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SAMPLE PAPER TEST – 06 (2020-21)
(SAMPLE ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : XII **DURATION : 3 HRS**

General Instruction:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections**- I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections**- III, IV and V.**
2. Section **III** comprises of 10 questions of **2 marks** each**.**
3. Section **IV** comprises of 7 questions of **3 marks** each**.**
4. Section **V** comprises of 3 questions of **5 marks** each**.**
5. Internal choice is provided in **3** questions of Section –III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART - A
SECTION-I
Questions 1 to 16 carry 1 mark each.

1. Find the angle between the vectors $\vec{a}$ and $\vec{b}$ if $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$.

We have, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$

Now, $\vec{a} \cdot \vec{b} = (2\hat{i} - \hat{j} + 2\hat{k}) \cdot (4\hat{i} + 4\hat{j} - 2\hat{k})$

$= 8 - 4 - 4 = 0.$

So, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$.

2. A matrix A of order 3 × 3 has determinant 5. What is the value of | 3A |?

Given, $|A| = 5$, order of A is 3×3.

$\therefore \quad |3A| = 3^3 |A| = 27 \times 5 = 135.$

OR

If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, then find the value of x.

Expanding the given determinant, we get

$x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta) = 8$

$\Rightarrow -x^3 - x + x = 8 \Rightarrow x^3 + 8 = 0 \Rightarrow (x + 2)(x^2 - 2x + 4) = 0$

$\Rightarrow x + 2 = 0 \quad [\because x^2 - 2x + 4 > 0 \ \forall x]$

$\Rightarrow x = -2$

3. Solve the differential equation $\frac{dy}{dx} = 2^{y-x}$

We have, $\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

Integrating both sides, we get

$$\frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$$

$\Rightarrow -2^{-y} + 2^{-x} = C\log 2 = k(\text{say}) \Rightarrow 2^{-x} - 2^{-y} = k$

OR

Solve the differential equation $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$

We have, $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3} \Rightarrow y^{-1/3}\,dy = x^{-1/3}\,dx$

Integrating both sides, we get

$\frac{3}{2}y^{2/3} = \frac{3}{2}x^{2/3} + k \Rightarrow y^{2/3} - x^{2/3} = \frac{2}{3}k = c\ (\text{say})$

Hence, required solution is $y^{2/3} - x^{2/3} = c$

4. Check whether the function f: N → N defined by f(x) = 4 – 3x is one-one or not.

We have, $f: N \to N, f(x) = 4 - 3x$

Let $f(x_1) = f(x_2) \Rightarrow 4 - 3x_1 = 4 - 3x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

5. If a line makes angles 90°, 60° and 30° with the positive directions of x, y and z-axis respectively, then find its direction cosines.

Let the direction cosines of the line be l, m, n. Then,

$l = \cos 90° = 0,\ m = \cos 60° = \frac{1}{2}$ and $n = \cos 30° = \frac{\sqrt{3}}{2}$

So, direction cosines are $< 0, \frac{1}{2}, \frac{\sqrt{3}}{2} >$.

OR

Find the direction cosines of the line passing through two points (2, 1, 0) and (1, –2, 3).

Here, $P(2, 1, 0)$ and $Q(1, -2, 3)$

So, $PQ = \sqrt{(1-2)^2 + (-2-1)^2 + (3-0)^2}$

$= \sqrt{1 + 9 + 9} = \sqrt{19}$

Thus, the direction cosines of the line joining two points

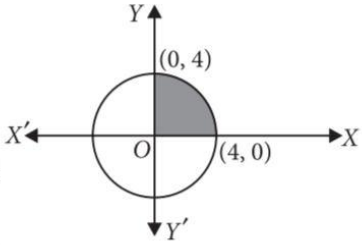
are $< \frac{1-2}{\sqrt{19}}, \frac{-2-1}{\sqrt{19}}, \frac{3-0}{\sqrt{19}} > i.e., < \frac{-1}{\sqrt{19}}, \frac{-3}{\sqrt{19}}, \frac{3}{\sqrt{19}} >$

6. Find the area enclosed between the curve $x^2 + y^2 = 16$ and the coordinate axes in the first quadrant.

Given curve is a circle with centre (0, 0) and radius 4.

$\therefore$ Required area

$$= \int_0^4 \sqrt{16 - x^2}\, dx$$



$$= \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4} \right]_0^4$$

$= 4\pi$ sq. units

7. Evaluate: $\int_0^{\pi/2} x\cos x dx$

$$\int_0^{\pi/2} x\cos x\, dx = [x\sin x]_0^{\pi/2} - \int_0^{\pi/2} 1\cdot \sin x\, dx \quad \text{[Integrating by parts]}$$

$$= \frac{\pi}{2} + [\cos x]_0^{\pi/2} = \frac{\pi}{2} - 1$$

OR

Evaluate: $\int \cos^3 x \sin x dx$

We have, $\int \cos^3 x \sin x\, dx$

Put $\cos x = t \Rightarrow \sin x\, dx = -dt$

$$\therefore \int \cos^3 x \sin x\, dx = -\int t^3 dt = -\frac{t^4}{4} + C = -\frac{1}{4}\cos^4 x + C$$

8. Check whether the relation R = {(1, 1), (2, 2) (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is an equivalence relation or not.

