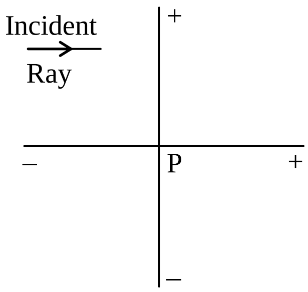
RAY OPTICS : CONCPTS of Lens by Dr. MUKESH SHRIMALI

Theory of Spherical refracting surfaces and Lens

1.1 Cartesian sign convention



(1) All distances are measured from the pole (P).

(2) Distances measured in the direction of incident rays are taken as positive.

(3) Distances above the principal axis are taken as positive.

(4) Angles measured from the normal in the anticlockwise sense are positive.

(Please see video lecture on you tube.com by Dr. Mukesh on basic concepts of lens https://youtu.be/oG_B9e5u-kY

➢ SPHERICAL LENS

- **Definition :** A piece of a transparent medium bounded by at least one spherical surface, is called a spherical lens.

- **Types :** There are two types of spherical lenses:

(i) **Convex or Converging Lenses :** These are thick in the middle and thin at the edges.

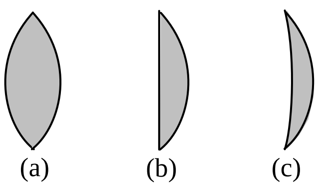


Fig. Three types of convex lenses

(a) **Double Convex Lens :** It has both the surfaces convex.

(b) **Plano–Convex Lens :** It has one surface plane and the other surface convex.

(c) **Concavo–Convex Lens :** It has one surface concave and the other surface convex.

(ii) **Concave or Diverging Lenses :** These are thin in the middle and thick at the edges.

There are three types of concave lenses :

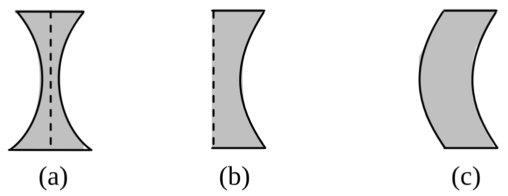


Fig. Three types of concave lenses

(a) **Double Concave Lens :** It has both the surfaces concave. (Fig.)

(b) **Plano–Concave Lens :** It has one surface plane and the other surface concave. (fig.)

(c) **Convexo–Concave Lens :** It has one surface convex and the other surface concave. (fig.)

❖ SOME ASSOCIATED TERMS :

(i) Centre of curvature (C) :

The centre of curvature of the surface of a lens is the centre of the sphere of which it forms a part, because a lens has two surfaces, so it has two centres of curvature. In figure (a) and (b) points, C_1 and C_2 are the centres of curvature.

(ii) Radius of curvature (R) :

The radius of curvature of the surface of a lens is the radius of the sphere of which the surface forms a part. R_1 and R_2 in the figure (a) and (b) represents radius of curvature.

(iii) Principal axis (C_1C_2) :

It is the line passing through the two centres of curvature (C_1 and C_2) of the lens.

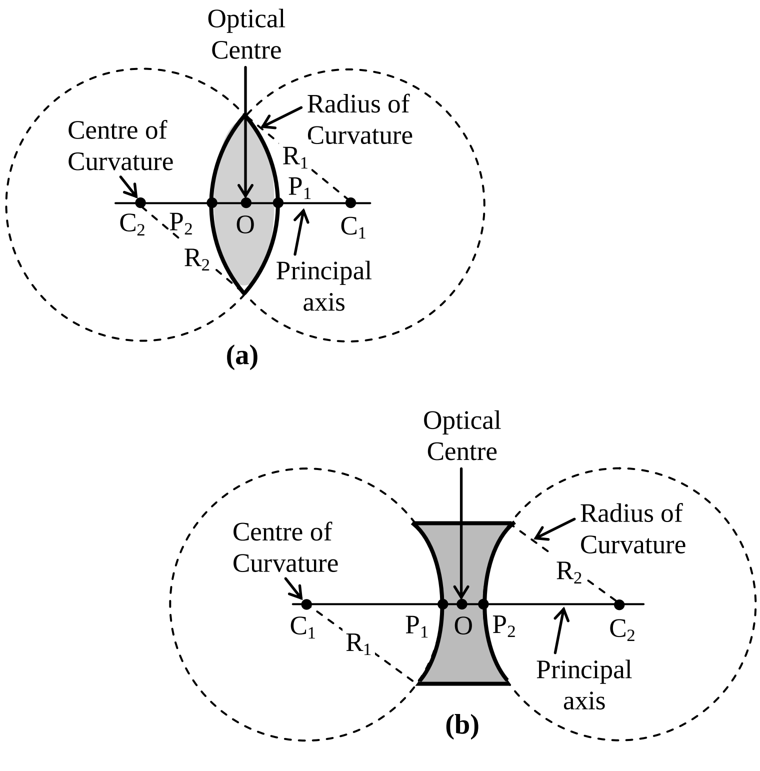


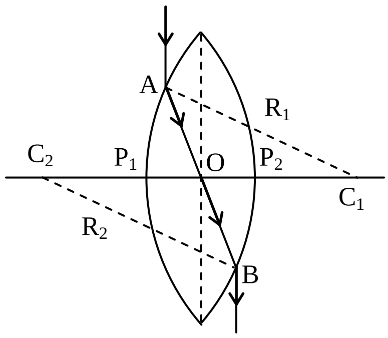
Figure : Characteristics of convex and concave lenses

(iv) Optical centre :

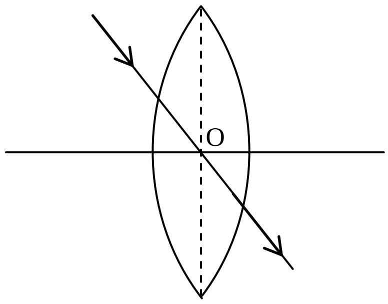
If a ray of light is incident on a lens such that after refraction through the lens the emergent ray is parallel to the incident ray, then the point at which the refracted ray intersects, the principal axis is called the optical centre of the lens.

$$\frac{OP_1}{OP_2} = \frac{P_1C_1}{P_2C_2} = \frac{R_1}{R_2}$$

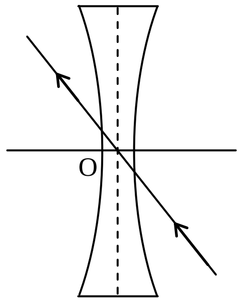
If the radii of curvature of the two surfaces are equal, then the optical centre coincides with the geometric centre of the lens.



