of no's following X

$\frac{X(X-1)}{2} = \frac{2450 - X^2 - X}{2}$

$\Rightarrow X^2 - X = 2450 - X^2 - X$

$\Rightarrow 2X^2 = 2450$

$\Rightarrow X^2 = 1225$

$\Rightarrow X = 35$

Hence, at *X* = 35, sum of number of houses preceding the house no. *X* is equal to sum of the number of houses following X. **Ans.**

119. A thief runs with a uniform speed of 100 m/minute. After one minute, a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief?

[Board Question]

Sol. Let total time be *n* minutes.
Since, policeman runs after 1 minute so he will catch the thief in (*n* – 1) minutes.
Total distance covered by thief
= 100 m/minute × *n* minute
= (100 *n*) m
Now, total distance covered by the policeman
= (100)m + (100 + 10)m + (100 + 10 + 10)m + ...+ (*n* – 1) terms
i.e., 100 + 110 + 120 + ... + (*n* – 1) terms

$\therefore S_{n-1} = \frac{n-1}{2}[2 \times 100 + (n-2)\,10]$

Now, $\frac{n-1}{2}[200 + (n-2)\,10] = 100\,n$.

$\Rightarrow (n-1)(200 + 10n - 20) = 200\,n$.

$\Rightarrow 200\,n - 200 + 10n^2 - 10n + 20 - 20n = 200n$

$\Rightarrow 10n^2 - 30n - 180 = 0$

$\Rightarrow n^2 - 3n - 18 = 0$

$\Rightarrow n^2 - (6-3)\,n - 18 = 0$

$\Rightarrow n^2 - 6n + 3n - 18 = 0$

$\Rightarrow n(n-6) + 3(n-6) = 0$

$\Rightarrow (n+3)(n-6) = 0$

$\therefore n = 6$ or $n = -3$ (neglect)

Hence, policeman will catch the thief in (6 – 1) *i.e.*, 5 minutes. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

120. Assertion: If the sum of n terms of a series is $2n^2 + 3n + 1$ then series is in A.P. with common difference 4.

Reason: If sum of n terms of an A.P is quadratic expression, then common difference is twice of the coefficient of quadratic term.

Ans. (a) Both the Assertion and the Reason are correct and Reason is the correct explanation of the Assertion.

Explanation :

As per the reason statement,

Sum of *n* terms of A.P. is

$S_n = \frac{n}{2}[2A + (n - 1)D]$

Where A = first term, D = common difference. Hence, the sum of *n* terms of an A.P. is always in the form of quadratic expression. Hence, it is proved that reason is true.

As per the assertion given,

$S_n = 2n^2 + 3n + 1$

$\therefore t_n = S_n - S_{n-1}$

$= 2[n^2 - (n - 1)^2] + 3[n - n + 1]$

$= 2[2n - 1] + 3$

$= 4n + 1$

$\therefore D = t_{n+1} - t_n = 4$

So, both assertion and reason are correct and reason is correct explanation of assertion.

121. Assertion: If sum of *n* terms of two arithmetic progressions are in the ratio $(3n + 8) : (7n + 15)$ then ratio of their n^{th} term is 3 : 16

Reason: If S_n is quadratic expression, then $t_n = S_n - S_{n-1}$.

Ans. (d) Assertion is false but the Reason is true.

Explanation :

$$\frac{S_n}{S'_n} = \frac{3n+8}{7n+15} = \frac{n(3n+8)}{n(7n+15)} = \frac{3n^2+8n}{7n^2+15n}$$

$$\Rightarrow \frac{t_n}{t'_n} = \frac{S_n - S_{n-1}}{S_n - S_{n-1}} = \frac{(3n^2+8n) - 3(n-1)^2 - 8(n-1)}{7n^2+15n-7(n-1)^2-15(n-1)}$$

$\Rightarrow \frac{t_n}{t'_n} = \frac{6n+5}{14n+8}$

$\Rightarrow t_n : t'_n = 6n + 5 : 14n + 8$

So, assertion is proved to be false and reason is true.

Case Based Questions

122. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) What is the production during first year?

(a) 5000

(b) 2200

(c) 10000

(d) None of these

Ans. (a) 5000

Explanation :

Let the production during first year be a and let d be the increase in productoin every year. Then,

$a_6 = 16000$

$\Rightarrow a + 5d = 16000$...(i)

and $a_9 = 22600$

$\Rightarrow a + 8d = 22600$...(ii)

On subracting (i) from (ii), we get

$3d = 6600$

$\Rightarrow d = 2200$

Putting $d = 2200$ in (i), we get $a + 5 \times 2200 = 16000$

$\Rightarrow a + 11000 = 16000$

$\Rightarrow a = 16000 - 11000 = 5000$.

Thus, $a = 5000$ and $d = 2200$.

(ii) Find the production during 8th year.

(a) 7200

(b) 22000

(c) 20400

(d) None of these

Ans. (c) 20400

Explanation :

Production during 8th year is given by a_8

$= (a + 7d)$

$= (5000 + 7(2200))$

$= (5000 + 15400)$

$= 20400$.

(iii) Find the production during first 3 years is :

(a) 21600

(b) 22000

(c) 20400

(d) none of these

Ans. (a) 21600

Explanation :

$a_2 = (a + d)$

$= (5000 + 2200)$

$= 7200$

$a_3 = (a_2 + d)$

$= 7200 + 2200$

$= 9400$

Production during first 3 years

$= 5000 + 7200 + 9400$

$= 21600$

(iv) In which year, the production is 29,200.

(a) 10

(b) 11

(c) 12

(d) 13

Ans. (c) 12.

Explanation :

$a_n = 5000 + (n - 1)2200 = 29200$

$(n - 1)2200 = 29200 - 5000$

$= 24200$

$\Rightarrow n - 1 = 11$

$\Rightarrow n = 12$

(v) Find the difference of the production during 7th year and 4th year is:

(a) 5000

(b) 2200

(c) 10000

(d) none of these

Ans. (d) none of these.

Explanation :

$a_4 = (a + 3d)$

$= (5000 + 3(2200))$

$= 5000 + 6600 = 11600.$

$a_7 = (a_6 + d)$

$= 16000 + 2200 = 18200.$

Difference = 18200 – 11600 = 6600

123. Amit was playing a number card game. In the game, some number cards (having both +ve or –ve numbers) are arranged in a row such that they are following an arithmetic progression. On his first turn, Amit picks up 6th and 14th card and finds their sum to be –76. On the second turn he picks up 8th and 16th card and finds their sum to be – 96.

Based on the above information, answer the following questions:

(i) What is the difference between the numbers on any two consecutive cards?

(a) 7

(b) – 5

(c) 11

(d) – 3

Ans. (b) – 5.

Explanation :

Let the numbers on the cards be a, $a + d$, $a + 2d$, ...

According to questions, we have

$(a + 5d) + (a + 13d) = -76$

$\Rightarrow 2a + 18d = -76$

$\Rightarrow a + 9d = -38$...(A)

And $(a + 7d) + (a + 15d) = -96$

$\Rightarrow 2a + 22d = -96 \Rightarrow a + 11d = -48$...(B)

On subtracting (A) from (B), we get

$2d = -10$

$\Rightarrow d = -5$.

(ii) The number on first card is :

(a) 12

(b) 3

(c) 5

(d) 7

Ans. (d) 7

Explanation :

From (A), $a + 9(-5) = -38 \Rightarrow a = 7$

Number on first card = $a = 7$

(iii) What is the number on the 19th card?

(a) – 88

(b) – 83

(c) – 92

(d) – 102

Ans. (b) – 83

Explanation :

Number on 19^{th} card, $a_{19} = a + 18d$

$= 7 + 18(–5) = – 83$

(iv) What is the number on the 23rd card?

(a) –103

(b) –122

(c) –108

(d) –117

Ans. (a) – 103

Explanation :

Number on 23rd card, $a_{23} = a + 22d$

$= 7 + 22(–5) = – 103$

(v) What is the sum of 9th and 15th card ?

(a) – 129

(b) – 122

(c) – 180

(d) – 171

Ans. (a) – 129

Explanation :

Number on 9th card, $a_9 = a + 8d$

$= 7 + 8 \times -5$

$= 7 - 40 = -33$

Number on 15th card, $a_{15} = a + 14d$

$= 7 + 14 \times -5$

$= 7 - 70 = -63$

Sum of 9^{th} and 15^{th} card $= -33 + (-63)$

$= -129$

124. While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an AP. If the number of the *n*th spot is 20 + 4*n*, then answer the following questions to help the player in spotting the clues.

(i) Which number is on the first spot?

(a) 20

(b) 24

(c) 16

(d) 28

Ans. (b) 24.

Explanation :

Number of n^{th} spot $= 20 + 4n$

i.e., $t_n = 20 + 4n$

Then, $t_1 = 20 + 4 \times 1 = 24$

(ii) Which number is on the $(n - 2)^{th}$ spot?

(a) $16 + 4n$

(b) $24 + 4n$

(c) $12 + 4n$

(d) $28 + 4n$

Ans. (c) $12 + 4n$.

Explanation :

Number on $(n - 2)^{th}$ spot = $t_{n-2} = 20 + 4(n - 2)$

$= 20 + 4n - 8 = 12 + 4n$

(iii) Which number is on the 34th spot?

(a) 156

(b) 116

(c) 120

(d) 160

Ans. (a) 156

Explanation :

Number on 34th spot = t_{34}

$= 20 + 4(34) = 156$

(iv) Which spot is numbered as 116?

(a) 5th

(b) 8th

(c) 9th

(d) 24th

Ans. (d) 24th

Explanation :

Let *n*th spot be numbered as 116.

$\therefore t_n = 116$

$\Rightarrow 20 + 4n = 116$

$\Rightarrow 4n = 96$

$\Rightarrow n = 24.$

125. Your friend Veer wants to participate in a 200 m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds to win it.

(i) Which of the following terms are in A.P. for the given situation.
(a) 51, 53, 55 ...
(b) 51, 49, 47 ....
(c) – 51, – 53, – 55 ...
(d) 51, 55, 59 ...
Ans. (b) 51, 49, 47 ...

Explanation :

A.P. = 51, (51 – 2), (51 – 2 – 2) .... (31)

A.P. = 51, 49, 47, ... 31

(ii) What is the minimum number of days he needs to practice till his goal is achieved.

(a) 10

(b) 12

(c) 11

(d) 9

Ans. (c) 11

Explanation :

$a = 51$, $d = -2$

Here, $a_n = 31$

$\Rightarrow a_n = a + (n - 1)d$

$\Rightarrow 31 = 51 + (n - 1)(-2)$

$\Rightarrow n = 11$ days

So, he will achieve his target in 11 days

(iii) Which of the following term is not in the A.P. of the above given situation.

(a) 41

(b) 30

(c) 37

(d) 39

Ans. (b) 30

Explanation :

As, 31 seconds is the last time he need to achieve.

(iv) If n^{th} term of an A.P. is given by $a_n = 2n + 3$, then common difference of an AP.

(a) 2

(b) 3

(c) 5

(d) 1

Ans. (a) 2

Explanation :

$a_n = 2n + 3$

$a_1 = 2 \times 1 + 3 = 5$

$a_2 = 2 \times 2 + 3 = 7$

$a_3 = 2 \times 3 + 3 = 9$

$a_4 = 2 \times 4 + 3 = 11$

A.P. = 5, 7, 9, 11

$d = 7 - 5 = 2$

(v) The value of x, for which $2x$, $x + 10$, $3x + 2$ are three consecutive terms of an AP:
(a) 6
(b) – 6
(c) 18
(d) – 18

Ans. (a) 6

Explanation :

$a = 2x$, $b = x + 10$, $c = 3x + 2$

Now, $a + c = 2b$

$\Rightarrow 2x + 3x + 2 = 2(x + 10)$

$\Rightarrow x = \frac{18}{3} = 6$

126. Aadita is celebrating her birthday. She invited her friends. She bought a packet of toffees/candies. She arranged the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Find the first term and common difference of A.P.

(a) $a = 2$, $d = 3$

(b) $a = 3$, $d = 2$

(c) $a = 2$, $d = -3$

(d) $a = 3$, $d = -2$

Ans. (b) $a = 3$, $d = 2$

Explanation :

As A.P. = 3, 5, 7 ...

$a = 3$, $d = 2$

(ii) How many candies are placed in the 9^{th} row?

(a) 22

(b) 19

(c) 24

(d) 18

Ans. (b) 19

Explanation :

Since, $a_n = a + (n - 1)d$

$a_9 = 3 + 8 \times 2 = 19$

(iii) Find the difference in number of candies placed in 7^{th} and 3^{rd} row.

(a) 8

(b) 10

(c) 12

(d) 14

Ans. (a) 8

Explanation :

$a_7 - a_3 = a + 6d - a - 2d = 4d$

$= 4 \times 2 = 8$

(iv) Find the number of candies in 12^{th} row.

(a) 21

(b) 30

(c) 25

(d) 19

Ans. (c) 25

Explanation :

Number of candies in 12^{th} row,

$a_{12} = a + 11d$

$= 3 + 11 \times 2 = 25$

(v) Find the number of candies in 15^{th} row.

(a) 30

(b) 32

(c) 28

(d) 31

Ans. (d) 31

Explanation :

Number of candies in 15^{th} row,

$a_{15} = a + 14d = 3 + 14 \times 2$

$= 31$.

127. Jack is much worried about his upcoming assessment on Arithmetic Progression. He was vigorously practicing for the exams but unable to solve some questions. One of these question is given below :

If the 3rd and 9th term of an A.P. are 4 and – 8 respectively, then help Jack in solving problem:

(i) What is the common difference of the AP?

(a) 2

(b) – 1

(c) – 2

(d) 4

Ans. (c) – 2

Explanation :

$a_3 = a + 2d = 4$...(i)

$a_9 = a + 8d = -8$...(ii)

Solving equations (i) and (ii)

$d = -2$

(ii) What is the first term of the A.P.?

(a) 6

(b) 2

(c) – 2

(d) 8

Ans. (d) 8

Explanation :

$a_3 = a + 2d = 4$...(i)

Substitute $d = -2$ in (i), we get

$a + 2(-2) = 4$

$\Rightarrow a = 8.$

(iii) Which term of the A.P. is – 160?

(a) 80^{th}

(b) 85^{th}

(c) 81^{st}

(d) 84^{th}

Ans. (b) 85^{th}

Explanation :

Let $t_n = -160 = a + (n-1)d$

$\Rightarrow -160 = 8 + (n-1)(-2)$

$\Rightarrow n = 85^{th}$ term

(iv) Which of the following is not the term of an A.P.

(a) – 123

(b) – 100

(c) 0

(d) – 200

Ans. (a) – 123

Explanation :

As, $a_n = a + (n-1)d$

$\Rightarrow -123 = 8 + (n-1) \times (-2)$

$\Rightarrow (n-1) \times -2 = -131$

$\Rightarrow n = \frac{-131}{-2} + 1$

= 65·5 + 1

= 66·5

Which is not a whole number.

(v) Which is the 75th term of an A.P.?

(a) – 140

(b) – 102

(c) – 150

(d) – 158

Ans. (a) – 140

Explanation :

$T_{75} = a + 74d = 8 + 74 \times (-2)$

= – 140.

Passage Based Questions

128. A man took a loan of ` 325,000 for his business. He repays it in instalments. In the first month he paid ` 2000, in the second month he paid ` 3500, in the third month he paid ` 5000, and so on.

Based on the given information, answer the following questions :

(i) How long will it take to clear the loan?

(ii) What amount he has to pay for his last instalment?

Sol. (i) Let, the number of months taken to clear the loan be *n*.

Clearly, the amounts form an A.P. with first term 2000 and the common difference

= 3500 – 2000 = 1500

Sum of amount = 325000

$\therefore S_n = 325000$

$\Rightarrow \frac{n}{2}[2 \times 2000 + (n - 1) 1500] = 325000$

$\Rightarrow \frac{n}{2}[4000 + 1500n - 1500] = 325000$

$\Rightarrow \frac{n}{2}[2500 + 1500n] = 325000$

$\Rightarrow n[3n + 5] = \frac{325000 \times 2}{500}$

$\Rightarrow 3n^2 + 5n - 1300 = 0$

$\Rightarrow 3n^2 + 65n - 60n - 1300 = 0$

$\Rightarrow n(3n + 65) - 20(3n + 65) = 0$

$\Rightarrow (3n + 65)(n - 20) = 0$

$\Rightarrow n = 20$ or $n = \frac{-65}{3}$

$\Rightarrow n = 20 \left[\because n \neq \frac{-65}{3}\right]$

So, it will take 20 months to clear the loan.

(ii) Amount of last instalment is

$a_n = a + (n - 1)d$

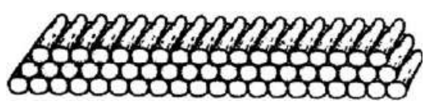
$a_{20} = 2000 + (20 - 1) \times 1500$

$= 2000 + 19 \times 1500$

$= 2000 + 28500 = 30500$

So, the amount of last instalment ` 30500. **Ans.**

129. 325 logs are stacked in the manner that there are 27 logs in the bottom row, 26 logs in the next row 25 logs in the row next to it and so on.



Base on the given information, answer the following questions:

(i) In how many rows are the 325 logs placed?

(ii) How many logs are in the top row?

(iii) which row is the middle row?

Sol. Clearly, logs stacked in each row form a sequence

$$27, 26, 25, 24, ...$$

It is an A.P. with $a = 27$, $d = 26 - 27 = -1$

(i) Let there are n rows

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 325 = \frac{n}{2}[2 \times 27 + (n - 1)(-1)]$

$\Rightarrow 750 = n[55 - n]$

$\Rightarrow n^2 - 55n + 750 = 0$

$\Rightarrow n^2 - 30n - 25n + 750 = 0$

$\Rightarrow n(n - 30) - 25(n - 3) = 0$

$\Rightarrow (n - 25)(n - 30) = 0$

$\Rightarrow n = 25$ or 30

For $n = 30$, we have

$a_{30} = 27 + 29 \times (-1)$

$= 27 - 29 = -2$

... Number of logs in the 30th row is – 2, which is not possible

So, $n = 25$

Thus there are, 25 rows.

(ii) Number of logs in the 25th row (top row) = a_{25}

$\Rightarrow a_{25} = a + 24d = 27 + 24 \times (-1)$

$= 27 - 24 = 3$ **Ans.**

(iii) Middle = $\left(\frac{25+1}{2}\right)^{\text{th}}$ row

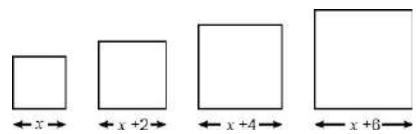
= 13th row. **Ans.**

130. The diagram shows a sequence of square wire frames. The lengths of the side of these frames are x cm, (x + 2) cm, (x + 4)

cm ...... respectively. The sum of the areas of the first three squares is 440 sq. cm.

Based on the following information answer the following questions:

(i) Express the length of side of the n^{th} frame in terms of x and n.



(ii) Find the value of x.

Sol. The length of the sides of the sequence of square frames are

$$x, x + 2, x + 4, ...$$

It is an A.P. with $a = x$, $d = 2$

(i) Side of the nth square = $a + (n - 1)d$

$= x + (n - 1)(2)$

$= x + 2n - 2.$

(ii) It is given that

$x^2 + (x + 2)^2 + (x + 4)^2 = 440$

$\Rightarrow 3x^2 + 12x + 20 = 440$

$\Rightarrow 3x^2 + 12x - 420 = 0$

$\Rightarrow x^2 + 4x - 140 = 0$

$\Rightarrow x^2 + 14x - 10x - 140 = 0$

$\Rightarrow x(x + 14) - 10(x + 14) = 0$

$\Rightarrow (x + 14)(x - 10) = 0$

$\Rightarrow x = 10$ (... $x \neq -14$)

131. Read the following passage and answer the questions that follows :

Ravi started to give home tuitions from the month of January. He charges ` 1000 from each student. He spends ` 1200 on rent, ` 2700 on his food, ` 980 on taxi fare and ` 1700 on electricity. In the first month he had 20 students and in each subsequent month, the number of students increases by 2.

(i) Do his savings form an arithmetic progression? If so, write the first three terms of this A.P.

(ii) How much did Ravi save till 11th month?
(iii) How much did he save in the month of July?
Sol. (i) Earnings from home tuition in the first month = ` (1000 × 20)
= ` 20,000.
Total expense = ` (1200 + 2700 + 980 + 1700)
= ` 6580

Ravi's savings in the 1st month
= (20,000 – 6580)
= ` 13,420

Ravi's savings in the 2nd month
= ` (13420 + 2000)
= ` 15,420

Ravi's savings in the 3rd month = (13,420 + 4000)
= 17,420
In each subsequent month, his savings increases by a fixed amount.
Therefore, his savings forms an A.P.
The A.P. is 13420, 15420, 17420, ...
(ii) The amount, Ravi saved till 11 months will be the sum of 11 terms of this A.P.
Here, a = 13420 and d = 2000

$S_{11} = \frac{11}{2}[2(13420) + 10(2000)]$

$= \frac{11}{2} \times 46,840$

= 2,57,620.

So, he saved ` 2,57,620 till 11th months.

(iii) Ravi's saving in July will be 7th term of this A.P.
a_7 = 13420 + 6 × 2000

[... $a_n = a + (n - 1)d$]

= 13,420 + 12,000
= 25,420
So, he saved ` 25,420 in the month of July.

Self-Assessment

132. A sum of ` 1000 is invested at 8% S.I. per annum. Calculate the rate of interest at the end of 1, 2, 3, … years. Is the sequence of the interests an A.P. ? Find the interest at the end of 30 years.

[NCERT]

Ans. The interest after 1 year = ` 80
The interest after 2 years = ` 160
The interest after 3 years = ` 240
Yes, the sequence of the interests is an A.P.

The interest after 30 years = ` 2400.

133. In a flower bed, there are 23 rose plants in the first row, 21 in the second row, 19 in the third row and so on. There are five plants in the last row. How many rows are there in the flower bed ?

[NCERT]

Ans. 10.

134. A person started working in 1995 at a salary of ` 5000 per month with a yearly increment of ` 200. In which year did his salary reach ` 7000 per month?

[NCERT]

Ans. In the year 2005.

135. A housewife saved ` 5 in the first week of the year and thereafter increased her weekly savings by ` 1.75. After how many weeks will her weekly savings be ` 20.75?

[NCERT]

Ans. 10 weeks.

136. Find the 12^{th} term from the end of the following arithmetic progression : 3, 8, 13, …, 253.

[NCERT]

Ans. 198.

137. The sum of the 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} term is 34. Find the first term and the common difference of the A.P.

[NCERT]

Ans. – 0.5, 2.5.

138. Find the A.P. whose third term is 16 and the seventh term exceeds its fifth term by 12.

[NCERT]

Ans. 4, 10, 16, 22, …

139. Two A.P.s have the same common difference. The difference between their 100^{th} terms is 100. What is the difference between their 1000^{th} terms?

[NCERT]

Ans. 100.

140. A manufacturer of TV sets produced 600 units in the 3^{rd} year and 700 units in the 7^{th} year. Assuming that the production increases uniformly by a fixed number every year, find the production in (i) the first year, (ii) the 10^{th} year.

[NCERT]

Ans. (i) 550, (ii) 775.

141. The contract of a construction job specifies a penalty for delay of completion beyond a certain date as follows : ` 200 for the first day, ` 250 for the second day, ` 300 for the third day and so on. How much does a delay of 30 days cost the contractor?

[NCERT]

Ans. ` 27,750.

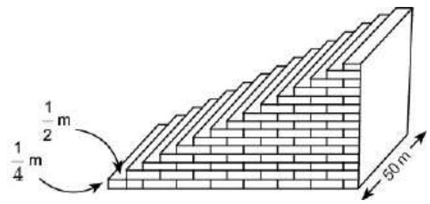
142. A sum of ` 280 is to be used to award four prizes. If each prize after the first is ` 20 less than its preceeding prize, find the value of each of the prizes.

[NCERT]

Ans. The first prize is `100, second prize is ` 80, third prize is ` 60 and fourth prize is ` 40.

143. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see Fig.). Calculate the total volume of concrete required to build the terrace.



Ans. 750 m^3

144. How many terms of the A.P. 9, 17, 25, … must be taken so that their sum is 636 ?

[NCERT]

Ans. 12.

145. Find the sum of the first 15 terms of the series where $t_n = 9 - 5n$.

[NCERT]

Ans. – 465.

146. Find the sum of the first 51 terms of an A.P. whose 2nd and 3rd terms are 14 and 18 respectively.

[NCERT]

Ans. 5610.

147. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

[NCERT]

Ans. 16, $\frac{8}{3}$.

148. Find the n^{th} term and the last term of an A.P. whose first term is 2, the common difference is 8 and the sum of all the terms is 90.

[NCERT]

Ans. $8n - 6$, 34.

149. If the sum of the first n terms of an A.P. is $4n - n^2$, find the first term, second term and sum of the first two terms.

[NCERT]

Ans. 3, 1, 4.

150. The first term of an A.P. is 17 and the last term is 350. If the common difference is 9, how many terms are there and what is their sum?

[NCERT]

Ans. 38, 6973.

151. The sum of n terms of an A.P. is $5n - n^2$. Find nth term of this A.P.

Ans. $6 - 2n$.

152. The sum of first n terms of an A.P. is $3n^2 + 4n$. Find the 25th term of this A.P.

Ans. 151.

153. The n^{th} term of an A.P. is $6n + 2$. Find the common difference.

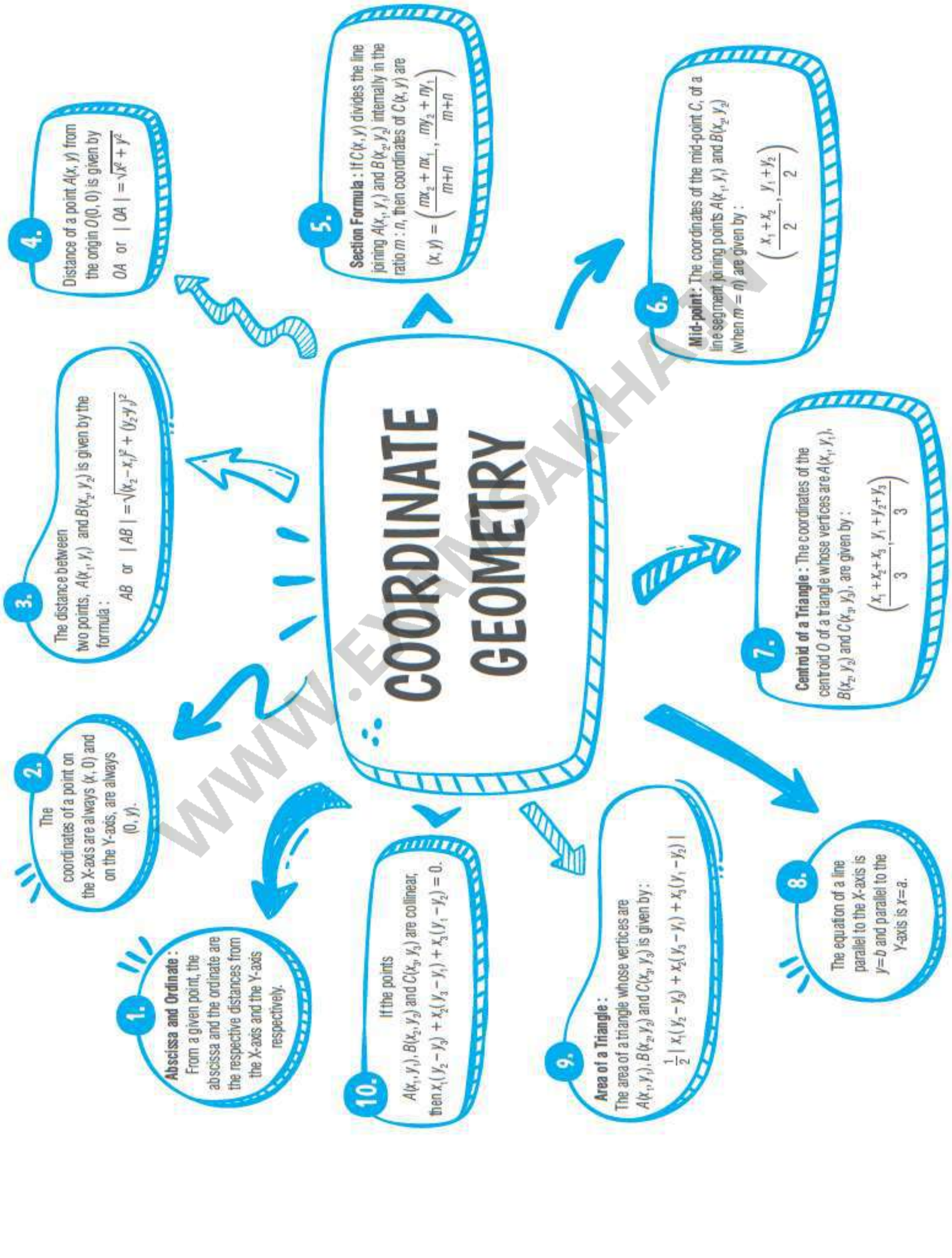
[NCERT]

Ans. 6.

Coordinate Geometry

Chapter
6

Basic Concepts



Multiple Choice Questions

1. **The mid-point of segment AB is the point P(0, 4). If the coordinates of B are (– 2, 3), then the coordinates of A are:**

(a) (2, 5)

(b) (– 2, – 5)

(c) (2, 9)

(d) (– 2, 11)

Ans. (a) (2, 5)

Explanation :

Given, P = (0, 4), B = (– 2, 3) and AP = PB

Let A = (x, y)

Thus, $P_x = \frac{x-2}{2}$ and $P_y = \frac{y+3}{2}$

$\Rightarrow 0 = \frac{x-2}{2}$ and $4 = \frac{y+3}{2}$

$\Rightarrow x = 2$ and $y = 5$

Thus, A = (2, 5).

2. **If the point P(6, 2) divides the line segment joining A(6, 5) and B(4, *y*) in the ratio 3 : 1, then the value of *y* is:**

[Board Question]

(a) 4

(b) 3

(c) 2

(d) 1

Ans. (d) 1

Explanation :

Using section formula,

y-coordinate of P = $\frac{3 \times y + 1 \times 5}{3+1}$

$\Rightarrow 2 = \frac{3y+5}{4}$

$\Rightarrow 8 = 3y + 5$

$\Rightarrow 3y = 3$

$\Rightarrow y = 1.$

3. **The distance between the points (*a* cos** q **+ *b* sin** q**, 0) and (0, *a* sin** q **– *b* cos** q**), is:**

[Board Question]

(a) $a^2 + b^2$

(b) $a^2 - b^2$

(c) $\sqrt{a^2+b^2}$

(d) $\sqrt{a^2-b^2}$

Ans. (c) $\sqrt{a^2+b^2}$

Explanation :

The distance formula,

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Here x_1 = *a* cos q + *b* sin q, y_1 = 0

x_2 = 0, y_2 = *a* sin q – *b* cos q

$d = \sqrt{[0-(a\cos\theta+b\sin\theta)]^2+[(a\sin\theta-b\cos\theta)-0]^2}$

$= \sqrt{(a\cos\theta+b\sin\theta)^2+(a\sin\theta-b\cos\theta)^2}$

$= \sqrt{a^2\cos^2\theta+b^2\sin^2\theta+2ab\sin\theta\cos\theta+a^2\sin^2\theta+b^2\cos^2\theta-2ab\sin\theta\cos\theta}$

$= \sqrt{a^2(\sin^2\theta+\cos^2\theta)+b^2(\sin^2\theta+\cos^2\theta)}$

$= \sqrt{a^2\times1+b^2\times1}$ {∵ $\sin^2$ q + $\cos^2$ q = 1}

$= \sqrt{a^2+b^2}$

4. **If the point P(k, 0) divides the line segment joining the points A(2, – 2) and B(– 7, 4) in the ratio 1 : 2, then the value of k is:**

[Board Question]

(a) 1

(b) 2

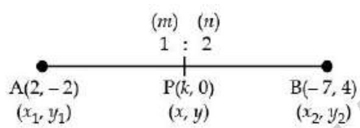
(c) – 2

(d) – 1

Ans. (d) – 1

Explanation :

By section formula,



$x = \frac{mx_2 + nx_1}{m+n}$,

$y = \frac{my_2 + ny_1}{m+n}$

Here $x = k$, $m = 1$, $n = 2$, $x_1 = 2$, $x_2 = -7$

we get,

$k = \frac{(1)(-7)+(2)(2)}{1+2}$

$\Rightarrow k = \frac{-7+4}{3}$

$\Rightarrow k = \frac{-3}{3}$

$\Rightarrow k = -1.$

5. **The value of p, for which the points A(3, 1), B(5, p) and C(7, – 5) are collinear, is:**

[Board Question]

(a) – 2

(b) 2

(c) – 1

(d) 1

Ans. (a) – 2

Explanation :

Given,

A(3, 1) ⇒ $x_1 = 3$, $y_1 = 1$

B(5, p) ⇒ $x_2 = 5$, $y_2 = p$

C(7, – 5) ⇒ $x_3 = 7$, $y_3 = -5$.

If A, B and C are collinear points, then

ar(DABC) = 0

⇒ $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

⇒ $\frac{1}{2}[3(p - (-5)) + 5(-5 - 1) + 7(1 - p)] = 0$

⇒ $\frac{1}{2}[3p + 15 - 25 - 5 + 7 - 7p] = 0$

⇒ $[-4p - 8] = 0$

⇒ $-2p - 4 = 0$

⇒ $p = \frac{4}{-2}$

⇒ $p = -2$.

6. The distance between the points (0, 5) and (– 5, 0) is:

(a) 5

(b) $5\sqrt{2}$

(c) – 5

(d) $6\sqrt{2}$

Ans. (b) $5\sqrt{2}$

Explanation :

Distance between (0, 5) and (– 5, 0) is

= $\sqrt{[0-(-5)]^2+(5-0)^2}$

= $\sqrt{25+25}$

= $\sqrt{50}$

= $5\sqrt{2}$

7. If the point C(*k*, 4) divides the joining of points A(2, 6) and B(5, 1) in the ratio 2: 3, then the value of *k* is:

(a) 16

(b) $\frac{28}{5}$

(c) $\frac{16}{5}$

(d) $\frac{8}{5}$

Ans. (c) $\frac{16}{5}$

Explanation :

$\because x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

$k = \frac{5\times2+3\times2}{2+3}$

= $\frac{10+6}{5} = \frac{16}{5}$

8. The distance of a point (– 5, 12) from the origin is:

(a) 17 units

(b) 7 units

(c) 13 units

(d) none of these

Ans. (c) 13 units

Explanation :

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(0+5)^2+(0-12)^2}$

$= \sqrt{25 + 144}$
$= \sqrt{169} = 13.$

9. .... is the point of intersection of the coordinate axes.

(a) X-axis

(b) Y-axis

(c) Origin

(d) None of these

Ans. (c) Origin

Explanation :

The coordinate plane has two axes, the horizontal axes and the vertical axes. The point of intersection of these two axes in origin (0, 0).

10. In the second quadrant, for a point, the abscissa is .... and the ordinate is ....

(a) positive, negative

(b) negative, positive

(c) negative, negative

(d) positive, positive

Ans. (b) negative, positive

Explanation :

In the second quadrant, X-axis is negative and Y-axis is positive. Therefore, for a point the abscissa is negative and the ordinate is positive.

11. The distance between the points (3, 4) and (– 5, 2) is ....

(a) $2\sqrt{18}$

(b) $\sqrt{17}$

(c) $\sqrt{24}$

(d) $2\sqrt{17}$

Ans. (d) $2\sqrt{17}$

Explanation :

The distance between (3, 4) and (– 5, 2)

= $\sqrt{(-5-3)^2+(2-4)^2}$

= $\sqrt{(-8)^2+(2)^2}$ = $\sqrt{64+4}$

= $\sqrt{68}$ = $2\sqrt{17}$

12. If three points are collinear, then the area of triangle formed by them is ....

(a) 1

(b) 2

(c) 0

(d) – 2

Ans. (c) 0

Explanation :

If three points are collinear, then they will lie on the same line so they can't form any triangle.

Area of triangle will be 0.

13. The ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the X-axis is ....

(a) 1 : 1

(b) 1 : 2

(c) 2 : 1

(d) 2 : 3

Ans. (a) 1 : 1

Explanation :

$y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

on x-axis, $y = 0$

$0 = \frac{m_1 \times 5 + m_2(-5)}{m_1 + m_2}$

$\Rightarrow 5m_1 - 5m_2 = 0$

$\Rightarrow m_1 = m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{1}{1}$

= 1 : 1.

14 AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0), B(5, 0). The length of its diagonal is:[NCERT Exemplar]

(a) $\sqrt{14}$

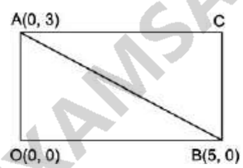
(b) $\sqrt{17}$

(c) $\sqrt{34}$

(d) $2\sqrt{17}$

Ans. (c) $\sqrt{34}$

Explanation :



Now length of the diagonal AB = Distance between the points A(0, 3) and B(5, 0).

∵ Distance between the points (x_1, y_1) and (x_2, y_2).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here, $x_1 = 0$, $y_1 = 3$ and $x_2 = 5$, $y_2 = 0$

Distance between the points A(0, 3) and B(5, 0) is

AB = $\sqrt{(5-0)^2 + (0-3)^2}$

= $\sqrt{25+9} = \sqrt{34}$

Hence, the required length of its diagonal is $\sqrt{34}$.

Very Short Answer Type Questions

15. ABCD is a rectangle whose vertices are B(4, 0), C(4, 3) and D(0, 3). Find the length of one of its diagonals.

Sol. In rectangle ABCD, B(4, 0), C(4, 3) and D(0, 3) are the coordinates.

Thus, BD is the diagonal.

Hence, length of BD = $\sqrt{(4-0)^2+(0-3)^2}$

= $\sqrt{16+9}$

= $\sqrt{25}$ = 5 units **Ans.**

16. If the points A(*x*, 2), B(– 3, – 4) and C(7, – 5) are collinear, then find the value of *x*.

Sol. Since A(x, 2), B(– 3, – 4) and C(7, – 5) are collinear,

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow x[-4-(-5)] + (-3)[(-5)-2] + 7[2-(-4)] = 0$

$\Rightarrow x + (-3)[-7] + 7[6] = 0$

$\Rightarrow x + 21 + 42 = 0$

$\Rightarrow x = -63$. **Ans.**

17. Find the perimeter of a triangle with vertices A(0, 4), B(0, 0) and C(3, 0).

Sol. Given, A(0, 4), B(0, 0) and C(3, 0)

Thus, Perimeter = AB + BC + CA

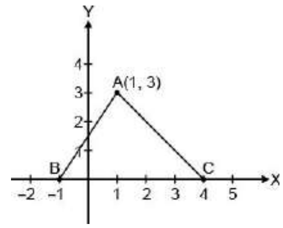
= $\left[\sqrt{(0-0)^2+(0-4)^2} + \sqrt{(3-0)^2+(0-0)^2} + \sqrt{(3-0)^2+(4-0)^2}\right]$

= $[\sqrt{16}+\sqrt{9}+\sqrt{9+16}]$

= (4 + 3 + 5) units

= 12 units. **Ans.**

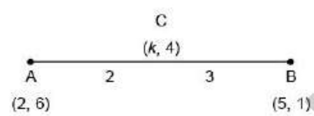
18. Find the area of the triangle in the given figure (in sq. units).



Sol. Given, A = (1, 3), B = (– 1, 0) and C = (4, 0).
Thus, area of the DABC
= $\frac{1}{2}$|1(0 – 0) – 1(0 – 3) + 4(3 – 0)|
= $\frac{1}{2}$|3 + 12|
= 7.5 sq. units. **Ans.**

19. If the point C(*k*, 4) divides the join of points A(2, 6) and B(5, 1) in the ratio 2 : 3, then find the value of *k*.

Sol. Given, A(2, 6), C(k, 4) and B(5, 1) and AC : BC = 2 : 3



Thus, by section formula,
$k = \frac{3\times2+2\times5}{2+3}$
$= \frac{6+10}{5}$
$= \frac{16}{5}$ **Ans.**

20. Find the distance of the point (– 3, 4) from the origin.

Sol. The given point is (– 3, 4).

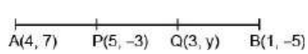
We know that,

Distance = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Thus, the distance = $\sqrt{(-3-0)^2+(4-0)^2}$

= 5 units. **Ans.**

21. In the given figure, P(5, – 3) and Q(3, *y*) are the points of trisection of the line segment joining A(4, 7) and B(1, – 5). Then find *y*.



Sol. Since P and Q are the points of trisection, thus, PQ = QB

⇒ $\sqrt{(5-3)^2+(-3-y)^2} = \sqrt{(3-1)^2+(y+5)^2}$

⇒ $(2)2 + (3 + y)2 = (2)2 + (y + 5)2$

⇒ $4 + 9 + 6y + y2 = 4 + 25 + 10y + y2$

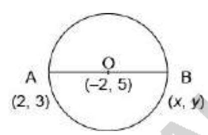
⇒ $6y + 13 = 29 + 10y$

⇒ $4y = -16$

⇒ $y = -4$. **Ans.**

22. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of the centre are (– 2, 5), then find the coordinates of the other end of the diameter.

Sol. Given, the coordinates of the centre O = (– 2, 5) and one end A(2, 3).



Let the coordinates of the other end be B (x, y)

Since, centre is the mid-point of the two end points

Thus, by mid point formula,

$-2 = \frac{2+x}{2}$ and $5 = \frac{3+y}{2}$

⇒ $-4 = 2 + x$ and $10 = 3 + y$

⇒ $x = -6$ and $y = 7$

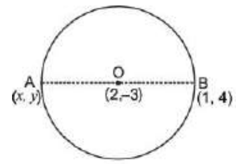
Hence, the coordinates are (– 6, 7). **Ans.**

23. Find the coordinates of point A, where AB is diameter of a circle whose centre is (2, – 3) and B is the point (1, 4).

[Board Question]

Sol. Let the co-ordinates of point A be (x, y) and point O (2, – 3) be the centre, then

By mid-point formula,



$\frac{x+1}{2}$ = 2 and $\frac{y+4}{2}$ = – 3

or x = 4 – 1 and y = – 6 – 4

⇒ x = 3 and y = – 10

The co-ordinates of point A are (3, – 10) **Ans.**

24. Find the coordinates of the point P dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1.

Sol. Given, A = (1, 3); B = (4, 6) and AP : PB = 2 : 1

Let the coordinates of point P be (x, y).

Then, by section formula

$x = \frac{2\times4+1\times1}{2+1}$ and $y = \frac{2\times6+1\times3}{2+1}$

⇒ $x = \frac{8+1}{3}$ and $y = \frac{12+3}{3}$

⇒ x = 3 and y = 5

(x, y) = (3, 5). **Ans.**

25. If $P\left(\frac{a}{2}, 4\right)$ is the mid-point of the line segment joining the points A(– 6, 5) and B(– 2, 3), then find the value of *a*.

Sol. Given, $P\left(\frac{a}{2}, 4\right)$ is the mid-point of the line segment joining the points A(– 6, 5) and B(– 2, 3).

Thus, $\frac{a}{2} = \frac{-6-2}{2}$

$= \frac{-8}{2} = -4$

or a = – 8. **Ans.**

26. If A and B are the points (– 6, 7) and (– 1, – 5) respectively, then find the distance 2AB.

Sol. Given, A = (– 6, 7) and B = (– 1, – 5).

By distance formula

AB = $\sqrt{(-6+1)^2+(7+5)^2}$

= $\sqrt{(-5)^2+(12)^2}$

= $\sqrt{25+144}$

= $\sqrt{169}$ = 13

Thus, 2AB = 2 × 13 = 26 units. **Ans.**

27. Find the area (in sq. units) of the triangle formed by the points A(*a*, 0), O(0, 0) and B(0, *b*).

Sol. Given,

A = (a, 0), O = (0, 0) and B = (0, b)

Thus, area of DABC

= $\frac{1}{2}|a(0-b)+0(b-0)+0(0-0)|$

= $\frac{1}{2}|-ab|$

= $\frac{1}{2}ab$ sq. units. **Ans.**

28. The point P which divides the line segment joining the points A(2, – 5) and B(5, 2) in the ratio 2 : 3 lies in which quadrant ?

Sol. Let the coordinates of the point P be (x, y).

Given, A = (2, – 5), B = (5, 2) and $m : n = 2 : 3$

By section formula,

Now, $P_x = \frac{2(5)+3(2)}{2+3}$ and $P_y = \frac{2(2)+3(-5)}{2+3}$

$\Rightarrow P_x = \frac{10+6}{5}$ and $P_y = \frac{4-15}{5}$

$\Rightarrow x = \frac{16}{5}$ and $y = -\frac{11}{5}$

$\Rightarrow P = \left(\frac{16}{5}, -\frac{11}{5}\right)$

Thus, P lies in quadrant IV. **Ans.**

29. Find the point on the X-axis which is equidistant from the point (– 1, 0) and (5, 0).

[Board Question]

Sol. Given, A = (– 1, 0) and B = (5, 0).

Let the point on the X-axis be P(x, 0).

It is given that AP = PB

⇒ $\sqrt{(x+1)^2+(0-0)^2} = \sqrt{(5-x)^2+(0-0)^2}$

⇒ $(x + 1)2 = (5 - x)2$

⇒ $x + 1 = 5 - x$

⇒ $2x = 4$

⇒ $x = 2$

Thus, the point is (2, 0). **Ans.**

30. Find the value of *k* if the point P(2, 4) is equidistant from the points A(5, *k*) and B(*k*, 7).

Sol. Given, A = (5, k), B = (k, 7) and P = (2, 4)

Also, AP = PB

⇒ $\sqrt{(5-2)^2+(k-4)^2} = \sqrt{(2-k)^2+(7-4)^2}$

⇒ $(3)2 + (k - 4)2 = (2 - k)2 + (3)2$

⇒ $9 + (k - 4)2 = (2 - k)2 + 9$

⇒ $(k - 4)^2 = (2 - k)^2$

⇒ $k - 4 = 2 - k$

⇒ $2k = 6$

⇒ $k = 3$. **Ans.**

31. Find a relation between *x* and *y* such that the points P(*x*, *y*) is equidistant from the points A(1, 4) and B(– 1, 2).

Sol. Given A = (1, 4), B = (– 1, 2) and P = (x, y)

Also, AP = PB

⇒ $\sqrt{(1-x)^2+(4-y)^2}$ = $\sqrt{(x+1)^2+(y-2)^2}$

⇒ $(1 - x)2 + (4 - y)2 = (x + 1)2 + (y - 2)2$

⇒ $(1 - x)2 - (1 + x)2 = (y - 2)2 - (4 - y)2$

⇒ $[(1 - x + 1 + x)(1 - x - 1 - x)] = [(y - 2 + 4 - y)(y - 2 - 4 + y)]$

⇒ $2(-2x) = [(2)(2y - 6)]$

⇒ $-4x = 4(y - 3)$

⇒ $-x = y - 3$

⇒ $x + y = 3$. **Ans.**

32. Find the value of *x* for which the distance between the points P(*x*, 4) and Q(9, 10) is 10 units.

Sol. Given, P = (x, 4), Q = (9, 10) and PQ = 10 units

By distance formula,

Now 10 = $\sqrt{(x-9)^2+(4-10)^2}$

⇒ $100 = (x - 9)2 + (-6)2$

⇒ $100 = x2 + 81 - 18x + 36$

⇒ $x2 - 18x + 17 = 0$

⇒ $x2 - 17x - x + 17 = 0$

⇒ $x(x - 17) - 1(x - 17) = 0$

⇒ $(x - 17)(x - 1) = 0$

⇒ $x = 1, 17$

Thus, the value of x is 1 or 17. **Ans.**

33. If the point P(*x*, *y*) is equidistant from the points A(5, 1) and B(– 1, 5), prove that 3*x* = 2*y*.

Sol. Given, A = (5, 1), B = (– 1, 5) and PA = PB

Thus $\sqrt{(5-x)^2+(1-y)^2}$ = $\sqrt{(-1-x)^2+(5-y)^2}$

$\Rightarrow 25 - 10x + x2 + 1 - 2y + y2$

$= 1 + 2x + x2 + 25 - 10y + y2$

$\Rightarrow 26 - 10x - 2y = 26 + 2x - 10y$

$\Rightarrow 2x + 10x = 10y - 2y$

$\Rightarrow 12x = 8y$

$\Rightarrow 3x = 2y$. **Hence Proved.**

34. Find the point on the *Y*-axis which is equidistant from the points (– 2, 5) and (– 2, 9).

Sol. As the point is on *Y*-axis, let its coordinates be (0, *y*)

Now, $\sqrt{(-2-0)^2+(5-y)^2} = \sqrt{(-2-0)^2+(y-9)^2}$

$\Rightarrow 4 + (5 - y)2 = 4 + (y - 9)2$

$\Rightarrow 5 - y = y - 9$

$\Rightarrow 2y = 14$

$\Rightarrow y = 7$

Thus, the point is (0, 7). **Ans.**

35. Write the coordinates of a point P on *x*-axis which is equidistant from the point A(– 2, 0) and B(6, 0).

[Board Question]

Sol. Let coordinates of P on *x*-axis be (*x*, 0)

Given, A(– 2, 0) and B(6, 0)

Here, PA = PB

$\sqrt{(x+2)^2+(0-0)^2} = \sqrt{(x-6)^2+(0-0)^2}$

$\Rightarrow \sqrt{(x+2)^2} = \sqrt{(x-6)^2}$

On squaring both sides, we get

$(x + 2)2 = (x - 6)2$

$\Rightarrow x2 + 4 + 4x = x2 + 36 - 12x$

⇒ $4 + 4x = 36 – 12x$

⇒ $16x = 32$

⇒ $x = 2$

Co-ordinates of P are (2, 0). **Ans.**

36. By the distance formula show that the points (1, – 1), (5, 2) and (9, 5) are collinear.

Sol. Let A(1, – 1), B(5, 2) and C(9, 5) be the given points.

Then by distance formula

AB = $\sqrt{(5-1)^2+(2+1)^2}$

= $\sqrt{16+9}$ = 5

BC = $\sqrt{(5-9)^2+(2-5)^2}$

= $\sqrt{16+9}$ = 5

AC = $\sqrt{(1-9)^2+(-1-5)^2}$

= $\sqrt{64+36}$

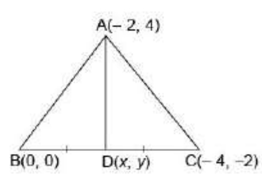
= 10

Clearly, AC = AB + BC

Hence A, B, C are collinear points. **Hence Proved.**

37. If A(– 2, 4), B(0, 0) and C(– 4, – 2) are the vertices of a D**ABC, then find the length of the median through the vertex A.**

Sol. Given, A(– 2, 4), B(0, 0) and C = (– 4, – 2)



Let AD be the median on BC through the vertex A.

Thus BD = DC

So D = $\left(\frac{0-4}{2}, \frac{0-2}{2}\right)$

⇒ D = (– 2, – 1)

Thus AD = $\sqrt{(-2+2)^2+(-1-4)^2}$

= 5 units

Hence, the length of the median is 5 units. **Ans.**

38. The line segment joining the points A(3, – 4) and B(1, 2) is trisected at the points P(*p*, – 2) and $Q\left(\frac{5}{3}, q\right)$ such that P is nearer to A. Find the value of *p* and *q*.

Sol. Given A(3, – 4), B(1, 2), P(*p*, – 2) and $Q\left(\frac{5}{3}, q\right)$

Now, AP = PQ = QB

Thus AP : PB = 1 : 2 and AQ : QB = 2 : 1

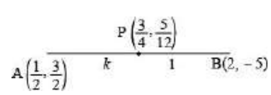
p = $\frac{1(1)+2(3)}{2+1}$

and *q* = $\frac{2(2)+1(-4)}{2+1}$

p = $\frac{7}{3}$ and *q* = 0. **Ans.**

39. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, –5).

Sol. Let point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line AB in ratio *k* : 1.



Then, by section formula,

Coordinates of P are

$\frac{2k+\frac{1}{2}}{k+1} = \frac{3}{4}$ and $\frac{-5k+\frac{3}{2}}{k+1} = \frac{5}{12}$

⇒ 8*k* + 2 = 3*k* + 3 and – 60*k* + 18 = 5*k* + 5

⇒ 8*k* – 3*k* = 3 – 2 and 65*k* = 18 – 5

⇒ 5*k* = 1 and 65*k* = 13

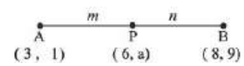
⇒ $k = \frac{1}{5}$ in each case.

Hence, the required ratio is 1 : 5. **Ans.**

40. Find the ratio in which the point P(–6, *a*) divides the line segment joining the points A(– 3, – 1) and B(– 8, 9). Also, find the value of *a*.

Sol. Given, A = (– 3, – 1), B = (– 8, 9) and P = (– 6, *a*)

Let the required ratio be *m* : *n*



Now, $-6 = \frac{n(-3)+m(-8)}{m+n}$

⇒ – 6(*m* + *n*) = – 3*n* – 8*m*

⇒ – 6*m* – 6*n* = – 3*n* – 8*m*

⇒ 2*m* = 3*n*

⇒ *m* : *n* = 3 : 2

Now, $a = \frac{2(-1)+3(9)}{2+3}$

$= \frac{25}{5} = 5$. **Ans.**

41. Find the centroid of DABC whose vertices are A(– 1, 0), B(5, – 2) and C(8, 2).

Sol. Given, A = (– 1, 0), B = (5, – 2) and C = (8, 2)

Let the centroid be G (*x*, *y*).

Thus (*x*, *y*) = $\left(\frac{-1+5+8}{3}, \frac{0-2+2}{3}\right)$

$= \left(\frac{12}{3}, \frac{0}{3}\right)$

= (4, 0). **Ans.**

42. Find the third vertex of a DABC if two of its vertices are B(– 3, 1) and C(0, – 2) and its centroid is at the origin.

Sol. Given, B = (– 3, 1), C = (0, – 2) and O = (0, 0)

Let the coordinates of A be (*x*, *y*).

Thus, (0, 0) = $\left(\frac{-3+0+x}{3}, \frac{1-2+y}{3}\right)$

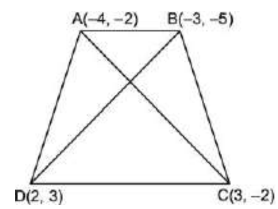
⇒ $x - 3 = 0$ and $y - 1 = 0$

⇒ $x = 3$ and $y = 1$

So, the coordinates of A = (3, 1). **Ans.**

43. Find the area of the quadrilateral ABCD whose vertices are A(– 4, – 2), B(– 3, – 5), C(3, – 2) and D(2, 3).

Sol. Given, A = (– 4, – 2), B = (– 3, – 5), C = (3, – 2) and D = (2, 3)



Area of ABCD = Area of DABC + Area of DADC

= $\frac{1}{2}$|– 4(– 5 + 2) – 3(– 2 + 2) + 3(– 2 + 5)|+ $\frac{1}{2}$| – 4(3 + 2) + 2(– 2 + 2) + 3(– 2 – 3)|

= $\frac{1}{2}$| 12 + 0 + 9| + $\frac{1}{2}$| – 20 + 0 – 15|

= $\frac{1}{2}$|21| + $\frac{1}{2}$| – 35|

= $\frac{21}{2} + \frac{35}{2}$

= 28 sq. units. **Ans.**

44. Find the area of DABC, whose vertices are A(– 5, 7), B(– 4, – 5) and C(4, 5).

[Board Question]

Sol. Area of DABC

= $\frac{1}{2}$|– 5(– 5 – 5) – 4(5 – 7) + 4(7 + 5)|

= $\frac{1}{2}$|50 + 8 + 48|

= 53 sq. units. **Ans.**

45. Find the value of *p* for which A(3, 2), B(4, *p*) and C(5, 3) are collinear.

Sol. Given, A(3, 2), B(4, *p*) and C(5, 3) are collinear

So, $|x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)| = 0$

Thus, $3(p - 3) + 4(3 - 2) + 5(2 - p) = 0$

$\Rightarrow 3p - 9 + 4 + 10 - 5p = 0$

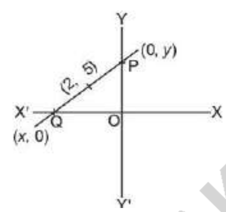
$\Rightarrow - 2p + 5 = 0$

$\Rightarrow 2p = 5$

$\Rightarrow p = \frac{5}{2}$. **Ans.**

46. A line intersects the *Y*-axis and *X*-axis at the points P and Q respectively. If (2, – 5) is the mid-point of PQ, then find the coordinates of P and Q.

Sol. Let the coordinate of P be (0, *y*) and coordinates of Q be (*x*, 0)



Since, mid-point is (2, –5)

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (2, -5)$

$\Rightarrow \left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$

$\Rightarrow \frac{x}{2} = 2 \; ; \; \frac{y}{2} = -5$

$\Rightarrow x = 4 \; ; \; y = -10$

Coordinates of P (0, –10)

Coordinates of Q (4, 0) **Ans.**

47. A(– 4, – 2), B(– 3, – 5), C(3, – 2) and D(2, *k*) are the vertices of a quadrilateral ABCD. Find the value of *k* if the area of the quadrilateral is 28 sq. units.

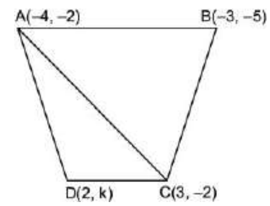
[Board Question]

Sol. Given, A = (– 4, – 2), B = (– 3, – 5), C = (3, – 2) and D = (2, *k*)

Also, area of ABCD = 28 sq. units

Now, area of ABCD

= Area of DABC + Area of DACD



⇒ 28 = $\frac{1}{2}$|– 4(– 5 + 2) – 3(– 2 + 2) + 3(– 2 + 5)|+ $\frac{1}{2}$|– 4(– 2 – *k*) + 3(*k* + 2) + 2(– 2 + 2)|

⇒ 56 = 12 + 9 + 8 + 4*k* + 3*k* + 6

⇒ 7*k* = 56 – 35

⇒ 7*k* = 21 ⇒ *k* = 3. **Ans.**

48. A(3, 2) and B(– 2, 1) are two vertices of a triangle ABC whose centroid G has the coordinates $\left(\frac{5}{3}, -\frac{1}{3}\right)$. Find the coordinates of the third vertex of the triangle.

Sol. Given, A = (3, 2), B = (– 2, 1) and centroid G = $\left(\frac{5}{3}, -\frac{1}{3}\right)$.

Let the coordinates of the third vertex be C(*x*, *y*).

Thus $\left(\frac{5}{3}, -\frac{1}{3}\right)$ = $\left(\frac{x+3-2}{3}, \frac{y+1+2}{3}\right)$

⇒ (5, – 1) = (*x* + 1, *y* + 3)

⇒ (*x*, *y*) = (4, – 4)

Hence, the coordinates of the third vertex are (4, – 4). **Ans.**

49. If the area of DABC formed by A(*x*, *y*), B(1, 2) and C(2, 1) is 6 sq. units, then prove that *x* + *y* = 15.

Sol. Given, A = (*x*, *y*), B = (1, 2), C = (2, 1) and area of DABC = 6 sq. units.

By, area of triangle's formula area of DABC

= $\frac{1}{2}$|*x*(2 – 1) + 1(1 – *y*) + 2(*y* – 2)|

$\Rightarrow 6 = \frac{1}{2}|x + 1 - y + 2y - 4|$

$\Rightarrow 12 = x + y - 3$

$\Rightarrow x + y = 15$. **Hence Proved.**

50. The *x*-coordinate of a point P is twice its *y*-coordinate. If P is equidistant from Q(2, – 5) and R(– 3, 6), find the coordinates of P.

Sol. Let the coordinates of point P be $(2y, y)$. Since, P is equidistant from Q and R

PQ = PR

$\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$

$\Rightarrow (2y - 2)2 + (y + 5)2 = (2y + 3)2 + (y - 6)2$

$\Rightarrow 4y2 + 4 - 8y + y2 + 25 + 10y$

$= 4y2 + 9 + 12y + y2 + 36 - 12y$

$\Rightarrow 2y + 29 = 45$

$\Rightarrow 2y = 45 - 29$

$\Rightarrow y = \frac{16}{2} = 8$

Hence, the coordinates of point P are (16, 8). **Ans.**

51. If (*x*, *y*) is on the line joining the points (1, – 3) and (– 4, 2), prove that $x + y + 2 = 0$.

Sol. Let A = (1, – 3) and B = (– 4, 2). Also, let P = (x, y) is on the line joining AB. Thus A, B and P are collinear.

So, $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

Hence, $1(2 - y) - 4(y + 3) + x(-3 - 2) = 0$

$\Rightarrow 2 - y - 4y - 12 - 5x = 0$

$\Rightarrow -5y - 10 - 5x = 0$

$\Rightarrow x + y + 2 = 0$.

Hence Proved.

52. If the point P(*m*, 3) lies on the line segment joining the points $A\left(-\frac{2}{5}, 6\right)$ and B (2, 8), find the value of *m*.

Sol. Given, A = $\left(-\frac{2}{5}, 6\right)$ and B = (2, 8). Also

P = (*m*, 3) is on the line joining AB. Thus A, B and P are collinear.

So, $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

Thus, $-\frac{2}{5}(8 - 3) + 2(3 - 6) + m(6 - 8) = 0$

$\Rightarrow -2 - 6 - 2m = 0$

$\Rightarrow -2m = 8$

$\Rightarrow m = -4.$

Ans.

53. If *R*(*x*, *y*) is a point on the line segment joining the points P(*a*, *b*) and Q(*b*, *a*), then prove that *x* + *y* = *a* + *b*.

Sol. Given, P = (*a*, *b*) and Q = (*b*, *a*) and R = (*x*, *y*)

So, $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

Thus , $x(b - a) + a(a - y) + b(y - b) = 0$

$\Rightarrow x(b - a) + a2 - ay + by - b2 = 0$

$\Rightarrow x(b - a) + y(b - a) - (b - a)(b + a) = 0$

$\Rightarrow (b - a) [x + y - (a + b)] = 0$

But $a \neq b$ as for $a = b$, point P and Q will be same.

$\Rightarrow x + y = a + b.$

Hence Proved.

54. What is the distance between the points A(*c*, 0) and B(0, – *c*) ?

Sol. Given, A = (*c*, 0) and B = (0, – *c*)

Distance, = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Thus, AB = $\sqrt{(c - 0)^2 + (0 + c)^2}$

= $\sqrt{(c)^2 + (c)^2}$

= $\sqrt{2}c$ units. **Ans.**

55. If the distance between the points (3, 0) and (0, *y*) is 5 units and *y* is positive then what is the value of *y*?

Sol. Given, the distance between the points (3, 0) and (0, *y*) is 5 units.

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Thus, 5 = $\sqrt{(3 - 0)^2 + (0 - y)^2}$

= $\sqrt{9 + y^2}$

⇒ 9 + *y*2 = 25 ⇒ *y*2 = 16

⇒ *y*2 = 42 ⇒ *y* = ± 4

But as *y* is positive so the value of *y* = 4. **Ans.**

56. The coordinates of houses of Sonu and Labhoo are (7, 3) and (4, 3) respectively. Coordinates of their school is (2, 2). If both leave their house at the same time in the morning and also reach school in time then who travels faster?

Sol. Distance between Sonu's house and school

= $\sqrt{(2 - 7)^2 + (2 - 3)^2}$

= $\sqrt{26}$ units

Distance between Labhoo's house and school

= $\sqrt{(2 - 4)^2 + (2 - 3)^2}$

= $\sqrt{5}$ units

... Distance of Sonu's house from school is more

Sonu travels faster. **Ans.**

Short Answer Type Questions

57. Find the ratio in which the line segment joining the points A(3, – 3) and B(– 2, 7) is divided by the X-axis. Also, find the coordinate of the point of division.

[Board Question]

Sol. Let AB be divided by the *X*-axis in the ratio *p* : 1 at the point K.

Thus, the coordinates of the point

K = $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

= $\left(\frac{p(-2)+3}{p+1}, \frac{7p+(-3)}{p+1}\right)$

= $\left(\frac{3-2p}{p+1}, \frac{7p-3}{p+1}\right)$

However as K lies in the *X*-axis, its coordinate is (*x*, 0).

$\Rightarrow \frac{7p-3}{p+1} = 0$

$\Rightarrow 7p - 3 = 0 \Rightarrow 7p = 3$

$\Rightarrow p = \frac{3}{7}$

Thus, the required ratio *p* = : 1 = 3 : 7

Now, $x = \frac{3-2p}{p+1}$

$\Rightarrow x = \frac{3-2\left(\frac{3}{7}\right)}{\left(\frac{3}{7}\right)+1}$

$= \frac{21-6}{3+7}$

$= \frac{15}{10} = \frac{3}{2}$

Hence, K = $\left(\frac{3}{2}, 0\right)$. **Ans.**

58. If the point P(*k* – 1, 2) is equidistant from the points A(3, *k*) and B(*k*, 5), find the value of *k*.

[Board Question]

Sol. As P is equidistant from A and B,

$\because$ AP = BP

$\Rightarrow \sqrt{[3-(k-1)]^2+(k-2)^2}$

$= \sqrt{(k-1-k)^2+(2-5)^2}$

$\Rightarrow (4 - k)^2 + (k - 2)2 = 1 + 9$

$\Rightarrow k^2 + 16 - 8k + 4 + k^2 - 4k = 10$

$\Rightarrow 2k^2 - 12k + 10 = 0$

$\Rightarrow k^2 - 6k + 5 = 0$

$\Rightarrow k^2 - 5k - k + 5 = 0$

$\Rightarrow k(k - 5) - 1(k - 5) = 0$

$\Rightarrow (k - 5)(k - 1) = 0$

$\Rightarrow k = 1, 5$

Thus, the value of *k* are 1 or 5. **Ans.**

59. Find the value of *p* for which the points (3*p* + 1, *p*), (*p* + 2, *p* – 5) and (*p* + 1, – *p*) are collinear.

Sol. Since the points (3*p* + 1, *p*), (*p* + 2, *p* – 5) and (*p* + 1, – *p*) are collinear, therefore, we have

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$\Rightarrow (3p + 1)(p - 5 + p) + (p + 2)(-p - p) + (p + 1)(p - p + 5) = 0$

$\Rightarrow (3p + 1)(2p - 5) + (p + 2)(-2p) + (p + 1)(5) = 0$

$\Rightarrow 6p^2 + 2p - 15p - 5 - 2p^2 - 4p + 5p + 5 = 0$

$\Rightarrow 4p^2 - 12p = 0$

$\Rightarrow 4p^2 = 12p$

$\Rightarrow 4p(p - 3) = 0$

Hence, the value of *p* is 3 or 0. **Ans.**

60. Find a relation between x and y if the points A(x, y), B(– 4, 6) and C(– 2, 3) are collinear.

[Board Question]

Sol. Given, A(x, y), B(– 4, 6), C(– 2, 3)

$x_1 = x,\ y_1 = y,\ x_2 = -4\ ,\ y_2 = 6,\ x_3 = -2\ ,\ y_3 = 3$

If these points are collinear, then area of triangle made by these points is 0.

$\frac{1}{2}[(x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}[x (6 - 3) + (-4) (3 - y) + (-2) (y - 6)] = 0 \Rightarrow 3x - 12 + 4y - 2y + 12 = 0 \Rightarrow 3x + 4y - 2y = 0$

$\Rightarrow 3x + 2y = 0 \Rightarrow 3x = -2y$ **Ans.**

61. Points A(– 1, y) and B(5, 7) lie on a circle with centre O(2, – 3y). Find the values of y. Hence find the radius of the circle.

[Board Question]

Sol. As A and B lie on a circle so AO and BO form the radii.

Thus, AO = BO

$\Rightarrow \sqrt{[2-(-1)]^2 + (-3y-y)^2}$

$= \sqrt{(2-5)^2 + (-3y-7)^2}$

$\Rightarrow (2 + 1)^2 + (-4y)^2 = (-3)^2 + (-3y - 7)^2$

$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow y^2 - 7y + y - 7 = 0$

$\Rightarrow y(y - 7) + 1(y - 7) = 0$

$\Rightarrow (y - 7) (y + 1) = 0$

$\Rightarrow y = -1, 7$

Thus, the value of y are – 1 or 7.

When $y = -1$,

AO = BO = $\sqrt{(2+1)^2 + (3+1)^2}$

$\Rightarrow$ AO = BO = $\sqrt{3^2 + 4^2}$

= $\sqrt{9+16}$ = 5 units

When $y = 7$,

AO = BO = $\sqrt{[2-(-1)]^2 + (-21-7)^2}$

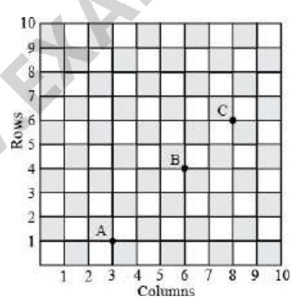
= $\sqrt{(3)^2 + (-28)^2}$ = $\sqrt{9+784}$

= $\sqrt{793}$ = 28.16 units

Hence, the radius of the circle is either 5 units or 28.16 units. **Ans.**

62. Given figure shows the arrangement of desks in a classroom. Ashima, Bharti and Camella are seated at A(3, 1), B(6, 4) and C(8, 6) respectively. Do you think they are seated in a line ? Give reason for your answer.

[Board Question]



Sol. By distance formula

AB = $\sqrt{(6-3)^2 + (4-1)^2}$

= $\sqrt{9+9}$ = $3\sqrt{2}$ units

and BC = $\sqrt{(8-6)^2 + (6-4)^2}$

= $\sqrt{4+4}$ = $2\sqrt{2}$ units

Now, AC = $\sqrt{(8-3)^2 + (6-1)^2}$

= $\sqrt{25+25}$

= $5\sqrt{2}$ units

Now, AB + BC = $3\sqrt{2}+2\sqrt{2}$ = $5\sqrt{2}$ = AC

Therefore, A, B and C are collinear.

Hence, Ashima, Bharti and Camella are seated in a line.**Ans.**

63. Three villages having nearly the same population are situated at the positions (1, 2) , (5, 4) and (3, 8). The Block Development Officer wants to set-up a community centre in such a way that it is equally convenient to reach it for the villagers of all the three villages. Find the position where the community centre should be set-up.

Sol. Suppose the position coordinate of the community centre be (x, y).

Since, three villages are situated at the positions (1, 2), (5, 4) and (3, 8).

So, distance between community centre and first village.

= Distance between community centre and second village

$\sqrt{(x-1)^2+(y-2)^2}$ = $\sqrt{(x-5)^2+(y-4)^2}$

After squaring on both sides, we get

$\Rightarrow (x-1)^2 + (y-2)^2 = (x-5)^2 + (y-4)^2$

$\Rightarrow x^2 + 1 - 2x + y^2 + 4 - 4y$

$= x^2 + 25 - 10x + y^2 + 16 - 8y$

$\Rightarrow 8x + 4y = 36$

$\Rightarrow 2x + y = 9$ ...(i)

Also, distance between community centre and second village

= Distance between community centre and third village

$\sqrt{(x-5)^2+(y-4)^2}$ = $\sqrt{(x-3)^2+(y-8)^2}$

$\Rightarrow x^2 + 25 - 10x + y^2 - 8y + 16$

$= x^2 + 9 - 6x + y^2 - 16y + 64$

$\Rightarrow 4x - 8y = -32$

$\Rightarrow x - 2y = -8$ ...(ii)

After solving (i) and (ii), we get

$x = 2, y = 5$

Hence, the position coordinate of the community centre will be (2, 5).**Ans.**

64. If the points P(– 3, 9), Q(*a*, *b*) and R(4, – 5) are collinear and *a* + *b* = 1, find the values of *a* and *b*.

[Board Question]

Sol. Since P(– 3, 9), Q(*a*, *b*) and R(4, – 5) are collinear

$\therefore x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow (-3)[b - (-5)] + a[(-5) - 9] + 4[9 - b] = 0$

$\Rightarrow (-3)[b + 5] - 14a + 36 - 4b = 0$

$\Rightarrow -3b - 15 - 14a + 36 - 4b = 0$

$\Rightarrow -14a - 7b = -21$

$\Rightarrow 2a + b = 3$ …(i)

$\Rightarrow 2a + b = 3(a + b)$ [$\because a + b = 1$]

$\Rightarrow 2a + b = 3a + 3b$

$\Rightarrow a + 2b = 0$

$\Rightarrow a = -2b$ …(ii)

Substituting $a = -2b$ in (i), we have

$-4b + b = 3 \Rightarrow -3b = 3$

$\Rightarrow b = -1$

and $a = 2$ [from (ii)] **Ans.**

65. Points P, Q, R and S divide the line segment joining the points A(1, 2) and B(6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

Sol. Let the points P, Q, R and S divide the join of A(1, 2) and B(6, 7) into five equal parts. Then P(x, y) divides AB in the ratio 1 : 4.

$\therefore$ P = $\left(\frac{1\times6+4\times1}{1+4}, \frac{1\times7+4\times2}{1+4}\right)$

$\Rightarrow$ P = $\left(\frac{10}{5}, \frac{15}{5}\right)$

$\Rightarrow$ P = (2, 3)

Also, Q(p, q) divides AB in the ratio 2 : 3.

$\therefore$ Q = $\left(\frac{2\times6+3\times1}{2+3}, \frac{2\times7+3\times2}{2+3}\right)$

$\Rightarrow$ Q = $\left(\frac{15}{5}, \frac{20}{5}\right)$

$\Rightarrow$ Q = (3, 4)

and R(r, s) divides AB in the ratio 3 : 2.

$\therefore$ R = $\left(\frac{3\times6+2\times1}{3+2}, \frac{3\times7+2\times2}{3+2}\right)$

$\Rightarrow$ R = $\left(\frac{20}{5}, \frac{25}{5}\right)$

$\Rightarrow$ R = (4, 5). **Ans.**

66. If the point P(2, 2) is equidistant from the points A(– 2, *k*) and B(– 2*k*, – 3), find *k*. Also, find the length of AP.

[Board Question]

Sol. As P is equidistant from both A and B,

AP = PB

$\Rightarrow$ $\sqrt{(2+2)^2+(2-k)^2} = \sqrt{(2+2k)^2+(2+3)^2}$

$\Rightarrow$ $16 + k^2 + 4 - 4k = 4k^2 + 4 + 8k + 25$

$\Rightarrow$ $3k^2 + 12k + 9 = 0$

$\Rightarrow k^2 + 4k + 3 = 0$

$\Rightarrow k^2 + 3k + k + 3 = 0$

$\Rightarrow k(k + 3) + 1(k + 3) = 0$

$\Rightarrow (k + 3)(k + 1) = 0$

$\Rightarrow k = -3, -1$

When $k = -1$, AP = $\sqrt{(2+2)^2 + (2+1)^2}$

$= \sqrt{(4)^2 + (3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

= 5 units

When $k = -3$, AP = $\sqrt{(2+2)^2 + (2+3)^2}$

$= \sqrt{(4)^2 + (5)^2}$

$= \sqrt{16+25}$

$= \sqrt{41}$ units

Thus, AP = 5 or $\sqrt{41}$ units. **Ans.**

67. Prove that the diagonals of a rectangle ABCD with vertices A(2, – 1), B(5, – 1), C(5, 6) and D(2, 6) are equal and bisect each other.

[Board Question]

Sol. Given, A(2, – 1), B(5, – 1), C(5, 6) and D(2, 6)

AC = $\sqrt{(5-2)^2 + (6+1)^2}$

$= \sqrt{(3)^2 + (7)^2}$

$= \sqrt{9+49} = \sqrt{58}$

BD = $\sqrt{(2-5)^2 + (6+1)^2}$

$= \sqrt{(-3)^2 + (7)^2}$

$= \sqrt{9+49}$

$= \sqrt{58}$

Thus, AC = BD.

Let E be the mid-point of AC and F be the mid-point of BD.

Thus, E = $\left(\frac{2+5}{2}, \frac{6-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$

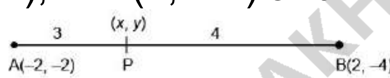
Also, F = $\left(\frac{5+2}{2}, \frac{6-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$

Hence, AC and BD bisect each other at the point $\left(\frac{7}{2}, \frac{5}{2}\right)$. **Hence Proved,**

68. Find the coordinates of a point P, which lies on the line segment joining the points A(–2, –2) and B(2, – 4) such that AP = $\frac{3}{7}$AB.

[Board Question]

Sol. Given, A = (– 2, – 2), B = (2, – 4) and AP = $\frac{3}{7}$AB



⇒ 7AP = 3AB = 3AP + 3BP

⇒ 4AP = 3BP

⇒ $\frac{AP}{BP} = \frac{3}{4}$

Thus, AP : PB = 3 : 4

Let the coordinates of P be (x, y).

So, $x = \frac{3(2)+4(-2)}{3+4}$ and $y = \frac{3(-4)+4(-2)}{3+4}$

⇒ $x = \frac{6-8}{7}$ and $y = \frac{-12-8}{7}$

⇒ $x = \frac{-2}{7}$ and $y = \frac{-20}{7}$

Hence, the coordinates of P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

Ans.

69. A point P divides the line segment joining the points A(3, – 5) and B(– 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line $x + y = 0$, then find the value of k.

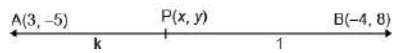
Sol. Given,

A = (3, – 5); B = (– 4, 8); $\frac{AP}{PB} = \frac{k}{1}$ and $x + y = 0$

As, $\frac{AP}{PB} = \frac{k}{1}$

So, AP : BP = k : 1

Let the coordinates of point P be (x, y).



Thus, $x = \frac{1(3)+k(-4)}{k+1}$ and $y = \frac{1(-5)+k(8)}{k+1}$

or $x = \frac{3-4k}{k+1}$ and $y = \frac{8k-5}{k+1}$

Now, $x + y = 0$

$\Rightarrow \frac{3-4k}{k+1} + \frac{8k-5}{k+1} = 0$

$\Rightarrow \frac{4k-2}{k+1} = 0$

$\Rightarrow 4k - 2 = 0$

$\Rightarrow 4k = 2$

$\Rightarrow k = \frac{1}{2}$. **Ans.**

70. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are *A*(2, 1), *B*(4, 3) and *C*(2, 5).

Sol. Let A(2, 1), B(4, 3) and C(2, 5) be the vertices of ∆ABC. Let D, E, F be the mid-point of sides BC, CA, AB respectively.

By mid-point formula,

Coordinates of D = $\left(\frac{4+2}{2}, \frac{3+5}{2}\right)$

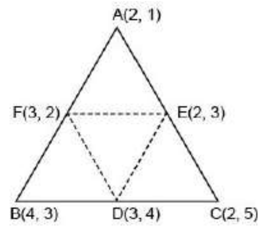
= $\left(\frac{6}{2}, \frac{8}{2}\right)$ = (3, 4)

Coordinates of E = $\left(\frac{2+2}{2}, \frac{1+5}{2}\right)$

= $\left(\frac{4}{2}, \frac{6}{2}\right)$ = (2, 3)

Coordinates of F = $\left(\frac{2+4}{2}, \frac{1+3}{2}\right)$

= $\left(\frac{6}{2}, \frac{4}{2}\right)$ = (3, 2)



Now, Area of ∆DEF

= $\frac{1}{2}$|3(3 – 2) + 2(2 – 4) + 3(4 – 3)|

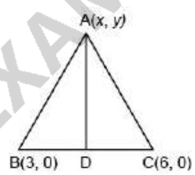
= $\frac{1}{2}$|3 – 4 + 3|

= $\frac{1}{2}$ × 2 = 1 sq. unit. **Ans.**

71. If two vertices of an equilateral triangle are (3, 0) and (6, 0), find the third vertex.

Sol. Given two vertices of an equilateral triangle are (3, 0) and (6, 0).

Let the third vertex be (x, y).



Thus, BC = $\sqrt{(6-3)^2+(0-0)^2}$ = 3

AB = $\sqrt{(x-3)^2+(y-0)^2}$

= $\sqrt{x^2-6x+9+y^2}$

AC = $\sqrt{(x-6)^2+(y-0)^2}$

= $\sqrt{x^2-12x+36+y^2}$

Now, AB = AC

or $AB^2 = AC^2$

So, $x^2 - 6x + 9 + y^2 = x^2 - 12x + 36 + y^2$

$\Rightarrow -2x + 3 = -4x + 12$

$\Rightarrow 2x = 9$

$\Rightarrow x = \frac{9}{2}$

Also, AB = BC

So, $x^2 - 6x + 9 + y^2 = 9$

$\Rightarrow x2 - 6x + y^2 = 0$

$\Rightarrow \left(\frac{9}{2}\right)^2 - 6\left(\frac{9}{2}\right) + y^2 = 0$

$\Rightarrow 81 - 108 + 4y^2 = 0$

$\Rightarrow -27 + 4y^2 = 0$

$\Rightarrow 4y^2 = 27$

$\Rightarrow y^2 = \frac{27}{4}$

$\Rightarrow y = \pm \frac{3\sqrt{3}}{2}$

Thus, A = $\left(\frac{9}{2}, \pm \frac{3\sqrt{3}}{2}\right)$. **Ans.**

72. Show that the points A(3, 0), B(6, 4) and C(– 1, 3) are the vertices of a right-angled triangle. Also, prove that these are the vertices of an isosceles triangle.

Sol. Given, A = (3, 0), B = (6, 4) and C = (– 1, 3)

Now, AB = $\sqrt{(6-3)^2 + (4-0)^2}$

= $\sqrt{(3)^2 + (4)^2}$ = 5 units

BC = $\sqrt{(6+1)^2 + (4-3)^2}$

= $\sqrt{(7)^2 + (1)^2}$

= $\sqrt{50}$ units

and AC = $\sqrt{(3+1)^2 + (0-3)^2}$

= $\sqrt{(4)^2 + (3)^2}$

= 5 units

Thus, AC = AB

Hence, it is proved that ∆ABC is an isosceles triangle.

Also, $AC^2 + AB^2 = 25 + 25 = 50$

and $BC^2 = (\sqrt{50})^2 = 50$

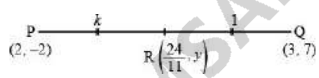
Thus, $BC^2 = AC^2 + AB^2$ follows the Pythagoras' theorem.

Hence, it is proved that ∆ABC is an isosceles right-angled triangle. **Hence Proved.**

73. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, – 2) and Q(3 , 7) ? Also find the value of *y*.

[Board Question]

Sol. Let point R$\left(\frac{24}{11}, y\right)$ divides PQ in the ratio *k* : 1



Thus, $\left(\frac{24}{11}, y\right) = \left(\frac{k(3)+1(2)}{k+1}, \frac{k(7)+1(-2)}{k+1}\right)$

$= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

$\Rightarrow \frac{3k+2}{k+1} = \frac{24}{11}$

$\Rightarrow 11(3k + 2) = 24 (k + 1)$

$\Rightarrow 33k + 22 = 24k + 24$

$\Rightarrow 33k – 24k = 24 – 22$

$\Rightarrow 9k = 2$

$\Rightarrow k = 2/9$

Thus, $k : 1 = \frac{2}{9} : 1$

= 2 : 9

Now, $y = \frac{7k-2}{k+1} = \frac{7\left(\frac{2}{9}\right)-2}{\frac{2}{9}+1}$

$= \frac{\frac{14}{9}-2}{\frac{2}{9}+1} = \frac{\frac{14-18}{9}}{\frac{2+9}{9}}$

$= \frac{-4}{11}$

Line PQ divides in the ratio 2 : 9 and value of

$y = \frac{-4}{11}$. **Ans.**

74. Prove that the points (*a*, *b* + *c*), (*b*, *c* + *a*) and (*c*, *a* + *b*) are collinear.

[Board Question]

Sol. Given,

A = (*a*, *b* + *c*), B = (*b*, *c* + *a*) and C = (*c*, *a* + *b*)

For A, B and C to be collinear, they must satisfy the condition of collinearity.

That, area of $\Delta ABC = 0$

$\frac{1}{2}|a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a)|$

$= a(c - b) + b(a - c) + c(b - a)$

$= ac - ab + ab - bc + bc - ac$

$= 0$

As area of DABC = 0, then the three points are collinear.**Hence Proved.**

75. If P(*x*, *y*) is any point on the line joining the points A(*a*, 0) and B(0, *b*), then show that $\frac{x}{a}+\frac{y}{b}$ = 1.

Sol. Given, A = (*a*, 0) and B = (0, *b*). Also

P = (*x*, *y*) is collinear with A and B.

Thus, area of ΔPAB = 0

$\Rightarrow \frac{1}{2}|x(0-b)+a(b-y)+0(y-0)|=0$

$\Rightarrow -bx+ab-ay=0$

$\Rightarrow ab=ay+bx$

$\Rightarrow \frac{ab}{ab} = \frac{ay}{ab}+\frac{bx}{ab}$

[dividing both sides by *ab*]

$\Rightarrow 1 = \frac{y}{b}+\frac{x}{a}$

$\Rightarrow \frac{x}{a}+\frac{y}{b} = 1$. **Hence Proved.**

76. If the points (*p*, *q*), (*m*, *n*) and (*p* – *m*, *q* – *n*) are collinear, show that *pn* = *qm*.

[Board Question]

Sol. Given, (*p*, *q*), (*m*, *n*) and (*p* – *m*, *q* – *n*) are collinear.

Thus,

$\frac{1}{2}|p[n-(q-n)]+m[(q-n)-q]+(p-m)(q-n)|$

$= 0$

$\Rightarrow p(2n-q)+m(-n)+pq-mq-pn+mn=0$

$\Rightarrow 2pn-pq-mn+pq-mq-pn+mn=0$

$\Rightarrow pn-mq=0$

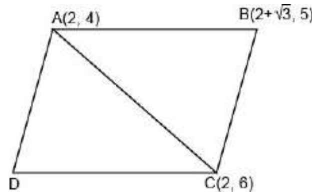
$\therefore pn=mq$. **Hence Proved.**

77. Find the area of a parallelogram ABCD if three of its vertices are A(2, 4), $B(2+\sqrt{3}, 5)$ and C(2, 6).

[Board Question]

Sol. Given, A = (2, 4), B = $(2+\sqrt{3}, 5)$ and C = (2, 6)

As AC is the diagonal of the parallelogram, so AC divides ABCD into two triangles of equal areas.



Thus, area of ABCD = 2 Area of ΔABC

= 2 × $\frac{1}{2}[2(5-6)+(2+\sqrt{3})(6-4)+2(4-5)]$

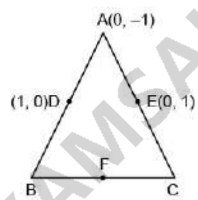
= $[2(-1)+2(2+\sqrt{3})+2(-1)]$

= $[-2+4+2\sqrt{3}-2]$

= $2\sqrt{3}$ sq. units. **Ans.**

78. In fig, ABC is a triangle coordinates of whose vertex A are (0, – 1). D and E respectively are the mid-points of the sides AB and AC and their coordinates are (1, 0) and (0, 1) respectively. If F is the mid-point of BC,find the areas of ΔABC and ΔDEF.

[Board Question]



Sol. Given, the coordinates of vertex A(0, – 1) and mid-points D(1, 0) and E(0, 1) respectively.

Let coordinates of B be (x, y)

Since, D is the mid-point of AB

then, $\left(\frac{x+0}{2}, \frac{y-1}{2}\right)$ = (1, 0)

which gives B(2, 1).

Similarly, E is the mid-point of AC.

Let coordinates of C are (x', y')

then, $\left(\frac{x'+0}{2}, \frac{y'-1}{2}\right)$ = (0, 1)

which gives C(0, 3)

Now, area of ΔABC = $\frac{1}{2}$| 0 (1 – 3) + 2 (3 + 1) + 0 (– 1 – 1)|

= 4 sq. units.

Now, F is the mid-point of BC.

⇒ Coordinates of F = $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ = (1, 2)

∴ Area of ΔDEF = $\frac{1}{2}$|1(1 – 2) + 0 (2 – 0) + 1 (0 –1)|

= $\frac{|-2|}{2}$ = 1 sq. unit. **Ans.**

79. If A(6, – 1), B(1, 3) and C(*k*, 8) are three points such that AB = BC, find the value of *k*.

Sol. Given, A(6, – 1), B(1, 3) and C(*k*, 8) and AB = BC

Thus, AB = $\sqrt{(6-1)^2+(-1-3)^2}$

= $\sqrt{(5)^2+(-4)^2} = \sqrt{41}$

Also BC = $\sqrt{(1-k)^2+(3-8)^2}$

= $\sqrt{(1-k)^2+(-5)^2}$

... AB = BC

⇒ $\sqrt{(1-k)^2+(-5)^2}$ = $\sqrt{41}$

⇒ (1 – *k*)2 + 25 = 41

⇒ (1 – *k*)2 = 41 – 25

⇒ (1 – *k*)2 = 16

⇒ 1 – *k* = ± 4

⇒ *k* = 5 or – 3

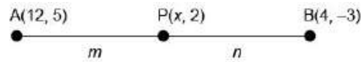
Hence, the value of *k* is 5 or – 3. **Ans.**

Long Answer Type Questions

80. Find the ratio in which the point P(*x*, 2) divides the line segment joining the points A(12, 5) and B(4, – 3). Also the value of *x*.

[Board Question]

Sol. Let the required ratio be $m : n$.



Now using section formula, we have

Coordinates of P = $\left(\frac{12n+4m}{m+n}, \frac{5n-3m}{m+n}\right)$

But it is given that

P = (x, 2)

$\Rightarrow \frac{12n+4m}{m+n} = x$

and $\frac{5n-3m}{m+n} = 2$

$\Rightarrow 5n - 3m = 2m + 2n$

$\Rightarrow 5m = 3n$

$\Rightarrow \frac{m}{n} = \frac{3}{5}$ …(i)

Required ratio is 3 : 5

Now, $\frac{12n+4m}{m+n} = x$

$\Rightarrow \frac{n\left(12+\frac{4m}{n}\right)}{n\left(\frac{m}{n}+1\right)} = x$[using (i)]

$\Rightarrow \frac{12+\frac{4\times3}{5}}{\frac{3}{5}+1} = x$

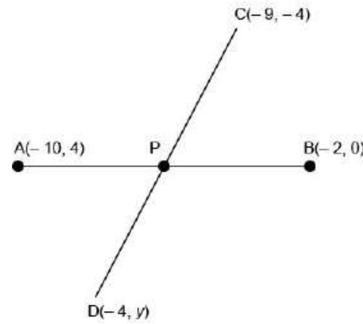
$\Rightarrow \frac{60+12}{3+5} = x$

$\Rightarrow \frac{72}{8} = x$

$\Rightarrow x = 9$. **Ans.**

81. The mid-point P of the line segment joining the points A(–10, 4) and B(– 2, 0) lies on the line segment joining the points C(– 9, – 4) and D(– 4, y). Find the ratio in which P divides CD. Also, find the value of y.

Sol. Since, P is the mid-point of AB



Thus, P = $\left(\frac{-10-2}{2}, \frac{4+0}{2}\right)$

= $\left(\frac{-12}{2}, \frac{4}{2}\right)$ = (– 6, 2)

Also let CD be divided in the ratio *s* : *t*.

Now, by section formula,

– 6 = $\frac{t(-9)+s(-4)}{s+t}$

⇒ – 6(*s* + *t*) = – 9*t* – 4*s*

⇒ 6*t* + 6*s* = 9*t* + 4*s*

⇒ 3*t* – 2*s* = 0

⇒ 2*s* = 3*t*

⇒ $\frac{s}{t}$ = $\frac{3}{2}$

⇒ *s* : *t* = 3 : 2

Hence, the required ratio is 3 : 2.

and 2 = $\frac{t(-4)+s \times y}{s+t}$

⇒ 2 = $\frac{2(-4)+3y}{2+3}$

⇒ 10 = – 8 + 3*y*

⇒ 3*y* = 18

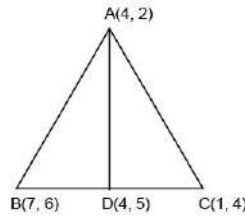
⇒ *y* = 6. **Ans.**

82. If A(4, 2), B(7, 6) and C(1, 4) are the vertices of ∆ABC and AD is its median, prove that the median AD divides ∆ABC into two

triangles of equal areas.

[Board Question]

Sol. Given, A(4, 2), B(7, 6) and C(1, 4) are the vertices of ΔABC and AD is its median.



Thus, AD bisects BC at D such that D is its mid-point.

So, the coordinates of D

$= \left(\frac{7+1}{2}, \frac{6+4}{2}\right) = (4, 5)$

Now, area of ΔABC

$= \frac{1}{2}|4(6 – 4) + 7(4 – 2) + 1(2 – 6)|$

$= \frac{1}{2}|4(2) + 7(2) + 1(– 4)|$

$= \frac{1}{2}|8 + 14 – 4| = 9$ sq. units

Also, area of ΔADB

$= \frac{1}{2}|4(6 – 5) + 7(5 – 2) + 4(2 – 6)|$

$= \frac{1}{2}|4(1) + 7(3) + 4(– 4)|$

$= \frac{1}{2}|4 + 21 – 16|$

$= 4.5$ sq. units

Also, area of ΔADC

$= \frac{1}{2}|4(4 – 5) + 1(5 – 2) + 4(2 – 4)|$

$= \frac{1}{2}|4(– 1) + 1(3) + 4(– 2)|$

$= \frac{1}{2}|– 4 + 3 – 8|$

= 4.5 sq. units

⇒ ar (ΔADB) = ar (ΔADC) = 4.5 sq. units

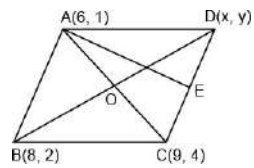
Hence Proved.

83. Three vertices of a parallelogram ABCD are A(6, 1), B(8, 2) and C(9, 4). If E is the mid-point of DC, find the area of ΔADE.

[Board Question]

Sol. As ABCD is a parallelogram, so

AB = CD, AD = BC



Area of ΔABC = Area of ΔADC

Let D = (x, y)

Thus, area of ΔABC

= $\frac{1}{2}$[6(2 – 4) + 8(4 – 1) + 9(1 – 2)]

= $\frac{1}{2}$[6(– 2) + 8(3) + 9(– 1)]

= $\frac{1}{2}$[– 12 + 24 – 9]

= $\frac{3}{2}$ sq. units

Hence, area of ΔABC

= Area of ΔADC

= $\frac{3}{2}$ sq. units

As E is the mid-point of DC, so

DE = EC

Area of ΔAEC = Area of ΔAED

and Area of ΔADC = Area of ΔAEC + Area of ΔAED

$\Rightarrow \frac{3}{2}$ sq. units = 2 × Area of ΔAED

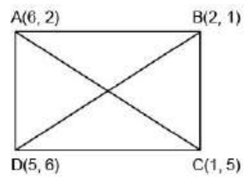
$\Rightarrow$ Area of ΔAED = $\frac{3}{4}$ sq. units. **Ans.**

84. Show that the points A(6, 2), B(2, 1), C(1, 5) and D(5, 6) are the vertices of a square.

[Board Question]

Sol. Given,

A = (6, 2), B = (2, 1), C = (1, 5) and D = (5, 6)



Thus AB = $\sqrt{(6-2)^2+(2-1)^2}$ = $\sqrt{(4)^2+(1)^2}$

= $\sqrt{17}$ units

BC = $\sqrt{(2-1)^2+(1-5)^2}$ = $\sqrt{(1)^2+(-4)^2}$

= $\sqrt{17}$ units

CD = $\sqrt{(1-5)^2+(5-6)^2}$ = $\sqrt{(-4)^2+(-1)^2}$

= $\sqrt{17}$ units

DA = $\sqrt{(6-5)^2+(2-6)^2}$ = $\sqrt{(1)^2+(-4)^2}$

= $\sqrt{17}$ units

AC = $\sqrt{(6-1)^2+(2-5)^2}$ = $\sqrt{(5)^2+(-3)^2}$

= $\sqrt{34}$ units

BD = $\sqrt{(2-5)^2+(1-6)^2}$ = $\sqrt{(-3)^2+(-5)^2}$

= $\sqrt{34}$ units

Hence, AB = CD = BC = DA and AC = BD

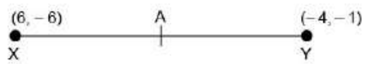
Thus, as all the sides are equal and the diagonals are also equal to each other, so the figure is a square.**Hence Proved.**

85. Point A lies on the line segment XY joining X(6, – 6) and Y (– 4, – 1) in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line 3*x* + *k*(*y* + 1) = 0, find the value of *k*.

[Board Question]

Sol. Given,

$\frac{XA}{XY} = \frac{2}{5}$



$\Rightarrow \frac{XA}{XA + AY} = \frac{2}{5}$

⇒ 5XA = 2XA + 2AY

⇒ 3XA = 2AY

$\Rightarrow \frac{XA}{AY} = \frac{2}{3}$

⇒ XA : AY = 2 : 3

So A divides XY in ratio 2 : 3

Here, $m = 2$, $n = 3$, $x_1 = 6$, $y_1 = -6$, $x_2 = -4$ and $y_2 = -1$

Coordinates of point A are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$\Rightarrow \left(\frac{2 \times (-4) + 3(6)}{2+3}, \frac{2(-1) + 3(-6)}{2+3}\right)$

$\Rightarrow \left(\frac{-8+18}{5}, \frac{-2-18}{5}\right) = (2, -4)$

Since, point A(2, – 4) lies on line 3*x* + *k* (*y* + 1) = 0.

Therefore, it will satisfy the equation.

On putting *x* = 2 and *y* = – 4 in the equation, we get

3 × 2 + *k* (– 4 + 1) = 0

⇒ 6 – 3*k* = 0 ⇒ 3*k* = 6

⇒ *k* = 2. **Ans.**

86. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points (– 2, – 5) and (6, 3). Find the coordinates of the point of intersection.

[Board Question]

Sol. Let required ratio be $k : 1$.

By section formula, we have

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

Here, $x_1 = -2$, $x_2 = 6$, $y_1 = -5$, $y_2 = 3$

$m = k$, $n = 1$

$\Rightarrow x = \frac{k(6)+(-2)}{k+1} = \frac{6k-2}{k+1}$

$\Rightarrow y = \frac{k(3)+(-5)}{k+1} = \frac{3k-5}{k+1}$

$\left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right)$ points lie on the line $x - 3y = 0$

$\therefore \left(\frac{6k-2}{k+1}\right) - 3\left(\frac{3k-5}{k+1}\right) = 0$

$\Rightarrow \frac{6k-2}{k+1} - \frac{(9k-15)}{k+1} = 0$

$\Rightarrow 6k - 2 - 9k + 15 = 0$

$\Rightarrow -3k + 13 = 0$

$\Rightarrow k = \frac{13}{3}$

Hence required ratio is $\left(\frac{13}{3} : 1\right)$ *i.e.,* (13 : 3).

Here, intersection point are,

$x = \frac{6k-2}{k+1} = \frac{\frac{6 \times 13}{3} - 2}{\frac{13}{3} + 1}$

$= \frac{(26-2) \times 3}{16}$

$= \frac{72}{16} = \frac{9}{2}$

$y = \frac{3k-5}{k+1} = \frac{3 \times \frac{13}{3} - 5}{\frac{13}{3}+1}$

$= \frac{(13-5) \times 3}{13+3} = \frac{24}{16}$

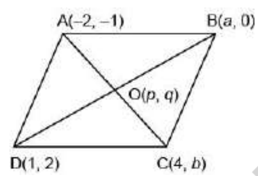
$= \frac{3}{2}$

∴ Intersection point is $\left(\frac{9}{2}, \frac{3}{2}\right)$. **Ans.**

87. If (– 2, – 1), (*a*, 0), (4, *b*) and (1, 2) are the vertices of parallelogram taken in order, find the value of *a* and *b*.

[Board Question]

Sol. Given, A = (– 2, – 1), B = (*a*, 0), C = (4, *b*) and D = (1, 2).



As ABCD is a parallelogram then its diagonals, AC and BD bisect each other.

Let the point of bisection of diagonals be O(*p*, *q*).

Thus, $p = \frac{1+a}{2} = \frac{4-2}{2}$

⇒ $p = 1 + a = 2$

⇒ $a = 1$

and $q = \frac{2+0}{2} = \frac{b-1}{2}$

⇒ $q = 2 = b - 1$

⇒ $q = b = 3$

Thus, $a = 1$ and $b = 3$. **Ans.**

88. Prove that the points A(3, 0), B(4, 5), C(– 1, 4) and D(– 2, – 1) taken in order, are the vertices of a rhombus. Also find its area.

[NCERT]

Sol. Given, A = (3, 0), B = (4, 5), C = (– 1, 4) and D = (– 2, – 1)

Now, AB = $\sqrt{(4-3)^2+(5-0)^2}$ = $\sqrt{(1)^2+(5)^2}$

= $\sqrt{26}$ units

BC = $\sqrt{(-1-4)^2+(4-5)^2}$

= $\sqrt{(-5)^2+(-1)^2}$

= $\sqrt{26}$ units

CD = $\sqrt{(-2+1)^2+(-1-4)^2}$

= $\sqrt{(-1)^2+(-5)^2}$

= $\sqrt{26}$ units

and DA = $\sqrt{(-2-3)^2+(-1-0)^2}$

= $\sqrt{(-5)^2+(-1)^2}$

= $\sqrt{26}$ units

Also, AC = $\sqrt{(3+1)^2+(0-4)^2}$

= $\sqrt{(4)^2+(-4)^2}$

= $\sqrt{32}$ units = $4\sqrt{2}$ units

and BD = $\sqrt{(4+2)^2+(5+1)^2}$ = $\sqrt{(6)^2+(6)^2}$

= $\sqrt{72}$ units = $6\sqrt{2}$ units

Thus, the sides of the quadrilateral are of equal length but the diagonals are unequal.

Hence, ABCD is proved to be a rhombus.

Now, Area of a rhombus

= $\frac{1}{2}[\sqrt{32}\times\sqrt{72}]$

= $\frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}$

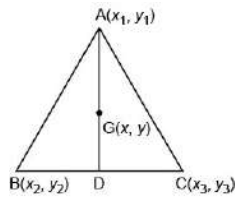
= 24 sq. units **Ans.**

89. Find the coordinates of the centroid of a triangle.

[NCERT]

[Board Question]

Sol. Let the vertices of ΔABC be A = (x_1, y_1), B = (x_2, y_2) and C = (x_3, y_3). Also, let the vertices of the centroid be G = (x, y).



Let AD be the median that bisects BC.

Thus BD = DC

Hence D = $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

If G is the centroid of ΔABC, then

AG : GD = 2 : 1

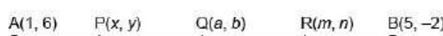
Thus, G(x, y)

= $\left(\frac{2\left(\frac{x_2+x_3}{2}\right)+1(x_1)}{2+1}, \frac{2\left(\frac{y_2+y_3}{2}\right)+1(y_1)}{2+1}\right)$

= $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. **Ans.**

90. Points P, Q and R divide the line segment joining the points A(1, 6) and B(5, – 2) in 4 equal parts. Find the co-ordinates of the points P, Q and R.