Reflexive : $(1, 1), (2, 2), (3, 3) \in R$, R is reflexive

Symmetric : $(1, 2) \in R$ but $(2, 1) \notin R$, R is not symmetric.

Transitive : $(1, 2) \in R$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$, R is transitive.

Since, R is not symmetric. So, R is not an equivalence relation.

OR

Find the domain of the function $f(x) = \dfrac{1}{\sqrt{\sin x + \sin(\pi + x)}}$ where $\{\cdot\}$ denotes fractional part.

$$f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi + x)\}}} = \frac{1}{\sqrt{\{\sin x\} + \{(-\sin x)\}}}$$

Now, $\{\sin x\} + \{-\sin x\} = \begin{cases} 0, & \sin x \text{ is integer} \\ 1, & \sin x \text{ is not integer} \end{cases}$

For $f(x)$ to be defined, $\{\sin x\} + \{-\sin x\} \neq 0$

$\Rightarrow \sin x \neq \text{integer} \Rightarrow \sin x \neq \pm 1, 0 \Rightarrow x \neq \dfrac{n\pi}{2}$

Hence, domain is $R - \left\{ \dfrac{n\pi}{2}, n \in I \right\}$.

9. Find the vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9.

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{a}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$$\therefore \text{Required vector} = \frac{9(\hat{i} - 2\hat{j} + 2\hat{k})}{3} = 3(\hat{i} - 2\hat{j} + 2\hat{k})$$

10. If E and F are events such that 0 < P(F) < 1, then prove that $P(E/F)+P(\bar{E}/F) = 1$

$P(E|F)+P(\bar{E}|F)$

$$=\frac{P(E\cap F)+P(\bar{E}\cap F)}{P(F)}=\frac{P((E\cup\bar{E})\cap F)}{P(F)}=\frac{P(F)}{P(F)}=1$$

11. Find the value of p for which $p(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

Let $\vec{a}=(\hat{i}+\hat{j}+\hat{k})$

So, unit vector of $\vec{a}=\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{1+1+1}}=\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

$\therefore$ The value of p is $\frac{1}{\sqrt{3}}$.

12. Let f: N → N be defined by $f(n)=\begin{cases}\frac{n+1}{2}, if \quad n \quad is \quad odd\\ \frac{n}{2}, if \quad n \quad is \quad even\end{cases}$ $\forall n\in N$. Find whether the function f is bijective or not.

Here, $f(1)=\frac{1+1}{2}=1, f(2)=\frac{2}{2}=1, f(3)=\frac{3+1}{2}=2, f(4)=\frac{4}{2}=2$

Thus, $f(2k-1)=\frac{(2k-1)+1}{2}=k$ and $f(2k)=\frac{2k}{2}=k$

$\Rightarrow f(2k-1)=f(2k)$, where $k\in N$

But, $2k-1\neq 2k \Rightarrow f$ is not one-one.

Hence, f is not bijective.

13. If P(A) = 7/13, P(B) = 9/13 and P(A ∪ B) = 12/13 then evaluate P(A|B).

$P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A\cup B)=\frac{12}{13}$

$\therefore \quad P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=\frac{7}{13}+\frac{9}{13}-\frac{12}{13} \Rightarrow P(A\cap B)=\frac{4}{13}$

$\therefore \quad P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{4/13}{9/13}=\frac{4}{9}$

14. Find the projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

We have, $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

$\therefore \quad \vec{a}\cdot\vec{b}=(2\hat{i}+3\hat{j}+2\hat{k})\cdot(\hat{i}+2\hat{j}+\hat{k}) = 2+6+2=10$

and $|\vec{b}|=\sqrt{1^2+2^2+1^2}=\sqrt{6}$

So, projection of $\vec{a}$ on $\vec{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{10}{\sqrt{6}}$

15. Simplify: $\tan\theta\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & -\sec\theta\end{bmatrix}+\sec\theta\begin{bmatrix}-\tan\theta & -\sec\theta\\ -\sec\theta & \tan\theta\end{bmatrix}$

$$\tan\theta\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & -\sec\theta\end{bmatrix}+\sec\theta\begin{bmatrix}-\tan\theta & -\sec\theta\\ -\sec\theta & \tan\theta\end{bmatrix}$$

$$=\begin{bmatrix}\tan\theta\sec\theta & \tan^2\theta\\ \tan^2\theta & -\tan\theta\sec\theta\end{bmatrix}+\begin{bmatrix}-\tan\theta\sec\theta & -\sec^2\theta\\ -\sec^2\theta & \tan\theta\sec\theta\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}.$$

16. Write a 2 × 2 matrix whose elements in the i th row and jth column are given by $a_{ij}=\frac{(2i-j)}{2}$.

$$a_{11}=\frac{2-1}{2}=\frac{1}{2},\ a_{12}=\frac{2-2}{2}=0,\ a_{21}=\frac{2(2)-1}{2}=\frac{3}{2},\ a_{22}=\frac{2(2)-2}{2}=\frac{2}{2}=1,$$

$$\therefore\ \text{Required matrix}=\begin{bmatrix}\frac{1}{2} & 0\\ \frac{3}{2} & 1\end{bmatrix}$$

SECTION-II

Case study-based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. In a particular school, Preboard examination is conducted in the month of January 2020 in which, 30% of the students failed in Chemistry, 25% failed in Mathematics and 12% failed in both Chemistry and Mathematics. A student is selected at random.