(a)



(b)

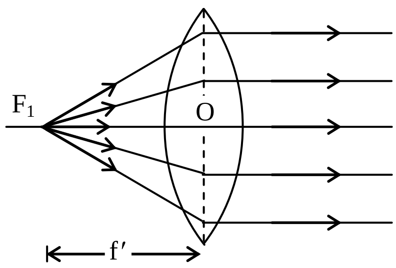


(c)

(v) Principal foci and focal length :

(A) First principal focus and first focal length :

It is a fixed point on the principal axis such that rays starting from this point (in convex lens) or appearing to go towards this point (concave lens), after refraction through the lens, become parallel to the principal axis. It is represented by F_1.



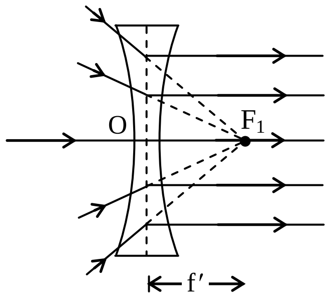
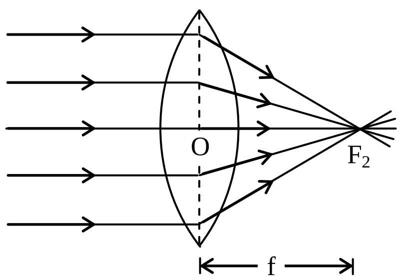


Figure : Ray diagram showing first principal focus

(B) Second principal focus and second focal length :

It is a fixed point on the principal axis such that the light rays incident parallel to the principal axis, after refraction through the lens, either converge to this point (in convex lens) or appear to diverge from this point (in concave lens). It is denoted by F_2.



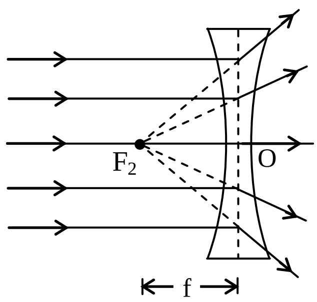


Figure : Ray diagram showing second principal focus

If the medium on both sides of a lens is same, then the numerical values of the first and second focal lengths are equal. Thus

$$f = f'$$

(vi) Aperture :

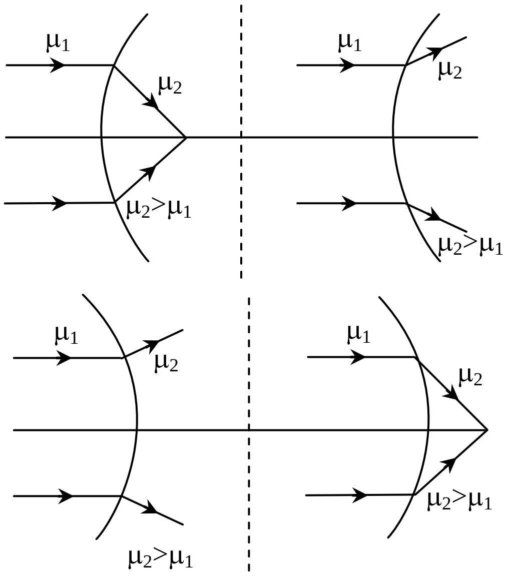
It is the diameter of the circular boundary of the lens.

Note :

(1) It is not always necessary that for convex boundary the parallel rays always converge. Similarly for concave boundary the incident parallel ray may converge or diverge depending upon the refractive index of two media,

(2) Laws of refraction are valid for spherical surface

(3) Pole, centre of curvature, Radius of curvature, Principal axis etc. are defined as spherical mirror except for the focus.



2. LATERAL MAGNIFICATION

It is defined as

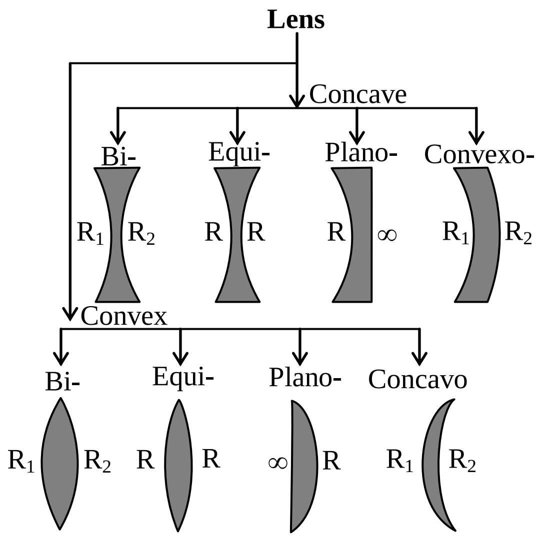
$$m = \frac{h_i}{h_o} = \frac{\text{height of image}}{\text{height of object}} = \frac{\mu_1}{\mu_2} \cdot \frac{v}{u}$$

3. LENS THEORY

3.1 Definition :

A lens is a piece of transparent material with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

3.2 Types of lenses :



Depending upon the shape of the refracting surfaces following types of lenses can be formed:

3.3 Terms related to thin spherical lenses :

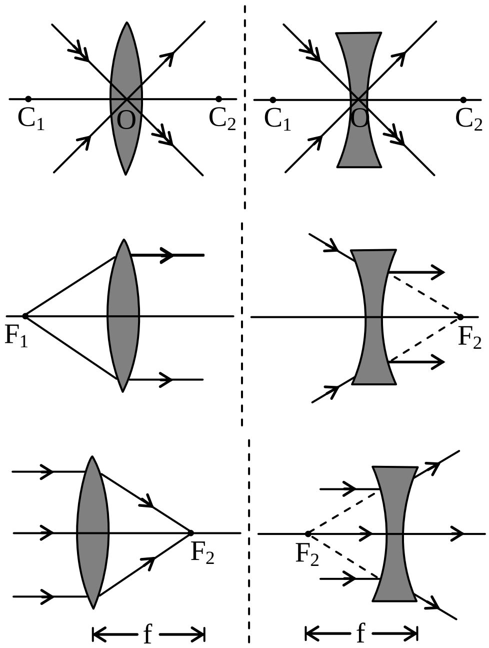
(A) Optical centre (O) is a point for a given lens through which any ray passes undeviated.