Sol. Given, A(1, 6) and B(5, – 2)



Let P = (x, y), Q = (a, b) and R = (m, n)

Now, AP = PQ = QR = RB

Thus AP : PB = 1 : 3

So $x = \frac{3\times1+1\times5}{1+3}$

and $y = \frac{3(6)+1(-2)}{1+3}$

⇒ $x = \frac{8}{4}$ and $y = \frac{16}{4}$

⇒ $x = 2$ and $y = 4$

⇒ P(x, y) = (2, 4)

Now, PQ : QB = 1 : 1

So, $a = \frac{1\times1+1\times5}{1+1}$

and $b = \frac{1(6)+1(-2)}{1+1}$

⇒ $a = \frac{6}{2}$ and $b = \frac{4}{2}$

⇒ $a = 3$ and $b = 2$

⇒ Q(a, b) = (3, 2)

and QR : RB = 1 : 1

So, $m = \frac{1(3)+1(5)}{1+1}$

and $n = \frac{1(2)+1(-2)}{1+1}$

⇒ $m = \frac{8}{2}$ and $n = \frac{0}{2}$

⇒ $m = 4$ and $n = 0$

⇒ R(m, n) = (4, 0). **Ans.**

91. Find the ratio in which the *y*-axis divides the line segment joining the points (– 1, – 4) and (5, – 6). Also find the coordinates of the point of intersection.

[Board Question]

Sol. Let the *y*-axis cuts the line joining point A(– 1, – 4) and point B(5, – 6) in the ratio k : 1 at the point P(0, y).

Then, by section fromula, we have $x = \frac{mx_2+nx_1}{m+n}$

⇒ $0 = \frac{k(5)+(-1)}{k+1}$

⇒ $0 = \frac{5k-1}{k+1}$

$\Rightarrow 5k - 1 = 0$

$\Rightarrow k = \frac{1}{5}$

Then the required ratio is $\left(\frac{1}{5} : 1\right)$ *i.e.,* (1 : 5)

Again, by section formula, we have $y = \frac{my_2 + ny_1}{m+n} = \frac{1(-6)+5(-4)}{1+5}$

$= \frac{-6-20}{6}$

$= \frac{-26}{6} = \frac{-13}{3}$

Hence, the intersection co-ordinates is $\left(0, -\frac{13}{3}\right)$.

Ans.

92. Determine the ratio in which the line *x* – *y* – 2 = 0 divides the line segment joining the points A(3, – 1) and B(8, 9).

Sol. Given, A = (3, – 1), B = (8, 9) and $x - y - 2 = 0$

Let the line $x - y - 2 = 0$ intersect the line segment AB at R in the ratio *p* : 1.

Now, $R_x = \frac{1(3)+p(8)}{p+1}$ and $R_y = \frac{1(-1)+p(9)}{p+1}$

$\Rightarrow R_x = \frac{3+8p}{p+1}$ and $R_y = \frac{-1+9p}{p+1}$

Thus, $R = \left(\frac{3+8p}{p+1}, \frac{9p-1}{p+1}\right)$

Now substituting the value of R in the given equation, we get

$\frac{3+8p}{p+1} - \frac{9p-1}{p+1} - 2 = 0$

$\Rightarrow 3 + 8p - 9p + 1 - 2(p + 1) = 0$

$\Rightarrow 4 - p - 2p - 2 = 0$

$\Rightarrow 3p = 2$

$\Rightarrow p = \frac{2}{3}$

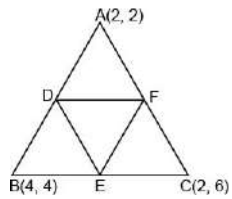
Thus, the ratio is $\frac{2}{3}$: 1 = 2 : 3. **Ans.**

93. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (2, 2) and (4, 4) and (2, 6).

[Board Question]

Sol. Given, A = (2, 2), B = (4, 4) and C = (2, 6).

Let D, E and F be the mid-points of AB, BC and CA respectively.



Thus, D = $\left(\frac{2+4}{2}, \frac{2+4}{2}\right)$ = (3, 3)

E = $\left(\frac{2+4}{2}, \frac{6+4}{2}\right)$ = (3, 5)

and F = $\left(\frac{2+2}{2}, \frac{2+6}{2}\right)$ = (2, 4)

Hence, area of ΔDEF

= $\frac{1}{2}$|3(5 – 4) + 3(4 – 3) + 2(3 – 5)|

= $\frac{1}{2}$|3 + 3 – 4|

= 1 sq. unit. **Ans.**

94. If $a \neq b \neq c$, prove that the points (a, a^2), (b, b^2) (c, c^2) will not be collinear.

[Board Question]

Sol. Given points are A(a, a^2), B(b, b^2) and C(c, c^2).

Now, area of ΔABC

= $\frac{1}{2}|a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)|$

= $\frac{1}{2}|a(b - c)(b + c) - a^2(b - c) - bc(b - c)|$

= $\frac{1}{2}$| ($b - c$) ($a(b + c) - a^2 - bc$)|

= $\frac{1}{2}$|($b - c$) ($ab + ac - a^2 - bc$)|

= $\frac{1}{2}$| ($b - c$) ($a - b$) ($c - a$) |

This can never be zero, as $a \neq b \neq c$.

Hence, these points can never be collinear.

Hence Proved.

95. If A(5, – 1), B(– 3, – 2) and C(– 1, 8) are the vertices of a triangle ABC, find the length of the median through A and the coordinates of the centroid.

Sol. Given, A = (5, – 1), B = (– 3, – 2) and C = (– 1, 8).

Let D be the point through which the median from *A* meets BC.

Thus, D is the mid-point of BC.

Hence D = $\left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$

= (– 2, 3)

So, AD = $\sqrt{(-2-5)^2+(3+1)^2}$

= $\sqrt{(-7)^2+(4)^2}$

= $\sqrt{65}$ units

Let O be the centroid of the triangle such that

O = (x, y)

Thus, (x, y) = $\left(\frac{5-3-1}{3}, \frac{-1-2+8}{3}\right)$

= $\left(\frac{1}{3}, \frac{5}{3}\right)$

Hence, the centroid of the triangle is $\left(\frac{1}{3}, \frac{5}{3}\right)$.

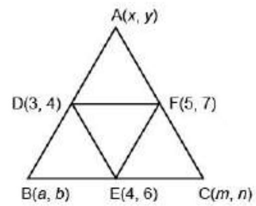
Ans.

96. If the coordinates of the mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7). Find its vertices.

[Board Question]

Sol. Given, D = (3, 4), E = (4, 6) and F = (5, 7)

Let A = (x, y), B = (a, b) and C = (m, n)



Now, $\frac{x+a}{2}$ = 3

⇒ $x + a = 6$ …(i)

$\frac{y+b}{2}$ = 4

⇒ $y + b = 8$ …(ii)

$\frac{a+m}{2}$ = 4

⇒ $a + m = 8$ …(iii)

$\frac{b+n}{2}$ = 6

⇒ $b + n = 12$ …(iv)

$\frac{x+m}{2}$ = 5

⇒ $x + m = 10$ …(v)

$\frac{y+n}{2}$ = 7

⇒ $y + n = 14$ …(vi)

Adding equations (i) and (v), we get

$x + a + x + m = 6 + 10$

⇒ $2x + (a + m) = 16$

⇒ $2x + 8 = 16$ [Using (iii)]

⇒ $2x = 8$

⇒ $x = 4$

Thus, substituting $x = 4$ in (i), we get

$4 + a = 6$

$\Rightarrow a = 2$

and substituting $x = 4$ in (v), we get

$4 + m = 10$

$\Rightarrow m = 6$

Similarly, adding (ii) and (vi), we get

$y + b + y + n = 8 + 14$

$\Rightarrow 2y + (b + n) = 22$

$\Rightarrow 2y + 12 = 22$ [Using (iv)]

$\Rightarrow 2y = 10 \Rightarrow y = 5$

Thus, substituting $y = 5$ in (ii), we get

$5 + b = 8 \Rightarrow b = 3$

and substituting $y = 5$ in (vi), we get

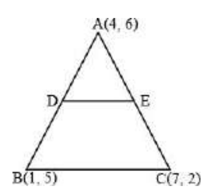
$5 + n = 14$

$\Rightarrow n = 9$

Hence, A = (x, y) = (4, 5); B = (a, b) = (2, 3) and C = (m, n) = (6, 9). **Ans.**

97. In fig, the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). ***A*** **line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.**

[Board Question]



Sol. We have, the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2) and $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

Then $\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{2}$

Then, coordinates of D

$= \left(\frac{1(1)+2(4)}{1+2}, \frac{1(5)+2(6)}{1+2}\right)$

$= \left(\frac{1+8}{3}, \frac{5+12}{3}\right) = \left(3, \frac{17}{3}\right)$

and coordinates of E

$= \left(\frac{1(7)+2(4)}{1+2}, \frac{1(2)+2(6)}{1+2}\right)$

$= \left(\frac{7+8}{3}, \frac{2+12}{3}\right) = \left(5, \frac{14}{3}\right)$

Now, area of ΔADE

$= \frac{1}{2}\left|4\left(\frac{17}{3}-\frac{14}{3}\right)+3\left(\frac{14}{3}-6\right)+5\left(6-\frac{17}{3}\right)\right|$

$= \frac{1}{2}\left|4(1)+3\left(-\frac{4}{3}\right)+5\left(\frac{1}{3}\right)\right|$

$= \frac{5}{6}$ sq. units

and area of ΔABC

$= \frac{1}{2}|4(5 – 2) + 1(2 – 6) + 7 (6 – 5)|$

$= \frac{1}{2}|4(3) + 1(– 4) + 7(1)|$

$= \frac{15}{2}$ sq. units

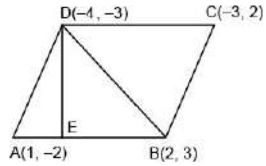
$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{5/6}{15/2} = \frac{1}{9}$

i.e., ar (ΔADE) : ar (ΔABC) = 1 : 9. **Ans.**

98. If the points A(1, – 2), B(2, 3), C(– 3, 2) and D(– 4, – 3) are the vertices of the parallelogram ABCD, then taking AB as the base, find the height of the parallelogram.

[Board Question]

Sol. Given, A = (1, – 2), B = (2, 3), C = (– 3, 2) and D = (– 4, – 3).



Let DE be the height of the parallelogram.

Now, area of $\Delta ABD = \frac{1}{2}[AB \times DE]$

$\Rightarrow DE = \frac{2 \times \text{Area of } \Delta ABD}{AB}$ …(i)

Also, area of ΔABD

$= \frac{1}{2}|1(3 + 3) + 2(-3 + 2) - 4(-2 - 3)|$

$= \frac{1}{2}|6 - 2 + 20|$

$= \frac{1}{2}|24|$

= 12 sq. units

and AB = $\sqrt{(2-1)^2 + (3+2)^2}$

$= \sqrt{(1)^2 + (5)^2}$

$= \sqrt{1 + 25}$

$= \sqrt{26}$ units

Thus, from (i),

DE = $\frac{2(12)}{\sqrt{26}}$ units

$= \frac{24}{\sqrt{26}}$ units. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

99. Assertion: The point (– 1, 6) divides the line segment joining the point (– 3, 10) and (6, – 8) in the ratio 2 : 7 internally.
Reason: Three points A, B and C are collinear if area of DABC = 0.
Ans. (b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

Using section formula, we have

$\Rightarrow -1 = \frac{k \times 6 + 1 \times (-3)}{k+1}$

$\Rightarrow -k - 1 = 6k - 3$

$\Rightarrow 7k = 2$

$\Rightarrow k = 2/7$

Ratio is 2 : 7 internally.

Also, if ar (DABC) = 0

A, B and C all there points are collinear.

Hence, Assertion and reason are true but reason is not the correct explanation of the assertion.
100. Assertion: By using graph we can locate (0,4) on the *y*-axis.
Reason: A point whose *y*-coordinate is zero lies on the *x*-axis.
Ans. (b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

In *xy*-plane, if *x*-coordinate is zero then the point lies on *y*-axis. So, by using graph we can locate (0, 4) on the *y*-axis and thus assertion is correct.

For the reason part, if a point whose *x*-coordinate is zero and *y*-coordinate is non-zero will lie on the *y*-axis. Similarly, a point whose *y*-coordinate is zero and *x*-coordinate is non-zero, it will definitely lie on x-axis. So, assertion is also true but reason is not explaining the assertion.

101. Assertion: C is the mid-point of PQ, if P is (4, x), C is (y, – 1) and Q is (– 2, 4), then x and y respectively are – 6 and 1.

Reason: The mid-point of the line segment joining the points P(x_1, y_1) and Q(x_2, y_2) is (x_1 + x_2)/2, (y_1 + y_2)/2.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

We know that the mid-point of the line segment joining the points P(x_1, y_1) and Q(x_2, y_2) is (x_1 + x_2)/2, (y_1 + y_2)/2.

So, Reason is correct.

Since, C(y, – 1) is the mid-point of P(4, x) and Q(– 2, 4).

We have $y = \frac{4-2}{2} = 1$

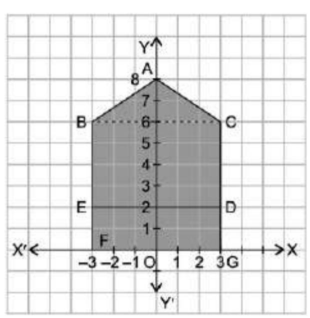
and $-1 = \frac{x+4}{2}$

$\Rightarrow x = -6$

So, Assertion is also true and Reason is the correct explanation of assertion.

Case Based Questions

102. Aditya asked carpenter to make front door of his guest house. The carpenter suggested him a design which is plotted on a graph as shown in figure given below:



Based on the information answer the following questions:

(i) What is the length of line segment AB?

(a) $\sqrt{10}$ units

(b) $\sqrt{11}$ units

(c) $\sqrt{13}$ units

(d) $\sqrt{14}$ units

Ans. (c) $\sqrt{13}$ units

Explanation :

AB = $\sqrt{(0+3)^2+(8-6)^2}$

= $\sqrt{13}$ units.

(ii) Is AB = AC?

(a) yes

(b) no

(c) can’t be determined

(d) None of these

Ans. (a) yes

Explanation :

AC = $\sqrt{(0-3)^2+(8-6)^2}$

= $\sqrt{13}$ units.

= AB.

(iii) The coordinates of the mid-point of BE are:

(a) (4, – 3)

(b) (– 3, 4)

(c) (– 3, – 4)

(d) (– 4, 4)

Ans. (b) (– 3, 4)

Explanation :

Co-ordinates of mid-point of BE are $\left(\frac{-3-3}{2}, \frac{6+2}{2}\right)$ = (– 3, 4).

(iv) Mid-point of ED will lie on:

(a) *x*-axis

(b) *y*-axis

(c) $x = y$

(d) $x = 2y$

Ans. (b) *y*-axis

Explanation :

Co-ordinate of E and D are (– 3, 2) and (3, 2)

$$\text{Mid-point of ED} = \left(\frac{-3+3}{2}, \frac{2+2}{2}\right) = (0, 2)$$

Point (0, 2) lies on *y*-axis.

(v) If we join BD, then the *y*-axis divides BD in the ratio:

(a) 1 : 1

(b) 2 : 1

(c) 1 : 2

(d) 2 : 3

Ans. (a) 1 : 1

Explanation :

Coordinate of B and D are (– 3, 6) and (3, 2)

$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

As on *y*-axis, *x*-coordinate is 0.

$\Rightarrow 3m_1 - 3m_2 = 0$

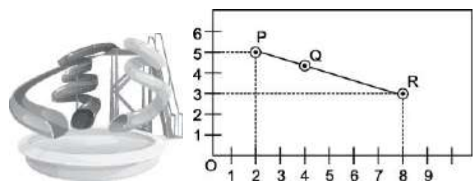
$\Rightarrow 3m_1 = 3m_2$

$\Rightarrow m_1 = m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{1}{1}$

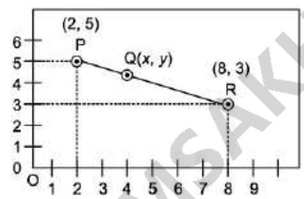
$m_1 : m_2 = 1 : 1.$

103. A group of Class X students goes to picnic during vacation, There were three different slides and three friends Ajay, Ram and Shyam are sliding in the three slides. The position of the three friends shown by P, Q and R in three different slides are given below:



Consider O as orgin, answer the below questions :

(i) The co-ordinates of the point '*Q*' which divides the line segment PR in the ratio 1 : 2 internally:



(a) $\left(4, \frac{13}{3}\right)$

(b) $\left(\frac{13}{3}, \frac{11}{3}\right)$

(c) $\left(\frac{10}{3}, \frac{13}{3}\right)$

(d) $\left(\frac{13}{3}, 4\right)$

Ans. (a) $\left(4, \frac{13}{3}\right)$

Explanation :

The co-ordinate of P and R are (2, 5) and (8, 3) is

$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1(8) + 2(2)}{3}$

$= \frac{12}{3} = 4$

$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1(3) + 2(5)}{3} = \frac{13}{3}$

(ii) Find the distance PR:

(a) $2\sqrt{10}$ units

(b) $\sqrt{36}$ units

(c) $\sqrt{38}$ units

(d) $\sqrt{20}$ units

Ans. (a) $2\sqrt{10}$ units

Explanation :

PR = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(8-2)^2 + (3-5)^2}$

= $\sqrt{6^2 + 2^2} = \sqrt{40}$ = $2\sqrt{10}$ units

(iii) The co-ordinates of point on *x*-axis which is at equal distance PQ is:

(a) $\left(\frac{11}{9}, 0\right)$

(b) (3, 0)

(c) $\left(\frac{13}{9}, 0\right)$

(d) (1, 3)

Ans. (c) $\left(\frac{13}{9}, 0\right)$

Explanation :

Let A(*x*, 0) be the point on *x*-axis which is equidistant from P and Q, then we have PA = QA

The coordinate point of Q = $\left(4, \frac{13}{3}\right)$

$\Rightarrow PA^2 = QA^2$

$(2 - x)^2 + (5 - 0)^2 = (4 - x)^2 + \left(\frac{13}{3} - 0\right)^2$

$\Rightarrow 4 + x^2 - 4x + 25 = 16 + x^2 - 8x + \frac{169}{9}$

$\Rightarrow 4x = 16 + \frac{169}{9} - 4 - 25$

$\Rightarrow x = \frac{13}{9}$

Hence the point $\left(\frac{13}{9}, 0\right)$.

(iv) The coordinates of the mid-point of PR is:

(a) (– 4, 5)

(b) (4, 5)

(c) (5, 4)

(d) (4, 4)

Ans. (c) (5, 4)

Explanation :

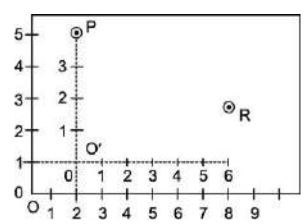
Mid-point of PR = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

= $\left(\frac{2+8}{2}, \frac{5+3}{2}\right)$

= $\left(\frac{10}{2}, \frac{8}{2}\right)$

= (5, 4).

(v) If we shift origin 'O' by 2 units towards right and 1 unit towards North. Then the co-ordinates of point is:



(a) (2, 6)

(b) (6, 2)

(c) (– 6, 2)

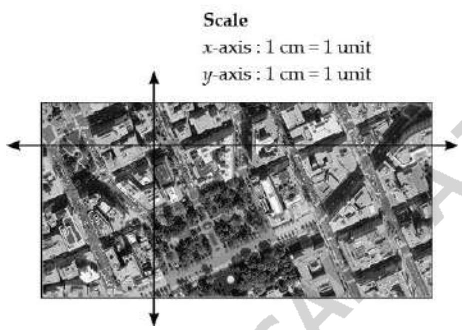
(d) (– 2, 6)

Ans. (b) (6, 2)

Explanation :

The co-ordinates of point R is (6, 2).

104. Satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of *x* and *y*-axis. If we go 2 cm east and 3 cm north from the house, then we reach to a grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to a electrician's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach to a bus stand.



Based on the above information, answer the following questions:

(i) The distance between grocery store and food cart is:

(a) 12 cm

(b) 15 cm

(c) 18 cm

(d) none of these

Ans. (d) none of these

Explanation :

Since, grocery store is at (2, 3) and food cart is at (6, –8)

Required distance

= $\sqrt{(6-2)^2+(-8-3)^2}$

= $\sqrt{4^2+11^2}$ = $\sqrt{16+121}$ = $\sqrt{137}$ cm

(ii) The distance of the bus stand from the house is:

(a) 5 cm

(b) 10 cm

(c) 12 cm

(d) 15 cm

Ans. (b) 10 cm

Explanation :

Since, house is at (0, 0) and bus stand is at (– 6, 8).

Required distance

= $\sqrt{(-6)^2 + 8^2}$ = $\sqrt{36 + 64}$ = $\sqrt{100}$

= 10 cm

(iii) If the grocery store and electrician's shop lie on a line, the ratio of distance of house from grocery store to that from electrician's shop, is:

(a) 3 : 2

(b) 2 : 3

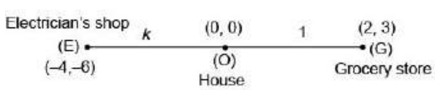
(c) 1 : 2

(d) 2 : 1

Ans. (c) 1 : 2

Explanation :

Let O divides EG in the ratio *k* : 1



then, 0 = $\frac{2k-4}{k+1}$

⇒ $2k = 4$

⇒ $k = 2$

Thus, O divides EG in the ratio 2 : 1

Hence, required ratio = OG : OE *i.e.*, 1 : 2

(iv) The ratio of distances of house from bus stand to food cart is:

(a) 1 : 2

(b) 2 : 1

(c) 1 : 1

(d) none of these

Ans. (c) 1 : 1

Explanation :

Since, (0,0) is the mid-point of (–6, 8) and (6, – 8), therefore both bus stand and food cart are at equal distances from the house. Hence, required ratio is 1 : 1.

(v) The coordinates of positions of bus stand, grocery store, food cart and electrician's shop form a:

(a) rectangle

(b) parallelogram

(c) square

(d) none of these

Ans. (d) none of these

Explanation :

Mid-point of grocery store and electrician's shop is $\left(\frac{2-4}{2}, \frac{3-6}{2}\right)$, *i.e.*, $\left(-1, \frac{-3}{2}\right)$

Mid-point of bus stand and food cart is $\left(\frac{-6+6}{2}, \frac{8-8}{2}\right)$ *i.e.*, 0, 0.

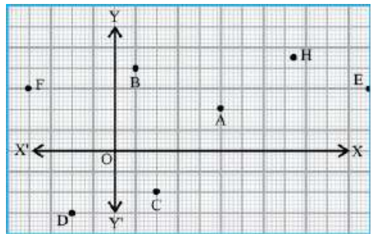
Thus, the diagonals does not bisect each other.

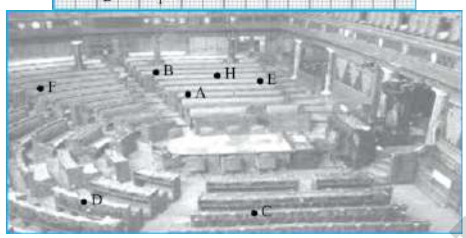
[$\because$ Mid-point are not same]

Hence, they form a quadrillateral.

105. Students of class X are on visit of Sansad Bhawan. Teacher assign them the activity to observe and take some pictures to analyses the seating arrangement between various MP and

speaker based on coordinate geometry. The staff tour guide explained various facts related to Math's of Sansad Bhawan to the students, students were surprised when teacher ask them you need to apply coordinate geometry on the seating arrangement of MP's and speaker.





Calculate the following refer to the above image and graph.
Answer the following questions:

(i) Refer to the points D and C, find the distance between the points C and D, if the co-ordinates of C is (2, – 2) and D is (– 2, – 3):

(a) $\sqrt{17}$ units

(b) 6.4 units

(c) 5 units

(d) 7 units

Ans. (a) $\sqrt{17}$ units

Explanation :

Distance between C(2, – 2) and D(– 2, – 3) is

D = $\sqrt{(-2-2)^2+(-3+2)^2}$

D = $\sqrt{16+1}$

D = $\sqrt{17}$ units **Ans.**

(ii) Refer to the points D, A and H, condition for collinearity of points DAH satisfy by relation:

(a) DA = AH = DH

(b) DA + AH = DH

(c) DA + AH **>** DH

(d) DA + AH **<** DH

Ans. (b) DA + AH = DH

Explanation :

From the given figure, it is clear that

DA + AH = DH **Ans.**

(iii) Refer to the points B and C, join BC. Mark a point P on BC and P divides the line segment B(1, 4) and C(2, – 2) such as $\frac{BP}{PC} = \frac{k}{1}$, if P lies on the line $x - y = -1$. Find the value of k.

(a) $\frac{3}{2}$

(b) $\frac{4}{5}$

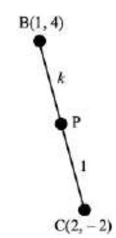
(c) $\frac{2}{5}$

(d) $\frac{5}{2}$

Ans. (c) $\frac{2}{5}$

Explanation :

Given that P divides the line segment joining B(1, 4) and C(2, – 2) in the ratio k : 1 so the coordinates of P are $\left(\frac{2k+1}{k+1}, \frac{-2k+4}{k+1}\right)$



$P\left(\frac{2k+1}{k+1}, \frac{-2k+4}{k+1}\right)$ lies on the line segment

$x - y = -1$

$\left(\frac{2k+1}{k+1}\right) - \left(\frac{-2k+4}{k+1}\right) = -1$

$\Rightarrow 2k + 1 + 2k - 4 = -k - 1$

$\Rightarrow 5k = 2$

$\Rightarrow k = \frac{2}{5}$ **Ans.**

(iv) Refer to the points F and E, Find the mid-point of the line segment joining by the points F(– 4, 3) and E(12, 3).

(a) (4, 4)

(b) (3, – 4)

(c) (– 4, 8)

(d) (4, 3)

Ans. (d) (4, 3)

Explanation :

Given point F(– 4, 3) and E(12, 3).

Coordinates of mid-point are

$\left(\frac{-4+12}{2}, \frac{3+3}{2}\right) \Rightarrow (4, 3)$ **Ans.**

(v) Refer to the point B and C, If B is, consider as origin then coordinates of C are :

(a) (1, 4)

(b) (1, – 6)

(c) (– 4, – 6)

(d) (0, 0)

Ans. (b) (1, – 6)

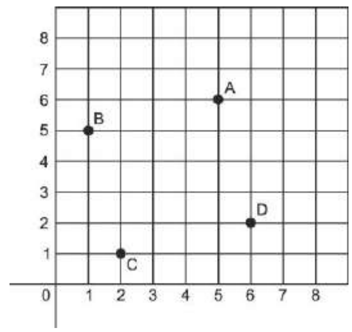
Explanation :

Coordinates of C are (1, – 6).**Ans.**

Passage Based Questions

106. In a colony's park there are three electric poles namely, A, B and C at the positions shown in the figure. Despite these three poles, some part of the park was still in dark. So, society decided to fix one more electric pole D in the park.

Based on the following information, answer the following questions:



(i) Write the coordinates of D.

(ii) Find the distance of D from pole A and pole C.

(iii) Is ABCD a square?

Sol. (i) Coordinates of D are (6, 2)

(ii) A(5, 6) and D(6, 2)

Distance AD = $\sqrt{(6-5)^2+(2-6)^2}$

= $\sqrt{1+16}=\sqrt{17}$ units

C(2, 1) and D(6, 2)

Distance CD = $\sqrt{(6-2)^2+(2-1)^2}$

= $\sqrt{16+1}=\sqrt{17}$ units.

(iii) A(5, 6), B(1, 5), C(2, 1) and D(6, 2)

AB = $\sqrt{(1-5)^2+(5-6)^2}$ = $\sqrt{16+1}$

= $\sqrt{17}$ units

BC = $\sqrt{(2-1)^2+(1-5)^2}$ = $\sqrt{1+16}$

= $\sqrt{17}$ units

CD = $\sqrt{(6-2)^2+(2-1)^2}$ = $\sqrt{16+1}$

= $\sqrt{17}$ units.

DA = $\sqrt{(5-6)^2+(6-2)^2}$ = $\sqrt{1+16}$

= $\sqrt{17}$ units.

AC = $\sqrt{(2-5)^2+(1-6)^2}$ = $\sqrt{9+25}$

= $\sqrt{34}$ units.

BD = $\sqrt{(6-1)^2+(2-5)^2} = \sqrt{25+9}$

= $\sqrt{34}$ units.

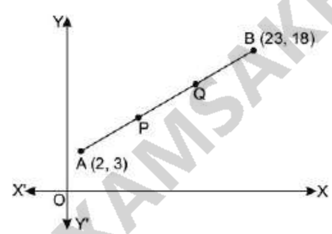
... AB = BC = CD = AD

Also, Diagonal AC = Diagonal BD

ABCD is a square. **Ans.**

107. A telephone company wants to fix two relay towers P and Q between two factories A and B in such away that AP = PQ = QB, points P and Q lies on the straight line joining A and B.

Based on the given information, answer the following questions:



(i) Find the ratio in which P and Q divides AB.

(ii) Find the coordinates of P.

(iii) Find the coordinates of Q.

Sol. (i) Let the distance of point P from A be x.

Then, distance of point Q from P is also x and the distance of point B from Q is also x

AQ = AP + PQ = 2x and PB = PQ + QB = 2x

⇒ AP : PB = x : 2x = 1 : 2

and AQ : QB = 2x : x = 2 : 1

The ratio in which P divides AB is 1 : 2 and the ratio in which Q divides AB is 2 : 1.

Ans.

(ii) Coordinates of P

$= \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$= \left(\frac{1\times23+2\times2}{1+2}, \frac{1\times18+3\times2}{1+2}\right)$

$= \left(\frac{23+4}{3}, \frac{18+6}{3}\right) = \left(\frac{27}{3}, \frac{24}{3}\right)$

= (9, 8)

(iii) Coordinates of Q

$= \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

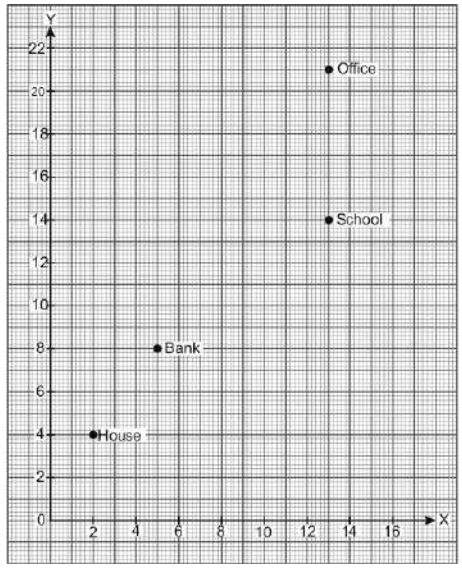
$= \left(\frac{2\times23+2\times1}{2+1}, \frac{18\times2+3\times1}{2+1}\right)$

$= \left(\frac{46+2}{3}, \frac{36+3}{3}\right) = \left(\frac{48}{3}, \frac{39}{3}\right)$

= (16, 13) **Ans.**

108. Sheenam starts walking from her house to office. Instead of going to the office directly, she goes to a bank first, from there to her daughter's school and then reaches the office. Assuming that all distances covered are in straight lines, answer the following questions.

Based on the given information, answer the following questions:



(i) What is the distance between her house and office?

(ii) Calculate the extra distance that is covered by Sheenam in reaching her office.

Sol. (i) House = (2, 4), Bank = (5, 8), School = (13, 14) and Office = (13, 21)

Distance between house and office

= $\sqrt{(13-2)^2+(21-4)^2}$ = $\sqrt{(11)^2+(17)^2}$

= $\sqrt{121+289}$ = $\sqrt{410}$ units

(ii) Distance covered from house to bank

= $\sqrt{(5-2)^2+(8-4)^2}$

= $\sqrt{9+16}$ = $\sqrt{25}$ = 5 units

Distance covered from bank to school

= $\sqrt{(13-5)^2+(14-8)^2}$

= $\sqrt{64+36}$ = $\sqrt{100}$ = 10 units

Distance covered from school to office

= $\sqrt{(13-13)^2+(21-14)^2}$

= $\sqrt{0+49}$ = $\sqrt{49}$ = 7 units

Extra distance covered

= (5 + 10 + 7) – $\sqrt{410}$

= $(22-\sqrt{410})$ units.

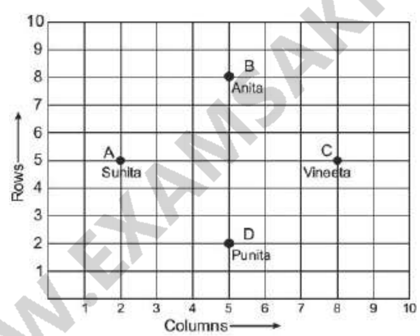
109. In an examination room, four students Sunita, Anita, Vineeta and Punita are sitting at point A(2, 5), B(5, 8), C(8, 5) and D(5, 2) respectively. Then a new student Anju enters the room.

Based on the given information, answer the following questions:

(i) The examiner tells Anju to sit in the middle of all the four girls sitting. Find the coordinates of the point, where she can sit.

(ii) Calculate the distance between Sunita and Punita.

(iii) Calculate the distance between Anita and Vineeta.



Sol. (i) A = (2, 5), B = (5, 8), C = (8, 5), D = (5, 2)

Coordinates of the position of Anju

= Coordinates of mid-point of AC

= Coordinates of mid-point of BD

Mid-point of AC = $\left(\frac{2+8}{2}, \frac{5+5}{2}\right)$

= $\left(\frac{10}{2}, \frac{10}{2}\right)$ = (5, 5)

Mid-point of BD = $\left(\frac{5+5}{2}, \frac{8+2}{2}\right)$

$= \left(\frac{10}{2}, \frac{10}{2}\right)$ = (5, 5)

So, the coordinates of the point, where Anju can sit are (5, 5). **Ans.**

(ii) Distance between Sunita and Punita = AD

$= \sqrt{(5-2)^2 + (2-5)^2} = \sqrt{(3)^2 + (-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units. **Ans.**

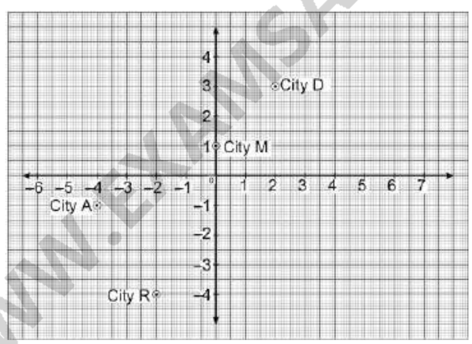
(iii) Distance between Anita and Vineeta = BC

$= \sqrt{(8-5)^2 + (5-8)^2} = \sqrt{(3)^2 + (-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units. **Ans.**

110. Three friends Aman, Raj and Deepak live in different cities. They decided to meet at a place in city M. Aman, Raj and Deepak lives in city A, city R and city D respectively. Coordinates of city M, city A, city R and city D are represented in the figure given below.

Based on the given situation, answer the following questions.



(i) Who will travel less distance, Aman, Raj or Deepak, to reach city M?

(ii) Find the distance between city A and city D.

(iii) Find the coordinates of city P, which is situated at point P, the mid-point of the line joining the points represented by city R and city D.

Sol. (i) A(– 4, – 1), R(– 2, – 4), D(2, 3) and M(0, 1)

Distance travelled by Aman

$= \sqrt{(0+4)^2+(1+1)^2}$

$= \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20}$ units

Distance travelled by Raj

$= \sqrt{(0+2)^2+(1+4)^2}$

$= \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$ units

Distance travelled by Deepak

$= \sqrt{(0-2)^2+(1-3)^2} = \sqrt{4+4}$

$= \sqrt{8}$ units

Deepak travels least distance among all. **Ans.**

(ii) Distance between city A and city D

$= \sqrt{(2+4)^2+(3+1)^2} = \sqrt{(6)^2+(4)^2}$

$= \sqrt{36+16} = \sqrt{52}$ units **Ans.**

(iii) Coordinates of P are $\left(\frac{-2+2}{2}, \frac{-4+3}{2}\right)$

$= \left(0, \frac{-1}{2}\right)$ **Ans.**

Self-Assessment

111. Find the point on the X-axis which is equidistant from the points $A(2, -5)$ and $B(-2, 9)$.

[Board Question & NCERT]

Ans. (– 7, 0).

112. Find the point on the Y-axis which is equidistant from the points (6, 5) and (– 4, 3).

[NCERT]

Ans. (0, 9).

113. Do the points $A(3, 2)$, $B(-2, -3)$ and $C(2, 3)$ form a triangle ? If so, name the type of triangle formed.

Ans. The triangle formed is a right-angled triangle.

114. Show that the points *A*(1, 7), *B*(4, 2), *C*(– 1, – 1) and *D*(– 4, 4) are the vertices of a square.

[NCERT]

115. If the point *Q*(0, 1) is equidistant from the points *P*(5, – 3) and *R*(*x*, 6), find the values of *x*. Also find the distances *QR* and *PR*.

[NCERT]

Ans. $x = \pm 4$, $QR = \sqrt{41}$, $PR = \sqrt{82}$ or $9\sqrt{2}$ units.

116. Find the values of *y* for which the distance between the points *P*(2, – 3) and *Q*(10, *y*) is 10 units.

[NCERT]

Ans. 3 or – 9.

117. Name the quadrilateral formed by the following points *A*(4, 5), *B*(7, 6), *C*(4, 3) and *D*(1, 2).

[NCERT]

Ans. Parallelogram.

118. Find the equation of the perpendicular bisector of the line segment joining the points (7, 1) and (3, 5).

[NCERT]

Ans. $x - y = 2$.

119. Find the relation between *x* and *y* such that the point (*x*, *y*) is equidistant from the points (3, 6) and (– 3, 4).

[NCERT]

Ans. $3x + y = 5$.

120. Find the coordinates of the points of trisection of the line segment joining the points *A*(2, – 2) and *B*(– 7, 4).

[NCERT]

Ans. (– 1, 0) and (– 4, 2)

121. If *A* and *B* are two points having coordinates (– 2, – 2) and (2, – 4) respectively, find the coordinates of *P* such that $AP = \frac{3}{7}AB$.

[Board Question & NCERT]

Ans. $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

122. Find the coordinates of the points which divide the line segment joining *A*(– 2, 2) and *B*(2, 8) into four equal parts.

[NCERT]

Ans. $\left(-1, \frac{7}{2}\right)$, (0, 5), $\left(1, \frac{13}{2}\right)$.

123. *ABCD* is a rectangle where *A*(– 1, – 1), *B*(– 1, 4), *C*(5, 4) and *D*(5, – 1) are the vertices. *P*, *Q*, *R*, *S* are the mid-points of sides *AB*, *BC*, *CD* and *AD* respectively. Is the quadrilateral *PQRS* a square or a rectangle or a rhombus ? Justify your answer.

[NCERT]

Ans. Rhombus because all the sides of quadrilateral are equal and diagonals are not equal to each other.

124. Find the ratio in which the point *P*(– 4, 6) divides the line segment joining the points *A*(– 6, 10) and *B*(3, – 8).

[NCERT]

Ans. 2 : 7.

125. Find the coordinates of a point *A* where *AB* is a diameter of a circle whose centre is (2, – 3) and *B* is (1, 4).

[NCERT]

Ans. (3, – 10).

126. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, – 1), (2, 1) and (0, 3). Find the ratio of the area of a triangle formed to the area of the given triangle.

[NCERT]

Ans. 1 sq. units, 1 : 4.

127. The vertices of Δ*ABC* are *A* = (4, 6), *B* = (1, 5) and *C* = (7, 2). A line is drawn to intersect sides *AB* and *AC* at *D* and *E* respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of Δ*ADE* and compare it with the area of Δ*ABC*.

[NCERT]

Ans. $\frac{15}{32}$ sq. units, 1 : 16.

128. Find the area of the quadrilateral *ABCD* whose vertices are *A*(– 4, – 2), *B*(– 3, – 5), *C*(3, – 2) and *D*(2, 3).

[NCERT]

Ans. 28 sq. units.

129. If the distance between (x, 3) and (1, 7) is 5. Calculate the value of x.

Ans. 4, – 2.

130. For what value of p will the points (22, 2), $\left(2, -\frac{1}{2}\right)$ and $\left(-\frac{p}{2}, 2\right)$ will be collinear ?

Ans. – 44.

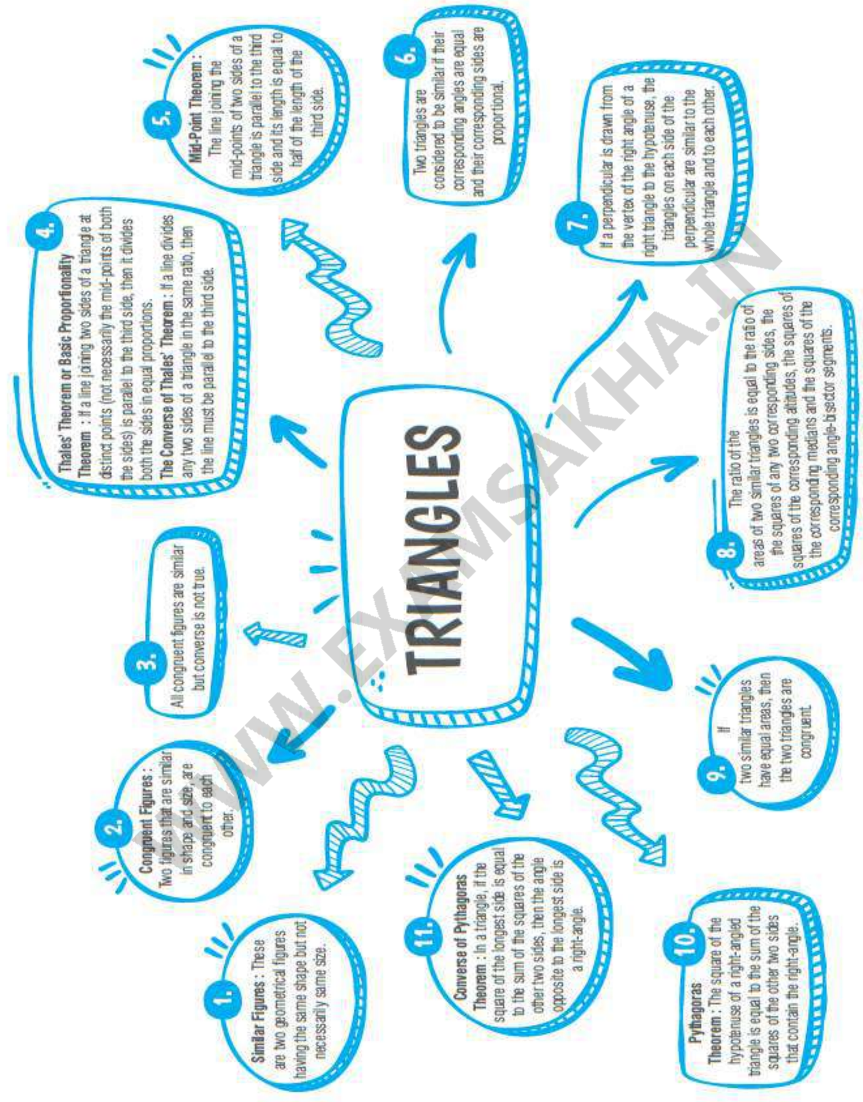
131. Determine the mid-point of the line segment joining the points (4, – 2) and (7, 5).

Ans. $\left(\frac{11}{2}, \frac{3}{2}\right)$.

Triangles

Chapter
7

Basic Concepts



Multiple Choice Questions

1. **In DPQR, if PS is the internal bisector of ∠P meeting QR at S and PQ = 15 cm, QS = (3 + *x*) cm, SR = (*x* – 3) cm and PR = 7 cm, then find the value of *x*.**

(a) 2.85 cm

(b) 8.25 cm

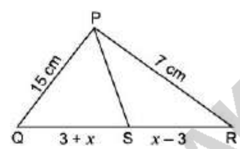
(c) 5.28 cm

(d) 8.52 cm

Ans. (b) 8.25 cm

Explanation :

Since PS is the internal bisector of ∠P and it meets QR at S.



$\therefore \frac{PQ}{QS} = \frac{PR}{SR}$

$\Rightarrow \frac{15}{3+x} = \frac{7}{x-3}$

$\Rightarrow 7(3 + x) = 15(x – 3)$

$\Rightarrow 21 + 7x = 15x – 45$

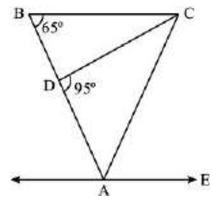
$\Rightarrow 15x – 7x = 45 + 21$

$\Rightarrow 8x = 66$

$\Rightarrow 4x = 33$

$\Rightarrow x = 8.25$ cm.

2. **In the figure given below, ABC is a triangle. BC is parallel to AE. If BC = AC, then what is the value of ∠CAE?**



(a) 20°

(b) 30°

(c) 40°

(d) 50°

Ans. (d) 50°

Explanation :

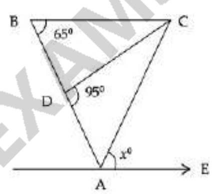
Given that, BC || AE

∠CBA + ∠EAB = 180°

⇒ ∠EAB = 180° – 65° = 115°

BC = AC

Hence, DABC is an isosceles triangle.



⇒ ∠CBA = ∠CAB

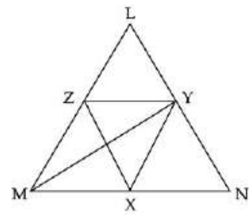
= 65°

Now, ∠EAB = ∠EAC + ∠CAB

⇒ 115° = x + 65°

⇒ x = 50°.

3. **In the figure given below, YZ is parallel to MN, XY is parallel to LM and XZ is parallel to LN. Then MY is:**



(a) The median of D LMN

(b) The angular bisector of ∠LMN

(c) Perpendicular to LN

(d) Perpendicular bisector of LN

Ans. (a) The median of D LMN

Explanation :

Given that, YZ || MN and XZ || LN

∴ XNYZ is a parallelogram.

ZX = YN... (i)

Also, ZX ||YN and XY || ZL

Hence, XYLZ is a parallelogram

∴ XZ = LY... (ii)

Now, from eqn, (i) and (ii),

∴ YN = LY

So, MY is a median of DLMN.

4. **The lengths of three sides (in cm) of a triangle is given. Which one of the following cases is not suitable to be the three sides of a triangle?**

(a) 2, 3, 4

(b) 2, 3, 5

(c) 2, 4, 5

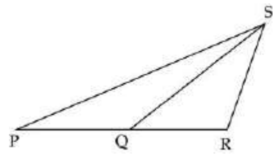
(d) 3, 4, 5

Ans. (b) 2, 3, 5

Explanation :

We know that, in any triangle the sum of two sides is always greater than its third side and the difference of two sides is always less than its third side. Only option (b) does not satisfy the above conditions.

5. **In the figure given below, PQ = QS and QR = RS. If ∠SRQ = 100°, then find the angle of ∠QPS?**



(a) 40°

(b) 30°

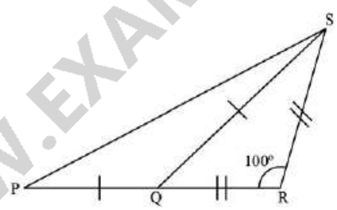
(c) 20°

(d) 15°

Ans. (c) 20°

Explanation :

DQRS is an isosceles triangle.



∠QSR = ∠SQR = 40°

∠PQS = 180° – ∠RQS

= 140°

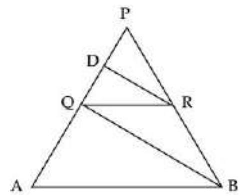
Again, D*PQS* is an isosceles triangle.

∠QPS = ∠QSP

$= \frac{180° - 140°}{2}$

$= \frac{40°}{2}$ = 20°

6. **In the given figure, QR is parallel to AB and DR is parallel to QB. What is the number of distinct pairs of similar triangles?**



(a) 1

(b) 2

(c) 3

(d) 4

Ans. (b) 2

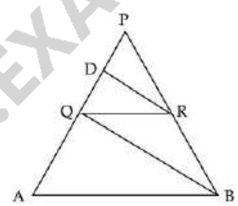
Explanation :

QR is parallel to AB.

According to basic proportionality theorem,

$\frac{PQ}{PA} = \frac{PR}{PB}$ ...(1)

DPQR ~ DPAB



DR is also parallel to QB.[Given]

According to basic proportionality theorem,

$\frac{PD}{PQ} = \frac{PR}{PB}$ ...(2)

DPDR ~ DPQB

Therefore, there are two distinct pair of similar triangles in the given figure.

7. **The vertical angle of an isosceles triangle is 15° more than each of its base angles. What is the vertical angle?**

(a) 35°

(b) 55°

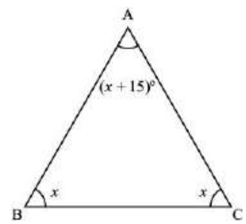
(c) 65°

(d) 70°

Ans. (d) 70°

Explanation :

Let each base angle of isosceles triangle be *x*.



Vertical angle of an isosceles triangle = $x + 15°$

We know that,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow x + 15° + x + x = 180°$

$\Rightarrow 3x = 165°$

$\Rightarrow x = 55°$

$x + 15 = 55 + 15$

$= 70°$.

8. **In DABC, D and E are points on sides AB and AC, such that DE || BC. If AD = *x*, DB = *x* – 2,AE = *x* + 2 and EC = *x* – 1, then the value of *x* is:**

(a) 4

(b) 2

(c) 1

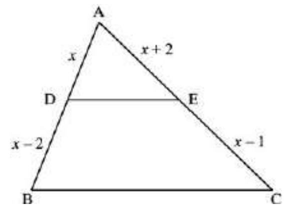
(d) 8

Ans. (a) 4

Explanation :

DE || BC

$\frac{AD}{DB} = \frac{AE}{EC}$



$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

[By basic proportionality theorem]

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow x = 4.$

9. **In △ABC, the angle bisector of ∠A cuts BC at E. Find the length of AC, if lengths of AB, BE and EC are 9 cm, 3.6 cm and 2.4 cm?**

(a) 5.4 cm

(b) 8 cm

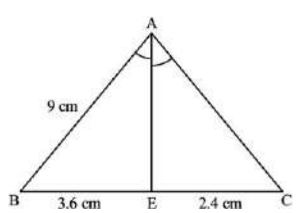
(c) 4.8 cm

(d) 6 cm

Ans. (d) 6 cm

Explanation :

In D ABC, if AE is the angle bisector of ∠A, then according to the angle bisector theorem,



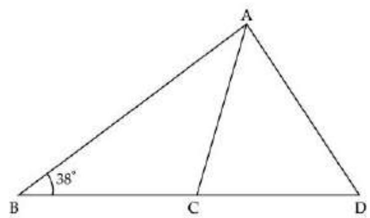
$\frac{AB}{AC} = \frac{BE}{EC}$

$\frac{9}{AC} = \frac{3.6}{2.4}$

AC = $\frac{9 \times 2.4}{3.6}$

= 6 cm.

10. **In the given figure, if ∠B = 38°, AC = BC and AD = CD, then ∠D equals to:**



(a) 26°

(b) 28°

(c) 38°

(d) 52°

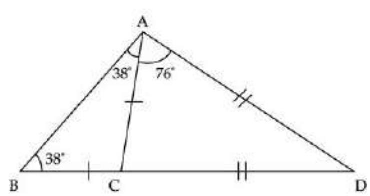
Ans. (b) 28°

Explanation :

Given,

Given,

AC = BC



In DABC, ∠ABC = ∠CAB

∠ABC = ∠*CAB* = 38°

∠ACB = 180° – (∠ABC + ∠CAB)

= 180° – (38° + 38°)

= 180° – 76° = 104°

∵ ∠ACB + ∠ACD = 180° (linear pair)

∠ACD = 180° – 104°

= 76°

And ∠ACD = ∠CAD = 76°(∵ CD = AD)

∠ADC = 180° – (∠ACD +∠CAD)

= 180° – (76° + 76°) = 28°.

11. If the areas of two similar triangles are equal, then these triangles are ....

(a) congruent

(b) equilateral

(c) equivalent

(d) none of these

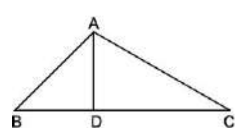
Ans. (a) congruent

Explanation :

If two triangles are equal in area, they are congruent.

12. In Fig. ∠BAC = 90° and AD ^ BC. then,

[NCERT Exemplar]



(a) BD . CD = BC^2

(b) AB . AC = BC^2

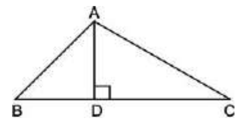
(c) BD . CD = AD^2

(d) AB . AC = AD^2

Ans. (c) BD . CD = AD^2

Explanation :

In DADB and DADC,



∠D = ∠D = 90°

∠DBA = ∠DAC

[each equal to 90° – ∠C]

Also, ∠BAD = ∠DCA

(∵ A = 90° and ∠DAC = 90° – ∠DCA)

∠BAD = ∠A – ∠DAC

= 90° – (90° – ∠DCA)

= ∠DCA

DADB ~ DADC

[by AAA similarity criterion]

$\frac{BD}{AD} = \frac{AD}{CD}$

⇒ BD.CD = AD^2.

13. If DABC ~ DEDF and DABC is not similar to DDEF, then which of the following is not true?

[NCERT Exemplar]

(a) BC . EF = AC . FD

(b) AB . EF = AC . DE

(c) BC . DE = AB . EF

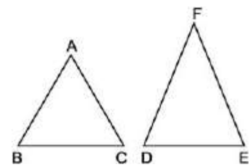
(d) BC . DE = AB . FD

Ans. (b) AB . EF = AC . DE

Explanation :

Given, DABC ~ DEDF

$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$



Taking first two terms, we get

$\frac{AB}{ED} = \frac{BC}{DF}$

⇒ AB · DF = ED · BC

or BC · DE = AB · DF

So, option (c) is true.

Taking last two terms, we get

$\frac{BC}{DF} = \frac{AC}{EF}$

⇒ BC · EF = AC · FD

So, option (a) is also true.

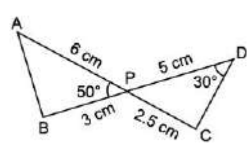
Taking first and last terms, we get

$\frac{AB}{ED} = \frac{AC}{EF}$

⇒ AB · EF = ED · AC

14. In Fig., two line segments, AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°. Then ∠PBA is equal to:

[NCERT Exemplar]



(a) 30°

(b) 60°

(c) 80°

(d) 100°

Ans. (d) 100°

Explanation :

In DAPB and DCPD,

∠APB = ∠CPD = 50°

[vertically opposite angles]

$\frac{AP}{PD} = \frac{6}{5}$…(i)

and $\frac{BP}{CP} = \frac{3}{2.5} = \frac{6}{5}$…(ii)

From equatons (i) and (ii),

$\frac{AP}{PD} = \frac{BP}{CP}$

DAPB ~ DDPC

[by SAS similarity criterion]

∠A = ∠D = 30°

[corresponding angles of similar triangles]

In DAPB,

∠A + ∠B + ∠APB = 180°

[sum of angles of a triangle = 180°]

⇒ 30° + ∠B + 50° = 180°

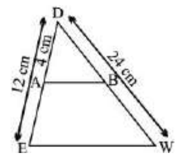
∠B = 180° – (50° + 30°)

= 100°

i.e., ∠PBA = 100°.

Very Short Answer Type Questions

15. In DDEW, AB || EW. If AD = 4 cm, DE = 12 cm and DW = 24 cm, then find the value of DB.



Sol. Given, AD = 4 cm, DE = 12 cm and DW = 24 cm.

Let BD = x cm

Since, BW = DW – BD

Then, BW = (24 – x) cm.

Similarly, AE = 12 – 4 = 8 cm

In DDEW, AB || EW

$\frac{AD}{AE} = \frac{BD}{BW}$ [Thales' theorem]

$\Rightarrow \frac{4}{12-4} = \frac{x}{24-x}$

$\Rightarrow 8x = 96 - 4x$

$\Rightarrow 12x = 96$

$\Rightarrow x = \frac{96}{12} = 8$

DB = 8 cm **Ans.**

16. In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. Find the radius of the circle inscribed in the triangle.

Sol. Given, DABC is right-angled at B.

Applying Pythagoras theorem,

AC2 = BC2 + AB2

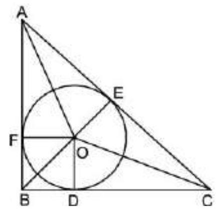
⇒ AC2 = 122 + 52

= 144 + 25

= 169

= 132

⇒ AC = 13 cm



Let O be the centre of the circle inscribed within DABC such that OD, OE and OF are the altitude of DBOC, DCOA and DAOB, respectively and OD = OE = OF = r.

∴ Area of DABC = Area of DBOC + Area of DCOA + Area of DAOB

⇒ $\frac{1}{2}$AB × BC = $\frac{1}{2}$OD × BC + $\frac{1}{2}$OE × AC + $\frac{1}{2}$OF × AB

⇒ $\frac{1}{2}(5 \times 12) = \frac{1}{2}r \times 12 + \frac{1}{2}r \times 13 + \frac{1}{2}r \times 5$

⇒ $60 = 12r + 13r + 5r$

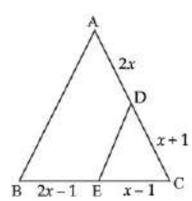
⇒ $60 = 30r$

⇒ $r = 2$ cm. **Ans.**

17. In DABC, D and E are mid-points of AC and BC respectively such that DE || AB. If AD = 2*x*, BE = 2*x* – 1, CD = *x* + 1 and CE = *x* – 1, then find the value of *x*.

Sol. Given, DE || AB

So, $\frac{AD}{CD} = \frac{BE}{EC}$ [by B.P.T.]



⇒ $\frac{2x}{x+1} = \frac{2x-1}{x-1}$

⇒ $2x(x - 1) = (x + 1)(2x - 1)$

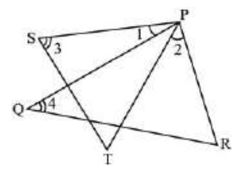
⇒ $2x2 - 2x = 2x2 + 2x - x - 1$

⇒ $-2x = x - 1$

⇒ 1 = 3x

or $x = \frac{1}{3}$ **Ans.**

18. On occasion of independence day, Mira made a Rangoli in a design as shown. If ∠1 = ∠2 , ∠3 = ∠4, show that PT. QR = PR. ST.



Sol. ∠1 = ∠2 [Given]

⇒ ∠1 + ∠QPT = ∠2 + ∠QPT

⇒ ∠SPT = ∠QPR

Also ∠3 = ∠4 [Given]

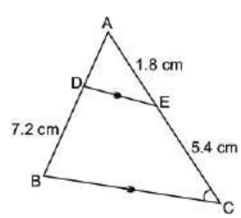
DTSP ~ DRQP [By AA similarity]

⇒ $\frac{ST}{QR} = \frac{PT}{PR}$ [c.p.c.t.]

⇒ ST. PR = PT. QR. **Hence Proved.**

19. In figure, DE || BC. Find the length of side AD, given that AE = 1·8 cm, BD = 7·2 cm and CE = 5·4 cm.

[Board Question]



Sol. Given, DE || BC

On applying, Thales theorem, we have

$\frac{AD}{AB} = \frac{AE}{AC}$

⇒ $\frac{AD}{AD+7.2} = \frac{1.8}{1.8+5.4}$

$\Rightarrow \frac{AD}{AD + 7.2} = \frac{1.8}{7.2}$

$\Rightarrow \frac{AD}{AD + 7.2} = \frac{1}{4}$

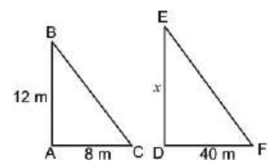
⇒ 4AD = AD + 7·2

⇒ 3AD = 7·2

⇒ AD = 2·4 cm **Ans.**

20. A vertical tree of 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts the shadow 40 m long on the ground. Determine the height of the tower.

Sol. Let AB be the tree and AC be its shadow.



Also, let DE be the vertical tower and DF be its shadow. Join BC and EF.

Let DE = x

We have, AB = 12 m, AC = 8 m

and DF = 40 m, DE = x m

In DABC and DDEF, we have

∠A = ∠D = 90°

∠C = ∠F

DABC ~ DDEF [AA criterion]

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$ [cpct]

$\Rightarrow \frac{12}{x} = \frac{8}{40}$

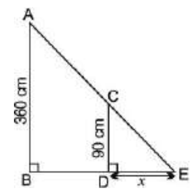
$\Rightarrow x = \frac{12 \times 40}{8}$

= 12 × 5 = 60

Thus, height of the tower is 60 m. **Ans.**

21. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Sol. Let AB denote the lamp-post and CD be the height of the girl after walking for 4 seconds away from the lamp-post.



From Fig. BD = 1.2 × 4

= 4.8 m

= 480 cm.

In DABE and DCDE,

∠B = ∠D [Each 90°]

∠E = ∠E [Common]

DABE ~ DCDE [By AA criterion]

⇒ $\frac{AB}{BE} = \frac{CD}{DE}$

⇒ $\frac{360}{480+x} = \frac{90}{x}$

⇒ 90 (480 + x) = 360x

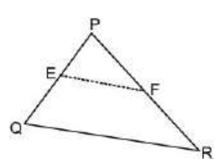
⇒ 43200 + 90x = 360x

⇒ 360x – 90x = 43200

⇒ 270x = 43200

⇒ x = 160 cm **Ans.**

22. Kitchen garden of Ms. Sanjana is in the form of a triangle as shown. She wants to divide it in two parts; one triangle and one trapezium.



She takes PE = 4 m, QE = 4.5 m, PF = 8 m and RF = 9 m. Is EF | | QR ? Justify your answer.

Sol. $\frac{PE}{QE} = \frac{4}{4.5} = \frac{8}{9}$ ...(i)

$\frac{PF}{RF} = \frac{8}{9}$ ...(ii)

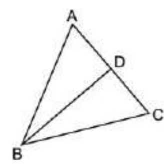
Form equations (i) and (ii),

$\frac{PE}{QE} = \frac{PF}{RF}$

EF | | QR (yes) [By converse of B.P.T.]

Ans.

23. Students of a school decided to participate in 'Save girl child' campaign. They decided to decorate a triangular path as shown. If AB = AC and BC2 = AC × CD, then prove that BD = BC.



Sol. In DABC and DBCD

BC2 = AC × CD [Given] ⇒ $\frac{BC}{CD} = \frac{AC}{BC}$

∠C = ∠C [Common]

DABC ~ DBDC [By SAS criterion]

⇒ $\frac{AB}{BD} = \frac{AC}{BC}$ [by c.p.c.t.]

But , AB = AC [Given]

BD = BC **Hence Proved.**

24. In DABC, AD is the bisector of ∠A. If AB = 5.6 cm, BD = 3.2 cm and BC = 6 cm, find AC.

Sol. Given, AD is the bisector of ∠A.

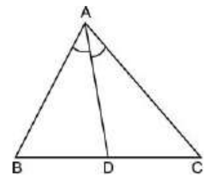
Thus, ∠BAD = ∠CAD

Now BC = BD + DC

⇒ 6 = 3.2 + DC

⇒ DC = 2.8 cm

Since AD is the angle bisector of ∠A,



Hence $\frac{DB}{DC} = \frac{AB}{AC}$ [Angle bisector theorem]

$\Rightarrow \frac{3.2}{2.8} = \frac{5.6}{AC}$

$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2}$

= 4.9 cm. **Ans.**

25. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, find the corresponding side of the second triangle.

Sol. Let ABC and DEF be the two similar triangles and their corresponding sides be AB and DE.

Thus, $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{\text{Length of side AB}}{\text{Length of side DE}}$

$\Rightarrow \frac{25}{15} = \frac{9}{\text{Length of side DE}}$

Thus, length of side DE = $\frac{15 \times 9}{25}$ = 5.4 cm

Thus, the length of the corresponding side is 5.4 cm. **Ans.**

26. X and Y are points on the sides AB and AC respectively of a triangle ABC such that $\frac{AX}{AB} = \frac{1}{4}$, AY = 2 cm and YC = 6 cm. Find whether XY || BC or not.

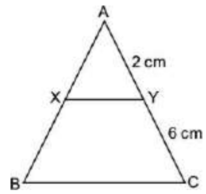
Sol. Given, $\frac{AX}{AB} = \frac{1}{4}$

i.e., AX = 1*k*, AB = 4*k* (K is constant)

BX = AB – AX

= 4*k* – 1*k*

= 3*k*



Now, $\frac{AX}{XB} = \frac{1k}{3k} = \frac{1}{3}$

and, $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

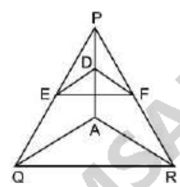
... $\frac{AX}{XB} = \frac{AY}{YC}$

XY || BC

[By converse of Thales' theorem]

Ans.

27. In the given figure, if DE || AQ and DF || AR. Prove that EF || QR.



Sol. In DPAQ,

DE || QA [Given]

$\frac{PE}{EQ} = \frac{PD}{DA}$ …(i)

In DPAR, DF || AR [Given]

$\frac{PD}{DA} = \frac{PF}{FR}$ …(ii)

From (i) and (ii),

$\frac{PE}{EQ} = \frac{PF}{FR}$

Thus in DPQR,

EF || QR.

[By converse of Thales' theorem]

Hence Proved.

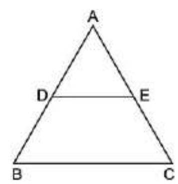
28. In DABC, D and E are points on the sides of AB and AC such that DE || BC. If AD = 2.5 cm, BD = 3 cm, AE = 3.75 cm, find the length of AC.

Sol. In DABC,

DE || BC [Given]

$\frac{AD}{BD} = \frac{AE}{EC}$

[Proportionality theorem]



$\Rightarrow \frac{2.5}{3} = \frac{3.75}{EC}$

$\Rightarrow$ EC = $\frac{3.75 \times 3}{2.5} = 4.5$ cm

Hence AC = AE + EC

= (3.75 + 4.5) cm

= 8.25 cm. **Ans.**

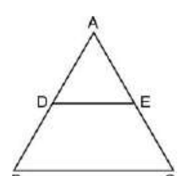
29. In ΔABC, D and E are points on AB and AC respectively such that DE || BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE.

Sol. In ΔABC,

DE || BC [Given]

So ΔADE ~ ΔABC

$\frac{AD}{AB} = \frac{DE}{BC}$ [cpct]



$\Rightarrow \frac{2.4}{AB} = \frac{2}{5}$

⇒ AB = (5 × 1.2) cm = 6 cm

So BD = AB – AD

= (6 – 2.4) cm = 3.6 cm

Also, $\frac{AE}{AC}$ = $\frac{DE}{BC}$ [by cpct]

⇒ $\frac{3.2}{AC}$ = $\frac{2}{5}$

⇒ AC = (5 × 1.6) cm = 8 cm

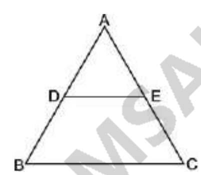
So EC = AC – AE = (8 – 3.2) cm

= 4.8 cm. **Ans.**

30. In D**ABC, D and E are points on AB and AC respectively such that DE || BC. If BD = CE. Prove that** D**ABC is isosceles.**

Sol. In DABC, it is given that

BD = CE



Since, DE || BC [Given]

Therefore, according to Thales' theorem,

$\frac{AD}{BD}$ = $\frac{AE}{EC}$

⇒ $\frac{AD}{EC}$ = $\frac{AE}{EC}$ (... BD = CE)

⇒ AD = AE

Thus, AB = AD + BD and AC = AE + EC

or AB = AC [... BD = CE (given)

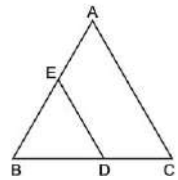
and AD = AE (proved above)]

So, DABC is isosceles. **Hence Proved.**

31. If ∆ABC and ∆BDE are equilateral triangles where D is the mid-point of BC, find the ratio of the area of ∆ABC and D**BDE.**

Sol. Let each side of ΔABC be *x*.

So Area of ΔABC = $\frac{\sqrt{3}}{4}x^2$



As D is the mid-point of BC,

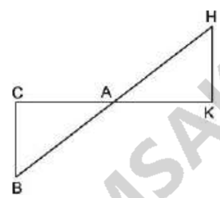
So, BD = $\frac{x}{2}$

So, Area of ΔBDE = $\frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{\sqrt{3}}{4} \times \frac{x^2}{4}$

Hence $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE}$ = $\frac{x^2}{\frac{x^2}{4}} = \frac{4}{1}$

Area of ΔABC : Area of ΔBDE = 4 : 1. **Ans.**

32. In the given figure, DAHK is similar to DABC. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC.



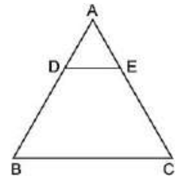
Sol. As DAHK ~ DABC [Given]

$\frac{AK}{HK}$ = $\frac{CA}{BC}$ [c.p.c.t.]

⇒ $\frac{10}{7}$ = $\frac{CA}{3.5}$

⇒ CA = 5 cm. **Ans.**

33. In the figure given below, DE || BC and AD = $\frac{1}{2}$BD. If BC = 4.5 cm, find DE.



Sol. In DABC,

AB = AD + BD

But AD = $\frac{1}{2}$BD

or BD = 2AD

Thus AB = AD + 2AD = 3AD

Now, Since DE || BC [Given]

$\frac{DE}{BC} = \frac{AD}{AB}$ [Thales' theorem]

or $\frac{DE}{4.5} = \frac{AD}{3AD}$

or DE = 1.5 cm. **Ans.**

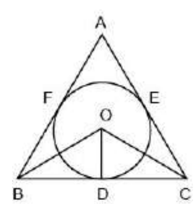
Short Answer Type Questions

34. The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC.

[Board Question]

Sol. Given that, ΔABC is isosceles with AB = AC. Also, the incircle touches the sides BC, CA and AB at D, E and F respectively. Let O be the centre of the circle. So, OD = OE = OF = *r*.
Now O is the point of intersection of angle bisectors of ΔABC. Thus, OB bisects ∠B and OC bisects ∠C.
In ΔABC, AB = AC [Given]
∠C = ∠B

[Angles opposite to equal sides are equal]



So $\frac{1}{2}$∠C = $\frac{1}{2}$∠B
∠OCD = ∠OBD

[OB bisects ∠B and OC bisects ∠C]

In ΔOBD and ΔOCD,
∠ODB = ∠ODC

[Radius of a circle is always perpendicular to the tangent at the point of contact]

Thus ∠OBD = ∠OCD [Proved above]

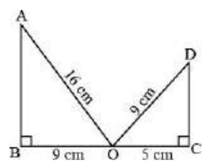
OD = OD [Common]

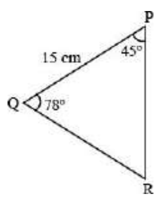
ΔOBD ≅ ΔOCD

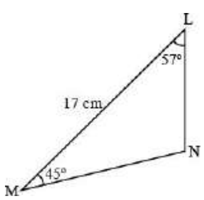
So BD = DC. [c.p.c.t.] **Hence Proved.**

35. State whether the given pairs of triangles are similar or not. In case of similarity, mention the criterion.

[Board Question]

(i) 

(ii) 



Sol.

(i) $\frac{AO}{DO} = \frac{16}{9}$ and $\frac{BO}{CO} = \frac{9}{5}$

$\because \frac{AO}{DO} \neq \frac{BO}{CO}$

∴ Given triangles are not similar. **Ans.**

(ii) In ΔPQR, ∠P + ∠Q + ∠R = 180°

[Angle-sum property of a Δ]

⇒ 45° + 78° + ∠R = 180°

⇒ ∠R = 180° – 45° – 78° = 57°

In ΔLMN, ∠L + ∠M + ∠N = 180°

[Angle-sum property of a Δ]

⇒ 57° + 45° + ∠N = 180°

⇒ ∠N = 180° – 57° – 45°

= 78°

Now, in ΔPQR and ΔLMN

∠P = ∠M [Each 45°]

∠Q = ∠N [Each 78°]

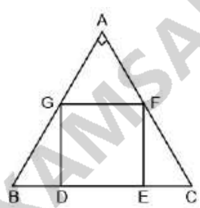
∠R = ∠L [Each 57°]

By AAA similarity criterion

ΔPQR ~ ΔMNL **Ans.**

36. In the given figure, DEFG is a square and ∠BAC = 90°. Prove that : (i) ΔAGF ~ ΔDBG, (ii) ΔAGF ~ ΔEFC, (iii) ΔDBG ~ ΔEFC, (iv) DE^2 = BD × EC.

[Board Question]



Sol, (i) In ΔAGF and ΔDBG, we have

∠GAF = ∠BDG = 90°

and ∠AGF = ∠GBD

[corresponding angles between parallel linesGF and BC with AB being the transversal]

Thus, by AA similarity criterion,

ΔAGF ~ ΔDBG.

(ii) In ΔAGF and ΔEFC, we have

∠GAF = ∠FEC = 90°

and ∠AFG = ∠FCE

[corresponding angles between parallel lines GF and BC with AC being the transversal]

Thus, $\Delta AGF \sim \Delta EFC$

(iii) From (i) and (ii), we have

$\Delta AGF \sim \Delta DBG$

and $\Delta AGF \sim \Delta EFC$

Thus $\Delta DBG \sim \Delta EFC$

(iv) From (iii), we have

$\Delta DBG \sim \Delta EFC$

Thus $\frac{BD}{FE} = \frac{DG}{EC}$ [cpct]

$\Rightarrow \frac{BD}{DE} = \frac{DE}{EC}$

[DE = DG = FE = GF as they are the sides of a square]

$\Rightarrow DE^2 = BD \times EC$. **Hence Proved.**

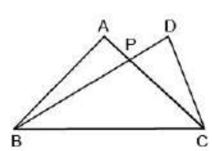
37. Two right-angled triangles, ABC and DBC, right angled at A and D respectively are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that AP × PC = PD × BP.

Sol. In ΔBAP and ΔCDP, we have

$\angle BAP = \angle CDP = 90°$ [given]

$\angle BPA = \angle CPD$

[vertically opposite angles]



By AA similarity criterion,

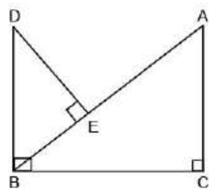
So, $\Delta BAP \sim \Delta CDP$

$\therefore \frac{AP}{DP} = \frac{BP}{CP}$ [cpct]

⇒ AP × CP = BP × DP

⇒ AP × PC = PD × BP. **Hence Proved.**

38. In the given figure, DB is perpendicular to BC, DE is perpendicular to AB and AC is perpendicular to BC. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Sol. In ΔBED and ΔCAB,

∠BED = ∠BCA = 90° [Given]

Here, BD || AC

∠DBE = ∠BAC [Alternate angles]

by AA similarity axiom,

So, ΔBED ~ ΔACB

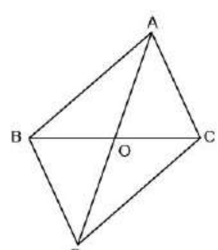
$\frac{BE}{AC} = \frac{DE}{BC}$ [c.p.c.t.]

⇒ $\frac{BE}{DE} = \frac{AC}{BC}$. **Hence Proved.**

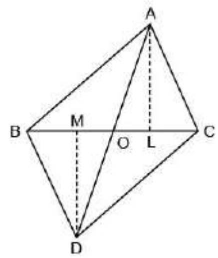
39. In the given figure, ΔABC and ΔDBC are on the BC. If AD intersects BC at O, prove that:

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DBC} = \frac{AO}{DO}.$$

[Board Question]



Sol. Let AL and DM be perpendicular to BC in ΔABC and ΔDBC respectively



Now, ∠AOL = ∠DOM

and ∠OMD = ∠OLA = 90°

∴ ΔALO ~ ΔDMO [AA similarity]

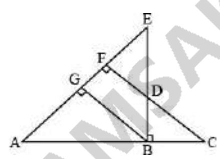
Hence $\frac{AL}{DM} = \frac{AO}{DO}$ …(i) [c.p.c.t.]

⇒ $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DBC} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM}$

= $\frac{AL}{DM} = \frac{AO}{DO}$ [using (i)]

⇒ $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DBC} = \frac{AO}{DO}$. **Hence Proved.**

40. In given figure, EB ⊥ AC, BG ⊥ AE and CF ⊥ AE Prove that :



(i) ΔABG ~ ΔDCB

(ii) $\frac{BC}{BD} = \frac{BE}{BA}$

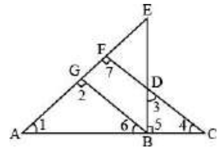
Sol. Given, EB ⊥ AC, BG ⊥ AE and CF ⊥ AE.

(i) In ΔABG and ΔDCB,

∠2 = ∠5 [Each 90°]

∠6 = ∠4 [Corresponding angles]

As GB || FC,



∴ ΔABG ~ ΔDCB [By AA similarity]

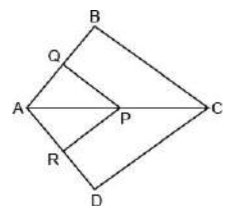
Hence Proved.

(ii) In ΔABE and ΔDBC

∠1 = ∠3 [∵ ∠ABG ~ ∠DCB]

∠ABE = ∠5 [Each 90°]
∴ ΔABE ~ ΔDBC [By AA similarity]
In similar Δs, corresponding sides are proportional
∴ $\frac{BC}{BD} = \frac{BE}{BA}$. **Hence Proved**

41. In the given figure, PQ || BC and PR || CD. Prove that : (i) $\frac{AR}{AD} = \frac{AQ}{AB}$, (ii) $\frac{QB}{AQ} = \frac{DR}{AR}$.



Sol. (i) In ΔADC,

PR || CD

∴ $\frac{AR}{AD} = \frac{AP}{AC}$ [Thales' theorem]…(i)

and in ΔABC,

PQ || BC

∴ $\frac{AP}{AC} = \frac{AQ}{AB}$ [Thales' theorem]…(ii)

From equations (i) and (ii),

$\frac{AR}{AD} = \frac{AQ}{AB}$. **Hence Proved.**

(ii) In ΔABC,

PQ || BC

∴ $\frac{QB}{AQ} = \frac{PC}{AP}$ [Thales' thereom]…(i)

and in ΔADC, PR || BC

∴ $\frac{PC}{AP} = \frac{DR}{AR}$ [Thales' theorem]…(ii)

From equation, (i) and (ii),

$\frac{QB}{AQ} = \frac{DR}{AR}$. **Hence Proved.**

42. In ΔABC, DE || BC. If $\frac{AD}{DB} = \frac{2}{3}$ find $\frac{BC}{DE}$.

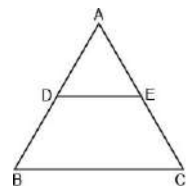
Sol. In ∆ABC and ∆ADE,

∠A = ∠A [common]

∠ADE = ∠B [Corresponding angles]

∠AED = ∠C [Corresponding angles]

Thus, ∆ABC ~ ∆ADE [AAA-criterion]



Now $\frac{AD}{DB} = \frac{2}{3}$

⇒ 3AD = 2DB

Also, AB = AD + DB

⇒ 2AB = 2AD + 2DB

⇒ 2AB = 2AD + 3AD [∵ 3AD = 2DB]

⇒ 2AB = 5AD

⇒ $\frac{AB}{AD} = \frac{5}{2}$

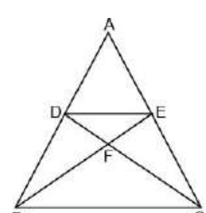
Since ∆ABC ~ ∆ADE,

Then $\frac{AB}{AD} = \frac{BC}{DE}$ [c.p.c.t.]

⇒ $\frac{BC}{DE} = \frac{5}{2}$ $\left[\because \frac{AB}{AD} = \frac{5}{2}\right]$

Ans.

43. In the given figure, DE || BC and AD : DB = 5 : 4. Find $\left(\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta CFB}\right)$.



Sol. Given, DE || BC

So ∠ADE = ∠ABC and ∠AED = ∠ACB

[Corresponding angles]

Thus, in ΔABC and ΔADE,

∠A = ∠A [Common]

∠ADE = ∠ABC

[Corresponding angles]

∠AED = ∠ACB

[Corresponding angles]

⇒ ΔABC ~ ΔADE

∴ $\frac{AD}{AB} = \frac{DE}{BC}$

We have $\frac{AD}{DB} = \frac{5}{4}$ [Given]

⇒ $\frac{DB}{AD} = \frac{4}{5}$

⇒ $\frac{DB}{AD} + 1 = \frac{4}{5} + 1$

⇒ $\frac{DB+AD}{AD} = \frac{4+5}{5}$

⇒ $\frac{AB}{AD} = \frac{9}{5}$

⇒ $\frac{AD}{AB} = \frac{5}{9} = \frac{DE}{BC}$

In ΔDFE and ΔCFB,

∠DEB = ∠EBC [Alternate angles]

and ∠DFE = ∠BFC

[Vertically opposite angles]

⇒ ΔDFE ~ ΔCFB [AA – Criteria]

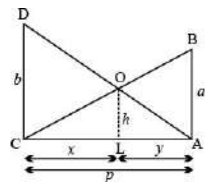
∴ $\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CFB} = \frac{DE^2}{BC^2}$

or $\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CFB} = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$. **Ans.**

Long Answer Type Questions

44. Two trees of height *a* and *b* are *p* metre apart. Prove that the height of the point of intersection of the lines joining the top of each tree to the foot of the opposite trees is given by $\frac{ab}{a+b}$ m.

Sol. Let AB and CD be the two trees of height *a* and *b* metre respectively that are *p* metre apart. Let the lines AD and BC meet at O such that OL = *h* metre



Let CL = x and LA= y

then $x + y = p$

In ΔABC and ΔLOC, we have

∠CAB = ∠CLO [Each 90º]

∠C = ∠C [Common]

⇒ ΔCAB ~ ΔCLO [By AA axiom]

∴ $\frac{CA}{CL} = \frac{AB}{LO}$ [c.p.c.t.]

⇒ $\frac{p}{x} = \frac{a}{h}$

⇒ $x = \frac{ph}{a}$ ...(i)

Also, in ΔALO and ΔACD

∠ALO = ∠ACD [Each 90º]

∠A = ∠A [Common]

ΔALO ~ ΔACD [By AA axiom]

∴ $\frac{AL}{AC} = \frac{OL}{DC}$ [c.p.c.t.]

⇒ $\frac{y}{p} = \frac{h}{b}$

⇒ $y = \frac{ph}{b}$ ...(ii)

Adding equations (i) and (ii), we get

$x + y = \frac{ph}{a} + \frac{ph}{b}$

$\Rightarrow x + y = ph\left(\frac{1}{a} + \frac{1}{b}\right)$

$\Rightarrow p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$ [$\because x + y = p$]

$\Rightarrow \frac{1}{h} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{1}{h} = \frac{a+b}{ab}$

$\Rightarrow h = \frac{ab}{a+b}$ m. **Hence Proved.**

45. In $\triangle$ABC, DE || BC such that AD = ($4x$ – 3) cm, AE = ($8x$ – 7) cm, BD = ($3x$ – 1) cm and CE = ($5x$ – 3) cm. Find the value of x.

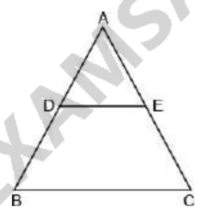
[Board Question]

Sol. In $\triangle$ABC, we have

DE || BC

So, $\frac{AD}{DB} = \frac{AE}{EC}$ [Thales' theorem]

$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$



$\Rightarrow (4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

$\Rightarrow 20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow 2x^2 - 2x + x - 1 = 0$

$\Rightarrow 2x(x - 1) + 1(x - 1) = 0$

$\Rightarrow (x - 1)(2x + 1) = 0$

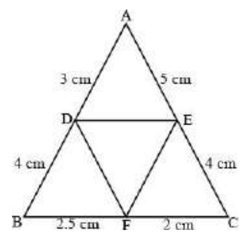
$\Rightarrow x = 1$

[as length cannot be negative]

So, the value of x = 1. **Ans.**

46. In the given figure, AD = 3 cm, AE = 5 cm, BD = 4 cm, CE = 4 cm, CF = 2 cm, BF = 2.5 cm, then find the pair of parallel lines and hence their lengths.

[Board Question]



Sol. We have,

AD = 3 cm, AE = 5 cm, BD = 4 cm, CE = 4 cm, CF = 2 cm and BF = 2.5 cm

So, $\frac{EC}{EA} = \frac{4}{5}$ and $\frac{CF}{FB} = \frac{2}{2.5} = \frac{4}{5}$

$\Rightarrow \frac{EC}{EA} = \frac{CF}{FB}$

Now in ΔABC,

... $\frac{EC}{EA} = \frac{CF}{FB}$

∴ EF || AB

[Converse of Thales' theorem]

Also, $\frac{CE}{CA} = \frac{4}{4+5} = \frac{4}{9}$ ...(i)

$\frac{CF}{CB} = \frac{2}{2+2.5} = \frac{2}{4.5} = \frac{4}{9}$

In ΔECF and ΔACB,

$\because \frac{CE}{CA} = \frac{CF}{CB}$

∠ECF = ∠ACB [Common]

∴ ΔCFE ~ ΔCBA [SAS similarity]

$\Rightarrow \frac{EF}{AB} = \frac{CE}{CA}$

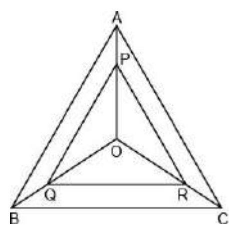
[In similar Δ's, corresponding sides are proportional]

$\Rightarrow \frac{EF}{7} = \frac{4}{9}$ [... AB = 3 + 4 = 7 cm]

∴ EF = $\frac{28}{9}$ cm and AB = 7 cm. **Ans.**

47. In the given figure, PQ || AB and PR || AC. Prove that QR || BC.

[Board Question]



Sol. In ΔOAB,

PQ || AB

∴ $\frac{OP}{PA} = \frac{OQ}{QB}$ [Thales' theorem] …(i)

In ΔOAC, PR || AC

∴ $\frac{OP}{PA} = \frac{OR}{RC}$ [Thales' theorem] …(ii)

From equations (i) and (ii), we get

$\frac{OQ}{QB} = \frac{OR}{RC}$ …(iii)

In ΔOBC, $\frac{OQ}{QB} = \frac{OR}{RC}$ [from (iii)]

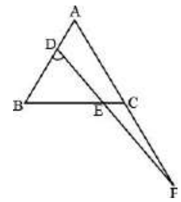
⇒ QR || BC

[Converse of Thales' theorem]

Hence Proved.

48. In the figure, ∠BED = ∠BDE and E divides BC in the ratio 2 : 1. Prove that AF × BE = 2 AD × CF.

[Board Question]



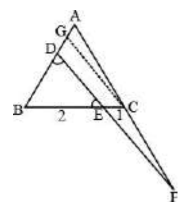
Sol. Since, E divides BC in the ratio 2 : 1

∴ $\frac{BE}{EC} = \frac{2}{1}$ ...(i)

∠BED = ∠BDE [Given]

⇒ BD = BE ...(ii)

[Sides opposite to equal angles]



In ΔCBG,

DE || CG [By construction]

⇒ $\frac{BD}{DG} = \frac{BE}{EC}$ [Thale's theorem]

⇒ $\frac{BD}{DG} = \frac{BE}{EC} = \frac{2}{1}$ [From eq. (i)]

⇒ $\frac{BD}{DG} = \frac{2}{1}$

⇒ 2DG = BD

⇒ 2DG = BE [From eq. (ii)]

⇒ DG = $\frac{1}{2}$BE ...(iii)

In ΔADF,

CG || DF [By construction]

⇒ $\frac{AG}{GD} = \frac{AC}{CF}$ [Thales' theorem]

⇒ $\frac{AG}{GD}+1 = \frac{AC}{CF}+1$

[Adding 1 on both sides]

$\Rightarrow \frac{AG+GD}{GD} = \frac{AC+CF}{CF}$

$\Rightarrow \frac{AD}{GD} = \frac{AF}{CF}$

⇒ AF × GD = AD × CF

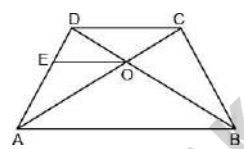
⇒ AF × $\frac{BE}{2}$ = AD × CF [From (iii)]

⇒ AF × BE = 2AD × CF. **Hence Proved.**

49. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{OC} = \frac{BO}{OD}$. Show that ABCD is a trapezium.

[Board Question]

Sol. Let EO || DC meet AD at E.



So, in ΔADC, EO || DC

So, $\frac{AO}{OC} = \frac{AE}{ED}$ [Thales' theorem] …(i)

But $\frac{AO}{OC} = \frac{BO}{OD}$ [Given] …(ii)

By (i) and (ii),

So, $\frac{BO}{OD} = \frac{AE}{ED}$

Hence, $\frac{BO}{OD} = \frac{AE}{ED}$ and BO, OD, AE and ED are segments of ΔDAB.

So, EO || AB

[By converse of Thales' theorem]

But EO || DC

Thus, AB || DC

As there are only two parallel sides in this quadrilateral, it is a trapezium. **Hence Proved.**

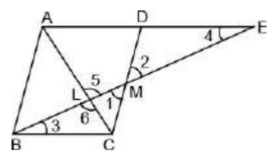
50. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC at L and AD produced at E and AD = DE. Prove that EL = 2BL.

[Board Question]

Sol. In ΔBMC and ΔEMD, we have

∠1 = ∠2

[Vertically opposite angles]



MC = MD

[M being the mid-point of CD]

∠BCM = ∠EDM [Alternate angles]

Thus, ΔBMC ≅ ΔEMD [by ASA]

⇒ BC = DE (cpct)

Again, BC = AD

[Opposite sides of the parallelogram ABCD]

∴ BC = AD = DE

So, AE = AD + DE = 2BC …(i)

Again, in ΔAEL and ΔCBL,

∠5 = ∠6

[Vertically opposite angles]

∠3 = ∠4 [Alternative angles]

So, ΔAEL ~ ΔCBL

∴ $\frac{EL}{BL} = \frac{AE}{BC} = \frac{2BC}{BC} = 2$ [From (i)]

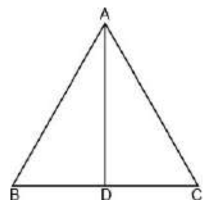
Thus, EL = 2BL. **Hence Proved.**

51. In an equilateral triangle with side *a*, prove that (i) altitude $\frac{\sqrt{3}}{2}$ = *a*, and (ii) area = $\frac{\sqrt{3}}{4}$ a^2.

[Board Question]

Sol. Let ΔABC be an equilateral triangle with side *a* and AD perpendicular to BC.

Then, AB = AC = BC = *a*



In ΔADB and ΔADC,

AB = AC [Given]

∠B = ∠C = 60°

∠ADB = ∠ADC = 90°

∴ ΔADB ≅ ΔADC

So, BD = DC = $\frac{a}{2}$

(i) In ΔADB, we have

$AB^2 = AD^2 + BD^2$

[Pythagoras theorem]

$\Rightarrow AD^2 = AB^2 - BD^2$

$\Rightarrow AD^2 = a^2 - \left(\frac{a}{2}\right)^2$

$\Rightarrow AD^2 = a^2 - \frac{a^2}{4}$

$\Rightarrow AD^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

$\Rightarrow AD = \frac{\sqrt{3}a}{2}$

Thus, altitude AD = $\frac{\sqrt{3}a}{2}$

(ii) Area of ΔABC = $\frac{1}{2}$ × base × height

= $\frac{1}{2}$ × BC × AD

= $\frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}$

= $\frac{\sqrt{3}a^2}{4}$ sq. units.**Ans.**

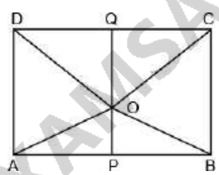
52. (i) O is any point inside a rectangle ABCD. Prove that : $OB^2 + OD^2 = OA^2 + OC^2$.

[Board Question]

(ii) If OB = 6 cm, OD = 8 cm, OA = 5 cm, find the length of OC.

[Board Question]

Sol. Let QP || AD || BC be perpendicular to AB and CD and pass through O.



So, QO is the altitude of ΔCOD and OP is the altitude of ΔBOA.

(i) Now, $OB^2 = OP^2 + PB^2$

[Pythagoras theorem] …(i)

$OA^2 = OP^2 + AP^2$

[Pythagoras theorem] …(ii)

$OC^2 = OQ^2 + QC^2$

[Pythagoras theorem] …(iii)

$OD^2 = OQ^2 + DQ^2$

[Pythagoras theorem] …(iv)

Thus, $OB^2 + OD^2 = OQ^2 + DQ^2 + OP^2 + PB^2$

$\Rightarrow OB^2 + OD^2 = OQ^2 + PB^2 + DQ^2 + OP^2$

$\Rightarrow OB^2 + OD^2 = OQ^2 + QC^2 + AP^2 + OP^2$

[... AP = DQ and PB = QC; opp sides of rectangle]

$\Rightarrow OB^2 + OD^2 = OC^2 + OA^2$

[From (ii) and (iii)]

Hence Proved.

(ii) ... $OB^2 + OD^2 = OC^2 + OA^2$ [Proved above]

Let OC = x cm

$\Rightarrow (6)^2 + (8)^2 = x^2 + (5)^2$

$\Rightarrow 36 + 64 = x^2 + 25$

$\Rightarrow x^2 + 25 = 100$

$\Rightarrow x^2 = 75$

$\Rightarrow x = 5\sqrt{3}$ cm. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

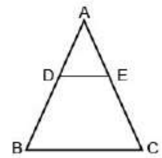
(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) Both the Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

53. Assertion: If in a DABC, a line DE || BC, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$.



Reason: If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

As per the Basic Proportionality Theorem of Thales, 'If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.' So, Reason is correct.

For assertion, DE || BC and now applying this basic proportionality theorem in the assertion, we get.

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow 1+\frac{DB}{AD} = 1+\frac{EC}{AE}$

$\Rightarrow \frac{AD+DB}{AD} = \frac{AE+EC}{AE}$

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$

Hence, assertion is also correct.

54. Assertion: ABC is an isosceles right triangle, right angled at C, then $AB^2 = 3AC^2$.

Reason: In an isosceles triangle ABC, if AC = BC and $AB^2 = 2AC^2$, then $\angle C = 90°$.

Ans. (d) Assertion is false but the Reason is true.

Explanation :

For the assertion, by applying the Pythagoras theorem in the right angle triangle ABC, we get.

$AB^2 = AC^2 + BC^2$

$= AC^2 + AC^2$ [∵ BC = AC]

$= 2AC^2$

So, the assertion is proved to be false.

For reason, by applying the Pythagoras theorem.

$AB^2 = 2AC^2 = AC^2 + AC^2$

$= AC^2 + BC^2$

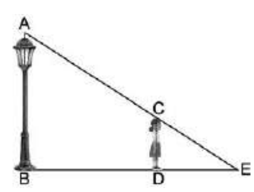
$\angle C = 90°$

(by converse of Pythagoras theroem)

So, the given statement in the reason is true.

Case Based Questions

55. On one day, a poor girl is looking for a lamp-post for completing her homework as in her area power is not there and she finds the same at some distance away from her home. After completing the homework, she is walking away from the base of a lamp-post at a speed of 1.2 m/s. The lamp post is 3.6 m above the ground and height of the girl is 90 cm (see below figure).



(i) The distance of the girl from the base of the lamp post after 4 seconds:

(a) 1·2

(b) 3·6 m

(c) 4·8 m

(d) none of these

Ans. (c) 4·8 m

Explanation :

Let AB denote the lamp-post and CD be the girl after walking for 4 seconds away from the lamp-post.

Now, her distance from the base of the lamp

BD = 1·2 m × 4

= 4·8 m.

(ii) The correct similarity criteria appliable for triangles ABE and CDE is:

(a) AA

(b) SAS

(c) SSS

(d) AAS

Ans. (a) AA

Explanation :

In Δ ABE and Δ CDE, $\angle B = \angle D$ (Each is of 90°)and $\angle E = \angle E$ (Same angle)

So, $\Delta ABE \sim \Delta CDE$ (AA similarity criterion)

(iii) The length of her shadow after 4 seconds is:

(a) 1·2 m

(b) 3·6 m

(c) 4·8 m

(d) none of these

Ans. (d) none of these

Explanation :

$\Delta ABE \sim \Delta CDE$

$\Rightarrow \frac{BE}{DE} = \frac{AB}{CD}$ (Let DE = x)

$\Rightarrow \frac{4.8 + x}{x} = \frac{3.6}{0.9}$

$\Rightarrow 4{\cdot}8 + x = 4x$

$\Rightarrow x = 1.6$

So, the shadow of the girl after walking for 4 seconds is 1·6 m long.

(iv) Sides of two similar triangles are in the ratio 9 : 16. The ratio of corresponding area of these triangles.

(a) 9 : 16

(b) 3 : 4

(c) 81 : 256

(d) 18 : 32

Ans. (c) 81 : 256

Explanation :

Since ratio of the area of two similar triangles is equal to the ratio of the square of their corresponding sides,

Ratio of areas of similar triangles

$= (9)^2 : (16)^2 = 81 : 256.$

(v) The ratio AC : CE. is:

(a) 1 : 3

(b) 3 : 1

(c) 1 : 4

(d) 4 : 1

Ans. (b) 3 : 1

Explanation :

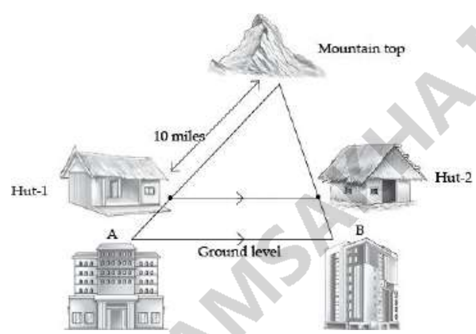
$\frac{AE}{CE} = \frac{BE}{DE} = \frac{4 \cdot 8 + 1 \cdot 6}{1 \cdot 6} = \frac{6 \cdot 4}{1 \cdot 6} = 4$

⇒ AE = 4CE

⇒ AC + CE = 4CE

⇒ AC = 3CE ⇒ $\frac{AC}{CE} = \frac{3}{1}$

56. Two hotels are at the ground level on either side of a mountain. On moving a certain distance towards the top of the mountain two huts are situated as shown in the figure. The ratio between the distance from hotel B to hut-2 and that of hut-2 to mountain top is 3 : 7.



(i) What is the ratio of the perimeters of the triangle formed by both hotels and mountain top to the triangle formed by both huts and mountain top?

(a) 5 : 2

(b) 10 : 7

(c) 7 : 3

(d) 3 : 10

Ans. (b) 10 : 7

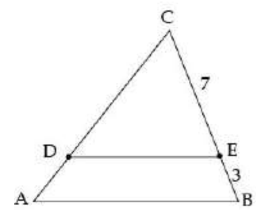
Explanation :

Let DABC is the triangle formed by both hotels and mountain top. DCDE is the triangle formed by both huts and mountain top.

Clearly DE ǁAB and so

D ABC ~ D DEC

[By AA-similarity criterion]



Now, required ratio = Ratio of their corresponding sides = $\frac{BC}{EC}$ = $\frac{10}{7}$ *i.e,* 10 : 7.

(ii) If distance between hut-1 and mountain top is 10 miles, then the distance between the hotel A and hut-1 is:

(a) 2.5 miles

(b) 29 miles

(c) 4.29 miles

(d) 1.5 miles

Ans. (c) 4.29 miles

Explanation :

Since, DEǁAB, therefore

$\frac{CD}{AD} = \frac{CE}{EB}$

$\Rightarrow \frac{10}{AD} = \frac{7}{3}$

$\Rightarrow$ AD = $\frac{10\times3}{7}$

= 4.29 miles.

(iii) If the horizontal distance between the hut-1 and hut-2 is 8 miles, then the distance between the two hotels is:

(a) 2.4 miles

(b) 11.43 miles

(c) 9 miles

(d) 7 miles

Ans. (b) 11.43 miles

Explanation :

Since, DABC ~ DDEC

$\frac{BC}{EC} = \frac{AB}{DE}$

[∵ Corresponding sides of similar triangles are proportional]

⇒ $\frac{10}{7} = \frac{AB}{8}$

⇒ AB = $\frac{80}{7}$

= 11.43 miles.

(iv) If the distance from mountain top to hut-1 is 5 miles more than that of distance from hotel B to mountain top, then what is the distance between hut-2 and mountain top?

(a) 3.5 miles

(b) 6 miles

(c) 5.5 miles

(d) 4 miles

Ans. (a) 3.5 miles

Explanation :

Given, DC = 5 + BC

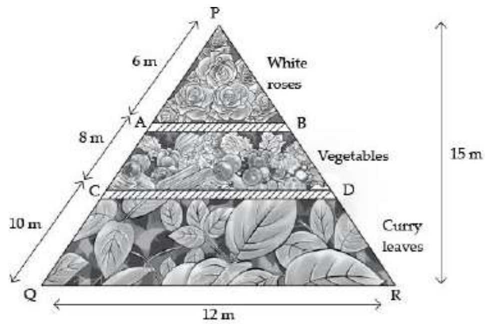
Clearly, BC = 10 – 5 = 5 miles

Now, CE = $\frac{7}{10}$ × BC = $\frac{7}{10}$ × 5

= 3.5 miles.

57. In the backyard of house, Shikha has some empty space in the shape of a DPQR. She decided to make it a garden. She divided the whole space into three parts by making boundaries

AB and CD using bricks to grow flowers and vegetables where AB||CD||QR as shown in figure.



Based on the above information, answer the following questions:

(i) The length of AB is:

(a) 3 m

(b) 4 m

(c) 5 m

(d) 6 m

Ans. (a) 3 m

Explanation :

In DPAB and DPQR,

∠P = ∠P (Common)

∠A = ∠Q

(Corresponding angles)

By AA similarity criterion, DPAB ~ DPQR

$\frac{AB}{QR} = \frac{PA}{PQ}$

$\Rightarrow \frac{AB}{12} = \frac{6}{24}$

⇒ AB = 3 m.

(ii) The length of CD is:

(a) 4 m

(b) 5 m

(c) 6 m

(d) 7 m

Ans. (d) 7 m

Explanation :

Similarly, DPCD and DPQR are similar.

$\frac{PC}{PQ} = \frac{CD}{QR}$

$\Rightarrow \frac{14}{24} = \frac{CD}{12}$

⇒ CD = 7 m.

(iii) Area of whole empty land is:

(a) 90 m^2

(b) 60 m^2

(c) 32 m^2

(d) 72 m^2

Ans. (a) 90 m^2

Explanation :

Area of whole empty land

= $\frac{1}{2}$ × base × height

= $\frac{1}{2}$ × 12 × 15

= 90 m^2

Passage Based Questions

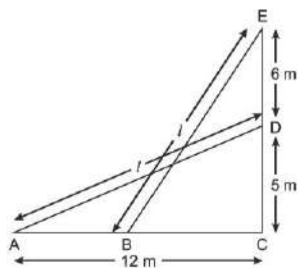
58. The foot of a ladder is 12 m away from a wall and its top reaches a window 5 m above the ground. The ladder is shifted

in such a way that its top touches the roof which is 6 m above the window.