(i) The probability that the selected student has failed in Chemistry, if it is known that he has failed in Mathematics, is

(a) 3/10 (b) 12/25 (c) 1/4 (d) 3/25

(ii) The probability that the selected student has failed in Mathematics, if it is known that he has failed in Chemistry, is

(a) 22/25 (b) 12/25 (c) 2/5 (d) 3/25

(iii) The probability that the selected student has passed in at least one of the two subjects, is

(a) 22/25 (b) 88/125 (c) 43/100 (d) 3/75

(iv) The probability that the selected student has failed in at least one of the two subjects, is

(a) 2/5 (b) 22/25 (c) 3/5 (d) 43/100

(v) The probability that the selected student has passed in Mathematics, if it is known that he has failed in Chemistry, is

(a) 2/5 (b) 3/5 (c) 1/5 (d) 4/5

Ans:
(i) (b) 12/25
(ii) (c) 2/5
(iii) (a) 22/25
(iv) (d) 43/100
(v) (b) 3/5

18. A farmer has a rectangular garden in his land. He wants to construct fencing using a rock wall on one side of the garden and wire fencing for the other three sides as shown in figure. He has 100 m of wire fencing.

Based on the above information, answer the following questions.

(i) To construct a garden using 100 m of fencing, we need to maximise its
(a) volume (b) area (c) perimeter (d) length of the side

(ii) If x denotes the length of side of garden perpendicular to rock wall and y denote the length of side parallel to rock wall, then find the relation representing total amount of fencing wall.
(a) $x + 2y = 100$ (b) $x + 2y = 50$ (c) $y + 2x = 100$ (d) $y + 2x = 50$

(iii) Area of the garden as a function of x i.e., A(x) can be represented as
(a) $100 + 2x^2$ (b) $x – 2x^2$ (c) $100x – 2x^2$ (d) $100 – x^2$

(iv) Maximum value of A(x) occurs at x equals
(a) 25 m (b) 30 m (c) 26 m (d) 31 m

(v) Maximum area of garden will be
(a) 1200 sq. m (b) 1000 sq. m (c) 1250 sq. m (d) 1500 sq. m

Ans:
(i) (b) area
(ii) (c) $y + 2x = 100$
(iii) (c) $100x – 2x^2$
(iv) (a) 25 m
(v) (c) 1250 sq. m

PART B
SECTION – III
Questions 19 to 28 carry 2 marks each.

19. Evaluate: $\int_2^4 \frac{(x^2+x)}{\sqrt{2x+1}}dx$

Using integration by parts, we get

$$\int_2^4 \frac{(x^2+x)}{\sqrt{2x+1}}dx = \left[(x^2+x)\cdot\sqrt{2x+1}\right]_2^4 - \int_2^4 (2x+1)\cdot\sqrt{2x+1}\,dx$$

$$= (60-6\sqrt{5}) - \int_2^4 (2x+1)^{3/2}dx \quad = (60-6\sqrt{5}) - \frac{1}{5}\cdot\left[(2x+1)^{5/2}\right]_2^4$$

$$= (60-6\sqrt{5}) - \left(\frac{243}{5} - 5\sqrt{5}\right) = \left(\frac{57}{5} - \sqrt{5}\right) = \left(\frac{57-5\sqrt{5}}{5}\right)$$

20. The equation of a line is 5x – 3 = 15y + 7 = 3 –10z. Write the direction cosines of the line.

The given line is $5x - 3 = 15y + 7 = 3 - 10z$

$$\Rightarrow \quad \frac{x-\frac{3}{5}}{\frac{1}{5}} = \frac{y+\frac{7}{15}}{\frac{1}{15}} = \frac{z-\frac{3}{10}}{-\frac{1}{10}}$$

Its direction ratios are $\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}$

i.e., its direction ratios are proportional to 6, 2, –3.

Now, $\sqrt{6^2+2^2+(-3)^2} = 7$

$\therefore$ Its direction cosines are $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$.