(B) Principal Axis (C_1C_2) is a line passing through optical centre and perpendicular to the lens. The centre of curvature of curved surfaces always lies on the principal axis .

(C) Principal Focus :

A lens has two surfaces causing two focul points

(i) **First focal point** is an object point on the principal axis whose image is formed at infinity.



(ii) **Second focal point** is an image point on the principal axis whose object lies at infinity.

(D) Focal length (f) is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appear to converge.

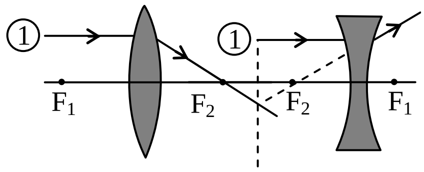
(E) Aperture : In reference to a lens, aperture means the effective diameter of its light transmitting area. So the brightness,i.e., intensity of image formed by a lens which depends on the light passing through the lens will depend on the square of aperture, i.e.,

$I \propto (\text{aperture})^2$

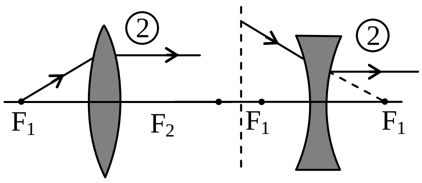
Ray diagram :

Graphically we can locate the position of image for a given object by drawing any two of the following three rays.

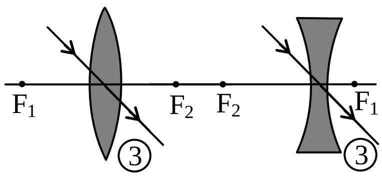
(A) A ray, initially parallel to the principal axis will pass (or appear to pass) through focus.



(B) A ray which initially passes (or appear to pass) through focus will emerge from the, lens parallel to the principal axis.



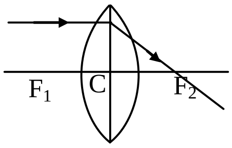
(C) A ray passing through the optical centre of the lens goes undeviated as it passes through the lens region.



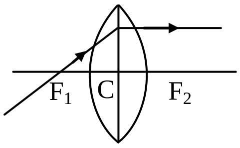
➢ RULE FOR IMAGE FORMATION BY RAY DIAGRAM METHOD

◈ THREE SPECIAL RAYS FOR CONVEX LENS

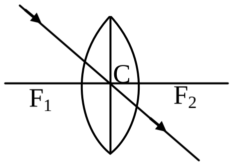
- When light ray incident parallel to principal axis.



- When light ray incident from focus.

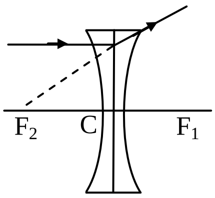


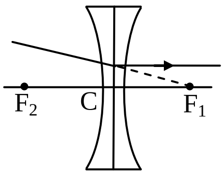
- When light ray incident on the pole.

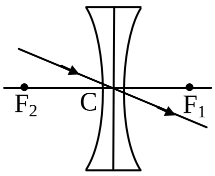


◈ THREE SPECIAL RAYS FOR CONCAVE LENS

- When light ray incident parallel to principal axis.







➢ IMAGE FORMATION BY LENS

- **Introduction :** From lens formula, we find that for a lens of a fixed focal length as object distance u changes, image distance ν also changes. Moreover, as u decreases or increases, this changes the position, the nature and the size of the image.

  Different cases, are as given below with their ray diagrams.

◈ CONVEX LENS IN DIFFERENT CASES

Case 1 : Object at Infinity

- **A point object lying on the principal axis**

  Rays come parallel to the principal axis and after refraction from the lens, actually meet at the second principal focus F_2.

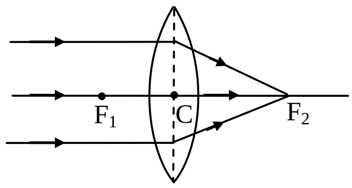


Fig. Convex lens point object at infinity, image at focus.

The image is formed at focus F_2. It is real and point sized.

- **A big size object with its foot on the principal axis**

  Parallel rays come inclined to the principal axis. Image of foot is formed at the focus.

  Image is formed at the second principal focus F_2. It is real inverted and diminished (smaller in size than the object). (Fig.)

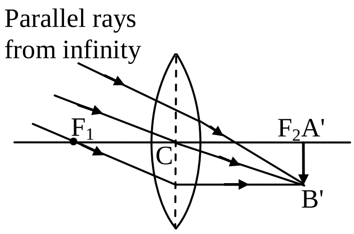


Fig. Convex lens : big size object at infinity, image at focus

Case 2 : Object at distance more than twice the Focal Length

Real object AB has its image A′B′ formed between distance f and 2f.

The image is real inverted and diminished (smaller in size than the object)

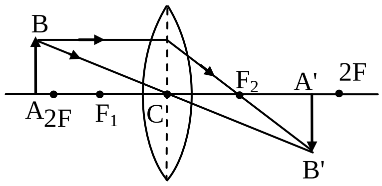


Fig. Convex lens : object beyond 2f, image between f and 2f.

Case 3. Object at distance twice the Focal Lengths

Real object AB has its image A′B′ formed at distance 2f.

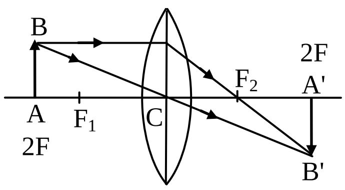


Fig. Convex lens : object at distance 2f, image at distance 2f.

The image is real, inverted and has same size as the object.

Case 4 : Object at distance more than Focal Length and less than twice is Focal Length

Real object AB has its image A′B′ formed beyond distance 2f.

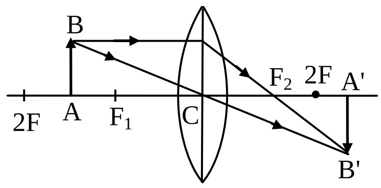


Fig. Convex lens : object at distance between f and 2f, image beyond 2f.

The image is real inverted and enlarged (bigger in size than the object).

Case 5 : Object at Focus

Real object AB has its image formed at infinity.

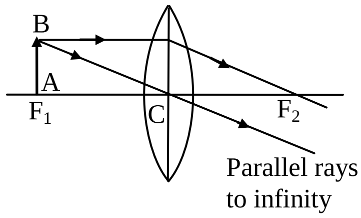
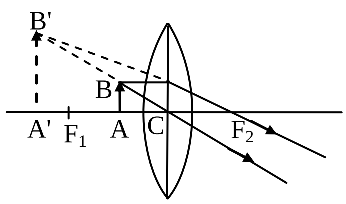


Fig. Convex lens : object at focus, image at infinity.