Based on the given information, answer the following questions :

(i) What is the length of the ladder?

(ii) How much the foot of ladder is shifted towards the wall, so that the top of ladder touches the roof?



Sol. (i) In ΔACD,

AC = Distance of foot of ladder from wall = 12 m

CD = Height of window = 5 m.

Let, l be the length of ladder.

Applying pythagoras theorem in ΔACD,we get

$l^2 = AC^2 + CD^2 \Rightarrow l^2 = (12)^2 + (5)^2$

$\Rightarrow l^2 = 144 + 25 \Rightarrow l^2 = 169$

$\Rightarrow l = 13$ m.

So, length of the ladder = 13 m. **Ans.**

(ii) Height of the roof = (5 + 6) m = 11 m.

Length of the ladder = 13 m.

Now, ΔBCE is a right angled triangle.

On applying pythagoras theorem, in ΔBCE, we get

$BE^2 = EC^2 + BC^2$

$\Rightarrow (13)^2 = (11)^2 + BC^2$

$\Rightarrow 169 - 121 = BC^2$

$\Rightarrow 48 = BC^2$

$\Rightarrow BC = 4\sqrt{3}$ m.

Required distance = AB = AC – BC

= $12 - 4\sqrt{3}$

= $4(3-\sqrt{3})$ m **Ans.**

59. An aeroplane leaves an airport and flies due North at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due West at a speed of 1200 km per hour.

Based on the following information, answer the following questions:

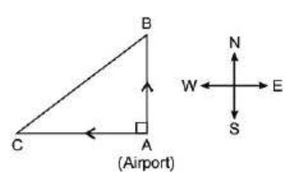
(i) How far apart will be the planes after $1\frac{1}{2}$ hour?

(ii) If the speed of first plane is 1600 km per hour, then find the distance between two planes after $1\frac{1}{2}$ hours?

(iii) If second plane flies in the South direction, then how far apart will be the planes after 1 hour?

Sol. (i) The distance travelled by first aeroplane in North direction = $1000 \times \frac{3}{2}$ km

AB = 1500 km



The distance travelled by second aeroplane in West direction

AC = $1200 \times \frac{3}{2}$ = 1800 km

In ΔBAC, using pythagoras theorem

$(BC)^2 = (1500)^2 + (1800)^2$

$\Rightarrow (BC)^2 = 2250000 + 3240000$

$\Rightarrow (BC)^2 = 5490000$

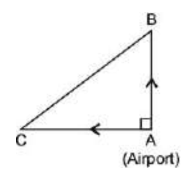
$\Rightarrow BC = 300\sqrt{61}$ km

Hence, the required distance = $300\sqrt{61}$ km

(ii) The distance travelled by first aeroplane

$AB = 1600 \times \frac{3}{2}$ km

$\Rightarrow AB = 2400$ km



The distance travelled by second aeroplane, AC = 1800 km

In ΔBAC, using pythagoras theorem

$(BC)^2 = (2400)^2 + (1800)^2$

$\Rightarrow BC^2 = 5760000 + 3240000$

$\Rightarrow BC = \sqrt{9000000}$

$\Rightarrow BC = 3000$ km

Hence, the required distance is 3000 km.

(iii) Distance travelled by first aeroplane is 1 hour

AB = 1000 km

Distance travelled by second aeroplane in 1 hour

AC = 1200 km

Hence, the required distance between two planes

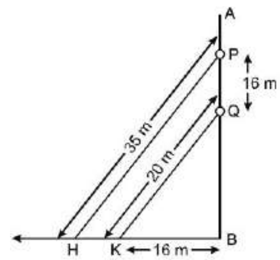
= AB + AC

= 1000 km + 1200 km

= 2200 km **Ans.**

60. Two spotlights, P and Q are mounted on a vertical pole AB, as shown below. Light beams from P and Q shine on two points on the ground, H and K respectively. Distance between two spotlights is 16 m, horizontal distance between first shine point and vertical pole is 16 m, distance of spotlight P and Q to its shine points on ground H and K is 35 m and 20 m

respectively.



Based on the following information, answer the following questions :

(i) The height above the ground at which the spotlight Q is mounted.

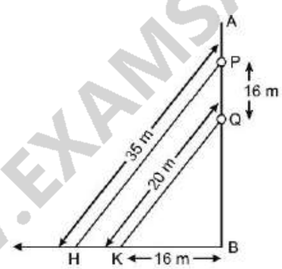
(ii) The distance between the projections of the light beams.

Sol. (i) In right angled ΔKBQ

BQ = $\sqrt{KQ^2 - KB^2}$ = $\sqrt{(20)^2 - (16)^2}$

= $\sqrt{400 - 256}$

= $\sqrt{144}$ = 12 m



(ii) Using (i), BP = (12 + 16)m = 28 m.

In right angled, ΔHBP

HB = $\sqrt{(HP)^2 - (PB)^2}$ = $\sqrt{(35)^2 - (28)^2}$

= $\sqrt{1225 - 784}$ = $\sqrt{441}$

= 21 m.

So, distance between the projections of the light beams =

HK = HB – KB

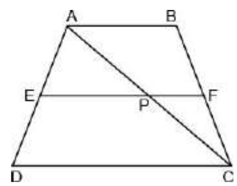
= (21 – 16) m

= 5 m. **Ans.**

Self-Assessment

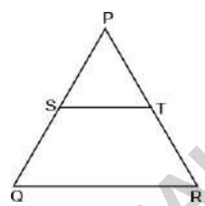
61. In the given figure, EF || AB || DC. Prove that : $\frac{AE}{ED} = \frac{BF}{FC}$.

[NCERT]



62. In the given figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ. Prove that ΔPQR is an isosceles triangles.

[NCERT]



63. Prove that the diagonals of a trapezium divide each other proportionally.

[NCERT]

64. Prove that the line joining the mid-points of two sides of a triangle is parallel to the third side.

65. Prove that two equiangular triangles are similar.

[NCERT]

66. If in two triangles, one pair of corresponding sides is proportional and the included angles are equal, then show that the two triangles are similar.

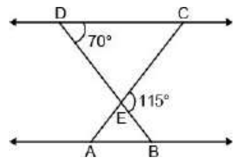
[NCERT]

67. If the corresponding sides of two triangles are proportional then show that they are similar.

[NCERT]

68. In the given figure, if ΔEDC ~ ΔEBA, ∠BEC = 115° and ∠EDC = 70°, find ∠DEC, ∠DCE, ∠EAB, ∠AEB and ∠EBA.

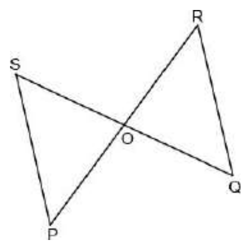
[NCERT]



Ans. ∠DEC = 65°, ∠DCE = 45°, ∠EAB = 45°, ∠AEB = 65° and ∠EBA = 70°.

69. In the given figure, if ΔPOS ~ ΔROQ, prove that PS || QR.

[NCERT]

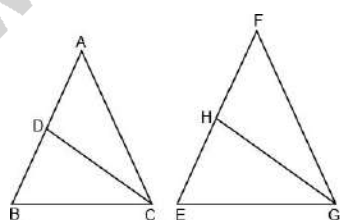


70. In the given figures, if CD and GH (D and H lie on AB and FE) are the bisectors of ∠ACB and ∠EGF respectively and ΔABC ~ ΔFEG, prove that:

(i) ΔDCA ~ ΔHGF, (ii) $\frac{CD}{GH}=\frac{AC}{FG}$,

(iii) ΔDCB ~ ΔHGE.

[NCERT]

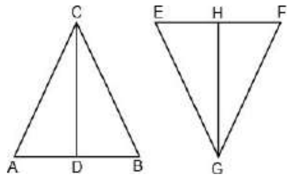


71. In the figures given below, CD and GH are respectively the medians of ΔABC and ΔFEG. If ΔABC ~ ΔFEG, prove that:

(i) ΔADC ~ ΔFGH, (ii) $\frac{CD}{GH}=\frac{AC}{FG}$,

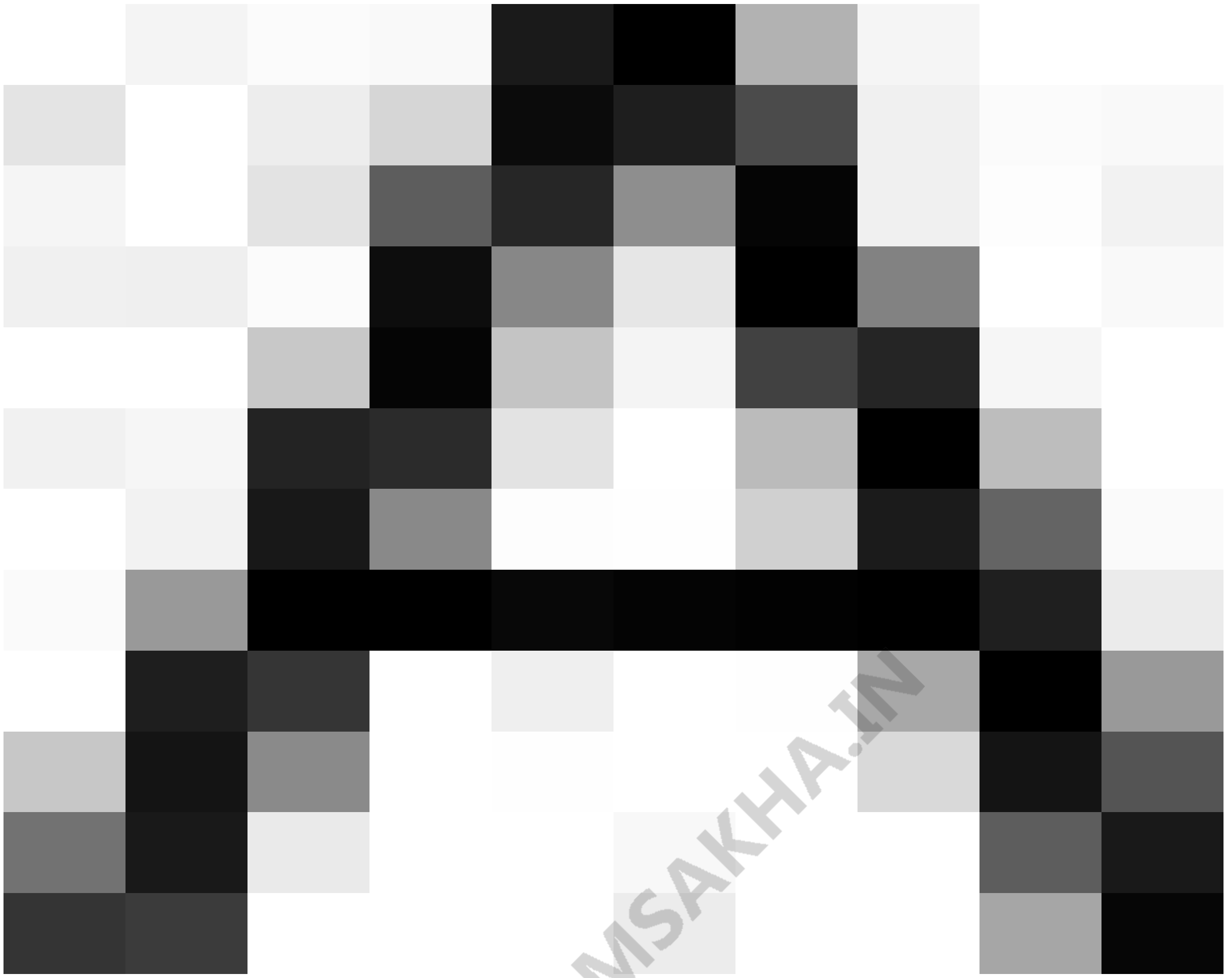
(iii) ΔCDB ~ ΔGHE.

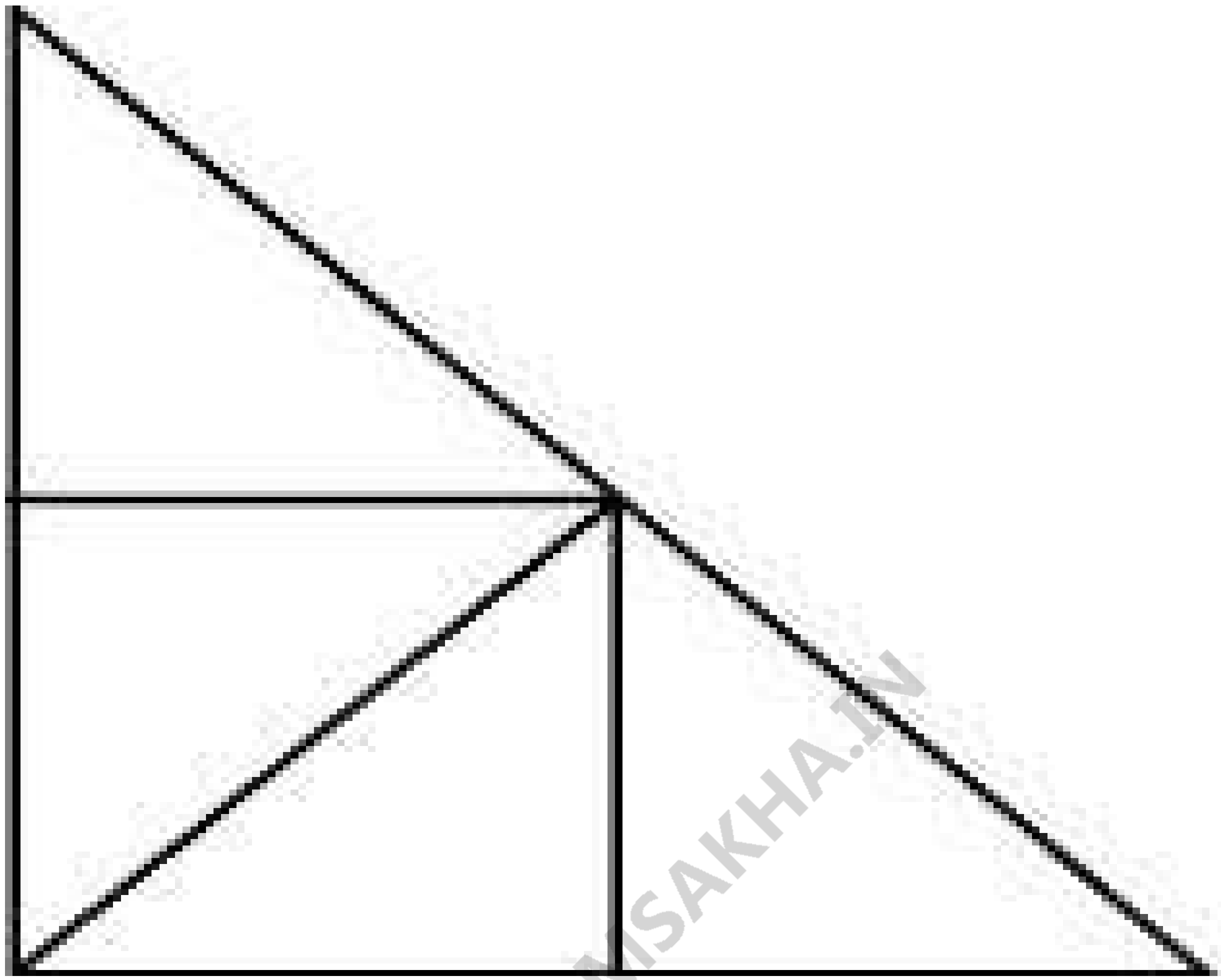
[NCERT]

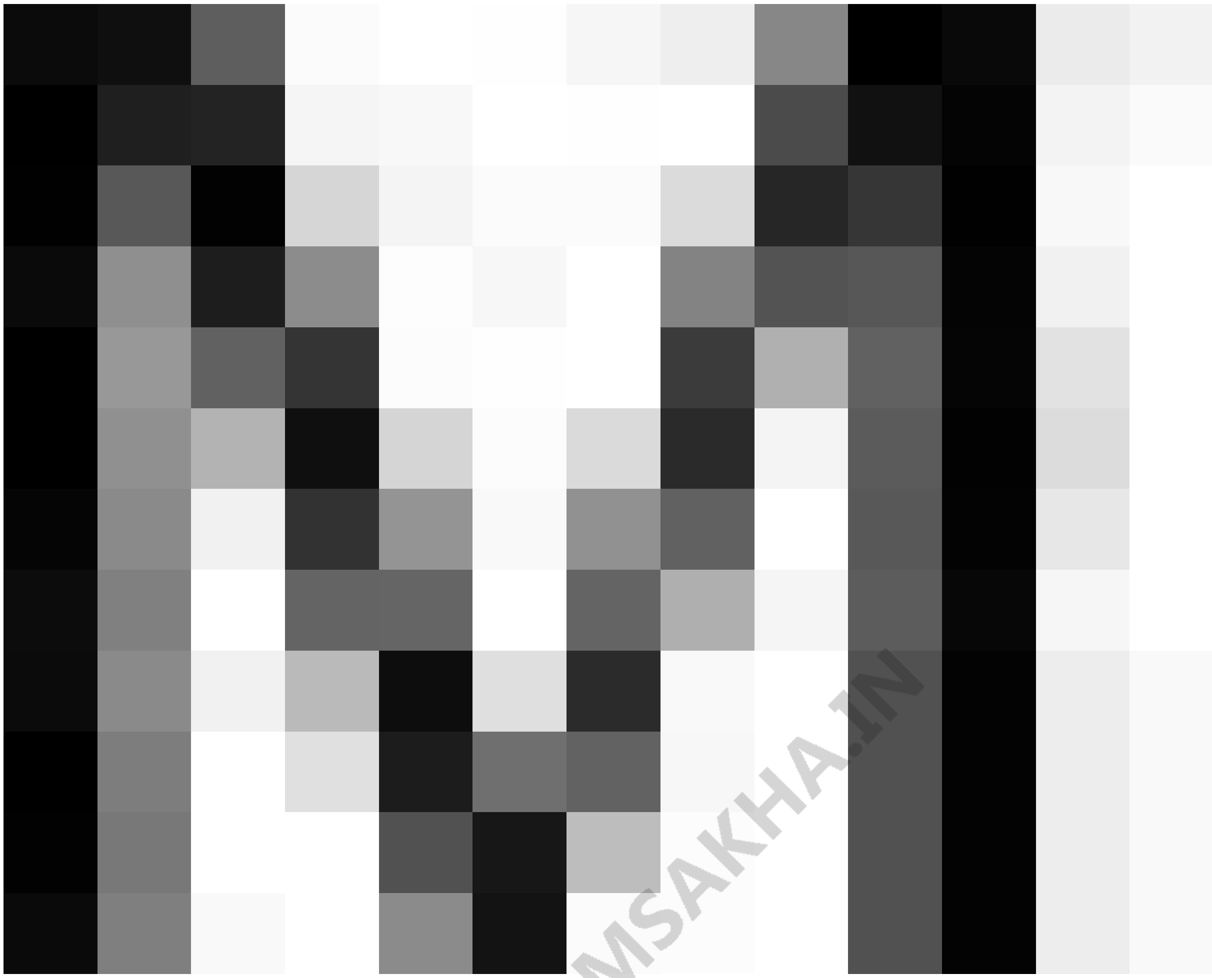


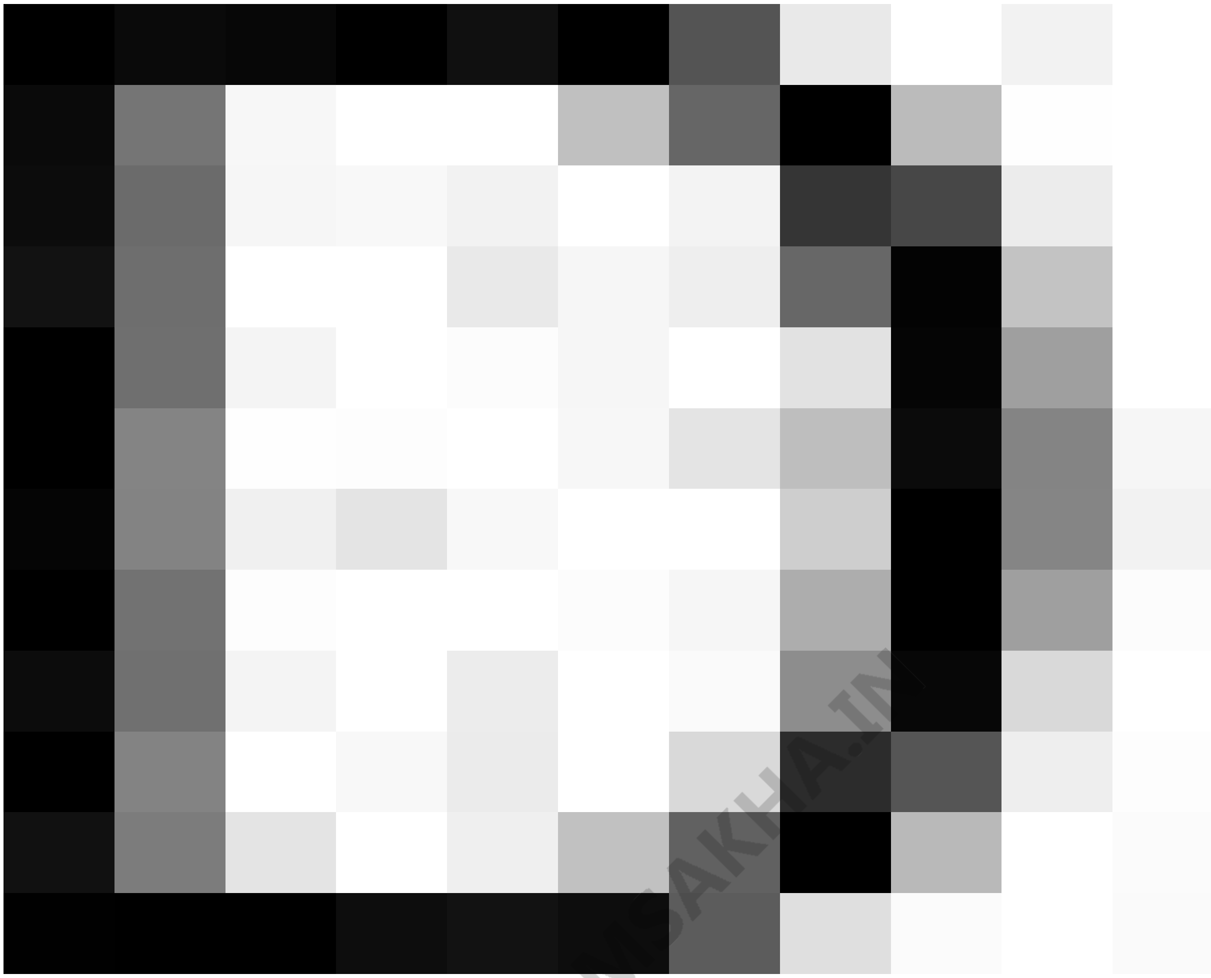
72. E is a point on side AD produced on a parallelogram ABCD and BE bisects CD at F. Prove that ΔABE ~ ΔCFB.

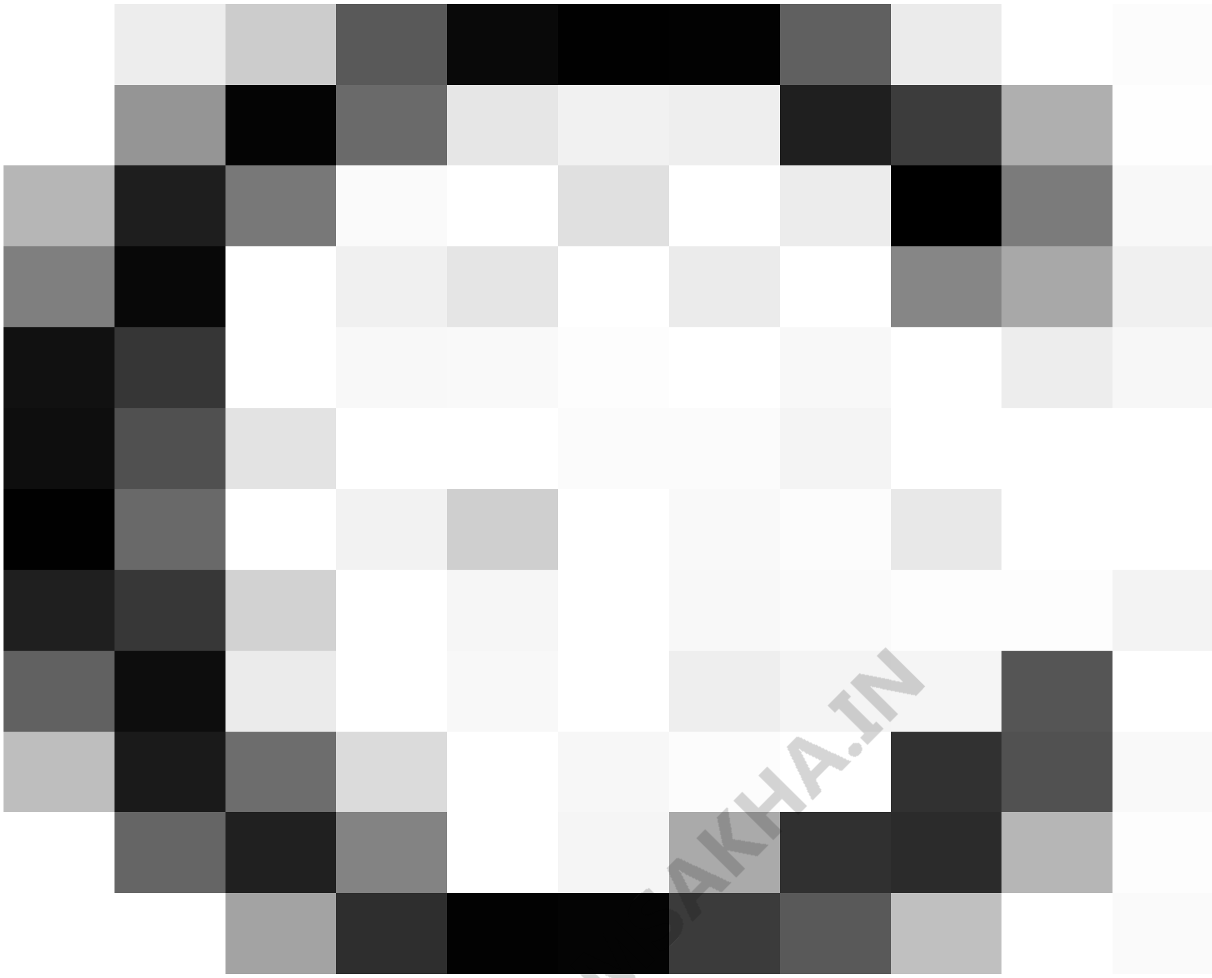
[NCERT]











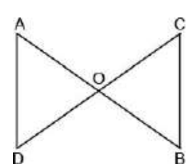
73. In the given figure, ABC is a right-angled triangle, where ∠B = 90° and D is the foot of the perpendicular drawn from B on AC. If DM is perpendicular to *BC* and DN to AB, prove that : (i) $DM^2 = DN \times MC$ and (ii) $DN^2 = DM \times AN$.

[NCERT]

74. In the figure given below, $\frac{OA}{OC} = \frac{OD}{OB}$.

Prove that : ∠A = ∠C and ∠B = ∠D.

[NCERT]



75. ABCD is a trapezium, in which AB || CD and AB = 2CD. Determine the ratio of the areas of ΔAOB and ΔCOD.

[NCERT]

Ans. 4 : 1.

76. D and E are points on the sides AB and AC respectively of ΔABC such that DE || BC and divides Δ*ABC* into two parts, equal in area. Find $\frac{BD}{AB}$.

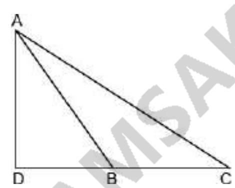
[NCERT]

Ans. $\frac{\sqrt{2}-1}{\sqrt{2}}$.

77. The given figure represents ΔABC, obtuse-angled at *B*. If AD is perpendicular to CB, prove that

$AC^2 = AB^2 + BC^2 + 2BC \times BD$.

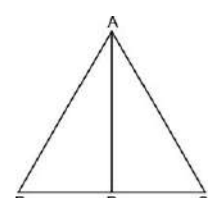
[NCERT]



78. The given figure represents ΔABC, acute-angled at B. If AD is perpendicular to CB, prove that

$AC^2 = AB^2 - BC^2 + 2BC \times BD$.

[NCERT]



79. A point *O* in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that $OB^2 + OD^2 = OC^2 + OA^2$.

[NCERT]

80. *ABCD* is a rhombus. Prove that

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$.

[NCERT]

81. In an equilateral ΔABC, the side BC is trisected at D. Prove that $9AD^2 = 7AB^2$.

[NCERT]

82. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

[NCERT]

83. A guy wire attached to a vertical pole of height 18 m is 24 m long hand has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

[NCERT]

Ans. $6\sqrt{7}$ m.

84. An aeroplane leaves an airport and flies due north at speed of 1000 km/hr. At the same time, another plane leaves due west at a speed of 1200 km/hr. How far apart will the two planes be after 1 hour and 30 minute?

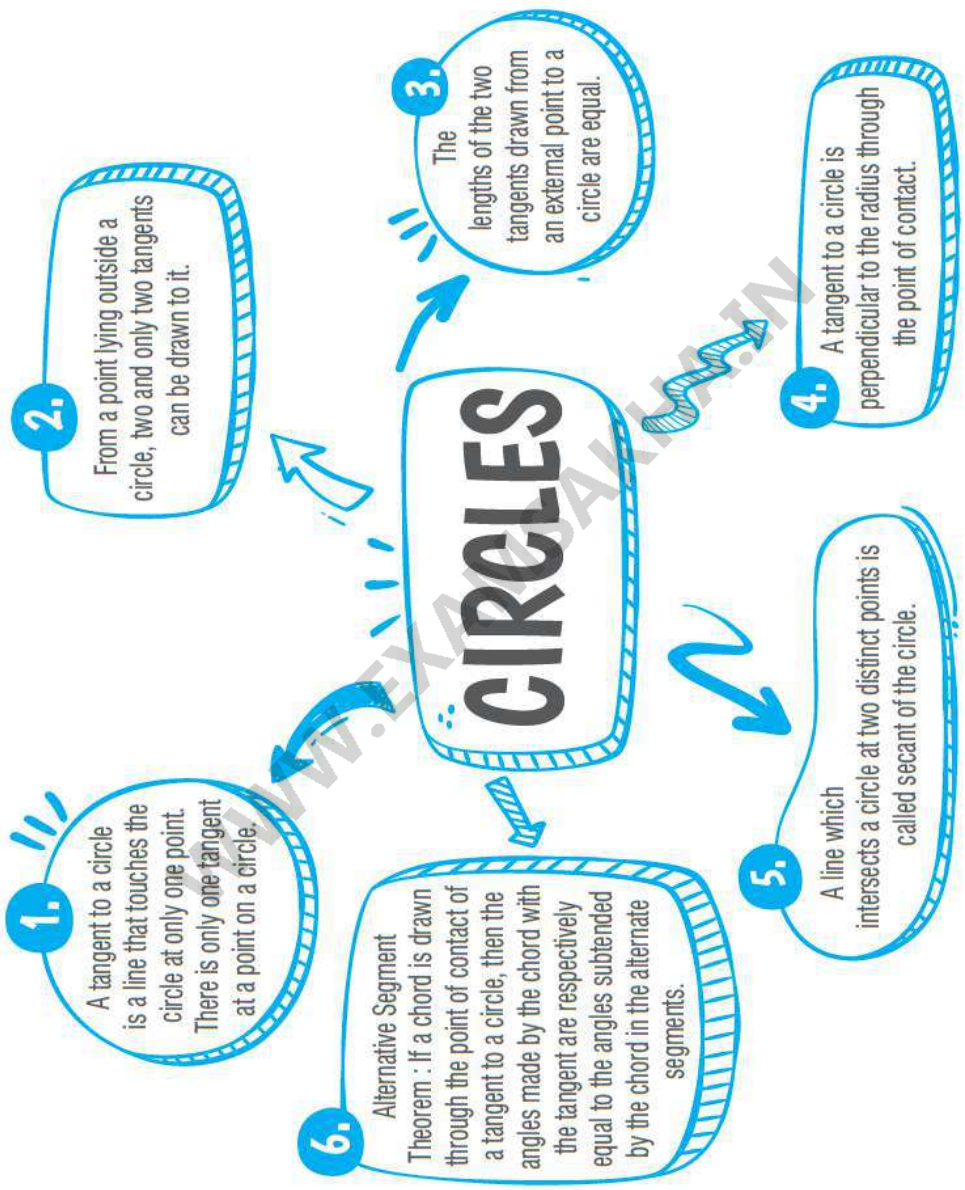
[NCERT]

Ans. 2343 km (approximately)

Chapter 8

Circles

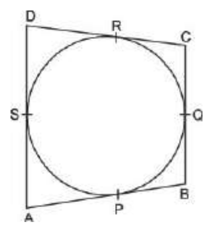
Basic Concepts



Multiple Choice Questions

1. In the given figure, a quadrilateral *ABCD* is drawn to circumscribe a circle such that its sides *AB*, *BC*, *CD* and *AD* touch the circle at *P*, *Q*, *R* and *S* respectively. If *AB* = *x* cm, *BC* = 7 cm, *CR* = 3 cm and *AS* = 5 cm, find *x*.

[Board Question]



(a) 10 cm

(b) 9 cm

(c) 8 cm

(d) 7 cm

Ans. (b) 9 cm

Explanation :

Given, *AB* = *x* cm, *BC* = 7 cm, *CR* = 3 cm and *AS* = 5 cm

Since, tangents from an external point are equal in length.

Hence, *CQ* = *CR* = 3 cm

[tangents from point *C*]

Thus, *BQ* = *BC* – *CQ*

= (7 – 3) cm = 4 cm

So, *PB* = *BQ* = 4 cm

[tangents from point *B*]

Also, *AS* = *AP* = 5 cm

[tangents from point *A*]

$\therefore$ $AB = AP + PB$

$= (5 + 4)$ cm $= 9$ cm

2. Two concentric circles have radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle (at a point).

[Board Question]

(a) 4 cm

(b) 5 cm

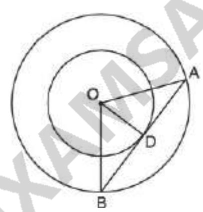
(c) 8 cm

(d) 10 cm

Ans. (c) 8 cm

Explanation :

Given, $OA = OB = 5$ cm; $OD = 3$ cm



AB is a tangent at D, smaller circle and chord for bigger circle.

$\angle ODB = \angle ODA = 90°$

[As the tangent at a point is perpendicular to

the radius of a circle through the point]

By pythagoras theorem

$OB^2 = BD^2 + OD^2$

$\Rightarrow BD^2 = OB^2 - OD^2$

$\Rightarrow BD^2 = (5)^2 - (3)^2$

$\Rightarrow BD^2 = 25 - 9 = 16$

$\Rightarrow BD = 4$ cm …(i)

Also, perpendicular from the centre bisects the chord

$AB = AD + BD$

$= 2BD = (2 \times 4)$ cm

= 8 cm.

3. Two circles touch each other externally at *P*. *AB* is a common tangent to the circles, touching them at *A* and *B*. The value of ∠*APB* is:

[Board Question]

(a) 30°

(b) 45°

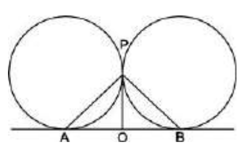
(c) 60°

(d) 90°

Ans. (d) 90°

Explanation :

Let *PO* meet *AB* at *O* such that *PO* is a tangent.



Thus, $AO = PO = OB$ and $\angle AOP = \angle BOP = 90°$

As D*AOP* and D*BOP* are right-angled isosceles triangles.

So, $\angle OAP = \angle OPA = \angle OBP = \angle OPB = 45°$

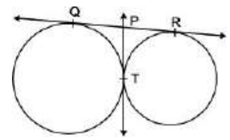
Thus, $\angle APB = \angle APO + \angle OPB$

$= 45° + 45° = 90°$

4. In the given figure, *QR* is a common tangent to the given

circles, touching externally at point *T*. The tangent at *T* meets *QR* at *P*. If *PT* = 3.8 cm, then the length of *QR* (in cm) is:

[Board Question]



(a) 3.8

(b) 7.6

(c) 5.7

(d) 1.9

Ans. (b) 7.6

Explanation :

Since *PT* intersects *QR* at *P*, it is the external point from two different tangents for the two circles are possible.

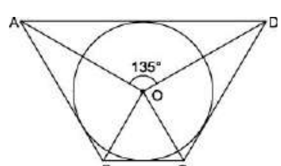
Thus, *PT* = *PQ* and *PT* = *PR*

So, *PT* = *PQ* = *PR* = 3.8 cm

Thus, *QR* = (3.8 + 3.8) cm = 7.6 cm.

5. In the given figure, if ∠*AOD* = 135°, then ∠*BOC* is equal to:

[Board Question]



(a) 52.5°

(b) 45°

(c) 62.5°

(d) 25°

Ans. (b) 45°

Explanation :

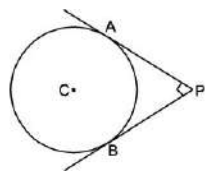
Given, $\angle AOD = 135°$

$\therefore \angle BOC = 180° - 135° = 45°$

Since, angle subtended at the centre by a pair of opposite sides are supplementary.

6. In the given figure, *PA* and *PB* are two tangents drawn from an external point *P* to a circle with centre *C* and radius 4 cm. If *PA* is perpendicular to *PB*, then the length of each tangent is:

[Board Question]



(a) 3 cm

(b) 4 cm

(c) 5 cm

(d) 6 cm

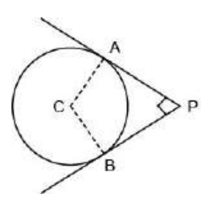
Ans. (b) 4 cm

Explanation :

Now, $AP = BP$

[Since, tagnents from an external point *P* are equal]

Join *CA* and *CB* and they are perpendicular to *AP* and *AB* respectively.



Given $CA = CB = 4$ cm (radius of circle)

Now, $\angle ACB = 360° - [90° + 90° + 90°]$

or $\angle ACB = 90°$

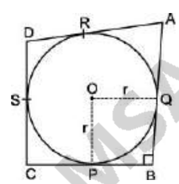
As *APBC* is a right-angled quadrilateral with its adjacent sides *CA* and *CB* being 4 cm.

So, *APBC* is a square.

$AP = PB = CA = CB = 4$ cm.

7. In the given figure, a circle with centre *O* is inscribed in a quadrilateral *ABCD* such that it touches the side *BC*, *AB*, *AD* and *CD* at points *P*, *Q*, *R* and *S* respectively. If *AB* = 29 cm, *AD* = 23 cm, ∠*B* = 90° and *DS* = 5 cm, then the radius of the circle (in cm) is:

[Board Question]



(a) 11

(b) 18

(c) 6

(d) 5

Ans. (a) 11

Explanation :

Given, $AB = 29$ cm, $AD = 23$ cm, $DS = 5$ cm and $\angle B = 90°$

Now, $DS = DR = 5$ cm

[tangents from point, *D*]

So, $AR = AD - DR = (23 - 5)$ cm $= 18$ cm

Again, $AR = AQ = 18$ cm

[tangents from points, A]

Now, $QB = AB - AQ = (29 - 18)$ cm = 11 cm

$\angle OPB = \angle OQB = 90°$

[$\because$ OQ and OP are radius of circle and

PB and BQ are tangents]

Thus, in the quadrilateral $OQBP$,

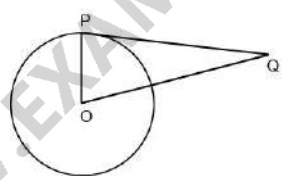
$\angle POQ = 360° - (90° + 90° + 90°) = 90°$

Hence, it is a right-angled quadrilateral with adjacent opposite sides are equal.

$\therefore$ $OQ = QB = 11$ cm $= r$

8. From a point *Q*, 13 cm away from the centre of a circle, the length of tangent *PQ* to the circle is 12 cm. The radius of the circle (in cm) is:

[Board Question]



(a) 25

(b) $\sqrt{313}$

(c) 5

(d) 1

Ans. (c) 5

Explanation :

Given,

$PQ = 13$ cm and $OQ = 12$ cm

In D*OPQ*, by pythagoras theorem

Thus, $r = PO = \sqrt{OQ^2 - PQ^2}$

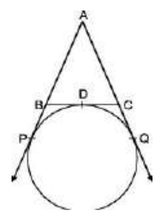
$= \sqrt{(13)^2 - (12)^2}$

$= \sqrt{169 - 144} = \sqrt{25} = 5$ cm

As length cannot be negative.

9. In the given figure, *AP*, *AQ* and *BC* are tangents to the circle. If *AB* = 5 cm, *AC* = 6 cm and *BC* = 4 cm, then the length of *AP* (in cm) is:

[Board Question]



(a) 7.5

(b) 15

(c) 10

(d) 9

Ans. (a) 7.5

Explanation :

Given, *AB* = 5 cm, *AC* = 6 cm and *BC* = 4 cm

Let *PB* = *x* cm, *CQ* = *y* cm and *d* be the point where the tangent *BC* touches the circle.

So, *AP* = (5 + *x*) cm and *AQ* = (6 + *y*) cm

Also *PB* = *BD* = *x* cm [tangents from *B*]

and *CQ* = *CD* = *y* cm [tangents from *C*]

Now, 5 + *x* = 6 + *y* [Q *AP* = *AQ*]

$\Rightarrow x - y = 1$ …(i)

and $x + y = 4$ [$\because$ $BC = 4$] …(ii)

Adding (i) and (ii), we get

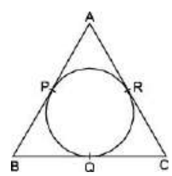
$2x = 5$

$\Rightarrow x = 2.5$

Thus, $AP = (5 + x)$ cm = 7.5 cm

10. In the given figure, the sides *AB*, *BC* and *CA* of a triangle *ABC* touch a circle at *P*, *Q* and *R* respectively. If *PA* = 4 cm, *BP* = 3 cm and *AC* = 11 cm, then the length of *BC* (in cm) is:

[Board Question]



(a) 11

(b) 10

(c) 14

(d) 15

Ans. (b) 10

Explanation :

Given, PA = 4 cm, PB = 3 cm and AC = 11 cm

Now, $BQ = BP$ = 3 cm

[Q BQ and BP are tangents from B]

and $AP = AR$ = 4 cm

[Q AP and AR are tangents from A]

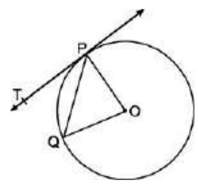
Thus, $RC = AC - AR$ = (11 – 4) cm = 7 cm

Also $RC = CQ$ = 7 cm [Q RC and CQ are tangents from C]

Hence, $BC = BQ + CQ = (3 + 7)$ cm = 10 cm.

11. In the given figure, *O* is the centre of the circle, *PQ* is a chord and *PT* is the tangent at *P*. If $\angle POQ = 70°$, then $\angle TPQ$ equals to:

[Board Question]



(a) 70°

(b) 45°

(c) 90°

(d) 35°

Ans. (d) 35°

Explanation :

Given,

$\angle POQ = 70°$

As *OP* and *OQ* are the radii of the circle so D*POQ* is isosceles.

Thus, $\angle OPQ = \angle OQP$

In D*POQ*,

$180° = \angle OPQ + \angle OQP + 70°$

$\Rightarrow 2\angle OPQ = 110°$

$\Rightarrow \angle OPQ = 55°$

Also as *OP* is perpendicular to *PT*, so

$\angle OPT = 90°$

Thus, $\angle TPQ = \angle OPT - \angle OPQ$

or $\angle TPQ = 90° - 55° = 35°$.

12. In the given figure, *AB* and *AC* are tangents to the circle with centre *O* such that ∠*BAC* = 40°. Then ∠*BOC* is equal to:

[Board Question]

(a) 40°

(b) 50°

(c) 140°

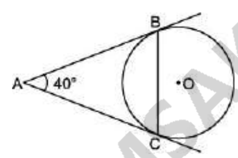
(d) 150°

Ans. (c) 140°

Explanation :

Given, *AB* = *AC*, *OB* = *OC* and ∠*A* = 40°

Join *AO* such that it is perpendicular to the chord *BC* and bisects ∠*A*.



Thus, ∠*BAO* = ∠*OAC* = 20°

Also, as *OB* and *OC* are perpendicular to *AB* and *AC* respectively.

∠*ABO* = ∠*ACO* = 90°

Now, in D*ABO*,

∠*BOA* = 180° – (90° + 20°)

or ∠*BOA* = 180° – 110° = 70°

Similarly in D*ACO*,

∠*COA* = 180° – (90° + 20°)

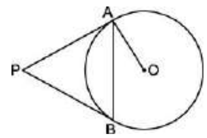
or ∠*COA* = 180° – 110° = 70°

Hence, ∠*BOC* = ∠*BOA* + ∠*COA*

= 70° + 70° = 140°

13. In the given figure, *PA* and *PB* are tangents to the circle with centre *O*. If ∠*APB* = 60°, then ∠*OAB* is:

[Board Question]



(a) 30°

(b) 60°

(c) 90°

(d) 15°

Ans. (a) 30°

Explanation :

Given,

∠*APB* = 60°

AP = *PB* [tangents from point *P*]

In D*PAB*, *PA* = *PB*

So, D*PAB* is isosceles and ∠*PAB* = ∠*PBA*

In D*PAB*,

Thus, ∠*PAB* + ∠*PBA* = 180° – ∠*APB*

⇒ 2∠*PAB* = 180° – 60°

= 120°

⇒ ∠*PAB* = 60° = ∠*PBA*

Now, ∠*PAO* = 90°

[Q *OA* is perpendicular to *AP*]

Thus, ∠*OAB* = ∠*PAO* – ∠*PAB*

= 90° – 60°

= 30°.

14. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is:

(a) 60 cm^2

(b) 65 cm^2

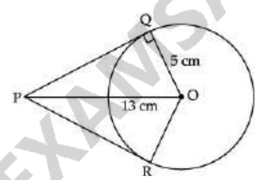
(c) 30 cm^2

(d) 32.5 cm^2

Ans. (a) 60 cm^2

Explanation :

Firstly, draw a circle of radius 5 cm having centre O, P is a point distance of 13 cm from O. A pair of tangents PQ and PR are drawn.



Thus, quadrilateral PQOR is formed.

Q OQ ⊥ QP [since, PQ is a tangent line]

In right angled ΔPQO,

$OP^2 = OQ^2 + PQ^2$

$\Rightarrow 13^2 = 5^2 + PQ^2$

$\Rightarrow PQ^2 = 169 - 25 = 144$

$\Rightarrow PQ = 12$ cm

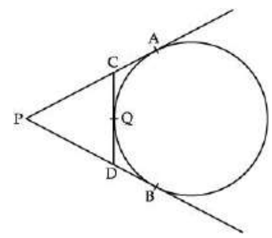
Now, area of ΔOQP = $\frac{1}{2}$ × QP × QO

$= \frac{1}{2} \times 12 \times 5 = 30\ cm^2$

Area of quadrilateral PQOR = 2 ΔOQP

$= 2 \times 30 = 60\ cm^2$

15. PA and PB are tangents to the circle drawn from an external point P. CD is the third tangent touching the circle at Q. If PB = 12 cm and CQ = 3 cm, then the length of PC will be:



(a) 8 cm

(b) 9 cm

(c) 6 cm

(d) 4 cm

Ans. (b) 9 cm

Explanation :

CA = CQ = 3 cm

PA = PB = 12 cm

[Q tangents drawn from an external

point have equal length]

PA = CA + PC ⇒ 12 = 3 + PC

⇒ PC = 9 cm

16. The tangent of a circle makes-angle with radius at point of contact:

(a) 45°

(b) 30°

(c) 90°

(d) 60°

Ans. (c) 90°

Explanation :

The angle between a tangent to a circle and the radius through the point of contact is always a right angle or 90°.

17. The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is:

(a) $\sqrt{7}$ cm

(b) 7 cm

(c) 5 cm

(d) 25 cm

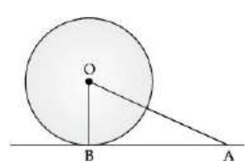
Ans. (c) 5 cm

Explanation :

Let AB be the tangent from A to the circle of centre O, then

OB = 3 cm

BA = 4 cm



and OB ^ BA

(∵ radius of a circle is ^r to the tangentat the point of contact)

In right DOBA,

$OA^2 = OB^2 + BA^2$

(Pythagoras Theorem)

$= (3)^2 + (4)^2 = 9 + 16 = 25 = (5)^2$

OA = 5

Thus, distance of A from the centre O = 5 cm.

18. If TP and TQ are two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to:

(a) 60°

(b) 70°

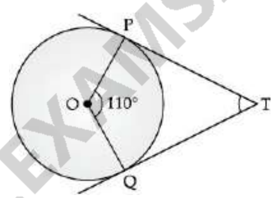
(c) 80°

(d) 90°

Ans. (b) 70°

Explanation :

TP and TQ are the tangents from T to the circle with centre O and OP, OQ are joined and ∠POQ = 110°



But ∠POQ + ∠PTQ = 180°

⇒ 110° + ∠PTQ = 180°

⇒ ∠PTQ = 180° – 110° = 70°

19. PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then ∠OQP is equal to:

(a) 30°

(b) 45°

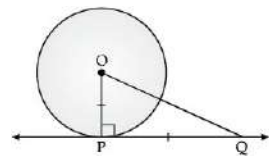
(c) 60°

(d) 90°

Ans. (b) 45°

Explanation :

In a circle with centre O, PQ is a tangent to the circle at P and DOPQ is an isosceles triangle such that OP = PQ



Here, OP is radius of the circle

But, OP ^ PQ

∠OPQ = 90°

and OP = PQ

∠POQ = ∠OQP

∵ ∠OPQ + ∠POQ + ∠OQP

= 180°(Angle sum property)

90° + ∠OQP + ∠OQP = 180°

⇒ ∠OQP = $\frac{90°}{2}$ = 45°

20. ABC is a right angled triangle, right angled at B such that BC = 6 cm and AB = 8 cm. A circle with centre O is incribed in ΔABC. The radius of the circle is:

(a) 1 cm

(b) 2 cm

(c) 3 cm

(d) 4 cm

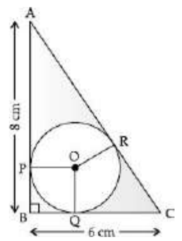
Ans. (b) 2 cm

Explanation :

In a right DABC, ∠B = 90°

BC = 6 cm,

AB = 8 cm



$AC^2 = AB^2 + BC^2$

(By Pythagoras Theorem)

$= (8)^2 + (6)^2 = 64 + 36$

$= 100 = (10)^2$

⇒ AC = 10 cm

An incircle is drawn with centre O which touches the sides of the triangle ABC at P, Q and R. OP, OQ and OR are radii and AB, BC and CA are the tangents to the circle.

OP ^ AB, OQ ^ BC and OR ^ CA

OPBQ is a square(∵ ∠B = 90°)

Let *r* be the radius of the circle

PB = BQ = *r*

AR = AP = 8 – *r*,

CQ = CR = 6 – *r*

AC = AR + CR

⇒ 10 = 8 – *r* + 6 – *r*

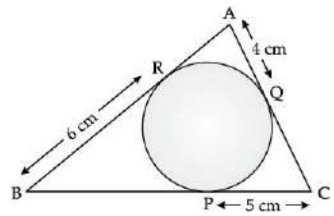
⇒ 10 = 14 – 2*r*

⇒ 2*r* = 14 – 10 = 4

⇒ *r* = 2

Radius of the incircle = 2 cm

21. In the figure, the perimeter of ∆ABC is:



(a) 30 cm

(b) 60 cm

(c) 45 cm

(d) 15 cm

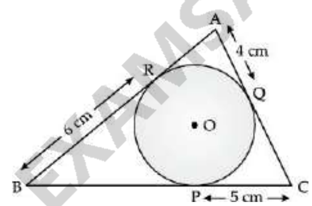
Ans. (a) 30 cm

Explanation :

Given : DABC is circumscirbed about a circle,

AQ = 4 cm, CP = 5 cm and BR = 6 cm.

AQ and AR the tangents to the circle AQ = AR = 4 cm



Similarly, BP and BR are tangents,

BP = BR = 6 cm

and CP and CQ are the tangents

CQ = CP = 5 cm

AB = AR + BR = 4 + 6 = 10 cm

BC = BP + CP = 6 + 5 = 11 cm

AC = AQ + CQ = 4 + 5 = 9 cm

Perimeter of DABC = AB + BC + AC

= 10 + 11 + 9 = 30 cm

22. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA is equal to:

(a) 50°

(b) 60°

(c) 70°

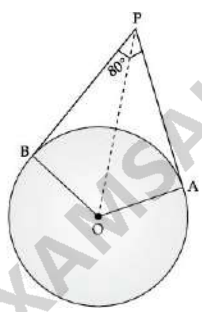
(d) 80°

Ans. (a) 50°

Explanation :

OA is the radius to tangent PA and OB is the radius to tangent PB.

So, OA is perpendicular to PA and OB is perpendicular to PB *i.e.*, OA ^ PA and OB ^ PB



So, ∠OBP = ∠OAP = 90°

Now, in the quadrilateral AOBP,

∠AOB + ∠OAP + ∠OBP + ∠APB = 360°

⇒ ∠AOB + 90° + 90° + 80° = 360°

⇒ ∠AOB + 260° = 360°

⇒ ∠AOB = 100°

Now, consider the triangles DOPB and DOPA. Here,

AP = BP

(Since, the tangents from a point are always equal)

Now, in DAOP and DBOP

OA = OB(radii of the circle)

OP = OP(common)

DOPB ≅ DOPA

(by SSS congruency)

So, ∠POB = ∠POA…(i)

Now, ∠AOB = ∠POA + ∠POB

2(∠POA) = ∠AOB [from (i)]

By putting the respective values, we get,

∠POA = $\frac{100°}{2}$ = 50°.

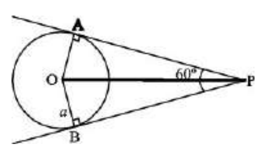
Very Short Answer Type Questions

23. If the angle between two tangents drawn from an external point *P* to a circle of radius *a* and centre *O*, is 60°, then find the length of *OP*.

Sol. Given, ∠*APB* = 60°

⇒ ∠*APO* = 30°

(since, tangents are inclined at an equal angle to line joining from the centre to the point outside the circle.)



Also, ∠*OAP* = 90°

[As, tangent is perpendicular to the radius]

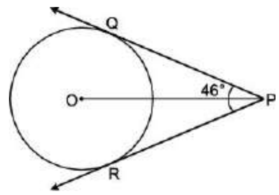
∴ In right angled D*OAP*,

⇒ $\frac{OP}{OA}$ = cosec 30°

⇒ $\frac{OP}{a}$ = 2

⇒ $OP = 2a$. **Ans.**

24. In the given figure, *PQ* and *PR* are two tangents to a circle with centre *O*. If ∠*QPR* = 46°, then find ∠*QOR*.



Sol. Join *OP* such that it bisects ∠*OPR*.

Thus ∠*OPQ* = ∠*OPR* = 23°

Also, ∠*OQP* = ∠*ORP* = 90°

[Tangent is ^r to radius]

So, ∠*QOP* = ∠*ROP*

= 180° – (90° + 23°)

= 180° – 113° = 67°

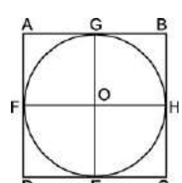
Hence ∠*QOR* = ∠*QOP* + ∠*ROP*

= 67° + 67°

= 134° **Ans.**

25. Find the perimeter of a square circumscribing a circle of radius *a* cm.

Sol. Let *ABCD* be the square that circumscribes a circle with centre *O*, touching it at the points *E*, *F*, *G* and *H*.



Given, the radius of the circle = *a* cm

Thus, *OE* = *OF* = *OG* = *OH* = *a* cm

Hence,

$OG + OE = AD = BC = 2a$

and $OF + OH = AB = DC = 2a$

So each side of the square = $2a$ cm

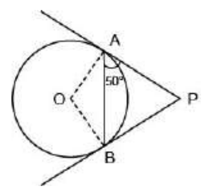
Hence, perimeter of the square

= $4(2a) = 8a$ cm. **Ans.**

26. From an external point *P*, tangents *PA* and *PB* are drawn to a circle with centre *O*. If $\angle PAB = 50°$, then find $\angle AOB$.

Sol. Since, tangents from an external point are equal

$\therefore AP = BP$



Given, $\angle PAB = 50°$

$\therefore \angle PBA = 50°$

[Angles opposite to equal sides are equal]

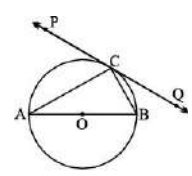
In ΔAPB,

$\angle APB = 180° - (50° + 50°) = 80°$

$\therefore \angle AOB = 180° - 80°$

= 100° **Ans.**

27. In the given figure, *PQ* is a tangent at a point *C* to a circle with centre *O*. If *AB* is a diameter and $\angle CAB = 30°$, find $\angle PCA$.

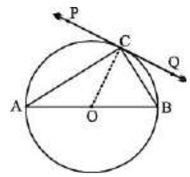


Sol. Given, $\angle CAB = 30°$ and *PQ* is a tangent at a point *C* to a circle with centre *O*.

Since, *AB* is a diameter

$\therefore \angle ACB = 90°$

Join *OC.*



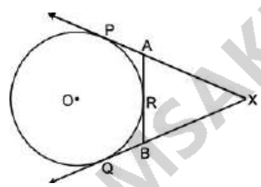
Now, $\angle CAO = \angle ACO = 30°$ [$\because OA = OC$]

and $\angle PCO = 90°$ (tangent is perpendicular to the radius through the point of contact)

$\therefore \angle PCA = \angle PCO - \angle ACO$

$= 90° - 30° = 60°$ **Ans.**

28. In the given figure, *XP* and *XQ* are two tangents to the circle with centre *O* drawn from an external point *X*. *ARB* is another tangent touching the circle at *R*. Prove that *XA* + *AR* = *XB* + *BR*.



Sol. As *XP* and *XQ* are two tangents from *X*, so

XP = *XQ* ...(i)

Also, *AP* = *AR* ...(ii)

[Tangents from the external point *A*]

and *BR* = *BQ* ...(iii)

[Tangents from the external point *B*]

Now, *XP* = *XQ* [from (i)]

⇒ *XA* + *AP* = *XB* + *BQ*

⇒ *XA* + *AR* = *XB* + *BR* [from (ii) and (iii)]

Hence Proved.

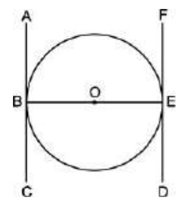
29. Prove that the tangents drawn at the end of any diameter are parallel.

Sol. As *AC* and *DF* are tangents to *B* and *E*, the opposite points of the diameter *BE*.

$\therefore \angle ABO = \angle OBC = \angle DEO = \angle OEF = 90°$

[the tangent at a point is always perpendicular

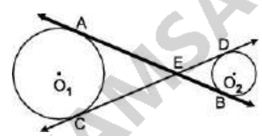
to the radius at that point]



Hence, $\angle ABO = \angle OED$, which are alternate interior angles.

AC || *DF*. **Hence Proved.**

30. In the given figure, *AB* and *CD* are common tangents to the two circles and intersect at *E*. Prove that *AB* = *CD*.



Sol. Given, *AB* and *CD* are tangents to circles with centres O_1 and O_2, respectively

$AB = AE + EB$ and $CD = CE + ED$

Now, as *E* is the external point from which *AE* and *CE* are tangents to the circle with centre O_1, so

$AE = CE$

Also, as *E* is the external point from which *ED* and *EB* are tangents to the circle with centre O_2, so

$EB = ED$

Thus, $AB = AE + EB = CE + ED = CD$.

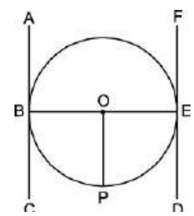
Hence Proved.

31. Prove that the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.

Sol. Given, *AC* || DC

Let *O* be the centre of the circle. Join *OB*, *OP*, *OE* such that

OB = *OP* = *OE* = *r*



Also, let *OP* || *BC* || *ED*

Thus, ∠*CBO* + ∠*BOP* = 180°

[co-interior angles]

or 90° + ∠*BOP* = 180°

[*OB* is perpendicular to *AC*]

or ∠*BOP* = 90°

Similarly, ∠*POE* = 90°

Thus, ∠*BOE* = ∠*POE* + ∠*POB*

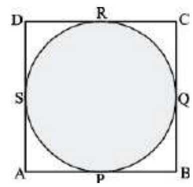
= 90° + 90°

= 180°

Hence, *BOE* is a striaght line passing through the centre *O*. **Hence Proved.**

32. A circle touches all the four sides of a quadrilateral *ABCD*. Prove that *AB* + *CD* = *BC* + *DA*.

Sol. Given : A quadrilateral *ABCD* and a circle touches its all the four sides at *P*, *Q*, *R*, and *S* respectively.



To prove : $AB + CD = BC + DA$

L.H.S. = $AB + CD$

= $AP + PB + CR + RD$

= $AS + BQ + CQ + DS$

[Tangents from same external points are always equal.]

= $(AS + SD) + (BQ + QC)$

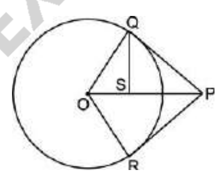
= $AD + BC$

= R.H.S. **Hence Proved.**

33. If from an external point *P* of a circle with centre *O*, two tangents *PQ* and *PR* are drawn such that ∠*QPR* = 120°, prove that 2*PQ* = *PO*.

Sol. Construct *QS* such that *QS* is perpendicular to *OP*. Let *OQ* be the radius such that *OQ* is perpendicular to *QP*. Hence

$\angle OQP = 90°$



But, $\angle QPR = 120°$

OP bisects ∠*QPR* such that

$\angle QPO = 60°$

Thus, $\angle QOP = 180° - (90° + 60°)$

= 30°

Now, in ΔOQP

$\sin 30° = \frac{QP}{OP}$

$\Rightarrow \frac{1}{2} = \frac{QP}{OP}$

$\Rightarrow OP = 2QP$

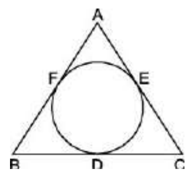
$\Rightarrow 2PQ = PO$. **Hence Proved.**

34. The incircle of an isosceles triangle *ABC*, in which *AB* = *AC*, touches the sides *BC*, *CA* and *AB* at *D*, *E* and *F* respectively. Prove that *BD* = *DC*.

Sol. Given,

$AB = AC$

As tangents from external points are equal in length.



$\therefore AF = AE$ [Tangents from same point *A*]

$BF = BD$ [Tangents from same point *B*]

and $DC = EC$ [Tangents from same point *C*]

As, $AB = AC$

$\Rightarrow AF + BF = AE + EC$

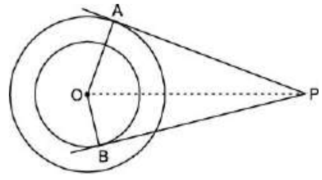
$\Rightarrow AF + BF = AF + EC$ [$\because AF = AE$]

$\Rightarrow BF = EC$

$\therefore BD = DC$[$\because BF = BD$ and $DC = EC$]

Hence Proved.

35. Tangents *PA* and *PB* are drawn from an external point *P* to two concentric circles with centre *O* and radii 8 cm and 5 cm respectively as shown in the given figure. If *AP* = 15 cm, find the length of *BP*.



Sol. Given, AP = 15 cm, OA = 8 cm and OB = 5 cm

Join OP.

In ΔAOP and ΔBOP,

Applying Pythagoras' theorem, we have

$AP^2 + OA^2 = OP^2 = OB^2 + BP^2$

$\Rightarrow (15)^2 + (8)^2 = (5)^2 + (BP)^2$

$\Rightarrow (BP)^2 = 225 + 64 - 25$

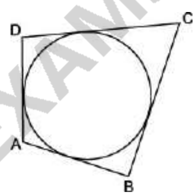
$\Rightarrow (BP)^2 = 225 + 39$

$\Rightarrow (BP)^2 = 264$

$\Rightarrow BP$ = 16.25 cm **Ans.**

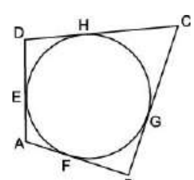
(as length cannot be negative)

36. In the given figure, a circle touches all the four sides of a quadrilateral *ABCD* whose sides are *AB* = 6 cm, *BC* = 9 cm and *CD* = 8 cm. Find the length of side *AD*.



Sol. Given, AB = 6 cm, BC = 9 cm and CD = 8 cm.

Let the circle touch AD at E, AB at F, BC at G and CD at H.



Also let $ED = x$ cm

$\Rightarrow DH = x$ cm

[Q DH and DE are tangents from point D]

$CH = (8 – x)$ cm

$\Rightarrow CG = (8 – x)$ cm

[Q CH and CG are tangents from point C]

$BG = [9 – (8 – x)]$ cm

$= (1 + x)$ cm

$FB = (1 + x)$ cm

[$\because$ BG and FB are tangents from point B]

$AF = [6 – (1 + x)]$ cm

$= (5 – x)$ cm

$AE = (5 – x)$ cm

[$\because$ AF and AE are tangents from point A]

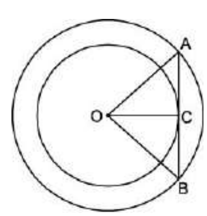
Now, $AD = AE + DE = [x + (5 – x)]$ cm

= 5 cm. **Ans.**

37. Two concentric circles are of radii 7 cm and *r* cm respectively, where *r* > 7. A chord of the larger circle of length 48 cm touches the smaller circle. Find the value of *r*.

Sol. Given, $OA = OB = r$ cm, $OC = 7$ cm and $AB = 48$ cm

Now AB, the chord of the larger circle is the tangent of the smaller circle.



So, OC is perpendicular to AB and bisects AB at C.

Hence, $AC = CB = \frac{48}{2} = 24$ cm

Thus, $AO^2 = AC^2 + OC^2$

$\Rightarrow (AO)^2 = 24^2 + 7^2$

$= 576 + 49$

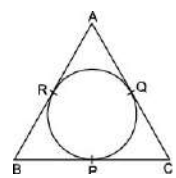
$= 625$

$\Rightarrow r = AO$

$= 25$ cm **Ans.**

38. A circle is inscribed in a ΔABC, touching *BC*, *CA* and *AB* at *P*, *Q* and *R* respectively. If *AB* = 10 cm, *AQ* = 7 cm and *CQ* = 5 cm, then find the length of *BC*.

Sol. Given, $AB = 10$ cm, $AQ = 7$ cm and $CQ = 5$ cm



Thus, $PC = CQ = 5$ cm

[As *PC* and *CQ* are tangents

from the same point *C*]

Also, $AQ = AR = 7$ cm

[As *AR* and *AQ* are tangents from

the same point *A*]

Then, $BR = (10 - 7)$ cm $= 3$ cm

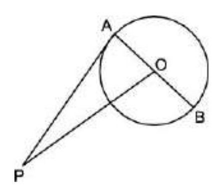
Also, $BP = BR = 3$ cm

[As *BP* and *BR* are tangents

from the same point *B*]

Hence, $BC = BP + PC = (3 + 5) = 8$ cm. **Ans.**

39. In the given figure, *PA* is a tangent from an external point *P* to a circle with centre *O*. If $\angle POB = 115°$, find $\angle APO$.



Sol. Given, *PA* is a tangent to the circle with centre *O*.

Thus, $\angle PAO = 90°$

[*OA*, the radius is perpendicular to *PA*]

Now, $\angle AOP = 180° - 115° = 65°$

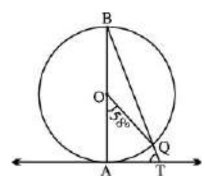
[Q $\angle AOP$ and $\angle BOP$ are supplementary angles]

In D*APO*,

$\angle APO = 180° - (90° + 65°) = 180° - 155°$

$= 25°$ **Ans.**

40. In given figure, *AB* is the diameter of a circle with centre *O* and *AT* is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$.



Sol. Given, *AB* is a diameter of a circle with centre *O* and *AT* is a tangent, then

$\angle BAT = 90°$

Now, $\angle ABQ = \frac{1}{2} \angle AOQ$

[∵ Angle subtended by an arc on the circle

is half of the angle subtended at centre]

$\angle ABQ = \frac{1}{2} \times 58° = 29°$

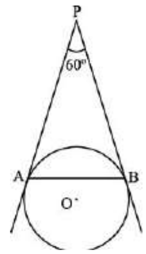
In D*ABT*,

$\angle ATQ = 180° - (\angle ABQ + \angle BAT)$

$= 180° - (29° + 90°)$

$\therefore \angle ATQ = 61°$ **Ans.**

41. In the given figure, *AP* and *BP* are tangents to a circle with centre *O*, such that *AP* = 5 cm and $\angle APB = 60°$. Find the length of chord *AB*.



Sol. Given, *AP* and *BP* are tangents to a circle with centre *O*.

∴ *AP* = *BP*

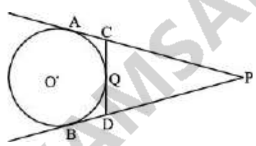
Now, ∠*APB* = 60° [given]

∴ ∠*PAB* = ∠*PBA* = 60° [∵AP= *BP*]

Thus, D*APB* is an equilateral triangle.

Hence, the length of chord *AB* is equal to the length of *AP i.e*. 5 cm.**Ans.**

42. In the given figure, *PA* and *PB* are tangents to the circle from an external point *P. CD* is another tangent touching the circle at *Q*. If *PA* = 12 cm, *QC* = *QD* = 3cm, then find *PC* + *PD.*



Sol. Given, *PA* = *PB* = 12 cm

[Tangents from same point *P*]

AC = *CQ* = 3 cm

[Tangents from same point *C*]

BD = *QD* = 3 cm

[Tangents from same point *D*]

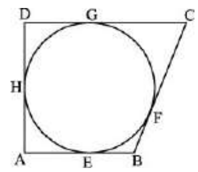
So, *PC* + *PD* = (*PA* – *AC*) + (*PB* – *BD*)

= (12 – 3) + (12 – 3)

= 9 + 9

= 18 cm. **Ans.**

43. A village Panchayt constructed a circular tank to serve as a bird bath. A fencing was made in the shape of a quadrilateral. Sides of the quadrilatieral touched the circle as shown in the figure.



If *AB* = 5 m, *CD* = 6 m, *BC* = 7 m, then find *AD*.

Sol. Since, tangent from an external point are equal

$AE = AH$

$BE = BF$

$GC = CF$

$DG = DH$

Then, $AB + CD = AE + BE + CG + DG$

$= AH + BF + CF + DH$

$= AD + BC$

$\Rightarrow AB + CD = BC + AD$

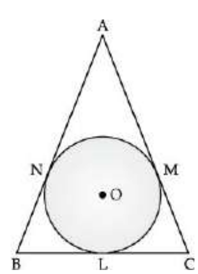
$\Rightarrow 5 + 6 = 7 + AD$

$\Rightarrow AD = 4$ m.**Ans.**

44. If D**ABC is isosceles with AB = AC and C (0, *r*) is the incircle of the** D**ABC touching BC at L. Prove that L bisects BC.**

Sol. Given : In ΔABC, AB = AC and a circle with centre O and radius *r* touches the side BC of ΔABC at L.

To Prove : L is mid-point of BC.



Proof :

AM and AN are the tangents to the circle from A.

So, AM = AN

But AB = AC (given)

AB – AN = AC – AM

⇒ BN = CM

Now BL and BN are the tangents from B to the circle

So, BL = BN

Similarly, CL = CM

But BN = CM (proved above)

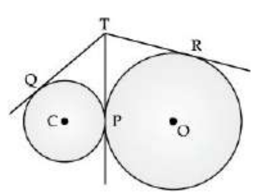
So, BL = CL

Therefore, L is mid-point of BC.

45. Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that TQ = TR.

Sol. Given : Two circles with centres O and C touch each other externally at P. PT is its common tangent. From point T, PT, TR and TQ are the tangents drawn to the circles.

To Prove : TQ = TR



Proof :

From T, TR and TP are two tangents to the circle with centre O

So, TR = TP ....(i)

Similarly, from point T, TQ and TP are two tangents to the circle with centre C

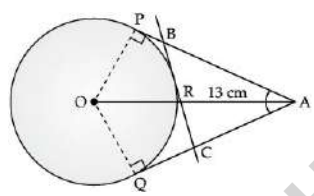
TQ = TP ....(ii)

From (i) and (ii), we get

TQ = TR **Hence proved.**

46. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC.

Sol. Two tangents are drawn from an external point A to the circle with centre O. Tangent BC is drawn at a point R and radius of circle = 5 cm.



We know that,

∠OPA = 90°

[Tangent at any point of a circle is perpendicular to the radius through the point of contact]

In ΔOAP,

$OA^2 = OP^2 + PA^2$

[By Pythagoras Theorem]

$\Rightarrow (13)^2 = 5^2 + PA^2$

$\Rightarrow PA^2 = 144 = 12^2$

$\Rightarrow PA = 12$ cm

Now, perimeter of ΔABC = AB + BC + CA

= (AB + BR) + (RC + CA)

= AB + BP + CQ + CA

[BR = BP, RC = CQ tangents from external point to a circle are equal]

= AP + AQ = 2AP = 2 × (12) = 24 cm

[AP = AQ tangent from external point to a circle are equal]

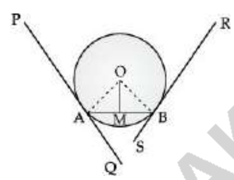
Therefore, the perimeter of ΔABC = 24 cm.

Short Answer Type Questions

47. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Sol. Given, a circle of radius *OA* and centred at *O* with chord *AB* and tangents *PQ* and *RS* are drawn from point *A* and *B* respectively.

Draw *OM* ⊥ *AB*, and join *OA* and *OB*.



Now, in Δ*OAM* and Δ*OMB*,

OA = *OB* [Radii]

OM = *OM* [Common]

∠*OMA* = ∠*OMB* [Each 90°]

∴ D*OAM* ≅ D*OMB* [R.H.S cong.]

∴ ∠*OAM* = ∠*OBM* ... (i) [By C.P.C.T.]

Also, ∠*OAP* = ∠*OBR* = 90° ...(ii)

[Line joining point of contact of tangent to centre is perpendicular on it].

On adding (i) and (ii),

∠*OAM* + ∠*OAP* = ∠*OBM* + ∠*OBR*

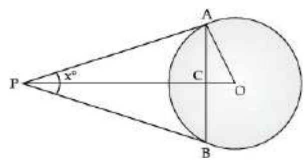
∴ ∠*PAB* = ∠*RBA* **Hence Proved.**

48. Two tangents *PA* and *PB* are drawn to a circle with centre *O* from an external point *P*. Prove that ∠*APB* = 2∠*OAB*.

[Board Question]

Sol. As *PA* and *PB* are two tangents drawn from an external point *P* on a circle with centre *O*, so

PA = *PB*



Construct *PO* such that *PO* is perpendicular to *AB* at *C* and bisects ∠*APB*.

Let ∠*APB* = $x°$

Thus, ∠*OPA* = $\frac{x°}{2}$

Also, as *AO* is perpendicular to *PA*, so

∠*OAP* = 90°

Hence, ∠*AOP* = $180° - \left(90° + \frac{x°}{2}\right)$

= $90° - \frac{x°}{2}$

But ∠*ACO* = 90°

[Q *CO* perpendicular to *AB*]

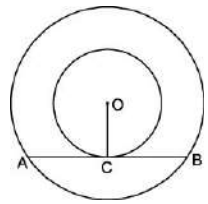
Thus, ∠*OAC* = $180° - \left[90° + 90° - \frac{x°}{2}\right]$

= $\frac{x°}{2}$ = ∠*OAB*

Hence, $\frac{\angle APB}{2}$ = ∠*OAB*

or ∠*APB* = 2∠*OAB*. **Hence Proved.**

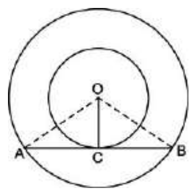
49. In the given figure, *O* is the centre of two concentric circles. *AB* is a chord of the larger circle touching the smaller circle at *C*. Prove that *AC* = *BC*.



Sol. Given, OC is the radius of the smaller circle and is perpendicular to AB because AB is a tangent to it.

Also, AO and OB are the radii of the larger circle.

Join, OA and OB.



Hence, in ΔOAC and ΔOBC,

$OC = OC$ [Common]

$OA = OB$ [Radii]

$\angle OCA = \angle OCB = 90°$ [Proved above]

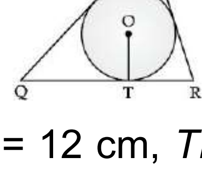
Hence $\Delta OAC \cong \Delta OBC$ [R.H.S.]

So, $AC = BC$

[Corresponding sides of congruent triangles]

Hence Proved.

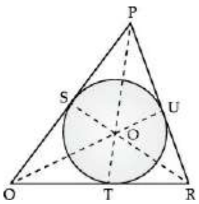
50. In the given figure, a D*PQR* is drawn to circumscribe a circle of radius 6 cm such that the segments *QT* and *TR* into which *QR* is divided by the point of contact are of lengths 12 cm and 9 cm respectively. If the area of D*PQR* = 189 cm^2, then find the lengths of sides *PQ* and *PR*.



Sol. Given, $OT = 6$ cm, $QT = 12$ cm, $TR = 9$ cm and area of ΔPQR

= 189 cm^2.

Let *PQ* and *PR* be tangents to the circle at *S* and *U* respectively.



Join *PO*, *QO* and *RO*.

So, *SO* = *UO* = *TO* = 6 cm

Now, *PS* = *PU* = *x* (say)

[Tangents from point *P*]

Also, *QS* = *QT* = 12 cm

[Tangents from point *Q*]

and *UR* = *RT* = 9 cm

[Tangents from point *R*]

Area of Δ*PQR* = Area of Δ*OQR* + Area of Δ*POQ* + Area of Δ*POR*

$\Rightarrow 189 = \frac{1}{2}[(12 + 9)6 + (12 + x)6 + (9 + x)6]$

$\Rightarrow 189 = \frac{6}{2}[(12 + 9) + (12 + x) + (9 + x)]$

$\Rightarrow 189 = 3[21 + 21 + 2x]$

$\Rightarrow 63 = 42 + 2x$

$\Rightarrow 2x = 21$

$\Rightarrow x = 10.5$ cm

Thus, the length of *PQ* = *PS* + *QS*

= (10.5 + 12) cm

= 22.5 cm

and the length of *PR* = *PU* + *UR*

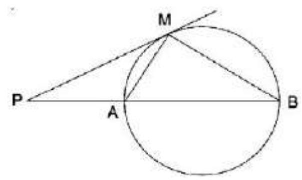
= (10.5 + 9) cm

= 19.5 cm. **Ans.**

51. In the figure, *PM* is a tangent to the circle and *PA* = *AM*. Prove that:

(i) Δ*PMB* is isosceles.

(ii) *PA* × *PB* = MB^2.



Sol. (i) In Δ*PAM*,

PA = *AM*(Given)

∠*APM* = ∠*AMP* …(i)

PA = *AM*

Also, ∠*ABM* = ∠*AMP*

(By alternate segment property of tangent) …(ii)

∴ ∠*APM* = ∠*ABM* [From (i) and (ii)]

∴ *PM* = *MB*

i.e., Δ*PMB* is an isosceles. **Hence Proved.**

(ii) By rectangle property of tangent and chord

$PM^2 = PA \times PB$

∴ $MB^2 = PA \times PB$ [∵ *PM* = *MB*]

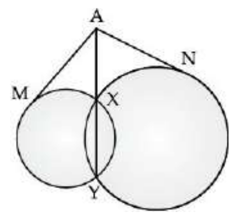
Hence Proved.

52. If from any point on the common chord of two intersecting circles, tangents be drawn to the circles, prove that they are equal.

Sol. Given : The two circles that intersect at points X and Y. XY is the common chord.

Suppose ‘A’ is a point on the common chord and AM and AN be the tangents drawn from A to the circle.

To Prove : AM = AN.



Proof :

In order to prove the above relation, following property has to be used.

Now AM is the tangent and AXY is a secant

$AM^2 = AX \times AY$ ....(i)

Similarly, AN is a tangent and AXY is a secant

$AN^2 = AX \times AY$ ....(ii)

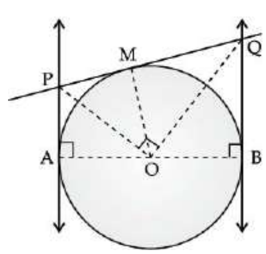
From (i) and (ii), we have $AM^2 = AN^2$

AM = AN

Therefore, tangents drawn from any point on the common chord of two intersecting circles are equal. **Hence Proved.**

53. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at centre.

Sol. Consider a circle with centre ‘O’ and has two parallel tangents through A and B at ends of diameter.



Let tangent through M intersect the parallel tangents at P and Q.

Then, we have to prove that ∠POQ = 90°.

From figure it is clear that ABQP is a quadrilateral

∠A + ∠B = 90° + 90° = 180° [At point of contact tangent and radius are perpendicular]

By angle sum property of quadrilateral

∠A + ∠B + ∠P + ∠Q = 360°

∠P + ∠Q = 360° – 180° = 180° ...(i)

Now, at points P and Q

∠APO = ∠OPQ = $\frac{1}{2}$ ∠P ...(ii)

∠BQO = ∠PQO = $\frac{1}{2}$ ∠Q ...(iii)

Using (ii) and (iii) in (i), we get

2∠OPQ + 2∠PQO = 180°

⇒ ∠OPQ + ∠PQO = 90° ...(iv)

In ΔOPQ,

∠OPQ + ∠PQO + ∠POQ = 180°

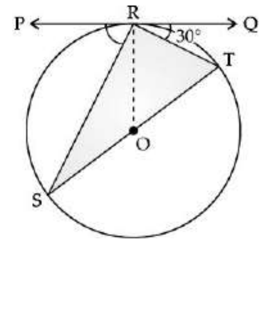
[Angle sum property]

90° + ∠POQ = 180° [from (iv)]

⇒ ∠POQ = 180° – 90° = 90°

Hence, ∠POQ = 90°.

54. In the given figure, PQ is tangent at point R of the circle with center O. If ∠TRQ = 30°, find ∠PRS.



Sol. Given, $\angle TRQ = 30°$

At point R, $OR \perp PQ$

So, $\angle ORQ = 90°$

$\Rightarrow \angle TRQ + \angle ORT = 90°$

$\Rightarrow \angle ORT = 90° - 30° = 60°$

Since ST is a diameter.

$\angle SRT = 90°$

Then,

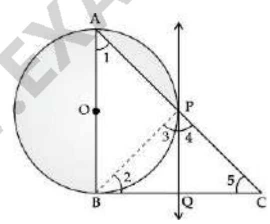
$\angle ORT + \angle SRO = 90° = \angle SRO + \angle PRS$

$\angle PRS = \angle ORT = 60°$.

55. In a right triangle ABC in which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.

Sol. Let O be the centre of the given circle. Suppose the tangent at P meets BC at Q. Join BP.

To Prove : BQ = QC



Proof :

Since, tangent at any point of circle is perpendicular to radius through the point of contact.

$\angle ABC = 90°$

In ΔABC, $\angle 1 + \angle 5 = 90°$

[by angle sum property, $\angle ABC = 90°$]

And, $\angle 3 = \angle 1$

[angle between tangent and the chord equals angle made by the chord in alternate segment]

Now, ∠3 + ∠5 = 90° ....(i)

Also, ∠APB = 90° [angle in semi-circle]

∠3 + ∠4 = 90° ...(ii)

[∠APB + ∠BPC = 180°, linear pair]

From (i) and (ii), we get

⇒ ∠3 + ∠5 = ∠3 + ∠4

or ∠5 = ∠4

⇒ PQ = QC

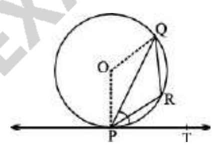
[sides opposite to equal angles are equal]

Also, QP = QB

[tangents drawn from an internal point to a circle are equal]

QB = QC **Hence Proved.**

56. In figure, *PQ* is a chord of a circle with centre *O* and *PT* is a tangent. If ∠*QPT* = 60°, find ∠*PRQ.*



Sol. Given, *O* is the centre of the given circle.

∴ *OQ* and *OP* are the radius of circle.

∵PT is a tangent

∴ *OP* ⊥ *PT*

So, ∠*OPT* = 90°

∴ ∠*OPQ* = 90° – ∠*QPT*

⇒ ∠*OPQ* = 90° – 60°[Given, ∠*QPT* = 60°]

⇒ ∠*OPQ* = 30°

$\therefore \angle OQP = 30°$

[$\because \angle OPQ$ is isosceles triangle]

Now, in D*OPQ*

$\angle POQ + \angle OPQ + \angle OQP = 180°$

$\Rightarrow \angle POQ + 30° + 30° = 180°$

$\Rightarrow \angle POQ = 120°$

reflex $\angle POQ = 360° - 120° = 240°$

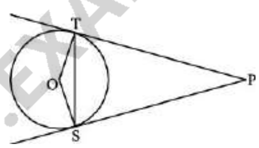
$\therefore \angle PRQ = \frac{1}{2} \angle POQ$

[$\because$ The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$\Rightarrow \angle PRQ = \frac{1}{2} \times 240°$

Hence, $\angle PRQ = 120°$. **Ans.**

57. In figure, from an external point *P*, two tangents *PT* and *PS* are drawn to a circle with centre *O* and radius *r*. If *OP* = 2*r*, show that ∠*OTS* = ∠*OST* = 30°.

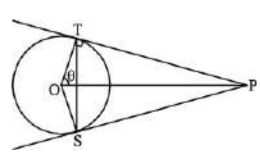


Sol. We have,

$OP = 2r$.

Let $\angle TOP = \theta$

In ΔOTP, $\cos \theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$



$\theta = 60°$

Hence, $\angle TOS = 2\theta$

$= 2 \times 60° = 120°$

In ΔTOS,

$\angle TOS + \angle OTS + \angle OST = 180°$

$\Rightarrow 120° + 2\angle OTS = 180°$

$[\because \angle OTS = \angle OST]$

$\Rightarrow 2\angle OTS = 180° - 120°$

$\Rightarrow \angle OTS = 30°$

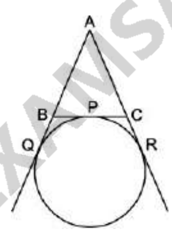
Hence, $\angle OTS = \angle OST = 30°$.

Hence Proved.

58. A circle is touching the side *BC* of ΔABC at *P* and touching *AB* and *AC* produced at *Q* and *R* respectively. Prove that $AQ = \frac{1}{2}$ (perimeter of ΔABC).

Sol. We have,

$AQ = AR$ [tangents from the point *A*]



$BQ = BP$

[tangents from the same point *B*]

and $PC = CR$

[tangents from the same point *C*]

Now, perimeter of ΔABC

$= AB + BC + AC$

$= AB + BP + PC + AC$

$= AB + BQ + CR + AC$

$= AQ + AR$

= 2*AQ*

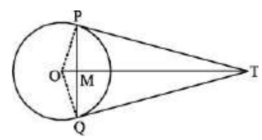
or $AQ = \frac{1}{2}$ (perimeter of ΔABC).

Hence Proved.

59. From a point *T* outside a circle of centre *O*, tangents *TP* and *TQ* are drawn to the circle. Prove that *OT* is the right bisector of line segment *PQ*.

Sol. Given : *TP* and *TQ* are the tangents drawn on a circle with centre *O*.

To prove : *OT* is the right bisector of *PQ*.



Proof : In ΔTPM and ΔTQM

$TP = TQ$ [Tangents drawn from external points are equal]

$TM = TM$ [Common]

[Angles opposite to equal sides]

$\angle TPM = \angle TQM$

$\therefore$ D*TPM* ≅ D*TQM* [By SAS congruence]

$\therefore PM = MQ$

and $\angle PMT = \angle QMT$ [By C.P.C.T.]

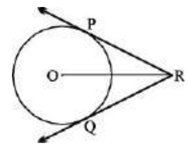
Since, *PMQ* is a straight line, then

$\angle PMT + \angle QMT = 180°$

$\therefore \angle PMT = \angle QMT = 90°$

$\therefore$ *OT* is the right bisector of *PQ*.**Hence Proved.**

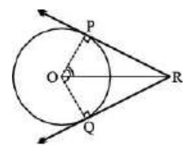
60. In figure, two tangents *RQ* and *RP* are drawn from an external point *R* to the circle with centre *O*. If $\angle PRQ = 120°$, then prove that *OR* = *PR* + *RQ*.



Sol. Given : *O* is the centre of the circle and ∠*PRQ* = 120°

Construction : Join *OP*, *OQ*

To prove : *OR* = *PR* + *RQ*.



Proof : We know that,

Tangent to a circle is perpendicular to the radius at the point of tangent *i.e.*, *OP* ⊥ *RP* and *OQ* ⊥ *RQ.*

∴ ∠*OPR* = ∠*OQR* = 90°

Now, in D*OPR* and D*OQR,*

OP = *OQ* [Radius of circle]

OR = *OR* [Common]

∠*OPR* = ∠*OQR* [Each 90°]

∴ D*OPR* ≅ D*OQR* [By RHS congruency]

So, *PR* = *QR* [By C.P.C.T.]

and ∠*ORP* = ∠*ORQ*

$= \frac{120°}{2} = 60°$

Now, in D*OPR*

$\cos 60° = \frac{PR}{OR}$ $\left[\because \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}\right]$

$\Rightarrow \frac{1}{2} = \frac{PR}{OR}$

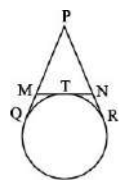
⇒ *OR* = 2*PR*

⇒ *OR* = *PR* + *PR*

⇒ $OR = PR + RQ$ [∵ PR = RQ]

Hence, $OR = PR + RQ$. **Hence Proved.**

61. Shipra prepared a project for rain water harvesting. diagrammatic representation of the project is given in the figure. *PQ* and *PR* are the pipes touching the circular pit. Length of these pipes is 5 m each. What is the perimeter of Δ*PMN*?



Sol. Here, $MT = MQ$ [Tangents from point *M*]

...(i)

and $NT = NR$ [Tangents from point *N*]

...(ii)

Now, $PQ + PR = PM + MQ + PN + NR$

$= PM + MT + PN + NT$ [Using eq. (i) and (ii)]

$= PM + PN + (MT + NT)$

$= PM + PN + MN$

= Perimeter of D*PMN*

∴ Perimeter of D*PMN* = 5 + 5

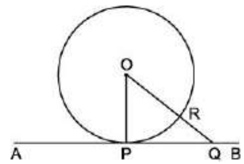
= 10 cm.**Ans.**

Long Answer Type Questions

62. Prove that the tangent at any point to a circle is perpendicular to the radius through the point of contact.

[Board Question]

Sol. Let *AB* be the tangent to the circle with centre *O* and let point of contact of the tangent and the circle be *P*.



Let Q be a point on AB, outside the circle and let R be the point of intersection of OQ with the circumference of the circle.

Thus, $OQ > OR$ …(i)

Also, $OR = OP$ …(ii)

[Both are radii of the circle with

with the same centre]

Now, $OQ > OP$ [From (i) and (ii)]

So, OP is perpendicular to AB as it is the shortest distance between the point O and the line AB.

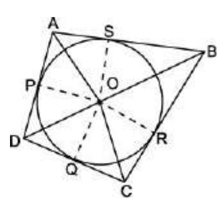
Hence Proved.

63. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

[NCERT]

[Board Question]

Sol. Given, a circle with centre O touches the opposite sides of a quadrilateral at points P, Q, R and S.



In order to prove the above, the following need to be satisfied :

$\angle AOB + \angle COD = 180°$ and $\angle AOD + \angle BOC = 180°$

Join OS, OP, OQ and OR.

In ΔOQC and ΔOCR,

$QC = CR$

[Tangents from external point C]

$OC = OC$ [Common]

$OQ = OR$ [Radii of circle]

Thus $\Delta OQC \cong \Delta OCR$ [by S.S.S. congruency]

Hence $\angle QOC = \angle COR$

[Corresponding angles of

congruent triangles]

Similarly,

$\angle ROB = \angle SOB$, $\angle SOA = \angle POA$

and $\angle POD = \angle QOD$

Now $\angle QOC + \angle COR + \angle ROB + \angle SOB + \angle SOA$

$+ \angle POA + \angle POD + \angle QOD = 360°$

$\Rightarrow 2(\angle QOC + \angle ROB + \angle SOA + \angle POD) = 360°$

and $2(\angle COR + \angle SOB + \angle POA + \angle QOD) = 360°$

$\Rightarrow \angle QOC + \angle ROB + \angle SOA + \angle POD = 180°$

and $\angle COR + \angle SOB + \angle POA + \angle QOD = 180°$

$\Rightarrow \angle QOC + \angle SOB + \angle SOA + \angle QOD = 180°$

and $\angle COR + \angle ROB + \angle POA + \angle POD = 180°$

$\Rightarrow \angle QOC + \angle QOD + \angle SOA + \angle SOB = 180°$

and $\angle COR + \angle ROB + \angle POA + \angle POD = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

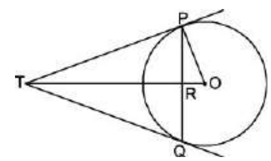
and $\angle AOD + \angle BOC = 180°$

Hence Proved.

64. In the given figure, *PQ* is a chord of length 16 cm and the radius of the circle is 10 cm. The tangents at *P* and *Q* intersect

at a point *T*. Find the length of *TP*.

[Board Question]



Sol. Given, $r = 10$ cm and length of chord $PQ = 16$ cm

As TO bisects PQ at R, so $PR = RQ = 8$ cm

Let $TP = x$ cm and $TR = y$ cm

Now $OR^2 = PO^2 - PR^2$

[By Pythagoras theorem as $\angle PRO = 90°$]

$\Rightarrow OR^2 = (10)^2 - (8)^2$

$\Rightarrow OR^2 = 100 - 64$

$\Rightarrow OR^2 = 36$

$\Rightarrow OR = 6$ cm

Thus, $TO = TR + RO = (y + 6)$ cm

Also $TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + 8^2$

$\Rightarrow x^2 = y^2 + 64$ ...(i)

Again $TO^2 = OP^2 + TP^2$

$\Rightarrow (y + 6)^2 = 10^2 + x^2$

$\Rightarrow (y + 6)^2 = 10^2 + y^2 + 64$ [From (i)]

$\Rightarrow y^2 + 36 + 12y = y^2 + 164$

$\Rightarrow 36 + 12y = 164$

$\Rightarrow 12y = 128$

$\Rightarrow 3y = 32$

$\Rightarrow y = \frac{32}{3}$ cm

Hence $TP^2 = x^2 = y^2 + 64$

$\Rightarrow TP^2 = x^2 = \left(\frac{32}{3}\right)^2 + 64$

$\Rightarrow x^2 = \frac{1024}{9} + 64$

$\Rightarrow x^2 = \frac{1024 + 576}{9}$

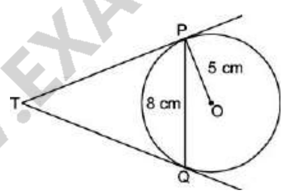
$= \frac{1600}{9}$

$\Rightarrow x = \frac{40}{3}$

Hence *TP* = 13.33 cm. **Ans.**

65. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.

[Board Question]



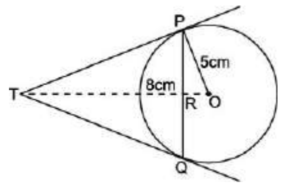
Sol. Join OT, let it intersect PQ at the point R

Now, ΔTPQ is an isosceles triangle and TO is the angle bisector of ∠PTQ. So, OT ⊥ PQ and therefore, OT bisects PQ

∴ PR = RQ = 4 cm

Also, OR = $\sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$

= $\sqrt{25 - 16}$ = $\sqrt{9} = 3$ cm



Now, ∠TPR + ∠RPO = 90° = ∠TPR + ∠PTR

[Q In DTRP, ∠TRP = 90°]

⇒ ∠RPO = ∠PTR

So, ΔTRP ~ ΔPRO (ByAA rule)

∴ $\frac{TP}{PO} = \frac{RP}{RO}$

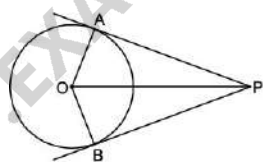
or $\frac{TP}{5} = \frac{4}{3}$ ⇒ TP = $\frac{20}{3}$ cm

Hence, the length of TP = $\frac{20}{3}$ cm **Ans.**

66. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

[Board Question]

Sol. Let *P* be the external point from which tangents *PA* and *PB* are drawn on the circle with centre *O*.



Thus, *OB* is perpendicular to *PB* and *OA* is perpendicular to *PA*.

Hence, ∠*OBP* = ∠*OAP* = 90°

In D*POB* and D*POA*,

OA = *OB* [Radii of the circle]

OP = *OP* [Common side]

∠*OBP* = ∠*OAP* [Each 90°]

∴ Δ*POB* ≅ Δ*POA* [S.A.S.]

So, *PA* = *PB* [By C.P.C.T.]

Hence Proved.

67. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

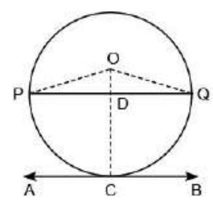
[Board Question]

Sol. Let *C* is the mid-point of the minor arc PQ and O is the centre of the circle and *AB* is tangent to the circle through point C.

To prove : PQ || AB

Proof : It is given that *C* is the mid-point of the arc PQ.

So, Minor arc PC = Minor arc QC



∠POC = ∠QOC

⇒ ∠POD = ∠QOD

In ΔPOD and ΔOQD,

OP = OQ = *r* (radius)

OD = OD (common)

∠POD = ∠QOD(proved above)

Thus, ΔOPD ≅ DOQD(By SAS congruency)

Hence ∠PDO = ∠QDO(By C.P.C.T.)

∠PDO + ∠QDO = 180° (linear pair)

⇒ 2∠PDO = 180°

⇒ ∠PDO = ∠QDO = 90°

⇒ ∠QDO = ∠OCB = 90°

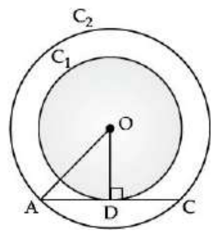
But they are corresponding angles.

Hence, PQ || AB.

68. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Sol. Let C_1 and C_2 be the two circles having same centre O.

And, AC is a chord which touches the circle C_1 at point D.



Join OD, So, OD ⊥ AC

[Since, perpendicular from centre bisects the chords]

AD = DC = 4 cm

Thus, in right angled ΔAOD,

$OA^2 = AD^2 + DO^2$

[By Pythagoras theorem]

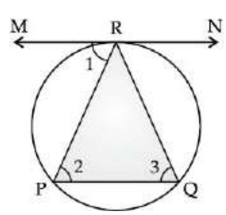
$\Rightarrow DO^2 = 5^2 - 4^2 = 25 - 16 = 9$

$\Rightarrow DO = 3$ cm

Therefore, the radius of the inner circle, OD = 3 cm.

69. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Sol. Given : Chord PQ is parallel to tangent at R.



To Prove : R bisects the arc PRQ.

Proof :

Since, PQ is parallel to tangent at R.

∠1 = ∠2 [alternate interior angles]

and ∠1 = ∠3

[Since, angle between tangent and chord is equal to angle made by chord in alternate segment]

Therefore, ∠2 = ∠3

⇒ PR = QR [Q sides opposite to equal

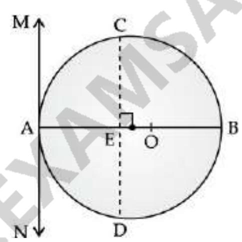
angles are equal]

Hence, clearly R bisects the are PRQ.

70. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

Sol. Given : AB is a diameter of the circle and a tangent is drawn from point A.

Construction : Draw a chord CD parallel to the tangent MAN.



So now, CD is a chord of the circle and OA is a radius of the circle.

Since, tangent at any point of a circle is perpendicular to the radius through the point of contact

∠MAO = 90°

∠CEO = ∠MAO [corresponding angles]

∠CEO = 90° (Since, ∠MAO = 90°)

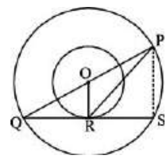
Therefore, OE bisects CD.

[As perpendicular from center of circle to chord bisects the chord]

Similarly, the diameter AB bisects all the chords which are parallel to the tangent at the point A.

71. The radii of two concentric circles are 13 cm and 8 cm. *PQ* is a diameter of the bigger circle. *QR* is a tangent to the smaller circle touching it at *R*. Find the length *PR*.

Sol. Let the line *QR* intersects the bigger circle at *S*.



Join *PS*.

O is the mid-point of *PQ*.

[∵PQ is a diameter of the bigger circle]

QR is a tangent to the smaller circle and *OR* is a radius through the point of contact *R*.

$\therefore OR \perp QR \Rightarrow OR \perp QS$

Since, *OR* is perpendicular to a chord *QS* of the bigger circle.

$\therefore QR = RS$

[∵ Perpendicular from the centre to a chord bisects the chord]

⇒ *R* is the mid-point of *QS*.

∴ In D*QSP*, *O* is the mid-point of *PQ* and *R* is the mid-point of *QS*.

$\therefore OR = \frac{1}{2}PS$

[∵ segment joining the mid-points of any two sides of a triangle is half of the third side]

$\Rightarrow PS = 2OR = 2 \times 8 \text{ cm} = 16 \text{ cm}$

In right D*OQR*,

$OR^2 + QR^2 = OQ^2$

$\Rightarrow 8^2 + QR^2 = 13^2$

$\Rightarrow 64 + QR^2 = 169$

$\Rightarrow QR^2 = 169 - 64 = 105$

$\Rightarrow QR = \sqrt{105}$

$\therefore RS = QR = \sqrt{105}$

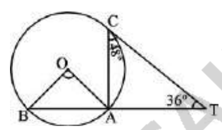
In D*PRS*,

$PR^2 = RS^2 + PS^2$

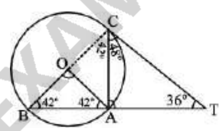
$= (\sqrt{105})^2 + 16^2$

$\Rightarrow$ *PR* = 19 cm. **Ans.**

72. *A*, *B* and *C* are three points on a circle. The tangent at *C* meets *BA* produced at *T*. Given that ∠*ATC* = 36° and that the ∠*ACT* = 48°, calculate the angle subtended by *AB* at the centre of the circle.



Sol. Let *O* be the centre of the circle. Join *OC*.