OR

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

Any point on the line

$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = r$ (say) is $(3r-1, 5r-3, 7r-5)$. ...(i)

Any point on the line

$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = k$ (say) is $(k+2, 3k+4, 5k+6)$ ...(ii)

For lines (i) and (ii) to intersect, we must have

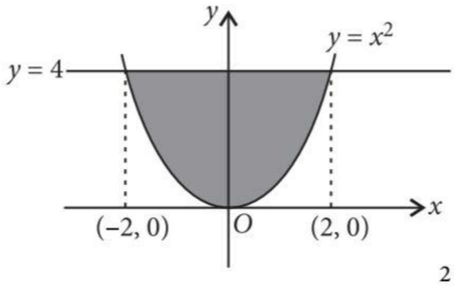
$3r - 1 = k + 2,\ 5r - 3 = 3k + 4,\ 7r - 5 = 5k + 6$

On solving these, we get $r = \frac{1}{2}$, $k = -\frac{3}{2}$

$\therefore$ Lines (i) and (ii) intersect and their point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

21. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

We have, $y = x^2$ and $y = 4$



Required area = area of shaded region $= 2\int_0^2 (4 - x^2)dx = 2\left(4x - \frac{x^3}{3}\right)\Big|_0^2 = \frac{32}{3}$ sq. units

22. Prove that: $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, $x \in [-1/2, 1/2]$

Put $\sin^{-1}x = \theta$. Then $x = \sin\theta$

Now, $\sin3\theta = (3\sin\theta - 4\sin^3\theta) = (3x - 4x^3)$

$\Rightarrow\ 3\theta = \sin^{-1}(3x - 4x^3)$

$\Rightarrow\ 3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ $[\because \theta = \sin^{-1}x]$

Hence, $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

23. An unbiased dice is thrown twice. Let the event A be ‘odd number on the first throw’ and B be the event ‘odd number on the second throw’. Check the independence of the events A and B.

$P(A) = \frac{18}{36} = \frac{1}{2}$ and $P(B) = \frac{18}{36} = \frac{1}{2}$

Also, $P(A \cap B) = P$(odd number on both throws)

$= \frac{9}{36} = \frac{1}{4}$

Now, $P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Clearly, $P(A \cap B) = P(A) \times P(B)$

Thus, A and B are independent events.

OR

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

Consider the following events.

E : Two balls drawn are white

A : There are 2 white balls in the bag

B : There are 3 white balls in the bag

C : There are 4 white balls in the bag

$P(A) = P(B) = P(C) = \frac{1}{3}$

$P(E/A) = \frac{^2C_2}{^4C_2} = \frac{1}{6}$, $P(E/B) = \frac{^3C_2}{^4C_2} = \frac{3}{6} = \frac{1}{2}$, $P(E/C) = \frac{^4C_2}{^4C_2} = 1$

$$\therefore\quad P(C/E) = \frac{P(C)\cdot P(E/C)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)} = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6} + \frac{1}{3}\times\frac{1}{2} + \frac{1}{3}\times 1} = \frac{3}{5}$$

24. Determine the value of ‘k’ for which the following function

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$ is continuous at x = 3.

Given, $f(x)$ is continuous at $x = 3$.

So, $\lim_{x \to 3} f(x) = f(3) \Rightarrow \lim_{x \to 3} \frac{(x+3)^2 - 36}{x-3} = k$

$\Rightarrow \lim_{x \to 3} \frac{(x+3)^2 - 6^2}{x-3} = k \Rightarrow \lim_{x \to 3} \frac{(x+3+6)\,(x+3-6)}{x-3} = k$

$\Rightarrow 3 + 3 + 6 = k \Rightarrow k = 12$

25. Find the solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$ satisfying y(0) = 1.

Given, $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$

$\Rightarrow 2xydy = (x^2 + y^2 + 1)dx \Rightarrow 2xydy - y^2dx = (x^2 + 1)dx$

$\Rightarrow xd(y^2) - y^2dx = (x^2 + 1)dx$

$\Rightarrow \frac{xd(y^2) - y^2dx}{x^2} = \left(1 + \frac{1}{x^2}\right)dx \Rightarrow d\left(\frac{y^2}{x}\right) = d\left(x - \frac{1}{x}\right)$

Integrating both sides, we get

$\frac{y^2}{x} = x - \frac{1}{x} + C \Rightarrow y^2 = x^2 - 1 + Cx$

Now, given that $y(1) = 1$

$\therefore\ 1 = 1 - 1 + C \Rightarrow C = 1$

Thus, curve becomes $y^2 = x^2 - 1 + x$

26. If the matrix A = $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmteric, then find the value of 'a' and 'b'.

Since, matrix A is skew symmetric matrix.

$\therefore\ A' = -A \qquad$ ...(i)

Now, as $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix} \quad \therefore \quad A' = \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$

From (i), $A + A' = O$

$\Rightarrow \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = O \Rightarrow \begin{bmatrix} 0 & 2+a & b-3 \\ a+2 & 0 & 0 \\ b-3 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore\ a + 2 = 0$ and $b - 3 = 0$

$\Rightarrow a = -2$ and $b = 3$

27. Find dy/dx at x = 1, y = $\pi/4$, if $\sin^2 y + \cos xy = k$.

We have, $\sin^2 y + \cos xy = K$

Differentiating w.r.t. x, we get

$$2\sin y \cos y \frac{dy}{dx} + (-\sin xy)\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{\left(1,\frac{\pi}{4}\right)} = \frac{\frac{\pi}{4}\sin\frac{\pi}{4}}{\sin\frac{\pi}{2} - \sin\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$$

28. Prove that the points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$.

The points A, B and C are collinear

$\Leftrightarrow$ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel vectors.