The image is imaginary inverted (refracted rays to downward) and must have very large size.

Case 6 : Object between Focus and Optical Centre

Real object AB has its image A′B′ formed in front of the lens.



◈ CONCAVE LENS IN DIFFERENT CASES

Case 1 : Object at infinity

- A point object lying on the principal axis.

  Rays come parallel to the principal axis and after refraction from the lens, appears to come from the second principal focus F_2.

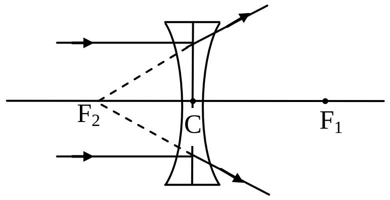


Fig. Concave lens point object at infinity, image at focus.

The image is formed at focus F_2. It is virtual and point sized (fig.)

- A big size object with its foot on the principal axis.

  Parallel rays come inclined to the principal axis. Image of foot is formed at focus.

  The image is formed at the second principal focus F_2.

  It is virtual–erect and diminished (fig.)

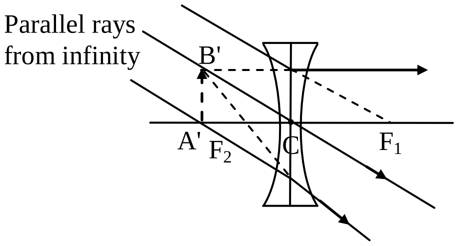


Fig. Concave lens : big size object at infinity image at focus.

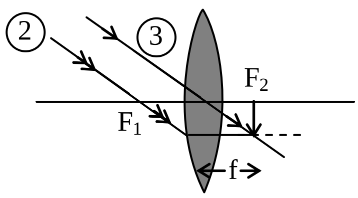
Case 2 : Object at a Finite Distance

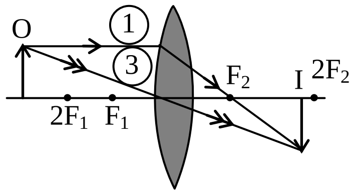
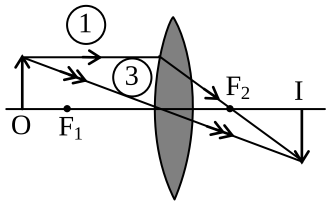
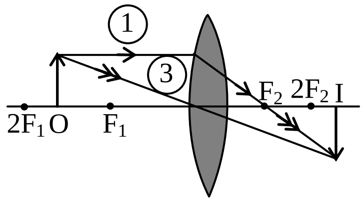
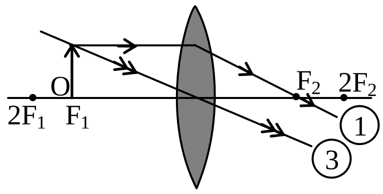
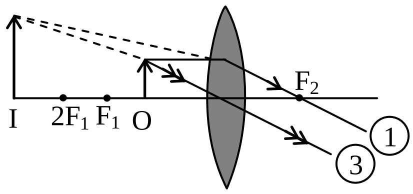
Real object AB has its image A′B′ formed between second principal focus F_2 optical centre C.

The image is virtual–erect and diminished.

(To understand difference between convex and concave lens please see video lecture by Dr. Mukesh Shrimali on youtube.com on link https://youtu.be/n6A-TiIRtvM

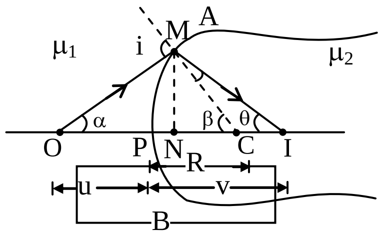
https://youtu.be/n6A-TiIRtvM

S.No.	**Position of Object**	**Position of image**	**Ray – Diagram**	***Nature (size)***
1.	At infinity	At focus		*Real, inverted, [Highly Diminished] (m << –1)*

2.	Between ∞ and 2F	Between F & 2F		*Real, inverted, [Diminished (m < –1)]*
3.	At 2F	At 2F		*Real, inverted, [Equal (m = –1)]*
4.	Between 2F & F	Between 2F & ∞		*Real, inverted, [Enlarged (m > –1)]*
5.	At F	At ∞		*Real, inverted, [Highly Enlarged (m >> –1)]*
6.	*Between F and P*	*Between ∞ & O on same side*		*Virtual, erect [Enlarged (m > +1)]*

REFRACTION ON SPHERICAL REFRACTING SURFACES

Suppose AB is the spherical surface which separates the two medium of refractive index μ_1 and μ_1 .O and I is the position of object and image in medium of R.I. μ_1 and μ_2 respectively.



If Object lies in rarer medium then For Rarer to denser medium $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

IF OBJECT LIES IN DENSER MEDIUM THEN FOR DENSER TO RARER MEDIUM

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

(For derivation of these formula please see video lecture of Dr. Mukesh Shrimali on youtube.com

https://youtu.be/2LtQEMp5L8M)

(A) LENS FORMULA

- **Definition :** The equation relating the object distance (u), the image distance (v) and the focal length (f) of the lens is called the lens formula.

- **Assumptions made :**

1. The lens is **thin.**
2. The lens has a **small aperture.**
3. The object lies **close** to principal axis.
4. The incident rays make **small angles** with the lens surface or the principal axis.

The object and image distances of a lens are related to each other as :

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ Where,

https://youtu.be/R4hrgV9pDiU

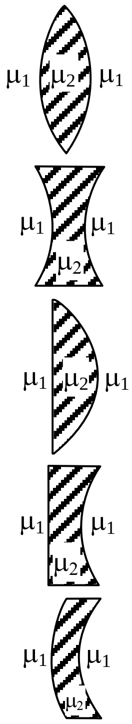
(B) Lens Makers' Formula

$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (Lens Maker's formula)

(For derivation of these formula please see video lectures of Dr. Mukesh Shrimali on youtube.com https://youtu.be/BMzhYZ5FTd8

Nature of various lenses depending upon their surroundings :

If μ_1 is the R.I of surrounding and μ_2 is that of lens then

Shape of lens	*Nature of lens*	
	For $\mu_1 < \mu_2$	*For* $\mu_1 > \mu_2$
	Converging	*Diverging*
	Diverging	*Converging*
	Converging	*Diverging*
	Diverging	*Converging*
	Depends on the radius of curvature of the first and second surface	

Power of a lens :

(C) POWER OF LENS

- **Definition :** It is the capacity or the ability of a lens to deviate (converge or diverge) the path of rays passing through it. A lens producing more converging or more diverging, is said to have more power

  Power of lens (in diopter) $\propto \dfrac{1}{\text{f (in metre)}}$

When focal length is written in metre . In Medium

P = $\frac{\mu}{f}$ D is known as the power of the lens.