Since angle between the radius and the tangent is 90°,

$\therefore \angle OCT = 90°$

$\Rightarrow \angle OCA = \angle OCT - \angle ACT$

$= 90° - 48° = 42°$

and *OA* = *OC* = *OB*

[Radii of the same circle]

$\Rightarrow \angle OAC = \angle OCA = 42°$

[Angles opposite to equal sides are equal]

In Δ*ACT*,

exterior $\angle CAB = \angle ACT + \angle CTA$

$= 48° + 36° = 84°$

$\therefore \angle OAB = \angle CAB - \angle OAC$

$\Rightarrow \angle OAB = 84° - 42° = 42°$

$\Rightarrow \angle OBA = \angle OAB = 42°$

[$\because$OA = OB = radii]

In isosceles ΔAOB,

$\angle OBA + \angle OAB + \angle AOB = 180°$

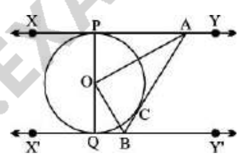
$\Rightarrow 42° + 42° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 180° - 84° = 96°$

Hence, the angle subtended by AB at centre is 96°.**Ans.**

73. In the given figure, *XY* and *X'Y'* are two parallel tangents to a circle with centre *O* and another tangent *AB* with point of contact *C* intersecting *XY* at *A* and *X'Y'* at *B*. Prove that ∠*AOB* = 90°.

[Board Question]



Sol. Join OC. In ΔAPO and ΔACO, we have

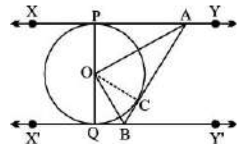
$AP = AC$

[Tangents drawn from external point A]

$AO = OA$ [Common]

$PO = OC$

[Radii of the same circle]



∴ $\Delta APO \cong \Delta ACO$ [By SSS criterion of congruence]

∴ $\angle PAO = \angle CAO$ [By C.P.C.T.]

⇒ $\angle PAC = 2\angle CAO$

Similarly, we can prove that

$\Delta OQB \cong \Delta OCB$

∴ $\angle QBO = \angle CBO$

⇒ $\angle CBQ = 2\angle CBO$

Now, $\angle PAC + \angle CBQ = 180°$

[Sum of interior angles on the same side of transversal is 180°]

⇒ $2\angle CAO + 2\angle CBO = 180°$

⇒ $\angle CAO + \angle CBO = 90°$

⇒ $180° - \angle AOB = 90°$

[$\because \angle CAO + \angle CBO + \angle AOB = 180°$]

⇒ $180° - 90° = \angle AOB$

⇒ $\angle AOB = 90°$. **Hence Proved.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the reason is true.

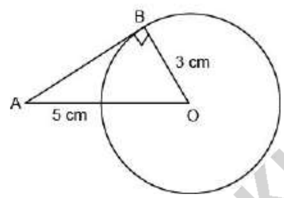
74. Assertion : If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent from that point will be 4 cm.

Reason : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

Let's construct a circle whose radius is 3 cm and distance of a point from the centre of a circle is5 cm



As per the above diagram, by joining the distant point A, we get DAOB and AB is the tangent to the given circle.

∵ tangent is perpendicular to the radius.

using the Pythagoras theorem,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular height})^2$

$\Rightarrow (OA)^2 = (AB)^2 + (OB)^2$

$\Rightarrow (AB)^2 = (OA)^2 - (OB)^2$

$= (5)^2 - (3)^2$

$= 25 - 9 = 16$

$\Rightarrow AB = \sqrt{16} = 4$ cm

75. Assertion : If two tangent segments are drawn to one circle from the same external point, then they are congruent.

Reason : If a line touches a circle, then perpendicular distance of the line from the centre of the circle is equal to the radius of the circle.

Ans. (b) Both the Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

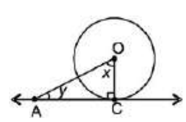
As per the two-tangent theorem, if two tangent segments are drawn to one circle from the same external point, then they are congruent. So, the assertion is proved to be correct.

For the reason, as per the tangent to a circle theorem 'A line is tangent to a circle if and only if the line is perpendicular to the radius drawn to the point of tangency.' So, the statement in the reason is also true. But they don't support or define each other.

Case Based Questions

76. For class 10 students, a teacher planned a game for the revision of chapter circles with some questions written on the board, which are to be answered by the students. For each correct answer, a student will get a reward. Some of the questions are given below.

(i) In the given figure, $x + y =$



(a) 60°

(b) 90°

(c) 120°

(d) 145°

Ans. (b) 90°.

Explanation :

∠OCA = 90°

[Since, radius at the point of contact is perpendicular to tangent]

In DOAC,

∠OCA + ∠OAC + ∠AOC = 180°

(Angle sum property)

⇒ 90° + ∠OAC + ∠AOC = 180°

⇒ ∠OAC + ∠AOC = 90°

⇒ $x + y = 90°$.

(ii) If PA and PB are two tangents drawn to a circle with centre O from P such that ∠PBA = 50°, then ∠OAB =

(a) 50°

(b) 25°

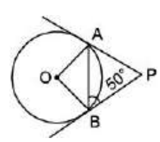
(c) 40°

(d) 130°

Ans. (c) 40°

Explanation :

Since, OB ^ PB [Since, radius at the point of contact is perpendicular to tangent]



and ∠PBA = 50° (Given)

∴ ∠OBA = 90° – 50°

= 40°

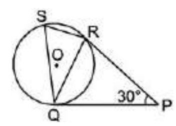
Also, OA = OB [Radii of circle]

∴ ∠OAB = ∠OBA

= 40°

[Angle opposite to equal sides are equal]

(iii) In the given, figure PQ and PR are two tangents to the circle, then ∠ROQ =



(a) 30°

(b) 60°

(c) 105°

(d) 150°

Ans. (d) 150°.

Explanation :

In quadrilateral OQPR,

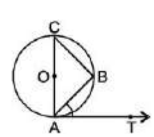
∠ROQ + ∠RPQ = 180°

[Angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the point of contact at the centre]

⇒ ∠ROQ = 180° – 30°

= 150°

(iv) In the adjoining figure, AB is a chord of the circle and AOC is its diameter such that ∠ACB = 55°, ∠BAT =



(a) 35°

(b) 55°

(c) 125°

(d) 110°

Ans. (b) 55°

Explanation :

Here, ∠ABC = 90° (Angle in a semicircle)

So, in DABC, ∠BAC = 180° – 90° – 55° = 35°

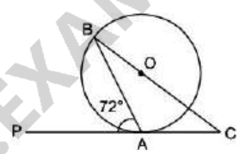
Also, ∠OAT = 90°

⇒ ∠BAT + ∠OAB = 90°

⇒ ∠BAT = 90° – 35°

= 55°

(v) In the adjoining figure, if PC is the tangent at A of the circle with ∠PAB = 72° and ∠AOB = 132°, then ∠ABC =



(a) 18°

(b) 30°

(c) 60°

(d) can't be determined

Ans. (b) 30°.

Explanation :

Here, ∠PAB = 72°

∴ ∠OAB = 90° – 72° = 18°

Also, $\angle AOB = 132°$ [Given]

Now, in DOAB,

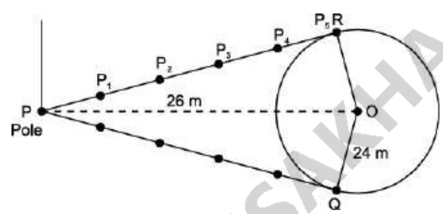
$\angle ABC = 180° - 132° - 18°$

$= 30°$.

Passage Based Questions

77. Read the following passage and answer the questions that follows :

There is a circular park of radius 24 m and a pole at a distance of 26 m from the centre of the park, as shown in the figure. From the pole, there are two paths PR and PQ, which are tangential to the park.



(i) Find the length of each path.

Sol. In right-angled triangle *PRO*

We have,

$PR = \sqrt{PO^2 - OR^2}$

$= \sqrt{(26)^2 - (24)^2}$

$= \sqrt{676 - 576} = \sqrt{100}$

$= 10$ m.

As *PR* and *PQ* are tangents from same external point so, *PR* = *PQ* = 10 m. **Ans.**

(ii) If five light poles has to be put along each tangential path at equal distances, find the distance between each consecutive light pole.

Sol. As five poles has to be put along *PQ* and *PR* each.

Let the distance between consecutive poles be *x*.

Hence,

$5x = 10$

$\Rightarrow x = 2$ m.

Each pole is at a distance of 2 m. **Ans.**

Self-Assessment

78. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 13 cm. Find the length of PQ.

[NCERT]

Ans. 12 cm

79. The radius of the incircle of a triangle is 8 cm and the segments into which one side is divided by the point of contact are 12cm and 16cm. Determine the other two sides of the triangle.

Ans. 26 cm, 30 cm.

80. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that:

$\angle PTQ = 2\angle OPQ$.

[NCERT]

81. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents P and Q intersect at a point T. Find the length of TP.

[NCERT]

Ans. 6.67 cm.

82. Two concentric circles are of radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the smaller circle.

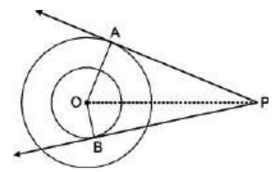
[NCERT]

Ans. 16 cm.

83. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of ΔPCD.

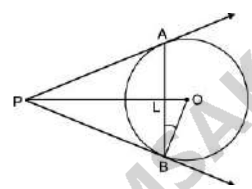
Ans. 28 cm.

84. In the given figure, there are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circle. If AP = 12 cm, find the length of BP.



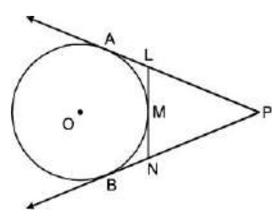
Ans. $4\sqrt{10}$ cm.

85. In the given figure, AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.



Ans. $\frac{40}{3}$ cm.

86. In Fig., PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at M. Prove that : PL + LM = PN + MN.



Introduction to Trigonometry

Chapter 10

Basic Concepts

INTRODUCTION TO TRIGONOMETRY

1. If triangle ABC is right-angled at B and $\angle BAC = \theta$, then with reference to the angle θ, we have

Base = AB, Perpendicular = BC, and Hypotenuse = AC, and

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$; $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$;

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$; $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}}$;

$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}}$; $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

2. $\frac{1}{\sin\theta} = \text{cosec}\,\theta$; $\sec\theta = \frac{1}{\cos\theta}$; $\cot\theta = \frac{1}{\tan\theta}$

3. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

4. The trigonometric ratios for angles 0°, 30°, 45°, 60°, and 90° are given in the table below :

Identity / Ratio	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

5. If θ is an acute angle, then

$\sin(90°-\theta) = \cos\theta$; $\cos(90°-\theta) = \sin\theta$;
$\tan(90°-\theta) = \cot\theta$; $\cot(90°-\theta) = \tan\theta$;
$\sec(90°-\theta) = \text{cosec}\,\theta$; $\text{cosec}(90°-\theta) = \sec\theta$;
$\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$;
$\tan(-\theta) = -\tan\theta$; $\sec(-\theta) = \sec\theta$;
$\text{cosec}(-\theta) = -\text{cosec}\,\theta$; $\cot(-\theta) = -\cot\theta$;

6. $\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\text{cosec}^2\theta - \cot^2\theta = 1$

Multiple Choice Questions

1. $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ **is equal to:**

[NCERT]

(a) tan 90°

(b) 1

(c) sin 45°

(d) sin 0°

Ans. (d) sin 0°

Explanation :

Given,

$\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-1}{1+1} = 0 = \sin 0^\circ$.

2. $\frac{1+\tan^2 A}{1+\cot^2 A}$ **equals to:**

[NCERT]

(a) $\sec^2 A$

(b) – 1

(c) $\cot^2 A$

(d) $\tan^2 A$

Ans. (d) $\tan^2 A$

Explanation :

Given,

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

$$= \frac{(\cos^2 A+\sin^2 A)\sin^2 A}{(\sin^2 A+\cos^2 A)\cos^2 A}$$

= $\frac{\sin^2 A}{\cos^2 A}$ ($\because$ $\sin^2 A + \cos^2 A = 1$)

= $\tan^2 A$.

3. $\frac{\sec 30^\circ}{\text{cosec } 60^\circ} = ?$

(a) $\frac{2}{\sqrt{3}}$

(b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

Ans. (d) 1

Explanation :

$$\frac{\sec 30^\circ}{\text{cosec } 60^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} = 1.$$

4. **If cos** θ **+ sin** q = $\sqrt{2}\cos\theta$**, then cos** θ **– sin** q **is:**

(a) – $\sqrt{2}\cos\theta$

(b) 2 sin θ

(c) $\sqrt{2}\sin\theta$

(d) $\sqrt{2}\tan\theta$

Ans. (c) $\sqrt{2}\sin\theta$

Explanation :

Given, cos θ + sin θ = $\sqrt{2}\cos\theta$...(i)

On squaring both sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta.\cos\theta = 2\cos^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

$\Rightarrow \cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{(\cos\theta + \sin\theta)}$

= $\frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta}$ [from Eq. (i)]

= $\sqrt{2}\sin\theta$

5. **If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $1 - 2\sin^2\theta$ is:**

(a) 0

(b) $\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $\frac{4}{3}$

Ans. (b) $\frac{2}{3}$

Explanation :

$\cos^4\theta - \sin^4\theta = \frac{2}{3}$
$\Rightarrow (\cos^2\theta)^2 - (\sin^2\theta)^2 = \frac{2}{3}$
$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$
$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{3}$ $(\because \sin^2\theta + \cos^2\theta = 1)$
$\Rightarrow 1 - \sin^2\theta - \sin^2\theta = \frac{2}{3}$

$(\because \cos^2\theta = 1 - \sin^2\theta)$

$1 - 2\sin^2\theta = \frac{2}{3}$.

6. **If $\sin\theta = \frac{a^2-1}{a^2+1}$, then the value of $\sec\theta + \tan\theta$ will be:**

(a) $\frac{a}{\sqrt{2}}$

(b) $\frac{a}{a^2+1}$

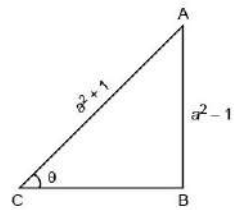
(c) $\sqrt{2a}$

(d) *a*

Ans. (d) *a*

Explanation :

$\sin\theta = \frac{a^2-1}{a^2+1}$



In DABC,

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$\Rightarrow BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{(a^2+1)^2 - (a^2-1)^2}$

$= \sqrt{a^4 + 1 + 2a^2 - a^4 - 1 + 2a^2}$

$= \sqrt{4a^2} = 2a$

$\sec\theta + \tan\theta = \frac{a^2+1}{2a} + \frac{a^2-1}{2a} = \frac{2a^2}{2a} = a$

7. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, then the value of $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ is:

(a) $\frac{25}{16}$

(b) $\frac{41}{9}$

(c) $\frac{41}{40}$

(d) $\frac{40}{41}$

Ans. (c) $\frac{41}{40}$

Explanation :

Given, $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$

$\Rightarrow \frac{\cos\theta\left(\frac{\sin\theta}{\cos\theta} + 1\right)}{\cos\theta\left(\frac{\sin\theta}{\cos\theta} - 1\right)} = \frac{5}{4}$

$\Rightarrow \frac{\tan\theta + 1}{\tan\theta - 1} = \frac{5}{4}$

⇒ $4 \tan \theta + 4 = 5 \tan \theta - 5$

⇒ $\tan \theta = 9$

On putting the value of tan θ, we get

$\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} = \frac{(9)^2 + 1}{(9)^2 - 1}$

$= \frac{81+1}{81-1} = \frac{82}{80}$

$= \frac{41}{40}$.

8. **If $\sec^2 \theta + \tan^2 \theta = \frac{7}{12}$, then $\sec^4 \theta - \tan^4 \theta$ is equal to:**

(a) $\frac{7}{12}$

(b) $\frac{1}{2}$

(c) $\frac{5}{12}$

(d) 1

Ans. (a) $\frac{7}{12}$

Explanation :

$\sec^2 q - \tan^2 q = 1$ [identity]
Also, $\sec^2 \theta + \tan^2 \theta = \frac{7}{12}$[given]

$\sec^4 \theta - \tan^4 \theta = (\sec^2 \theta - \tan^2 \theta)(\sec^2 \theta + \tan^2 \theta)$
$= 1 \times \frac{7}{12}$
$= \frac{7}{12}$.

9. **The numerical value of $\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right) \left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$ is:**

(a) 0

(b) – 1

(c) 1

(d) 2

Ans. (c) 1

Explanation :

$\left(\frac{1}{\cos\theta}+\frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta}-\frac{1}{\cot\theta}\right)$

$= \left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)$

$= \left(\frac{1+\sin\theta}{\cos\theta}\right)\left(\frac{1-\sin\theta}{\cos\theta}\right)$

$= \frac{1-\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta} = 1.$

10. **$\cot^2 A \cos^2 A$ is equal to:**

(a) $\cot^2 A + \cos^2 A$

(b) $\tan^2 A - \cos^2 A$

(c) $\cot^2 A - \cos^2 A$

(d) $\tan^2 A + \cos^2 A$

Ans. (c) $\cot^2 A - \cos^2 A$

Explanation :

$\cot^2 A \cos^2 A = \frac{\cos^2 A}{\sin^2 A} \times \cos^2 A$

$= \frac{\cos^4 A}{\sin^2 A} = \frac{\cos^2 A \times \cos^2 A}{\sin^2 A}$

$= \frac{\cos^2 A\,(1-\sin^2 A)}{\sin^2 A}$

$= \frac{\cos^2 A - \sin^2 A \cos^2 A}{\sin^2 A}$

$= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A$

$= \cot^2 A - \cos^2 A.$

11. If tan θ + cot θ = 2, then the value of $\tan^2\theta + \cos^2\theta$ is ....

(a) – 2

(b) 3

(c) + 1

(d) 2

Ans. (d) 2

Explanation :

tan θ + cot θ = 2

Squaring both sides,

$(\tan\theta + \cot\theta)^2 = 2^2$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta.\cot\theta = 4$

$\Rightarrow \tan^2\theta + \cot^2\theta = 4 - 2$

= 2

$\left(\because \tan\theta = \frac{1}{\cot\theta}\right)$

12. **If cos A = $\frac{4}{5}$, then the value of tan A is:**

[NCERT Exemplar]

(a) $\frac{3}{5}$

(b) $\frac{3}{4}$

(c) $\frac{4}{3}$

(d) $\frac{5}{3}$

Ans. (b) $\frac{3}{4}$

Explanation :

Given, $\cos A = \frac{4}{5}$

$\sin A = \sqrt{1-\cos^2 A}$

$\left[\because \sin^2 A + \cos^2 A = 1 \atop \therefore \sin A = \sqrt{1-\cos^2 A}\right]$

$= \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Now, $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$.

13. **Given that $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to:**

[NCERT Exemplar]

(a) $\frac{b}{\sqrt{b^2-a^2}}$

(b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2-a^2}}{b}$

(d) $\frac{a}{\sqrt{b^2-a^2}}$

Ans. (c) $\frac{\sqrt{b^2-a^2}}{b}$

Explanation :

Given, $\sin\theta = \frac{a}{b}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}]$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\left(\frac{a}{b}\right)^2} = \sqrt{1-\frac{a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$

Very Short Answer Type Questions

14. Find the value of $\frac{2\tan 30°}{1+\tan^2 30°}$.

[NCERT]

Sol. We have,

$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ = $\frac{2\frac{1}{\sqrt{3}}}{1+\frac{1}{3}}$

= $\frac{2\times 3}{\sqrt{3}\times 4}$ = $\frac{\sqrt{3}}{2}$ **Ans.**

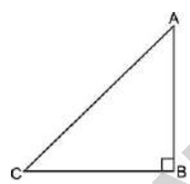
15. What happens to value of cos θ when θ increases from 0° to 90°?

Sol. cos 0° = 1, cos 90° = 0

When θ increases from 0° to 90°, the value of cos θ decreases from 1 to 0. **Ans.**

16. If DABC is right angled at B, what is the value of sin (A + C)?

Sol. ∠B = 90° [Given]



We know that in DABC,

∠A + ∠B + ∠C = 180°

[Angle sum property of a **D**]

⇒ ∠*A* + 90° + ∠C = 180°

⇒ ∠A + ∠C = 180° – 90°

⇒ ∠A + ∠C = 90°

⇒ sin (A + C) = sin 90°

= 1. **Ans.**

17. If $\sqrt{3}$ sin θ = cos θ, find the value of

$$\frac{3\cos^2\theta+2\cos\theta}{3\cos\theta+2}.$$

Sol. $\sqrt{3}$ sin θ = cos q [Given]

⇒ $\frac{\sin\theta}{\cos\theta}$ = $\frac{1}{\sqrt{3}}$ or $\tan\theta=\frac{1}{\sqrt{3}}$

⇒ tan θ = tan 30°

⇒ θ = 30°

Now,

$\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2} = \frac{\cos\theta(3\cos\theta + 2)}{(3\cos\theta + 2)} = \cos\theta$

Put θ = 30°

cos 30° = $\frac{\sqrt{3}}{2}$ **Ans.**

18. Find the value of 9 sec2 A – 9 tan2 A.

[NCERT]

Sol. Given,

9 sec2 A – 9 tan2 A = 9(sec2 A – tan2 A) = 9(1) [... sec2 θ – tan2 θ = 1]

= 9. **Ans.**

19. Find the value of

(1 + tan θ + sec θ) (1 + cot θ – cosec θ)

[NCERT]

Sol. Given,

(1 + tan θ + sec θ) (1 + cot θ – cosec θ)

= [1 + tan θ + sec θ + cot θ + tan θ cot θ + sec θ cot θ– cosec θ – tan θ cosec θ – sec θ cosec θ]

= [1 + tan θ + sec θ + cot θ + 1 + cosec θ – cosec θ – sec θ – sec θ cosec θ]

= [2 + tan θ + cot θ – sec θ cosec θ]

= 2 + $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta\cos\theta}$

= 2 + $\frac{\sin^2\theta + \cos^2\theta - 1}{\sin\theta\cos\theta}$

= 2 + $\frac{1-1}{\sin\theta\cos\theta}$ [... sin2 θ + cos2 θ = 1]

= 2. **Ans.**

20. Find (sec A + tan A) (1 – sin A).

[NCERT]

Sol. Given,

(sec A + tan A) (1 – sin A)

= $\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$(1 – sin A)

= $\left(\frac{1+\sin A}{\cos A}\right)$(1 – sin A)

= $\left(\frac{1-\sin^2 A}{\cos A}\right)$

= $\frac{\cos^2 A}{\cos A}$ [... 1 – sin2 A = cos2 A]

= cos A. **Ans.**

21. If *x* = 3 sin θ and *y* = 4 cos θ, find the value of $\sqrt{16x^2+9y^2}$.

Sol. Given, *x* = 3 sin θ

⇒ *x*2 = 9 sin2 θ

⇒ sin2 θ = $\frac{x^2}{9}$ ...(i)

and *y* = 4 cos θ

⇒ *y*2 = 16 cos2 θ

⇒ cos2 θ = $\frac{y^2}{16}$ ...(ii)

On adding equations (i) and (ii),

sin2 θ + cos2 θ = $\frac{x^2}{9}+\frac{y^2}{16}$

⇒ 1 = $\frac{x^2}{9}+\frac{y^2}{16}$ [... sin2 θ + cos2 θ =1]

⇒ 1 = $\frac{16x^2+9y^2}{144}$

⇒ 16*x*2 + 9*y*2 = 144

Taking square root of both sides

$\sqrt{16x^2+9y^2} = \sqrt{144}$

Thus, $\sqrt{16x^2+9y^2}$ = 12 **Ans.**

22. If cot θ = $\frac{15}{8}$, then evaluate

$$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}.$$

Sol. Given, cot θ = $\frac{15}{8}$

Now, $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

= $\frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$

= $\frac{1-\sin^2\theta}{1-\cos^2\theta}$

= $\frac{\cos^2\theta}{\sin^2\theta}$ (... $\sin^2\theta + \cos^2\theta = 1$)

= cot2 θ

= $\left(\frac{15}{8}\right)^2 = \frac{225}{64}$. **Ans.**

23. If sin A = $\frac{3}{4}$, calculate sec A.

[Board Question]

Sol. Given,

sin A = $\frac{3}{4}$

We know,

sin A = $\frac{P}{H} = \frac{3}{4}$

P = 3*k* and H = 4*k*

P2 + B2 = H2

[Applying Pythagoras theorem]

⇒ 9*k*2 + B2 = 16*k*2

⇒ B2 = 7*k*2

⇒ B = $\sqrt{7}k$

sec A = $\frac{H}{B} = \frac{4k}{\sqrt{7}k} = \frac{4}{\sqrt{7}}$ **Ans.**

24. If tan (A + B) = $\sqrt{3}$ and tan (A – B) = $\frac{1}{\sqrt{3}}$, find the value of A and B if 0° < A + B < 90° and A > B.

Sol. Given,

tan (A + B) = $\sqrt{3}$ = tan 60°

or A + B = 60° …(i)

and tan (A – B) = $\frac{1}{\sqrt{3}}$ = tan 30°

or (A – B) = 30° …(ii)

Adding (i) and (ii),

2A = 90°

or A = 45°

Thus in (i), A + B = 60°

or B = 60° – 45° = 15°

Hence, A = 45° and B = 15°. **Ans.**

25. Prove that: 2 cos2 θ + $\frac{2}{1+\cot^2\theta} = 2.$

Sol. Consider,

L.H.S. = 2 cos2 θ + $\frac{2}{1+\cot^2\theta}$

= 2 cos2 θ + $\frac{2\sin^2\theta}{\sin^2\theta+\cos^2\theta}$

$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

= 2(sin2 θ + cos2 θ) = 2 = R.H.S.

Hence Proved.

26. Prove that:

$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}$ **+ sin θ cos θ = 1.**

Sol. Consider,

L.H.S. = $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}$ + sin θ cos θ

= (sin2 θ + cos2 θ – sin θ cos θ) + sin θ cos θ

$\left[\because \frac{a^3+b^3}{a+b}=a^2+b^2-ab\right]$

= 1 = R.H.S. **Hence Proved.**

27. Prove that:

$$\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2\sec^2\theta}{\tan^2\theta-1}$$

Sol. Consider,

L.H.S. = $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

= $\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{\sin^2\theta-\cos^2\theta}$

= $\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

= $\frac{2(\sin^2\theta+\cos^2\theta)}{\sin^2\theta-\cos^2\theta}$

= $\frac{2}{\sin^2\theta-\cos^2\theta}$ = $\frac{2\sec^2\theta}{\tan^2\theta-1}$ [dividing both the numerator and denominator by cos2 θ]

= R.H.S **Hence Proved.**

28. Prove that:

$$\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$$

Sol. Consider,

L.H.S. = $\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right)$

= (1 + cot2 A) (1 + tan2 A)

= cosec^2 A sec^2 A

= $\frac{1}{\sin^2 A\cos^2 A}$

= $\frac{1}{\sin^2 A(1-\sin^2 A)}$

$= \frac{1}{\sin^2 A - \sin^4 A} = \text{R.H.S.}$ **Hence Proved.**

29. Prove that:

$$\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} \text{ = } \mathbf{cos}\ \theta \text{ + } \mathbf{sin}\ q.$$

Sol. Consider,

L.H.S. = $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$

$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta - \sin\theta}$

= cos θ + sin q = R.H.S.

Hence Proved.

30. Prove that:

$$\frac{\text{cosec}\,\theta}{\text{cosec}\,\theta - 1} + \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta + 1} = 2\sec 2\,\theta.$$

Sol. Consider,

L.H.S. = $\frac{\text{cosec}\,\theta}{\text{cosec}\,\theta - 1} + \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta + 1} = \frac{\text{cosec}\,\theta(\text{cosec}\,\theta + 1) + \text{cosec}\,\theta(\text{cosec}\,\theta - 1)}{\text{cosec}^2\theta - 1}$

$= \frac{2\,\text{cosec}^2\theta}{\text{cosec}^2\theta - 1} = \frac{\frac{2}{\sin^2\theta}}{\frac{1}{\sin^2\theta} - 1}$

$= \frac{2}{1 - \sin^2\theta} = \frac{2}{\cos^2\theta}$

= 2 sec2 θ = R.H.S. **Hence Proved.**

31. Prove that:

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta.$$

Sol. Consider,

L.H.S. = $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

= $\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$

= $\frac{\sin\theta(\sin^2\theta+\cos^2\theta-2\sin^2\theta)}{\cos\theta[(2\cos^2\theta-(\cos^2\theta+\sin^2\theta)]}$

= $\frac{\sin\theta(\cos^2\theta-\sin^2\theta)}{\cos\theta(\cos^2\theta-\sin^2\theta)}$ = tan θ = R.H.S. **Hence Proved.**

32. Prove that:

$$(\text{cosec}\,\theta - \cot\theta)2 = \frac{1-\cos\theta}{1+\cos\theta}.$$

Sol. Consider,

L.H.S. = (cosec θ – cot θ)2

= $\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

= $\frac{(1-\cos\theta)^2}{\sin^2\theta}$

= $\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$ [... sin2 θ = 1 – cos2 θ]

= $\frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$ = $\frac{(1-\cos\theta)}{(1+\cos\theta)}$ = R.H.S. **Hence Proved.**

33. Prove that:

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta.$$

Sol. Consider,

L.H.S. = $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ = $\sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$

= $\sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{1-\sin^2\theta}}$

= $\sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{\cos^2\theta}}$

= $\frac{(1+\sin\theta)}{\cos\theta} + \frac{(1-\sin\theta)}{\cos\theta}$

= 2 sec θ = R.H.S. **Hence Proved.**

34. Prove that:

sec θ (1 – sin θ) (sec θ + tan θ) = 1.

Sol. We have, sec θ (1 – sin θ) (sec θ + tan θ)

= (sec θ – sin θ sec θ) (sec θ + tan θ)

= (sec θ – tan θ) (sec θ + tan θ)

= sec2 θ – tan2 θ

= 1= R.H.S. [... sec2 θ – tan2 θ = 1]

Hence Proved.

35. Prove that:

sin θ (1 + tan θ) + cos θ (1 + cot θ) = sec θ + cosec θ.

Sol. Consider,

L.H.S. = sin θ (1 + tan θ) + cos θ (1 + cot θ)

= sin θ + sin θ tan θ + cos θ + cos θ cot θ

= sin θ + cos θ + $\frac{\sin^2\theta}{\cos\theta}+\frac{\cos^2\theta}{\sin\theta}$

= sin θ + cos θ + $\frac{1-\cos^2\theta}{\cos\theta}+\frac{1-\sin^2\theta}{\sin\theta}$

= sin θ + cos θ + $\frac{1}{\cos\theta}+\frac{1}{\sin\theta}$ – sin θ – cos θ

= sec θ + cosec θ = R.H.S. **Hence Proved.**

36. Prove that:

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta} = 2 \text{ cosec } \theta.$$

Sol. Consider,

L.H.S. = $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$

= $\frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta\,(1+\cos\theta)}$

= $\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta\,(1+\cos\theta)}$

= $\frac{2+2\cos\theta}{\sin\theta\,(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)} = \frac{2}{\sin\theta}$

= 2 cosec θ = R.H.S. **Hence Proved.**

37. Prove that:

[Board Question]

sin6 θ + cos6 θ = 1 – 3 sin2 θ cos2 θ.

Sol. Consider,

L.H.S. = sin6 θ + cos6 θ

= (sin2 θ)3 + (cos2 θ)3

= (sin2 θ + cos2 θ)3 – 3 sin2 θ cos2 θ (sin2 θ + cos2 θ)

= 13 – 3 sin2 θ cos2 q (1)

= 1 – 3 sin2 θ cos2 θ

= R.H.S. **Hence Proved.**

38. Prove that:

(1 – sin θ + cos θ)2 = 2(1 + cos θ)(1 – sin θ).

Sol. Consider,

L.H.S. = (1 – sin θ + cos θ)2

= [1 – (sin θ – cos θ)]2

= 1 + (sin θ – cos θ)2 – 2(sin θ – cos θ)

= 1 + sin2 θ + cos2 θ – 2 sin θ cos θ – 2(sin θ – cos θ)

= 2 – 2 sin θ cos θ – 2(sin θ – cos θ)

= 2[1 – sin θ cos θ – (sin θ – cos θ)]

R.H.S. = 2(1 + cos θ) (1 – sin θ)

= 2[1 + cos θ – sin θ – sin θ cos θ]

= 2[1 – sin θ cos θ – (sin θ – cos θ)]

Hence L.H.S. = R.H.S. **Hence Proved.**

39. Prove that:

$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$ **= cot θ.**

Sol. Consider,

L.H.S. = $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\cos\theta+\cos^2\theta}{\sin\theta\,(1+\cos\theta)}$

(∵ 1 – $\sin^2\theta = \cos^2\theta$)

= $\frac{\cos\theta\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)}$

= cot θ = R.H.S. **Hence Proved.**

40. If *x* = *a* sin θ + *b* cos θ and *y* = *a* cos θ – *b* sin θ, prove that *x*2 + *y*2 = *a*2 + *b*2.

Sol. Given,

x = *a* sin θ + *b* cos θ

and *y* = *a* cos θ – *b* sin θ

Now *x*2 + *y*2 = (*a* sin θ + *b* cos θ)2 + (*a* cos θ – *b* sin θ)2

= *a*2 sin2 θ + *b*2 cos2 θ + 2*ab* sin θ cos θ + *a*2 cos2 θ + *b*2 sin2 θ – 2*ab* sin θ cos θ

= *a*2(sin2 θ + cos2 θ) + *b*2(sin2 θ + cos2 θ)

= *a*2 + *b*2. **Hence Proved.**

41. If sec θ + tan θ = *m*, show that:

$\frac{m^2-1}{m^2+1}$ **= sin θ.**

Sol. Given, sec θ + tan θ = *m*

Thus, $\frac{m^2-1}{m^2+1}$

= $\frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1} = \frac{(1+\sin\theta)^2-\cos^2\theta}{(1+\sin\theta)^2+\cos^2\theta}$

= $\frac{1+\sin^2\theta+2\sin\theta-\cos^2\theta}{1+\sin^2\theta+2\sin\theta+\cos^2\theta}$

= $\frac{\sin^2\theta+\cos^2\theta+\sin^2\theta+2\sin\theta-\cos^2\theta}{2+2\sin\theta}$

[... sin2 **θ** + cos2 **θ** = 1]

$= \frac{2\sin^2\theta + 2\sin\theta}{2(1+\sin\theta)} = \frac{2\sin\theta\,(1+\sin\theta)}{2(1+\sin\theta)}$

= sin θ. **Hence Proved.**

42. If sin θ + sin2 θ = 1, prove that:

cos2 θ + cos4 θ = 1.

Sol. Given,

sin θ + sin2 θ = 1

⇒ sin θ + (1 – cos2 θ) = 1

⇒ sin θ = cos2 θ

⇒ sin2 θ = cos4 θ

⇒ 1 – cos2 θ = cos4 θ

⇒ cos2 θ + cos4 θ = 1. **Hence Proved.**

43. Evaluate : sin2 60° + 2 tan 45° – cos2 30°.

[Board Question]

Sol. We know,

sin2 60° = $\frac{\sqrt{3}}{2}$, tan 45° = 1 and cos 30° = $\frac{\sqrt{3}}{2}$

sin2 60° + 2 tan 45° – cos2 30°

$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4} + 2 - \frac{3}{4}$ = 2 **Ans.**

44. Solve:

$\frac{2}{3}$(cos^4 30° – sin^4 45°) – 3(sin^2 60° – sec^2 45°) + $\frac{1}{4}$(cot2 30°).

Sol. Consider,

$\frac{2}{3}$(cos4 30° – sin4 45°) – 3(sin2 60° – sec2 45°) + $\frac{1}{4}$(cot2 30°)= $\frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right] - 3\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{2}}{1}\right)^2\right] + \frac{1}{4}(\sqrt{3})^2$

$= \frac{2}{3}\left(\frac{9}{16}-\frac{1}{4}\right)-3\left(\frac{3}{4}-2\right)+\frac{3}{4}$

$= \frac{5}{24}+\frac{15}{4}+\frac{3}{4}=\frac{5}{24}+\frac{9}{2}=\frac{5+108}{24}=\frac{113}{24}$. **Ans.**

45. Prove that:

2 sec2 θ – sec4 θ – 2 cosec2 θ + cosec4 θ = cot4 θ – tan4 θ

Sol. Consider,

2 sec2 θ – sec4 θ – 2 cosec2 θ + cosec4 θ

= cosec4 θ – 2 cosec2 θ – (sec4 θ – 2 sec2 θ)

= cosec4 θ – 2 cosec2 θ + 1 – (sec4 θ – 2 sec2 θ + 1)

= (cosec2 θ – 1)2 – (sec2 θ – 1)2

= cot4 θ – tan4 θ. **Hence Proved.**

46. Prove that:

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$$

Sol. Consider, $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$

$= \frac{(\tan\theta+\sec\theta)+(\tan^2\theta-\sec^2\theta)}{\tan\theta-\sec\theta+1}$

$= \frac{(\tan\theta+\sec\theta)+(\tan\theta+\sec\theta)(\tan\theta-\sec\theta)}{\tan\theta-\sec\theta+1}$

$= \frac{(\tan\theta+\sec\theta)[1+(\tan\theta-\sec\theta)]}{\tan\theta-\sec\theta+1}$

= (tan θ + sec θ) = $\frac{1+\sin\theta}{\cos\theta}$ · **Hence Proved.**

47. Prove that:

(1 – sin θ + cos θ)2 = 2(1 + cos θ) (1 – sin θ).

Sol. Consider,

L.H.S. = (1 – sin θ + cos θ)2

= [(1– sin θ) + cos θ]2

= (1 – sin θ)2 + cos2 θ + 2 cos θ (1 – sin θ)

= 1 + sin2 θ – 2 sin q + cos2 θ + 2 cos θ – 2 sin θ cos θ

= 1 + ($\sin^2 \theta + \cos^2 \theta$) – 2 sin θ + 2 cos θ– 2 sin θ cos θ

= 2 – 2 sin θ + 2 cos q – 2 sin q cos θ

= 2(1 – sin θ) + 2 cos θ (1 – sin θ)

= 2(1 + cos θ) (1 – sin θ). **Hence Proved.**

48. If sec2 θ (1 + sin θ) (1 – sin θ) = *k*, find *k*.

Sol. Given, sec2 θ (1 + sin θ) (1 – sin θ) = *k*

⇒ sec2 θ (1 – sin2 θ) = *k*

⇒ sec2 θ cos2 θ = *k* $\left(\because \sec\theta = \frac{1}{\cos\theta}\right)$

⇒ *k* = 1. **Ans.**

49. Show that

cos2 θ – sin2 θ = $\frac{2\tan\theta}{(1-\tan^2\theta)}$ is not an identity.

Sol. Taking θ = 30°

L.H.S. = cos2 θ – sin2θ = cos2 30° – sin2 30°

= $\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$ = $\frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{1}{2}$

R.H.S. = $\frac{2\tan\theta}{1-\tan^2\theta}$

= $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ = $\frac{2\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

= $\frac{3}{\sqrt{3}} = \sqrt{3}$

... L.H.S. ≠ R.H.S.

Hence given expresion is not an identity.

Hence Proved.

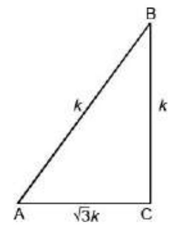
Short Answer Type Questions

50. In △ABC, it is given that ∠C = 90° and tan A $\frac{1}{\sqrt{3}}$. Find the value of (sin A cos B + cos A sin B).

Sol. Given,

tan A = $\frac{1}{\sqrt{3}}$ = $\frac{\text{Perpendicular}}{\text{Base}}$ = $\frac{BC}{AC}$

Let, AC = $\sqrt{3}k$ and BC = k



Applying Pythagoras' theorem

$AB^2 = BC^2 + AC^2$

$\Rightarrow AB^2 = (1k)^2 + (\sqrt{3}k)^2$

$= 1k^2 + 3k^2 = 4k^2$

$\Rightarrow AB = 2k$

So sin A = $\frac{BC}{AB} = \frac{1k}{2k} = \frac{1}{2};$

cos A = $\frac{AC}{AB} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2};$

sin B = $\frac{AC}{AB} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

and cos B = $\frac{BC}{AB} = \frac{1k}{2k}$ = $\frac{1}{2}$

Hence, (sin A cos B + cos A sin B)

= $\left[\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right]$

= $\frac{1}{4} + \frac{3}{4}$ = 1. **Ans.**

51. If 7 sin2 θ + 3 cos2 θ = 4, show that tan θ = $\frac{1}{\sqrt{3}}$.

Sol. Given

7 sin2 θ + 3 $\cos^2$ θ = 4

$\Rightarrow 4 \sin^2 \theta + 3(\sin^2 \theta + \cos^2 \theta) = 4$

$\Rightarrow 4 \sin^2 \theta + 3 = 4$

$\Rightarrow 4 \sin^2 \theta = 4 - 3 = 1$

$\Rightarrow \sin^2 \theta = \frac{1}{4}$

$\therefore \cos^2 \theta = 1 - \sin^2 \theta$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Hence, $\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$

$= \frac{1/4}{3/4} = \frac{1}{3}$

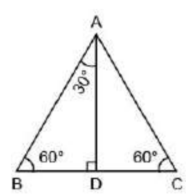
$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$. **Hence Proved.**

52. Find the value of sin 30° geometrically.

[Board Question]

Sol. To find the value of sin 30° geometrically, let us consider an equilateral ΔABC with each side having a length of 2*a*. So each angle equals 60°. Let AD be a perpendicular from A on BC. Since the triangle is equilateral, AD is the bisector of ∠A and D is the mid-point of BC.

Thus BD = DC = *a* and ∠BAD = 30°



Thus in ΔABD, ∠D is a right angle, AB = 2*a* is the hypotenuse and BD = *a* is the base.

Hence by using Pythagoras' theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + a^2$

$\Rightarrow AD^2 = 3a^2$

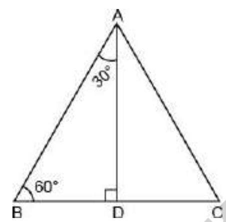
$\Rightarrow AD = \sqrt{3}a$

Thus $\sin 30° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$. **Ans.**

53. Find the value of sin 60° geometrically.

[Board Question]

Sol. To find the value of sin 60° geometrically, let us consider an equilateral ΔABC with each side having a length of 2*a*. So each angle equals 60°. Let AD be a perpendicular from A on BC. Since the triangle is equilateral, AD is the bisector of ∠A and D is the mid-point of BC.



Thus, BD = DC = *a* and ∠BAD = 30°

Thus in ΔABD, ∠D is a right angle, AB = 2*a* is the hypotenuse and BD = *a* is the base.

Hence by using Pythagoras' theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + a^2$

$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = \sqrt{3}a$

Thus, $\sin 60° = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$. **Ans.**

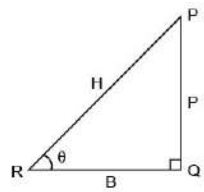
54. If $\sin\theta = \frac{12}{13}$, $0° < \theta < 90°$, find the value of:

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \frac{1}{\tan^2\theta}.$$

[Board Question]

Sol. Given, $\sin\theta = \frac{12}{13}$

$\Rightarrow \frac{P}{H} = \frac{12}{13}$



Let, P = 12K, H = 13K

$P^2 + B^2 = H^2$ [Pythagoras' theorem]

$\Rightarrow (12K)^2 + B^2 = (13K)^2$

$\Rightarrow 144K^2 + B^2 = 169K^2$

$\Rightarrow B^2 = 169\,K^2 - 144K^2$

$= 25\,K^2$

$\Rightarrow B = 5K$

$\therefore \cos\theta = \frac{B}{H} = \frac{5K}{13K} = \frac{5}{13}$

and $\tan\theta = \frac{P}{B} = \frac{12K}{5K} = \frac{12}{5}$

Now , $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \frac{1}{\tan^2\theta}$

$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$

$= \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144}$

$= \frac{119}{120} \times \frac{25}{144}$

$= \frac{595}{3456}$. **Ans.**

55. Prove that:

(1 + cot θ – cosec θ) (1 + tan θ + sec θ) = 2.

[Board Question]

Sol. Consider,

L.H.S. = (1 + cot θ – cosec θ) (1 + tan θ + sec θ)

$= \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$

$= \frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$

$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$

$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$

= R.H.S. **Hence Proved.**

56. A school organised maths quiz to reward students participating in co-curricular activities group and a group with 100% attendence.

One of the question asked is as follows:

If $\sin\theta + \sin^2\theta = 1$, then find the value of $\cos^2\theta + \cos^4\theta$.

Sol. Given,

$\sin\theta + \sin^2\theta = 1$ ...(i)

$\Rightarrow \sin\theta = 1 - \sin^2\theta$ ...(ii)

Now, $\cos^2\theta + \cos^4\theta$

$= \cos^2\theta(1 + \cos^2\theta)$

$= (1 - \sin^2\theta)(1 + 1 - \sin^2\theta)$

$= \sin\theta(1 + \sin\theta)$[Using (ii)]

= sin θ + $\sin^2\theta$ = 1[From (i)]

Ans.

57. Prove that:

$$\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta+\cot\theta}.$$

Sol. Consider,

L.H.S. = $\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}$

= $\frac{\sin\theta}{1-\cos\theta}-\frac{1}{\sin\theta}$

= $\frac{\sin^2\theta-1+\cos\theta}{(1-\cos\theta)\sin\theta}$

= $\frac{\cos\theta-(1-\sin^2\theta)}{(1-\cos\theta)\sin\theta}$

= $\frac{\cos\theta-\cos^2\theta}{(1-\cos\theta)\sin\theta}$

= $\frac{(1-\cos\theta)\cos\theta}{(1-\cos\theta)\sin\theta}$

= cot θ

R.H.S. = $\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta+\cot\theta}$

= $\frac{1}{\sin\theta}-\frac{\sin\theta}{1+\cos\theta}$

= $\frac{1+\cos\theta-\sin^2\theta}{(1+\cos\theta)\sin\theta}$

= $\frac{\cos\theta+(1-\sin^2\theta)}{(1+\cos\theta)\sin\theta}$

= $\frac{\cos\theta+\cos^2\theta}{(1+\cos\theta)\sin\theta}$

= $\frac{(1+\cos\theta)\cos\theta}{(1+\cos\theta)\sin\theta}$

= cot θ

Thus, L.H.S. = R.H.S. **Hence Proved.**

58. Prove that:

$(\tan^2 A - \tan^2 B) = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$.

Sol. Consider,

$(\tan^2 A - \tan^2 B)$

$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$= \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A(1 - \sin^2 B) - \sin^2 B(1 - \sin^2 A)}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$ …(i)

Also, $(\tan^2 A - \tan^2 B)$

$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$= \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B}$

$= \frac{\cos^2 B(1 - \cos^2 A) - \cos^2 A(1 - \cos^2 B)}{\cos^2 A \cos^2 B}$

$= \frac{\cos^2 B - \cos^2 A \cos^2 B - \cos^2 A + \cos^2 A \cos^2 B}{\cos^2 A \cos^2 B}$

$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$ …(ii)

Thus, from equations (i) and (ii),

$(\tan^2 A - \tan^2 B) = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$.

Hence Proved.

59. Prove that:

$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$.

Sol. Consider,

L.H.S. = $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

= $\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

= $\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

= $\frac{1-\cos^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{1-\sin^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

= $\frac{1}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos\theta}{\sin\theta-\cos\theta}$

$+\frac{1}{\sin\theta(\cos\theta-\sin\theta)}-\frac{\sin\theta}{\cos\theta-\sin\theta}$

= $\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{1}{\cos\theta}-\frac{1}{\sin\theta}\right]+\frac{\sin\theta-\cos\theta}{(\sin\theta-\cos\theta)}$

= $\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin\theta-\cos\theta}{\sin\theta\cos\theta}\right]+1$

= $\left[\frac{1}{\sin\theta\cos\theta}\right]+1$

= 1 + sec θ cosec θ

= R.H.S. **Hence Proved.**

60. Prove that:

$\frac{1-\sin\theta}{1+\sin\theta}$ **= (sec θ – tan θ)^2.**

Sol. Consider,

L.H.S. = $\frac{1-\sin\theta}{1+\sin\theta}$ = $\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

= $\frac{(1-\sin\theta)(1-\sin\theta)}{1-\sin^2\theta}$

= $\frac{(1-\sin\theta)^2}{\cos^2\theta}$

R.H.S. = (sec θ – tan θ)^2

= $\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2$

= $\frac{(1-\sin\theta)^2}{\cos^2\theta}$

Thus, L.H.S. = R.H.S. **Hence Proved.**

61. Prove that:

$\frac{\text{cosec }\theta + \cot\theta}{\text{cosec }\theta - \cot\theta}$ **= (cosec θ + cot θ)^{2} = 1 + 2 cot^{2} θ + 2 cosec θ cot θ.**

Sol. Consider,

$$\frac{\text{cosec }\theta + \cot\theta}{\text{cosec }\theta - \cot\theta}$$

$$= \frac{(\text{cosec }\theta + \cot\theta)(\text{cosec }\theta + \cot\theta)}{(\text{cosec }\theta - \cot\theta)(\text{cosec }\theta + \cot\theta)}$$

$$= \frac{(\text{cosec }\theta + \cot\theta)^2}{\text{cosec}^2\theta - \cot^2\theta}$$

= (cosec θ + cot θ)^{2}

[∵ cosec^{2} **θ** = 1 + cot^{2} **θ**] …(i)

1 + 2 cot^{2} θ + 2 cosec θ cot θ

= cosec^{2} θ + cot^{2} θ + 2 cosec θ cot θ

= (cosec θ + cot θ)^{2} ...(ii)

From equations (i) and (ii),

$\frac{\text{cosec }\theta + \cot\theta}{\text{cosec }\theta - \cot\theta}$ = (cosec θ + cot θ)^{2}

= 1 + 2 cot^{2} θ + 2 cosec θ cot θ.

Hence Proved.

62. Prove that:

$$\frac{\sin\theta + 1 - \cos\theta}{\cos\theta - 1 + \sin\theta} = \frac{1 + \sin\theta}{\cos\theta}.$$

Sol. Consider,

L.H.S. = $\frac{\sin\theta + 1 - \cos\theta}{\cos\theta - 1 + \sin\theta}$

$$= \frac{(\sin\theta - \cos\theta) + 1}{(\cos\theta + \sin\theta) - 1}$$

$$= \frac{[(\sin\theta - \cos\theta) + 1][(\cos\theta + \sin\theta) + 1]}{[(\cos\theta + \sin\theta) - 1][(\cos\theta + \sin\theta) + 1]}$$

(sin θ – cos θ) (sin θ + cos θ)= $\frac{+ (\sin\theta + \cos\theta) + (\sin\theta - \cos\theta) + 1}{(\cos\theta + \sin\theta)^2 - 1}$

$= \frac{\sin^2\theta - \cos^2\theta + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - \sin^2\theta - \cos^2\theta}$

[∵ $\sin^2 \theta + \cos^2 \theta = 1$]

$= \frac{2\sin^2\theta + 2\sin\theta}{2\sin\theta\cos\theta}$

$= \frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$. **Hence Proved.**

63. If tan θ + sin θ = *m* and tan θ – sin θ = *n*, show that ($m^2 - n^2$) = $\sqrt{mn}$.

Sol. Given,

tan θ + sin θ = *m* and tan θ – sin θ = *n*

Now, $(m^2 - n^2) = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta$

$= 4\tan\theta\sin\theta$

and $4\sqrt{mn} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{\tan^2\theta\left(1 - \frac{\sin^2\theta}{\tan^2\theta}\right)}$

$= 4\tan\theta\sqrt{1 - \frac{\sin^2\theta \times \cos^2\theta}{\sin^2\theta}}$

$= 4\tan\theta\sqrt{1-\cos^2\theta}$

$= 4\tan\theta\sqrt{\sin^2\theta}$

$= 4\tan\theta\sin\theta$

Thus, $(m^2 - n^2) = 4\sqrt{mn}$. **Hence Proved.**

64. If *a* = sin θ + cos θ; *b* = $\sin^3\theta + \cos^3\theta$, then show that (3*a* – 2*b*) = a^3.

Sol. Given,

$a = \sin\theta + \cos\theta$ ...(i)

$b = \sin^3\theta + \cos^3\theta$ ...(ii)

Here $(\sin\theta + \cos\theta)^2 = a^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = a^2$

$\Rightarrow \sin\theta\cos\theta = \frac{a^2-1}{2}$ ...(iii)

$[\because \sin^2\theta + \cos^2\theta = 1]$

Also, $(\sin\theta + \cos\theta)^3 = a^3$

$\Rightarrow \sin^3\theta + \cos^3\theta + 3\sin\theta\cos\theta(\sin\theta + \cos\theta) = a^3$

$\Rightarrow \sin^3\theta + \cos^3\theta + 3\left(\frac{a^2-1}{2}\right) a = a^3$

[From (i) and (iii)]

$\Rightarrow b + \frac{3a}{2}(a^2 - 1) = a^3$ [From (ii)]

$\Rightarrow 2b + 3a(a^2 - 1) = 2a^3$

$\Rightarrow 2b + 3a^3 - 3a = 2a^3$

$\Rightarrow (3a - 2b) = a^3$ **Hence Proved.**

65. If $a\cos\theta - b\sin\theta = c$, prove that

$(a\sin\theta + b\cos\theta) =$ $\sqrt{a^2+b^2-c^2}$.

Sol. Given, $a\cos\theta - b\sin\theta = c$ ...(i)

Now, $a^2 + b^2 - c^2 = a^2 + b^2 - (a\cos\theta - b\sin\theta)^2$

[From (i)]

$= a^2 + b^2 - (a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta)$

$= a^2 + b^2 - [a^2(1 - \sin^2\theta) + b^2(1 - \cos^2\theta) - 2ab\sin\theta\cos\theta]$

$= a^2 + b^2 - [a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\sin\theta\cos\theta]$

$= a^2 + b^2 - a^2 + a^2 \sin^2 \theta - b^2 + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta]$

$= a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta$

$= (a \sin \theta + b \cos \theta)^2$

Thus, $a^2 + b^2 - c^2 = (a \sin \theta + b \cos \theta)^2$

or $\sqrt{a^2 + b^2 - c^2} = (a \sin \theta + b \cos \theta)$

$\Rightarrow (a \sin \theta + b \cos \theta) = \sqrt{a^2 + b^2 - c^2}$.

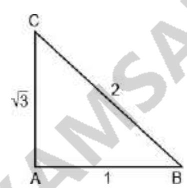
Hence Proved.

66. In a $\triangle ABC$, right-angled at A, if tan C = $\sqrt{3}$, find the value of sin B cos C + cos B sin C.

Sol. Given,

$\tan C = \sqrt{3} = \frac{AB}{AC}$

or $AB = \sqrt{3}AC$



Thus, $CB^2 = AB^2 + AC^2$

[Pythagoras' theorem]

$\Rightarrow CB^2 = (\sqrt{3}AC)^2 + AC^2 = 4AC^2$

$\Rightarrow CB = 2AC$

Hence $\sin B = \frac{AC}{BC} = \frac{\sqrt{3}}{2}$, $\cos C = \frac{AC}{BC} = \frac{\sqrt{3}}{2}$,

$\cos B = \frac{AB}{BC} = \frac{1}{2}$ and $\sin C = \frac{AB}{BC} = \frac{1}{2}$

So, sin B cos C + cos B sin C

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$. **Ans.**

67. If sec θ + tan θ = *p*, prove that sin θ = $\frac{p^2-1}{p^2+1}$.

[Board Question]

Sol. Given,

sec θ + tan θ = *p*

Now, R.H.S. = $\frac{p^2-1}{p^2+1}$

$= \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$

$= \frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}$

[∵ $(a+b)^2 = a^2 + b^2 + 2ab$]

$= \frac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+(1+\tan^2\theta)+2\sec\theta\tan\theta}$

$= \frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\sec^2\theta+2\sec\theta\tan\theta}$

$\left[\because \begin{array}{l}\sec^2\theta-1=\tan^2\theta\\ \sec^2\theta=1+\tan^2\theta\end{array}\right]$

$= \frac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$

$= \frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)} = \frac{\tan\theta}{\sec\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

= sin θ = L.H.S. **Hence Proved.**

68. If *x* = *a* sec θ + *b* tan θ and *y* = *a* tan θ + *b* sec θ, prove that $x^2 - y^2 = a^2 - b^2$.

[Board Question]

Sol. L.H.S. = $x^2 - y^2$

$= (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

$= a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta - 2ab \sec \theta \tan \theta$

$= a^2 \sec^2 \theta - a^2 \tan^2 \theta + b^2 \tan^2 \theta - b^2 \sec^2 \theta$

$= a^2 (\sec^2 \theta - \tan^2 \theta) + b^2 (\tan^2 \theta - \sec^2 \theta)$

$= a^2 - b^2$ = R.H.S. **Hence Proved.**

69. If cos A = $\frac{7}{25}$, find the value of tan A + cot A.

Sol. Given,

$\cos A = \frac{7}{25}$

$\Rightarrow \cos^2 A = \frac{49}{625}$

We know, $\sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin^2 A = 1 - \frac{49}{625} = \frac{625 - 49}{625} = \frac{576}{625}$

$\Rightarrow \sin A = \frac{24}{25}$

Also, $\tan A = \frac{\sin A}{\cos A}$

$= \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$

and $\cot A = \frac{1}{\tan A} = \frac{7}{24}$

Hence, $\tan A + \cot A = \frac{24}{7} + \frac{7}{24} = \frac{625}{168}$. **Ans.**

70. Prove that:

(sin θ + cosec θ)2 + (cos θ + sec θ)2 = 7 + tan2 θ + cot2 θ

[Board Question]

Sol. Consider,

L.H.S. = (sin θ + cosec θ)2 + (cos θ + sec θ)2

= sin2 θ + cosec2 θ + 2 sin θ cosec q + cos2 θ + sec2 θ + 2 cos θ sec θ

= (sin2 θ + cos2 θ) + (1 + cot2 θ) + (1 + tan2 θ) + 2(sin θ cosec θ + cos θ sec θ) = 1 + 1 + cot2 θ +1 + tan2 θ + 4

= 7 + cot2 θ + tan2 θ

= R.H.S. **Hence Proved.**

71. Prove that:

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A.$$

Sol. Consider,

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$$

$$= \left(\frac{\sin^2 A+\cos^2 A}{\sin^2 A+\cos^2 A}\right)\frac{\sin^2 A}{\cos^2 A}$$

= tan2 A …(i)

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2$$

$$= \left[\left(\frac{\cos A-\sin A}{\sin A-\cos A}\right)\left(\frac{\sin A}{\cos A}\right)\right]^2$$

$$= \left[\left(-\frac{\sin A-\cos A}{\sin A-\cos A}\right)(\tan A)\right]^2$$

= tan2 A …(ii)

From (i) and (ii),

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2$$

= $\tan^2$ A.

Hence Proved.

Long Answer Type Questions

72. In ∆OPQ right angled at P, OP = 7 cm , OQ – PQ = 1 cm. Determine the values of sin Q and cos Q.

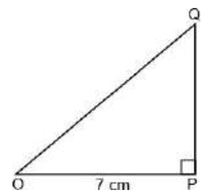
[NCERT]

Sol. In ΔOPQ, we have

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow (PQ + 1)^2 = OP^2 + PQ^2$

$[\because OQ - PQ = 1 \Rightarrow OQ = 1 + PQ]$



$\Rightarrow PQ^2 + 2PQ + 1 = OP^2 + PQ^2$

$\Rightarrow 2PQ + 1 = 7^2$

$\Rightarrow 2PQ + 1 = 49$

$\Rightarrow 2PQ = 48$

$\Rightarrow PQ = 24$ cm

and $OQ - PQ = 1$ cm

$OQ = (PQ + 1)$ cm

$= 25$ cm

Thus, $\sin Q = \frac{OP}{OQ} = \frac{7}{25}$

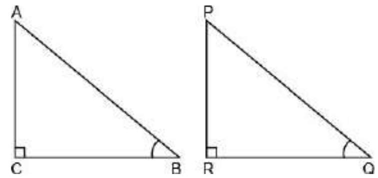
and $\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$. **Ans.**

73. If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.

[NCERT]

Sol. Consider two right angled triangles ABC and PQR such that $\sin B = \sin Q$.

We have, $\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$



... sin B = sin Q

⇒ $\frac{AC}{AB} = \frac{PR}{PQ}$ (∵ sin B = sin Q)

⇒ $\frac{AC}{PR} = \frac{AB}{PQ}$ = *k* (say), ...(i)

⇒ AC = *k* PR and AB = *k* PQ

In ∆ABC and ∆PQR by Pythagoras theorem, we have

$AB^2 = AC^2 + BC^2$

and $PQ^2 = PR^2 + QR^2$

⇒ BC = $\sqrt{AB^2 - AC^2}$

and QR = $\sqrt{PQ^2 - PR^2}$

⇒ $\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$

$= \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$

⇒ $\frac{BC}{QR} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}}$ = *k* ...(ii)

From (i) and (ii), we have

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

⇒ ∆ACB ~ ∆PRQ

⇒ ∠B = ∠Q. **Hence Proved.**

74. Prove the following trigonometric identity:

sin A (1 + tan A) + cos A (1 + cot A) = sec A + cosec A.

Sol. L.H.S. = sin *A* (1 + tan A) + cos A (1 + cot A)

$= \sin A\left(1+\frac{\sin A}{\cos A}\right)+\cos A\left(1+\frac{\cos A}{\sin A}\right)$

$= \sin A\left(\frac{\cos A + \sin A}{\cos A}\right) + \cos A\left(\frac{\sin A + \cos A}{\sin A}\right)$

$= (\sin A + \cos A)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= (\sin A + \cos A)\left(\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}\right)$

$= \frac{\sin A + \cos A}{\cos A \sin A}$ [... $\sin^2 A + \cos^2 A = 1$]

$= \frac{\sin A}{\cos A \sin A} + \frac{\cos A}{\sin A \cos A}$

$= \frac{1}{\cos A} + \frac{1}{\sin A}$ = sec A + cosec A

= R.H.S. **Hence Proved.**

75. Prove that:

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}.$$

[Board Question]

Sol. L.H.S. = $\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$

$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$

$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\sin^2 A \cdot \cos^2 A}}$

$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{1}{\sin^2 A} \times \frac{\sin^2 A \cdot \cos^2 A}{\sin^2 A - \cos^2 A}$

$= \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A}$

$= \frac{1}{\sin^2 A - \cos^2 A}$

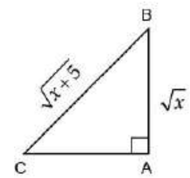
$= \frac{1}{1 - \cos^2 A - \cos^2 A}$

$= \frac{1}{1 - 2\cos^2 A}$

= R.H. S. [... $\sin^2 A = 1 - \cos^2 A$]

Hence Proved.

76. In the ΔABC (see figure), $\angle$A = right angle, AB = $\sqrt{x}$ and BC = $\sqrt{x+5}$. Evaluate



sin C. cos C. tan C + $\cos^2$ C. sin A.

[Board Question]

Sol. In ΔABC, by Pythagoras theorem,

$(\sqrt{x+5})^2 = (\sqrt{x})^2 + AC^2$

$\Rightarrow x + 5 = x + AC^2$

$\Rightarrow 5 = AC^2$

$\Rightarrow AC = \sqrt{5}$

sin C = $\frac{\sqrt{x}}{\sqrt{x+5}}$; cos C = $\frac{\sqrt{5}}{\sqrt{x+5}}$;

tan C = $\frac{\sqrt{x}}{\sqrt{5}}$

and sin A = sin 90°

= 1

Thus, sin C cos C tan C + $\cos^2$ C sin A

$= \frac{\sqrt{x}}{\sqrt{x+5}} \frac{\sqrt{5}}{\sqrt{x+5}} \frac{\sqrt{x}}{\sqrt{5}} + \left(\frac{\sqrt{5}}{\sqrt{x+5}}\right)^2 .1$

$= \frac{x}{x+5} + \frac{5}{x+5} = \frac{x+5}{x+5} = 1$. **Ans.**

77. If $\frac{\cos B}{\sin A} = n$ and $\frac{\cos B}{\cos A} = m$, then show that $(m^2 + n^2) \cos^2 A = n^2$.

[Board Question]

Sol. Given , $m = \frac{\cos B}{\cos A}$; $n = \frac{\cos B}{\sin A}$

So, $m^2 = \frac{\cos^2 B}{\cos^2 A}$; $n^2 = \frac{\cos^2 B}{\sin^2 A}$

L.H.S. = $(m^2 + n^2)\cos^2 A$

$= \left(\frac{\cos^2 B}{\cos^2 A} + \frac{\cos^2 B}{\sin^2 A}\right)\cos^2 A$

$= \frac{(\sin^2 A\cos^2 B + \cos^2 A\cos^2 B)}{\cos^2 A\sin^2 A} \times \cos^2 A$

$= \frac{\cos^2 B(\sin^2 A + \cos^2 A)}{\sin^2 A}$

$= \frac{\cos^2 B}{\sin^2 A}$ [... $\sin^2 A + \cos^2 A = 1$]

$= n^2$ = R.H.S. **Hence Proved.**

78. $(\tan A + \text{cosec } B)^2 - (\cot B - \sec A)^2 = 2\tan A\cot B(\text{cosec } A + \sec B)$.

Sol. L.H.S. = $(\tan A + \text{cosec } B)^2 - (\cot B - \sec A)^2$

$= \tan^2 A + \text{cosec}^2 B + 2\tan A.\text{cosec } B - (\cot^2 B + \sec^2 A - 2\cot B.\sec A) = \tan^2 A + \text{cosec}^2 B + 2\tan A.\text{cosec } B - \cot^2 B - \sec^2 A + 2\cot B\sec A$

$= (\text{cosec}^2 B - \cot^2 B) - (\sec^2 A - \tan^2 A) + 2\tan A.\text{cosec } B + 2\cot B\sec A$

$= 1 - 1 + 2\tan A.\text{cosec } B + 2\cot B.\sec A$

$= 2\tan A.\text{cosec } B + 2\cot B.\sec A$

$= 2\left(\frac{\sin A}{\cos A} \times \frac{1}{\sin B} + \frac{\cos B}{\sin B} \times \frac{1}{\cos A}\right)$

$= 2\left(\frac{\sin A + \cos B}{\cos A.\sin B}\right)$ ...(i)

R.H.S. = $2\tan A.\cot B.(\text{cosec } A + \sec B)$

$= 2\tan A.\cot B.\text{cosec } A + 2\tan A.\cot B.\sec B$

$= 2\frac{\sin A}{\cos A}\cdot\frac{\cos B}{\sin B}\cdot\frac{1}{\sin A} + \frac{2\sin A}{\cos A}\cdot\frac{\cos B}{\sin B}\cdot\frac{1}{\cos B}$

$= 2\left(\frac{\cos B}{\cos A.\sin B} + \frac{\sin A}{\cos A.\sin B}\right)$

$= 2\left(\frac{\cos B + \sin A}{\cos A . \sin A}\right)$ ...(ii)

From (i) and (ii),

L.H.S. = R.H.S. **Hence Proved.**

79. Prove that:

[Board Question]

$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ **= 1 + sec** q **cos** θ**.**

Sol. L.H.S. = $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ = $\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$

$= \frac{1}{\sin\theta - \cos\theta}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$

$= \frac{1}{\sin\theta - \cos\theta}\left[\frac{\sin^3\theta - \cos^3\theta}{\cos\theta . \sin\theta}\right]$

$= \frac{[\sin\theta - \cos\theta][\sin^2\theta + \cos^2\theta + \sin\theta . \cos\theta]}{(\sin\theta - \cos\theta)\cdot(\cos\theta \times \sin\theta)}$

[∵ $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$]

$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta\cdot\cos\theta}{(\cos\theta \times \sin\theta)}$

[∵ $\sin^2\theta + \cos^2\theta = 1$]

$= \frac{1 + \sin\theta . \cos\theta}{\cos\theta \times \sin\theta}$

$= \frac{1}{\cos\theta . \sin\theta} + \frac{\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$

= 1 + sec θ. cosec θ

$\left(\because \frac{1}{\cos\theta} = \sec\theta, \frac{1}{\sin\theta} = \text{cosec}\ \theta\right)$

Hence Proved.

80. If $\frac{x}{a}$ cos θ + $\frac{y}{b}$ sin θ = 1 and $\frac{x}{a}$ sin θ – $\frac{y}{b}$ cos θ = 1, prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 2.

[Board Question]

Sol. Given,

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta$ = 1

Squaring both sides, we get

$\frac{x^2}{a^2}\cos^2\theta+\frac{y^2}{b^2}\sin^2\theta+\frac{2xy}{ab}\cos\theta.\sin\theta=1$ ...(i)

and $\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta$ = 1(given)

Squaring both sides, we get

$\frac{x^2}{a^2}\sin^2\theta+\frac{y^2}{b^2}\cos^2\theta-\frac{2xy}{ab}\cos\theta\sin\theta=1$ …(ii)

Adding equations (i) and (ii), we get

$\frac{x^2}{a^2}(\cos^2\theta+\sin^2\theta)+\frac{y^2}{b^2}(\sin^2\theta+\cos^2\theta)=2$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 2 [∵ $\sin^2\theta+\cos^2\theta=1$]

Hence Proved.

81. If sec θ = *x* + $\frac{1}{4x}$, prove that

sec θ + tan θ = 2*x* or $\frac{1}{2x}$.

Sol. Given,

sec θ = $x+\frac{1}{4x}$

On squaring both sides, we get

$\sec^2\theta=\left(x+\frac{1}{4x}\right)^2$

We know that

$\tan^2\theta=\sec^2\theta-1$

$\Rightarrow \tan^2\theta=\left(x+\frac{1}{4x}\right)^2-1$

$\Rightarrow \tan^2\theta=x^2+\frac{1}{16x^2}+\frac{1}{2}-1$

⇒ $\tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$

⇒ $\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

⇒ $\tan\theta = \pm\left(x - \frac{1}{4x}\right)$

⇒ $\tan\theta = x - \frac{1}{4x}$ or $\tan\theta = -\left(x - \frac{1}{4x}\right)$

When $\tan\theta = \left(x - \frac{1}{4x}\right)$

Then, $\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$

When $\tan\theta = -\left(x - \frac{1}{4x}\right)$

Then $\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) - \left(x - \frac{1}{4x}\right) = \frac{2}{4x}$

$= \frac{1}{2x}$

Hence, $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$. **Hence Proved.**

82. If cosec θ – sin θ = *m* and sec θ – cos θ = *n*, prove that $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$.

Sol. Given,

$\text{cosec}\,\theta - \sin\theta = m$

⇒ $\frac{1}{\sin\theta} - \sin\theta = m$

⇒ $\frac{1-\sin^2\theta}{\sin\theta} = m$

⇒ $\frac{\cos^2\theta}{\sin\theta} = m$

Also, $\sec\theta - \cos\theta = n$

⇒ $\frac{1}{\cos\theta} - \cos\theta = n$

⇒ $\frac{1-\cos^2\theta}{\cos\theta} = n$

⇒ $\frac{\sin^2\theta}{\cos\theta} = n$

Now, consider,

L.H.S. = $(m^2 n)^{2/3} + (mn^2)^{2/3}$

$= \left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \frac{\sin^2\theta}{\cos\theta}\right\}^{2/3} + \left\{\frac{\cos^2\theta}{\sin\theta} \times \left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right\}^{2/3}$

$= \left(\frac{\cos^4\theta.\sin^2\theta}{\sin^2\theta.\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\right)^{2/3}$

$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3}$

$= \cos^2\theta + \sin^2\theta$

= 1 = R.H.S. **Hence Proved.**

83. Prove that :

[Board Question]

$$\frac{\sec A - 1}{\sec A + 1} = \left(\frac{\sin A}{1 + \cos A}\right)^2 = (\cot A - \operatorname{cosec} A)^2$$

Sol. L.H.S. = $\frac{\sec A - 1}{\sec A + 1} = \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$

$= \frac{\frac{1 - \cos A}{\cos A}}{\frac{1 + \cos A}{\cos A}}$

$= \frac{1 - \cos A}{1 + \cos A}$

$= \frac{(1 - \cos A)(1 + \cos A)}{(1 + \cos A)(1 + \cos A)}$

$= \frac{1 - \cos^2 A}{(1 + \cos A)^2}$

$= \frac{\sin^2 A}{(1 + \cos A)^2}$ [... $1 - \cos^2 A = \sin^2 A$]

$= \left(\frac{\sin A}{1 + \cos A}\right)^2$

and $\left(\frac{\sin A}{1 + \cos A}\right)^2 = \left[\left(\frac{\sin A}{1 + \cos A}\right) \times \frac{(1 - \cos A)}{(1 - \cos A)}\right]^2$

$= \left[\frac{\sin A(1 - \cos A)}{1 - \cos^2 A}\right]^2$

$= \left[\frac{\sin A(1 - \cos A)}{\sin^2 A}\right]^2$

$= \left[\frac{1-\cos A}{\sin A}\right]^2$

$= (\text{cosec } A - \cot A)^2$

$= (-1)^2 (\cot A - \text{cosec } A)^2$

$= (\cot A - \text{cosec } A)^2$ = R.H.S.

Hence Proved.

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

84. Assertion: In a right angled triangle, if cos θ = $\frac{1}{2}$ and sin θ = $\frac{\sqrt{3}}{2}$, then tan θ = $\sqrt{3}$

Reason: tan θ = $\frac{\sin\theta}{\cos\theta}$

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

The trigonometric formula given in reason states that

tan θ = $\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{\sin\theta}{\cos\theta}$

So, the reason is correct.

Now applying this formula in assertion, we deduce that

tan θ = $\frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$

85. Assertion: In a right angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.

Reason: $(\text{greatest side})^2 = (\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

The formula mentioned in the Reason is the Pythagoras theorem according to which the sum of the squares of the two sides of a right triangle is equal to the square of its hypotenuse or the greatest side of the triangle. Hence, reason is correct.

So applying the Pythagoras theorem in the right angled triangle given in the assertion,

$(\text{greatest side})^2 = (3)^2 + (4)^2 = 9 + 16 = 25$

greatest side = $\sqrt{25}$ = 5 units

Thus the assertion is correct.

86. Assertion: If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then the value of $x + y = 3$.

Reason: For any value of θ, $\sin^2\theta + \cos^2\theta = 1$.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

We know that for any value of θ, $\sin^2\theta + \cos^2\theta = 1$

So, Reason is correct.

Now applying this formula in assertion, we deduce that

$x + y = 2\sin^2\theta + 2\cos^2\theta + 1$

$= 2(\sin^2\theta + \cos^2\theta) + 1$

$= 2 \times 1 + 1[\because \sin^2 \theta + \cos^2 \theta = 1]$

$= 2 + 1 = 3$

Hence, Assertion is also correct.
87. Assertion: sin A is the product of sin and A.
Reason: The value of sin θ increases as θ increases.
Ans. (d) Assertion is false but the Reason is true.

Explanation :

For assertion: sin A is not the product of sin A and A.

It is the sine of ∠A.

Assertion is not correct.

For reason : The value of sin θ increases as θ increases in interval of $0° < \theta < 90°$.

So, Reason is correct.
88. Assertion : In a right DABC, right-angled at B, if tan A = 1, then 2 sin A · cos A = 1.
Reason : cosec A is the abbreviation used for cosecant of angle A.
Ans. (b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

We know that cosec A is the abbreviation used for cosecant of angle A.

So, Reason is correct.

For Assertion, we have, tan A = 1

$\Rightarrow \sin A/\cos A = 1$

$\Rightarrow \sin A = \cos A$

$\Rightarrow \sin A - \cos A = 0$

Squaring both sides, we get

$(\sin A - \cos A)^2 = 0$

⇒ $\sin^2 A + \cos^2 A - 2 \sin A \cdot \cos A = 0$

⇒ $1 - 2 \sin A \cdot \cos A = 0$

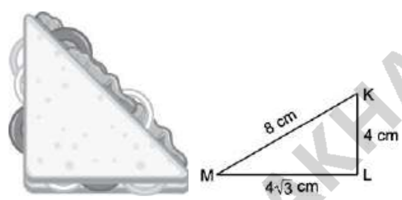
⇒ $2 \sin A \cdot \cos A = 1$

So, Assertion is also correct.

But reason is not the correct explanation of the assertion.

Case Based Questions

89. Ritu's daughter is feeling so hungry and so thought to eat something. She looked into fridge and found some bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right angled triangle, with sides 4 cm, $4\sqrt{3}$ cm and 8 cm.



(i) Find the value of tan M:

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) 1

(d) None of these

Ans. (b) $\frac{1}{\sqrt{3}}$

Explanation :

tan M = $\frac{KL}{LM} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$

(ii) The value of ∠M =

(a) 30°

(b) 60°

(c) 45°

(d) None of these

Ans. (a) 30°

Explanation :

$\because$ tan M = $\frac{1}{\sqrt{3}}$

tan M = tan 30°

⇒ ∠M = 30°

(iii) The value of ∠K =

(a) 45°

(b) 30°

(c) 60°

(d) None of these

Ans. (c) 60°

Explanation :

tan K = $\frac{ML}{KL} = \frac{4\sqrt{3}}{4} = \sqrt{3}$ = tan 60°

⇒ ∠K = 60°.

(iv) $\sec^2$ M – 1 =

(a) tan M

(b) $\cot^2$ M

(c) $\tan^2$ M

(d) None of these

Ans. (c) $\tan^2$ M

Explanation :

$\because$ $\sec^2$ M – $\tan^2$ M = 1

$\sec^2 M - 1 = \tan^2 M$.

(v) The value of $\frac{\tan^2 45° - 1}{\tan^2 45° + 1}$ is :

(a) 0

(b) 1

(c) 2

(d) – 1

Ans. (a) 0

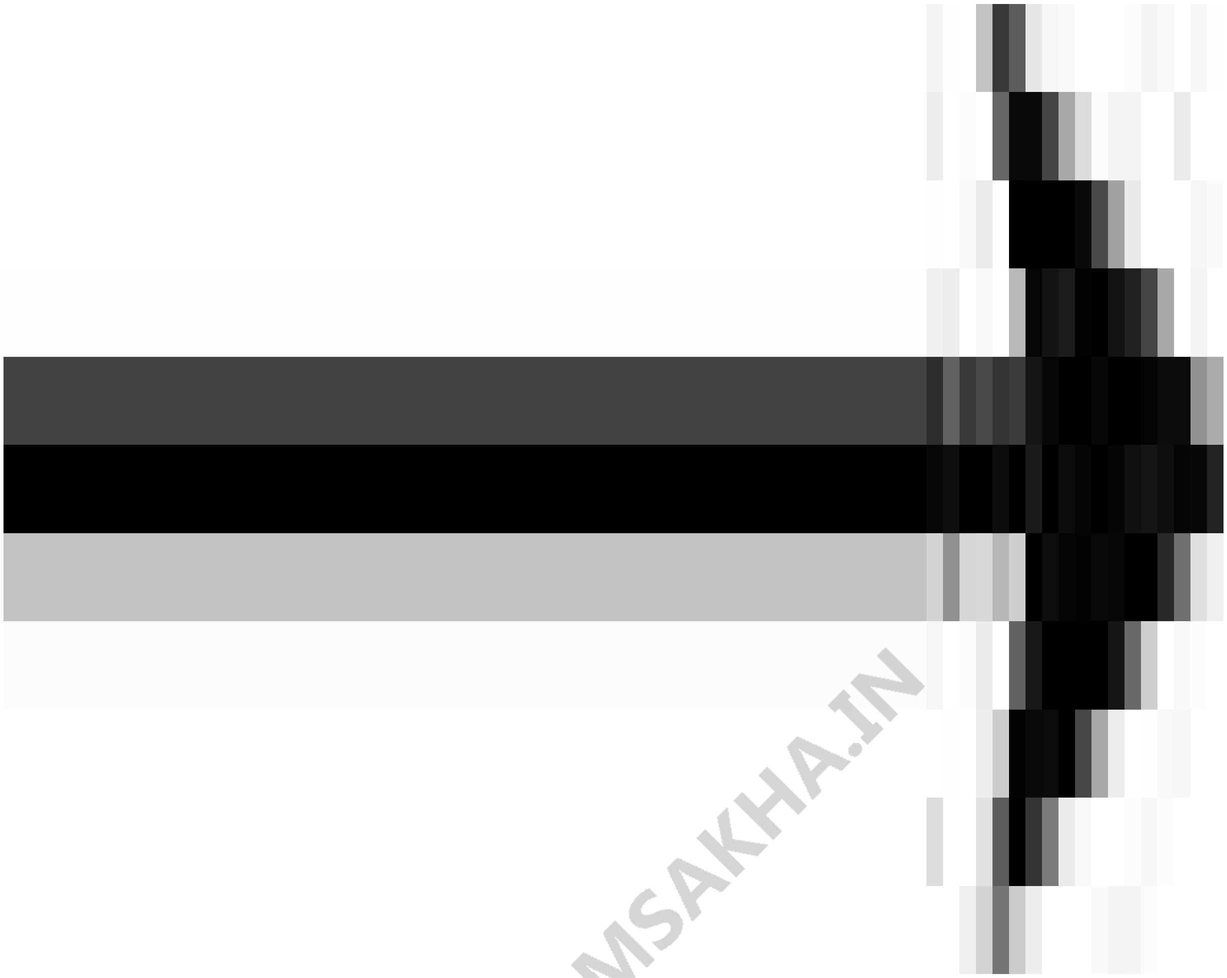
Explanation :

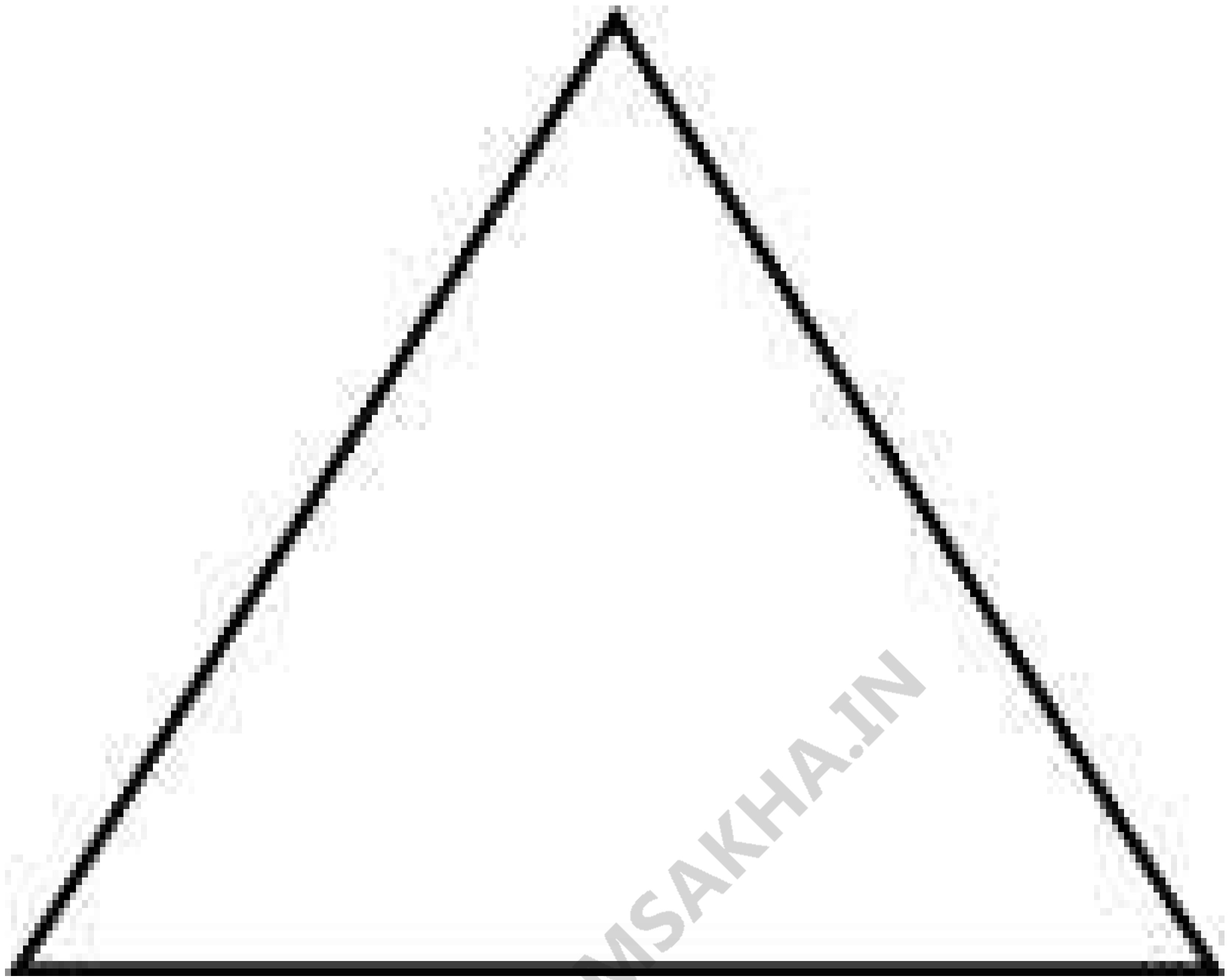
$\frac{\tan^2 45° - 1}{\tan^2 45° + 1} = \frac{(1)^2 - 1}{1^2 + 1} = \frac{0}{2} = 0$.

90. Aanya went to meet her friend Juhi for a party. When they reached to Juhi's place, Aanya saw the roof of the house, which is triangular in shape. If she imagined the dimensions of the roof as given in the figure, then answer the following questions.

(i) If D is the mid point of AC, then BD =

A





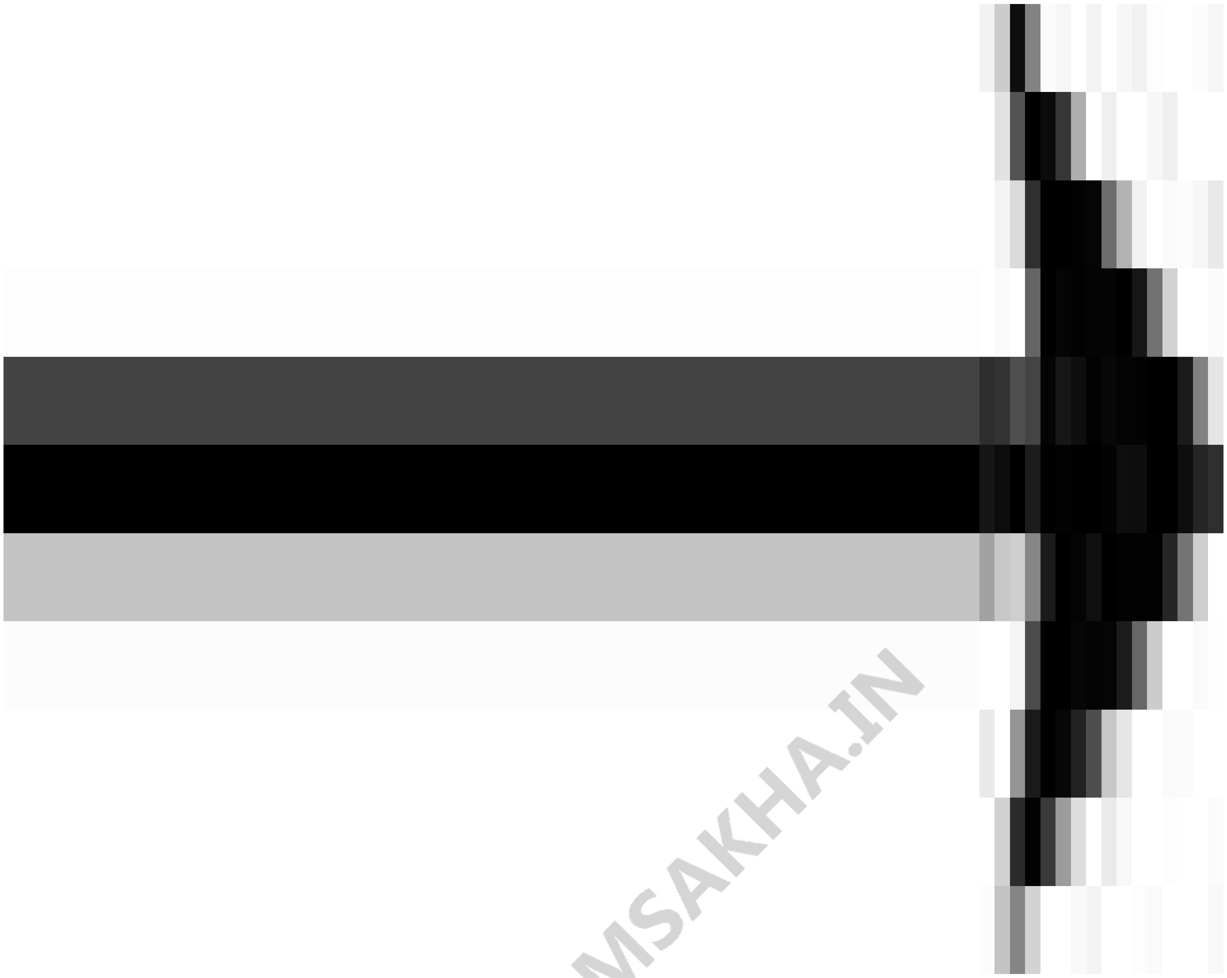
$6\sqrt{2}$ m

12 m

B

D

$6\sqrt{2}$ m



C

(a) 2 m

(b) 3 m

(c) 4 m

(d) 6 m

Ans. (d) 6 m

Explanation :

We have

AB = BC = $6\sqrt{2}$ m and AC = 12 m

∵ D is mid point of AC.

∴ AD = DC = 6 m

Now, $AB^2 = BD^2 + AD^2$

(∵ ΔABD is a right triangle)

⇒ $BD^2 = (6\sqrt{2})^2 - 6^2$

= 72 – 36

= 36

⇒ BD = 6 m ...(1)

(ii) Measure of ∠A =

(a) 30°

(b) 60°

(c) 45°

(d) None of these

Ans. (c) 45°

Explanation :

In ΔABD,

sin A = $\frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ [Using (1)]

⇒ sin A = sin 45°

⇒ ∠A = 45°.

(iii) Measure of ∠C =

(a) 30°

(b) 60°

(c) 45°

(d) None of these

Ans. (c) 45°

Explanation :

In ΔBDC,

tan C = $\frac{BD}{DC} = \frac{6}{6}$ [Using (1)]

⇒ tan C = 1

= tan 45°

⇒ ∠C = 45°.

(iv) Find the value of sin A + cos C :

(a) 0

(b) 1

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

Ans. (d) $\sqrt{2}$

Explanation :

sin A = $\frac{1}{\sqrt{2}}$, cos C = cos 45° = $\frac{1}{\sqrt{2}}$

∴ sin A + cos C = $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(v) Find the value of $\tan^2 C + \tan^2 A$:

(a) 0

(b) 1

(c) 2

(d) $\frac{1}{2}$

Ans. (c) 2

Explanation :

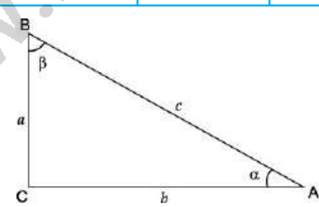
tan C = 1, tan A = tan 45° = 1

$\tan^2 C + \tan^2 A = 1 + 1$

= 2.

91. There are in total six trigonometric ratios, namely sine (sin), cosine(cos), tangent (tan), cosecant (cosec), secant (sec) and cotangent (cot). The trigonometric functions cosecant, secant and cotangent are simply the reciprocals of the trigonometric functions sine, cosine and tangent for the angles of a triangle. The values of these trigonometric ratios gives a certain rational for some values of angle (say a**). Some such values for the angle of triangle are shown in the table below:**

Angle-ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cosec θ	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cot θ	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0



(i) What is the value of sin a + cos b, when the values of angle a and b are respectively 30° and 60°?

(a) 1

(b) 1/2

(c) 2

(d) 0

Ans. (a) 1

Explanation :

We have a = 30° and that of b = 60°

Thus, sin a + cos b = sin 30° + cos 60°

= $\frac{1}{2}+\frac{1}{2}$ = 1 **Ans.**

(ii) Find the value of ($\sin^2$ 30° – sin 0°)/($\cos^2$ 90°– $\cos^2$ 60°).

(a) 0

(b) –1

(c) 1

(d) 1/2

Ans. (b) –1

Explanation :

Here ($\sin^2$ 30° – sin 0°) = $\left(\frac{1}{2}\right)^2$ – 0 = $\frac{1}{4}$

and $\cos^2$ 90°– $\cos^2$ 60° = 0 – $\frac{1}{4}$ = $-\frac{1}{4}$

Thus, ($\sin^2$ 30° – sin 0°)/($\cos^2$ 90° – $\cos^2$60°) = $\left(\frac{1}{4}\right)/\left(\frac{1}{-4}\right)$ = – 1 **Ans.**

(iii) If a = 90° and b = 60°, determine the value of $\sin^2$ a + $\cos^2$ b.

(a) 4/5

(b) 3/5

(c) 5/4

(d) 5/3

Ans. (c) 5/4

Explanation :

Here, $\sin^2$ 90°+ $\cos^2$60° = 1^2 + $\left(\frac{1}{2}\right)^2$

$= 1 + \frac{1}{4}$

$= \frac{5}{4}$ **Ans.**

(iv) If both a and b = 60°, find the value of $\sin^2 a + \cos^2 b$.

(a) 1

(b) 2

(c) –1

(d) –2

Ans. (a) 1

Explanation :

Here, $\sin^2 60° + \cos^2 60° = \frac{3}{4} + \frac{1}{4}$

= 1. **Ans.**

(v) If in the triangle ABC the lengths of sides AC and BC are in the ratio of 13 : 5, find the value of cosec b.

(a) 5/13

(b) 13/8

(c) 8/13

(d) 13/5

Ans. (d) 13/5

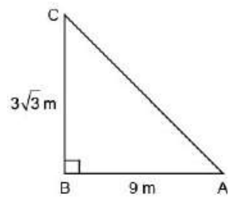
Explanation :

From the table we can find that cosec b $= \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$ **Ans.**

Passage Based Questions

92. Three friends Nikita, Palak and Kanika are playing hide and seek in a park. Nikita, Palak hide in the shrubs and Kanika have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right

angled triangle such that AB = 9 cm, BC = $3\sqrt{3}$m and ∠B = 90°, then answer the following question:



(i) Find the measure of ∠A and ∠C.

(ii) What is the distance between Nikita and Kanika?

Sol. (i) We have AB = 9 m, BC = $3\sqrt{3}$ m

In DABC, we have

tan A = $\frac{BC}{AB}$

= $\frac{3\sqrt{3}}{9}$ = $\frac{1}{\sqrt{3}}$

⇒ tan A = tan 30°

⇒ A = 30°

Similarly, tan C = $\frac{AB}{BC}$

= $\frac{9}{3\sqrt{3}} = \sqrt{3}$

⇒ tan C = tan 60°

⇒ ∠C = 60°.

(ii) Since sin A = $\frac{BC}{AC}$

⇒ sin 30° = $\frac{BC}{AC}$ ⇒ $\frac{1}{2}$ = $\frac{3\sqrt{3}}{AC}$

⇒ AC = $6\sqrt{3}$ m

Hence the distance between Nikita and Kanika is $6\sqrt{3}$ m.

Self-Assessment

93. In a right triangle ABC, right-angled at C, if tan A = 1, then verify that 2 sin A cos A = 1.

[NCERT]

94. In ΔABC right – angled at C, AB = 29 units, BC = 21 units and ∠ABC = θ. Determine the values of (i) $\cos^2 \theta + \sin^2 \theta$, and (ii) $\cos^2 \theta - \sin^2 \theta$.

[NCERT]

Ans. (i) 1, (ii) $\frac{41}{841}$

95. In ΔOPQ, right-angled at P, OP = 7 cm, OQ – PQ = 1 cm. Determine the values of sin Q and cos Q.

[NCERT]

Ans. sin Q = $\frac{7}{25}$, cos Q = $\frac{24}{25}$

96. In ΔPQR, right-angled at Q, PQ = 5 cm, PR + QR = 25 cm. Determine the values of sin P, cos P and tan P.

[NCERT]

Ans. sin P = $\frac{12}{13}$, cos P = $\frac{5}{13}$ and tan P = $\frac{12}{5}$.

97. In ΔABC, right-angled at B, if tan A = $\frac{1}{\sqrt{3}}$, find the value of (i) sin A cos C + cos A sin C, and (ii) cos A cos C – sin A sin C.

[NCERT]

Ans. (i) sin A cos C + cos A sin C = 1

(ii) cos A cos C – sin A cos C = 0.

98. If ∠B and ∠Q are acute angles such that sin B = sin Q, then prove ∠B = ∠Q.

[NCERT]

99. In a ΔABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine (i) sin A, cos A and (ii) sin C, cos C.

[NCERT]

Ans. (i) $\sin A = \frac{7}{25}$ and $\cos A = \frac{24}{25}$

(ii) $\sin C = \frac{24}{25}$ and $\cos C = \frac{7}{25}$

100. Given 15 cot A = 8. Find sin A and sec A.

[NCERT]

Ans. $\sin A = \frac{15}{17}$, $\sec A = \frac{17}{8}$.

101. If $\cot \theta = \frac{7}{8}$ evaluate (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ and (ii) $\cot^2 \theta$.

[NCERT]

Ans. (i) $\frac{49}{64}$, (ii) $\frac{49}{64}$.

102. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

[NCERT]

Ans. Both are equal to $\frac{7}{25}$.

103. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

[NCERT]

104. If $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$, $0° < A + B \leq 90°$ and A > B, find A and B.

[NCERT]

Ans. A = 45° and B = 15°.

105. If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, $0° < A + B \leq 90°$ and A > B, find A and B.

[NCERT]

Ans. A = 45° and B = 15°.

106. In ∆PQR, right-angled at Q, PQ = 3 cm and PR = 6 cm. Determine ∠P and ∠R.

[NCERT]

Ans. ∠P = 60° and ∠R = 30°.

107. Prove that: $\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$

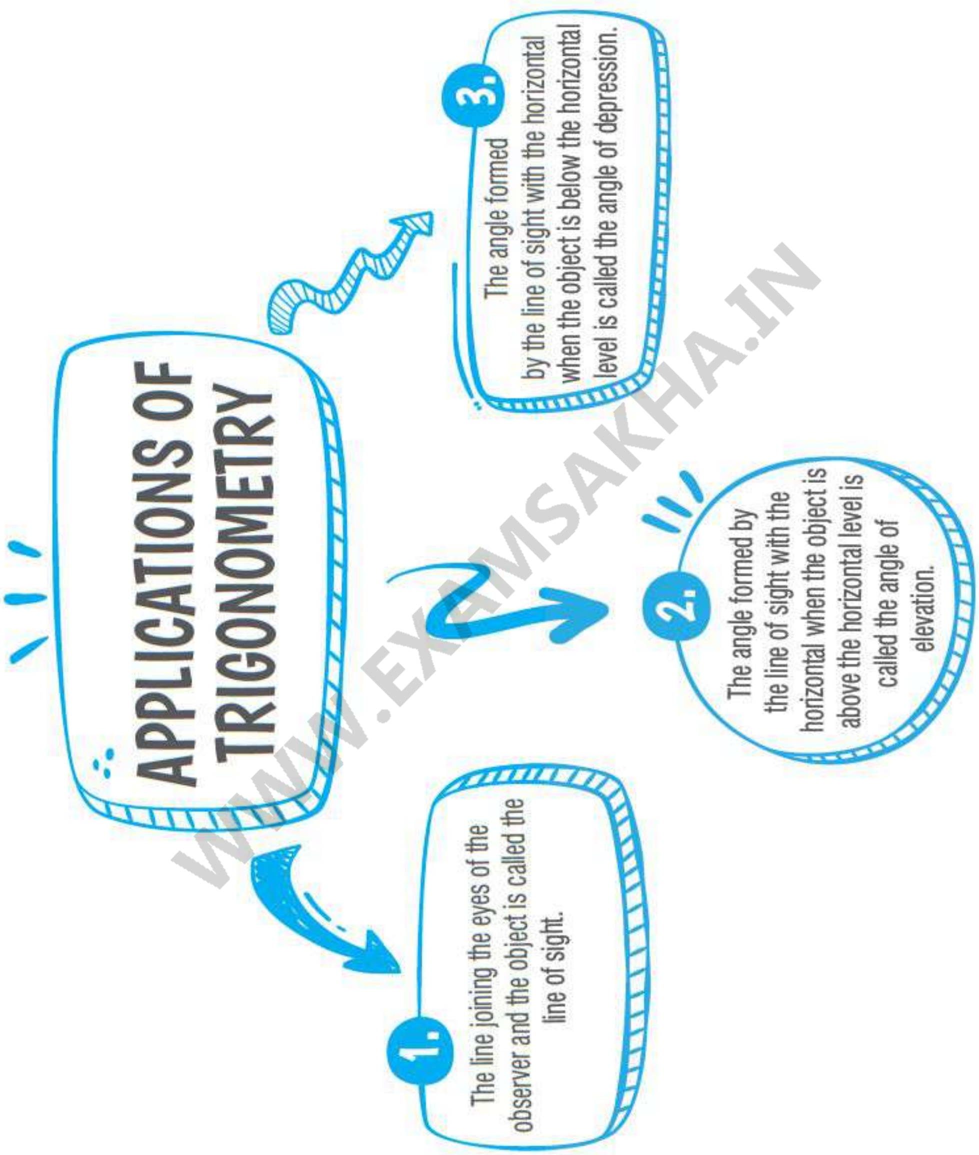
108. Prove that: (cosec A – sin A) (sec A – cos A) $\sec^2$ A = tan A.

Applications of Trigonometry

Chapter

11

Basic Concepts



Multiple Choice Questions

1. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then the angle of elevation of the Sun at that time is:

[Board Question]

(a) 30°

(b) 60°

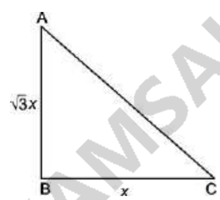
(c) 45°

(d) 75°

Ans. (b) 60°

Explanation :

Let the length of the shadow be *x*.



Thus, the height of the vertical pole is $\sqrt{3}x$.

tan θ = $\frac{\text{Perpendicular}}{\text{Base}}$ = $\frac{AB}{BC}$

⇒ tan θ = $\frac{\sqrt{3}x}{x}$

⇒ tan q = $\sqrt{3}$ = tan 60° ⇒ θ = 60°

Thus, the angle of elevation is 60°.

2. From the top of a cliff 20 m high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is:

[Board Question]

(a) 20 m

(b) 40 m

(c) 60 m

(d) 80 m

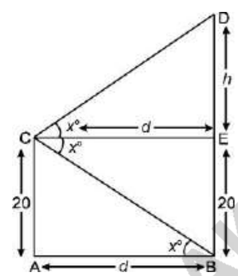
Ans. (b) 40 m

Explanation :

Let the angle of elevation = angle of depression = $x°$

Also, let the distance between the tower (BD) and the cliff (AC) be d m.

Thus, in ΔABC, tan x = $\frac{AC}{AB} = \frac{20}{d}$ …(i)



Also, in ΔCDE,

tan x = $\frac{ED}{CE} = \frac{h}{d}$ …(ii)

From equations (i) and (ii),

$\frac{h}{d} = \frac{20}{d}$

⇒ h = 20 m

⇒ Height of the tower is 20 + 20 = 40 m.

3. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder, in metres is:

[Board Question]

(a) $\frac{2}{\sqrt{3}}$

(b) $2\sqrt{3}$

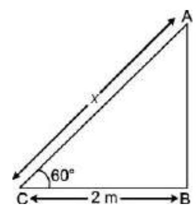
(c) $2\sqrt{2}$

(d) 4

Ans. (d) 4

Explanation :

Given, the distance of the ladder AC from the base of the wall AB is 2 m.



So, cos 60° = $\frac{BC}{AC}$ = $\frac{2}{x}$

⇒ $\frac{1}{2}$ = $\frac{2}{x}$

⇒ $x = 2 \times 2$

The height of the ladder = 4 m.

4. The angle of depression of a car standing on the ground from the top of a 75 m high tower is 30°. The distance of the car from the base of the tower (in m) is:

[Board Question]

(a) $25\sqrt{3}$

(b) $50\sqrt{3}$

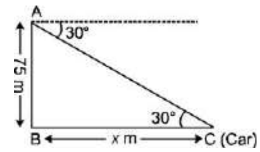
(c) $75\sqrt{3}$

(d) 150

Ans.(c) $75\sqrt{3}$

Explanation :

Let the distance of the car from the base of the tower be *x* m.



Height of tower = 75 m

tan 30° = $\frac{\text{Perpendicular}}{\text{Base}}$ = $\frac{AB}{BC}$

⇒ $\frac{1}{\sqrt{3}} = \frac{75}{x}$

⇒ $x = 75\sqrt{3}$

Thus, distance of the parked car from the base of the tower is $75\sqrt{3}$ m.

5. A kite is flying at a height of 30 m from the ground. The length of a string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is:

[Board Question]

(a) 45°

(b) 30°

(c) 60°

(d) 90°

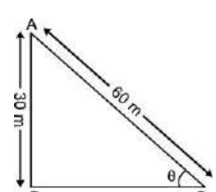
Ans. (b) 30°

Explanation :

Given,

Perpendicular, AB = 30 m and

hypotenuse AC = 60 m



So, $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC}$

$= \frac{30}{60} = \frac{1}{2} = \sin 30°$

Thus, $\theta = 30°$.

6. At some point of time in the day, the length of the shadow of a tower is equal to its height. Then the sun's altitude at that time is:

[Board Question]

(a) 30°

(b) 60°

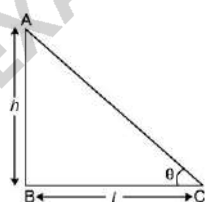
(c) 90°

(d) 45°

Ans. (d) 45°

Explanation :

Let the height of the tower be *h* m and the length of the shadow be *l* m.



Now, $h = l$

Thus, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$ $(\because h = l)$

$= \frac{h}{l}$

$= \tan 45°$

Hence, $\theta = 45°$

7. A tower stands vertically on the ground. From a point on the ground which is 25 m away from the foot of the tower, the angle

of elevation of the top of the tower is found to be 45°. Then the height (in metres) of the tower is:

[Board Question]

(a) $25\sqrt{2}$

(b) $25\sqrt{3}$

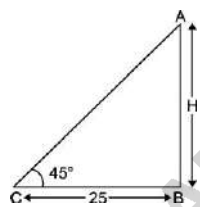
(c) 25

(d) 12·5

Ans. (c) 25

Explanation :

Let the height of the tower be H m.



Thus, tan 45° = $\frac{\text{Perpendicular}}{\text{Base}}$

or $\frac{H}{25}$ = 1

Hence, H = 25 m

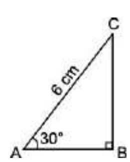
8. In the adjoining figure, the length of BC is:

(a) $2\sqrt{3}$ cm

(b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm

(d) 3 cm



Ans. (d) 3 cm

Explanation :

In right angled ΔABC,

$\sin 30° = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{BC}{6}$

$\Rightarrow$ BC = 3 cm.

9. The tops of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is:

(a) $8\sqrt{3}$ m

(b) 8 m

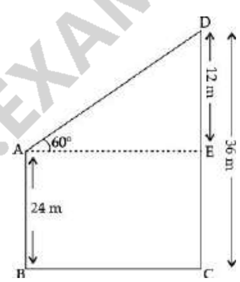
(c) $6\sqrt{3}$ m

(d) 6 m

Ans. (a) $8\sqrt{3}$ m

Explanation :

Let AB and DC be two poles and AD be the wire



In Δ ADE,

$\sin 60° = \frac{DE}{AD}$ [AD = Length of wire]

$\Rightarrow AD = \frac{DE}{\sin 60°} = \frac{12}{\frac{\sqrt{3}}{2}}$

$= \frac{12 \times 2}{\sqrt{3}} = \frac{24 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

[On multiplying and dividing by $\sqrt{3}$]

= $\frac{24\sqrt{3}}{3}$ = $8\sqrt{3}$ m

10. The length of shadow of a tree is 16 m when the angle of elevation of the Sun is 60°. What is the height of the tree?

(a) 8 m

(b) 16 m

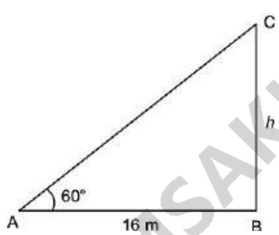
(c) $16\sqrt{3}$ m

(d) $\frac{16}{\sqrt{3}}$ m

Ans. (c) $16\sqrt{3}$ m

Explanation :

Let the height of the tree is *h* m.



tan 60° = $\frac{h}{16}$

⇒ $\sqrt{3}$ = $\frac{h}{16}$

⇒ h = $16\sqrt{3}$ m

11. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 9 m from the surface of river, then find the width of the river.

(a) 24 m

(b) 25 m

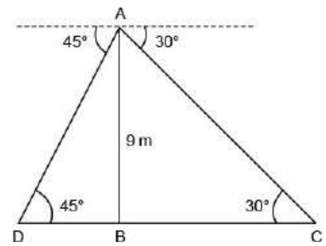
(c) 27 m

(d) 29 m

Ans. (b) 25 m

Explanation :

Here, width of the river = DC



In Δ ABC,

tan 30° = $\frac{AB}{BC}$

$\Rightarrow$ BC = $\frac{9}{\tan 30°}$ = $9\sqrt{3}$ m

Now, in Δ ABD

tan 45° = $\frac{AB}{BD}$

$\Rightarrow$ 1 = $\frac{AB}{BD}$

$\Rightarrow$ AB = BD

BD = 9 m[... AB = 9 m]

Hence, DC = DB + BC = 9 + $9\sqrt{3}$

= $9(\sqrt{3}+1)$

= 24.588 m $[\sqrt{3} = 1.732]$

$\simeq$ 25 m.

12. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30°, then when it was 60°. What is the height of the tower?

(a) 25 m

(b) 25 $\sqrt{3}$ m

(c) $\frac{25}{\sqrt{3}}$ m

(d) 30 m

Ans. (b) $25\sqrt{3}$ m

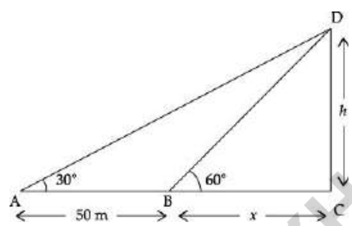
Explanation :

Let the height of tower be *h* and BC = *x* m.

In ΔBCD, tan 60° = $\frac{h}{x}$

⇒ $\sqrt{3}$ = $\frac{h}{x}$

⇒ *h* = $x\sqrt{3}$ ...(i)



In ΔACD, tan 30° = $\frac{h}{50+x}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{x\sqrt{3}}{50+x}$ [from (i)]

⇒ 50 + *x* = 3*x*

⇒ *x* = 25 m

Now put the value of *x* in equation (i),

∴ *h* = $25\sqrt{3}$ m

13. From the top of a building 60 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60°. The height of the tower is:

(a) 40 m

(b) 45 m

(c) 50 m

(d) 55 m

Ans. (a) 40 m

Explanation :

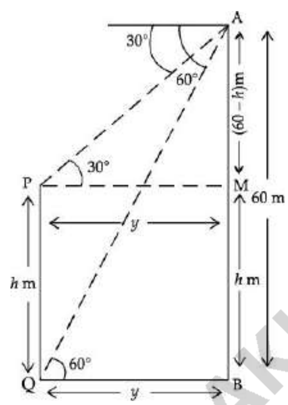
Let AB be a building and PQ be a tower.

Let height of tower PQ = *h* m

AM = (60 – *h*)

BQ = *y* = PM,

AB = 60m, BM = *h* m



Now, in ∆AMP ,

$\tan 30° = \frac{AM}{PM} = \frac{60-h}{y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{y}$

$\Rightarrow y = (60 - h)\sqrt{3}$ ...(i)

Now, in ∆ AQB,

$\tan 60° = \frac{AB}{BQ} = \frac{60}{y}$

$\Rightarrow \sqrt{3} = \frac{60}{y}$

$\Rightarrow y = \frac{60}{\sqrt{3}}$ ....(ii)

From equations (i) and (ii), we get

$(60 - h)\sqrt{3} = \frac{60}{\sqrt{3}}$

$\Rightarrow 3(60 - h) = 60$

$\Rightarrow 180 - 3h = 60$

$\Rightarrow 3h = 180 - 60$

$\Rightarrow 3h = 120$

$\Rightarrow h = 40$

Hence, height of tower = 40 m

14. The angles of depression of two ships from the top of a light - house are 60° and 45° towards east. If the ships are 300 m apart, the height of the light - house is:

(a) $200(3+\sqrt{3})\text{m}$

(b) $250(3+\sqrt{3})\text{m}$

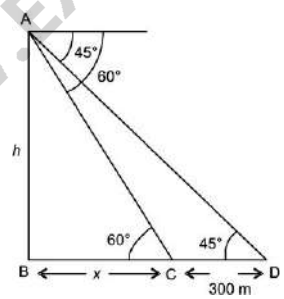
(c) $150(3+\sqrt{3})\text{m}$

(d) $160(3+\sqrt{3})\text{m}$

Ans. (c) $150(3+\sqrt{3})\text{m}$

Explanation :

Let x be the distance BC.



In Δ ABC,

$\frac{h}{x}$ = tan 60°

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$ ....(i)

In Δ ABD,

$\frac{h}{x+300}$ = tan 45°

⇒ $h = x + 300$

⇒ $h = \frac{h}{\sqrt{3}} + 300$

[From equations (i)]

⇒ $h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$ = 300

⇒ $h = \frac{300\sqrt{3}}{\sqrt{3}-1}$

⇒ $h = \frac{300\times\sqrt{3}\times(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

⇒ $h = \frac{300\times(3+\sqrt{3})}{(3-1)}$

⇒ $h = \frac{300\times(3+\sqrt{3})}{2}$

$h = 150\ (3 + \sqrt{3})$

Height of the light house = 150 (3 + $\sqrt{3}$) m

15. An observer, 1·5 m tall, is 28·5 m away from a 30 m high tower, then the angle of elevation of the top of the tower from the eye of the observer is:

(a) 60°

(b) 30°

(c) 15°

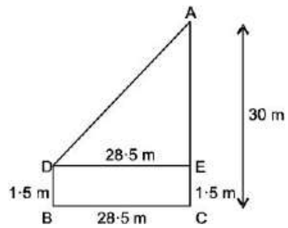
(d) 45°

Ans. (d) 45°

Explanation :

Here, AC = Tower

BD = Observer



BD = EC = 7·5

AE = AC – EC

= 30 – 1·5

= 28·5

In ΔADE,

tan θ = $\frac{AE}{DE}$ = $\frac{28.5}{28.5}$

= 1

⇒ q = 45°.

16. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°, then the distance of the car from the tower (in meter) is:

(a) $150\sqrt{3}$

(b) $100\sqrt{3}$

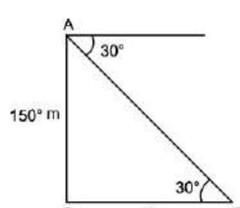
(c) 50

(d) $\sqrt{3}$

Ans. (a) $150\sqrt{3}$

Explanation :

Let distance of car = *x*



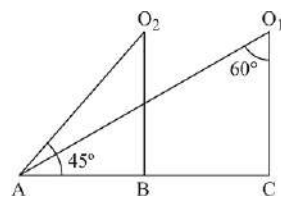
In ∆ABC,

tan 30° = $\frac{AB}{BC}$ = $\frac{150}{x}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{150}{x}$

⇒ x = $150\sqrt{3}$

17. In figure, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are ...., ...



(a) 30°, 60°

(b) 30°, 90°

(c) 45°, 30°

(d) 30°, 45°

Ans. (d) 30°, 45°

Explanation :

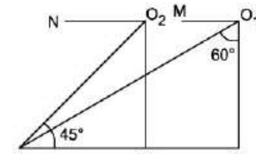
In DO_1AC,

$\angle O_1AC$ = 180° – (90° + 60°)

= 180° – 150°

= 30° = $\angle MO_1A$

(Vertical opposite angle)



In DO_2AB,

$\angle O_2AB = \angle NO_2A = 45°$

(Alternate angles)

⇒ Angles of depression are 30° and 45°.

18. The height of the tower is 10 m. Then the length of its shadow when sun's altitude is 45° is ....

(a) 15 m

(b) 10 m

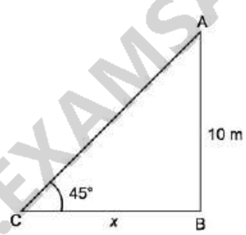
(c) 5 m

(d) 2 m

Ans. (b) 10 m

Explanation :

Let, shadow = *x* = BC



From DABC,

$\tan 45° = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{AB}{BC}$

AB = BC = 10 m.

19. If the angles of elevation of a tower from two points distant *a* and *b* (*a* > *b*) from its foot and in the same straight line from it are 30° and 60° respectively, then the height of the tower is:

(a) $\sqrt{a+b}$

(b) $\sqrt{ab}$

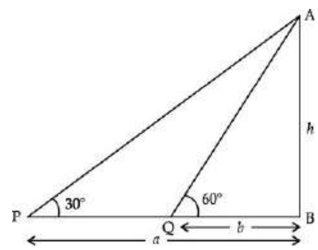
(c) $\sqrt{a-b}$

(d) $\sqrt{a}+\sqrt{b}$

Ans. (b) $\sqrt{ab}$

Explanation :

Let AB be the tower and P and Q are such points that PB = *a*, QB = *b* and angles of elevation at P and Q are 30° and 60° respectively.



Let AB = *h*

Now in right DAPB,

tan q = $\frac{\text{Perpendicular}}{\text{Base}}$ = $\frac{\text{AB}}{\text{PB}}$

⇒ tan 30° = $\frac{h}{a}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{h}{a}$ ...(i)

Similarly, in right DAQB,

tan 60° = $\frac{\text{AB}}{\text{QB}}$

⇒ $\sqrt{3}$ = $\frac{h}{b}$...(ii)

Multiplying (i) and (ii),

$\frac{1}{\sqrt{3}}$ × $\sqrt{3}$ = $\frac{h}{a}\times\frac{h}{b}$

⇒ 1 = $\frac{h^2}{ab}$

⇒ $h^2 = ab$

⇒ h = $\sqrt{ab}$

Height of the tower = $\sqrt{ab}$

20. The shadow of a 5 m long stick is 2 m long. At the same time the length of the shadow of a 12.5 m high tree is:

(a) 3 m

(b) 3.5 m

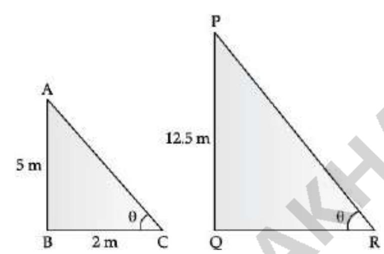
(c) 4.5 m

(d) 5 m

Ans. (d) 5 m

Explanation :

Let AB be the stick and BC be its shadow and PQ be the tree and QR be its shadow.



We have,

AB = 5 m, BC = 2 m, PQ = 12.5 m

In DABC,

tan q = $\frac{AB}{BC}$

⇒ tan q = $\frac{5}{2}$ …(i)

In DPQR,

tan q = $\frac{PQ}{QR}$

⇒ $\frac{5}{2}$ = $\frac{12.5}{QR}$ [Using (i)]

⇒ QR = 12.5 × $\frac{2}{5}$

⇒ QR = 5 m.

21. If a l.5 m tall girl stands at a distance of 3 m from a lamp

post and casts a shadow of length 4.5 m on the ground, then the height of the lamp post is:

(a) 1.5 m

(b) 2 m

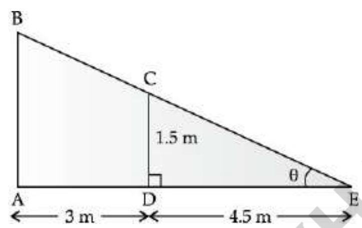
(c) 2.5 m

(d) 2.8 m

Ans. (c) 2.5 m

Explanation :

Let AB be the lamp-post; CD be the girl and DE be her shadow.



We have,

CD = 1.5 m, AD = 3 m, DE = 4.5 m

Let ∠AEB = ∠DEC = q

In DCDE,

tan q = $\frac{CD}{DE}$

⇒ tan q = $\frac{1.5}{4.5}$

⇒ tanθ = $\frac{1}{3}$…(i)

Now, in DABE,

tan q = $\frac{AB}{AE}$

⇒ $\frac{1}{3}$ = $\frac{AB}{AD+DE}$ [Using (i)]

⇒ $\frac{1}{3}$ = $\frac{AB}{3+4.5}$ ⇒ AB = $\frac{7.5}{3}$

⇒ AB = 2.5 m.

22. From a light house the angles of depression of two ships on opposite sides of the light house are observed to be 30° and 45°. If the height of the light house is *h* metres, the distance between the ships is:

(a) $(\sqrt{3}+1)h$ metres

(b) $(\sqrt{3}-1)h$ metres

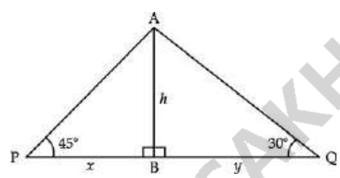
(c) $\sqrt{3}h$ metres

(d) $(1-\sqrt{3})h$ metres

Ans. (a) $(\sqrt{3}+1)h$ metres

Explanation :

Let AB be light house and P and Q are two ships on its opposite sides which form angle of elevation of A as 45° and 30° respectively.



Let PB = x, QB = y and AB = h

Now in right DAPB,

tan q = $\frac{\text{Perpendicular}}{\text{Base}}$ = $\frac{AB}{PB}$

⇒ tan 45° = $\frac{h}{x}$

⇒ 1 = $\frac{h}{x}$

⇒ $x = h$ ...(i)

Similarly in right DAQB,

tan 30° = $\frac{AB}{QB} = \frac{h}{y}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{h}{y}$

⇒ y = $\sqrt{3}h$ ...(ii)

Adding (i) and (ii),

PQ = $x + y = h + \sqrt{3}h$

= $(\sqrt{3}+1)h$ meters

23. A tower subtends an angle of 30° at a point on the same level as its foot. At a second point *'h'* metres above the first, the depression of the foot of the tower is 60°. The height of the tower is:

(a) $2h^2$ m

(b) $\sqrt{3}h$ m

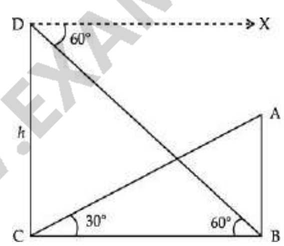
(c) $\frac{h}{3}$ m

(d) $h\sqrt{3}\text{m}$

Ans. (c) $\frac{h}{3}$ m

Explanation :

Let AB be the tower and C is a point on the same level as its foot such that ∠ACB = 30°



The given situation can be represented as the above figure.

Here D is a point h m above the point C.

In DBCD,

tan B = $\frac{CD}{BC}$

⇒ tan 60° = $\frac{h}{BC}$

⇒ $\sqrt{3}$ = $\frac{h}{BC}$

⇒ CB = $\frac{h}{\sqrt{3}}$ ...(i)

Again in DABC,

tan C = $\frac{AB}{CB}$

⇒ tan 30° = $\frac{AB}{h\sqrt{3}}$ [Using (i)]

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{AB}{h\sqrt{3}}$

⇒ AB = $\frac{h}{3}$.

24. The length of the shadow of a tower standing on level ground is found to be 2*x* metres longer when the sun's elevation is 30° than when it was 45°. The height of the tower is:

(a) $(2\sqrt{3}x)$ m

(b) $(3\sqrt{2}x)$ m

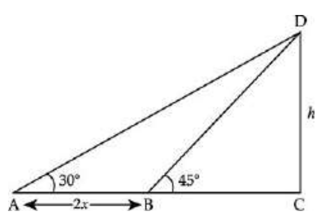
(c) $(\sqrt{3}-1)x$ m

(d) $(\sqrt{3}+1)x$ m

Ans. (d) $(\sqrt{3}+1)x$ m

Explanation :

Let CD = *h* be the height of the tower.



We have,

AB = 2*x*, ∠DAC = 30° and ∠DBC = 45°

In DBCD,

tan 45° = $\frac{CD}{BC}$

$\Rightarrow 1 = \frac{h}{BC}$

$\Rightarrow BC = h$

Now, in DACD,

$\tan 30° = \frac{CD}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AB+BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{2x+h}$

$\Rightarrow 2x + h = h\sqrt{3}$

$\Rightarrow h\sqrt{3} - h = 2x$

$\Rightarrow h(\sqrt{3}-1) = 2x$

$\Rightarrow h = \frac{2x}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h = \frac{2x(\sqrt{3}+1)}{2}$

$\Rightarrow h = x(\sqrt{3}+1)$ m

25. In a rectangle, the angle between a diagonal and a longer side is 30° and the length of this diagonal is 8 cm. The area of the rectangle is:

(a) 16 cm^2

(b) $6\sqrt{3}$ cm^2

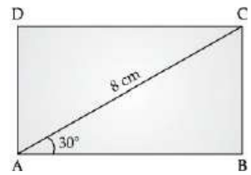
(c) $16\sqrt{3}$ cm^2

(d) $8\sqrt{3}$ cm^2

Ans. (c) $16\sqrt{3}\,cm^2$

Explanation :

Let ABCD be the rectangle in which ∠BAC = 30° and AC = 8 cm.



In DBAC,

We have,

$\frac{AB}{AC}$ = cos 30° = $\frac{\sqrt{3}}{2}$

⇒ $\frac{AB}{8}$ = $\frac{\sqrt{3}}{2}$

Þ AB = $4\sqrt{3}$ cm

Also, sin 30° = $\frac{BC}{AC}$

⇒ $\frac{1}{2}$ = $\frac{BC}{8}$

⇒ BC = $\frac{8}{2}$ = 4 cm

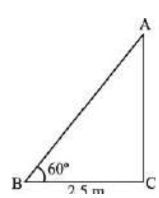
Area of the rectangle = (AB × BC) = $(4\sqrt{3} \times 4)$

= $16\sqrt{3}$ cm^2

Very Short Answer Type Questions

26. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Sol. Let AB be the ladder leaning against a wall AC.



Then, cos 60° = $\frac{BC}{AB}$

⇒ $\frac{1}{2}$ = $\frac{2.5}{AB}$

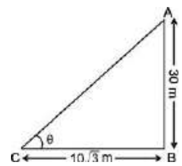
AB = 2.5 × 2 = 5 m

∴ Length of ladder is 5 m.**Ans.**

27. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the Sun?

Sol. In ΔABC,

tan θ = $\frac{AB}{BC}$



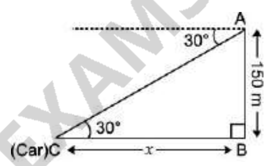
tan θ = $\frac{30}{10\sqrt{3}}$ = $\frac{3}{\sqrt{3}}$ = $\sqrt{3}$

⇒ tan θ = tan 60° ⇒ θ = 60°

Hence, angle of elevation is 60°.**Ans.**

28. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower (in metres).

Sol. Let the distance of the parked car from the base of the tower be *x* m.



Height of tower = 150 m

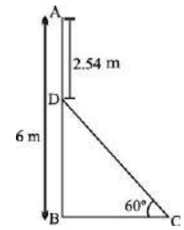
tan 30° = $\frac{\text{Perpendicular}}{\text{Base}}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{150}{x}$

⇒ x = $150\sqrt{3}$

Thus, distance of the parked car from the base of the tower is $150\sqrt{3}$ m.**Ans.**

29. In the given figure AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD = 2·54 m, find the length of the ladder. (Use $\sqrt{3}$ = 1·73)



Sol. Given, AB = 6 m and AD = 2.54 m.

∴ DB = (6 – 2.54) m

= 3·46 m

In ∆DBC,

sin 60° = $\frac{DB}{DC}$ Þ $\frac{\sqrt{3}}{2}$ = $\frac{3.46}{DC}$

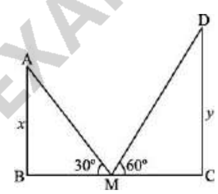
⇒ DC = $\frac{3.46 \times 2}{1.73}$

= 4 m.

∴ The length of the ladder is 4 m**Ans.**

30. The tops of two towers of height *x* and *y*, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find *x* : *y*.

Sol. Let AB and CD be two towers of height *x* and *y* respectively.



M is the mid-point of BC *i.e.,* BM = MC

In ∆ABM, we have

$\frac{AB}{BM}$ = tan 30°

Þ BH = $\frac{x}{\tan 30°}$ ...(i)

In ∆CDM, we have

$\frac{DC}{MC}$ = tan 60° Þ $\frac{y}{MC}$ = tan 60°

Þ MC = $\frac{y}{\tan 60°}$ ...(ii)

From equations (i) and (ii), we get

$\frac{x}{\tan 30°} = \frac{y}{\tan 60°}$ (... BM = MC)

Þ $\frac{x}{y} = \frac{\tan 30°}{\tan 60°}$ Þ $\frac{x}{y} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

Þ $x : y = 1 : 3$.**Ans.**

Short Answer Type Questions

31. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Sol. Let AB = x be the height of the transmission tower.

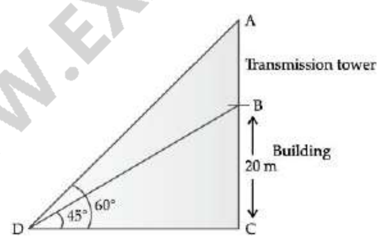
If BC = 20 m is the height of the building.

In DBCD,

$\tan 45° = \frac{BC}{CD}$

Þ $1 = \frac{20}{CD}$

Þ CD = 20 m.



In DACD,

$\tan 60° = \frac{AC}{CD}$

Þ $\sqrt{3} = \frac{AB + BC}{CD}$

Þ $\sqrt{3} = \frac{x + 20}{20}$ (Let AB = x)

Þ $20\sqrt{3} = x + 20$

Þ $x = 20\sqrt{3} - 20$

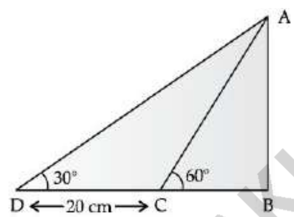
Þ $x = 20(\sqrt{3} - 1)$

Height of the transmission tower = $20(\sqrt{3} - 1)$ m.

32. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

Sol. Let AB = *h* be the height of the tower.

BC = *x* be the width of the canal.



In DABC,

tan 60° = $\frac{AB}{BC}$

Þ $\sqrt{3} = \frac{h}{x}$

Þ $h = \sqrt{3}x$ m ...(i)

In DABD,

tan 30° = $\frac{AB}{BD}$

Þ $\frac{1}{\sqrt{3}} = \frac{h}{BC + CD}$

Þ $\frac{1}{\sqrt{3}} = \frac{h}{x + 20}$

Þ $h = \frac{x + 20}{\sqrt{3}}$ ...(ii)

From (i) and (ii),

Þ $\frac{x + 20}{\sqrt{3}} = \sqrt{3}x$

Þ $x + 20 = 3x$

Þ $2x = 20$

Þ $x = 10$ m

substitute $x = 10$ m in (i),

$h = \sqrt{3}x$

$= \sqrt{3}(10)$

$= 10\sqrt{3}$

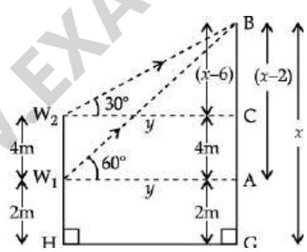
= 10(1.732)

= 17.32 m

33. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30°, respectively. Find the height of the balloon above the ground.

Sol. Let B be a balloon at a height GB = x m.

Let W_1 be the window, which is 2 m above the ground H.



W_1H = 2 m = AG

Let W_2 be the second window, which is the 4 m above the window W_1.

Þ W_2W_1 = AC = 4 m

The angles of elevation of balloon B from W_1 and W_2 are 60° and 30° respectively.

BA = (x – 2)m

BC = x – 2 – 4

= (x – 6)m

In DW$_2$CB, we have

tan 30° = $\frac{x-6}{y}$

Þ $\frac{1}{\sqrt{3}}$ = $\frac{x-6}{y}$

Þ y = $\sqrt{3}(x-6)$...(i)

Now, in DW$_1$AB,

tan 60° = $\frac{x-2}{y}$

Þ $\sqrt{3}$ = $\frac{x-2}{y}$

Þ $\sqrt{3}y$ = (x − 2) ...(ii)

Þ $\sqrt{3}\times\sqrt{3}(x-6)$ = x − 2 [From (i)]

Þ 3x − 18 = x − 2

Þ 3x − x = 18 − 2

Þ 2x = 16

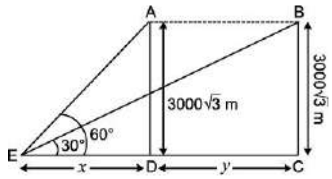
Þ x = 8 m

Hence, the height of the balloon above the ground is 8m.

34. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds, the angle of elevation become 30°. If the aeroplane is flying at a constant height of 3000$\sqrt{3}$ m, find the speed of the aeroplane.

[Board Question]

Sol. Let the ground distance between the aeroplane A and the point E be x m.



Given, height AD is $3000\sqrt{3}$ m and the angle of elevation is 60°.

So, in ∆AED,

tan 60° = $\frac{3000\sqrt{3}}{x}$

Þ $\sqrt{3}$ = $\frac{3000\sqrt{3}}{x}$

⇒ x = 3000 m

Let the new distance (DC) covered by aeroplane in 30 seconds be y m.

So tan 30° = $\frac{3000\sqrt{3}}{3000+y}$

Þ $\frac{1}{\sqrt{3}}$ = $\frac{3000\sqrt{3}}{3000+y}$

⇒ 3000 + y = $(3000\sqrt{3})\sqrt{3}$

⇒ 3000 + y = 9000

⇒ y = 9000 – 3000

= 6000 m

Thus, distance covered in 30 seconds = 6000 m

Hence, Speed = $\frac{6000}{30}$

= 200 m/sec

= 200 × $\frac{18}{5}$ km/hr

= 720 km/hr

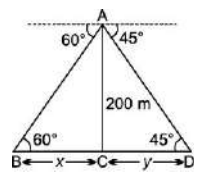
Hence, the speed of the aeroplane is 720 km/hr.

Ans.

35. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45°. If the height of the lighthouse is 200 m, find the distance between the two ships. [Use $\sqrt{3}$ = 1.73]

[Board Question]

Sol. Given, the height of the lighthouse AC is 200 m. Let the distance of the first ship B from the light house be *x* m and the distance of the second ship D from the lighthouse be *y* m.



Since, the angle of depression is 60° for the first ship.

In ∆ABC, we have

tan 60° = $\frac{200}{x}$

⇒ $\sqrt{3}$ = $\frac{200}{x}$

⇒ x = $\frac{200}{\sqrt{3}}$ …(i)

Also, as the angle of depression is 45° with the second ship.

In ∆ACD, we have

tan 45° = $\frac{200}{y}$

⇒ 1 = $\frac{200}{y}$

⇒ y = 200 m…(ii)

Thus, from equations (i) and (ii), the distance between the two ships is

$x + y = \frac{200}{\sqrt{3}} + 200$

= $200\left(1+\frac{1}{\sqrt{3}}\right)$ = $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$

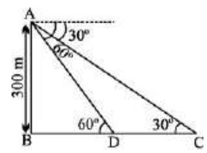
$= 200\left(\frac{1.73+1}{1.73}\right) = 200\left(\frac{2.73}{1.73}\right)$

= 200(1.578)

= 315.6 m.**Ans.**

36. A highway leads to the foot of 300 m high tower. An observatory is set at the top of the tower. It sees a car moving towards it at an angle of depression of 30°. After 15 seconds, angle of depression becomes 60°. Find the distance travelled by the car during this time.

Sol. Let AB = 300 m is the tower. Initially, car is at C and after 15 seconds, it reaches at D.



In right $\triangle ABC$,

$\frac{AB}{BC}$ = tan 30°

$\Rightarrow \frac{300}{BC} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ BC = $300\sqrt{3}$ m

In right $\triangle ABD$,

$\frac{AB}{BD}$ = tan 60°

$\frac{300}{BD} = \sqrt{3}$

$\Rightarrow$ BD = $\frac{300}{\sqrt{3}}$ m

$\therefore$ Distance covered by car

= DC = BC – BD

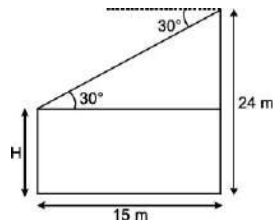
$= 300\sqrt{3} - \frac{300}{\sqrt{3}} = \frac{600}{\sqrt{3}}$

= $200\sqrt{3}$ m**Ans.**

37. The horizontal distance between two poles is 15 m. The angle of depression of the top of the first pole as seen from the top of the second pole is 30°. If the height of the second pole is 24 m, find the height of the first pole.

[Use $\sqrt{3}$ = 1.732]

Sol. Let the height of the first pole be H m.



Thus, tan 30° = $\frac{24-H}{15}=\frac{1}{\sqrt{3}}$

⇒ $\sqrt{3}(24-H)$ = 15

⇒ 24 – H = $5\sqrt{3}$

⇒ H = $24-5\sqrt{3}$

= [24 – 5(1.732)] m

= (24 – 8.66 m)

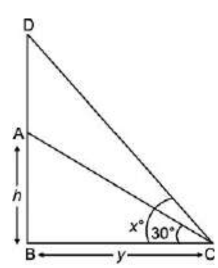
= 15.34 m

So, the height of the first pole is 15.34 m.**Ans.**

38. The angle of elevation of the top of a tower at a point on the ground is 30°. If the height of the tower is tripled, find the angle of elevation of the top of the same point.

[Board Question]

Sol. Let the height of the tower AB be *h* m and the distance between the tower and the point of observation C on the ground be *y* m.



So, $\tan 30° = \frac{h}{y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{y}$

$\Rightarrow y = h\sqrt{3}$ ...(i)

Let the new height be DB and the new angle of elevation be $x°$.

So $\tan x° = \frac{3h}{y}$

$\Rightarrow \tan x° = \frac{3h}{h\sqrt{3}}$ [from (i)]

$= \sqrt{3}$

$= \tan 60°$

$\Rightarrow x° = 60°$

Thus, the angle of elevation is 60° from the same point when the height of the tower is tripled.

Ans.

39. A contractor was assigned to construct a vertical pillar at a horizontal distance of 100 m from a fixed point. It was decided that angle of elevation of the top of the complete pillar from this point to be 60°. Contractor finished the job by making a pillar such that angle of elevation of its top was 45°. Find the height of the pillar to be increased as per the terms of contract.

Sol. Let AB is the required pillar and BC = 100 m such that

$\angle ACB = 60°$

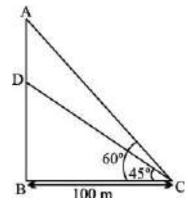
Let contractor constructed pillar BD.

In right ΔDBC,

$\frac{DB}{BC} = \tan 45°$

$\Rightarrow \frac{DB}{100} = 1$

$\Rightarrow DB = 100$ m



In right ΔABC,

$\frac{AB}{BC}$ = tan 60°

$\Rightarrow \frac{AB}{100} = \sqrt{3}$

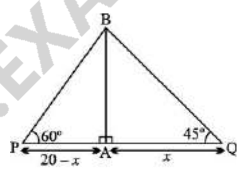
$\Rightarrow$ AB = $100\sqrt{3}$ m

Height of pillar to be increase = $100\sqrt{3} - 100$

= $100(\sqrt{3} - 1)$ m. **Ans.**

40. A fire in a building AB is reported on telephone to fire stations P and Q, 20 km apart from each other on a straight road. P observes that the fire is at angle of 60° to the road and Q observes that it is at an angle of 45° to the road. Which station should send its team and how much will this team have to travel?

Sol. Let BA $\perp$ PQ. A is a point on the road PQ (P, A and Q are points on the ground and AB is the height of the building).



Let distance between Q and A be *x.*

Then, distance between P and A = 20 – *x.*

In right ΔBAQ,

$\frac{BA}{AQ}$ = tan 45°

$\Rightarrow$ BA = AQ = x km

In right ΔBAP,

$\frac{BA}{AP}$ = tan 60°

$\Rightarrow \frac{x}{20-x} = \sqrt{3}$

$\Rightarrow x = \sqrt{3}(20 – x)$

$\Rightarrow x + \sqrt{3}x = 20\sqrt{3}$

$\Rightarrow x = \frac{20\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow x = \frac{20(\sqrt{3})(\sqrt{3}-1)}{3-1}$

$= 30 – 10\sqrt{3}$

= 30 – 10 × 1.732

= 12.68 km

Thus, AQ = 12.68 km

and AP = 20 – x

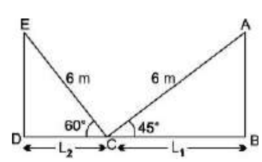
= 20 – 12.68

= 7.32 km

Hence, station P should send its team and distance covered = 7.32 km **Ans.**

41. A ladder of length 6 m makes an angle of 45° with the floor while leaning against one wall of a room. If the foot of the ladder is kept fixed on the floor and it is made to lean against the opposite wall of the room, it makes an angle of 60° with the floor. Find the distance between these two walls of the room.

Sol. Let the distance between the first wall and the ladder be L_1 and the distance between the ladder and the second wall be L_2.



In ΔABC,

$\cos 45° = \frac{BC}{AC} = \frac{L_1}{6}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{L_1}{6}$

⇒ $L_1 = 3\sqrt{2}$ m

In ∆EDC,

cos 60° = $\frac{CD}{EC} = \frac{L_2}{6}$

⇒ $\frac{1}{2} = \frac{L_2}{6}$

⇒ L_2 = 3 m

Thus, the total distance between the walls

= $(3+3\sqrt{2})$ m

= $3(1+\sqrt{2})$ m

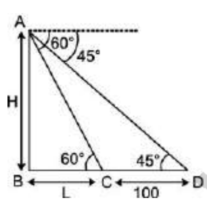
= 3(2.414) m $[\because \sqrt{2} = 1.414]$

= 7.242 m

Thus, the distance between the walls is 7.242 m.

Ans.

42. From the top of a vertical tower, the angles of depression of two cars in the same straight line with the base of the tower at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [Use $\sqrt{3}$ = 1.73]

Sol. 

Here, AB is a tower and position of the cars are at C and D.

We have, tan 45° = $\frac{AB}{BD} = \frac{H}{L+100}$

⇒ $1 = \frac{H}{L+100}$

⇒ L + 100 = H…(i)

and tan 60° = $\frac{AB}{BC} = \frac{H}{L}$

⇒ $\sqrt{3} = \frac{H}{L}$

⇒ $\sqrt{3}L$ = H…(ii)

Substituting equation (ii) in equation (i), we have

$\sqrt{3}L$ = L + 100

⇒ $\sqrt{3}L - L$ = 100

⇒ $(\sqrt{3}-1)L$ = 100

⇒ L = $\frac{100}{0.732}$

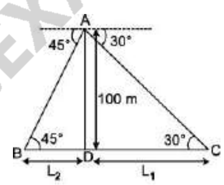
= 136.61 m

Thus H = 236·61 m[From (i)]

Hence, the height of the tower is 236.61 m.**Ans.**

43. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars. [Use $\sqrt{3}$ = 1.73]

Sol. Let the distance of one car from the base of the tower be L_1 and the distance of the other car be L_2.



In ΔADC,

tan 30° = $\frac{AD}{CD} = \frac{100}{L_1}$

⇒ $\frac{1}{\sqrt{3}} = \frac{100}{L_1}$

⇒ L_1 = $100\sqrt{3}$…(i)

In ΔADB,

tan 45° = $\frac{AD}{BD} = \frac{100}{L_2}$

$\Rightarrow 1 = \frac{100}{L_2}$

$\Rightarrow L_2 = 100$…(ii)

Adding equations (i) and (ii),

$L_1 + L_2 = 100\sqrt{3} + 100$

$= 100(\sqrt{3}+1)$

= 100(1·732 + 1)

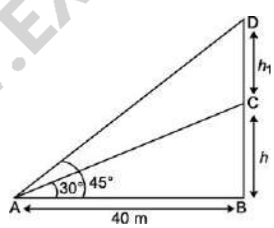
= 100 (2·732)

= 273·2

Thus, the distance between the two cars is 273·2 m.**Ans.**

44. From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The angle of elevation of the top of a water tank (on the top of the tower) is 45°. Find the (i) height of the tower, (ii) the depth of the tank.

Sol. Let BC be the tower of height h m and CD be the water tank of height h_1 m. Let A be a point on the ground at a distance of 40 m away from the foot B of the tower.



In ΔABD, $\tan 45° = \frac{BD}{AB}$

$\Rightarrow 1 = \frac{h+h_1}{40}$

$\Rightarrow h + h_1 = 40$ m…(i)

In ΔABC, we have

$\tan 30° = \frac{BC}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{40}$

$\Rightarrow h = \frac{40}{\sqrt{3}}\text{m}$

$= \frac{40\sqrt{3}}{3}\text{m}$

= 23.1 m

On putting the value of *h* in equation (i), we have

$23.1 + h_1 = 40$

$\Rightarrow h_1 = (40 – 23.1)$ m

⇒ = 16.9 m

Thus, the height of the tower is *h* = 23.1 m and the depth of the tank is h_1 = 16.9 m. **Ans.**

45. A man is standing on the deck of a ship, which is 10 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

[Board Question]

Sol. Let the man is standing on the deck of a ship at point A and let CD be the hill.

Given, ∠EAD = 60°, ∠BCA = 30°

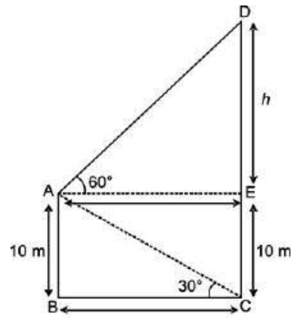
AB = 10 m

In ΔAED, we have

$\tan 60° = \frac{\text{DE}}{\text{EA}}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$ …(i)



In ΔABC, we have

tan 30° = $\frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$

$\Rightarrow x = 10\sqrt{3}$ …(ii)

On putting $x = 10\sqrt{3}$ in equation (i), we get

$h = \sqrt{3} \times 10\sqrt{3} = 30$

DE = 30 m

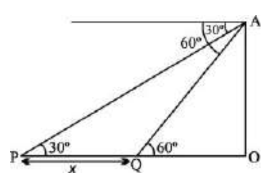
Thus, CD = CE + ED = 10 + 30

= 40 m

The distance of the hill from the ship is $10\sqrt{3}$ m and the height of the hill is 40 m.**Ans.**

46. A man on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60°. Find the time taken by the boat to reach the shore.

Sol. Suppose OA be the cliff and P be the initial position of the boat where the angle of depression is 30°. After 6 minutes, the boat reaches to Q where the angle of depression at Q is 60°.



Suppose, PQ = x m.

In ΔPOA and ΔQOA,

tan 30° = $\frac{OA}{OP}$

and tan 60° = $\frac{OA}{OQ}$

$\therefore$ $\frac{1}{\sqrt{3}}$ = $\frac{OA}{OP}$ and $\sqrt{3}$ = $\frac{OA}{OQ}$

$\Rightarrow$ OA = $\frac{OP}{\sqrt{3}}$ and OA = $\sqrt{3}\,OQ$

$\therefore$ $\frac{OP}{\sqrt{3}}$ = $\sqrt{3}\,OQ$

$\Rightarrow$ OP = 3 OQ

PQ = OP – OQ

= $OP - \frac{OP}{3}\left[OQ = \frac{1}{3}OP\right]$

= $\frac{2}{3}OP$

Consider the speed of the boat of *v* m/minute, so, PQ = Distance covered by the boat in 6 minutes.

PQ = 6*v*

$\Rightarrow$ $\frac{2}{3}$(OP) = 6*v*

$\Rightarrow$ OP = 9*v*

So, time elapsed by the boat to reach at the shore is given by,

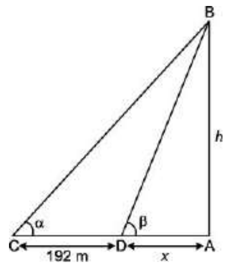
T = $\frac{OP}{v}$

$\Rightarrow$ *T* = $\frac{9v}{v}$ minutes

= 9 minutes**Ans.**

47. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is 5/12. On walking 192 metres towards the tower, the tangent of the angle of elevation is 3/4. Find the height of the tower.

Sol. Let AB be the tower and let the angle of elevation of its top at C be α. Let D be a point at a distance of 192 metres from C such that the angle of elevation of the top of the tower at D be β.



Let h be the height of the tower and AD = x.

Given that, tan α = $\frac{5}{12}$ and tan β = $\frac{3}{4}$

In ΔCAB, we have

tan α = $\frac{AB}{AC}$

⇒ $\frac{5}{12} = \frac{h}{x+192}$ …(i)

In ΔDAB, we have

tan β = $\frac{AB}{AD}$

⇒ $\frac{3}{4} = \frac{h}{x}$

⇒ $x = \frac{4h}{3}$

On putting the value of x in equation (i), we get

$\frac{5}{12} = \frac{h}{\frac{4h}{3}+192}$

⇒ $5\left(\frac{4h}{3}+192\right) = 12h$

⇒ $\frac{5(4h+576)}{3} = 12h$

⇒ $20h + 2880 = 36h$

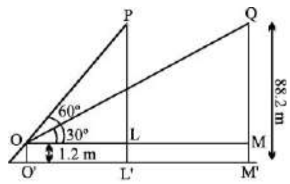
⇒ $16h = 2880$

⇒ $h = 180$

Thus, the height of the tower is 180 m.**Ans.**

48. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

Sol. Suppose P be the position of the balloon if its angle of elevation from the eyes of the girl is 60° and Q be the position if angle of elevation is 30°.



In ΔOLP, $\tan 60° = \frac{PL}{OL}$

$\Rightarrow \sqrt{3} = \frac{PL' - LL'}{OL}$

$\sqrt{3} = \frac{88.2 - 1.2}{OL}$

$= \frac{87}{OL}$

So, OL = $\frac{87}{\sqrt{3}}$

In ΔOMQ, $\tan 30° = \frac{QM}{OM}$

$= \frac{QM' - MM'}{OM}$

$\therefore \frac{1}{\sqrt{3}} = \frac{88.2 - 1.2}{OM}$

So, OM = $87\sqrt{3}$

Therefore, distance covered by the balloon,

PQ = OM – OL

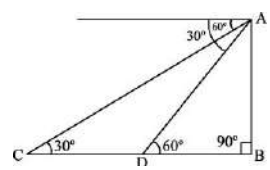
$= \left(87\sqrt{3} - \frac{87}{\sqrt{3}}\right) m$

$= 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) m$

= $\frac{174}{\sqrt{3}}$ m = $58\sqrt{3}$ m **Ans.**

49. A straight highway leads to the foot of a tower. A watchman standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Two minutes later, the angle of depression was found to be 60°. The watchman suspects that some terrorists are approaching the tower. It needs half a minute for the watchman to inform the security staff to be on the alert. How much time the car will take to reach the foot of the tower?

Sol. Let AB be the tower. Suppose C and D are the two positions of the car.



∴ In △CBA,

tan 30° = $\frac{AB}{BC}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{AB}{BC}$ ...(i)

and in △DBA,

tan 60° = $\frac{AB}{BD}$

⇒ $\sqrt{3}$ = $\frac{AB}{BD}$ ...(ii)

After dividing equation (ii) by equation (i), we get

3 = $\frac{BC}{BD}$

⇒ BC = 3 BD

⇒ BD + CD = 3BD[... BC = BD + CD]

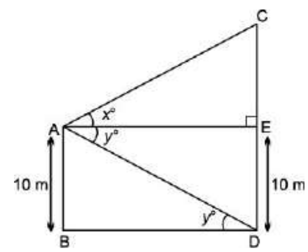
⇒ BD = $\frac{1}{2}$CD

As time elasped by car in travelling from C to D is 2 minutes. So, the time taken by car in travelling from D to B will be 1 minute because

BD = $\frac{1}{2}CD$.

Therefore, the car will take one minute to reach the foot of the tower. **Ans.**

50. From a window A, 10 m above ground, the angle of elevation of the top C to a tower is $x°$, where tan $x°$ = 5/2 and the angle of depression of the foot D of the tower is $y°$, where tan $y°$ = 1/4. Calculate the height CD of the tower in meters.



Sol. From figure,

∠BDA = ∠DAE = $y°$ [Alternate angles]

and AB = DE = 10 m

In ΔABD, tan $y°$ = $\frac{AB}{BD}$

⇒ $\frac{1}{4}$ = $\frac{10}{BD}$

or BD = 40 m = AE

In ΔAEC, tan $x°$ = $\frac{CE}{AE}$

⇒ $\frac{5}{2}$ = $\frac{CE}{40}$

⇒ CE = 5 × 20 = 100 m

Thus, CD = CE + DE

= 100 m + 10 m

= 110 m.

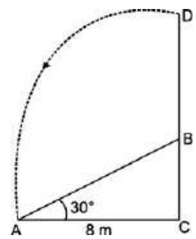
Thus, the height of tower is 110 m.**Ans.**

51. A pole being broken by the wind, the top struck the ground making an angle of 30° and at a distance of 8 m from the foot of

the pole. Find the complete height of the pole.

Sol. Let ABC be the pole. When broken at B by the wind, let its top A strike the ground.

Given, ∠CAB = 30° and AC = 8 m



In ΔACB, we have

tan 30° = $\frac{BC}{AC}$

⇒ $\frac{1}{\sqrt{3}} = \frac{BC}{8}$

⇒ BC = $\frac{8}{\sqrt{3}}$m

Also, cos 30° = $\frac{AC}{AB}$

⇒ $\frac{\sqrt{3}}{2} = \frac{8}{AB}$

⇒ AB = $\frac{16}{\sqrt{3}}$m

∴ Height of the pole = AB + BC

= $\left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right)$m

= $\frac{24}{\sqrt{3}}$m = $8\sqrt{3}$ m

= 8 × 1.732

= 13.856 m.**Ans.**

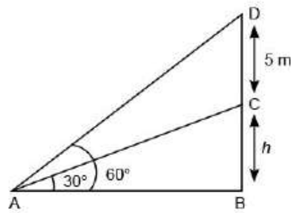
52. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 5 metres. At a point on the plane, the angles of elevation of the bottom and the top of the flagstaff are respectively 30° and 60°. Find the height of the tower.

Sol. Let height of the tower = BC = h m

Height of flagstaff, DC = 5 m

$\therefore \angle BAC = 30°$

and $\angle BAD = 60°$



In ΔABC, we have

$\Rightarrow \tan 30° = \frac{BC}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AB}$

$\Rightarrow AB = \sqrt{3}h$ ...(i)

In ΔABD, we have

$\tan 60° = \frac{BD}{AB}$

$\Rightarrow \sqrt{3} = \frac{h+5}{AB}$

$\Rightarrow AB = \frac{h+5}{\sqrt{3}}$ ...(ii)

From equations (i) and (ii),

$\sqrt{3}h = \frac{h+5}{\sqrt{3}}$

$\Rightarrow 3h = h + 5$

$\Rightarrow 2h = 5$

$\Rightarrow h = 2.5$ m.**Ans.**

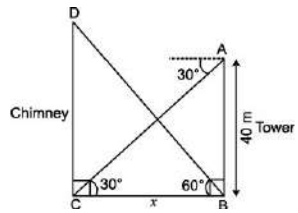
Long Answer Type Questions

53. The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the

chimney from the top of the tower is 30°. If the height of the tower is 40 m, find the height of the chimney.

[Board Question]

Sol. Let AB be the tower of height 40 m and CD be the chimney of height *h* m.



Let the distance between the foot of the tower and the foot of the chimney be *x* m.

∴ As per the question,

In ΔABC, $\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{x}$

$\Rightarrow x = 40\sqrt{3}$

In ΔBCD, $\tan 60° = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x = \sqrt{3} \times 40\sqrt{3}$

= 120 m

Thus, the height of the chimney is 120 m. **Ans.**

54. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.

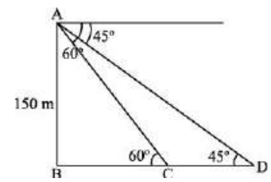
[Board Question]

Sol. Let AB be the cliff and C, D be the two position of the boat during a period of 2 minutes.

So, AB = 150 m

In ΔABC, $\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{150}{BC} \Rightarrow BC = \frac{150}{\sqrt{3}}$ m



In ΔABD, $\tan 45° = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{AB}{BD}$ [tan 45° = 1]

$\Rightarrow AB = BD$

$\Rightarrow BD = 150$ m

Distance covered in 2 min = BD – BC

$= 150 - \frac{150}{\sqrt{3}}$

$= \frac{150\sqrt{3} - 150}{\sqrt{3}}$

Distance covered in 1 hour = $\frac{150(\sqrt{3}-1)}{\sqrt{3} \times 2} \times 60$ m

Speed = $\frac{4500(\sqrt{3}-1)}{\sqrt{3}}$

$= 4500 - 1500\sqrt{3}$

$= 4500 - 2598$

$= 1902$ m/hr

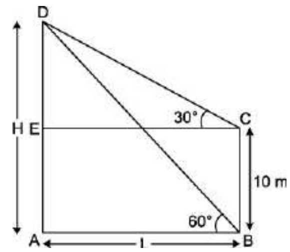
Hence, the speed of boat is 1902 m/hr. **Ans.**

55. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 30°. Find the height of the tower.

[Board Question]

Sol. Let AD be the tower and B, C be the two observation points such that C is 10 m above B.

So, BC = 10 m



Let AD = H m and AB = L m.

In ΔABD, $\tan 60° = \frac{AD}{AB}$

$\Rightarrow \sqrt{3} = \frac{H}{L}$

$\Rightarrow L = \frac{H}{\sqrt{3}}$ …(i)

In ΔDEC, $\tan 30° = \frac{ED}{EC}$

and $\frac{1}{\sqrt{3}} = \frac{H-10}{L}$

$\Rightarrow L = \sqrt{3}(H-10)$ …(ii)

From equations (i) and (ii),

$\frac{H}{\sqrt{3}} = \sqrt{3}(H-10)$

$\Rightarrow H = 3(H - 10)$

$\Rightarrow H = 3H - 30$

$\Rightarrow 2H = 30$

$\Rightarrow H = 15$ m

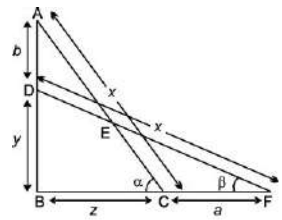
Thus, the height of the tower is 15 m.

56. A ladder rests against a wall at an angle having measure α with the ground. Its foot is pulled away from the wall by *a* m keeping ladder on the ground. By doing this, its upper end on the wall slides down by *b* m. Now the ladder makes an angle of measure β with the ground. Prove that $\frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

Sol. Let AB be the wall and AC be the ladder.

∴ AC = DF = x m (say)

and AD = b m, CF = a m, BD = y and BC = z m (say)



In ΔABC, $\cos\alpha = \frac{BC}{AC} = \frac{z}{x}$

and $\sin\alpha = \frac{AB}{AC} = \frac{y+b}{x}$

In ΔDB*F,* $\cos\beta = \frac{BF}{DF} = \frac{z+a}{x}$

and $\sin\beta = \frac{BD}{DF} = \frac{y}{x}$

So, $\cos\alpha - \cos\beta = \frac{z}{x} - \frac{z+a}{x}$

$= \frac{z-z-a}{x}$

$= -\frac{a}{x}$

∴ $\cos\beta - \cos\alpha = \frac{a}{x}$

and $\sin\alpha - \sin\beta = \frac{y+b}{x} - \frac{y}{x} = \frac{y+b-y}{x}$

⇒ $\sin\alpha - \sin\beta = \frac{b}{x}$

∴ $\frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta} = \frac{\frac{a}{x}}{\frac{b}{x}} = \frac{a}{b}$

⇒ $\frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta} = \frac{a}{b}$ **Hence Proved.**

57. A flagstaff of height *h* is surmounted on a pole. From a point at the base of the pole and some distance away from it, the angle of elevation of the top of the pole is α and that of the flagstaff is β. If the height of the pole is *l* m and the distance of the point from the base is *d*, prove that *h* = *d*(tan β – tan α).

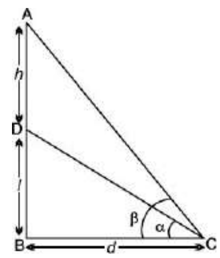
Sol. Let BD be the pole and AD be the flagstaff.

Let BD = l

Now, in ΔBDC,

$\tan \alpha = \frac{BD}{BC}$

$\Rightarrow \tan \alpha = \frac{l}{d}$



$\Rightarrow l = d \tan \alpha$ …(i)

and in ΔABC, $\tan \beta = \frac{AB}{BC}$

$\Rightarrow \tan \beta = \frac{l+h}{d}$

$\Rightarrow l + h = d \tan \beta$…(ii)

Substituting the value of l from eq. (i) in eq. (ii), we have

$d \tan \alpha + h = d \tan \beta$

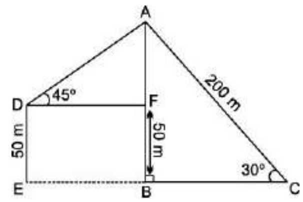
$\Rightarrow h = d(\tan \beta - \tan \alpha)$

Hence Proved.

58. Amit, standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, finds the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

[Board Question]

Sol. Let Amit be at C point and bird is at A point, such that □ACB = 30°. AB is the height of bird from point B on ground and deepak is at D point, DE is the building of height 50m.



Now, in right ΔABC, we have

sin 30° = $\frac{P}{H} = \frac{AB}{AC}$

⇒ $\frac{1}{2}$ = $\frac{AB}{200}$

⇒ AB = 100 m

In right □AFD, we have

sin 45° = $\frac{AF}{AD}$

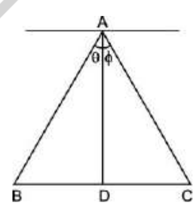
⇒ $\frac{1}{\sqrt{2}}$ = $\frac{50}{AD}$

(∵ AB = AF + BF, 100 = AF + 50, AF = 50)

⇒ AD = $50\sqrt{2}$ m

Hence, the distance of bird from Deepak is $50\sqrt{2}$ m.

59. In the given figure, ΔABC is right-angled at D and BD : DC is 2 : 3. Show that $\frac{\tan\theta}{\tan\phi} = \frac{2}{3}$.



Sol. Given, BD : DC = 2 : 3

Thus, in ΔABD,

tan θ = $\frac{BD}{AD}$

⇒ BD = AD tan θ…(i)

and in ΔADC,

tan φ = $\frac{DC}{AD}$

⇒ DC = AD tan φ…(ii)

Now, BD : DC = 2 : 3

⇒ $\frac{BD}{DC} = \frac{2}{3}$

⇒ 3BD = 2DC

⇒ 3AD tan θ = 2AD tan φ [Using (i) and (ii)]

⇒ 3 tan θ = 2 tan φ

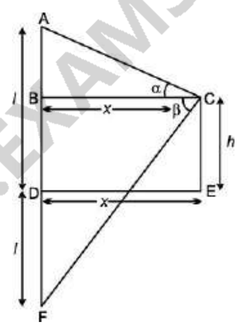
⇒ $\frac{\tan\theta}{\tan\phi} = \frac{2}{3}$ ·**Hence Proved.**

60. The angle of elevation of a cloud from a point *h* m above a lake is α and the angle of depression of the reflection of the cloud in the lake is β. Prove that the distance of the cloud from the point of observation is $\frac{2h \sec\alpha}{\tan\beta - \tan\alpha}$ m.

[Board Question]

Sol. Let DE be the surface of the lake.

Thus, DE = BC = x m (say)



Also let AD be the height (l) of the cloud from the surface of the lake, DF be the length of the image of the cloud below the surface of the lake and AC be the distance of the cloud from the point of observation C.

Thus, AD = DF

[From the laws of plane mirrors, an image is as far behind a plane mirror as the object is in front of it]

So AD = DF = l m

Given, ∠BCF = β and ∠BCA = α

Now, in ΔABC,

tan α = $\frac{AB}{BC}$

⇒ tan α = $\frac{l-h}{x}$

⇒ x = $\frac{l-h}{\tan\alpha}$m …(i)

Similarly, in ΔBCF,

tan β = $\frac{FB}{BC}$

⇒ tan β = $\frac{l+h}{x}$

⇒ x = $\frac{l+h}{\tan\beta}$m …(ii)

From equations (i) and (ii), we get

$\frac{l-h}{\tan\alpha} = \frac{l+h}{\tan\beta}$

⇒ (l – h) tan β = (l + h) tan α

⇒ l tan β – h tan β = l tan α + h tan α

⇒ l tan β – l tan α = h tan β + h tan α

⇒ l (tan β – tan α) = h (tan β + tan α)

⇒ l = $\frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}$

Put value of l in equation (i),

Now, x = $\frac{\frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}-h}{\tan\alpha}$ [From eq. (i)]

⇒ x = $\frac{h(\tan\beta+\tan\alpha)-h(\tan\beta-\tan\alpha)}{\tan\alpha(\tan\beta-\tan\alpha)}$

⇒ x = $h\left[\frac{\tan\beta+\tan\alpha-\tan\beta+\tan\alpha}{\tan\alpha(\tan\beta-\tan\alpha)}\right]$

⇒ x = $h\left[\frac{2\tan\alpha}{\tan\alpha(\tan\beta-\tan\alpha)}\right]$

⇒ $x = \frac{2h}{\tan\beta - \tan\alpha}$ …(iii)

Thus, in ΔABC, sec α = $\frac{AC}{BC}$

⇒ sec α = $\frac{AC}{x}$

⇒ AC = x sec α

⇒ AC = $\frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$

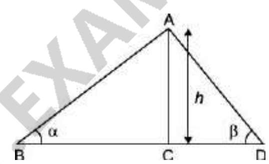
[from (iii)]

Hence Proved.

61. From an aeroplane, vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of the aeroplane above the road is given by $\frac{\tan\beta\tan\alpha}{\tan\beta+\tan\alpha}$.

[Board Question]

Sol. Let A be the position of aeroplane and B, D be the two consecutive milestone.



Let AC = h m

Given, BD = BC + CD = 1 mile

⇒ BC = 1 – CD

Now, in ΔABC

tan α = $\frac{AC}{BC}$

⇒ AC = BC tan α …(i)

and in ΔADC, tan β = $\frac{AC}{CD}$

⇒ AC = CD tan β …(ii)

Thus, from equations (i) and (ii), we have

BC tan α = CD tan β

⇒ 1 – CD = $\frac{\text{CD}\tan\beta}{\tan\alpha}$

⇒ $\frac{\text{CD}\tan\beta}{\tan\alpha}$ + CD = 1

⇒ CD $\left(\frac{\tan\alpha+\tan\beta}{\tan\alpha}\right)$ = 1

⇒ CD = $\frac{\tan\alpha}{\tan\alpha+\tan\beta}$

Hence, from equation (ii), we get

AC = CD tan β

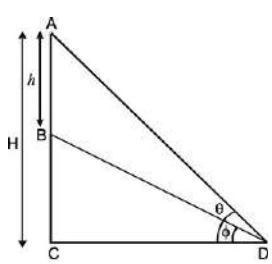
⇒ AC = $\frac{\tan\alpha}{\tan\alpha+\tan\beta}$ tan β

⇒ AC = $\frac{\tan\beta\tan\alpha}{\tan\beta+\tan\alpha}$ miles.

Hence Proved.

62. An aeroplane when flying at a height of *H* m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the point on the ground are θ and φ respectively. Prove that the vertical distance between the two planes is $\frac{H(\tan\theta-\tan\phi)}{\tan\theta}$.

Sol. Let AB be the distance between the two planes and D be the point of observation on the ground.



So, AB = *h* m

Let BC be the height of the second plane from the ground and BC = (*H* – *h*) m

Now, in ΔBDC,

$\tan \varphi = \frac{BC}{CD} = \frac{H-h}{CD}$

$\Rightarrow CD = (H - h) \cot \varphi$…(i)

and, in ΔADC

$\tan \theta = \frac{AC}{CD} = \frac{H}{CD}$

$\Rightarrow CD = H \cot \theta$…(ii)

From equations (i) and (ii), we have

$(H - h) \cot \varphi = H \cot \theta$

$\Rightarrow H(\cot \varphi - \cot \theta) = h \cot \varphi$

$\Rightarrow h = \frac{H(\cot \phi - \cot \theta)}{\cot \phi}$

$\Rightarrow h = \frac{H\left(\frac{1}{\tan \phi} - \frac{1}{\tan \theta}\right)}{\frac{1}{\tan \phi}}$

$\Rightarrow h = \frac{H(\tan \theta - \tan \phi) \tan \phi}{\tan \theta . \tan \phi}$

$\Rightarrow h = \frac{H(\tan \theta - \tan \phi)}{\tan \theta}$.

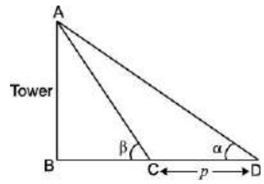
Hence Proved.

63. The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is α. On advacing '*p*' m towards the foot of the tower, the angle of elevation becomes β. Show that the height '*h*' of the tower is given by

$$h = \left(\frac{p \tan \alpha . \tan \beta}{\tan \beta - \tan \alpha}\right) m$$

Also determine the height of the tower when *p* = 150 m, α = 30° and β = 60°.

Sol. Let AB be the tower of height *h* m and C, D be the observation points.



Given, CD = p m

Thus, in ΔABD,

tan α = $\frac{AB}{BD} = \frac{AB}{BC + CD} = \frac{h}{BC + p}$

⇒ h = (BC + p) tan α…(i)

and in ΔABC,

tan β = $\frac{AB}{BC} = \frac{h}{BC}$

⇒ h = BC tan β

⇒ BC = h cot β…(ii)

Substituting equation (ii) in (i), we get

h = (h cot β + p) tan α

⇒ h = h cot β. tan α + p tan α

⇒ h – h cot β. tan α = p tan α

⇒ h(1 – cot β. tan α) = p tan α

⇒ $h = \frac{p \tan \alpha}{(1 - \cot \beta . \tan \alpha)}$

⇒ $h = \frac{p \tan \alpha . \tan \beta}{\tan \beta - \tan \alpha}$ m.

Hence Proved.

Now substituting p = 150 m, α = 30° and β = 60° in

$h = \frac{p \tan \alpha \tan \beta}{(\tan \beta - \tan \alpha)}$, we have

⇒ $h = \frac{150 \tan 30° \tan 60°}{(\tan 60° - \tan 30°)}$

$\Rightarrow h = \frac{150\left(\frac{1}{\sqrt{3}}\right)(\sqrt{3})}{\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)}$

$\Rightarrow h = \frac{150\sqrt{3}}{(3-1)}$

$\Rightarrow h = 75\sqrt{3}$

$\Rightarrow h = 75(1.732)$

$\Rightarrow h = 129.9$ m

Hence, the height of the tower is 129.9 m.**Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

64. Assertion : In a right-angled triangle ABC, right angled at B if BC = 20 m and ∠ACB = 30°, then height AB is 11.56 m.

Reason : tan θ = $\frac{AB}{BC} = \frac{\text{Perpendicular}}{\text{Base}}$, where θ is the ∠ACB, in right triangle ACB.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

Reason is correct as it is the derivation formula from the Pythagoras theorem.

Now applying it in assertion, we get

$\tan\theta = \frac{AB}{BC} = \frac{\text{Perpendicular}}{\text{Base}}$

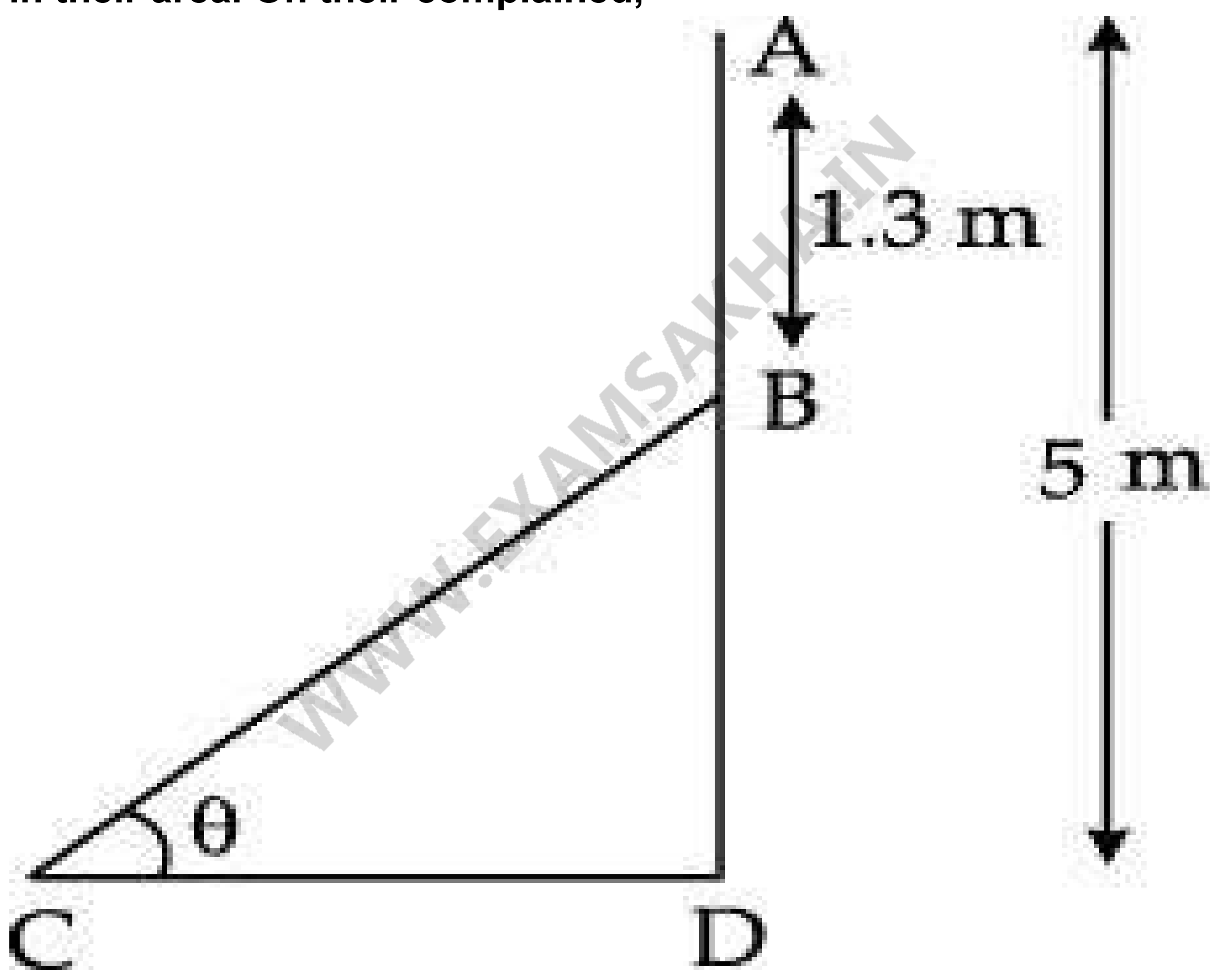
$\Rightarrow \tan 30° = \frac{AB}{20}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{20}$

$\Rightarrow AB = \frac{20}{\sqrt{3}} = 11.56\text{cm}$

Case Based Questions

65. In a village, group of people complained for an electric fault in their area. On their complained,



an electrician reached village to repair an electric fault on a pole of height 5 m. She needs to reach a point 1·3 m below the top of the pole to complete the repair

work (see the adjoining figure). She used ladder, inclined

at an angle of θ to the horizontal such that cos θ = 0·5, to reach the required position.

$(\sqrt{3} = 1.73)$

(i) The angle of elevation θ is:

(a) 60°

(b) 45°

(c) 30°

(d) 90°

Ans. (a) 60°

Explanation :

cos θ = 0·5 = $\frac{1}{2}$

= cos 60°

⇒ θ = 60°

(ii) The length BD is:

(a) 3 m

(b) 3·5 m

(c) 3·7 m

(d) 4 m

Ans. (c) 3·7 m

Explanation :

BD = AD – AB

= (5 – 1·3) m

= 3·7 m.

(iii) The length of the ladder. (take $\sqrt{3}$ = 1·73) is:

(a) 4 m

(b) 4·5 m

(c) 4·2 m

(d) 4·28 m

Ans. (d) 4·28 m

Explanation :

$\frac{BD}{BC}$ = sin 60°

⇒ $\frac{3\cdot7}{BC} = \frac{\sqrt{3}}{2}$

⇒ BC = $\frac{3\cdot7\times2}{\sqrt{3}}$

= 4·28 m (approx.)

(iv) How far from the foot of the pole should she place the foot of the ladder?

(a) 2 m

(b) 2·14 m

(c) 2·2 m

(d) 2·28 m

Ans. (b) 2·14 m

Explanation :

$\frac{DC}{BD}$ = cot 60° = $\frac{1}{\sqrt{3}}$

⇒ DC = $\frac{3\cdot7}{\sqrt{3}}$ = 2·14 m (approx.)

(v) If the height of pole and distance BC is doubled, then what should be the length of the ladder.

(a) 8 m

(b) 8·73 m

(c) 8·56 m

(d) 8·28 m

Ans. (c) 8·56 m

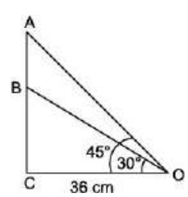
Explanation :

$\frac{BD}{BC}$ = sin 60° or $\frac{7 \cdot 4}{BC} = \frac{\sqrt{3}}{2}$

BC = $\frac{7.4 \times 2}{\sqrt{3}}$ = 8·56 m(approx.)

66. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself and support one more antenna on its structure including microwave dishes. They are among the tallest human made structurs. There are 2 main types gauged and self-supporting structures.

On a similar concept, a radio station tower was built in two sections. A and B tower is supported by wires from a point O. Distance between the base of the tower and point is 36 m. From point O, the angle of elevation of the top of section B is 30° and the angle of elevation of the top of section A is 45°.



(i) What is the height of section B?

(a) 12$\sqrt{3}$m

(b) 12$\sqrt{2}$m

(c) 8$\sqrt{3}$m

(d) 4$\sqrt{2}$m

Ans. (a) 12$\sqrt{3}$m

Explanation :

In ΔBOC,

tan 30° = $\frac{BC}{OC}$

⇒ BC = OC tan 30°

⇒ BC = 36 × $\frac{1}{\sqrt{3}}$ ⇒ $12\sqrt{3}m$

(ii) What is the height of the section A?

(a) 12 (2 – $\sqrt{2}$) m

(b) 24 (2– $\sqrt{2}$) m

(c) 12 (3 – $\sqrt{3}$) m

(d) 24 (3 – $\sqrt{3}$) m

Ans. (c) 12 (3 – $\sqrt{3}$) m

Explanation :

In ΔACO,

tan 45° = $\frac{AC}{OC}$ = 1

⇒ AC = OC = 36 m

AB = AC – BC

= 36 – 12$\sqrt{3}$

= 12 (3 – $\sqrt{3}$)m

(iii) What is the length of the wire structure from the point O to the section A's top?

(a) 32$\sqrt{2}$m

(b) 24 – $\sqrt{3}$m

(c) 28$\sqrt{3}$m

(d) 36$\sqrt{2}$m

Ans. (d) $36\sqrt{2}$ m

Explanation :

In ΔACO,

$\cos 45° = \frac{OC}{OA}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{36}{OA}$

$\Rightarrow OA = 36\sqrt{2}$ m

(iv) What is the length of the wire structure from the point O to the point B?

(a) $12\sqrt{3}$ m

(b) $28\sqrt{3}$ m

(c) $24\sqrt{3}$ m

(d) $16\sqrt{3}$ m

Ans. (c) $24\sqrt{3}$ m

Explanation :

In Δ BCO, $\cos 30° = \frac{OC}{OB}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{OB}$

$OB = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24 \times 3 \times \sqrt{3}}{3} = 24\sqrt{3}$ m

(v) What is the angle of depression from top of tower to point O?

(a) 30°

(b) 45°

(c) 15°

(d) 75°

Ans. (b) 45°

Explanation :

It is clear from figure that angle of elevation from point O to top of tower is 45°. This is equal to the angle of depression from top of tower to point O.

67. A house painter is a tradesman responsible for the painting and decorating of buildings. The purpose of painting is to improve the appearance of a building and to protect it from damages. Your father hired a painter to paint your house. The painter leans a ladder against a wall of the room shown in the picture below. The two opposite walls are $3\sqrt{3}$ m apart and the ladder reaches 9 m up the wall. A person of height 1.8m was standing below the ladder to help the painter. (Use $\sqrt{3} = 1.73$)

(i) What is the angle made by the ladder with the floor of the room:
(a) 60°
(b) 30°
(c) 45°
(d) 90°

Ans. (a) 60°

Explanation :

tan θ = Opposite side / Adjacent side

$= \frac{9}{3\sqrt{3}}$

$= \sqrt{3}$ = tan 60°

⇒ θ = 60°

(ii) How close the person can walk towards left hand wall without bumping his head:

(a) 1.808 m

(b) 4.164 m

(c) 1.038 m

(d) 2.585 m

Ans. (c) 1.038 m

Explanation :

tan 60° = $\frac{1.8}{x}$

x = $\frac{1.8}{\sqrt{3}}$

= 0.6 × $\sqrt{3}$

= 1.038 m

(iii) What is the length of the ladder:

(a) 8.38 m

(b) 10.38 m

(c) 6.58 m

(d) 12.48 m

Ans. (b) 10.38 m

Explanation :

sin 60° = $\frac{9}{l}$

⇒ $\frac{\sqrt{3}}{2}$ = $\frac{9}{l}$

⇒ l = 9 × $\frac{2}{\sqrt{3}}$ = 10.38 m

(iv) The angle made by the ladder with the wall is:

(a) 60°

(b) 45°

(c) 30°
(d) 90°

Ans. (c) 30°

Explanation :

θ = 180° – 90° – 60° = 30°

(v) The ladder is moved 1.1 m towards right hand wall. What is the tangent of an angle made by the ladder with the floor? (Use $\sqrt{3}$ = 1.7)

(a) 2.525

(b) 3.606

(c) 1.500

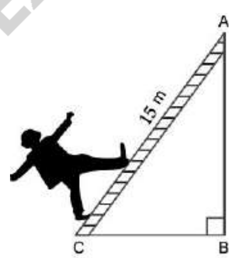
(d) 2.455

Ans. (a) 2.525

Explanation :

$$\tan\theta = \frac{(9+1.1)}{3\sqrt{3}-1.1} = \frac{10.1}{5.1-1.1} = \frac{10.1}{4} = 2.525 \text{ m}$$

68. A circus artist is showing stunts in a show climbing through a 15 m long rope which is highly stretched and tied from the top of a vertical pole to the ground as shown below:



(i) What is the height of the pole, if angle made by rope to the ground level is 45°:
(a) 15 m
(b) $15\sqrt{2}$ m
(c) $\frac{15}{\sqrt{3}}$ m
(d) $\frac{15}{\sqrt{2}}$ m

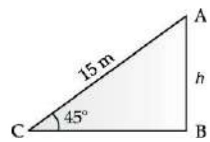
Ans. (d) $\frac{15}{\sqrt{2}}$ m

Explanation :

Consider *h* be the height of the pole.

In DABC,

sin 45° = $\frac{h}{15}$



⇒ $\frac{1}{\sqrt{2}}$ = $\frac{h}{15}$

⇒ h = $\frac{15}{\sqrt{2}}$m

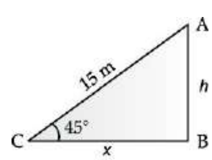
(ii) If the angle made by the rope to the ground level remains same, then find the distance between artist and pole at ground level.

(a) $\frac{15}{\sqrt{2}}$ m
(b) $15\sqrt{2}$ m
(c) 15 m
(d) $15\sqrt{3}$ m

Ans. (a) $\frac{15}{\sqrt{2}}$m

Explanation :

Let *x* be the required distance.



In DABC,

⇒ $\frac{x}{15}$ = cos 45°

⇒ $\frac{x}{15}$ = $\frac{1}{\sqrt{2}}$

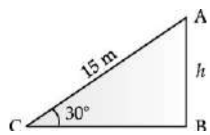
⇒ x = $\frac{15}{\sqrt{2}}$m

(iii) Find the height of the pole if the angle made by the rope to the ground level changes to 30°:
(a) 2.5 m
(b) 5 m
(c) 7.5 m
(d) 10 m

Ans. (c) 7.5 m

Explanation :

Let *h* be the height of the pole.



In right DABC,

$\frac{h}{15}$ = sin 30° = $\frac{1}{2}$

⇒ h = $\frac{15}{2} = 7.5$ m

(iv) If the angle made by the rope to the ground level changes to 30° and 3 m rope is broken, then find the height of the pole.

(a) 2 m

(b) 4 m

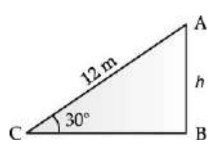
(c) 5 m

(d) 6 m

Ans. (d) 6 m

Explanation :

If 3 m rope is broken, then the length of the rope is 12 m.



In DABC,

$\frac{h}{12} = \sin 30° = \frac{1}{2}$

$\Rightarrow h = \frac{12}{2} = 6$ m

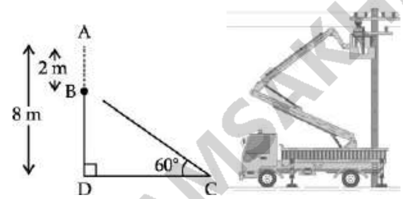
(v) Which mathematical concept is used here?
(a) Similar triangles
(b) Pythagoras Theorem
(c) Application of Trigonometry
(d) None of the above

Ans. (c) Application of Trigonometry

Explanation :

Trignometric ratios and value of standard angles are used to find the answers.

69. An electrician was given a task to repair an electric fault on the pole of height is 8 m. He needs to reach a point 2 m below the top of the pole to undertake the repair work.



(i) What is the length of BD?
(a) 10 m
(b) 6 m
(c) 4 m
(d) 4 m

Ans. (b) 6 m

Explanation :

Total height of pole = 8 m

BD = AD – AB

= (8 – 2) m

= 6 m.

(ii) What should be the length of ladder, so that it makes an angle of 60° with the ground?

(a) $4\sqrt{3}$ m

(b) $2\sqrt{3}$ m

(c) $3\sqrt{3}$ m

(d) $5\sqrt{3}$ m

Ans. (a) $4\sqrt{3}$ m

Explanation :

In DBDC,

$\frac{BD}{BC}$ = sin 30°

⇒ $\frac{6}{BC}$ = $\frac{\sqrt{3}}{2}$

⇒ BC = $\frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

= $4\sqrt{3}$ m

(iii) What is the distance between the foot of ladder and pole is?

(a) $6\sqrt{3}$ m
(b) $4\sqrt{3}$ m
(c) $3\sqrt{3}$ m
(d) $2\sqrt{3}$ m

Ans. (d) $2\sqrt{3}$ m

Explanation :

In DBDC,

$\frac{BD}{CD}$ = tan 60°

⇒ $\frac{6}{CD}$ = $\sqrt{3}$

⇒ CD = $\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

= $2\sqrt{3}$ m

(iv) What will be the measure of ÐBCD when BD and CD are equal?
(a) 30°
(b) 45°
(c) 60°
(d) 75°

Ans. (b) 45°

Explanation :

In DBCD,

$\frac{BD}{CD}$ = tan q

⇒ 1 = tan q[... BD = CD]

⇒ q = 45°

(v) What is the measure of ÐDBC.
(a) 15°
(b) 60°
(c) 30°
(d) 45°

Ans. (c) 30°

Explanation :

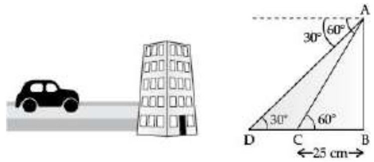
In DBDC,

ÐB + ÐD + ÐC = 180°

ÐB = 180° – 60° – 90°

= 30°.

70. Shravan is standing at the top of the building and observes a car at an angle of 30°, which is approaching the foot of the building with an uniform speed. 6 seconds later, angle of depression of car found to be 60°, whose distance at that instant from the building is 25 m.



(i) Height of the building is:
(a) $25\sqrt{2}$ m
(b) 50 m
(c) $25\sqrt{3}$ m
(d) 25 m

Ans. (c) $25\sqrt{3}$ m

Explanation :

In DABC,

$\frac{AB}{BC}$ = tan 60°

⇒ AB = $25 \times \sqrt{3}$

Height of building is $25\sqrt{3}$ m.

(ii) What is the distance between two positions of the car?
(a) 40 m
(b) 50 m
(c) 60 m
(d) 75 m

Ans. (b) 50 m

Explanation :

In DABD,

$\frac{AB}{BD}$ = tan 60°

⇒ $\frac{25\sqrt{3}}{BD}$ = $\frac{1}{\sqrt{3}}$

⇒ BD = 75 m

Distance between two positions of car

= (75 – 25) m = 50 m

(iii) What is the total time taken by the car to reach the foot of the building from starting point?
(a) 4 sec.
(b) 3 sec.
(c) 6 sec.
(d) 9 sec.

Ans. (d) 9 sec.

Explanation :

Time taken to cover 50 m distance

= 6 sec.

Time taken to cover 25 m distance = 3 sec.

Total time taken by car = 6 sec. + 3 sec = 9 sec

(iv) What is the distance of the observer from the car when it makes an angle of 60°?

(a) 25 m

(b) 45 m

(c) 50 m

(d) 75 m

Ans. (c) 50 m

Explanation :

In DABC,

$\frac{BC}{AC}$ = cos 60°

⇒ $\frac{25}{AC} = \frac{1}{2}$

⇒ AC = 50 m

(v) The angle of elevation increases:

(a) when point of observation moves towards the object.

(b) when point of observation moves away from the object.

(c) when object moves away from the obsever.

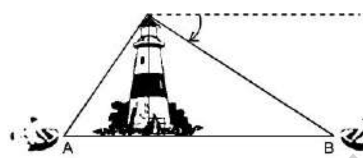
(d) none of the above

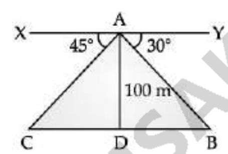
Ans. (a) when point of observation moves towards the object.

Explanation :

As we move toward the foot of vertical object the angle of elevation gradually increases.

71. A boy is standing on the top of light house. He observed that boat A and boat B are approaching to light house from opposite directions. He finds that angle of depression of boat A is 45° and angle of depression of boat B is 30°. He also knows that height of the light house is 100 m.





Answer the following questions.

(i) What is the ÐACD?

(a) 30°

(b) 45°

(c) 60°

(d) 90°

Ans. (b) 45°

Explanation :

ÐXAC = 45° (Given)

ÐACD = 45° [Alternate interior angles]

(ii) If ÐYAB = 30°, then ÐABD is also 30°, why?

(a) vertically opposite angles

(b) alternate interior angles
(c) alternate exterior angles
(d) corresponding angles

Ans. (b) alternate interior an gles

Explanation :

By property of parallel lines.

(iii) What is the length of CD?

(a) 90 m

(b) 60 m

(c) 100 m

(d) 80 m

Ans. (c) 100 m

Explanation :

In DACD,

$\frac{AD}{DC}$ = tan 45°

⇒ $\frac{100}{DC}$ = 1

⇒ DC = 100 m

(iv) What is the length of BD?

(a) 50 m

(b) 100 m

(c) $100\sqrt{2}$ m

(d) $100\sqrt{3}$ m

Ans. (d) $100\sqrt{3}$ m

Explanation :

In DABD,

$\frac{AD}{BD}$ = tan 30°

⇒ $\frac{100}{BD} = \frac{1}{\sqrt{3}}$

⇒ BD = $100\sqrt{3}$ m

(v) What is the length of AC?

(a) $100\sqrt{2}$ m

(b) $100\sqrt{3}$ m

(c) 50 m

(d) 100 m

Ans. (a) $100\sqrt{2}$ m

Explanation :

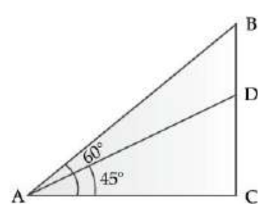
In DADC,

$\frac{AD}{AC}$ = sin 45°

⇒ $\frac{100}{AC} = \frac{1}{\sqrt{2}}$

⇒ AC = $100\sqrt{2}$ m

72. In an exhibition, a statue stands on the top of a pedestal. From the point on ground where a girl is clicking the photograph of the statue the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of pedestal is 45°.



(i) If the height of the pedestal is 20 m, the distance between girl and the foot of the pedestal is:

(a) 20 m

(b) 40 m

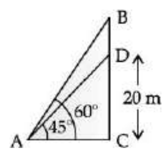
(c) 60 m

(d) 80 m

Ans. (a) 20 m

Explanation :

In DACD,



tan 45° = $\frac{CD}{AC}$ = 1

AC = CD

= 20 m...(i)

(ii) If the height of the pedestal is 20 m, then what is the height of the statue?

(a) $20\sqrt{3}$ m

(b) $20(\sqrt{3}-1)$ m

(c) $20(\sqrt{3}+1)$m

(d) $10(\sqrt{3}-1)$m

Ans. (b) $20(\sqrt{3}-1)$m

Explanation :

Let, BD = *h* m be the height of the statue.

In DABC, tan 60° = $\frac{BC}{AC}$

⇒ $\frac{BD+CD}{AC}$ = $\sqrt{3}$

⇒ $\frac{20+h}{20}$ = $\sqrt{3}$ [Using (i)]

⇒ *h* = $20(\sqrt{3}-1)$m

(iii) If the height of the statue is 1.6 m, then what is the height of the pedestal?

(a) $0.8(\sqrt{3}-1)\,\text{m}$

(b) $1.6(\sqrt{3}+1)\,\text{m}$

(c) $0.8(\sqrt{3})\,\text{m}$

(d) $0.8(\sqrt{3}+1)\,\text{m}$

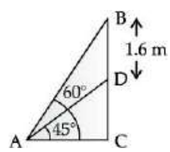
Ans. (d) $0.8(\sqrt{3}+1)\,\text{m}$

Explanation :

Since, in DACD,

ÐDAC = 45°

AC = CD (say *x*)



In DBAC, tan 60° = $\frac{BC}{AC}$

⇒ $\frac{1.6+x}{x} = \sqrt{3}$

⇒ 1.6 = $x(\sqrt{3}-1)$

⇒ *x* = $\frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $0.8(\sqrt{3}+1)\,\text{m}$

(iv) If the total height of the statue and pedestal is 39 m, then find the length of AC.

(a) 13 m

(b) $12\sqrt{3}\,\text{m}$

(c) $13\sqrt{3}\,\text{m}$

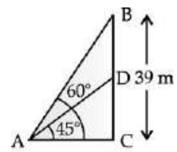
(d) $15\sqrt{3}\,\text{m}$

Ans. (c) $13\sqrt{3}\,\text{m}$

Explanation :

In DABC,

tan 60° = $\frac{BC}{AC}$



⇒ $\frac{39}{AC} = \sqrt{3}$

⇒ AC = $\frac{39}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 13\sqrt{3}\,m$

(v) What is the length of AD, the height of the pedestal is 35 m?

(a) $35\sqrt{2}\,m$

(b) $40\sqrt{2}\,m$

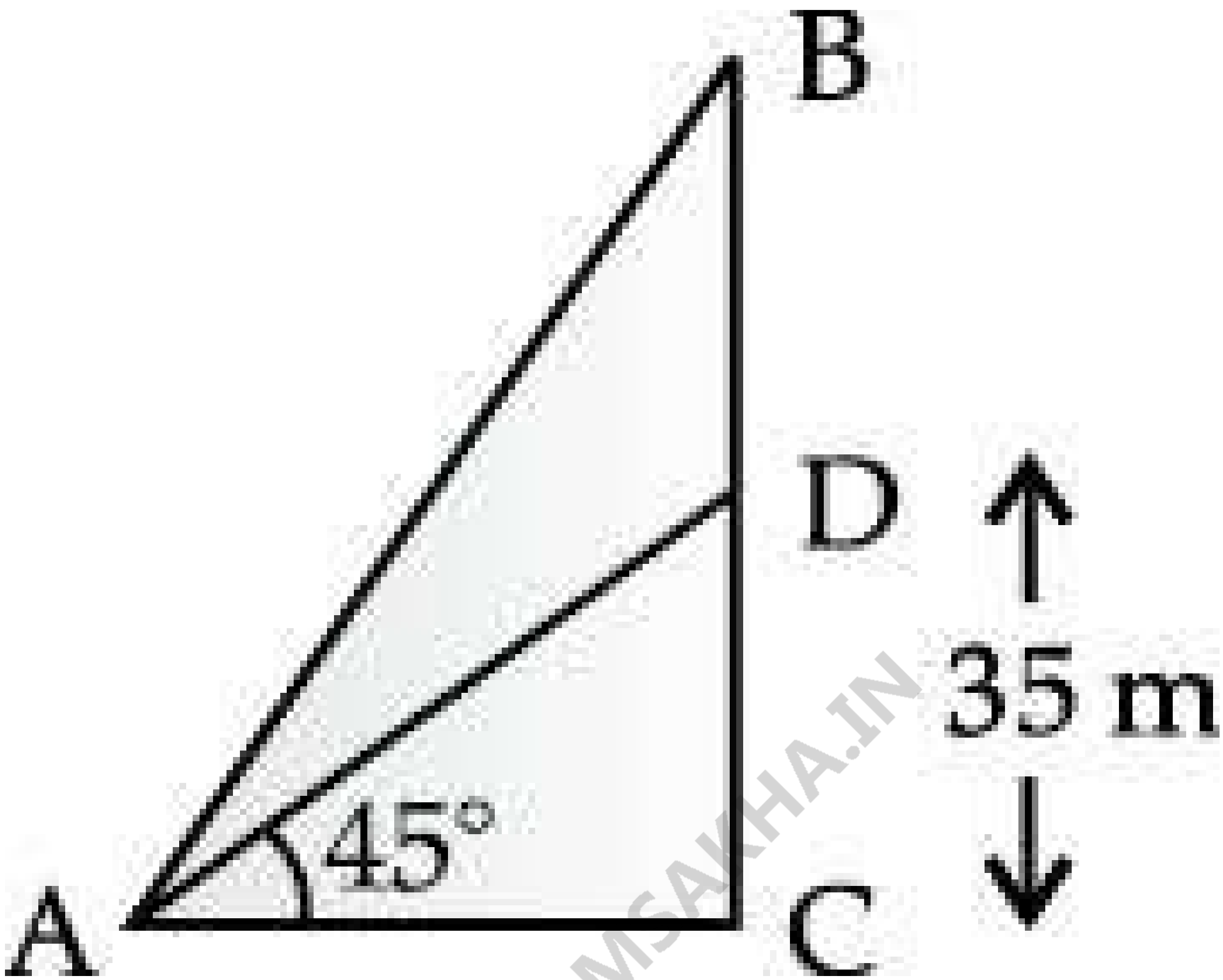
(c) $35(\sqrt{2}+1)\,m$

(d) $35(\sqrt{2}-1)\,m$

Ans. (a) $35\sqrt{2}\,m$

Explanation :

In DACD,

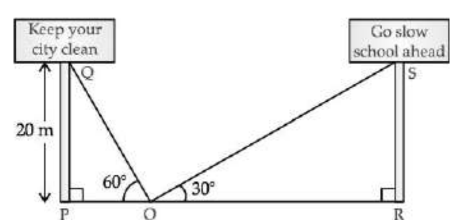


sin 45° = $\frac{CD}{AD}$

⇒ $\frac{35}{AD} = \frac{1}{\sqrt{2}}$

⇒ AD = $35\sqrt{2}$ m

73. Two hoardings are put on two poles of equal heights standing on either side of the road. From a point between them on the road (not the mid point) the angle of elevation of the top of poles are 60° and 30° respectively. Height of the each pole is 20 m



(Take $\sqrt{3}$ = 1.73)

(i) Find the length of PO:
(a) 20 m
(b) $20\sqrt{3}$ m
(c) $\frac{20}{\sqrt{3}}$ m
(d) none of these

Ans. (c) $\frac{20}{\sqrt{3}}$ m

Explanation :

In DOPQ, we have

tan 60° = $\frac{PQ}{PO}$

⇒ $\sqrt{3}$ = $\frac{20}{PO}$

⇒ PO = $\frac{20}{\sqrt{3}}$ m

(ii) Find the length of RO:
(a) 20 m
(b) $20\sqrt{3}$ m
(c) $\frac{20}{\sqrt{3}}$ m
(d) none of these

Ans. (b) $20\sqrt{3}$ m

Explanation :

In DORS, we have

tan 30° = $\frac{RS}{OR}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{20}{OR}$

⇒ OR = $20\sqrt{3}$ m

(iii) The width of the road is:

(a) 31.23 m

(b) 35.68 m

(c) 39.73 m

(d) 46.24 m

Ans. (d) 46.24 m

Explanation :

Clearly,

width of road = PR

= PO + OR = $\left(\frac{20}{\sqrt{3}} + 20\sqrt{3}\right)$m

= $20\left(\frac{4}{\sqrt{3}}\right)$m

= $\frac{80}{\sqrt{3}}$m = 46.24 m

(iv) If the angle of elevation made by pole PQ is 45°, then the length of PO =

(a) 20 m

(b) $20\sqrt{3}$ m

(c) $\frac{20}{\sqrt{3}}$m

(d) None of these

Ans. (a) 20 m

Explanation :

In DOPQ,

tan 45° = $\frac{PQ}{PO}$

⇒ 1 = $\frac{20}{PO}$

⇒ PO = 20 m.

(v) Angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is known as:

(a) angle of depression

(b) angle of elevation

(c) right angle

(d) reflex angle

Ans. (b) angle of elevation

Explanation :

The angle formed by the line of sight with the horizontal when the object is above the horizontal level is called the angle of elevation.

74. Suppose a straight vertical tree is broken at some point due to storm and the broken part is inclined at a certain distance from the foot of the tree.

And when the tree bends, it forms a right angled triangle. Answer the following questions.

(i) If the top of upper part of broken tree touches ground at a distance of 30 m (from the foot of the tree) and makes an angle of inclination 30°, then the height of remaining part of the tree is:

(a) $\sqrt{3}$ m

(b) $30\sqrt{3}$ m

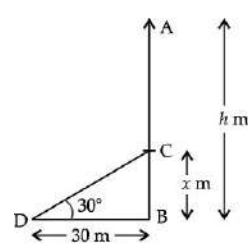
(c) $\frac{30}{\sqrt{3}}$ m

(d) 30 m

Ans. (c) $\frac{30}{\sqrt{3}}$m

Explanation :

Let AB be the tree of height *h* m and let it broken at height of *x* m, as shown in figure.



Clearly CD = AC = ($h - x$) m

Now, in DCBD, we have

tan 30° = $\frac{x}{30}$

⇒ x = $\frac{30}{\sqrt{3}}$m

Thus, the height of remaining part of the tree is $\frac{30}{\sqrt{3}}$m.

(ii) If the top of broken part of a tree touches the ground at a point whose distance from foot of the tree is equal to height of remaining part, then its angle of inclination is:

(a) 30°

(b) 60°

(c) 45°

(d) none of these

Ans. (c) 45°

Explanation :

In this case, BD = BC = x m

If q be the angle of inclination, then

tan q = $\frac{BC}{BD}$ = 1

⇒ tan q = tan 45° ⇒ q = 45°.

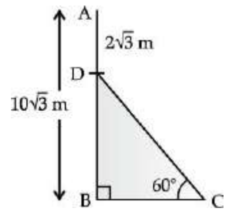
(iii) The angle of elevation is always:
(a) obtuse angle
(b) acute angle
(c) right angle
(d) reflex angle

Ans. (b) acute angle

Explanation :

The angle of elevation and depression are always acute angles.

(iv) If AB = $10\sqrt{3}$m, AD = $2\sqrt{3}$m, then CD =



(a) 9 m

(b) 11 m

(c) 14 m

(d) 16 m

Ans. (d) 16 m

Explanation :

Clearly, BD = AB – AD

= $(10\sqrt{3} - 2\sqrt{3})\text{m}$

= $8\sqrt{3}\,\text{m}$

Now, in DBCD, we have

sin 60° = $\frac{BD}{DC}$

⇒ $\frac{\sqrt{3}}{2}$ = $\frac{8\sqrt{3}}{DC}$

⇒ DC = 16 m

(v) If the height of a tree is 6 m, which is broken by wind is such a way that its top touches the ground and makes an angles 30° with the ground. At what height from the bottom of the tree is broken by the wind?

(a) 2 m

(b) 4 m

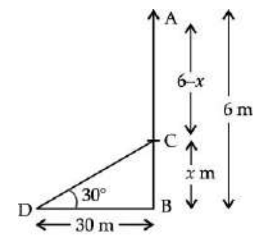
(c) 8 m

(d) 10 m

Ans. (a) 2 m

Explanation :

Now, in DBCD, we have



sin 30° = $\frac{BC}{DC}$

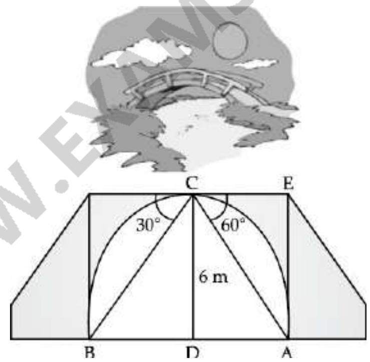
⇒ $\frac{1}{2}$ = $\frac{x}{6-x}$

⇒ 6 – x = 2x

⇒ 3x = 6

⇒ x = 2

75. One day while sitting on the bridge across a river Aaradhya observes the angles of depression of the banks on opposite sides of the river to be 30° and 60° respectively as shown in the figure.(Take $\sqrt{3}$ = 1.73)



Based on the above information, answer the following questions.