$\Leftrightarrow \overrightarrow{AB}\times\overrightarrow{BC}=\vec{0}$

$\Leftrightarrow (\vec{b}-\vec{a})\times(\vec{c}-\vec{b})=\vec{0} \Leftrightarrow (\vec{b}-\vec{a})\times\vec{c}-(\vec{b}-\vec{a})\times\vec{b}=\vec{0}$

$\Leftrightarrow (\vec{b}\times\vec{c}-\vec{a}\times\vec{c})-(\vec{b}\times\vec{b}-\vec{a}\times\vec{b})=\vec{0}$

$\Leftrightarrow (\vec{b}\times\vec{c}+\vec{c}\times\vec{a})-(\vec{0}-\vec{a}\times\vec{b})=\vec{0}$ $[\because \vec{a}\times\vec{c}=-(\vec{c}\times\vec{a})]$

$\Leftrightarrow \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

OR

If $\vec{a}=7\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$, then find the projection of $\vec{b}$ on $\vec{a}$.

Given, $\vec{a}=7\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$

$\therefore$ Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}$

$$= \frac{14+6-12}{\sqrt{49+1+16}} = \frac{8}{\sqrt{66}}$$

SECTION – IV

Questions 29 to 35 carry 3 marks each.

29. Show that the relation R in the set of real numbers, defined as R = {(a, b): $a \le b^2$} is neither reflexive, nor symmetric, nor transitive.

Given relation is $R = \{(a, b): a \le b^2\}$

Reflexivity: Let $a \in$ real numbers.

$aRa \Rightarrow a \le a^2$ but if $a < 1$, then $a \not\le a^2$

For example, $a = \frac{1}{2} \Rightarrow a^2 = \frac{1}{4}$ so, $\frac{1}{2} \not\le \frac{1}{4}$

Hence, R is not reflexive.

Symmetricity : $aRb \Rightarrow a \le b^2$

But then $b \le a^2$ is not true

$\therefore$ $aRb \not\Rightarrow bRa$

For example, $a = 2, b = 5$ then $2 \le 5^2$ but $5 \le 2^2$ is not true.

Hence, R is not symmetric.

Transitivity : Let $a, b, c \in$ real numbers

Considering aRb and bRc

$aRb \Rightarrow a \le b^2$ and $bRc \Rightarrow b \le c^2 \Rightarrow a \le c^4 \not\Rightarrow aRc$

For example, if $a = 2, b = -3, c = 1$

$aRb \Rightarrow 2 \le 9$

$bRc \Rightarrow -3 \le 1$

$aRc \Rightarrow 2 \le 1$ is not true.

Hence, R is not transitive

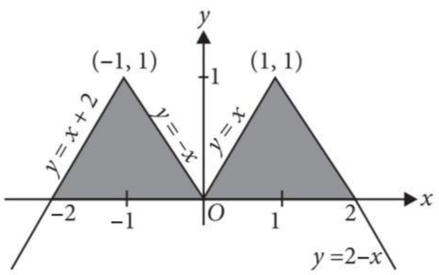
30. Find the area bounded by the lines y = 1 – ||x| – 1| and the x- axis.

We have, $y = 1 - |x - 1|$ if $x \ge 0$

$$= \begin{cases} 1-(x-1), & \text{if } x \ge 1 \\ 1+(x-1), & \text{if } x < 1 \end{cases} = \begin{cases} 2-x, & \text{if } x \ge 1 \\ x, & \text{if } x < 1 \end{cases}$$

and $y = 1 - |-x - 1| = 1 - |x + 1|$, if $x < 0$

$$= \begin{cases} 1-(x+1), & \text{if } x \ge -1 \\ 1+(x+1), & \text{if } x < -1 \end{cases} = \begin{cases} -x, & \text{if } x \ge -1 \\ x+2, & \text{if } x < -1 \end{cases}$$



$$\text{Required area} = 2\left[\int_0^1 x\,dx + \int_1^2 (2-x)\,dx\right]$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 + 2\left[2x - \frac{x^2}{2}\right]_1^2 = 1 + 1 = 2 \text{ sq. units}$$

31. Find the equation of tangent to the curve x = sin 3t, y = cos 2t at t = π/4

The given curve is $x = \sin 3t$; $y = \cos 2t$

$$\Rightarrow \frac{dx}{dt} = 3\cos 3t\,;\ \frac{dy}{dt} = -2\sin 2t \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\frac{2\sin 2t}{3\cos 3t}$$

$$\text{At } t = \frac{\pi}{4},\ x = \sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$y = \cos\frac{\pi}{2} = 0 \text{ and } \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = -\frac{2\sin\frac{\pi}{2}}{3\cos\frac{3\pi}{4}} = -\frac{2\cdot 1}{-\frac{3}{\sqrt{2}}} = \frac{2\sqrt{2}}{3}$$