Where D is (diopter) unit of power and μ is the refractive index of the medium in which the lens is placed.

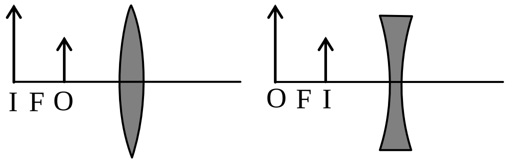
For calculation purpose if focal length in cm then

P = $\dfrac{100}{f}$ D

Important concepts of lens :

Converging lens is represent as (A)

while diverging lens in represented as (B)



(A) (B)

(A) For real extended objects if the image formed by a single lens is erect (i.e. m is positive) it is always virtual. In this situation if the image is enlarged the lens is converging (i.e. convex) with object between focus and optical centre and if diminished the lens is diverging (i.e. concave) with image between focus and optical centre.

(B) As every part of a lens forms complete image, if a portion (say lower half) is obstructed (say covered with black paper) full image will be formed but brightness i.e. intensity will be reduced (to half). Also if a lens is painted with black strips and a donkey is seen through it, the donkey will not appear as a zebra but will remain a donkey with reduced intensity.

(C) If L is the distance between a real object and its real image formed by a lens, then as

$L = (|u| + |v|) = (\sqrt{u} - \sqrt{v})^2 + 2\sqrt{uv}$

So L will be minimum when

$(\sqrt{u} - \sqrt{v})^2 = \min = 0$ i.e. u = v

On substituting u = – u and v = +u in lens formula, we get

$\frac{1}{u} - \frac{1}{-u} = \frac{1}{f}$ i.e. u = 2f

So that $(L)_{min} = 2\sqrt{2f \times 2f} = 4f$

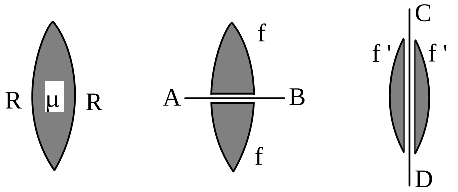
[as L_{min}, u = v]

i. e. the minimum distance between a real object and its real image formed by a single lens is 4f.

(D) If an object is moved at constant speed towards a convex lens from infinity to focus, the image will move slower in the beginning and faster later on, away from the lens. This is because in the time the object moves from infinity to 2F, the image will move from F to 2F and when the object moves from 2F to F, the image will move from 2F to infinity. At 2F the speed of object and image will be equal. It can be shown that in case of lens, speed of image.

$V_i = V_0 \left[\frac{f}{u+f}\right]^2$

(E) If an equi-convex lens of focal length f is cut into two equal parts by a horizontal plane AB, then as none of μ, R_1 and R_2 will change, the focal length of each part be equal to that of initial lens, i.e. (**For detail explination of this concept please see video lecture on youtube.com by Dr. Mukesh Shrimali)**



$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{-R}\right] = \frac{2(\mu - 1)}{R}$

However in this situation as light transmitting area of each part becomes half of initial, so intensity will be reduced to half and aperture to $(1/\sqrt{2})$ times of its initial value

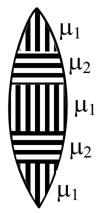
However, if the same lens is cut into two equal parts by a vertical plane CD, the focal length of each part will become.

$\frac{1}{f'} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] = \frac{(\mu - 1)}{R} = \frac{1}{2f}$

i.e., $f' = 2f$

i.e., focal length of each part will be double of initial value. In this situation as the light transmitting area of each part of lens remains equal to initial, intensity and aperture will not change.

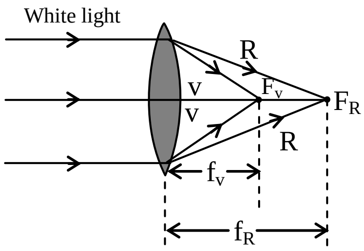
(F) If a lens is made of a number of layers of different refractive indices as shown in figure for a given wavelength of light it will have as many focal

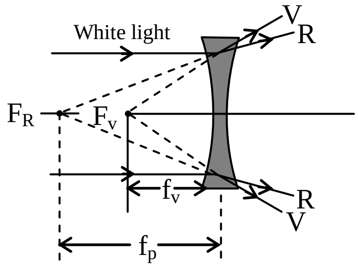


lengths or will form as many

images as there are μ's as $\frac{1}{f} \propto (\mu - 1)$

(G) As focal length of a lens depends on μ, i.e. $(1/f) \propto (\mu - 1)$, the focal length of a given lens is different for different wavelengths and maximum for red and minimum for violet whatever be the nature of lens.





(H) If a lens of glass ($\mu = 3/2$) is shifted from air ($\mu = 1$) to water ($\mu = 4/3$) then as :

$$\frac{1}{f_A} = \left[\frac{3/2}{1} - 1\right] K \text{ and}$$

$$\frac{1}{f_V} = \left[\frac{(3/2)}{(4/3)} - 1\right] K$$

with $K = \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

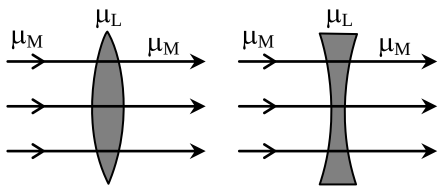
$$\frac{f_W}{f_A} = \left[\frac{8}{K}\right] \times \left[\frac{K}{2}\right] \text{ i.e. } f_W = 4f_A$$

i. e. focal length of a lens in water becomes 4 times of its value in air.

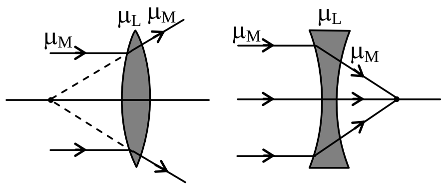
(I) If a lens is shifted from one medium to the other, depending on the refractive index of the lens and medium, the following three situations are possible.