(i) If the bridge is at a height of 6 m, then AD =

(a) 6 m

(b) $\frac{\sqrt{3}}{6}$m

(c) $6\sqrt{3}$ m

(d) $2\sqrt{3}$ m

Ans. (d) $2\sqrt{3}$ m

Explanation :

Clearly, ∠DAC = ∠ACE = 60°

So, in DADC, we have

tan 60° = $\frac{CD}{AD}$

⇒ $\sqrt{3}$ = $\frac{6}{AD}$

⇒ AD = $\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ = $2\sqrt{3}$ m

(ii) Now, BD =
(a) 6 m
(b) $6\sqrt{3}$ m
(c) $\sqrt{3}$ m
(d) $10\sqrt{3}$ m

Ans. (b) $6\sqrt{3}$ m

Explanation :

Clearly, ∠DBC = 30°

So, in DBDC, we have

tan 30° = $\frac{CD}{BD}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{6}{BD}$

⇒ BD = $6\sqrt{3}$ m

(iii) What is the width of the river?
(a) 10.85 m
(b) 13.87 m
(c) 15.85 m
(d) 19.85 m

Ans. (b) 13.87 m

Explanation :

Width of the river = AB = AD + BD

$= \frac{6}{\sqrt{3}} + 6\sqrt{3}$

$= 6\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)$

$= 6\left(\frac{4}{\sqrt{3}}\right) = \frac{24}{\sqrt{3}}\text{m}$

= 13.87 m

(iv) The angles of elevation and depression are always:

(a) acute angles

(b) obtuse angles

(c) right angles

(d) straight angles

Ans. (a) acute angles

Explanation :

The angle of elevation and angle of depression are always acute angles.

(v) If BD = 21 m, then height of the bridge is:

(a) 7 m

(b) 21 m

(c) $7\sqrt{3}\,\text{m}$

(d) $\frac{7}{\sqrt{3}}\text{m}$

Ans. (c) $7\sqrt{3}\,\text{m}$

Explanation :

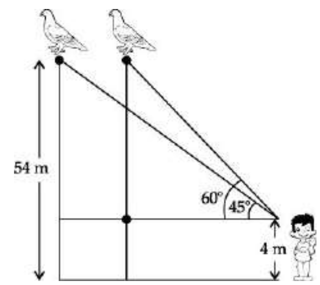
In DBCD, if BD = 21 m, then

tan 30° = $\frac{CD}{BD}$

⇒ $\frac{1}{\sqrt{3}} = \frac{CD}{21}$

⇒ CD = $\frac{21\sqrt{3}}{3} = 7\sqrt{3}\,\text{m}$

76. Raju 4 m tall spots a pigeon sitting on the top of a pole of height 54 m from the ground. The angle of elevation of the pigeon from the eyes of boy at any instant is 60°. The pigeon flies away horizontally in such a way that it remained at a constant height from the ground. After 8 seconds, the angle of elevation of the pigeon from the same point is 45°.



Now, answer the following questions. (Take $\sqrt{3}$ = 1.73)

(i) Find the distance of first position of the pigeon from the eyes of the boy.

(a) 54 m

(b) 100 m

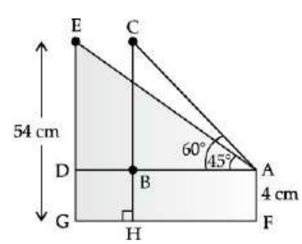
(c) $\frac{100}{\sqrt{3}}$ m

(d) $100\sqrt{3}$ m

Ans. (c) $\frac{100}{\sqrt{3}}$ m

Explanation :

Distance of first position of pigeon from the eyes of boy = AC



In DABC,

sin 60° = $\frac{BC}{AC}$

⇒ AC = $\frac{CH-BH}{\sin 60°}$

$= \frac{54-4}{\sqrt{3}/2} = \frac{100}{\sqrt{3}}m$

(ii) If the distance between the position of pigeon increases, then the angle of elevation:

(a) Increases (b) decreases

(c) remains unchanged (d) can't say

Ans. (b) decreases

Explanation :

If the distance between positions increases, then the angle of elevation decreases.

(iii) What is the distance between Raju and the pole.

(a) 50 m

(b) $\frac{50}{\sqrt{3}}m$

(c) $50\sqrt{3}m$

(d) $60\sqrt{3}m$

Ans. (b) $\frac{50}{\sqrt{3}}m$

Explanation :

Distance between Raju and pole = AB

Now in DABC,

tan 60° = $\frac{BC}{AB}$

⇒ $\sqrt{3}AB$ = 50

⇒ AB = $\frac{50}{\sqrt{3}}m$

(iv) How much distance the pigeon covers in 8 seconds?

(a) 12.13 m

(b) 19.60 m

(c) 21.09 m

(d) 26.32 m

Ans. (c) 21.09 m

Explanation :

In DAED,

tan 45° = $\frac{ED}{AD}$

⇒ AD = BC = 50 m(... ED = BC)

Now, distance between two positions of pigeon

= EC

= BD = AD – AB

= $\left(50 - \frac{50}{\sqrt{3}}\right)$m

= $\frac{50(1.73 - 1)}{1.73}$

= 21.09 m

(v) What is the speed of the pigeon.

(a) 2.63 m/sec

(b) 3.88 m/sec

(c) 6.7 m/sec

(d) 9.3 m/sec

Ans. (a) 2.63 m/sec

Explanation :

Speed of pigeon = $\frac{\text{Distance covered}}{\text{Time taken}}$

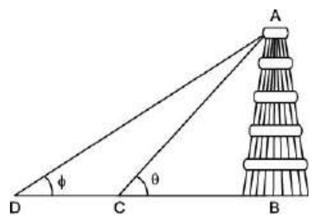
= $\left(\frac{21.09}{8}\right)$m / sec

= 2.63 m/sec

Passage Based Questions

77. Read the passage and answer the questions that follows:

The students of a school are visiting India's tallest monument Qutub Minar. Students roam around the Minar and start discussing that height of Minar seems different as observed from the different points.



(i) If a student is standing at a distance of 75 m from the base of Minar and the angle of elevation of the top of Minar is 45°, then find the height of Qutub Minar.

(ii) If two students, standing on same side, are looking at the Minar, at the same time, making angles of elevation of 30° and 60° respectively and height of the minar is 75 m, then find the distance between them.

Sol. (i) Let the height of Minar be *h*.

tan 45° = $\frac{h}{75}$

⇒ 1 = $\frac{h}{75}$

⇒ h = 75

Hence, the height of minar is 75 m.

(ii) From the figure,

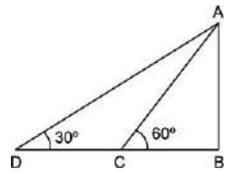
Distance between two students = DC

In ΔADB,

tan 30° = $\frac{AB}{BD}$ = $\frac{75}{BD}$

⇒ $\frac{1}{\sqrt{3}}$ = $\frac{75}{BD}$

⇒ BD = $75\sqrt{3}$



In ΔABC,

tan 60° = $\frac{75}{BC}$

$\Rightarrow \sqrt{3} = \frac{75}{BC}$

$\Rightarrow$ BC = $\frac{75}{\sqrt{3}}$

$\therefore$ DC = BD – BC

= $75\sqrt{3} - \frac{75}{\sqrt{3}}$

= $75\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$ = $75\left(\frac{3-1}{\sqrt{3}}\right)$

= $\frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ = $50\sqrt{3}$ m

Hence, the distance between two students is $50\sqrt{3}$ m.

Self-Assessment

78. A boy is standing at some distance from a 30 m tall building and his eye level from the ground is 1.5 m. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Ans. $19\sqrt{3}$ m.

79. A ramp for unloading a moving truck, has an angle of elevation of 30°. If the top of the ramp is 0.9 m above the ground level, then find the length of the ramp.

Ans. 1·8 m.

80. A girl standing on a light-house built on a cliff near the sea shore, observes two boats due East of the light-house. The angles of depression of the two boats are 30° and 60°. The distance between two boats is 300 m. Find the distance of the top of the light-house from the sea level. (Boats and foot of the light-house are in a straight line).

Ans. $150\sqrt{3}$ m.

81. Two crows A and B are sitting at a height of 15 m and 10 m on two different trees vertically opposite to each other. They view a vadai (an eatable) on the ground at an angle of depression 45° and 60° respectively. They start at the same time and fly at the same speed along the shortest path to pick up the vadai. Which bird will succeed in it?

[**Hint :** Foot of two trees and vadai (an eatable) are in a straight line]

Ans. Crow B.

82. A vertical wall and a tower are on the ground. As seen from the top of the tower, the angles of depression of the top and bottom of the wall are 45° and 60° respectively. Find the height of the wall if the height of the tower is 90 m.

(Take $\sqrt{3}$ = 1.732)

Ans. 38.04 m.

83. A lamp-post stands at the centre of a circular park. Let P and Q be two points on the boundary such that PQ subtends an angle 90° at the foot of the lamp-post and the angle of elevation of the top of the lamp from P is 30°. If PQ = 30 m, then find the height of the lamp-post.

Ans. $5\sqrt{6}$ m.

84. A person in an helicopter flying at a height of 700 m, observes two objects lying opposite to each other on either bank of a river. The angles of depression of the objects are 30° and 45°. Find the width of the river. (Take $\sqrt{3}$ = 1.732)

Ans. 1912.40 m.

85. A person X standing on a horizontal plane, observes a bird flying at a distance of 100 m from him at an angle of elevation of 30°. Another person Y, standing on the roof of a 20 m high building, observes the bird at the same time at an angle of elevation of 45°. If points X and Y are on the opposite sides of the bird, then find the distance of the bird from Y.

Ans. $30\sqrt{2}$ m.

86. The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, then find the height of the hill.

Ans. 150 m.

87. Suppose two insects A and B can hear each other upto a range of 2 m. The insect A is on the ground and 1 m away from a wall and sees her friend B on the wall, about to be eaten by a spider. If A sounds a warning to B and if the angle of elevation of B from A is 30°, will the spider have a meal or not ? (Assume that B escapes if she hears A calling)

Ans. Spider will not have a meal.

88. A simple pendulum of length 40 cm subtends 60° at the vertex in one full oscillation. What will be the shortest distance between the initial position and the final position of the bob?

Ans. 40 cm.

89. A straight highway leads to the foot of a tower. A man standing on the top of the tower spots a van at an angle of depression of 30°. The van is approaching the tower with a uniform speed. After 6 minutes, the angle of depression of the van is found to be 60°. How many more minutes will it take for the van to reach the tower?

Ans. 3 minutes.

90. The angles of elevation of an artificial earth satellite measured from two earth stations, situated on the same side of the satellite are found to be 30° and 60°. The two earth stations and the satellite are in the same vertical plane. If the distance between the earth stations is 4000 km, find the distance between the satellite and earth.

Ans. 3464 km (Use $\sqrt{3}$ = 1.732)

91. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds horizontally, the angle of elevation changes to 30°. If the aeroplane is flying at a speed of 200 m/s, then find the constant height at which the aeroplane is flying.

Ans. $1500\sqrt{3}$ m.

92. A student sitting in a classroom sees a picture on the black board at a height of 1.5 m from the horizontal level of sight. The angle of elevation of the picture is 30°. As the picture is not clear to him, he moves straight towards the black board and sees the picture at an angle of elevation of 45°. Find the distance moved by the student.

Ans. 1.098 m.

93. A boy standing on the ground, spots a balloon moving with the wind in a horizontal line at a constant height. The angle of elevation of the balloon from the boy at an instant is 60°. After 2 minutes, from the same point of observation, the angle of elevation reduces to 30°. If the speed of wind is $29\sqrt{3}$ m/min, then find the height of the balloon from the ground level.

Ans. 87 m.

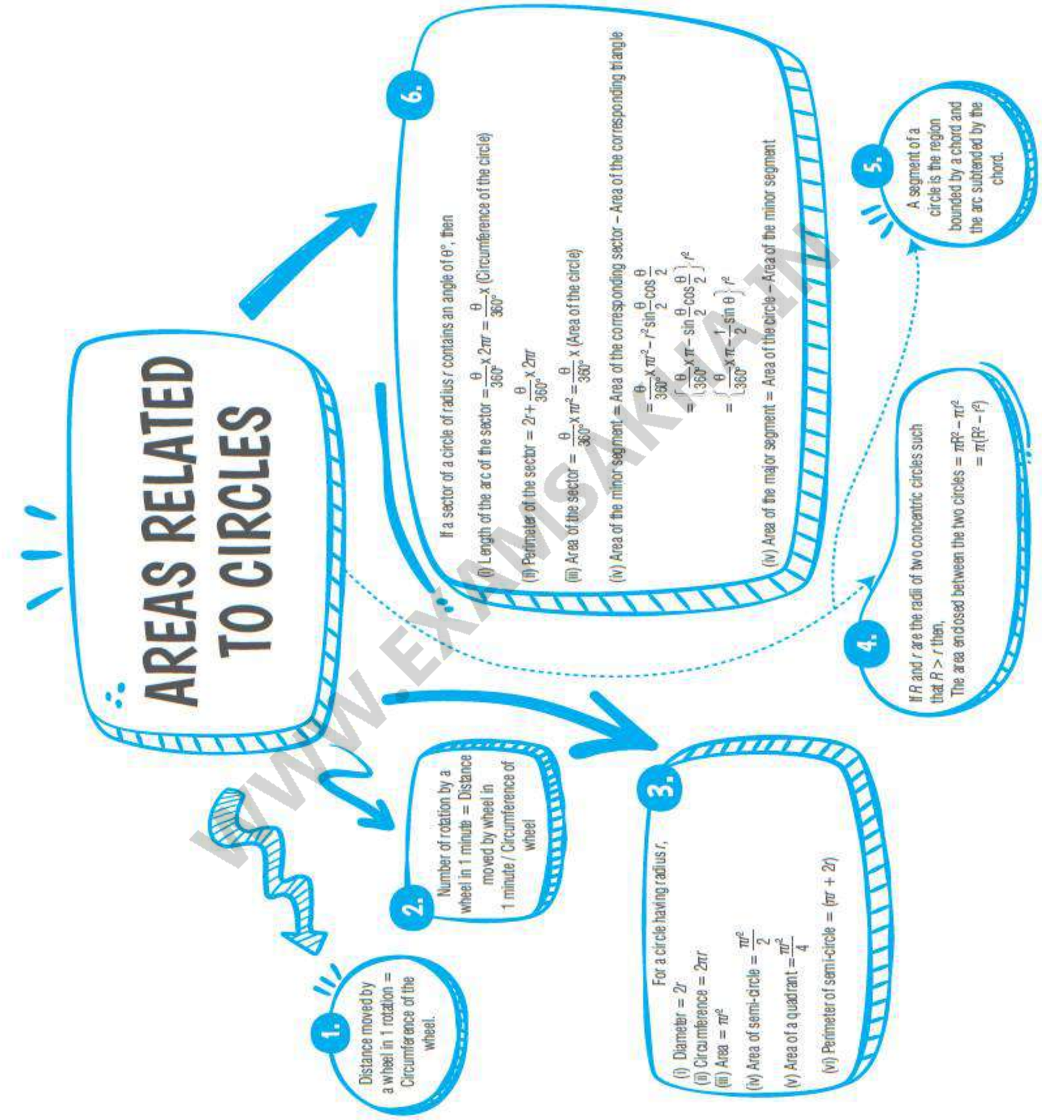
94. The angle of elevation of a hovering helicopter as seen from a point 45 m above a lake is 30° and the angle of depression of its reflection in the lake, as seen from the same point and at the same time is 60°. Find the distance of the helicopter from the surface of the lake.

Ans. 90 m.

Chapter 12

Areas Related to Circles

Basic Concepts



Multiple Choice Questions

1. **If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, the radius of the circle (in cm) is:**

(a) 154

(b) 44

(c) 14

(d) 7

Ans. (d) 7

Explanation :

Let the radius be r cm.

Thus, circumference = $2\pi r$ cm

Now, $2\pi r - r = 37$

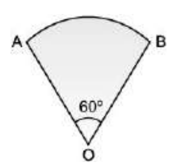
$\Rightarrow r\left(\frac{44}{7} - 1\right) = 37$

$\Rightarrow r\left(\frac{44-7}{7}\right) = 37$

$\Rightarrow r = \frac{7 \times 37}{37} = 7$ cm.

2. **In the given figure AOB is a sector of circle of radius 10.5 cm. The perimeter of the sector (in cm) is:**

$\left(\text{Take } \pi = \frac{22}{7}\right)$



[Board Question]

(a) 32

(b) 21

(c) 11

(d) 35

Ans. (a) 32

Explanation :

Given : OA = OB = 10.5 cm

and ∠AOB = 60°

Perimeter of the sector = OA + OB + AB

= $10.5 + 10.5 + \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 10.5 \left[\because l = \frac{\theta}{360°} \times 2\pi r\right]$

= 21 + 11 = 32 cm.

3. **The area of the sector of a circle of radius 6 cm whose central angle is 30°. (Take** p **= 3.14)**

[Board Question]

(a) 9.42 cm^2

(b) 7.42 cm^2

(c) 8.42 cm^2

(d) 9.42 cm^2

Ans. (a) 9.42 cm^2

Explanation :

Given : Radius of sector (*r*) = 6 cm and central angle (q) = 30°.

Area of sector of a circle = $\frac{\theta}{360°} \times \pi r^2$

= $\frac{30°}{360°} \times 3.14 \times 6^2$

= 3 × 3.14

= 9.42 cm^2

4. **If** π **is taken as** $\frac{22}{7}$, **the distance (in meters) covered by a wheel of diameter 35 cm, in one revolution is:**

(a) 2.2

(b) 1.1

(c) 9.625

(d) 96.25

Ans. (b) 1.1

Explanation :

Given, diameter = 35 cm

Now, one revolution = circumference

= πd = $\frac{22 \times 35}{7}$

= 110 cm

= 1.1 m.

5. The circumference of a circular field is 528 cm. Then the radius will be:

(a) 84 cm

(b) 64 cm

(c) 55 cm

(d) 45 cm

Ans. (a) 84 cm

Explanation :

Circumference of a circular field = 528

⇒ 2p*r* = 528

⇒ $2 \times \frac{22}{7} \times r$ = 528

⇒ r = $\frac{528 \times 7}{2 \times 22}$

= 84 cm.

6. The length of arc of a sector of angle q° **of a circle with radius R is:**

(a) $\frac{2\pi R\theta}{180°}$

(b) $\frac{2\pi R\theta}{360°}$

(c) $\frac{\pi R^2\theta}{180°}$

(d) $\frac{\pi R^2\theta}{360°}$

Ans. (b) $\frac{2\pi R\theta}{360°}$

Explanation :

Length of arc of a sector

= $\frac{\theta}{360°}$ × 2pR

= $\frac{2\pi R\theta}{360°}$.

7. If the circumference and the area of a circle are numerically equal, then diameter of the circle is:

(a) $\frac{r}{2}$ (b) 2p

(c) 2

(d) 4

Ans. (d) 4

Explanation :

$2\pi r = \pi r^2 \Rightarrow 2 = r$
$\therefore d = 2r = 4$.

8. **The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 kmph?**

(a) 200

(b) 240

(c) 250

(d) 260

Ans. (c) 250

Explanation :

Distance to be covered in 1 min = $\frac{66 \times 1000}{60}$ m
= 1100 m
Circumference of the wheel

$= \left(2 \times \frac{22}{7} \times 0.70\right)$ = 4.4 m

∴ Number of revolutions per min = $\left(\frac{1100}{4.4}\right)$
= 250.

9. **A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel:**
(a) 11 m
(b) 14 m
(c) 12 m
(d) 10 m

Ans. (b) 14 m

Explanation :

Distance covered in one revolution

$= \frac{88 \times 1000}{1000}$ m = 88 m

2pR = 88 ⇒ $2 \times \frac{22}{7}$ × R = 88 m

R = $\left(88 \times \frac{7}{44}\right)$= 14 m.

10. **The inner circumference of a circular race track, 14 m wide, is 440 m. Find the radius of the outer circle:**

(a) 85 m

(b) 82 m

(c) 80 m

(d) 84 m

Ans. (d) 84 m

Explanation :

Let inner radius be *r* metres. Then,

2p*r* = 440 ⇒ *r* = $\left(440 \times \frac{7}{44}\right)$= 70 m

Radius of the outer circle = (70 + 14) m = 84 m.

11. **Two concentric circles form a ring. The inner and outer circumference of the ring are $50\frac{2}{7}$ m and $75\frac{3}{7}$ m respectively. Find the width of the ring.**

(a) 1 m

(b) 2 m

(c) 3 m

(d) 4 m

Ans. (d) 4 m

Explanation :

Let the inner and outer radii be *r* and R metres.

Then, 2p*r* = $\frac{352}{7}$

⇒ *r* = $\left(\frac{352}{7}\times\frac{7}{22}\times\frac{1}{2}\right)$= 8 m

and 2pR = $\frac{528}{7}$

⇒ R = $\left(\frac{528}{7}\times\frac{7}{22}\times\frac{1}{2}\right)$= 12 m

Width of the ring = (R – *r*) = (12 – 8) m = 4 m.

12. **A sector of 120° cut out from a circle, has an area of $9\frac{3}{7}$ sq. cm. Find the radius of the circle.**

(a) 1 cm

(b) 1 m

(c) 3 cm

(d) 3 m

Ans. (c) 3 cm

Explanation :

Let the radius of the circle be *r* cm. Then,

$\frac{\pi r^2\theta}{360°} = \frac{66}{7}$

⇒ $\frac{22}{7}\times r^2\times\frac{120°}{360°} = \frac{66}{7}$

⇒ $r^2 = \left(\frac{66}{7}\times\frac{7}{22}\times 3\right)$= 9

⇒ *r* = 3

Hence, radius = 3 cm.

13. **If the radius of a circle is increased by 75%, then its circumference will increase by:**

(a) 25%

(b) 50%

(c) 75%

(d) 100%

Ans. (c) 75%

Explanation :

Let original radius be R cm. Then, original circumference = (2pR) cm
New radius = (175% of R) cm
= $\left(\frac{175}{100}\times R\right)$cm = $\frac{7R}{4}$ cm
New circumference = $\left(2\pi\times\frac{7R}{4}\right)$cm = $\frac{7\pi R}{2}$ cm
Increase in circumference
= $\left(\frac{7\pi R}{2}-2\pi R\right)$ = $\frac{3\pi R}{2}$ cm
Increase % = $\left(\frac{3\pi R}{2}\times\frac{1}{2\pi R}\times 100\right)\%$
= 75%.

14. **A can go around a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, then the time required by A to go around the new path once, travelling at the same speed as before is:**

(a) 20 min

(b) 25 min

(c) 50 min

(d) 100 min

Ans. (c) 50 min

Explanation :

Let original diameter be *d* meters. Then, its circumference = (p*d*) metres

Time taken to cover (8p*d*) m = 40 min

New diameter = (10 *d*) m.

Then, its circumference

= (p × 10*d*) m

∴ Time taken to go around it once

= $\left(\frac{40}{8\pi d} \times 10\pi d\right)$m = 50 min.

15. **If the radius of a circle is doubled, its area is increased by:**

(a) 100%

(b) 200%

(c) 300%

(d) 400%

Ans. (c) 300%

Explanation :

Let the original radius be R cm. New radius = 2R

Original area = pR^2, New area = $p(2R)^2$ = $4pR^2$

Increase in area = ($4pR^2$ – pR^2) = $3pR^2$

Increase % = $\left(\frac{3\pi R^2}{\pi R^2} \times 100\right)$= 300%.

16. **If the circumference of a circle increases from 4**p **to 8**p**, what change occurs in its area?**

(a) It is halved

(b) It doubles

(c) It triples

(d) It quadruples

Ans. (d) It quadruples

Explanation :

$4pR_1 = 4p$ and $4pR_2 = 8p$

$R_1 = 2$ and $R_2 = 4$

Original area = $(p \times 2^2) = 4p$

Increased area = $(p \times 4^2) = 16p$

Thus, the area quadruples.

17. Number of rounds that a wheel of diameter $\frac{7}{11}$ metre will make in moving a distance of 4 km is ....

(a) 1000 rounds

(b) 2000 rounds

(c) 3000 rounds

(d) none of these

Ans. (b) 2000 rounds

Explanation :

Circumference = $2pr = p \times d$

$= \frac{22}{7} \times \frac{7}{11}$

= 2

No. of rounds = $\frac{\text{Distance travelled}}{\text{Circumference of circle}}$

$= \frac{4000}{2}$ = 2000.

18. A pendulum swings through an angle of 30º and describes an arc 6·6 cm in length then the length of pendulum is ....

(a) 12.6 cm

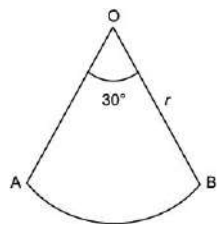
(b) 10.6 cm

(c) 5 cm

(d) 10 cm

Ans. (a) 12.6 cm

Explanation :



Let r be the length of pendulum length of arc
= 6·6 cm
∠AOB = 30°

Length of an arc = $\frac{\theta}{360^\circ} \times 2\pi r$

⇒ 6·6 = $\frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r$

⇒ r = $\frac{6 \cdot 6 \times 360 \times 7}{2 \times 30 \times 22}$

= $\frac{6 \cdot 6 \times 12 \times 7}{44}$

= $\frac{6 \times 12 \times 7}{40}$

r = 12·6 cm.

19. If the radius of a circle is diminished by 10%, then its area is diminished by ....

(a) 29%

(b) 19%

(c) 15%

(d) 9%

Ans. (b) 19%

Explanation :

Let radius = x
Given that the radius is dimished by 10%, so

Radius = $x - \frac{x}{10}$ = $\frac{9x}{10}$

Area = px^2

New Area = $\pi\left(\frac{9x}{10}\right)^2$

= $\pi \times \frac{81x^2}{100}$

Change in area = $\pi x^2 - \pi \frac{81}{100} x^2$

= $\pi x^2 - \frac{\pi x^2 \times 81}{100}$

= $\frac{19}{100} \pi x^2$ = 19%.

20. If the area of a semi circular region is 308 sq cm, then its perimeter is ....

(a) 27 cm

(b) 75 cm

(c) 80 cm

(d) 72 cm

Ans. (d) 72 cm

Explanation :

Area of semicircular region = 308

$\frac{\pi r^2}{2}$ = 308

⇒ $\frac{22}{7} \times \frac{r^2}{2}$ = 308

⇒ r^2 = $\frac{308 \times 7 \times 2}{22}$

⇒ r^2 = 14 × 7 × 2

⇒ r = $\sqrt{7 \times 2 \times 7 \times 2}$ = 14 cm

Perimeter = pr + 2r = $\frac{22}{7} \times 14 + 2 \times 14$

= 44 + 28 = 72 cm.

21. If a chord of a circle of radius 14 cm makes an angle of 90° at the centre, then the area of major segment is ....

(a) 560 cm^2

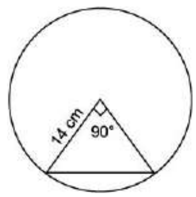
(b) 300 cm^2

(c) 160 cm^2

(d) none of these

Ans. (a) 560 cm^2

Explanation :



Area of major segment

$= \frac{\theta}{360°} \times \pi r^2 + \frac{r^2 \sin \theta}{2}$

$= \frac{270°}{360°} \times \pi(14)^2 + \frac{14 \times 14}{2} \times \sin 90°$

$= \frac{3 \times 14 \times 14\pi}{4} + \frac{14 \times 14}{2}$

= 462 + 98

= 560 cm^2.

22. **Area of a sector of angle *p*(in degrees) of a circle with radius R is:**

[NCERT Exemplar]

(a) $\frac{p}{180°} \times 2\pi R$

(b) $\frac{p}{180°} \times \pi R^2$

(c) $\frac{p}{360°} \times 2\pi R$

(d) $\frac{p}{720°} \times 2\pi R^2$

Ans. (d) $\frac{p}{720°} \times 2\pi R^2$

Explanation :

Sector angle is *p* in degrees

Radius of the circle = R

Area of the sector = $\frac{\pi R^2 p}{360^\circ} = \frac{(\pi R^2 p)2}{720^\circ}$

= $\frac{p}{720^\circ} \times 2\pi R^2$.

Very Short Answer Type Questions

23. The circumference of a circle is 22 cm. Find the area of its quadrant (in cm2).

Sol. Given, circumference of circle

= 2p*r* = 22 cm

or *r* = $\frac{11}{\pi}$ cm

Area of the quadrant

= $\frac{\pi r^2}{4} = \frac{\pi}{4} \times \frac{11}{\pi} \times \frac{11}{\pi}$

= $\frac{11 \times 11 \times 7}{4 \times 22}$ = $\frac{77}{8}$ cm^2. **Ans.**

24. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, then find the diameter of the larger circle (in cm).

Sol. Given, the diameters of the two smaller circles = 10 cm and 24 cm.

Thus, radii of the two smaller circles = 5 cm and 12 cm, respectively.

Let the radius of the larger circle be *R*′.

Now, area of the larger circle

= Sum of the area of the two smaller circles

= p(R2 + *r*2)

= p[(12)2 + (5)2]

= p[144 + 25]

= p[169]

= p(13)2

⇒ p(R′)2 = p(13)2

⇒ (R′)2 = (13)2

⇒ R′ = 13 cm

Hence, diameter of the larger circle = 26 cm.

Ans.

25. If the area of a circle is numerically equal to twice its circumference then find the diameter of the circle.

Sol. Let the radius of the circle be *r*.

Given, the area of a circle is numerically equal to twice its circumference.

Thus p*r*2 = 2(2p*r*)

or *r* = 4 units

Thus, the diameter of the circle = 8 units. **Ans.**

26. Find the area of a quadrant of a circle whose circumference is 88 cm.

Sol. Given,

Circumference of circle = 88 cm

⇒ 2p*r* = 88 cm

⇒ *r* = $\frac{88}{2\times\frac{22}{7}}=\frac{88\times7}{2\times22}$

⇒ *r* = 14 cm

Area of quadrant = $\frac{1}{4}$ p*r*2

= $\frac{1}{4}\times\frac{22}{7}\times14\times14$

= 154 cm2. **Ans.**

27. Find the area of annulus whose inner and outer radii are 6 cm and 8 cm.

Sol. Given,

Inner radius of annulus = 6 cm

Outer radius of annulus = 8 cm

Area of annulus = p(R2 – *r*2)

= p(82 – 62)

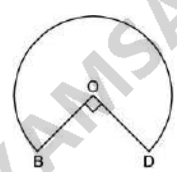
= p(64 – 36)

= 28p

= 28 × $\frac{22}{7}$

= 88 cm2 **Ans.**

28. In the given figure, the top of a table in a restaurant is that of a segment of a circle with centre O and ∠BOD = 90°, BO = OD = 60 cm, then find the area of the top of the table. (Use π = 3.14)



Sol. Area of the top of table

= Area of 3 quadrants of circle

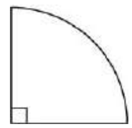
= $\frac{3}{4}$p*r*2

= $\frac{3}{4}$p(60)2

= $\frac{3}{4}$× 3.14 × 3600

= 8478 cm2. **Ans.**

29. The perimeter of a sheet of tin in the shape of quadrant of a circle is 12.5 cm then find its area.



Sol. If *r* be the radius of the circle.

Then, the perimeter of a quadrant of the circle

= $\frac{1}{4}$× 2p*r* + 2*r*

= $\left(\frac{\pi}{2}+2\right)r$

= 12.5 (Given)

⇒ $\left(\frac{1}{2}\times\frac{22}{7}+2\right)r$ = $\frac{25}{2}$

⇒ $\frac{25r}{7}$ = $\frac{25}{2}$

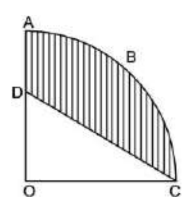
⇒ *r* = $\frac{7}{2}$

Area of quadrant = $\frac{1}{4}$p*r*2

= $\frac{1}{4}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2 \text{cm}^2$

= 9.625 cm2 **Ans.**

30. In the given figure, OABC is a quadrant of a circle of radius 7 cm. If OD = 4 cm, find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Sol. Given, radius = OA = OC = 7 cm and DO = 4 cm

Thus, area of the quadrant ABCO

= $\frac{\pi r^2}{4} = \frac{(7)^2}{4}\times\frac{22}{7}$

= $\frac{77}{2}\text{cm}^2$

Also, area of DCOD

$= \frac{1}{2} \times 7 \times 4 = 14$ cm2

Thus, area of the shaded region ABCD

= area of the quadrant ABCO– area of DCOD

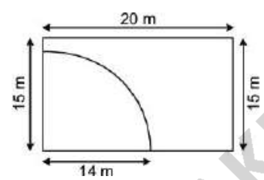
or Area of ABCD = $\frac{77}{2} - 14$

$= \frac{77-28}{2} = \frac{49}{2}$

= 24.5 cm2. **Ans.**

31. A cow is tied with a rope of length 14 m at one corner of a rectangular field of dimensions 20 m × 15 m. Find the area of the field in which the cow cannot graze. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Area of the field = 20 m × 15 m = 300 m2



Area of the quadrant = $\frac{\pi r^2}{4} = \frac{1}{4} \times \frac{22}{7} \times (14)^2$

$= \frac{22 \times 14 \times 14}{7 \times 4}$

= 154 cm2

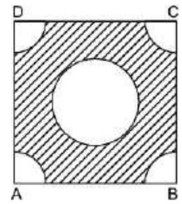
Thus, area of the field left ungrazed

= Area of the rectangle– Area of the quadrant

= (300 – 154) cm2

= 146 cm2.**Ans.**

32. In the given figure, ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each corner of the square and a circle of diameter 2 cm is drawn at the centre. Find the area of the shaded region. (Use π = 3.14)



Sol. Area of the square

= 4 × 4

= 16 cm2

Area of each quadrant

= $\frac{\pi \times 1^2}{4}$

= $\frac{3.14 \times 1}{4}$

= 0.785 cm2

Given, diameter of the circle = 2 cm

Thus, radius = 1 cm

Hence area of the circle = p(1)2 = 3.14 cm2

Area of the shaded region

= Area of the square– [4 × Area of a quadrant+ Area of the circle]

∴ Area of the shaded region

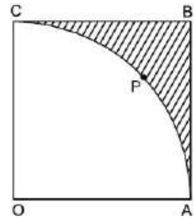
= 16 – [4 × (0.785) + 3.14] cm2

= 16 – (3.14 + 3.14) cm2

= (16 – 6.28) cm2

= 9.72 cm2. **Ans.**

33. In the given figure, OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Sol. Given, OA = AB = BC = CO = 7 cm

Also, CO = OA = Radius of the quadrant

Thus, area of the shaded region

= Area of square – Area of quadrant

= (7)2 – $\frac{1}{4}\pi(7)^2$

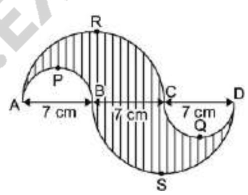
= 49 – $\frac{1}{4}\times\frac{22}{7}\times(7)^2$

= 49 – $\frac{77}{2}$

= $\frac{98-77}{2}=\frac{21}{2}$

= 10.5 cm2. **Ans.**

34. In the given figure, APB and CQD are semi-circles of diameter 7 cm each, while ARC and BSD are semi-circles of diameter 14 cm each. Find the perimeter of the figure. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Sol. Given, *d* = 7 cm and D = 14 cm

Thus, *r* = 3.5 cm and R = 7 cm

Hence, the perimeter of the figure

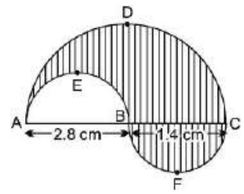
= Perimeter of (APB + BSD + DQC + CRA)

= p(3.5 + 7 + 3.5 + 7)

= $\frac{22}{7}$(21)

= 66 cm. **Ans.**

35. Find the perimeter of the shaded region, where ADC, AEB and BFC are semi-circles on diameters AC, AB and BC respectively.



Sol. Given, diameter of ADC

= (2.8 + 1.4) cm

= 4.2 cm

Thus, radius of ADC = 2.1 cm

Diameter of AEB = 2.8 cm

Thus, radius of AEB = 1.4 cm

Diameter of BFC = 1.4 cm

Thus, radius of BFC = 0.7 cm

Hence, Perimeter of the shaded region

= perimeter of (ADC + BFC + AEB)

= p(2.1 + 0.7 + 1.4) cm

= $\frac{22}{7}$(4.2) cm

= 13.2 cm. **Ans.**

36. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. Given, length of the minute hand

= *r* = 14 cm

Total number of divisions in the clock = 12

Hence the angle subtended at the centre between two digits covering five minutes = $\frac{360°}{12} = 30°$

Hence, area swept by the minute hand in covering 5 minutes

$= \frac{\theta}{360°}$ p*r*2

$= \frac{30°}{360°} \times \frac{22}{7} \times (14)^2$

$= \frac{154}{3}$

= 51.33 cm2. **Ans.**

37. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find (i) the length of the arc, and (ii) area of the sector formed by the arc. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Given, *r* = 21 cm and q = 60°

(i) Length of an arc = $\frac{\theta}{360°}$ × 2p*r*

$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$

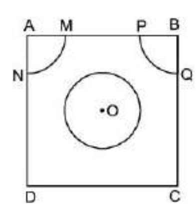
= 22 cm

(ii) Area of the sector = $\frac{\theta}{360°}$ × p*r*2

$= \frac{60°}{360°} \times \frac{22}{7}(21)^2$

= 231 cm2. **Ans.**

38. From each of the two adjacent corners of a square of side 8 cm, a quadrant of a circle of radius 1.4 cm is cut. Another circle of radius 4.2 cm is also cut from the centre as shown in the given figure. Find the area of the remaining portion of the square. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Sol. Given, AB = BC = CD = DA = 8 cm,

AM = AN = PB = QB = 1.4 cm and radius of circle with centre O = 4.2 cm

Thus, area of the remaining portion

= Area of ABCD – [Area of circle + 2 Area of quadrants]

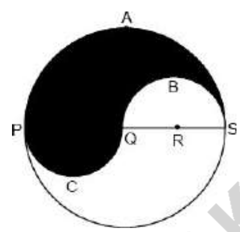
= 8 × 8 – $\left\{\frac{22}{7}(4.2)^2 + 2 \times \frac{1}{4} \times \frac{22}{7}(1.4)^2\right\}$

= 64 – (55.44 + 3.08) cm2

= 5.48 cm2. **Ans.**

Short Answer Type Questions

39. PS is the diameter of a circle of radius 6 cm. Q and R are points on the diameter such that PQ, QR and RS are equal. Semicircles are drawn with PQ and QS as diameters as shown in the figure. Find the perimeter of the shaded region. [Use π = 3.14]



Sol. Given,

r = 6 cm, PQ = QR = RS

and PS = PQ + QR + RS

Hence, PS = 2*r* = 12 cm

Now, PS = PQ + QR + RS

⇒ 12 = 3PQ = 3QR = 3RS

⇒ PQ = QR = RS = 4 cm

Thus, PQ is the diameter of the smaller semicircle

and (QR + RS) is the diameter of the larger semicircle.

So, the perimeter of the shaded region

= Perimeter of (PAS + PCQ + QBS)

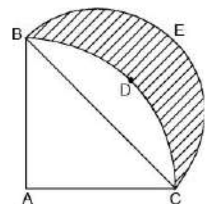
= [π(6) + π(2) + π(4)] cm

= 12π cm

= 12(3.14) cm

= 37.68 cm. **Ans.**

40. In the given figure, ABDC is a quadrant of a circle of radius 28 cm and a semi-circle BEC is drawn with BC as diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Sol. Given, radius = 28 cm

Thus, AB = AC = 28 cm

Now, area of quadrant ABDC

= $\frac{\pi r^2}{4} = \frac{22}{7} \times \frac{(28)^2}{4}$

= 616 cm^2

Also, applying Pythagoras' theorem, we get

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = (28)^2 + (28)^2$

$\Rightarrow BC^2 = 2(28)^2$

$\Rightarrow$ BC = $28\sqrt{2}$ cm

Thus, radius of semi-circle BEC = $14\sqrt{2}$ cm.

Hence, area of semi-circle BEC

= $\frac{\pi r^2}{2}$

= $\frac{22}{7} \times \frac{(14\sqrt{2})^2}{2}$

= 616 cm^2

and Area of ΔABC = $\frac{1}{2}$ × 28 × 28

= 392 cm^2

Thus, the area of the shaded region

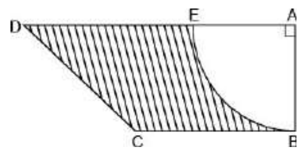
= Area of semi-circle BEC + Area of ΔABC– Area of quadrant ABCD

= 616 cm^2 + 392 cm^2 – 616 cm^2

= 392 cm^2. **Ans.**

41. In the given figure, ABCD is a trapezium of area 24.5 sq. cm. AD || BC, ∠DAB = 90°, AD = 10 cm and BC = 4 cm. If ABE is a quadrant of a circle, find the area of the shaded region.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Sol. Given,

Area of ABCD = 24.5 sq. cm, AD || BC,

∠DAB = 90°, AD = 10 cm and BC = 4 cm

Now, area of trapezium ABCD

= $\frac{1}{2}$ × (Sum of parallel sides)× (Distance between parallel sides)

⇒ 24.5 = $\frac{1}{2}$ × (10 + 4) × (AB)

⇒ 24.5 = $\frac{1}{2}$ × (14) × (AB)

⇒ AB = $\frac{24.5}{7}$

= 3.5 cm

Again, AB is the radius of the quadrant of the circle.

So, area of quadrant ABE = $\frac{\pi r^2}{4}$

= $\frac{22}{7} \times \frac{(3\cdot5)^2}{4}$ = 9·625 cm^2

Thus, area of the shaded region

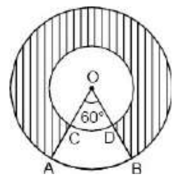
= Area of trapezium ABCD – Area of quadrant ABE

= (24.5 – 9·625) cm^2

= 14.875 cm^2. **Ans.**

42. In the given figure, two concentric circles with centre O have radii of 21 cm and 42 cm. If ∠AOB = 60°, find the area of the shaded region.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



[Board Question]

Sol. Given, AO = OB = 42 cm, CO = OD = 21 cm and ∠AOB = 60°.

Now, area of the larger circle

$= \frac{22}{7}(42)^2$

$= \frac{22 \times 42 \times 42}{7}$

= 22 × 42 × 6

= 5544 cm^2

Area of the smaller circle

$= \frac{22}{7}(21)^2$

$= \frac{22 \times 21 \times 21}{7}$

= 22 × 21 × 3

= 1386 cm^2

Area of ACDB = $\frac{\theta}{360°} \times \pi(R^2 - r^2)$

$= \frac{60°}{360°} \times \frac{22}{7}\{(42)^2 - (21)^2\}$

$= \frac{11}{21}\{(63)(21)\}$

= 693 cm^2

Thus, the area of the shaded region

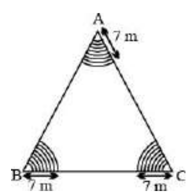
= Area of the larger circle– [Area of the smaller circle + Area of ACDB]

= [5544 – (1386 + 693)] cm^2

= (5544 – 2079) cm^2

= 3465 cm^2. **Ans.**

43. A farmer has a field in the form of an equila-teral triangle. He makes pits for rainwater harvesting in three corners of his field as shown in shaded part. Find the area of shaded part. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Sol. Area of one sector = $\frac{\theta}{360°} \times \pi r^2$
Here θ = 60°, r = 7 m

∴ Area of the sector = $\frac{60°}{360°} \times \frac{22}{7} \times 7 \times 7$

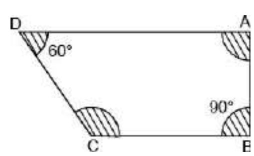
= $\frac{1}{6}$ × 22 × 7
= $\frac{77}{3}$ m^2

∴ Area of 3 sectors = 3 × $\frac{77}{3}$ = 77 m^2

Hence, total area of pits (shaded part) = 77 m^2.

Ans.

44. ABCD is a field in the shape of a trapezium. AD || BC, ∠ABC = 90° and ∠ADC = 60°. Four sectors are formed with centres A, B, C and D as shown in figure. The radius of each sector is 14 m.



Find the following :

(i) Total area of the four sectors.

(ii) Area of the remaining portion given that AD = 55 m, BC = 45 m and AB = 30 m.

Sol. Area of quadrant A

= Area of quadrant B

= $\frac{\pi r^2}{4} = \frac{1}{4} \times \frac{22}{7} \times (14)^2$

= 154 cm^2

Area of sector D = $\frac{\theta}{360°} \times \pi r^2$

= $\frac{60°}{360°} \times \frac{22}{7} \times (14)^2$

= $\frac{1}{6} \times \frac{22}{7} \times (14)^2$

= 102.67 cm^2

Angle of sector C = 360° – (90° + 90° + 60°)

= 120°

∴ Area of sector C = $\frac{\theta}{360°} \times \pi r^2$

= $\frac{120°}{360°} \times \frac{22}{7} \times (14)^2$

= $\frac{1}{3} \times \frac{22}{7} \times (14)^2$

= 205.33 cm^2

(i) Total area of the four sectors

= (154 + 154 + 102.67 + 205.33) cm^2

= 616 cm^2

(ii) Area of the remaining portion

= Area of the trapezium– Total area of the four sectors

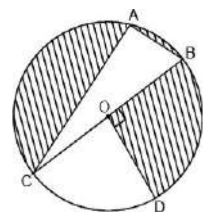
= $\left[\frac{1}{2} \times (55+45) \times (30) - 616\right]$ cm^2

= $\left[\frac{1}{2} \times (100) \times (30) - 616\right]$ cm^2

$= [1500 - 616]\ cm^2$

$= 884\ cm^2$. **Ans.**

45. In the given figure, O is the centre of the circle, AC = 24 cm, AB = 7 cm and ∠BOD = 90°. Find the area of the shaded region. [Use π = 3.14].



[Board Question]

Sol. Given, AC = 24 cm, AB = 7 cm and ∠BOD = 90°

Also, the angle in a semi-circle is always 90°. So

∠CAB = 90°

Thus, ΔABC is a right-angled triangle.

Hence, Area of ΔABC = $\frac{1}{2} \times 24 \times 7 = 84\ cm^2$

Now, $CB^2 = AB^2 + AC^2$

[By Pythagoras' Theorem]

$\Rightarrow CB^2 = (7)^2 + (24)^2$

$\Rightarrow CB^2 = 49 + 576 = 625$

$\Rightarrow CB = 25$ cm

[∵ length cannot be negative]

As CB passes through the centre and touches opposite ends of the circle, it is the diameter.

So, radius = 12.5 cm = OD

Hence, area of the circle

$= \pi(12.5)^2$

= 3.14 × 12.5 × 12.5

= 490.63 cm^2

and area of quadrant COD

= $\frac{\pi r^2}{4}$ = $\frac{1}{4} \times 3.14 \times (12.5)^2$

= 122.66 cm^2

Hence, area of the shaded region

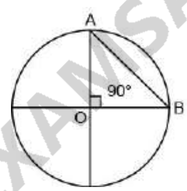
= Area of the circle – [Area of the ΔABC + Area of quadrant COD]

= 490.63 – [84 + 122.66] cm^2

= 283.97 cm^2 ≈ 284 cm^2. **Ans.**

46. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment and hence find the area of the major segment.[Use π = 3.14]

Sol. Given, *r* = AO = BO = 10 cm and ∠AOB = 90°



Thus, the area of the minor segment

= $\left\{\frac{90°}{360°} \times \pi - \frac{1}{2} \sin 90°\right\}$ $(10)^2$

= $\left\{\frac{3.14}{4} - \frac{1}{2}\right\}$ $(10)^2$

= $\left\{\frac{1.57}{2} - \frac{1}{2}\right\}$ $(10)^2$

= $\left\{\frac{0.57}{2}\right\}$100

= $\frac{57}{2}$ = 28.5 cm^2

Thus, the area of the major segment

= Area of the circle – Area of the minor segment

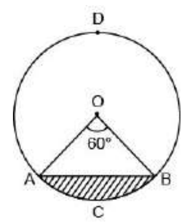
$= (\pi r^2 - 28.5)\ cm^2 = [3.14(10)^2 - 28.5]\ cm^2$

$= [314 - 28.5]\ cm^2$

$= 285.5\ cm^2$. **Ans.**

47. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Let ACB be the given arc subtending an angle of 60° at the centre.



Here, r = 14 cm and θ = 60°.

Area of the minor segment ACBA

= (Area of the sector *O*ACBO) – (area of ΔOAB)

$= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin \theta$

$= \frac{22}{7} \times 14 \times 14 \times \frac{60°}{360°} - \frac{1}{2} \times 14 \times 14 \times \sin 60°$

$= \frac{308}{3} - 7 \times 14 \times \frac{\sqrt{3}}{2} = \frac{308}{3} - 49\sqrt{3}$

$= 17.89\ cm^2$

Area of the major segment BDAB

= Area of circle – Area of minor segment ACBA

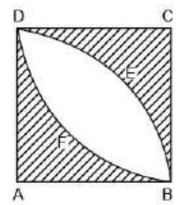
$= \pi r^2 - 17.89$

$= \frac{22}{7} \times 14 \times 14 - 17.89$

$= 616 - 17.89$

$= 598.11 \approx 598\ cm^2$. **Ans.**

48. Calculate the area other than the area common between two quadrants of circles of radius 16 cm each which is shown as the shaded region in the given figure.



Sol. Area of ABCD = (16 × 16) cm^2 = 256 cm^2

Area of quadrant DFBC

= Area of quadrant DEBA

= $\frac{\pi}{4}r^2 = \frac{1}{4} \times \frac{22}{7}(16)^2$

= 201.14 cm^2

Now, area of the shaded region

= [Area of square ABCD – Area of quadrant DFBC]

+ [Area of square *A*BCD – Area of quadrant DEBA]

= [(256 – 201.14) + (256 – 201.14)] cm^2

= 109.72 cm^2. **Ans.**

Long Answer Type Questions

49. Find the area of the minor segment of a circle of radius 5 cm formed by a chord subtending an angle of 90° at the centre. [Use π = 3.14]

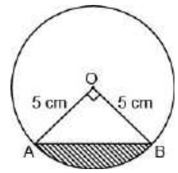
[Board Question]

Sol. Given, radius of the circle = 5 cm and θ = 90°

Now,

Area of the segment

= Area of the corresponding sector – Area of the corresponding triangle



Area of the sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times \pi(5)^2$

= $\frac{1}{4}\pi(25)$ = $\frac{25\pi}{4}$

Area of the triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

= $\frac{1}{2} \times 5 \times 5$ = $\frac{25}{2}$

⇒ Area of the segment = $\frac{25\pi}{4} - \frac{25}{2}$

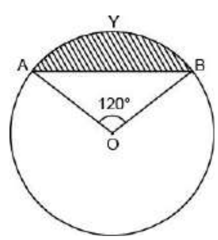
= $25\left(\frac{3.14}{4} - \frac{1}{2}\right)$ = $25\left(\frac{1.57}{2} - \frac{1}{2}\right)$

= $25\left(\frac{0.57}{2}\right)$

= 7.125 cm^2. **Ans.**

50. Find the area of the segment shown in Fig. if radius of the circle is 21 cm and ∠AOB = 120° .

$\left(\text{Use } \pi = \frac{22}{7}\right)$



[Board Question]

Sol. Given, radius of the circle = 21 cm and
∠AOB = 120°

Area of the segment AYB = Area of sector AOB – Area of ΔAOB

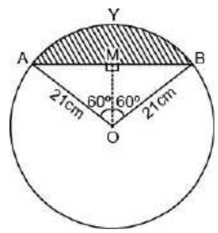
Area of sector AOB = $\frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21$

= 462 cm^2

To find the area of ΔOAB, draw OM ⊥ AB

ΔAMO ≅ ΔBMO (by R.H.S.) ∠AOM = ∠BOM = $\frac{1}{2} \times 120°$

= 60°



In ΔOMA, $\frac{OM}{OA}$ = cos 60° ⇒ $\frac{OM}{21}$ = $\frac{1}{2}$

⇒ OM = $\frac{21}{2}$ cm

Also, $\frac{AM}{OA}$ = sin 60° ⇒ AM = $21 \times \frac{\sqrt{3}}{2}$

AB = 2 × AM

= $21\sqrt{3}$ cm

since perpendicular from the centre of the chord bisects its.

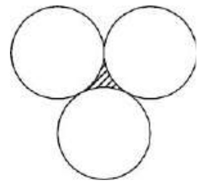
So, area of ΔOAB = $\frac{1}{2} \times AB \times OM$ = $\frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$ = $\frac{441}{4}\sqrt{3}$ cm^2

∴ Area of segment = $\left(462 - \frac{441}{4}\sqrt{3}\right)$ cm^2

= $\frac{21}{4}(88 - 21\sqrt{3})$ cm^2 = 271·04 cm^2 **Ans.**

51. In the given figure, three circles each of radius 3.5 cm are drawn in such a way that each of them just touches the other two. Find the area enclosed between these three circles (shaded region). $\left[\text{Use } \pi = \frac{22}{7}\right]$

[Board Question]



Sol. Given, $r = 3.5$ cm

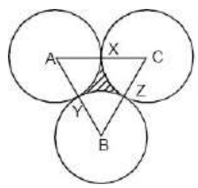
Join the three centres size of the circles such that

$$AB = BC = CA = (3.5 + 3.5)\text{ cm} = 7\text{ cm}$$

Hence, ΔABC is an equilateral triangle where

$\angle A = \angle B = \angle C = 60°$

Let the points where the circles touch each other be X, Y and Z.



Thus XAY, YBZ and ZCX are the three sectors of the three circles.

Now, area of $\Delta ABC = \frac{\sqrt{3}}{4}(7)^2$ cm^2

$\Rightarrow$ Area of $\Delta ABC = \frac{49\sqrt{3}}{4}$ cm^2 = 21.22 cm^2

Area of sector XAY

= Area of sector YBZ = Area of sector ZCX

$= \frac{60°}{360°} \times \frac{22}{7}(3.5)^2$ cm^2

$= \frac{1}{6} \times \frac{22}{7}\left(\frac{7}{2}\right)^2$ cm^2

= 6·42 cm^2

Thus, area of the shaded region

= Area of ΔABC – 3 × Area of sector XAY

= [21.22 – 3(6.42)] cm^2

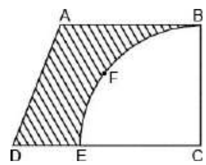
= [21.22 – 19.26] cm^2

$= 1.96\ cm^2$

$\approx 2\ cm^2$. **Ans.**

52. From a thin metallic piece in the shape of a trapezium ABCD in which AB || CD and ∠BCD = 90°, a quarter circle BFEC is removed as shown in the figure. Given, AB = BC = 3.5 cm and DE = 2 cm, calculate the area of the remaining part of the metal sheet.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



[Board Question]

Sol. Given, AB = BC = 3.5 cm and DE = 2 cm

As BC and EC are the radii of the circle, so

BC = EC

Thus, DC = (3.5 + 2) cm

= 5.5 cm

Hence, area of the remaining part of the metal sheet

= Area of the trapezium – Area of the quadrant

$= \frac{1}{2} \times$ (sum of parallel sides) × (distance between parallel sides) – $\frac{1}{4}\pi r^2$

$= \frac{1}{2} \times (3.5 + 5.5) \times 3.5 - \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$

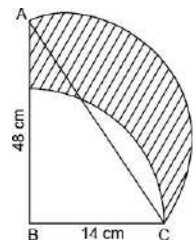
$= \frac{1}{2} \times 9 \times 3.5 - \frac{11 \times 0.5 \times 3.5}{2}$

$= \frac{31.5}{2} - \frac{19.25}{2}$

$= \frac{12.25}{2}$

$= 6.125\ cm^2$. **Ans.**

53. In the given figure, ΔABC is a right-angled triangle with $\angle B = 90°$, AB = 48 cm and BC = 14 cm. With AC as diameter, a semi-circle is drawn and with BC as radius, a quarter of a circle is drawn. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



[Board Question]

Sol. Given, AB = 48 cm and BC = 14 cm

Now, $AC^2 = AB^2 + BC^2$

[Pythagoras theorem]

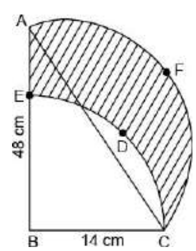
$\Rightarrow AC^2 = (48)^2 + (14)^2$

$\Rightarrow AC^2 = 2304 + 196$

$= 2500$

$\Rightarrow AC = 50$ cm

Thus, area of $\Delta ABC = \frac{1}{2} \times 48 \times 14 = 336$ cm^2



Area of quadrant BCE

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154$ cm^2

Area of semi-circle ACF

$= \frac{1}{2} \times \frac{22}{7} \times 25 \times 25$

$= 982.14\ cm^2$

Hence, area of AFCDE

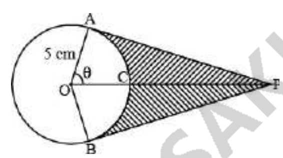
= Area of semi-circle ACF+ Area of ΔABC – Area of quadrant BCE

$= (982.14 + 336 – 154)\ cm^2$

$= 1164.14\ cm^2$. **Ans.**

54. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

[Use π = 3.14 and $\sqrt{3}$ = 1.73]



[Board Question]

Sol. Given, a circular pulley of radius 5 cm with centre O.

∴ AO = OB = OC = 5 cm

and OP = 10 cm

Now, in right ΔAOP,

$\cos\theta = \frac{AO}{OP} = \frac{5}{10} = \frac{1}{2}$

$\therefore \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

∴ ∠AOB = 2θ = 120°

⇒ Reflex ∠AOB = 360° – 120° = 240°

Length of major arc $\widehat{AB}$

$= \frac{2\pi r}{360°}$ reflex ∠AOB

$= \frac{2 \times 3.14 \times 5 \times 240^\circ}{360^\circ}$

= 20.93 cm

Hence, length of the belt that is still in contact with pulley = 20.93 cm.

Now, by Pythagoras theorem,

$\Rightarrow (AP)^2 = (OP)^2 - (OA)^2$

$\Rightarrow (AP)^2 = (10)^2 - (5)^2$

$\Rightarrow AP = \sqrt{100 - 25}$

$= \sqrt{75} = 5\sqrt{3}$ cm

$\therefore$ Area of $\Delta AOP = \frac{1}{2} \times 5 \times 5\sqrt{3}$

$= \frac{25\sqrt{3}}{2}$ cm

and Area of ΔBOP = Area of ΔAOP

and, area of quadrilateral AOBP

= 2(Area of ΔAOP)

$= 2 \times \frac{25\sqrt{3}}{2} = 25\sqrt{3}$ cm^2

= 43.25 cm^2

Area of sector ACBO $= \frac{\angle AOB}{360^\circ} \pi r^2$

$= \frac{120^\circ}{360^\circ} \times 3.14 \times 5 \times 5$

= 26.16 cm^2

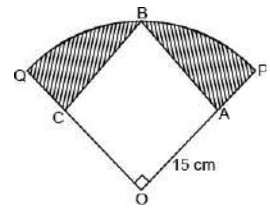
$\therefore$ Area of shaded region

= Area of quad. AOBP – Area of sector ACBO

= (43.25 – 26.16) cm^2

= 17.09 cm^2 **Ans.**

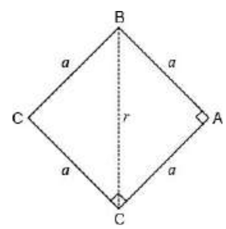
55. In figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 15 cm, find the area of the shaded region.[Use π = 3·14]



[Board Question]

Sol. Given, *O*ABC is a square with OA = 15 cm

OB = radius = *r*



Let side of square be *a* then,

$a^2 + a^2 = r^2$

$\Rightarrow 2a^2 = r^2$

$\Rightarrow r = \sqrt{2}a$

$\Rightarrow r = 15\sqrt{2}$ cm (... a = 15 cm)

Area of square = Side × Side

= 15 × 15

= 225 cm^2

Area of quadrant OPBQ

$= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times 3{\cdot}14 \times 15\sqrt{2} \times 15\sqrt{2}$

$= \frac{225 \times 2 \times 3{\cdot}14}{4}$

= 225 × 1·57

$= 353 \cdot 25\ cm^2$

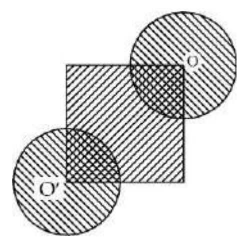
Area of shaded region

= Area of quadrant OPBQ – Area of square

$= 353 \cdot 25 - 225$

$= 128 \cdot 25\ cm^2$ **Ans.**

56. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where O and O' are centres of the circles. Find the area of shaded region.



[Board Question]

Sol. Area of shaded region

= 2 × (Area of circle) + Area of square – 2 × (Area of quadrant)

$= 2 \times \pi r^2 + (\text{Side})^2 - 2\left(\frac{1}{4} \times \pi r^2\right)$

$= 2\pi r^2 - \frac{1}{2}\pi r^2 + (\text{Side})^2$

$= \frac{3}{2}\pi r^2 + (\text{Side})^2$

Now, $r = \frac{1}{2}(\text{Side})$

= 14 cm [∵ Side = 28 cm]

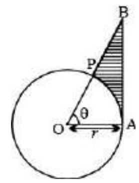
∴ Area of shaded region

$= \frac{3}{2} \times \frac{22}{7} \times 14 \times 14 + 28 \times 28$

$= 924 + 784 = 1708\ cm^2$ **Ans.**

57. In the given, a sector OAP of a circle is shown with centre O, containing ∠θ. AB is perpendicular to the radius *OA* and meets OP produced at B. Prove that the perimeter of

shaded region is
$r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right]$.



[Board Question]

Sol. Given, the radius of circle with centre O is r and ∠POA = θ.

Then, length of the arc $\widehat{PA} = \frac{\theta}{360^\circ}2\pi r = \frac{\theta}{180^\circ}\pi r$

and $\tan\theta = \frac{AB}{r}$

⇒ AB = $r\tan\theta$

and $\sec\theta = \frac{OB}{r}$

⇒ OB = $r\sec\theta$

Now, PB = OB – OP

= $r\sec\theta - r$

∴ Perimeter of shaded region

= AB + PB + $\widehat{PA}$

= $r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180^\circ}$

= $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right]$ **Hence Proved.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

58. Assertion: Sector is the region between the chord and its corresponding arc.

Reason: To define an arc we need at least 3 points.

Ans. (d) Assertion is false but the Reason is true.

Explanation :

For Assertion, sector is the region between an arc and two radii joining the centre to the end points of the arc. So, the assertion is false.
For Reason, to define an arc we need minimum three points which are the starting, ending points as well as the point on the boundary of the circle which passes through it.

59. Assertion: The area enclosed by a chord and the major arc is major segment.

Reason: If a circle is divided into three equal arcs, then each is a major arc.

Ans. (c) Assertion is true but the Reason is false.

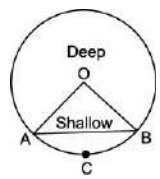
Explanation :

For assertion, the area enclosed by a sector is proportional to the arc length of the sector.

So, $A=\frac{RL}{2}$, A = area, R= radius and L = arc length
Hence, the resulting enclosed area is a major segment if it is a major arc.
So assertion is true.
For reason,
We basically denote an arc as a major arc when it is greater than the semicircle and if we divide a circle into 3 arc each of it will be less than a semicircle so it is a minor arc.

Case Based Questions

60. Ramesh went to stadium everyday to enjoy his summer vacation. In stadium, there is a circular swimming pool with centre O. The radius of pool is 7m. There are 2 points on the wall of the pool separated by equal distance.
These 2 points are named A and B. A rope is attached between A and B. This rope separates the shallow section of pool from deep section of pool such that ∠AOB = 90°. The shallow section is the smaller section.



(i) The area of DAOB is:

(a) 49 m^2

(b) 24.5 m^2

(c) 98 m^2

(d) 140 m^2

Ans. (b) 24.5 m^2

Explanation :

Area of DAOB = $\frac{1}{2} \times b \times h$

= $\frac{1}{2} \times 7 \times 7$

= 24.5 m.

(ii) The area of minor sector AOB is:

$\left(\text{Use} : \pi = \frac{22}{7}\right)$

(a) 77 m^2

(b) 38.5 m^2

(c) 154 m^2

(d) 70 m^2

Ans. (b) 38.5 m^2

Explanation :

Area of sector AOB = $\frac{1}{4}$ pr^2

= $\frac{1}{4} \times \frac{22}{7}$ × 7 × 7

= 38.5 m^2.

(iii) The area of shallow region in swimming pool is:

(a) 28 m^2

(b) 56 m^2

(c) 14 m^2

(d) 7 m^2

Ans. (c) 14 m^2

Explanation :

Area of shallow = Area of sector AOB – Area of DAOB

= 38.5 – 24.5

= 14 m^2.

(iv) The area of swimming pool is:

(a) 77 m^2

(b) 38.5 m^2

(c) 154 m^2

(d) 70 m^2

Ans. (c) 154 m^2

Explanation :

Area of swimming pool = p × r^2

$= \frac{22}{7} \times 7 \times 7$

$= 154\ m^2$.

(v) The area of deeper section of swimming pool is :

(a) 77 m^2

(b) 150 m^2

(c) 154 m^2

(d) 140 m^2

Ans. (d) 140 m^2

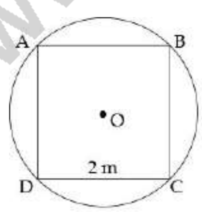
Explanation :

Area of deeper section

= Area of swimming – Area of shallow section

$= 154 - 14 = 140\ m^2$.

61. Your mother has bought a circular wooden table top. But by accident the edge of the table top has been broken. Now your mother asked you to cut the circular table top in the shape of the square (Side = 2m). Considering you manage to draw a perfect square on the top of circular table-top shown below. Answer the questions given below.



Circular table

(i) The radius of the circular wooden table top is:

(a) $2\sqrt{2}$ m

(b) 2.62 m

(c) $\sqrt{2}$m

(d) 2 m

Ans. (c) $2\sqrt{2}$ m

Explanation :

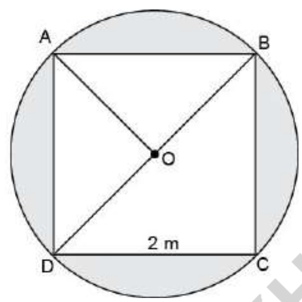
$BD^2 = AB^2 + AD^2$

$\Rightarrow BD^2 = 2^2 + 2^2$

$\Rightarrow BD = 2\sqrt{2}$ m

OB = $\frac{2\sqrt{2}}{2}$

= $\sqrt{2}$ m



(ii) The area of the circular wooden table top is:

(a) 6 m^2

(b) 12.5 m^2

(c) 6.28 m^2

(d) 25.14 m^2

Ans. (c) 6.28 m^2

Explanation :

Area = pr^2

= $\left(\frac{22}{7}\right) \times (\sqrt{2})^2$

= 6.28 m^2.

(iii) What is the segment cutted to form a square top?

(a) 6m^2

(b) 1.5 m^2

(c) 2.5 m^2

(d) 0.57 m^2

Ans. (d) 0.57 m^2

Explanation :

Area of segment = area of sector – area of triangle

= $\frac{\theta}{360°}$ × pr^2 – $\frac{1}{2}$× *b* × *h*

= $\frac{90°}{360°} \times \pi r^2 - \frac{1}{2} \times b \times h$

= $\frac{\pi r^2}{4}$ – $\frac{1}{2}$× *b* × *h*

= $\frac{6.28}{4}$ – $\frac{1}{2} \times \sqrt{2} \times \sqrt{2}$

= 0.57 m^2.

(iv) The total area to be cut from the circular table top to form a square table top:

(a) 2.28 m^2

(b) 6 m^2

(c) 10 m^2

(d) 4.24 m^2

Ans. (a) 2.28 m^2

Explanation :

Total area = 4 × area of one segment

= 4 × 0.57

= 2.28 m^2.

(v) What is the new area of the table top?

(a) 2 m^2

(b) 4 m^2

(c) 8 m^2

(d) 6 m^2

Ans. (b) 4 m^2

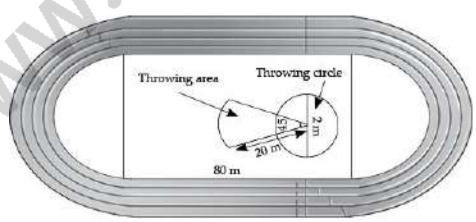
Explanation :

New area of table top = side × side

= 2 × 2

= 4 m^2.

62. Your school's sports day is scheduled to take place next week in a stadium near your school. For the event of shot put the stadium authority has allotted a rectangular area. The total area and the length of the rectangular area is 4480 m^2 and 80 m respectively. The throwing circle of diameter 2 m in which the participants has to stand to throw the ball is also drawn on the rectangular field. With the help of the diagram given below answer the following questions.



(i) Total area of the stadium is:

(a) 9856 m^2

(b) 6944 m^2

(c) 14306 m^2

(d) 8500 m^2

Ans. (b) 6944 m^2

Explanation :

Breadth of rectangle = $\frac{\text{area}}{\text{length}}$

= $\frac{4480}{80}$

= 56 m

Radius of circular part = $\frac{56}{2}$

= 28 m

Total area of the stadium

= area of 2 semicircle + area of rectangle

= $\frac{\pi r^2}{2} + \frac{\pi r^2}{2}$ + 4480

= pr^2 + 4480

= $\frac{22}{7}$ × 28 × 28 + 4480

= 2464 + 4480

= 6944 m^2

(ii) The area of the ground covered by the throwing circle is:

(a) 3.14 m^2

(b) 6.28 m^2

(c) 2.81 m^2

(d) 4.52 m^2

Ans. (a) 3.14 m^2

Explanation :

Area of throwing circle = pr^2

$= \frac{22}{7} \times 1 \times 1 = 3.14\ m^2$

(iii) The area of the ground covered by the throwing area (*i.e.*, sector) is:

(a) $152.14\ m^2$

(b) $162.14\ m^2$

(c) $157.14\ m^2$

(d) $150.20\ m^2$

Ans. (c) $157.14\ m^2$

Explanation :

Area of throwing area (Area of sector)

$= \frac{\theta}{360} \times pr^2$

$= \left(\frac{45°}{360°}\right) \times \left(\frac{22}{7}\right) \times 20 \times 20$

$= 157{\cdot}14\ m^2$

(iv) The length of the arc of the throwing area is:

(a) 15.71 m

(b) 157.1 m

(c) 6.28 m

(d) 162.2 m

Ans. (a) 15.71 m

Explanation :

Length of the arc $= \frac{\theta}{360°} \times 2pr$

$= \frac{45°}{360°} \times 2 \times \frac{22}{7} \times 20$

$= 15.71$ m

(v) The area of the ground in the semi-circular shape is:

(a) 2464 m^2

(b) 4480 m^2

(c) 2440 m^2

(d) 4400 m^2

Ans. (a) 2464 m^2

Explanation :

Area of semi-circular ground

= $\frac{\pi r^2}{2}+\frac{\pi r^2}{2}$ = pr^2

= $\frac{22}{7}$ × 28 × 28

= 2464 m^2.

Passage Based Questions

63. Aman is cycling such that the wheels of the cycle are making 180 revolution per minute. If the diameter of the wheel is 70 cm.

Based on the given information, answer the following questions:

(i) Find the speed (in km/hr) with which the Aman is cycling.

(ii) Distance travelled by Aman in $\frac{1}{2}$ hour.

Sol. (i) Speed = Distance covered by the wheel in one hour.

Clearly, distance covered by wheel in one revolution is circumference of wheel.

Radius of wheel = $\frac{70}{2}$ = 35 cm.

Circumference of wheel = 2pr = $2\times\frac{22}{7}\times 35$
= 220 cm

So, distance covered by wheel in one revolution

= 220 cm

∴ Distance covered by wheel in 180 revolutions

= 220 × 180 = 39600 cm

= $\frac{39600}{100}$ m

= 396 m = $\frac{396}{1000}$ km

∴ Distance covered by wheel in one minute

= $\frac{396}{1000}$ km

∴ Distance covered by wheel in one hour *i.e.,* 60 minutes

= $\frac{396}{1000} \times 60$ km = 23.76 km

Hence, the speed of the cycle is 23.76 km/hr.

Ans.

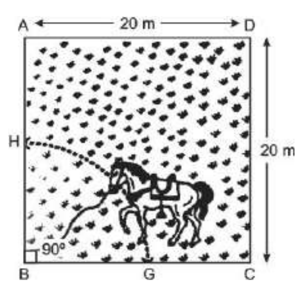
(ii) Distance coverd by Aman in one hour = 23.76 km

So, distance covered by Aman in $\frac{1}{2}$ hour

= $\frac{1}{2} \times 23.76$ = 11.88 km **Ans.**

64. Overgrazing occurs when plants are exposed to intensive grazing for extended periods of time, or without sufficient recovery periods. Overgrazing reduces the usefulness, productivity and biodiversity of the land and is one cause of desertification and erosion.

Based on the following figure related to overgrazing, answer the questions:



(i) If the length of a rope is 7 m, then find the area of that part of the field in which the horse can graze.

(ii) If the length of rope is 14 m, then find the area of the field in which the horse can graze.

(iii) Find the increase in the grazing area, if the rope is 14 m long instead of 7 m.

Sol. (i) Required area A_1 = $\frac{\theta}{360^\circ}\pi r^2$

= $\frac{90^\circ}{360^\circ}\times\left(\frac{22}{7}\right)\times(7)^2$

= $\frac{1}{4}\times\frac{22}{7}\times7\times7$

= 38.5 m^2

(ii) Required area A_2 = $\frac{\theta}{360^\circ}\pi r^2$

= $\frac{90^\circ}{360^\circ}\times\frac{22}{7}\times14\times14$

= $\frac{1}{4}\times\frac{22}{7}\times14\times14$

= 154 m^2

(iii) Increase in grazing area = $A_2 - A_1$

= (154 – 38.5) m^2 = 115.5 m^2 **Ans.**

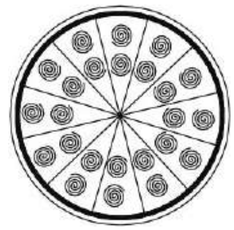
65. A wheel is made for a school project with silver wire in the form of circle as shown in the figure. The diameter of the circle is 42 mm. The cost of each mm of silver wire is ` 30. The wire is also used in making 6 diameters which divide the circle into 12 equal sectors.

Based on the following figure answer the questions :

(i) How much silver wire will be required to make the wheel?

(ii) What is the area of each sector of the wheel?

(iii) What will be the cost of silver wire used to make the wheel?



Sol. (i) Total length of wire

= Length of 6 diameters + circumference of the circle

= $\left[(6 \times 42) + \left(2 \times \frac{22}{7} \times \frac{42}{2}\right)\right]$mm

= (252 + 132) mm = 384 mm.

(ii) Area of each sector = $\frac{\theta}{360^\circ} \times \pi r^2$

Here, q = $\frac{360^\circ}{12} = 30^\circ$

= $\frac{30^\circ}{360^\circ} \times \frac{22}{7} \times \left(\frac{42}{2}\right)^2$ mm^2

= $\frac{1}{12} \times \frac{22}{7} \times \frac{42}{2} \times \frac{42}{2}$ mm^2

= 115.5 mm^2

(iii) Required amount = ` (384 × 30)

= ` 11520. **Ans.**

Self-Assessment

66. The radii of two circles are 19 cm and 9 cm respectively. Find the radius and the area of the circle which has its circumference equal to the sum of the circumference of the two circles.

[NCERT]

Ans. 28 cm, 2464 cm^2.

67. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned in each sweep of the blades.

[NCERT]

Ans. 1254.96 cm^2.

68. To warn ships about underwater rocks, a lighthouse spread a red-coloured light over a sector of 80° angle to a distance of 16.5 km. Find the area of the sea over which the ships are warned. [Use $\pi = 3.14$]

[NCERT]

Ans. 189.97 km^2.

69. An umbrella has 8 ribs. Assuming the umbrella to be a flat circle of radius 45 cm, find the area between two consecutive ribs of the umbrella.

[NCERT]

Ans. 795.53 cm^2.

70. A circular brooch is made of silver wire and consists of 5 diameters (35 mm each) forming of equal dimensions. Calculate the total length of silver wire required and the area of each sector that is formed.

[NCERT]

Ans. 285 mm, 96.25 mm^2.

71. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find : (i) the length of the arc, (ii) area of the sector formed by the arc, (iii) area of the segment formed by the corresponding chord of the arc.

[NCERT]

Ans. (i) 22 cm, (ii) 231 cm^2, (iii) 40.04 cm^2.

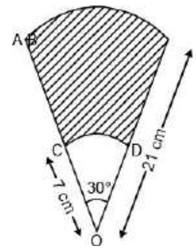
72. A chord PQ of length 12 cm subtends an arc of 120° at the centre of a circle. Find the area of the minor segment cut off by the chord PQ.

[NCERT]

Ans. 88.44 cm^2.

73. AB and CD respectively are the arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If ∠AOB = 30°, find the area of the shaded region.

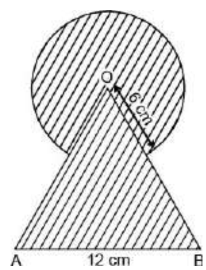
[NCERT]



Ans. 102.67 cm^2.

74. Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with the vertex O of an equilateral triangle OAB of side 12 cm as centre.

[NCERT]



Ans. 156.6 cm^2.

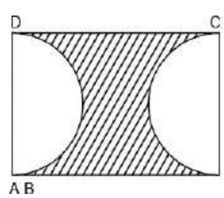
75. A circular field has a perimeter of 650 m. A square plot having its vertices on the circumference of the field is marked in the field. Calculate the area of the square plot.

Ans. 21387 m^2.

76. In Fig., ABCD is a rectangle having AB = 20 cm and BC = 14 cm. Two sectors of 180° have been cut off. Calculate :

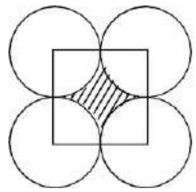
(i) the area of the shaded region.

(ii) the length of the boundary of the shaded region.



Ans. (i) 126 cm^2, (ii) 84 cm.

77. The diameter of a coin is 1 cm. If four such coins be placed on a table so that rim of each touches that of the other two, find the area of the shaded region. (Take π = 3.1416)



Ans. 0.2146 cm^2.

78. A circular pend is of diameter 17.5 m. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of ` 25 per square metre.(Use π = 3.14)

[Board Question]

Ans. ` 3061.50.

Chapter 13

Surface Areas and Volumes

Basic Concepts

SURFACE AREAS AND VOLUMES

1. Cuboid : In a cuboid, let l be the length, b be the breadth and h be the height, then

(i) Total surface area $= 2(lb + bh + lh)$ sq. units

(ii) Volume $= l \times b \times h$ cubic units

(iii) Diagonal of a cuboid $= \sqrt{l^2+b^2+h^2}$ units

(iv) Area of the four walls $= 2(l+b)h$ sq. units

2. Cube : If the side of a cube is 'a' units, then

(i) Total surface area $= 6a^2$ sq. units

(ii) Volume $= a^3$ cubic units

(iii) Diagonal of a cube $= \sqrt{3}a$ units

3. Right-Circular Cylinder : For a right-circular cylinder of base r and height h,

(i) Area of each end $= \pi r^2$ sq. units

(ii) Curved surface area $= 2\pi rh$ sq. units

(iii) Total surface area $= 2\pi r(h+r)$ sq. units

(iv) Volume $= \pi r^2 h$ cubic units

4. Hollow Cylinder : Let R and r be the external radius and the internal radius, respectively and h be the height of hollow cylinder, then

(i) Area of each end $= \pi(R^2 - r^2)$ sq. units

(ii) Curved surface area $= 2\pi(R + r)h$ sq. units

(iii) Total surface area $= 2\pi(R + r)(R + h - r)$ sq. units

(iv) Volume $= \pi(R^2 - r^2)h$ cubic units

5. Right-Circular Cone : If r, h and l denote the radius of the base, height and slant height of a cone, respectively, then

(i) $l^2 = r^2 + h^2$

(ii) Curved surface area $= \pi rl$ sq. units

(iii) Total surface area $= \pi r^2 + \pi rl$ sq. units

(iv) Volume $= \frac{1}{3}\pi r^2 h$ cubic units

6. Sphere : For a sphere of radius r,

(i) Surface area $= 4\pi r^2$ sq. units

(ii) Volume $= \frac{4}{3}\pi r^3$ cubic units

7. Hemisphere : For a hemisphere of radius r,

(i) Curved surface area $= 2\pi r^2$ sq. units

(ii) Total surface area $= 3\pi r^2$ sq. units

(iii) Volume $= \frac{2}{3}\pi r^3$ cubic units

8. Spherical Shell : For a spherical shell of outer radius R and inner radius r,

(i) Surface area $= 4\pi(R^2 - r^2)$ sq. units

(ii) Volume of material $= \frac{4}{3}\pi(R^3 - r^3)$ cubic units

9. Frustum of a Cone : If h, l, r_1 and r_2 denote the height, slant height and radii of the bases of a frustum of a cone, respectively, then

(i) Volume $= \frac{\pi}{3}(r_1^2 + r_1 r_2 + r_2^2)$ h cubic units

(ii) Lateral surface area $= \pi(r_1 + r_2)\, l$ sq. units

(iii) Total surface area $= \pi\{(r_1 + r_2)l + (r_1^2 + r_2^2)\}$ sq. units

(iv) Slant height of the frustum $= \sqrt{h^2 + (r_1 - r_2)^2}$

(v) Height of the cone of which the frustum is a part $= \frac{hr_1}{r_1 - r_1}$

(vi) Slant height of the cone of which the frustum is a part $= \frac{lr_1}{r_1 - r_1}$

Multiple Choice Questions

1. A rectangular sheet of paper 40 cm × 22 cm is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is:

[Board Question]

(a) 3.5

(b) 5·3

(c) 2·5

(d) 5

Ans. (a) 3.5

Explanation :

Given, Area of the sheet of paper

= (40 × 22) cm^2

Height of the cylinder = 40 cm

Let *r* be the radius of the cylinder.

Now, curved surface area of the cylinder = 2π*rh*

∵ Area of rectangular sheet is equal to curved surface area of cylinder.

$2\left(\frac{22}{7}\right)40r$ = 40 × 22

⇒ $2\left(\frac{1}{7}\right)r$ = 1 × 1

⇒ $r = \frac{7}{2}$ cm

= 3.5 cm

2. The surface area of two spheres are in the ratio 16 : 9. The ratios of their volumes is:

[Board Question]

(a) 64 : 27

(b) 16 : 9

(c) 4 : 3

(d) $16^3 : 9^3$

Ans. (a) 64 : 27

Explanation :

Let the radius of the larger sphere be R and the smaller be *r*.

Given, $4\pi R^2 : 4\pi r^2 = 16 : 9$

$\Rightarrow R^2 : r^2 = 4^2 : 3^2$

$\Rightarrow R : r = 4 : 3$

Thus, $V_1 : V_2 = \frac{4\pi}{3}R^3 : \frac{4\pi}{3}r^3 = 4^3 : 3^3$

or $V_1 : V_2 = 64 : 27$

3. A solid right-circular cone is cut into two parts at the middle of its height by a plane parallel to its base. The ratio of the volume of the smaller cone to the whole cone is:

[Board Question]

(a) 1 : 2

(b) 1 : 4

(c) 1 : 6

(d) 1 : 8

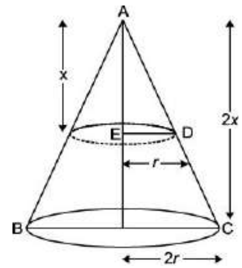
Ans. (d) 1 : 8

Explanation :

Let the height of the bigger cone be 2*x* cm.

Thus, height of the smaller cone = *x* cm

Also let the radius of the larger cone be 2*r* cm.



Thus, radius of the smaller cone

= r cm

(by mid-point theorem)

Now, volume of the large cone

V = $\frac{1}{3} \times \pi(2r)^2 (2x)$

= $\frac{8\pi}{3} r^2 x$

and volume of the smaller cone

= $\frac{1}{3}\pi(r)^2 (x) = \frac{\pi}{3} r^2 x$

Thus, $\frac{\text{Volume of the smaller cone}}{\text{Volume of the larger cone}}$

= $\frac{\frac{\pi}{3} r^2 x}{\frac{8\pi}{3} r^2 x} = \frac{1}{8}$

4. If the radius of the base of a right-circular cylinder is halved keeping the height same then the ratio of the volume of the cylinder thus obtained to the volume of the original cylinder is:

[Board Question]

(a) 1 : 2

(b) 2 : 1

(c) 1 : 4

(d) 4 : 1

Ans. (c) 1 : 4

Explanation :

Let the original radius of the base be $2r$ cm and the height be h.

Thus, the new radius of the base = r cm

Now, Original volume = $\pi(2r)^2h$

New volume = $\pi(r)^2h$

Thus, $\frac{\text{Original volume}}{\text{New volume}} = \frac{\pi \times 4r^2h}{\pi r^2h}$

$= \frac{4}{1}$

$\Rightarrow \frac{\text{New volume}}{\text{Original volume}} = \frac{1}{4}$

5. The radius of the largest circular cone that can be cut out from a cube of edge 4.2 cm is:

[Board Question]

(a) 4.2 cm

(b) 2.1 cm

(c) 8.4 cm

(d) 1.05 cm

Ans. (b) 2.1 cm

Explanation :

Given, the edge of cube = 4.2 cm

Now, the largest circular cone will have diameter equal to the measure of the edge.

Thus, diameter = 4.2 cm

Hence, radius = 2.1 cm

6. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm partly filled with water. If the sphere is completely submerged then the water level rises (in cm) by:

(a) 3

(b) 4

(c) 5

(d) 6

Ans. (a) 3

Explanation :

Given,

Diameter of sphere = 18 cm

Diameter of cylinder = 36 cm

Let the increase in height of water in the cylinder be h cm.

Radius of sphere = 9 cm

Radius of cylinder = 18 cm

Thus, volume of sphere = $\frac{4}{3}\pi(9)^3$ cm^3

= $\frac{4}{3}\pi$ × (81) (9) cm^3

= 81 × 12π cm^3

Thus, increase in volume of water level in the cylinder = p $(18)^2 h$ cm^3

Hence, $\pi(18)^2h = (81)(12)\pi$

⇒ h = 3 cm

7. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface area of the two are equal then the ratio of the radius and the slant height of the conical part is:

[Board Question]

(a) 2 : 1

(b) 1 : 2

(c) 1 : 4

(d) 4 : 1

Ans. (b) 1 : 2

Explanation :

Given, Radius of hemisphere = Radius of cone

Also, curved surface area of hemisphere

= Curved surface area of cone

$\Rightarrow 2\pi r^2 = \pi r l$

[where l = slant height of the cone]

$\Rightarrow 2r = l$

$\Rightarrow r : l = 1 : 2$

8. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. If the total height of the toy is 31 cm, then the height of the cone is:

(a) 31 cm

(b) 38 cm

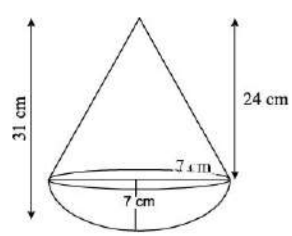
(c) 7 cm

(d) 24 cm

Ans. (d) 24 cm

Explanation :

Radius of sphere = Radius of cone



Height of cone = Total height of toy– Radius of sphere

= (31 – 7)cm

= 24 cm.

9. Volumes of two spheres are in the ratio 125 : 216. The ratio of their surface areas is:

(a) 5 : 6

(b) 25 : 36

(c) 1 : 2

(d) 5 : 2

Ans. (b) 25 : 36

Explanation :

Let radius of spheres be r_1 and r_2

Volume (V_1) = $\frac{4}{3}\pi r_1^3$

and Volume (V_2) = $\frac{4}{3}\pi r_2^3$

According to the question,

$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{125}{216}$

$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{(5)^3}{(6)^3}$

$\Rightarrow \frac{r_1}{r_2} = \frac{5}{6}$ ...(i)

Now ratio of surface area

$= \frac{4\pi r_1^2}{4\pi r_2^2}$

$= \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{5}{6}\right)^2$ **=** $\frac{25}{36}$[From (i)]

10. If the surface area of a sphere is 616 cm^2, then the radius of the sphere is:

(a) 14 cm

(b) 7 cm

(c) 3.5 cm

(d) None of these

Ans. (b) 7 cm

Explanation :

Surface area of sphere = 616 cm^2
$\Rightarrow 4\pi r^2 = 616$
$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 616$
$\Rightarrow r^2 = \frac{616 \times 7}{22 \times 4}$
$\Rightarrow r^2 = 49$
$\Rightarrow r = 7$ cm.

11. A right circular cylinder of radius *r* cm and height *h* cm (*h* > 2*r*) just encloses a sphere of diameter equals to:

(a) *r* cm

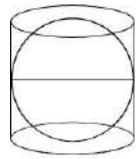
(b) 2*r* cm

(c) 3*r* cm

(d) 4*r* cm

Ans. (b) 2*r* cm

Explanation :



Here, maximum diameter of sphere is equal to the diameter of cylinder.

∵ radius of cylinder = r

diameter of cylinder = diameter of sphere = $2r$

12. The diameter of the Moon is approximately one-fourth of the diameter of the Earth. What is the ratio (approximate) of their volumes?

(a) 1 : 16

(b) 1 : 64

(c) 1 : 4

(d) 1 : 128

Ans. (b) 1 : 64

Explanation :

The diameter of Moon is approximately one-fourth of the diameter of Earth.

Let, Radius of Moon = r,

Then, Radius of Earth = $4r$

Required ratio = $\frac{\text{Volume of Moon}}{\text{Volume of Earth}}$

= $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (4r)^3}$

= $\frac{r^3}{64r^3}$

= $\frac{1}{64}$

= 1 : 64.

13. If the volume of a cube is 729 cm^3, what is the length of its diagonal?

(a) $9\sqrt{2}$ cm

(b) $9\sqrt{3}$ cm

(c) 18 cm

(d) $18\sqrt{3}$ cm

Ans. (b) $9\sqrt{3}$ cm

Explanation :

Volume of cube = (Side)^3

⇒ 729 = a^3

⇒ a = 9 cm

Diagonal of cube = $\text{Side} \times \sqrt{3}$

= $9 \times \sqrt{3} = 9\sqrt{3}$ cm

14. The curved surface area of a right circular cone of radius 14 cm is 440 sq. cm. what is the slant height of the cone?

(a) 10 cm

(b) 11 cm

(c) 12 cm

(d) 13 cm

Ans. (a) 10 cm

Explanation :

Curved surface area of right circular cone

= $\pi r l$

⇒ 440 = $\frac{22}{7} \times 14 \times l$

$\Rightarrow l = \frac{440 \times 7}{22 \times 14} = 10$ cm

15. If the volumes of two cones are in the ratio of 1 : 4 and their diameters are in the ratio of 4 : 5, then the ratio of their heights is:

(a) 1 : 5

(b) 5 : 4

(c) 5 : 16

(d) 25 : 64

Ans. (d) 25 : 64

Explanation :

Let the radii of cones be 4*x* and 5*x* and their heights be *h* and H respectively. Then,

According to the question,

$$\frac{\frac{1}{3} \times \pi \times (4x)^2 \times h}{\frac{1}{3} \times \pi \times (5x)^2 \times H} = \frac{1}{4}$$

$$\Rightarrow \frac{h}{H} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}$$

16. A cone of height 7 cm and base radius 3 cm is curved from a rectangular block of wood 10 cm × 5 cm × 2 cm.

The percentage of wood wasted is:

(a) 34%

(b) 46%

(c) 54%

(d) 66%

Ans. (a) 34%

Explanation :

Volume of the block = (10 × 5 × 2) cm^3

= 100 cm^3

Volume of the cone carved out

= $\left(\frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7\right) cm^3$

= 66 cm^3

Wood wasted = (100 – 66) cm^3

= 34 cm^3

Required % = $\frac{34}{100}$ × 100

= 34%.

17. The total surface area of a hemisphere of radius *r* is:

(a) πr^2

(b) 2 πr^2

(c) $3\pi r^2$

(d) $\frac{2}{3}\pi r^2$

Ans. (c) 3pr^2

Explanation :

Total **a**rea of hemisphere = Area of curved surface + Area of base

= $2\pi r^2 + \pi r^2 = 3\pi r^2$

18. A cylindrical vessel 32 cm high and 18 cm as the radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, the radius of its base is:

(a) 12 cm

(b) 24 cm

(c) 36 cm

(d) 48 cm

Ans. (c) 36 cm

Explanation :

Radius of a cylindrical vessel (r_1) = 18 cm

and height of a cylindrical vessel (h_1) = 32 cm

Volume of sand filled in it

= p$(r_1)^2$ h_1

= p$(18)^2$ × 32

= p × 324 × 32 cm^3

= 10368 p cm^3

Now height of the conical heap (h_2) = 24 cm

Let r_2 be its radius, then

$\frac{1}{3}$pr_2^2 h_2 = 10368p

⇒ $\frac{1}{3}\pi r_2^2$ × 24 = 10368p

⇒ 8pr_2^2 = 10368p

⇒ r_2^2 = 1296

r_2 = $\sqrt{1296}$ = 36

Hence, radius of the base of the heap = 36 cm.

19. If each edge of a cube is increased by 50%, the percentage increase in the surface area is:

(a) 50%

(b) 75%

(c) 100%

(d) 125%

Ans. (d) 125%

Explanation :

Let each edge of cube be *x*.

So, the surface area of the cube = $6x^2$

Since, the edge of the cube is increased by 50%

New edge = $x + \frac{x}{2} = \frac{3x}{2}$

So, the new surface area = $6\left(\frac{3x}{2}\right)^2$

$= 6 \times \frac{9x^2}{4}$

$= \frac{27x^2}{2}$

Incre ase in the surface area

$= \frac{27x^2}{2} - 6x^2 = \frac{15x^2}{2}$

Percentage increase = $\frac{15\frac{x^2}{2} \times 100}{6x^2}$

$= \frac{15}{12} \times 100$

= 125%.

20. A cylindrical pencil sharpened at one edge is the combination of:

[NCERT Exemplar]

(a) a cone and a cylinder.

(b) frustum of a cone and a cylinder.

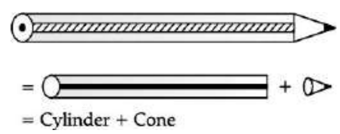
(c) a hemisphere and a cylinder.

(d) two cylinders.

Ans. (a) a cone and a cylinder.

Explanation :

Because the shape of sharpened pencil is,



21. The shape of a gilli, in the gilli-danda game see the given figure is a combination of:

[NCERT Exemplar]

(a) two cylinders

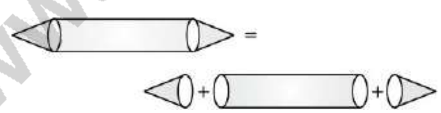
(b) a cone and a cylinder

(c) two cones and a cylinder.

(d) two cylinders and a cone.

Ans. (c) two cones and a cylinder.

Explanation :



= Cone + Cylinder + Cone

= Two cones and a cylinder

22. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is:

(a) 12p cm^3

(b) 15p cm^3

(c) 16p cm^3

(d) 20p cm^3

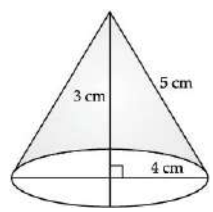
Ans. (c) 16p cm^3

Explanation :

A cone is formed by rotating the right angled triangle above the side 3 cm

Height of cone (h) = 3 cm

and radius (r) = 4 cm



V = $\frac{1}{3}$pr^2h = $\frac{1}{3}$p × $(4)^2$ × 3 cm^3

= $\frac{1}{3}$p × 16 × 3 cm^3

= 16p cm^3

23. A solid consists of a circular cylinder surmounted by a right circular cone. The height of the cone is *h*. If the total volume of the solid is 3 times the volume of the cone, then the height of the cylinder is:

(a) $\frac{2}{3}h$

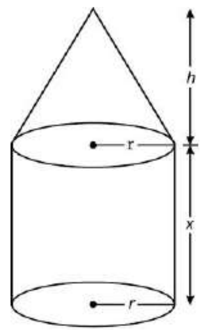
(b) $\frac{3}{2}h$

(c) h

(d) $2h$

Ans. (a) $\frac{2}{3}h$

Explanation :



Let x be the height of cylinder.

Since, volume of the total solid is equal to 3 times the volume of the cone.

So,

$\frac{1}{3}pr^2h + pr^2x = 3\left(\frac{1}{3}\pi r^2h\right)$

$\Rightarrow \frac{1}{3}pr^2h - pr^2h + pr^2x = 0$

$\Rightarrow pr^2x = \frac{2}{3}pr^2h$

$\Rightarrow x = \frac{2}{3}h$

24. If two solid-hemispheres of same base radius *r* are joined together along their bases, then curved surface area of this new solid is:

(a) $4pr^2$

(b) $6pr^2$

(c) $3pr^2$

(d) $8pr^2$

Ans. (a) $4pr^2$

Explanation :

Because curved surface area of a hemisphere is $2pr^2$ each and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.

Hence, the curved surface area of new solid = $2\text{p}r^2 + 2\text{p}r^2 = 4\text{p}r^2$

25. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is:

(a) 60p cm^2

(b) 68p cm^2

(c) 120p cm^2

(d) 136p cm^2

Ans. (d) 136p cm^2

Explanation :

Height of cone, h = 15 cm

Base radius, $r = \frac{16}{2}$ = 8 cm

Slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225}$

$= \sqrt{289}$

$\Rightarrow l$ = 17 cm

C.S.A. of cone = prl

= p × 8 × 17 = 136p cm^2.

26. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is:

(a) 12 m

(b) 18 m

(c) 36 m

(d) 66 m

Ans. (d) 36 m

Explanation :

Diameter of sphere = 6 cm

Radius (r) = $\frac{6}{2}$ = 3 cm

Volume = $\frac{4}{3}$pr^3

= $\frac{4}{3}$ p × $(3)^3$ cm^3

= $\frac{4}{3}$p × 3 × 3 × 3

= 36p cm^3

Diameter of wire = 2 mm

Radius (R) = 1 mm = $\frac{1}{10}$cm

Let h be its length, then

pR^2h = 36p

⇒ p × $\left(\frac{1}{10}\right)^2$ h = 36p

⇒ p × $\frac{1}{100}$ h = 36p

⇒ h = 36 × 100

= 3600 cm

Height or length of wire = 3600 cm

= 36 m.

27. A medicine capsule is in the shape of a cylinder of diameter 0.5 cm with a hemisphere stucked at each end. The length of the entire capsule is 2 cm. The capacity of the capsule is:

[NCERT Exemplar]

(a) 0.33 cm^3

(b) 0.34 cm^3

(c) 0.35 cm^3

(d) 0.36 cm^3

Ans. (d) 0.36 cm^3

Explanation :

Radius of the capsule = 0.25 cm

Let the length of the cylindrical part of the capsule be *x* cm.

So, 0.25 + *x* + 0.25 = 2

⇒ 0.5 + *x* = 2 ⇒ *x* = 1.5

Capacity of the capsule

= 2 × (Volume of the hemisphere) + (Volume of the cylinder)

$= 2 \times \left(\frac{2}{3}\pi r^3\right) + (pr^2h)$

$= 2 \times \left(\frac{2}{3} \times \frac{22}{7} \times (0.25)^3\right) + \left(\frac{22}{7} \times (0.25)^2 \times 1.5\right)$

= 0.36 cm^3.

28. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is:

(a) 27 : 20

(b) 20 : 27

(c) 4 : 9

(d) 9 : 4

Ans. (b) 20 : 27

Explanation :

Let the radii of the two cylinders be 2*x* and 3*x*, and the heights of the two cylinders be 5y and 3y respectively.

Ratio of the volume of the cylinders

$= \frac{\pi(2x)^2(5y)}{\pi(3x)^2(3y)}$ = 20 : 27

That is, the ratio of their volume is 20 : 27.

29. Water flows at the rate of 10 metre per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

(a) 48 minutes 15 sec

(b) 51 minutes 12 sec

(c) 52 minutes 1 sec

(d) 55 minutes

Ans. (b) 51 minutes 12 sec

Explanation :

The radius of cylindrical pipe

$r = \frac{5}{2}$ mm = 0.25 cm

The volume per minute of water flow from the pipe

= p × $(0.25)^2$ × 1000

(∵ 10 m = 1000 cm)

= 62·5p cm^3/minute

The radius of cone = $\frac{40}{2}$ = 20 cm

Depth of cone = 24 cm

Volume of cone = $\frac{1}{3}$p$(20)^2$ × 24

= 3200p cm^3

The time it will take to fill up a conical vessel

$= \frac{3200\pi}{62.5\pi} = \frac{32000}{625}$

= 51.2 minutes or 51 min 12 sec.

30. What is the formula required to use for T.S.A. of an article which is made by dragging out a hemisphere from each end of a soild cylinder?

(a) C.S.A. of the cylinder – 2(C.S.A. of the hemisphere)

(b) C.S.A. of the cylinder

+ C.S.A. of the hemisphere

(c) C.S.A. of the cylinder + 2(C.S.A. of the hemisphere)

(d) C.S.A. of the cylinder – C.S.A. of the hemisphere

Ans. (c) C.S.A. of the cylinder + 2(C.S.A. of the hemisphere

Explanation :

To find the T.S.A. of an article which is made by digging out a hemisphere from each end of a solid cylinder, we need C.S.A. of the cylinder and C.S.A. of the hemisphere.

T.S.A. of the article = C.S.A. of the cylinder + 2(C.S.A. of the hemisphere)

Very Short Answer Type Questions

31. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Sol. Let radius of hemisphere be *r* units.

Volume of hemisphere = S.A. of hemisphere

(Given)

$\Rightarrow \frac{2}{3}\pi r^3 = 3\pi r^2$

$\Rightarrow r = \frac{9}{2}$ or diameter = 9 units

Ans.

32. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Given, total surface area of a solid hemisphere

= 462 cm^2

Let its radius be *r*.

Thus $3\pi r^2 = 462$

$\Rightarrow \left(\frac{22}{7}\right)r^2 = 154$

$\Rightarrow \left(\frac{1}{7}\right)r^2 = 7$

$\Rightarrow r^2 = (7)^2$

$\Rightarrow r = 7$ cm

So, volume of the hemisphere

$= \frac{2}{3}\pi r^3 = \frac{2}{3}\left(\frac{22}{7}\right)(7)^3$ cm^3

$= \frac{2}{3}\left(\frac{22}{1}\right)(7)^2$ cm^3

$= \frac{44}{3}(49)$ cm^3

= 718.67 cm^3. **Ans.**

33. The volume of a hemisphere is 2425.5 cm^3. Find its curved surface area. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Given, volume = 2425.5 cm^3

Let the radius of hemisphere be *r*.

Thus, $\frac{2}{3}\pi r^3 = 2425.5$

$\frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{24255}{10}$

$\Rightarrow \frac{44}{21}r^3 = \frac{4851}{2}$

$\Rightarrow \frac{4}{21}r^3 = \frac{441}{2}$

$\Rightarrow 8r^3 = 441 \times 21$

$\Rightarrow 8r^3 = 21^3$

$\Rightarrow (2r)^3 = (21)^3$

$\Rightarrow r = \frac{21}{2}$ cm

Thus, its curved surface area

$= 2\pi\left(\frac{21}{2}\right)^2 \text{ cm}^2$

$= 2\left(\frac{22}{7}\right)\left(\frac{21}{2}\right)\left(\frac{21}{2}\right) \text{cm}^2$

$= (33)(21) \text{ cm}^2$

$= 693 \text{ cm}^2$. **Ans.**

34. Two cubes, each of volume 27 cm^3 are joined end to end to form a solid. Find the surface area of the resulting cuboid.