$\therefore$ Equation of the tangent to the given curve at $t = \frac{\pi}{4}$ is

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right) \Rightarrow 3y = 2\sqrt{2}x - 2$$

OR

Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (a) strictly increasing (b) strictly decreasing

We have, $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2)$

$\Rightarrow f'(x) = 12x(x+1)(x-2)$

Now, $f'(x) = 0$

$\Rightarrow 12x(x+1)(x-2) = 0 \Rightarrow x = -1, x = 0$ or $x = 2$

Hence these points divide the whole real line into four disjoint open intervals namely $(-\infty, -1)$, $(-1, 0)$, $(0, 2)$ and $(2, \infty)$

Interval	**Sign of $f'(x)$**	**Nature of function**
$(-\infty, -1)$	$(-)(-)(-) < 0$	Strictly decreasing
$(-1, 0)$	$(-)(+)(-) > 0$	Strictly increasing
$(0, 2)$	$(+)(+)(-) < 0$	Strictly decreasing
$(2, \infty)$	$(+)(+)(+) > 0$	Strictly increasing

(a) $f(x)$ is strictly increasing in $(-1, 0) \cup (2, \infty)$

(b) $f(x)$ is strictly decreasing in $(-\infty, -1) \cup (0, 2)$

32. Evaluate: $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$

Let $I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}} \Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \text{...(i)}$

By the property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2}\left[\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}+\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right]dx = \int_0^{\pi/2} 1\cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

33. Let $f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b, & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x, & \frac{\pi}{2} < x \le \pi \end{cases}$ be continuous in [0, p], then find the value of a + b.

Since, $f(x)$ is continuous at $x = \pi/4$.

$\therefore$ L.H.L. $\left(\text{at } x = \frac{\pi}{4}\right) = f\left(\frac{\pi}{4}\right)$ = R.H.L. $\left(\text{at } x = \frac{\pi}{4}\right)$

$$\lim_{x\to\frac{\pi}{4}}\left(x+a\sqrt{2}\sin x\right)=2\times\frac{\pi}{4}\cot\frac{\pi}{4}+b \Rightarrow \frac{\pi}{4}+a=\frac{\pi}{2}+b \Rightarrow a-b=\frac{\pi}{4} \quad ...(i)$$

Also, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$\therefore$ L.H.L. $\left(\text{at } x=\frac{\pi}{2}\right)=f\left(\frac{\pi}{2}\right)$ = R.H.L. $\left(\text{at } x=\frac{\pi}{2}\right)$

$$\lim_{x\to\frac{\pi}{2}}(2\times x\cot x+b)=\lim_{x\to\frac{\pi}{2}}(a\cos 2x-b\sin x)$$

$$\Rightarrow 2\times\frac{\pi}{2}\cot\frac{\pi}{2}+b=a\cos 2\times\frac{\pi}{2}-b\sin\frac{\pi}{2}$$

$$\Rightarrow b=-a-b \Rightarrow 2b=-a \quad ...(ii)$$

From (i) and (ii), we get $a + b = \frac{\pi}{12}$.

34. Solve: $(x\sqrt{x^2+y^2}-y^2)dx+xydy=0$

The given equation can be written as

$\frac{dy}{dx}=\frac{y^2-x\sqrt{x^2+y^2}}{xy}$, which is clearly homogeneous.

Putting $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$$v+x\frac{dv}{dx}=\frac{v^2x^2-x\sqrt{x^2+v^2x^2}}{vx^2}$$

$$\Rightarrow x\frac{dv}{dx}=\left(\frac{v^2-\sqrt{1+v^2}}{v}-v\right)\Rightarrow x\frac{dv}{dx}=\frac{-\sqrt{1+v^2}}{v}$$

$$\Rightarrow \int\frac{v}{\sqrt{1+v^2}}dv=-\int\frac{dx}{x}\Rightarrow \sqrt{1+v^2}=-\log|x|+C$$

$$\Rightarrow \sqrt{x^2+y^2}+x\log|x|=Cx$$

OR

Solve the differential equation $x\frac{dy}{dx}+y=x\cos x+\sin x$, given that y = 1 when x = $\pi/2$.

We have, $x\frac{dy}{dx}+y=x\cos x+\sin x \Rightarrow \frac{dy}{dx}+\frac{y}{x}=\cos x+\frac{\sin x}{x}$

It is a linear differential equation.

$$\text{I.F.}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$$

$$\therefore y\cdot x=\int x\left(\cos x+\frac{\sin x}{x}\right)dx+c=\int[x\cos x+\sin x]dx+c$$

$$=x\sin x-\int\sin x\,dx+\int\sin x\,dx+c=x\sin x+c \Rightarrow y=\sin x+\frac{c}{x}$$

Given that, $y = 1$ when $x=\frac{\pi}{2}$

$\therefore \quad 1 = 1 + \frac{c}{\pi/2} \quad \Rightarrow \quad c = 0$

$\therefore \quad y = \sin x$ is the required solution.