(i) **$\mu_M < \mu_L$ but μ_M increases :** In this situation $\mu = (\mu_L/\mu_M)$ will remain greater than unity but will decrease and as $(1/f) \propto (\mu - 1)$, $(1/f)$ will decrease i.e., f will increase (without change in nature of lens)

(ii) **$\mu_M = \mu_L$:** In this situation $\mu = (\mu_L/\mu_M) = 1$, so that $(1/f) \propto (\mu - 1) = 0$, i.e., $f = \infty$, i.e., lens will neither converge nor diverge but will behave as a plane glass plate.



(iii) **$\mu_M > \mu_L$:** In this situation $\mu = (\mu_L / \mu_M) < 1$, so in **Lens-maker's formula** sign of f and hence nature of lens will change, i.e. a converging lens will behave as divergent and vice-versa.



4. COMBINATION OF LENSES

4.1 When lenses are in contact with each other (For Derivation of these formula please see video lecture by Dr. Mukesh Shrimali on youtube.com https://youtu.be/7DSNRDWbV1o

When several lenses are kept co-axially , the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens, the image formed by the second acts as an object the third and so on .

(A) Net magnification , m = $m_1 \times m_2 \times m_3 \times .....$

(B) If thin lenses are kept close together with their principal axis coinside then,

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots\ldots}$$

$$\boxed{\text{or } P = P_1 + P_2 + P_3 + \ldots\ldots\ldots}$$

1. If the two thin lenses are separated by a distance ' d ' then, $P = P_1 + P_2 - dP_1P_2$

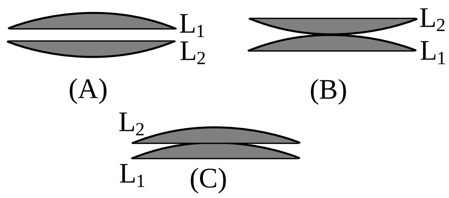
2. If a lens of focal length f is divided into two equal parts as shown in Fig.(A) and each part has a focal length f ' then as

$$\frac{1}{f} = \frac{1}{f'} + \frac{1}{f'} \qquad \text{i.e., } f' = 2f$$

i.e., each part will have focal length 2f.

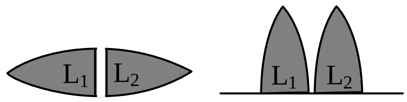
Now if these parts are put in contact as in Fig. (B) or (C) the resultant focal length of the combination will be

$$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f} \qquad \text{i.e., } F = f \text{ (= initial value)}$$



3. If a lens of focal length f is cut in two equal parts as shown in figure (A) each part will have focal length f. Now if these parths are put in contact as shown in figure (B) the resultant focal length will be.

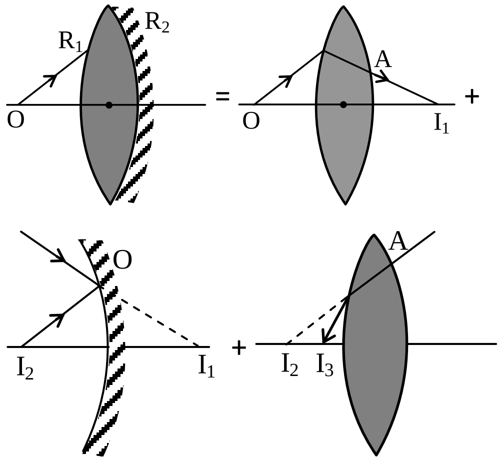
$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} \quad \text{i.e. } F = (f/2)$$



(A) (B)

5. LENS WITH ONE SURFACE SILVERED

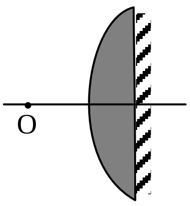
A . Silvered convex lens :



$$\boxed{P = \frac{2}{R}[2\mu - 1]} \qquad \boxed{f = \frac{-R}{2[2\mu - 1]}}$$

B. When the plane surface is silvered:

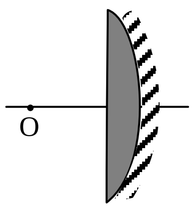
i.e., $\boxed{P = 2\frac{(\mu - 1)}{R} + 0 = \frac{2(\mu - 1)}{R}}$



$$\boxed{f = -\frac{1}{P} = -\frac{R}{2(\mu - 1)}} \qquad ...(2)$$

C. When the curved surface is silvered :

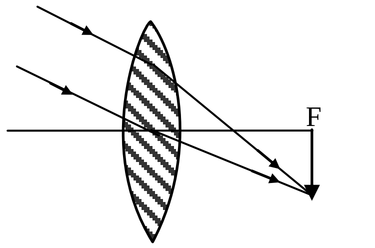
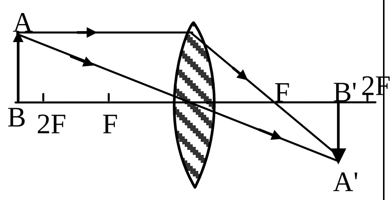
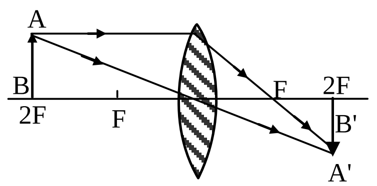
Here the object is in front of plane surface In this situation

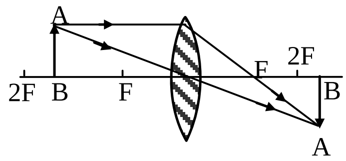
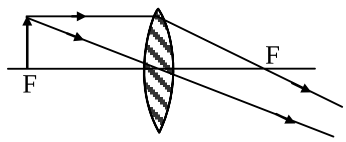
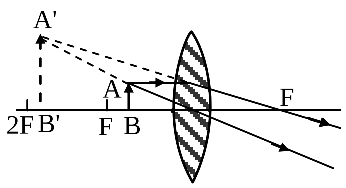


i.e., $\boxed{P = \frac{2(\mu-1)}{R} + \frac{2}{R} = \frac{2\mu}{R}}$ .....(3)

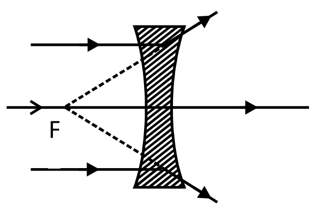
So $\boxed{f = -\frac{1}{P} = -\frac{R}{2\mu}}$ .....(4)