Sol. Given, volume of the each cube = 27 cm^3

$\Rightarrow a^3 = 27$

$\Rightarrow a^3 = (3)^3$

$\Rightarrow a = 3$ cm

Thus, length of the larger solid

= (3 + 3) cm = 6 cm

Width of the larger solid = 3 cm

Height of the larger solid = 3 cm

Thus, surface area of the resultant cuboid

$= 2(6 \times 3 + 3 \times 3 + 6 \times 3) \text{ cm}^2$

$= 2(18 + 9 + 18) \text{ cm}^2$

$= 2(45) \text{ cm}^2$

$= 90 \text{ cm}^2$. **Ans.**

35. A conical vessel, whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the water rises in the cylindrical vessel.

Sol. Given, Height of conical vessel = 24 cm

Radius of conical vessel = 5 cm

Radius of cylindrical vessel = 10 cm

Let the height of the water-level in the cylindrical vessel be *h* cm.

Thus, volume of water in the cylindrical vessel

= Volume of water in the conical vessel

$\Rightarrow \pi(10)^2 h = \frac{1}{3}\pi(5)^2\ 24$

$\Rightarrow (10)^2 h = (5)^2 8$

$\Rightarrow 100h = 25(8)$

$\Rightarrow 4h = 8$

$\Rightarrow h = 2$ cm

Thus, the water will rise to a height of 2 cm in the cylindrical vessel. **Ans.**

36. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is filled into small cylindrical bottles, each of diameter 3 cm and height 4 cm. How many bottles are needed to empty the bowl?

Sol. Given, internal radius of hemispherical bowl

= 9 cm

Diameter of cylindrical bottles = 3 cm

Thus, radius of cylindrical bottles = 1.5 cm

Height of cylindrical bottles = 4 cm

Now, number of bottles

$= \frac{\text{Volume of hemispherical bowl}}{\text{Volume of cylindrical bottle}}$

$= \frac{\frac{2}{3}\pi(9)^3}{\pi(1.5)^2 4} = \frac{729}{6 \times 2.25}$

$= \frac{243}{2 \times 2.25}$ = 54. **Ans.**

37. A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.51 litres per minute. Find the rate of flow in the pipe in km/hr.

Sol. Given, inner diameter of cylindrical pipe = 7 cm = 0.07 m.

Thus, inner radius of pipe = 3.5 cm = 0.035 m.

Let the length of the column of the water flowing through the pipe be *h* m.

Since, rate of flow of water

= 192.51 litre per minute

So, in 1 minute, volume of water that flows

= 192.5 *l*

Thus in 1 hour (60 minutes), the volume of water that flows

= 192.5(60) = 11550 *l*

= 11.55 m^3

Now, $\pi(0.035)^2 h = 11.55$

or h = 3000 m

Thus, in 1 hour, 3000 m of water flows through the pipe.

Hence, the rate of flow of water = 3000 m/hr

= 3 km/hr. **Ans.**

38. A cylinder and a cone are of the same base radius and height. Calculate the ratio of the volume of the cylinder and the cone.

Sol. Given, Radius of cylinder = Radius of cone = *r* cm

and Height of cylinder = Height of cone = *h* cm

Thus, $\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h}$

$= \frac{3}{1}$

Hence, Volume of cylinder : Volume of cone = 3 : 1.**Ans.**

39. Two types of water tankers are available in a shop. One is in a cubic form of dimensions 1 m × 1 m × 1 m and another is in the cylindrical form of height 1 m and diameter 1 m. Calculate the volume of both the tankers. (Use π = 3.14)

Sol. Dimensions of cubic tank,

$l = 1$ m, $b = 1$ m, $h = 1$ m

∴ Volume of cubic tank = $lbh = 1 \times 1 \times 1 = 1\ m^3$

Height of cylindrical tank = 1 m.

Radius of cylindrical tank = $\frac{1}{2}$m

∴ Volume of cylindrical tank = $\pi r^2 h$

$= 3.14 \times \frac{1}{2} \times \frac{1}{2} \times 1$

$= 0.785\ m^3$ **Ans.**

Short Answer Type Questions

40. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of ` 25 per m.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Given, width of cloth = 5 m

Base diameter of tent = 14 m

∴ Radius = $\frac{14}{2}$ = 7 cm

Height = 24 m

Slant height of cone,

l = $\sqrt{h^2 + r^2}$

= $\sqrt{(24)^2 + (7)^2}$

= $\sqrt{576 + 49}$

= $\sqrt{625}$

= 25 cm

Let the length of the cloth be *l¢* m.

∴ Curved surface area of cone

= Area of cloth

⇒ π*rl* = *l¢* × *b*

⇒ $\frac{22}{7} \times 7 \times 25$ = *l¢* × 5

⇒ 22 × 25 = *l* × 5

⇒ *l¢* = $\frac{22 \times 25}{5}$ m

⇒ *l¢* = 110 m

Thus, total cost of the cloth = ` (25 × 110) = ` 2750.

Ans.

41. The largest possible sphere is carved out of a solid wooden cube of side 7 cm. Find the volume of the wood left. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Length of each side of the cube = 7 cm

∴ Largest possible diameter of the sphere = 7 cm

∴ Radius of the sphere = 3.5 cm

Thus, volume of cube = $(7)^3$ cm^3 = 343 cm^3

and volume of sphere = $\frac{4}{3}\times\frac{22}{7}(3.5)^3$ cm^3

= $\frac{539}{3}$ cm^3

∴ Remaining volume of wood

= $\left(343-\frac{539}{3}\right)$ cm^3 = $\frac{1029-539}{3}$ cm^3

= 163.33 cm^3. **Ans.**

42. Due to heavy flood in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ` 120 per sq. m, find the amount shared by each school to set-up the tents.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Radius of the base of cylinder (r) = 2.8 m

Radius of the base of the cone (r) = 2.8 m

Height of the cylinder (h) = 3.5 m

Height of the cone (H) = 2.1 m

Slant height of conical part (l)

= $\sqrt{r^2+H^2}$

= $\sqrt{(2.8)^2+(2.1)^2}$

= $\sqrt{7.84+4.41}$

= $\sqrt{12.25}$

= 3.5 m

Area of canvas used to make tent

= CSA of cylinder + CSA of cone

= $2\pi rh + \pi rl$

= $2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5$

= 61.6 + 30.8

= 92.4 m^2

Cost of 1500 tents at ` 120 per sq. m

= 1500 × 120 × 92.4

= ` 16,632,000

Share of each school to set-up the tents

= $\frac{16632000}{50}$

= ` 332,640 **Ans.**

43. A hemispherical bowl of internal radius 9 cm is full of water. Its contents are emptied in a cylindrical vessel of internal radius 6 cm. Find the height of the water in the cylindrical vessel.

Sol. Given, internal radius of hemispherical bowl

= 9 cm

Internal radius of cylindrical vessel = 6 cm

Let the height of the water in the cylindrical vessel be *h* cm.

Now, Volume of the hemispherical vessel= Volume of water in the cylindrical vessel

$\Rightarrow \frac{2}{3}\pi R^3 = \pi r^2 h$

$\Rightarrow \frac{2}{3}\pi(9)^3 = \pi(6)^2 h$

$\Rightarrow \frac{2}{3}(9)^3 = (6)^2 h$

$\Rightarrow h = \frac{2\times9\times9\times9}{3\times6\times6}$

= $\frac{27}{2}$ cm

⇒ h = 13.5 cm

Hence, the height of the water level in the cylindrical vessel = 13.5 cm. **Ans.**

44. From a solid cylinder of height 7 cm and base diameter of 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Given, height of cylinder

= Height of cone

= 7 cm

Base diameter of cylinder

= Base diameter of cone

= 12 cm

Thus, base radius = 6 cm

Thus, total surface area of the remaining solid

= Total surface area of cylinder

+ Curved surface area of cone – Area of the base of the cylinder and cone

$= 2\pi r(h + r) + \pi rl - \pi r^2$

$= 2\pi \times 6(7 + 6) + \pi \times 6(\sqrt{6^2 + 7^2}) - \pi(6)^2 \text{ cm}^2$

$= \pi[12(13) + 6(\sqrt{36 + 49}) - 36] \text{ cm}^2$

$= \pi[156 + 6(\sqrt{85}) - 36] \text{ cm}^2$

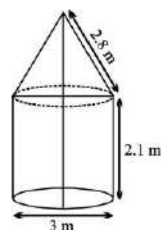
$= \pi[156 + 55.317 - 36] \text{ cm}^2$

$= \pi[175.317] \text{ cm}^2$

$= 550.997 \text{ cm}^2$. **Ans.**

45. In given figure a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ` 500 per sq. metre.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Sol. We have, height (*h*) and diameter (*d*) of cylinder as 2.1 m and 3 m respectively.

∴ Radius = $\frac{3}{2}$m

and, slant height of concial part is 2.8 m.

Area of canvas needed

= C.S.A. of (cylinder + cone)

= $2\pi rh + \pi rl$

= $2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1 + \frac{22}{7} \times \frac{3}{2} \times 2.8$

= $\frac{22}{7}$ (6.3 + 4.2)

= $\frac{22}{7} \times 10.5 = 33 \text{ m}^2$

∴ Cost of canvas needed at the rate of ` 500 per m^2

= ` (33 × 500)

= `16500 **Ans.**

46. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer.

Sol. Internal diameter of hemispherical bowl = 36 cm

∴ Radius of hemispherical bowl (r) = 18 cm

Volume of liquid, V = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times \pi \times (18)^3$

Now, diameter of bottle = 6 cm

∴ Radius of bottle = 3 cm

Now,

Volume of cylindrical bottle = $\pi R^2 h$

= $\pi \times 3^2 \times h$

= $9\pi h$

Volume of liquid transferred

= Total volume of liquid – 10% wasted liquid

= $\frac{2}{3}\pi (18)^3 - \frac{10}{100}\left(\frac{2}{3}\pi (18)^3\right)$

= $\frac{2}{3}\pi (18)^3 \left(1 - \frac{10}{100}\right)$

= $\frac{2}{3}\pi (18)^3 \times \frac{9}{10}$

Volume of liquid transferred = $\pi \times (18)^3 \times \frac{3}{5}$

Number of cylindrical bottles

= $\frac{\text{Volume of liquid to be transferred}}{\text{Volume of a bottle}}$

$\Rightarrow 72 = \frac{\pi \times 18 \times 18 \times 18 \times \frac{3}{5}}{9\pi h}$

$\Rightarrow h = \frac{27}{5}$ = 5.4 cm

Hence, height of each bottle will be 5.4 cm. **Ans.**

47. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere

can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of ` 5 per 100 sq. cm.

[Use π = 3.14]

Sol. Side of the cubical block (*a*) = 10 cm.

Since, the cube is surmounted by a hemisphere, therefore, the greatest diameter of the hemisphere should be equal to the side of the cube.

∴ Diameter of the sphere = 10 cm

⇒ Radius of the sphere (*r*) = 5 cm

Total surface area of solid

= T.S.A. of the cube + C.S.A. of hemisphere – Inner cross-section area of hemisphere

$= 6a^2 + 2\pi r^2 - \pi r^2 = 6a^2 + \pi r^2$

$= 6(10)^2 + 3.14(5)^2$

$= 600 + 25 \times 3.14$

$= 600 + 78.5$

$= 678.5 \text{ cm}^2$

Cost of painting per square metre is ` 5

Total cost for painting = $\frac{678.5}{100} \times 5$

= ` 33. 925 » ` 34

Hence, total cost for painting will be ` 34.**Ans.**

48. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Let the radius of base and height of a solid cylinder be *r* and *h* respectively.

Now, we have

$r + h = 37$ cm...(i)

and T.S.A. of solid cylinder = $2\pi r(r + h)$

= 1628 cm^2

$\Rightarrow 2\pi r(37) = 1628$ [Using (i)]

$\Rightarrow r = \frac{1628}{37 \times 2 \times \frac{22}{7}}$

$\Rightarrow r = 7$ cm

$\therefore$ Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 30$

[Using eq. (i), $h = 30$]

= 4620 cm^3 **Ans.**

49. The difference between the outer and inner curved surface area of a hollow right-circular cylinder 14 cm long is 88 cm^2. If the volume of metal used in making the cylinder is 176 cm^3, find the outer and inner diameters of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Given, volume of the cylinder = 176 cm^3

Height of the cylinder = 14 cm

and difference between outer and inner curved surface area = 88 cm^2

Now, let the outer radius be R cm and the inner radius be r cm

Thus, $\frac{22}{7}(R^2 - r^2)14 = 176$

$\Rightarrow (R^2 - r^2) = 4$

$\Rightarrow (R - r)(R + r) = 4$ …(i)

Now, outer curved surface area – Inner curved surface area = 88

cm^2

$\Rightarrow 2\pi R(14) - 2\pi r(14) = 88$

$\Rightarrow 2 \times \frac{22}{7} \times (14)(R - r) = 88$

$\Rightarrow (R - r) = 1$ …(ii)

Substituting equation (ii) in (i), we get

$(R + r) = 4$ …(iii)

Adding equations (ii) and (iii), we get

$2R = 5$

or R = 2.5 cm

and r = 1.5 cm

Thus, the outer diameter = 5 cm

and the inner diameter = 3 cm. **Ans.**

50. Cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in the vessel. Find the volume of water displaced and the volume remaining. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Given, diameter of cylindrical vessel = 10 cm

Height of cylindrical vessel = 10.5 cm

Diameter of cone = 7 cm

∴ Radius of cone = 3.5 cm

Height of cone = 6 cm

Total volume of water in the cylinder

= Volume of cylinder

$= \frac{22}{7}(5)^2 (10.5) \text{ cm}^3$

$= \frac{22}{1}(25)(1.5) \text{ cm}^3$

= 33(25) cm^3

= 825 cm^3

Volume of water displaced

= Volume of the cone

= $\frac{1}{3}\left(\frac{22}{7}\right)(3.5)^2 6 \text{ cm}^3$

= 77 cm^3

Thus, volume of water left

= (825 – 77) cm^3

= 748 cm^3. **Ans.**

51. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

[Board Question]

Sol. Given, width of canal = 5.4 m

Depth of canal = 1.8 m

Speed of flowing water = 25 km/h

∴ Length of water in canal in 1 hr

= 25 km

= 25000 m

Volume of water flown out from canal in 1 hr

= $l \times b \times h$

= 5.4 × 1.8 × 25000

= 243000 m^3

Volume of water in 40 min = 243000 × $\frac{40}{60}$

$= 162000\ m^3$

Thus, area irrigated with 10 cm standing water in field

$= \frac{\text{Volume}}{\text{Height}} = \frac{162000}{0.10} m^2$

$= 1620000\ m^2$

= 162 hectare **Ans.**

52. A juice-seller serves his customers using a glass whose inner diameter is 5 cm but the bottom of the glass has a raised hemispherical portion that reduces its capacity. If the height of the glass is 10 cm, find the apparent and actual capacities of the glass. [Use π = 3.14]

[NCERT]

[Board Question]

Sol. Given, height of glass = 10 cm

Diameter of glass = Diameter of the hemisphere

= 5 cm

Thus, radius of glass = Radius of the hemisphere

= 2.5 cm

Now, apparent capacity of the glass

$= \pi(2.5)^2\ 10\ cm^3$

$= 196.25\ cm^3$

And, actual capacity of the glass

= Apparent capacity of the glass– Volume of the hemisphere

$= \left[196.25 - \frac{2}{3}\pi(2.5)^3\right] cm^3$

$= [196.25 - 32.71]\ cm^3$

$= 163.54\ cm^3$ **Ans.**

Long Answer Type Questions

53. Sushant has a vessel in the shape of an inverted cone that is open at the top. Its height is 11 cm and the radius of the top is 2.5 cm. It is full of water and metallic spherical balls of diameter 0.5 cm are put in the vessel such that $\frac{2}{5}$ th of the water flows out. Find the number of balls that were put in the vessel.

[Board Question]

Sol. Given, Height of the cone = 11 cm

Radius of the top of the cone = 2.5 cm

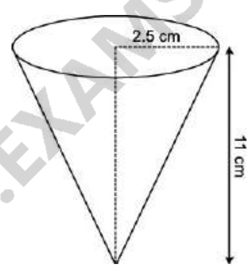
Diameter of each ball = 0.5 cm

Thus, radius of each ball = 0.25 cm

Volume of water that flows out

= $\frac{2}{5}$(Volume of the cone)

Now, volume of the water in the cone = $\frac{1}{3}\pi r^2 h$



= $\frac{\pi}{3}(2.5)^2 (11)$ cm^3

= $\frac{11\pi}{3}(2.5 \times 2.5)$ cm^3

Thus, the volume of water that flows out

= $\frac{11}{3}\pi(2.5 \times 2.5)\left(\frac{2}{5}\right)$cm^3

= Total volume of all the spherical balls

Now, volume of 1 spherical ball

$= \left(\frac{4}{3}\right)\pi(0.25)^3 \text{ cm}^3$

$= \left(\frac{4}{3}\right)\pi(0.25 \times 0.25 \times 0.25) \text{ cm}^3$

Hence, the number of spherical balls

$= \frac{\left(\frac{1}{3}\right)11\pi(2.5\times2.5)\left(\frac{2}{5}\right)}{\left(\frac{4}{3}\right)\pi(0.25\times0.25\times0.25)}$

$= \frac{11(2.5\times2.5)}{10(0.25\times0.25\times0.25)}$

= 440

Thus, number of balls = 440. **Ans.**

54. Water is flowing through a cylindrical pipe of internal diameter 2 cm into a cylindrical tank of base radius 40 cm at the rate of 0.4 m/s. Determine the rise in the water level in the tank in half an hour.

[Board Question]

Sol. Given, internal diameter of cylindrical pipe = 2 cm

Thus, radius of the cylindrical pipe = 1 cm

Rate of flow of water through the pipe

= 0.4 m/s = 40 cm/s

Base radius of the tank = 40 cm

Time = 30 min = (30 × 60) sec = 1800 sec

Let the rise in water level = *h* m

Now, volume of water that flows through the pipe in 1 sec

$= \pi(1)^2 (40) \text{ cm}^3$

$= 40\pi \text{ cm}^3$

Thus, volume of water that flows through the pipe in 1800 sec = $(1800)40\pi \text{ cm}^3$

Thus, volume of water that collects in the tank in 1800 sec

= (1800)40π cm^3

∴ $\pi(40)^2 h = (1800)40\pi$

⇒ $(40)^2 h = (1800)40$

⇒ $40h = 1800$

⇒ $h = 45$ cm

Thus, rise in the water level in the tank in half an hour is 45 cm. **Ans.**

55. A hemispherical tank full of water is emptied by a pipe at the rate of $\frac{25}{7}$ *l*/s. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m?

Sol. Given, diameter of the tank = 3 m

∴ Radius = 1.5 m

Rate of flow of water = $\frac{25}{7} l/s$

Thus, volume = $\frac{2}{3}\pi(1.5)^3$ m^3

= 2.25π m^3

= 2250π *l* [...1 m^3 = 1000 *l*]

Thus, half of the volume of the tank = 1125π *l*

Thus, the time taken to empty half of the tank

= $\frac{1125\pi}{\frac{25}{7}}$ sec

= $\frac{1125 \times \frac{22}{7}}{\frac{25}{7}}$ sec

= $\frac{1125 \times 22}{25}$ sec

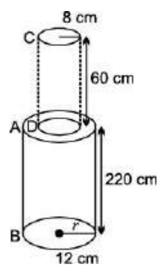
= 990 sec

= 16 minutes 30 seconds. **Ans.**

56. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 gm mass. (Use π = 3·14)

[Board Question]

Sol. Let AB be the iron pole of height 220 cm with base radius 12 cm and there is an other cylinder CD of height 60 cm whose base radius is 8 cm.



Volume of AB pole = $\pi r_1^2 h_1$ = 3·14 × 12 × 12 × 220

= 99475·2 cm^3

Volume of CD pole = $\pi r_2^2 h_2$ = 3·14 × 8 × 8 × 60

= 12057·6 cm^3

Total volume of the poles = 99475.2 + 12057.6

= 111532·8 cm^3

It is given that,

Mass of 1 cm^3 of iron = 8 gm

Thus mass of 111532.8 cm^3 of iron = 111532.8 × 8 gm

Thus, total mass of the pole is

= 111532.8 × 8 gm

= 892262.4 gm

= 892.2624 kg **Ans.**

57. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

[Board Question]

Sol. We have, diameter of well = 4 m

∴ Radius, $r = 2$ m

and Height, $h = 14$ m.

Volume of earth taken out after digging the well

$= pr^2h$

$= \frac{22}{7} \times 2 \times 2 \times 14$

$= 176 \text{ m}^3$

Let x be the width of the embankment formed by the earth taken out.

Then, volume of embankment

$\Rightarrow \frac{22}{7}[(2 + x)^2 - (2)^2] \times \frac{40}{100} = 176$

$\Rightarrow \frac{22}{7}[4 + x^2 + 4x - 4] \times \frac{2}{5} = 176$

$\Rightarrow x^2 + 4x = \frac{176 \times 5 \times 7}{22 \times 2}$

$\Rightarrow x^2 + 4x - 140 = 0$

$\Rightarrow x^2 + 14x - 10x - 140 = 0$

$\Rightarrow x(x + 14) - 10(x + 14) = 0$

$\Rightarrow (x + 14)(x - 10) = 0$

$\Rightarrow x = -14$ or 10

Since, width cannot be negative.

∴ $x = 10$

Hence, width of embankment = 10 m. **Ans.**

58. A gulabjamun when ready for eating contains sugar syrup of about 30% of its volume. Find approximately how much syrup would be found in 45 such gulabjamuns if each of them is shaped like a cylinder with two hemispherical ends. The complete length of each of them is 5 cm and the diameter is 2.8 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[NCERT]

[Board Question]

Sol. Given, length of each gulabjamun = 5 cm

diameter of each gulabjamun = 2.8 cm

Thus, radius of each gulabjamun, r = 1.4 cm

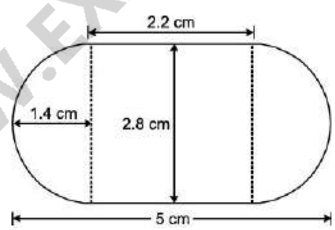
Total number of gulabjamuns = 45

Percentage of syrup in each gulabjamun = 30%

and length of the cylindrical part

= [5 – (1.4 + 1.4)] cm

= (5 – 2.8) cm = 2.2 cm



∴ Volume of 45 gulabjamun

= 45[Volume of (cylindrical portion + 2 hemispherical ends)]

= $45\left[\pi(1.4)^2 2.2 + 2\left\{\frac{2}{3}\pi(1.4)^3\right\}\right]$ cm^3

= $45\pi(1.4)^2 \left[2.2 + \frac{4}{3}(1.4)\right]$ cm^3

= $(45)\frac{22}{7}(1.4)(1.4)\left[2.2 + \frac{5.6}{3}\right]$ cm^3

$= (45)\frac{44}{100}(14)\left[\frac{6.6+5.6}{3}\right]$ cm^3

$= (45)\frac{44}{100}(14)\left[\frac{12.2}{3}\right] cm^3$

$= 1127.28\ cm^3$

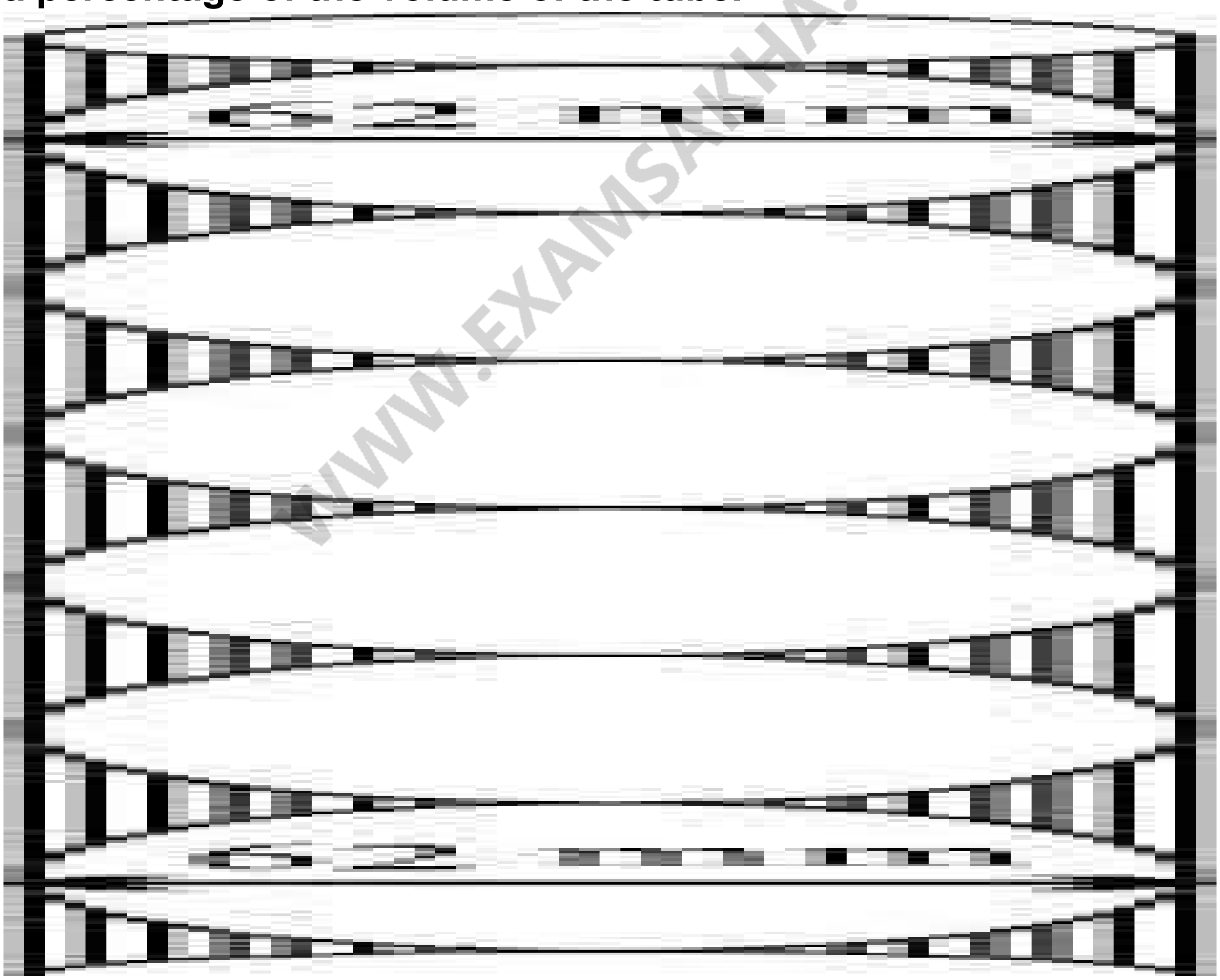
Hence, volume of syrup in 45 gulabjamuns

$= 30\%$ of $1127.28\ cm^3$

$= \frac{30}{100} \times 1127.28\ cm^3$

$= 338.18\ cm^3$. **Ans.**

59. Six tennis balls of diameter 62 mm are placed in cylindrical tube given in the figure. Find the volume of the six balls and the internal volume of unfilled space in the tube and express this as a percentage of the volume of the tube.



Sol. Diameter of the tennis balls = 62 mm.

... Radius of the balls and tube is half the diameter

∴ $r = \frac{1}{2} \times 62 = 31$ mm

Volume of one ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 31 \times 31 \times 31$

$= 124,838.476$ mm^3

$= 124.838$ cm^3

∴ Volume of 6 tennis balls = 6 × 124.838

$= 749.028$ cm^3

Height of tube (h) is 6 times the diameter of the ball

∴ $h = 6 \times 62$

= 372 mm.

∴ Volume of the tube

$= \pi r^2 h = \frac{22}{7} \times 31 \times 31 \times 372$

$= 1123,546.29$ mm^3

$= 1123.54$ cm^3

∴Volume of unfilled space (shaded area) in the tube

= 1123.54 – 749.028

$= 374.512$ cm^3

Space as a percentage of the volume of the tube

$= \frac{374.512}{1123.54} \times 100$

= 33.33%

Hence, volume of unfilled space is 33.33% of tube.**Ans.**

60. A sector of a circle of radius 6 cm has an angle of 120°. It is rolled up so that the two bounding radii are joined together to form a cone. Find volume of cone and T.S.A. of the cone.

Sol. On rolling up the sector, we get a cone whose slant height = radius of sector (R¢) = 6 cm.

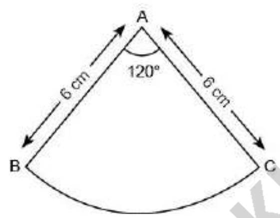
Circumference of the base of cone

= Length of sector

⇒ $2\pi R = \frac{\theta}{360°} \times 2\pi R¢ = \frac{120°}{360°} \times 2 \times \pi \times 6$

⇒ $2\pi R = 4\pi$ cm.

⇒ R = 2 cm



T.S.A. of cone = C.S.A. of cone + Area of base of cone

= Area of sector + $\pi(R)^2$

= $\frac{\theta}{360°}\pi r^2 + \pi \times (2)^2$

= $\frac{120°}{360°} \times \pi \times (6)^2 + 4\pi$

= $12\pi + 4\pi = 16\pi$ cm^2

Height of cone, $h = \sqrt{l^2 - R^2} = \sqrt{6^2 - 2^2}$

= $\sqrt{36-4} = \sqrt{32}$

= $4\sqrt{2}$ cm.

Volume of cone = $\frac{1}{3}\pi R^2 h$

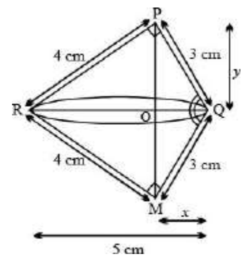
= $\frac{1}{3}\pi(2)^2 \times (4\sqrt{2})$

= $\frac{16\sqrt{2}}{3}\pi$ cm^3 **Ans.**

61. A right triangle with side 3 cm and 4 cm is revolved around its hypotenuse. Find the volume of double cone thus generated.

(Use π = 3.14)

Sol. In the given figure, ΔPQR is a right triangle, where PQ = 3 cm and PR = 4 cm



Now in ΔPQR,

$QR^2 = PQ^2 + PR^2$

$= 3^2 + 4^2$

= 9 + 16

= 25

Þ QR = 5 cm

Let $OQ = x$ Þ $OR = 5 – x$ and $OP = y$

Now, in right angled-triangle POQ, we have

$PQ^2 = OQ^2 + OP^2$

Þ $(3)^2 = x^2 + y^2$

Þ $y^2 = 9 – x^2$ ...(i)

Also from right angled triangle POR, we have

$OP^2 + OR^2 = PR^2$

Þ $y^2 + (5 – x)^2 = (4)^2$

Þ $y^2 = 16 – (5 – x)^2$ ...(ii)

From equation (i) and (ii), we get

$9 - x^2 = 16 - (5 - x)^2$

Þ $9 - x^2 = 16 - (25 + x^2 - 10x)$

Þ $9 - x^2 = -9 - x^2 + 10x$

Þ $10x = 18$

Þ $x = \frac{9}{5}$

... $OR = 5 - x = 5 - \frac{9}{5} = \frac{16}{5}$

Now putting $x = \frac{9}{5}$ in (i), we get

$y^2 = 9 - \left(\frac{9}{5}\right)^2$

$= 9 - \frac{81}{25} = \frac{144}{25}$

Þ $y = \frac{12}{5}$

$OP = y = \frac{12}{5}$

Now, for the cone *PQM*

Radius, $OP = \frac{12}{5}$ cm

Height, $OQ = \frac{9}{5}$ cm

Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{12}{5}\right)^2 \times \frac{9}{5}$

$= \frac{432\pi}{125}$ cm^3

Also for the cone *PRM*, we have

Radius, $OP = \frac{12}{5}$ cm,

Height, $OR = \frac{16}{5}$ cm

Volume = $\frac{1}{3}\pi\left(\frac{12}{5}\right)^2 \times \frac{16}{5}$ cm^3

$= \frac{768\pi}{125}$ cm^3

Hence, total volume, *i.e.,* volume of the double cone

$= \left(\frac{432\pi}{125} + \frac{768\pi}{125}\right)$ cm^3

$= \frac{1200\pi}{125}$ cm^3

= 9.6 × 3.14 cm^3

= 30.144 cm^3. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

85. Assertion : A hemisphere of radius 7 cm is to be painted outside on the surface. The total cost of painting at it ` 5 per cm$^2$ is ` 2300.

Reason : The total surface area hemisphere is $3\pi r^2$.

Ans. (d) Assertion is false but the Reason is true.

Explanation :

Total surface area of the hemisphere is

$\pi r^2 + 2\pi r^2 = 3pr^2 = 3 \times \frac{22}{7} \times 7 \times 7$

= 462 cm^2

Cost of painting = 462 × 5

= ` 2310

So, assertion is false but reason is true.

Case Based Questions

62. Mathematics teacher of a school took her 10th standard students to show Red fort. It was a part of their Educational trip. The teacher had interest in history as well. She narrated the facts of Red fort to students. Then the teacher said in this monument one can find combination of solid figures. There are 2 pillars which are cylindrical in shape. Also 2 domes at the corners which are hemispherical 7 smaller domes at the centre. Flag hoisting ceremony on Independence Day takes place near these domes.

(i) How much cloth material will be required to cover 2 big domes each of radius 2·5 metres? (Take π = 22 / 7)

(a) 75 m^2

(b) 78.57 m^2

(c) 87.47 m^2

(d) 25.8 m^2

Ans. (b) 78.57 m^2

Explanation :

Radius of a dome, r = 2·5 cm

The dome is hemispherical in shape.

Then, cloth material required

= 2 × Surface area of hemisphere

$= 2 \times 2\pi r^2$

$= 4 \times \frac{22}{7} \times 2{\cdot}5 \times 2{\cdot}5$

$= 78{\cdot}57$ m^2

(ii) The formula to find the volume of a cylindrical pillar:

(a) $\pi r^2 h$

(b) πrl

(c) $\pi r(l + r)$

(d) $2\pi r$

Ans. (a) $\pi r^2 h$

Explanation :

The formula to find the volume of a cylindrical pillar is pr^2h

(iii) The lateral surface area of two pillars if height of the pillar is 7 m and radius of the base is 1·4 m is:

(a) 112.3 m^2

(b) 123.2 m^2

(c) 90 m^2

(d) 345.2 m^2

Ans. (b) 123.2 m^2

Explanation :

Height of each pillar, $h = 7$ m

Radius of base,

$r = 1.4$ m

Lateral surface area or curved surface area of 2 pillars = $2 \times 2\pi rh$

$= 4 \times \frac{22}{7} \times 1.4 \times 7$

$= 123.2\ m^2$

(iv) The volume of a hemisphere if the radius of the base is 3.5 m, is:

(a) $85.9\ m^3$

(b) $80\ m^3$

(c) $98\ m^3$

(d) $89.83\ m^3$

Ans. (d) $89.83\ m^3$

Explanation :

Radius of hemisphere, $r = 3.5$ m

Then, volume of a hemisphere,

$V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$

$= 89.83\ m^3$

(v) The ratio of sum of volumes of two hemispheres of radius 1 cm each to the volume of a sphere of radius 2 cm?

(a) 1 : 1

(b) 1 : 8

(c) 8 : 1

(d) 1 : 16

Ans. (b) 1 : 8

Explanation :

Volume of 2 hemispheres of radius 1 cm

$= 2 \times \frac{2}{3}\pi r^3 = \frac{4}{3}\pi(1)^3$

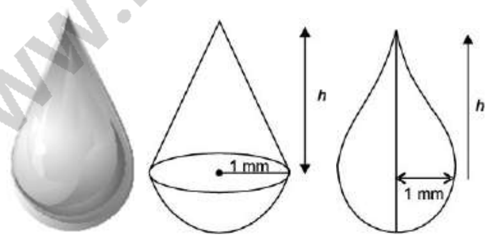
$= \frac{4}{3}\pi \text{ cm}^3$

Volume of a sphere of radius 2 cm

$= \frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi \text{cm}^3$

Then, required ratio = $\frac{\frac{4}{3}\pi}{\frac{32}{3}\pi} = \frac{1}{8}$ = 1 : 8

62. In the month of December 2020, it rained heavily throughout the day over the city of Hyderabad. Anil observed the raindrops as they reached him. Each raindrop was in the shape of a hemisphere surmounted by a cone of the same radius of 1 mm. Volume of one of such drops is 3·14 mm^3. Anil collected the rain water in a pot having a capacity of 1099 cm^3. [Use $\sqrt{2}$ = 1·4].



Based on the above situation, answer the following questions.

(i) The total height of the drop is:

(a) 1 mm

(b) 2 mm

(c) 3 mm

(d) 4 mm

Ans. (b) 2 mm

Explanation :

Radius of hemispherical part

= Radius of conical part, $r = 1$ mm.

Let the height of the conical part be h mm.

Volume of a rain drop

= Volume of hemispherical part + Volume of conical part

$\Rightarrow 3{\cdot}14 = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$\Rightarrow 3{\cdot}14 = \frac{1}{3}\pi r^2 (2r + h)$

$\Rightarrow 3{\cdot}14 = \frac{1}{3} \times 3{\cdot}14 \times 1 \times 1 (2 \times 1 + h)$

$\Rightarrow 2 + h = 3$

$\Rightarrow h = 1$ mm

Height of a rain drop

= Height of hemispherical part+ Height of conical part

= (1 + 1) mm = 2 mm.

(ii) The curved surface area of the drop is:

(a) 8·74 mm^2

(b) 9·12 mm^2

(c) 10·68 mm^2

(d) 12·54 mm^2

Ans. (c) 10·68 mm^2

Explanation :

Slant height of conical part,

$l = \sqrt{r^2 + h^2} = \sqrt{1^2 + 1^2}$ mm

= $\sqrt{2}$ mm

= 1·4 mm.

CSA of a rain drop

= CSA of hemispherical part + CSA of conical part = $2\pi r^2 + \pi r l = \pi r (2r + l)$

= 3·14 × 1 × (2 × 1 + 1·4) mm^2

= (3·14 × 3·4) mm^2

= 10·68 mm^2

(iii) As the drop fell into the pot, it changed into a sphere. What was the radius of this sphere?

(a) $(3/4)^{1/3}$

(b) $(4/3)^{1/3}$

(c) $3^{1/3}$

(d) $4^{1/3}$

Ans. (a) $\left(\frac{3}{4}\right)^{1/3}$

Explanation :

Let the radius of the sphere be R mm.

Then, volume of sphere = Volume of a rain drop

⇒ $\frac{4}{3}\pi R^3 = 3.14$

⇒ $\frac{4}{3} \times 3.14 \times R^3 = 3.14$

$\Rightarrow R^3 = \frac{3}{4}$

$\Rightarrow R = \left(\frac{3}{4}\right)^{1/3}$

(iv) How many drops will fill the pot completely:

(a) 260000

(b) 280000

(c) 320000

(d) 350000

Ans. (d) 350000

Explanation :

Volume of pot = 1099 cm^3

= (1099 × 10 × 10 × 10) mm^3

= 1099000 mm^3.

Let *n* rain drops be needed to fill the pot.

Then, volume of *n* rain drops

= Volume of pot

$\Rightarrow n \times 3 \cdot 14 = 1099000$

$\Rightarrow n = \frac{1099000}{3 \cdot 14} = 350000.$

(v) The total surface area of a hemisphere of radius *r* is:

(a) 2/3 πr^3

(b) 4/3 πr^3

(c) $2\pi r^2$

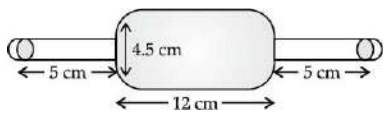
(d) $3\pi r^2$

Ans. (d) $3\pi r^2$

Explanation :

Total surface area of a hemisphere of radius r is given by $3\pi r^2$.

63. Aparna is studying in X standard. While helping her mother in kitchen, she saw rolling pin made of steel and empty from inner side, with two small hemispherical ends as shown in the figure.



(i) Find the curved surface area of two identical cylindrical parts, if the diameter is 2.5 cm and length of each part is 5 cm:

(a) 475 cm^2

(b) 78.57 cm^2

(c) 877 cm^2

(d) 259.19 cm^2

Ans. (b) 78.57 cm^2

Explanation :

Curved surface area of two identical cylindrical parts,

= 2 × 2p*rh*

= $2\times2\times\frac{22}{7}\times\frac{2.5}{2}\times5$

= 78.57 cm^2

(ii) Find the volume of big cylindrical part:

(a) 190.93 cm^3

(b) 75 cm^3

(c) 77 cm^3

(d) 83.5 cm^3

Ans. (a) 190.93 cm^3

Explanation :

Volume of big cylindrical part = pr^2h

= $\frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 12$

= 190.93 cm^3

(iii) Volume of two hemispherical ends having diameter 2.5 cm, is:

(a) 4.75 cm^3

(b) 8.18 cm^3

(c) 2.76 cm^2

(d) 75 cm^3

Ans. (b) 8.18 cm^3

Explanation :

Volume of two hemispherical ends

= 2 × $\frac{2}{3}$pr^3

= $2 \times \frac{2}{3} \times \frac{22}{7} \times \left(\frac{2.5}{2}\right)^3$

= 8.18 cm^3

(iv) Curved surface area of two hemispherical ends, is:

(a) 17.5 cm^2

(b) 7.9 cm^2

(c) 19.64 cm^2

(d) 15.5 cm^2

Ans. (c) 19.64 cm^2

Explanation :

Curved surface area of two hemispherical ends,

= 2 × 2pr^2

= 2 × 2 × $\frac{22}{7}$ × $\frac{2.5}{2}$ × $\frac{2.5}{2}$

= 19.64 cm^2

(v) Find the difference of volumes of bigger cylindrical part and total volume of the two small hemispherical ends:

(a) 175.50 cm^3

(b) 182.75 cm^3

(c) 76.85 cm^3

(d) 96 cm^3

Ans. (b) 182.75 cm^3

Explanation :

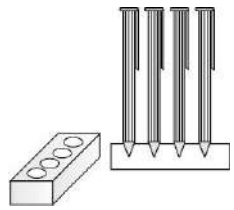
Difference of volume of bigger cylinder to two small hemispherical ends

= 190.93 – 8.18

= 182.75 cm^3

64. A carpenter used to make and sell different kinds of wooden pen stands like rectangular, cuboidal, cylindrical, conical. Aarav went to his shop and asked him to make a pen stand as explained below. Pen stand must be of the cuboidal

shape with three conical depressions, which can hold 3 pens. The dimensions of the cuboidal part must be 20 cm × 15 cm × 5 cm and the radius and depth of each conical depression must be 0.6 cm and 2.1 cm respectively. Based on the above information, answer the following questions.



(i) The volume of the cuboidal part is:

(a) 1250 cm^3

(b) 1500 cm^3

(c) 1625 cm^3

(d) 1600 cm^3

Ans. (b) 1500 cm^3

Explanation :

Volume of cuboidal part

$= l \times b \times h$

$= (20 \times 15 \times 5)\ cm^3 = 1500\ cm^3$

(ii) Total volume of conical depressions is:

(a) 2.508 cm^3

(b) 1.5 cm^3

(c) 2.376 cm^3

(d) 3.6 cm^3

Ans. (c) 2.376 cm^3

Explanation :

Radius of conical depression,

$r = 0.6$ cm

Height of conical depression,

$h = 2.1$ cm

Total volume of conical depressions

$= 3 \times \frac{1}{3} p r^2 h = \frac{22}{7} \times 0.6 \times 0.6 \times 2.1$

$= \frac{2376}{1000} = 2.376$ cm^3

(iii) Volume of the wood used in the entire stand is:

(a) 631.31 cm^3

(b) 3564 cm^3

(c) 1502.376 cm^3

(d) 1497.624 cm^3

Ans. (d) 1497.624 cm^3

Explanation :

Volume of wood used in the entire stand

= Volume of cuboidal part – Total volume of conical depressions

= 1500 – 2.376 = 1497.624 cm^3

(iv) Total surface area of cone of radius r is given by:

(a) $prl + pr^2$

(b) $2prl + pr^2$

(c) $pr^2l + pr^2$

(d) $prl + 2pr^3$

Ans. (a) p*rl* + pr^2

Explanation :

Total surface area of cone of radius *r* is given by p*rl* + pr^2.

(v) If the cost of wood used is ` 5 per cm^3, then the total cost of making the pen stand is:

(a) ` 8450.50

(b) ` 7480

(c) ` 9962.14

(d) ` 7488.12

Ans. (d) ` 7488.12

Explanation :

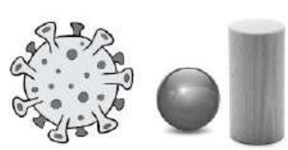
Cost of wood per cm^3 = ` 5

Total cost of making the pen stand,

= ` (5 × 1497.624)

= ` 7488.12

65. Arun a X standard student makes a project on corona virus in science for an exhibition in his school. In this project, he picks a sphere which has volume 38808 cm^3 and 11 cylindrical shapes, each of volume 1540 cm^3 with length 10 cm.



Based on the above information, answer the following questions.

(i) Diameter of the base of the cylinder is:

(a) 7 cm

(b) 14 cm

(c) 12 cm

(d) 16 cm

Ans. (b) 14 cm

Explanation :

We know that,

Volume of cylinder = pr^2h

⇒ 1540 = $\frac{22}{7}$ × r^2 × 10

⇒ r^2 = $\frac{154 \times 7}{22}$ = 49

⇒ r = 7 cm

Diameter of the base of cylinder

= 2r = 2 × 7 = 14 cm

(ii) Diameter of the sphere is:

(a) 40 cm

(b) 42 cm

(c) 21 cm

(d) 20 cm

Ans. (b) 42 cm

Explanation :

We know that,

Volume of sphere = $\frac{4}{3}$pr^3

⇒ 38808 = $\frac{4}{3}$ × $\frac{22}{7}$ × r^3

⇒ r^3 = $\frac{38808 \times 3 \times 7}{4 \times 22}$

= 441 × 21 = $(21)^3$

⇒ $r = 21$ cm

Diameter of sphere = 42 cm

(iii) Total volume of the shape formed is:

(a) 85541 cm^3

(b) 45738 cm^3

(c) 24625 cm^3

(d) 55748 cm^3

Ans. (d) 55748 cm^3

Explanation :

Total volume of shape formed

= Volume of cylindrical shape + Volume of sphere

= 11 × 1540 + 38808

= 16940 + 38808

= 55748 cm^3

(iv) Curved surface area of the one cylindrical shape is:

(a) 850 cm^2

(b) 221 cm^2

(c) 440 cm^2

(d) 540 cm^2

Ans. (c) 440 cm^2

Explanation :

Curved surface area of one cylindrical shape

= 2p*rh*

$= 2 \times \frac{22}{7} \times 7 \times 10$

$= 440\ cm^2$

(v) Total area covered by cylindrical shapes on the surface of sphere is:

(a) 1694 cm^2

(b) 1580 cm^2

(c) 1896 cm^2

(d) 1740 cm^2

Ans. (a) 1694 cm^2

Explanation :

Area covered by cylindrical shapes on the surface of sphere

$= 11 \times pr^2$

$= 11 \times \frac{22}{7} \times 7 \times 7$

$= 1694\ cm^2$

Passage Based Questions

66. Vijay has rain water harvesting plant on his roof. After rain, all the water that is collected on the roof of 22 m × 20 m is drained into the cylindrical tank having diameter of base 2 m and height 3.5 m. It rained heavily last night and in the morning the tank is just full.

(i) How much water is collected in the tank (in litres)?

(ii) Find the rainfall in cm ?

Sol. (i) We have,

Radius of cylindrical tank, $r = \frac{2}{2} = 1$ m

Height of cylindrical tank, $h = 3.5$ m

Volume of cylindrical tank = $\pi r^2 h$

$= \frac{22}{7} \times (1)^2 \times 3.5$

$= 11 \text{ m}^3$

Since, $1 \text{ m}^3 = 1000$ *l*

$11 \text{ m}^3 = 11000$ litres.

11000 litres of water is collected in tank.**Ans.**

(ii) Let the rainfall be x m,

Volume of water = Volume of cuboid

$= 22 \text{ m} \times 20 \text{ m} \times x \text{ m}$.

$= (22 \times 20 \times x) \text{ m}^3$

$= 440x \text{ m}^3$

Since, the tank is just full of water that drains out of the roof into the tank.

∴ Volume of water

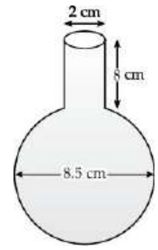
= Volume of the cylindrical vessel

$\Rightarrow 22 \times 20 \times x = 11$

$\Rightarrow x = \frac{11}{440} = \frac{1}{40} \text{m}$

$\Rightarrow = \frac{100}{40} \text{cm} = 2.5$ cm **Ans.**

67. A thirsty crow saw a spherical glass vessel with cylindrical neck 8 cm long and 2 cm in diameter. The diameter of spherical part is 8.5 cm. The vessel is half filled with water. But due to long neck of vessel crow was not able to drink water from it. He saw few spherical marbles of diameter 1 cm lying near by the vessel. He start dropping them inside the vessel one by one.



Based on the given information, answer the following questions :

(i) What is the total volume of the vessel?

(ii) How many marbles crow has to drop inside the vessel to drink the water?

Sol. (i) We have,

Radius of cylindrical neck (r) = $\frac{2}{2}$ cm = 1 cm

Length of cylindrical neck (h) = 8 cm

Radius of spherical part (R) = $\frac{8.5}{2}$ = 4.25 cm

Volume of cylindrical part = $\pi r^2 h = \pi \times (1)^2 \times 8$

= 8π cm^3

Volume of spherical part = $\frac{4}{3}\pi R^3$

= $\frac{4}{3}\pi(4.25)^3$

= 102.354π cm^2

Total volume of vessel

= Volume of cylindrical part + Volume of spherical part

= $8\pi + 102.354\pi$ cm^3

= 110.354π cm^3**Ans.**

(ii) Since, vessel is half filled.

∴ Volume of water to be filled = $\frac{110.354\pi}{2}$ cm^3

$= 55.177\pi\ cm^3$

Volume of each marble of radius $\left(\frac{1}{2} cm\right)$

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\left(\frac{1}{2}\right)^3 cm^3$

$= \frac{\pi}{6} cm^3$

No. of marbles

$= \frac{\text{Volume of water to be filled}}{\text{Volume of each marble}}$

$= \frac{55.177\pi}{\frac{\pi}{6}} = 331{\cdot}06$

≈ 331

Crow has to drop 331 marbles so it can drink water from it. **Ans.**

Self-Assessment

68. If the diameter of a sphere is 14 cm, then what is its curved surface area?

Ans. 616 cm^2

69. The radius and height of a right-circular cone are 12 cm and 9 cm respectively. What is the curved surface area of that cone?

Ans. 565.71 cm^2

70. The volume of a right-circular cylinder is 352 cm^3 and the height is 7 cm. Find the radius of the base.

Ans. 4 cm

71. If the radii of a sphere and a right-circular cylinder are 3 cm each and if their volumes are equal as well, then find the height of the cylinder.

Ans. 4 cm

72. The curved surface area and the volume of a right circular cylinder are numerically equal. Find the radius of the cylinder.

Ans. 2 cm

73. The volumes of a sphere and cylinder are equal. The diameter of the base of the cylinder is equal to the diameter of the sphere. Find the ratio between the radius of the base and the height of the cylinder.

Ans. 3 : 4

74. 77 m^2 of canvas is required to make a conical tent of slant height 7 m. Find the area of the base.

Ans. $\frac{77}{2}$ m^2.

75. The volume of the right-circular cone of height 24 cm is 1232 cm^3. Find the lateral surface area of the cone.

Ans. 550 cm^2.

76. Two cubes each of volume 64 cm^3 are joined end to end to form a solid. Find the surface area and volume of the resulting cuboid.

[NCERT]

Ans. 160 cm^2, 128 cm^3.

77. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform of 22 m by 14 m. Find the height of the platform.

[NCERT]

Ans. 2.5 m.

78. A copper wire 3 mm in diameter is wound around a cylinder whose length is 12 m and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of the copper wire to be 8.88 g/cm.

[NCERT]

Ans. 12.57 m, 789.41 g.

79. Selvi's house has an overhead tank in the shape of a cylinder. It is filled up by pumping water from an underground tank that is cuboid in shape. The dimensions of the cuboid are 1.57 m × 1.44 m × 0.95 m. The radius of the overhead tank is 60 cm and its height is 95 cm. Find the height of the water-level in the underground tank after the overhead tank has been filled up completely. Compare the capacities of both the tanks. (Use π = 3.14)

[NCERT]

Ans. 47.5 cm, 1 : 2.

80. Water in a canal, 6 m deep and 1.5 m wide is flowing at a speed of 10 km/hr. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed for irrigation ?

[NCERT]

Ans. 562500 m^2.

81. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 3 km/hr then in how much time will the tank be completely filled ?

[NCERT]

Ans. 1 hour and 40 minutes.

82. A wooden toy rocket is in the shape of a cone mounted on a cylinder. The height of the entire rocket is 26 cm while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm while the diameter of the cylinder is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. [Use π = 3.14]

[NCERT]

Ans. 195.47 cm^2.

83. A wooden toy was made from the rest of the solid cylinder after scooping out a hemisphere of same radius from each of it. If the height of the cylinder is 10 cm and its base radius is 3.5 cm, find the total surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[NCERT]

Ans. 374 cm^2.

84. A tent is in the form of a right-circular cylinder of base diameter 4 m and height 2.1 m surmounted by a right circular cone of the same base radius and slant height 2.8 m. Find the area of the canvas used and the cost of canvas at ` 500 per square metre.

[NCERT]

Ans. ` 22000.

85. A solid is in the shape of a cone mounted on a hemisphere, the radius of each of them being 1 cm and the total height of the cone equal to its radius. Find the volume of the solid in terms of π.

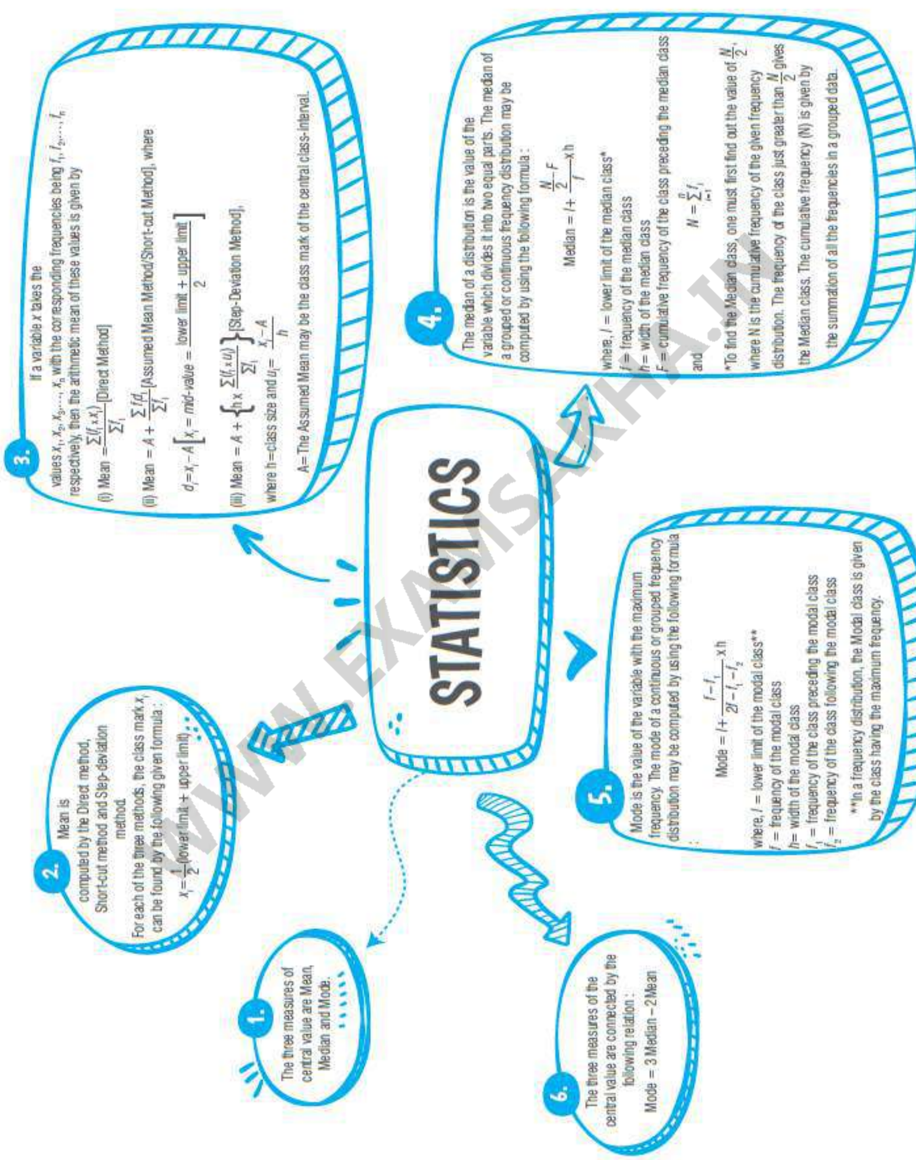
[NCERT]

Ans. π cm^3.

Statistics

Chapter 14

Basic Concepts



Multiple Choice Questions

1. Find the class marks of class 10 – 25 and 35 – 55:

[Board Question]

(a) 1.75 and 45

(b) 17.5 and 4.5

(c) 1.75 and 4.5

(d) 17.5 and 45

Ans. (d) 17.5 and 45

Explanation :

We derive the class mark x_i by the following formula :

$x_i = \frac{1}{2}$(lower limit + upper limit)

Thus, $x_i = \frac{1}{2}(10 + 25)$

$= \frac{1}{2}(35) = 17.5$

and $x_{ii} = \frac{1}{2}$(lower limit + upper limit)

Thus, $x_{ii} = \frac{1}{2}(35 + 55)$

$= \frac{1}{2}(90) = 45$

2. Write down the median class of the following frequency distribution:

[Board Question]

Class Interval	Frequency
0 – 10	4
10 – 20	4
20 – 30	8

30 – 40	**10**
40 – 50	**12**
50 – 60	**8**
60 – 70	**4**

(a) 20 – 30

(b) 30 – 40

(c) 40 – 50

(d) 50 – 60

Ans. (b) 30 – 40

Explanation :

Class Interval	Frequency (f_i)	Cumulative Frequency
0 – 10	4	4
10 – 20	4	8
20 – 30	8	16
30 – 40	10	26
40 – 50	12	38
50 – 60	8	46
60 – 70	4	50

N = Σf_i = 50

Thus, $\frac{N}{2}$ = 25

The cumulative frequency just above 25 is 26.

Hence, the median class is 30 – 40.

3. Calculate the value of *p* from the following data:

Class	Frequency
0 – 20	8
20 – 40	15
40 – 60	p
60 – 80	12
80 – 100	5
	$N = \Sigma f_i = 60$

(a) 20

(b) 30

(c) 45

(d) 50

Ans. (a) 20

Explanation :

Given, $N = \Sigma f_i = 60$

$\Rightarrow 8 + 15 + p + 12 + 5 = 60$

$\Rightarrow 40 + p = 60$

$\Rightarrow p = 20$

4. $\Sigma f_i = 15$, $\Sigma f_i x_i = 3p + 36$ and mean of the distribution is 3, then p will be:

(a) 2

(b) 3

(c) 1

(d) 6

Ans. (b) 3

Explanation :

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow 3 = \frac{3p + 36}{15}$

$\Rightarrow 45 = 3p + 36 \Rightarrow 3p = 9$

$\Rightarrow p = 3$.

5. If the value of mean and mode are 30 and 15, respectively, then median will be:

(a) 25

(b) 24

(c) 23.5

(d) 26

Ans. (a) 25

Explanation :

We know that

Mode = 3 median – 2 mean

Þ 15 = 3 median – 60

Þ 3 median = 75

Þ Median = 25.

6. The mean of the first 10 natural numbers is:

(a) 0

(b) 5.5

(c) 7

(d) 5

Ans. (b) 5.5

Explanation :

∵ The first 10 natural numbers are

1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

∴ Mean = $\frac{1+2+3+4+5+6+7+8+9+10}{10}$

= $\frac{55}{10}$

= 5.5.

7. The relation between mean, median and mode is:

(a) mode = 3 mean – 2 median

(b) mode = 3 median – 2 mean

(c) median = 3 mean – 2 mode

(d) mean = 3 median – 2 mode

Ans. (b) mode = 3 median – 2 mean

Explanation :

This is called an empirical relation between mean, median and mode.

8. The median of the following frequency distribution will be:

x	6	7	5	2	10	9	3

y	**9**	**12**	**8**	**13**	**11**	**14**	**7**

(a) 7

(b) 4

(c) 5

(d) 6

Ans. (d) 6

Explanation :

x	*f*	*c.f.*	*x*	*f*	*c.f.*
2	13	13	7	12	49
3	7	20	9	14	63
5	8	28	10	11	74
6	9	37			

Here $\frac{N}{2}$ = $\frac{74}{2}$ = 37th observation

Median = 37th observation = 6

9. If Σf_i = 17, $\Sigma f_i x_i$ = 4*p* + 63 and mean = 7, then *p* is:

(a) 14

(b) 13

(c) 12

(d) 11

Ans. (a) 14

Explanation :

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

Þ 7 = $\frac{4p+63}{17}$

Þ p = 14

10. What is the mean of the following data:

Class interval	Frequency
50 – 60	**8**
60 – 70	**6**
70 – 80	**12**
80 – 90	**11**
90 – 100	**13**

(a) 78

(b) 68

(c) 48

(d) 58

Ans. (a) 78

Explanation :

Class Interval	f	x	$f \times x$
50 – 60	8	55	440
60 – 70	6	65	390
70 – 80	12	75	900
80 – 90	11	85	935
90 – 100	13	95	1235
	$\Sigma f = 50$		$\Sigma f_i x_i = 3900$

Mean = $\bar{x}$ = $\frac{3900}{50}$

= 78.

11. If the mean of observations x_1, x_2, x_3, ... x_n is, $\bar{x}$, then the mean of ax_1, ax_2, ax_3, ..., ax_n is:

(a) $\bar{x}$

(b) $a+\bar{x}$

(c) $a\bar{x}$

(d) None of these

Ans. (c) $a\bar{x}$

Explanation :

Given, $\frac{x_1 + x_2 + \ldots + x_n}{n}$ = $\bar{x}$ …(i)

Now, Mean of ax_1 + ax_2, ax_3 + …, ax_n

= $\frac{ax_1 + ax_2 + \ldots + ax_n}{n}$ = $a\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)$

= $a\bar{x}$ [from (i)]

12. Which of the following cannot be determined graphically?

(a) Mean

(b) Median

(c) Mode

(d) None of these

Ans. (a) Mean

Explanation :

Mean cannot be determined graphically.

13. If the mean of first *n* natural number is 15, then *n* =

(a) 15

(b) 30

(c) 14

(d) 29

Ans. (d) 29

Explanation :

Mean of first *n* natural number = 15

$= \frac{n(n+1)}{2 \times n}$

Þ $\frac{(n+1)}{2 \times n}$ = 15

Þ $n + 1 = 30$ Þ $n = 29$

14. For the following distribution:

Class	**Frequency**
0 – 5	**10**
5 – 10	**15**
10 – 15	**12**
15 – 20	**20**
20 – 25	**9**

The sum of lower limits of the median class and modal class is:

(a) 15

(b) 25

(c) 30

(d) 35

Ans. (b) 25

Explanation :

Here,

Class	**Frequency**	**Cumulative Frequency**
0 – 5	10	10
5 – 10	15	25
10 – 15	12	37
15 – 20	20	57
20 – 25	9	66

Now, $\frac{N}{2} = \frac{66}{2}$ = 33, which lies in the interval 10 – 15. Therefore, lower limit of the median class is 10.

The highest frequency is 20, which ties in the interval 15 – 20. Therefore, lower limit of modal class is 15.

Hence, requried sum is 10 + 15 = 25.

15. Consider the following frequency distribution:

Class	**Frequency**
0 – 5	**13**
6 – 11	**10**
12 – 17	**15**
18 – 23	**8**
24– 29	**11**

The upper limit of the median class is:

(a) 11.5

(b) 17.5

(c) 23.5

(d) 29.5

Ans. (b) 17.5

Explanation :

Here,

Class	**Frequency**	**Cumulative Frequency**
0.5 – 5.5	13	13
5.5 – 11.5	10	23
11.5 – 17.5	15	38
17.5 – 23.5	8	46
23.5 – 29.5	11	57

Here, N = 57

Therefore, $\frac{N}{2} = \frac{57}{2}$

= 28.5

Median class is 11.5 – 17.5. So upper limit is 17.5.

16. While computing mean of grouped data, we assume that the frequencies are:

(a) Evenly distributed over all the classes

(b) Centred at the classmarks of the classes

(c) Centred at the upper limits of the classes

(d) Centred at the lower limits of the classes

Ans. (b) Centred at the class marks of the classes

Explanation :

In computing the mean of grouped data, the frequencies are centred at the class marks of the classes.

17. In the formula $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$ finding the mean of grouped data d_i's are deviations from:

(a) lower limits of classes

(b) upper limits of classes

(c) mid-points of classes

(d) frequency of the class marks

Ans. (c) mid-points of classes.

Explanation :

We know that, $d_i = x_i - a$

i.e., d_i's are the deviation from the mid-points of the classes.

18. For a symmetrical frequency distribution, we have:

(a) Mean < Mode < Median

(b) Mean < Mode > Median

(c) Mean = Mode = Median

(d) Mode = 12 + 12 (Mean + Median)

Ans. (c) Mean = Mode = Medain

Explanation :

For a symmetrical distribution, we have

Mean = Mode = Median

19. The algebraic sum of the deviations of a frequency distribution from its mean is:

(a) Always positive

(b) Always negative

(c) 0

(d) A non-zero number

Ans. (c) 0

Explanation :

The algebraic sum of the deviations of a frequency distribution from its mean is zero.

Let x_1, x_2, x_3, ... x_n are observations and $\bar{x}$ is the mean

$$\left(\because \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x} \right)$$

$(\bar{x} - x_1) + (\bar{x} - x_2) + (\bar{x} - x_3) + ... + (\bar{x} - x_n)$

$= n\bar{x} - (x_1 + x_2 + x_3 + ... + x_n)$

$= n\bar{x} - (x_1 + x_2 + x_3 + ... + x_n)$

$= n\bar{x} - n\bar{x} = 0$

20. If mode of a series exceeds its mean by 12, then mode exceeds the median by:

(a) 4

(b) 8

(c) 6

(d) 10

Ans. (b) 8

Explanation :

Given : Mode – Mean = 12…(i)

We know that Mode = 3 Median – 2 Mean

Mode – Mean = 3(Median – Mean)

Þ 12 = 3(Median – Mean)

Þ Median – Mean = 4...(ii)

On subtracting (ii) from (i), we get

Mode – Median = 12 – 4

= 8.

21. Consider the following distribution:

Marks obtained	**Number of students**
More than or equal to 0	**63**
More than or equal to 10	**58**
More than or equal to 20	**55**
More than or equal to 30	**51**

More than or equal to 40	**48**
More than or equal to 50	**42**

the frequency of the class 30 – 40 is:

(a) 3

(b) 4

(c) 48

(d) 51

Ans. (a) 3

Explanation :

Marks obtained	**Number of students**
0 – 10	(63 – 58) = 5
10 – 20	(58 – 55) = 3
20 – 30	(55 – 51) = 4
30 – 40	(51 – 48) = 3
40 – 50	(48 – 42) = 6
50	42 = 42

The frequency is the class interval 30 – 40 is 3.

22. If the mean of the following distribution is 2.6, then the value of *y* is:

Variable (*x*)	**Frequency**
1	**4**

2	**5**
3	***y***
4	**1**
5	**2**

(a) 3

(b) 8

(c) 13

(d) 24

Ans. (b) 8

Explanation :

Mean = 2.6

Variable (*x*)	**Frequency (*f*)**	***fx***
1	4	4
2	5	10
3	*y*	3*y*
4	1	4
5	2	10
Total	12 + *y*	28 + 3*y*

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{28+3y}{12+y}$ Þ 2.6 = $\frac{28+3y}{12+y}$

Þ $2.6(12 + y) = 28 + 3y$ Þ $31.2 + 2.6y = 28 + 3y$

Þ $0.4y = 3.2$ Þ $y = 8$.

23. The times, in seconds, taken by 150 atheletes to run a 110

m hurdle race are tabulated below:

Class	Frequency
13.8 – 14·0	2
14·0 – 14.2	4
14.2 – 14.4	5
14.4 – 14.6	71
14.6 – 14.8	48
14.8 – 15.0	20

The number of atheletes who completed the race in less than 14.6 seconds is:

(a) 11

(b) 71

(c) 82

(d) 130

Ans. (c) 82

Explanation :

The number of atheletes who completed the race in less than 14.6 seconds = 2 + 4 + 5 + 71 = 82

24. Consider the frequency distribution of the heights of 60 students of a class:

Height (in cm.)	No. of students	Cumulative frequency
150 – 155	16	16

155 – 160	**12**	**28**
160 – 165	**9**	**37**
165 – 170	**7**	**44**
170 – 175	**10**	**54**
175 – 180	**6**	**60**

The sum of the lower limit of the modal class and the upper limit of the median class is:

(a) 310

(b) 315

(c) 320

(d) 330

Ans. (b) 315

Explanation :

Class having maximum frequency is the modal class.

Hence, modal class is 150 – 155.

Lower limit of the modal class = 150

Now, $\frac{N}{2} = \frac{60}{2} = 30$

The cumulative frequency just greater than 30 is 37.

Hence, the median class is 160 – 165.

Upper limit of the median class = 165

Required sum = 150 + 165 = 315.

25. Match the following columns:

	Column I		**Column II**

a.	**The most frequent value in a data is known as ....**	**p.**	**Standard deviation**
b.	**Which of the following cannot be determined graphically out of mean, mode and median?**	**q.**	**Median**
c.	**An ogive is used to determine ....**	**r.**	**Mean**
d.	**Out of mean, mode, median and standard deviation, which is not a measure of central tendency?**	**s.**	**Mode**

Ans.

	Column I		Column II
a.	The most frequent value in a data is known as	s.	Mode
b.	Which of the following cannot be determined graphically out of mean, mode and median?	r.	Mean
c.	An ogive is used to determine	q.	Median
d.	Out of mean, mode, median and standard deviation, which is not a measure of central tendency?	p.	Standard deviation

26. Look at the cumulative frequency distribution table given below:

Monthly income	Number of families
More than ` 10000	**100**

More than ` 14000	**85**
More than ` 18000	**69**
More than ` 20000	**50**
More than ` 25000	**37**
More than ` 30000	**15**

Number of families having income range ` 20000 to ` 25000 is :

(a) 19

(b) 16

(c) 13

(d) 22

Ans. (c) 13

Explanation :

Number of families having income more than ` 20000 = 50

Number of families having income than ` 25000 = 37

Hence, number of families having income range 20000 to 25000 = 50 – 37 = 13.

Very Short Answer Type Questions

27. Write down the median class of the following frequency distribution:

Class Interval	Frequency
0 – 10	4
10 – 20	4

20 – 30	**8**
30 – 40	**10**
40 – 50	**12**
50 – 60	**8**
60 – 70	**4**

Sol.

Class Interval	Frequency (f_i)	Cumulative Frequency
0 – 10	4	4
10 – 20	4	8
20 – 30	8	16
30 – 40	10	26
40 – 50	12	38
50 – 60	8	46
60 – 70	4	50
	N = Sf_i = 50	

Thus, $\frac{N}{2}$ = 25

The cumulative frequency just above 25 is 26.

Hence, the median class is 30 – 40. **Ans.**

28. From the following probability distribution, find the median class.

Cost of living index	Number of weeks
1400-1550	**8**
1550-1700	**15**
1700-1850	**21**
1850-2000	**8**

Sol.

Cost of living index	No. of weeks (f)	*c.f.*
1400 – 1550	8	8
1550 – 1700	15	23
1700 – 1850	21	44
1850 – 2000	8	52
	$\Sigma f = 52$	

Here, N = 52

$\Rightarrow \frac{N}{2} = \frac{52}{2} = 26$

26 will lie in the class interval 1700 – 1850.

∴ Median class is 1700 – 1850. **Ans.**

29. If empirical relationship between mean, median and mode is expressed as Mean = *k*(3 Median – Mode), then find the value of *k*.

Sol. Given, Mean = *k*(3 Median – Mode)

As we know,

Mode = 3 Median – 2 Mean

∴ Mean = k[3 Median – (3 Median – 2 Mean)]

⇒ Mean = k[3 Median – 3 Median + 2 Mean]

⇒ Mean = 2 k Mean

⇒ 2k Mean – Mean = 0

⇒ Mean [2k – 1] = 0

⇒ 2k – 1 = 0

⇒ 2k = 1

⇒ k = 1/2. **Ans.**

30. The following table shows the weights (in gm) of a sample of 100 potatoes taken from a large consignment.

Weight (in gm)	Frequency
50 – 60	8
60 – 70	10
70 – 80	12
80 – 90	16
90 – 100	18
100 – 110	14
110 – 120	12
120 – 130	10

Calculate the cumulative frequency and determine the median class.

Sol. The cumulative frequency is as shown below:

Weight in (gm)	**Frequency**	**Cumulative Frequency**
50 – 60	8	8
60 – 70	10	18
70 – 80	12	30
80 – 90	16	46
90 – 100	18	64
100 – 110	14	78
110 – 120	12	90
120 – 130	10	100
	N = Sf_i = 100	

Now, N = Σf_i = 100

⇒ $\frac{N}{2}$ = 50

The cumulative frequency just above 50 is 64.

Hence, the median class is 90 – 100. **Ans.**

31. The contents of 100 matchboxes were checked to determine the number of matchsticks they contained.

Matchboxes	**Matchstick**
35	**6**
36	**10**

37	**18**
38	**25**
39	**21**
40	**12**
41	**8**

Calculate the mean of the number of matchsticks per box and determine how many extra matchsticks would have to be added to the total contents of the 100 boxes to bring up the mean to 39?

Sol.

Class Interval (x_i)	**Frequency (f_i)**	**($f_i \times x_i$)**
35	6	210
36	10	360
37	18	666
38	25	950
39	21	819
40	12	480
41	8	328
	Sf_i = 100	S($f_i \times x_i$) = 3813

Thus, Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$ = $\frac{3813}{100}$ = 38.13

Now, to make the mean = 39, $\Sigma(f_i \times x_i)$ should be 3900.

So, the number of matchsticks to be added

= 3900 – 3813

= 87. **Ans.**

32. The table below shows the distribution of marks obtained by students in an examination. Calculate the median marks.

Marks less than	Number of Students
10	5
20	10
30	30
40	60
50	105
60	180
70	270
80	355
90	390
100	400

Sol.

Class Interval	Cumulative Frequency	Frequency (f_i)
0 – 10	5	5

10 – 20	10	10 – 5 = 5
20 – 30	30	30 – 10 = 20
30 – 40	60	60 – 30 = 30
40 – 50	105	105 – 60 = 45
50 – 60	180	180 – 105 = 75
60 – 70	270	270 – 180 = 90
70 – 80	355	355 – 270 = 85
80 – 90	390	390 – 355 = 35
90 – 100	400	400 – 390 = 10
		N = Sf_i = 400

Now, N = Σf_i = 400

⇒ $\frac{N}{2}$ = 200

The cumulative frequency just above 200 is 270.

Hence, the median class is 60 – 70.

So, l = 60, h = 10, f = 90, F = 180 and $\frac{N}{2}$ = 200

Median = $l+\frac{\frac{N}{2}-F}{f}\times h$

= $60+\frac{200-180}{90}\times 10$

= 60 + 2.22

= 62.22 **Ans.**

33. The table below shows the daily profits (in `) of 100 shops. Calculate the mode.

Profit	Number of Shops
0 – 100	**12**
100 – 200	**18**
200 – 300	**27**
300 – 400	**20**
400 – 500	**17**
500 – 600	**6**

Sol.

Profit	Number of Shops
0 – 100	12
100 – 200	18
200 – 300	27
300 – 400	20
400 – 500	17
500 – 600	6
	$N = Sf_i = 100$

Since 200 – 300 has the highest frequency, it is the modal class.

So, $l = 200$, $h = 100$, $f = 27$, $f_1 = 18$, $f_2 = 20$

Thus, Mode = $l+\frac{f-f_1}{2f-f_1-f_2}\times h$

= $200+\frac{27-18}{54-18-20}\times 100$

= $200+\frac{9}{16}\times 100$

= 200 + 56.25

= ` 256.25 **Ans.**

34. The table below shows the distribution of the daily wages (in `) of 160 workers. Calculate the median wage.

Wages	**Number of Workers**
0 – 10	**12**
10 – 20	**20**
20 – 30	**30**
30 – 40	**38**
40 – 50	**24**
50 – 60	**16**
60 – 70	**12**
70 – 80	**8**

Sol.

Class Interval	**Frequency (f_i)**	**Cumulative Frequency**
0 – 10	12	12
10 – 20	20	32

20 – 30	30	62
30 – 40	38	100
40 – 50	24	124
50 – 60	16	140
60 – 70	12	152
70 – 80	8	160
	$N = Sf_i = 160$	

Now, $N = \Sigma f_i = 160$

So, $\frac{N}{2} = 80$

The cumulative frequency just above 80 is 100.

Hence, the median class is 30 – 40.

So, $l = 30$, $h = 10$, $f = 38$, $F = 62$ and $\frac{N}{2} = 80$

Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

= $30 + \frac{80 - 62}{38} \times 10$

= 30 + 4.7

= ` 34.70. **Ans.**

35. The table below shows the distribution of marks obtained by students in an examination. Calculate the value of *x* if the mean mark is 18.

Marks	No. of Students
5	**6**

10	**4**
15	**6**
20	**12**
25	***x***
30	**4**

Sol. Given, Mean = 18

Marks (x_i)	**No. of students (f_i)**	**Mid-value $f_i \times x_i$**
5	6	30
10	4	40
15	6	90
20	12	240
25	x	$25x$
30	4	120
Total	$32 + x$	$520 + 25x$

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

or $\Sigma f_i \times \text{Mean} = \Sigma f_i x_i$

$\Rightarrow (32 + x)18 = 520 + 25x$

$\Rightarrow 520 + 25x = 576 + 18x$

$\Rightarrow 25x - 18x = 576 - 520$

$\Rightarrow x = \frac{56}{7} = 8$ **Ans.**

36. The table gives the frequency distribution of the heights (in cm) of a group of people. Determine the median height.

Height	**People**
150 – 155	**6**
155 – 160	**12**
160 – 165	**18**
165 – 170	**20**
170 – 175	**13**
175 – 180	**8**
180 – 185	**6**

Sol.

Class Interval	**Frequency (f_i)**	**Cumulative Frequency**
150 – 155	6	6
155 – 160	12	18
160 – 165	18	36
165 – 170	20	56
170 – 175	13	69
175 – 180	8	77
180 – 185	6	83

	$N = Sf_i = 83$	

Now $N = \Sigma f_i = 83$

$\Rightarrow \frac{N}{2} = 41.5$

The cumulative frequency just above 41.5 is 56.

Hence, the median class is 165 – 170.

So, $l = 165$, $h = 5$, $f = 20$, $F = 36$ and $\frac{N}{2} = 41.5$

Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$= 165 + \frac{41.5 - 36}{20} \times 5$

= 165 + 1.375

= 166.375 cm **Ans.**

37. The following table provides data about the weekly wages (in `) of workers in a factory. Calculate the mean and the modal Class.

Wages	Number of Workers
50 – 55	5
55 – 60	20
60 – 65	10
65 – 70	10
70 – 75	9
75 – 80	6
80 – 85	12
85 – 90	8

Sol.

Class Interval	**Frequency (f_i)**	**Class Marks**	**($f_i \times x_i$)**
50 – 55	5	52.5	262.5
55 – 60	20	57.5	1150
60 – 65	10	62.5	625
65 – 70	10	67.5	675
70 – 75	9	72.5	652.5
75 – 80	6	77.5	465
80 – 85	12	82.5	990
85 – 90	8	87.5	700
	N = Sf_i = 80		S($f_i \times x_i$) = 5520

Thus Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$ = $\frac{5520}{80}$ = ` 69

Since 55 – 60 has the highest frequency 20, so it is the Modal Class. **Ans.**

38. The marks obtained by 120 students in a Mathematics test are given below:

Marks	**Number of Students**
0 – 10	**5**
10 – 20	**9**

20 – 30	**16**
30 – 40	**22**
40 – 50	**26**
50 – 60	**18**
60 – 70	**11**
70 – 80	**6**
80 – 90	**4**
90 – 100	**3**

Calculate the median.

Sol.

Class Interval	Frequency (f_i)	Cumulative Frequency
0 – 10	5	5
10 – 20	9	14
20 – 30	16	30
30 – 40	22	52
40 – 50	26	78
50 – 60	18	96
60 – 70	11	107
70 – 80	6	113

80 – 90	4	117
90 – 100	3	120
	N = Sf_i = 120	

Now N = Σf_i = 120

So $\frac{N}{2}$ = 60

The cumulative frequency just above 60 is 78.

Hence, the median class is 40 – 50.

So, l = 40, h = 10, f = 26, F = 52 and $\frac{N}{2}$ = 60

Median = $l+\frac{\frac{N}{2}-F}{f}\times h$

= $40+\frac{60-52}{26}\times 10$

= 40 + 3.08

= 43.08 **Ans.**

39. Find the mean of the following frequency distribution:

Class Interval	Frequency
0 – 50	**4**
50 – 100	**8**
100 – 150	**16**
150 – 200	**13**
200 – 250	**6**
250 – 300	**3**

Sol.

Class Interval	Frequency (f_i)	Class Marks (x_i)	($f_i \times x_i$)
0 – 50	4	25	100
50 – 100	8	75	600
100 – 150	16	125	2000
150 – 200	13	175	2275
200 – 250	6	225	1350
250 – 300	3	275	825
	N = Sf_i = 50		S($f_i \times x_i$) = 7150

Thus Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$

= $\frac{7150}{50}$

= 143. **Ans.**

40. Find the mode of the following frequency distribution:

Class Interval	Frequency
0 – 10	**2**
10 – 20	**8**
20 – 30	**10**
30 – 40	**5**
40 – 50	**4**
50 – 60	**3**

Sol.

Class Interval	Frequency (f_i)

0 – 10	2
10 – 20	8
20 – 30	10
30 – 40	5
40 – 50	4
50 – 60	3
	$N = Sf_i = 32$

Since, 20 – 30 has the highest frequency, it is the modal class.

So, $l = 20$, $f = 10$, $f_1 = 8$, $f_2 = 5$, $h = 10$

Thus, Mode = $l + \frac{f - f_1}{2f - f_1 - f_2} \times h$

$= 20 + \frac{10 - 8}{20 - 8 - 5} \times 10$

$= 20 + \frac{2}{7} \times 10$

= 20 + 2.86 = 22.86 **Ans.**

41. Find the mean of the following distribution:

Class Interval	Frequency
20 – 30	**10**
30 – 40	**6**
40 – 50	**8**
50 –60	**12**
60 – 70	**5**
70 – 80	**9**

Sol.

Class Interval	Frqeuency (f_i)	Class Marks (x_i)	($u_i = x_i - A$)	$f_i u_i$
20 – 30	10	25	– 20	– 200
30 – 40	6	35	– 10	– 60
40 – 50	8	45 = A	0	0
50 – 60	12	55	10	120
60 – 70	5	65	20	100
70 – 80	9	75	30	270
	N = Sf_i = 50			

Thus, Mean = $A + \frac{\Sigma f_i u_i}{\Sigma f_i}$

= $45 + \frac{230}{50}$

= 45 + 4.6

= 49.6 **Ans.**

42. Given below is a cumulative frequency distribution table. Corresponding to it, make an ordinary frequency distribution table.

x	***c.f.***
More than equal to 0	**45**
More than equal to 10	**38**
More than equal to 20	**29**
More than equal to 30	**17**
More than equal to 40	**11**
More than equal to 50	**6**

Sol.

C.I.	Frequency
0 – 10	07(45 – 38)
10 – 20	09(38 – 29)
20 – 30	12(29 – 17)
30 – 40	6(17 – 11)
40 – 50	5(11 – 6)
50 – 60	6(6 – 0)

Short Answer Type Questions

43. The marks attained by 40 students in a short assessment is given below where *a* and *b* are missing. If the mean of the distribution is 7.2, find *a* and *b*.

Marks	**5**	**6**	**7**	**8**	**9**
No. of students	**6**	***a***	**16**	**13**	***b***

Sol.

Class Interval	Frequency	($f_i \times x_i$)
5	6	30
6	a	$6a$
7	16	112
8	13	104
9	b	$9b$
	$Sf_i = 35 + a + b$	$S(f_i \times x_i)$

	= 40	= 246 + 6*a* + 9*b*

We know that, Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$

$7.2 = \frac{246+6a+9b}{40}$

$\Rightarrow 246 + 6a + 9b = 40(7.2)$

$\Rightarrow 246 + 6a + 9b = 288$

$\Rightarrow 6a + 9b = 42$

$\Rightarrow 2a + 3b = 14$…(i)

Also $35 + a + b = 40$

$\Rightarrow a + b = 5$

$\Rightarrow 2a + 2b = 10$…(ii)

Subtracting equation (ii) from equation (i),

$b = 4$

and $a = 5 – 4 = 1$. **Ans.**

44. Find the mean of children per family from the data given below :

No. of children	**0**	**1**	**2**	**3**	**4**	**5**
No. of family	**5**	**11**	**25**	**12**	**5**	**2**

Sol.

No. of children (x_i)	**No. of families (f_i)**	**$f_i x_i$**
0	5	0
1	11	11
2	25	50
3	12	36
4	5	20

5	2	10
Total	$Sf_i = 60$	$Sf_ix_i = 127$

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

= $\frac{127}{60}$

= 2.12 (approx.) **Ans.**

45. If the arithmetic mean of the following frequency distribution is 54, determine the value of *p*.

Class	Frequency
0 – 20	**7**
20 – 40	***p***
40 – 60	**10**
60 – 80	**9**
80 – 100	**13**

Sol.

Class Interval	Frequency (f_i)	Class Marks (x_i)	f_ix_i
0 – 20	7	10	70
20 – 40	*p*	30	30*p*
40 – 60	10	50	500
60 – 80	9	70	630
80 – 100	13	90	1170

	Sf_i = 39 + p		S(f_i × x_i)= 2370 + 30p

We know that,

Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$

Þ 54 = $\frac{2370 + 30p}{39 + p}$

Þ 2370 + 30p = 54(39 + p)

Þ 2370 + 30p = 2106 + 54p

Þ 24p = 264

Þ p = 11. **Ans.**

46. If the arithmetic mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50, determine the value of f_1 and f_2.

Class	**Fequency**
0 – 20	**5**
20 – 40	f_1
40 – 60	**10**
60 – 80	f_2
80 – 100	**7**
100 – 120	**8**
Total	**50**

Sol. We have,

5 + f_1 + 10 + f_2 + 7 + 8 = 50

⇒ $30 + f_1 + f_2 = 50$

⇒ $f_1 + f_2 = 20$

⇒ $f_2 = 20 - f_1$

Class Interval	Frequency (f_i)	Class Marks (x_i)	($f_i \times x_i$)
0 – 20	5	10	50
20 – 40	f_1	30	$30f_1$
40 – 60	10	50	500
60 – 80	$f_2 = 20 - f_1$	70	$1400 - 70f_1$
80 – 100	7	90	630
100 – 120	8	110	880
	$Sf_i = 30 + f_1 + f_2 = 50$		$S(f_i \times x_i) = 3460 - 40f_1$

We know that,

Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$

$62.8 = \frac{3460 - 40f_1}{50}$

Þ $3460 - 40f_1 = 50(62.8)$

Þ $3460 - 40f_1 = 3140$

Þ $40f_1 = 320$

Þ $f_1 = 8$

and f_2 = 20 – 8 = 12. **Ans.**

47. Find the mean and median for the following data:

Class	Frequency
0 – 4	**3**
4 – 8	**5**
8 – 12	**9**
12 – 16	**5**
16 – 20	**3**

Sol.

Class	Frequency (f_i)	X_i	f_iX_i	$c.f_i$
0 – 4	3	2	6	3
4 – 8	5	6	30	cf = 8
8 – 12	f = 9	10	90	17
12 – 16	5	14	70	22
16 – 20	3	18	54	25
	Sf_i = 25		Sf_iX_i = 250	

Mean $(\bar{x})$ = $\frac{\Sigma f_i X_i}{\Sigma f_i}$

= $\frac{250}{25}$ = 10

Now, N = $\sum f_i$ = 25

$\frac{N}{2}$ = $\frac{25}{2}$ = 12.5

Cumulative frequency just above 12.5 is 17 which lies in class interval 8 – 12.

∴ Median class is 8 – 12.

Here, $l = 8$, $h = 4$, $cf = 8$, $f = 9$

Median = $l+\frac{\frac{N}{2}-c.f}{f}\times h$

= $8+\frac{12.5-8}{9}\times 4$

= $8+\frac{4.5}{9}\times 4$

= 8 + 2 = 10 **Ans.**

48. Find the mean of the following distribution by assumed mean method:

Class	Frequency
10 – 20	8
20 – 30	7
30 – 40	12
40 – 50	23
50 – 60	11
60 – 70	13
70 – 80	8
80 – 90	6
90 – 100	12

Sol.

Class interval	Frequency (f_i)	X_i	$d_i = X_i - 55$	$f_i d_i$

10 – 20	8	15	– 40	– 320
20 – 30	7	25	– 30	– 210
30 – 40	12	35	– 20	– 240
40 – 50	23	45	– 10	– 230
50 – 60	11	55	0	0
60 – 70	13	65	10	130
70 – 80	8	75	20	160
80 – 90	6	85	30	180
90 – 100	12	95	40	480
	$Sf_i = 100$			$Sf_id_i = -50$

Let A = 55

Mean = $A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

= $55 + \left(\frac{-50}{100}\right)$

= $55 - \frac{5}{10}$

= 55 – 0.5

= 54.5 **Ans.**

49. The average score of boys in the examination of a school is 71 and that of the girls is 73. The average score of the school in the examination is 71.8. Find the ratio of number of boys of the number of girls who appeared in the examination.

Sol. Let the number of boys = n_1

and number of girls = n_2

Average boys score = 71 = $\overline{X}_1$ (Let)

Average girls score = 73 = $\overline{X}_2$ (Let)

Combined mean = $\frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$

71.8 = $\frac{n_1(71) + n_2(73)}{n_1 + n_2}$

$71n_1 + 73n_2 = 71.8n_1 + 71.8n_2$

Þ $71n_1 – 71.8n_1 = 71.8n_2 – 73n_2$

Þ $–0.8n_1 = –1.2n_2$

Þ $\frac{n_1}{n_2} = \frac{1.2}{0.8}$

$= \frac{3}{2}$

Þ $n_1 : n_2 = 3 : 2$

∴ No. of boys : No. of girls = 3 : 2 **Ans.**

50. Following table gives the ages in years of militants operating in a certain area of a country.

Age (in years)	Number of militants
40 – 43	**31**
43 – 46	**58**
46 – 49	**60**
49 – 52	***k***
52 – 54	**27**

If mean of the above distribution is 47.2, find how many militants in the age groups 49-52 are active in the area?

Sol.

Class Interval	Frequency (f_i)	x_i	f_ix_i
40 – 43	31	41.5	1286.5
43 – 46	58	44.5	2581
46 – 49	60	47.5	2850
49 – 52	k	50.5	$50.5k$
52 – 55	27	53.5	1444.5
	$Sf_i = 176 + k$		$Sf_ix_i = 8162+ 50.5k$

Mean $(\bar{x})$ = 47.2[Given]

We know that, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow 47.2 = \frac{8162 + 50.5k}{176 + k}$

$\Rightarrow 47.2\ (176 + k) = 8162 + 50.5k$

$\Rightarrow 8307.2 + 47.2k = 8162 + 50.5k$

$\Rightarrow 8307.2 – 8162 = 50.5k – 47.2k$

$\Rightarrow 145.2 = 3.3k$

$\Rightarrow k = \frac{145.2}{3.3} = 44$

Thus, there are 44 militants operating in the age group 49 – 52. **Ans.**

51. The weekly pocket money (in Rupees) of 50 students is given below. Calculate the mode.

Pocket money	Frequency
40 – 50	**2**

50 – 60	**8**
60 – 70	**12**
70 – 80	**14**
80 – 90	**8**
90 – 100	**6**

Sol.

Class Interval	**Frequency (f_i)**
40 – 50	2
50 – 60	8
60 – 70	12
70 – 80	14
80 – 90	8
90 – 100	6

Since class interval 70 – 80 has the highest frequency, therefore, it is the modal class.

Thus $l = 70$, $h = 10$, $f_1 = 14$, $f_0 = 12$, $f_2 = 8$

We know that,

Mode = $l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$

= $70+\frac{14-12}{28-12-8}\times 10$

= $70+\frac{2}{8}\times 10$

= 70 + 2.5

= ` 72.5 **Ans.**

52. The following table shows the age distribution of cases of a certain disease admitted during a year in a particular hospital:

Class	Frequency
5 – 14	**6**
15 – 24	**11**
25 – 34	**21**
35 – 44	**23**
45 – 54	**14**
55 – 64	**5**

Find the average age for which maximum cases occurred.

Sol.

Age (in years)	No. of cases	
4.5 – 14.5	6	
14.5 – 24.5	11	
24.5 – 34.5	21	
34.5 – 44.5	23	→ Modal Class
44.5 – 54.5	14	
54.5 – 64.5	5	

Here, highest frequency group = 34.5 – 44.5

∴ Modal class = 34.5 – 44.5

Thus, l = 34.5, h = 10, f_1 = 23, f_0 = 21, f_2 = 14

Mode = $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

= $34.5+\left(\frac{23-21}{46-21-14}\right)\times 10$

= 34.5 + $\frac{2}{11}$× 10

= 34·5 + 1.81

= 36.31 **Ans.**

53. For the month of February, a class teacher of Class IX has the following absentee record for 45 students. Find the mean number of days, a student was absent.

Number of days of absent	**Number of students**
0 – 4	**18**
4 – 8	**3**
8 – 12	**6**
12 – 16	**2**
16 – 20	**0**
20 – 24	**1**

Sol.

C.I.	f_i	x_i **(mid-value)**	$d = x_i - A$	$f_i \times d_i$
0 – 4	18	2	– 12	– 216
4 – 8	3	6	– 8	– 24

8 – 12	6	10	– 4	– 24
12 – 16	2	A = 14	0	00
16 – 20	0	18	4	00
20 – 24	1	22	8	08
	Sf_i = 30			S$f_i d_i$ = – 256

Mean = $A+\frac{\Sigma f_i d_i}{\Sigma f_i}$

= $14+\left(\frac{-256}{30}\right)$

= 14 – 8.53

= 5.47 **Ans.**

54. Find the missing frequency (*x*) of the following distribution, if mode is 34.5.

Marks obtained	Name of students
0 – 10	**4**
10 – 20	**8**
20 – 30	**10**
30 – 40	***x***
40 – 50	**8**

Sol.

C.I.	Frequency
0 – 10	4
10 – 20	8

20 – 30	$10 = f_0$
l = 30 – 40	$x = f_1$
40 – 50	$8 = f_2$

We have, mode = 34.5

∴ Modal class = 30 – 40

We know,

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

Þ 34.5 = $30 + \left(\frac{x-10}{2x-10-8}\right) \times 10$

Þ 4.5 = $\left(\frac{x-10}{2x-18}\right) \times 10$

Þ 4.5 = $\left(\frac{x-10}{x-9}\right)5$

Þ 0.9 = $\frac{x-10}{x-9}$

Þ 0.9 (x – 9) = x – 10

Þ 0.9x – 8.1 = x – 10

Þ x – 0.9x = 10 – 8.1

Þ 0.1x = 1.9

Þ x = 19 **Ans.**

55. Calculate the arithmetic mean of the following frequency distribution by the step-deviation method:

Class	**0 – 50**	**50 – 100**	**100 – 150**	**150 – 200**	**200 – 250**	**250 – 300**
Frequency	**17**	**35**	**43**	**40**	**21**	**24**

Sol.

Class Interval	Frequency (f_i)	Mid-Value (x_i)	$u_i = \frac{x_i - A}{h}$; h = [upper limit – lower limit]	$f_i \times u_i$
0 – 50	17	25	– 2	– 34
50 – 100	35	75	– 1	– 35
100 – 150	43	125 = A	0	0
150 – 200	40	175	1	40
200 – 250	21	225	2	42
250 – 300	24	275	3	72
	Sf_i = 180			S($f_i \times u_i$) = 85

Thus A = 125, h = 50, Sf_i = 180, S($f_i \times u_i$) = 85

We know that,

Mean = A + $\left\{h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right\}$ = 125 + $\left\{50 \times \frac{85}{180}\right\}$

= 125 + 23.61

= 148.61. **Ans.**

56. The table below gives the distribution of villages under different heights from sea level in a certain region.

Heights	No. of village
200	**142**

600	**265**
1000	**560**
1400	**271**
1800	**89**
2200	**16**

Compute the mean height of the region.

Sol.

Height (x_i)	**No. of villages**	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
200	142	– 3	– 426
600	265	– 2	– 530
1000	560	– 1	– 560
A = 1400	271	0	0
1800	89	1	89
2200	16	2	32
	$Sf_i = 1343$		$Sf_i u_i = -1395$

Here A = 1400, h = 400

We know that,

Mean = A + $\left\{\frac{\Sigma f_i u_i}{\Sigma f_i}\right\} \times h$

= 1400 + $\frac{-1395}{1343}$ × 400

= 1400 – 415.49

= 984.51. **Ans.**

Long Answer Type Questions

57. The arithmetic mean of the following frequency distribution is 53. Find the value of *k*.

[Board Question]

Class	**Frequency**
0 – 20	**12**
20 – 40	**15**
40 – 60	**32**
60 – 80	***k***
80 – 100	**13**

Sol. Given, median = 53

Class	**Frequency (f_i)**	**Mid-value (x_i)**	**$f_i x_i$**
0 – 20	12	10	120
20 – 40	15	30	450
40 – 60	32	50	1600
60 – 80	k	70	$70k$
80 – 100	13	90	1170
	$72 + k$		$3340 + 70k$

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

53 = $\frac{3340 + 70k}{72 + k}$

⇒ 53 (72 + k) = 3340 + 70k

⇒ 3816 + 53k = 3340 + 70k

⇒ 17k = 476 ⇒ k = 28 **Ans.**

58. The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure of food.

[Board Question]

Daily Expenditure (in `) :	**Number of Households**
100 – 150	**4**
150 – 200	**5**
200 – 250	**12**
250 – 300	**2**
300 – 350	**2**

Sol.

Daily Expendi-ture	**No. of Households (f_i)**	**Mid-value (x_i)**	**$f_i x_i$**
100 – 150	4	125	500

150 – 200	5	175	875
200 – 250	12	225	2700
250 – 300	2	275	550
300 – 350	2	325	650
	$\Sigma f_i = 25$		$\Sigma f_i x_i = 5275$

Mean $(\bar{x})$ = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

= $\frac{5275}{25}$

= 211

Hence, Mean = 211. **Ans.**

59. Find the mean, median and mode of the following data:

[Board Question]

Class	Frequency
0 – 10	**5**
10 – 20	**10**
20 – 30	**18**
30 – 40	**30**
40 – 50	**20**
50 – 60	**12**
60 – 70	**5**

Sol.

Class Interval	Frequ-ency (f_i)	Cumulative Frequency (*c.f.*)	Class Marks (x_i)	($f_i \times x_i$)
0 – 10	5	5	5	25
10 – 20	10	15	15	150
20 – 30	18	33	25	450
30 – 40	30	63	35	1050
40 – 50	20	83	45	900
50 – 60	12	95	55	660
60 – 70	5	100	65	325
	N = Sf_i = 100			S(f_i × x_i) = 3560

(i) Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$

= $\frac{3560}{100}$ = 35.6

(ii) Now, N = Σf_i = 100

So, $\frac{N}{2}$ = 50

The cumulative frequency just above 50 is 63.

Hence, the median class is 30 – 40.

So, l = 30, h = 10, f = 30, F = 33 and $\frac{N}{2}$ = 50

Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$= 30+\frac{50-33}{30}\times 10$

= 30 + 5.67

= 35.67

(iii) We know,

Mode = 3 Median – 2 Mean

⇒ Mode = 3(35.67) – 2(35.6)

= 107.01 – 71.2

= 35.81. **Ans.**

60. Find the mode of following frequency distribution.

Class	Frequency
0 – 10	**8**
10 – 20	**10**
20 – 30	**10**
30 – 40	**16**
40 – 50	**12**
50 – 60	**6**
60 – 70	**7**

Sol. The given frequency distribution table is

Class	Frequency
0 – 10	8
10 – 20	10

20 – 30	10
30 – 40	16
40 – 50	12
50 – 60	6
60 – 70	7

Here, the maximum class frequency is 16.

∴ Modal class = 30 – 40

∴ Lower limit (l) of modal class = 30

Class size (h) = 10

Frequency (f_1) of the modal class = 16

Frequency (f_0) of preceding class = 10

Frequency (f_2) of succeeding class = 12

Mode = $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

= $30+\left(\frac{16-10}{32-10-12}\right)\times 10$

= $30+\frac{6}{32-22}\times 10$

= $30+\frac{6}{10}\times 10$

= 30 + 6

= 36

Hence, Mode = 36. **Ans.**

61. 100 surnames were randomly picked from a local telephone directory and the distribution of the number of letters of the English alphabet in the surnames was obtained as follows:

Letters	Surnames

1 – 4	**6**
4 – 7	**30**
7 – 10	**40**
10 – 13	**16**
13 – 16	**4**
16 – 19	**4**

Determine the median and mean number of letters in the surname. Also find the modal size of the surnames.

Sol.

Class Interval	**Frequ-ency (f_i)**	**Cumulative Frequency**	**Class Mark (x_i)**	**(f_i × x_i)**
1 – 4	6	6	2.5	15
4 – 7	30	36	5.5	165
7 – 10	40	76	8.5	340
10 – 13	16	92	11.5	184
13 – 16	4	96	14.5	58
16 – 19	4	100	17.5	70
	N = Sf_i = 100	100		S(f_i × x_i) = 832

(i) Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$

= $\frac{832}{100}$

= 8.32

(ii) Now, N = Σf_i = 100

So, $\frac{N}{2}$ = 50

The cumulative frequency just above 50 is 76.

Hence, the median class is 7 – 10.