35. Show that the function f(x) = |x – 1| + |x + 1|, for all x ∈ R, is not differentiable at the points x = –1 and x = 1.

The given function is $f(x) = |x-1| + |x+1| = \begin{cases} -(x-1)-(x+1), & x < -1 \\ -(x-1)+x+1, & -1 \le x < 1 \\ x-1+x+1, & x \ge 1 \end{cases} = \begin{cases} -2x, & x < -1 \\ 2, & -1 \le x < 1 \\ 2x, & x \ge 1 \end{cases}$

At $x = 1$,

$$f'(1^-) = \lim_{h \to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h \to 0} \frac{2-2}{-h} = 0$$

$$f'(1^+) = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0} \frac{2(1+h)-2}{h} = \lim_{h \to 0} \frac{2h}{h} = 2$$

$\therefore \quad f'(1^-) \ne f'(1^+) \Rightarrow f$ is not differentiable at $x = 1$.

At $x = -1$,

$$f'(-1^-) = \lim_{h \to 0} \frac{f(-1-h)-f(-1)}{-h} = \lim_{h \to 0} \frac{-2(-1-h)-(2)}{-h} = \lim_{h \to 0} \frac{2h}{-h} = -2$$

$$f'(-1^+) = \lim_{h \to 0} \frac{f(-1+h)-f(-1)}{h} = \lim_{h \to 0} \frac{2-2}{h} = 0$$

$\therefore \quad f'(-1^-) \ne f'(-1^+)$

$\Rightarrow \quad f$ is not differentiable at $x = -1$.

SECTION – V

Questions 36 to 38 carry 5 marks each.

36. Find the product BA of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of linear equations: x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

We have, $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

So, $BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow BA = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad BA = 4I \quad \Rightarrow \quad B^{-1}(BA) = 4B^{-1}I \qquad$ [Pre multiplying by B^{-1}]

$\Rightarrow \quad 4B^{-1} = IA \quad \Rightarrow \quad B^{-1} = \frac{1}{4}A \qquad \ldots$(i)

Now, given system of equations can be written as $BX = C$, where

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{and } C = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

or $X = B^{-1}C \Rightarrow X = \frac{1}{4}AC$ [Using (i)]

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$\therefore \quad x = 2, y = 1, z = -1.$

OR

If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Here, $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

$\Rightarrow \ |A| = -11$ and $|B| = 1$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj } A = -\frac{1}{11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} \text{and } B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$\Rightarrow$ R.H.S. $= B^{-1} A^{-1}$

$$= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \cdot \left(-\frac{1}{11}\right)\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \left(-\frac{1}{11}\right)\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} \quad ...(i)$$

$$\text{Now, } A \cdot B = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix} \Rightarrow |AB| = 14 - 25 = -11$$

$$\therefore \quad \text{L.H.S.} = (AB)^{-1} = \left(-\frac{1}{11}\right)\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} \quad ...(ii)$$

From (i) and (ii), $(AB)^{-1} = B^{-1} A^{-1}$.

37. Find the equation of the plane passing through the points (2, 2, –1) and (3, 4, 2) and parallel to the line whose direction ratios are 7, 0, 6.

The equation of a plane passing through (2, 2, –1) is

$a(x - 2) + b(y - 2) + c(z + 1) = 0$ ...(i)

This plane also passes through (3, 4, 2).

$\therefore \quad a(3 - 2) + b(4 - 2) + c(2 + 1) = 0$

$\Rightarrow \quad a + 2b + 3c = 0$ ...(ii)

Now, plane (i) is parallel to the line whose direction ratios are 7, 0, 6

Therefore, $7a + 0(b) + 6c = 0$ ...(iii)

Solving (ii) and (iii) by cross-multiplication method, we get

$$\frac{a}{(2)(6) - (0)(3)} = \frac{b}{(7)(3) - (6)(1)} = \frac{c}{(0)(1) - (2)(7)} \Rightarrow \frac{a}{12} = \frac{b}{15} = \frac{c}{-14} = \lambda \text{ (say)}$$

$\Rightarrow \quad a = 12\lambda, b = 15\lambda, c = -14\lambda$

Substituting the values of a, b, c in (i), we get

$$12\lambda(x-2) + 15\lambda(y-2) - 14\lambda(z+1) = 0$$

$\Rightarrow$ $12x - 24 + 15y - 30 - 14z - 14 = 0$ $[\because \lambda \neq 0]$

$\Rightarrow$ $12x + 15y - 14z = 68$

This is the required equation of plane.

OR

Find the distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines $\frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$.

Any plane through (–1, –2, –1) is $a(x+1) + b(y+2) + c(z+1) = 0$ ...(i)

D.R.'s of any normal to (i) are $< a, b, c >$.