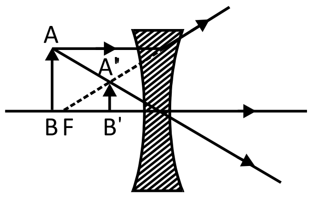
Convex Lens

Position of object	Position of image	Real/ virtual	Inverted/ erect	Magnification and size of image	Sign of magnification	RAY DIAGRAM
at infinity ($u = \infty$)	at focus ($v = f$)	real	inverted	m < 1 greatly diminished	negative	
beyond 2f($u > 2f$)	between f and 2f ($f < v < 2f$)	real	inverted	m < 1 diminished	negative	
at 2f ($u = 2f$)	at 2f ($v = 2f$)	real	inverted	m = 1 same size	negative	

between f and 2f ($f < u < 2f$)	beyond 2f($v > 2f$)	real	inverted	$m > 1$ magnified	negative	
at f($u = f$)	at infinity ($v = \infty$)	real	inverted	$m = \infty$ magnified	negative	
between optical centre and focus ($u < f$)	at a distance greater than the object distance and on the same side as object ($v > u$)	virtual	erect	$m > 1$ magnified	positive	

Concave Lens

at infinity ($u = \infty$)	at focus ($v = f$)	virtual	erect	$m < 1$ diminished	positive 

between infinity and optical centre	between optical centre and focus	virtual	erect	m < 1 diminished	positive 

Solved numerical problems on lens and related phenomena

Ex.1 An object is placed 36 cm from a convex lens. A real image is formed 24 cm from the lens. Calculate the focal length of the lens.

Sol. According to the sign convention the object is placed on the left-hand side of the lens. So object distance (u) is negative. Real image is formed on the other side of the lens. So the image distance (v) is positive. Thus, u = –36 cm, v = +24 cm, f = ?

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{+24} - \frac{1}{-36} = \frac{1}{f}$$

or $$\frac{1}{f} = \frac{1}{24} + \frac{1}{36} = \frac{5}{72}$$

$\therefore$ $$f = \frac{72}{5} = 14.4 \text{ cm}$$

Ex.2 A 2 cm long pin is placed perpendicular to the principal axis of a lens of focal length 15 cm at distance of 25 cm from the lens. Find the position of image and its size.

Sol. Here, u = –25 cm, f = +15

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

or $$\frac{1}{v} - \frac{1}{-25} = \frac{1}{+15}$$

or $$\frac{1}{v} = \frac{1}{15} - \frac{1}{25} = \frac{2}{75}$$

or $$v = \frac{75}{2} = 37.5 \text{ cm}$$

The positive sign shows that the image is formed on the right-hand side of the lens.

Magnification is given by

$$m = \frac{h'}{h} = \frac{v}{u}$$

or $= \frac{h'}{h} = \frac{37.5}{-25} = -1.5$

$\therefore$ $h = -1.5 \times h = -1.5 \times 2$ cm

$= -3$ cm

The image of the pin is 3 cm long. The negative sign shows that it is formed below the principal axis, i.e. the image is inverted.

Ex.3 A point object is placed at a distance of 18 cm from a convex lens on its principal axis. Its image is formed on the other side of the lens at 27 cm. Calculate the focal length of the lens.

Sol. According to the sign convention, the object is placed on the left-hand side of the lens, therefore object-distance is negative,

i.e. u = –18 cm. Since the image is formed on the other side, the image-distance is positive, i.e., v = +27 cm. Using lens formula,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have

$$\frac{1}{+27} - \frac{1}{-18} = \frac{1}{f}$$

or $\frac{1}{27} + \frac{1}{18} = \frac{5}{54} = \frac{1}{f}$

or $f = \frac{54}{5} = 10.8$ cm

Ex.4 A convex lens forms an image of the same size as the object at a distance of 30 cm from the lens. Find the focal length of the lens. Also find power of the lens. What is the distance of the object from the lens ?

Sol. A convex lens forms the image of the same size as the object only when the object is placed at a distance 2f from the lens. In this case the image is also equal to 2f from the lens.

Hence, 2f = 30 cm

or f = 15 cm = 0.15 m

Power of the lens, $P = \frac{1}{f} = \frac{1}{0.15}$ D = 6.6D

The distance of the object from the lens is also 2f = 30 cm.

Ex.5 A 3 cm high object is placed at a distance of 80 cm from a concave lens of focal length 20 cm. Find the position and size of the image.

Sol. Here, u = –80 cm, f = – 20 cm

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

or $\frac{1}{v} - \frac{1}{-80} = \frac{1}{-20}$

or $\frac{1}{v} = -\frac{1}{20} - \frac{1}{80} = \frac{-5}{80} = -\frac{1}{16}$

or $v = -16$ cm

Magnification, $m = \frac{h'}{h} = \frac{v}{u} = \frac{-16}{-80} = \frac{1}{5}$

or $h' = \frac{h}{5} = \frac{3.0}{5} = 0.6$ cm

Length of image is 0.6 cm. Positive sign shows that the image is erect.

Ex.6 An object is placed on the principal axis of a concave lens at a distance of 40 cm from it. If the focal length of the lens is also 40 cm, find the location of the image and the magnification.

Sol. For a concave lens focal length f is negative, i.e. f = –40 cm. Since by convention, object is placed on the left of the lens, so u = – 40 cm.

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

or $\frac{1}{v} - \frac{1}{-40} = \frac{1}{-40}$

or $\frac{1}{v} = -\frac{1}{40} - \frac{1}{40} = -\frac{1}{20}$

or $v = -20$ cm

The image is formed 20 cm from the lens. Minus sign shows that the image is formed on the same side of the lens as the object.

Now, magnification, $m = \frac{h'}{h} = \frac{v}{u} = \frac{-20}{-40} = \frac{1}{2}$

Positive sign shows that the image is erect.

Ex.7 A beam of light travelling parallel to the principal axis of a concave lens appears to diverge from a point 25 cm behind the lens after refraction. Calculate the power of the lens.

Sol. When a parallel beam after refraction through the lens is incident on a concave lens, it appears to diverge from the focus of the lens. Hence, the focal length of the lens is 25 cm. According to sign convention, focal length of a concave lens is negative.