So, l = 7, h = 3, f = 40, F = 36 and $\frac{N}{2}$ = 50

Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

= $7 + \frac{50-36}{40} \times 3$

= 7 + 1.05 = 8.05

(iii) Mode = 3 Median – 2 Mean

⇒ Mode = 3(8.05) – 2(8.32)

= 24.15 – 16.64

= 7.51 **Ans.**

62. From the following data, find the median age of 100 residents of a colony who took part in Swachh Bharat Abhiyan:

Age (in yrs.) More than or equal to	No. of residents
0	50
10	46
20	40
30	20

40	**10**
50	**3**

Sol. First convert the given table into C.I. Table.

C.I.	**Frequency**	***c.f.***
0 – 10	4(50 – 46)	4
10 – 20	6(46 – 40)	10
20 – 30	20(40 – 20)	30
30 – 40	10(20 – 10)	40
40 – 50	7(10 – 7)	47
50 – 60	3	50

Here, $\frac{N}{2} = \frac{50}{2} = 25$

Cumulative frequency just above 25 is 30.

Hence, the median class is 20 – 30.

So, $l = 20$, $h = 10$, $f = 20$, $F = 10$, $\frac{N}{2} = 25$

Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$= 20 + \frac{25 - 10}{20} \times 10$

$= 20 + \frac{15}{2}$

= 27.5 **Ans.**

63. The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.

Marks	No. of Students
50 – 60	**4**
60 – 70	**8**
70 – 80	**14**
80 – 90	**19**
90 – 100	**5**

Sol.

Class Interval	Frequ-ency (f_i)	Cumulative Frequency	Class Mark (x_i)	($f_i \times x_i$)
50 – 60	4	4	55	220
60 – 70	8	12	65	520
70 – 80	14	26	75	1050
80 – 90	19	45	85	1615
90 – 100	5	50	95	475
	$N = Sf_i$ = 50			$S(f_i \times x_i)$ = 3880

(i) Mean = $\frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$

= $\frac{3880}{50}$

= 77.6 marks

» 78 marks

(ii) Now, N = Σf_i = 50

So, $\frac{N}{2}$ = 25

The cumulative frequency just above 25 is 26.

Hence, the median class is 70 – 80.

So l = 70, h = 10, f = 14, F = 12 and $\frac{N}{2}$ = 25

Median = $l + \frac{\frac{N}{2} - F}{f} \times 10$

= 70 + $\frac{25-12}{14}$ × 10

= 70 + 9.29

= 79.29 marks

» 79 marks.

(iii) Since the highest frequency is 19, so the modal class is 80 – 90.

So, l = 80, h = 10, f = 19, f_1 = 14, f_2 = 5

Mode = $l + \frac{f - f_1}{2f - f_1 - f_2} \times h$

= 80 + $\frac{19-14}{38-14-5}$ × 10

= 80 + $\frac{5}{19}$ × 10

= 80 + 2.63

= 82.63 marks » 83 marks. **Ans.**

64. Mode of the following frequency distribution is 65 and sum of all the frequencies is 70. Find the missing frequencies *x* and *y*.

Class Interval	Frequency

0 – 20	**8**
20 – 40	**11**
40 – 60	***x***
60 – 80	**12**
80 – 100	***y***
100 – 120	**9**
120 – 140	**9**
140 – 160	**5**

Sol.

Class Interval	Frequency
0 – 20	8
20 – 40	11
40 – 60	$x(f_0)$
60 – 80	$12(f_1)$
80 – 100	$y(f_2)$
100 – 120	9
120 – 140	9
140 – 160	5
	$Sf = 70$

Here, $8 + 11 + x + 12 + y + 9 + 9 + 5 = 70$[Given]

Þ 54 + x + y = 70

Þ x + y = 70 – 54 = 16...(i)

Mode = 65[Given]

∴ Modal Class is 60 – 80

So, l = 60, h = 20, f_0 = x, f_1 = 12, f_2 = y

Mode = $l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$

∴ 65 = $60+\frac{12-x}{2(12)-x-y}\times 20$

Þ 65 – 60 = $\frac{12-x}{24-(x+y)}$ × 20

Þ 5 = $\frac{12-x}{24-16}\times 20$

[From equation (i)]

Þ 5 = $\frac{12-x}{8}\times 20$

⇒ $5\times\frac{2}{5}$ = 12 – x

Þ 2 = 12 – x

∴ x = 12 – 2

= 10

x + y = 16[From equation (i)]

Þ 10 + y = 16

⇒ y = 16 – 10 = 6

∴ x = 10, y = 6 **Ans.**

65. Find the Arithmetic mean for the following frequency distribution:

Class	Frequency
25 – 29	14

30 – 34	**22**
35 – 39	**16**
40 – 44	**6**
45 – 49	**5**
50 – 54	**3**
55 – 59	**4**

Sol. As this series is an inclusive one, we can make it exclusive by adding 0.5 to the upper limit and subtracting 0.5 from the lower limit of each class interval. Thus, we have

Class Interval	**Frequ-ency (f_i)**	**Mid Value (x_i)**	**$u_i = \frac{x_i - A}{h}$; h = upper limit – lower limit**	**$f_i \times u_i$**
24.5-29.5	14	27	– 3	– 42
29.5-34.5	22	32	– 2	– 44
34.5-39.5	16	37	– 1	– 16
39.5-44.5	6	42 = A	0	0
44.5-49.5	5	47	1	5

49.5-54.5	3	52	2	6
54.5-59.5	4	57	3	12
	Sf_i = 70			S(f_i × u_i) = – 79

Thus, A = 42, h = 5, Sf_i = 70, S(f_i × u_i) = – 79

Mean = A + $\left\{h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right\}$

= 42 + $\left\{5 \times \frac{(-79)}{70}\right\}$

= 42 – 5.64

= 36.36. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

66. Each question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer : use the following code :

Assertion (A)

Consider the following distribution :

Class interval	Frequency
3 – 6	2
6 – 9	5
9 – 12	21
12 – 15	23
15 – 18	10
18 – 21	12

The mode of the above is 12.4.

Reason (R)

The value of the variable which occurs most often is the mode.

Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

Maximum frequency = 23

Hence, modal class is 12 – 15

Now, mode = $l + \frac{(f_k - f_{k-1})}{2f_k - f_{k-1} - f_{k+1}} \times h$

= $12 + \left(\frac{23-21}{46-21-10}\right) \times 3$

= $12 + \frac{6}{46-31}$

= $12 + \frac{6}{15}$

= 12.4

Case Based Questions

67. Mr. Madhu Sudhan is a Maths teacher who is working in Pearl Public School in Banglore. In class X, total 80 students are there. He decided to teach them as per their capabilities. So, he conducted one revision test on the basis of class IX result. The maximum marks were 50. There were 12 students who scored less than 10 marks. Shruthi who got 3 marks was handed over a red card as an intimation to work hard for 1 month and show improvement, as she scored the least in the class. Rishank was presented a badge of honour for scoring the highest in the class. He scored 48 marks and a best performer badge was given to him. Mr. Madhu Sudhan prepared a frequency distribution table for the data of the marks obtained by the students in the revision test as follows:

Marks	Number of Students	*c.f.*
0 – 10	12	12
10 – 20	16	28
20 – 30	21	49
30 – 40	13	62
40 – 50	18	80

(i) The lower limit of modal class of the frequency distribution obtained by Mr. Madhu Sudhan is:

(a) 10

(b) 20

(c) 30

(d) 40

Ans. (b) 20

Explanation :

Highest frequency is 21.

Modal class is 20 – 30.

Hence lower limit is 20.

(ii) The median class of the ditribution is:

(a) 10 – 20

(b) 20 – 30

(c) 30 – 40

(d) 40 – 50

Ans. (b) 20 – 30

Explanation :

N = 80

$\frac{N}{2}$ = 40

The class having cumulative frequency just above 40 is 49. So, 20 – 30 is the median class.

(iii) The mean marks obtained by the students is:

(a) 23.25

(b) 24.25

(c) 26.125

(d) 31.375

Ans. (c) 26.125

Explanation :

Marks	**Numbers of Students (*f*)**	***x***	***f* × *x***
0 – 10	12	5	60

10 – 20	16	15	240
20 – 30	21	25	525
30 – 40	13	35	455
40 – 50	18	45	810
			Σfx = 2090

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2090}{80}$

= 26.125.

(iv) The range of the marks obtained by the students is :

(a) 31

(b) 37.25

(c) 41.25

(d) 45

Ans. (d) 45

Explanation :

Range of marks obtained by the students

= Highest Marks – Lowest marks.

= 48 – 3

= 45.

(v) Mr. Madhu Shudhan formed Section A for those who scored above 40, Section B for those who scored between 30 and 40, Section C between 20 and 30 and Section D for those who scored below 20. How many students were there in Section D?

(a) 12

(b) 16

(c) 28

(d) 49

Ans. (c) 28

Explanation :

Number of students in section D

= Number of students who scored less than 20 marks

= cumulative frequency of the class 10 – 20

= 28.

68. The COVID-19 pandemic, also known as the corona virus pandemic, is an going pandemic of corona disease 2019 (COVID-19) caused by severe acute respiratory syndrome corona virus 2 [SARS - COV-2]. It was first identified in December 2019 in Wuhan, China.

During survey, the ages of 80 patients between 4 and 65, infected by COVID and admitted in the one of the city hospital were recorded and the collected data is represented in the less than cumulative frequency distribution table.

Age (in years)	No. of Patients
Below 15	6
Below 25	17
Below 35	38
Below 45	61
Below 55	75

Below 65	**80**

Based on the above information, answer these question:

(i) The modal class interval is:

(a) 45 – 55

(b) 35 – 45

(c) 25 – 35

(d) 15 – 25

Ans. (b) 35 – 45

Explanation :

Age (in years)	No. of Patients	*c.f.*
5 – 15	6	6
15 – 25	11	17
25 – 35	21	38
35 – 45	23	61
45 – 55	14	75
55 – 65	5	80

Since the highest frequency is 23 which belongs to 35 – 45.

∴ Modal class is 35 – 45.

(ii) The median class interval is:

(a) 45 – 55

(b) 35 – 45

(c) 25 – 35

(d) 15 – 25

Ans. (b) 35 – 45

Explanation :

Here, n = 80, $\frac{n}{2}$ = 40, which is in 35 – 45.

∴ Median class is 35 – 45.

(iii) Which age group was affected the most?

(a) 35 – 45

(b) 25 – 35

(c) 15 – 25

(d) 45 – 55

Ans. (a) 35 – 45

Explanation :

Modal class is 35 – 45.

Therefore, they are affected the most.

(iv) The modal age of the patients admitted in the hospital is:

(a) 38.6 years

(b) 35.8 years

(c) 36.8 years

(d) 38.5 years

Ans. (c) 36.8 years

Explanation :

Here l = 35, f_0 = 21, f_1 = 23, f_2 = 14, h = 10

Mode = $l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$

= $35+\frac{23-21}{46-21-14}\times 10$

$= 35 + \frac{2}{11} \times 10 = 36.8$ years.

(v) How many patients of the age 45 years and above were admitted?

(a) 61

(b) 19

(c) 14

(d) 23

Ans. (b) 19

Explanation :

Number of patients of the age 45 and above =

14 + 5 = 19.

69. A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec)	No. of Student
0 – 20	8
20 – 40	10
40 – 60	13
60 – 80	6
80 – 100	3

(i) The mean time taken by a student to finish the race is:

(a) 54 sec

(b) 63 sec

(c) 43 sec

(d) 50 sec

Ans. (c) 43 sec

Explanation :

Time (in sec)	No. of (*f*) Students	(*x*)	*c.f.*	*fx*
0 – 20	8	10	8	80
20 – 40	10	30	18	300
40 – 60	13	50	31	650
60 – 80	6	70	37	420
80 – 100	3	90	40	270
	$f = 40$			$\Sigma fx = 1720$

Mean = $\frac{1720}{40}$ = 43 sec.

(ii) The upper limit of the modal class is:

(a) 20

(b) 40

(c) 60

(d) 80

Ans. (c) 60

Explanation :

Here, modal class is 40 – 60; its upper limit is 60.

(iii) The construction of cumulative frequency table is useful in determining the:

(a) Mean

(b) Median

(c) Mode

(d) All of these

Ans. (b) Median

Explanation :

Median is determined using cumulative frequency.

(iv) The sum of lower limits of median class and modal class is:

(a) 60

(b) 100

(c) 80

(d) 140

Ans. (c) 80

Explanation :

Median class = 40 – 60

Modal class = 40 – 60

Sum of lower limits of median class and modal class

= 40 + 40

= 80.

(v) How many students finished the race within 1 minute?

(a) 18

(b) 37

(c) 31

(d) 8

Ans. (c) 31

Explanation :

Students finished the race within 1 minute

= Students between 0 – 20 + Student between 20 – 40 + Students between 40 – 60

= 8 + 10 + 13 = 31.

70. A bread manufacturer wants to know the lifetime of the product. For this, he tested the life time of 400 packets of bread. The following tables gives the distribution of the life time of 400 packets.

Lifetime (in hours)	Number of packets (Cumulative frequency)
150 – 200	14
200 – 250	70
250 – 300	130
300 – 350	216
350 – 400	290
400 – 450	352
450 – 500	400

Based on the above information, answer the following questions :

(i) If *m* be the class mark and *b* the upper limit of a class in a continuous frequency distribution, then lower limit of the class is:

(a) $2m + b$

(b) $2m + \sqrt{b}$

(c) $m - b$

(d) $2m - b$

Ans. (d) $2m - b$

Explanation :

We know that,

Class mark = $\frac{\text{Lower limit} + \text{Upper limit}}{2}$

$\Rightarrow m = \frac{\text{Lower limit} + b}{2}$

$\Rightarrow$ Lower limit = $2m - b$

(ii) The average lifetime of a packet is:

(a) 341 hrs

(b) 300 hrs

(c) 340 hrs

(d) 301 hrs

Ans. (a) 341 hrs

Explanation :

Lifetime (in hours)	**Class mark x_i**	f_i	$d_i = x_i - A$	$f_i d_i$
150 – 200	175	14	–150	–2100
200 – 250	225	56	–100	–5600
250 – 300	275	60	–50	–3000
300 – 350	325 = A	86	0	0
350 – 400	375	74	50	3700

400 – 450	425	62	100	6200
450 – 500	475	48	150	7200
Total		400		6400

∴ Average lifetime of a packet

$\bar{x}$ = $A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

= $325 + \frac{6400}{400}$

= 341 hrs.

(iii) The median lifetime of a packet is:

(a) 347 hrs

(b) 310 hrs

(c) 346 hrs

(d) 342 hrs

Ans. (b) 341 hrs

Explanation :

Here, N = 400

⇒ $\frac{N}{2}$ = 200

Also, cumulative frequency for the given distribution are 14, 70, 130, 216, 290, 352, 400.

∴ *c.f.* just greater than 200 is 216, which is corresponding to the interval 300–350.

l = 300, f = 86, *c.f.* = 130, h = 50

Median = $l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$

$= 300+\left(\frac{200-130}{86}\right)\times 50$

= 300 + 40.697

= 340.697

= 341 hrs (approx.)

(iv) If empirical formula is used, then modal lifetime of a packet is:

(a) 340 hrs

(b) 341 hrs

(c) 338 hrs

(d) 349 hrs

Ans. (b) 341 hrs

Explanation :

We know that Mode = 3 Median – 2 Mean

= 3(341) – 2(341)

= 1023 – 682

= 341 hrs.

(v) Manufacturer should claim that the lifetime of a packet is:

(a) 346 hrs

(b) 341 hrs

(c) 340 hrs

(d) 347 hrs

Ans. (b) 341 hrs

Explanation :

Since, mean, median and mode are approximately 341 hrs. So, manufacturer should claim that lifetime of a packet is 341 hrs.

71. As the demand for the products grew, a manufacturing

company decided to hire more employees. For which they want to know the mean time required to complete the work for a worker.

The following table shows the frequency distribution of the time required for each worker to complete a work.

Time (in hours)	**Number of workers**
15 – 19	**10**
20 – 24	**15**
25 – 29	**12**
30 – 34	**8**
35 – 39	**5**

Based on the above information, answer the following questions.

(i) The class mark of the class 25 – 29 is:

(a) 17

(b) 22

(c) 27

(d) 32

Ans. (c) 27

Explanation :

Class mark of class 25 – 29

= $\frac{25+29}{2}=\frac{54}{2}$

= 27.

(ii) If x_i's denotes the class marks and f_i's denotes the corresponding frequencies for the given data, then the value of S$x_i f_i$ equals to:

(a) 1200

(b) 1205

(c) 1260

(d) 1265

Ans. (d) 1265

Explanation :

Class	**Class mark (x_i)**	**Frequency (f_i)**	$x_i f_i$
15 – 19	17	10	170
20 – 24	22	15	330
25 – 29	27	12	324
30 – 34	32	8	256
35 – 39	37	5	185
Total		Sfi = 50	S$x_i f_i$ = 1265

(iii) The mean time required to complete the work for a worker is:

(a) 22 hrs

(b) 23 hrs

(c) 24 hrs

(d) None of these

Ans. (d) None of these

Explanation :

Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1265}{50}$ = 25.3

(iv) If a worker works for 8 hrs in a day, then approximate time required to complete the work for a worker is:

(a) 3 days

(b) 4 days

(c) 5 days

(d) 6 days

Ans. (a) 3 days

Explanation :

If worker works for 8 hours in a days

No. of days = $\frac{25.3}{8}$ = 3.16 ~ 3

Therefore, it is 3 days approx.

(v) The measure of central tendency is:

(a) Mean

(b) Median

(c) Mode

(d) All of these

Ans. (d) All of these

Explanation :

We know the measure of central tendency are mean, median and mode.

72. A group of 71 people visited to a museum on a certain day.

The following table shows their ages.

Ages (in years)	Number of persons

Less than 10	**3**
Less than 20	**10**
Less than 30	**22**
Less than 40	**40**
Less than 50	**54**
Less than 60	**71**

Based on the above information, answer the following questions.

(i) If true class limits have been decided by making the classes of interval 10, then first class must be:

(a) 4 – 15

(b) 0 – 10

(c) 10 – 20

(d) None of these

Ans. (b) 0 – 10

Explanation :

The age of any person is a positive number, so the first class must be 0 – 10.

(ii) The median class for the given data will be:

(a) 20 – 30

(b) 10 – 20

(c) 30 – 40

(d) 40 – 50

Ans. (c) 30 – 40

Explanation :

Let us consider the following table:

Age Class Frequencies Cumulative (in years)	Class interval (x_i)	Frequencies (f_i)	Cumulative frequency *c.f.*
Less than 10	0 – 10	3	3
Less than 20	10 – 20	10 – 3 = 7	10
Less than 30	20 – 30	22 – 10 = 12	22
Less than 40	30 – 40	40 – 22 = 18	40
Less than 50	40 – 50	54 – 40 = 14	54
Less than 60	50 – 60	71 – 54 = 17	71

Here, N = 71

therefore, $\frac{N}{2}$ = 35.5

Now, the class interval whose cumulative frequency is just greater than 35.5 is 30 – 40.

Median class = 30 – 40.

(iii) The cumulative frequency of class preceding the median class is:

(a) 22

(b) 13

(c) 25

(d) 35

Ans. (a) 22

Explanation :

Clearly, the cumulative frequency of the class preceding the median class is 22.

(iv) The median age of the persons visited the musem is:

(a) 30 years

(b) 32.5 years

(c) 34 years

(d) 37.5 years

Ans. (d) 37.5 years

Explanation :

Median = $l+\left(\frac{\frac{N}{2}-cf}{f}\right)\times h$

= $30+\left(\frac{35.5-22}{18}\right)\times 10$

= $30+\left(\frac{13.5}{18}\right)\times 10$

= $30+\frac{135}{18}$

= 30 + 7.5

= 37.5

Thus, the median age of the persons visited the museum is 37.5 years.

(v) If the price of a ticket for the age group 30 – 40 is ` 30, then the total amount spent by this age group is:

(a) ` 360

(b) ` 420

(c) ` 540

(d) ` 340

Ans. (c) ` 540

Explanation :

Number of persons, whose age lying in 30 – 40

= 18.

Total amount spent by people of this group

= ` (30 × 18)

= ` 540

73. Household income in India was drastically impacted due to the COVID–19 lockdown. Most of the companies decided to bring down the salaries of the employees by 50%

The following table shows the salaries (in percent) received by 25 employees during lockdown.

Salaries received (in percent)	Number of employees
50 – 60	9
60 – 70	6
70 – 80	8
80 – 90	2

Based on the above information, answer the following questions.

(i) Total number of persons whose salary is reduced by more than 30% is:

(a) 10

(b) 20

(c) 25

(d) 15

Ans. (d) 15

Explanation :

Required number of persons = 9 + 6 = 15

(ii) Total number of persons whose salary is reduced by almost 40% is:

(a) 15

(b) 10

(c) 16

(d) 8

Ans. (c) 16

Explanation :

Required number of persons = 6 + 8 + 2 = 16

(iii) The modal class is:

(a) 50 – 60

(b) 60 – 70

(c) 70 – 80

(d) 80 – 90

Ans. (a) 50 – 60

Explanation :

50 – 60 is the modal class as the maximum frequency is 9.

(iv) The median class of the given data is:

(a) 50 – 60

(b) 60 – 70

(c) 70 – 80

(d) 80 – 90

Ans. (b) 60 – 70

Explanation :

The cumulative frequency distribution table for the given data can be drawn as:

Salaries received (in percent)	**Number of employees (f_i)**	**Cumulative frequency (*c.f.*)**
50 – 60	9	9
60 – 70	6	9 + 6 = 15
70 – 80	8	15 + 8 = 23
80 – 90	2	23 + 2 = 25
Total	Sf_i = 25	

Here, $\frac{N}{2} = \frac{25}{2}$

= 12.5

The cumulative frequency just greater than 12.5 lies in the interval 60 – 70.

Hence, the median class is 60 – 70.

(v) The empiricial relationship between mean, median and mode is:

(a) 3 Median = Mode + 2 Mean

(b) 3 Median = Mode – 2 Mean

(c) Median = 3 Mode – 2 Mean

(d) Median = 3 Mode + 2 Mean

Ans. (a) 3 Median = Mode + 2 Mean

Explanation :

We know, Mode = 3 Median – 2 Mean

3 Median = Mode + 2 Mean

74. Transport department of a city wants to buy some electric buses for the city. For which they wants to analyse the distance travelled by existing public transport buses in a day.

The following data shows the distance travelled by 60 existing public transport buses in a day.

Daily distance travelled (in km)	Number of buses
200 – 209	4
210 – 219	14
220 – 229	26
230 – 239	10
240 – 249	6

Based on the above information, answer the following questions.

(i) The upper limit of a class and lower limit of its succeeding class is differ by:

(a) 9

(b) 1

(c) 10

(d) None of these

Ans. (b) 1

Explanation :

The upper limit of a class and the lower class of its succeeding class differ by 1.

(ii) The median class is:

(a) 229.5 – 239.5

(b) 230 – 239

(c) 220 – 229

(d) 219.5 – 229.5

Ans. (d) 219.5 – 229.5

Explanation :

Here, class intervals are in inclusive from. So, we first convert them in exclusive form. The frequency distribution table in exclusive form is as follows:

Class interval	**Frequency (f_i)**	**Frequency cumulative *c.f.***
199.5 – 209.5	4	4
209.5 – 219.5	14	18
219.5 – 229.5	26	44
229.5 – 239.5	10	54
239.5 – 249.5	6	60

Here, Sf_i = N = 60

$\frac{N}{2} = \frac{60}{2}$

= 30

Now, the class interval whose cumulative frequency is just greater than 30 is 219.5 – 229.5

Median class is 219.5 – 229.5

(iii) The cumulative frequency of the class preceeding the median class is:

(a) 10

(b) 18

(c) 26

(d) 10

Ans. (b) 18

Explanation :

Clearly, the cumulative frequency of the class preceeding the median class is 18.

(iv) The median of the distance travelled is:

(a) 222 km

(b) 225 km

(c) 223 km

(d) None of the above

Ans. (d) None of the above

Explanation :

Median = $l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$

= $219.5 + \frac{\left(\frac{60}{2} - 18\right)}{26} \times 10$

= 224.12

Median of the distance travelled is 224.12 km.

(v) If the mode of the distance travelled is 223.78 km, then mean of the distance travelled by the bus is:

(a) 225 km

(b) 220 km

(c) 230.29 km

(d) 224.29 km

Ans. (d) 224.29 km

Explanation :

We know, Mode = 3 Median – 2 Mean

Mean = $\frac{(3 \text{ Median} - \text{Mode})}{2}$

= $\frac{(672.36 - 223.78)}{2}$

= 224.29 km.

75. A petrol pump owner wants to analyse the daily need to diesel at the pump. For this he collected the data of vehicles visited in 1hr. The following frequency distribution table shows the classification of the number of vehicles and quantity of diesel filled in them.

Diesel filled (in Litres)	Number of vehicles
3 – 5	5
5 – 7	10
7 – 9	10
9 – 11	7
11 – 13	8

Based on the above data, answer the following questions.

(i) Which of the following is correct?

(a) If x_i and f_i are sufficiently small, then direct method is appropriate choice for calculating mean.

(b) If x_i and f_i are sufficiently large, then direct method is appropriate choice for calculating mean.

(c) If x_i and f_i are sufficiently small, then assumed mean method is appropriate choice for calculating mean.

(d) None of the above.

Ans. (a) If x_i and f_i are sufficiently small, then direct method is appropriate choice for calculating mean.

Explanation :

If f_i and x_i are very small, then direct method is appropriate method for calculating mean.

(ii) Average diesel required for a vehicle is:

(a) 8.15 litres

(b) 6 litre

(c) 7 litres

(d) 5.5 litres

Ans. (a) 8.15 litres

Explanation :

The frequency distribution table from the given data can be drawn as :

Class	**Class mark (x_i)**	**Frequency (f_i)**	**f_ix_i**
3 – 5	4	5	20
5 – 7	6	10	60

7 – 9	8	10	80
9 – 11	10	7	70
11 – 13	12	8	96
Total		**40**	**326**

Median or Average = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

= $\frac{326}{40}$

= 8.15 litres.

(iii) If approximately 2000 vehicles comes daily at the petrol pump, then how much litres of diesel the pump should have?

(a) 16200 litres

(b) 16300 litres

(c) 10600 litres

(d) 15000 litres

Ans. (b) 16300 litres

Explanation :

If 2000 vehicles comes daily and average quantity of diesel required for a vehicle is 8.15 litres, then total quantity of diesel required,

= 2000 × 8.15 = 16300 litres

(iv) The sum of upper and lower limit of median class is:

(a) 22

(b) 10

(c) 16

(d) None of these

Ans. (c) 16

Explanation :

Here, N = 40 therefore $\frac{N}{2}$ = 20

c.f. for the distribution are 5, 15, 25, 32, 40

Now, *c.f.* just greater than 20 is 25 which is corresponding to the class interval 7 – 9.

So median class is 7 – 9.

Required sum of upper limit and lower limit

= 7 + 9

= 16

(v) If the median of given data is 8 litres, then mode will be equal to:

(a) 7.5 litres

(b) 7.7 litres

(c) 5.7 litres

(d) 8 litres

Ans. (b) 7.7 litres

Explanation :

We know, Mode = 3 Median – 2 Mean

= 3 × 8 – 2 × 8.15

= 24 – 16.3 = 7.7

76. An inspector in an enforcement squad of electricity department visit to a locality of 100 families and record their monthly consumption of electricity, on the basis of family members, electronic items in the house and wastage of electricity, which is summarised in the following table:

Monthly consumption (in kwh)	**Number of families**

0 – 100	**2**
100 – 200	**5**
200 – 300	***x***
300 – 400	**12**
400 – 500	**17**
500 – 600	**20**
600 – 700	***y***
700 – 800	**9**
800 – 900	**7**
900 – 1000	**4**

Based on the above, data, answer the following questions.

(i) The value of $x + y$ is:

(a) 100

(b) 42

(c) 24

(d) 200

Ans. (c) 24

Explanation :

Here, it is given that total frequency = 100

$76 + x + y = 100$

Þ $x + y = 24$

(ii) If the median of the above data is 525, then x is equal to:

(a) 10

(b) 8

(c) 9

(d) None of these

Ans. (c) 9

Explanation :

Here, $\frac{N}{2} = \frac{100}{2} = 50$

Also, median = 525

Median class is 500 – 600.

Now, Median = $l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$

Þ 525 = $500 + \left(\frac{50 - 36 - x}{20}\right) \times 100$

Þ 25 = 5(14 – x)

Þ x = 9

(iii) What will be the upper limit of the modal class?

(a) 400

(b) 600

(c) 650

(d) 700

Ans. (b) 600

Explanation :

Since, maximum frequency is 20, so modal class is 500 – 600.

Hence, upper limit of modal class is 600.

(iv) The average monthly consumption of a family of this locality is approximately?

(a) 540 kwh

(b) 522 kwh

(c) 540 kwh

(d) None of these

Ans. (b) 522 kwh

Explanation :

Since, $x + y = 24$

Þ $y = 24 – 9$

= 15

Class Interval	**Class Mark (x_i)**	**Frequency (f_i)**	**$x_i f_i$**
0 – 100	50	2	100
100 – 200	150	5	750
200 – 300	250	9	2250
300 – 400	350	12	4200
400 – 500	450	17	7650
500 – 600	550	20	11000
600 – 700	650	15	9750
700 – 800	750	9	6750
800 – 900	850	7	5950
900 – 1000	950	4	3800
Total		100	52200

Required average consumption.

$= \frac{52200}{100}$

= 522

(v) If A be the assumed mean, then A is always.

(a) > (Actual mean)

(b) < (Actual Mean)

(c) (Actual Mean)

(d) Can‘t say

Ans. (d) Can‘t say

Explanation :

Since exact value of A is not known, thus assumed mean can’t be determined

Self-Assessment

77. A survey was conducted by a group of students as a part of their environmental awareness programme, in which they collected the following data regarding the number of plants in 20 houses of a locality. Find the mean number of plants per house.

[NCERT]

Number of Plants	Number of Houses
0 – 2	1
2 – 4	2
4 – 6	1
6 – 8	5

8 – 10	6
10 – 12	2
12 – 14	3

Ans. 8 plants.

78. Thirty women were examined in a hospital by a doctor and the number of heart-beats per minute were recorded and summarised as follows. Find the mean heart-beats per minute.

[NCERT]

Heart-beats per minute	Number of Women
65 – 68	2
68 – 71	4
71 – 74	3
74 – 77	8
77 – 80	7
80 – 83	4
83 – 86	2

Ans. 75.9 heart-beats per minute.

79. The following distribution shows the daily pocket allowance given to the children of a multi-storey building. The average pocket allowance is ` 18. Find out the missing frequency.

[NCERT]

Class Interval	Frequency

11 – 13	7
13 – 15	6
15 – 17	9
17 – 19	13
19 –21	*f*
21 – 23	5
23 – 25	4

Ans. 20.

80. In the retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was their distribution according to the number of boxes.

No. of Mangoes	No. of Boxes
50 – 52	15
53 – 55	110
56 – 58	135
59 – 61	115
62 – 64	25

Find the mean number of mangoes kept in a packing box.

[NCERT]

Ans. 57.

81. The table below shows the daily expenditure on food of 25 households in a locality. Find the mean expenditure.

[NCERT]

Expenditure	Number of Households
100 – 150	4
150 – 200	5
200 – 250	12
250 – 300	2
300 – 350	2

Ans. ` 211.

82. A class teacher has the following absentee record of 40 students for the whole term. Find the mean number of days a student remained absent.

[NCERT]

Number of Days	Number of Students
0 – 6	11
6 – 10	10
10 – 14	7
14 – 20	4
20 – 28	4
28 – 38	3

38 – 40	1

Ans. 12.475 days.

83. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

[NCERT]

Literacy %	Cities
45 – 55	3
55 – 65	10
65 – 75	11
75 – 85	8
85 – 95	3

Ans. 69.43%.

84. The following is the distribution of the marks obtained by 74 students in a class test. Find the mean of the distribution.

Marks	Students
4 – 6	5
6 – 12	6
12 – 15	15
15 – 27	12
27 – 32	15
32 – 40	21

Ans. 19.77 ≈ 20.

85. Find the mean of the following distribution :

Class	Frequency
24 – 36	15
36 – 42	15
42 – 47	51
47 – 52	21
52 – 70	18

Ans. 45.35

86. If the median of the distribution given below is 28.5 and $\Sigma f_i = 60$, find the value of x and y.

[NCERT]

Class Interval	Frequency
0 – 10	5
10 – 20	x
20 – 30	20
30 – 40	15
40 – 50	y
50 – 60	5

Ans. $x = 8$, $y = 7$.

87. A survey was conducted related to the heights (in cm) of 51 girls of class X and the following data was obtained :

Heights	Number of Girls

135 – 140	4
140 – 145	7
145 – 150	18
150 – 155	11
155 – 160	6
160 – 165	5

Determine the median height.

Ans. 149.03 cm.

88. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons in the age group of 18 to 60 years.

[NCERT]

Age (in years)	Number of Policy-holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98

Below 60	100

Ans. 35.76 years.

89. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre and the data obtained is represented in the following table. Find the median length of the leaves.

[NCERT]

Length (in mm)	Number of Leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Ans. 146.75 mm.

90. The following table gives the distribution of the life time (in hours) of 400 neon lamps. Find the median life time of a lamp.

[NCERT]

Life time (in hours)	Number of Lamps
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	86

3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Ans. 3406.98 hours.

91. The following distribution gives the weight (in kg) of 30 students in a class. Find the median weight of the students.

[NCERT]

Weight (in kg)	Number of Students
40 – 45	2
45 – 50	3
50 – 55	8
55 – 60	6
60 – 65	6
65 – 70	3
70 – 75	2

Ans. 56.67 kg.

92. The following data gives the distribution of the total household expenditure (in rupees) of manual workers in a city:

Expenditure (in Rupees)	Frequency
1000 – 1500	24
1500 – 2000	40

2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

Find the mode of the average expenditure of the families.

[NCERT]

Ans. ` 1847.83.

93. The following table shows the ages of the patients admitted to a hospital during a year.

Ages (in years)	Number of Patients
5 – 15	6
15 – 25	11
25 – 35	21
35 – 45	23
45 – 55	14
55 – 65	5

Find the mean and mode of the data given above.

[NCERT]

Ans. Mean = 35.375, Mode = 36.8

94. The following distribution provides information about the observed lifetime (in hours) of 225 electrical components. Determine the modal lifetime of the components.

[NCERT]

Lifetime (in years)	Number of Components
0 – 20	10
20 – 40	35
40 – 60	52
60 – 80	61
80 – 100	38
100 – 120	29

Ans. 65.625 hours.

95. The following distribution gives the state-wise teacher-student ratio in higher-secondary schools of India. Find the mean and mode from the data.

[NCERT]

Number of Students/Teachers	Number of States/U.T.s
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0

45 – 50	0
50 – 55	2

Ans. Mean = 29.2, Mode = 30.6

96. The following data shows the number of runs scored in ODI cricket by some of the top batsmen of the world:

Runs Scored	Number of Batsmen
3000 – 4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	7
7000 – 8000	6
8000 – 9000	3
9000 – 10000	1
10000 – 11000	1

Find the mode of the data.

[NCERT]

Ans. 4608.7 runs.

97. A student noted the number of cars passing through a spot on a road for 100 periods, each of 3 minutes and summarized it in the table given below. Find the mode.

[NCERT]

Number of Cars	Frequency

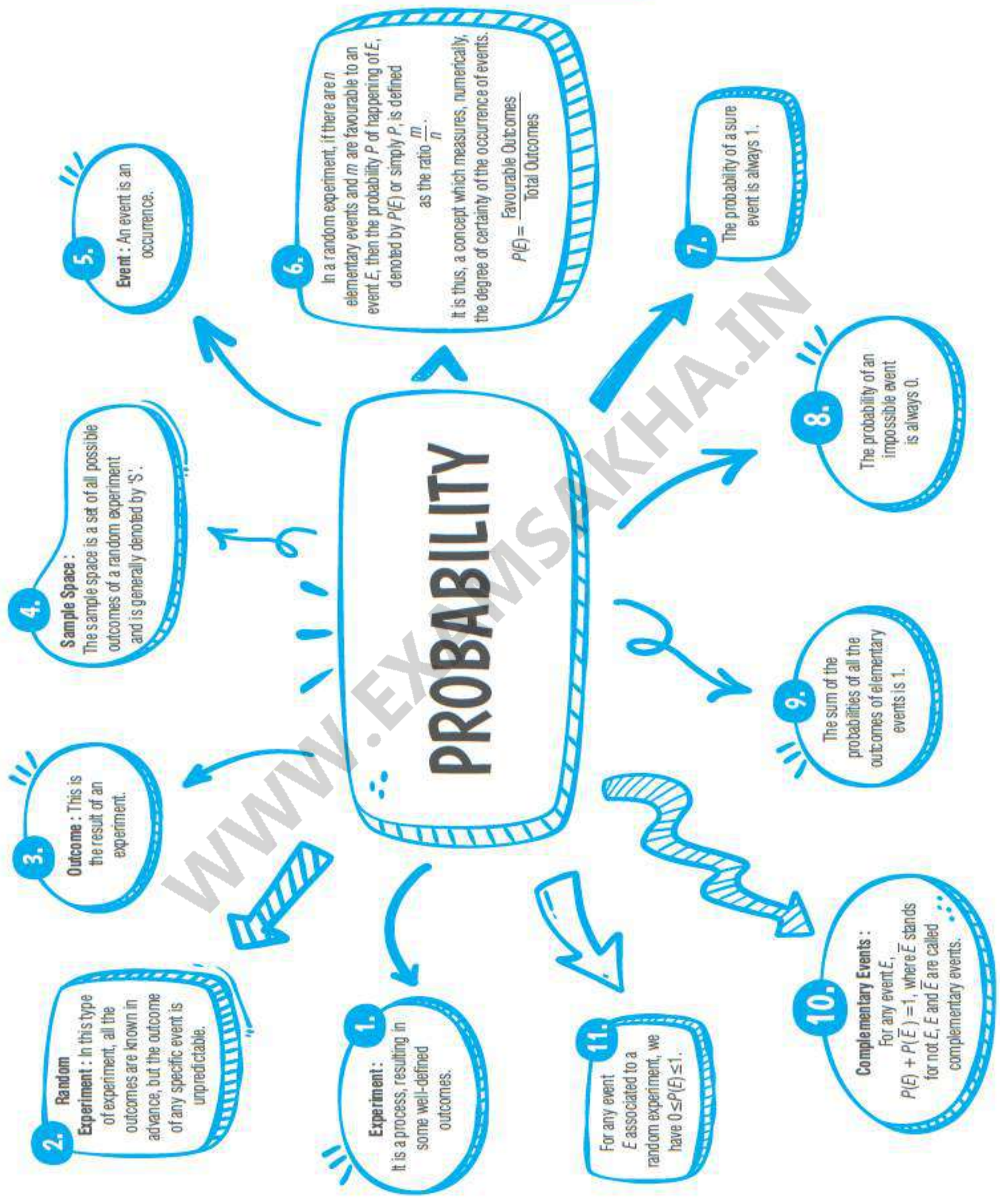
0 – 10	7
10 – 20	14
20 – 30	13
30 – 40	12
40 – 50	20
50 – 60	11
60 – 70	15
70 – 80	8

Ans. Mode = 44.71 cars ≈ 45 cars.

Probability

Chapter
15

Basic Concepts



Multiple Choice Questions

1. **In a family of three children, the probability of having at least one boy is:**

(a) $\frac{7}{8}$

(b) $\frac{1}{8}$

(c) $\frac{5}{8}$

(d) $\frac{3}{4}$

Ans. (d) $\frac{3}{4}$

Explanation :

Total number of children = 3

All possible outcomes = GGG, GGB, GBB, BBB.

Let the event of getting at least one boy be E.

Then, the favourable outcomes are

GGB, GBB, BBB.

Hence, number of favourable outcomes = 3

$\therefore$ P(E) = $\frac{3}{4}$ ·

2. **The probability that a number selected at random from the numbers 1, 2, 3, …, 15 is a multiple of 4 is:**

(a) $\frac{4}{15}$

(b) $\frac{2}{15}$

(c) $\frac{1}{5}$

(d) $\frac{1}{3}$

Ans. (c) $\frac{1}{5}$

Explanation :

Total number = 15

Let the event of getting a number multiple of 4 be E.

Then, the favourable outcomes are 4, 8, 12

Hence, number of favourable outcomes = 3

∴ P(E) = $\frac{3}{15}$ = $\frac{1}{5}$

3. **A bag contains cards numbered 1 to 25. A card is drawn at random from the bag. The probability that the number on the card is divisible by both 2 and 3 is:**

(a) $\frac{1}{5}$

(b) $\frac{3}{25}$

(c) $\frac{4}{25}$

(d) $\frac{2}{25}$

Ans. (c) $\frac{4}{25}$

Explanation :

Total number = 25

Let the event of getting a number divisible by 2 and 3 be E.

Then, the favourable outcomes are 6, 12, 18, 24
Hence, number of favourable outcomes = 4
∴ P(E) = $\frac{4}{25}$

4. **The probability of getting an even number when a die is thrown once is:**

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{6}$

(d) $\frac{5}{6}$

Ans. (a) $\frac{1}{2}$

Explanation :

Total number of possible outcomes
= 1, 2, 3, 4, 5, 6
Thus, the favourable outcomes are1, 3, 5
Hence, the total number of favourable outcomes
= 3
∴ P(E) = $\frac{3}{6}$ = $\frac{1}{2}$

5. **A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a prime number less than 23 is:**

(a) $\frac{7}{90}$

(b) $\frac{10}{90}$

(c) $\frac{4}{45}$

(d) $\frac{9}{89}$

Ans. (c) $\frac{4}{45}$

Explanation :

Total number of possible outcomes = 90

Thus, the favourable outcomes are 2, 3, 5, 7, 11, 13, 17, 19

Hence, the total number of favourable outcomes

= 8

∴ P(E) = $\frac{8}{90}$ = $\frac{4}{45}$

6. **If P(A) denotes the probability of an event A, then:**

(a) P(A) < 0

(b) P(A) > 1

(c) 0 ≤ P(A) ≤ 1

(d) – 1 ≤ P(A) ≤ 1

Ans. (c) 0 ≤ P(A) ≤ 1

Explanation :

The probability of a sure event is always 1.

The probability of an impossible event is always 0.
The sum of the probabilities of all the outcomes of elementary events is 1.
Thus, P(A) lies between 0 and 1, and could be 0 or 1 also.

7. **One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number that is a multiple of 7 is:**

(a) $\frac{1}{7}$

(b) $\frac{1}{8}$

(c) $\frac{1}{5}$

(d) $\frac{7}{40}$

Ans. (b) $\frac{1}{8}$

Explanation :

The total number of possible outcomes = 40

The favourable outcomes are 7, 14, 21, 28, 35

Hence, the number of favourable outcomes = 5

Thus, P(E) = $\frac{5}{40}$ = $\frac{1}{8}$

8. **Two dice are thrown together. The probability of getting the same number on both the dice is:**

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{6}$

(d) $\frac{1}{12}$

Ans. (c) $\frac{1}{6}$

Explanation :

Total number of possible outcomes = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 36

Thus, the favourable outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6).
Hence, the total number of favourable outcomes
= 6
∴ P(E) = $\frac{6}{36}$ = $\frac{1}{6}$

9. **Cards bearing numbers 2, 3, 4, …, 11 are kept in a bag. A card is drawn at random from it. The probability of getting a card with a prime number is:**

(a) $\frac{1}{2}$

(b) $\frac{2}{5}$

(c) $\frac{3}{10}$

(d) $\frac{5}{9}$

Ans. (a) $\frac{1}{2}$

Explanation :

Total number of possible outcomes = 10

Thus, the favourable outcomes are 2, 3, 5, 7, 11

Hence, the total number of favourable outcomes

= 5

∴ P(E) = $\frac{5}{10}$ = $\frac{1}{2}$

10. **The probability of throwing a number greater than 2 with a single die is:**

(a) $\frac{2}{3}$

(b) $\frac{5}{6}$

(c) $\frac{1}{3}$

(d) $\frac{2}{5}$

Ans. (a) $\frac{2}{3}$

Explanation :

Total number of possible outcomes = 6

Thus, the favourable outcomes = 3, 4, 5, 6

Hence, the total number of favourable outcomes

= 4

∴ P(E) = $\frac{4}{6}$ = $\frac{2}{3}$

11. **A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the card will not be an ace is:**

(a) $\frac{1}{13}$

(b) $\frac{1}{4}$

(c) $\frac{12}{13}$

(d) $\frac{3}{4}$

Ans. (c) $\frac{12}{13}$

Explanation :

Total number of possible outcomes = 52

Number of aces in the pack = 4

Thus, the probability of not drawing an ace

∴ P(E) = $\frac{52-4}{52}$ = $\frac{48}{52}$ = $\frac{12}{13}$

12. **Which of the following cannot be the probability of an event:**

(a) 1.5

(b) $\frac{3}{5}$

(c) 1/4

(d) 0.3

Ans. (a) 1.5

Explanation :

The sum of the probabilities of all the outcomes of elementary events is 1.

Thus, P(A) cannot be less than 0 and more than 1.

13. **A die thrown once. What is the probability of getting a number less than 3?**

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{6}$

(d) $\frac{1}{4}$

Ans. (b) $\frac{1}{3}$

Explanation :

Possible outcomes are 1, 2, 3, 4, 5, 6

Number of possible outcomes = 6

The favourable outcomes are 1, 2

Number of the favourable outcomes = 2

∴ P(E) = $\frac{2}{6}$ = $\frac{1}{3}$

14. **Cards bearing numbers 3 to 20 are placed in a bag and mixed thoroughly. A card is taken out of the bag at random. What is the probability that the number on the card taken out is an even number?**

(a) $\frac{9}{17}$

(b) $\frac{1}{2}$

(c) $\frac{5}{9}$

(d) $\frac{7}{18}$

Ans. (b) $\frac{1}{2}$

Explanation :

Total number of possible outcomes = 18

Then, favourable outcomes are 4, 6, 8, 10, 12, 14, 16, 18, 20

Number of favourable outcomes = 9

∴ P(E) = $\frac{9}{18}$ = $\frac{1}{2}$

15. **A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. What is the probability of getting a black ball?**

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{1}{10}$

(d) None of these

Ans. (b) $\frac{3}{5}$

Explanation :

Total number of possible outcomes = 10

Total number of favourable outcomes = 6

∴ P(E) = $\frac{6}{10}$ = $\frac{3}{5}$

16. **A card is drawn out from a well-shuffled deck of 52 cards. What is the probability of getting a black king?**

(a) $\frac{1}{13}$

(b) $\frac{1}{26}$

(c) $\frac{1}{28}$

(d) None of these

Ans. (b) $\frac{1}{26}$

Explanation :

Total number of possible outcomes = 52

The total number of favourable outcomes = 2

∴ P(E) = $\frac{2}{52}$ = $\frac{1}{26}$

17. **Two friends were born in the year 2000. What is the probability that they have the same birthday?**

(a) $\frac{1}{365}$

(b) $\frac{1}{366}$

(c) $\frac{2}{365}$

(d) $\frac{1}{183}$

Ans. (b) $\frac{1}{366}$

Explanation :

Since, year 2000 is a leap year.

Total possible cases = 366

Favourable case = 1

∴ P(E) = $\frac{1}{366}$

18. **What is the probability that two friends have different birthdays?**

(a) $\frac{1}{365}$

(b) $\frac{2}{365}$

(c) $\frac{364}{365}$

(d) $\frac{363}{365}$

Ans. (c) $\frac{364}{365}$

Explanation :

Total number of possible outcomes

= 365 × 365

Total number of favourable outcomes

= 365 × (365 – 1) = 365 × 364

∴ P(E) = $\frac{365 \times 364}{365 \times 365}$

= $\frac{364}{365}$

19. **A die is thrown once. The probability of getting a prime number is:**

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$

Ans. (c) $\frac{1}{2}$

Explanation :

The maximum number of possible outcomes = 6

The favourable outcomes = 2, 3, 5

Total number of the favoruable outcomes = 3

∴ P(E) = $\frac{3}{6}$ = $\frac{1}{2}$

20. What is the probability of an impossible event?

(a) $\frac{1}{2}$

(b) 1

(c) 0

(d) More than 1

Ans. (c) 0

Explanation :

The probability of an impossible event is always zero.

21. The probability of getting a number four or more in throwing a die is:

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

Ans. (c) $\frac{1}{2}$

Explanation :

Total No. of possible outcomes = 6

Thus, the favourable outcomes are 4, 5, 6

Required probability $= \frac{3}{6} = \frac{1}{2}$.

22. The probability of an event that is certain to happen is:

(a) – 1

(b) 1

(c) 0

(d) None of the above

Ans. (b) 1

Explanation :

The probability of a sure event is always 1.

23. A number is selected at random from first 50 natural number, then the probability of getting a perfect square is ....

(a) $\frac{7}{50}$

(b) $\frac{1}{50}$

(c) $\frac{9}{50}$

(d) $\frac{11}{50}$

Ans. (a) $\frac{7}{50}$

Explanation :

1, 4, 9, 16, 25, 36, 49

Let the event of getting a perfect square be E.

Then the no. of favourable outcomes = 7

P(E) = $\frac{7}{50}$

24. The sum of the probability of all the elementary events of an experiment is ....

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) 0

(d) 1

Ans. (d) 1

Explanation :

The sum of the probabilities of all the outcomes of elementary event is one.

Very Short Answer Type Questions

25. If two dice are rolled together, find the probability of getting an even number on both the dice.

Sol. Total number of possible outcomes =
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 36

Let E be the event of getting an even number on both the dice.

Then, the favourable outcomes are (2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6).

Hence, the total number of favourable outcomes = 9

P(E) = $\frac{9}{36} = \frac{1}{4}$. **Ans.**

26. When a number is selected at random from numbers 1 to 30, what is the probability that it is a prime number?

Sol. Total numbers = 30

Let the event of getting prime number be E.

Then the favourable outcomes are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Hence number of favourable outcomes = 10

P(E) = $\frac{10}{30} = \frac{1}{3}$. **Ans.**

27. Two different coins are tossed simultaneously. Find the probability of getting at least one head.

Sol. Total number of possible outcomes

= HH, HT, TH, TT = 4

Let E be the event of getting at least one head.

Then, the favourable outcomes are HH, HT, TH

Hence, the total number of favourable outcomes

= 3

P(E) = $\frac{3}{4}$. **Ans.**

28. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

Sol. Total number of all possible outcomes = 26

Number of consonants = 21

Let E be the event of getting a consonant

P (getting a consonant) = P(E) = $\frac{21}{26}$. **Ans.**

29. A card is drawn from a well-shuffled deck of 52 playing cards. Then what is the probability that the card will not be a diamond?

Sol. Total number of possible outcomes = 52

Number of diamonds in the pack = 13

Thus, the probability of not drawing a diamond

$= \frac{52-13}{52} = \frac{39}{52} = \frac{3}{4}$ **Ans.**

30. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap?

Sol. Total apples = 900

P(E) = 0.18

$\Rightarrow \frac{\text{No. of rotten apples}}{\text{Total no. of apples}} = 0.18$

$\Rightarrow \frac{\text{No. of rotten apples}}{900} = 0.18$

⇒ No. of rotten apples = 900 × 0.18

= 162 **Ans.**

31. A dice is thrown once. What is the probability of getting a number less than 4?

Sol. The possible outcomes are 1, 2, 3, 4, 5, 6.

Total number of possible outcomes = 6

Let E be the event of getting a number less than 4.

Then, favourable outcomes are 1, 2, 3

Hence, total number of favourable outcomes = 3

P(E) = $\frac{3}{6} = \frac{1}{2}$ **Ans.**

32. Cards bearing numbers 2 to 21 are placed in a bag and mixed thoroughly. A card is taken out of the bag at random. What is the probability that the number on the card taken out is an even number?

Sol. Total number of possible outcomes = 20

Let E be the event of getting an even number on the card.

The favourable outcomes are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.

Sum of favourable outcomes = 10

P(E) = $\frac{10}{20} = \frac{1}{2}$ **Ans.**

33. Cards marked with numbers 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.

Sol. Total outcomes = 3, 4, 5, ..., 50

Total no. of outcomes = 48

Let E be the event of getting a perfect square number

Favourable outcomes = 4, 9, 16, 25, 36, 49.

No. of favourable outcomes = 6

P(E) = $\frac{6}{48} = \frac{1}{8}$ **Ans.**

34. A card is drawn out from a well-shuffled deck of 52 cards. What is the probability of getting a red queen?

Sol. Total number of possible outcomes = 52

Let E be the event of getting a red queen.

The total number of favourable outcomes = 2

P(E) = $\frac{2}{52} = \frac{1}{26}$ **Ans.**

35. A dice is thrown once. Find the probability of getting an even number.

Sol. The maximum number of possible outcomes = 6

Let E be the event of getting an even number on the dice.

Favourable outcomes = 2, 4, 6.

Total number of the favourable outcomes = 3

P(E) = $\frac{3}{6} = \frac{1}{2}$ **Ans.**

36. Two different dice are tossed together. Find the probability that (i) the number on each dice is odd, and (ii) the sum on the numbers, appearing on the two dice, is 5.

Sol. Total number of possible outcomes = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 36

(i) Favourable outcomes are (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5).

Hence, total number of favourable outcomes = 9

P(E) = $\frac{9}{36}=\frac{1}{4}$

(ii) Favourable outcomes are (1, 4), (2, 3), (3, 2), (4, 1)

Hence, total number of the favourable outcomes = 4

P(E) = $\frac{4}{36}=\frac{1}{9}$. **Ans.**

37. Rahim tosses two different coins simulta-neously. Find the probability of getting at least one tail.

Sol. Total number of possible outcomes = HH, HT, TH, TT

Favourable outcomes are HT, TH, TT

Hence, the total number of favourable outcomes = 3

P(E) = $\frac{3}{4}$. **Ans.**

38. Two different dice are rolled simultaneously. Find the probability that the sum of the numbers appearing on the two dice is 10.

Sol. Total number of possible outcomes =

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 36

Favourable outcomes are (4, 6), (5, 5), (6, 4)

Hence, the total number of favourable outcomes = 3

P(E) = $\frac{3}{36}=\frac{1}{12}$. **Ans.**

39. A card is drawn from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.

Sol. Total number of cards = 52

Number of kings and queens = 4 + 4 = 8

Thus, number of favourable outcomes
= 52 – 8 = 44

P(E) = $\frac{44}{52} = \frac{11}{13}$. **Ans.**

40. A card is drawn from a well shuffled pack of 52 playing cards. Find the probability of getting (i) a red-faced card and (ii) a black-king.

Sol. Total number of cards = 52

(i) Number of red-faced cards = 3 + 3 = 6

P(E) = $\frac{6}{52} = \frac{3}{26}$

(ii) Number of black queens = 2

P(E) = $\frac{2}{52} = \frac{1}{26}$. **Ans.**

41. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting (i) a red king, and (ii) a queen or a jack.

Sol. Total number of cards = 52

(i) The number of favourable outcomes of drawing a red king = 2

P(E) = $\frac{2}{52} = \frac{1}{26}$

(ii) The number of favourable outcomes of drawing a queen or a jack = 8

P(E) = $\frac{8}{52} = \frac{2}{13}$. **Ans.**

42. A ticket is drawn at random from a bag containing tickets numbered from 1 to 40. Find the probability that the selected ticket has a number which is a multiple of 5.

Sol. Total number of tickets = 40

Favourable outcomes are 5, 10, 15, 20, 25, 30, 35, 40

Total number of favourable outcomes = 8

P(E) = $\frac{8}{40}=\frac{1}{5}$. **Ans.**

43. Cards each marked with one of the numbers between 6, 7, 8, …, 15 are placed in a box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting a card with a number less than 10 ?

Sol. The possible outcomes are 6, 7, 8, …, 15

Thus, total number of possible outcomes = 10

The favourable outcomes are 6, 7, 8, 9.

Hence, sum of the number of favourable outcomes = 4

P(E) = $\frac{4}{10}=\frac{2}{5}$. **Ans.**

44. A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn out at random from the box, what is the probability that it will not be a white marble?

Sol. The sum of the possible outcomes

= 3 + 2 + 4 = 9

If the marble drawn out is not to be white then the favourable outcomes are 3 blue marbles and 4 red marbles.

Hence, the sum of the favourable conditions

= 3 + 4 = 7

P(E) = $\frac{7}{9}$. **Ans.**

45. Two dice are thrown simultaneously. Find the probability that the sum of the numbers appearing on the two dice is more than 9.

[Board Question]

Sol. Total number of possible outcomes = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (14, 5), (4, 6), (5, 1), (15, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (16, 4), (6, 5), (6, 6) = 36

Favourable outcomes are (4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)

Hence, the total number of favourable outcomes
= 6

P(E) = $\frac{6}{36}$ = $\frac{1}{6}$. **Ans.**

46. Find the probability that a number selected at random from the numbers 1 to 25 is not a prime number.

Sol. The total number of possible outcomes = 25

The favourable outcomes are 1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25
Hence, the sum of the numbers of favourable outcomes = 16

P(E) = $\frac{16}{25}$. **Ans.**

47. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that the card is neither a red card nor a jack.

Sol. Total number of cards = 52

The number of favourable outcomes of drawing neither a red card nor a jack card = 52 – (26 + 2) = 24

P(E) = $\frac{24}{52} = \frac{6}{13}$. **Ans.**

48. A card is drawn at random from a shuffled pack of 52 cards. Find the probability of drawing a (i) face card, and (ii) a card which is neither a king nor a red card.

Sol. Total numbers of cards = 52

(i) The number of favourable outcomes of drawing a face card = 12

P(E) = $\frac{12}{52} = \frac{3}{13}$

(ii) The number of favourable outcomes of drawing neither a king nor a red card

= 52 – (2 + 26)

= 24

P(E) = $\frac{24}{52} = \frac{6}{13}$. **Ans.**

49. A dice is thrown once. Find the probability of getting a number (i) less than three, and (ii) more than four.

Sol. Maximum possible outcomes = 6

(i) Favourable outcomes for a number less than 3 are 1, 2

Thus, the number of the favourable outcomes

= 2

P(E) = $\frac{2}{6} = \frac{1}{3}$

(ii) Favourable outcomes for a number more than 4 are 5, 6.

Thus, the number of the favourable outcomes = 2

P(E) = $\frac{2}{6} = \frac{1}{3}$. **Ans.**

50. What is the probability that a number selected at random from the number 3, 4, 5, 6, 7, 8, 9 is a multiple of 4 ?

Sol. The maximum possible outcomes = 7

Favourable outcomes are 4, 8.

Thus, the number of the favourable outcomes = 2

P(E) = $\frac{2}{7}$. **Ans.**

51. Samsung Electronics has launched two new mobile hand sets : Set-I and Set -II. Set-I is cheaper as compared to Set-II. But Set-II has an built-in device to recharge the battery with auto-cut power supply when it is fully charged. In a lot there are 250 pieces of Set-I and 100 of Set-II. If a mobile is picked at random :

(a) Find the probability of getting Set-I

(b) Find the probability of getting Set-II.

Sol. Total no. of mobile sets = 250 + 100 = 350

(a) Probability of getting set-I = $\frac{250}{350}$ = $\frac{5}{7}$

(b) Probability of getting set-II = $\frac{100}{350} = \frac{2}{7}$ **Ans.**

52. In answering a question of MCQ test with 4 choices per question, one of them being correct, a student knows the answer, guesses or copies the answer. Suppose a student guesses the answer. What is the probability that his answer is correct?

Sol. Number of choices = 4

Number of correct choice = 1

Probability that answer is correct = $\frac{1}{4}$ **Ans.**

53. Honey goes to school either by a car driven by his driver or uses his bicycle. Probability that he will use the car is $\frac{3}{7}$. What is the probability that he will use his bicycle for going to the school?

Sol. Probability that he will use his bicycle for going to the school = 1 – Probability that he will use the car = 1 – $\frac{3}{7} = \frac{4}{7}$ **Ans.**

Short Answer Type Questions

54. A box contains 100 red cards, 200 yellow cards and 50 blue cards. If a card is drawn at random from the box, then find the probability that it will be :

(i) a blue card,

(ii) not a yellow card, and

(iii) neither yellow nor a blue card.

Sol. Total number of cards

= 100 + 200 + 50 = 350

(i) Possible outcomes of drawing a blue card = 50

Hence, P(E) = $\frac{50}{350}$ = $\frac{1}{7}$.

(ii) Possible outcomes of not drawing a yellow card = 350 – 200 = 150.

Hence, P(E) = $\frac{150}{350}$ = $\frac{3}{7}$.

(iii) Possible outcomes of drawing neither yellow nor a blue card = 350 – (200 + 50) = 350 – 250 = 100

Hence, P(E) = $\frac{100}{350}$ = $\frac{2}{7}$. **Ans.**

55. All kings, queens and aces are removed from a pack of 52 cards. The remaining cards are well-shuffled and then a card is drawn from it. Find the probability that the drawn card is (i) a black face card, and (ii) a red card.

Sol. Total number of cards = 52

Number of cards removed = 4 + 4 + 4 = 12

Hence, total number of remaining cards

= 52 – 12 = 40.

(i) Now, number of black face cards (Jack card)

= 1 + 1 = 2

Hence, P(E) = $\frac{2}{40}$ = $\frac{1}{20}$.

(ii) And number of red cards = 13 of hearts + 13 of diamond – (1 king of hearts + 1 king of diamonds + 1 queen of hearts + 1 queen of diamonds + 1 ace of hearts + 1 ace of diamonds)

= 20

Hence, P(E) = $\frac{20}{40}$ = $\frac{1}{2}$. **Ans.**

56. Cards marked with numbers 5, 6, 7, …, 74 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that the number on the card is a perfect square.

Sol. Total number of possible outcomes

= 5, 6, 7, 8, 9, …, 74 = 70

Now, the favourable outcomes = 9, 16, 25, 36, 49, 64

Hence, the total number of favourable outcomes = 6

∴ P(E) = $\frac{6}{70}$ = $\frac{3}{35}$. **Ans.**

57. A box contains 80 discs numbered from 1 to 80. If one disc is drawn at random from the box, find the probability that the number it bears is a perfect square.

Sol. Total number of possible outcomes = 1, 2, 3, 4, 5, 6, 7, 8, 9, …, 80 = 80

Now, the favourable outcomes = 1, 4, 9, 16, 25, 36, 49, 64

Hence, the total number of favourable outcomes

= 8

∴ P(E) = $\frac{8}{80}$ = $\frac{1}{10}$. **Ans.**

58. Two dice are rolled together. Find the probability of getting such number on the two dice whose product is a perfect square.

Sol. Total number of possible outcomes = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 36

Now, the favourable outcomes = (1, 1), (2, 2),(3, 3), (4, 4), (5, 5), (6, 6), (1, 4), (4, 1)

Thus, the total number of favourable outcomes = 8

∴ P(E) = $\frac{8}{36}$ = $\frac{2}{9}$. **Ans.**

59. A game consists of tossing a coin three times and noting the outcome each time. Hanif wins if he gets three heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.

[NCERT & Board Question]

Sol. The possibilities are HHH, HHT, HTT, THT, TTH, TTT, HTH, THH

Thus, the total number of possibilities = 8

Favourable outcomes for Hanif's loss are HHT, HTT, THT, TTH, HTH, THH

Thus, the total number of favourable outcomes = 6

∴ P(E) = $\frac{6}{8}$ = $\frac{3}{4}$. **Ans.**

60. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card (i) is divisible by 9 and is a perfect square (ii) is a prime number greater than 80.

[Board Question]

Sol. Number of possible outcomes = 100

(i) Let E_1 be the event of getting a number divisible by 9 and is a perfect square.

∴ Favourable outcomes = {9, 36, 81}

Number of favourable outcomes = 3

∴ $P(E_1)$ = $\frac{3}{100}$ **Ans.**

(ii) Let E_2 be the event of getting a prime number greater than 80.

∴ Favourable outcomes = {83, 89, 97}

Number of favourable outcomes = 3

∴ $P(E_2)$ = $\frac{3}{100}$. **Ans.**

61. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of a red ball, find the number of blue balls in the bag.

Sol. Let the number of blue balls be *x*.

Thus, sum of the possible outcomes = 5 + x

Now, the sum of the favourable outcomes if the red ball is drawn = 5.

∴ P(R) = $\frac{5}{5+x}$

Again, the sum of the favourable outcomes if the blue ball is drawn = x.

∴ P(B) = $\frac{x}{5+x}$

According to the question,

P(B) = 3P(R) [Given]

⇒ $\frac{x}{5+x} = 3\left(\frac{5}{5+x}\right)$

⇒ $x = 15$

Hence, the number of blue balls = 15. **Ans.**

62. From a pack of 52 playing cards, jacks, queens and kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is :

(i) a black king (ii) a card of red colour (iii) a card of black colour

Sol. Since, jacks, queens and kings of red colour are removed. Then,

Total number of possible outcomes = 52 – 6 = 46

(i) Let E_1 be the event of getting a black king.

∴ Favourable outcomes = king of spade and king of club.

No. of favourable outcomes = 2

P(E_1) = $\frac{2}{46} = \frac{1}{23}$. **Ans.**

(ii) Let E_2 be the event of getting a card of red colour

∴ Favourable outcomes = 10 cards of heart and 10 cards of diamond.

No. of favouable outcomes = 20

$P(E_2) = \frac{20}{46} = \frac{10}{23}$. **Ans.**

(iii) Let E_3 be the event of getting a card of black colour

∴ Favourable outcomes = 13 cards of spade and 13 cards of club.

No. of favourable outcomes = 26

$P(E_3) = \frac{26}{46} = \frac{13}{23}$. **Ans.**

63. It is known that a box of 200 electric bulbs contains 16 defective bulbs. One bulb is taken out at random from the box. What is the probability that the bulb drawn is (i) defective, and (ii) non-defective?

Sol. The total number of possible outcomes = 200

(i) Total number of defective bulbs = 16

∴ $P(E) = \frac{16}{200} = \frac{2}{25}$. **Ans.**

(ii) Total number of non-defective bulbs

= 200 – 16 = 184

∴ $P(E) = \frac{184}{200} = \frac{23}{25}$. **Ans.**

64. A bag contains cards numbered from 1 to 30. If a card is drawn at random from the bag after mixing them thoroughly, find the probability that the number on the card is not divisible by 3.

[Board Question]

Sol. All possible outcomes = 1, 2, 3, …, 30

Total number of all possible outcomes = 30

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Total number of favourable outcomes

= (30 – 10) = 20

∴ $P(E) = \frac{20}{30} = \frac{2}{3}$. **Ans.**

65. Find the probability of getting 53 Fridays in a leap year.

Sol. Total number of days in a leap year = 366

Total number of days in a week = 7 days

Thus, the number of weeks and days in a leap year

= $\frac{366}{7}$ = 52 weeks 2 days

Thus, the two days can be Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday

So, the number of the possible outcomes = 7

and the favourable outcomes are Thursday and Friday; Friday and Saturday

Therefore, the number of the favourable outcomes = 2

∴ P(E) = $\frac{2}{7}$. **Ans.**

66. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting (i) a card of spades or an ace, (ii) a red king, (iii) either a king or a queen, and (iv) neither a king nor a queen.

Sol. Total number of cards = 52

(i) Number of favourable outcomes of drawing a card of spades or an ace = 13 + 3 = 16

∴ P(E) = $\frac{16}{52}$ = $\frac{4}{13}$ · **Ans.**

(ii) The number of favourable outcomes of drawing a red king = 2

∴ P(E) = $\frac{2}{52}$ = $\frac{1}{26}$ ·**Ans.**

(iii) The number of favourable outcomes of drawing a king or a queen = 4 + 4 = 8

∴ P(E) = $\frac{8}{52}$ = $\frac{2}{13}$· **Ans.**

(iv) The number of favourable outcomes of drawing neither a king nor a queen = 52 – (4 + 4)

= 44

∴ P(E) = $\frac{44}{52}$ = $\frac{11}{13}$ · **Ans.**

67. A bag contains tickets numbered 11, 12, 13, …, 30. A ticket is drawn at random from the bag. Find the probability that the number on the ticket is a (i) multiple of 7, and (ii) is greater than 15 and a multiple of 5.

Sol. The number of possible outcomes = 20

(i) The favourable outcomes for the number to be a multiple of 7 are 14, 21, 28.

Hence, the sum of the number of outcomes of favourable conditions = 3.

∴ P(E) = $\frac{3}{20}$. **Ans.**

(ii) The favourable outcomes for the number to be greater than 15 and a multiple of 5 are 20, 25, 30.

Hence, the sum of the number of outcomes of favourable conditions = 3.

∴ P(E) = $\frac{3}{20}$. **Ans.**

68. A die is thrown twice. Find the probability that:

(i) 5 will come up at least once.

(ii) 5 will not come up either time.

Sol. When two dice are thrown simultaneously, all possible outcomes are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Total number of outcomes = 36

Favourable outcomes where 5 comes up at least once = 11

(i) Probability that 5 will come up at least once

$= \frac{\text{Total outcomes where 5 will come up}}{\text{Total number of outcomes}}$

$= \frac{11}{36}$

(ii) Outcomes where 5 will not come up

= 36 – 11 = 25

Probability that 5 will not come up either time

$= \frac{\text{Total outcomes where 5 will not come up}}{\text{Total number of outcomes}}$

$= \frac{25}{36}$. **Ans.**

69. The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.

[Board Question]

Sol. Let probability of selecting a blue marble, black marble and green marble are P(x), P(y), P(z) respectively.

P(x) = $\frac{1}{5}$, P(y) = $\frac{1}{4}$ (Given)

We know, P(x) + P(y) + P(z) = 1

$\Rightarrow \frac{1}{5} + \frac{1}{4} + P(z) = 1$

$\Rightarrow \frac{4+5}{20} + P(z) = 1$

$\Rightarrow \frac{9}{20} + P(z) = 1$

$\Rightarrow P(z) = 1 - \frac{9}{20}$

$= \frac{20-9}{20}$

$\Rightarrow P(z) = \frac{11}{20}$

$\frac{\text{No. of green marbles}}{\text{Total no. of marbles}} = \frac{11}{20}$

$\Rightarrow \frac{11}{\text{Total no. of marbles}} = \frac{11}{20}$

(... No. of green marbles = 11)

⇒ Total no. of marbles = 20

 There are 20 marbles in the jar. **Ans.**

70. A group consists of 12 persons out of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest.

Sol. Number of extremely patient persons = 3

Number of extremely honest persons = 6

Number of extremely kind persons = 12 – (6 + 3)

= 3

Total number of ways to select a person = 12

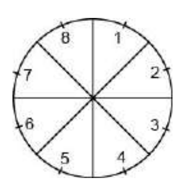
(i) Number of ways to select one extremely patient person = 3

∴ Probability of selecting one extremely patient person = $\frac{3}{12} = \frac{1}{4}$. **Ans.**

(ii) Number of ways of selecting one extremely kind or honest person = 6 + 3 = 9

∴ Probability of selecting a person who is extremely kind or honest = $\frac{9}{12} = \frac{3}{4}$. **Ans.**

71. A game of chance consists of spinning an arrow which comes to rest pointing at one of the number 1, 2, 3, 4, 5, 6, 7, 8 (see figure) and these are equally likely outcomes. What is the probability that it will point at:



(i) 8 ?

(ii) an odd number ?

(iii) a number greater than 2 ?

(iv) a number less than 9 ?

Sol. (i) Here, total number of points = 8

As, number of favourable outcomes when an arrow will point at 8 = 1

So, P(arrow will point at 8)

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{1}{8}$. **Ans.**

(ii) Here, number of odd number points = 4

So, P(arrow will point at odd number)

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{4}{8} = \frac{1}{2}$. **Ans.**

(iii) Here, number of points greater than 2 = 6

So, P(arrows will point at a number greater than 2)

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{6}{8} = \frac{3}{4}$. **Ans.**

(iv) Here, number of point less than 9 = 8.

So, P(arrow will point at a number less than 9)

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{8}{8} = 1$. **Ans.**

Long Answer Type Questions

72. Five cards the ten, jack, queen, king and ace of diamonds are well shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the drawn card is a queen? (ii) If the queen is drawn and put aside and a

second card is drawn, find the probability that the second card is (a) an ace, and (b) a queen.

Sol. Total number of cards = 5

(i) Possible outcomes of drawing a queen = 1

Hence, P(E) = $\frac{1}{5}$

(ii) Now the queen is drawn and set aside.

So, the total number of cards = 4

(a) Possible outcomes of drawing an ace = 1

Hence P(E) = $\frac{1}{4}$

(b) Possible outcomes of drawing a queen = 0

Hence, P(E) = 0

This is an impossible event. **Ans.**

73. A piggy bank contains hundred 50 p coins, fifty ` 1 coins, twenty ` 2 coins and ten ` 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, find the probability that the coin which fell (i) will be a 50p coin, (ii) will be of value more than ` 1, (iii) will be value less than ` 5, and (iv) will be a ` 1 or ` 2 coin.

[NCERT & Board Question]

Sol. Total number of coins = (100 + 50 + 20 + 10) = 180

(i) Possible number of outcomes of a 50 p coin falling out = 100

Hence, P(E) = $\frac{100}{180} = \frac{5}{9}$

(ii) Possible number of outcomes of a coin falling out having value more than ` 1 = (20 + 10) = 30

Hence, P(E) = $\frac{30}{180} = \frac{1}{6}$

(iii) Possible number of outcomes of a coin falling out having value less than ` 5 = (100 + 50 + 20) = 170

Hence, P(E) = $\frac{170}{180} = \frac{17}{18}$

(iv) Possible number of outcomes of a coin falling out having value either ` 1 or ` 2 = (50 + 20) = 70

Hence, P(E) = $\frac{70}{180} = \frac{7}{18}$ **Ans.**

74. Two different dice are thrown together. Find the probability that the numbers obtained have

(i) even sum, and

(ii) even product.

[Board Question]

Sol. Total possible outcomes, when 2 dice are thrown = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1),(2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(i) **For even sum :** Favourable outcomes are

(1, 1), (1, 3), (1, 5), (2, 2), (2, 4),(2, 6), (3, 1),(3, 3),(3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)

No. of favourable outcomes = 18

P(even sum) = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

= $\frac{18}{36}$ = $\frac{1}{2}$. **Ans.**

(ii) **For even product :** Favourable outcomes are

(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3) ,(4, 4), (4, 5), (4, 6), (5, 2), (5 ,4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

No. of favourable outcomes = 27

P(even product) = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

= $\frac{27}{36}$ = $\frac{3}{4}$. **Ans.**

75. Find the probability that a number selected at random from the numbers 1, 2, 3, …, 34, 35 is a (i) prime, (ii) multiple of 7, and (iii) divisible by 3 or 5.

[Board Question]

Sol. All possible outcomes = {1, 2, 3, …, 34, 35}

∴ Total number of all possible outcomes = 35

(i) Favourable outcomes = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31}

Thus, total number of favourable outcomes = 11

Hence, P(E) = $\frac{11}{35}$

(ii) Favourable outcomes = {7, 14, 21, 28, 35}

Thus, total number of favourable outcomes = 5

Hence, P(E) = $\frac{5}{35}$ = $\frac{1}{7}$

(iii) Favourable outcomes = {3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30, 33, 35}

Thus, total number of favourable outcomes = 16

Hence P(E) = $\frac{16}{35}$. **Ans.**

76. A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn out from the bag is (i) white, (ii) red or black, (iii) not green, and (iv) neither white nor black.

[Board Question]

Sol. Total outcomes = 6 + 8 + 5 + 3 = 22

(i) Favourable outcomes = 8

∴ P(E) = $\frac{8}{22}$ = $\frac{4}{11}$

(ii) Total number of favourable outcomes

= 6 + 3 = 9

∴ P(E) = $\frac{9}{22}$

(iii) Total number of the favourable outcomes

= 6 + 8 + 3 = 17

∴ P(E) = $\frac{17}{22}$

(iv) Total number of the favourable outcomes

= 6 + 5 = 11

∴ P(E) = $\frac{11}{22}$ = $\frac{1}{2}$. **Ans.**

77. Two different dice are thrown together. Find the probability that the numbers obtained

(i) have a sum less than 7.

(ii) have a product less than 16.

(iii) is a doublet of odd numbers.

[Board Question]

Sol. Total number of possible outcomes in each case

= 6 × 6 = 36

(i) Having a sum less than 7

Possible outcomes are,

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5)

(2 ,1), (2, 2), (2, 3), (2, 4)

(3, 1), (3, 2), (3, 3)

(4, 1), (4, 2)

(5, 1)

∴ n(E) = 15

So, required probability = $\frac{15}{36} = \frac{5}{12}$

(ii) Having a product less than 16

Possible outcomes are,

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5),

(4, 1), (4, 2), (4, 3),

(5, 1), (5, 2), (5, 3),

(6, 1), (6, 2),

∴ $n(E) = 25$

So, probability = $\frac{25}{36}$

(iii) A doublet of odd numbers

Possible outcomes are (1, 1), (3, 3), (5, 5)

∴ $n(E) = 3$

So, probability = $\frac{3}{36} = \frac{1}{12}$. **Ans.**

78. A box contains 20 balls bearing numbers from 1 to 20. A ball is drawn at random from the box. Find the probability that the number on the ball is (i) an odd number, (ii) divisible by 2 or 3, (iii) not divisible by 10, and (iv) a prime number.

[Board Question]

Sol. Total number of possible outcomes = 20

(i) Favourable outcomes for numbers that are odd are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

Hence, the number of favourable outcomes = 10

∴ $P(E) = \frac{10}{20} = \frac{1}{2}$

(ii) Favourable outcomes for numbers to be divisible by 2 or 3 are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

Hence, number of favourable outcomes = 13

∴ P(E) = $\frac{13}{20}$

(iii) Favourable outcomes for numbers that are not divisible by 10 are 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19.

Hence, the number of favourable outcomes = 18.

∴ P(E) = $\frac{18}{20}$ = $\frac{9}{10}$

(iv) Favourable outcomes for getting a prime numbers are 2, 3, 5, 7, 11, 13, 17, 19.

Hence, the number of favourable outcomes = 8

∴ P(E) = $\frac{8}{20}$ = $\frac{2}{5}$. **Ans.**

79. A bag contains 6 black balls and some white balls. If the probability of drawing a white ball from the bag is double that of a black ball, find the number of white balls in the bag.

[Board Question]

Sol. Let the number of white balls be *x*.

Thus, the total number of the possible outcomes

= 6 + *x*

Now, the favourable outcomes if the black ball is drawn = 6

∴ P(B) = $\frac{6}{6+x}$

Again, the favourable outcomes if the white ball is drawn = *x*

∴ P(W) = $\frac{x}{6+x}$

According to the question,

$\frac{x}{6+x} = 2\left(\frac{6}{6+x}\right)$

⇒ $x = 12$

Hence, the number of white balls = 12. **Ans.**

Assertion and Reasoning Based Questions

Mark the option which is most suitable:

(a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) Assertion is false but the Reason is true.

80. Assertion: In a cricket match a batsman hits a boundary 9 times out of 45 balls he plays. The probability that in a given throw he does not hit the boundary is 4/5.
Reason: P(E) + P (not E) = 1.
Ans. (a) Both the Assertion and the Reason are correct and the Reason is the correct explanation of the Assertion.

Explanation :

For reason, as per the empirical formula

The sum of all the probabilities of all possible outcomes of experiment is 1.

P(Event) + P (not an Event) = 1

So, reason is true.

For assertion,

As per the description

P(boundary) or P(E) = 9/45 = 1/5

So P(no boundary) or P(not E) = 1 – 1/5 = 4/5

So, assertion is also true and Reason is the correct explanation of the assertion.
81. Assertion: The probability of a sure event is 1.
Reason: Let E be an event. Then $0 \le P(E) \le 1$.

Ans. (b) The Assertion and the Reason are correct but the Reason is not the correct explanation of the Assertion.

Explanation :

For Assertion,

The set of all possible outcomes which are certain to occur form sure event, thus the entire sample space is a sure event. For example, in a random experiment of tossing of a coin the event of getting a headon tail is a sure event. So, the probability of the sure event is 1 *i.e.*, P(S) = 1.

So, assertion is true.

For reason,

For any event E associated to a random experiment, we have

$$0 \le P(E) \le 1$$

So, both assertion and reason are correct, but reason is not an explanation for assertion.

82. Assertion: It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992, then the probability that the 2 students have the same birthday is 0.128.
Reason: If *n*(A) = 1 and *n*(S) = 13, then P(A) = 1/3.
Ans. (d) Assertion is false but the Reason is true.

Explanation :

For assertion,

Let E be the event of having same birthday.

E' be the event of not having the same birthday.

Given P(E') = 0.992

Since we know that

P(E) + P(E') = 1

⇒ P(E) = 1 – P(E')

⇒ P(E) = 1 – 0.992 = 0.008.

So, assertion is false.

For reason,

Given n(A) = 1 and n(S) = 13,

then P(A) = $\frac{n(A)}{n(S)} = \frac{1}{13}$

So, reason is true.

Case Based Questions

83. One day, during games period four friends A, B, C and D planned to play game using numbers cards. They prepared 20 numbered cards with labelled 1 to 20 and then they put all the number cards in the empty chalk box available in the classroom. In this game, every friend was asked to pick the card randomly and after each drawn, card was replaced back in the chalk box.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

(i) The probability, first boy pick the card and he get the card with an even number is:

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{6}$

(d) $\frac{3}{8}$

Ans. (b) $\frac{1}{2}$

Explanation :

Number of possible outcomes = 20

Number of favourable outcomes

= {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} *i.e.*, 10

P(even number) = $\frac{10}{20} = \frac{1}{2}$.

(ii) If the card drawn in first case is replaced and the second boy draws a card. What is the probability of getting a prime number?

(a) $\frac{2}{5}$

(b) $\frac{4}{5}$

(c) $\frac{7}{8}$

(d) $\frac{9}{11}$

Ans. (a) $\frac{2}{5}$

Explanation :

Number of favourable outcomes

= {2, 3, 5, 7, 11, 13, 17, 19} *i.e.*, 8

P(prime number) = $\frac{8}{20} = \frac{2}{5}$

(iii) If the card drawn, is not replaced in the second drawn, the probability that he got a multiple of 3 greater than 4 is:

(a) $\frac{1}{11}$

(b) $\frac{7}{20}$

(c) $\frac{6}{19}$

(d) $\frac{5}{19}$

Ans. (d) $\frac{5}{19}$

Explanation :

Number of possible outcomes = 20 – 1 = 19

Favourable outcomes = {6, 9, 12, 15, 18} *i.e.*, 5

P(multiple of 3 greater than 4) = $\frac{5}{19}$

(iv) For a sure event A, P(A) = ?

(a) 1

(b) 0

(c) – 1

(d) 2

Ans. (a) 1

Explanation :

The probability of sure event is 1.

(v) If all cards drawn are replaced then the probability of getting a multiple of 3 and 5 is:

(a) $\frac{1}{2}$

(b) $\frac{1}{5}$

(c) $\frac{1}{20}$

(d) $\frac{1}{18}$

Ans. (c) $\frac{1}{20}$

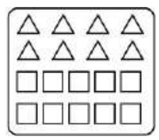
Explanation :

Number of possible outcomes = 20

Favourable cases = {15} *i.e.*, 1

P(mutliple of 3 and 5) = $\frac{1}{20}$

84. Aditya went to shop to purchase a child's game along with his friend. He selected one child's game which had 8 triangles out of which 3 are blue and rest are red and 10 squares of which 6 are blue and rest are red. While checking the game, one piece is lost at random.



(i) How many triangles are of red colour and how many squares are of red colour?

(a) 5, 4

(b) 4, 5

(c) 5, 5

(d) 8, 6

Ans. (a) 5, 4

Explanation :

Number of red colour triangles

= 8 – 3

= 5

Number of red colour squares

= 10 – 6

= 4.

(ii) Find the probability that lost piece is square:

(a) $\frac{4}{9}$

(b) $\frac{5}{9}$

(c) $\frac{1}{3}$

(d) $\frac{5}{18}$

Ans. (b) $\frac{5}{9}$

Explanation :

Total number of figure

= 8 triangles + 10 squares

= 18

P(lost piece is a square) = $\frac{10}{18} = \frac{5}{9}$.

(iii) Find the probability that lost piece is triangle:

(a) $\frac{4}{9}$

(b) $\frac{5}{9}$

(c) $\frac{1}{3}$

(d) $\frac{5}{18}$

Ans. (a) $\frac{4}{9}$

Explanation :

P(lost piece is triangle) = $\frac{8}{18}$

= $\frac{4}{9}$.

(iv) Find the probability that lost piece is square of blue colour:

(a) $\frac{4}{9}$

(b) $\frac{5}{9}$

(c) $\frac{1}{3}$

(d) $\frac{5}{18}$

Ans. (c) $\frac{1}{3}$

Explanation :

P(square of blue colour) = $\frac{6}{18}=\frac{1}{3}$.

(v) Find the probability that lost piece is triangle of red colour:

(a) $\frac{4}{9}$

(b) $\frac{5}{9}$

(c) $\frac{1}{3}$

(d) $\frac{5}{18}$

Ans. (d) $\frac{5}{18}$

Explanation :

P(triangle of red colour) = $\frac{5}{18}$.

85. Rishank and Riyank are best friends. They stay in the same colony. Both studies in the same class and in the same school. During winter vacation Rishank visited Riyank's house to play Ludo. They decided to play Ludo with 2 dice.

(i) To win a game, Rishank wanted a total of 7. what is the probability of winning a game by Rishank?

(a) $\frac{1}{6}$

(b) $\frac{7}{2}$

(c) $\frac{5}{18}$

(d) $\frac{1}{9}$

Ans. (a) $\frac{1}{6}$

Explanation :

(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

Favourable outcomes = 6

Total outcomes = 36

Probability = $\frac{6}{36}$

= $\frac{1}{6}$.

(ii) To win a game, Riyank wanted a total of 8 as the sum. What is the probability of winning a game by Riyank?

(a) $\frac{1}{12}$

(b) $\frac{7}{36}$

(c) $\frac{5}{36}$

(d) $\frac{1}{6}$

Ans. (c) $\frac{5}{36}$

Explanation :

Favourable outcomes = (2, 6), (6, 2), (3, 5), (5, 3), (4, 4) = 5

Total outcomes = 36

Probability = $\frac{5}{36}$.

(iii) The probability that the sum of the numbers on the both the dice is divisible by 10 is:

(a) $\frac{1}{12}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{1}{4}$

Ans. (a) $\frac{1}{12}$

Explanation :

Favourable outcomes = (4, 6), (6, 4), (5, 5)

Required probability = $\frac{3}{36}$

= $\frac{1}{12}$

(iv) What is the probability that the sum of the number on both the dice is divisible by 4 or 6?

(a) $\frac{7}{18}$

(b) $\frac{7}{15}$

(c) $\frac{5}{18}$

(d) $\frac{2}{9}$

Ans. (a) $\frac{7}{18}$

Explanation :

For sum to be divisible by 4 outcomes are (1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (6, 2), (5, 3), (6, 6).

For sum to be divisible by 6 outcomes are (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6).

Required probability = $\frac{14}{36}$

= $\frac{7}{18}$

(v) The probability that 5 will come up at least in die is:

(a) $\frac{7}{36}$

(b) $\frac{11}{36}$

(c) $\frac{25}{36}$

(d) $\frac{2}{9}$

Ans. (b) $\frac{11}{36}$

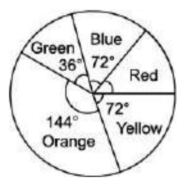
Explanation :

Outcomes = (1, 5), (5, 1), (2, 5), (5, 2), (3, 5), (5, 3) (4, 5), (5, 4), (5, 5), (6, 5), (5, 6)

Number of favourable outcomes = 11

Probability = $\frac{11}{36}$.

86. A survey was taken at a high school and the results were put in a circular graph. The students were asked to list their favourite colour. The measurement of each central angle is shown. If a person is chosen at random from the school find the probability of each response.



(i) The probability of favourite colour being red is:

(a) 0.1

(b) 0.2

(c) 0.3

(d) 0.4

Ans. (a) 0.1

Explanation :

Central angle for colour being red

= 360° – (36° + 72° + 72° + 144°)

= 36°

Probability = $\frac{\text{Area of region}}{\text{Area of circle}}$

= $\frac{\text{Total angle in region}}{360°}$

P(red) = $\frac{36^\circ}{360^\circ}$ = $\frac{1}{10}$ = 0.1

(ii) The probability of favourite colour being blue or green is:

(a) 0.1

(b) 0.2

(c) 0.3

(d) 0.4

Ans. (c) 0.3

Explanation :

P(blue of green) = $\frac{72^\circ + 36^\circ}{360^\circ}$ = $\frac{108^\circ}{360^\circ}$ = $\frac{3}{10}$

= 0.3

(iii) The probability of favourite colour not being red or blue is:

(a) 0.35

(b) 0.70

(c) 0.15

(d) 0.50

Ans. (b) 0.70

Explanation :

P(not red or blue) = 1 – P(red or blue)

= $1 - \frac{36^\circ + 72^\circ}{360^\circ}$ = $1 - \frac{108^\circ}{360^\circ}$

= $1 - \frac{3}{10}$

= 1 – 0.3

= 0.7

(iv) The probability of favourite colour not being orange or green?

(a) 0.65

(b) 0.75

(c) 0.25

(d) 0.50

Ans. (d) 0.50

Explanation :

P(not orange or green)

= 1 – P(orange or green)

= $1 - \frac{144° + 36°}{360°}$

= $1 - \frac{180°}{360°}$

= $1 - \frac{1}{2}$

= 0.50

(v) The probability of favourite colour being red or blue is:

(a) 0.2

(b) 0.3

(c) 0.1

(d) 0.4

Ans. (b) 0.3

Explanation :

P(red or blue) = 1 – P(not red or blue)

= 1 – 0.7 = 0.3

Passage Based Questions

87. In a game at a fair the entry fee is of ` 10. The game consists of tossing a coin 3 times. If it shows one or two heads, Pooja will get her entry fee back. If she throws 3 heads, she will get double the amount of entry fee, other- wise she will lose.

For tossing a coin three times, find the probability that she

(i) loses the entry fee,

(ii) gets double entry fee

(iii) just gets her entry fee back.

Sol. When a coin is tossed three times, possible outcomes are

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Total numbers of elementary events = 8

(i) Let A be the event that she will lose the entry fee

A = {TTT}

$n(A) = 1$

$P(A) = \frac{1}{8}$

(ii) Let B be the event that she will gets double the amount of entry fee.

B = {HHH}

$n(B) = 1$

$P(B) = \frac{1}{8}$ **Ans.**

(iii) Let C be the event that she will just get her entry fee back.

C = {HHT, HTH, HTT, THH, THT, TTH}

$n(C) = 6$

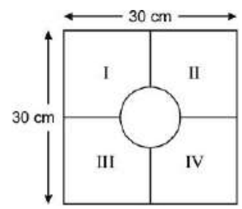
$P(C) = \frac{6}{8} = \frac{3}{4}$ **Ans.**

88. Raju was playing ludo with his sister. Everytime, he throws a die, it always landed on the board and the ludo pieces get displaced from their positions. So, his sister scolded him and told him to throw the die on the circular part of the board only.

Based on the given situation and the following figure, answer the questions.

(i) If the diameter of the circular portion is 14 cm, what is the probability that die will land inside the circular portion ?

(ii) What is the probability of getting a prime number on a single throw of a die?



Sol. (i) Area of circular portion = pr^2

= $\frac{22}{7}$ × 7 × 7 = 154 cm^2

Area of board (square) = $(side)^2$

= 30 × 30

= 900 cm^2

P (die will land inside circular portion)

= $\frac{154}{900}$

= $\frac{77}{450}$ **Ans.**

(ii) Total number of outcomes = 6

And prime numbers from 1 to 6 are {2, 3, 5}

⇒ Number of favourable outcomes = 3

P (Prime number) = $\frac{3}{6} = \frac{1}{2}$. **Ans.**

89. In a class, there are 20 girls and 18 boys. The class teacher wants to choose one student for class monitor and she don't want to be partial. So, she writes the name of each student on a card and puts them into a basket and mixes them thoroughly. A child is asked to pick one card from the basket.

Based on the given information, answer the following questions:

(i) What is the probability, the name written on the card is the name of a girl?

(ii) What is the probability, the name written on the card is the name of a boy?

Sol. Total no. of students in the class = 20 + 18 = 38

(i) No. of girls in the class = 20

Probability (name of a girl) = $\frac{20}{38} = \frac{10}{19}$ **Ans.**

(ii) No. of boys in the class = 18.

Probability (name of a boy) = $\frac{18}{38} = \frac{9}{19}$ **Ans.**

90. A lot consists of 50 mobile phones of which 44 are good, 3 have only minor defects and 3 have major defects. Vanshika will buy a phone if it is good but the trader will only buy a mobile if it has no major defect. One phone is selected at random from the lot.

Based on the given information, answer the following questions:

(i) What is the probability that it is acceptable to Vanshika?

(ii) What is the probability that it is acceptable to trader?

(iii) What is the probability that it is neither acceptable to trader

nor to Vanshika?

Sol. Total number of phones in the lot = 50

(i) No. of mobile phones acceptable to Vanshika

= 44.

Probability that mobile phone is acceptable to Vanshika

= $\frac{44}{50} = \frac{22}{25}$ **Ans.**

(ii) No. of mobile phones acceptable to trader

= 44 + 3 = 47

Probability that mobile phone is acceptable to trader

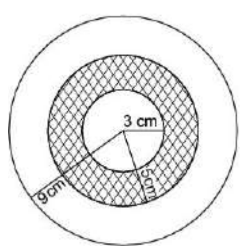
= $\frac{47}{50}$ **Ans.**

(iii) No. of mobile phones not acceptable to both trader and Vanshika = 3

Required probability = $\frac{3}{50}$ **Ans.**

91. A target game consist of three concentric circles of radii 3 cm, 5 cm and 9 cm respectively. A person wins the game if dart hits on the shaded region, and gets a second chance if dart hits the inner most circle. And loses the game if dart hits the outer most circle.

Based on the given information, answer the following questions:



(i) What is the probability that a person will win the game ?

(ii) What is the probability that a person will get the second chance?

(iii) What is the probability that a person will loose the game?

Sol. Area of inner most circle with radius 3 cm

$A_1 = pr^2 = p(3)^2 = 9p$

Area of circle of with radius 5 cm

$A_2 = p(5)^2 = 25p$

Area of circle with radius 9 cm

$A_3 = p(9)^2 = 81p$

Area of shaded region = $A_2 – A_1$

= 25p – 9p = 16p

Area of outer ring = $A_3 – A_2$ = 81p – 25p

= 56p

(i) Probability of winning the game

= $\frac{16\pi}{81\pi} = \frac{16}{81}$ **Ans.**

(ii) Probability of getting the second chance

= $\frac{9\pi}{81\pi} = \frac{1}{9}$ **Ans.**

(iii) Probability of lossing the game

= $\frac{56\pi}{81\pi} = \frac{56}{81}$ **Ans.**

Self-Assessment

92. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is double that of a red ball, find the number of blue balls in the bag.

[NCERT]

Ans. 10.

93. A bag contains 12 balls out of which x are white. If one ball is drawn at random what is the probability that it will be white ? If six more white balls are put in the bag the probability of drawing a white ball doubles. Find x.

[NCERT]

Ans. $\frac{1}{4}$, 3.

94. A child has a block in the shape of a cube with a letter written on each face (A, B, C, D, E, A).The cube is thrown once. What is the probability of getting (i) an A and (ii) a D?

[NCERT]

Ans. (i) $\frac{1}{3}$, (ii) $\frac{1}{6}$.

95. There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher, in order to select a class representative, writes down the names of all the students on same-sized pieces of cardboard and puts them in a box. Before drawing out a name, she stirs it well. What is the probability of the name being that of a (i) girl's and (ii) a boy's?

[NCERT]

Ans. (i) $\frac{5}{8}$, (ii) $\frac{3}{8}$.

96. A carton consists of 100 shirts of which, 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader will accept only the good shirts but Sujatha another trader will reject only the shirts with major defects. One shirt is drawn at random from the carton. What is the probability that it is acceptable to (i) Jimmy and (ii) to Sujatha?

Ans. (i) 0.88 (ii) 0.96.

97. Gopi buys a fish from a shop for his aquarium. The shopkeeper picks one at random from a tank containing 8 female fish and 5 male fish. What is the probability that the fish taken out is a male fish?

[NCERT]

Ans. $\frac{5}{13}$.

98. A jar contains 24 marbles of which some are green and the rest are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

[NCERT]

Ans. 8.

99. A die is thrown. Find the probability of getting (i) a prime number, (ii) an even prime number, (iii) a number greater than 5 and (iv) a number lying between 2 and 6.

[NCERT]

Ans. (i) $\frac{1}{2}$ (ii) $\frac{1}{6}$

(iii) $\frac{1}{6}$ (iv) $\frac{1}{2}$

100. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a red king, (ii) a face card, (iii) a red face card, (iv) a black queen, (v) a jack of hearts, and (vi) a spade.

[NCERT]

Ans. (i) $\frac{1}{26}$ (ii) $\frac{3}{13}$ (iii) $\frac{3}{26}$

(iv) $\frac{1}{26}$ (v) $\frac{1}{52}$ (vi) $\frac{1}{4}$.

101. A bag contains 3 red and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball is (i) red, (ii) black?

[NCERT]

Ans. (i) $\frac{3}{8}$ (ii) $\frac{5}{8}$.

102. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any day as on another. What is the probability that both will visit the shop on (i) the same day, (ii) on different days and (iii) on consecutive days?

[NCERT]

Ans. (i) $\frac{1}{6}$ (ii) $\frac{5}{6}$ (iii) $\frac{5}{36}$.

103. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

[NCERT]

Ans. 0.008.

104. (i) A lot of 20 electric bulbs contains 4 defective bulbs. One bulb is taken out at random from the box. What is the probability that the bulb drawn is defective ? (ii) Suppose the bulb is not defective and not replaced and another bulb is drawn from the lot, what is the probability that the second bulb is also not defective?

[NCERT]

Ans. (i) $\frac{1}{5}$ (ii) $\frac{15}{19}$.

105. A box contains 90 discs that are numbered from 1 to 90. If a disc is drawn at a random from the box, find the probability that it bears (i) a two-digit number, (ii) a perfect square number and (iii) a number divisible by 5.

[NCERT]

Ans. (i) $\frac{9}{10}$ (ii) $\frac{1}{10}$ (iii) $\frac{1}{5}$.

106. A lot consists of 144 ball pens of which 20 are defective and others are good. Nuri will buy a pen if it is good but will not buy one if it is defective. The shopkeeper draws one pen at a random and gives it to her. Find the probability of Nuri's buying and not buying it.

[NCERT]

Ans. $\frac{31}{36}$, $\frac{5}{36}$.

107. A lot consists of 144 pens of which 12 are defective and others are good. One pen is drawn out at random. Find the probability of the pen being a good one.

[NCERT]

Ans. $\frac{19}{18}$.

108. An unbiased die is thrown. What is the probability of getting (i) a multiple of 3, (ii) an even number or a multiple of 3 and (iii) an odd number?

[Board Question]

[NCERT]

Ans. (i) $\frac{1}{3}$ (ii) $\frac{2}{3}$ (iii) $\frac{1}{2}$.

109. A box contains 4 green, 8 white and 5 red marbles. If a marble is drawn out at random from the box, what is the probability that it will be (i) red, (ii) white, and (iii) not green marble?

[NCERT]

Ans. (i) $\frac{5}{17}$ (ii) $\frac{8}{17}$ (iii) $\frac{13}{17}$.

110. What is the probability of having 53 sundays in a leap year?

Ans. $\frac{2}{7}$.

111. Two dice are thrown together. What is the probability of having one number being twice the other?

Ans. $\frac{1}{12}$.

112. Two dice are thrown together. What is probability of having a difference of 3 between the two numbers?

Ans. $\frac{1}{6}$.

113. Two dice are thrown together. What is the probability of having a product of 6 between the two numbers?

Ans. $\frac{1}{9}$.

114. A card is drawn from a deck of 52 cards. What is the probability of drawing a red king or a black jack?

Ans. $\frac{1}{13}$.

115. From a deck of 52 cards the face cards are removed and they are replaced by two jokers and two blank cards. What is the probability of drawing a joker or a blank card?

Ans. $\frac{1}{11}$.

116. For a game, the entry fee is ` 5. The game consists of tossing a coin three times. If one or two heads show up then Shweta gets back her entry free. If she tosses three heads, she receives double the entry fee or else she loses. After tossing the coin thrice, find the probabilities that she (i) loses the entry fee, (ii) gets double the entry fee and (iii) gets her entry free back.

Ans. (i) $\frac{1}{8}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{4}$.

117. A die has six faces marked as 0, 1, 1, 1, 6, 6. Two such dices are thrown together and the total score is recorded. (i) How many different scores are possible ? (ii) What is the probability of getting a total of 7?

Ans. (i) 6 (ii) $\frac{1}{3}$.

118. A bag contains white, black and red balls only. A ball is drawn at random from the bag. The probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$. Find the probability of getting a red ball if the bag contains 20 black balls. Also find the total number of balls.

Ans. $\frac{3}{10}$, 50.

119. A bag contains 24 balls of which *x* are red, 2*x* are white and 3*x* are blue. A ball is selected at random. What is the probability that the ball drawn is (i) not white, (ii) blue?

Ans. (i) $\frac{2}{3}$ (ii) $\frac{1}{2}$.

120. The probability of guessing the correct answer to a certain test is $\frac{P}{12}$. If the probability of not guessing the correct answer to this question is $\frac{1}{3}$. Find the value of P.

Ans. 8.

121. A number is selected at random from the numbers 3, 5, 5, 7, 7, 7, 9, 9, 9, 9. Find the probability that the selected number is their average.

Ans. $\frac{3}{10}$.

122. If a number *x* is chosen from the sequence 1, 2, 3 and a number *y* is chosen from the sequence 1, 4, 9, find the probability that $xy = 10$.

Ans. 0.

123. If 65% of the population has black eyes, 25% have brown eyes and the remaining have blue eyes, what is the probability that a person selected at random has (i) blue eyes, (ii) brown or black eyes, (iii) blue or black eyes and (iv) neither blue nor brown eyes?

Ans. (i) $\frac{1}{10}$ (ii) $\frac{9}{10}$

(iii) $\frac{3}{4}$ (iv) $\frac{13}{20}$.

124. If a number *x* is chosen from the sequence 1, 2, 3 and a number *y* is chosen from the sequence 1, 4, 9, find the probability that *xy* will be less than 9?

Ans. $\frac{5}{9}$.

125. A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble at random is $\frac{1}{3}$ and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does the jar contain?

Ans. 12.

126. In a game of musical chair, the person playing the music has been asked to stop the music within 2 minutes. What is the probability that she would stop the music within the first half minute?

Ans. $\frac{1}{4}$.

127. What is the probability that the month of june will have 5 Mondays in a (i) Leap Year and (ii) non-Leap Year?

Ans. (i) $\frac{2}{7}$, (ii) $\frac{2}{7}$.

128. What is the probability that the month of February will have 5 wednesdays in a (i) Leap Year and (ii) non-Leap Year?

Ans. (i) $\frac{1}{7}$ (ii) 0.

129. A number x is chosen from the numbers – 4, – 3 – 2, – 1, 0, 1, 2, 3, 4. Find the probability that $x < 3$.

Ans. $\frac{7}{9}$.