As this normal is perpendicular to both L_1 and L_2,

therefore, $3a + 1b + 2c = 0$ ...(ii)

$1a + 2b + 3c = 0$ ...(iii)

Eliminating a, b, c between (i), (ii) and (iii), we obtain

$$\begin{vmatrix} x+1 & y+2 & z+1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow (x+1)(3-4) - (y+2)(9-2) + (z+1)(6-1) = 0$$

$$\Rightarrow -(x+1) - 7(y+2) + 5(z+1) = 0$$

or $x + 7y - 5z + 10 = 0$ ...(iv)

This is the required equation of plane.

$\therefore$ Distance of (1, 1, 1) from the plane (iv) $= \frac{|1+7-5+10|}{\sqrt{1^2+7^2+(-5)^2}} = \frac{13}{\sqrt{75}}$ units.

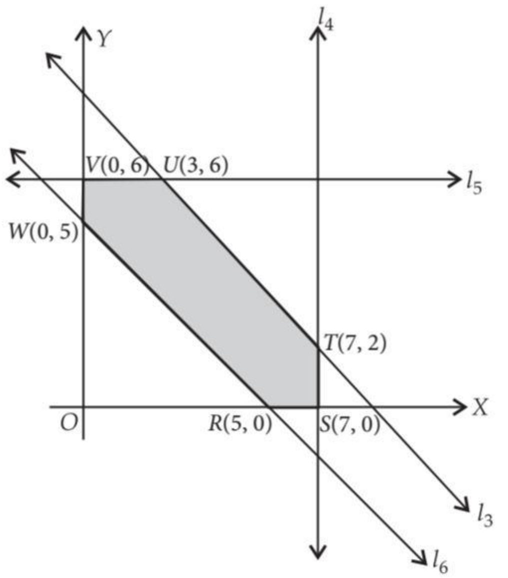
38. Solve the following linear programming problem graphically.
Minimize Z = x – 7y + 227 subject to constraints:
$x + y \leq 9$; $x \leq 7$; $y \leq 6$; $x + y \geq 5$; $x, y \geq 0$

We have Minimize $Z = x - 7y + 227$, subject to the constraints

$x \geq 0, y \geq 0, x + y \leq 9, x \leq 7, y \leq 6$ and $x + y \geq 5$.

We draw the graphs of the lines

$l_1 : x = 0$, $l_2 : y = 0$, $l_3 : x + y = 9$, $l_4 : x = 7$, $l_5 : y = 6$ and $l_6 : x + y = 5$ as shown below.



Now, the intersection point of l_3 and l_4 is (7, 2).
Similarly, the intersection point of l_3 and l_5 is (3, 6).
Thus, the shaded region represents the feasible region whose vertices are R, S, T, U, V and W.

Corner points	Value of $Z = x - 7y + 227$
R (5, 0)	5 – 0 + 227 = 232
S (7, 0)	7 – 0 + 227 = 234
T (7, 2)	7 – 7 × 2 + 227 = 220
U (3, 6)	3 – 7 × 6 + 227 = 188
V (0, 6)	0 – 7 × 6 + 227 = 185 (Minimum)
W (0, 5)	0 – 7 × 5 + 227 = 192

Thus, Z is minimum at $x = 0$ and $y = 6$ and minimum value of z is 185.

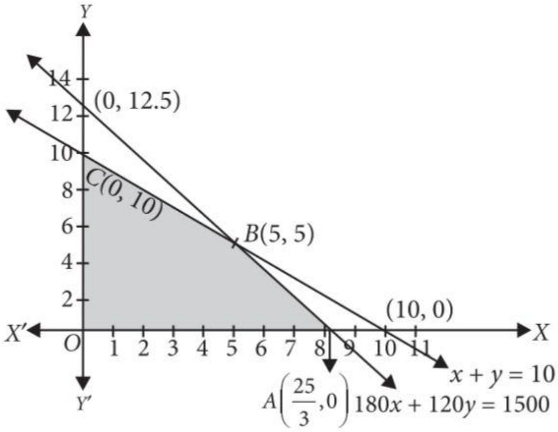
OR

Solved the following linear programming problem graphically.
Maximize Z = 11x + 9y subject to constraints:
$180x + 120y \le 1500$; $x + y \le 10$; $x, y \ge 0$

We have, Maximize $Z = 11x + 9y$ Subject to the constraints,

$180x + 120y \le 1500$ ... (i)
$x + y \le 10$ ... (ii)
$x, y \ge 0$ ... (iii)

Now, plotting the graph of (i), (ii) and (iii), we get the required feasible region (shaded) as shown below. We observe that the feasible region is bounded.



We have corner points as, $A\left(\frac{25}{3},0\right)$, $B(5,5)$ and $C(0,10)$

Corner points	Value of $Z = 11x + 9y$
$A\left(\frac{25}{3},0\right)$	$\left(11\times\frac{25}{3}\right)+(9\times 0)=91.67$
$B(5, 5)$	$(11 \times 5) + (9 \times 5) = 100$ (Maximum)
C(0, 10)	$(11 \times 0) + (9 \times 10) = 90$

Thus, Z is maximum at $x = 5$ and $y = 5$ and maximum value of Z is 100.