$\therefore$ f = –25 cm = –0.25 m

$\therefore$ Power, $P = \frac{1}{f} = \frac{1}{-0.25} = -4D$

Ex.8 A convex lens of power 5D is placed at a distance of 30 cm from a screen. At what distance from the lens should the screen be placed so that its image is formed on the screen?

Sol. Power of the lens, $P = +5D$

$\therefore$ Focal length, $f = \frac{1}{5D} = \frac{1}{5} = 0.20$ m = 20 cm

Here, the screen is placed 30 cm from the lens.

$\therefore$ $v = +30$ cm, $f = +20$ cm, $u = ?$

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{30} - \frac{1}{u} = \frac{1}{20}$$

or $$\frac{1}{u} = \frac{1}{30} - \frac{1}{20} = -\frac{1}{60}$$

or $$u = -60 \text{ cm}$$

Therefore, the screen should be placed at 60 cm from the lens.

Ex.9 A pin 3 cm long is placed at a distance of
24 cm from a convex lens of focal length 18 cm. The pin is placed perpendicular to the principal axis. Find the position, size and nature of the image.

Sol. Here, $u = -24$ cm, $f = +18$ cm, $v = ?$

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{-24} = \frac{1}{+18}$$

or $$\frac{1}{v} = \frac{1}{18} - \frac{1}{24} = \frac{1}{72}$$

or $$v = 72 \text{ cm}$$

The image is formed 72 cm from the lens on the other side. So the image is real.

Magnification, $m = \frac{h'}{h} = \frac{v}{u} = \frac{72}{-24} = -3$

or $h' = -3 \times h = -3 \times 3.0 = -9$ cm

The image is 9 cm in size. Negative sign shows that the image is inverted.

Ex.10 A convex lens of focal length 40 cm and a concave lens of focal length 25 cm are placed in contact in such a way that they have the common principal axis. Find the power of the combination.

Sol. Focal length of the convex lens,

$f_1 = 40$ cm = +0.4 m

$\therefore$ Power of the convex lens,

$$P_1 = \frac{1}{+0.40} = +2.5D$$

Focal length of the concave lens,

$$f_2 = -25 \text{ cm} = -0.25 \text{ m}$$

$\therefore$ Power of the concave lens,

$$P_2 = \frac{1}{-0.25} = -4D$$

Power of the combination,

$$P = P_1 + P_2 = 2.5 - 4D = -1.5D$$

Ex.11 A concave lens has a focal length of 15 cm. At what distance should the object be from the lens placed so that it forms an image 10 cm from the lens ? Also find the magnification.

Sol. A concave lens always forms a virtual, erect image on the same side as the object.

Image distance, $v = -10$ cm

Focal length $f = -15$ cm

Object distance, $u = ?$

Using, the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

or $$\frac{1}{-10} - \frac{1}{u} = \frac{1}{-15}$$

or $$-\frac{1}{10} = \frac{2-3}{30} = -\frac{1}{30}$$

or $$u = -30 \text{ cm}$$

Thus, the object should be placed 30 cm on the lens.

Magnification, $$m = \frac{v}{u} = \frac{-10}{-30} = \frac{1}{3} = 0.33$$

The positive sign shows that the image is erect and virtual. The size of the image is one-third of that of the object.

Ex.12 A 2 cm tall object is placed perpendicular to the principal axis of a convex lens of focal length 10 cm. The distance of object from the lens is 15 cm. Find the position, nature and size of the image. Calculate the magnification of the lens.

Sol. Object distance, $u = -15$ cm

Focal length, $f = +10$ cm

Object height, $h = +2$ cm

Image distance, $v = ?$

Image height, $h' = ?$

Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} = \frac{1}{-15} = \frac{1}{+10}$$

or $$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{+1}{30}$$

or $$v = +30 \text{ cm}$$

Positive sign of v shows that the image is formed at a distance of 30 cm on the right side of the lens. Therefore the image is real and inverted

Magnification, $m = \frac{h'}{h} = \frac{v}{u}$

$$\frac{h'}{2.0} = \frac{+30}{-15} = -2$$

or $h' = -2 \times 2 = -4$ cm

Magnification, $m = \frac{v}{u} = \frac{30}{-15} = -2$

Negative sign with the magnification and height of the image shows that the image is inverted and real. Thus, a real image of height 4 cm is formed at a distance of 30 cm on the right side of the lens. Image is inverted and twice the size of the object.

Ex 13. A bi-convex lens is made from glass of refractive index 1.5 and radius of curvature of both surfaces of the lens is 20 cm. The incident ray parallel to principal axis will be focussed at a distance L cm from lens on principal axis where :

(A) L = 10 (B) L = 20

(C) L = 40 (D) L = 20/3

Sol. From lens formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= 0.5\left[\frac{1}{R} - \left(\frac{1}{-R}\right)\right] = \frac{0.5 \times 2}{20}$$

f = 20 cm

Therefore rays coming parallel to axis will form image at 20 cm.

Hence correct answer is (B).

Ex14. A convex lens of power 4D is kept in contact with a concave lens of power 3D, the effective power of combination will be :

(A) 7D (B) 4D/3(C) 1D (D) 3D/4

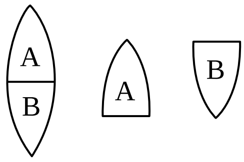
Sol. Effective power P is

$= P_1 + P_2$

$= 4 - 3 = 1D$

Hence correct answer is (C).

Ex.15 The power of a plano-convex lens is P. If this lens is cut longitudinally along its principal axis into tow equal parts and then they are joined as given in the figure. The power of combination will be :



(A) P (B) 2P (C) P/2 (D) zero

Sol. One part of combination will behave as converging lens and the other as diverging lens of same focal length. As such total power will be zero.

Hence correct answer is (D).

Ex16. A convex lens is made out of a substance of 1.2 refractive index. The two surfaces of lens are convex. If this lens is placed in water whose refractive index is 1.33, it will behave as :

(A) convergent lens (B) divergent lens

(C) plane glass plate (D) like a prism

Sol. The focal length of lens in water is given by

$$f_l = \frac{{}_a\mu_g - 1}{\frac{{}_a\mu_g}{{}_a\mu_\ell} - 1} f_a = \frac{1.2 - 1}{\frac{1.2}{1.33} - 1} f_a$$

$$f_l = -\frac{0.2 \times 1.33}{0.13} f_a$$

Hence f_l is negative and as such it behaves as a divergent lens.

Hence correct answer is (B).